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Equilibrium Yield Curves and the Interest Rate Lower Bound*

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Abstract

We present a calibrated DSGE model with an occasionally binding effective lower bound (ELB) constraint on the short-term nominal rate that matches key features of the macroeconomy and the term structure of interest rates in the United States. We show that the ELB constraint induces state dependency in term premiums by affecting macroeconomic uncertainty and interest rate sensitivity to economic activity, typically lowering the absolute size of term premiums and generating distinct dynamics near the ELB. The central bank's forward guidance at the ELB lowers the expected short-rate path, but increases or decreases term premiums depending on whether demand or supply shocks are dominant.

JEL: E12, E32, E43, E44, E52, G12

Keywords: Term Structure of Interest Rates, Term Premiums, Effective Lower Bound, Forward Guidance, New Keynesian Model, Recursive Preferences

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1 Introduction

Since the onset of the 2007–2009 global financial crisis, many central banks have been forced to implement monetary policy under the effective lower bound (ELB) constraint on the short-term nominal interest rate. In particular, central banks have implemented various "unconventional" policies such as those aimed at providing more transparency in the likely path of the policy rate (so-called forward guidance policies) and purchases of long-term government bonds, to stimulate the economy through their effects on the term structure of interest rates. Accordingly, if we are to explain the dynamics of interest rates and evaluate the efficacy of these unconventional policies, we should have a coherent understanding of the interaction between the macroeconomy, the term structure of interest rates, and monetary policy at the ELB. While many studies have contributed to this effort, the effect of the ELB on term premiums—a crucial component of the term structure—appears to be particularly understudied within an explicit macroeconomic framework.[1]

In this paper, we study how the ELB constraint jointly affects the macroeconomy, the term structure of interest rates, and monetary policy in a structural general equilibrium (DSGE) model. Our model is a variant of a standard New Keynesian model featuring nominal price rigidities in the product market and an interest rate feedback rule. Explicit microfoundations set our model apart from the existing term structure models with the ELB—which are mostly statistical models with limited economic structure. We offer two contributions. First, we clarify how the ELB constraint can affect the dynamics of term premiums using a stylized model that can be solved in closed form. Second, we analyze a fully nonlinear model which augments the stylized model with additional features to match key properties of the macroeconomy and the term structure of interest rates in the United States.

Our main finding from the stylized model is that the ELB constraint generates state

[1]See the literature review for references. Policymakers have also reiterated the importance of term premiums while the economy was at the ELB. For example, former Fed Governor Jeremy Stein stated in a speech, "When policy works by moving term premiums, as opposed to moving expectations about the path of short rates, the transmission to the real economy may be altered in subtle yet important ways that can have implications for the benefits of a policy action, its costs, and even its consequences for financial stability." (Stein (2012)). Also, former Fed Chairman Ben Bernanke pointed to the decline in term premiums as an important factor behind the decline in longer-term U.S. interest rates (Bernanke (2015)).

dependency in term premiums through two forces affecting term premiums—macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations. On the one hand, macroeconomic uncertainty is higher when the policy rate is constrained by the ELB than when it is not, as the ELB constraint prevents the central bank from counteracting the effects of exogenous shocks to demand on output and inflation. This increased macroeconomic uncertainty at and near the ELB *amplifies* the absolute size of term premiums. On the other hand, the sensitivity of interest rates to macroeconomic fluctuations can be smaller when the policy rate is at or near the ELB than when it is not, as the central bank faces restrictions in adjusting its policy rate in the near term. This reduced sensitivity of interest rates at and near the ELB can *compress* the absolute size of term premiums. While the compression effect of the ELB may be widely understood, the amplification effect, which arises from the endogenous response of the macroeconomy to the ELB constraint, can only be uncovered through an equilibrium macro-finance model such as ours.

We further show that the two forces can be conflicting, and which of these dominates depends on the state of the economy and yield maturity. When the economy is in a deep recession and the policy rate is expected to be at the ELB for a long time, the second compression effect typically dominates the first amplification effect, and the size of the term premium is smaller than when the policy rate is comfortably above the ELB. When the economy is in a milder recession and the policy rate is expected to be at the ELB for a shorter time, the first effect is likely to dominate the second and the size of term premiums is larger than when the policy rate is comfortably above the ELB. The compressing effect on term premiums induced by the reduced sensitivity of interest rates is stronger for shorter-maturity yields that are more strongly affected by the presence of the ELB than for longer-maturity yields. Overall, our analysis shows that the ELB constraint can add a unique source of variation in term premiums, which suggests that term premiums may warrant more attention in a severe recession involving the ELB than under normal circumstances.

We next extend the stylized model in several directions to improve its ability to quantitatively match key features of output, inflation, and the term structure of interest rates in the U.S. First, we introduce recursive utility. Second, we add policy inertia in the monetary policy rule. Third, we add productivity shocks with stochastic volatility, which makes the conditional moments of our model while the policy rate is above the ELB consistent with those in the data. In particular, yield curves are on average upward sloping, and term premiums are positive, large, and volatile while the policy rate is above the ELB in our model. In addition, our yield curve is steeper when the policy rate is constrained at the ELB than otherwise, as in the data. Fourth, we include stochastic volatility in the demand shock process. By allowing the volatility of demand shocks to increase as demand falls, we generate term premiums that are on average negative at the ELB. This is because the volatility of demand shocks is heightened around the ELB where demand is particularly depressed, magnifying negative term premiums that arise from a given level of demand shock volatility. Negative term premiums are consistent with survey-based estimates of term premiums—as well as term premium estimates based on some available term structure models—largely from the period in which the federal funds rate was at the ELB. Based on a moment-matching exercise for the first ELB episode, we find that the compression effect of the ELB on term premiums was strong.

Our quantitative model is used to interpret the term structure dynamics during the first ELB episode in the U.S. following the financial crisis. In particular, we show how shorter-maturity term premiums were compressed as the expected ELB duration spiked upon the introduction of forward guidance, and later became negative as the liftoff approached. We also provide one of the first analyses on how to interpret term structure dynamics during the Covid-19 crisis—the latest period when the ELB was binding—using a structural macro-finance model. There, we find that in order to explain the increase in term premiums, it is important to consider not only negative demand and supply shocks, but also the possibility of a notable increase in the volatility of the supply shocks.

A key benefit of structural models is that they allow counterfactual policy experiments that are robust to the Lucas critique. In our final exercise, we use the quantitative model to study how forward guidance—the central bank's announcement to keep the policy rate at the

ELB for longer than previously expected—affects the macroeconomy and the term structure of interest rates. We find that this forward guidance not only reduces the expected short-rate path but also the absolute size of term premiums as a result of the strengthened compression effect. In our benchmark specification in which average term premiums are negative at the ELB, this result means that forward guidance increases term premiums, partially offsetting the declines in the expected short-rate path. However, this result also implies that if average term premiums are positive even at the ELB, forward guidance reduces term premiums, amplifying the decline in the expected short-rate path.

One caveat in our analysis is that our model does not have an explicit role for the central bank balance sheet, an ingredient that is likely to be important in understanding the effects of unconventional policy such as quantitative easing (QE). Introducing such an ingredient into our model can be an important extension. However, we believe that, because unconventional policy was typically undertaken while the policy rate was constrained at the ELB, clarifying how the ELB constraint affects the term structure of interest rates is an essential first step toward a comprehensive understanding of unconventional monetary policy. Moreover, if balance sheet policies work mainly by signaling to the private sector about the central bank's commitment to an accommodative policy stance and thus affecting the expected path of the policy rate, as suggested by studies such as Woodford (2012), the analysis of forward guidance provided in our paper may capture part of the effects of balance sheet policies.

After staying at the ELB for a few years since the pandemic crisis, interest rates have recently risen notably above the ELB in the U.S. However, measures of the natural rate of interest (or, the "r-star") remain historically low possibly due to various structural factors, suggesting that the long-run trend of low interest rates may not change significantly. Thus, the ELB is likely to remain a relevant constraint for future monetary policy.

Literature Review: Our contribution is to present the first structural analysis of the yield curve and term premiums using a DSGE model with an occasionally binding ELB constraint that is calibrated to match key features of U.S. data. Naturally, our work is related to several strands of the literature. First, our paper is closely related to papers analyzing the implications of the ELB on the macroeconomy in fully nonlinear New Keynesian models with an occasionally binding ELB constraint. Examples are Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez (2015), Gavin, Keen, Richter, and Throckmorton (2015), Gust, Herbst, López-Salido, and Smith (2017), and Nakata (2017), among many others. The key difference between these papers and ours is our focus on the term structure of interest rates. Reflecting this difference, our model includes some features absent in these papers, such as recursive preferences and stochastic volatilities in exogenous shocks.

Second, our work is related to the literature on term structure models with the ELB. Examples are Ichiue and Ueno (2007), Kim and Singleton (2012), Krippner (2012), Bauer and Rudebusch (2016), Christensen and Rudebusch (2016), Wu and Xia (2016), and Kim and Priebsch (2024). As noted above, the existing models are largely reduced form in the sense that they impose very limited economic structure, such as lack of arbitrage opportunities.[2] In contrast, our general equilibrium model features explicit microfoundations and includes a description of monetary policy, allowing us to give economic interpretations to the term structure of interest rates.

Third, our paper extends the literature on equilibrium term structure models to incorporate the ELB constraint. In particular, our paper is closely related to papers analyzing the term structure of interest rates in a production economy. Examples are Rudebusch and Swanson (2008, 2012), Andreasen (2012a,b), Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), Dew-Becker (2014), Kung (2015), Lopez, Lopez-Salido, and Vazquez-Grande (2015), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018), Swanson (2019), Bretscher, Hsu, and Tamoni (2020), Campbell, Pflueger, and Viceira (2020), Hsu, Li, and Palomino (2021), Nguyen (2022), Pflueger and Rinaldi (2022), and Pflueger (2025).[3] Our main departure from these models is to incorporate an occasionally binding ELB constraint. Even though these papers have made substantial progress in fitting macro and yield

[2]These papers, in turn, build on the vast literature on term structure models without the ELB constraint, which we do not review here. See, for example, Gürkaynak and Wright (2012) and references therein for more information.

[3]These papers in turn build on earlier work by Piazzesi and Schneider (2007) and many others that analyze equilibrium yield curves in endowment economies.

data jointly, they usually find it difficult to generate endogenous volatility in term premiums. In our model, endogenous volatility in term premiums can arise naturally due to the ELB constraint.

Fourth, our paper belongs to the nascent literature on equilibrium asset pricing models with the ELB constraint, such as Datta, Johannsen, Kwon, and Vigfusson (2021). Within this literature, papers on equilibrium term structure models with the ELB are particularly scarce. Branger, Schlag, Shaliastovich, and Song (2015) and Katagiri (2022) introduce the ELB constraint into an endowment economy model. In contrast, we study a production economy with a realistic description for monetary policy. Sakurai (2016) develops a New Keynesian bond pricing model with the ELB constraint, but his pricing kernel is reduced form and the discussion on term premiums is brief. The pricing kernel in our model builds explicitly on household behavior. Most closely related to our paper are contemporaneous studies by Bilal (2017) and Gourio and Ngo (2020) that examine asset-pricing implications of a DSGE model with the ELB constraint.[4] While they focus on the correlation between stock and bond returns, we focus on the term structure of interest rates and analyze term premiums in more detail. Specifically, we characterize the state dependency of term premiums in closed form, and our quantitative model, which features time-varying volatility, produces term premiums that are significantly negative at the ELB, while remaining positive above the ELB.

Finally, since the periods of QE policy implementation largely coincide with the ELB episodes, our work complements studies that analyze QE policy and its impact on the term structure. Examples include Chen, Cúrdia, and Ferrero (2012), Gertler and Karadi (2011), King (2019), and Sims and Wu (2021). In contrast to these papers, our model does not incorporate QE. However, we use a fully nonlinear DSGE model with an occasionally binding ELB constraint to study the implications of the ELB on the term structure in greater depth.

The paper is structured as follows. Section 2 presents the stylized model. We then extend it to a quantitative model to match certain features of U.S. data in Section 3. We use

[4]Our first publicly available working paper was Nakata and Tanaka (2016).

this model to interpret term structure dynamics during the two ELB episodes in Section 4. Section 5 conducts monetary policy experiments at the ELB based on the model. Section 6 concludes.

2 Equilibrium Yield Curves in the Stylized Model

We start out by analyzing the dynamics of equilibrium yields and term premiums in a term structure model that builds on a stylized New Keynesian core subject to the ELB constraint. A key feature of the model is that the variables follow a 3-state Markov chain, which makes it very tractable. In fact, our model is simple enough to be solved in closed form despite the nonlinearity arising from the occasionally binding ELB constraint. The goal of this section is to describe how the ELB affects macroeconomic variables and the yield curve in a transparent way. We build a more elaborate quantitative model in Section 3, where we further explore how the mechanisms highlighted in this section can help us better understand the data.

2.1 Model Specification

The Macro Block: As in the canonical New Keynesian model, we assume the economy consists of a representative household, monopolistically competitive firms that face nominal price rigidities, and a central bank which sets monetary policy based on a Taylor-type rule. The agents' decisions constitute the standard "macro block", which is a log-linear system except for the nonlinearity due to the ELB constraint.[5] Specifically, the macro block consists of the following three equations:

$$\hat{y}_t = \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \frac{1}{\gamma}\left(r_t^{(1)} - \mathbb{E}_t[\hat{\pi}_{t+1}] - \bar{r}\right) + \delta_t \tag{1}$$

$$\hat{\pi}_t = \kappa \hat{y}_t \tag{2}$$

$$r_t^{(1)} = \max[0, \bar{r} + \phi\hat{\pi}_t], \tag{3}$$

[5]Similar semi-log-linear models with an occasionally binding ELB constraint have been studied in detail by Nakata and Schmidt (2019), among others.

where y_t is (log) output, π_t is the inflation rate, and $r_t^{(1)}$ is the 1-period nominal interest rate (the policy rate or the short rate). $\bar{x}$ denotes the deterministic steady state of x, and $\hat{x} \equiv x - \bar{x}$ indicates the deviation of x from its steady state. Equation (1) is the log-linearized Euler equation and δ_t is an exogenous shock to be specified below. The exact origin of δ_t is not important for the discussion in this section, and we interpret δ_t as a generic "demand" shock that moves output and inflation in the same direction and occasionally pushes the economy to the ELB. However, we revisit the microfoundation of (1) in more detail when we formulate the quantitative model in Section 3. Equation (2) is a log-linearized static Phillips curve. We choose this specification to obtain tractable closed-form solutions, following studies such as Bilbiie (2019).[6] Equation (3) is the monetary policy rule (with $\phi > 1$) subject to the ELB constraint. The ELB is set to zero for simplicity (zero lower bound).

The Asset Pricing Block: We assume there exists another representative agent called the "investor", who receives funds from the household and invests in default-free nominal bonds of arbitrary maturity that are in zero net supply. The investor takes macroeconomic dynamics determined by the macro block as given, and prices bonds based on the following recursive equation:

$$Q_t^{(n)} = \mathbb{E}_t[M_{t+1} Q_{t+1}^{(n-1)}], \tag{4}$$

where $Q_t^{(n)}$ $(n \geq 1)$ is the price of an n-period zero-coupon nominal bond that pays \$1 at maturity, and $Q_t^{(0)} = 1$. M_t is the nominal pricing kernel. We assume $m_{t+1} \equiv \ln M_{t+1}$ has the following form consistent with standard power utility:

$$m_{t+1} = -\bar{r} - \gamma\delta_t - \gamma(\hat{y}_{t+1} - \hat{y}_t) - \hat{\pi}_{t+1}. \tag{5}$$

Note (5) and (4) reduce to (1) when linearized for $n = 1$. Thus, we can interpret the pricing equation as a result of the investor's optimal strategy which coincides with the household's consumption-saving decision, except that the investor takes risk premiums into account

[6]The static Phillips curve can be derived from the dynamic optimization of Rotemberg pricing firms by setting $P_{t-1}(i) = P_{t-1}$ in the objective function (26).

whereas the household does not. A similar setup is proposed by Bianchi, Lettau, and Ludvigson (2022), though they do not impose the ELB constraint.[7] Such a separation of households and investors could be justified if the latter can make more sophisticated investment decisions. Alternatively, we can view the framework as a tractable approximation of the fully nonlinear model with a single representative agent, which allows us to highlight its key implications in closed form by abstracting from nonlinearities that are not crucial for our purposes. We do not make this separation in the quantitative model in Section 3; the household's and investor's Euler equations for bonds (and hence their attitudes towards risk) are consistent with each other.

The continuously compounded yield to maturity of the n-period bond is $r_t^{(n)} = -\frac{1}{n}\ln Q_t^{(n)}$. The term premium of this bond is defined in the standard way as the difference between the yield and its "risk-neutral" counterpart $r_t^{(n)\mathbb{Q}}$:

$$tp_t^{(n)} \equiv r_t^{(n)} - r_t^{(n)\mathbb{Q}}, \tag{6}$$

where $r_t^{(n)\mathbb{Q}} = -\frac{1}{n}\ln Q_t^{(n)\mathbb{Q}}$. $Q_t^{(n)\mathbb{Q}}$ is the risk-neutral price of an n-period zero-coupon nominal bond that is computed analogously to (4) by replacing M_{t+1} with the 1-period rate; $Q_t^{(n)\mathbb{Q}} = \exp(-r_t^{(1)})\mathbb{E}_t[Q_{t+1}^{(n-1)\mathbb{Q}}]$, where $Q_t^{(0)\mathbb{Q}} = 1$.[8]

While we can solve for yields and term premiums of arbitrary maturity, we focus on the 2-period maturity ($n = 2$) in the rest of this section. We further approximate (4) to obtain a simple expression for the 2-period yield:

$$r_t^{(2)} = -\frac{1}{2}\left[\mathbb{E}_t[m_{t+1} - r_{t+1}^{(1)}] + \frac{1}{2}\mathbb{V}_t[m_{t+1} - r_{t+1}^{(1)}]\right]. \tag{7}$$

Then, the 2-period term premium follows from (6):

$$tp_t^{(2)} = \frac{1}{2}\mathbb{C}\text{ov}_t[m_{t+1}, r_{t+1}^{(1)}], \tag{8}$$

[7] Instead, they impose heterogeneous beliefs and use it mainly for pricing equities.

[8] In other words, it is the present value of \$1 discounted by the expected path of the 1-period rate.

where $\mathbb{V}_t[\cdot]$ and $\mathbb{C}\text{ov}_t[\cdot]$ denote the conditional variance and covariance, respectively.[9]

State Space and Equilibrium: Finally, we specify δ_t as a 3-state Markov chain with the state space $\{\delta_H, \delta_M, \delta_L\} = \{\delta, 0, -\delta\}$, an initial distribution, and transition matrix:

$$\begin{bmatrix} p_{HH} & p_{HM} & p_{HL} \\ p_{MH} & p_{MM} & p_{ML} \\ p_{LH} & p_{LM} & p_{LL} \end{bmatrix} = \begin{bmatrix} p & 1-p & 0 \\ \frac{p(1-p)}{2} & 1-p(1-p) & \frac{p(1-p)}{2} \\ 0 & 1-p & p \end{bmatrix},$$

where $p_{ij} \equiv \Pr(\delta_{t+1} = \delta_j | \delta_t = \delta_i)$. This specification resembles a discretized AR(1) process in that δ_t is stationary, has an unconditional mean of zero with a first-order autocorrelation of p, and is conditionally homoskedastic.[10] We exclude a jump from H to L or vice versa in one step to obtain tractable closed-form solutions. While a 2-state process is more common in equilibrium models with the ELB (e.g., Eggertsson and Woodford (2003), Bilbiie (2019), and Nakata and Schmidt (2019)), we need at least three states to demonstrate state dependency of term premiums. Given the specification of δ_t, the equilibrium is characterized by the vector $\{y_s, \pi_s, r_s^{(1)}, r_s^{(2)}, tp_s^{(2)}\}_{s\in\{H,M,L\}}$ that solves the system of equations (1), (2), (3), (5), (7), and (8).

To capture the key implications of a more realistic setup with continuous states, we further impose the following assumptions on the state space:

$$\bar{r} + \phi\hat{\pi}_H > 0, \quad \bar{r} + \phi\hat{\pi}_L < 0, \quad \bar{r} + \phi\hat{\pi}_M = 0. \tag{9}$$

The first two assumptions are fairly standard in 2-state models with the ELB, and simply say that the state space covers the two states of interest; state H, in which the economy is strictly above the ELB, and state L, in which the ELB is strictly binding. The last assumption means

[9]To derive the expressions for the yield and the term premium, we use the approximation $\ln \mathbb{E}[\exp(X)] \approx \mathbb{E}[X] + \frac{1}{2}\mathbb{V}[X]$ for an arbitrary random variable X. Wu and Xia (2016) also use this approximation to build a term structure model with an ELB constraint. See Appendix A.1 for more details on the derivation.

[10]We want the exogenous process to be homoskedastic in order to isolate the role of endogenous heteroskedasticity arising from the ELB constraint in bond pricing. We relax this condition in the quantitative model, which improves the fit to data.

that the size of the decline in the policy rate in state M is such that the policy rate is exactly at the ELB in M.[11] The main results hold without this assumption, but the specific choice of the intermediate state simplifies the problem considerably.

2.2 Results

We summarize the main results in a few propositions based on closed-form solutions, but it is still useful to first characterize the equilibrium visually. For this purpose, we assign standard parameter values as shown in Table 1:

Table 1: **Parameter Values for the Stylized Model**

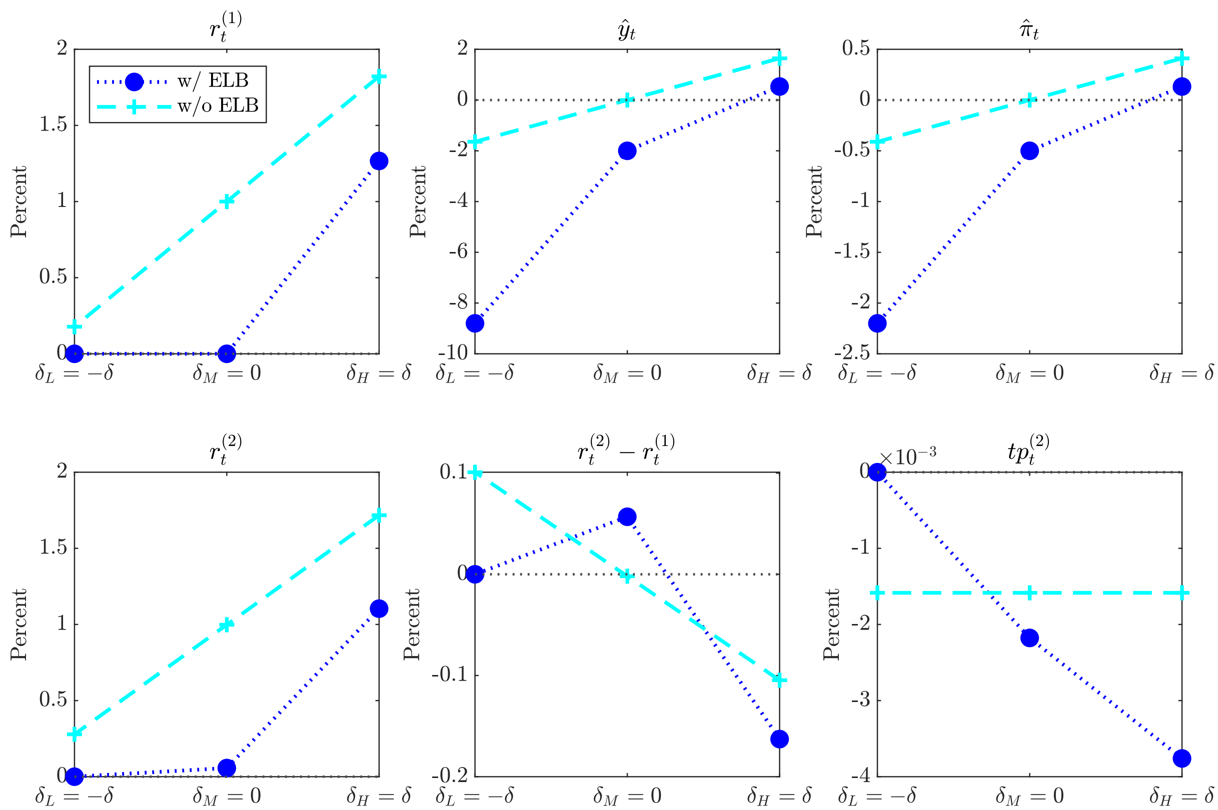
Parameter	Value	Description
γ	1	1/Elasticity of intertemporal substitution, Risk aversion
κ	0.25	Phillips curve slope
$\bar{r}$	0.01	Steady state policy rate
ϕ	2	Coefficient on inflation in the Taylor rule
p	0.75	AR(1) coefficient of the demand shock

The top panels of Figure 1 plot the equilibrium policy rate, output and inflation (deviation from their steady states for the latter two) as functions of the demand shock δ_t. In addition to the functions for the model with the ELB (blue lines with circle markers), we also show the corresponding functions for the model without the ELB (light blue lines with plus markers) for comparison.

The policy rate, output and inflation all decrease as δ_t decreases. Weaker demand for the final good by the households leads to lower marginal costs and inflation. According to our interest rate feedback rule, the policy rate declines in response to lower inflation (we set the coefficient on output to zero), partially offsetting the contractionary effects of the negative demand shock. The levels of all three variables from the model with the ELB are lower than those from the model without the ELB. This is because, even at state H, in which the policy rate is above the ELB, the anticipation of becoming constrained by the ELB in the future reduces today's inflation and policy rate by lowering inflation expectations (see the discussion

[11] This assumption amounts to a restriction on δ; it is satisfied by setting $\delta = \tilde{\delta}$ where $\tilde{\delta}$ is a function of the other parameters such that the equality $\bar{r} + \phi\hat{\pi}_M = 0$ holds.

Figure 1: **Equilibrium Functions of the 3-State Model**



* Blue lines with circle markers indicate variables of the model with the ELB constraint, and light blue lines with plus markers indicate variables of the model without the ELB constraint.

of Nakata and Schmidt (2019), among others). For a sufficiently small realization of δ_t (δ_M and δ_L), the policy rate hits the ELB. When the policy rate is constrained at the ELB, the contractionary effects of a decrease in δ_t cannot be offset by a decline in the policy rate. As a result, a further decrease in demand leads to larger falls in inflation and output when the policy rate is at the ELB than when it is not.

