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The Information Content of the Embedded Deflation Option in TIPS

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Abstract

In this paper we estimate the value of the embedded option in U.S. Treasury Inflation Protected Securities (TIPS). The option value exhibits significant time variation that is correlated with periods of deflationary expectations. We use our estimated option values to construct an embedded option price index and an embedded option return index. We then use our embedded option indices as independent variables and examine their statistical and economic significance for explaining the future inflation rate. In most of our regressions, our embedded option return index is significant even in the presence of traditional inflation variables, such as the yield spread between nominal Treasuries and TIPS, the return on gold bullion, the VIX index return, and the lagged inflation rate. We conduct several robustness tests, including alternative weighting schemes, alternative variable specifications, and alternative data samples. We conclude that the embedded option in TIPS contains useful information for future inflation, both in-sample and out-of-sample. Our results should be of value to anyone interested in assessing inflationary expectations at a point in time or in tracking changes in those expectations over time.

JEL Classification: E31, G12, E43, E44
Keywords: TIPS, embedded option, inflation, deflation, term structure

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In this paper we estimate the value of the embedded option in U.S. Treasury Inflation Protected Securities (TIPS). The option value exhibits significant time variation that is correlated with periods of deflationary expectations. We use our estimated option values to construct an embedded option price index and an embedded option return index. We then use our embedded option indices as independent variables and examine their statistical and economic significance for explaining the future inflation rate. In most of our regressions, our embedded option return index is significant even in the presence of traditional inflation variables, such as the yield spread between nominal Treasuries and TIPS, the return on gold bullion, the VIX index return, and the lagged inflation rate. We conduct several robustness tests, including alternative weighting schemes, alternative variable specifications, and alternative data samples. We conclude that the embedded option in TIPS contains useful information for future inflation, both in-sample and out-of-sample. Our results should be of value to anyone interested in assessing inflationary expectations at a point in time or in tracking changes in those expectations over time.

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1 Introduction

The market for U. S. Treasury Inflation Protected Securities (TIPS) has experienced significant growth since its inception in 1997. As of September 2009, the face amount of outstanding TIPS was about $530 billion, which is roughly 10% of the size of the nominal U. S. Treasury market. The TIPS market averages about $70 billion in new issuances each year and has about $8 billion of average daily turnover.[1] The main advantage of TIPS over nominal Treasuries is that an investor who holds TIPS is hedged against inflation risk.[2] Although there are costs to issuing TIPS (Roush, 2008), there appears to be widespread agreement that the benefits of TIPS outweigh the costs. Campbell, Chan, and Viceira (2003), Kothari and Shanken (2004), Roll (2004), Mamun and Visaltanachoti (2006), Dudley, Roush, and Ezer (2009), Barnes, Bodie, Triest, and Wang (2010), Huang and Zhong (2010), and Bekaert and Wang (2010) all conclude that TIPS offer significant diversification and hedging benefits to risk averse investors.

The main contribution of our paper is to point out an informational benefit of TIPS that has been overlooked in the literature. At the maturity date of a TIPS, the TIPS owner receives the greater of the original principal or the inflation adjusted principal. This contractual feature is an embedded put option since a TIPS investor can force the U.S. Treasury to redeem the TIPS at par if the cumulative inflation over the life of the TIPS is negative (i.e., deflation). The first TIPS auction in 1997 was for a 10-year note. Prior to the auction, Roll (1996) dismissed the importance of the embedded option since the United States had not experienced a decade of deflation for more than 100 years. Our paper directly examines the embedded deflation option in TIPS. Using a sample of 10-year TIPS from 1997 to 2010, we estimate that the value of the embedded option does not exceed $0.0615 per $100 principal amount. If we amortize $0.0615 over the 10-year life of a TIPS, the impact

[1] See the Report to the Secretary of the Treasury, "Treasury Inflation Protected Securities Should Play a Heightened Role in Addressing Debt Management Challenges," U. S. Government Accountability Office, GAO-09-932, September 2009. A copy of the report can be found at http://www.gao.gov/new.items/d09932.pdf.

[2] The coupon payments and the principal amount of a TIPS are indexed to inflation using the Consumer Price Index (CPI), which protects an investor's purchasing power.

on the TIPS yield is very small, which appears to justify Roll's (1996) comment. However, when we add 5-year TIPS to our sample, we find that the estimated embedded option value is much larger, up to \$1.4447 per \$100 principal amount. If we amortize \$1.4447 over the 5-year life of a TIPS, the impact on the yield is about 29 basis points. Most important, we find significant time variation in the embedded option values, for both 5-year and 10-year TIPS. We show that this time variation is useful for explaining future inflation, even in the presence of widely used inflation variables such as the price of gold, lagged inflation, and the yield spread between nominal and real bonds.[3] We call this the informational content of the embedded option in TIPS.

To value the embedded option in TIPS, we use a dynamic term structure model that has two factors, the nominal interest rate and the inflation rate. Since our two factors are jointly Gaussian, we obtain a closed-form solution for the price of a TIPS and for the price of a nominal Treasury Note (T-Note). We include nominal T-Notes in our analysis to avoid overfitting the TIPS market, which helps to address the issues of TIPS mispricing and illiquidity that are raised by Fleming and Krishnan (2009) and Fleckenstein, Longstaff, and Lustig (2010). Our TIPS pricing model includes a closed-form solution for the embedded deflation option. Thus when we estimate our model, we can decompose each TIPS price into two parts, a part that corresponds to the embedded option value and a part that corresponds to the inflation-adjusted coupons and the inflation-adjusted principal. This makes our approach different from what is found in Sun (1992), Bakshi and Chen (1996), Jarrow and Yildirim (2003), Buraschi and Jiltsov (2005), Lioui and Poncet (2005), Chen, Liu, and Cheng (2010), Ang, Bekaert, and Wei (2008), and Haubrich, Pennacchi, and Ritchken (2011). These papers show how to value real bonds, but they ignore the embedded deflation option that is found in TIPS. To the best of our knowledge, we are the first to price the embedded option in TIPS and to use its time variation to explain future inflation.[4]

[3] The yield spread is the difference between nominal Treasury yields and real Treasury yields, using matched maturities. The yield spread is sometimes called a break-even rate (Grishchenko and Huang, 2010) or an inflation compensation rate (Ang, Bekaert, and Wei, 2008).

[4] Christensen, Lopez, and Rudebusch (2011) also estimate the value of the embedded option in TIPS. However, unlike our paper, they do not use the time variation in the embedded option value to explain future inflation.

When we fit our model to the data, we find the embedded option values are close to zero prior to 2002. From 2002 through 2004, the option values have considerable time variation. The overall trend during this time period is increasing option values followed by decreasing option values, with a peak around November 2003. From 2005 through the first half of 2008, there is some variation in option values, but mostly the values are close to zero. Finally, during the second half of 2008 and all of 2009, there is a surge in option values, which outstrips the previous peak value from 2003. We argue that the time variation in option values is capturing the deflation scare period of 2003-2004 and the deflationary expectations that were associated with the financial crisis in 2008-2009. Our results are consistent with those in Campbell, Shiller, and Viceira (2009), Wright (2009), and Christensen, Lopez, and Rudebusch (2010). However, our approach is different since we explicitly value the embedded option in TIPS and we quantify its time variation.

Although our estimated option values for 10-year TIPS are small economically, the option returns are very large. When we stack our option returns into a vector and perform a Wald test, we strongly reject the null hypothesis that the returns are jointly equal to zero (p-value is less than 0.0001). When we add 5-year TIPS to our sample, we not only reject the null hypothesis that the option returns are jointly equal to zero, but we also reject the null hypothesis that the option values are jointly equal to zero (both p-values are less than 0.0001). This is consistent with our earlier statement that the embedded option in 5-year TIPS is worth more than its counterpart in 10-year TIPS.

To quantify the informational content of the embedded option in TIPS, we construct several explanatory variables that we use in a regression analysis. We use our estimated option values from 10-year TIPS to construct two value-weighted indices, one for the embedded option price level and one for the embedded option return. We show that the coefficient on each index is significant at the 1% level for explaining the future inflation rate (Table 5). The embedded option return index remains significant at the 1% level even when we include control variables such as lagged inflation, the return on gold, the VIX index, and the yield spread. By itself, the embedded option return index explains more than 24% of the variation in the one-month ahead inflation rate (Table 5). When we include our control variables, this number increases to about 35%. Using our regression point estimate, we find that a 100% embedded option return is consistent with a 0.52% decrease in the one-month ahead annualized inflation rate. Thus our results are economically significant as well as statistically significant. For completeness, we also analyze the significance of our indices for explaining the one-month forward inflation rate and the out-of-sample inflation rate. For many of these regressions, one or both of our embedded option indices is significant while more common variables, such as the return on gold and the yield spread, are insignificant. This is true both in-sample (Table 5) and out-of-sample (Table 6).

We verify our results by performing several robustness checks. First, we construct a new explanatory variable (ORF, option return fraction) that captures the fraction of embedded options in each month that has a positive return. This variable is less sensitive to model specification since any other pricing model that produces the same sign for the embedded option returns will produce the same explanatory variable. This variable is statistically significant using data for 10-year TIPS (Table 7) and when we include 5-year TIPS (Table 11). Second, we alter the weighting scheme that is used to construct the embedded option indices. Instead of using value weights, we construct the indices with weights that favor shorter term options, longer term options, options that are nearer-the-money, and options that are further out-of-the-money. We do this for both our in-sample regressions (Table 8) and our out-of-sample regressions (Table 9). Third, using our value-weighted option price and option return indices, we analyze how the informational content of the embedded option is altered when 5-year TIPS are included in the sample (Table 10). Lastly, we examine the informational content of the embedded option using 5-year TIPS alone, without 10-year TIPS. For all of our robustness checks, we find that our main conclusions are not altered.

Our embedded option return index and our robustness variable ORF are statistically significant for explaining both the one-month ahead inflation rate and the one-month forward inflation rate. In other words, these variables contain relevant information for explaining future inflation out to a horizon of at least two months. We use a two month horizon to dispel any concerns that our results may be driven by timing differences between measuring inflation and reporting inflation. If timing matters at all, it should matter for the current month or one-month ahead, but not for one-month forward. We also show that liquidity is not a likely explanation for our results (section 4.9). We use a traditional asset pricing model that does not account for liquidity. Thus if liquidity is present in the market prices of TIPS, our pricing errors should reflect liquidity. However, the sample correlations between our TIPS pricing errors and our option-based explanatory variables are always less than 0.1, and in many cases are less than 0.05. Thus is it not likely that our estimated option values are proxying for liquidity in the TIPS market. Instead, the evidence suggests that the estimated option values are capturing the possibility of deflation.

Explaining future inflation has received a considerable amount of attention in the literature. Many explanatory variables for future inflation have been proposed, such as the interest rate level and lagged inflation (Fama and Gibbons, 1984), the unemployment rate (Stock and Watson, 1999), the money supply (Stock and Watson, 1999; Stockton and Glassman, 1987), inflation surveys (Mehra, 2002; Ang, Bekaert, and Wei, 2007; Chernov and Mueller, 2011; Chun, 2011), the price of gold (Bekaert and Wang, 2010), and the spread between nominal Treasury yields and TIPS yields (Stock and Watson, 1999; Shen and Corning, 2001; Roll, 2004; Christensen, Lopez, and Rudebusch, 2010; Gurkaynak, Sack, and Wright, 2010; D'Amico, Kim, and Wei, 2009; Pflueger and Viceira, 2011). Our paper is different since we focus on the dynamics of the embedded option in TIPS rather than on traditional variables such as the price of gold or the yield spread. However, we include these traditional variables as control variables in our regressions. This allows us to analyze the marginal contribution of the variables.

In summary, our paper uncovers the informational content of the embedded deflation option in TIPS. While this feature of TIPS has been largely ignored in the literature, we develop a model to value the embedded option explicitly. We show that the time variation in the embedded option value is correlated with periods of deflationary expectations. We also show that the embedded option return is economically important and statistically significant, even in the presence of standard explanatory inflation variables. We argue that our results should be useful to anyone interested in assessing inflationary expectations.

The remainder of our paper is organized as follows. Section 2 introduces our model and derives a closed form solution for TIPS and for nominal Treasury securities. Section 3 describes the data. Section 4 presents our empirical methodology, our model estimation results, and our regression results. We focus on in-sample results, out-of-sample results, and robustness checks. Section 5 gives our concluding remarks. The Appendix provides technical details on our pricing model and discusses how we chose the initial values for our model estimation.

2 The model

We use a continuous time model in which bond prices are driven by two state variables, the nominal interest rate r_t and the inflation rate i_t. The evolution of r_t and i_t is described by the Gaussian system of stochastic processes

$$dr_t = (a_1 + A_{11} r_t + A_{12} i_t)\, dt + B_{11} dz_{1t}^Q + B_{12} dz_{2t}^Q, \tag{1}$$

$$di_t = (a_2 + A_{21} r_t + A_{22} i_t)\, dt + B_{21} dz_{1t}^Q + B_{22} dz_{2t}^Q, \tag{2}$$

where Q is a risk neutral probability measure, z_{1t}^Q and z_{2t}^Q are independent Brownian motions under Q, and a_1, a_2, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{12}, B_{21}, and B_{22} are parameters. Ang and Piazzesi (2003) show that the inflation rate impacts the mean of the short term nominal interest rate. We use their result as motivation for including the parameters A_{12} and A_{21} in equations (1)-(2). This makes each of the processes in (1)-(2) more complex than the Vasicek (1977) process, but it allows for a richer set of dynamics between r_t and i_t. In Appendix C we derive the constant long run means of r_t and i_t under Q.

In our empirical analysis below, we use both TIPS and nominal T-Notes. Section 2.1 describes our pricing model for TIPS, while section 2.2 describes our pricing model for nominal Treasuries. Both of our pricing models are derived under the Q probability measure, which eliminates the need to be specific about the functional form of the risk premia. For example, the inflation risk premium may be time varying, as shown in Evans (1998) and Grishchenko and Huang (2010), for the UK and US Treasury markets, respectively. Furthermore, if the risk premia happen to be affine functions of r_t and i_t, then (1)-(2) are consistent with Barr and Campbell (1997), who show that the expected real interest rate in the UK is highly variable at short horizons, but it is comparatively stable at long horizons. Our model can support many functional forms for the risk premia since we can always describe the evolution of r_t and i_t under the true probability measure and then use a prudent change of measure to arrive at (1)-(2). Thus the risk premia are subsumed by Q.

The advantage of specifying the model under Q is that the number of parameters is reduced, which makes our model parsimonious. Once we constrain the volatility matrix in (1)-(2) to be lower triangular, as in Chun (2011), our model has only 9 parameters. In contrast, Sun (1992, p. 603) uses a model with 13 parameters, Lioui and Poncet (2005, pp. 1269-1270) use 17 parameters, and Christensen, Lopez, and Rudebusch (2010) use 28 to 40 parameters (see their Table 7). Given the limited data for TIPS, it is crucial that we keep the number of parameters as small as possible. To avoid overfitting our model to the TIPS market, we use matching nominal T-Notes in our sample. We also examine model robustness by constructing an alternative explanatory variable (ORF, option return fraction) that is less sensitive to model specification. We describe these items later in detail.

2.1 TIPS pricing

Consider a TIPS that is issued at time u and matures at time T. We want to determine the price P_t of the TIPS at time t, where $u < t < T$. The principal amount of the TIPS is F and the coupon rate is c. Suppose there are n coupons yet to be paid, where the coupon payments occur at $t_1, t_2, \ldots, t_n$. If we let $u < t < t_1 < t_2 < \cdots < t_{n-1} < t_n = T$, we can write the TIPS price as

$$P_t = \mathbb{E}_t^Q \left[\sum_{k=1}^{n} cFe^{\int_u^{t_k} i_s ds} e^{-\int_t^{t_k} r_s ds} + e^{-\int_t^{t_n} r_s ds} \left[Fe^{\int_u^{t_n} i_s ds} + \max\left(0, F - Fe^{\int_u^{t_n} i_s ds}\right) \right] \right] \quad (3)$$

where $\mathbb{E}_t^Q[\cdot]$ denotes expectation at time t under Q. The right-hand side of (3) has three terms. The first term is the value of the inflation-adjusted coupon payments, the second term is the value of the inflation-adjusted principal amount, and the third term is the value of the embedded option. The inflation adjustment in (3) is captured by the exponential term

$$e^{\int_u^{t_k} i_s ds} \tag{4}$$

for $k = 1, 2, \ldots, n$. In our empirical specification, we use the U.S. Treasury's CPI index ratio to capture the known part of the inflation adjustment.[5] The unknown inflation adjustment depends on the stochastic process in (2).

Using (1)-(2), the random variables $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$ for $k = 1, 2, \ldots, n$ have a joint Gaussian distribution. Thus we can evaluate the expectation in (3) to get a closed-form solution for the TIPS price. Our solution depends on the moments $\mathbb{E}_t^Q[\int_t^{t_k} r_s ds]$, $\mathbb{E}_t^Q[\int_t^{t_k} i_s ds]$, $Var_t^Q[\int_t^{t_k} r_s ds]$, $Var_t^Q[\int_t^{t_k} i_s ds]$, and $Cov_t^Q[\int_t^{t_k} r_s ds, \int_t^{t_k} i_s ds]$ for $k = 1, 2, \ldots, n$, which are also available in closed-form. We give details in Appendix A.

2.2 Pricing nominal Treasury Notes

Consider a nominal T-Note that is issued at time u and matures at time T. We want to determine the T-Note's price $\bar{P}_t$ at time t, where $u < t < T$. The principal amount is F, the coupon rate is $\bar{c}$, and there are n coupon payments yet to be paid, at times $t_1, t_2, \ldots, t_n$. As before, we let $u < t < t_1 < t_2 < \cdots < t_{n-1} < t_n = T$ and thus we can write the T-Note's price as

$$\bar{P}_t = \mathbb{E}_t^Q \left[\sum_{k=1}^{n} \bar{c} F e^{-\int_t^{t_k} r_s ds} + F e^{-\int_t^{t_n} r_s ds} \right]. \tag{5}$$

The price in (5) contains two terms. The first term is the value of the nominal coupon payments, while the second term is the value of the principal amount. Since we are pricing a nominal T-Note, there is no explicit inflation adjustment in (5). However, since A_{12} in

[5] The U.S. Treasury constructs the CPI index ratio using the lagged CPI. The impact of the index lag is small economically. Grishchenko and Huang (2010) estimate that it does not exceed four basis points in the TIPS real yield.

