Structural Shocks and the Comovements Between Output and Interest Rates

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Structural Shocks and the Comovements between Output and Interest Rates

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Abstract

Stylized facts on U.S. output and interest rates have so far proved hard to match with DSGE models. But model predictions hinge on the joint specification of economic structure and a set of driving processes. In a model, different shocks often induce different comovements, such that the overall pattern depends as much on the specified transmission mechanisms from shocks to outcomes, as well as on the composition of these driving processes. I estimate covariances between output, nominal and real interest rate conditional on several shocks, since such evidence has largely been lacking in previous discussions of the output-interest rate puzzle.

Conditional on shocks to neutral technology and monetary policy, the results square with simple models, like the standard RBC model or a textbook version of the New Keynesian model. In addition, news about future productivity help to explain the overall counter-cyclical behavior of the real rate.

A sub-sample analysis documents also interesting changes in these pattern. During the Great Inflation (1959–1979), permanent shocks to inflation accounted for the counter-cyclical behavior of the real rate and its inverted leading indicator property. Over the Great Moderation (1982–2006), neutral technology shocks were more dominant in explaining comovements between output and interest rates, and the real rate has been pro-cyclical.

JEL Classification: C32, E32 and E43.

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1 Introduction

Understanding the relationship between output and interest rates is important to macroeconomists and policymakers alike. But basic stylized facts on their comovements in U.S. data have proved difficult to match within a variety of simple DSGE models. For instance, King and Watson (1996) study three models: a real business cycle model, a sticky price model, and a portfolio adjustment cost model. They report that this battery of modern dynamic models fails to match the business cycle comovements of real and nominal interest rates with output:

While the models have diverse successes and failures, none can account for the fact that real and nominal interest rates are “inverted leading indicators” of real economic activity.\(^1\)

Calling interest rates inverted leading indicators refers to their negative correlation with future output. These correlations are typically measured once the series have been passed through a business cycle filter.\(^2\) Amongst the diverse failures mentioned by King and Watson, RBC models generate mostly a pro-cyclical real rate.

But in the data, the real rate is clearly counter-cyclical, it is negatively correlated with current output. As mentioned already, it is also a negative leading indicator. This commonly found pattern of correlation between bc-filtered output and short-term interest rates is depicted in Figure \(^1\).\(^3\)

What is the correct conclusion from a mismatch between implications from a dynamic model and stylized facts? Modern dynamic models always involve a joint specification of fundamental economic structure and driving processes. Model outcomes, such as the output-interest rate corre-

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\(^1\) King and Watson (1996) p.35. The inverted leading indicator property has been the subject of various empirical studies, for example Sims (1992) and Bernanke and Blinder (1992). The expression “negative leading indicator” is synonymous.

\(^2\) When it can be applied without confusion, I use the phrase “business cycle filter”, or short “bc-filter”, to describe the bandpass filters developed and applied in Baxter and King (1999) and Stock and Watson (1999) or the filter of Hodrick and Prescott (1997, “HP”) since each eliminates nonstationary and other low frequency components from a time series. These filters differ mainly in that the typical bandpass filters eliminates not only cycles longer than 32 quarters but also those shorter than 6 quarters, while this latter high-frequency component is retained in the HP filter.

\(^3\) This evidence is broadly in line with previous studies, see for instance the stylized facts collected by Stock and Watson (1999, Table 2) for bandpass-filtered U.S. data. The facts are also significant as can be seen from the confidence intervals plotted in Panel b) of Figure 3.
Figure 1: Lead-lag Correlations for Output and Interest Rates

Note: $\text{cor}(\tilde{y}_t, \tilde{x}_{t-k})$ where $\tilde{y}_t$ is bandpass-filtered per-capita output and $\tilde{x}_t$ is bandpass-filtered nominal, respectively real rate. This ex-ante real rate is constructed from the VAR described in Section 3 as $r_t = i_t - E_t\pi_{t+1}$. Quarterly lags on the x-axis. U.S. data 1959–2006.

lation, involve the compound effect of these two features. Yet, when “puzzling” findings are taken as evidence against a particular structural feature – such as sticky prices or portfolio adjustment costs – it is typically not acknowledged that the economy might alternatively be driven by different types of shocks that yield different effects within the given structure. Yet, more carefully, it is simply unclear whether dynamic models fail (or succeed) because of their transmission mechanisms or because of the nature of their driving forces.

To shed more light on this important issue, I provide empirical evidence about output-interest rate comovement conditional on various types of shocks: Neutral technology shocks, monetary shocks, investment specific shocks and news about future productivity and permanent inflation shocks. The first two of these also drive the models of King and Watson (1996). The decomposition is applied both to a continuous sample of postwar data ranging from 1959–2006 as well as as
to two sub-samples, commonly associated with the Great Inflation (1959–1979) and the Great Moderation (1982–2006), extending the original dataset of King and Watson (1996) by more than ten years. There are striking results of my decomposition, which are reported in Section 4 using plots analogous to Figure 1.

- After conditioning on neutral technology shocks, the real rate is pro-cyclical and a positive leading indicator – just the opposite of its unconditional behavior. In response to such permanent growth shocks, this is a common outcome for variants of the neoclassical growth model, be they of the RBC or the New Keynesian variety (King and Watson, 1996; Gali, 2003; Walsh, 2003; Woodford, 2003).

- Conditional on monetary shocks, the real rate is counter-cyclical and a negative leading indicator, which squares with simple New-Keynesian models, too.

- A strong counter-cyclical, and negatively leading behavior of the real rate is attributed to news about future productivity; and to a lesser extent also to investment specific shocks.

- As in the full sample, the real rate has been counter-cyclical during the Great Inflation, mostly due to permanent inflation shocks. However, during the Great Moderation the real rate has been pro-cyclical, with technology shocks playing a more prominent role in accounting for its comovements with output.

Thus, for the full sample the “output-interest rate puzzle” is already defused by conditioning on two widely-studied shocks: Technology and monetary shocks, which counteract each other. Such opposing effects of shocks to “supply” and “demand” are a general theme in Keynesian models (Bénassy, 1995). This result also carries over to the sub-sample analysis, which is motivated by growing evidence, documenting substantial changes in the behavior of macroeconomic data since

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4 Neither sample includes more data for 2007 and later, since the disruptions in financial markets introduce numerous issues which are beyond the scope of this paper.
6 Money is neutral in RBC models, so they have not much to say here. Conditional on monetary shocks, output remains in steady state and correlations are zero.
Figure 2: Cyclical behavior of Output and Interest Rates

Note: Time series of bandpass-filtered output, nominal and real interest rates. The real rate is constructed from the VAR described in Section 3 as \( r_t = i_t - E_t \pi_{t+1} \).

The late 1970s and early 1980s.\(^7\) In particular, since the beginning of the 1980s—and at least until 2006—macroeconomic data has been characterized by a decrease in overall volatility, called the Great Moderation (Bernanke, 2004), whereas the 1970s have been characterized by high and rising inflation rates, also known as the Great Inflation (Sargent, 1999; Stock and Watson, 2007).

To foreshadow some of the sub-sample results, consider the time series of bandpass-filtered output and interest rates shown in Figure 2. As can be seen from the figure’s upper panel, the counter-cyclicality of the real rate is most prevalent over the Great Inflation, while it displays mostly positive comovements with output over the later part of the sample. The nominal rate behaves consistently pro-cyclical throughout the entire postwar period. What remains to be seen

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\(^7\)Amongst others, these changes have been attributed to differences in the conduct of monetary policy (Clarida et al., 2000; Lubik and Schorfheide, 2004) or variations in the nature of shocks hitting the economy (Sims and Zha, 2006).
from the sub-sample decompositions presented in Section 4 is how these changes can mostly be attributed to changes in the relative importance of shocks, rather than substantial changes in the conditional comovements. This result is consistent with the study of Sims and Zha (2006), who attribute changes in macroeconomic dynamics over the postwar period mostly to changes in the amplitudes of shocks rather than changes in their transmission mechanism.