As is evident from the panels, the equilibrium variables are linear with respect to δ_t when the model does not include the ELB. This implies that the variables are homoskedastic. In contrast, the variables are nonlinear with respect to δ_t when the model incorporates the ELB, which implies heteroskedasticity. We explicitly characterize these features of output, inflation and the policy rate in the proposition below:

Proposition 1 (**Heteroskedasticity of output, inflation, and the policy rate**). *The equilibrium output* y_t*, inflation* π_t*, and policy rate* $r_t^{(1)}$ *are:*

1. *Conditionally homoskedastic without the ELB constraint:*

$$\mathbb{V}_{U,H}[x] = \mathbb{V}_{U,M}[x] = \mathbb{V}_{U,L}[x] \equiv \mathbb{V}_U[x] \qquad \textit{for} \quad x \in \{y, \pi, r^{(1)}\} \tag{10}$$

2. *Conditionally heteroskedastic with the ELB constraint:*

$$\mathbb{V}_{E,H}[x] < \mathbb{V}_{E,M}[x] < \mathbb{V}_{E,L}[x] \qquad \textit{for} \quad x \in \{y, \pi\} \tag{11}$$

$$\mathbb{V}_{E,H}[r^{(1)}] > \mathbb{V}_{E,M}[r^{(1)}] > \mathbb{V}_{E,L}[r^{(1)}] = 0 \tag{12}$$

3. *The conditional volatility of* y_t*,* π_t*, and* $r_t^{(1)}$ *at the high state is larger for the model with the ELB, compared to the model without the ELB:*

$$\mathbb{V}_U[x] < \mathbb{V}_{E,H}[x] \qquad \textit{for} \quad x \in \{y, \pi, r^{(1)}\}, \tag{13}$$

where $\mathbb{V}_{m,s}[x]$ *is the 1-period-ahead variance of* x *conditional on being at state* $s \in \{H, M, L\}$ *for model* $m \in \{U\textit{(unconstrained model without the ELB)}, E\textit{(model with the ELB)}\}$*.*

Proof. See Appendix A.2. □

In particular, Proposition 1.2 highlights how the conditional volatilities of output and inflation *increase* as the economy approaches the ELB, while the volatility of the policy rate *decreases*. The volatility of the policy rate decreases naturally due to the ELB constraint. However, precisely because the policy rate cannot be adjusted to mitigate the effects of shocks, the volatilities of output and inflation increase. This contrasting behavior of volatilities induced by the ELB constraint is key to understanding how the ELB affects term structure dynamics, and in particular, understanding how term premiums can become time-varying, as we show next.[12]

[12]Endogenous volatility due to the ELB constraint has also been studied by Basu and Bundick (2016)

The bottom left and middle panels of Figure 1 show the equilibrium 2-period nominal yield and its spread with the policy rate. A couple features are worth highlighting when the model is not subject to the ELB constraint. First, the 2-period yield increases with δ_t similar to the policy rate. Second, the yield spread decreases with δ_t, i.e., the yield spread is *countercyclical*. These two features also imply that the 2-period yield is less sensitive to changes in δ_t, reflecting the natural consequence of stationarity that is built into the model, i.e., the term structure of yield volatility is downward sloping. These are all well-documented features of U.S. Treasury yield data.[13]

When the model is subject to the ELB constraint, the 2-period yield does not hit the ELB at δ_M, contrary to the policy rate; the ELB leads to a positive yield spread at δ_M. This is because the yield accounts for the possibility of moving to state H in the future. Interestingly, the yield spread is no longer linear in δ_t; it is *increasing* in δ_t for $\delta_t \in \{\delta_L, \delta_M\}$, showing *cyclicality* in part of the state space when the ELB is binding. This also implies that yield volatility is no longer necessarily downward sloping but potentially upward sloping, especially near or at the ELB.

The bottom right panel plots the 2-period nominal term premiums. Here, we highlight two features. First, term premiums are (weakly) negative regardless of the level of δ_t for both models with and without the ELB. Second, the ELB introduces a unique state dependency (or time variation) in the term premium, which is constant in the model without the ELB constraint. Specifically, at δ_H, where the economy is away from the ELB but still close enough to bind in the near future, the term premium is more negative (its absolute value is larger) compared to what it would be without the ELB. In other words, the term premium is *amplified* by the ELB. However, at δ_L, where the economy is at the ELB and is expected to remain there in the next period, the term premium is zero, and hence smaller in absolute value compared to what it would be without the ELB. In other words, the term premium is *compressed* by the ELB. Note that the ELB affects term premiums even when the policy

and Plante, Richter, and Throckmorton (2018). However, their results are numerical and focus on macro dynamics, whereas we offer an analytical characterization and uncover the link to the term structure.

[13]For a recent survey of the properties of yield spreads, see, for example, Haubrich (2021). For an empirical analysis of yield volatilities, see, for example, Cieslak and Povala (2016).

rate is away from the ELB because it binds stochastically in our model. We summarize these results in the following two propositions. The first proposition derives explicit expressions of the term premiums:

Proposition 2 (Term premiums). *The 2-period term premium is:*

1. *For the model without the ELB:*

$$tp^{(2)}_{U,H} = tp^{(2)}_{U,M} = tp^{(2)}_{U,L} \equiv tp^{(2)}_{U} = -\frac{1}{2}(\gamma+\kappa)\kappa\phi\mathbb{V}_U[y]. \tag{14}$$

2. *For the model with the ELB:*

$$tp^{(2)}_{E,H} = -\frac{1}{2}(\gamma+\kappa)\kappa\phi\mathbb{V}_{E,H}[y] \tag{15}$$

$$tp^{(2)}_{E,M} = -\frac{1}{2}(\gamma+\kappa)\kappa\phi\left(\mathbb{V}_{E,M}[y]-\xi\right) \tag{16}$$

$$tp^{(2)}_{E,L} = 0, \tag{17}$$

where $tp^{(2)}_{m,s}$ is the 2-period term premium at state $s \in \{H, M, L\}$ for model $m \in \{U, E\}$, $\xi \equiv \frac{1}{2}\left[(1-\tilde{p})\mathbb{V}_{E,L}[y] + \tilde{p}\sqrt{\mathbb{V}_{E,H}[y]}\sqrt{\mathbb{V}_{E,L}[y]}\right] > 0$, and $\mathbb{V}_{E,M}[y] > \xi$.

Proof. See Appendix A.3. □

This proposition shows that term premiums are indeed negative, except at δ_L, where they are zero in the model with the ELB. Also, it is evident that the time variation of the term premium comes from the heteroskedasticity of the macro variables, which we showed was induced by the ELB. The next proposition characterizes the amplification and compression effects of the ELB on term premiums:

Proposition 3 (Amplification/compression of the term premium). *For the high and low states in the model with the ELB:*

$$|tp^{(2)}_{E,H}| > |tp^{(2)}_{U}| \quad (Amplification) \tag{18}$$

$$|tp^{(2)}_{E,L}| < |tp^{(2)}_{U}| \quad (Compression) \tag{19}$$

For the medium state:

$$|tp^{(2)}_{E,M}| \lesseqgtr |tp^{(2)}_{U}| \quad \textit{iff} \quad \Delta^{y}_{E,ML} \lesseqgtr \tilde{\Delta}, \tag{20}$$

where $\Delta^{y}_{E,ML} \equiv y_{E,M} - y_{E,L}$*, and* $\tilde{\Delta}$ *is a function of parameters.*

Proof. See Appendix A.4. □

According to this proposition, the ELB unambiguously amplifies the term premium in state H and compresses the term premium in state L. Whether the ELB amplifies or compresses the term premium in state M depends on the difference in output between states M and L.

To better understand the force behind the dynamics of term premiums just described, it is useful to decompose the 2-period term premium into its macroeconomic determinants. From (8):

$$\begin{aligned} tp^{(2)}_t &= \frac{1}{2}\mathbb{C}\text{ov}_t[m_{t+1}, r^{(1)}_{t+1}] \\ &\propto -\mathbb{C}\text{ov}_t(\gamma\Delta y_{t+1}, r^{(1)}_{t+1}) - \mathbb{C}\text{ov}_t(\pi_{t+1}, r^{(1)}_{t+1}) \\ &= -\sum_{x\in\{\Delta y,\pi\}} \gamma_x \times \underbrace{\sigma_t[x]}_{\substack{\text{macro}\\ \text{uncertainty}}} \times \underbrace{\sigma_t[r^{(1)}] \times \rho_t[x, r^{(1)}]}_{\substack{\text{interest rate}\\ \text{sensitivity}}}, \end{aligned} \tag{21}$$

where $\sigma_t[x]$ and $\rho_t[x, y]$ denote the 1-period-ahead conditional standard deviation of variable x and conditional correlation of variables x and y, respectively. Also, $\gamma_{\Delta y} = \gamma$ and $\gamma_\pi = 1$. Since inflation is proportional to output in our model with a static Phillips curve, the decomposition further simplifies to:

$$tp^{(2)}_t \propto -(\gamma + \kappa)\sigma_t[y]\sigma_t[r^{(1)}]\rho_t[y, r^{(1)}]. \tag{22}$$

Thus, term premiums fluctuate based on two time-varying components—*macroeconomic uncertainty* and the *sensitivity of interest rates to macroeconomic fluctuations*. The latter can further be decomposed into interest rate volatility and the correlation between interest rates and a macro variable (output growth or inflation).[14]

According to this decomposition, the negative term premium shown in Proposition 2 follows from the fact that the correlations between the policy rate and inflation/output growth are both positive for a given δ_t.[15] Intuitively, the correlations reflect the risk exposure of bonds. Suppose we expect a negative shock to δ_t next period. A negative δ_t shock implies a decrease in output growth and inflation next period. According to the monetary policy rule, the expected decline in output and inflation means that the policy rate is expected to fall, or, equivalently, the price of the 2-period bond next period—which becomes the price of a 1-period bond—is expected to rise. Thus, the 2-period bond price next period is expected to rise when output is expected to fall, implying that the 2-period bond works as a hedge and the term premium must be negative.

We can also understand the state dependency of term premiums through this decomposition by analyzing how macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations vary with δ_t.

First, consider state H. In this state, the policy rate is above the ELB, but output volatility is higher compared to the model without the ELB due to the possibility of the ELB binding in the future (Proposition 1.3).[16] The same proposition also implies that policy-rate volatility is also higher with the ELB, since the policy rate is still sufficiently away from the ELB that it can aggressively counter a recession that leads to the ELB. Relatedly, output and the policy rate are perfectly correlated, i.e., $\rho_t[y, r^{(1)}] = \rho_{m,H}[y, r^{(1)}] = 1$ for both models $m \in \{U, E\}$. Thus, *both* macro uncertainty and interest rate sensitivity are higher when the ELB is present. This leads to an unambiguous increase in the term premium (in absolute terms).

Next, consider state M. On the one hand, output volatility in M is even higher than in

[14] Although we think this decomposition is intuitive, alternatives are conceivable. For instance, macro uncertainty can be defined as the *variance* of macro variables and the interest rate sensitivity as the coefficient of the regression of macro variables on interest rates (or the "beta"). Another way would be to decompose the term premium into "risk price" and "risk exposure of bonds." This decomposition may be suitable if, for example, one assumes an important role of time-varying risk aversion.

[15] Note that the conditional second moments of output and its growth rate 1 period ahead coincide.

[16] Although we highlight output volatility in the next few paragraphs based on the simplified decomposition (22), it equally applies to inflation. Hence we can safely interpret this as "macro" volatility.

state H in the model with the ELB, and is thus higher than that in M in the model without the ELB, as monetary policy is at the threshold of the ELB with less room to mitigate the effects of shocks (Proposition 1.2 and 1.3). On the other hand, interest rate volatility in M is lower than in state H (Proposition 1.2) in the model with the ELB, precisely because it is more likely that the policy rate will remain at the ELB in the future. We can also show that interest rate sensitivity can decline accordingly, and more than offset the increase in macro uncertainty, leading to the ambiguous effect of the ELB on the term premium in this state.[17]

Finally, consider state L. Output volatility is the highest in this state in the model with the ELB (Proposition 1.2). However, since the policy rate is at the ELB and expected to stay there in the next period, interest rate sensitivity is zero. Hence, the term premium is zero, which is smaller (in absolute terms) compared to the term premium in state L in the model without the ELB.

To better understand the robustness of our results, we conduct a few additional exercises. First, in Appendix B, we analyze a version of the model with government spending shocks instead of the δ_t shock, which is an interesting case where consumption dynamics decouple from output dynamics. We show that even in this model, the ELB constraint can amplify and compress term premiums. Second, in Appendix C, we confirm that the results carry over to a version of the stylized model in which the states are continuous and the macro block is nonlinear with a forward-looking Phillips curve. We also analyze the effects of the ELB constraint across yield maturities in more detail, where we show the relative magnitudes of the amplification and compression effects vary across maturities.

In sum, whether term premiums increase or decrease due to the ELB depends on two forces—macro uncertainty and interest rate sensitivity—which can conflict with each other as the economy falls deeper into the ELB state. In the model, the ELB increases both macro uncertainty and interest rate sensitivity at δ_H. For this case, the ELB leads to a more negative term premium (amplification). By contrast, when the demand shock is sufficiently negative

[17]While the bottom right panel suggests a monotonic relation between δ_t and the term premium, this in fact depends on parameter values. It can be shown that under a sufficiently large $\Delta^y_{E,ML}$, the term premium in state M can be *more negative* than in state H. This point is easier to understand when the state space is continuous; see Appendix C.

at δ_L, the increase in macro uncertainty is more than offset by the decrease in interest rate sensitivity, and the size of the term premium decreases and approaches zero (compression).[18]

3 Equilibrium Yield Curves in the Quantitative Model

We now turn to the analysis of a quantitative model that incorporates additional features into the stylized model so as to better capture key features of U.S. data. First, we formulate the model and calibrate its parameters to match some key features of output, inflation, yield data and term premium estimates at and above the ELB (this section). The exercise offers a coherent interpretation of macro and term structure dynamics of the past two decades including the recent U.S. ELB episodes through the lens of our model (Section 4). Second, we examine how alternative monetary policy strategies affect the dynamics of yield curves and their decomposition into the expected short-rate path and term premiums at the ELB (Section 5).

3.1 Model Specification

The quantitative model extends the stylized model to a standard New Keynesian model with continuous state variables, a forward-looking Phillips curve and a more elaborate monetary policy rule with policy inertia. Importantly, the quantitative model introduces Epstein-Zin preferences and a richer shock structure—a shock to the preference for holding default-free bonds and a shock to productivity, both exhibiting countercyclical volatility. While these additional features help the model to jointly fit the macroeconomic and term structure data significantly better, we still keep the model relatively parsimonious such that its mechanism remains transparent.

[18] Note this clear tension between macro uncertainty and interest rate sensitivity depends somewhat on our assumption that the model contains only a demand shock. Output and inflation near and at the ELB can respond differently to a "supply" shock. However, we do not feature the case where the policy rate hits the ELB *exclusively* due to a supply shock, mainly because it is unclear whether it is empirically relevant; higher productivity becomes contractionary at the ELB. Nevertheless, accounting for supply shocks is important to understand the data, and we analyze a model with both demand and supply shocks in Section 3.

Household: The representative household's value function V_t takes the following recursive form proposed by Epstein and Zin (1989) (EZ preferences):[19]

$$V_t = \begin{cases} U_t(C_t, N_t) + \beta \left\{ \mathbb{E}_t \left[V_{t+1}^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \geq 0 \\ U_t(C_t, N_t) - \beta \left\{ \mathbb{E}_t \left[(-V_{t+1})^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \leq 0. \end{cases} \tag{23}$$

β is the time discount rate. $\tilde{\gamma}$ is a measure of risk aversion. Note that a larger $\tilde{\gamma}$ implies higher risk aversion when $U \geq 0$ and lower risk aversion when $U \leq 0$. $\tilde{\gamma} = 0$ implies power utility. The period utility function $U_t(C_t, N_t)$ takes a standard form:

$$U_t(C_t, N_t) = \frac{C_t^{1-\chi_c}}{1-\chi_c} - Z_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{1+\chi_n}, \tag{24}$$

where $\chi_c > 0$ is the inverse elasticity of intertemporal substitution and $\chi_n > 0$ is the inverse Frisch elasticity. Z_t is a deterministic trend in total factor productivity (TFP) that grows at a rate of ζ (i.e., $\zeta = \frac{Z_t}{Z_{t-1}}$). The scaling of labor disutility by Z_t ensures the existence of a balanced growth path.

$C_t = \left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$ is the household's aggregate consumption of final goods based on a CES aggregator of intermediate goods, where $\theta > 1$ is the elasticity of demand across the intermediate goods. $N_t = \int_0^1 N_t(i) di$ denotes the household's total supply of labor, which is the integral of labor $N_t(i)$ supplied to each intermediate good producer i in a perfectly competitive labor market given nominal wage W_t.

The household faces the following sequence of budget constraints:

$$P_t C_t + \exp(-\delta_t) \mathbb{E}_t \left[\widetilde{M}_{t+1} \mathcal{W}_{t+1} \right] \leq W_t N_t + \mathcal{W}_t + \Xi_t + T_t, \tag{25}$$

where the aggregate price level of the consumption basket $P_t \equiv \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is implied by the household's cost minimization problem. Ξ_t is firms' profits rebated back to the household, T_t denotes lump-sum government taxes and/or transfers and $\mathcal{W}_t$ is the household's wealth portfolio consisting of default-free bonds of arbitrary maturity chosen by the

[19] The way we formulate EZ preferences is based on Rudebusch and Swanson (2012).

end of period t. Given this flow of income, the household maximizes (23) by choosing C_t, N_t and bond holdings next period $\mathcal{W}_{t+1}$. $\widetilde{M}_{t+1}$ is the nominal pricing kernel based on EZ preferences: $\widetilde{M}_{t+1} = \beta \left(\frac{U_{c,t+1}}{U_{c,t}} \right) \left[\frac{V_{t+1}}{\left[\mathbb{E}_t\left[V_{t+1}^{1-\tilde{\gamma}}\right]\right]^{\frac{1}{1-\tilde{\gamma}}}} \right]^{-\tilde{\gamma}} \frac{1}{\Pi_{t+1}}$ (if $U_t \geq 0$), where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the gross inflation rate.

In contrast to the stylized model in which we assumed the household was risk-neutral and the investor was risk-averse, here there is no such discrepancy; a representative risk-averse household makes an integrated optimal consumption-investment decision. This internal consistency may be theoretically appealing and is in line with many equilibrium asset pricing models.

We assume the household has a particular preference for holding bonds, which is represented by the time-varying parameter δ_t. A positive shock to δ_t can be interpreted as a *decrease* in preference for holding bonds over other (risky) assets priced by $\widetilde{M}_{t+1}$, and thus we call it a "risk preference" shock. As we confirm below, this shock represents a "demand" shock similar to the shock in the stylized model. Such a shock is also considered to be an important driver of business cycles in the macroeconomic literature.[20]

Intermediate Goods Producers: There is a continuum of monopolistically competitive intermediate goods producers indexed by $i \in [0, 1]$. Each producer faces costly price adjustments in the form proposed by Rotemberg (1982). The expected discounted value of each producer i's profit stream is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \widetilde{M}_{0,t} \left[P_t(i)Y_t(i) - W_t N_t(i) - \frac{\varphi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)\bar{\Pi}} - 1 \right)^2 P_t Y_t \right], \tag{26}$$

where $\widetilde{M}_{0,0} = 1$, $\widetilde{M}_{0,t} \equiv \prod_{s=1}^{t} \widetilde{M}_{s-1,s}$, and $\varphi > 0$ measures the degree of costly price adjustment. We assume the producers index their prices on steady-state inflation $\bar{\Pi}$ (or the central bank's inflation target). Each producer maximizes (26) by choosing $\{P_t(i), Y_t(i), N_t(i)\}$ sub-

[20]See for example, Smets and Wouters (2007) and Gust, Herbst, López-Salido, and Smith (2017). Fisher (2015) shows that this shock can indeed be interpreted as a structural shock to the demand for safe and liquid assets. Another way to introduce demand shocks is through shocks to the time discount rate in the utility function. For example, Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Creal and Wu (2020) apply this idea to analyze bond prices.

ject to the demand and production functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t,$$

$$Y_t(i) = \exp(a_t) Z_t N_t(i),$$

where a_t and Z_t are two exogenous components of TFP. We assume a_t is stationary, while Z_t is a deterministic trend as described above. The details of the process a_t are described below.

Monetary Policy: The central bank sets the nominal one-period interest rate, $R_t^{(1)} \equiv \exp(r_t^{(1)})$, following a Taylor rule with an occasionally binding ELB constraint:

$$R_t^{(1)} = \max\left[R_{ELB}, R_t^*\right], \tag{27}$$

where R_{ELB} is the ELB and the nominal *shadow rate* R_t^* follows:

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_r} \left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\pi} \left[\frac{Y_t}{\bar{Y} Z_t}\right]^{\phi_y}\right)^{1-\rho_r}, \tag{28}$$

where $\bar{R}$ and $\bar{Y}$ denote the steady state of $R_t^{(1)}$ and normalized output $\hat{Y}_t \equiv \frac{Y_t}{Z_t}$, respectively.[21] In the stylized model, $\rho_r = \phi_y = 0$, so that there was neither interest rate smoothing nor a response to the output gap. In the quantitative model, $\rho_r, \phi_y > 0$.

Exogenous Stochastic Processes: In the stylized model, a_t was constant and the only exogenous process was δ_t, which followed a 3-state Markov chain. Instead, for the quantitative model, we assume the state space is continuous and both variables follow an AR(1) process

[21] The inertial rule is specified such that the shadow policy rate today depends on the lagged shadow policy rate, as opposed to the lagged actual policy rate. This specification is known to have empirical support (e.g., Gust, Herbst, López-Salido, and Smith (2017)), as well as theoretical appeal in that the rule resembles optimal commitment policy (e.g., Hills and Nakata (2018)).

with time-varying volatility. In other words:

$$x_t = \rho_x x_{t-1} + \sigma_{x,t-1}\varepsilon_{x,t}, \qquad \varepsilon_{x,t} \sim \mathcal{N}(0,1), \tag{29}$$

for each exogenous process $x_t \in \{\delta_t, a_t\}$. The two processes can be interpreted as a "demand" and "supply" shock, respectively, and are standard in DSGE models fitted to macro variables. In our model, each process also serves an important role in generating plausible term structure dynamics at and above the ELB. In particular, (trend) stationarity of productivity is critical in obtaining an upward sloping term structure.[22] Meanwhile, δ_t can generate a negative term premium, as we saw in the stylized model.

We further specify time-varying volatility $\sigma_{x,t}$ as a logistic function of x_t as follows:

$$\sigma_{x,t} = \frac{\theta_{1,x}}{1+\exp(\theta_{2,x}(x_t - \theta_{3,x}))}\bar{\sigma}_x, \tag{30}$$

where $\bar{\sigma}_x$ is the conditional standard deviation of x_t when x_t is at its average level of zero. The term multiplying $\bar{\sigma}_x$ controls how much it is scaled depending on the deviation of x_t from its average. $\theta_{1,x} > 1$ sets the *upper bound* of the scaling and $\theta_{2,x}$ controls the *sensitivity* of the scaling with respect to x_t. Once $\theta_{1,x}$ and $\theta_{2,x}$ are chosen, $\theta_{3,x}$ is set to guarantee that the scaling term equals 1 and hence $\sigma_{x,t} = \bar{\sigma}_x$ when $x = \bar{x}$.[23] Our specification is computationally convenient as it does not add additional state variables to the model. Note $\sigma_{x,t}$ is a smooth monotonic function of x_t similar to $\sigma_{x,t} = \exp(-\theta x_t)$, but the logistic form ensures that the volatility is bounded at the tails, which leads to better numerical behavior.

$\sigma_{x,t}$ is increasing (decreasing) in x when $\theta_{2,x}$ is negative (positive). We set $\theta_{2,x} > 0$ to capture countercyclical volatility of each shock, which is consistent with empirical evidence of economic uncertainty rising in recessions (Bloom (2014) and references within). As shown below, this specification turns out to be useful in jointly fitting macroeconomic data and

[22]For similar specifications, see, for example, Andreasen (2012a,b), Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), Dew-Becker (2014), and Rudebusch and Swanson (2008, 2012). A productivity shock generates positive inflation risk premium by moving consumption growth and inflation in the opposite direction. When the productivity shock is trend stationary, the real term premium is also positive.

[23]$\theta_{1,x}$ and $\bar{\sigma}_x$ are separately identified, as we set $\theta_{3,x}$ as a function of $\theta_{1,x}$ and not $\bar{\sigma}_x$. In essence, two free parameters are added to the model when we introduce heteroskedasticity to each shock.

features of the term structure in the U.S., including the ELB periods.[24]

Term Structure of Default-free Interest Rates: Consistent with the household's optimization, we price the term structure of default-free interest rates using the pricing kernel that is adjusted for the risk preference shock. The pricing equation is the same as the one in the stylized model (4), except that we replace the pricing kernel with $M_{t+1} = \exp(-\delta_t)\widetilde{M}_{t+1}$ for the nominal term structure.[25] The definition of yields and term premiums are also the same as in the stylized model.

Equilibrium Characterization and Solution Method: To close the model, we impose the goods market clearing condition $Y_t = C_t$. Instead of the common assumption that the price adjustment cost entails a loss of aggregate resources, we assume that the cost is rebated back to the household, following studies such as Ascari and Rossi (2012).[26] We also integrate the supply of intermediate goods to obtain $Y_t = \exp(a_t)Z_tN_t$. Finally, we assume that the bonds are in zero net supply. The equilibrium is defined in the standard way and we relegate further details to Appendix D.

In solving models with an occasionally binding ELB constraint, standard local approximation methods such as perturbation can lead to significant fitting errors. Hence, we solve the model globally using a time-iteration method. This is important for our purposes since asset prices can be sensitive to the strong nonlinearity arising from the combination of highly risk averse agents, the ELB constraint, and time-varying volatility. The details of the solution method are described in Appendix E.

[24] To the extent that it captures asymmetric effects on volatility, our specification is analogous to a simplified EGARCH setup (Nelson (1991)). Another convenient feature is that as productivity increases, its conditional volatility becomes exponentially small, making it unlikely for a large *increase* in productivity to push the policy rate to the ELB (by lowering inflation). As discussed earlier, a productivity-driven ELB episode seems empirically uninteresting. For other DSGE term structure models with stochastic volatility in productivity, see, e.g., Andreasen (2012b) and Kung (2015).

[25] We use $M_{t+1} = \exp(-\delta_t)\widetilde{M}_{t+1}\Pi_{t+1}$ to price the real term structure.

[26] Miao and Ngo (2021) also show that this specification has some benefits in explaining macro dynamics in a model with the ELB constraint.

