Fiscal consolidation using the example of Germany

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Fiscal consolidation using the example of Germany *

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Abstract

After the run up in debt-to-GDP ratios around the world in the aftermath of the financial crisis and the associated lower fiscal space, the question of prudent fiscal consolidation is back on the agenda. In this paper, I study the macroeconomic implications of fiscal consolidation triggered by the newly introduced "debt brake" in Germany, which dampens the accumulation of debt. I address this question using a medium-size new Keynesian DSGE model for Germany. The model includes the government debt-to-GDP ratio, government transfers, labour income tax, consumption tax and capital tax revenues. I find that the "debt brake" enforces fiscal consolidation in times of economic expansions without constraining fiscal policy makers in times of recessions. I also find that the debt brake raises the government spending multiplier initially but not over time. Finally, the debt brake, with a fiscal consolidation on the government spending and transfers side, leads to a significant stabilization of the private sector without increasing the volatility of the fiscal instruments.

Keywords: DSGE modelling, Bayesian Estimation, Fiscal policy, Fiscal consolidation, Debt brake

JEL-Codes: C11, E62, H63

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1 Introduction

During the financial crisis, advanced economies have taken sizable measures to stabilize the banking sector and domestic demand.\(^1\) This has led to a large build up of debt. In Germany, the debt-to-GDP ratio has increased to 83.2\% of GDP after 65.3\% in the end of 2007, exceeding the reference value of the Maastricht Treaty by 23\%.\(^2\) In the United States, the general government gross debt-to-GDP ratio increased from 62.1\% in 2007 to 97.6\% in 2011.

Blanchard et al. [2010] propose in their paper "Rethinking Macroeconomic Policy" to reduce debt targets. They write: "Still, the lesson from the crisis is clearly that target levels should be lower than those observed before the crisis. The policy implications for the next decade or two are that, when cyclical conditions permit, major fiscal adjustment is necessary and, should economic growth recover rapidly, it should be used to reduce debt-to-GDP ratios substantially, rather than to finance expenditure increases or tax cuts."

To anchor the indebtedness, Germany’s government introduced a "debt brake" in 2011.\(^3\) This "debt brake" has five characteristics. First, it has no explicit debt target. It is a deficit rule, attempting to lower the government deficit and allowing the debt-to-GDP ratio to decline through GDP growth. Second, the rule enforces a maximum average structural deficit of 0.35\% of GDP. The target is the structural deficit not the actual deficit. At the expense of complication, this concept has the advantage of allowing a countercyclical fiscal stance in recessions. Third, deviations of the structural deficit from the target level of 0.35\% are booked on an adjustment account, which must be cleared over time. This smooths the fiscal consolidation over a longer period, leading to less abrupt changes in fiscal policy instruments. Fourth, fiscal consolidation triggered by the debt brake is capped to allow a maximum adjustment of fiscal instruments of 0.35\% of GDP annually. And last, the consolidation is state-dependent, so that the German government will only reduce spending and transfers or increase tax rates when the economy is not in recession.

In this paper, I study the macroeconomic implications of fiscal consolidation triggered by the debt brake described above. I construct a DSGE model with a detailed government sector. The model builds on New-Keynesian DSGE models à la Smets and Wouters [2007] or Christiano et al. [2005], which have shown to fit the data well. Because the focus of this paper is on government debt, I allow for rich dynamics induced by the interactions between policy variables and debt. Following Kollmann [1998] and Leeper et al. [2010], government spending, transfers, income tax, capital tax,

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\(^1\)See Cwik and Wieland [2009] for an overview of the fiscal packages of the eleven biggest euro area countries.

\(^2\)The Maastricht Treaty obliges Member States of the European Union to avoid deficits exceeding 3\% of GDP and debt levels above 60\% of GDP.

\(^3\)The German debt brake was proposed by the "Föderalismuskommission II", signed into law from the German parliament on the 29th May 2009 and becomes operative in 2011. However, the rule will be legally binding for the federal government from 2016 onwards and for the state governments from 2020 onwards.
and consumption tax can respond to government indebtedness. In addition, the income and capital tax rates adjust to the state of the economy to allow progressive taxation. Since the literature on fiscal policy has emphasized the importance of non-Ricardian consumers in DSGE models, I introduce a share of rule-of-thumb consumers as described in Gali et al. [2007] together with forward-looking households in the model. The model is estimated on German data from 1983Q1 up to 2009Q4 with Bayesian techniques using 12 time series. My estimation sample ends before the introduction of the debt brake. To analyze the implications of the debt brake, I supplement the estimated fiscal reaction functions with the debt brake policy.

I have three main findings. First, the debt brake works asymmetrically. In general, after expansionary shocks, government revenues increase, whereas spending and transfers initially drop. This improves the government budget and leads government debt to fall. But this additional fiscal space has a second round effect on the fiscal instruments. Lower debt creates incentive for fiscal policy makers to lower tax rates and increase structural spending. Both lead to an increase in the structural deficit. The debt brake enforces a faster fiscal consolidation in times of economic expansion without constraining fiscal policy makers in times of contractionary shocks. Second, the debt brake initially increases the government spending multiplier independent of the type of fiscal consolidation. Although a consolidation of spending and transfers leads to a more output favourable outcome. Agents expect that through the debt brake, the increase in discretionary spending will be offset faster in the future by lower spending and transfers or by higher taxes, both of which lowers the wealth effect and encourages private consumption. But after two years, when fiscal consolidation takes place, the government spending multiplier is lower with the debt brake. And third, the debt brake stabilizes the economy, reducing the volatility of key macroeconomic variables without increasing that of fiscal instruments. The stabilization is more pronounced for a debt brake with spending-based consolidation, which is state-dependent and therefore only leads to lower government spending and lower transfers to households when the economy is growing.

The remainder of the paper is organized as follows. Section 2 provides a description of the DSGE model. Section 3 explains the estimation procedure, section 4 shows the out-of-sample forecast for the debt-to-GDP ratio and section 5 discusses the implications of the debt brake. Section 6 concludes.

2 Model

In this section, I develop a New-Keynesian DSGE model that includes the main transmission channels of fiscal policy. Most of the model features are standard and familiar from so-called medium scale DSGE models, as put forward, for instance, in Christiano et al. [2005] or Smets and Wouters [2007]. A fraction of the representative households are forward-looking, smooth their consumption over time by buying domestic government bonds, own the capital stock, which is rented together with
labor services to intermediate goods producers on a period-by-period basis, and pay sales tax, labour income tax and capital income tax. The other fraction, the rule-of-thumb consumers, consume their after tax labour income. They work an equal proportion of their time as Ricardian households and receive the same wage. Adjusting investment is costly.

I assume there is a continuum of intermediate good producers operating under monopolistic competition and being constrained in price setting à la Calvo. They produce using a Cobb-Douglas production function and rent capital and labour services in competitive factor markets. Labour services are provided by an intermediate labour union sector, which pools labour services from Ricardian and non Ricardian households and set wages. Wages are sticky à la Calvo. Final goods firms combine intermediate goods to provide final goods for private consumption, investment and government consumption.

I introduce policy rules for all government instruments in the model. Fiscal authorities are assumed to adjust the tax rates, government spending, and transfers with respect to the current state of the economy and government indebtedness. The debt brake supplements the policy rules in the following way. I define the structural government deficit as the deficit, which is present when there is no adjustment of fiscal instruments to output and the tax base is at their equilibrium value. If the structural government deficit is greater than zero, the amount is booked on an adjustment account, which is cleared over time by appropriately adjusting government revenues or spending. I close the model with a characterization of monetary policy in terms of an interest rate feedback rule. In the following I give a formal exposition of the model.

