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Firm Networks and Asset Returns

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Firm Networks and Asset Returns

ABSTRACT

This paper argues that changes in the propagation of idiosyncratic shocks along firm networks are important to understanding variations in asset returns. When calibrated to match key features of supplier–customer networks in the United States, an equilibrium model in which investors have recursive preferences and firms are interlinked via enduring relationships generates long-run consumption risks. Additionally, the model matches cross-sectional patterns of portfolio returns sorted by network centrality, a feature unaccounted for by standard asset pricing models.

Firms do not function as isolated entities. Instead, they are interlinked via a variety of material relationships, such as strategic alliances, joint ventures, research and development (R&D) partnerships, and supplier–customer relationships. As shown by recent empirical evidence, these relationships may serve as propagation mechanisms of shocks to individual firms and, in doing so, potentially alter asset returns.[1] Despite this evidence, the asset pricing implications of such shock propagation remains, at best, imperfectly understood. In this paper, I develop an equilibrium model to study the asset pricing properties that stem from the propagation of idiosyncratic shocks along firm networks and the extent to which such shock propagation quantitatively explains asset market phenomena.

I show that changes in the propagation of idiosyncratic shocks along firm networks are important to understanding variations in asset returns, both in the aggregate and in the cross section. In particular, the model generates long-run consumption risk when calibrated to match key characteristics of supplier–customer networks in the United States. Consequently, the model replicates prime characteristics of asset market data, such as a high and volatile risk premium and a low and stable risk-free rate. Additionally, the model matches cross-sectional patterns of portfolio returns sorted by network centrality.

The model has two main features. First, idiosyncratic shocks propagate via long-lasting relationships. As a consequence, firms' cash-flow growth rates are related via a firm network. Second, investors have a preference for early resolution of uncertainty and, thus, care about uncertainty regarding firms' long-term growth prospects.

Aside from aggregate shocks, the distribution of aggregate consumption growth is shaped by two characteristics within the model: (a) the topology of the firm network and (b) the propensities of relationships to transmit idiosyncratic shocks, henceforth referred to as propensities. Propensities are assumed to vary over time. Such variation captures temporal changes in relationship-specific characteristics that make firms more susceptible to shocks

[1]See Hertzel et al. (2008), Jorion and Zhang (2009), Boone and Ivanov (2012), Carvalho et al. (2014), Boyarchenko and Costello (2015), Todo et al. (2015), Boehm et al. (2015) and Barrot and Sauvagnat (2016) among others. Using French firm-level data from 1990 to 2007, Di Giovanni et al. (2014) provide empirical evidence of the importance of firm-specific shocks in generating aggregate fluctuations.

affecting their neighbors. As propensities vary over time, the connectivity of the firm network also varies over time. This variation introduces a time-varying correlation structure among firms' cash-flow growth rates, which in equilibrium generates stochastic volatility in consumption growth.

In the calibrated model, changes in network connectivity are infrequent because firms tend to engage in enduring and stable relationships with their major customers. Then, the nature of these relationships generates long-lasting interdependencies among firms' cash-flow growth rates. In such an economy, idiosyncratic shocks to one firm have the potential not only to change the current cash flows of every neighboring firm, but also to change the long-term growth prospects of all such firms. Such infrequent changes in network connectivity are what fundamentally drive low-frequency movements in aggregate output growth, which, in equilibrium, generate a persistent component in expected aggregate consumption growth. As a result of investors having preferences for early resolution of uncertainty, the model generates long-run consumption risks. The model accounts for sizable risk premiums because investors fear that extended periods of low economic growth coincide with low asset prices. The model generates a small risk-free rate as a result of investors saving for long periods of low economic growth.

Besides generating long-run consumption risk, the calibrated model matches cross-sectional patterns of portfolio returns sorted by network centrality. Central firms command lower risk premiums than peripheral firms because, in the data, relationships of peripheral firms tend to exhibit higher propensities than relationships of central firms, as peripheral firms tend to rely more heavily on their major customers. As a consequence, central firms are less exposed to contagion risk than peripheral firms, commanding lower risk premiums. The model generates a realistic monthly return spread of 0.8% between firms in the lowest and firms in the highest decile of centrality. This economically and statistically significant return spread arises naturally in equilibrium as compensation for contagion risk, a feature unaccounted for by standard asset pricing models.