The backbone of my calculations is a VAR for the joint process of (unfiltered) output, nominal and real interest rate. The VAR serves both as a platform for identifying the structural shocks and to model the bc-filtered covariances and correlations. The identified shocks are shocks to the unfiltered data. For instance, both technology shocks have permanent effects on output but they might also have important effects on economic fluctuations. The point of bc-filtered statistics is to judge models solely on those cyclical properties, not on their implications for growth (Prescott 1986). In this vein, the VAR is used to trace out the effects of shocks to the bc-filtered components of output and interest rates. This is done analytically using a frequency domain representation for the VAR and the bc-filters. Rotemberg (1996), Galí (1999) and den Haan (2000) stress the importance of looking at conditional comovements in the context of the comovements of output with either prices or hours. In applying this general idea to output and interest rates, my specific approach is motivated by the fact that the “puzzle” in this area is typically expressed in terms of bc-filtered data.

The remainder of this paper is structured as follows: Related literature is briefly discussed in Section 2. Section 3 lays out my VAR framework for the identification of shocks as well as for decomposing the filtered covariances. Results are presented in Sections 4. Concluding comments are given in Section 5.

2 Related Literature

To overcome the output-interest rate puzzle, Beaudry and Guay (1996) and Boldrin et al. (2001) propose models with habit preferences and frictions to capital accumulation respectively sectoral
factor immobility. This matches the real rate evidence by tweaking the transmission mechanism for a single kind of shock, namely technology. But the evidence presented in this study, suggests that the standard RBC mechanism for technology works fine. It is rather the interaction of several shocks leading to the “puzzling” evidence.

In this spirit, Rotemberg and Woodford (1997) report success with decision lags in a sticky price model. The only structural shock they identify are disturbances to monetary policy. But their solution to the output-interest rate puzzle is based on the interaction with other shocks, which are left unidentified. This is revealed by their impulse response functions (Rotemberg and Woodford, 1997, Figure 1). Following a monetary shock, their model’s output responses are negative (respectively zero) at all lags whilst they are positive for the nominal rate. Since conditional lead-lag covariances are just convoluted impulse responses, they are negative (respectively zero) at all leads and lags. This contrasts with the changing signs in the unconditional covariances depicted in my Figure respectively their Figure 2.

Likewise, Fuhrer and Moore (1995) model the inverted leading indicator property of interest rates with multiple, non-structural shocks and couch their analysis just in terms of unconditional statistics. This paper is an empirical attempt to disentangle the underlying interaction of the various structural shocks.

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8Beaudry and Guay (1996) recognize the importance of conditioning on technology, too. They use cointegrating properties between output, consumption and investment derived by King et al. (1991), which are similar in spirit to my specification described in Section 3.1. When conditioning on these permanent shocks, they report negative correlations between output growth and the unfiltered real rate. Since growth rates amplify high-frequency fluctuations instead of focusing on business cycle characteristics, these results are not directly comparable to my approach and the puzzle framed by King and Watson (1996).

9Another line of attack in this area has been opened by Dotsey et al. (2003) by pointing out that the real rate evidence is sensitive to the choice of price deflator used for constructing the real rate. The widely reported counter-cyclicality of the real rate is particularly strong when deflating with the CPI which is used in this paper. It is a-cyclical or weakly pro-cyclical using the deflator for personal consumption expenditures (PCE). I can replicate this with my VAR, too. However, the basic results for conditional comovements between output and real rate remain valid. These alternative results for the PCE deflator are available from the author upon request.

10Rotemberg and Woodford (1997) look only at output and the nominal rate. They use linear detrending instead of the stochastic procedures considered here. Still they find similar patterns of covariation and juxtapose their results to the puzzle posed by King and Watson (1996).
3 Empirical Methodology

The variables of interest for this paper are the logs of per-capita output, the nominal as well as the real interest rate: \( Y_t = \begin{bmatrix} y_t \ t_t \ r_t \end{bmatrix} \). Let us call their be-filtered component \( \tilde{Y}_t \). The goal is to model and estimate how structural shocks induce comovements between the elements of \( \tilde{Y}_t \).

The backbone of the calculations in this paper is a VAR. Since the real interest rate, \( Y_t \) is not fully observable, the VAR cannot be run directly over \( Y_t \) but rather over a vector of observables \( X_t \). To sufficiently capture a wide-ranging set of structural shocks, the logs of the following eleven variables are included in \( X_t \): the change in the relative price of investments, the growth rate in labor productivity, inflation, the ratios of consumption and investment to output, hours worked, wage markups, capacity utilization, the nominal interest rate, the velocity of money and the spread between nominal long- and short-term interest rates:

\[
X_t = \begin{bmatrix} \Delta p_t^I \ \Delta a_t \ \pi_t \ c_t - y_t \ x_t - y_t \ l_t \ \mu_t \ u_t \ i_t \ v_t \ i_t^L - i_t \end{bmatrix}
\] (1)

where \( a_t = y_t - l_t \). Apart from adding the long-term bond spread to the list of observables, this specification corresponds to the VAR used by [Altig et al. (2005)]. Details of the data construction are described in Section 4. The dynamics of \( X_t \) are captured by a \( p \)-th order VAR:

\[
A(L)X_t = e_t = Q \varepsilon_t
\] (2)

where \( A(L) = \sum_{k=0}^{p} A_k L^k, A_0 = I \) and \( E_{t-1} \varepsilon_t = 0, E_{t-1} \varepsilon_t \varepsilon_t' = I \). The coefficients \( A_k \) and forecast errors \( e_t \) can be estimated using OLS. Identification of the structural shocks \( \varepsilon_t \) will be concerned with pinning down \( Q \). Since fewer shocks are identified than the VAR has equations, there remains an unidentified component without structural interpretation.

The real rate is computed from the Fisher equation \( r_t = i_t - E_t \pi_{t+1} \) where inflation expectations

\[11\] All quantity variables shall be per-capita without further mention.

\[12\] For convenience, I dropped the constants such that \( X_t \) is mean zero. This is without loss of generality since estimating a VAR from demeaned data is equivalent to running a VAR with constants.
are given by the VAR. So \( Y_t \) can be constructed from \( X_t \) by applying a linear filter:

\[
Y_t = \begin{bmatrix} y_t \\ i_t \\ r_t \end{bmatrix} = H(L)X_t \text{ where } H(L) = \begin{bmatrix} (1 - L)^{-1}h_a + h_i \\ h_i \\ h_i - h_\pi \left( \sum_{k=1}^{p} A_k L^{k-1} \right) \end{bmatrix}
\]

and where \( h_a, h_i \) and \( h_\pi \) are selection vectors such that \( \Delta a_t = h_a X_t \) and so on.

The remainder of this section describes the following: First, how the structural shocks are identified (Section 3.1). This gives us \( Q \) and the conditional dynamics of the unfiltered variables can be computed from \( Y_t = H(L)A(L)^{-1}Q \varepsilon_t \). Second, how to apply a bc-filter to the structural components of \( Y_t \) to obtain the decomposition of their auto-covariances (Section 3.2).

### 3.1 Identification of Structural Shocks

In order to identify a variety of structural shocks, which are commonly investigated by theoretical and empirical studies, this paper applies long-, medium- and short-run restrictions to the estimated VAR model. For the benchmark VAR, whose eleven variables are specified in (1), four structural shocks are identified: Investment-specific and neutral technology shocks, monetary policy shocks and news shocks about future productivity. When studying the sub-sample of the data associated with the Great Inflation (1959–1979), the VAR is respecified to allow for a common trend in inflation and nominal interest rates. For this model, innovations to this nominal trend are estimated as a fifth structural shock. Since all VAR models estimated here have more observables than identified shocks, there remains also an unidentified component, which is orthogonal to the identified shocks. These identification schemes are described below.

#### 3.1.1 Investment-Specific and Neutral Technology Shocks

Two types of technology shocks are identified from long-run restrictions as it has been done before by [Fisher (2006), Altig et al. (2005), Gali and Gambetti (2009)]. Investment-specific shocks

\[^{13}\text{See for example Rotemberg and Woodford (1997), Altig et al. (2005) or Smets and Wouters (2007).}\]
are identified as the sole source of variations in the permanent component of the relative price of investments, $p_t^I$. Neutral technology shocks are identified as driving the permanent component of labor-productivity (output per hour) beyond what is explained by the investment-specific technology shocks\textsuperscript{14} Since hours are assumed to be stationary, the identification scheme used here is actually equivalent to identifying (neutral) technology shocks from the permanent component of output as in \cite{ShapiroWatson1988}\textsuperscript{15}.

