

International Finance Discussion Papers

Number 174

January 1981

EXCHANGE INTERVENTION POLICY IN A MULTIPLE COUNTRY WORLD

by

Matthew B. Canzoneri

I would like to thank Dale Henderson for several helpful discussions of the material presented here; Kenneth Rogoff ought to be considered a coauthor of the appendix on uniqueness problems; and Robert Flood and Guy Stevens asked questions that led to useful clarifications. However, the views expressed must be considered those of the author; they do not necessarily represent the views of the Board or any other member of its staff.

Many economists have argued that countries experiencing large financial disturbances would enjoy more employment stability if exchange rates were fixed, while countries experiencing large goods market disturbances would have more employment stability if exchange rates were flexible.<sup>1/</sup> These arguments are usually made within the framework of a one or two country model, and the policy options considered are generally the two extreme cases of fixed and flexible exchange rates.<sup>2/</sup> So how are these arguments to be interpreted in a multicountry world that eschews either policy extreme? Do they tell us which countries should form a currency union and stabilize a bilateral exchange rate? Do they explain when a single country should intervene against, say, a basket of currencies? Or do they only apply to a world stabilization effort?

This paper develops two and three country models within which these matters can be investigated. The two country model is quite similar to Henderson's (1980), though the policy design is different. The countries' goods and bonds are imperfect substitutes; monetary policy affects employment in each country because of nominal wage contracts in the labor markets; each country uses open market operations to peg its interest rate; and exchange rate policy consists of sterilized interventions linked to exchange rate movements.

The two country model is described in the next section, and the factors that determine the optimal amount of intervention are identified. It turns out that this does not involve a simple dichotomy between "real" and "monetary" disturbances; only disturbances that affect the exchange

rate matter. A three country extension is discussed in the following section. Bilateral exchange rate stabilization is seen to have spillover effects that can either stabilize or destabilize the other exchange rates. The two country results do not give a good indication of which countries ought to form currency unions; in fact, it turns out that if employment stability is the goal, it is rather difficult to make a strong case for a currency union. The two country logic and results carry over much more readily to the question of unilateral intervention against a basket of currencies.

Discussing fixed interest rate regimes is one way of highlighting intervention policy, and it also allows for very clean results. However, pegging nominal interest rates can lead to indeterminate price levels and exchange rates in very standard specifications of models of international trade. Uniqueness problems are discussed in an appendix.

#### I. TWO COUNTRY MODEL

The model presented here is log-linear. The original specification and its log-linearization are discussed in an appendix. The model consists of the following equations:

Supply Equations

$$y_t = \theta(p_t - p_{t|t-1}) \quad y_t^a = \theta(p_t^a - p_{t|t-1}^a) \quad (1)$$

Demand Equations

$$y_t = \delta(p_t^a + e_t^a - p_t) - \Delta[i_t - (pi_{t+1|t-1}^a - pi_t^a)] \\ - \Delta[i_t^a - (pi_{t+1|t-1}^a - pi_t^a)] - \partial(p_t + p_t^a) + u_t - z_t^a \quad (2)$$

$$y_t^a = -\delta(p_t^a + e_t^a - p_t) - \Delta[i_t - (pi_{t+1|t-1} - pi_t)] \\ - \Delta[i_t^a - (pi_{t+1|t}^a - pi_t^a)] - \partial(p_t + p_t^a) + u_t + z_t^a$$

Money Market Equations

$$m_t - pi_t = -\alpha i_t - \beta[i_t^a + (e_{t+1|t-1}^a - e_t^a)] + [(p_t - pi_t) + y_t] - v_t \\ m_t^a - pi_t^a = -\alpha i_t^a - \beta[i_t^a - (e_{t+1|t-1}^a - e_t^a)] + [(p_t^a - pi_t^a) + y_t^a] - v_t^a \quad (3)$$

Bond Market Equations

$$b_t = -\gamma[i_t^a + (e_{t+1|t-1}^a - e_t^a) - i_t] + \frac{1}{2}[a_t - m_t - (m_t^a + e_t^a)] + \frac{1}{2}(v_t - v_t^a) - w_t^a \\ b_t^a + e_t^a = \gamma[i_t^a + (e_{t+1|t-1}^a - e_t^a) - i_t] + \frac{1}{2}[a_t - m_t - (m_t^a + e_t^a)] + \frac{1}{2}(v_t^a - v_t) + w_t^a \quad (4)$$

Central Bank Stock Constraint

$$m_t + b_t + m_t^a + b_t^a = 0 \quad (5)$$

There are two goods in the model.  $y$  is the log of home output;  $y^a$  is the log of country A output.  $p$  is the log of the domestic price of home output, and  $p_{t|t-1}$  is the expected value of  $p_t$  given information available at the end of period  $t - 1$ ;  $p^a$  and  $p_{t|t-1}^a$  are their country A counterparts. Expectations are assumed to be "rational". The supply equations (1) assert that the levels of production depend upon price prediction errors; this is the "Natural Rate" hypothesis.

The "Natural Rate" hypothesis can be motivated in a number of ways. The present paper takes a contracting approach. Nominal wages for period  $t$

are specified in labor contracts that are negotiated at the end of period  $t - 1$ . In period  $t$ , firms take prices from the market, wages from the contracts and maximize profits. Consider figure 1 where the labor supply curve is assumed to be vertical. When the contract is negotiated, the nominal wage is set at the value that is expected to clear the labor market. That is,  $w_t$  is set so that  $w_t - p_t|_{t-1} = v$ . The real wage that will actually prevail in period  $t$  depends upon the price prediction error.<sup>3/</sup> If there is no prediction error, then the real wage will be  $v$  and employment will be at its equilibrium on "natural" rate,  $\bar{n}$ . If prices turn out to be say lower than was predicted, then the real wage will be too high and employment will be below  $\bar{n}$ . So employment and output fluctuate with the price prediction errors.<sup>4/ 5/</sup> Units have been chosen so that the "natural" rates of output in each country are equal to one, and their logs are equal to zero.

The demand curves (2) represent world demand for the two products.  $e^a$  is the home currency price of foreign exchange (expressed in logs), and  $p^a + e^a - p$  is the terms of trade.  $i$  and  $i^a$  are nominal interest rates, and

$$pi \equiv \frac{1}{2}p + \frac{1}{2}(p^a + e^a), \quad pi^a \equiv \frac{1}{2}(p - e^a) + \frac{1}{2}p^a$$

are price indices. Since the "natural" rate of output is equal to one in both countries, the two prices are given equal weight. Demand for each good depends upon relative prices, expected real interest rates, and real world wealth.<sup>6/</sup>  $u$  and  $z^a$  are disturbance terms. All of the disturbance terms in this model are serially uncorrelated random variables with zero

mean.  $u$  represents a dissavings that is split equally between the two goods;  $z^a$  is a shift in demand from the home good to country A's good.