3.2 Nominal Yields and Term Premiums

As we discuss in detail below, we calibrate the model parameters based on U.S. data. However, before presenting the calibration, we discuss key features of nominal yields and term premium estimates that we are interested in fitting our model to.[27] The top-left panel of Figure 2 shows 3-month, 1-, 2-, and 5-year Treasury yields from January 1999 until June 2019.[28] We focus on maturities up to 5 years for two reasons. First, since the expected ELB duration throughout the (first) ELB episode did not exceed 3 years (Section 4), the effect of the ELB on yields beyond the 5-year maturity is likely to be small. Swanson and Williams (2014) shows empirically that this was indeed the case. Second, our model abstracts from QE policy, which can be an important driver of the term structure since the financial crisis. Nevertheless, a series of empirical work such as Vissing-Jorgensen and Krishnamurthy (2011) and D'Amico and King (2013) suggests that the effects of QE are larger for longer-term yields, and in particular, yields beyond the 5-year maturity. Thus, we view our model's abstraction from QE to be reasonable insofar as explaining the term structure up to 5 years, and useful for sharpening our discussion on how the ELB alone can affect yields and term premiums.[29]

Nominal yields are procyclical. They are higher during the economic booms preceding the 2001 recession and the 2007–2009 recession than during those recessions and the subsequent periods of sluggish recovery. The slope of the yield curve is positive on average and countercyclical. Finally, as discussed further in Section 3.4, the term structure of yield volatility above the ELB is downward sloping, except for the shortest maturities. However, at the ELB, yield volatility is increasing in maturity.

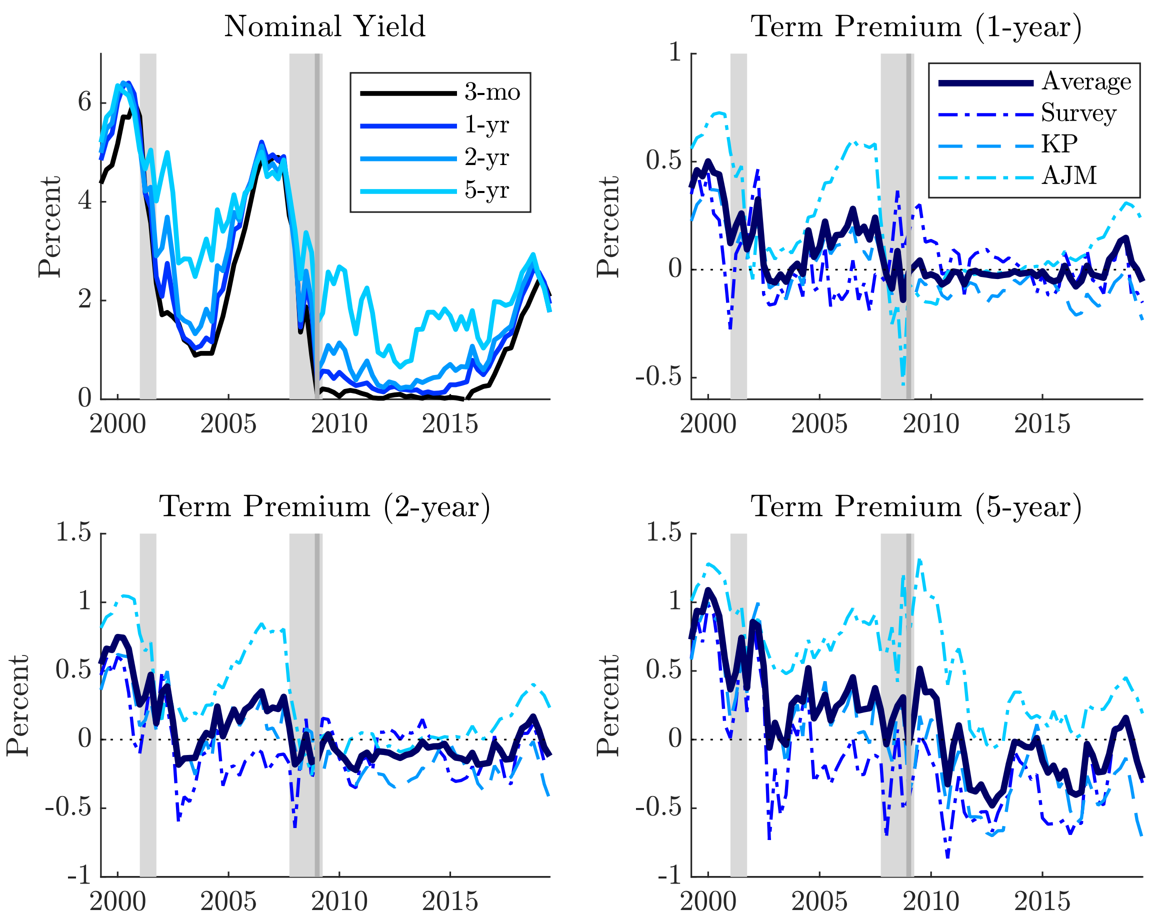
The top-right and bottom two panels show estimates of term premiums for 1-, 2- and 5-year bonds, respectively. Since term premiums are not clearly observed, we must derive them

[27]We defer details of the source as well as details of other U.S. data used for our calibration to Appendix F.

[28]The starting date is motivated by (i) empirical evidence suggesting distinct changes in U.S. Treasury bond risk with respect to the aggregate stock market since the late 1990s (e.g., Campbell, Sunderam, and Viceira (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020)) and (ii) the relative stabilization of trend inflation over a similar time period (e.g., Mertens (2016)). Restricting our analysis to this shorter period makes it less susceptible to misspecification from structural breaks in the economy.

[29]As discussed in Section 3.1, δ_t, which represents *net* demand for safe assets, could, in principle, be influenced by QE. Thus, our calibration may capture part of the effects of such policies, albeit in a reduced form manner.

Figure 2: **Nominal Yields and Term Premiums**



* For term premiums: Survey—based on 3-month T-bill forecasts from Blue Chip surveys; KP—from the shadow-rate model of Kim and Priebsch (2024), AJM—from the linear-rational square-root model of Andreasen, Jørgensen, and Meldrum (2025); Average—average of the three term premium measures. The solid vertical line indicates 2008:Q4 (start of the ELB period), and the shaded gray areas correspond to NBER recession periods. Data are at a quarterly frequency.

from either survey forecasts or statistical models. In this paper, we rely on three particular estimates, as shown in the figure. One is calculated using 3-month T-bill forecasts from Blue Chip surveys as a proxy for future short-rate expectations (blue dash-dotted lines). In terms of model-based estimates, the popular measures such as those by Kim and Wright (2005) or Adrian, Crump, and Moench (2013) do not account for the ELB. Hence, we refer to two alternative measures that explicitly account for it. The first measure is based on the "shadow-rate" term structure model of Kim and Priebsch (2024) (KP, blue dashed lines), which adds an

ELB constraint to a standard no-arbitrage affine term structure model through a shadow rate specification of the short rate.[30] The second measure is from the linear-rational square-root model of Andreasen, Jørgensen, and Meldrum (2025) (AJM, light-blue dash-dotted lines), which enforces the ELB and also allows for unspanned stochastic volatility.

Given the model- and survey-based measures broadly comove, we focus on a summary measure of term premiums for the rest of our analysis, which is a simple average of the three measures (dark-blue solid lines), computed for each maturity. We calibrate our quantitative model to fit this measure. Our construction of the measure is admittedly somewhat ad hoc, but to our knowledge, there are few term premium series that take into account the ELB, and we believe the three measures we consider are a good representation of plausible estimates. Averaging the three measures is a simple and transparent way to summarize that information.[31] That said, we acknowledge that any term premium estimate is subject to measurement uncertainty; the estimates can diverge, and the levels can be quite different for longer maturities.

According to the average measure, term premiums were positive and generally increasing in maturity in the early part of the sample. Afterwards, term premiums appeared to trend downward, and following the financial crisis—when the ELB became a binding constraint—term premiums became smaller and even negative. Meanwhile, term premiums generally remained volatile both above and at the ELB, especially for longer maturities, but for shorter maturities, term premiums became more compressed towards zero, starting shortly after the ELB became binding, until the liftoff of the policy rate approached.

[30] These models contain latent factors as drivers of yields, which make them very flexible and fit yield data accurately. We use the benchmark estimates of KP, which is derived from their fixed-parameter model. Their alternative estimates from a model which allows for a structural break imply similar means and standard deviations of term premiums.

[31] KP uses Blue Chip survey forecasts as input for estimation, while (the baseline) AJM does not. Thus, the average of the three measures overweights survey forecasts to some extent. We think this is reasonable, as survey forecasts, despite their potential noisiness, are model-free measures, and hence survey-based term premiums are closest to what we can call observable data.

3.3 Calibration

The calibrated parameter values for the quantitative model are summarized in Table 2. For ease of exposition, we group the parameters into two broad categories; the first category includes parameters that are calibrated from external sources, and the second category includes parameters that are chosen to match selected moments of the data.

In terms of the parameters in the first category, the deterministic trend growth rate of productivity (ζ) is set to 2.1 percent per annum, which is consistent with the average per capita output growth from 1999:Q1 to 2015:Q4. The inverse of the elasticity of intertemporal substitution (1/EIS or χ_c) is set to 10, close to Rudebusch and Swanson (2012). An EIS significantly below 1 is consistent with seminal macroeconomic studies (Hall (1988)), which has been confirmed more recently by Yogo (2004) and Havránek (2015).[32] The inverse Frisch elasticity (χ_n) of 2 is in line with existing studies.[33] The demand elasticity for intermediate goods (θ) is also set to a standard value of 6. Moving on to the parameters for the monetary policy rule, the ELB rate ($R_{ELB} - 1$) is an annualized 0.13 percent based on the middle of the lowest federal funds rate target range of 0 to 0.25 percent. The coefficient on inflation (ϕ_π) is set to 4, which is the value estimated by Plante, Richter, and Throckmorton (2018), and allows the model to fit the relatively stable inflation over the sample period.[34] The coefficient on output (ϕ_y) is set to 0.5, well within the range of common values. The interest rate smoothing parameter (ρ_r) is fixed at 0.3. Introducing policy inertia makes ELB episodes in the model longer and thus more realistic, while also improving the fit to average yields at the ELB. That said, to retain some volatility in short-maturity term premiums at the ELB, we set the value to the lower end of the range used in the literature, as suggested by Rudebusch (2006). Finally, we calibrate the persistence of the shock processes based on existing studies. The persistence of productivity (ρ_a) is set to 0.95, following Rudebusch

[32]Models explaining the equity risk premium typically set the EIS to be above 1, but this is not necessarily without controversy, as suggested by Beeler and Campbell (2012).

[33]Frisch elasticities using microeconomic data tend to come out closer to zero, while those using aggregate data can be notably higher (see, for example, Peterman (2016)). Our choice lies in between.

[34]It is not uncommon to assume a high inflation coefficient as it also helps achieve equilibrium existence in the model with occasionally binding ELB constraints. See also, Erceg and Lindé (2014), Hills, Nakata, and Schmidt (2019), and the 2012 working paper of Gust, Herbst, López-Salido, and Smith (2017), whose model is closer to ours than the published version.

and Swanson (2012). The persistence of risk preference (ρ_δ) is set to 0.85, following Gust, Herbst, López-Salido, and Smith (2017). These values fall within the range typically used in the literature.

Given the externally calibrated parameters, the parameters in the second category are chosen to match the average and standard deviations of output, inflation, yields and term premiums both above and at the ELB. Since the number of targeted moments far exceeds the number of parameters (overidentified), it would be ideal to conduct a systematic optimization over a sufficiently wide parameter space. However, this remains quite challenging for our highly nonlinear model which includes both macro variables and yields with multiple maturities. Thus, as explained below, we determine the values through a coarse grid search over a parameter space guided by our understanding of the model mechanism as well as insights from previous studies.

Although, in general, there is no straightforward mapping between a parameter and a moment(s), some parameters predominantly affect certain moments. Based on this observation, the time discount rate accounting for trend productivity growth ($\tilde{\beta} \equiv \beta\zeta^{-\chi_c}$) is set primarily to match the average 1-year nominal yield above the ELB of 3.5 percent.[35] The steady-state inflation rate ($\bar{\Pi}$) is calibrated largely to match an average inflation rate of 2.3 percent. The firms' price-adjustment-cost parameter (φ) is chosen primarily to match the volatility of inflation above and at the ELB, conditional on the other parameters. Consistent with studies such as Hills, Nakata, and Schmidt (2019), a large φ implies a flat Phillips curve and plays a key role in capturing the lack of sizable deflation during the ELB period, as well as ensuring inflation does not become counterfactually volatile at the ELB.[36]

The coefficient of relative risk aversion (γ) is set to 339, according to the formula provided by Swanson (2018).[37] This choice is in line with other equilibrium term structure models which typically use values that are substantially larger than those suggested by microeconomic

[35]Given choices for ζ and χ_c, we set β to hit our target for $\tilde{\beta}$. The focus on $\tilde{\beta}$ is justified by Kocherlakota (1990).

[36]For reference, our value of φ corresponds to a 95 percent probability of Calvo-style firms not changing prices each quarter in a log-linear economy, which is only somewhat higher than the values reported in Hills, Nakata, and Schmidt (2019) (90 percent) or Del Negro, Giannoni, and Schorfheide (2015) (87 percent).

[37]We set $\tilde{\gamma} = -150$ and convert this using the formula: $\gamma = \chi_c/(1+\chi_c/\chi_n)+\tilde{\gamma}(1-\chi_c)/(1+(\chi_c-1)/(1+\chi_n))$.

studies.[38] The high risk aversion—which we interpret as an aggregate measure that may also capture preferences toward ambiguity or early resolution of uncertainty in a concise way—helps the model generate sizable term premiums while fitting macro data.

The rest of the parameters characterize the (stochastic) volatility of productivity (a_t) and risk preferences (δ_t). For the model to fit the data, we find that the above-ELB dynamics must be mostly due to variation in a_t, while the variation in δ_t plays an important role at the ELB. This is because, in the data, the slope of the yield curve and term premiums are positive on average above the ELB (Section 3.2), which requires a_t to be the dominant driver (Section 3.1, footnote 22), while term premiums are significantly lower and even negative at the ELB, which can be generated from moves in δ_t (Section 2). As a result, when both a_t and δ_t are at their means, the conditional volatility of a_t ($\bar{\sigma}_a$) is set to be significantly larger than the volatility of δ_t ($\bar{\sigma}_\delta$). Meanwhile, stochastic volatility parameters for δ_t ($\theta_{1,\delta}$ and $\theta_{2,\delta}$) are chosen to generate meaningful volatility of δ_t following large negative shocks to itself that takes the economy to the ELB.[39]

The stochastic volatility parameters also serve a key role in fitting the volatility of term premiums, which is consistent with other related studies (Section 3.1). In particular, the parameters for a_t ($\theta_{1,a}$ and $\theta_{2,a}$) are chosen primarily to match term premium volatility above the ELB, while $\theta_{1,\delta}$ and $\theta_{2,\delta}$ help match term premium volatility at the ELB. For additional discipline, we also check whether the degree of time variation in volatility has plausible implications for output and inflation dynamics, although we do not explicitly attempt to match them.

[38]For example, Rudebusch and Swanson (2012) use 110, Bretscher, Hsu, and Tamoni (2020) use 188 and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) use 616 in their benchmark model.

[39]When the volatility of δ_t increases, the relative impact of δ_t on term premiums temporarily increases. Thus, the negative correlation of output and inflation caused by TFP shocks is dampened, making term premiums less positive, or even negative.

Table 2: **Parameter Values for the Quantitative Model**

Calibrated from External Sources

Parameter	Value	Description	Source
$\zeta - 1$	2.1	Trend productivity growth ($\times 400$)	Labor productivity growth
χ_c	10	1/Elasticity of intertemporal substitution	Rudebusch and Swanson (2012)
χ_n	2	1/Frisch elasticity	Standard
θ	6	Demand elasticity for intermediate goods	Standard
ϕ_π	4	Coeff. on inflation in the Taylor rule	Plante et al. (2018)
ϕ_y	0.5	Coeff. on the output gap in the Taylor rule	Standard
ρ_r	0.3	Interest rate smoothing in the Taylor rule	Rudebusch (2006)
ρ_a	0.95	AR(1) coeff. of productivity	Rudebusch and Swanson (2012)
ρ_δ	0.85	AR(1) coeff. of risk preference	Gust et al. (2017)
$R_{ELB} - 1$	0.13	Effective interest rate lower bound ($\times 400$)	Midpoint of 0 - 0.25%

Calibrated to Match Moments

Parameter	Value	Description	Comment
β	0.994	Time discount rate	Target ave. 1-yr nom. rate above ELB
$\bar{\Pi}$	1.007	Steady-state inflation rate	Target ave. inflation above ELB
γ	339	Risk aversion	Term structure model calibrations
φ	2000	Price adjustment cost	Calvo prob. $= 0.95$; Hills et al. (2019)
$\bar{\sigma}_a$	1	Stdv. of prod. shock at $a_t = 0$ ($\times 100$)	Target moments above ELB
$\theta_{1,a}$	4.5	Upper bound of prod. volatility	
$\theta_{2,a}$	22	Sensitivity of prod. volatility	
$\bar{\sigma}_\delta$	0.074	Stdv. of r.p. shock at $\delta_t = 0$ ($\times 100$)	Target moments at ELB
$\theta_{1,\delta}$	30	Upper bound of r.p. volatility	
$\theta_{2,\delta}$	250	Sensitivity of r.p. volatility	

3.4 Results

3.4.1 Moments above and at the ELB

We now describe the ability of our model to fit the data by looking at selected moments of (detrended) output, inflation and the term structure of interest rates both above and at the ELB. The moments of interest are conditional averages and conditional standard deviations for "above the ELB" and "at the ELB" samples.[40] To compute moments for the data counterparts, we focus on the period until the end of the first ELB episode; we define the period above the ELB to be from 1999:Q1 to 2008:Q3, and the ELB period to be from

[40]The model implied moments are computed by simulating the model and computing moments of interest conditional on the economy being above or at the ELB, i.e., $\mathbb{E}[X_t \mid R_t > R_{ELB}]$ or $\mathbb{E}[X_t \mid R_t = R_{ELB}]$, where X_t is the equilibrium variable of interest.

2008:Q4 to 2015:Q4, as the effective federal funds rate hit the ELB in the middle of December 2008.

Table 3: **Macro and Yield Curve Moments above and at the ELB —Model versus Data—**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
	A. Macro Variables (Mean)				
Output†	—	—	-2.83	-2.33	-1.51
Inflation	2.33	2.46	1.40	0.58	1.27
	B.1. Macro Variables (Volatility)				
Output	0.85	0.98	1.77	2.07	1.32
Inflation	0.76	0.86	0.91	0.93	0.65
	B.2. Macro Variables (Vol. of Vol.)				
Output	0.21	0.19	0.56	0.65	0.42
Inflation	0.26	0.16	0.34	0.30	0.21
	C. Nominal Yields (Mean)				
3-month	3.16	3.40	0.08	0.13	-3.86
1-year	3.50	3.44	0.30	0.65	-3.03
2-year	3.70	3.50	0.57	1.26	-1.93
5-year	4.24	3.72	1.60	2.24	0.21
	D. Nominal Yields (Volatility)				
3-month	1.71	1.53	0.06	0.00	3.76
1-year	1.72	1.50	0.16	0.35	3.36
2-year	1.54	1.46	0.27	0.52	2.78
5-year	1.10	1.35	0.56	0.59	1.75
	E. Nominal Term Premium (Mean)				
1-year	0.16	0.05	-0.02	-0.08	-0.32
2-year	0.22	0.13	-0.10	-0.17	-0.55
5-year	0.41	0.37	-0.10	-0.09	-0.52
	F. Nominal Term Premium (Volatility)				
1-year	0.19	0.08	0.07	0.05	0.11
2-year	0.27	0.19	0.10	0.08	0.21
5-year	0.30	0.41	0.29	0.17	0.32

* This table contains summary statistics for selected macroeconomic and term structure variables comparing data versus model counterparts. The sample period is 1999:Q1–2008:Q3 for data above the ELB, and 2008:Q4–2015:Q4 for data at the ELB. All variables are in annualized percentage terms except for output, which is not annualized.

The "mean" reported for output at/below the ELB is the average deviation from trend (for data) or the model-implied trend (for the model) in percentage points. We do not report statistics above the ELB, as the mean roughly coincides with the trend.

Table 3 presents the moments of the data and from the model. In each table, the first two columns show the moments when the policy rate is above the ELB and the next two columns show the moments when the policy rate is at the ELB. The fifth (final) column shows the conditional moments when the policy rate is below the ELB rate from a version of our model without the ELB constraint. As discussed earlier, our ELB episodes are largely driven by a fall in δ_t. However, this shock is also associated with an increase in the volatility of δ_t. The comparison of the final column with the fourth column helps us isolate the effects of the ELB on the model's moments from the effects associated with the increased volatility of δ_t.

Following the discussion in Section 3.2, we focus on the nominal term structure of interest rates up to the 5-year maturity and use the average of term premium measures as the data counterpart for term premiums. However, we briefly discuss our model's implications for the 10-year maturity as well as the real term structure in Appendix G.

Moments above the ELB: As shown in the first two columns of Table 3, panels A and B.1, the average inflation as well as volatilities of output and inflation in the data are in line with those from our model when the policy rate is above the ELB. In the data, yields are upward sloping and the term structure of yield volatility is downward sloping on average when the policy rate is above the ELB (panels C and D, first column). Our model successfully replicates these patterns qualitatively. Quantitatively, the average yield slope is shallower in the model, but the model-implied yield volatilities are broadly in line with the data (panels C and D, second column).

Panels E and F compare term premiums. The mean of the empirical estimates is positive and increases with maturity (Panel E, first column). The volatility of the empirical estimates also increases with maturity (Panel F, first column). Term premiums from our model are consistent with these patterns. Quantitatively, the mean term premiums from our model are somewhat smaller up to the 2-year maturity, but fairly close to the empirical counterparts at the 5-year maturity. While DSGE models tend to generate little variation in risk premiums, term premiums are significantly volatile in our model and within the ballpark of the empirical estimates. As discussed in the previous section, this is largely due to the introduction of stochastic volatility in a_t. However, we also check whether the degree of time variation in volatility has reasonable implications for output and inflation dynamics. Indeed, the first two columns of Panel B.2 show that the model-implied volatility of historical volatility is roughly similar to the data, both for output and inflation.[41] Overall, our model's term premium dynamics appear quantitatively reasonable.

Moments at the ELB: We turn to the comparison of our model with the data at the ELB. According to the first and third columns of Panel A of Table 3, output and inflation are on average about 2.8 percent and 0.9 percentage points lower, respectively, at the ELB than above the ELB in the data. Our model broadly captures these declines in output and inflation observed at the ELB, although the drop in inflation is more pronounced (second and fourth columns). In the data, the volatility of output is higher at the ELB than above the ELB, while the volatility of inflation is only modestly higher at the ELB than above the ELB (Panel B, first and third columns). The output and inflation volatilities in our model are in line with the data (Panel B, second and fourth columns). The comparison of the fourth column with the fifth (final) column, where we show the moments from the model without the ELB, indicates that the existence of the ELB makes the recession more severe. Based on the model, the output gap and inflation were about 0.8 and 0.7 percentage points lower, respectively, due to the ELB constraint.[42]

In the data, nominal yields are lower and the yield curve is steeper on average at the ELB than above the ELB (Panel C of Table 3, first and third columns). Moreover, the volatilities of yields are lower at the ELB than above the ELB across maturities (Panel D, first and third columns). While the yield volatility declines with maturity when the policy rate is above the ELB, the yield volatility increases with maturity when the policy rate is at the ELB. Our model captures these features reasonably well.[43]

[41]To compute historical volatility, we compute the volatility of the past 4-quarter observations.

[42]There are few studies that rigorously quantify the effect of the ELB on macro variables, but Gust, Herbst, López-Salido, and Smith (2017) offer one such analysis. According to their mean estimates, output and inflation were 1.2 percent and 0.3 percentage points lower on average, respectively, over the 2008-2012 period due to the ELB. Our estimates of the effect of the ELB are in a similar range.

[43]It turns out that the pure effect of the ELB is to increase average yields (Panel C, fourth and final columns). However, the upward slope of the volatility term structure is a consequence of the ELB. In the absence of the ELB, the volatility decreases with maturity during a severe recession in which the policy rate is negative (Panel D, fourth and final columns).

Panels E and F compare term premium moments. When the policy rate is constrained at the ELB, the empirical term premiums are on average significantly smaller than those above the ELB, and in fact, negative across maturities (Panel E, third column). Our model-implied averages (Panel E, fourth column) capture these features well. Turning to volatilities, the empirical term premiums remain volatile at the ELB (Panel F, third column), despite the significant difference in the average levels. Our model is also consistent with this pattern (Panel F, fourth column). While the 5-year volatility is somewhat smaller than the empirical estimate, volatilities for shorter maturities are close to their empirical counterparts. Similar to the case above the ELB, the third and fourth columns of Panel B.2 show that the model-implied volatility of historical volatility is roughly similar to the data, both for output and inflation. Hence, we conclude that the model-implied volatility of term premiums does not result from an implausible degree of stochastic volatility in δ_t and a_t. Term premium dynamics at the ELB are largely determined by two forces in our quantitative model. The first force is the countercyclical volatility of δ_t, as discussed in Section 3.3. With elevated demand volatility, the correlation between output growth and inflation becomes less negative and, as a result, term premiums fall. This force is present regardless of the ELB constraint. Indeed, term premiums are negative even in the model without the ELB constraint (Panel E, last column).

The second force is the ELB constraint. As discussed in detail in Section 2, the ELB constraint can either increase or decrease the size of term premiums depending on how much the ELB decreases the sensitivity of interest rates to macroeconomic fluctuations and how much it increases macroeconomic uncertainty. According to the fourth and final columns of Panel E, the compression effect of the ELB associated with reduced sensitivity of interest rates dominates the amplification effect associated with increased macroeconomic uncertainty, and the ELB constraint reduces the absolute size of term premiums (this can also be seen based on the volatility of term premiums, by comparing the fourth and final columns of Panel F). In our model, in which term premiums are negative at the ELB, the reduction in the absolute size of term premiums means an increase in term premiums.

The detailed account of how our model matches the averages and volatilities of existing term premium estimates, both above and at the ELB, marks a clear departure from related analysis by Gourio and Ngo (2020). They only report the model-implied 10-year term premium which is on average positive but shows little variability, and do not discuss the negative term premiums that are observed up to intermediate maturities.

Comparative Statics: Before closing the analysis of this section, we study alternative parameterizations of the model to clarify the role of several key parameters. Specifically, we focus on the parameters that control the stochastic volatility of δ_t and a_t, respectively, as well as the inflation and inertia coefficient in the monetary policy rule. To save space, we summarize the takeaways regarding the volatility parameters here in the main text, and delegate the rest of the details, including a table of the results, to Appendix H.