(1) may not be zero, the price $\bar{P}_t$ depends not only on r_t and the parameters in (1), but also on i_t and the parameters in (2). This sets our model apart from Vasicek (1977).

Since (1)-(2) are jointly Gaussian processes, we can evaluate (5) to get a closed-form solution for $\bar{P}_t$. Like (3), our closed-form solution for (5) depends on the moments $\mathbb{E}_t^Q[\int_t^{t_k} r_s ds]$, $\mathbb{E}_t^Q[\int_t^{t_k} i_s ds]$, $Var_t^Q[\int_t^{t_k} r_s ds]$, $Var_t^Q[\int_t^{t_k} i_s ds]$, and $Cov_t^Q[\int_t^{t_k} r_s ds, \int_t^{t_k} i_s ds]$ for $k = 1, 2, \ldots, n$. We give details in Appendix B.

3 The data

To estimate our model, we construct a monthly time series for the nominal interest rate and for the inflation rate. We obtain our data from the Federal Reserve Economic Database (FRED) at the Federal Reserve Bank of St. Louis. We use the 3-month Treasury Bill rate as a proxy for the nominal interest rate. To construct a monthly time series for the inflation rate, we use the non-seasonally adjusted Consumer Price Index for All Urban Consumers (CPI-U), which is released monthly by the U.S. Bureau of Labor Statistics. This is the same index that is used for inflation adjustments to TIPS. Suppose Π_τ is the value of the CPI-U that corresponds to month τ. We define the annualized inflation rate for month $\tau + 1$ as $i_{\tau,\tau+1} = (12)\ln(\Pi_{\tau+1}/\Pi_\tau)$, where 12 is the annualization factor. Thus the inflation rate is the annualized log change in the price level, which is consistent with (4).

We use Datastream to obtain daily price data for all of the 5-year and 10-year TIPS that have been auctioned by the U.S. Treasury through May 2010. We use 10-year TIPS since it gives us the longest possible sample period, from January 1997 (the first ever TIPS auction) through May 2010. However, we include 5-year TIPS since the embedded option values for these TIPS are larger due to the lower cumulative inflation. Each TIPS in Datastream is identified by its International Securities Identification Number (ISIN). To verify the ISIN, we match it with the corresponding CUSIP in Treasury Direct. We use abbreviations to simplify the exposition. For example, the ISIN for the 10-year TIPS that was auctioned in January 1997 is US9128272M3. Since US9128 is common to all of the TIPS, we drop these characters and use the abbreviation 272M3. For each TIPS, we obtain the clean price, settlement date, coupon rate, issue date, maturity date, next coupon date, and number of coupons left. We construct the gross market price of a TIPS as

$$\text{Gross Market Price} = (\text{Clean Price} + \text{Accrued Interest}) \times \text{Index Ratio}. \tag{6}$$

In (6), the accrued interest is calculated using the coupon rate, the settlement date, the previous coupon date, and the next coupon date, while the index ratio is the CPI-U inflation adjustment term that is reported on Treasury Direct. To match our TIPS data with our monthly interest rate and inflation data, we use the TIPS gross market price on the last day of each month.

In addition to our sample of 5-year and 10-year TIPS, we also use data on 5-year and 10-year nominal T-Notes. There are 21 ten-year TIPS and 7 five-year TIPS in our sample. For each TIPS, we search for a nominal T-Note with approximately the same issue and maturity dates. We are able to match all but one of our TIPS (the exception is for January 1999, for which we cannot identify a matching 10-year nominal T-Note). Thus our sample includes 21 ten-year TIPS and 7 five-year TIPS, plus 20 ten-year matching nominal T-Notes and 7 five-year matching nominal T-Notes. For the matching nominal T-Notes, we obtain our data from Datastream.

We include nominal T-Notes in our sample for several reasons. First, nominal Treasury securities are an important input to any term structure model that is used to assess inflationary expectations. For example, see Campbell and Viceira (2001), Brennan and Xia (2002), Ang and Piazzesi (2003), Sangvinatsos and Wachter (2005), and Kim (2009), to name just a few. Second, by including nominal T-Notes in our estimation, we effectively double our sample size in each month, which helps to deliver more precise parameter estimates. Lastly, since the TIPS market is only about 10% of the size of the nominal Treasury market, we avoid overfitting the TIPS market by including nominal Treasury securities. This helps to control for the trading differences between TIPS and nominal Treasuries

(Fleming and Krishnan, 2009) and it helps to address the relative overpricing in the TIPS market (Fleckenstein, Longstaff, and Lustig, 2010). In other words, by including nominal Treasuries in our sample, it is less likely that our fitted parameters are capturing TIPS market imperfections that are present in the data.

Our final data set includes monthly interest rates, monthly inflation rates, and monthly gross prices for TIPS and matching nominal T-Notes. Table 1 shows the TIPS and the nominal T-Notes that are included in our sample. There are 1,405 monthly observations for 10-year TIPS, 1,268 monthly observations for 10-year nominal T-Notes, 256 monthly observations for 5-year TIPS, and 250 monthly observations for 5-year nominal T-Notes. We use these data to estimate the parameters of our model in (1)-(2).

4 Empirical results

Our empirical approach involves several steps. First, we use the data from Section 3 and we minimize the sum of squared pricing errors for our sample of TIPS and nominal T-Notes. The solution to this minimization problem provides an estimate of the parameters in (1)-(2). Second, we use our estimated parameters and our formula for the TIPS embedded option (see equations (44)-(46) in Appendix A) to calculate a times series of embedded option values for each TIPS in our sample. We use these time series to construct a value-weighted embedded option price index and a value-weighted embedded option return index. These two option indices, along with various controls, are then used as explanatory variables for in-sample and out-of-sample inflation regressions. We show that our embedded option return index is highly statistically significant for explaining next month's inflation rate, both in-sample and out-of-sample. We also consider several robustness checks, such as alternative weighting schemes and alternative variable specifications.

4.1 Parameter estimation

We estimate the parameters in (1)-(2) by minimizing the sum of the squared errors between our model prices and the true market prices. A similar technique is used in Bakshi, Cao, and Chen (1997, p. 2016). Specifically, for the Treasury securities shown in Table 1, we solve the problem

$$\min_{\Theta}\ SSE(\Theta) = \sum_{t=1}^{T}\left[\sum_{n=1}^{N_t}\left(P^*_{nt} - P_{nt}\right)^2 + \sum_{n=1}^{\bar{N}_t}\left(\bar{P}^*_{nt} - \bar{P}_{nt}\right)^2\right], \tag{7}$$

where T is the total number of months in our sample, N_t is the number of TIPS in our sample for month t, $\bar{N}_t$ is the number of nominal T-Notes in our sample for month t, P^*_{nt} is the gross market price of the nth TIPS for month t, $\bar{P}^*_{nt}$ is the gross market price of the nth nominal T-Note for month t, P_{nt} is the model price of the nth TIPS for month t, and $\bar{P}_{nt}$ is the model price of the nth nominal T-Note for month t. The model prices P_{nt} and $\bar{P}_{nt}$ are given by (3) and (5), respectively. The parameter vector is $\Theta = (a_1, a_2, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, B_{22})^\top$, where we have set $B_{12} = 0$. The variance-covariance matrix for r_t and i_t is symmetric, so it involves only three quantities. However our specification in (1)-(2) involves the four quantities B_{11}, B_{12}, B_{21}, and B_{22}. Thus we follow Chun (2011) and we set B_{12} equal to zero, which makes the volatility matrix in (1)-(2) lower triangular.[6]

To solve (7), we use Newton's method in the nonlinear least squares (NLIN) routine in SAS. Appendix D discusses how we chose the initial parameter values. Since (7) is sensitive to the choice of initial conditions, we double check our results by re-solving the problem using the Marquardt method, which is known to be less sensitive to the choice of initial values. In particular, we use a two-step procedure, first using the Marquardt method and then polishing the estimated parameter values using Newton's method. This robustness check provides the same result as using Newton's method alone. For our reported estimates,

[6] We performed a robustness check by including B_{12} in our parameter vector Θ. This had no impact on our empirical results. The estimated value for B_{12} was close to zero and no other parameter estimate was affected.

we verify a global minimum for (7) by checking that the first-order derivatives are zero and all eigenvalues of the Hessian are positive, which implies a positive definite Hessian.

Table 2 shows our parameter estimates and pricing errors. Panel A shows our results when we estimate the model using only 10-year TIPS and 10-year matching T-Notes. We present these estimates separately because 10-year TIPS are the largest segment of the TIPS market. Panel B shows our results when we estimate the model using all of the TIPS and matching T-Notes from Table 1. In Panel A, the root mean square pricing error (RMSE) across all TIPS and nominal T-Notes is \$3.836 per \$100 face amount. If we amortize this amount over a ten year period using semi-annual compounding, we get about 40 basis points per annum, which is consistent with the pricing errors for the $\mathbb{A}_2(3)_{DS}$ model of Dai and Singleton (2000, Table IV). Our RMSE in both panels is higher than what is reported in Chen, Liu, and Cheng (2010), but since our sample period is longer than theirs, our model is fit to a wider variation in economic conditions. Our root mean square yield error across all TIPS and nominal T-Notes is 71 basis points in Panel A and 73 basis points in Panel B. Although these numbers are higher than those reported in Dai and Singleton (2000), they appear to be reasonable given that we are using a parsimonious model that simultaneously fits two markets, TIPS and nominal Treasuries.

4.2 Time variation in embedded option values

In Panel A of Table 2, the maximum estimated value of the embedded deflation option is \$0.0615 per \$100 face amount. The minimum estimated option value is close to zero. If we amortize \$0.0615 using semi-annual compounding over the 10-year life of a TIPS, we get about 0.6 basis points. Thus on average, ignoring the embedded option on any given trading day has very little impact on the yield of a 10-year TIPS. This may explain why the existing TIPS literature does not focus on the embedded option. However, in Panel B of Table 2, the maximum estimated value of the embedded option is \$1.4447 per \$100 face amount, which corresponds to a 5-year TIPS. Thus the embedded option value is not necessarily small.[7] Over the life of a 5-year TIPS, it accounts for up to 29 basis points of the TIPS yield. This is similar to what is reported in Christensen, Lopez, and Rudebusch (2011), who find that the average value of the TIPS embedded option during 2009 is about 41 basis points.

We find that the estimated value of the embedded deflation option exhibits substantial time variation. Panel A of Figure 1 shows time series of the estimated option values for all 21 ten-year TIPS in our sample. We find a large spike in option values at the end of 2008 and the beginning of 2009. This corresponds to the period of the financial crisis, which was marked by deflationary expectations and negative changes in the CPI index for the second half of 2008. We also find a smaller spike in option values during the 2003-2004 period, which was also marked by deflationary pressure (Ip, 2004). The variation during 2003-2004 is difficult to see in Panel A, but it is more evident in Panel C, which is a zoomed version of Panel A. During most other time periods, the embedded option values are closer to zero. The fact that we have two spikes in option values (during 2003-2004 and during 2008-2009) tells us that it is probably deflationary expectations that are driving our results and not liquidity issues surrounding the financial crisis. We further explore this point later.

We find similar results when we estimate our model using both 10-year TIPS and 5-year TIPS. Panel A of Figure 2 shows the estimated option values for all 7 five-year TIPS in our sample, while Panel B of Figure 2 shows the estimated option values for all 21 ten-year TIPS.[8] We again find a large spike in option values during the financial crisis (both Panels A and B) and we also find a second spike during the 2003-2004 period (Panel B). Thus including 5-year TIPS does not alter the time variation in the option values.

Our results in Figures 1 and 2 are consistent with the existing literature. Wright (2009) and Christensen (2009) use TIPS to infer the probability of deflation. During the later part of 2008, Wright (2009, Figure 2) shows that the probability of deflation was greater than 0.6,

[7] If we estimate our model using only 5-year TIPS and 5-year matching nominal T-Notes, the maximum estimated embedded option value is $1.3134 per $100 face amount, which is similar to what we report in Panel B of Table 2.

[8] In Panel A of Figure 2, the time series has a gap since there were no outstanding 5-year TIPS from August 2002 through September 2004.

while Christensen (2009, Figure 3) shows that it was closer to 1.0. Christensen, Lopez, and Rudebusch (2010, Figure 11) also document expected deflation during the fourth quarter of 2008. In addition, Christensen (2009, Figure 3) shows that the probability of deflation during the last half of 2009 was 0.20 to 0.30.

4.3 Joint significance of embedded option values and returns

We use our estimated option values to calculate a time series of option returns for each TIPS in our sample. Although the estimated option values are sometimes small (see Panel A of Table 2), the option returns are economically larger. For example, in Panel A of Figure 1, when the embedded option value increases from \$0.01 to \$0.06 during the 2008-2009 period, the return is 500%. To test the statistical significance of the estimated option values and the option returns, we perform several Wald tests. In Panel A of Table 3, none of the option values in 10-year TIPS are individually statistically different than zero. Furthermore, we cannot reject the null hypothesis that the option values are jointly equal to zero. However, the results are different for the option returns. We find that 131 out of 1,143 (about 11.5%) of the option returns are individually statistically different than zero. We also strongly reject the null hypothesis that the option returns are jointly equal to zero. The Wald test statistic is 2,498.6 with a corresponding p-value of less than 0.0001.

In Panel B of Table 3, we include the 5-year TIPS. In this case, only one option value is individually statistically different than zero. However, unlike Panel A, we strongly reject the null hypothesis that the option values are jointly equal to zero (the p-value is less than 0.0001). We also find that 575 out of 1,504 (about 38.2%) of the option returns are individually statistically different than zero. Furthermore, we again strongly reject the null hypothesis that the option returns are jointly equal to zero (the p-value is less than 0.0001).

When we calculate option returns, we lose 21 observations in Panel A and 28 observations in Panel B, which equals the number of TIPS, respectively, for each panel. In addition, to avoid numerical issues with calculating our option return test statistics, we eliminate other observations for which the option values are too close to zero to calculate a meaningful return.[9] Removing the smallest option values from our sample has the effect of trimming outlier returns. Thus our option return tests in Panels A and B of Table 3 are not driven by outliers. The fact that we remove these outliers explains why the sample sizes are different for our option value tests (which includes all options) and our option return tests (which removes outlier returns).

4.4 Option-based explanatory variables

We use our estimated option values and option returns to construct explanatory variables for our empirical analysis. For the nth TIPS in month t, let O_{nt} denote the estimated value of the embedded option. Thus the option return in month t for the nth TIPS is $R_{nt} = O_{nt}/O_{n,t-1} - 1$. We construct a value-weighted index for the embedded option price level and a value-weighted index for the embedded option return. The weight W_{nt} for the nth TIPS in month t is

$$W_{nt} = \frac{O_{nt}}{\sum_{n=1}^{N_t} O_{nt}}, \tag{8}$$

where N_t is the number of TIPS in the sample for month t. Thus the value-weighted embedded option price index in month t is

$$OP_t = \sum_{n=1}^{N_t} W_{n,t-1} O_{nt} \tag{9}$$

and the value-weighted embedded option return index for month t is

$$OR_{t-1,t} = \sum_{n=1}^{N_t} W_{n,t-1} R_{nt}. \tag{10}$$

For robustness, we also used an alternative definition of the option return index, namely $OR_{t-1,t} = OP_t/OP_{t-1} - 1$. Under this alternative definition, we found no material impact on our empirical results. We construct (9)-(10) in two ways: (i) using only the 10-year

[9] To be precise, we discard option values that are smaller than 10^{-18}. We tried other cutoff values and obtained similar results. We use a cutoff of 10^{-18} since it produces the largest sample size while avoiding numerical issues with calculating the option return test statistics.

TIPS from Panel A of Table 1; and (ii) using all of the TIPS in Panels A and C of Table 1. The resulting embedded option indices are used as explanatory variables in our regression analysis below.

4.5 In-sample results

We examine the informational content of our variables OP_t and $OR_{t-1,t}$ for explaining the future inflation rate. Suppose Π_t is the value of the CPI-U that corresponds to month t. We define the inflation rate from month $t+\tau$ to month $t+\tau+1$ as

$$i_{t+\tau,t+\tau+1} = 12\ln\left[\frac{\Pi_{t+\tau+1}}{\Pi_{t+\tau}}\right], \tag{11}$$

where 12 is an annualization factor. Substituting $\tau = 0$ in (11) gives the one-month ahead inflation rate, while substituting $\tau = 1$ in (11) gives the one-month forward inflation rate. These inflation rates are used as the dependent variable in our regression analysis. In addition to OP_t and $OR_{t-1,t}$, our explanatory variables include: (i) the yield spread YS_t, which is the difference between the average yields of the nominal T-Notes and the TIPS in our sample; (ii) the one-month lagged inflation rate, $i_{t-1,t}$; (iii) the return on gold, $GoldRet_{t-1,t}$, which we calculate using gold prices from the London Bullion Market Association; (iv) the return on VIX, $VIXRet_{t-1,t}$, which is the return on the S&P 500 implied volatility index; and (v) the value-weighted return on the TIPS in our sample, $BondRet_{t-1,t}$.

We include YS_t as an explanatory variable since it is a common measure of inflation expectations. Hunter and Simon (2005) have also shown that the yield spread is correlated with TIPS returns. We include $GoldRet_{t-1,t}$ since the fluctuation in the price of gold has long been associated with inflation expectations. Bekaert and Wang (2010) show that the inflation beta for gold in North America is about 1.45. We include $VIXRet_{t-1,t}$ since its time variation captures the uncertainty associated with many macroeconomic variables, as described in Bloom (2009). Lastly, we include $BondRet_{t-1,t}$ as a control variable to see if the TIPS total return has incremental explanatory power beyond that of the embedded option. This allows us to compare the informational content of the embedded option, which is the focus of our study, to that of the TIPS itself, which is examined by Chu, Pittman, and Chen (2007), D'Amico, Kim, and Wei (20009), and Chu, Pittman, and Yu (2011).