There are four assets: home money and home bonds ( $m$  and  $b$ ), and country A money and country A bonds ( $m^a$  and  $b^a$ ). The residents of each country hold all of their own money stock, primarily for transactions purposes. The remainder of each countries' wealth is split between the two bond assets. The money market equations (3) explain how the residents of each country tradeoff the benefits of liquidity with the expected earnings on the two bond assets. Money is assumed to be a closer substitute for the bond of like denomination ( $\alpha > \beta$ ), and the income elasticity of demand is unitary.<sup>7/</sup> The bond market equations (4) show how the residents of the world split the remainder of their wealth between the two bond assets.  $\gamma$  measures the substitutability of the two bonds; as  $\gamma \rightarrow \infty$ , the two become perfect substitutes.<sup>8/</sup>  $a$  is the home currency value of world wealth; so  $a - m - (m^a + e)$  is the amount of wealth to be split between the two bond assets. The  $v$ 's and  $w$ 's are stochastic disturbances;  $v$ 's represent shifts between assets with the same currency denomination while  $w$ 's represent shifts between assets of different denomination.

Walras' Law implies that the two equations in (4) do not represent separate restrictions on the endogenous variables. They can be collapsed, by subtraction, into a single restriction.

$$b_t^a + e_t^a - b_t = 2\gamma[i_t^a + (e_{t+1}^a |_{t-1} - e_t^a) - i_t] + v_t^a - v_t + 2w_t^a \quad (4)'$$

Finally, the stock constraint (5) asserts that monetary policy is not conducted via helicopter operations or confiscations. Instead, the two governments are constrained to free market swaps that leave the nominal value

of private sector wealth intact. It should be noted that all of the equations have been log-linearized around an equilibrium in which prices, the exchange rate, both monies and both bonds are equal to one (and their logs are equal to zero); the details of this are spelled out in an appendix.

Now suppose each country uses open market operations (swaps of money and bonds) to peg its nominal interest rate, and suppose one or both uses sterilized interventions (essentially swaps of foreign and domestic bonds) to counter exchange rate movements. This can be summarized by setting  $i_t$  and  $i_t^a$  equal to zero in the equations above and letting<sup>9/</sup>

$$b_t = -m_t - \rho e_t^a \text{ and } b_t^a = -m_t^a + \rho e_t^a$$

or

$$b_t^a - b_t = m_t - m_t^a + 2\rho e_t^a \tag{6}$$

A positive value for  $\rho$  implies a policy of "leaning against the wind;" a depreciating exchange rate is met with a purchase of the depreciating asset by one (or both) of the central banks. Larger values of  $\rho$  represent more vigorous stabilization efforts.

The next step is to solve the model to determine the effect of  $\rho$  on the stability of exchange rates and employment. A trick that is sometimes used in solving models like this is to simply assert that all of the expectations are time independent constants (zero in the present case). This has the effect of reducing the model to a set of static relationships that can be solved in a straight forward (if tedious) manner. Later, one

can verify that the expectations are indeed stationary, as was originally asserted.<sup>10/</sup>

The trouble with this procedure is that it tends to hide uniqueness problems. It is well known (in the closed economy literature anyway) that pegging interest rates can result in an indeterminate price level.<sup>11/</sup> Uniqueness is not a problem in the present model because (i) wealth effects appear in the demand curves ( $\partial > 0$ ), (ii) foreign and domestic assets are not perfect substitutes ( $\gamma < \infty$ ,  $\beta < \infty$ ), and (iii) intervention policy leans against the wind ( $\rho \geq 0$ ).<sup>12/</sup> As is shown in an appendix, (i) insures that the two price levels are determinant, and (ii) and (iii) insure that the exchange rate is determinant. This appendix is intended as a justification for using the simple solution technique described above.

Setting  $p_t|_{t-1}$ ,  $p_t^a|_{t-1}$ ,  $pi_{t+1}|_{t-1}$ ,  $pi_{t+1}^a|_{t-1}$  and  $e_{t+1}^a|_{t-1}$  equal to zero, the model is easily solved. Adding and subtracting the equations in (1) and (2) results in

$$\begin{aligned} p_t + p_t^a &= \theta^{-1}(y_t + y_t^a) = \theta^{-1}\phi 2u_t \\ p_t - p_t^a &= \theta^{-1}(y_t - y_t^a) = \theta^{-1}\lambda(2\delta e_t^a - 2z_t^a) \end{aligned} \tag{7}$$

where

$$\lambda \equiv \theta(\theta + 2\delta)^{-1} \text{ and } \phi \equiv \theta(\theta + 2\partial + 4\Delta)^{-1}$$

And adding and subtracting the equations in (7),

$$\begin{aligned}
 y_t &= \theta p_t = \lambda \delta e_t^a + \phi u_t - \lambda z_t^a \\
 y_t^a &= \theta p_t^a = -\lambda \delta e_t^a + \phi u_t + \lambda z_t^a
 \end{aligned}
 \tag{8}$$

The equations in (8) are semi-reduced forms for output and employment;  $e_t^a$  is of course an endogenous variable. An appreciation of the exchange rate (that is, a decrease in  $e_t^a$ ) raises the relative price of the home good and shifts demand in the same direction as  $z_t^a$ ;  $u_t$  raises demand for output and employment in both countries. Now suppose  $e_t^a$  turns out to be uncorrelated with either  $u_t$  or  $z_t^a$ . Both countries will want to limit its fluctuations since, in this case, exchange rate fluctuations are an independent source of variation in output and employment. On the other hand, fluctuations in  $e_t^a$  may tend to offset the effects of  $u_t$  and  $z_t^a$  in (8), in which case they will be deemed beneficial.<sup>13/</sup>

So what causes the fluctuations in  $e_t^a$ ? With interest rates pegged, the exchange rate is determined by the ratios of supplies and demands for home and foreign assets;<sup>14/</sup> this can be seen in equations (4)' and (6). Using (3) to calculate  $m_t - m_t^a$ , these equations imply

$$\begin{aligned}
 e_t^a &= (\rho + \eta)^{-1} x_t^a \quad \text{where} \quad x_t^a \equiv w_t^a + \theta^{-1}(1 + \theta)\lambda z_t^a \\
 \eta &\equiv \frac{1}{2} + \gamma + \beta + \theta^{-1}(1 + \theta)\lambda \delta
 \end{aligned}
 \tag{9}$$

$w^a$  shifts demand from home bonds to foreign bonds, while  $z^a$  shifts demand from home money to foreign money (through its effect on transactions demand in each country). Either has the effect of depreciating the home currency. A central bank purchase of home bonds in exchange for foreign bonds goes

the other way; so the intervention policy summarized in (6) does indeed "lean against the wind." The  $u$ ,  $v$  and  $v^a$  disturbances have no effect upon the exchange rate. This is because they do not affect the relative demands for home and foreign assets;  $u$  increases the transactions demands for both monies, while the  $v$ 's shift demand between assets of like denomination.