The small and persistent component in expected consumption growth generated by low-frequency movements in network connectivity provides an equilibrium foundation for long-run risk models in the spirit of Bansal and Yaron (2004). Moreover, the model helps explain the cross section of expected returns, as it provides a mapping between firms' importance in the network and their contagion risk. Overall, these results suggest that extending standard asset pricing models to take into account how idiosyncratic shocks propagate along firm networks can make significant progress toward generating a unifying framework that simultaneously captures dynamics of the aggregate and the cross section of stock returns.

This paper contributes to three strands of the literature. First, the paper develops a new theoretical framework that adds to a growing body of work focused on understanding the effects of economic linkages in asset pricing, for example, Buraschi and Porchia (2012), Ahern (2013), and Herskovic (2017).[2] Unlike these papers, however, this model emphasizes relationships at the firm level to explore the asset pricing properties that stem from the propagation of idiosyncratic shocks along firm networks.

Second, this paper adds to a body of work that explores how granular shocks may lead to aggregate fluctuations in the presence of linkages among different sectors of the economy, for example, Carvalho (2010), Gabaix (2011), Acemoglu et al. (2012, 2015), Oberfield (2013), Carvalho and Gabaix (2013), Blume et al. (2013), Elliott et al. (2014), Chaney (2014, 2016), and Lim (2016). This paper contributes to this literature by exploring the asset pricing implications of linkages at the firm level and studying how changes in the propagation of idiosyncratic shocks affect not only aggregate variables but also asset returns and aggregate risk premia.

Third, this paper adds to recent research that examines the potential sources of long-run risks, for example, Kaltenbrunner and Lochstoer (2010), Kung and Schmid (2015), Bidder

[2] Buraschi and Porchia (2012) show that more central firms in a market-based network have lower price dividend ratios and higher expected returns. Using the network of intersectoral trade, Ahern (2013) provides evidence that firms in more central industries have greater exposure to systematic risk. Herskovic (2017) focuses on efficiency gains that come from changes in the input-output network and how those changes are priced in equilibrium. My paper, on the other hand, focuses on how changes in the propagation of shocks within a fixed network alter equilibrium asset prices and risk premia.

and Dew-Becker (2016), and Collin-Dufresne et al. (2016).[3] This paper contributes to this literature by showing that changes in the propagation of idiosyncratic shocks along firm networks can generate long-run risks.

I. Baseline Model

Though stylized, the baseline model conveys the main intuition for how changes in the propagation of idiosyncratic shocks along firm networks, in combination with recursive preferences, generates long-run consumption risks and implications for the cross section of asset returns. To facilitate exposition, the baseline model abstracts from firms' production decisions and considers a single-good economy in which firm cash flows are related via a network of long-lasting relationships. Internet Appendix A shows that under some conditions, the main intuition continues to hold within an equilibrium framework where production is explicitly modeled.

A. The environment

Consider an economy with one perishable good and an infinite time horizon. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. In each period, the single good is produced by n infinitely lived firms, with n being potentially large. Firms' outputs, henceforth cash flows, are related via a network of long-lasting relationships.[4] Because I focus on the effect of the firm network on asset returns rather than on strategic network formation, relationships are

[3] Kaltenbrunner and Lochstoer (2010) shows that long-run risks endogenously arise in a standard production economy model, even when technology growth is i.i.d., because of consumption smoothing. Kung and Schmid (2015) shows that a model of endogenous innovation and R&D is able to generate long-run risks, while Bidder and Dew-Becker (2016) shows that long-run risks arise in an economy in which investors are pessimistic and not sure about the true model driving the economy. Collin-Dufresne et al. (2016) shows that parameter learning generates long-lasting risks that help explain standard asset pricing puzzles in an economy where investors are uncertain about the structural parameters governing the model economy.