The identifying restriction for both technology shocks imposes zero restrictions on $A(1)^{-1}Q$:\textsuperscript{16}

$$A(1)^{-1}Q = \begin{bmatrix}
a_{11} & 0 & \ldots & \ldots & 0 \\
a_{21} & a_{22} & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\end{bmatrix} \quad (3)$$

The shocks are signed by imposing $a_{11} < 0$ and $a_{22} > 0$; $a_{21}$ is unrestricted.\textsuperscript{17} Together with the orthogonality of the structural shocks, this identifies the first two column of $Q$, which is then computed as in \cite{BlanchardQuah1989}\textsuperscript{18}. The standardized shocks to investment specific and neutral technology will be denoted $\varepsilon^I_t$ and $\varepsilon^a_t$, respectively.

\textsuperscript{14}This builds on the identification strategy of \cite{GalÌ1999} who identified technology shocks as the sole driver of variations in the permanent component of labor productivity.

\textsuperscript{15}This is easy to see from $y_t = (y_t - l_t) + l_t$ where $l_t$ are log hours (per capita). A stochastic trend in output will be identical to the one of labor productivity if $l_t \sim I(0)$. While both the measurement of hours and the treatment of their stationarity have been found to be contentious issues, see for example \cite{FrancisRamey2005}, \cite{Christianoetal2003} and \cite{GaliRabanal2004}, my results for bc-filtered comovements are robust to whether hours are included in levels or differences. The covariate-augmented Dickey-Fuller test of \cite{Hansen1995} also rejects the unit-root hypothesis for hours in my sample at the 1\% level, when using the VAR data as covariates.

\textsuperscript{16}Dots represent otherwise unrestricted numbers.

\textsuperscript{17}The negative sign of $a_{11}$ associates the investment specific shock with a permanent decrease in the relative price of investments.

\textsuperscript{18}An alternative method, yielding the same results, would be the instrumental variables regressions of \cite{ShapiroWatson1988}, which are also used by \cite{Altigetal2005}. 

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3.1.2 Monetary Policy Shocks

Shocks to monetary policy are defined as unexpected deviations from endogenous policy. As in Christiano et al. (1999) or Rotemberg and Woodford (1997), the short-term interest rate is assumed to follow a linear policy rule

\[ i_t = \gamma_Z Z_t + \gamma_a \varepsilon^a_t + \gamma_I \varepsilon^I_t + \theta(L) X_{t-1} + \sigma_m \varepsilon^m_t \]

where \( Z_t \) is a vector of containing all observable variables \( X_{t,t} \) which appear before \( i_t \) in (1). The standardized policy shock \( \varepsilon^m_t \) is assumed to be uncorrelated with the other variables in the policy rule. \( \varepsilon^m_t \) corresponds to the projection of the VAR’s forecast error in the interest rate equation off the innovations of \( Z_t \) and off the two technology shocks. The velocity of money, \( v_t \), and the interest rate spread, \( i^L_t - i_t \), have been included in the VAR specification (1) to ensure that this nullspace has (at least) a rank of one. A previous working paper version of this paper (Mertens, 2007) used the shock series constructed by Romer and Romer (2004) for each FOMC meeting from 1966–1996 and found results similar to those reported here.

3.1.3 News Shocks about Future Productivity

News shocks have lately attracted considerable interest in theoretical and empirical work, with at least parts of the literature suggesting that they might play an important role in business cycle fluctuations (Beaudry and Portier, 2006; Schmitt-Grohe and Uribe, 2008; Jaimovich and Rebelo, 2009). Barsky and Sims (2009) and Sims (2009) identify news shock as “the shock orthogonal to technology innovations that best explains future variation in technology”. While they use direct data on total factor productivity, their definition is applied here to labor productivity, as measured by the second element in the vector of VAR observables defined in (1). In addition to requiring that news have no contemporaneous impact on labor productivity, news shocks are assumed to be

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19 The vector \( X_t \) defined in (1) can thus be written as \( X_t = [Z'_t \quad i_t \quad v_t \quad i^L_t - i_T]^\prime \) and \( Z_t \) contains \( \Delta p_t, \Delta \sigma_t, \pi_t, c_t - y_t, x_t - y_t, l_t, \mu_t \) and \( u_t \).

20 Absent an exact collinearity in the VAR residuals, the nullspace of \( [Z'_t \quad \varepsilon^I_t \quad \varepsilon^a_t] \) is exactly of rank one.
orthogonal to the other shocks identified in this study. Computational details of the identification strategy are described in Appendix B.

3.1.4 Nominal Trend Shocks

When estimating the VAR over the “Great Inflation” sample (1959–1979), nominal trend shocks are identified as well. As described in Section 4.3 below, for this sub-sample the VAR is adapted to allow for a common trend in inflation and nominal rates by replacing \( \pi_t \) in (1) with \( \Delta \pi_t \) and \( i_t \) by \( i_t - \pi_t \). The nominal trend shocks are then identified as the third element in the Blanchard-Quah factorization of the spectral density at frequency zero shown in (3). Such a trend could capture slow-moving fluctuations in policymakers’ preferences for inflation—be these variations in their actual inflation target as in Ireland (2007) or self-fulfilling market perceptions as in Albanesi et al. (2003) or Sargent (1999).

3.2 Decomposition of BC-Filtered Covariances

Summarizing the previous discussion, the impulse responses of the unfiltered variables in \( Y_t \) are given by \( H(L)A(L)^{-1}Q \). These do not only trace out the business cycle responses of \( Y_t \) to the structural shocks \( \varepsilon_t \), but also how the shocks induce growth as well as high-frequency variations. The motivation for bc-filtering is to focus only on the business cycle effects. Formally, it remains

\[ H(L)A(L)^{-1}Q \]

This statistics can be perfectly justified from the perspective of model evaluation in the frequency domain: The goal is not to match data and model over all spectral frequencies, but only over a subset which is associated with “business cycles”. For the United States this is typically taken to be 6 to 32 quarters following the

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21 The identification of Blanchard and Quah (1989) relies on a Choleski factorization of the spectral density of \( X_t \) at frequency zero. In (3), the two technology shocks are identified as the first two elements of this lower triangular factorization. This factorization allows both technology shocks to have permanent effects on the third VAR element — inflation in this case — as well. However, the long-run effects of both technology effects on inflation are insignificant. This has been verified by two types of statistical tests, both relying on instrumental variables regressions as in Shapiro and Watson (1988). First, a Wald test fails to reject that both technology coefficients are jointly zero with a p-value of 67%, when estimating the unrestricted inflation equation as in Blanchard and Quah (1989). Secondly, the inflation equation could also be estimated while imposing the over-identifying of zero long-run effects from technology shocks on inflation. The J-statistic of Hansen (1982) fails to reject the overidentifying restrictions with a p-value of 42%. To ensure that the covariance decompositions add up exactly, I employ the Blanchard-Quah factorization which is equivalent to the unrestricted inflation equation, for details on this equivalence see for example Francis et al. (2003).