2.1 Firms

Final goods are composites of intermediate goods produced by a continuum of monopolistic competitive firms and are used for domestic consumption, $C_t$, investment, $I_t$, and government spending, $G_t$. I use $i \in [0, 1]$ to index intermediate good firms as well as their products and prices. Final goods firms operate under perfect competition and purchase intermediate goods, $Y_t(i)$. They use the following Kimball aggregation technology with the elasticity of demand being an increasing function of the relative price of the intermediate good $P_t(i)$ and the final good $P_t$.

$$\int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \lambda_{p,t}\right) di = 1$$

$G$ is a concave and increasing function such that $G(1) = 1$ and $(1 + \lambda_{p,t})$ is the time-varying price markup with

$$\ln \lambda_{p,t} = (1 - \rho_p) \ln \lambda_p + \rho_p \ln \lambda_{p,t-1} + \theta_p \epsilon_{p,t-1} + \epsilon_{p,t}$$
The representative final goods firms produce $Y_t$ while minimizing expenditures. The resulting demand function for an individual intermediate good $i$ is given by

$$Y_t(i) = Y_t G' \left[ \frac{P_t(i)}{P_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) dY_t \right]$$  \hspace{1cm} (3)$$

The production of intermediate goods, $Y_t(i)$, is governed by a Cobb-Douglas production function

$$Y_t(i) = Z_t K_s^\alpha_t(i) \left[ \gamma_t L_t^\gamma(i) \right]^{1-\alpha-\gamma} \Phi,$$  \hspace{1cm} (4)$$

where $L_t(i)$ and $K_s^\alpha_t(i)$ respectively denote labour and capital employed by firm $i$ and $\Phi$ denote fixed costs of production, which are set to ensure that profits are zero in steady state. $\gamma_t$ represents the labour-augmenting deterministic growth rate in the economy and $Z_t$ governs total factor productivity, which is given by the following shock process

$$\ln Z_t = \rho_z \ln Z_t - 1 + \epsilon_z,t$$  \hspace{1cm} (5)$$

where $\epsilon_z,t$ is iid distributed. Let $W_t$ and $R_t^k$ denote the nominal wage rate and the rental rate of capital, respectively. Minimizing the costs of producing intermediate goods implies for (nominal) marginal costs

$$MC_t = \alpha - \alpha(1-\alpha)W_t^{1-\alpha}(R_t^k)\gamma - (1-\alpha)\Phi,$$  \hspace{1cm} (6)$$

which are independent of the level of production and identical across firms, because both factors of production can be adjusted freely across firms.

I assume that price setting is constrained exogenously by a discrete time version of the mechanism suggested by Calvo [1983]. Each firm has the opportunity to change its price with a given probability $1 - \xi_p$. Firms that do not reoptimize in a certain period index their price to steady state inflation $\pi_s$.

The optimal price $\widetilde{P}_t(i)$ set by the firm that is allowed to re-optimize at time $t$ results from the following optimization problem

$$\max E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \frac{\Xi_{t+s}}{\Xi_t P_t(i)} \left[ \tilde{P}_t(i)(\pi_s)^s - MC_{t+s} \right] Y_{t+s}(i)$$  \hspace{1cm} (7)$$

subject to the demand function defined by (3).

### 2.2 Households

There is a continuum of households indexed by $h \in [0, 1]$. A share $1 - \omega$ of these households makes optimizing, forward-looking decision. They are indexed by $j \in [0, 1 - \omega)$. These households have access to financial markets. They buy and sell government bonds $B_t(j)$ and accumulate physical

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4Profits are discounted with the stochastic discount factor of the Ricardian households, $\beta^s \Xi_{t+s} P_t(i)$, who are the owners of the firms.
capital $K_t(j)$. Given capital utilization costs $a(u_t(j))$, they decide how much of the accumulated capital they rent to firms. They receive wage income $W^h_t(j)L_t(j)$, dividend payments from the firms, and profits from the labour unions and pay capital tax $\tau^k_t$, consumption tax $\tau^c_t$ and labour income tax $\tau^n_t$ to the government. Their decisions are made so as to maximize a utility function that is separable in consumption $C_t(j)$ and labour supply $L_t(j)$. Specifically, I use the log utility function for consumption to ensure a balanced steady state growth path as put forward in King et al. [1988].

\[
E_t \sum_{s=0}^{\infty} \beta^s \log(C_{t+s}(j)) = \frac{1}{1+\sigma_t} L_{t+s}(j)^{1+\sigma_t} 
\]  

subject to the budget constraint

\[
(1 + \tau^c_{t+s})C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\delta_{t+s}P_{t+s}} \leq (1 - \tau^n_{t+s}) \frac{W^h_{t+s}(j)L_{t+s}(j)}{P_{t+s}} 
\] 

\[
+ (1 - \tau^k_{t+s}) \left[ \frac{R_{t+s}^k}{P_{t+s}} u_{t+s}(j)K_{t+s-1}(j) - a(u_{t+s}(j))K_{t+s-1}(j) \right] 
\] 

\[
+ \frac{B_{t+s-1}(j)R_{t+s-1}}{P_{t+s}} + \tau^k_{t+s}\delta K_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}} 
\]

the capital accumulation equation

\[
K_t(j) = (1 - \delta)K_{t-1}(j) + \mu_t \left[ 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j) 
\]  

and the definition of employed capital

\[
K^a_t(j) = u_t(j)K_{t-1}(j) 
\]

where $\delta$ denotes the depreciation rate. I assume for simplicity that the utilisation costs of physical capital and physical capital depreciation are exempted from taxation as in Coenen et al. [2008]. It may be costly to adjust the level of investment, $I_t$. Here $S$ is the adjustment cost function, with $S(\gamma) = S'(\gamma) = 0$ and $S'' > 0$ ensures that the steady state capital stock is independent of the investment adjustment costs.\(^6\) $d_t$ is an exogenous premium in the return to bonds as in Edge et al. [2008] given by

\[
\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} 
\]

and $\mu_t$ is a stochastic shock to the price of investment relative to consumption goods of the following form

\[
\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t} 
\]

\(^5\)I estimated the model with external habits, price indexation and wage indexation and found values close to 0.1. To keep the parameter space sparse I excluded the exogenous persistence parameters. Kiley [2010] finds that introducing rule-of-thumb consumers can explain a lower habit parameter.

\(^6\)See Christiano et al. [2005].
The remaining share $\omega$ of non Ricardian households is indexed by $s \in [1 - \omega, 1)$. They simply consume their disposable income, which is given by the after-tax wage income and the transfer payments from the government.

$$(1 + \tau_c^t)C_t(s) = (1 - \tau^n_t) \frac{W^h_t L_t(s)}{P_t} + T_t(s)$$  \hspace{1cm} (13)$$

Aggregating over all households implies that overall consumption is a weighted average of the consumption function of rational and rule-of-thumb consumers

$$C_t = \int_0^1 C_{h,t} dh = (1 - \omega)C_{j,t} + \omega C_{s,t}$$  \hspace{1cm} (14)$$

Labour packers buy the labour supplied by the households from labour unions $l \in [0, 1]$, package $L_t$ using the following aggregation technology and resell it to the intermediate goods producers.

$$L_t = \left[ \int_0^1 L_t(l) \frac{1}{1 + \lambda w,t} \, dl \right]^{1 + \lambda w,t}$$  \hspace{1cm} (15)$$

$(1 + \lambda w,t)$ denotes the time varying wage markup in the labour market driven by the moving average process of the form

$$\ln \lambda w,t = (1 - \rho_w) \ln \lambda w + \rho_w \ln \lambda w,t-1 + \theta_w \epsilon w,t-1 + \epsilon w,t$$  \hspace{1cm} (16)$$

The labour packers maximize profits in a perfectly competitive environment leading to the following demand for individual labour $l$.

$$L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{1 + \lambda w,t} L_t$$  \hspace{1cm} (17)$$

where $W_t$ is the aggregate wage index. Labour unions allocate and differentiate the labour services from the households and set the same nominal wage rate for both types of households. They choose the wage given nominal rigidities à la Calvo subject to the labour demand equation (17) and the labour supply decision of the Ricardian households

$$\frac{W^h_t}{P_t} = \frac{(1 + \tau^F_t)}{(1 - \tau^n_t)} C_t(j)L_t^\sigma_l$$  \hspace{1cm} (18)$$