[4] Long-lasting relationships potentially allow firms to circumvent difficulties in contracting due to unforeseen contingencies, asymmetries of information, and specificity on firms' investments, for example, Williamson (1979, 1983).

assumed to be exogenously determined and fixed before $t = 0$.[5] Besides firms, there is a large number of identical, infinitely lived individuals who are aggregated into a representative investor with Epstein-Zin-Weil preferences who owns all assets in the economy.

B. The firm network and firms' cash flows

Firms' cash flows vary stochastically over time and depend on aggregate and firm-level shocks. Input, labor, and capital decisions are deliberately normalized to 1. Firm i's cash flow at t, $y_{i,t}$, follows

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) \equiv \widetilde{a}_t + \widetilde{z}_{i,t} \ , \quad i \in \{1, \cdots, n\} \, , \tag{1}$$

where Y_{t-1} denotes the aggregate output of the economy at $t-1$, $\widetilde{a}_t \xrightarrow{d}$ i.i.d. $\mathcal{N}(0, 2\sigma_a^2)$ is a shock that affects all firms in the economy at t, and $\widetilde{z}_{it}$ is a shock that affects firm i at t.

A key feature of this model is that the firm network determines the dependence structure among shocks to individual firms. In particular, long-lasting relationships have a dual nature within the model. While relationships may increase firms' growth opportunities via efficiency gains, relationships may also have additional consequences as they increase a firm's reliance on its neighbors and, thus, increase a firm's exposure to negative idiosyncratic shocks affecting a broader set of firms in the economy. To capture such a trade-off, $\widetilde{z}_{i,t+1}$ is assumed to follow

$$\widetilde{z}_{i,t+1} = \alpha_1 d_i - \alpha_2 \widetilde{\varepsilon}_{i,t+1} \ , \quad i \in \{1, \cdots, n\} \ , \tag{2}$$

where parameters α_1 and α_2 are non-negative and equal across firms. Parameter d_i represents the number of direct relationships of firm i—which may differ across firms. Uncertainty on $\widetilde{z}_{i,t+1}$ is introduced by a Bernoulli random variable $\widetilde{\varepsilon}_{i,t+1}$. If firm i is either directly affected by

[5] See Demange and Wooders (2005), Goyal (2007), and Jackson (2008) for a detailed description of network formation models. For models of endogenous formation of production networks, see Oberfield (2013), Chaney (2014, 2016), and Lim (2016), among others.

an idiosyncratic shock or affected by an idiosyncratic shock that affects one of its neighbors, then $\widetilde{\varepsilon}_{i,t+1} = 1$. Otherwise, $\widetilde{\varepsilon}_{i,t+1} = 0$.

To simplify the modeling, the distribution of $\widetilde{\varepsilon}_{i,t+1}$ is determined by the following stochastic process—which abstracts from the temporal propagation of idiosyncratic shocks. At the beginning of $t+1$, each firm faces a negative shock independently of other firms with probability $0 < q < 1$, which is equal across firms and time invariant. A negative idiosyncratic shock to firm i at $t+1$ also affects firm j at $t+1$, and, thus, $\widetilde{\varepsilon}_{i,t+1} = \widetilde{\varepsilon}_{j,t+1} = 1$ if two things happen: (1) there exists a sequence of relationships that connects i and j in the firm network and (2) each relationship in that sequence transmits shocks at $t+1$.[6] The relationship between firms i and j either transmits shocks at $t+1$ or does not, independently of all other relationships, with probability $\widetilde{p}_{ij,t+1}$. For simplicity, relationships are assumed to be undirected, and, thus, $\widetilde{p}_{ijt} = \widetilde{p}_{jit}$, $\forall (i,j)$, $\forall\, t$. Consequently, $\widetilde{p}_{ij,t+1}$ measures the propensity of relation (i,j) to transmit idiosyncratic shocks from firm i (j) to j (i) at $t+1$.[7]

At a fundamental level, the value of $\widetilde{p}_{ij,t+1}$ captures interdependencies between the cash flows of firm i and firm j at $t+1$. Such interdependencies, which cannot be mitigated through