A vigorous intervention policy will dampen exchange rate fluctuations no matter what their source. If  $w^a$  is the primary source of exchange rate fluctuations, then a strong intervention policy will be desirable. The exchange rate carries  $w^a$  disturbances from bond markets to labor markets. If on the other hand  $z^a$  is causing the fluctuations, then they should be allowed to occur.  $z^a$  shifts demand from the home good to the foreign good, and this increases transactions demand for foreign money and decreases transactions demand for home money; the resulting depreciation produces a terms of trade effect that partially offsets the original shift in demand. The offset is only partial since

$$\delta(\rho + \eta)^{-1} \theta^{-1} (1 + \theta) \lambda < \lambda$$

for all  $\rho \geq 0$ . So even a freely floating exchange rate does not go far enough; an active policy of employment stabilization would have to destabilize the exchange rate.<sup>15/</sup>

So free market exchange rate movements partially absorb the effects of shifts in demand for home and foreign goods, but they carry shifts between home and foreign assets into the labor markets of both countries. The choice of an intervention policy will have to reflect a

balancing of these two factors; the best  $\rho$  will depend upon the relative sizes of the  $w^a$ 's and the  $z^a$ 's. Note that the sizes of the  $u$ ,  $v$  and  $v^a$  disturbances are irrelevant in this regard. These disturbances do not affect relative demands for home and foreign assets or the exchange rate; so exchange rate policy will not alter their effects on employment.<sup>16/</sup> So it is not a simple dichotomy between real and financial disturbances; only disturbances that affect relative demands for home and foreign assets matter.

## II. THREE COUNTRY MODEL

One might be tempted to look among the results of the last section for the criteria a country ought to consider when deciding whether or not to join a currency union. Those results seem to suggest that countries with volatile capital movements but rather stable trade flows will have an incentive to join. An alternative is for the country to intervene unilaterally against a basket of currencies. These matters can be investigated in a three country version of the model developed in the last section.

### Supply Equations

$$y_t = \theta(p_t - p_{t|t-1}), y_t^a = \theta(p_t^a - p_{t|t-1}^a), y_t^b = \theta(p_t^b - p_{t|t-1}^b) \quad (1)$$

### Demand Equations

$$y_t = \delta(p_t^a + e_t^a - p_t) + \delta(p_t^b + e_t^b - p_t) - \Delta(r_t + r_t^a + r_t^b) \\ - \partial(p_t + p_t^a + p_t^b) + u_t - z_t^a - z_t^b \quad (2)$$

$$\begin{aligned}
 y_t^a &= -\delta(p_t^a + e_t^a - p_t) + \delta(p_t^b + e_t^b - p_t^a - e_t^a) - \Delta(r_t + r_t^a + r_t^b) \\
 &\quad - \partial(p_t + p_t^a + p_t^b) + u_t + z_t^a - z_t^{ab} \\
 y_t^b &= -\delta(p_t^b + e_t^b - p_t) - \delta(p_t^b + e_t^b - p_t^a - e_t^a) - \Delta(r_t + r_t^a + r_t^b) \\
 &\quad - \partial(p_t + p_t^a + p_t^b) + u_t + z_t^b + z_t^{ab}
 \end{aligned}$$

Money Market Equations

$$\begin{aligned}
 m_t - pi_t &= -\alpha i_t - \beta[i_t^a + (e_{t+1}^a|_{t-1} - e_t^a)] \\
 &\quad - \beta[i_t^b + (e_{t+1}^b|_{t-1} - e_t^b)] + [(p_t - pi_t) + y_t] - v_t \\
 m_t^a - pi_t^a &= -\alpha i_t^a - \beta[i_t^b + (e_{t+1}^b|_{t-1} - e_t^b) - (e_{t+1}^a|_{t-1} - e_t^a)] \\
 &\quad - \beta[i_t^a - (e_{t+1}^a|_{t-1} - e_t^a)] + [(p_t^a - pi_t^a) + y_t^a] - v_t^a \\
 m_t^b - pi_t^b &= -\alpha i_t^b - \beta[i_t^a + (e_{t+1}^a|_{t-1} - e_t^a) - (e_{t+1}^b|_{t-1} - e_t^b)] \\
 &\quad - \beta[i_t^b - (e_{t+1}^b|_{t-1} - e_t^b)] + [(p_t^b - pi_t^b) + y_t^b] - v_t^b
 \end{aligned} \tag{3}$$

Bond Market Equations

$$\begin{aligned}
 b_t &= \gamma[i_t - (i_t^a + e_{t+1}^a|_{t-1} - e_t^a)] + \gamma[i_t - (i_t^b + e_{t+1}^b|_{t-1} - e_t^b)] \\
 &\quad + 1/3(b_t + b_t^a + e_t^a + b_t^b + e_t^b) + v_t - w_t^a - w_t^b \\
 b_t^a + e_t^a &= \gamma[(i_t^a + e_{t+1}^a|_{t-1} - e_t^a) - (i_t^b + e_{t+1}^b|_{t-1} - e_t^b)] \\
 &\quad + \gamma[(i_t^a + e_{t+1}^a|_{t-1} - e_t^a) - i_t] + 1/3(\cdot) + v_t^a + w_t^a - w_t^{ab}
 \end{aligned} \tag{4}$$

$$b_t^b + e_t^b = \gamma [(i_t^b + e_{t+1}^b |_{t-1} - e_t^b) - (i_t^a + e_{t+1}^a |_{t-1} - e_t^a)] \\ + \gamma [i_t^b + e_{t+1}^b |_{t-1} - e_t^b) - i_t] + 1/3(\cdot) + v_t^b + w_t^b + w_t^{ab}$$