Labour unions that cannot adjust the wage in a given period index their wage to the deterministic growth rate $\gamma$ and steady state inflation. The wage setting problem for a union that can adjust its wage in period $t$, $\tilde{W}_t(l)$, becomes

$$\max E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \frac{\xi_{t+s} P_t}{\tilde{W}_t(l) \gamma_{s+1}} \left[ \tilde{W}_t(l)(\gamma \pi_s)^s - W^h_{t+s} \right] L_{t+s}(l)$$  \hspace{1cm} (19)$$
2.3 Fiscal authorities

The government raises consumption, capital, and labour income taxes and decides on government expenditures subject to the following budget constraint

\[ G_t + T_t + \frac{R_{t-1}B_{t-1}}{P_t} = \Psi_t + \frac{B_t}{P_t} \]  

where real government revenues are given by

\[ \Psi_t = \tau^n_t \frac{W^n_t L_t}{P_t} + \tau^k_t K_{t-1} \left( \frac{R^k_t u_t}{P_t} - a(u_t) - \delta \right) + \tau^C_t C_t \]  

As in Kollmann [1998], I assume that taxes are adjusted to stabilize the government debt-to-gdp ratio. Additionally, the labour income and capital tax rates respond to the state of the economy to allow for automatic stabilization i.e. progressive taxation. For illustrative purposes, I linearize the reaction functions, where \( \hat{x}_t = \frac{x_t - x}{x} \) defines the percentage deviation of a variable from trend.  

\[ \hat{\tau}^n_t = \rho^n \hat{\tau}^n_{t-1} + \phi_b^n \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) + \phi^y_b \left( \hat{Y}_t \right) + \hat{\tau}^{n,e}_t + \frac{(1 - \tau^n)Y}{\tau^n W^n L} \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]  

\[ \hat{\tau}^k_t = \rho^k \hat{\tau}^k_{t-1} + \phi_b^k \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) + \phi^y_k \left( \hat{Y}_t \right) + \hat{\tau}^{k,e}_t + \frac{\gamma Y}{\tau^k K (R^K - \delta)} \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]  

\[ \hat{\tau}^c_t = \rho^c \hat{\tau}^c_{t-1} + \phi_b^c \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) + \hat{\tau}^{c,e}_t + \frac{Y}{\tau^c C} \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]

where \( \phi^b, \phi^y_b \) and \( \phi^k \) measure the elasticity of labour income, consumption and capital tax to government debt and \( \phi^y_b \) and \( \phi^y_k \) the elasticity of the tax rates w.r.t. output. \( \rho^n, \rho^k \) and \( \rho^c \) capture the persistence in the adjustment of tax rates by policy makers. \( \hat{\tau}^{n,e}_t, \hat{\tau}^{k,e}_t \) and \( \hat{\tau}^{c,e}_t \) are iid shocks to pick up the residual volatility in the tax rates. The function \( \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \) governs fiscal consolidation triggered by the debt brake, which depends on the value on the adjustment account \( \hat{AC}_{t-1} \), output growth \( \Delta \hat{Y}_t \) and the speed of clearing on the adjustment account \( \rho \). The term in front of the function \( \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \) is the inverse of the tax elasticity of the respective revenues in steady state or in other words by how much policy makers need to adjust the respective tax rates to increase tax revenues by 1% of GDP. I will explain the debt brake mechanism in more detail in the next subsection.

The fiscal rules for government spending and transfer payments are defined in a similar way.

\[ \hat{G}_t = \rho^G \hat{G}_{t-1} - \phi_b^G \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) - \phi^G_y \left( \hat{Y}_t \right) + \hat{G}^*_t - \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]  

\[ \hat{T}_t = \rho^T \hat{T}_{t-1} - \phi_b^T \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) - \phi^T_y \left( \hat{Y}_t \right) + \hat{T}^*_t - \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]  

\[ ^7 \text{Although the reaction functions of fiscal policy makers are expressed in terms of linearized variables, the upper and lower bound on the adjustment account and the state-dependent clearing患有 the adjustment of the government instruments nonlinear, as shown in the next subsection.} \]
Transfers and spending are reduced, when debt increases and are increased when output falls below trend to capture higher transfer payments in recessions i.e. higher unemployment benefits. Again, I account for persistence in the adjustment of the fiscal instruments and there is possible downward pressure on spending and transfers through fiscal consolidation.

2.4 Structural government deficit

For the evaluation of the fiscal stance, the debt brake uses the structural government deficit, that is the deficit which would be present when the economy was on the balanced growth path, instead of the actual deficit. The advantage of this concept is that it allows for countercyclical fiscal policy but restrains excessive deficits in the medium run. The model outlined in the subsections above provides a natural measure of the structural deficit

\[ D^S_t = G^S_t + T^S_t + \frac{R_{t-1}B_{t-1}}{P_{t-1}} - \Psi^S_t - \frac{B_{t-1}}{P_{t-1}} \]

(27)

Structural government spending, transfers, and tax revenues \((G^S_t, T^S_t, \Psi^S_t)\) are obtained by setting the output-elasticities of taxes, spending and transfers to zero \((\phi^n_y = 0, \phi^k_y = 0, \phi^g_y = 0, \phi^t_y = 0)\). For example,

\[ \hat{G}^S_t = \rho^G \hat{G}^S_{t-1} - \phi^G_y \left( \frac{Y}{B} \hat{B}_{t-1} - \hat{Y}_{t-1} \right) + \hat{G}^\varphi_t - \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) \]

(28)

Additionally, I assume that the tax base is in steady state.\(^8\) Structural revenues in percent deviation from steady state are then defined as follows

\[ \hat{\Psi}^S_t = \frac{\tau^n W^L}{Y} \hat{z}^{S,n}_{t} + \frac{\tau^k K(R^k - \delta)}{Y^\gamma} \hat{z}^{S,k}_{t} + \frac{\tau^c C}{Y^\gamma} \hat{z}^{S,c}_{t}. \]

(29)

2.5 The debt brake

The debt brake incorporates an adjustment account, which is defined as the sum of all past and contemporaneous deviations of the structural deficit from zero minus the adjustment account clearing.

\[ 0 \leq \hat{AC}_t = (1 - \rho)\hat{AC}_{t-1} + \hat{D}^S_t \leq 1.5 \]

(30)

The lower bound is introduced to correct only excessive structural deficits not surpluses, whereas the upper bound caps the maximum amount of fiscal consolidation every period. The parameter \(\rho\) determines the fraction of the adjustment account, which is cleared every period and calibrated to allow for a maximum clearing of 0.35% of GDP annually, which is stated explicitly in the relevant law. The function \(\Phi\) is therefore given by

\[ \Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) = \rho \hat{AC}_{t-1}(\hat{Y}_t - \hat{Y}_{t-4}) \geq 0. \]

(31)

\(^8\)As I will discuss in section 3, the steady state enforces long-run budget balance.
I multiply the clearing amount by the Q4-Q4 GDP growth rate. Together with the non-negativity constraint, this ensures that there is only fiscal consolidation taking place when the economy is not in a recession. Excluding the last term \( \hat{Y}_t - \hat{Y}_{t-4} \) in the equation allows for a stricter version of the debt brake and triggers a fiscal consolidation independent of the state of the economy.