[6]Within the baseline model, only negative idiosyncratic shocks are allowed to propagate in a probabilistic manner. However, the baseline model can be easily extended to allow positive and negative shocks to propagate along the network. To do so, define $\widetilde{\psi}_{i,t+1} \equiv \widetilde{\varepsilon}_{i,t+1} - 1/2$ so that shocks can be positive and negative. Then, redefine equation (2) so that

$$\begin{aligned} \widetilde{z}_{i,t+1} &= \alpha_1 d_i - \alpha_2 \widetilde{\psi}_{i,t} \\ &= \alpha_2/2 + \alpha_1 d_i - \alpha_2 \widetilde{\varepsilon}_{i,t+1}, \end{aligned}$$

which is similar to equation (2). The cross sectional results in this paper continue to hold as long as the decrease in firms' cash flow growth due to negative shocks is larger than the increase in firms' cash flow growth due to positive shocks.

[7]In each period, this stochastic process can be thought of as a variation of either a reliability network or a bond percolation model. In a typical reliability network model, the edges of a given network are independently removed with some probability. The remaining edges are assumed to transmit a message. A message from node i to j is transmitted as long as there is at least one path from i to j after edge removal—see Colbourn (1987) for more details. Similarly, in a bond percolation model, edges of a given network are removed at random with some probability. Edges that are not removed are assumed to percolate a liquid. The question in percolation is whether the liquid percolates from one node to another in the network—which is similar to the problem of transmitting a message in a reliability context. For more details, see Grimmett (1989), Stauffer and Aharony (1994), and Newman (2010, Chapter 16.1). Blume et al. (2013) analyze a propagation mechanism similar to the one analyzed here. They focus, however, on strategic network formation issues in a static environment. They provide asymptotic bounds on the welfare of both optimal and stable networks and show that small amounts of "over-linking" may impose large losses in welfare to networks' participants.

contractual protections, may be driven by the characteristics of the relationship between i and j. Intuitively, the higher the value of $\widetilde{p}_{ij,t+1}$, the higher the likelihood that disruptions affecting the cash flow of firm i (j) also affect the cash flow of firm j (i) at $t+1$.[8]

Probabilities $\{\widetilde{p}_{ij,t+1}\}_{(i,j)}$ are drawn from a Beta distribution with parameters $\zeta_{1,t+1} > 0$ and $\zeta_{2,t+1} > 0$ at the beginning of period $t+1$. Parameters $\zeta_{i,t+1} > 0$, $i = \{1, 2\}$, which are drawn prior to drawing from the Beta distribution, determine the shape of the distribution of propensities across relationships at period $t+1$. The model timeline at period t is depicted in figure 1.

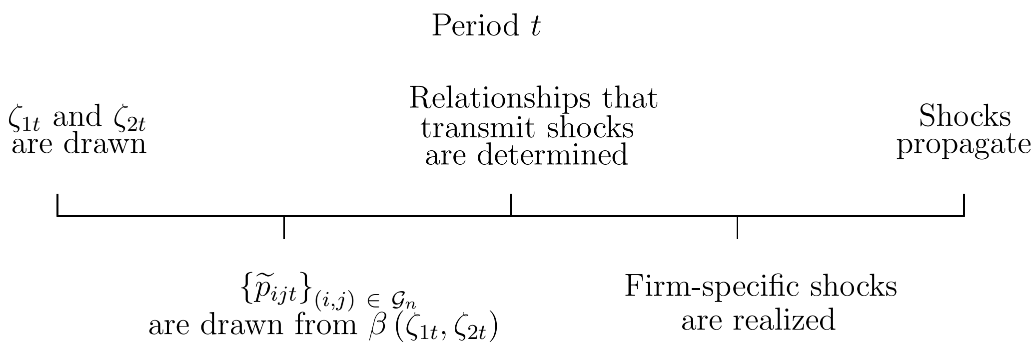

Figure 1. Model timeline in period t.