Table 4 shows summary statistics for our regression variables. In Panel A, the mean of our embedded option return index is 1.36, which corresponds to a 136% monthly average value-weighted option return. The standard deviation of the embedded option return index is 4.529, which coincides with our earlier statement about the substantial time variation in the option returns. Panel B shows our sample correlation matrix. The first eight rows of the correlation matrix correspond to the explanatory variables in our regression analysis, while the last two rows are used as dependent variables. The one-month ahead inflation rate is in the ninth row ($Inflation$, $lead1$). If we examine the p-values for this row, we find that all of the correlations are statistically different than zero at the 5% level, and all except the gold return correlation are significant at the 1% level. In particular, our three option-based variables all have p-values that are 0.0005 or smaller. The last row in Panel B shows the one-month forward inflation rate ($Inflation$, $forward1$). Here the story is different since only the option return index, the VIX return, and the one-month ahead inflation ($Inflation$, $lead1$) have correlations that are statistically different than zero. Although this is not a multivariate analysis, it suggests that the information content of some traditional inflation variables, such as the yield spread and the return on gold, may be short-lived.

Table 4 uses only 10-year TIPS, but our sample correlation matrix does not change very much if instead we use 5-year TIPS and 10-year TIPS. The correlation between our option value index using only 10-year TIPS and our option value index using both 5-year and 10-year TIPS is 0.942 (p-value is less than 0.0001). Likewise, the correlation between the two option return indices (i.e., with and without 5-year TIPS) is 0.961 (again the p-value is less than 0.0001). The option values for 5-year TIPS are larger than those for 10-year TIPS since the cumulative inflation, which determines the option's strike price, is lower for a 5-year TIPS relative to a 10-year TIPS. However, the sample correlation matrix changes very little when we include 5-year TIPS.

4.5.1 Inflation regressions

Our first regression is

$$\begin{aligned} i_{t,t+1} &= \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} \\ &\quad + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+1}, \end{aligned} \tag{12}$$

which is shown in Panel A of Table 5. In (12), the variables OP_t and $OR_{t-1,t}$ are constructed using only 10-year TIPS. As Panel A shows, our variable $OR_{t-1,t}$ is always statistically significant at the 1% level.[10] This is true even when we include other variables that are known to capture inflation, such as lagged inflation and the yield spread. Our variable OP_t is significant in columns 1 and 5, but in column 11 it is driven out by the full set of explanatory variables. Thus $OR_{t-1,t}$ appears to be a more important explanatory variable than OP_t.

If we examine the adjusted-R^2 values in Panel A, we find that $OR_{t-1,t}$ explains 24.4% of the variation in the one-month ahead inflation rate (column 2), while the yield spread explains only 4.7% (column 3) and the lagged inflation rate explains 21.3% (column 4). When the full set of variables is used, we explain 35.3% (column 11). For all of our regressions, the sign of the coefficient on $OR_{t-1,t}$ is negative. This is consistent with our economic intuition. Since the embedded TIPS option is a deflation option, a higher option return this month (as captured by $OR_{t-1,t}$) is associated with a lower inflation rate next month.

We find that our results are not only statistically significant, but also economically significant. In Panel A of Table 5, column 2, the coefficient on $OR_{t-1,t}$ is -0.0052. Thus a 100% embedded option return predicts a decrease of 52 basis points in the one-month ahead annualized rate of inflation. The other columns that include $OR_{t-1,t}$ are similar, but columns 7 and 11 are slightly smaller, with coefficients of 37 basis points and 31 basis points, respectively. Comparing our results to the other variables in Panel A, our variable

[10] For all of our regressions, Newey and West (1987) t-statistics with four lags are reported. We also calculated standard errors using 3, 5, and 6 lags. This had no impact on our results.

$OR_{t-1,t}$ is at least as important economically as traditional variables such as the yield spread (coefficient of 0.83 in column 6) or the lagged inflation (coefficient of 0.29 in column 7). A one percentage point increase in the yield spread (lagged inflation rate) predicts a 83 basis point (29 basis point) increase in the one-month ahead annualized rate of inflation.

We further analyze the informational content of the embedded option by exploring whether OP_t and $OR_{t-1,t}$ can explain the one-month forward rate of inflation. As shown in Panel B of Table 5, we run the regression

$$\begin{aligned} i_{t+1,t+2} &= \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} \\ &\quad + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+2}, \end{aligned} \tag{13}$$

where $i_{t+1,t+2}$ is the one-month forward inflation rate. The coefficient on OP_t is significant in columns 1 and 5, but the significance vanishes when we include all of the variables (column 11). The coefficient on $OR_{t-1,t}$ in Panel B of Table 5 is statistically significant at the 5% level, including column 11. Thus $OR_{t-1,t}$ continues to matter for the one-month forward inflation rate, which shows that our results are not driven by timing differences between measuring inflation and reporting inflation (i.e., CPI-U announcements). The adjusted-R^2 values in Panel B are lower than those in Panel A. We can explain 35.3% of the variation in the one-month ahead inflation rate (Panel A of Table 5, column 11), but only 6.8% of the variation in the one-month forward inflation rate (Panel B of Table 5, column 11). The economic significance in Panel B is also lower. In Panel A of Table 5, a 100% embedded option return predicts a decrease of 52 basis points in the inflation rate. In Panel B of Table 5, the comparable number is about 20 basis points (column 2). In spite of the lower economic significance, it appears that $OR_{t-1,t}$ contains relevant information for future inflation out to a horizon of at least two months.

4.6 Out-of-sample results

In Section 4.5 we examined the ability of OP_t and $OR_{t-1,t}$ to explain the future rate of inflation. Since our parameter estimates in Table 2 use the full sample of data, the variables OP_t and $OR_{t-1,t}$ from Section 4.4 have a forward looking bias. Thus our results in Section 4.5 should not be interpreted as inflation forecasts – they are in-sample results. We now address this issue by using a rolling window approach. Using the securities in Panels A and B of Table 1, we estimate our model parameters using rolling subsamples. For each subsample, we calculate the embedded option values and the embedded option returns. We then use the option values and the option returns to explain the future inflation rate, which is a true out-of-sample analysis.

Our full sample period is January 1997 through May 2010, which is 161 months. We use a rolling 80-month window, which allows us to construct 82 subsamples. The first subsample spans January 1997 through August 2003, the second subsample spans February 1997 through September 2003, and so forth. For each subsample, we use the initial values described in Appendix D and we solve the optimization problem (7) to get a set of estimated parameters. Upon examining the sets of estimated parameters, we find nine subsamples that do not produce a positive long run mean for the inflation rate. We eliminate these nine subsamples from our analysis. For each of the remaining subsamples, we use the embedded option values from the last month of the subsample and from the next to the last month of the subsample to calculate OP_t and $OR_{t-1,t}$ according to (8)-(10). In the subsample that spans January 1997 - August 2003, we use the embedded option values from July-August 2003 to calculate OP_t and $OR_{t-1,t}$ for August 2003; in the subsample that spans February 1997 - September 2003, we use the embedded option values from August-September 2003 to calculate OP_t and $OR_{t-1,t}$ for September 2003; and so forth. This gives us a new time series for OP_t and a new time series for $OR_{t-1,t}$ that do not suffer from the forward looking bias in Section 4.5.

Upon examining the time series for $OR_{t-1,t}$, we find several months with abnormally high returns. These abnormal returns originate in months where the beginning and ending option values have different orders of magnitude, yet both values are economically close to zero. For example, if an option value moves from 10^{-12} to 10^{-10}, the monthly return is very large, but both values are approximately zero. We remove these outlier returns by reconstructing the time series for OP_t and $OR_{t-1,t}$, this time ignoring individual embedded option values that are smaller than 10^{-8}. We use a cutoff level of 10^{-8} since it preserves the time variation in $OR_{t-1,t}$ while effectively eliminating the monthly outlier returns.[11] For robustness, we applied the same cutoff to our in-sample results in Section 4.5 and we found no impact on the results. Thus the outliers in the subsamples are probably due to estimation error, given that our rolling window is shorter than our full sample period.

4.6.1 Inflation regressions

Table 6 re-estimates the regressions in (12)-(13) using our out-of-sample approach. Panel A shows our estimation results for the one-month ahead out-of-sample inflation rate, while Panel B shows our results for the one-month forward out-of-sample inflation rate. In Panel A of Table 6, $OR_{t-1,t}$ is statistically significant at the 1% level in the presence of the VIX return (column 9), the return on gold (column 8), and the yield spread (column 6). D'Amico, Kim, and Wei (2009) show that the yield spread is a useful measure of inflation expectations, but only after controlling for liquidity in the TIPS market. We do not directly control for TIPS liquidity, but our out-of-sample analysis focuses on the second half of our sample period, where TIPS liquidity was much less of a concern relative to the early years of TIPS trading. As column 6 in Panel A shows, even during this relatively liquid period for TIPS, YS_t is insignificant in the presence of $OR_{t-1,t}$. The yield spread YS_t is the difference between the average yields of the 10-year nominal Treasury Notes and the 10-year TIPS in our sample. Thus the insignificance of YS_t is probably due to the fact that it captures inflation expectations over a relatively longer horizon while our regressions in Panel A of Table 6 focus on a relatively shorter horizon (i.e., the one-month ahead inflation rate).

Column 10 in Panel A of Table 6 verifies that the informational content of TIPS is

[11] We chose the cuofff level to be just below the minimum of the option price index in Table 4. We also tried other cutoff levels, such as 10^{-6} and 10^{-10}, but it did not impact the significance of our variable $OR_{t-1,t}$.

coming from the embedded option return and not from the TIPS total return.[12] However, as shown in column 7 of Panel A, $OR_{t-1,t}$ is not significant in the presence of lagged inflation. When we include all of the variables in our regression (see column 11 of Panel A in Table 6), only the lagged inflation and the VIX return are significant. Although the lagged inflation drives out the significance of $OR_{t-1,t}$, the adjusted-R^2 is 44.3%, which is larger than the 35.3% adjusted-R^2 in column 11 of Panel A in Table 5.

While the out-of-sample one-month ahead results in Panel A of Table 6 are fairly strong, our one-month forward results in Panel B of Table 6 are relatively weak. The option return is insignificant and the R^2 values in many of the columns are close to zero, which produces negative adjusted-R^2 values. The highest adjusted-R^2 is 16.4%, which occurs in column 9, where the VIX return is significant at the 5% level. Columns 1-4 of Panel B show that none of the variables, including lagged inflation, has an ability to explain the one-month forward out-of-sample inflation rate.

4.6.2 Robustness

In our earlier regressions, we constructed the variables OP_t and $OR_{t-1,t}$ by assuming that the interest rate and the inflation rate follow (1)-(2). In this section, we explore an alternative explanatory variable that is less sensitive to model specification. We use the embedded option returns in the last month of each of the rolling subsamples to compute a new variable, ORF_t, which we define as the fraction of options in month t with a positive return. To calculate ORF_t, we divide the number of embedded options with a positive return in month t by the total number of embedded options in month t. Using ORF_t instead of $OR_{t-1,t}$ allows us to investigate the robustness of our modeling assumptions. Any other model that produces positive (negative) embedded option returns when our model produces positive (negative) embedded option returns will give the same time series for ORF_t and thus the same regression results.

Table 7 reproduces the out-of-sample regressions from Table 6, this time using ORF_t in

[12] This result is different from what is found in Chu, Pittman, and Chen (2007), who show that the gross market price of maturing TIPS contains information about inflation expectations.

place of $OR_{t-1,t}$. If we compare each column in Panel A of Table 7 with each column in Panel A of Table 6, we find that ORF_t in Table 7 is statistically significant whenever $OR_{t-1,t}$ in Table 6 is statistically significant. The coefficients in Table 7 are significant at the 5% level, which is less than the 1% level in Table 6. In column 7 in Panel A of Table 7, we find that lagged inflation drives out the significance of ORF_t, which mirrors our conclusion from Table 6. This result continues to hold when we include other explanatory variables (Panel A of Table 7, column 11). We find that ORF_t does nearly as well as $OR_{t-1,t}$ for explaining the out-of-sample variation in the one-month ahead inflation rate. The adjusted-R^2 in column 11 in Panel A of Table 7 is 41.2%, which is only slightly lower than the adjusted-R^2 of 44.3% from column 11 in Panel A of Table 6.

In Panel B of Table 7, we examine the ability of ORF_t to explain the one-month forward out-of-sample inflation rate. The coefficient on ORF_t is significant at the 10% level in columns 2, 5-6, and 9-10. However, it is not significant in column 11, which includes all of the explanatory variables. When we examine the adjusted-R^2 values, we find that column 9 in Panel B of Table 7 explains 21.1% of the variation in the one-month forward inflation rate, while column 11 in the same panel explains only 17.4%. There are only two variables in column 9, ORF_t and the VIX return, which are significant at the 10% and 5% levels, respectively. Apparently, the VIX return is proxying for economic uncertainty (Bloom, 2009) that is not captured by our variable ORF_t.

4.6.3 Discussion

The regressions in Tables 6-7 show how our variables $OR_{t-1,t}$ and ORF_t help to explain the out-of-sample inflation rate. For the one-month ahead inflation rate, the significance of our variables is not driven away by common inflation variables such as the yield spread, the return on gold, or the VIX return (Tables 6-7, Panel A, columns 6, 8, and 9). For the one-month forward inflation rate, the results are mixed. Sometimes our variables remain significant (Table 7, Panel B, columns 2, 5, 6, 9, and 10), while other times they are insignificant (Table 6, Panel B, all columns and Table 7, Panel B, columns 7, 8, and 11).

Overall, our out-of-sample results in Tables 6-7 are somewhat weaker than our in-sample results in Table 5. There are several contributing reasons. First, our rolling subsample is only 80 months long, which is half as long as our full sample (161 months). Thus our parameter estimates and our embedded option estimates are noisier in the subsamples, which makes for noisier embedded option explanatory variables. Second, given the short length of our rolling window, some of our subsamples do not include periods with deflationary expectations. Thus the embedded option values in these subsamples are close to zero and exhibit little variation. Lastly, the short length of our window decreases not only the time length of each subsample, but it can also decrease the number of securities that is included in each subsample. For example, in our early subsamples, the number of TIPS and matching nominal Treasuries is reduced since some of these securities have not yet been auctioned. The smaller number of securities implies that there are fewer observations within the subsample for estimating our model parameters.

4.7 Alternative weighting schemes

In this section we perform an additional robustness check by using alternative weighting schemes to construct the variables OP_t and $OR_{t-1,t}$. We then re-estimate our in-sample and out-of-sample regressions using these alternative variables. We conclude that our results in Tables 5-6 are robust to using different weighting schemes.

4.7.1 Maturity weights and moneyness weights

In (8)-(10), we used value weights to construct the variables OP_t and $OR_{t-1,t}$. We now consider weighting schemes that are based on maturity or moneyness. Following Section 4.4, let N_t denote the number of TIPS in our sample in month t. Suppose the nth TIPS in month t has time to maturity V_{nt}, which is measured in years. We use V_{nt} to construct a set of maturity weights, where the weight assigned to the nth TIPS in month t is

$$W_{nt} = \frac{V_{nt}}{\sum_{n=1}^{N_t} V_{nt}}. \tag{14}$$

Upon substituting (14) into the right-hand side of (9)-(10), we get a new pair of explanatory variables, $OptValLT_t$ and $OptRetLT_{t-1,t}$. The variable $OptValLT_t$ is a maturity-weighted option price index while the variable $OptRetLT_{t-1,t}$ is a maturity-weighted option return index. Given the weighting scheme in (14), longer term options are assigned larger weights.

We also construct a pair of explanatory variables that favors shorter term options. To do this, the weight assigned to the nth TIPS in month t is

$$W_{nt} = \frac{10 - V_{nt}}{\sum_{n=1}^{N_t} (10 - V_{nt})}, \tag{15}$$

where the number 10 is used to reflect the 10-year maturity of the TIPS. Upon substituting (15) into the right-hand side of (9)-(10), we get a new pair of explanatory variables, $OptValST_t$ and $OptRetST_{t-1,t}$. The variable $OptValST_t$ ($OptRetST_{t-1,t}$) is an option price (option return) index that favors shorter term options.

Using equation (44) in Appendix A, the embedded option's strike price divided by the inflation adjusted face value for the nth TIPS in month t is

$$M_{nt} = \frac{F}{Fe^{\int_u^t i_s ds}}, \tag{16}$$

where the exponential term in (16) is the inflation adjustment factor. As discussed in Section 2.1, we substitute the U.S. Treasury's CPI-U index ratio for the inflation adjustment factor. Thus M_{nt} in (16) describes the moneyness of the embedded option. Inflation in our sample is usually positive, so all of the embedded options are out-of-the-money. However, we can use M_{nt} to construct explanatory variables that favor nearer-to-the-money (NTM) options. To do this, the weight assigned to the nth TIPS in month t is

$$W_{nt} = \frac{M_{nt}}{\sum_{n=1}^{N_t} M_{nt}}. \tag{17}$$

Alternatively, we can construct explanatory variables that favor deeper out-of-the-money

(OTM) options. In this case, the weight assigned to the nth TIPS in month t is

$$W_{nt} = \frac{1 - M_{nt}}{\sum_{n=1}^{N_t} (1 - M_{nt})}, \tag{18}$$

where the number 1 represents an at-the-money option. Upon substituting (17) into the right-hand side of (9)-(10), we get a new pair of explanatory variables, $OptValNTM_t$ and $OptRetNTM_{t-1,t}$. These are the moneyness-weighted option price and option return indices that favor NTM options. Similarly, upon substituting (18) into the right-hand side of (9)-(10), we get a new pair of explanatory variables, $OptValOTM_t$ and $OptRetOTM_{t-1,t}$. These are the moneyness-weighted option price and option return indices that favor deeper OTM options. We use all of these new variables in the next section.

4.7.2 Empirical results with alternative weights

Table 8 shows the in-sample regressions using our alternative weighting schemes. Panel A shows our estimation results for the one-month ahead inflation rate, while Panel B shows our results for the one-month forward inflation rate. Columns 1, 3, 5, and 7 are univariate regressions that use $OptRetLT_{t-1,t}$, $OptRetST_{t-1,t}$, $OptRetNTM_{t-1,t}$, and $OptRetOTM_{t-1,t}$, respectively, as the explanatory variable. In both Panels A and B, the coefficients on these variables have the correct sign and are statistically significant at the 1% level. In columns 2, 4, 6, and 8 we add several additional explanatory variables. These additional variables do not drive out the significance of our alternative option return indices. The gold return and the embedded option price index are not significant in any of the columns of Table 8, which is consistent with our findings in column 11 of Table 5. Likewise, the lagged inflation and the VIX return remain important in Panel A of Table 8, but they are not significant in Panel B. Again, these results mimic those in column 11 of Table 5. Overall, Table 8 shows that our in-sample results in Table 5 are robust to different weighting schemes.