22 Business cycle filters have also been criticized for creating spurious cycles, originally by Harvey and Jaeger (1993) and followed by Cogley and Nason (1995) as well as in the discussion between Canova (1998a, 1998b) and Burnside (1998). Whilst most of these papers focused on the HP filter, their analysis also applies to the bandpass filter. But the bc-filtered statistics employed here can be perfectly justified from the perspective of model evaluation in the frequency domain: The goal is not to match data and model over all spectral frequencies, but only over a subset which is associated with “business cycles”. For the United States this is typically taken to be 6 to 32 quarters following the
to apply a bc-filter and to decompose the filtered lead-lag covariances into the contributions of the structural shocks. The computations are straightforward to perform in the frequency domain and a brief overview is given in [A]

Since the bc-filtered variables in $\tilde{Y}_t$ are covariance-stationary, their lead-lag covariances exist and so does their spectrum. They can be computed from the VAR parameters and the filters $H(L)$ and $B(L)$. To ease notation, the impulse responses of $Y$ after applying the bc-filter are written as

$$\tilde{C}(L) \equiv B(L)H(L)A(L)^{-1}Q$$

so that the bc-filtered spectrum can be expressed as $S_{\tilde{Y}}(\omega) = \tilde{C}(e^{-i\omega})\tilde{C}(e^{-i\omega})'$. For each frequency $\omega$, this is simply a product of complex-numbered matrices. The lead-lag covariance matrices of $\tilde{Y}_t$ can be recovered from the spectrum in what is known as an inverse Fourier transformation

$$\Gamma_{\tilde{Y}}^k \equiv E\tilde{Y}_t\tilde{Y}_{t-k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\tilde{Y}}(\omega) e^{i\omega k} d\omega$$

(4)

Since the structural shocks are orthogonal to each other, the decomposition of the covariances $\Gamma_{\tilde{Y}}^k$ is straightforward. First, the spectrum is computed conditional on each shock. Then, the conditional lead-lag covariances follow from an inverse Fourier transformation, analogously to equation (4). To fix notation, the shocks are indexed by $s$ and $J_s$ is a square matrix, full of zeros except for a unit entry in its $s$'th diagonal element. The spectrum conditioned on shock $s$ is $S_{\tilde{Y}|s}(\omega) = \tilde{C}(e^{-i\omega}) J_s \tilde{C}(e^{-i\omega})'$. Since $\sum_s J_s = I$ the conditional spectra add up to $S_{\tilde{Y}}(\omega)$. This carries over to the coefficients $\Gamma_{\tilde{Y}|s}^k \equiv E(\tilde{Y}_t\tilde{Y}_{t-k}|s)$ from the inverse Fourier transformation of $S_{\tilde{Y}|s}(\omega)$ such that $\sum_s \Gamma_{\tilde{Y}|s}^k = \Gamma_{\tilde{Y}}^k$.

NBER definitions of Burns and Mitchell ([Baxter and King](1999) [Stock and Watson](1999)). Formal concepts of model evaluation in this vein have been advanced by [Watson](1993), [Diebold et al.](1998), as well as [Christiano and Vigfusson](2003). Using the concept of the pseudo-spectrum this extends also to nonstationary variables, notwithstanding the analysis of [Harvey and Jaeger](1993).
4 Decomposition Results

This section presents the results for the VAR described in the previous section. The VAR is estimated using quarterly data for the United States from 1959 to 2006. The construction of the data is described in C. The lag-length of each VAR used in this paper has been selected based on a comparison of information criteria, bootstrapped Portmanteau tests and the ability of the VAR to replicate bc-correlation computed from applying the Bandpass filter of Baxter and King (1999) directly to the data. Details can be found in the web-appendix for this paper, which has been attached at the end of this document.

To assess the statistical significance of the results, bootstrapped confidence intervals are computed for each shock. As discussed by (Sims and Zha, 1999) these are best interpreted as the posterior distributions from a Bayesian estimation with flat prior. The small sample adjustment of Kilian (1998) is used to handle the strong persistence of the VAR. In a first round, the small sample bias of the VAR coefficients is estimated from 1,000 Monte Carlo draws. In the second round, the posterior distribution is constructed from 2,000 draws using the VAR adjusted for the small sample bias.

The web-appendix documents the robustness of some of the key results reported below, when applying the decomposition to a much smaller, trivariate VAR, using only data on output, inflation and the nominal interest rate:

- Conditional on (neutral) technology, the real rate is pro-cyclical and a positive leading indicator.
- Technology shocks play a more substantial role over the Great Moderation, accounting for an unconditionally pro-cyclical real rate.
- The counter-cyclical, negatively leading behavior of the real rate during the Great Inflation is explained by permanent shocks to inflation.

\footnote{Estimating the VAR over the full sample of postwar data, the largest root of the VAR amounts to 0.9905, see. When performing the bootstraps, a rejectance sampling is applied considering only stable VARs such that the long run restrictions can be applied.}
4.1 Full Sample

A result which consistently holds over the various samples considered here is that neutral technology shocks induce a strongly pro-cyclical real rate which is also a positive leading indicator for up to one year. For the full sample of data (1959–2006), this is depicted in the upper panel of Figure 3 which decomposes the filtered covariances between output and the real rate. Covariances add linearly, so they are a natural measure for the decomposition. The total covariances in Figure 3 are just a rescaling of the correlations reported in Figure 1 above. Figure 3 shows further that monetary shocks and news about future productivity induce negative covariances at leads between zero and up to two years. (To a much lesser, and largely insignificant extent, the same is true for investment specific technology shocks.) Turning to the nominal rate, all identified shocks contribute to its pro-cyclical and positively leading comovement with output. For brevity, these results are documented in a web-appendix, attached at the end of this document.

When adding up the conditional covariances induced by the four structural shocks, there remains an unidentified component, inducing a counter-cyclical real rate, comoving negatively with output at leads and lags of up to a year. Whilst monetary policy and news shocks help to account for the counter-cyclical, negatively leading real rate, their strength does at best offset the procyclical and positively leading behavior induced by technology shocks. Still, the results clearly demonstrate that conditional comovements are far from being as puzzling as suggested by King and Watson (1996).

The comovements induced by monetary policy and news shocks are also significant as can be seen from the lower panel of Figure 3. There is much larger uncertainty and small sample bias associated with the technology shocks, reflecting that they are identified from the more uncertain, long run behavior of the data (Christiano et al., 2006). The small bias becomes apparent when comparing the point estimates of the VAR (dashed line) with the mean of the bootstrapped small sample distribution. When considering shape and location of the bootstrapped distributions the

\footnote{Unfortunately, the small sample adjustment used from Kilian (1998) is only partially successful here. The bootstraps are generating by treating the point estimates as true values. As a counterfactual exercise, the bootstraps have also been repeated, generating longer samples, with the result that the mean of the bootstrapped distributions becomes...}
### Table 1: Variance Decomposition

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mon. Policy</th>
<th>N-Tech</th>
<th>I-Tech</th>
<th>News</th>
<th>Nom. Trend</th>
<th>Rest</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>6.36</td>
<td>15.09</td>
<td>7.29</td>
<td>26.70</td>
<td></td>
<td>44.56</td>
</tr>
<tr>
<td>Great Moderation</td>
<td>0.88</td>
<td>56.50</td>
<td>5.39</td>
<td>15.23</td>
<td></td>
<td>22.00</td>
</tr>
<tr>
<td>Great Inflation</td>
<td>3.74</td>
<td>5.57</td>
<td>12.41</td>
<td>18.16</td>
<td>30.59</td>
<td>29.53</td>
</tr>
<tr>
<td><strong>Nom. rate</strong></td>
<td>11.24</td>
<td>11.04</td>
<td>3.72</td>
<td>43.91</td>
<td></td>
<td>30.09</td>
</tr>
<tr>
<td><strong>Real rate</strong></td>
<td>9.92</td>
<td>31.43</td>
<td>3.27</td>
<td>17.71</td>
<td></td>
<td>37.67</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>4.30</td>
<td>2.60</td>
<td>13.24</td>
<td>29.06</td>
<td></td>
<td>50.80</td>
</tr>
<tr>
<td><strong>Hours</strong></td>
<td>5.11</td>
<td>16.39</td>
<td>6.93</td>
<td>31.16</td>
<td></td>
<td>40.40</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>4.30</td>
<td>2.60</td>
<td>13.24</td>
<td>29.06</td>
<td></td>
<td>50.80</td>
</tr>
<tr>
<td>Great Moderation</td>
<td>0.52</td>
<td>18.22</td>
<td>1.43</td>
<td>14.81</td>
<td></td>
<td>65.02</td>
</tr>
<tr>
<td>Great Inflation</td>
<td>3.54</td>
<td>28.49</td>
<td>4.39</td>
<td>37.06</td>
<td></td>
<td>26.52</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>3.71</td>
<td>4.83</td>
<td>15.82</td>
<td>14.36</td>
<td></td>
<td>36.20</td>
</tr>
<tr>
<td>Great Moderation</td>
<td>11.76</td>
<td>4.25</td>
<td>22.75</td>
<td>6.84</td>
<td>28.06</td>
<td>26.33</td>
</tr>
<tr>
<td>Great Inflation</td>
<td>4.74</td>
<td>2.73</td>
<td>11.58</td>
<td>22.67</td>
<td>30.83</td>
<td>27.44</td>
</tr>
</tbody>
</table>

Note: Variance decomposition for bandpass-filtered variables computed from the VARs described in Sections 3 (Full Sample and Great Moderation) and 4.3 (Great Inflation). “Mon. Policy” is the monetary policy shock, “N-Tech” and “I-Tech” refer to the neutral, respectively investment specific technology shocks and “News” refers to the news shocks about future productivity whose identification is described in Section 3.1. As described in Section 4.3, the VAR estimated for the Great Inflation allows for a common trend in inflation and real rates. Shocks to this trend are labeled “Nom. Trend” above.

conditional correlations for neutral technology shocks, it is however reassuring that they are tilted towards pro-cyclical point estimates.