Central Bank Stock Constraint

$$m_t + b_t + m_t^a + b_t^a + m_t^b + b_t^b = 0 \quad (5)$$

There are three countries: the home country, country A, and country B. Each supplies a good and a bond to world markets; as before, the residents of each country hold all of their own money stock and split the rest of their wealth between the various bond assets. The home currency price of country A exchange is  $e^a$ , and the home currency price of country B exchange is  $e^b$ ; so the country A currency price of country B exchange is  $-e^a + e^b$ . The  $r$ 's in (2) are expected real rates of interest; for example,

$$r_t^a \equiv i_t^a - (pi_{t+1}^a |_{t-1} - pi_t^a)$$

The  $p$ 's are the domestic currency prices of the three goods. The  $pi$ 's are price indices; for example

$$pi_t^a \equiv (1/3)p_t^a + (1/3)(p_t - e_t^a) + (1/3)(p_t^b - e_t^a + e_t^b)$$

As before, the supply of each good depends on price prediction errors, and demand depends on relative prices, expected real interest rates and wealth effects;  $z^b$  is a shift in demand from home output to country B output, and  $z^{ab}$  is a shift from country A output to country B output. Money demands depend upon transaction needs and the expected earnings on the three bonds;

bond demands depend upon expected earnings differentials.  $w^b$  and  $w^{ab}$  are shifts in world preferences from home and country A bonds to country B bonds. Bond market equilibrium implies only two independent restrictions; they can be expressed as

$$\begin{aligned}
 b_t^a + e_t^a - b_t &= 3\gamma[(i_t^a + e_{t+1}^a|_{t-1} - e_t^a) - i_t] \\
 &\quad + v_t^a - v_t + 2w_t^a + w_t^b - w_t^{ab} \\
 b_t^b + e_t^b - b_t &= 3\gamma[(i_t^b + e_{t+1}^b|_{t-1} - e_t^b) - i_t] \\
 &\quad + v_t^b - v_t + 2w_t^b + w_t^a + w_t^{ab}
 \end{aligned}
 \tag{4}'$$

With interest rates pegged, the exchange rates  $e^a$  and  $e^b$  are determined by ratios of supplies and demands for home and foreign assets.

Setting the  $i$ 's and the  $\pi_{t+1|t-1}$ 's equal to zero, equations (1) and (2) can be solved for semi-reduced forms for output and (implicitly) employment.

$$\begin{aligned}
 y_t &= \lambda\delta(e_t^a + e_t^b) + \phi u_t - \lambda z_t^a - \lambda z_t^b \\
 y_t^a &= \lambda\delta(e_t^b - 2e_t^a) + \phi u_t + \lambda z_t^a - \lambda z_t^{ab} \\
 y_t^b &= \lambda\delta(e_t^a - 2e_t^b) + \phi u_t + \lambda z_t^b + \lambda z_t^{ab}
 \end{aligned}
 \tag{6}$$

where

$$\lambda \equiv \theta(\theta + 3\delta)^{-1} \quad \text{and} \quad \phi \equiv \theta(\theta + 3\delta + 9\Delta)^{-1}$$

The exchange rate configurations in (6) can be given a simple interpretation.

Let the "Sally" be a basket of currencies, a basket containing one unit of

each of the three currencies. It turns out that  $(e^a + e^b)/3$  is the home currency price of a Sally;  $(e^b - 2e^a)/3$  is the country A currency price, and  $(e^a - 2e^b)/3$  is the country B currency price.<sup>17/</sup> So it is just the three Sally rates that appear in the semi-reduced forms (6).

#### Currency Unions:

Suppose the home country and country A form a currency union; that is, they agree on an intervention policy to stabilize  $e^a$ . As before, each country uses open market operations to peg its interest rate. So for the union  $i_t$  and  $i_t^a$  equal zero and

$$b_t = -m_t - \rho e_t^a, \quad b_t^a = -m_t^a + \rho e_t^a$$

and for country B,  $i_t^b$  equals zero and  $b_t^b$  equals  $-m_t^b$ . Country B does not intervene in foreign exchange markets.<sup>18/</sup> So finally,

$$b_t^a - b_t = m_t - m_t^a + 2\rho e_t^a$$

$$b_t^b - b_t = m_t - m_t^b + \rho e_t^a$$

$$b_t^b - b_t^a = m_t^a - m_t^b - \rho e_t^a$$

(7)

The union intervention policy affects the relative supplies of its members assets in a way that will be seen to stabilize  $e^a$ . But notice that it also affects the relative supplies of each union member's assets and country B assets. This means that the union's policy of stabilizing  $e^a$  will have spillover effects on both  $e^b$  and  $-e^a + e^b$ .

Using (3) to calculate  $m_t - m_t^a$  and  $m_t - m_t^b$ , one can solve (4)' and (7) for

$$\begin{aligned}
 e_t^a &= (\rho + \eta)^{-1} (x_t^a + \frac{1}{2}x_t^b - \frac{1}{2}x_t^{ab}) \\
 e_t^b &= (1/2\eta)[1 - \rho(\rho + \eta)^{-1}]x_t^a + (1/\eta)[1 - (\frac{1}{2})\rho(\rho + \eta)^{-1}]x_t^b \\
 &\quad + (1/2\eta)[1 + (1/2)\rho(\rho + \eta)^{-1}]x_t^{ab}
 \end{aligned} \tag{8}$$

where

$$\eta \equiv (\frac{1}{2}) + (3/2)[\gamma + \beta + \theta^{-1}(1 + \theta)\lambda\delta]$$

$$x_t^i \equiv w_t^i + \theta^{-1}(1 + \theta)\lambda z_t^i \quad i = a, b, ab$$

These exchange rate fluctuations are depicted in figure 2. The exchange rate along one side of the triangle is determined by the relative supplies and demands for the assets of the countries at adjacent vortices; the  $x$ 's represent relative demand shifts. If, for example, the demand for country A assets rises relative to the demand for home assets,  $e^a$  will depreciate. And if the supply of country A assets rises relative to the supply of home assets,  $e^a$  will appreciate.<sup>19/</sup>

The  $x$  disturbances have a direct effect on a particular exchange rate and indirect effects on the others. Consider  $x^a$ , and suppose for the moment that  $\rho = 0$ .  $x^a$  represents real ( $z^a$ ) and financial ( $w^a$ ) disturbances that have the direct effect of shifting demand from home assets to country A assets and depreciating  $e^a$ . But this shift also has the indirect effect of

throwing country B assets out of balance. It raises the demand for country B assets relative to home assets, depreciating  $e^b$ , and it raises the demand for country A assets relative to country B assets, appreciating  $-e^a + e^b$ . The indirect effects are only half as large as the direct effect, so the movements in  $e^b$  and  $-e^a + e^b$  are only half as large as the movement in  $e^a$ . A union intervention in response to say a depreciation in  $e^a$  works the other way, like a negative  $x^a$ ; it appreciates  $e^a$  and the indirect effects on the other two exchange rates are the spillover effects referred to earlier.

Not surprisingly, the union's intervention policy stabilizes  $e^a$ . The effect of the spillovers onto the other exchange rates is less obvious.  $e^b$ , for example, is stabilized against  $x^a$  and  $x^b$  disturbances, but the effects of  $x^{ab}$  disturbances are magnified.<sup>20/</sup> So the spillovers are a potential source of conflict between country B and the union.