### 2.6 Monetary authorities

I assume that monetary policy is characterized by an interest rate feedback rule as in Smets and Wouters [2007]. Specifically, I assume for the interest rate

\[
\frac{R_t}{R^*_s} = \left( \frac{R_{t-1}}{R_s} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi^*_s} \right)^{\psi_1} \left( \frac{Y_t}{Y^*_t} \right)^{\psi_2} \left( \frac{Y_t/\hat{Y}_{t-1}}{Y^*_t/\hat{Y}^*_{t-1}} \right)^{\psi_3} \right] r_t
\]  

where \( R_s \) is the steady state gross nominal interest rate and \( Y^*_t \) is the natural output level defined as the output level in absence of nominal frictions. The parameter \( \rho_R \in [0, 1] \) captures interest rate smoothing. Finally, \( r_t \) represents a shock to the short-term interest rate not-accounted for by the systematic feedback rule. It thus represents a monetary policy shock

\[
\ln r_t = \rho_r \ln r_{t-1} + \epsilon_{r,t}.
\]

The following resource constraint closes the model

\[
Y_t = C_t + I_t + G_t + N_{ex_t} + a(u_t)K_{t-1}
\]

where \( N_{ex_t} \) is an autocorrelated stochastic shock reflecting the effects of changes in foreign demand on net exports, which is given by

\[
N_{ex_t} = \rho_{nex} N_{ex_{t-1}} + \epsilon_{nex,t}.
\]

I assume that debt, government revenues, transfers and spending grow in steady state at the same rate as output. Therefore I detrend the variables including the fiscal policy variables with a fixed common growth trend \( \gamma^t \). Then the equations are linearized around the deterministic steady state.

### 3 Estimation

I use quarterly German data for the period 1983Q1-2009Q4 to estimate the model with Bayesian estimation techniques and match the following 12 variables: real consumption, real investment, real GDP, real wages, hours worked, the GDP-Deflator, the short-run interest rate, government debt, government labour income tax revenues and social security contributions received by the government, consumption tax revenues, capital tax revenues, and social security benefits paid by the government.\(^9\)

\(^9\)A detailed description of the data and the exact measurement equations are stated in Appendix A.
The debt brake is operative since 2011 and was not affecting the decisions of the German government until 2010. Therefore I will assume during estimation that the adjustment account is zero and also $\Phi(\Delta \hat{Y}_t, \hat{AC}_{t-1}; \rho) = 0$. The estimated model can thus be interpreted as a description of the behavior of the government, private agents and firms before the introduction of the debt brake and serves as a good benchmark to compare macroeconomic outcomes before and after the debt brake.

I estimate the mode of the posterior distribution by maximizing the log posterior kernel, which combines the prior information on the parameters with the likelihood of the data. Then the Metropolis-Hastings algorithm is used to evaluate the whole posterior distribution and the marginal likelihood of the model.

A few parameters are kept fixed throughout the estimation. The depreciation rate, $\delta$, is set to 2.5\% and the steady state wage markup, $(1+\lambda_w)$, to 1.5. The curvature parameter of the Kimball aggregator in the goods market $\eta$, which is given by $\eta = 1/\lambda_p \left[ \frac{2 + \frac{G'''}{G''}}{1 + \frac{G''}{G'}} - 1 \right]$, is not identified. Therefore I fix it to 10 analogous to Smets and Wouters [2007]. I set the quarterly trend growth rate $\gamma$ to 0.3, which is equivalent to the average growth rate of real GDP in the data.

A sample of 250,000 Metropolis-Hastings draws was created (neglecting the first 50,000 draws).

### Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$(1 + \lambda_w)$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>10</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.32</td>
</tr>
<tr>
<td>$b_*$</td>
<td>0.52</td>
</tr>
<tr>
<td>$\psi_\alpha^*$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\psi_c^*$</td>
<td>0.11</td>
</tr>
<tr>
<td>$\psi_k^*$</td>
<td>0.004</td>
</tr>
<tr>
<td>$t_*$</td>
<td>0.18</td>
</tr>
<tr>
<td>$\tau^{nl}$</td>
<td>0.46</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.10</td>
</tr>
<tr>
<td>$G_*$</td>
<td>0.21</td>
</tr>
</tbody>
</table>

The steady-state government debt to GDP ratio, $b_*$, is calibrated to 0.52, which corresponds to the mean of this variable using the whole estimation sample from 1983 up to now. I calibrate the steady state tax revenues to GDP ratios in the same way. The income, consumption and capital tax rates in steady state are calculated to match the tax revenues to GDP ratios stated above. Finally, the government spending to GDP ratio in steady state is pinned down to ensure that the government...
budget constraint is fulfilled. See Table 1 for an overview of the calibrated parameters.

### Table 2: Prior and posterior distributions of the structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_w ) Calvo wages</td>
<td>beta</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \xi_p ) Calvo prices</td>
<td>beta</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>SS price markup</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( S^\pi ) Investment adj. cost</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \omega ) non Ricardian cons.</td>
<td>beta</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \sigma_l ) Labour supply elas.</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \chi ) Capital utilization</td>
<td>beta</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \alpha ) Capital share</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \rho_R ) Int. rate smoothing</td>
<td>beta</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \psi_1 ) Inflation response</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \psi_2 ) Outputgap response</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \psi_3 ) Diff. outputgap resp</td>
<td>norm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_b^g ) Government spending</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_b^l ) Labour tax</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_b^c ) Consumption tax</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_b^k ) Capital tax</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_b^t ) Transfers</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_y^g ) Government spending</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_y^l ) Labour tax</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_y^c ) Capital tax</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \phi_y^k ) Transfers</td>
<td>invg</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \beta ) Discount rate</td>
<td>gamm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
<tr>
<td>( \pi ) SS inflation rate</td>
<td>gamm</td>
<td>type mean std.dev. mode std.dev. mean 5 % 95 %</td>
</tr>
</tbody>
</table>


### 3.1 Prior distributions of the estimated parameters

I estimate 37 parameters and 12 standard deviations of shock innovations. The parameter \( \chi \) governing the capital utilisation costs is given by \( \chi = \frac{1}{1 + \omega} \), where \( \omega \) and \( \omega' \) are the first and second order derivatives of the capital utilisation cost function w.r.t. capital utilisation evaluated at the steady state.

The estimated parameter \( \beta \), which drives the discount factor is defined as follows \( \beta = 100(\frac{\omega}{\rho} - 1). \) Tables 2 and 3 show the assumptions for the prior distributions, which are similar to Smets and Wouters [2007] for the standard model parameters. For the share of rule-of-thumb consumers \( \omega \), I assume a beta distribution with mean 0.3 and standard deviation 0.1, with the mean being close to the finding of Coenen and Straub [2005], who estimated the share of rule-of-thumb consumers in the
euro area. I assume for the debt elasticities of the tax rates an inverse gamma distribution with mean 0.05 and standard deviation 2, and for the output elasticities an inverse gamma distribution with mean 0.1 and standard deviation 2, implying sluggish adjustment by the government.