C. Propagation of idiosyncratic shocks and the distribution of $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$

To fix notation, let $\mathcal{G}_n$ denote the network of relationships among n firms—where nodes represent firms and edges represent relationships. Given how idiosyncratic shocks propagate along the network, the joint distribution of $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$ is determined by $\mathcal{G}_n$, q, and the process driving the stochastic propensity matrix $\widetilde{p}_t \equiv [\widetilde{p}_{ijt}]_{(i,j)}$. The marginal distribution of $\widetilde{\varepsilon}_{i,t}$, conditional on $\widetilde{p}_t$, depends on q, the network $\mathcal{G}_n$, and the location of firm i in $\mathcal{G}_n$. In other

[8]In the context of supply chains, $\widetilde{p}_{ij,t+1}$ may capture restrictions on firm i's and j's use of alternative inputs at $t+1$. The higher the value of $\widetilde{p}_{ij,t+1}$, the higher the switching costs firms i or j may face at $t+1$ and, thus, the higher the likelihood that a negative shock to firm i (j) also affects firm j (i), provided that firm j (i) may not be able to restructure its production sufficiently fast to overcome firm i (j)'s disruption in production.

words,

$$\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right) \quad = \quad f\left(q,\mathcal{G}_n,\text{location of firm } i \text{ in } \mathcal{G}_n\right),$$

where $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=0\middle|\widetilde{p}_t\right)=1-\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right)$, and $f(\cdot)$ is a mapping characterized by the stochastic process described in section I.B, which generates a time-varying correlation structure among firms' cash-flow growth as $\widetilde{p}_t$ varies over time.

Despite the fact that the mapping $f(\cdot)$ is hard to characterize for large n, its properties are easy to describe given the formulation of the stochastic process that generates it. First, in the absence of relationships, $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right)=\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\right)=q$, $\forall\ i$ and $\forall\ t$, so firm-level shocks are independent and identically distributed across firms over time. Second, if only one sequence of relationships exists between two firms, the longer the sequence, the smaller the correlation between firm-level shocks.[9]

D. *Temporal changes in shock propagation*

To capture temporal changes in relationship-specific characteristics, the shape parameters ζ_{it}, $i=\{1,2\}$, are allowed to vary over time. Variation in the shape parameters may arise from changes in complementarities among firms' activities or the arrival of new technologies that reshape the economy's long-term growth prospects. For simplicity, ζ_{it} takes two values, ζ_{iL} or ζ_{iH}, with $\zeta_{iL}<\zeta_{iH}$, and the shape parameter vector $\zeta_t\equiv[\zeta_{1t}\ \zeta_{2t}]$ follows a four-state ergodic Markov process with transition matrix Ω and states $\zeta_{LL}\equiv[\zeta_{1L}\ \zeta_{2L}]$, $\zeta_{LH}\equiv[\zeta_{1L}\ \zeta_{2H}]$, $\zeta_{HL}\equiv[\zeta_{1H}\ \zeta_{2L}]$, and $\zeta_{HH}\equiv[\zeta_{1H}\ \zeta_{2H}]$.[10]

[9] Having this feature—which is sometimes called correlation decay, as in, for example, Gamarnik (2013)—greatly helps obtain numerical solutions of the model relatively fast when n is large.

[10] The main results are robust to variations in the number of values that ζ_{it} can take. In unreported results, I allow ζ_{it} to take K values, with $K=\{3,4,5\}$, and, hence, the vector ζ_t follows a 9-, 16-, and 25-ergodic Markov process. In all those cases, the main results continue to hold.

II. Aggregate Consumption Growth

Aside from aggregate shocks, two features of the model are important to understanding the distribution of aggregate consumption growth: (a) the topology of $\mathcal{G}_n$ and (b) how idiosyncratic shocks propagate along $\mathcal{G}_n$, captured by the propensity matrix $\widetilde{p}_t$ and its dynamics. In this section, I study how changes in these two features affect the distribution of aggregate consumption growth and, thus, alter the distribution of the pricing kernel.