Removing stochastic volatility from a_t largely affects moments above the ELB, since at the ELB, the impact of δ_t is dominant and thus the specification of a_t matters less. In particular, the removal leads to less macroeconomic volatility and higher average yields above the ELB, compared to the baseline. Meanwhile, both the averages and volatilities of term premiums are significantly smaller, to the point of little time variation. The result that stochastic volatility is an important source of term premium variability is in line with the literature, as discussed in Section 3.3.

Next, consider removing stochastic volatility from δ_t instead. In this case, the variation in δ_t is never large enough for the model to hit the ELB in the first place due to the small average volatility $\bar{\sigma}_\delta$. For the model to be occasionally constrained by the ELB, we could increase $\bar{\sigma}_\delta$. However, this comes at a cost of counterfactual dynamics above the ELB, such as more volatile output, inflation and yields.

Finally, consider removing stochastic volatility from both a_t and δ_t. In this case, the ELB constraint binds occasionally, but mostly due to *positive* shocks to a_t, which seems clearly counterfactual. Similar to the previous parameterization where only a_t has stochastic volatility, increasing $\bar{\sigma}_\delta$ leads to average term premiums above the ELB that are too small. In addition, term premium volatility becomes negligible both above and at the ELB.

3.4.2 Dynamics

We now describe the dynamics of our model in response to shocks that send the policy rate to the ELB to gain further insights on how the ELB constraint affects the term structure of interest rates. We are particularly interested in how the term structure behaves when the economy is near the entry to or exit from the ELB state, insights that cannot be gleaned from comparing moments above and at the ELB.

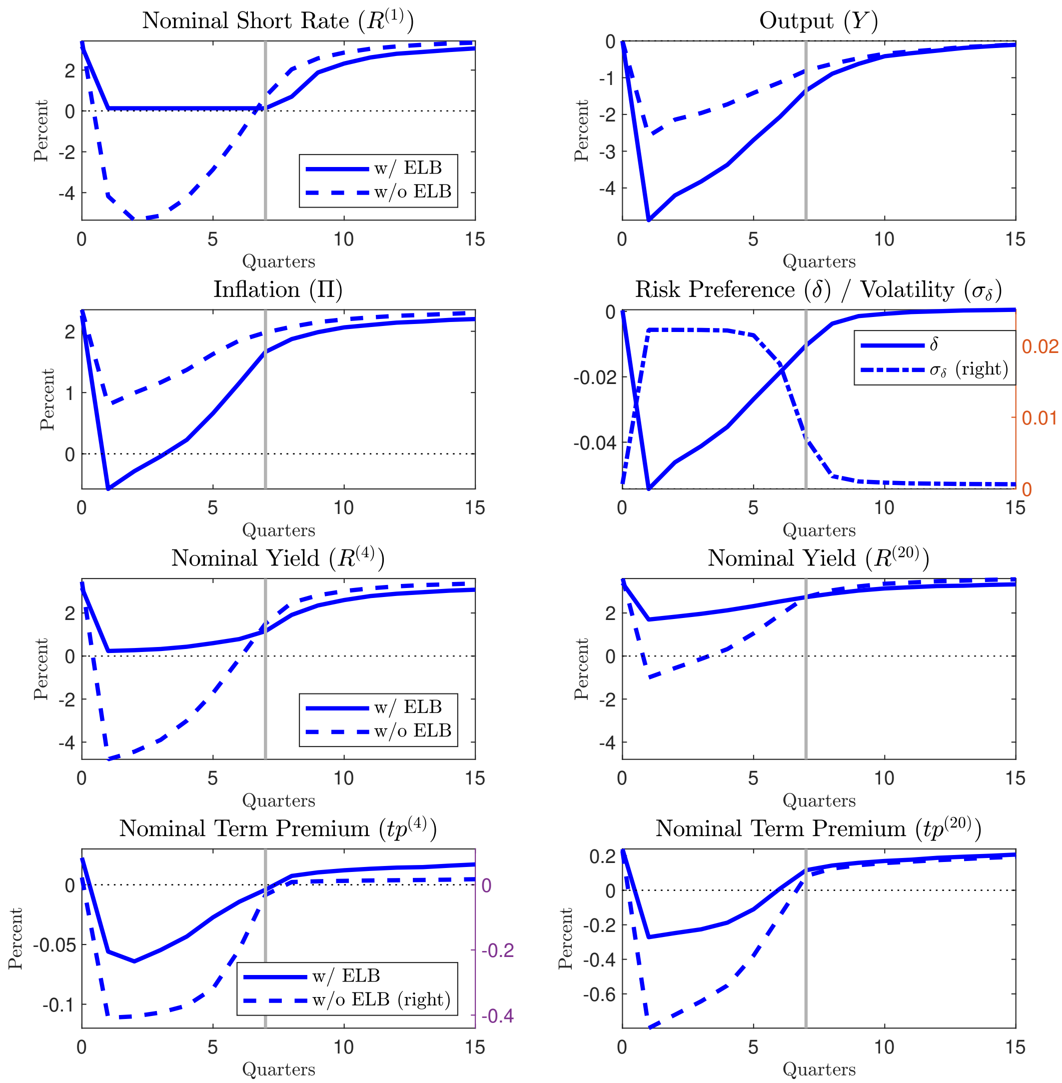
Figure 3 plots the median responses of macro and term structure variables to a negative δ_t shock. The size of the shock is set such that the initial declines in output and inflation in the model with the ELB constraint broadly coincide with the decline in their data counterparts between the business cycle peak of 2007:Q4 and trough of 2009:Q2, defined by the NBER. For each variable, the blue solid and dashed lines are for the models with and without the ELB constraint, respectively.

In response to the decrease in δ_t (second row, right, blue line), the nominal short rate falls from its average level and reaches the ELB (first row, left). Output (first row, right) and inflation (second row, left) initially drop by about 5 percent and 3 percentage points, respectively. As the initial shock fades, output and inflation gradually return to their steady states, supported by accommodative monetary policy. The liftoff occurs at quarter 7. As explained in Section 2, the declines in output and inflation are notably larger in the presence of the ELB constraint than in the absence of it.

The responses of select nominal yields are shown in the third row. After the negative shock, nominal yields decline for all maturities. The 1-year (or 4-quarter) yield drops to the ELB, and the 5-year (or 20-quarter) yield falls to about 2 percent, before gradually increasing. Note that, while the 5-year yield is comfortably above the ELB throughout the horizon, it is still affected by the presence of the ELB; were it not for the ELB, the 5-year yield would be lower, as shown by the dashed line (third row, right).

Turning to term premium dynamics, we first note that, due to countercyclical volatility in δ_t, the negative shock to δ_t increases its own conditional volatility in the initial period, which stays constant at a higher level for an extended period of time (second row, right, dash-

Figure 3: **Impulse Responses to δ_t Shocks**



* We plot $median[X_{t+h} \mid \varepsilon_{\delta,t} = -\vartheta_\delta, \bar{\Omega}]$ for a given variable X_t. The blue solid lines are the responses from the model with the ELB constraint, and the blue dashed lines are the responses from the model without the ELB constraint. The gray vertical lines indicate the timing of liftoff from the ELB.

dotted line). The heightened volatility of δ_t leads to negative term premiums as nominal bonds become more attractive as hedging instruments when volatility in demand increases. This effect can be seen most clearly from the responses of term premiums in the economy without the ELB (bottom row, blue dashed lines).

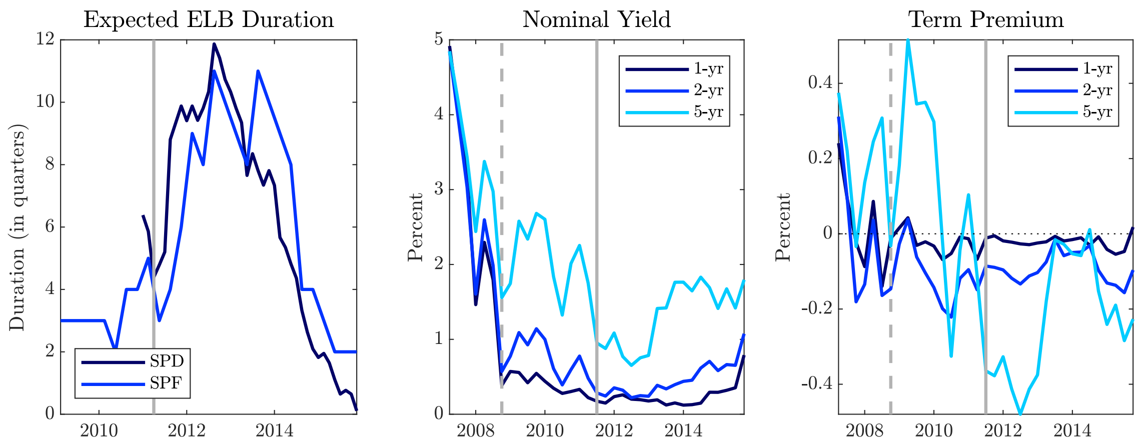
However, the ELB constraint introduces additional dynamics. As shown in the bottom left panel, the 1-year term premium shows an (inverse) hump-shaped response, in which it declines for another quarter after the initial decline, before rising back to its average (positive) level. The response is in contrast to the 5-year term premium (bottom right), which declines to its trough immediately after the initial shock. The reason is, since the ELB constraint is expected to bind in the near term, the 1-year term premium is compressed towards zero more strongly than its 5-year counterpart in the initial quarter. In the second quarter, the 1-year term premium falls further because the expected ELB duration is now shorter, and thus less binding, which implies future policy rates can react to macroeconomic risk more easily and hence bonds become a better hedging instrument. Later on, as δ_t returns closer to its average and the economy recovers, the volatility of δ_t ($\sigma_{\delta,t}$) declines (second row, right) and the effect of productivity shocks starts to dominate. As a result, nominal bonds become riskier and the term premium increases. The 5-year term premium (bottom right) incorporates the expected decline in $\sigma_{\delta,t}$ quickly, and turns from positive to negative right after the initial shock. However, even at the 5-year horizon, term premiums are compressed by the ELB. We revisit these results in the context of the recent U.S. experience in Section 4 and show that a sequence of shocks can generate further variation in the short-maturity term premium in ways that help us understand the data better.

Overall, the ELB constraint exacerbates the recession induced by the negative δ_t shocks and generally dampens the negative response of yields and term premiums that would have occurred in the absence of the ELB constraint. At the same time, the compression effect of the ELB introduces interesting nonlinear term premium dynamics at the ELB.

4 Model-based Account of Two ELB Episodes in the U.S.

To further understand term structure dynamics around the ELB, we zoom in on the first ELB episode in the U.S. following the financial crisis. In particular, we investigate how yields and term premiums evolved in conjunction with the expected ELB duration throughout the episode and how our model can account for it. Figure 4 shows survey-based expectations for the time until liftoff (left panel), nominal yields (middle panel), and term premiums

Figure 4: **Nominal Yields and Term Premiums at the ELB —The First ELB Episode—**



* The expected ELB duration is computed as the time to the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. Data are at the FOMC frequency for the SPD and otherwise quarterly. The term premiums are the average of the survey- and model-based term premiums. The gray vertical lines indicate 2008:Q4 (dashed) and 2011:Q3 (solid).

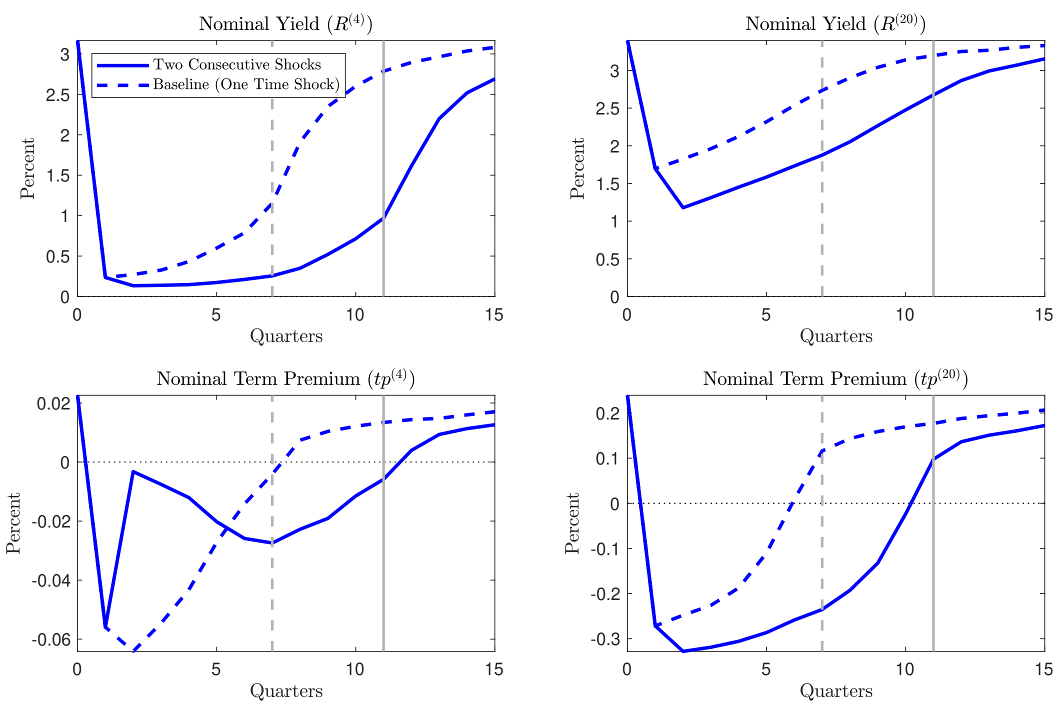
(right panel) of 1-, 2-, and 5-year maturities since 2007:Q2, just before the onset of the financial crisis.[44] Nominal yields were well above the ELB, and term premiums were largely positive across maturities at the time. However, as the financial crisis unfolded, yields declined sharply. Shorter-maturity term premiums declined, while the 5-year term premium fluctuated and rose initially until about mid-2009, before turning negative. Around the introduction of "calendar-based" forward guidance in August 2011 (the solid vertical lines), the expected ELB duration began to increase and reached about 2.5 years (10 quarters) by the end of 2012. Thereafter, the expected ELB duration fell.[45] The policy rate lifted off from the ELB in December 2015. Consistent with this evolution of the expected ELB duration, yields declined and remained low for a period of time as the expected duration was elevated around its peak. The 1-year yield started to increase around mid-2014, while the 2-year yield gradually rose beginning as early as mid-2013. Meanwhile, the 5-year yield increased sharply circa mid-2013.

[44] As a measure of term premiums, we plot the average measure used in Section 3.4.1. We plot the survey-based measure in Figure 6 since not all model-based measures are available beyond 2019.

[45] The FOMC explicitly stated that it was likely to keep the policy rate at the ELB at least through mid-2013.

1- and 2-year term premiums were negative until the second quarter of 2011. However, around the introduction of forward guidance, both 1- and 2-year term premiums compressed towards zero, as the expected duration of the ELB jumped. Both term premiums hovered around zero until mid-2013, after which they gradually turned negative as the expected duration of the ELB decreased. Meanwhile, the 5-year term premium was volatile, but broadly declined from its peak in 2009, and did not compress when the expected ELB duration increased sharply in the second quarter of 2011.[46]

Figure 5: **Impulse Responses to Two Consecutive δ_t Shocks**



* The blue solid lines are the responses to two consecutive shocks of the same size (adjusted by standard deviation); $median[X_{t+h} \mid \varepsilon_{\delta,t} = \varepsilon_{\delta,t+1} = -\vartheta_\delta, \bar{\Omega}]$ for a given variable X_t, and the blue dashed lines are the baseline responses to a one-time shock; $median[X_{t+h} \mid \varepsilon_{\delta,t} = -\vartheta_\delta, \bar{\Omega}]$ for a given variable X_t. The vertical lines indicate the timing of liftoff from the ELB for each response.

From the impulse responses of our model in the previous section, we do not see short-maturity term premiums compressed from negative values towards zero as expected ELB duration rises, and term premiums later becoming negative again, as expected ELB duration

[46]The sharp rise in the 5-year term premium observed after mid-2013 corresponds to the "taper tantrum" episode where term premiums of longer-term bonds reportedly became volatile as a result of uncertainty about the tapering of LSAPs amplified by reduced market liquidity.

falls. However, by simply assuming *two* consecutive negative shocks that hit δ_t unexpectedly instead of one shock in the initial period, we can replicate a similar pattern in short-maturity term premium dynamics. Indeed, as shown in the bottom-left panel of Figure 5, after the initial shock makes the 1-year term premium negative, the second shock compresses the term premium toward zero (blue solid line), as the expected ELB duration increases (implied by the difference between the location of the solid and dashed vertical lines). Afterwards, the term premium drops again (before rising) as the economy approaches liftoff and the compression effect of the ELB constraint fades. In contrast, the 5-year term premium is not as compressed, since longer-maturity yields are less sensitive to the current ELB episode, consistent with the data (right panel, blue solid line). One could loosely interpret the first shock as representing a shock that led to the financial crisis, and the second one as the shock that extended the expected ELB duration further around the timing of the date-based forward guidance.[47]

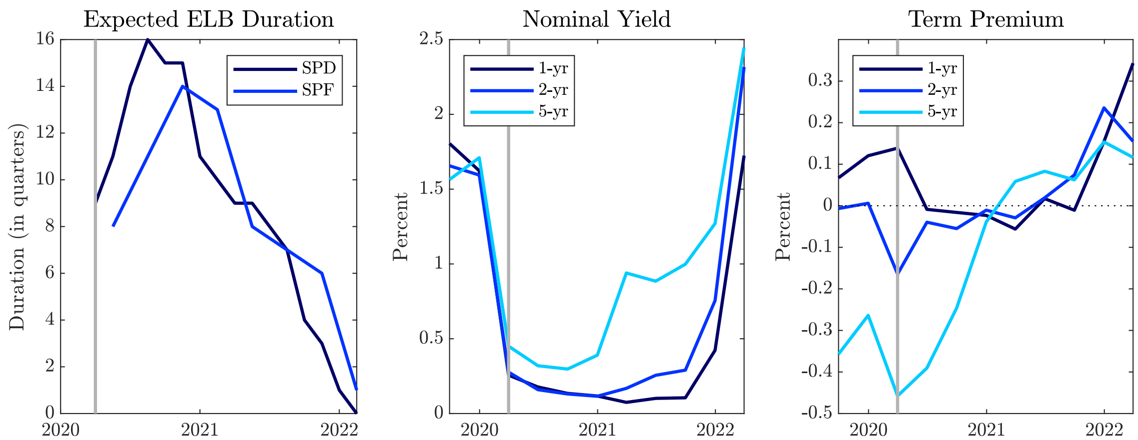
The U.S. experienced another ELB episode following the Covid-19 crisis in early 2020. Figure 6 summarizes expected ELB duration, yields and term premiums during this episode, analogous to Figure 4. Similar to 2011, the expected ELB duration increased sharply after the policy rate hit the ELB, but the pace of decline afterwards was significantly faster (left panel). Reflecting these expectations, yields fell discretely in March 2020 as the policy rate reached the ELB, but the 1-year yield began to retrace after about a year and a half, much earlier than the first ELB episode. The 2- and 5-year yields started to increase earlier accounting for the future liftoff, similar to developments before the liftoff in 2015 (middle panel).

2- and 5-year term premiums were generally negative towards the end of 2019 and dropped notably at the end of the first quarter of 2020 after the policy rate hit the ELB (right panel). However, term premiums of shorter maturities quickly reverted towards zero and stayed there until around mid-2021. These developments appear to be broadly similar to the first ELB episode. However, in sharp contrast to the first ELB episode, term premiums in 2021 *increased* as the policy rate approached liftoff. In addition, while the 5-year term premium remained negative in the first ELB episode, it increased steadily to a positive level in the

[47] The second shock associated with forward guidance could represent a "Fed information" effect.

second episode.

Figure 6: **Nominal Yields and Term Premiums at the ELB**
—Covid-19 Crisis Episode—



* The expected ELB duration is computed as the time to the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. Data are at the FOMC frequency for the SPD and otherwise quarterly. The term premiums are computed based on the Blue Chip survey. The gray vertical lines indicate 2020:Q2.

Though our model is calibrated to the economy before the pandemic, we can conduct some simple counterfactual model simulations that help us interpret such term structure developments since the onset of the pandemic crisis. To this end, we hit the baseline model with both a negative TFP shock and a negative δ_t shock in the initial period. In addition, we hit the model with the same shocks again in the next period, similar to the experiment above.[48] The combination of negative demand and supply shocks to describe the initial impact of the pandemic is consistent with Chen, Del Negro, Goyal, Johnson, and Tambalotti (2021) and Eichenbaum, Rebelo, and Trabandt (2022). Our contribution is to analyze its impact on the term structure. The impulse responses for selected yields and term premiums are shown in Figure 7.[49] The blue dashed lines are the responses from the baseline model with the δ_t shock only, identical to the solid lines in Figure 5. The light blue dash-dotted lines are the responses that include the negative TFP shock. When the model is hit with negative

[48]Similar to the previous experiment, the two shocks allow the model to generate a more realistic path for the short-maturity term premiums which were broadly negative before the start of the pandemic crisis.

[49]Output and inflation both fall in response to the shocks, consistent with the data. The addition of negative TFP shocks leads to a smaller drop in inflation compared to the baseline, as one might expect.

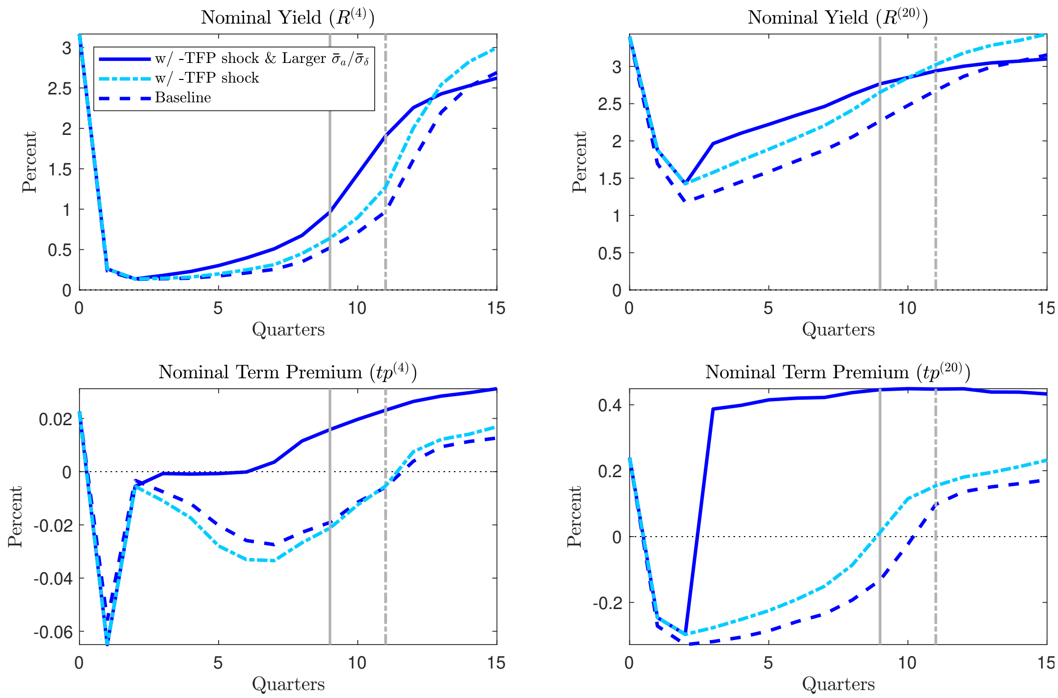
TFP shocks which raise inflation, the policy rate is raised earlier than when the shocks are absent. Thus, the shorter-term yield—represented by the 1-year yield in the figure—rises faster compared to the baseline (top left panel). The longer-term yield—represented by the 5-year yield in the figure—is also higher compared to the baseline (top right panel). Such model dynamics are broadly consistent with the shorter duration of the second ELB episode and the associated faster increase in yields.

Interestingly, shorter-maturity term premiums *decline* in response to the addition of negative TFP shocks (bottom left panel). This decline is, in fact, consistent with a faster pace of liftoff from the ELB due to the TFP shock (top left panel); the *weakening* of the compression effect of the ELB leads to more negative term premiums. Meanwhile, longer-maturity term premiums increase slightly (bottom right panel). This increase is because the negative TFP shocks raise the volatility of TFP and increase term premiums in our model. While this effect is also present for shorter maturities, it is outweighed by the effect of the ELB constraint.

The term premium dynamics from the simulations are somewhat at odds with what we observed during the second ELB episode in which (1) shorter-maturity term premiums did not decrease approaching liftoff, and (2) longer-maturity term premiums showed a notable increase to a positive level during the ELB period. One simple way to justify these observations is to further assume that the relative volatility of TFP with respect to δ_t ($\bar{\sigma}_a/\bar{\sigma}_\delta$) shifted up (unexpectedly) following the initial TFP shock. This assumption is motivated by the empirical evidence of shifts in economic volatility following the pandemic (e.g., Carriero, Clark, Marcellino, and Mertens (2024)).[50] The responses under this scenario are shown in blue solid lines. An increase in TFP volatility pulls forward the timing of liftoff (solid vertical line) and generally increases yields, similar to the negative TFP shock (top panels). More importantly, the effects on term premiums are very different. First, the shorter-maturity term premium is *higher* compared to the baseline response after the initial drop (bottom-left panel). In particular, the term premium increases from zero to positive as the liftoff

[50]Specifically, we allow for a one-time increase in $\bar{\sigma}_a/\bar{\sigma}_\delta$ by setting $\bar{\sigma}_a = 1.1/100$ and $\bar{\sigma}_\delta = 0.04/100$ in the third period. This permanent increase is assumed only for simplicity and is intended to parsimoniously capture a persistent increase in volatility which may have gradually built up since the pandemic.

Figure 7: **Impulse Responses to δ_t Shocks**
—Adding TFP Shocks and Increased Volatility of TFP—



* The blue dashed lines are the median impulse responses from the baseline model with the ELB constraint, the light blue dash-dotted lines are the responses where negative TFP shocks are added on impact, and the blue solid lines are the responses where a further increase in $\bar{\sigma}_a/\bar{\sigma}_\delta$ occurs. The gray vertical lines indicate the timing of liftoff from the ELB corresponding to each scenario (the dashed and dash-dotted lines coincidentally overlap).

approaches. This is because the permanent increase in TFP volatility exerts a strong force in raising the term premium which more than offsets the downward force from the weakening of the ELB constraint mentioned earlier. Second, by a similar mechanism, the longer-maturity term premium increases sharply and becomes positive (bottom-right panel).[51]

So far, modeling time-varying volatility seems to have received little attention in equilibrium models that focus on post-pandemic macroeconomic developments. However, second moments are a key driver of risk premiums. Our analysis shows that the changes in volatility may indeed be crucial in understanding term structure dynamics since the pandemic crisis.