Table 3: Prior and posterior distributions of the shock processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution type</th>
<th>Prior distribution mean</th>
<th>Prior distribution std. dev.</th>
<th>Prior distribution mode</th>
<th>Posterior distribution mean</th>
<th>Posterior distribution 5 % 95 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_z$ Technology shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.98</td>
<td>0.00</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho_p$ Price mark-up shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.90</td>
<td>0.04</td>
<td>0.89</td>
</tr>
<tr>
<td>$\rho_\mu$ Investment shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.23</td>
<td>0.10</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_d$ Risk premium shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.93</td>
<td>0.03</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho_r$ MP shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.24</td>
<td>0.07</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_w$ Wage mark-up shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.96</td>
<td>0.03</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho_u$ Gov. spend. shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.70</td>
<td>0.05</td>
<td>0.69</td>
</tr>
<tr>
<td>$\rho_{nex}$ Net exports shock</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.96</td>
<td>0.02</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_{r^n}$ Labour tax</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.92</td>
<td>0.02</td>
<td>0.91</td>
</tr>
<tr>
<td>$\rho_{r^c}$ Consumption tax</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.89</td>
<td>0.04</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho_{r^h}$ Capital tax</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.95</td>
<td>0.02</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho_T$ Transfers</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.94</td>
<td>0.03</td>
<td>0.93</td>
</tr>
<tr>
<td>$\theta_p$ MA price mark-up</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.93</td>
<td>0.02</td>
<td>0.93</td>
</tr>
<tr>
<td>$\theta_w$ MA wage mark-up</td>
<td>beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.67</td>
<td>0.10</td>
<td>0.61</td>
</tr>
<tr>
<td>$\epsilon_z$ Technology shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.50</td>
<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td>$\epsilon_p$ Price mark-up shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.49</td>
<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td>$\epsilon_\mu$ Investment shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>1.48</td>
<td>0.16</td>
<td>1.45</td>
</tr>
<tr>
<td>$\epsilon_d$ Risk premium shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.14</td>
<td>0.03</td>
<td>0.15</td>
</tr>
<tr>
<td>$\epsilon_r$ MP shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.14</td>
<td>0.01</td>
<td>0.15</td>
</tr>
<tr>
<td>$\epsilon_w$ Wage mark-up shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.55</td>
<td>0.09</td>
<td>0.58</td>
</tr>
<tr>
<td>$\epsilon_u$ Gov. spend. shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.87</td>
<td>0.06</td>
<td>0.89</td>
</tr>
<tr>
<td>$\epsilon_{nex}$ Net exports shock</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.82</td>
<td>0.06</td>
<td>0.83</td>
</tr>
<tr>
<td>$\epsilon_{r^n}$ Labour tax</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.58</td>
<td>0.04</td>
<td>0.61</td>
</tr>
<tr>
<td>$\epsilon_{r^c}$ Consumption tax</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>1.14</td>
<td>0.08</td>
<td>1.17</td>
</tr>
<tr>
<td>$\epsilon_{r^h}$ Capital tax</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>2.26</td>
<td>0.16</td>
<td>2.31</td>
</tr>
<tr>
<td>$\epsilon_T$ Transfers</td>
<td>invg</td>
<td>0.10</td>
<td>2.00</td>
<td>0.26</td>
<td>0.02</td>
<td>0.28</td>
</tr>
</tbody>
</table>


3.2 Posterior distributions of the estimated parameters

The results of the posterior maximization can be find in Tables 2 and 3, which display the posterior mode and mean together with the 90% confidence bands for all 49 estimated parameters. Figures 4 and 5 in the Appendix show graphically the prior and posterior distributions of the structural model parameters and the parameters of the shock processes. The AR(1) coefficients of the fiscal instru-
ments are quite high with values around 0.9, which capture the persistence in the decision making of government policy. The standard structural parameters of the model also look reasonable. The Calvo parameter in the goods market has a posterior mean of 0.91 despite strategic complementarities through a Kimball type aggregator for final goods. But Smets and Wouters [2003] find with 0.91 a value of similar magnitude in their estimated model for the euro area and inflation persistence was even higher in Germany compared to other euro area countries. The estimate for the coefficient on inflation in the monetary policy rule is 1.59. Thus monetary policy reacts moderately to movements in inflation. I estimate with a 27% share of non Ricardian consumers, in line with estimates of Coenen and Straub [2005] for the euro area and Cogan et al. [2010] for the US. The debt elasticities of tax rates, spending, and transfers are estimated to be below 0.03, i.e. tax rates are adjusted slowly to the debt-to-GDP ratio in Germany. An increase in the debt-to-GDP ratio of 10% of GDP, for example, would cause a reduction in government spending of 0.45% of GDP after one year.\textsuperscript{12} I find higher estimates for the output elasticities of tax rates and spending, especially for the labour income tax rate with a posterior mean of 0.09 and for the capital tax rate with a value of 0.08. This points to significant progressivity in both tax rates. The dynamics of fiscal policy in the model are quite complex, with almost all fiscal instruments reacting to debt and the business cycle. As Uhlig [2010] points out, the effect of discretionary fiscal policy depends very strongly on the reaction of the other fiscal instruments to a higher level of government spending. I will show in the next section how this fiscal set up interacts with the debt brake.

\section{Forecasting Performance}

I first evaluate the model’s out-of-sample forecasting ability. In particular, I use the estimated model described above to construct an out-of-sample density forecast for the debt-to-GDP ratio from the first quarter of 2010 until 2016 and compare it to the actual debt-to-GDP ratio in 2010 and 2011 and the projection of the German government provided in the "German stability Programme 2012" for the EU commission. The adjustment of fiscal instruments in practise depends on political negotiations and results of the elections, which leads to variability in the parameters of the fiscal policy rules over time. I take this uncertainty regarding the parameters of the model into account and pick randomly 2,500 Metropolis-Hastings draws of the posterior parameter distributions (Figure 4 in the Appendix). For each parameter combination, a forecast for the debt-to-GDP ratio is computed using the linear model solution. Additionally policy makers don’t have perfect foresight and don’t know certainly how the economy will evolve in the future. To cope with this uncertainty I create every period random draws

\textsuperscript{12}Taking the smoothing of government spending into account and assuming all else equal, one can compute the adjustment of government spending to a 10\% increase in the debt-to-GDP ratio as follows \(0.45 = 0.182 + 0.182 \times 0.6911 + 0.182 \times 0.6911^2 + 0.182 \times 0.6911^3\).
Figure 1: Forecast debt-to-GDP ratio

Notes: 2500 subdraws of the Metropolis Hastings algorithm and draws from the shock distributions are used to generate density forecast. Historic debt-to-GDP series from 2005 until 2010 is displayed with a solid black line, the mean forecast by a dashed-dotted black line, the 10th up to the 90th percentiles are displayed by grey shaded areas and the mean forecast with additional discretionary fiscal package in 2010 is shown by the dashed blue line. The actual debt-to-GDP ratio in 2010 and 2011 is plotted in a red solid line. The projections of the German government out of the "German stability Programme 2012" provided for the EU commission with and without european stability measures are shown by a red dashed line and a red dashed-dotted line.

from the distributions of the 12 shock innovations in the model and add them to the model solution while computing the forecasts.

Figure 1 shows the historical debt-to-GDP series up to 2010 (black solid line), followed by the mean model forecast (black dashed-dotted line) and the corresponding 10th up to the 90th percentiles (grey shaded areas). The model predicts an increase in the debt-to-GDP ratio from 73.4% in the last quarter of 2009 until 79% in 2010 and afterwards a gradual decline due to the expected economic recovery leading to higher expected labour income and a higher average labour income tax rate both stimulating tax revenues.

Some parts of the fiscal stimulus packages enacted by the German government became operative in 2010. Therefore these measures, which increase the debt-to-GDP ratio in 2010, are not included in the baseline forecasts. In Appendix B I list in detail the fiscal measures and the size of both programs and group them to the fiscal instruments in the model. Table 4 contains an overview. Measures to lower the income tax wedge, which sum up to 29.6 billion euro or 1.24% of 2009 GDP, build the major part of the fiscal package in 2010, which consists of the reduction in health insurance contributions, tax deductibility of professional commute or direct income tax cuts. Additional government spending has a size of 10.1 billion euro mainly due to the infrastructure investment program. The major part of
The "Growth Acceleration Act" are higher tax exemptions for dependent children and child benefits with a value of 4.6 billion euro, which I grouped as transfers. Changes in the legislation of business tax, which are estimated to lower capital tax revenues by 2.4 billion euro in 2010 are the second biggest part. Overall both programs are expected to lower the tax base by 57.7 billion euro or 2.41% of 2009 GDP in 2010. To take the effect of these fiscal changes on the forecast for the debt-to-GDP ratio into account I compute conditional forecasts. I add in every quarter in 2010 a discretionary government spending shock in the size of 0.105% of GDP and a transfer shock in the size of 0.10% of GDP to the model solution, together with the random shock realisations, while computing the forecasts. Additionally, I use the baseline forecasts for the different tax revenues in 2010 and lower the respective tax rates to decrease the labour income tax, consumption tax and capital tax revenues in the amount of 0.31%, 0.015% and 0.075% of GDP every quarter in 2010. The conditional forecast is shown with a blue dashed line in Figure 1. One can see that after adding the fiscal package in 2010, the model forecast predicts the peak in the debt-to-GDP ratio of 83% in the end of 2010 right, when comparing the conditional forecast to the actual data of the German government (red solid line). The medium-term projection of the German government also lies within a reasonable confidence band especially when european stability measures are excluded (red dashed-dotted line). Overall the model seems to predict the evolution of the debt-to-GDP ratio in Germany well.