To see the effect of the intervention policy on output and employment, one has to calculate the three Sally rates that appear in (6).

$$\begin{aligned}
 e_t^a + e_t^b &= (\rho + \eta)^{-1} \{ (3/2)x_t^a + [(3/2) + (3/4\eta)\rho]x_t^b - (3/4\eta)\rho x_t^{ab} \} \\
 e_t^b - 2e_t^a &= (\rho + \eta)^{-1} \{ - (3/2)x_t^a - (3/4\eta)\rho x_t^b + [(3/2) + (3/4\eta)\rho]x_t^{ab} \} \quad (9) \\
 e_t^a - 2e_t^b &= - (3/2\eta) (x_t^b + x_t^{ab})
 \end{aligned}$$

The first and perhaps most interesting thing to note is that country B's Sally rate,  $e^a - 2e^b$ , is unaffected by the size of  $\rho$ . The union's intervention policy has no effect at all on country B's employment. Spillover

effects may or may not destabilize  $e^b$  and  $-e^a + e^b$ , but they simply cancel out in terms of the exchange rate that matters for country B's employment, its Sally rate.<sup>21/</sup> If employment stability is the primary goal of country B's monetary authorities, they should not worry about the spillover effects from the union's attempts to stabilize  $e^a$ .

Interventions do affect the Sally rates of both union members, but for them the results are rather mixed. For financial disturbances between themselves, both will benefit from a strong intervention policy.  $w^a$  shocks make both Sally rates fluctuate, and these fluctuations carry the financial disturbances into the labor markets; a strong intervention policy will stabilize both Sally rates against  $w^a$  disturbances. But consider financial disturbances between country A and country B. For  $w^{ab}$  shocks, country A will benefit from a strong intervention policy, but the home country will not. In fact, the home Sally rate will not even be affected by  $w^{ab}$  shocks if  $\rho = 0$ ;  $w^{ab}$  shocks move  $e^a$  and  $e^b$  in equal and opposite directions. But since interventions do affect the home Sally rate, a policy designed to offset fluctuations in  $e^a$  will make the home Sally rate fluctuate with  $w^{ab}$ , destabilizing home employment. Similarly, for financial disturbances between the home country and country B, the home country will benefit from an intervention policy, but its partner will be made worse off. Financial disturbances between union members and country B cause conflicts within the union.

As in the two country model, real disturbances are partially absorbed by the exchange rates if they are allowed to fluctuate. The Sally rates do not completely offset the  $z$  effects in (6) even if  $\rho = 0$  (since  $\delta(3/2\eta)\theta^{-1}(1+\theta)\lambda$

is less than 1), but the less intervention the better. In fact,  $\rho > 0$  implies that the Sally rates will carry new  $z$  disturbances to the labor markets of union members; intervention makes home employment fluctuate with  $z^{ab}$  and it makes country A employment fluctuate with  $z^b$ .

The two country results suggested that countries with volatile capital movements and stable trade flows would have an incentive to unite, but the analysis above suggests that these criteria are insufficient and perhaps seriously misleading. The union intervention has the unfortunate effect of transmitting new disturbances, both real and financial, to the labor markets of its members. In fact, the only disturbances for which the policy is unambiguously beneficial are financial shifts between the member countries.

Different conclusions can be drawn from this analysis. My own is that it is difficult to make a case for currency unions, at least on the basis of employment stability. Bilateral interventions make sense if they are used in response to identified shifts in preference for the two assets in question, but the general smoothing of bilateral exchange rates is seldom called for.

#### Intervention Against a Basket:

It is not difficult to see why the two country logic and results did not carry over in a more straight forward manner. It is the Sally rates, and not  $e^a$ , that appear in the semi-reduced forms (6). One might expect that if, say, the home country intervened unilaterally against the Sally, the results would be more symmetric, and perhaps more beneficial to the home country.

Suppose the home country uses sterilized interventions against the Sally to stabilize its Sally rate; then

$$b_t = -m_t - \rho(e_t^a + e_t^b)$$

$$b_t^a = -m_t^a + \frac{1}{2}\rho(e_t^a + e_t^b)$$

$$b_t^b = -m_t^b + \frac{1}{2}\rho(e_t^a + e_t^b)$$

As before, each country uses open market operations to peg its own interest rate. The semi reduced forms (6) are still valid, but (9) becomes

$$e_t^a + e_t^b = (\tilde{\rho} + \eta)^{-1} (3/2) (x_t^a + x_t^b)$$

$$e_t^b - 2e_t^a = (\tilde{\rho} + \eta)^{-1} (3/2\eta) [\tilde{\rho}x_t^b - (\tilde{\rho} + \eta)(x_t^a - x_t^{ab})] \quad (9)'$$

$$e_t^a - 2e_t^b = (\tilde{\rho} + \eta)^{-1} (3/2\eta) [\tilde{\rho}x_t^a - (\tilde{\rho} + \eta)(x_t^b + x_t^{ab})]$$

where  $\tilde{\rho} = (4/3)\rho$ .

For the home country, the results are quite analogous to the two country results. The financial disturbances  $w^a$  and  $w^b$  cause fluctuations in the Sally rate that will be carried to the labor market unless the Sally rate is stabilized; on the other hand, fluctuations in the Sally rate partially absorb the real disturbances  $z^a$  and  $z^b$ . And this intervention policy does not carry  $x^{ab}$  disturbances to the home labor market.

But unlike the currency union case, the home country's intervention policy will affect employment in the other countries, and not always beneficially. For example, the home country interventions carry  $x^b$  shocks

to country A's labor market where they had no effect before (that is, under a free float). Again, this fact should not be too surprising; stabilizing the home Sally rate is not equivalent to stabilizing country A's Sally rate.

So what would happen if the other countries retaliated by stabilizing their Sally rates. For simplicity, suppose all three countries have the same intervention parameters  $\rho$ , then

$$b_t = -m_t - \rho(e_t^a + e_t^b) + \frac{1}{2}\rho(e_t^b - 2e_t^a) + \frac{1}{2}\rho(e_t^a - 2e_t^b)$$

$$b_t^a = -m_t^a - \rho(e_t^b - 2e_t^a) + \frac{1}{2}\rho(e_t^a + e_t^b) + \frac{1}{2}\rho(e_t^a - 2e_t^b)$$

$$b_t^b = -m_t^b - \rho(e_t^a - 2e_t^b) + \frac{1}{2}\rho(e_t^a + e_t^b) + \frac{1}{2}\rho(e_t^b - 2e_t^a)$$

A lot of cancellation occurs, and

$$b_t = -\tilde{\rho}(3/2)(e_t^a + e_t^b) - m_t$$

$$b_t^a = -\tilde{\rho}(3/2)(e_t^b - 2e_t^a) - m_t^a$$

$$b_t^b = -\tilde{\rho}(3/2)(e_t^a - 2e_t^b) - m_t^b$$

Equation (9)' becomes

$$e_t^a + e_t^b = (\tilde{\rho} + \eta)^{-1} (3/2) (x_t^a + x_t^b)$$

$$e_t^b - 2e_t^a = (\tilde{\rho} + \eta)^{-1} (3/2) (x_t^{ab} - x_t^a)$$

$$e_t^a - 2e_t^b = -(\tilde{\rho} + \eta)^{-1} (3/2) (x_t^b + x_t^{ab})$$

where  $\tilde{\rho} = (4/3)\rho$ , and the results are completely analogous to the two country case.