[51] As mentioned above, the baseline model already includes a mechanism where a negative TFP shock increases TFP volatility. However, since the increase is transitory, it does not have as strong of an impact as a permanent shift.

5 Monetary Policy and Equilibrium Yield Curves at the ELB

The structural feature of our model makes it a natural framework to study how monetary policy affects the dynamics of yields and term premiums in a coherent manner. In this section, we focus on how a commitment to keep the policy rate at the ELB for longer than what would be warranted above the ELB (or, accommodative "forward guidance") affects the term structure of interest rates as well as the macroeconomy in our quantitative model.

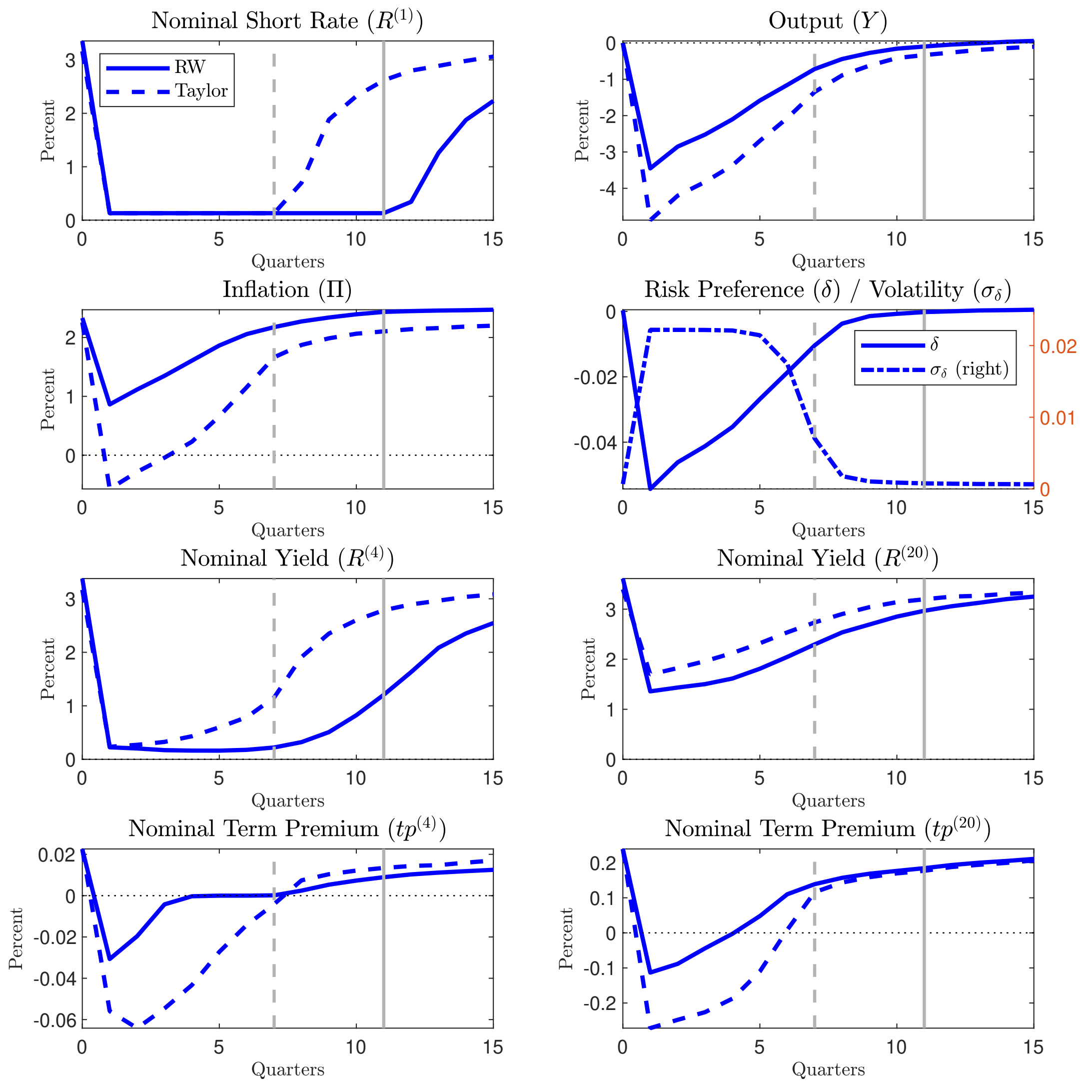
We model this alternative policy based on the work of Reifschneider and Williams (2000)—the "RW rule"—, where we augment the baseline monetary policy rule as follows:

$$\begin{aligned} R_t^{(1)} &= \max\left[R_{ELB}, R_t^* - \phi_{RW} J_t\right], \\ J_t &= J_{t-1} + (R_{t-1}^{(1)} - R_{t-1}^*). \end{aligned} \tag{31}$$

The shadow rate R_t^* is the same as in (28) and $\phi_{RW} \geq 0$ controls the degree of extra accommodation when past policy rates have been constrained by the ELB. J_t is the cumulative past deviation of the policy rate from the shadow rate. This rule implies that the timing of liftoff depends more strongly on the cumulative shortfalls in inflation and output than our baseline rule. Note that in the model without the ELB constraint, this rule is identical to our baseline rule, and when $\phi_{RW} = 0$, the RW rule collapses to the baseline rule with the ELB constraint (27).[52] We set $\phi_{RW} = 0.5$, a value taken from Reifschneider and Roberts (2006), and conduct a similar experiment where we compare the impulse responses of the economy to a negative demand shock under the baseline rule and the RW rule, respectively. The results are shown in Figure 8. As shown in the top-left panel, the policy rate is kept at the ELB for longer—by about 4 quarters—under the RW rule (blue solid line) than under the baseline rule (blue dashed line). With the nominal rate staying at the ELB for longer, the path of real rates is lower (not shown), which stimulates consumption of forward-looking households.

[52]Some papers on the effects of forward guidance (e.g., Campbell, Evans, Fisher, and Justiniano (2012); Del Negro, Giannoni, and Patterson (2023)) model forward guidance as exogenous "news" shocks to the policy rule. We prefer our formulation for two reasons: (1) the timing of liftoff and the evolution of the policy rate thereafter are state contingent under the RW rule, consistent with the data-dependent nature of the policy decisions repeatedly emphasized by FOMC participants, and (2) computational advantage; modeling forward guidance on the policy rate path by news shocks adds many state variables to an already computationally burdensome model, whereas the RW rule adds only one state variable (J_{t-1}).

Figure 8: **Effect of Accommodative Forward Guidance at the ELB**



* We plot $median[X_{t+h} \mid \varepsilon_{\delta,t} = -\vartheta_\delta, \bar{\Omega}]$ for a given variable X_t. The blue solid lines are the responses from the model under the RW rule, and the blue dashed lines are the responses from the model under the Taylor rule with the ELB constraint. The gray vertical lines indicate the timing of liftoff from the ELB corresponding to each rule.

An increase in consumption is associated with an increase in output (top right), which raises marginal costs of production and thus inflation (second row, left). The mechanism through which the RW rule stimulates economic activities at the ELB is the same as the mechanism through which the central bank stimulates economic activities at the ELB under optimal commitment policy and is well known in the macro literature.[53]

Consistent with the lower path of short-term interest rates, the paths of other nominal yields are lower under the RW rule than under the baseline rule (third row). Note that, while the difference in the short-term rate between these two policy rules emerges well after the initial shock, the difference in longer-term yields shows up at the onset of the crisis, reflecting the forward-looking nature of longer-term yields.

The panels in the last row plot the effects of adopting the RW rule on nominal term premiums. In contrast to the effects on yields and the expected path of future short rates, term premiums at the ELB are higher under the RW rule than under the baseline rule for both maturities. The intuition can be understood through our discussion in Section 2. The RW rule extends the period in which the policy rate is at the ELB and thus the interest rate sensitivity becomes even lower, compressing term premiums. Also, the more accommodative policy achieved by the RW rule leads to smaller macroeconomic volatility, further compressing term premiums. Accordingly, the absolute size of term premiums is lower, or, in this case, term premiums are less negative, under the RW rule than under the baseline rule.

Note that this result implies that the policy accommodation achieved through the decline in the expected short-rate path is partially offset by an increase in term premiums. However, since the effect of forward guidance is to compress the *absolute* size of term premiums, this result depends crucially on the risk exposure of bonds. For example, contrary to our baseline model in which demand shocks play a dominant role at the ELB, suppose TFP shocks play a dominant role, making average term premiums positive even at the ELB. Then, forward guidance could lower term premiums, amplifying the decline in the expected short-rate path. We present this case in Appendix I.

We used the forward guidance policy to motivate our analysis of the effects of adopting the RW rule. However, our analysis may be useful in understanding the effects of the central bank's other unconventional policies, such as QE. Some have argued that QE stimulates

[53]Classic references discussing optimal commitment policy at the ELB include Eggertsson and Woodford (2003) and Adam and Billi (2006). Jung, Teranishi, and Watanabe (2005) and Nakov (2008) compare the performance of the RW rule and optimal commitment policy.

economic activities partly by signaling the central bank's commitment to an extended period of more accommodative policies in the future (e.g., Woodford (2012), Bauer and Rudebusch (2014), and Bhattarai, Eggertsson, and Gafarov (2023)). If that is the case, then the RW rule can be seen as capturing this signaling effect of QE policy.

6 Conclusion

We have studied the term structure of interest rates in a New Keynesian model with an occasionally binding ELB constraint. Using a stylized model, we showed that the ELB constraint induced state dependency in term premiums by affecting macroeconomic uncertainty and the sensitivity of interest rates to economic activity. Further extending the model to match key features of output, inflation, yield curves, and term premium estimates in the U.S., we found that the compression effect of the ELB constraint was strong, reducing the absolute level of term premiums on average, while generating distinct dynamics around liftoff that would not arise without the constraint. We also used the model to provide a structural interpretation of term structure dynamics during the two recent ELB episodes.

We additionally investigated how the central bank's forward guidance to keep the policy rate at the ELB longer than previously expected affects the term structure of interest rates. We demonstrated that the adoption of such a policy reduced the expected short-rate path and the absolute size of term premiums. That is, if demand shocks are dominant and bonds are a hedge against economic downturns, term premiums are negative on average, and forward guidance increases term premiums. By contrast, if supply shocks are dominant and bonds are risky, term premiums are positive on average, and forward guidance decreases term premiums.

At a time when central banks in advanced economies have been providing various unconventional policies at or near the ELB, we believe that studying how the ELB constraint affects the dynamics of the term structure of interest rates is an essential step towards a deeper understanding of these policies. Extending our model to explicitly include the central bank's balance sheet is an interesting avenue for future research.

References

Adam, K., and R. M. Billi (2006): "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 38(7), 1877–1905.

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110(1), 110–138.

Albuquerque, R., M. Eichenbaum, V. X. Luo, and S. Rebelo (2016): "Valuation risk and asset pricing," *Journal of Finance*, 71(6), 2861–2904.

Andreasen, M. M. (2012a): "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, Real Term Premia, and Inflation Risk Premia," *European Economic Review*, 56(8), 1656–1674.

——— (2012b): "On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-linear DSGE Models," *Review of Economic Dynamics*, 15(3), 295–316.

Andreasen, M. M., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2018): "The pruned state-space system for non-linear DSGE models: Theory and empirical applications," *Review of Economic Studies*, 85(1), 1–49.

Andreasen, M. M., K. Jørgensen, and A. Meldrum (2025): "Bond risk premiums at the zero lower bound," *Journal of Econometrics*, 247, 105939.

Ascari, G., and L. Rossi (2012): "Trend inflation and firms price-setting: Rotemberg versus Calvo," *Economic Journal*, 122(563), 1115–1141.

Basu, S., and B. Bundick (2016): "Endogenous volatility at the zero lower bound: Implications for stabilization policy," *NBER Working Papers*, 21838.

Bauer, M. D., and G. D. Rudebusch (2014): "The Signaling Channel for Federal Reserve Bond Purchases," *International Journal of Central Banking*, 10(3), 233–289.

——— (2016): "Monetary policy expectations at the zero lower bound," *Journal of Money, Credit and Banking*, 48(7), 1439–1465.

Beeler, J., and J. Y. Campbell (2012): "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1(1), 141–182.

Bernanke, B. S. (2015): "Why are Interest Rates so Low, Part 4: Term Premiums," *Ben Bernanke's Blog, April 13*.

Bhattarai, S., G. B. Eggertsson, and B. Gafarov (2023): "Time consistency and duration of government debt: A model of quantitative easing," *Review of Economic Studies*, 90(4), 1759–1799.

Bianchi, F., M. Lettau, and S. C. Ludvigson (2022): "Monetary policy and asset valuation," *Journal of Finance*, 77(2), 967–1017.

Bilal, M. (2017): "Zeroing in: Asset pricing near the zero lower bound," *Working Paper*, SSRN 2897926.

Bilbiie, F. O. (2019): "Optimal forward guidance," *American Economic Journal: Macroeconomics*, 11(4), 310–345.

Bloom, N. (2014): "Fluctuations in uncertainty," *Journal of Economic Perspectives*, 28(2), 153–76.

Branger, N., C. Schlag, I. Shaliastovich, and D. Song (2015): “Macroeconomic Bond Risks at the Zero Lower Bound,” *Working Paper*.

Bretscher, L., A. Hsu, and A. Tamoni (2020): “Fiscal policy driven bond risk premia,” *Journal of Financial Economics*, 138(1), 53–73.

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012): “Macroeconomic Effects of Federal Reserve Forward Guidance,” *Brookings Papers on Economic Activity*, pp. 1–80.

Campbell, J. Y., C. Pflueger, and L. M. Viceira (2020): “Macroeconomic drivers of bond and equity risks,” *Journal of Political Economy*, 128(8), 3148–3185.

Campbell, J. Y., A. Sunderam, and L. M. Viceira (2013): “Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds,” *NBER Working Papers*, 14701.

Carriero, A., T. E. Clark, M. Marcellino, and E. Mertens (2024): “Addressing COVID-19 outliers in BVARs with stochastic volatility,” *Review of Economics and Statistics*, 106(5), 1403–1417.

Chen, H., V. Cúrdia, and A. Ferrero (2012): “The macroeconomic effects of large-scale asset purchase programmes,” *Economic Journal*, 122(564), F289–F315.

Chen, W., M. Del Negro, S. Goyal, A. Johnson, and A. Tambalotti (2021): “The New York Fed DSGE Model Forecast–March 2021,” *Federal Reserve Bank of New York Liberty Street Economics*.

Christensen, J. H., and G. D. Rudebusch (2016): “Modeling yields at the zero lower bound: Are shadow rates the solution?,” in *Dynamic Factor Models*, vol. 35, pp. 75–125. Emerald Group Publishing Limited.

Cieslak, A., and P. Povala (2016): “Information in the term structure of yield curve volatility,” *Journal of Finance*, 71(3), 1393–1436.

Creal, D. D., and J. C. Wu (2020): “Bond risk premia in consumption-based models,” *Quantitative Economics*, 11(4), 1461–1484.

D’Amico, S., and T. B. King (2013): “Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply,” *Journal of Financial Economics*, 108(2), 425–448.

Datta, D. D., B. K. Johannsen, H. Kwon, and R. J. Vigfusson (2021): “Oil, equities, and the zero lower bound,” *American Economic Journal: Macroeconomics*, 13(2), 214–253.

Del Negro, M., M. P. Giannoni, and C. Patterson (2023): “The forward guidance puzzle,” *Journal of Political Economy Macroeconomics*, 1(1), 43–79.

Del Negro, M., M. P. Giannoni, and F. Schorfheide (2015): “Inflation in the great recession and new keynesian models,” *American Economic Journal: Macroeconomics*, 7(1), 168–96.

Dew-Becker, I. (2014): “Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale Bayesian DSGE Model,” *Journal of Money, Credit and Banking*, 46(5), 837–888.

Eggertsson, G. B., and M. Woodford (2003): “Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, pp. 139–233.

Eichenbaum, M. S., S. Rebelo, and M. Trabandt (2022): “Epidemics in the New Keynesian model,” *Journal of Economic Dynamics and Control*, 140, 104334.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57(4), 937–969.

Erceg, C., and J. Lindé (2014): "Is there a fiscal free lunch in a liquidity trap?," *Journal of the European Economic Association*, 12(1), 73–107.

Fernández-Villaverde, J., G. Gordon, P. Guerrón-Quintana, and J. F. Rubio-Ramirez (2015): "Nonlinear Adventures at the Zero Lower Bound," *Journal of Economic Dynamics and Control*, 57, 182–204.

Fisher, J. D. (2015): "On the Structural Interpretation of the Smets–Wouters "Risk Premium" Shock," *Journal of Money, Credit and Banking*, 47(2-3), 511–516.

Gavin, W. T., B. D. Keen, A. W. Richter, and N. A. Throckmorton (2015): "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics," *Journal of Economic Dynamics and Control*, 55, 14–38.

Gertler, M., and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58(1), 17–34.

Gourio, F., and P. Ngo (2020): "Risk premia at the ZLB: a macroeconomic interpretation," *FRB of Chicago Working Paper*, WP-2020-01.

Gürkaynak, R. S., and J. H. Wright (2012): "Macroeconomics and the Term Structure," *Journal of Economic Literature*, 50(2), 331–367.

Gust, C., E. Herbst, D. López-Salido, and M. E. Smith (2017): "The empirical implications of the interest-rate lower bound," *American Economic Review*, 107(7), 1971–2006.

Hall, R. E. (1988): "Intertemporal substitution in consumption," *Journal of Political Economy*, 96(2), 339–357.

Haubrich, J. G. (2021): "Does the yield curve predict output?," *Annual Review of Financial Economics*, 13(1), 341–362.

Havránek, T. (2015): "Measuring intertemporal substitution: The importance of method choices and selective reporting," *Journal of the European Economic Association*, 13(6), 1180–1204.

Hills, T. S., and T. Nakata (2018): "Fiscal multipliers at the zero lower bound: the role of policy inertia," *Journal of Money, Credit and Banking*, 50(1), 155–172.

Hills, T. S., T. Nakata, and S. Schmidt (2019): "Effective lower bound risk," *European Economic Review*, 120, 103321.

Hsu, A., E. X. Li, and F. Palomino (2021): "Real and nominal equilibrium yield curves," *Management Science*, 67(2), 1138–1158.

Ichiue, H., and Y. Ueno (2007): "Equilibrium Interest Rate and the Yield Curve in a Low Interest Rate Environment," *Working Paper*.

Jung, T., Y. Teranishi, and T. Watanabe (2005): "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit, and Banking*, 37(5), 813–835.

Katagiri, M. (2022): "Equilibrium Yield Curve, the Phillips Curve, and Monetary Policy," *Journal of Money, Credit and Banking*, 54(8), 2235–2272.

Kim, D. H., and M. A. Priebsch (2024): "Are Shadow Rate Models of the Treasury Yield Curve Structurally Stable?," *Journal of Financial and Quantitative Analysis*, 59(7), 3500–3530.

Kim, D. H., and K. J. Singleton (2012): "Term Structure Models and the Zero Bound: an Empirical Investigation of Japanese Yields," *Journal of Econometrics*, 170(1), 32–49.

Kim, D. H., and J. H. Wright (2005): "An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates," *Finance and Economics Discussion Series*, 2005-33.

King, T. B. (2019): "Expectation and duration at the effective lower bound," *Journal of Financial Economics*, 134(3), 736–760.

Kocherlakota, N. R. (1990): "On the discount factor in growth economies," *Journal of Monetary Economics*, 25(1), 43–47.

Krippner, L. (2012): "Modifying Gaussian Term Structure Models when Interest Rates are near the Zero Lower Bound," *Reserve Bank of New Zealand Discussion Paper*, 2012/02.

Kung, H. (2015): "Macroeconomic Linkages between Monetary Policy and the Term Structure of Interest Rates," *Journal of Financial Economics*, 115(1), 42–57.

Lopez, P., D. Lopez-Salido, and F. Vazquez-Grande (2015): "Nominal Rigidities and the Term Structures of Equity and Bond Returns," *Finance and Economics Discussion Series*, 2015-64.

Mertens, E. (2016): "Measuring the level and uncertainty of trend inflation," *Review of Economics and Statistics*, 98(5), 950–967.

Miao, J., and P. V. Ngo (2021): "Does Calvo Meet Rotemberg at the Zero Lower Bound?," *Macroeconomic Dynamics*, 25(4), 1090–1111.

Nakata, T. (2017): "Uncertainty at the zero lower bound," *American Economic Journal: Macroeconomics*, 9(3), 186–221.

Nakata, T., and S. Schmidt (2019): "Conservatism and liquidity traps," *Journal of Monetary Economics*, 104, 37–47.

Nakata, T., and H. Tanaka (2016): "Equilibrium Yield Curves and the Interest Rate Lower Bound," *Finance and Economics Discussion Series*, 2016-85.

Nakov, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4(2), 73–127.

Nelson, D. B. (1991): "Conditional heteroskedasticity in asset returns: A new approach," *Econometrica*, 59(2), 347–370.

Nguyen, T. T. (2022): "Public debt, consumption growth, and the slope of the term structure," *Review of Financial Studies*, 35(8), 3742–3776.

Peterman, W. B. (2016): "Reconciling micro and macro estimates of the Frisch labor supply elasticity," *Economic Inquiry*, 54(1), 100–120.

Pflueger, C. (2025): "Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds," *Journal of Financial Economics*, 167, 104027.

Pflueger, C., and G. Rinaldi (2022): "Why does the fed move markets so much? a model of monetary policy and time-varying risk aversion," *Journal of Financial Economics*, 146(1), 71–89.

Piazzesi, M., and M. Schneider (2007): "Equilibrium Yield Curves," *NBER Macroeconomics Annual 2006*, 21, 389–472.

Plante, M., A. W. Richter, and N. A. Throckmorton (2018): "The zero lower bound and endogenous uncertainty," *Economic Journal*, 128(611), 1730–1757.

Reifschneider, D., and J. C. Williams (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit and Banking*, pp. 936–966.

Reifschneider, D. L., and J. M. Roberts (2006): "Expectations Formation and the Effectiveness of Strategies for Limiting the Consequences of the Zero Bound," *Journal of the Japanese and International Economies*, 20(3), 314–337.

Rotemberg, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49(4), 517–531.

Rudebusch, G. D. (2006): "Monetary Policy Inertia: Fact or Fiction?," *International Journal of Central Banking*, 2(4), 85–135.

Rudebusch, G. D., and E. T. Swanson (2008): "Examining the Bond Premium Puzzle with a DSGE Model," *Journal of Monetary Economics*, 55, S111–S126.

——— (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal," *American Economic Journal: Macroeconomics*, 4(1), 105–143.

Sakurai, Y. (2016): "How does the Bond Market Perceive Macroeconomic Risks under Zero Lower Bound?," *Working Paper*.

Sims, E., and J. C. Wu (2021): "Evaluating central banks tool kit: Past, present, and future," *Journal of Monetary Economics*, 118, 135–160.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Song, D. (2017): "Bond market exposures to macroeconomic and monetary policy risks," *Review of Financial Studies*, 30(8), 2761–2817.

Stein, J. C. (2012): "Evaluating Large-Scale Asset Purchases," *Remarks at the Brookings Institution, Washington D.C.*

Swanson, E. T. (2018): "Risk aversion, risk premia, and the labor margin with generalized recursive preferences," *Review of Economic Dynamics*, 28, 290–321.

——— (2019): "A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt," *Working Paper*.

Swanson, E. T., and J. C. Williams (2014): "Measuring the effect of the zero lower bound on medium-and longer-term interest rates," *American Economic Review*, 104(10), 3154–3185.

Van Binsbergen, J. H., J. Fernández-Villaverde, R. S. Koijen, and J. Rubio-Ramírez (2012): "The Term Structure of Interest Rates in a DSGE model with Recursive Preferences," *Journal of Monetary Economics*, 59(7), 634–648.

Vissing-Jorgensen, A., and A. Krishnamurthy (2011): "The effects of quantitative easing on interest rates: Channels and implications for policy," *Brookings Papers on Economic Activity*, pp. 215–287.

WOODFORD, M. (2012): "Methods of Policy Accommodation at the Interest-Rate Lower Bound," in *The Changing Policy Landscape: 2012 Jackson Hole Symposium*. Federal Reserve Bank of Kansas City.

WU, J. C., AND F. D. XIA (2016): "Measuring the macroeconomic impact of monetary policy at the zero lower bound," *Journal of Money, Credit and Banking*, 48(2-3), 253–291.

YOGO, M. (2004): "Estimating the elasticity of intertemporal substitution when instruments are weak," *Review of Economics and Statistics*, 86(3), 797–810.

Appendix to "Equilibrium Yield Curves and the Interest Rate Lower Bound"

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A Derivations and Proofs for the Stylized Model

A.1 Derivation of the Term Premium

To derive (8), we use the definition of $r_t^{(n)\mathbb{Q}}$ and obtain an analogous expression to (7):

$$r_t^{(2)\mathbb{Q}} = -\frac{1}{2}\left[\mathbb{E}_t[-r_t^{(1)} - r_{t+1}^{(1)}] + \frac{1}{2}\mathbb{V}_t[r_{t+1}^{(1)}]\right]. \tag{A.1}$$

While the equilibrium 1-period interest rate is derived from the macro block, we assume the investor accounts for risk when pricing n-period bonds ($n \geq 2$), and thus, the 1-period rate used for calculating n-period yields and term premiums is computed as $r_t^{(1)} = -\mathbb{E}_t[m_{t+1}] - \frac{1}{2}\mathbb{V}_t[m_{t+1}]$ (Meanwhile, when we compute the 2-period yield in Figure 1, we assume the investor computes the *moments* in the formula using dynamics from the macro block). These assumptions lead to a clean characterization of term premiums, which is convenient for our closed-form analysis. As we mention in the main text, such assumptions will not be imposed on the stylized model with continuous states, as well as the quantitative model. Finally, by subtracting (A.1) from (7), we obtain (8).

A.2 Proof of Proposition 1

We first solve the macro block for $\{y_s, \pi_s, r_s^{(1)}\}_{s\in\{H,M,L\}}$ in closed form. Then, we compute the conditional variance of each variable. To simplify notation, here $x_s \in \{y_s, \pi_s\}$ indicates the log *deviation* of x from its deterministic steady state, i.e., $x_s \equiv \hat{x}_s$, while r_s indicates the *level* of the policy rate $r_s^{(1)}$.