5 Implications of the Debt Brake

5.1 Implications for the responses to a positive demand shock

To shed more light on the model dynamics, I compute the Impulse Responses to a one standard deviation exogenous increase in demand of 0.83% of GDP. Figure 2 shows the reaction of key macroeco-
nomic variables to this expansionary shock in the model without the debt brake. The increase in demand makes labour income increase and generates higher labour income tax revenues (middle panel second row). Due to a progressive labour income tax rate, an increase in labour income also raises the average labour income tax rate in the economy (middle panel first row), which further fosters the improvement of the government budget. On the other hand, the expansion in GDP and the subsequent upward pressure on prices leads monetary policy to raise the interest rate, creating incentives for Ricardian households to save more and consume less. The crowding-out of Ricardian household’s consumption dominates the consumption increase of non-Ricardian consumers, who react to a higher labour income by spending more. Therefore consumption tax revenues are slightly subdued. As might be expected given the estimates of $\phi^G_y$ and $\phi^T_y$, government spending and transfers react countercyclically in the first 5-10 quarters. Both higher overall tax revenues and lower spending lead to a budget surplus and lower government debt. The reduction in government indebtedness causes a second-round effect in the fiscal instruments. The greater fiscal space creates incentives for fiscal authorities to increase spending and lower tax rates. Therefore the average labour income tax rate, and the consumption tax rate fall and spending and transfers increase after 5 quarters. These second-round effects lead to an increase in the cyclical adjusted or structural deficit (bottom left panel). The model
observations are in line with empirical results by Gali and Perotti [2003], who estimate the output gap elasticities of the cyclical component of the government budget deficit and the cyclical adjusted component for a sample of OECD countries from 1980-2002. They find a procyclical structural deficit and a countercyclical fiscal stance on automatic stabilizers in Germany.

In Figure 3, I compare the baseline results with two consolidation scenarios. The empirical literature typically treats fiscal consolidation through cuts in spending or through tax increases separately, as in Alesina and Ardagna [2010]. I take the same approach and first look at a spending-based consolidation with 50% of the consolidation taking place through cuts in transfers and 50% through cuts in government purchases (blue lines) and second, a tax-based scenario with 71% of the consolidation taking place through an increase in the labour income tax, 28% through an increase in the consumption tax and 1% through an increase in the capital tax (red lines). Taxes are raised according to their corresponding steady-state revenue-to-GDP shares. The dashed lines correspond to the baseline specification of the debt brake, where fiscal consolidation is capped at a maximum of 0.35% of GDP. The dotted lines show a more ambitious case, where the adjustment account is capped at 3% of GDP.
leading to a maximum fiscal consolidation of 0.70% of GDP annually; the speed of adjustment \( \rho \) is kept unchanged. The nonlinear model equations associated with the implementation of the debt brake require nonlinear solution techniques; I use the methodology described in Juillard [1996].

Under either approach to consolidation following a demand shock, the structural deficit leads to an accumulation on the adjustment account until it reaches the maximum of 1.5% (3%) of GDP at around 10 (18) quarters. With spending-based consolidation, government spending and transfers are below baseline starting around the fifth quarter. In scenario 2, the income tax rate and consumption tax rate are increased compared to baseline. In both scenarios, this improves the budget deficit or leads to a higher budget surplus (last panel third row) and debt falls substantially more than in the baseline case. With the spending-based consolidation and the baseline debt brake specification, debt peaks at -1.35 percent, 0.25% of GDP lower than in the case without debt brake. The greater fall in debt creates second-round effects in the fiscal instruments. In scenario 1, fiscal policy lowers taxes after around 10 quarters, whereas in the tax-based consolidation case spending and transfers are increased.

These fiscal dynamics have important implications for the private sector. As can be seen in the second panel on the left side, the drop in private consumption after the exogenous demand shock is lower in the case of a spending-based consolidation and larger in scenario 2. With log utility and no consumption habits, the linearized euler equation of the Ricardian households can be simply written as a stream of future real interest rates and changes in the consumption tax rate.\(^{15}\)

\[
\hat{C}_t = \sum_{s=0}^{\infty} -E_t(\hat{R}_{t+s} - \hat{\pi}_{t+1+s}) + \frac{\tau^c}{1 + \tau^c} E_t(\hat{\tau}^c_{t+1+s} - \hat{\tau}^c_{t+s})
\]

Higher contemporaneous and expected future real induces households to consume more today. Here the first effect dominates the last. In scenario 1 (2) the lower (higher) expected real interest rates compared to baseline overcompensates the expected drop (increase) in the consumption tax rate. A smaller drop in consumption in scenario 1 associated with a lower marginal utility of consumption for Ricardian households compared to baseline and scenario 2 raises their desired real wage. This leads to the higher-than-baseline income tax revenues (middle panel second row) in the case of spending-based consolidation in the first 10 quarters. Afterwards, the movements in the tax rates determine the evolution of wages. Both the increase in the income tax rate and the consumption tax rate in scenario 2 raises the wedge between the marginal rate of substitution of leisure for consumption and the real wage in the labor supply decision of the Ricardian households and induces households to demand higher wages. Therefore in the medium-run wages are higher in scenario 2, which pushes

\(^{15}\)Departing from the assumption of log utility and assuming a higher coefficient of relative risk aversion in a nonseparable utility function a la King et al. [1988], doesn’t change the qualitative results. Consumption falls less in all scenarios due to agent’s lower intertemporal elasticity of substitution and debt falls more due to greater income tax revenues generated through higher wages in the economy. Assuming external habits in consumption doesn’t change the qualitative results either, although the reaction of Ricardian household’s consumption is more hump-shaped and persistent.
marginal costs and inflation up and leads monetary policy to raise interest rates. This in turn raises the financing costs of government debt and leads to a more subdued drop in government debt compared to the spending consolidation case. Overall, the debt brake with spending-based consolidation reduces government debt most given the same amount of adjustment account clearing relative to the case of tax-based consolidation and leads to slightly higher GDP due to less of a drop in private consumption, which is partly offset by lower public demand. Allowing for more fiscal consolidation (dotted lines) reinforces the effects described above. Angeloni et al. [2011] come to the same conclusions, when looking at different exit strategies out of debt in the aftermath of a recession. In particular, they find that pre-announced spending based fiscal consolidation is most promising.

5.2 Implications for the responses to a negative demand shock

How do the results change if a contractionary shock occurs? In the case of a negative demand shock, government revenues drop due to a lower tax base, and spending and transfers increase, leading to a jump up in the budget deficit and debt, but to a structural budget surplus. The results are the mirror image of Figure 2. The debt brake is set up in a way that it works asymmetrically i.e. only structural deficits are balanced through tax increases or spending cuts but no structural surplus is cleared. As a consequence, the adjustment account stays at zero and fiscal instruments are not modified. Thus, the additional fiscal consolidation associated with the debt brake takes place only in economic expansions without constraining fiscal policy in recessions and even creates room for additional fiscal measures in recessions.