Table 9 shows a similar analysis for our out-of-sample results. In columns 1, 3, 5, and 7 of Panel A, the variables $OptRetLT_{t-1,t}$, $OptRetST_{t-1,t}$, $OptRetNTM_{t-1,t}$, and

$OptRetOTM_{t-1,t}$ are significant at the 1% level and explain about 11% of the variation in the one-month ahead out-of-sample inflation rate, which is similar to Table 6, Panel A, column 2. However, once we add the additional explanatory variables (see columns 2, 4, 6, and 8), the embedded option return index is insignificant. Only the lagged inflation and the VIX return are significant, which mirrors our result in Table 6, Panel A, column 11. In Panel B of Table 9, our variables are statistically insignificant, which is consistent with our earlier results in Panel B of Table 6. Overall, Table 9 shows that our out-of-sample results in Table 6 are robust to different weighting schemes.

4.8 Five-year TIPS

In sections 4.5-4.6, we used 10-year TIPS to construct our option-based explanatory variables. We now re-examine our empirical results by including 5-year TIPS. For completeness, we do this two ways. First, in section 4.8.1 below, we estimate our model using all of the securities in Table 1, i.e., we use 10-year TIPS, 5-year TIPS, and all matching nominal T-Notes. Second, in section 4.8.2, we estimate our model using only the securities in Panels C and D of Table 1, i.e., we use 5-year TIPS and 5-year matching nominal T-Notes.

4.8.1 Empirical results with 5-year and 10-year TIPS

Table 10 shows our in-sample regressions when we estimate our model using all of the securities in Table 1. In this case, the variables OP_t and $OR_{t-1,t}$ are constructed using both 5-year TIPS and 10-year TIPS. As Panel A of Table 10 shows, our variable $OR_{t-1,t}$ is always statistically significant at the 1% level, which mirrors our result from Panel A of Table 5. The adjusted-R^2 values in Panel A of Table 10 are similar in magnitude to their counterparts in Table 5. Furthermore, the sign of the coefficient on $OR_{t-1,t}$ is negative, which is consistent with economic intuition. Thus the inclusion of 5-year TIPS supports our earlier claim that the embedded deflation option contains useful information for explaining the one-month ahead inflation rate.

The main difference between Panel A of Table 10 and Panel A of Table 5 is the economic significance of $OR_{t-1,t}$. In column 2 of Panel A in Table 5, recall that the coefficient on $OR_{t-1,t}$ is -0.0052. Thus a 100% option return predicts a decrease of 52 basis points in the one-month ahead annualized rate of inflation. In contrast, in column 2 of Panel A in Table 10, the coefficient on $OR_{t-1,t}$ is -0.021, which differs from the coefficient in Table 5 by a factor of four. The reason for this is that the estimated option values tend to be larger when we include 5-year TIPS in our sample. In Panel A of Table 2, the embedded option value in 10-year TIPS does not exceed \$0.0615 per \$100 face amount. However, when we include 5-year TIPS (Panel B of Table 2), the embedded option value is as large as \$1.4447 per \$100 face amount. These larger option values have less variation and produce lower option returns. Thus our option return index, which is value-weighted, is lower when we include 5-year TIPS. In other words, 100% option return using 10-year TIPS is equivalent to a less than 100% return using 5-year TIPS and 10-year TIPS. Thus the economic significance of Tables 5 and 10 is closer than it appears.

In Panel B of Table 10, we find that $OR_{t-1,t}$ is significant at the 5% level in columns 7 and 9-11 and is significant at the 10% level otherwise. This is slightly less than the significance in Panel B of Table 5. If we compare column 11 in Panel B of Table 10 to its counterpart in Panel B of Table 5, we find that in both cases $OR_{t-1,t}$ is the only significant explanatory variable (at the 5% level) and the adjusted-R^2 values are similar (5.5% vs. 6.8%). Furthermore, the coefficients on $OR_{t-1,t}$ differ by approximately a factor of 4, which is consistent with our discussion in the previous paragraph. Overall, the inclusion of 5-year TIPS supports our earlier claim that the embedded option in TIPS contains useful information for explaining the one-month forward inflation rate.

Table 11 shows our out-of-sample regressions when we use ORF_t as an explanatory variable. Recall that ORF_t is robust to model specification since any other pricing model that produces the same signs for the embedded option returns will produce the same variable ORF_t. If we compare Panel A of Table 7 to Panel A of Table 11, we find very similar results. Thus the inclusion of 5-year TIPS does not alter our earlier claim that ORF_t is a useful variable for explaining the one-month ahead inflation rate. In Panel B of Table 11, we find that the coefficient on ORF_t is more significant than its counterpart in Panel B of Table 7. For example, in column 11 of Panel B in Table 7, ORF_t is insignificant and the adjusted-R^2 is 17.4%. In contrast, in column 11 of Panel B in Table 11, ORF_t is significant at the 10% level and the adjusted-R^2 is 22.3%. The highest adjusted-R^2 in Panel B of Table 11 is 23.9%, which occurs in column 9. In this column, ORF_t is significant at the 1% level and the VIX return is significant at the 5% level. Overall, the inclusion of 5-year TIPS strengthens our earlier result that ORF_t is a useful variable for explaining the one-month forward inflation rate.

4.8.2 Empirical results with only 5-year TIPS

As a robustness check, we also estimated our model using only 5-year TIPS and 5-year matching nominal T-Notes. We did not use this case in our main analysis since the number of 5-year TIPS during our sample period is one-third the number of 10-year TIPS. Furthermore, the number of monthly observations for 5-year TIPS is about one-fifth the number of monthly observations for 10-year TIPS (Table 1). There is also a gap in the data using 5-year TIPS since the 5-year TIPS that was issued in July 1997 matured in July 2002, and the next auction of 5-year TIPS occurred in October 2004. However, in spite of these issues, we went ahead and estimated our model using the available monthly 5-year TIPS data from July 1997 - May 2010. To avoid overfitting the TIPS market, we include matching nominal 5-year T-Notes, which mirrors our approach using 10-year TIPS.

We estimate that the value of the embedded option in 5-year TIPS does not exceed \$1.3134 per \$100 principal amount, which is similar to what we report in Panel B of Table 2.[13] This is much higher than the \$0.0615 per \$100 principal amount that we found for 10-year TIPS, but it makes sense because most of the 5-year TIPS were outstanding during the deflationary period in the second half of 2008. In addition, the probability of experiencing

[13] After estimating our model using 5-year TIPS and 5-year matching nominal T-Notes, we tested the joint statistical significance of the option values and the option returns by performing Wald tests. For both tests, the p-value is less than 0.0001. Thus we reject the null hypotheses that the estimated option values and the estimated option returns from 5-year TIPS are jointly equal to zero. This confirms our results in Panel B of Table 3.

cumulative deflation over a 5-year period is likely higher than the probability of experiencing cumulative deflation over a 10-year period. At the margin, this may be contributing to a higher embedded option value in 5-year TIPS relative to 10-year TIPS. When we examine the time variation in the embedded option value in 5-year TIPS, the pattern is similar to what we observe in Panel A of Figure 2. Thus we do not think that our earlier results are driven by our choice of 10-year TIPS, or 10-year and 5-year TIPS, instead of 5-year TIPS alone.

Lastly, when we reconstruct our embedded option indices using only 5-year TIPS, we find that our regression results are weaker statistically but stronger economically than our earlier results. For example, using 10-year TIPS, our embedded option return index is significant at the 1% level in columns 2 and 5-11 in Panel A of Table 5. If we construct an otherwise similar table using only 5-year TIPS, the statistical significance drops to 5% in columns 2, 5-6, and 8-11, while column 7 is significant at the 10% level. In addition, the coefficient on the embedded option return index is always negative, which is the correct sign. In column 2 of Panel A in Table 5, the coefficient on $OR_{t-1,t}$ is -0.0052. Thus a 100% embedded option return predicts a decrease of 52 basis points in the one-month ahead inflation rate. If we instead use only 5-year TIPS, the coefficient on $OR_{t-1,t}$ in column 2 is -0.011, which implies a decrease of 110 basis points. Thus the economic significance is greater when we estimate our model using 5-year TIPS instead of 10-year TIPS. In summary, given the evidence from 5-year TIPS, our overall conclusion remains the same. The embedded option in TIPS contains useful information about future inflation.

4.9 Liquidity

Table 4 shows that the sample correlation between the option price index and the yield spread is -0.3768 (p-value is less than 0.0001). A possible explanation for this correlation, which we refute below, is that the option price index might be capturing illiquidity in the TIPS market. For example, as illiquidity increases, the TIPS market price should decrease. All else equal, as the TIPS market price decreases, the real yield will increase and thus the yield spread between nominal and real bonds will decrease. The correlation of -0.3768 shows that as the yield spread decreases, the embedded option value tends to increase even though the TIPS market price tends to decrease. This raises the question of whether the option price index is capturing a lower break-even inflation rate, which would make the option more valuable, or whether the option price index is capturing the increase in illiquidity. We argue in favor of the former explanation, i.e., the embedded option value is capturing the possibility of deflation and is not directly related to liquidity in the TIPS market.

Our TIPS pricing model in (1)-(3) is a traditional asset pricing model since it does not explicitly account for liquidity. Thus if liquidity is present in the market prices of TIPS, the TIPS pricing errors from solving (7) should reflect liquidity. However, when we examine the sample correlations between the TIPS pricing error and our embedded option indices, we cannot reject the null hypothesis that these correlations are zero. The sample correlation between the TIPS pricing error and our variable OP_t is -0.042 (p-value is 0.599), while the sample correlation between the root mean square TIPS pricing error and OP_t is 0.012 (p-value is 0.879). Likewise, the sample correlation between the TIPS pricing error and our variable $OR_{t-1,t}$ is 0.095 (p-value is 0.230), while the sample correlation between the root mean square TIPS pricing error and $OR_{t-1,t}$ is 0.052 (p-value is 0.512). Thus it is unlikely that our option-based variables are proxying for liquidity in the TIPS market. Instead, the empirical evidence suggests that the estimated option values are capturing the possibility of deflation.[14]

Our results are consistent with Wright (2009, Figure 1), which shows the yields on two TIPS with comparable maturity dates but different issue dates. The two TIPS are the 1.875% 10-year TIPS with ISIN ending in 28BD1 and the 0.625% 5-year TIPS with ISIN ending in 28HW3. In spite of the higher real coupon rate on the 10-year TIPS, Wright's Figure 1 shows that the 10-year TIPS yield is higher than the 5-year TIPS yield during the

[14] We get a similar result if we use ORF_t instead of OP_t or $OR_{t-1,t}$. Specifically, the sample correlation between the TIPS pricing error and our variable ORF_t is 0.066 (p-value is 0.408), while the sample correlation between the root mean square TIPS pricing error and ORF_t is -0.045 (p-value is 0.571).

last few months of 2008 and the first half of 2009. Wright (2009, pp. 128-129) argues that the yield difference between these two TIPS is mostly due to differences in the deflation option value and not due to liquidity. In other words, the embedded deflation option in the 5-year TIPS is worth more than the embedded deflation option in the 10-year TIPS.

We use our TIPS pricing model to directly verify Wright's (2009) conclusions. Using monthly data, we reproduce Wright's Figure 1 in Panel A of our Figure 3. Panel B of our Figure 3 shows the yield difference, which we calculate as the 10-year TIPS yield minus the 5-year TIPS yield. From the estimation using both 5-year TIPS and 10-year TIPS along with matching nominal T-Notes, we extract the embedded option values for these two TIPS, which we plot in Panel C of Figure 3. Lastly, Panel D of Figure 3 shows the option value difference, which we calculate as the option value in 5-year TIPS minus the option value in 10-year TIPS. Comparing Panels B and D, we find that the option value difference closely tracks the yield difference. The biggest difference in yields and option values occurs in the fall of 2008, which was a deflationary period. When we regress the yield difference in Panel B onto the option value difference in Panel D, we get an adjusted-R^2 of 75.5%. Thus our results confirm Wright's (2009) conjecture that the yield difference between on-the-run and off-the-run TIPS is mostly due to different embedded option values, and not due to liquidity.

5 Conclusion

We contribute to the literature by uncovering the informational content of the embedded deflation option in TIPS. While the previous literature has ignored the embedded option in TIPS, we value the option explicitly. To the best of our knowledge, we are the first to study the dynamics of the embedded option value. We argue that the embedded option return contains important information for explaining the future inflation rate, even in the presence of standard inflation variables. We show that the informational content of the embedded option is statistically and economically important and thus it should not be ignored. Our paper should be valuable to anyone interested in assessing inflationary expectations.

We find that the embedded option values, across all months and all 10-year TIPS in our sample, vary from approximately zero to a maximum of \$0.0615 per \$100 face value. When we add 5-year TIPS to our sample, the maximum option value increases to almost \$1.45 per \$100 face amount. For both samples, we show that the time variation in the option values coincides with periods that are marked by deflationary expectations (2003-2004 and 2008-2009). We use our estimated option values to construct several explanatory variables, which we then use to explain the in-sample and the out-of-sample one-month ahead inflation rate and the one-month forward inflation rate.

There are several important findings in our paper. First, we conclude that the embedded option return index is a significant variable for explaining the one-month ahead inflation rate, both in-sample and out-of-sample. Our results suggest that a 100% embedded option return is consistent with a 52 basis point (61 basis point) decrease in next month's in-sample (out-of-sample) annualized rate of inflation. Both of these results are statistically significant at the 1% level. For most of our regressions, the traditional inflation variables such as the yield spread and the return on gold are insignificant in the presence of our embedded option return index. However, the lagged inflation rate and the return on the VIX index continue to be important variables. Presumably, these variables capture additional uncertainty beyond what is contained in the embedded option return. Second, our main conclusions are not altered when we use alternative weighting schemes to construct our embedded option indices. Our results are robust to using value weights, moneyness weights, or maturity weights. Third, we find that the fraction of positive option returns, as captured by our variable ORF_t, is also significant for explaining future inflation. The significance of ORF_t for explaining the one-month forward inflation rate increases when we include 5-year TIPS in our sample (Table 11). Since ORF_t is less sensitive to model specification than $OR_{t-1,t}$, our results appear to be robust. Lastly, we find that the inclusion of 5-year TIPS in our sample does not alter our main conclusions. In summary, our paper shows that the embedded deflation option in TIPS is informationally relevant for explaining future inflation, both in-sample and out-of-sample.

Appendix

A Pricing model for TIPS

We stack the nominal interest rate r_t and the inflation rate i_t into a vector $X_t = [r_t \;\; i_t]^\top$, where $\top$ denotes the transpose. Thus we can rewrite (1)-(2) as

$$dX_t = (a + AX_t)\,dt + Bdz_t^Q, \tag{19}$$

where $a = [a_1 \;\; a_2]^\top$, $z_t^Q = [z_{1t}^Q \;\; z_{2t}^Q]^\top$, and A and B are the matrices

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

Since A is not a diagonal matrix, (19) is a coupled system of equations. Changes in r_t depend on both r_t and i_t, while changes in i_t depend on both i_t and r_t. Instead of working with X_t directly, we work with a decoupled system that is related to (19). Define Λ as

$$\Lambda = \begin{bmatrix} 1 & \frac{A_{12}}{\lambda_2 - A_{11}} \\ \frac{A_{21}}{\lambda_1 - A_{22}} & 1 \end{bmatrix},$$

where λ_1 and λ_2 are

$$\begin{aligned} \lambda_1 &= \frac{1}{2}(A_{11} + A_{22}) + \frac{1}{2}\sqrt{(A_{11} - A_{22})^2 + 4A_{12}A_{21}}\,, \\ \lambda_2 &= \frac{1}{2}(A_{11} + A_{22}) - \frac{1}{2}\sqrt{(A_{11} - A_{22})^2 + 4A_{12}A_{21}}\,. \end{aligned}$$

The constants λ_1 and λ_2 are the eigenvalues of A, while the columns of Λ are the associated eigenvectors. It is easily verified that $\Lambda^{-1}A\Lambda = D$, where D is the diagonal matrix

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

We now define a new set of variables $Y_t = \Lambda^{-1}X_t$, where $Y_t = [Y_{1t} \quad Y_{2t}]^\top$. Also define $b = \Lambda^{-1}a$ and $\Sigma = \Lambda^{-1}B$, where $b = [b_1 \;\; b_2]^\top$ and where

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}.$$

Using Itô's lemma, the process for Y_t is

$$dY_t = (b + DY_t)\, dt + \Sigma dz_t^Q, \tag{20}$$

which is an uncoupled system since D is diagonal. We solve (3) using the variables Y_{1t} and Y_{2t}. We then recover the TIPS price in terms of r_t and i_t by noting that $X_t = \Lambda Y_t$, i.e.,

$$\begin{bmatrix} r_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{A_{12}}{\lambda_2 - A_{11}} \\ \frac{A_{21}}{\lambda_1 - A_{22}} & 1 \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} Y_{1t} + \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right) Y_{2t} \\ \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right) Y_{1t} + Y_{2t} \end{bmatrix}. \tag{21}$$

To get the moments for Y_{1t} and Y_{2t}, we solve (20) to get

$$Y_{1s} = e^{\lambda_1(s-t)}Y_{1t} + \frac{b_1}{\lambda_1}\left[e^{\lambda_1(s-t)} - 1\right] + e^{\lambda_1 s}\int_t^s e^{-\lambda_1 u}\left(\sigma_{11}dz_{1u}^Q + \sigma_{12}dz_{2u}^Q\right), \tag{22}$$

$$Y_{2s} = e^{\lambda_2(s-t)}Y_{2t} + \frac{b_2}{\lambda_2}\left[e^{\lambda_2(s-t)} - 1\right] + e^{\lambda_2 s}\int_t^s e^{-\lambda_2 u}\left(\sigma_{21}dz_{1u}^Q + \sigma_{22}dz_{2u}^Q\right), \tag{23}$$

for $s \geq t$. Taking expectations of (22)-(23) gives

$$\mathbb{E}_t^Q\left[Y_{1s}\right] = e^{\lambda_1(s-t)}Y_{1t} + \frac{b_1}{\lambda_1}\left[e^{\lambda_1(s-t)} - 1\right], \tag{24}$$

$$\mathbb{E}_t^Q\left[Y_{2s}\right] = e^{\lambda_2(s-t)}Y_{2t} + \frac{b_2}{\lambda_2}\left[e^{\lambda_2(s-t)} - 1\right]. \tag{25}$$

To get the variance of Y_{1s}, note that

$$\begin{aligned} Var_t^Q\left[Y_{1s}\right] &= \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q\left[Y_{1s}\right]\right)^2\right] = e^{2\lambda_1 s}\int_t^s e^{-2\lambda_1 u}\left(\sigma_{11}^2 + \sigma_{12}^2\right) du \\ &= \frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1}\left[e^{2\lambda_1(s-t)} - 1\right]. \end{aligned} \tag{26}$$

A similar calculation gives

$$Var_t^Q[Y_{2s}] = \frac{\sigma_{21}^2 + \sigma_{22}^2}{2\lambda_2}\left[e^{2\lambda_2(s-t)} - 1\right]. \tag{27}$$

To get the covariance between Y_{1t} and Y_{2t}, note that

$$\begin{aligned}
Cov_t^Q[Y_{1s}, Y_{2s}] &= \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q[Y_{1s}]\right)\left(Y_{2s} - \mathbb{E}_t^Q[Y_{2s}]\right)\right] \\
&= e^{(\lambda_1+\lambda_2)s}\left(\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}\right)\int_t^s e^{-(\lambda_1+\lambda_2)u}du \\
&= \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1+\lambda_2}\left[e^{(\lambda_1+\lambda_2)(s-t)} - 1\right]. \qquad (28)
\end{aligned}$$

Given (20), Y_{1s} and Y_{2s} are bivariate normal with conditional moments (24)-(25), (26)-(27), and (28). To evaluate the TIPS price, we need to know the joint distribution of $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$ for $k = 1, 2, \ldots, n$. Using (21), note that

$$\begin{aligned}
\int_t^{t_k} r_s ds &= \int_t^{t_k} Y_{1s}ds + \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s}ds, \\
\int_t^{t_k} i_s ds &= \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right)\int_t^{t_k} Y_{1s}ds + \int_t^{t_k} Y_{2s}ds.
\end{aligned}$$

Thus to get the joint distribution of $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$, it is sufficient to characterize the joint distribution of $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$. Since Y_{1s} and Y_{2s} are jointly normal, $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$ are also jointly normal. This follows since the sum of normally distributed random variables is also normally distributed. Thus we only need to characterize the first two moments of $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$.