Model without the ELB: The system of equations for each state is:

$$y_H = [py_H + (1-p)\, y_M] + \frac{1}{\gamma}\left[p\pi_H + (1-p)\,\pi_M - r_H + \bar{r}\right] + \delta_H \tag{A.2}$$

$$\pi_H = \kappa y_H \tag{A.3}$$

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$$r_H = \bar{r} + \phi\pi_H \tag{A.4}$$

$$\begin{aligned} y_M = & \left[\frac{(1-p)\,p}{2} y_H + (1-(1-p)\,p)\, y_M + \frac{(1-p)\,p}{2} y_L\right] \\ & + \frac{1}{\gamma}\left[\frac{(1-p)\,p}{2}\pi_H + (1-(1-p)\,p)\,\pi_M + \frac{(1-p)\,p}{2}\pi_L - r_M + \bar{r}\right] + \delta_M \end{aligned} \tag{A.5}$$

$$\pi_M = \kappa y_M \tag{A.6}$$

$$r_M = \bar{r} + \phi\pi_M \tag{A.7}$$

$$y_L = [(1-p)\, y_M + p y_L] + \frac{1}{\gamma}[(1-p)\,\pi_M + p\pi_L - r_L + \bar{r}] + \delta_L \tag{A.8}$$

$$\pi_L = \kappa y_L \tag{A.9}$$

$$r_L = \bar{r} + \phi\pi_L. \tag{A.10}$$

Substitute out r and rearrange for state H:

$$y_H = [p y_H + (1-p)\, y_M] + \frac{1}{\gamma}[p\pi_H + (1-p)\,\pi_M - r_H + \bar{r}] + \delta_H$$

$$\Leftrightarrow (1-p)\,(y_H - y_M) = \frac{1}{\gamma}[(p-\phi)\,\pi_H + (1-p)\,\pi_M] + \delta_H \tag{A.11}$$

$$\pi_H = \kappa y_H \Leftrightarrow y_H = \frac{\pi_H}{\kappa}. \tag{A.12}$$

Substitute out r and rearrange for state M:

$$\begin{aligned} y_M = & \left[\frac{(1-p)\,p}{2} y_H + (1-(1-p)\,p)\, y_M + \frac{(1-p)\,p}{2} y_L\right] \\ & + \frac{1}{\gamma}\left[\frac{(1-p)\,p}{2}\pi_H + (1-(1-p)\,p)\,\pi_M + \frac{(1-p)\,p}{2}\pi_L - r_M + \bar{r}\right] + \delta_M \\ \Leftrightarrow & (1-p)\,p\left(-\frac{y_H}{2} + y_M - \frac{y_L}{2}\right) = \frac{1}{\gamma}\left[\frac{(1-p)\,p}{2}\pi_H + (1-(1-p)\,p-\phi)\,\pi_M + \frac{(1-p)\,p}{2}\pi_L\right] + \delta_M \end{aligned} \tag{A.13}$$

$$\pi_M = \kappa y_M \Leftrightarrow y_M = \frac{\pi_M}{\kappa}. \tag{A.14}$$

Substitute out r and rearrange for state L:

$$y_L = [(1-p)\, y_M + p y_L] + \frac{1}{\gamma}[(1-p)\,\pi_M + p\pi_L - r_L + \bar{r}] + \delta_L$$

$$\Leftrightarrow (1-p)\,(y_L - y_M) = \frac{1}{\gamma}[(1-p)\,\pi_M + (p-\phi)\,\pi_L] + \delta_L \tag{A.15}$$

$$\pi_L = \kappa y_L \Leftrightarrow y_L = \frac{\pi_L}{\kappa}. \tag{A.16}$$

Write the equilibrium conditions with respect to $\{\pi_H, \pi_M, \pi_L\}$. To do so, first, plug in (A.12) and (A.14) into (A.11) and rearrange:

$$(1-p)(y_H - y_M) = \frac{1}{\gamma}[(p-\phi)\pi_H + (1-p)\pi_M] + \delta_H$$

$$\Leftrightarrow A\pi_H + B\pi_M + C\pi_L = \delta_H, \tag{A.17}$$

where $A = \frac{(1-p)-\kappa\frac{1}{\gamma}(p-\phi)}{\kappa}$, $B = \frac{(1-p)\left(-1-\kappa\frac{1}{\gamma}\right)}{\kappa}$, and $C = 0$.
Next, plug in (A.12), (A.14), and (A.16) into (A.13) and rearrange:

$$(1-p)p\left(-\frac{y_H}{2} + y_M - \frac{y_L}{2}\right) = \frac{1}{\gamma}\left[\frac{(1-p)p}{2}\pi_H + (1-(1-p)p)\pi_M + \frac{(1-p)p}{2}\pi_L - r_M + \bar{r}\right] + \delta_M$$

$$\Leftrightarrow D\pi_H + E\pi_M + F\pi_L = \delta_M, \tag{A.18}$$

where $D = \frac{(1-p)p\left(-1-\kappa\frac{1}{\gamma}\right)}{2\kappa}$, $E(\phi) = \frac{(1-p)p-\kappa\frac{1}{\gamma}(1-(1-p)p-\phi)}{\kappa}$, and $F = D$.
Finally, plug in (A.14) and (A.16) into (A.15) and rearrange:

$$(1-p)(y_L - y_M) = \frac{1}{\gamma}[(1-p)\pi_M + (p-\phi)\pi_L] + \delta_L$$

$$\Leftrightarrow G\pi_H + H\pi_M + I\pi_L = \delta_L, \tag{A.19}$$

where $G = 0$, $H = \frac{(1-p)\left(-1-\kappa\frac{1}{\gamma}\right)}{\kappa}$ (note this notation should not be confused with state "H"), and $I = \frac{(1-p)-\kappa\frac{1}{\gamma}(p-\phi)}{\kappa}$. Therefore, we can stack the equilibrium conditions and solve for $\{\pi_H, \pi_M, \pi_L\}$:

$$\mathbf{A}\pi = \vartheta$$

$$\Leftrightarrow \pi = \mathbf{A}^{-1}\vartheta,$$

where $\mathbf{A} = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$, $\pi = \begin{bmatrix} \pi_H \\ \pi_M \\ \pi_L \end{bmatrix}$, and $\vartheta = \begin{bmatrix} \delta_H \\ \delta_M \\ \delta_L \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \\ -\delta \end{bmatrix}$. Then, we can solve for $y_s = \frac{1}{\kappa}\pi_s$ and $r_s = \bar{r} + \phi\pi_s$. For example, the explicit solution for output is:

$$y_{U,H} = \frac{1}{1-p+\frac{\kappa}{\gamma}(\phi-p)}\delta = -y_{U,L} \tag{A.20}$$

$$y_{U,M} = 0. \tag{A.21}$$

Model with the ELB: The system of equations is the same as the model without the ELB for states H and M ((A.2) through (A.7)). The only difference is for state L, where $r_L = 0$:

$$y_L = [(1-p)y_M + py_L] + \frac{1}{\gamma}[(1-p)\pi_M + p\pi_L - r_L + \bar{r}] + \delta_L$$

$$\pi_L = \kappa y_L$$

$$r_L = 0.$$

Substitute out r and rearrange:

$$y_L = [(1-p)\, y_M + p y_L] + \frac{1}{\gamma}\,[(1-p)\,\pi_M + p\pi_L - r_L + \bar{r}] + \delta_L$$

$$\Leftrightarrow (1-p)\,(y_L - y_M) = \frac{1}{\gamma}\,[(1-p)\,\pi_M + p\pi_L + \bar{r}] + \delta_L \tag{A.22}$$

$$\pi_L = \kappa y_L \Leftrightarrow y_L = \frac{\pi_L}{\kappa}. \tag{A.23}$$

Hence, the equilibrium conditions boil down to (A.17), (A.18), and (A.22):

$$(1-p)\,(y_L - y_M) = \frac{1}{\gamma}\,[(1-p)\,\pi_M + p\pi_L + \bar{r}] + \delta_L$$

$$\Leftrightarrow G\pi_H + H\pi_M + I^*\pi_L = \frac{1}{\gamma}\bar{r} + \delta_L, \tag{A.24}$$

where $G = 0$, $H = \frac{(1-p)(-1)-\kappa\frac{1}{\gamma}(1-p)}{\kappa}$, and $I^* = \frac{1-p-\kappa\frac{1}{\gamma}p}{\kappa}$. Therefore, we can stack the equilibrium conditions and solve for $\{\pi_H, \pi_M, \pi_L\}$:

$$\mathbf{A}^*\pi = \vartheta^*$$

$$\Leftrightarrow \pi = \mathbf{A}^{*-1}\vartheta^*,$$

where $\mathbf{A}^* = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I^* \end{bmatrix}$, $\pi = \begin{bmatrix} \pi_H \\ \pi_M \\ \pi_L \end{bmatrix}$, and $\vartheta^* = \begin{bmatrix} \delta_H \\ \delta_M \\ \frac{1}{\gamma}\bar{r} + \delta_L \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \\ \frac{1}{\gamma}\bar{r} - \delta \end{bmatrix}$. Note δ is set to $\tilde{\delta}$ such that it solves $\bar{r} + \phi\hat{\pi}_M = 0$. Hence,

$$\tilde{\delta} = \frac{\bar{r}(\phi-1)\{2(1-p(1+\kappa\frac{1}{\gamma}))(1-p^2(1+\kappa\frac{1}{\gamma})) + \phi\kappa\frac{1}{\gamma}(2-p(1+p)(1+\kappa\frac{1}{\gamma}))\}}{p(1-p)\phi^2\kappa(1+\kappa\frac{1}{\gamma})}.$$

We assume $\tilde{\delta} > 0$, which is satisfied if, for example, $p < 1/(1+\frac{\kappa}{\gamma})$ (a loose constraint based on standard empirical estimates suggesting κ/γ is small). Then, we can solve for $y_s = \frac{1}{\kappa}\pi_s$ for $s \in \{H, M, L\}$, $r_s = \bar{r} + \phi\pi_s$ for $s \in \{H, M\}$, and $r_L = 0$. For example, the explicit solution for output is:

$$y_{E,H} = \frac{\bar{r}(2\phi - 2 - p(1+\kappa\frac{1}{\gamma})(p(\phi-2)+\phi))}{\kappa p(1-p)(1+\kappa\frac{1}{\gamma})\phi^2} \tag{A.25}$$

$$y_{E,M} = -\frac{\bar{r}}{\kappa\phi} \tag{A.26}$$

$$y_{E,L} = \frac{\bar{r}(p^2(1+\kappa\frac{1}{\gamma})(3\phi-2) - 2(\phi-1)(1+\kappa\frac{1}{\gamma}\phi) - p\phi(1+\kappa\frac{1}{\gamma}))}{\kappa p(1-p)(1+\kappa\frac{1}{\gamma})\phi^2}. \tag{A.27}$$

Conditional Variances: Given the transition matrix, for any $\{x_H, x_M, x_L\}$,

$$\mathbb{V}_H[x] = p(1-p)(\Delta^x_{HM})^2 \tag{A.28}$$

$$\mathbb{V}_L[x] = p(1-p)(\Delta^x_{ML})^2 \tag{A.29}$$

$$\mathbb{V}_M[x] = p(1-p)\left[\frac{2-p(1-p)}{4}[(\Delta^x_{HM})^2 + (\Delta^x_{ML})^2] + \frac{p(1-p)}{2}\Delta^x_{HM}\Delta^x_{ML}\right], \tag{A.30}$$

where $\Delta^x_{HM} \equiv x_H - x_M$ and $\Delta^x_{ML} \equiv x_M - x_L$. Given the solution for $\{y_{U,H}, y_{U,M}, y_{U,L}\}$, $\Delta^y_{U,HM} = \Delta^y_{U,ML}$. Hence, (10) holds. Given the solution for $\{y_{E,H}, y_{E,M}, y_{E,L}\}$,

$$\Delta^y_{E,HM} = \frac{r^{(1)}_{E,H}}{\phi\kappa} > 0 \tag{A.31}$$

$$\Delta^y_{E,ML} = \Delta^y_{E,HM} + \frac{2\bar{r}\frac{1}{\gamma}(\phi-1)}{p(1-p)(1+\kappa\frac{1}{\gamma})\phi} > \Delta^y_{E,HM}, \tag{A.32}$$

since, by assumption, $\phi > 1$. Combining these inequalities with the above expressions for conditional variance, it can be shown that (11) holds for y. Then, (11) for π is immediate from $\hat{\pi} = \kappa\hat{y}$. Given the solution for $\{r^{(1)}_{E,H}, r^{(1)}_{E,M}, r^{(1)}_{E,L}\}$, $\Delta^r_{E,HM} = r^{(1)}_{E,H} > 0$ and $\Delta^r_{E,ML} = 0$. Hence (12) holds.

To show (13) holds, note $\Delta^y_{U,HM} > 0$ and:

$$\Delta^y_{E,HM} - \Delta^y_{U,HM} = \frac{\bar{r}\frac{1}{\gamma}(\phi-1)}{\phi(1-p+\kappa\frac{1}{\gamma}(\phi-p))} > 0, \tag{A.33}$$

since, by assumption, $\phi > 1$. Also, $\hat{\pi}_s = \kappa\hat{y}_s$ and $r^{(1)}_s = \bar{r} + \phi\kappa\hat{y}_s$, for $s \in \{H, M\}$. Thus, (13) holds.

A.3 Proof of Proposition 2

We obtain the expressions for term premiums by plugging in the equilibrium inflation, output, and policy rate derived in Proposition 1 into the definition of the term premium (8). Specifically, we plug in the expressions for inflation, output, and the policy rate into the following formulas for the conditional covariances of the Markov chain, which are the key components of the term premium:

Conditional Covariances: Given the transition matrix, for any $\{x_H, x_M, x_L\}$ and $\{z_H, z_M, z_L\}$,

$$\mathbb{C}\text{ov}_H[x, z] = 2\tilde{p}\Delta^x_{HM}\Delta^z_{HM} \tag{A.34}$$

$$\mathbb{C}\text{ov}_L[x, z] = 2\tilde{p}\Delta^x_{ML}\Delta^z_{ML} \tag{A.35}$$

$$\mathbb{C}\text{ov}_M[x, z] = \tilde{p}\left[(1-\tilde{p})(\Delta^x_{HM}\Delta^z_{HM} + \Delta^x_{ML}\Delta^z_{ML}) + \tilde{p}(\Delta^x_{HM}\Delta^z_{ML} + \Delta^z_{HM}\Delta^x_{ML})\right], \tag{A.36}$$

where $\Delta^v_{HM} \equiv v_H - v_M$, $\Delta^v_{ML} \equiv v_M - v_L$, and $\tilde{p} \equiv \frac{p(1-p)}{2}$.

A.4 Proof of Proposition 3

Amplification (18) follows from Propositions 1 and 2 ((13), (14) and (15)). Compression (19) follows from Proposition 2 ((14) and (17)). To characterize the term premium for the medium state M (20), we take the difference between the term premium in state M with the ELB (16) and the term premium without the ELB (14), and check the condition for each inequality to hold. Note $\tilde{\Delta} \equiv \frac{2(\Delta^y_{U,HM})^2-(1-\tilde{p})(\Delta^y_{E,HM})^2}{\tilde{p}\Delta^y_{E,HM}}$, where $\tilde{p} \equiv \frac{p(1-p)}{2}$.

B Stylized Model with Government Spending Shocks

In this section, we analyze a version of the 3-state Markov model with government spending shocks, and show that even in this setup, the ELB constraint can amplify and compress term premiums.

The model remains similar to the stylized model described by equations (1) through (3) in the main text, but a crucial difference is that government spending drives a wedge between consumption and output, so the two variables must be treated separately. In addition, it is well known that the qualitative effect of government spending on consumption dynamics is sensitive to modeling assumptions, which, for our purposes, can affect the term structure of interest rates as well. Thus, it is helpful to clarify some microfoundations of the model. In particular, following Monacelli and Perotti (2008) and Bilbiie (2011), we assume the representative household in the macro block has GHH (Greenwood, Hercowitz, and Huffman (1988)) preferences, and firms face sufficiently large nominal price rigidities. These assumptions, which are common in the literature, ensure positive government spending raises consumption, a feature that is widely supported empirically (e.g., Blanchard and Perotti (2002) and Galí, López-Salido, and Vallés (2007)).

The linearized model is characterized by the following equations:

$$\hat{c}_t = \mathbb{E}_t\left[\hat{c}_{t+1}\right] - \frac{1}{\tilde{\gamma}}\left(r^{(1)}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \bar{r}\right) + \psi g_t \tag{B.1}$$

$$\hat{\pi}_t = \kappa \hat{y}_t \tag{B.2}$$

$$r^{(1)}_t = \max[0, \bar{r} + \phi\hat{\pi}_t] \tag{B.3}$$

$$\bar{c}\hat{c}_t + g_t = \hat{y}_t, \tag{B.4}$$

where $\tilde{\gamma}, \psi > 0$, and $\bar{c}$ is the consumption-output ratio at the steady state. $\tilde{\gamma}$ and ψ are each functions of primitive parameters characterizing utility from consumption and labor (the details of which are not essential for the discussion here), as well as the exogenous government spending process g_t. We assume g_t follows a 3-state Markov chain with the state space $\{g_H, g_M, g_L\} = \{g, 0, -g\}$ and the same transition matrix assumed for δ_t. We also assume conditions (9) hold. For simplicity and to keep the changes from the baseline model as minimal as possible, we assume the asset pricing block is essentially the same as the baseline, i.e., the stochastic discount factor used to price bonds is characterized by equation (5) without the δ_t term. In this setup, we can prove the following proposition:

Proposition B.1 (**Amplification/compression of the term premium in a model with government spending shocks**). *For the high and low states in the model with the ELB:*

$$|tp^{(2)}_{E,H}| > |tp^{(2)}_{U}| \quad (Amplification) \tag{B.5}$$

$$|tp^{(2)}_{E,L}| < |tp^{(2)}_{U}| \quad (Compression) \tag{B.6}$$

Proof. First, we rewrite equation (B.1) in terms of output using equation (B.4). Then, the system composed of the rewritten equation (B.1) through equation (B.3) can be recast as the 3-state system in the same way as the baseline model. In particular, the solution for $\{y_s, \pi_s, r^{(1)}_s\}_{s\in\{H,M,L\}}$ is exactly the same as in the baseline model up to the parameters γ, κ, and the size of the shock. Thus, following the proof of Proposition 1, $\Delta^y_{E,HM} > \Delta^y_U > 0$ and $\mathbb{V}_U[y] < \mathbb{V}_{E,H}[y]$.

Second, with the solution in hand, we compute the 2-period term premium. The 2-period term premium for the model without the ELB is:

$$\begin{aligned} tp^{(2)}_{U,H} = tp^{(2)}_{U,M} = tp^{(2)}_{U,L} \equiv tp^{(2)}_{U} &= -\frac{1}{2}\left[\mathbb{C}\mathrm{ov}_t(\gamma\Delta c_{t+1}, r^{(1)}_{t+1}) + \mathbb{C}\mathrm{ov}_t(\pi_{t+1}, r^{(1)}_{t+1})\right] \\ &= -\frac{1}{2}\left[\gamma\mathbb{C}\mathrm{ov}_t(c_{t+1}, r^{(1)}_{t+1}) + \kappa\mathbb{C}\mathrm{ov}_t(y_{t+1}, r^{(1)}_{t+1})\right] \\ &= -\frac{1}{2}\left[\gamma\mathbb{C}\mathrm{ov}_t(\frac{1}{\bar{c}}(y_{t+1} - g_{t+1}), r^{(1)}_{t+1}) + \kappa\mathbb{C}\mathrm{ov}_t(y_{t+1}, r^{(1)}_{t+1})\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\mathbb{C}\mathrm{ov}_t(y_{t+1} - g_{t+1}, r^{(1)}_{t+1}) + \kappa\mathbb{C}\mathrm{ov}_t(y_{t+1}, r^{(1)}_{t+1})\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa\mathbb{C}\mathrm{ov}_t(y_{t+1} - g_{t+1}, y_{t+1}) + \phi\kappa^2\mathbb{V}_t(y_{t+1})\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa[\mathbb{V}_U[y] - \mathbb{C}\mathrm{ov}_U[g, y]] + \phi\kappa^2\mathbb{V}_U[y]\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa p(1-p)\Delta^y_U(\Delta^y_U - g) + \phi\kappa^2\mathbb{V}_U[y]\right]. \end{aligned} \tag{B.7}$$

Importantly, with GHH preferences and sufficiently sticky prices (a small enough κ), we can show that $\Delta^y_U - g > 0$. Since $c_t = \frac{1}{\bar{c}}(y_t - g_t)$, this condition ensures consumption increases with government spending. Thus, $tp^{(2)}_U < 0$.

The term premiums for the model with the ELB for state H and L are:

$$\begin{aligned} tp^{(2)}_{E,H} &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa\mathbb{C}\mathrm{ov}_t(y_{t+1} - g_{t+1}, y_{t+1}) + \phi\kappa^2\mathbb{V}_t(y_{t+1})\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa[\mathbb{V}_{E,H}[y] - \mathbb{C}\mathrm{ov}_{E,H}[g, y]] + \phi\kappa^2\mathbb{V}_{E,H}[y]\right] \\ &= -\frac{1}{2}\left[\frac{\gamma}{\bar{c}}\phi\kappa p(1-p)\Delta^y_{E,HM}(\Delta^y_{E,HM} - g) + \phi\kappa^2\mathbb{V}_{E,H}[y]\right] \end{aligned} \tag{B.8}$$

$$tp^{(2)}_{E,L} = 0. \tag{B.9}$$

Given $\Delta^y_U - g > 0$, $\Delta^y_{E,HM} > \Delta^y_U$ implies $\Delta^y_{E,HM} - g > 0$. Finally, the proposition follows from comparing the expressions (B.7) through (B.9) using $\Delta^y_{E,HM} > \Delta^y_U$ and $\mathbb{V}_{E,H}[y] >$

$\mathbb{V}_U[y]$. □

Why do we obtain term premium dynamics similar to the baseline model despite the government spending shock driving a wedge between consumption and output? The intuitive reason is that the shock is analogous to other types of demand shocks such as preference shocks insofar as moving consumption and inflation in the same direction.

C Stylized Model with Continuous States

C.1 Motivation

In Section 2, we showed how the ELB constraint generated endogenous heteroskedasticity in output, inflation and the policy rate, which affected term structure dynamics in interesting ways. Importantly, we argued that the ELB constraint can make term premiums time varying through the amplification and compression effects. We presented these results using a stylized New Keynesian term structure model with three states to obtain simple closed-form solutions.

In this section, we show that the results hold even if we relax some of the simplifying assumptions. First, we expand the state space to a continuous space. Second, we allow the Phillips curve to include a forward looking term, as in standard New Keynesian models. Third, instead of using a linearized macro block, we use a nonlinear one in which the Euler equation is fully consistent with the asset pricing block and solve the model with a global solution method. The last nonlinear modification is to show that our results do not rely on either agent heterogeneity or the approximations used in our derivation. We also provide further analysis on how the ELB constraint affects yields and term premiums across different maturities.

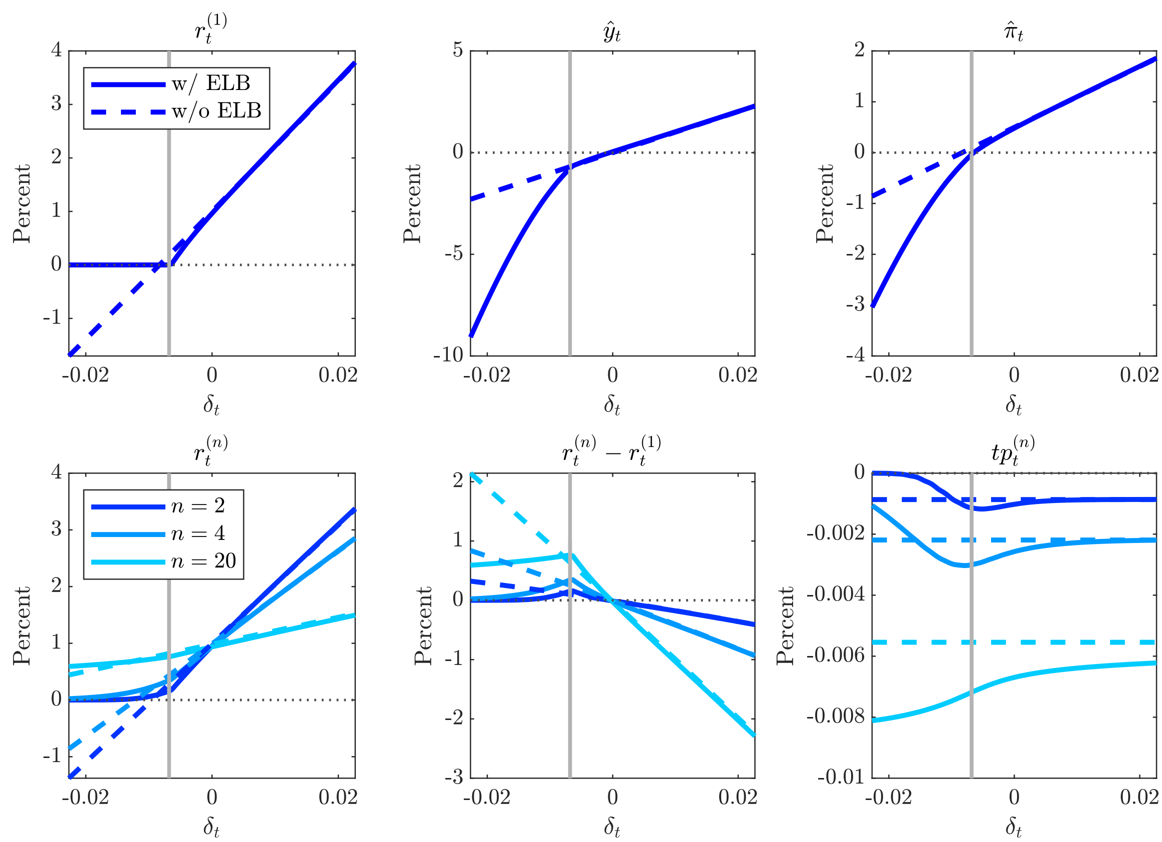
C.2 Model

Specifically, the model boils down to a nonlinear macro block with the same asset pricing block as in the 3-state model. The new macro block consists of the nonlinear Euler equation (4) with the stochastic discount factor (5), a nonlinear forward looking Phillips curve, and the monetary policy rule (3), where the exogenous process δ_t is continuous across states and follows an AR(1) process with standard-normal shocks. If linearized, the Euler equation and the Phillips curve will coincide with (1) and (2), respectively (up to a normalization of δ_t and an addition of the forward looking term in the Phillips curve). We delegate the details to Appendix D.2, where we recast this model in terms of a stripped-down version of the quantitative model. Since this model cannot be solved analytically, we solve it numerically with the time-iteration method used to solve our quantitative model.