Let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)$ and $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ denote log consumption and output growth at $t+1$, respectively. Rather than assuming that aggregate consumption is the dividend on the portfolio of all invested wealth, I follow Campbell (1986), Cecchetti et al. (1993), and Abel (1999), and make the slightly more general assumption that the dividend on the aggregate stock market equals aggregate consumption raised to a power. Thus $\Delta\widetilde{c}_{t+1}$ and $\widetilde{x}_{t+1}$ satisfy

$$\widetilde{x}_{t+1} \quad = \quad \left(\frac{1}{\tau}\right)\Delta\widetilde{c}_{t+1}, \tag{3}$$

where τ is a constant. Hence, the representative investor is assumed to have access to labor income. As in Abel (1999), $(1/\tau)$ represents the leverage ratio on equity. If $\tau = 1$, then the market portfolio is a claim to total wealth. For tractability, consider $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$. It then follows from equations (1), (2), and (3) that

$$\begin{aligned}
\Delta\widetilde{c}_{t+1} \quad = \quad \tau\widetilde{x}_{t+1} \quad &= \quad \tau\log\left(\prod_{i=1}^{n}\left(\frac{y_{i,t+1}}{Y_t}\right)^{1/n}\right) \\
&= \quad \tau\left(\widetilde{a}_{t+1} + \alpha_1 \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} d_i\right)} - \alpha_2 \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}\widetilde{\varepsilon}_{i,t+1}\right)}\right) \\
&= \quad \tau\left(\widetilde{a}_{t+1} + \alpha_1 \quad \bar{d} \quad - \alpha_2 \quad \widetilde{W}_{n,t+1} \quad\right),
\end{aligned} \tag{4}$$

where $\bar{d}$ denotes the average number of relationships per firm in the economy, whereas $\widetilde{W}_{n,t+1}$ denotes the average number of firms affected by idiosyncratic shocks at $t+1$. It follows from equation (4) that the distribution of $\Delta\widetilde{c}_{t+1}$ critically depends on $\widetilde{W}_{n,t+1}$. Given how idiosyncratic shocks propagate along the network, the distribution of $\widetilde{W}_{n,t+1}$ is affected by $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$. As a result, these two features affect the distribution of $\Delta\widetilde{c}_{t+1}$.

To appreciate the importance of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ in determining the distribution of $\Delta\widetilde{c}_{t+1}$, consider two cases. First, suppose there are no relationships. Then, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ is a sequence of i.i.d. Bernoulli random variables and $n\widetilde{W}_{n,t+1}$ follows a Binomial distribution. By the Central Limit Theorem (CLT), $\sqrt{n}(\widetilde{W}_{n,t+1} - q)$ is normally distributed as n grows large. Provided the absence of relationships, the matrix $\widetilde{p}_{t+1}$ is irrelevant to determining the distribution of $\Delta\widetilde{c}_{t+1}$, as the unconditional mean and variance of $\sqrt{n}\widetilde{W}_{n,t+1}$ are q and $\frac{q(1-q)}{n}$, respectively. Second, suppose every firm has two relationships and each relationship has propensity p, which does not vary over time. Then, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ is a sequence of dependent Bernoulli random variables and $n\widetilde{W}_{n,t+1}$ approximately follows a Binomial distribution if p is sufficiently small—see Soon (1996). In this case, p affects the distribution of consumption growth, as the unconditional mean and variance of $\widetilde{W}_{n,t+1}$ are approximately π and $\frac{\pi(1-\pi)}{n}$, respectively, where $\pi \in [0, 1]$ solves the following equation:

$$\pi = q + (1-q)\pi p \left(\pi p + 2\left[p(1-\pi) + \pi(1-p)\right]\right).$$

Despite the fact that $\widetilde{W}_{n,t+1}$ is the aggregation of shocks to individual firms, there is no guarantee that $\Delta\widetilde{c}_{t+1}$ is normally distributed, as $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ is a sequence of dependent Bernoulli random variables in the presence of relationships. Figure 2 illustrates the previous point. Figure 2(a) depicts a star network in an economy with $n = 5$ firms, whereas figure 2(b) depicts the empirical probability density function of $\widetilde{W}_{n,t+1}$ for the star network depicted in figure 2(a). As figure 2(b) shows, the distribution of $\widetilde{W}_{n,t+1}$ may differ from a normal distribution if the elements of the matrix $\widetilde{p}_{t+1}$ are sufficiently close to 1. In particular, as some components in $\widetilde{p}_{t+1}$ tend toward one, the distribution of $\widetilde{W}_{n,t+1}$ tends to be bimodal.[11]