Suppose $k = n$ and recall that $t_n = T$. We focus on the case of time T, but our results apply for any t_k in the upper limit of integration. Using (22)-(23), we have

$$\begin{aligned}
\int_t^T Y_{1s}ds &= \int_t^T e^{\lambda_1(s-t)}Y_{1t}ds + \frac{b_1}{\lambda_1}\int_t^T \left[e^{\lambda_1(s-t)} - 1\right]ds \qquad (29) \\
&\quad + \int_t^T e^{\lambda_1 s}\int_t^s e^{-\lambda_1 u}\left(\sigma_{11}dz_{1u}^Q + \sigma_{12}dz_{2u}^Q\right)ds
\end{aligned}$$

and

$$\int_t^T Y_{2s} ds = \int_t^T e^{\lambda_2(s-t)} Y_{2t} ds + \frac{b_2}{\lambda_2} \int_t^T \left[e^{\lambda_2(s-t)} - 1 \right] ds \tag{30}$$
$$+ \int_t^T e^{\lambda_2 s} \int_t^s e^{-\lambda_2 u} \left(\sigma_{21} dz_{1u}^Q + \sigma_{22} dz_{2u}^Q \right) ds.$$

Thus

$$\mathbb{E}_t^Q \left[\int_t^T Y_{1s} ds \right] = \left(Y_{1t} + \frac{b_1}{\lambda_1} \right) \frac{1}{\lambda_1} \left[e^{\lambda_1(T-t)} - 1 \right] - \frac{b_1}{\lambda_1} (T-t), \tag{31}$$

$$\mathbb{E}_t^Q \left[\int_t^T Y_{2s} ds \right] = \left(Y_{2t} + \frac{b_2}{\lambda_2} \right) \frac{1}{\lambda_2} \left[e^{\lambda_2(T-t)} - 1 \right] - \frac{b_2}{\lambda_2} (T-t). \tag{32}$$

To get the variance of $\int_t^T Y_{1s} ds$ note that

$$Var_t^Q \left[\int_t^T Y_{1s} ds \right] = Cov_t^Q \left[\int_t^T Y_{1s} ds, \int_t^T Y_{1u} du \right] \tag{33}$$
$$= \int_t^T Cov_t^Q \left[Y_{1s}, \int_t^s Y_{1u} du \right] ds + \int_t^T Cov_t^Q \left[Y_{1s}, \int_s^T Y_{1u} du \right] ds.$$

The last line of (33) includes two terms. The first term is

$$\int_t^T Cov_t^Q \left[Y_{1s}, \int_t^s Y_{1u} du \right] ds = \int_t^T \left(\int_t^s Cov_t^Q \left[Y_{1s}, Y_{1u} \right] du \right) ds. \tag{34}$$

We need to calculate $Cov_t^Q \left[Y_{1s}, Y_{1u} \right]$ which is

$$Cov_t^Q \left[Y_{1s}, Y_{1u} \right] = \mathbb{E}_t^Q \left[\left(Y_{1s} - \mathbb{E}_t^Q \left[Y_{1s} \right] \right) \left(Y_{1u} - \mathbb{E}_t^Q \left[Y_{1u} \right] \right) \right]$$
$$= e^{\lambda_1 s} e^{\lambda_1 u} \int_t^u e^{-2\lambda_1 v} \left(\sigma_{11}^2 + \sigma_{12}^2 \right) dv$$
$$= e^{\lambda_1 s} e^{\lambda_1 u} \frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1} \left[e^{-2\lambda_1 t} - e^{-2\lambda_1 u} \right]. \tag{35}$$

Substituting (35) into the right-hand side of (34), we get

$$\frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1} \int_t^T \left(\int_t^s e^{\lambda_1 s} e^{\lambda_1 u} \left[e^{-2\lambda_1 t} - e^{-2\lambda_1 u} \right] du \right) ds \tag{36}$$

which is easy to evaluate. The second term in the last line of (33) is

$$\int_t^T Cov_t^Q \left[Y_{1s}, \int_s^T Y_{1u} du \right] ds.$$

Using (29), note that

$$\begin{aligned}\int_s^T Y_{1u} du &= \int_s^T e^{\lambda_1 (u-s)} Y_{1s} du + \frac{b_1}{\lambda_1} \int_s^T \left[e^{\lambda_1 (u-s)} - 1 \right] du \\ &+ \int_s^T e^{\lambda_1 u} \int_s^u e^{-\lambda_1 v} \left(\sigma_{11} dz_{1v}^Q + \sigma_{12} dz_{2v}^Q \right) du.\end{aligned}$$

The right hand side of the above expression has three terms, but only the first term on the right hand side has non-zero correlation with Y_{1s}. Thus

$$\begin{aligned}\int_t^T Cov_t^Q \left[Y_{1s}, \int_s^T Y_{1u} du \right] ds &= \int_t^T Cov_t^Q \left[Y_{1s}, \int_s^T e^{\lambda_1 (u-s)} Y_{1s} du \right] ds \\ &= \int_t^T Var_t^Q \left[Y_{1s} \right] \left[\int_s^T e^{\lambda_1 (u-s)} du \right] ds \qquad (37)\end{aligned}$$

which can be evaluated using (26). Combining (36) and (37) gives the result

$$Var_t^Q \left[\int_t^T Y_{1s} ds \right] = \frac{\sigma_{11}^2 + \sigma_{12}^2}{\lambda_1^2} (T-t) + \frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1^3} \left[e^{2\lambda_1 (T-t)} - 1 \right] + \frac{\sigma_{11}^2 + \sigma_{12}^2}{\lambda_1^3} \left[2 - 2e^{\lambda_1 (T-t)} \right].$$

A similar calculation gives

$$Var_t^Q \left[\int_t^T Y_{2s} ds \right] = \frac{\sigma_{21}^2 + \sigma_{22}^2}{\lambda_2^2} (T-t) + \frac{\sigma_{21}^2 + \sigma_{22}^2}{2\lambda_2^3} \left[e^{2\lambda_2 (T-t)} - 1 \right] + \frac{\sigma_{21}^2 + \sigma_{22}^2}{\lambda_2^3} \left[2 - 2e^{\lambda_2 (T-t)} \right].$$

To get the covariance between $\int_t^T Y_{1s} ds$ and $\int_t^T Y_{2s} ds$, note that

$$\begin{aligned}Cov_t^Q \left[\int_t^T Y_{1s} ds, \int_t^T Y_{2u} du \right] &= \int_t^T Cov_t^Q \left[Y_{1s}, \int_t^T Y_{2u} du \right] ds \\ &= \int_t^T Cov_t^Q \left[Y_{1s}, \int_t^s Y_{2u} du \right] ds \qquad (38) \\ &+ \int_t^T Cov_t^Q \left[Y_{1s}, \int_s^T Y_{2u} du \right] ds.\end{aligned}$$

Like equation (33), there are two terms in (38) that must be evaluated. The first term is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_t^s Y_{2u}du\right]ds = \int_t^T\left[\int_t^s Cov_t^Q\left[Y_{1s}, Y_{2u}\right]du\right]ds. \tag{39}$$

Since $u \leq s$ we have,

$$\begin{aligned} Cov_t^Q\left[Y_{1s}, Y_{2u}\right] &= \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q\left[Y_{1s}\right]\right)\left(Y_{2u} - \mathbb{E}_t^Q\left[Y_{2u}\right]\right)\right] \\ &= e^{\lambda_1 s}e^{\lambda_1 u}\frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1 + \lambda_2}\left[e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_1+\lambda_2)u}\right], \end{aligned}$$

and thus the right-hand side of (39) is easy to evaluate. The second term in (38) is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{2u}du\right]ds. \tag{40}$$

Using (30), we have

$$\begin{aligned} \int_s^T Y_{2u}du &= \int_s^T e^{\lambda_2(u-s)}Y_{2s}du + \frac{b_2}{\lambda_2}\int_s^T\left[e^{\lambda_2(u-s)} - 1\right]du \\ &\quad + \int_s^T e^{\lambda_2 u}\int_s^u e^{-\lambda_2 v}\left(\sigma_{21}dz_{1v}^Q + \sigma_{22}dz_{2v}^Q\right)du. \end{aligned}$$

The right hand side of the above expression has three terms, but only the first term on the right hand side has non-zero correlation with Y_{1s}. Thus (40) is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{2u}du\right]ds = \int_t^T Cov_t^Q\left[Y_{1s}, Y_{2s}\right]\left[\int_s^T e^{\lambda_2(u-s)}du\right]ds \tag{41}$$

which can be evaluated using (28). Combining (39) and (41) gives the result

$$Cov_t^Q\left[\int_t^T Y_{1s}ds, \int_t^T Y_{2u}du\right] = \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1 + \lambda_2}\left\{\begin{array}{c} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)(T-t) \\ +\frac{1}{\lambda_1}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\left[1 - e^{\lambda_1(T-t)}\right] \\ +\frac{1}{\lambda_2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\left[1 - e^{\lambda_2(T-t)}\right] \\ +\frac{1}{\lambda_1\lambda_2}\left[e^{(\lambda_1+\lambda_2)(T-t)} - 1\right] \end{array}\right\}.$$

We now return to (3) to evaluate the TIPS price. The first term in (3) is

$$\sum_{k=1}^{n} cF\, \mathbb{E}_t^Q \left[e^{\int_u^{t_k} i_s ds} e^{-\int_t^{t_k} r_s ds} \right].$$

Note that

$$\begin{aligned} \mathbb{E}_t^Q \left[e^{\int_u^{t_k} i_s ds} e^{-\int_t^{t_k} r_s ds} \right] &= e^{\int_u^t i_s ds} \mathbb{E}_t^Q \left[e^{\left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right) \int_t^{t_k} Y_{1s} ds + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right) \int_t^{t_k} Y_{2s} ds} \right] \\ &= e^{\int_u^t i_s ds} e^{G(Y_{1t}, Y_{2t}, t, t_k)}, \end{aligned} \tag{42}$$

where $G = G(Y_{1t}, Y_{2t}, t, t_k)$ is

$$\begin{aligned} G &= \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1 \right) \mathbb{E}_t^Q \left[\int_t^{t_k} Y_{1s} ds \right] + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}} \right) \mathbb{E}_t^Q \left[\int_t^{t_k} Y_{2s} ds \right] \\ &\quad + \frac{1}{2} \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1 \right)^2 Var_t^Q \left[\int_t^{t_k} Y_{1s} ds \right] + \frac{1}{2} \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}} \right)^2 Var_t^Q \left[\int_t^{t_k} Y_{2s} ds \right] \\ &\quad + \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1 \right) \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}} \right) Cov_t^Q \left[\int_t^{t_k} Y_{1s} ds, \int_t^{t_k} Y_{2s} ds \right]. \end{aligned} \tag{43}$$

In (42), we have used the property that for any normally distributed random variable Z, $\mathbb{E}[e^Z] = e^{\mathbb{E}(Z) + 0.5 Var(Z)}$. The second term in (3) is

$$\mathbb{E}_t^Q \left[F e^{\int_u^{t_n} i_s ds} e^{-\int_t^{t_n} r_s ds} \right] = F e^{\int_u^t i_s ds} e^{G(Y_{1t}, Y_{2t}, t, t_n)}.$$

where G is given in (43). The third term in (3) is

$$\begin{aligned} &\mathbb{E}_t^Q \left[e^{-\int_t^{t_n} r_s ds} \max\left(0, F - F e^{\int_u^{t_n} i_s ds}\right) \right] \\ = \; & F e^{\int_u^t i_s ds} \mathbb{E}_t^Q \left[e^{-\int_t^{t_n} r_s ds} \left(e^{-\int_u^t i_s ds} - e^{\int_t^{t_n} i_s ds} \right) 1_{\left\{ -\int_u^t i_s ds > \int_t^{t_n} i_s ds \right\}} \right] \end{aligned} \tag{44}$$

where $1_{\{\cdot\}}$ is the indicator function for the event in curly brackets. Equation (44) involves two expectations, where each expectation is of the form

$$\mathbb{E}\left[e^{Z_1} 1_{\{d > Z_2\}} \right], \tag{45}$$

where Z_1 and Z_2 are bivariate normal random variables and d is a constant. The joint distribution of Z_1 and Z_2 is characterized by $\mathbb{E}(Z_1)$, $\mathbb{E}(Z_2)$, $Var(Z_1)$, $Var(Z_2)$, and $Cov(Z_1, Z_2)$. A direct calculation reveals that (45) is equal to

$$\mathbb{E}\left[e^{Z_1} 1_{\{d>Z_2\}}\right] = e^{\mathbb{E}(Z_1)+\frac{1}{2}Var(Z_1)} N\left(\frac{d - \mathbb{E}(Z_2) - Cov(Z_1, Z_2)}{\sqrt{Var(Z_2)}}\right), \tag{46}$$

where $N(\cdot)$ is the standard normal cumulative distribution function. To analyze the first expectation in (44), we use (46) and we let

$$\begin{aligned} Z_1 &= -\int_t^{t_n} r_s ds = -\int_t^{t_n} Y_{1s} ds - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_n} Y_{2s} ds, & (47)\\ Z_2 &= \int_t^{t_n} i_s ds = \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right)\int_t^{t_n} Y_{1s} ds + \int_t^{t_n} Y_{2s} ds, & (48)\\ d &= -\int_u^t i_s ds. & (49)\end{aligned}$$

To analyze the second expectation in (44), we use (46) and we let

$$Z_1 = -\int_t^{t_n} r_s ds + \int_t^{t_n} i_s ds = \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)\int_t^{t_n} Y_{1s} ds + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_n} Y_{2s} ds,$$

where Z_2 and d are given by (48) and (49), respectively. Thus (44) depends on $\mathbb{E}_t^Q[\int_t^{t_n} Y_{1s} ds]$, $\mathbb{E}_t^Q[\int_t^{t_n} Y_{2s} ds]$, $Var_t^Q[\int_t^{t_n} Y_{1s} ds]$, $Var_t^Q[\int_t^{t_n} Y_{2s} ds]$, and $Cov_t^Q[\int_t^{t_n} Y_{1s} ds, \int_t^{t_n} Y_{2s} ds]$, which are given above. This completes the derivation of the TIPS price in (3).

B Pricing model for nominal Treasuries

We now derive the price of a nominal Treasury Note. Using equation (21), the first term in (5) can be written as

$$\sum_{k=1}^{n} \bar{c} F\, \mathbb{E}_t^Q\left[e^{-\int_t^{t_k} r_s ds}\right] = \sum_{k=1}^{n} \bar{c} F\, \mathbb{E}_t^Q\left[e^{-\int_t^{t_k} Y_{1s} ds - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s} ds}\right]$$

Note that

$$\mathbb{E}_t^Q\left[e^{-\int_t^{t_k} Y_{1s}ds-\left(\frac{A_{12}}{\lambda_2-A_{11}}\right)\int_t^{t_k} Y_{2s}ds}\right]=e^{H(Y_{1t},Y_{2t},t,t_k)},$$

where $H=H\left(Y_{1t},Y_{2t,}t,t_k\right)$ is

$$\begin{aligned} H &= -\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{1s}ds\right]-\left(\frac{A_{12}}{\lambda_2-A_{11}}\right)\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{2s}ds\right] \\ &\quad +\frac{1}{2}Var_t^Q\left[\int_t^{t_k} Y_{1s}ds\right]+\frac{1}{2}\left(\frac{A_{12}}{\lambda_2-A_{11}}\right)^2 Var_t^Q\left[\int_t^{t_k} Y_{2s}ds\right] \\ &\quad +\left(\frac{A_{12}}{\lambda_2-A_{11}}\right)Cov_t^Q\left[\int_t^{t_k} Y_{1s}ds,\int_t^{t_k} Y_{2s}ds\right]. \end{aligned} \tag{50}$$

Like equation (43), (50) uses the property that for any normally distributed random variable Z, $\mathbb{E}[e^Z]=e^{\mathbb{E}(Z)+0.5Var(Z)}$. Similarly, the second term in (5) is

$$\mathbb{E}_t^Q\left[Fe^{-\int_t^{t_n} r_s ds}\right]=Fe^{H(Y_{1t},Y_{2t},t,t_n)},$$

where the function $H\left(Y_{1t},Y_{2t,}t,t_n\right)$ is obtained by substituting t_n for t_k in (50). This completes the derivation of the nominal Treasury Note price in (5).