### III. SUMMARY AND CONCLUDING COMMENTS

Two and three country models were developed to investigate the effects of unilateral intervention policies and currency unions on employment stability. In each case interest rates were pegged via open market operations, and an exchange rate was stabilized via sterilized interventions linked to the exchange rate's movements.

With interest rates pegged, exchange rates were determined by relative supplies and demands for home and foreign assets, and each country's employment could be expressed in semi-reduced form as a function of goods market disturbances and an exchange rate. (In the three country model, the exchange rate was the domestic currency price of a basket of currencies called the "Sally".) This regime afforded sharp, clear-cut results; if the interest rates had not been pegged, the results would not have been so clean. .

In the two country model, intervention stabilized employment in both countries when demand shifts between the two countries' bonds were large and when shifts between the two countries' goods were small. As can be seen from the semi-reduced forms for employment, exchange rate fluctuations tend to absorb the goods market shifts but they also tend to carry financial shifts from the bonds market to the labor markets in both countries. These results carried over to the three country model in the case of a single country intervening against the Sally. Its employment was stabilized if financial shifts to and from the other countries were large and if goods market shifts were small. There was a complication though; stabilizing one country's Sally rate can have the effect of destabilizing others, and

this could provoke a response. However, it was also shown that the two country results carried over exactly to the case of all three countries intervening unilaterally to stabilize their own Sally rates.

The currency union case proved to be somewhat different. Intervening to stabilize a bilateral exchange rate was seen to produce spillover effects that could either stabilize or destabilize the other exchange rates. This may be a source of conflict with non-member countries, though it ought not to be if they are only worried about employment stability. It was shown that the spillover effects cancelled out in terms of the exchange rate that was important for the non-member's employment, its Sally rate. However, the results were very mixed for the union members themselves. For any kind of shift between a union member and the outside world, the union intervention policy stabilized one member's employment and destabilized the others. The only disturbances for which the policy was unambiguously good were financial shifts within the union.

Two concluding comments may be in order. First, the models used here collapse everything into a one period analysis. However, completely analogous results could be obtained in a multi-period setting with overlapping labor contracts, lagged feedback rules for intervention policy and serially correlated disturbances.<sup>22/</sup> Second, one might reasonably argue that some of the coefficients in the model are not independent of the intervention policy in effect. For example,  $\gamma$  and  $\beta$  in the two country model may be inversely related to the conditional variance of the exchange rate  $e^a$ , which in the one period setup depends upon the value of  $\rho$ .<sup>23/</sup> When calculating the optimal value of  $\rho$ , cross equation restrictions on the coefficients can be taken into account,<sup>24/</sup> but these observations really point to the need for more research into the microeconomic foundations for the models postulated above.

## FOOTNOTES

1/ Using a small country model with perfect capital mobility, Mundell (1968) showed that monetary policy was effective (in the sense that it affected output) with flexible rates, but not with fixed rates, and that fiscal policy was effective with fixed rates, but not with flexible rates. Kaminow (1979) reinterpreted these results in the way they are presented above. Henderson (1979, 1980) produced analogous results in one and two country models with imperfect capital mobility. Related recent studies include Bryant (1980), Buiter and Eaton (1980), and Flood and Marion (1980).

2/ Exceptions include Marston (1979), who discusses currency unions in a three country setting, and Boyer (1978) and Buiter and Eaton (1980), who consider feedback intervention policies.

3/ The real wage is

$$w_t - p_t = w_t - p_t|_{t-1} - (p_t - p_t|_{t-1}) = v - (p_t - p_t|_{t-1})$$

4/ Adding productivity disturbances complicates matters somewhat. The labor demand curve and the full employment real wage ( $v$ ) shift up and down with the disturbances, and employment and output fluctuate with productivity prediction errors in addition to price prediction errors. Stabilizing employment is no longer equivalent to stabilizing output. To keep employment constant, one has to have a positive productivity disturbances matched by a negative price prediction error. In fact, stabilizing employment is equivalent to stabilizing nominal output,  $p + y$ . Productivity disturbances are interesting in the present context, but the role they play is quite similar to demand disturbances. In particular, if productivity disturbances in the two countries are uncorrelated, then for present purposes they act like demand shifts,  $z^a$ ; if they are perfectly correlated, they act like savings shocks,  $u$ .

5/ Letting labor supply depend upon a real wage defined in terms of a basket of goods would complicate the analysis. In figure 1, the labor supply curve would have a positive slope, and it would fluctuate with the terms of trade,  $p_t^a + e_t^a - p_t$ . This means that the "natural" rate of employment and the equilibrium real wage would fluctuate with the terms of trade.

6/ The  $\alpha(p + p^a)$  terms are the wealth effects. The exchange rate falls out because the share of foreign assets in world wealth is equal to the share of foreign goods in the world consumption bundle in the equilibrium that was linearized about; see the appendix for a more detailed account.

7/ This assumption achieves some algebraic simplicity, but it is not necessary for the results that follow.

- 8/ The present study assumes that  $\gamma < \infty$ . If bonds are perfect substitutes then sterilized interventions have no effect on exchange rates (on anything else, for that matter), and fixed interest rate regimes result in an indeterminate exchange rate. The validity of this assumption can be questioned on both theoretical and empirical grounds; Henderson (1980) provides a brief discussion of this issue and some references.
- 9/ Note that these equations satisfy the stock constraint (5).
- 10/ Wallich and Gray (1980) use this trick, as does Henderson (1980).
- 11/ Wicksell noted this problem in nonstochastic neoclassical models; Sargent and Wallace (1975) and Canzoneri (1980) discussed it in rational expectations models incorporating the "natural" rate hypothesis.
- 12/ Actually,  $\rho$  can be negative if it is not too big in absolute value; see the appendix.
- 13/ It is assumed that employment stability, rather than exchange rate stability, is the primary goal of the central banks in each country. This need not be the case; there are costs associated with exchange rate fluctuations that are not considered here.
- 14/ Of course, demands for the various assets depend upon prices and outputs;  $e_t^a$  is not determined in financial markets alone.
- 15/ Buiter and Eaton (1980) made the same point. One has to be careful here. The value of  $\rho$  that makes the terms of trade effect completely offset the original shift is  $\rho = -(\frac{1}{2} + \gamma + \beta)$ , but this policy can result in indeterminate exchange rates if  $\gamma + \beta \geq \frac{1}{2}$ . See the appendix.
- 16/ The strength of this result is due to the pegging of interest rates. Henderson's (1980) choice between a "rates constant" policy and an "aggregates constant" policy will depend upon these disturbances as well.
- 17/ See the appendix on log-linearization.
- 18/ Note that all of this is consistent with the stock constraint (5).
- 19/ Notice that  $u$  and  $v$  disturbances play no role here since they do not affect the relative demands for countries' assets. As in the two country model, this strong result is due to the pegging of interest rates.