C.3 Main Results

Figure C.1 summarizes the equilibrium of the model as functions of the demand shock δ_t, analogous to the 3-state case in Figure 1. The top panels of Figure C.1 plot the equilibrium policy rate, output and inflation (deviation from their steady states for the latter two) as functions of the demand shock δ_t. In addition to the functions for the model with the ELB (blue solid lines), we also show the corresponding functions for the model without the ELB (blue dashed lines) for comparison.

Figure C.1: **Equilibrium Functions of the Continuous-State Model**

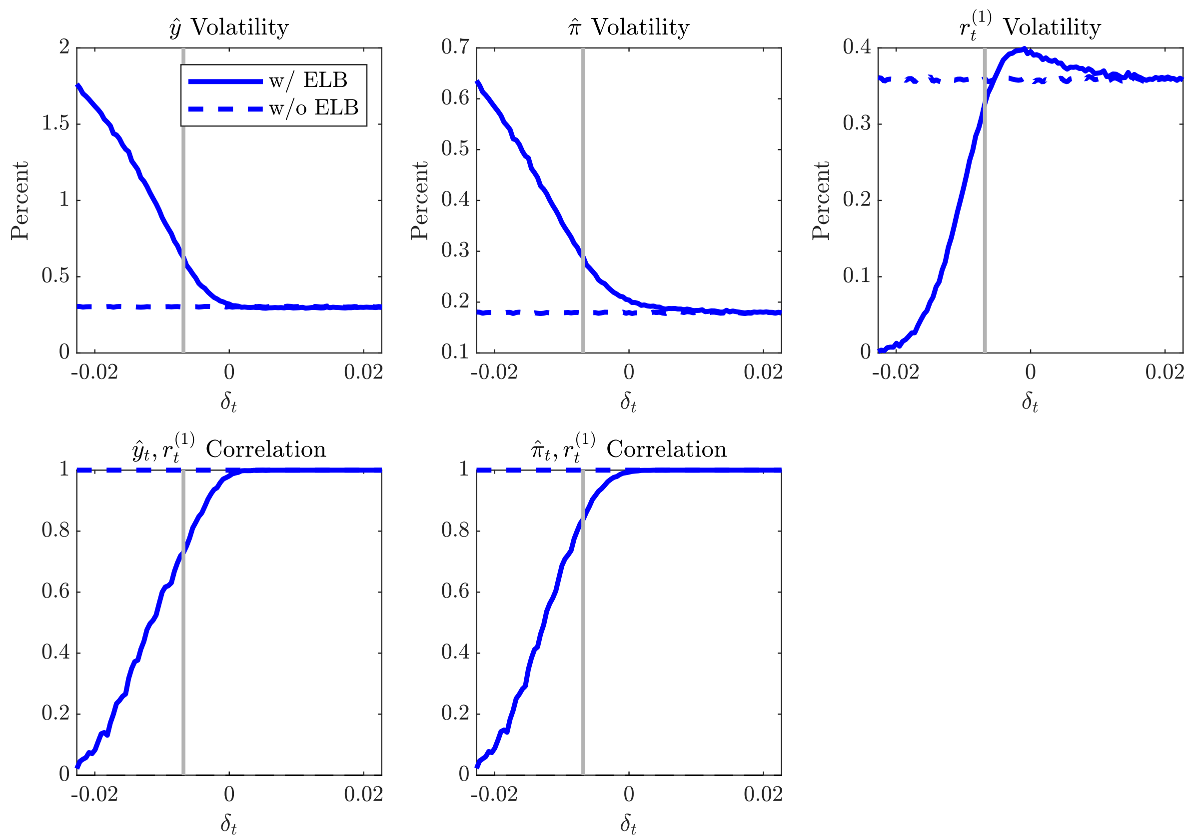


* Blue solid lines indicate variables of the model with the ELB constraint, and blue dashed lines indicate variables of the model without the ELB constraint. The gray vertical lines indicate the threshold state where the ELB binds.

The policy rate, output, and inflation all decrease as δ_t decreases, similar to the 3-state model. In addition, when monetary policy is subject to the ELB constraint, the decreases in output and inflation are larger at the ELB compared to when monetary policy is unconstrained, as we saw for the 3-state model. There are no such kinks in the equilibrium functions without the ELB constraint.

Based on a similar logic for the 3-state model, the equilibrium variables are basically linear with respect to δ_t when the model does not include the ELB, which implies homoskedasticity. In contrast, the variables are nonlinear with respect to δ_t when the model incorporates the ELB, which implies heteroskedasticity. While we cannot provide an analytical characterization as in Proposition 1 for the model with continuous states, we can still numerically compute the conditional volatility of each variable, as shown in the top panels of Figure C.2. Indeed, consistent with the results in Proposition 1, the conditional volatilities of output and inflation increase as the economy approaches the ELB (top-left and top-middle panels), while the volatility of the policy rate decreases (top-right panel). Analogous to the 3-state model, this endogenous heteroskedasticity is key to generate time-varying term premiums, as we discuss in detail below.

Figure C.2: **Conditional Volatilities and Correlations**



* Blue solid lines indicate variables of the model with the ELB constraint, and blue dashed lines indicate variables of the model without the ELB constraint. The gray vertical lines indicate the threshold state where the ELB binds.

The bottom-left and middle panels of Figure C.1 show the equilibrium yield and its spread with the policy rate for three different maturities (2, 4, and 20 periods). Again, the features highlighted using the 3-state model carry over to the model with continuous states.

When the model is not subject to the ELB constraint, yields increase with δ_t for all maturities. By contrast, the yield spread decreases with δ_t (countercyclicality). Also, as shown in the bottom-left panel, yields are less sensitive to changes in δ_t the longer the maturity, and thus the term structure of yield volatility is downward sloping. When the model is subject to the ELB constraint, yield spreads are no longer linear in δ_t. In particular, yield spreads are cyclical at the ELB, as in the 3-state model.

By computing yields for multiple maturities, it becomes evident that the ELB constraint affects yield dynamics for all maturities. Not surprisingly, shorter-maturity yields are more affected by the presence of the ELB than longer-maturity yields. Notice that, even when a longer-maturity yield is not constrained at the ELB, it can be influenced by the ELB constraint if the shorter-maturity yield is constrained or is expected to be constrained at the

ELB. For example, the 20-period yield is above the ELB at the lowest δ_t shown in the figure. However, at low realizations of δ_t, the 20-period yield is somewhat higher in the model with the ELB constraint than in the model without the ELB, reflecting the fact that the policy rate in the model with the ELB does not fall as much as it would in the model without the ELB. Relatedly, since shorter-maturity yields are more constrained than longer-maturity yields, the term structure of yield volatility is no longer necessarily downward sloping but potentially upward sloping at the ELB.

The bottom-right panel of Figure C.1 plots nominal term premiums. We confirm that the two features we highlighted in the 3-state model are present in the continuous state model. First, term premiums are negative regardless of the level of δ_t for both models with and without the ELB. We also verify that this holds regardless of yield maturity. Second, while term premiums are virtually constant in the model without the ELB, they are state dependent (or time varying) in the model with the ELB.

The continuous state model, however, sheds further light on the state dependency of term premiums, which was not necessarily clear from the 3-state model. To fix ideas, we focus on the 4-period term premium for the moment. When δ_t is at its lowest level, the term premium from the model with the ELB (solid line) is compressed near zero compared to the term premium from the model without the ELB (dashed line). However, as δ_t increases towards the state where the ELB is no longer binding (gray vertical line), the term premium with the ELB becomes more negative (amplified) compared to the term premium from the model without the ELB. These results correspond to what we showed for the 3-state model. In addition, the continuous state model shows that if δ_t increases further to the point where the probability of the ELB binding in the future becomes negligible, the term premium becomes increasing in δ_t, and eventually converges to the term premium without the ELB. Note that the ELB constraint affects term premiums even when the policy rate is away from the ELB because it binds stochastically in our model.

The mechanism behind the two features can be explained with basically what we described in Section 2. Recall that we can decompose the term premium into two components—macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations, as shown in equation (21). The latter can further be decomposed into interest rate volatility and the correlation between interest rates and a macro variable (output growth or inflation). According to this decomposition, the sign of the term premium is determined by the correlation between the policy rate and inflation/output growth. As shown in the bottom panels of Figure C.2, both correlations are positive for δ_t in both models with and without the ELB constraint.[54] As a result, the term premium is negative for any δ_t in both models with and without the ELB constraint.

We can also understand the state dependency of term premiums through this decomposition by analyzing how macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations vary with respect to δ_t. As shown in the top-left and top-middle panels of Figure C.2, two ingredients of macroeconomic uncertainty—output (growth) and inflation volatilities—do not depend on the level of δ_t as long as the policy rate is sufficiently above zero. However, when the policy rate is near or at the ELB, output (growth) and inflation volatilities increase as δ_t decreases, since the policy rate cannot be adjusted to mitigate the effects of shocks. Thus, macro uncertainty components of term premiums increase near

[54] Note that the conditional second moments of output and output growth 1 period ahead coincide.

and at the ELB.

The sensitivity of interest rates to macro fluctuations decreases, however, near and at the ELB, as both interest rate volatility and the correlation between the interest rate and macro variables decrease—as shown in the top-right and bottom panels of Figure C.2. As a result, the reduction in the sensitivity of interest rates to macro fluctuations near and at the ELB acts as a force that can reduce the absolute size of term premiums.

Whether nominal term premiums increase or decrease at and near the ELB thus depends on which of the two conflicting forces—an increase in macro uncertainty versus a decrease in interest rate sensitivity—dominates. In our stylized model, the amplification effect of higher macro uncertainty dominates the compression effect of lower interest rate sensitivity near the ELB, and the size of the term premiums increases for a certain range of δ_t. However, for a sufficiently small δ_t, the compression effect dominates the amplification effect, and the size of the term premiums decreases and approaches zero.

C.4 Term Premiums across Maturities

The bottom-right panel of Figure C.1 shows that term premium dynamics differ with respect to maturity. The forces that shape term premiums of longer maturities are more complicated than those of 2-period maturities. Yet, the basic tension is similar. We illustrate the mechanism of how the $n-$period term premiums are determined following a logic similar to what we used to understand the 2-period case. We continue to focus on nominal term premiums under power utility, but those derived from other utility functions or real term premiums could be understood analogously. Starting from the definition of the n-period term premium (6), we obtain a generalization of the decomposition (21) as follows:

$$
\begin{aligned}
tp_t^{(n)} &= \mathbb{E}_t\left[\sum_{i=0}^{n-2}\frac{n-i-1}{n}\mathbb{C}\mathrm{ov}_{t+i}(m_{t+i+1}, r_{t+i+1}^{(n-i-1)})\right] + u_t^{(n)} \\
&= \frac{n-1}{2}\mathbb{E}_t\left[\sum_{i=0}^{n-2}\omega_{n-i-1}(-\mathbb{C}\mathrm{ov}_{t+i}(\gamma\Delta y_{t+i+1}, r_{t+i+1}^{(n-i-1)}) - \mathbb{C}\mathrm{ov}_{t+i}(\pi_{t+i+1}, r_{t+i+1}^{(n-i-1)}))\right] + u_t^{(n)} \\
&= \frac{n-1}{2}\mathbb{E}_t\left[\sum_{i=0}^{n-2}\omega_{n-i-1}\left(-\sum_{x\in\{\Delta y,\pi\}}\gamma_x \times \sigma_{t+i}[x] \times \sigma_{t+i}[r^{(n-i-1)}] \times \rho_{t+i}[x, r^{(n-i-1)}]\right)\right] + u_t^{(n)},
\end{aligned}
\tag{C.1}
$$

where, as in (21), $\sigma_t[x]$ and $\rho_t[x,y]$ denote the 1-period-ahead conditional standard deviation of variable x and conditional correlation of variables x and y as of t, respectively. Also, $\gamma_{\Delta y} = \gamma$ and $\gamma_\pi = 1$. The second equality follows by defining $\omega_{n-i-1} \equiv \frac{2(n-i-1)}{n(n-1)} > 0$ such that $\sum_i \omega_{n-i-1} = 1$, and from the specification of the stochastic discount factor (5). Finally, $u_t^{(n)}$ collects all terms arising from a Jensen's inequality effect, which we ignore here.[55]

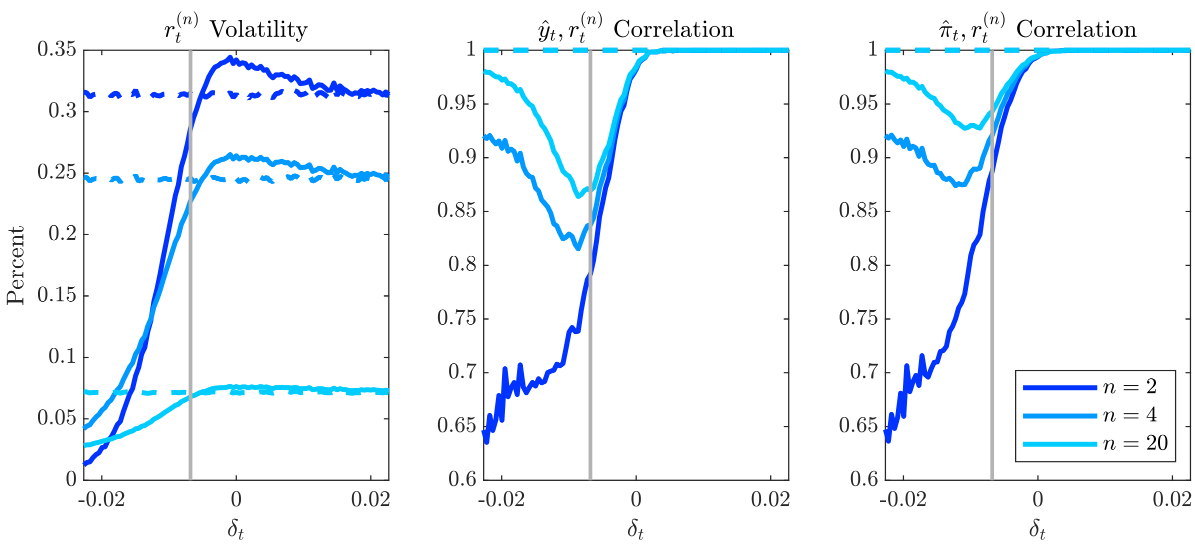
According to equation (C.1), the n-period term premium can be decomposed into the conditional expectation of a weighted average of several components. These components include the current and future 1-period-ahead conditional volatilities of output, inflation and *yields up to maturity* $n-1$, as well as the 1-period-ahead conditional correlation between

[55]Since $u_t^{(2)} = 0$, this term did not appear in (21).

output/inflation and *the same yields*. In other words, what matters for longer-maturity term premiums are (expectations about) the sensitivity of *longer-term yields* (instead of the short-term policy rate) to macro fluctuations.[56]

In Figure C.3, we plot the 1-period-ahead conditional yield volatilities as well as the correlation between yields and output/inflation. Given $\rho_{t+i}[x, r^{(n-i-1)}] > 0$ for $x \in \{\Delta y, \pi\}$ as implied by the middle and right panels of the figure, the negative term premiums of n-period maturities follow naturally.

Figure C.3: **Conditional Volatilities and Correlations**



* Solid lines indicate variables of the model with the ELB constraint and dashed lines indicate variables of the model without the ELB constraint. The gray vertical lines indicate the threshold state where the ELB binds.

Compared to shorter-term yields, conditional moments of longer-term yields depend less on the current state, as they largely reflect the economy's eventual convergence to its ergodic distribution. Thus, the ELB is typically less binding for longer-term yields, and reduces the sensitivity of longer-term yields by less than the sensitivity of short rates. Indeed, Figure C.3 shows that the volatility of yields as well as the correlation between yields and output/inflation are less affected by the ELB for longer maturities. The weak state dependency of conditional moments of longer-term yields, in turn, implies that the change in longer-maturity term premiums near and at the ELB will be characterized mostly by the changes in macro uncertainty. For instance, as δ_t decreases, the fall in the 20-period term premium is larger and more consistent compared to its 2-period counterpart (bottom-right panel of Figure C.1). This is because the compression effect from a reduction in the yield sensitivity is dominated by the amplification effect of an increase in macro uncertainty, and the absolute size of the term premium is higher at the ELB than away from the ELB. Only when δ_t is sufficiently small does the former effect dominate the latter and the absolute size of the term

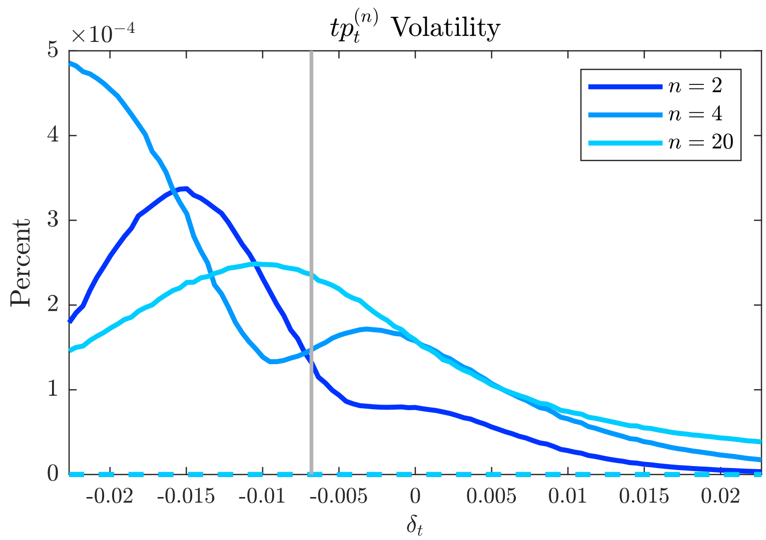
[56]To be more precise, the entire term structure matters, but the coefficients ωs put more weight on longer-term maturities.

premium declines (the state space shown in the figure is too small to show this eventual compression to zero). Interestingly, even when δ_t is high and the policy rate is far above the ELB, long-maturity term premiums in the model with the ELB are different from those in the model without the ELB. In sum, the amplification and compression effects of the ELB constraint can affect yields differently depending on their maturity, leading to distinct term premium dynamics across maturities.

The nonlinearity of term premiums as a function of δ_t points to time variation in term premium volatility that is unlike the time-variation in yield volatility. We plot term premium volatilities as a function of δ_t in Figure C.4. Naturally, yield volatility is constant (Figure C.3, left panel, dashed lines) and term premium volatility is virtually zero in the model without the ELB (Figure C.4, dashed lines). However, term premium volatility is positive and time varying in the model with the ELB (Figure C.4, solid lines). Specifically, for shorter maturities, term premium volatility increases as δ_t decreases from the steady state, but starts declining when δ_t is sufficiently low and the policy rate is expected to be at the ELB for a long time. That is, term premium volatility peaks when the short rate is currently at the ELB but is expected to be positive in the near future. For longer maturities, volatility peaks around the level of δ_t where the ELB starts binding. In contrast, yield volatility is on average lower while the policy rate is at the ELB than when it is not. These results suggest that term premiums may warrant more attention in a severe recession involving the ELB than under normal circumstances.

The analysis in this section can be further extended to incorporate Epstein-Zin preferences and applied to the real term structure as well as inflation compensation. The interested reader may refer to our earlier working paper Nakata and Tanaka (2016).

Figure C.4: **Term Premium Volatility**



* Solid lines indicate variables of the model with the ELB constraint and dashed lines indicate variables of the model without the ELB constraint. The gray vertical lines indicate the threshold state where the ELB binds.

D Equilibrium Conditions

D.1 Quantitative Model

In this section, we provide further details on the equilibrium of the quantitative model. Given the initial condition $\{R^*_{-1}\}$ and the exogenous processes $\{\delta_t, a_t, Z_t\}_{t\geq 0}$, a monopolistically competitive equilibrium is defined in a standard way as a set of stochastic processes $\{C_t(i)$, $N_t(i)$, $Y_t(i)$, C_t, N_t, Y_t, W_t, P_t, $P_t(i)$, $R_t^{(n)}$, $R_t^*\}_{t\geq 0}$, such that (a) households maximize utility; (b) firms maximize profits; (c) monetary policy follows the interest rate rule; (d) fiscal policies satisfy the budget constraint; and (e) goods, labor, and asset markets clear. To obtain a stationary equilibrium, we normalize relevant variables by the deterministic productivity trend Z_t. We list the normalized equilibrium conditions below:

$$\hat{V}_t = \begin{cases} \frac{\hat{C}_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} + \zeta\tilde{\beta}\left\{\mathbb{E}_t\left[\hat{V}_{t+1}^{1-\tilde{\gamma}}\right]\right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \geq 0 \\ \frac{\hat{C}_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} - \zeta\tilde{\beta}\left\{\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \leq 0. \end{cases} \tag{D.1}$$

$$\widetilde{M}_{t+1} = \begin{cases} \tilde{\beta}\left(\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)^{-\chi_c}\left[\frac{\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right]^{\frac{1}{1-\tilde{\gamma}}}}\right]^{-\tilde{\gamma}}\frac{1}{\Pi_{t+1}} & \text{for } U_t \geq 0 \\ \tilde{\beta}\left(\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)^{-\chi_c}\left[\frac{-\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right]^{\frac{1}{1-\tilde{\gamma}}}}\right]^{-\tilde{\gamma}}\frac{1}{\Pi_{t+1}} & \text{for } U_t \leq 0 \end{cases} \tag{D.2}$$

$$\mathbb{E}_t\left[\exp(-\delta_t)\widetilde{M}_{t+1}R_t^{(1)}\right] = 1, \tag{D.3}$$

$$Q_t^{(n)} = \mathbb{E}_t[\exp(-\delta_t)\widetilde{M}_{t+1}Q_{t+1}^{(n-1)}] \qquad (\text{for } n \geq 2), \tag{D.4}$$

$$\ln R_t^{(n)} = -\frac{1}{n}\ln Q_t^{(n)}, \tag{D.5}$$

$$\hat{w}_t = N_t^{\chi_n}\hat{C}_t^{\chi_c}, \tag{D.6}$$

$$\hat{Y}_t\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} - (1-\theta) - \theta\frac{\hat{w}_t}{\exp(a_t)}\right] = \mathbb{E}_t\left[\zeta\widetilde{M}_{t+1}\Pi_{t+1}\hat{Y}_{t+1}\varphi\left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\right], \tag{D.7}$$

$$R_t^{(1)} = \max\left[R_{ELB}, R_t^*\right], \tag{D.8}$$

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_r}\left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{\hat{Y}_t}{\bar{Y}}\right]^{\phi_y}\right)^{1-\rho_r}, \tag{D.9}$$

$$\hat{Y}_t = \exp(a_t)N_t, \tag{D.10}$$

$$\hat{Y}_t = \hat{C}_t, \tag{D.11}$$

$$\delta_t = \rho_\delta\delta_{t-1} + \sigma_{\delta,t-1}\varepsilon_{\delta,t}, \qquad \varepsilon_{\delta,t} \sim \mathcal{N}(0,1), \tag{D.12}$$

$$a_t = \rho_a a_{t-1} + \sigma_{a,t-1}\varepsilon_{a,t}, \qquad \varepsilon_{a,t} \sim \mathcal{N}(0,1), \tag{D.13}$$

$$\sigma_{\delta,t} = \frac{\theta_{1,\delta}}{1+\exp(\theta_{2,\delta}(\delta_t - \theta_{3,\delta}))}\bar{\sigma}_\delta, \tag{D.14}$$

$$\sigma_{a,t} = \frac{\theta_{1,a}}{1+\exp(\theta_{2,a}(a_t - \theta_{3,a}))}\bar{\sigma}_a, \tag{D.15}$$

where "hat" variables indicate that they are normalized ($\hat{X}_t \equiv \frac{X_t}{Z_t}$ for all variables except for $\hat{V}_t \equiv \frac{V_t}{Z_t^{1-\chi_c}}$); note this notation is distinct from the "hat" used for indicating deviations from the steady state in Section 2). Also, $\tilde{\beta} \equiv \beta\zeta^{-\chi_c}$.

D.2 Stylized Model with Continuous States

The stylized model with continuous states analyzed in Appendix C can be interpreted as a stripped-down version of the quantitative model. Since the asset pricing block is the same as in the 3-state model, we only present the equilibrium conditions for the macro block:

$$\mathbb{E}_t\left[\exp(-\delta_t)\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\frac{R_t^{(1)}}{\Pi_{t+1}}\right] = 1, \tag{D.16}$$

$$Y_t\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)\frac{\Pi_t}{\bar{\Pi}} - (1-\theta) - \theta Y_t^{\gamma+\chi_n}\right] = \mathbb{E}_t\left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma} Y_{t+1}\varphi\left(\frac{\Pi_{t+1}}{\bar{\Pi}}-1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\right], \tag{D.17}$$

$$R_t^{(1)} = \max\left[1, \bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi}\right], \tag{D.18}$$

$$\delta_t = \rho_\delta\delta_{t-1} + \sigma_\delta\varepsilon_{\delta,t}, \qquad \varepsilon_{\delta,t} \sim \mathcal{N}(0,1). \tag{D.19}$$

Linearizing (D.16) gives us (1) up to a normalization of δ_t, linearizing (D.17) gives us (2) with a forward-looking term, and (D.18) is the same as (3). Because of the extended state space and the additional nonlinearity, we assign (standard) values to a few additional parameters; $\chi_n = 2$, $\theta = 6$, $\varphi = 100$, $\bar{\Pi} = 1 + 2/400$, $\sigma_\delta = 0.3/100$. Note $\beta = \frac{\bar{\Pi}}{\bar{R}}$, where $\bar{R} = \exp(\bar{r})$.

E Solution Method

We solve the quantitative model globally using a time-iteration method based on Coleman (1991). Similar methods are used in studies of the occasionally binding ELB constraint, such as Gavin, Keen, Richter, and Throckmorton (2015) and Nakata (2017). In addition to the iteration on decision rules conducted by these papers, we also iterate on the value function since our model includes recursive utility.

The equilibrium conditions of our model laid out in Appendix D can be cast in the general form $\mathbb{E}_t[f(\mathbf{Y}_{t+1}, \mathbf{Y}_t, \mathbf{X}_{t+1}, \mathbf{X}_t)] = 0$, where $\mathbf{Y}_t$ is a vector of non-predetermined variables and $\mathbf{X}_t$ is a vector of predetermined variables. Note that $\mathbf{X}_t$ can be decomposed into an endogenous state vector $\mathbf{X}_{t-1}^{END}$ and an exogenous state vector $\mathbf{X}_t^{EXO}$ such that $\mathbf{X}_t \equiv \{\mathbf{X}_{t-1}^{END}, \mathbf{X}_t^{EXO}\}$. In our quantitative model, $\mathbf{X}_{t-1}^{END} \equiv \{R_{t-1}^*(\mathbf{X}_{t-1})\}$ ($\mathbf{X}_{t-1}^{END} \equiv \{R_{t-1}^*(\mathbf{X}_{t-1}), J_{t-1}(\mathbf{X}_{t-1})\}$ in our extension to the RW rule) and $\mathbf{X}_t^{EXO} \equiv \{\delta_t, a_t\}$. To solve our model with recursive utility, it is convenient to expand the set of non-predetermined states $\mathbf{Y}_t$ to include the value function V_t. Thus, $\mathbf{Y}_t \equiv \{C_t(\mathbf{X}_t), Y_t(\mathbf{X}_t), N_t(\mathbf{X}_t), \Pi_t(\mathbf{X}_t), W_t(\mathbf{X}_t), R_t(\mathbf{X}_t), V_t(\mathbf{X}_t)\}$ (excluding such variables as the stochastic discount factor and the term structure, as they are unnecessary to obtain other equilibrium objects).