C Long run means

In this section we show how to derive the long run means and the speeds of mean reversion for r_t and i_t. We can rewrite (19) as $dX_t=-A\left(-A^{-1}a-X_t\right)dt+Bdz_t^Q$, where we define $\kappa=-A$ and $\pi=-A^{-1}a=[\pi_r\ \ \pi_i]^\top$. Upon substituting we get $dX_t=\kappa\left(\pi-X_t\right)dt+Bdz_t^Q$, which is a more traditional form. The long run means are

$$\pi_r = \frac{a_2A_{12}-a_1A_{22}}{A_{11}A_{22}-A_{12}A_{21}}, \tag{51}$$

$$\pi_i = \frac{a_1A_{21}-a_2A_{11}}{A_{11}A_{22}-A_{12}A_{21}}. \tag{52}$$

Our empirical estimates for (51)-(52) are shown in Table 2.

D Choice of initial values for model estimation

To minimize the sum of the squared pricing errors in (7), we need to choose initial values for the parameters. Since (7) is a nonlinear function of the parameters, the minimized value of (7) depends on the choice of the initial parameter values. To ensure that we converge to a global minimum, we construct multiple sets of initial parameter values.

We use three methods to derive 84 sets of initial parameter values. For the first method, we discretize (1)-(2) using a time step of Δt to get

$$r_{t+\Delta t} - r_t = \alpha_1 + \beta_{11} r_t + \beta_{12} i_t + \epsilon_{1,t+\Delta t}, \tag{53}$$

$$i_{t+\Delta t} - i_t = \alpha_2 + \beta_{21} r_t + \beta_{22} i_t + \epsilon_{2,t+\Delta t}, \tag{54}$$

where $\alpha_i = a_i \Delta t$ and $\beta_{ij} = A_{ij} \Delta t$ for $j = 1, 2$. Following Section 4, we set $B_{12} = 0$. Thus we have $Var(\epsilon_{1,t+\Delta t}) = B_{11}^2 \Delta t$, $Var(\epsilon_{2,t+\Delta t}) = (B_{21}^2 + B_{22}^2)\Delta t$, and $Cov(\epsilon_{1,t+\Delta t}, \epsilon_{2,t+\Delta t}) = B_{11} B_{21} \Delta t$. Using the monthly interest rate and the monthly inflation data that are described in Section 3, we estimate (53)-(54) using OLS for the sample period January 15, 1997 through May 31, 2010. This is the full sample period for all 10-year TIPS. We then use the OLS estimates and the time step Δt to back out our first set of initial parameter values. We repeat this procedure using monthly data to obtain 21 additional sets of parameter values, where the 21 sample periods correspond to the lives of the 21 TIPS in Table 1. The entire procedure is then repeated using daily data, where the daily inflation rate uses a linearly interpolated CPI. Thus our first method constructs a total of 44 sets of parameter values.

Our second method uses an iterative approach to generate 20 additional sets of parameter values. We first estimate (53)-(54) using OLS with monthly data for two sample periods, January 15, 1997 through August 31, 2009 and October 15, 2004 through August 31, 2009. These sample periods cover the majority of 10-year TIPS and 5-year TIPS, respectively. The results of the OLS regressions give us 2 sets of parameter values. We use the set corresponding to January 15, 1997 through August 31, 2009 as a starting point to minimize (7) using the Marquardt method, where we use data for 10-year TIPS, at both monthly and weekly frequency, and with and without matching nominal 10-year Treasury Notes. We then repeat the process by using the set corresponding to October 15, 2004 through August 31, 2009 as a starting point to minimize (7) using the Marquardt method, where we use data for 5-year TIPS, at both monthly and weekly frequency, and with and without matching nominal 5-year Treasury Notes. The outcome of this process gives us 8 additional sets of parameter values. We then use each of these 8 sets as a starting point to again minimize (7), where we now focus on 10-year TIPS at monthly frequency with matching nominal 10-year Treasury Notes. This produces additional sets of parameter values. We select the most economically reasonable set and we use it as a starting point to again minimize (7), where we use data for 5-year TIPS and 10-year TIPS, at both monthly and weekly frequency, and with and without matching nominal Treasury Notes. This produces another 8 sets of parameter values. We then examine all of our sets of parameter values and we eliminate any set that is economically unreasonable. Upon doing so, our iterative procedure produces 20 sets of parameter values.

Our third method constructs 20 additional sets of parameter values. We re-parameterize our pricing model in terms of λ_1, λ_2, b_1, b_2, A_{11}, A_{12}, σ_{11}, σ_{21}, and σ_{22}. We estimate these nine quantities directly and then we back out the original parameters, i.e., the vector a and the matrices A and B. By re-parameterizing the model, we get a different nonlinear functional form for (7). We use our 20 sets of parameters from the second method to minimize the new nonlinear form (7), using monthly data on 10-year TIPS with matching nominal 10-year Treasury Notes. This gives 20 new sets of parameter values.

When we combine the three methods, we have 84 sets of parameter values. We use these 84 sets as the initial parameter values in our main empirical analysis, for both in-sample and out-of-sample estimations (see Section 4). For our in-sample estimation, 9 of the 84 sets lead to the global minimum in (7). This is true for both Panels A and B of Table 2. For our out-of-sample estimation, the number of sets that lead to a global minimum varies by subsample. For example, 4 out of the 84 sets lead to the global minimum in subsample 1, 6 out of the 84 sets lead to the global minimum in subsample 2, and so forth. Across all of the subsamples, 19 of the 84 sets on average lead to the global minimum. In all cases, we verify the global minimum by checking that the first-order derivatives are zero and that all eigenvalues of the Hessian are positive, which gives a positive definite Hessian at the global minimum.

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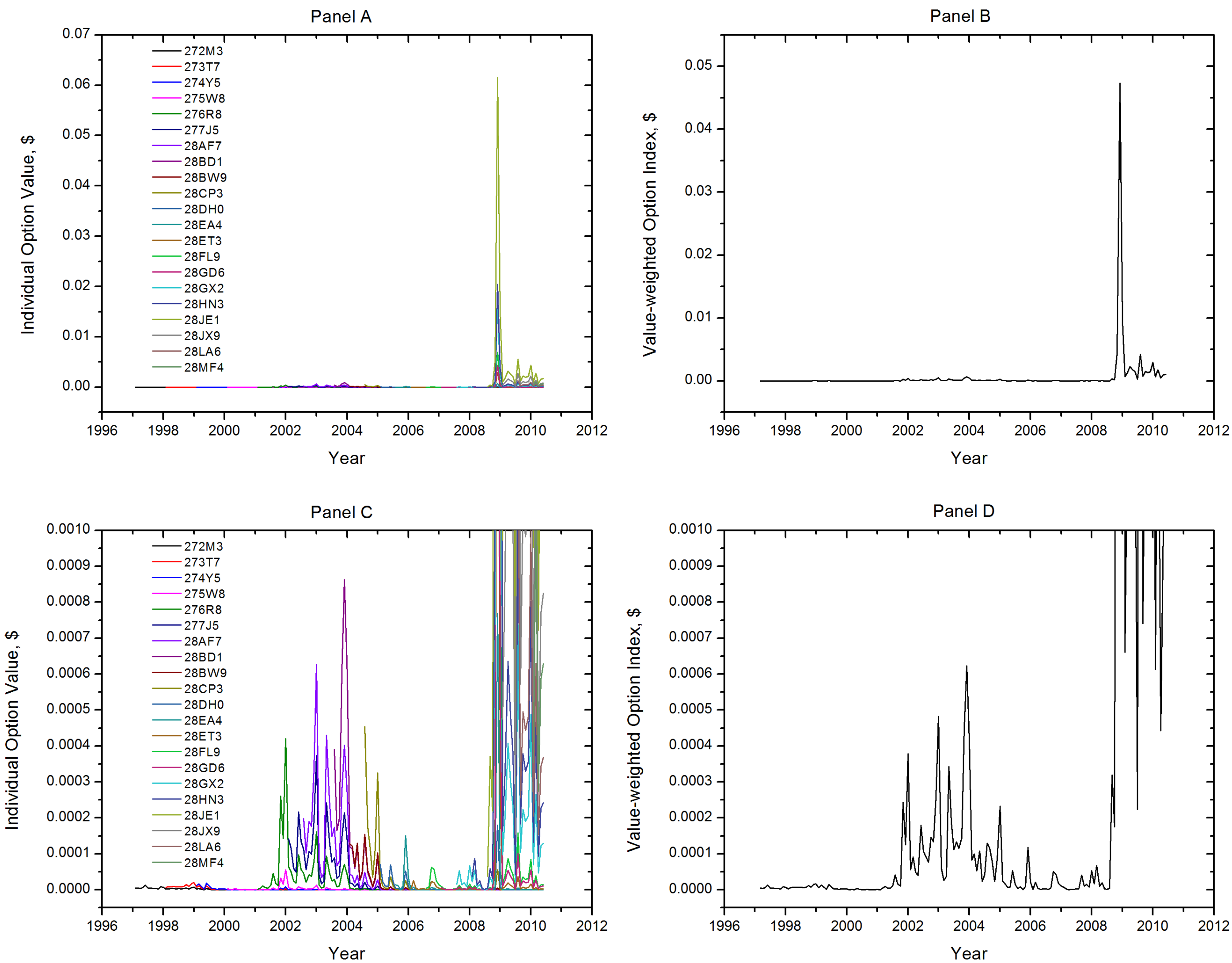

Figure 1: Deflation Option Values for 10-year TIPS

The figure presents the embedded deflation option values for 10-year TIPS. The deflation options are estimated using 10-year TIPS and 10-year matching T-Notes. Top two panels (A and B) represent individual options' series and a value-weighted option index. Bottom two panels (C and D) represent individual options' series and a value-weighted option index on the zoomed scale with the maximum value of $0.0010. Sample period is January 1997 - May 2010, monthly frequency.

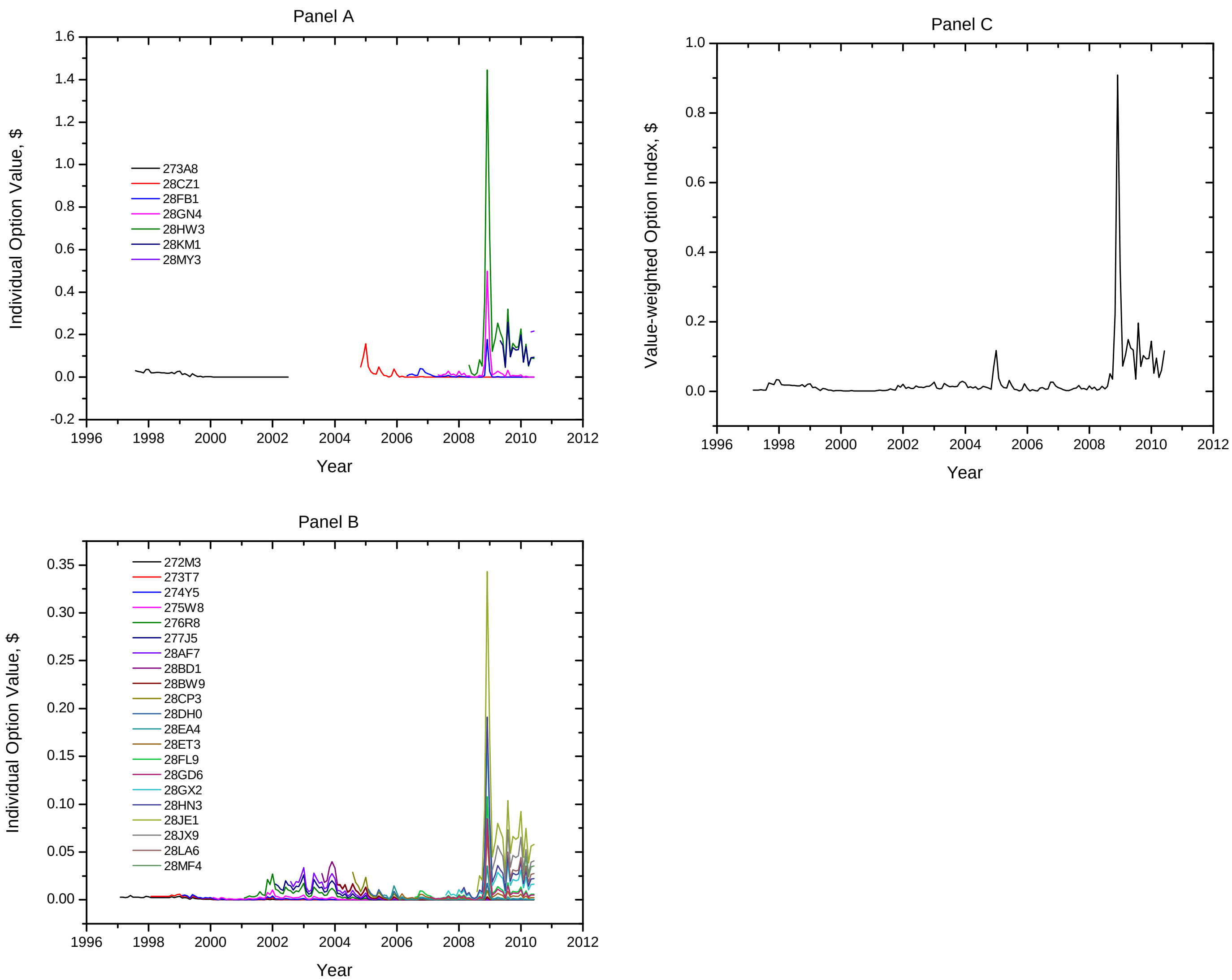

Figure 2: Deflation Option Values for 5-year and 10-year TIPS

The figure presents the embedded deflation option values for 5-year and 10-year TIPS. The deflation option values are estimated using both 5-year and 10-year TIPS and matching T-Notes. For clarity, the individual options' series for 5-year and 10-year TIPS are plotted separately in Panels A and B, respectively. Panel C represents a value-weighted option index based on both options' series. Sample period is January 1997 - May 2010, monthly frequency. There were no outstanding 5-year TIPS from August 2002 through September 2004.

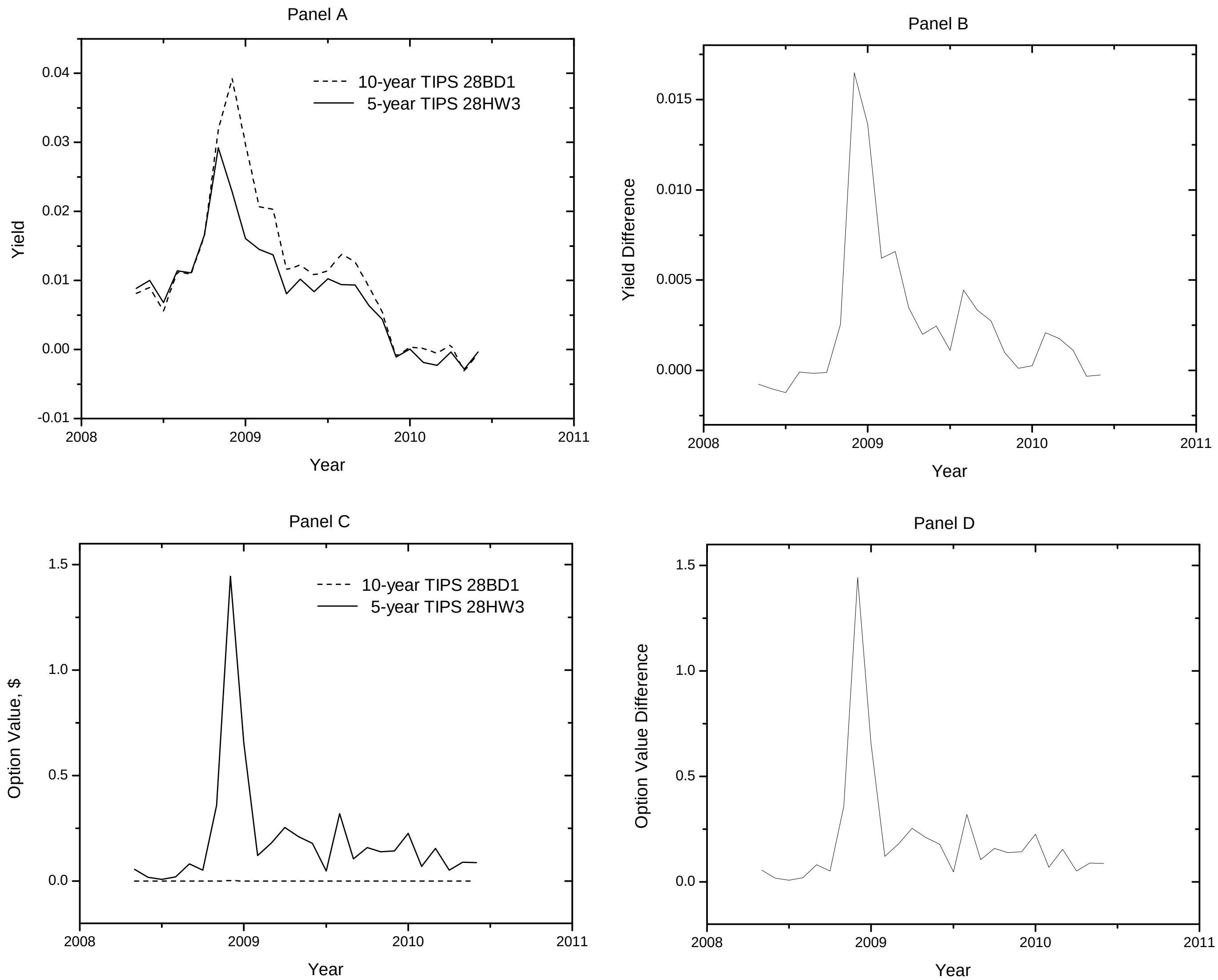

Figure 3: Yield Difference versus Option Value Difference

This figure shows yield difference and option value difference of two very similar TIPS. One is the 10-year TIPS 28BD1 with maturity on July 15, 2013; the other is the 5-year TIPS 28HW3 with maturity on April 15, 2013. The yields and the yield difference for the two TIPS are plotted in Panels A and B, respectively. The option values and the option value difference for the two TIPS are plotted in Panels C and D, respectively. The OLS regression of the yield difference on the option value difference generates an adjusted R^2, 75.5%. The option values are obtained from the estimation using both 5-year and 10-year TIPS and matching T-Notes. Sample period is January 1997 - May 2010, monthly frequency.