20/  $x^{ab}$  disturbances shift demand from country A assets to country B assets, depreciating  $-e^a + e^b$ . This is the direct effect. The indirect effects are to increase demand for country B assets with respect to home assets and to decrease demand for country A assets with respect to home assets, depreciating  $e^b$  and appreciating  $e^a$ . To stabilize  $e^a$ , the union monetary authority buys country A bonds and sells home bonds. This has the indirect effect of increasing the excess demand for country B bonds with respect to home bonds, further depreciating  $e^b$ .

21/ Going back to figure 2, country B's Sally rate gives equal weight to  $e^b$  and  $-e^a + e^b$ . A union intervention consisting of say a purchase of country A bonds and a sale of home bonds will appreciate  $-e^a + e^b$  by the same amount that it depreciates  $e^b$ .

22/ See Canzoneri (1980) for an example of this kind of symmetry of results.

23/ See, for example, Muth's (1961) derivation of a speculative demand for inventories.

24/ Suppose the analysis called for some stabilizing of  $e^a$ . From equation (9) it is apparent that the parameters  $\rho$ ,  $\gamma$  and  $\beta$  play a symmetrical role in the stabilization of  $e^a$ . If the values of  $\gamma$  and  $\beta$  rise with  $\rho$ , then a smaller value of  $\rho$  will achieve the optimal amount of stabilization.

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FIGURE 1

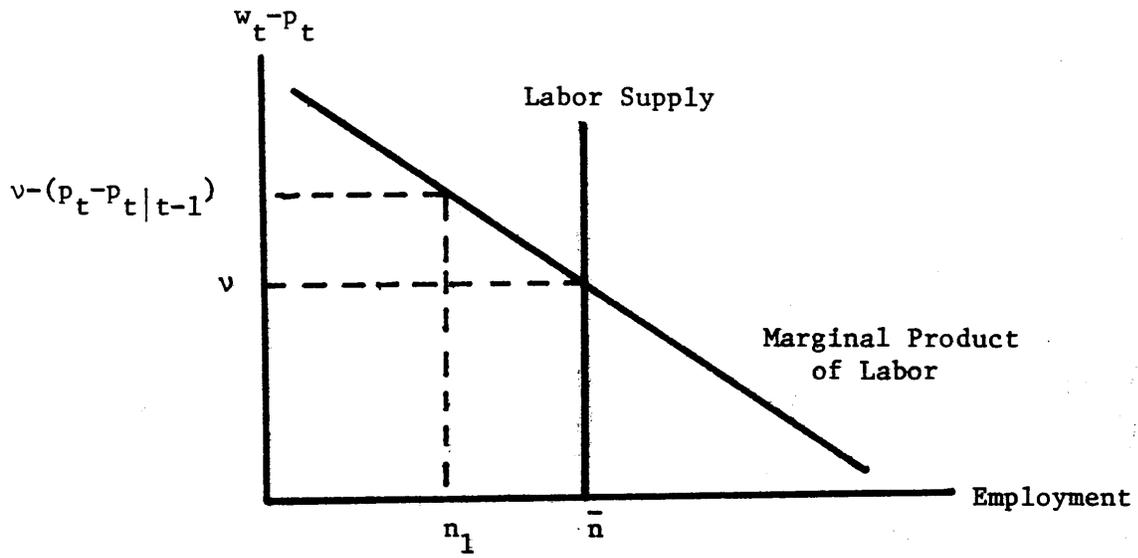
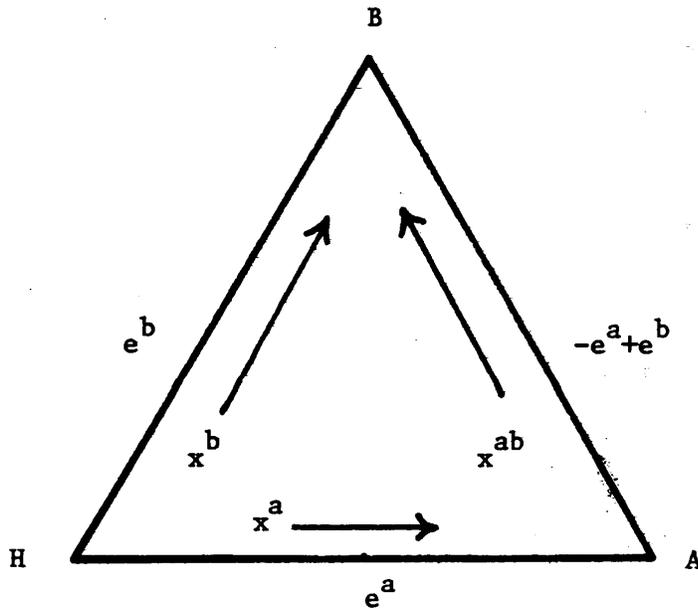


FIGURE 2



Appendix: Simplifying Rational Expectations Models and Uniqueness Problems

Suppose the model can be represented in matrix form by

$$z_t = Az_{t+1|t-1} + B(z_t - z_{t|t-1}) + C\bar{x} + u_t \quad (1)$$

where  $z$  is a vector of endogenous variables,  $\bar{x}$  is a fixed vector of exogenous variables, and  $u$  is a vector of serially uncorrelated random disturbances.

The main purpose of this appendix is to show that

$$z_{t+1|t-1} = z_{t|t-1} = \bar{z} \equiv (I - A)^{-1}C\bar{x} \quad (2)$$

if the roots of  $A$  are less than one (in absolute value) and if "speculative bubbles" are ruled out. The force of this result is that once it is established that the roots of  $A$  are less than one, one might as well work with the simpler static expression

$$z_t = A\bar{z} + B(z_t - \bar{z}) + C\bar{x} + u_t \quad (3)$$

Note that  $\bar{z}$  is just the unconditional expected value of  $z_t$ ; it can be found by taking expectations of both sides of (1).