Since covariances can be negative as well as positive, it is ambiguous to put a number like “percentage explained” on their decomposition. Table 1 reports variance shares for the bandpass-filtered fluctuations induced by each shocks for output, inflation, interest rates and hours. For the full sample of postwar data, the identified shocks account for about half the business cycle fluctuations in output, inflation and hours and up to two-thirds of the variations in nominal and real interest rates — mostly due to neutral technology shocks and news about future productivity.
In the sub-sample analysis discussed below, the unexplained share of variations will mostly drop below a third of the total.

So far the cyclical behavior of output and interest rates has been described in terms of bandpass-filtered covariances. Figure 4 documents the unfiltered impulse responses estimated for the full sample, and the results resonate very well with the filtered comovements discussed above. Because of the non-trivial effects of bc-filtering\textsuperscript{25} it is however not a foregone conclusion, that the picture emerging from the impulse responses should mirror the results for the bc-filtered comovements.

A monetary policy shocks increases the nominal rate for about a year, causing a prolonged contraction in output for up to four years as well as a reduction in inflation.\textsuperscript{26} In sum, the real rate rises while the economy contracts after a monetary shock. By construction, the neutral technology shock raises output permanently. As would be predicted by standard neoclassical growth models, e.g. such as the ones considered in \cite{King and Watson 1996}, the technology induced growth in output is accompanied by an increase in the real rate for up to two years. Corroborating the results of \cite{Sims 2009}, output and hours drop in response to a news shock while the consumption output ratio increases.\textsuperscript{27} After an initial increase, the real rate drops as output recovers, in line with their counter-cyclical comovements documented for the business cycle frequencies in Figure 3. The full sample responses of real rate and output to investment specific shocks are largely insignificant.\textsuperscript{28}

\textsuperscript{25}For a critical discussion see for instance \cite{Canova 1998a} or \cite{King and Rebelo 1993}.
\textsuperscript{26}The reduction in inflation follows after an initial upwards blip in the first period after the shocks, mirroring previous results of a price puzzle, which is for example evident in the estimates of \cite{Altig et al. 2005} as well.
\textsuperscript{27}By construction, the response of output per hours (labor productivity) to a news shocks is zero. In addition, the sign of the news shock is identified such that “positive” news lift the consumption output ratio. The sign of the (identical) impact responses of output and hours is however unrestricted.
\textsuperscript{28}After an initial increase in output, the estimated output responses to an investment specific shock displays even a prolonged, but temporary decline in real activity. The long-run effect of an investment specific shock on output is positive, but only at horizons beyond the ten years shown in Figure 4. Re-estimating separate impulse responses for the Great Moderation and the Great Inflation, the output responses to investment specific shocks are generally more positive, mirroring also the results reported by \cite{Fisher 2006} who uses a similar sub-sample split.
Figure 3: Output and Real Rates: Conditional Comovements

(a) Covariance Decomposition

(b) Conditional Correlations

Note: Bandpass-filtered moments, Cov(\(\tilde{y}_t, \tilde{r}_{t-k}\)), for U.S. data (1959–2006) computed from the VAR described in Section 3. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
Figure 4: Impulse Responses

Note: Estimates for U.S. data (1959–2006) from VAR, equation (1) described in Section 3. Responses of unfiltered variables to a one-standard deviation shock \((H(L)A(L)^{-1}Q)\). Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
4.2 Great Moderation

Reestimating the VAR described in Section 3 with data from the Great Moderation (1982–2006) yields considerably different results than what has been found above for the full sample of postwar data. However, just as in the full sample analysis, neutral technology shocks induce a pro-cyclical and positively leading real rate. Strikingly, the unconditional behavior of the real rate has been pro-cyclical and a positive leading indicator of output during the Great Moderation—just the opposite of what King and Watson (1996) have been puzzled about using data from 1959–1992.

To a good extent the shift in the overall cyclicality of real rates can be attributed to an increase in the relative importance of neutral technology shocks as a source of business cycle fluctuations (see the middle panel of Table 1). Similarly, shocks to investment specific technology and monetary policy induce a pro-cyclical real rate, however without being quantitatively relevant.

The second factor behind the change in the unconditional comovements is a different behavior in the responses to news shocks, which are estimated to induce a pro-cyclical real rate over the Great Moderation. Similar to the full sample results, the unfiltered impulse responses to news shocks still display a drop in aggregate activity while the real rate increases on impact before it drops shortly thereafter. However, in subsequent periods the movements in the real rate follow more closely the subsequent recovery in aggregate activity, compared to the more prolonged drop of the real rate after a news shock in the full sample estimates shown in Figure 4. (The impulse responses for the sub-samples are reported in a web-appendix attached at the end of this document.) These subtle changes in the unfiltered responses cause a significant shift in the cyclical components of real rate and output as witnessed by the decomposition results shown in Figure 5.
Figure 5: Output and Real Rates: Conditional Comovements (Great Moderation)

Note: Bandpass-filtered moments, Cov (\(\tilde{y}_t, \tilde{r}_{t-k}\)), for U.S. data (1982–2006) computed from VAR described in Section 3. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
4.3 Great Inflation

A notable feature of the Great Inflation (1959–1979) is the highly persistent rise in inflation, consistent with a common stochastic trend in inflation and nominal interest rates. Accordingly, the VAR specification described in Section 3 has been adapted to allow for such a common trend, by replacing the inflation rate with the change in inflation, and by replacing the nominal interest rate with the spread between the nominal interest rate and the contemporaneous inflation rate. This allows to identify permanent shocks to the common trend in inflation and nominal interest rates as described in Section 3.1.

In addition, the velocity of money, capacity utilization and wage markups have been dropped from the VAR, reflecting their very persistent behavior over this limited sample. The vector of VAR observables used for the Great Inflation sample is thus

\[ X_t = [\Delta p_I t, \Delta a_t, \Delta \pi_t, \Delta (c - y) t, \Delta (x - y) t, \Delta (l - \pi) t, \Delta (i - \pi) L t, \Delta (i_l - \pi) L t] \]

In general, the real rate decomposition for the Great Inflation resembles what has been found above for the full sample: The real rate has been pro-cyclical and a negative leading indicator, conditional on neutral technology shocks it has been pro-cyclical and positively leading, and vice versa when conditioning on monetary policy shocks. However, both shocks play quantitatively a smaller role in accounting for the comovements between output and the real rate over the Great Inflation. As before, part of the real rate’s counter-cyclical behavior is accounted for by news shocks.