5.3 Implications for government spending multipliers

Does the debt brake change the effectiveness of discretionary government spending? I compute the impulse responses to a 1% of GDP government spending shock setting the output and debt elasticities of government spending to zero \( \phi_g = \phi_b = 0 \) to render the process for government spending exogenous and to make the results comparable to the literature, which usually studies an autocorrelated 1% increase in government spending. Table 5 shows the initial multipliers and the present value multipliers after 2 years for the baseline case without debt brake, the spending-based and tax-based scenarios equivalent to the subsection above and a mixed scenario, which corresponds to the savings package the German government announced in 2010 to fulfill the debt brake.\footnote{The Present value multipliers are calculated using the following formula, where government spending and output are discounted with the SS nominal interest rate: \( PV = \sum_{i=0}^{\infty} (R^{-i}) \Delta Y_{t+i}, \Delta G_{t+i} \).} 17

The main finding in Table 5 is that the implications of the debt brake for government spending

\footnote{The savings package contains mainly transfer cuts and spending cuts, but also additional taxes like a ticket tax for flights. The weights for the fiscal instruments associated with this package are: Transfers 45%, government spending 25%, income tax revenues 21.3%, consumption tax revenues 8.4% and capital tax revenues 0.3%.}
multipliers are small. Perhaps surprisingly, the initial multiplier increases when the debt brake is in place, no matter which consolidation scheme the government chooses. But the stimulation in GDP is greatest with a spending-based consolidation. This finding supports the results by Corsetti et al. [2012], who show that the initial government spending multiplier increases if agents expect a spending reversal in the future i.e. the government announces spending cuts in the future. The mechanism is similar with the debt brake in place. Agents know that the government is credibly committed to keeping the structural deficit stable over time, and that a discretionary increase in government spending raises the structural deficit. Therefore they will expect lower public spending or higher taxes in the near future and less of an increase in government debt. This lowers the wealth effect and leads agents to increase their consumption compared to the case without the debt brake.

But the present value multipliers after two years are lower with the debt brake in place than without, independent of the consolidation scheme. If the economy is not in a recessionary episode, the fiscal consolidation will kick in immediately after the worsening in fiscal conditions and will lead either to an increase in distortions in the economy through a tax increase or to lower public demand or to lower household disposable income.

### 5.4 Stabilization

Does the debt brake stabilize or destabilize the economy? To answer this question, I simulate the model with and without the debt brake for 120,000 periods, dropping the first 20,000 observations as a training period. Due to the nonlinearities associated with the debt brake, I cannot simply iterate the policy function of the linear model forward adding a draw from the shock distribution each period; instead I use an extended shooting algorithm described in Braun and Koerber [2011]. Starting from steady state in period 0, agents experience a draw from the shock distribution in period 1 and given these shocks, solve the set of nonlinear equations that describe their respective decision rules forward for 500 periods until the economy is back at steady state. Agents use the outcome in period 1 as initial conditions and experience a new set of shocks in period 2 and again solve the model forward.
for 500 periods. This is repeated for a total of 120,000 periods. Afterwards I compute the variances of key macroeconomic variables and derive the following measure of stabilization $S$ to compare the volatility of the variable $(x)$ in the case with and without debt brake:

$$S = \left( \frac{\text{var}(x)(\text{with}_\text{debt}\_\text{brake})}{\text{var}(x)(\text{without}_\text{debt}\_\text{brake})} - 1 \right) \times 100.$$ (37)

Two results emerge from Table 6. First, a spending-based consolidation is preferable in terms of stabilization. Although a tax-based consolidation still leads to lower volatility in consumption and partly in output compared to the baseline scenario without the debt brake, a spending-based consolidation clearly outperforms both. Second, a state-dependent consolidation (Columns 3 and 5), which only triggers an adjustment account clearing in the absence of a recession, is preferred over a state-independent consolidation (Columns 2 and 4).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Spending (State-dependent)</th>
<th>Spending (State-dependent)</th>
<th>Tax (State-dependent)</th>
<th>Tax (State-dependent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-2.11</td>
<td>-2.81</td>
<td>0.00</td>
<td>-0.35</td>
</tr>
<tr>
<td>Hours</td>
<td>-1.31</td>
<td>-1.31</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>-4.20</td>
<td>-4.66</td>
<td>-0.47</td>
<td>-0.94</td>
</tr>
<tr>
<td>Investment</td>
<td>-2.24</td>
<td>-2.61</td>
<td>0.00</td>
<td>-0.37</td>
</tr>
<tr>
<td>Structural deficit</td>
<td>-1.75</td>
<td>-2.44</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>-0.13</td>
<td>-0.19</td>
<td>0.57</td>
<td>1.09</td>
</tr>
<tr>
<td>Revenues</td>
<td>-0.98</td>
<td>-2.92</td>
<td>0.98</td>
<td>1.97</td>
</tr>
<tr>
<td>Spending</td>
<td>-2.41</td>
<td>-3.60</td>
<td>0.00</td>
<td>1.22</td>
</tr>
<tr>
<td>Transfers</td>
<td>-1.69</td>
<td>-1.69</td>
<td>1.70</td>
<td>3.52</td>
</tr>
<tr>
<td>Income tax rate</td>
<td>-5.28</td>
<td>-6.31</td>
<td>2.15</td>
<td>2.15</td>
</tr>
</tbody>
</table>

Notes: Model simulation with and without debt brake for 120,000 periods dropping the first 20,000 observations. The table shows the percent change in the variance of the indicated variables, when moving from the baseline case without debt brake to the respective debt brake scenarios.

Surprisingly, in the case of a spending-based consolidation, the debt brake stabilizes the economy without leading to more volatility in the fiscal instruments. One might expect that more adjustment in the fiscal instruments is necessary to achieve a stabilization of the private sector. There are three forces at work. First, the consolidation triggered by the debt brake through the adjustment account clearing (function $\Phi$) creates additional volatility in the instruments. But this additional volatility is eased by the smoothed consolidation via the adjustment account. The overall impact of this first force depends therefore on the maximal clearing and on the smoothing parameter in the adjustment account $\rho$. Second, the debt brake makes debt itself more volatile but not necessarily the debt-to-GDP ratio, and the fiscal instruments are all a function of the debt-to-GDP ratio. In the case of a spending-based consolidation, the output stabilization dominates the debt destabilization and the debt-to-GDP
ratio becomes less volatile; in the case of tax based consolidation, it’s the other way round. Third, the automatic stabilizers react endogenously to the state of the economy. If there is less volatility in output, tax revenues, spending, and transfers are also less volatile. In the case of spending-based consolidation, the latter two forces clearly dominate the first. In the case of tax-based consolidation, the first two effects dominate the last.

6 Conclusion

In this paper, I study the macroeconomic implications of Germany’s recently introduced "debt brake" in a New Keynesian DSGE model à la Christiano et al. [2005] or Smets and Wouters [2007] integrating non-Ricardian consumers as well as Ricardian consumers, adding distortionary taxes like consumption tax, capital tax, and labor income tax, and fiscal rules for transfers and spending. I estimate the model on 12 German data series including the debt-to-GDP ratio, tax revenues, and transfers. The debt brake constrains fiscal policy makers in booms but not in recessions, forcing them to reduce the government budget deficit through increases in taxes or expenditure cuts. Additionally, the debt brake raises initial government spending multipliers independent of the type of fiscal consolidation and stabilizes the private sector without leading to a greater volatility in the fiscal instruments. That is especially true when the fiscal consolidation triggered by the debt brake is executed through expenditure cuts and the fiscal consolidation takes place only in times of positive output growth.