Table 1: Summary of Treasury Security Data

This table shows our sample of 10-year TIPS (Panel A), 10-year matching T-Notes (Panel B), 5-year TIPS (Panel C), and 5-year matching T-Notes (Panel D). The ISIN numbers are abbreviated. The full ISIN coding is preceded by US9128. “N/A” refers to “not available”. Sample period is January 1997 - May 2010, monthly frequency. There are 1,405 (1,268) observations for 10-year TIPS (T-Notes). And there are 256 (250) observations for 5-year TIPS (T-Notes). Source: Datastream Advance 4.0.

ISIN	Issue Date	Maturity Date	Coupon	Obs.
Panel A: 10-year Treasury Inflation Protected Securities				
272M3	1/15/1997	1/15/2007	3.375	120
273T7	1/15/1998	1/15/2008	3.625	120
274Y5	1/15/1999	1/15/2009	3.875	120
275W8	1/15/2000	1/15/2010	4.25	120
276R8	1/15/2001	1/15/2011	3.5	113
277J5	1/15/2002	1/15/2012	3.375	101
28AF7	7/15/2002	7/15/2012	3	95
28BD1	7/15/2003	7/15/2013	1.875	83
28BW9	1/15/2004	1/15/2014	2	77
28CP3	7/15/2004	7/15/2014	2	71
28DH0	1/15/2005	1/15/2015	1.625	65
28EA4	7/15/2005	7/15/2015	1.875	59
28ET3	1/15/2006	1/15/2016	2	53
28FL9	7/15/2006	7/15/2016	2.5	47
28GD6	1/15/2007	1/15/2017	2.375	41
28GX2	7/15/2007	7/15/2017	2.625	35
28HN3	1/15/2008	1/15/2018	1.625	29
28JE1	7/15/2008	7/15/2018	1.375	23
28JX9	1/15/2009	1/15/2019	2.125	17
28LA6	7/15/2009	7/15/2019	1.875	11
28MF4	1/15/2010	1/15/2020	1.375	5
Panel C: 5-year Treasury Inflation Protected Securities				
273A8	7/15/1997	7/15/2002	3.625	60
28CZ1	10/15/2004	4/15/2010	0.875	66
28FB1	4/15/2006	4/15/2011	2.375	50
28GN4	4/15/2007	4/15/2012	2	38
28HW3	4/15/2008	4/15/2013	0.625	26
28KM1	4/15/2009	4/15/2014	1.25	14
28MY3	4/15/2010	4/15/2015	0.5	2

ISIN	Issue Date	Maturity Date	Coupon	Obs.
Panel B: 10-year matching nominal Treasury Notes				
272J0	2/15/1997	2/15/2007	6.25	120
273X8	2/15/1998	2/15/2008	5.5	120
N/A				
275Z1	2/15/2000	2/15/2010	6.5	120
276T4	2/15/2001	2/15/2011	5	112
277L0	2/15/2002	2/15/2012	4.875	100
28AJ9	8/15/2002	8/15/2012	4.375	94
28BH2	8/15/2003	8/15/2013	4.25	82
28CA6	2/15/2004	2/15/2014	4	76
28CT5	8/15/2004	8/15/2014	4.25	70
28DM9	2/15/2005	2/15/2015	4	64
28EE6	8/15/2005	8/15/2015	4.25	58
28EW6	2/15/2006	2/15/2016	4.5	52
28FQ8	8/15/2006	8/15/2016	4.875	46
28GH7	2/15/2007	2/15/2017	4.625	40
28HA1	8/15/2007	8/15/2017	4.75	34
28HR4	2/15/2008	2/15/2018	3.5	28
28JH4	8/15/2008	8/15/2018	4	22
28KD1	2/15/2009	2/15/2019	2.75	16
28LJ7	8/15/2009	8/15/2019	3.625	10
28MP2	2/15/2010	2/15/2020	3.625	4
Panel D: 5-year matching nominal Treasury Notes				
273C4	7/31/1997	7/31/2002	6	61
28CX6	10/15/2004	10/15/2009	3.375	60
28FD7	4/30/2006	4/30/2011	4.875	49
28GQ7	4/30/2007	4/30/2012	4.5	38
28HY9	4/30/2008	4/30/2013	3.125	26
28KN9	4/30/2009	4/30/2014	1.875	14
28MZ0	4/30/2010	4/30/2015	2.5	2

Table 2: Two-factor Model Estimation Results

This table reports parameter estimates and pricing errors for our two-factor term structure model used to price TIPS and nominal T-Notes. Newton's method is used to minimize the sum of squared errors (SSE) between model prices and observed market prices. We minimize

$$SSE(\Theta) = \sum_{t=1}^{T} \left[\sum_{n=1}^{N_t} \left(P^*_{nt} - P_{nt}\right)^2 + \sum_{n=1}^{\bar{N}_t} \left(\bar{P}^*_{nt} - \bar{P}_{nt}\right)^2 \right],$$

where T is the total number of months in our sample, N_t is the number of TIPS in our sample for month t, $\bar{N}_t$ is the number of nominal T-Notes in our sample for month t, P^*_{nt} is the gross market price of the nth TIPS for month t, $\bar{P}^*_{nt}$ is the gross market price of the nth nominal T-Note for month t, P_{nt} is the model price of the nth TIPS for month t, and $\bar{P}_{nt}$ is the model price of the nth nominal T-Note for month t. The 9-dimensional parameter vector is $\Theta = (a_1, a_2, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, B_{22})^\top$. π_r is the implied long-run mean of interest rate; π_i is the implied long-run mean of inflation rate; meP is the mean dollar pricing error; $rmseP$ is the root mean squared dollar pricing error; meY is the mean yield error between actual and fitted bond yields; $rmseY$ is the root mean squared yield error. Our sample of Treasury securities, which is shown in Table 1, contains monthly gross bond prices covering the period from January 1997 to May 2010. Standard errors are given in parentheses.

Panel A: 10-year TIPS and matching T-Notes								
a_1	a_2	A_{11}	A_{12}	A_{21}	A_{22}	B_{11}	B_{21}	B_{22}
0.0146	0.0181	-0.2867	0.0328	0.9475	-2.9718	0.0025	0.0446	0.0000
(0.0004)	(0.0030)	(0.0114)	(0.0209)	(0.0960)	(0.1907)	(0.0095)	(0.0784)	(0.1696)
Implied long-run mean		Model pricing errors				Other statistics		
π_r	π_i	meP	rmseP	meY	rmseY	OptionMax	Obs.	
0.0534	0.0231	\$0.1775	\$3.8362	-0.0003	0.0071	\$0.0615	2,673	
(0.0006)	(0.0006)							

Panel B: 5- and 10-year TIPS and matching T-Notes								
a_1	a_2	A_{11}	A_{12}	A_{21}	A_{22}	B_{11}	B_{21}	B_{22}
0.0144	0.0171	-0.2806	0.0289	1.0020	-3.0528	0.0027	0.0714	0.0000
(0.0003)	(0.0029)	(0.0104)	(0.0190)	(0.0887)	(0.1734)	(0.0075)	(0.0338)	(0.2123)
Implied long-run mean		Model pricing errors				Other statistics		
π_r	π_i	meP	rmseP	meY	rmseY	OptionMax	Obs.	
0.0537	0.0232	\$0.1559	\$3.6068	-0.0002	0.0073	\$1.4447	3,179	
(0.0006)	(0.0007)							

Table 3: Significance Tests of Option Values and Option Returns

This table tests whether the estimated option values (or returns) are individually (or jointly) significantly different from zero. The significance criteria for individual tests are (1) (option / option std. err.) > 1.64; (2) abs (option return / std. err. of option returns) > 1.64. The standard errors are obtained based on Delta method. The significance criteria for joint tests are based on the Wald test value, which has a Chi-squared distribution. Panel A(B) reports significance tests of option values and returns based on a sample including 10-year (both 5-year and 10-year) TIPS and matching T-Notes. An option value cutoff of $1E-18$ is applied in the significance tests of option returns.

	Individual test		Joint test		
	Number of significant values	Sample size	Wald test value	Degree of freedom	p-value
Panel A: 10-year TIPS and matching Treasury notes					
Option value	0	1,405	335.7	1,396	1
Option return	131	1,143	2,498.6	1,134	<0.0001
Panel B: 5- and 10-year TIPS and matching Treasury notes					
Option value	1	1,661	73,766.1	1,652	<0.0001
Option return	575	1,504	15,439.9	1,495	<0.0001

Table 4: Summary Statistics and Correlations

This table reports descriptive statistics and correlations of all variables involved in the 10-year data. *Option Val* (*Ret*) is the monthly option value (return) index constructed as a value-weighted average of all option values (returns) available in each month, *ORF* is a fraction of the number of positive option returns over the total number of available option returns in each month, *Yield Spread* is the yield spread between the average 10-year nominal and real yields, *Gold Ret* is the return on Gold Bullion LBM U$/Troy Ounce, *VIX Ret* is return on the S&P500 implied volatility (VIX) index, and *Bond Ret* is a value-weighted average of individual TIPS gross price returns. These variables are used in the inflation forecast in Table 5. In Panel B, the p-values for the null hypothesis that the correlation coefficient is zero are reported in parentheses.

Panel A: Descriptive statistics

Variable	Mean	Median	Std. Dev.	Minimum	Maximum	Obs.
Option Val	0.0006	1.4960E−5	0.0038	6.9969E−8	0.0474	160
Option Ret	1.3608	−0.0893	4.5293	−0.9855	26.8318	160
ORF	0.4374	0	0.4829	0	1.0000	160
Yield Spread	0.0178	0.0196	0.0099	−0.0321	0.0374	160
Gold Ret	0.0089	0.0067	0.0471	−0.1698	0.1797	160
VIX Ret	0.0187	−0.0115	0.1918	−0.3150	0.9075	160
Bond Ret	0.0027	0.0029	0.0149	−0.0823	0.0502	160
Inflation, lag1	0.0237	0.0236	0.0468	−0.2321	0.1458	160
Inflation, lead1	0.0236	0.0236	0.0470	−0.2321	0.1458	159
Inflation, forward1	0.0236	0.0234	0.0471	−0.2321	0.1458	158

Panel B: Correlations

	Option Val	*Option Ret*	*ORF*	*Yield Spread*	*Gold Ret*	*VIX Ret*	*Bond Ret*	*Inflation, Lag1*	*Inflation, Lead1*	*Inflation, Forward1*
Option Val	1.0000									
Option Ret	0.2710	1.0000								
	(0.0005)									
ORF	0.1066	0.4813	1.0000							
	(0.1799)	(<0.0001)								
Yield Spread	−0.3768	−0.1129	0.0381	1.0000						
	(<0.0001)	(0.1551)	(0.6326)							
Gold Ret	0.1688	−0.2449	−0.1867	−0.0886	1.0000					
	(0.0329)	(0.0018)	(0.0181)	(0.2652)						
VIX Ret	−0.0553	0.0643	−0.0276	−0.0574	−0.0697	1.0000				
	(0.4876)	(0.4193)	(0.7288)	(0.4707)	(0.3812)					
Bond Ret	−0.0402	−0.2505	−0.0991	0.1166	0.3369	−0.0293	1.0000			
	(0.6140)	(0.0014)	(0.2125)	(0.1420)	(<0.0001)	(0.7131)				
Inflation, lag1	−0.5124	−0.4899	−0.4174	0.2950	0.0637	0.0626	0.1532	1.0000		
	(<0.0001)	(<0.0001)	(<0.0001)	(0.0002)	(0.4237)	(0.4319)	(0.0531)			
Inflation, lead1	−0.2737	−0.4991	−0.3110	0.2298	0.1635	−0.2216	0.2383	0.4670	1.0000	
	(0.0005)	(<0.0001)	(<0.0001)	(0.0036)	(0.0395)	(0.0050)	(0.0025)	(<0.0001)		
Inflation, forward1	0.0335	−0.1970	−0.0131	−0.0783	0.0862	−0.2510	0.0401	0.0318	0.4669	1.0000
	(0.6757)	(0.0131)	(0.8703)	(0.3280)	(0.2818)	(0.0015)	(0.6166)	(0.6921)	(<0.0001)	

Table 5: Full Sample Inflation Regressions

This table reports the results of the in-sample regressions using the 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, $OP_t(OR_{t-1,t})$ is the monthly option value (return) index constructed as a value-weighted average of all option values (returns) available at the end of month t, YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports full sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample period is from January 1997 to May 2010. Sample size is 159(158) monthly observations for Panel A(B). *- stat. sign. at 10% level; **- stat. sign. at 5% level; *** - stat. sign. at 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$											
Option Val	-3.33*** (-10.90)				-1.82*** (-5.81)						-0.43 (-0.90)
Option Ret		-0.0052*** (-3.48)			-0.0047*** (-3.54)	-0.0050*** (-3.81)	-0.0037*** (-2.83)	-0.0050*** (-3.36)	-0.0050*** (-3.72)	-0.0048*** (-4.15)	-0.0031*** (-2.99)
Yield Spread			1.09 (1.59)			0.83** (2.03)					0.39 (1.43)
Inflation, lag1				0.47*** (4.83)			0.29*** (4.02)				0.28*** (3.41)
Gold Ret								0.044 (0.69)			0.040 (0.52)
VIX Ret									-0.047* (-1.75)		-0.053** (-2.26)
Bond Ret										0.38 (1.06)	0.29 (0.93)
Constant	0.026*** (6.10)	0.031*** (8.43)	0.0042 (0.28)	0.012*** (2.67)	0.031*** (8.53)	0.016* (1.89)	0.022*** (5.54)	0.030*** (7.96)	0.031*** (8.60)	0.029*** (7.03)	0.014** (2.41)
Adj-R^2	0.069	0.244	0.047	0.213	0.260	0.270	0.305	0.241	0.275	0.253	0.353
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$											
Option Val	0.41* (1.69)				1.14*** (2.73)						0.41 (0.42)
Option Ret		-0.0020** (-2.19)			-0.0023** (-2.04)	-0.0022** (-2.10)	-0.0025** (-2.53)	-0.0019** (-2.23)	-0.0019** (-2.40)	-0.0021** (-2.58)	-0.0021** (-2.41)
Yield Spread			-0.37 (-0.92)			-0.48* (-1.66)					-0.47 (-1.60)
Inflation, lag1				0.032 (0.30)			-0.085 (-0.87)				-0.0039 (-0.03)
Gold Ret								0.040 (0.47)			0.0042 (0.05)
VIX Ret									-0.059 (-1.44)		-0.060 (-1.38)
Bond Ret										-0.032 (-0.08)	-0.023 (-0.07)
Constant	0.023*** (4.75)	0.026*** (6.48)	0.030*** (2.82)	0.023*** (3.76)	0.026*** (6.28)	0.035*** (4.67)	0.029*** (5.73)	0.026*** (5.95)	0.027*** (7.04)	0.026*** (5.52)	0.036*** (3.95)
Adj-R^2	-0.005	0.033	-0.000	-0.005	0.035	0.037	0.032	0.028	0.084	0.026	0.068

Table 6: Out-of-Sample Inflation Regressions

This table reports the results of the out-of-sample regressions using the 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, $OP_t(OR_{t-1,t})$ is the monthly option value (return) index constructed as a value-weighted average of all option values (returns) available at the end of month t, YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U\$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports out-of-sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample is a rolling window constructed as a half-sample of January 1997 to May 2010. Every month, half-sample preceding this month is used to estimate the model and compute the embedded deflation option value in the last two months of the rolling window, then the τ-month forward inflation rate is forecasted. The sample has 62(61) monthly observations, Panel A(B). $1E-8$ option value cutoff is imposed. *- stat. sign. at 10% level; **- stat. sign. at 5% level; ***- stat. sign. at 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$											
Option Val	-0.0042				-0.011						-0.0074
	(-0.19)				(-0.53)						(-0.37)
Option Ret		-0.0061***			-0.0061***	-0.0050***	-0.0013	-0.0064***	-0.0064***	-0.0053***	-0.0013
		(-3.19)			(-3.24)	(-3.31)	(-0.79)	(-2.82)	(-3.58)	(-3.43)	(-0.99)
Yield Spread			1.29			0.90					0.33
			(1.54)			(1.49)					(1.13)
Inflation, lag1				0.53***			0.49***				0.46***
				(4.42)			(3.45)				(4.92)
Gold Ret								0.26			0.11
								(1.27)			(1.09)
VIX Ret									-0.098*		-0.092***
									(-1.80)		(-3.23)
Bond Ret										1.05	0.57
										(1.28)	(1.30)
Constant	0.025**	0.030***	0.0026	0.012	0.031***	0.013	0.014	0.025**	0.033***	0.027**	0.0090
	(2.02)	(3.19)	(0.12)	(1.38)	(2.97)	(0.76)	(1.42)	(2.18)	(3.94)	(2.54)	(1.02)
Adj-R^2	-0.016	0.114	0.074	0.305	0.101	0.140	0.298	0.155	0.220	0.193	0.443
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$											
Option Val	0.0064				0.0063						0.0067
	(0.26)				(0.25)						(0.26)
Option Ret		-0.000093			-0.000062	-0.00059	0.0011	-0.00026	-0.00072	0.00052	0.00032
		(-0.12)			(-0.07)	(-0.55)	(0.47)	(-0.40)	(-0.72)	(0.44)	(0.16)
Yield Spread			-0.36			-0.41					-0.83***
			(-0.76)			(-0.80)					(-3.15)
Inflation, lag1				0.089			0.12				0.17
				(0.61)			(0.57)				(0.93)
Gold Ret								0.16			0.026
								(0.91)			(0.22)
VIX Ret									-0.13**		-0.13**
									(-2.14)		(-2.37)
Bond Ret										0.76	0.45
										(1.14)	(1.30)
Constant	0.023*	0.024**	0.030*	0.022*	0.023*	0.031	0.020	0.021	0.027***	0.022*	0.034**
	(1.81)	(2.00)	(1.68)	(1.73)	(1.73)	(1.62)	(1.28)	(1.54)	(2.76)	(1.71)	(2.52)
Adj-R^2	-0.016	-0.017	-0.010	-0.008	-0.034	-0.027	-0.023	-0.015	0.164	0.011	0.152