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30 Since there are less observables, and an additional long-run shock, one cannot impose as many zero restrictions on the impact responses to monetary policy as in the larger VAR specified in equation (1). As a compromise, the impact responses of the price of investments, as well as the ratios of consumption and investment to output have been left unrestricted (as before, the response of the term spread remained unrestricted as well). For the Great Inflation, the identified shocks may thus also reflect some of the systematic responses of monetary policy to aggregate activity. But the impulse responses reported in the web-appendix, still display the typically contractionary behavior of activity and inflation after a policy shocks.
Figure 6: Output and Real Rates: Conditional Comovements (Great Inflation)

(a) Covariance Decomposition

(b) Conditional Correlations

Note: Bandpass-filtered moments, $\text{Cov} (\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1959–1979) computed from VAR allowing for a common trend in inflation and nominal rates as described in Section 4.3. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
What is new, is that a further part of the hitherto unexplained, counter-cyclical remainder of
the real rate, can now be attributed to permanent inflation shocks. In response to a permanent
increase in inflation, output rises temporarily whilst nominal rates do not sufficiently increase with
inflation, causing a drop in real rates. These result are consistent with the (unfiltered) impulse
responses found by Ireland (2007) when estimating a New-Keynesian DSGE model with random
walk shocks to an exogenous inflation target. The web-appendix reports similar decomposition
results when allowing for a nominal trend over the entire sample of postwar data.

5 Conclusions

An economic model specifies restrictions on how the economy responds to exogenous forces. Data
may not conform to these predictions, either because the specified responses are wrong, or because
the set of forces considered in the model does not sufficiently capture those impinging on the
real world (or both). King and Watson (1996) report an output-interest rate puzzle, because of
discrepancies in the unconditional correlations of output and interest rates in U.S. data and a variety
of calibrated models. But it appears in a different light, once the bc-statistics are conditioned on
structural shocks. At the root of the “puzzle” are not so much the transmission mechanisms of
their models, but rather the interaction of several shocks and their relative importance in different
sub-samples of the data. Three points stand out:

    Conditional on technology shocks, the comovements between output and real rate lines up
fairly well with standard models, be it the standard RBC model or the technology channel of
For all specifications considered, the contemporaneous correlation between (bc-filtered) real rate
and output is positive. Likewise, the real rate is a positive leading indicator of output for almost one

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32 A case in point is how Christiano and Eichenbaum (1992) add government spending shocks to RBC theory to
resolve the Dunlop (1938), Tarshis (1939), Keynes (1939) debate on the overall cyclicity of real wages, the issue is
also summarized by Sargent (1987, p. 487). In a similar vein, Baxter and King (1991) enrich the RBC model with
demand shocks.
year. Unconditionally, the real rate is widely reported to be just the opposite – namely counter- or a-cyclical and a negative leading indicator. Attempts to match this with technology shocks appear to be going in the wrong direction.\footnote{See for instance the RBC modifications of Beaudry and Guay (1996) and Boldrin et al. (2001) with habit preferences and frictions like capital accumulation and sectoral factor immobility. Beaudry and Guay (1996) recognize the importance of conditioning on technology. But since they use a quite different detrending method their results of a counter-cyclical real rate even after conditioning on technology are hard to compare with the results in this study. See also Footnote 8.}

The overall behavior of the real rate must be the outcome of an interaction of several shocks. Indeed: When conditioning on monetary shocks, the real rate is counter-cyclical and a negative leading indicator as predicted by the simple New-Keynesian models. In addition, news about future productivity contribute to the counter-cyclical behavior of the real rate over the postwar period. Such opposing responses to “supply” and “demand” shocks are a general theme in Keynesian models (Bénassy, 1995).

The comovements between output and the real rate differ when comparing the Great Inflation (1959–1979) with the Great Moderation (1982–2006). As in the full sample, the real rate has been counter-cyclical during the Great Inflation, mostly due to permanent inflation shocks. However, during the Great Moderation the real rate has been pro-cyclical, with technology shocks playing a more prominent role in accounting for its comovements with output.

Acknowledgments

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and I would like to thank the Center for its generous support during my dissertation. This research has been carried out within the National Center of Competence in Research “Financial Valuation and Risk Management” (NCCR FINRISK). NCCR FINRISK is a research program supported by the Swiss National Science Foundation. Earlier drafts of this paper have also been circulated under the title “Puzzling Comovements between Output and Interest Rates? Multiple Shocks are the Answer.”

Appendix

A Applying BC-Filters to VAR Model

Conceptually, the analysis in this paper is applicable to a wide class of bc-filters, including the HP-Filter, the approximate bandpass filter of Baxter and King (1999) as well as the exact bandpass filter. A classic reference for the necessary tools is Priestley (1981), and similar techniques are employed by Altig et al. (2005) and Chari et al. (2007). For the computations it is key that the bc-filter can be written as a linear, two-sided, infinite horizon moving average whose coefficients sum to zero:

\[ \tilde{Y}_t \equiv B(L)Y_t \]  

where

\[ B(L) = \sum_{k=-\infty}^{\infty} B_k L^k \]  

and \( B(1) = 0 \).

The bandpass-filter is a such a symmetric moving average. It is explicitly defined in the frequency domain and most of my calculations are carried out in the frequency domain. For frequencies \( \omega \in [-\pi, \pi] \), evaluate the filter at the complex number \( e^{-i\omega} \) instead of the lag operator \( L \). This is also known as the Fourier transform of the filter which represents it as a series of complex numbers (one for each frequency \( \omega \)). Requiring \( B(1) = 0 \) sets the zero-frequency component of the filtered time series to zero. For instance, the bandpass filter passes only cycles between two

34Of course, some coefficients \( B_k \) can be zero. So \( B(L) \) could also be the first-difference filter. But meaningful bc-filters should also be symmetric, such that they have a zero phase shift. Otherwise, comovements over one frequency band, say business cycles, could be attributed by the filter to other frequencies, like growth.
and a half and eight years. For monthly data, it is specified as follows:

\[ B(e^{-i\omega}) = \begin{cases} 
1 & \forall |\omega| \in \left[ \frac{2\pi}{8.12}, \frac{2\pi}{2.5.12} \right] \\
0 & \text{otherwise} 
\end{cases} \]

This VAR framework is also capable of handling unit roots in \( Y_t \). By construction, \( H(L) \) and thus \( Y_t \) has a unit root such that \( H(1) \) is infinite. When computing the bc-filtered spectrum \( S_{Y'}(\omega) \), \( B(1) = 0 \) takes precedence over this unit root. It is straightforward to check that \( H(e^{i\omega}) \) is well defined everywhere, except at frequency zero. So we can think of the nonstationary vector \( Y_t \) as having a pseudo-spectrum \( S_Y(\omega) = C(e^{-i\omega}) C(e^{-i\omega})' \) where \( C(L) = H(L) A(L)^{-1} Q \) and which exists for every frequency on the unit circle except zero.

### B Identification of News Shocks

This section describes the identification of news shocks about future productivity. The identification strategy follows Barsky and Sims (2009) and Sims (2009) who identify news shock as “the shock orthogonal to technology innovations that best explains future variation in technology”. While they use direct data on total factor productivity, their definition is applied here to labor productivity, as measured by the second element in the VAR observables defined in (1). In addition to requiring that news have no contemporaneous impact on labor productivity, they are assumed to be orthogonal to the other shocks identified in this study (neutral and investment-specific technology shocks as well as monetary policy shocks). This orthogonality constraint can be easily accommodated by extending the method of Uhlig (2004) as explained below.

For this method it is convenient to express the identification in terms of an orthonormal matrix \( \tilde{Q} \) and not in terms of the matrix of impact coefficients \( Q \) defined in equation (2) above. These two are related via the Cholesky decomposition of the VAR’s forecast error variance, \( \Sigma = Ee_t e_t' \):

---

35When applying the VAR decomposition to the Great Inflation as discussed in Section 4.3 the news shocks are also required to be orthogonal to the nominal trend shocks.
\[ \Psi \equiv \text{chol}(\Sigma) \text{ and } \tilde{Q} \equiv \Psi^{-1}Q. \] By construction we have \( \tilde{Q}\tilde{Q}' = I. \) We seek the column of \( \tilde{Q}, \) associated with the news shock. Denoting this column as \( \tilde{q}, \) it solves the following variance maximization problem

\[
\max_{\tilde{q}} \quad h_a' \left( \sum_{k=0}^{50} C_k \Psi \tilde{q} \Psi' C_k' \right) h_a = \tilde{q}' \left( \sum_{k=0}^{50} C_k' \Psi' h_a' h_a \Psi C_k \right) \tilde{q} \equiv S
\]

subject to \( \tilde{q}'\tilde{q} = 1 \) and \( (\Psi^{-1} \tilde{Q})' \tilde{q} = 0 \)

where \( C_k \) are the coefficients of the VAR’s vector moving average representation, \( C(L) = A(L)^{-1}, \) \( h_a \) selects the growth rate of labor productivity from the VAR defined in (1). \( \tilde{Q} \) is a matrix with four columns, containing the previously identified three columns of \( Q \) as well as a fourth column ensuring that news shocks have no contemporaneous effect on labor productivity. This fourth column contains the slopes from regression the VARs forecast errors on the projection of the innovation in labor productivity off the space of forecast errors spanned by the previously identified three shocks.\(^{36}\)

\(^{36}\)The fourth column of \( \tilde{Q} \) thus contains the slopes from regressing the forecast errors \( e_t \) onto the residual \( \tilde{e}_t^a \) in the regression \( e_t^i = \beta_1 e_t^j + \beta_2 e_t^a + \beta_3 e_t^m + \tilde{e}_t^a. \)

[Uhlig (2004)] solves the above problem without the orthogonality constraint (6). In this case the problem reduces to finding the largest eigenvector of the positive definite matrix \( S \) defined in (5) with \( \tilde{q} \) being its normalized eigenvector.