In the model, I do not take into account how the evolution of debt affects the interest rate. Laubach [2010] estimates a significant positive impact of expected budget deficits and the expected debt-to-GDP ratio on long-term real interest rates in the US. Taking this channel into account, a faster consolidation of the government budget through the debt brake would lower future interest payments on outstanding government debt further and would have even more favourable outcomes in terms of stabilization and government spending multipliers.
A Appendix estimation

Data

The data I employ are from the OECD Economic Outlook Database. For the period 1983:1-1991:1 I use data for West Germany, which are scaled so that the observation in the first quarter of 1991 matches the observation for reunified Germany in this quarter. Precisely I use the following OECD time series: Gross domestic product, volume, market prices (DEUGDPV, WGRGDPV), Gross domestic product, deflator, market prices (DEUPGDP, WGRPGDP), Private final consumption expenditure, value (DEUCPAA, WGRCPAA), Private non-residential gross fixed capital formation, volume (DEUIBV, WGRIBV), Private non-residential fixed capital formation, deflator (DEUPIB, WGRPIB), Total employment (DEUET, WGRET), Short-term interest rate (DEUIRS, WGRIRS), Trend labour force (DEULFS, WGRLFS), Hours worked per employee, total economy (DEUHRS, WGRHRS), Compensation rate, total economy (DEUWSST, WGRWSST), Capital tax and transfers receipts, value (DEUTKTRG, WGRTKTRG), Indirect taxes, value (DEUTIND, WGRIND), Total direct taxes, value (DEUTY, WGRTY), Social security contribution received by general government, value (DEUSSRG, WGRSSRG), Social security benefits paid by general government, value (DEUSSPG, WGRSSPG), General government gross financial liabilities, as a percentage of GDP (WGRGGFLQ) for the period 1983:1-1994:4 and Gross public debt according to the Maastricht criterion as a percentage of GDP (DEUGGFLMQ) for the period 1995:1-2009:4. Data on Gross debt according to the Maastricht criterion are not available for Germany before 1995. The concepts differ in two respects. Gross debt according to the Maastricht criterion does not include trade credits and advances and government bonds are valued at their market value not nominal value. Times series in monthly frequency are converted to quarterly frequency by using the average of the monthly rates. Annual fiscal data are converted to quarterly data by assuming that the variable grows with the same rate during the year.

Measurement equations

I use quarterly German data for the period 1983Q1-2009Q4 to estimate the model with Bayesian estimation techniques and match the following 12 variables: the log difference of real consumption ($dlCons_t$), real investment ($dlInv_t$), real GDP ($dlGDP_t$), real wages ($dlWage_t$) and hours worked ($dlHours_t$), the log difference of the GDP-Deflator ($dlP_t$), the quarterly short-run interest rate ($Interest_t$), the log of government debt ($ldebt_t$), the Q4-Q4 log difference of government labour income tax revenues and social security benefits received ($dlinctax_t$), consumption tax revenues ($dlnconstax_t$), capital tax revenues ($dlcaptax_t$) and social security benefits paid ($dltransfer_t$). All government variables are expressed relative to GDP. I removed a linear trend in hours worked to
make the data series stationary and consistent with the model assumptions. The following measurement equations are employed to link the model variables to the data.

\[
\begin{bmatrix}
    dlGDP_t \\
    dlCons_t \\
    dlInv_t \\
    dlWage_t \\
    dlHours_t \\
    dlP_t \\
    Interest_t \\
    ldebt_t \\
    dlinctax_t \\
    dlconstax_t \\
    dlcaptax_t \\
    dltransfer_t
\end{bmatrix}
= \begin{bmatrix}
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
    \gamma \\
\end{bmatrix}
\begin{bmatrix}
    \hat{y}_t - \hat{y}_{t-1} \\
    \hat{c}_t - \hat{c}_{t-1} \\
    \hat{i}_t - \hat{i}_{t-1} \\
    \hat{w}_t - \hat{w}_{t-1} \\
    \hat{l}_t - \hat{l}_{t-1} \\
    \hat{\pi}_t \\
    \hat{r}_t \\
    \hat{\beta}_s - \hat{\gamma}_t \\
    \hat{\Psi}_l^n / \Psi^n_s - \hat{y}_t - \left( \hat{\Psi}_l^{n-4} / \Psi^{n}_s - \hat{y}_{t-4} \right) \\
    \hat{\Psi}^c_l / \Psi^c_s - \hat{y}_t - \left( \hat{\Psi}^c_{l-4} / \Psi^c_s - \hat{y}_{t-4} \right) \\
    \hat{\Psi}^k_l / \Psi^k_s - \hat{y}_t - \left( \hat{\Psi}^k_{l-4} / \Psi^k_s - \hat{y}_{t-4} \right) \\
    \left( \hat{f}_t / t_s - \hat{y}_t \right) - \left( \hat{f}_{t-4} / t_s - \hat{y}_{t-4} \right)
\end{bmatrix}
\]

where \( dl \) stands for 100 times the log difference and \( l \) for 100 times the log. \( \hat{x}_t = \frac{y_t - \bar{x}}{\bar{x}} \) defines the percentage deviation of a variable from trend. \( \bar{\gamma} = 100(\gamma - 1) \) denotes the common real quarterly trend growth rate, \( \bar{\pi} \) the quarterly steady state inflation rate, \( \bar{\pi} \) the steady state nominal interest rate, \( b_s \) the steady state debt-to-GDP ratio, with \( \bar{b} = 100 \ln (b_s) \), \( \Psi^a_s \) the steady state ratio of labour income tax revenues to GDP, \( \Psi^c_s \) the steady state ratio of consumption tax revenues to GDP, \( \Psi^k_s \) the steady state ratio of capital tax revenues to GDP and \( t_s \) the ratio of steady state social security benefits paid by the government to GDP.
Figure 4: Prior and posterior distributions of the structural parameters. Notes: Prior (solid grey) vs. posterior (solid black) distributions for the estimated structural parameters. Estimates obtained from Bayesian estimation of the DSGE model using German data from 1983:1-2009:4.
Figure 5: Prior and posterior distributions of the shock processes. Notes: Prior (solid grey) vs. posterior (solid black) distributions for the estimated shocks processes. Estimates obtained from Bayesian estimation of the DSGE model using German data from 1983:1-2009:4.
### B Details of Germany´s fiscal stimulus measures in 2010

#### Stimulus packages

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Measure</th>
<th>(bln Euro)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tax deductibility of professional commute</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>Package for tax burden reduction, stabilisation of social security contributions and investment in families</td>
<td>11.91</td>
</tr>
<tr>
<td>Income tax</td>
<td>Income tax cut</td>
<td>6.04</td>
</tr>
<tr>
<td></td>
<td>State payment of 50% social insurance for short-time workers for short-time workers</td>
<td>1.15</td>
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<tr>
<td></td>
<td>Reduction in health insurance contributions</td>
<td>6.50</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>Suspension of car tax on new vehicles</td>
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</tr>
<tr>
<td></td>
<td>Reform of car tax</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Decrease of tax on Biodiesel</td>
<td>0.13</td>
</tr>
<tr>
<td>Capital tax</td>
<td>Higher tax-free allowances for companies</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>Degressive depreciation deduction</td>
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</tr>
<tr>
<td>Spending</td>
<td>Investments into transport infrastructure</td>
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<td></td>
<td>Infrastructure investment programme</td>
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<tr>
<td></td>
<td>Innovation support programme</td>
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</tr>
<tr>
<td>Transfers</td>
<td>Retraining and stronger job service</td>
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</tr>
<tr>
<td></td>
<td>Increased child benefits</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>Increased housing benefits</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>49.31</strong></td>
</tr>
</tbody>
</table>

*Source: Brot und Butter Brief “Der Wirtschaftskrise entgegensteuern”, German Treasury*

#### Growth Acceleration Act

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Measure</th>
<th>(bln Euro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption tax</td>
<td>Decrease of VAT on Hotels and restaurants</td>
<td>1.00</td>
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<tr>
<td>Capital tax</td>
<td>Changes in the legislation of business tax</td>
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</tr>
<tr>
<td>Transfers</td>
<td>Higher tax exemption for dependent children and child benefits</td>
<td>4.60</td>
</tr>
<tr>
<td></td>
<td>Lower inheritance tax</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>8.40</strong></td>
</tr>
</tbody>
</table>

*Source: Wachstumsbeschleunigungsgesetz, German Treasury*
References