To prove this assertion, forward (1) by  $j$  periods and take the conditional expected value of the resulting expression to obtain

$$z_{t+j|t-1} = Az_{t+j+1|t-1} + C\bar{x}$$

This difference equation can be "solved forward" to obtain an expression for  $z_{t+1|t-1}$ ; that is,

$$\begin{aligned} z_{t+1|t-1} &= Az_{t+2|t-1} + C\bar{x} \\ &= A^2 z_{t+3|t-1} + (I + A)C\bar{x} \\ &= \dots \\ &= \lim_{T \rightarrow \infty} A^{T-1} z_{t+T|t-1} + \left( \sum_{i=0}^{\infty} A^i \right) C\bar{x} \end{aligned} \quad (4)$$

Now if the roots of A are less than one (so that  $A^{T-1} \rightarrow 0$  as  $T \rightarrow \infty$ ) and if there are no "speculative bubbles" (so that  $z_{t+\infty|t-1}$  is finite), then the first term vanishes and the second converges to  $(I - A)^{-1}Cx$ . This establishes that  $z_{t+1|t-1} = \bar{z}$ . To find  $z_t|_{t-1}$ , one has only to replace  $z_{t+1|t-1}$  with  $\bar{z}$  in (1) and take expected values.

In the two country model with interest rates fixed, (1) can be written as

$$\begin{bmatrix} e_t^a \\ p_t^+ \\ p_t^- \end{bmatrix} = \begin{bmatrix} (\gamma + \beta)/(\gamma + \beta + \rho + 1) & 0 & 0 \\ 0 & 2\Delta/(2\Delta + \rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{t+1|t-1} \\ p_{t+1|t-1}^+ \\ p_{t+1|t-1}^- \end{bmatrix} + B(z_t - z_t|_{t-1}) + u_t$$

where  $p_t^+ \equiv p_t + p_t^a$  and  $p_t^- \equiv p_t - p_t^a$ . The roots of the A matrix are less than one if (i)  $\rho > 0$ , (ii)  $\gamma < \infty$ ,  $\beta < \infty$ , and (iii)  $\rho > -1$ .

Suppose for example that wealth effects did not appear in the demand equations ( $\rho = 0$ ); then the root in the second column of the A matrix is equal to one. From equation (4), it is apparent that  $p_{t+1|t-1}^+ = p_{t+\infty|t-1}^+$  and unless one can make a case for a particular value of  $p_{t+\infty|t-1}^+$ ,  $p_{t+1|t-1}^+$  is not pinned down; the world price level is indeterminate. When the root is equal to one, ruling out "speculative bubbles" (that is, assuming  $p_{t+\infty|t-1}^+$  is finite) is not sufficient for uniqueness. This problem is the open economy counterpart of a problem that is fairly well known in the closed economy literature. See Sargent and Wallace (1977) and Canzoneri (1980).

Similarly, if the two countries' bonds are perfect substitutes ( $\gamma \rightarrow \infty$ ), exchange rates are indeterminant. Discussions of fixed interest rate regimes in models with perfect capital mobility require a stronger assumption than "no speculative bubbles". Either  $e_{t+\infty|t-1}$  must be specified or something like Taylor's (1977) minimum variance criterion must be adopted.

Finally, if intervention policy is too accomodative ( $\rho \leq -1$ ) uniqueness problems can arise. Fischer Black (1974) discussed this kind of problem in the framework of monetary growth models.

Appendix: Log-linearization.

The models used in the main body of the paper were written in terms of percentage deviations from a fixed equilibrium. This appendix furnishes some of the details of that log-linearization. Only the two country model is discussed here; moving to three countries is straight forward.

The home currency value of world private sector wealth is

$$A = M + B + E^a(M^a + B^a) \quad (1)$$

where capital letters represent the actual values of variables. For example, M is the home money supply and

$$m \equiv d \log M = (dM)/\bar{M}$$

is its percentage (or log) deviation from  $\bar{M}$ , the value of M in the equilibrium we are log-linearizing about. Taking the total differential of (1) and dividing by  $\bar{A}$ ,

$$\begin{aligned} a &\equiv dA/\bar{A} \\ &= (\bar{M}/\bar{A})(dM/\bar{M}) + (\bar{B}/\bar{A})(dB/\bar{B}) \\ &\quad + (\bar{E}^a \bar{M}^a / \bar{A})(dM^a / \bar{M}^a) + (\bar{E}^a \bar{B}^a / \bar{A})(dB^a / \bar{B}^a) \\ &\quad + [(\bar{E}^a \bar{M}^a + \bar{E}^a \bar{B}^a) / \bar{A}](dE^a / \bar{E}^a) \end{aligned} \quad (2)$$

Now it is assumed that in the equilibrium we are log-linearizing about

$$\bar{M} = \bar{M}^a = \bar{B} = \bar{B}^a = \bar{E}^a = \bar{P} = \bar{P}^a = 1 \quad (3)$$

So  $\bar{A} = 4$ , and (2) becomes

$$a = \frac{1}{4}(m + b + m^a + b^a) + \frac{1}{2}e^a \quad (4)$$

The home currency price index is

$$PI = P^{\frac{1}{2}}(E^a P^a)^{\frac{1}{2}} \text{ or } pi = \frac{1}{2}(p + e^a + p^a)$$

So the real value of private sector wealth is

$$\begin{aligned} a - pi &= (dA/\bar{A}) / (dPI/\bar{PI}) \\ &= \frac{1}{4}(m + b + m^a + b^a) - \frac{1}{2}(p + p^a) \end{aligned} \quad (5)$$

The exchange rate drops out of  $a - pi$  because foreign wealth is the same proportion of world wealth as foreign goods are of the world market basket in the equilibrium we are log-linearizing about.

The central bank stock constraint is

$$dM + dB + \bar{E}^a(dM^a + dB^a) = 0$$

So dividing by  $\bar{A}$ ,

$$\begin{aligned} (\bar{M}/\bar{A})(dM/\bar{M}) + (\bar{B}/\bar{A})(dB/\bar{B}) \\ + \bar{E}^a [ (\bar{M}^a/\bar{A})(dM^a/\bar{M}^a) + (\bar{B}^a/\bar{A})(dB^a/\bar{B}^a) ] = 0 \end{aligned}$$

and using (3)

$$(\frac{1}{4})(m + b + m^a + b^a) = 0$$

Multiplying by 4, we obtain the stock constraint used in the main text, and using this constraint in (5), we have

$$a - p_i = - \frac{1}{2}(p + p^a) \tag{6}$$

This explains the real wealth effect,  $\partial(p + p^a)$ , that appears in the commodity demand equations.