A previous working paper version of this paper ([Mertens, 2007]) has extended Uhlig’s computations to handle the orthogonality constraint (6) as follows.\(^{37}\) Let \( B \) be an orthonormal basis for the nullspace of \( \Psi^{-1} \tilde{Q}. \) The set of permissible vectors \( \tilde{q} \) is then \( \{ \tilde{q} : \tilde{q} = Bz \forall z \in \mathbb{R}^n \} \) where \( n \) is the dimension of the nullspace. Reparametrized in terms of \( z, \) the problem reduces to set \( z \) equal to the normalized eigenvector of of \( B'SB \) associated with its largest eigenvalue, denoted \( z^*. \) The sign of \( z^* \) (and thus the sign of the news shock) is determined by making it raise the consumption-output ratio on impact. It should be noted that this sign restriction only affects the sign of first moments

\(^{37}\)Barsky and Sims (2009) proceed similarly.
(e.g. impulse responses), but not the sign of second moments (e.g. conditional covariances), which are at the heart of this paper.

\section*{C Data}

The dataset used for this study has been collected from FRED\textsuperscript{38} and the Federal Reserve Board’s G17 releases, which are publicly available. All data is quarterly and expressed in logs\textsuperscript{39} Quantity variables have been converted to per-capita units using the Bureau Labor Statistics (BLS) measure of the civilian population over 16, interest rates and inflation rates have been annualized.

The data is largely identical to the time series used by Altig et al.\textsuperscript{(2005)} for the sample 1959 to 2006. The only major difference is the use of NIPA data to construct the relative price of investments as in Gali and Gambetti\textsuperscript{(2009)} and Justiniano and Primiceri\textsuperscript{(2008)}, who construct the (log) relative price of investments as a weighted average of the (log) deflators of nondurables and services consumption minus the weighted average of the (log) deflators for investment and durable consumption, with the weights given by the relative (nominal) shares of each spending category\textsuperscript{40} whereas Altig et al.\textsuperscript{(2005)} use data constructed by Fisher\textsuperscript{(2006)}.

In detail, output \((y_t)\) is measured by real GDP per-capita and inflation is measured by the log-difference in the CPI index. As in Altig et al.\textsuperscript{(2005)}, the (log-) ratios of consumption and investment to output, \(c_t/y_t\) and \(x_t/y_t\), are computed from nominal consumption and investments as well as nominal GDP\textsuperscript{41}. Hours per capita, \(l_t\) are calculated from the BLS measure of nonfarm business hours. Wage markups, \(\mu_t = y_t^g - h_t - w_t^g\), are computed from the difference of nominal GDP less hours worked, less the BLS measure of nominal compensation per hour in the nonfarm business sector. Capacity utilization, \(u_t\), is measured for the manufacturing industry as reported

\footnotesize
\begin{itemize}
    \item \textsuperscript{38}Federal Reserve Economic Data, maintained by the Federal Reserve Bank of St. Louis.
    \item \textsuperscript{39}The nominal interest rate is computed from the log of the annualized gross interest rate. \(i_t = \log(1 + I_t/100)\) where \(I_t\) is the annualized percentage rate quoted by FRED.
    \item \textsuperscript{40}The deflators are constructed as the ratios of nominal to real expenditure in each category, using the following formulas: PCDG/PCDGCC96 (durables), PCND/PCNDGC96 (nondurables), PCESV/PCESVC96 (services) and FPI/FPIC1 (investment).
    \item \textsuperscript{41}Consumption is the the sum of nondurables, services and government consumption. Investment is the sum of household consumption of durables and gross private domestic investment.
\end{itemize}
by the Federal Reserve Board’s G17 release. The quarterly average of nominal yields on three month Treasury Bills is used for the nominal short-term interest rate, $i_t$. The term spread of nominal interest rates is computed from the short-term interest rate and the 10-year Treasury constant maturity rate. The velocity of money, $v_t = y_t^8 - m_t$, is computed from the log-difference between nominal GDP and the money-at-zero-maturity measure computed by the Federal Reserve Bank of St. Louis.

Web-Appendix

This is the web-appendix with supplementary details of the VAR lag-length selection (Section I), additional results for the large VARs, which are also used in the main paper (Section II) as well as results for smaller, trivariate VARs, which use only data on output, inflation and the nominal short-term interest rate (Section III).

I Lag-Length Selection

Specification of the VAR’s lag-length is based on a comparison of information criteria, bootstrapped Portmanteau tests and the ability of the VAR to replicate bc-correlation computed from applying the Bandpass filter of Baxter and King (1999) directly to the data. Lag-Length selection tables and plots of the autocovariance functions for the VARs used in the main paper are presented below.

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42An index of total capacity utilization is only available since 1967.

43Since the large sample distribution of the Portmanteau tests tend to over-reject for U.S. macro data, the critical values are bootstrapped as in Altig et al. (2005).
<table>
<thead>
<tr>
<th>lag</th>
<th>T-K</th>
<th>llf/T max root</th>
<th>AIC</th>
<th>HQIC</th>
<th>SIC</th>
<th>Portmanteau Tests for lags ...</th>
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<td>−11.05</td>
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<tr>
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<td>1.0030</td>
<td>18.51†</td>
<td>29.07</td>
<td>44.56</td>
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</tbody>
</table>

Note: Model chosen with lag-length 3. † denotes minimum IC. Q-statistics for Portmanteau test. * denotes significance at the 10%, ** at the 5%, *** at the 1% level of bootstrapped distribution (2000 draws). The column labeled $T - K$ reports the degrees of freedom in each estimation, measured by the number of observations after dropping initial values ($T$) less the number of VAR coefficients to be estimated ($K$). llf/T reports the log-likelihood of each VAR scaled by the number of observations. AIC, HQIC and SIC denote the Akaike Information Criteria, Hannan-Quinn and Schwarz Information Criteria, respectively.
Figure A.7: BC-Moments: Filtered VAR versus Data (Large VAR, Full Sample)

Note: Bandpass-filtered moments, \( \hat{E}_t \hat{Y}'_{t-k} \). The thick lines plot unconditional correlations computed from filtering the VAR-Spectrum. The thin lines are their analogues computed from filtering the data first and then taking sample correlations. Data for the ex-ante real rate is constructed from fitted values of the VAR for expected inflation. The exact bandpass was used for the VAR-based correlations, and the Baxter-King approximation for the data (Baxter and King (1999) recommend a lag truncation of 12 in quarterly data, with monthly data, \( 12 \times 4 = 48 \) is used here). (The thin lines plot mostly underneath the thick ones). Correlations on upper diagonal.
Table A.3: Lag-Length Selection Criteria for (Large VAR, Great Moderation)