Table 7: Out-of-Sample Inflation Regressions, ORF Variable

This table reports the results of the out-of-sample regressions using the 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 ORF_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, OP_t is the monthly option value index constructed as a value-weighted average of all option values available at the end of month t, $ORF_{t-1,t}$ is a fraction of the number of positive option returns over the total number of available option returns at the end of month t, YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U\$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports out-of-sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample is a rolling window constructed as a half-sample of January 1997 to May 2010. Every month, half-sample preceding this month is used to estimate the model and compute the embedded deflation option value in the last two months of the rolling window, then the τ-month forward inflation rate is forecasted. The sample has 71(70) monthly observations, Panel A(B). *- stat. sign. at 10% level; **- stat. sign. at 5% level; ***- stat. sign. at 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$											
Option Val	-0.0036				-0.0097						-0.0094
	(-0.18)				(-0.64)						(-0.52)
ORF		-0.043**			-0.043**	-0.045***	-0.013	-0.038**	-0.047**	-0.040**	-0.018
		(-2.21)			(-2.24)	(-2.69)	(-0.75)	(-2.19)	(-2.29)	(-2.60)	(-1.12)
Yield Spread			1.27			1.35*					0.54*
			(1.58)			(1.92)					(1.82)
Inflation, lag1				0.52***			0.48***				0.42***
				(4.60)			(3.82)				(4.79)
Gold Ret								0.16			0.087
								(1.08)			(0.70)
VIX Ret									-0.10**		-0.096***
									(-2.04)		(-3.69)
Bond Ret										1.14	0.65
										(1.55)	(1.51)
Constant	0.025**	0.040***	0.0027	0.012	0.041***	0.018	0.018*	0.036***	0.044***	0.037***	0.012
	(2.26)	(5.67)	(0.13)	(1.49)	(5.19)	(1.15)	(1.77)	(3.65)	(6.30)	(4.20)	(1.15)
Adj-R^2	-0.014	0.078	0.065	0.263	0.065	0.155	0.258	0.082	0.185	0.169	0.412
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$											
Option Val	0.0053				0.0030						0.0021
	(0.23)				(0.14)						(0.09)
ORF		-0.018*			-0.018*	-0.018*	-0.014	-0.015	-0.025*	-0.016*	-0.015
		(-1.83)			(-1.81)	(-1.75)	(-1.14)	(-1.49)	(-1.99)	(-1.70)	(-1.00)
Yield Spread			-0.33			-0.30					-0.66**
			(-0.71)			(-0.71)					(-2.32)
Inflation, lag1				0.099			0.055				0.12
				(0.77)			(0.37)				(0.85)
Gold Ret								0.094			0.00010
								(0.53)			(0.00)
VIX Ret									-0.14**		-0.14**
									(-2.53)		(-2.64)
Bond Ret										0.52	0.25
										(0.79)	(0.74)
Constant	0.024**	0.031***	0.030*	0.022*	0.030***	0.036**	0.028**	0.028**	0.037***	0.029**	0.041***
	(2.15)	(3.05)	(1.85)	(1.98)	(2.71)	(2.37)	(2.03)	(2.09)	(4.16)	(2.55)	(3.02)
Adj-R^2	-0.014	0.001	-0.010	-0.005	-0.014	-0.010	-0.012	-0.008	0.211	0.006	0.174

Table 8: Full Sample Inflation Regressions, Alternative Weighting Schemes

This table reports the results of the in-sample regressions using the 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, $OP_t(OR_{t-1,t})$ is the monthly option value (return) index computed as: $OptValLT(OptRetLT)$ - maturity-weighted option value (return) index favoring long time-to-maturity options; $OptValST(OptRetST)$ - maturity-weighted option value (return) index favoring short time-to-maturity options; $OptValNTM(OptRetNTM)$ - moneyness-weighted option value (return) index favoring near-the-money options; $OptValOTM(OptRetOTM)$ - moneyness-weighted option value (return) index favoring deeper-out-of-the-money options. YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports full sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample period is from January 1997 to May 2010. Sample size is 159(158) monthly observations for Panel A(B). $1E-8$ option value cutoff is imposed. *- 10% stat. sign.;**- 5% stat. sign.; ***- 1% stat. sign.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$								
OptValLT		1.65						
		(0.67)						
OptRetlLT	-0.0022***	-0.0013**						
	(-3.57)	(-2.25)						
OptValST				16.4				
				(1.26)				
OptRetST			-0.0014***	-0.00086**				
			(-3.69)	(-2.24)				
OptValNTM						2.47		
						(0.77)		
OptRetNTM					-0.0020***	-0.0012**		
					(-3.53)	(-2.18)		
OptValOTM								3.04
								(0.23)
OptRetOTM							-0.0014***	-0.00073*
							(-2.90)	(-1.66)
Yield Spread		0.28		0.27		0.28		0.25
		(1.08)		(1.05)		(1.07)		(0.93)
Inflation, lag1		0.31***		0.33***		0.31***		0.33***
		(3.93)		(4.22)		(4.01)		(4.26)
Gold Ret		0.041		0.042		0.042		0.056
		(0.54)		(0.57)		(0.56)		(0.72)
VIX Ret		-0.051**		-0.051**		-0.051**		-0.053**
		(-2.30)		(-2.30)		(-2.31)		(-2.35)
Bond Ret		0.23		0.22		0.24		0.27
		(0.81)		(0.78)		(0.81)		(0.89)
Constant	0.030***	0.014**	0.029***	0.014**	0.030***	0.014**	0.029***	0.014**
	(8.17)	(2.40)	(8.13)	(2.26)	(8.15)	(2.36)	(8.00)	(2.32)
Adj-R^2	0.256	0.342	0.247	0.338	0.252	0.340	0.226	0.328
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$								
OptValLT		4.89						
		(1.36)						
OptRetlLT	-0.00092***	-0.0013***						
	(-3.27)	(-3.79)						
OptValST				28.3				
				(1.34)				
OptRetST			-0.00057***	-0.00086***				
			(-3.16)	(-3.75)				
OptValNTM						6.36		
						(1.40)		
OptRetNTM					-0.00083***	-0.0012***		
					(-3.24)	(-3.78)		
OptValOTM								21.3
								(1.48)
OptRetOTM							-0.00056***	-0.00081***
							(-2.83)	(-3.08)
Yield Spread		-0.56**		-0.59**		-0.56**		-0.59**
		(-2.01)		(-2.13)		(-2.03)		(-2.20)
Inflation, lag1		-0.011		-0.0073		-0.0092		0.0054
		(-0.08)		(-0.05)		(-0.06)		(0.04)
Gold Ret		-0.0094		-0.0051		-0.0080		-0.0030
		(-0.12)		(-0.07)		(-0.10)		(-0.04)
VIX Ret		-0.056		-0.056		-0.056		-0.058
		(-1.33)		(-1.32)		(-1.33)		(-1.36)
Bond Ret		-0.11		-0.13		-0.11		-0.092
		(-0.41)		(-0.46)		(-0.41)		(-0.32)
Constant	0.026***	0.038***	0.026***	0.038***	0.026***	0.038***	0.026***	0.037***
	(6.27)	(4.36)	(6.12)	(4.25)	(6.22)	(4.31)	(6.09)	(4.37)
Adj-R^2	0.039	0.089	0.034	0.088	0.037	0.088	0.031	0.084

Table 9: Out-of-Sample Inflation Regressions, Alternative Weighting Schemes

This table reports the results of the out-of-sample regressions using the 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, $OP_t(OR_{t-1,t})$ is the monthly option value (return) index computed as: $OptValLT(OptRetLT)$ - maturity-weighted option value (return) index favoring long time-to-maturity options; $OptValST(OptRetST)$ - maturity-weighted option value (return) index favoring short time-to-maturity options; $OptValNTM(OptRetNTM)$ - moneyness-weighted option value (return) index favoring near-the-money options; $OptValOTM(OptRetOTM)$ - moneyness-weighted option value (return) index favoring deeper-out-of-the-money options. YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports out-of-sample forecasting results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample is a rolling window constructed as a half-sample of January 1997 to May 2010. Every month, half-sample preceding this month is used to estimate the model and compute the embedded deflation option value in the last two months of the rolling window, then the τ-month forward inflation rate is forecasted. The sample has 62(61) monthly observations for Panel A(B). $1E-8$ option value cutoff is imposed. *- 10% stat. sign.;**- 5% stat. sign.; ***- 1% stat. sign.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$								
OptValLT		-0.0056						
		(-0.23)						
OptRetLT	-0.0012***	-0.00029						
	(-12.24)	(-1.51)						
OptValST				0.0013				
				(0.03)				
OptRetST			-0.00063***	-0.00015				
			(-13.02)	(-1.50)				
OptValNTM						-0.0059		
						(-0.21)		
OptRetNTM					-0.0010***	-0.00025		
					(-12.41)	(-1.52)		
OptValOTM								0.0044
								(0.09)
OptRetOTM							-0.00067***	-0.00013
							(-12.39)	(-1.25)
Yield Spread		0.31		0.30		0.31		0.28
		(1.08)		(1.05)		(1.07)		(0.87)
Inflation, lag1		0.46***		0.46***		0.46***		0.46***
		(4.92)		(4.95)		(4.93)		(4.90)
Gold Ret		0.12		0.12		0.12		0.070
		(1.19)		(1.20)		(1.19)		(0.61)
VIX Ret		-0.092***		-0.092***		-0.092***		-0.087***
		(-3.20)		(-3.19)		(-3.20)		(-3.10)
Bond Ret		0.57		0.57		0.57		0.74
		(1.25)		(1.25)		(1.25)		(1.39)
Constant	0.028***	0.0087	0.028***	0.0083	0.028***	0.0087	0.026**	0.0083
	(2.90)	(1.07)	(2.86)	(1.02)	(2.89)	(1.06)	(2.53)	(0.86)
Adj-R^2	0.111	0.443	0.108	0.442	0.111	0.443	0.108	0.438
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$								
OptValLT		0.012						
		(0.41)						
OptRetLT	0.00011	0.00025						
	(1.16)	(0.68)						
OptValST				0.043				
				(0.78)				
OptRetST			0.000069	0.00015				
			(1.44)	(0.76)				
OptValNTM						0.015		
						(0.44)		
OptRetNTM					0.00010	0.00022		
					(1.23)	(0.70)		
OptValOTM								0.042
								(0.73)
OptRetOTM							0.000077	0.00013
							(1.42)	(0.65)
Yield Spread		-0.80***		-0.80***		-0.80***		-0.86***
		(-2.96)		(-2.94)		(-2.95)		(-3.14)
Inflation, lag1		0.20		0.20		0.20		0.20
		(1.03)		(1.05)		(1.04)		(1.04)
Gold Ret		0.010		0.0091		0.0097		0.041
		(0.09)		(0.08)		(0.08)		(0.31)
VIX Ret		-0.13**		-0.13**		-0.13**		-0.13**
		(-2.38)		(-2.38)		(-2.38)		(-2.41)
Bond Ret		0.47		0.47		0.47		0.50
		(1.38)		(1.40)		(1.39)		(1.19)
Constant	0.023*	0.033**	0.023*	0.032**	0.023*	0.033**	0.022*	0.034**
	(1.97)	(2.51)	(1.97)	(2.52)	(1.97)	(2.51)	(1.77)	(2.47)
Adj-R^2	-0.016	0.156	-0.016	0.160	-0.016	0.157	-0.017	0.168

Table 10: Full Sample Inflation Regressions, 5-year and 10-year Data

This table reports the results of the in-sample regressions using both 5-year and 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 OR_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, $OP_t(OR_{t-1,t})$ is the monthly option value (return) index constructed as a value-weighted average of all option values (returns) available at the end of month t, YS_t is the yield spread between the average nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports full sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample period is from January 1997 to May 2010. Sample size is 159(158) monthly observations for Panel A(B). *- stat. sign. at 10% level; **- stat. sign. at 5% level; *** - stat. sign. at 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$											
Option Val	-0.17***				-0.085***						-0.0069
	(-4.44)				(-3.45)						(-0.25)
Option Ret		-0.021***			-0.019***	-0.020***	-0.015***	-0.020***	-0.020***	-0.019***	-0.013***
		(-3.67)			(-3.82)	(-4.17)	(-2.78)	(-3.53)	(-3.91)	(-4.43)	(-2.90)
Yield Spread			1.22*			0.99**					0.58*
			(1.79)			(2.34)					(1.91)
Inflation, lag1				0.47***			0.27***				0.24***
				(4.83)			(3.32)				(2.67)
Gold Ret								0.034			0.026
								(0.50)			(0.34)
VIX Ret									-0.050*		-0.054**
									(-1.79)		(-2.31)
Bond Ret										0.44	0.34
										(1.11)	(1.06)
Constant	0.029***	0.029***	0.0025	0.012***	0.031***	0.011	0.021***	0.028***	0.030***	0.027***	0.011*
	(7.38)	(8.50)	(0.18)	(2.67)	(8.90)	(1.37)	(5.50)	(8.01)	(8.87)	(6.80)	(1.75)
Adj-R^2	0.086	0.250	0.061	0.213	0.265	0.290	0.296	0.246	0.287	0.262	0.356
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$											
Option Val	-0.00051				0.038						-0.0053
	(-0.02)				(1.31)						(-0.10)
Option Ret		-0.0073*			-0.0083*	-0.0076*	-0.0096**	-0.0069*	-0.0070**	-0.0073**	-0.0080**
		(-1.93)			(-1.80)	(-1.91)	(-2.33)	(-1.95)	(-2.07)	(-2.23)	(-2.04)
Yield Spread			-0.18			-0.27					-0.25
			(-0.42)			(-0.80)					(-0.65)
Inflation, lag1				0.032			-0.098				-0.048
				(0.30)			(-0.93)				(-0.35)
Gold Ret								0.042			0.015
								(0.48)			(0.19)
VIX Ret									-0.061		-0.060
									(-1.45)		(-1.39)
Bond Ret										-0.00072	-0.013
										(-0.00)	(-0.04)
Constant	0.024***	0.025***	0.027**	0.023***	0.025***	0.030***	0.028***	0.025***	0.026***	0.025***	0.032***
	(4.87)	(6.12)	(2.42)	(3.76)	(5.49)	(3.56)	(5.42)	(5.60)	(6.86)	(5.15)	(2.88)
Adj-R^2	-0.006	0.026	-0.005	-0.005	0.023	0.023	0.026	0.021	0.080	0.019	0.055

Table 11: Out-of-Sample Inflation Regressions, 5-year and 10-year Data, ORF Variable

This table reports the results of the out-of-sample regressions using both 5-year and 10-year data:

$$i_{t+\tau,t+\tau+1} = \beta_0 + \beta_1 OP_t + \beta_2 ORF_{t-1,t} + \beta_3 YS_t + \beta_4 i_{t-1,t} + \beta_5 GoldRet_{t-1,t} + \beta_6 VIXRet_{t-1,t} + \beta_7 BondRet_{t-1,t} + \epsilon_{t+\tau+1},$$

where $i_{t+\tau,t+\tau+1}$ is a τ-period forward seasonally-unadjusted CPI-based annualized log inflation rate, OP_t is the monthly option value index constructed as a value-weighted average of all option values available at the end of month t, $ORF_{t-1,t}$ is a fraction of the number of positive option returns over the total number of available option returns at the end of month t, YS_t is the yield spread between the average 10-year nominal and real yields, $GoldRet$ is the return on Gold Bullion LBM U\$/Troy Ounce, $VIXRet$ is return on the S&P500 implied volatility (VIX) index, and $BondRet$ is a value-weighted average of individual TIPS gross price returns. Panel A(B) reports out-of-sample regression results for one-month ahead (forward) inflation regressions, $\tau = 0$ ($\tau = 1$). The $t-$statistics based on four lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. Sample is a rolling window constructed as a half-sample of January 1997 to May 2010. Every month, half-sample preceding this month is used to estimate the model and compute the embedded deflation option value in the last two months of the rolling window, then the τ-month forward inflation rate is forecasted. The sample has 69(68) monthly observations, Panel A(B). *- stat. sign. at 10% level; **- stat. sign. at 5% level; ***- stat. sign. at 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: Dependent variable is one-month ahead inflation, $\tau = 0$											
Option Val	-0.030				-0.021						-0.0037
	(-0.88)				(-0.71)						(-0.19)
ORF		-0.042**			-0.040**	-0.042**	-0.010	-0.038**	-0.048**	-0.036**	-0.015
		(-2.21)			(-2.14)	(-2.64)	(-0.63)	(-2.23)	(-2.32)	(-2.31)	(-0.98)
Yield Spread			1.36			1.38*					0.49*
			(1.67)			(1.90)					(1.70)
Inflation, lag1				0.57***			0.54***				0.47***
				(5.30)			(4.48)				(6.45)
Gold Ret								0.12			0.11
								(0.70)			(0.85)
VIX Ret									-0.11**		-0.093***
									(-2.02)		(-2.97)
Bond Ret										0.95	0.39
										(1.14)	(0.79)
Constant	0.026***	0.038***	0.00069	0.0097	0.041***	0.016	0.014	0.036***	0.042***	0.033***	0.0090
	(2.73)	(5.20)	(0.03)	(1.33)	(5.18)	(1.06)	(1.45)	(3.57)	(5.76)	(3.30)	(0.94)
Adj-R^2	-0.001	0.085	0.083	0.313	0.078	0.174	0.308	0.081	0.211	0.150	0.436
Panel B: Dependent variable is one-month forward inflation, $\tau = 1$											
Option Val	0.023				0.029						0.033
	(0.78)				(0.97)						(1.12)
ORF		-0.025**			-0.027***	-0.025**	-0.017	-0.022**	-0.036***	-0.022*	-0.021*
		(-2.59)			(-2.65)	(-2.57)	(-1.36)	(-2.21)	(-3.09)	(-1.83)	(-1.87)
Yield Spread			-0.083			-0.074					-0.65**
			(-0.17)			(-0.18)					(-2.18)
Inflation, lag1				0.20*			0.15				0.23*
				(1.83)			(1.14)				(1.75)
Gold Ret								0.12			0.067
								(0.73)			(0.54)
VIX Ret									-0.14**		-0.14**
									(-2.32)		(-2.37)
Bond Ret										0.50	0.092
										(0.72)	(0.21)
Constant	0.022*	0.035***	0.027	0.021**	0.032**	0.036**	0.028**	0.032**	0.040***	0.032**	0.035***
	(1.74)	(3.41)	(1.63)	(2.08)	(2.50)	(2.31)	(2.12)	(2.48)	(5.04)	(2.48)	(2.98)
Adj-R^2	-0.008	0.020	-0.015	0.024	0.017	0.005	0.022	0.015	0.239	0.026	0.223