The time iteration starts by discretizing the state space and providing an initial guess of the decision rules and the value function for each node. We use the deterministic steady state as the initial guess $\mathbf{Y}_{t+1}^{(1)}$. Given the guess $\mathbf{Y}_{t+1}^{(1)}$, we can solve for $\mathbf{Y}_t^{(1)}$ and $\mathbf{X}_t^{END,(1)}$ using $\mathbb{E}_t[f(\mathbf{Y}_{t+1}^{(1)}, \mathbf{Y}_t^{(1)}, \mathbf{X}_{t+1}^{(1)}, \mathbf{X}_t)] = 0$ for each node in the discretized state space of $\mathbf{X}_t \equiv \{\mathbf{X}_{t-1}^{END}, \mathbf{X}_t^{EXO}\}$. This functional equation is solved by combining numerical integration with a nonlinear equation solver. The normally distributed shocks are discretized using the Gaussian quadrature rule. Via piecewise linear interpolation across the nodes and linear extrapolation beyond, we obtain a continuous decision rule vector $\mathbf{Y}_t^{(1)}$ (Richter, Throckmorton, and Walker (2014) also support this interpolation approach). We iterate this procedure of using a new guess $\mathbf{Y}_{t+1}^{(\tau+1)} = \mathbf{Y}_t^{(\tau)}$ to obtain $\mathbf{Y}_t^{(\tau+1)}$ and $\mathbf{X}_t^{END,(\tau+1)}$ for $\tau = 1, 2, 3, ...$ until convergence.

Once the solutions (decision rules) for the macroeconomic variables are obtained, we can solve for the decision rules of the stochastic discount factor. The solution for the stochastic discount factor allows us to solve for equilibrium yields and term premiums.

We assess the numerical accuracy of our solution through the standard practice of examining Euler equation errors (for the consumption Euler equation). The Euler equation error is defined in the common form, which is normalized with respect to consumption: $EE_t \equiv \log_{10}|1 - \tilde{C}_t/\hat{C}_t|$, where $\hat{C}_t$ denotes the consumption (normalized by trend) directly from the decision rule, and $\tilde{C}_t$ is the consumption *implied* by the Euler equation. Table E.1 summarizes the quantiles of the error distribution generated from 200,000 simulated samples of our quantitative model. We find that the errors from the model—despite featuring strong nonlinearities due to high risk aversion, stochastic volatility and the ELB constraint—are small in economic magnitude and comparable to what are reported in the literature. For example, they are comparable to Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez (2015), who report errors with a median of -3.2 and a 99.9 percent quantile of -2.2 for their New Keynesian model with an occasionally binding ELB constraint.

Table E.1: **Euler Equation Errors**

Percentile	25	50	75	99.9
Error	-4.1	-3.9	-3.8	-3.4

* Errors are for the consumption Euler equation and expressed in base 10 logarithms.

F Data

For the short-term nominal interest rate, we use the 3-month T-bill rates from the Federal Reserve Board's H.15 statistical release. For nominal yields of 1-, 2-, 5-, and 10-year maturities, we use the zero-coupon yields from Gürkaynak, Sack, and Wright (2007). For TIPS yields of 5- and 10-year maturities, we use the zero-coupon yields from Gürkaynak, Sack, and Wright (2010). However, to account for the potentially large liquidity premiums embedded in TIPS yields, we subtract the liquidity premiums estimated by D'Amico, Kim, and Wei (2018) (DKW). We use quarterly averages from 1999:Q1 to 2015:Q4 to calibrate our model. These yield data are widely used in the literature.

For survey-based term premium estimates, we use the 3-month T-bill forecasts from the Blue Chip surveys. For each survey, we linearly interpolate across maturities assuming the forecast for a specific quarter represents the forecast for the midpoint date of that quarter to obtain a constant-maturity measure. We use long-range forecasts when available and interpolate across surveys to obtain a quarterly measure of constant-maturity interest rate forecasts. We subtract the survey-based expectations from the yields of corresponding maturity to obtain quarterly term premiums. For model-based empirical measures, we use the estimates from Kim and Priebsch (2024) and Andreasen, Jørgensen, and Meldrum (2025), as we explained in Section 3.2. For real term premiums, we use the estimates from DKW.

For output, we use data from Haver Analytics. We compute real GDP per capita by dividing seasonally adjusted real GDP data (GDPH@USECON) by the sum of the civilian labor force (LF@USECON) and the population not in the labor force (LH@USECON). We detrend real GDP per capita with an HP filter for the period above the ELB (1999:Q1 to 2008:Q3). For the ELB period, we detrend it using a linear trend that implies the gap between output and its trend has closed in 2015:Q4. We assume the HP-filtered output trend shifts to this linear trend from 2008:Q4. Our trend at the ELB is admittedly ad hoc, but is simple and appears sensible. We use the quarterly change in the GDP deflator (seasonally adjusted, JGDP@USECON) for our measure of inflation. Our choice of output and inflation data is standard.

For the policy rate forecasts necessary to compute the expected ELB duration, we use the median of the federal funds rate forecasts from the Survey of Primary Dealers and the 3-month T-bill forecasts from the Survey of Professional Forecasters, respectively.

G Longer-Maturity / Real Term Structure

We start by comparing the moments of the 10-year yield and term premium implied by our model with their data counterparts and their empirical estimates, respectively. The results are reported in Table G.1. We find that above the ELB, the average slope in the model is shallower than the data, but the term premiums are sufficiently positive and volatile in the model, consistent with empirical estimates. At the ELB, the model-implied yield and term premium dynamics are broadly consistent with the data, but the model-implied term premium volatility is notably smaller than its empirical counterpart. Interestingly, the term structure of average term premiums from the empirical estimates appears inversely *hump-shaped*, as the term premium becomes more negative as the maturity increases up to 5 years, but the 10-year term premium turns positive. Our model generates this pattern quite accurately. Meanwhile, the lack of 10-year term premium volatility may be partly due to the fact that yields and term premiums in the model do not react to (expectations about) an explicit QE policy.

We also compare the moments of real yields and term premiums implied by our model with their data counterparts and their empirical estimates, respectively. The results are in Table G.2. Due to poor market liquidity of shorter-maturity TIPS, we report the comparison for 5-year and 10-year maturities, following standard practice in the literature. As described in Appendix F, the TIPS yields used to compute the data moments are adjusted for liquidity premiums. For empirical estimates of real term premiums, we use the measures from D'Amico, Kim, and Wei (2018) (DKW). However, this measure does not explicitly account

Table G.1: **Term Structure Moments above and at the ELB —Model versus Data—**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
	A. Nominal Yields (10-year)				
Mean	4.92	3.99	2.77	3.04	1.93
Volatility	0.76	1.14	0.75	0.55	1.08
	B. Nominal Term Premium (10-year)				
Mean	0.79	0.68	0.35	0.28	-0.04
Volatility	0.36	0.52	0.56	0.26	0.35

* This table contains summary statistics for selected term structure variables comparing data versus model counterparts. The sample period is 1999:Q1–2008:Q3 for data above the ELB, and 2008:Q4–2015:Q4 for data at the ELB. All variables are in annualized percentage terms.

for the ELB, while alternative real term premium measures are difficult to find. Thus, we add the caveat that we use this measure only as a rough reference point (hence the values in parentheses).

Moments above the ELB: The average real yield slope is positive in the data, which our model can match qualitatively. The average 5-year yield in our model is about 1.1 percent, which is close to the data, but the model-implied yield slope is shallower (first two columns in Panel A). The volatility of real yields declines with maturity both in the data and in our model (first two columns of Panel B). The 10-year real yield volatility in our model is close to the data, but the model-implied volatility slope is, again, shallower.

Turning to real term premiums, in our model, the average premiums are positive and increase with maturity, but are smaller than nominal term premiums. The DKW measures also increase with maturity, but they are larger than our model across maturities (first two columns of Panel C). The volatility of real term premiums in our model increases with maturity and is somewhat smaller than the volatility of nominal term premiums. The size is broadly consistent with the DKW measures (first two columns of Panel D).

Table G.2: **Term Structure Moments above and at the ELB —Model versus Data—**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
	A. Real Yields (Mean)				
5-year	1.35	1.07	-0.58	0.51	-1.92
10-year	1.99	1.24	0.38	0.87	-0.52
	B. Real Yields (Volatility)				
5-year	0.90	0.59	0.31	0.12	1.50
10-year	0.58	0.52	0.47	0.16	0.85
	C. Real Term Premium† (Mean)				
5-year	(0.51)	0.15	(0.14)	0.02	-0.56
10-year	(0.81)	0.32	(0.41)	0.19	-0.25
	D. Real Term Premium† (Volatility)				
5-year	(0.23)	0.18	(0.22)	0.04	0.24
10-year	(0.26)	0.25	(0.30)	0.08	0.21

* This table contains summary statistics for selected term structure variables comparing data versus model counterparts. The sample period is 1999:Q1–2008:Q3 for data above the ELB, and 2008:Q4–2015:Q4 for data at the ELB. All variables are in annualized percentage terms.
†The term premium measure corresponding to the "Data" columns are from D'Amico, Kim, and Wei (2018).

Moments at the ELB: Real yields are on average lower at the ELB than above the ELB, both in the data and in our model. Also, similar to nominal yields, the average spread between the 10-year and 5-year yields is notably larger at the ELB both in the data and in our model (third and fourth columns of Table G.2, Panel A). Similar to nominal yields, the ELB increases average real yields (the fourth and last columns of Panel A), since monetary policy cannot lower rates further at the ELB despite the fall in inflation. Real yield volatility is upward sloping at the ELB in the data, which our model also captures qualitatively. However, the model-implied volatility is smaller than the data (third and fourth columns of Panel B).

In our model, real term premiums at the ELB are on average positive and increasing in maturity, similar to the DKW measures. However, the model-implied premiums are smaller (third and fourth columns of Panel C). Term premium volatility in our model is also increasing in maturity, consistent with the DKW measures, but the levels of the model-implied volatilities are smaller than their DKW counterparts (third and fourth columns of Panel D).

H Comparative Statics

In this section, we analyze the role of several key parameters in the quantitative model. Specifically, we focus on the parameters that control the stochastic volatility of the risk preference (δ_t) and productivity (a_t) processes, respectively, as well as the inertia coefficient and inflation coefficient in the monetary policy rule. We study the role of these parameters by comparing the moments from five alternative parameterizations of the model with the moments from the baseline model. The results are summarized in Tables H.1 and H.2. The takeaway: stochastic volatility in both δ_t and a_t, together with the monetary policy rule coefficients, are all important for the model to fit the data. We provide details below.

Model (Ia) refers to the parameterization where we assume the a_t process has no stochastic volatility. This is achieved by setting $\theta_{1,a} = 2$ and $\theta_{2,a} = 0$. Removing stochastic volatility from a_t results in less macroeconomic volatility and higher average yields above the ELB, compared to the baseline. The term premium implications are striking. Average term premiums are significantly smaller across maturities. Furthermore, the volatilities of term premiums are sharply reduced. The result that stochastic volatility is an important source of time-variation in term premiums is in line with the literature, as we discussed in Section 3.3. At the ELB, removing the stochastic volatility of a_t has mixed effects on the macroeconomy, and has generally smaller effects on the term structure compared to the effects above the ELB, since the impact of δ_t is dominant and thus the specification of a_t matters less.

Model (Ib) refers to the parameterization where we assume the δ_t process has no stochastic volatility. This is achieved by setting $\theta_{1,\delta} = 2$ and $\theta_{2,\delta} = 0$. In addition, we increase the steady state volatility of δ_t ($\bar{\sigma}_\delta$) to $\bar{\sigma}_\delta = 0.005$. This increase in $\bar{\sigma}_\delta$ is necessary because $\bar{\sigma}_\delta$ is too small for the model to hit the ELB without stochastic volatility. The size of the increase is determined such that the frequency at the ELB is similar to the baseline. This alternative parameterization leads to more volatile output and inflation. Also, yields are on average lower and more volatile above the ELB, compared to the baseline. Meanwhile, the term premium implications are similar to the baseline, as the removal of stochastic volatility from δ_t and the increase in $\bar{\sigma}_\delta$ have offsetting effects. Moving on to the effects of removing the stochastic volatility of δ_t at the ELB, we find that, despite increasing $\bar{\sigma}_\delta$, the drops in average output and inflation from their steady states are less severe, and the volatility of output, inflation and yields are significantly lower, compared to the baseline. In addition, the average term premiums are positive in contrast to the negative values in the baseline and the data. While further increasing $\bar{\sigma}_\delta$ can improve the model's fit at the ELB in terms of macro variables and the average term premium, this comes at a cost of excessive macro volatility and depressed term premiums above the ELB, counter to the data.

Model (Ic) refers to the parameterization where we assume no stochastic volatility in *both* the a_t and δ_t processes. This is achieved by setting $\theta_{1,a} = \theta_{1,\delta} = 2$ and $\theta_{2,a} = \theta_{2,\delta} = 0$. Also, we increase $\bar{\sigma}_\delta$ to the same level as in Model (Ib) because otherwise the ELB binds mostly due to *positive* supply shocks, which is counterfactual. Additionally, it makes for an easier comparison with Model (Ib), as this parameterization amounts to shutting down stochastic volatility of a_t from Model (Ib). This change further increases the impact of δ_t relative to a_t, lowering average term premiums even more than Model (Ib) when the economy is above the ELB. In addition, by removing both sources of stochastic volatility from the model, term premium volatility becomes negligible above the ELB. At the ELB, term premiums are compressed but slightly positive on average, and their volatility is negligible. Overall, the term premiums from this parameterization are even farther from their empirical counterparts compared to Model (Ib).

Model (II) refers to the parameterization where we assume no inertia in the monetary policy rule ($\rho_r = 0$). Above the ELB, assuming no inertia does not have material effects on model fit. At the ELB, however, assuming no inertia produces higher average yields and yield volatility, especially for shorter maturities, compared to the baseline. This is because the absence of inertia leads to less accommodative monetary policy at the ELB, and the model-implied average duration at the ELB is shorter for Model (II) than in the baseline. As a result, average output and inflation are lower compared to the baseline at the ELB. Also, since the loosening of the ELB constraint affects shorter-maturity yields the most, term premiums for shorter maturities are notably less compressed by the constraint and thus significantly more negative and somewhat more volatile compared to the baseline.

Model (III) refers to the parameterization with a lower inflation coefficient in the monetary policy rule of $\phi_\pi = 2.5$. Above the ELB, less emphasis on the stabilization of inflation leads to less volatile output and significantly larger volatility in inflation. In turn, average yields and yield volatility are higher compared to the baseline. The significant increase in inflation volatility leads to larger average term premiums, which are more volatile. At the ELB, the fall of average inflation is much more severe. Meanwhile, average yields are somewhat lower and yield volatility is higher across maturities compared to the baseline. Average term premiums are somewhat more negative for shorter maturities compared to the baseline. At the ELB where demand shocks become more dominant, the increase in inflation volatility acts more as a force to lower term premiums.

Table H.1: **Macro Moments above and at the ELB**
—Comparative Statics—

	Above the ELB							At the ELB						
	Data	Model						Data	Model					
		Base	(Ia)	(Ib)	(Ic)	(II)	(III)		Base	(Ia)	(Ib)	(Ic)	(II)	(III)
A. Macro Variables (Mean)														
Output†	—	—	—	—	—	—	—	-2.83	-2.33	-0.55	-1.23	0.33	-2.60	-3.15
Inflation	2.33	2.46	2.64	2.06	2.44	2.43	2.49	1.40	0.58	0.26	0.77	0.22	0.41	0.14
B.1 Macro Variables (Volatility)														
Output	0.85	0.98	0.52	1.05	0.62	0.98	0.87	1.77	2.07	2.36	0.95	0.56	2.09	2.05
Inflation	0.76	0.86	0.73	1.02	0.85	0.87	1.57	0.91	0.93	0.68	0.54	0.48	0.93	1.01
B.2 Macro Variables (Vol. of Vol.)														
Output	0.21	0.19	0.07	0.19	0.11	0.19	0.17	0.56	0.65	0.80	0.46	0.26	0.61	0.63
Inflation	0.26	0.16	0.10	0.18	0.13	0.16	0.30	0.34	0.30	0.27	0.25	0.16	0.29	0.34

* This table contains summary statistics for selected macroeconomic variables comparing data versus model counterparts. The sample period is 1999:Q1–2008:Q3 for data above the ELB, and 2008:Q4–2015:Q4 for data at the ELB. All variables are in annualized percentage terms except for output, which is not annualized.

The "mean" reported for output at/below the ELB is the average deviation from trend (for data) or the model-implied trend (for the model) in percentage points. We do not report statistics away from the ELB, as the mean roughly coincides with the trend.

Table H.2: **Yield and Term Premium Moments above and at the ELB —Comparative Statics—**

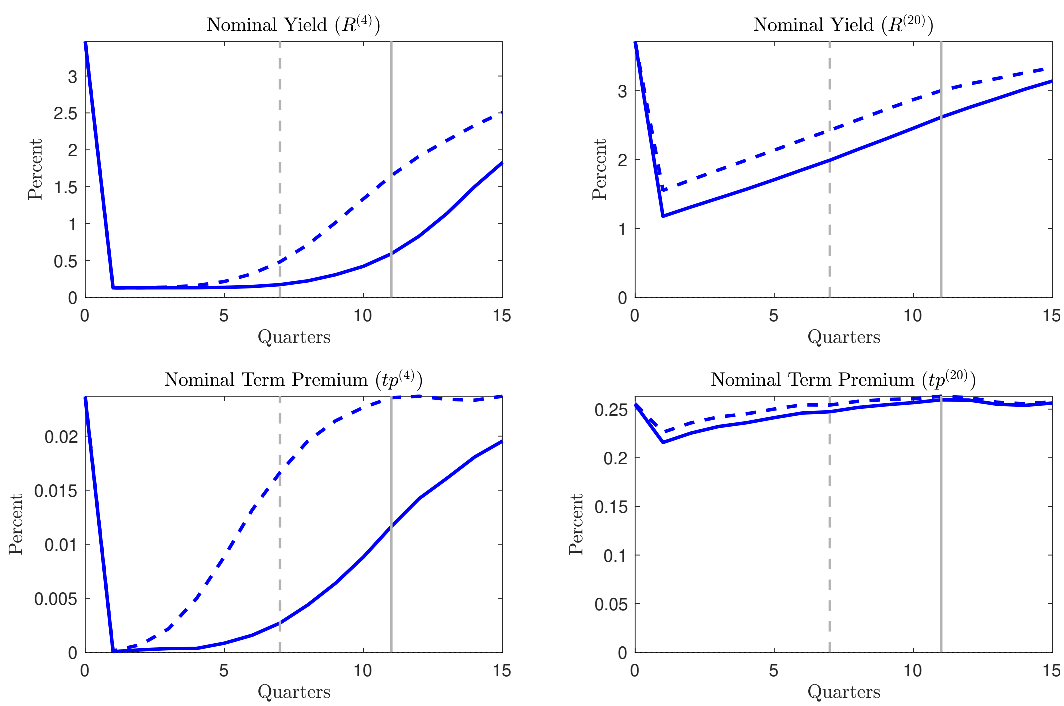
	Above the ELB							At the ELB						
	Data	Model						Data	Model					
		Base	(Ia)	(Ib)	(Ic)	(II)	(III)		Base	(Ia)	(Ib)	(Ic)	(II)	(III)
C. Nominal Yields (Mean)														
3-month	3.16	3.40	4.48	2.43	3.87	3.32	3.60	0.08	0.13	0.13	0.13	0.13	0.13	0.13
1-year	3.50	3.44	4.49	2.71	4.04	3.37	3.66	0.30	0.65	0.48	0.41	0.49	0.89	0.58
2-year	3.70	3.50	4.51	2.90	4.13	3.45	3.77	0.57	1.26	0.98	0.88	0.96	1.46	1.17
5-year	4.24	3.72	4.54	3.20	4.19	3.68	4.05	1.60	2.24	1.99	1.84	1.86	2.34	2.24
D. Nominal Yields (Volatility)														
3-month	1.71	1.53	1.85	2.10	2.40	1.57	2.19	0.06	0.00	0.00	0.00	0.00	0.00	0.00
1-year	1.72	1.50	1.71	1.89	2.06	1.52	2.16	0.16	0.35	0.36	0.16	0.15	0.40	0.34
2-year	1.54	1.46	1.52	1.71	1.72	1.47	2.09	0.27	0.52	0.59	0.31	0.24	0.55	0.54
5-year	1.10	1.35	1.13	1.43	1.15	1.35	1.87	0.56	0.59	0.69	0.49	0.28	0.62	0.68
E. Nominal Term Premium (Mean)														
1-year	0.16	0.05	0.02	0.04	0.01	0.06	0.07	-0.02	-0.08	-0.05	0.00	0.00	-0.16	-0.09
2-year	0.22	0.13	0.05	0.10	0.02	0.14	0.18	-0.10	-0.17	-0.12	0.02	0.01	-0.23	-0.19
5-year	0.41	0.37	0.12	0.33	0.06	0.39	0.49	-0.10	-0.09	-0.12	0.16	0.04	-0.10	-0.08
F. Nominal Term Premium (Volatility)														
1-year	0.19	0.08	0.01	0.09	0.00	0.10	0.11	0.07	0.05	0.09	0.00	0.00	0.08	0.06
2-year	0.27	0.19	0.02	0.20	0.00	0.21	0.25	0.10	0.08	0.17	0.05	0.00	0.09	0.07
5-year	0.30	0.41	0.02	0.41	0.01	0.42	0.52	0.29	0.17	0.25	0.17	0.00	0.18	0.17

* This table contains summary statistics for selected term premiums comparing estimates versus model counterparts. The sample period is 1999:Q1–2008:Q3 for estimates above the ELB, and 2008:Q4–2015:Q4 for estimates at the ELB. All variables are in annualized percentage terms.

I Alternative Forward Guidance Scenario

The results in Section 5 showed that the RW rule provided further policy accommodation compared to the baseline Taylor rule through the decline in the expected short-rate path, but this accommodation was partially offset by an increase in term premiums. However, since the effect of forward guidance is to compress the *absolute* size of term premiums, this result depends crucially on the risk exposure of bonds. To highlight this point, we show an alternative parameterization of the model in which productivity shocks play a dominant role at the ELB, contrary to our baseline model in which demand shocks play a dominant role. Specifically, we set the stochastic volatility parameters for the risk preference process δ_t to $\theta_{1,\delta} = 2$ and $\theta_{2,\delta} = 0$, such that there is no stochastic volatility in δ_t; volatility of δ_t can no longer increase significantly at the ELB. The impulse responses of yields and term premiums are summarized in Figure I.1. In this alternative case, the introduction of the RW rule leads

Figure I.1: **Effect of Accommodative Forward Guidance at the ELB —The Case of Positive Term Premiums at the ELB—**



* We plot $median[X_{t+h} \mid \varepsilon_{\delta,t} = -\vartheta_\delta, \bar{\Omega}]$ for a given variable X_t. The blue solid lines are the responses from the model under the RW rule, and the blue dashed lines are the responses from the model under the Taylor rule with the ELB constraint. The gray vertical lines indicate the timing of liftoff from the ELB corresponding to each rule.

to a decline in yields (top panels), similar to the baseline case. The difference lies in its effects on term premiums. Since bonds are more exposed to productivity shocks, average term premiums are positive even at the ELB in the alternative case. Then, the introduction of the RW rule lowers term premiums, amplifying the decline in the expected short-rate path.

In sum, the flattening effects of forward guidance on the expectations of the short-rate path can be either mitigated or amplified by changes in term premiums, depending crucially on the risk exposure of bonds.

References for Appendix

Andreasen, M. M., K. Jørgensen, and A. Meldrum (2025): "Bond risk premiums at the zero lower bound," *Journal of Econometrics*, 247, 105939.

Bilbiie, F. O. (2011): "Nonseparable preferences, frisch labor supply, and the consumption multiplier of government spending: One solution to a fiscal policy puzzle," *Journal of Money, Credit and Banking*, 43(1), 221–251.

Blanchard, O., and R. Perotti (2002): "An empirical characterization of the dynamic effects of changes in government spending and taxes on output," *Quarterly Journal of Economics*, 117(4), 1329–1368.

Coleman, W. J. (1991): "Equilibrium in a Production Economy with an Income Tax," *Econometrica*, 59(4), 1091–1104.

D'Amico, S., D. H. Kim, and M. Wei (2018): "Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices," *Journal of Financial and Quantitative Analysis*, 53(1), 395–436.

Fernández-Villaverde, J., G. Gordon, P. Guerrón-Quintana, and J. F. Rubio-Ramirez (2015): "Nonlinear Adventures at the Zero Lower Bound," *Journal of Economic Dynamics and Control*, 57, 182–204.

Galí, J., J. D. López-Salido, and J. Vallés (2007): "Understanding the effects of government spending on consumption," *Journal of the European Economic Association*, 5(1), 227–270.

Gavin, W. T., B. D. Keen, A. W. Richter, and N. A. Throckmorton (2015): "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics," *Journal of Economic Dynamics and Control*, 55, 14–38.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78(3), 402–417.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The US Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics*, 54(8), 2291–2304.

——— (2010): "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, pp. 70–92.

Kim, D. H., and M. A. Priebsch (2024): "Are Shadow Rate Models of the Treasury Yield Curve Structurally Stable?," *Journal of Financial and Quantitative Analysis*, 59(7), 3500–3530.

Monacelli, T., and R. Perotti (2008): "Fiscal policy, wealth effects, and markups," *NBER Working Papers*, 14584.

Nakata, T. (2017): "Uncertainty at the zero lower bound," *American Economic Journal: Macroeconomics*, 9(3), 186–221.

Nakata, T., and H. Tanaka (2016): "Equilibrium Yield Curves and the Interest Rate Lower Bound," *Finance and Economics Discussion Series*, 2016-85.

Richter, A. W., N. A. Throckmorton, and T. B. Walker (2014): "Accuracy, speed and robustness of policy function iteration," *Computational Economics*, 44(4), 445–476.