<table>
<thead>
<tr>
<th>lag</th>
<th>T-K</th>
<th>llf/T</th>
<th>max root</th>
<th>AIC</th>
<th>HQIC</th>
<th>SIC</th>
<th>Portmanteau Tests for lags…</th>
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<td>19.53</td>
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<td>634.86</td>
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Note: Model chosen with lag-length 2. † denotes minimum IC. Q-statistics for Portmanteau test. * denotes significance at the 10%, ** at the 5%, *** at the 1% level of bootstrapped distribution (2000 draws). The column labeled $T - K$ reports the degrees of freedom in each estimation, measured by the number of observations after dropping initial values ($T$) less the number of VAR coefficients to be estimated ($K$). llf/T reports the log-likelihood of each VAR scaled by the number of observations. AIC, HQIC and SIC denote the Akaike Information Criteria, Hannan-Quinn and Schwarz Information Criteria, respectively.
Figure A.8: BC-Moments: Filtered VAR versus Data (Large VAR, Great Moderation)

Note: Bandpass-filtered moments, $\tilde{\mathbb{Y}}_t \tilde{\mathbb{Y}}'_{t-k}$. The thick lines plot unconditional correlations computed from filtering the VAR-Spectrum. The thin lines are their analogues computed from filtering the data first and then taking sample correlations. Data for the ex-ante real rate is constructed from fitted values of the VAR for expected inflation. The exact bandpass was used for the VAR-based correlations, and the Baxter-King approximation for the data (Baxter and King (1999) recommend a lag truncation of 12 in quarterly data, with monthly data, $12 \times 4 = 48$ is used here). (The thin lines plot mostly underneath the thick ones). Correlations on upper diagonal.
<table>
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<th>max root</th>
<th>VAR Information Criteria</th>
<th>Portmanteau Tests for lags . . .</th>
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Note: Model chosen with lag-length 2. † denotes minimum IC. Q-statistics for Portmanteau test. * denotes significance at the 10%, ** at the 5%, *** at the 1% level of bootstrapped distribution (2000 draws). The column labeled \( T - K \) reports the degrees of freedom in each estimation, measured by the number of observations after dropping initial values \( T \) less the number of VAR coefficients to be estimated \( K \). llf/T reports the log-likelihood of each VAR scaled by the number of observations. AIC, HQIC and SIC denote the Akaike Information Criteria, Hannan-Quinn and Schwarz Information Criteria, respectively.
Figure A.9: BC-Moments: Filtered VAR versus Data (Large VAR, Great Inflation)

Note: Bandpass-filtered moments, $E\tilde{Y}_t\tilde{Y}'_{t-k}$. The thick lines plot unconditional correlations computed from filtering the VAR-Spectrum. The thin lines are their analogues computed from filtering the data first and then taking sample correlations. Data for the ex-ante real rate is constructed from fitted values of the VAR for expected inflation. The exact bandpass was used for the VAR-based correlations, and the Baxter-King approximation for the data (Baxter and King (1999) recommend a lag truncation of 12 in quarterly data, with monthly data, $12 \times 4 = 48$ is used here). (The thin lines plot mostly underneath the thick ones). Correlations on upper diagonal.
II Additional Results for Large VARs

This section reports the following results:

- Impulse Responses estimated for the Great Moderation and the Great Inflation
- Decompositions for comovements between output and the *nominal* rate
- Results obtained from estimating a VAR allowing for a common stochastic trend in inflation and nominal rates over the full sample of postwar data. In addition to what is described in Section 4.3 of the main paper, the VAR includes also the velocity of money as well as the ratio of total reserves to the price level.
Figure A.10: Impulse Responses (Large VAR, Great Moderation)

Note: Estimates for U.S. data (1982–2006) from VAR, equation (1) in Section 3 of the main paper. Responses of unfiltered variables to a one-standard deviation shock ($H(L)A(L)^{-1}Q$). Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
Figure A.11: Impulse Responses (Large VAR, Great Inflation)

Note: Estimates for U.S. data (1959–1979) from VAR with nominal trend, described in Section 4.3 of the main paper. Responses of unfiltered variables to a one-standard deviation shock ($H(L)A(L)^{-1}Q$). Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
Figure A.12: Output and Nominal Rates (Large VAR, Full Sample)

(a) Covariance Decomposition

(b) Conditional Correlations

Note: Bandpass-filtered moments, $\text{Cov}(\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1959–2006) computed from VAR described in Section 3 of the main paper. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)'s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
Figure A.13: Output and Nominal Rates (Large VAR, Great Moderation)

Note: Bandpass-filtered moments, $Cov(\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1982–2006) computed from VAR described in Section 3 of the main paper. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
Figure A.14: Output and Nominal Rates (Large VAR, Great Inflation)

(a) Covariance Decomposition

(b) Conditional Correlations

Note: Bandpass-filtered moments, $\text{Cov} (\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1959–1979) computed from VAR allowing for a common trend in inflation and nominal rates as described in Section 4.3 of the main paper. Lower panel: Bootstrapped standard-errors bands with Kilian’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
Figure A.15: Impulse Responses (Full Sample w/Nominal Trend)

Note: Estimates for U.S. data (1959–1979) from VAR with nominal trend. Responses of unfiltered variables to a one-standard deviation shock \((H(L)A(L)^{-1}Q)\). Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
**Figure A.16:** Output and Real Rates (Full Sample w/Nominal Trend)

Note: Bandpass-filtered moments, $\text{Cov} (\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1982–2006) computed from VAR with nominal trend. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian’s (1998) small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
Figure A.17: Output and Nominal Rates (Full Sample w/Nominal Trend)

Note: Bandpass-filtered moments, $\text{Cov}(\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1982–2006) computed from VAR with nominal trend. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.
III Results for Small VARs

As a robustness check, this appendix reports decomposition results obtained from smaller, trivariate VARs, using the bare minimum of variables, which are needed to model output and the real interest rate: output growth, inflation and the nominal interest rate. As in the main paper, a common trend in inflation and nominal rate is allowed for when estimating the VAR for the Great Inflation. The trivariate VAR cannot be used to identify as many shocks as with the larger VAR used in the main paper. Attention is limited here to the identification of neutral technology shocks, as being the sole driver of the permanent component in output, as well as nominal trend shocks (when estimating the VAR for the Great Inflation). Confirming results from the main paper, the key findings from this exercise are that the real rate is pro-cyclical when conditioned on technology shocks, and counter-cyclical when conditioned on nominal trend shocks during the Great Inflation.

In the main paper, the permanent component of labor productivity is driven by neutral and investment-specific technology shocks. Since hours are assumed to be stationary, the permanent component of labor productivity is identical to the permanent component of output and the technology shocks identified here correspond to a mixture of both types of technology shocks identified in the main paper. In all three samples considered (Full, Great Moderation and Great Inflation), the real rate is pro-cyclical when conditioned on technology shocks.

For the VAR specification with stationary inflation, results are also reported for “Monetary Policy” which are simply identified by projecting the innovation of the nominal rate off the technology shocks and the innovation in inflation, which is unlikely to properly account for endogenous policy responses to variations in aggregate activity and inflation. Still, the results mirror the conditionally counter-cyclical behavior of the real rate reported in the main paper for the larger VARs.

References


David Altig, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde. Firm-specific capital,
Figure A.18: Output and Real Rate decomposed with Small VAR (Full Sample)

Note: Bandpass-filtered moments, $\text{Cov} (\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1959–2006) computed from a trivariate VAR using output growth, inflation and the nominal interest rate. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
Figure A.19: Output and Real Rate decomposed with Small VAR (Great Moderation)

Note: Bandpass-filtered moments, $\text{Cov}(\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1982–2006) computed from a trivariate VAR using output growth, inflation and the nominal interest rate. Lower panel: Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The sold line is the mean of the bootstraps. The dashed line is the point estimate of the VAR. Quarterly lags on the x-axis.
Figure A.20: Output and Real Rate decomposed with Small VAR (Great Inflation)

Note: Bandpass-filtered moments, $Cov(\tilde{y}_t, \tilde{r}_{t-k})$, for U.S. data (1959–1979) computed from trivariate VAR using output growth, the change inflation and the difference between the nominal interest rate and inflation, which allows for a common trend in inflation and the nominal rate. Quarterly lags on the x-axis. Lower panel: Bootstrapped standard-errors bands with Kilian’s (1998) small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light) and 68% (dark). The solid line is the mean of the bootstraps. The dashed line is the point estimate of the VAR.


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