[11]For a large variety of network topologies, simulation shows that the distribution of $\Delta\widetilde{c}_{t+1}$ may differ from a normal distribution. In particular, if some elements of the matrix $\widetilde{p}_{t+1}$ are sufficiently close to one and $\mathcal{G}_n$ is locally connected—i.e., there is at least one sequence of relationships between any two firms in an

Despite the existence of relationships—and the convoluted dependencies they may generate among firm-level shocks—the topology of $\mathcal{G}_n$ and matrix $\widetilde{p}_{t+1}$ can be restricted so that (1) the distribution of $\widetilde{W}_{n,t+1}$ can be approximated by well-known distributions, and (2) $\Delta\widetilde{c}_{t+1}$ is normally distributed as the economy grows large. If $\Delta\widetilde{c}_{t+1}$ is normally distributed, keeping track of temporal changes in the distribution of $\Delta\widetilde{c}_{t+1}$ is equivalent to keeping track of temporal changes in averages and standard deviations. Then, the dynamics of consumption growth can be recast as a version of Hamilton (1989) Markov-switching model. Internet Appendix B provides conditions under which $\widetilde{W}_{n,t+1}$ follows a Poisson distribution when n is finite and conditions under which $\widetilde{W}_{n,t+1}$ follows a normal distribution when n grows large.

III. Asset Pricing

To see what $\mathcal{G}_n$ and ζ_t imply for asset returns, I embed the output correlation structure generated by the firm network into a standard asset pricing framework. The representative investor has Epstein-Zin-Weil recursive preferences to account for asset pricing phenomena that are challenging to address with power utility preferences. The asset pricing restrictions on the gross return of firm i, $\widetilde{R}_{i,t+1}$, are

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{R}_{i,t+1}\right) \;=\; 1, \tag{5}$$

where $\widetilde{M}_{t+1} \equiv \left[\beta\left(e^{\Delta\widetilde{c}_{t+1}}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}$ represents the pricing kernel at $t+1$ and $\widetilde{R}_{a,t+1}$ denotes the gross return on aggregate wealth—an asset that delivers aggregate consumption as its dividend each period.

To solve the model, I look for equilibrium asset prices so that price–dividend ratios are stationary, as in Mehra and Prescott (1985), Weil (1989), and Kandel and Stambaugh (1991). Because equilibrium values are time-invariant functions of the state of the economy,

arbitrarily large neighborhood around any given firm—then a non-negligible fraction of firms in the economy are almost surely affected by negative shocks. Therefore, the distribution of $\Delta\widetilde{c}_{t+1}$ may exhibit thicker tails than a normal distribution would.

which is determined by the state of the vector ζ_t, index t can be eliminated. Hereinafter, $s \in \mathcal{S} \equiv \{\mathrm{LL}, \mathrm{LH}, \mathrm{HL}, \mathrm{HH}\}$ denotes the current state of vector ζ.

The expected gross return of aggregate wealth in the current state is (see Appendix A for detailed derivations)

$$\mathbb{E}\left(R_a|s\right) = e^{\tau(\alpha_1 \bar{d}+\tau\sigma_a^2)} \sum_{s'\in\mathcal{S}} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a}\right) \mathbb{E}\left(e^{-\tau\alpha_2 \widetilde{W}_{n,t+1}} \middle| s'\right), \tag{6}$$

where w_s^a is the current price of aggregate wealth and is the solution of the following system of equations,

$$w_s^a = \beta e^{\tau(1-\rho)(\tau(1-\gamma)\sigma_a^2+\alpha_1\bar{d})} \left(\sum_{s'\in\mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{-\tau(1-\gamma)\alpha_2 \widetilde{W}_{n,t+1}} \middle| s'\right) \left(w_{s'}^a+1\right)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}}.$$

It follows from the above equations that the expected return and price of aggregate wealth are affected by (a) aggregate shocks, parameterized by σ_a^2, (b) the topology of $\mathcal{G}_n$, which determines $\bar{d}$; and (c) the dynamics of ζ_t, which jointly with $\mathcal{G}_n$, determines the distribution of $\widetilde{W}_{n,t+1}$. The dynamics of ζ_t, parameterized by Ω, affect the price and the expected return of aggregate wealth, as Ω determines the persistence of changes in network connectivity.

Next, I consider the risk-free asset, which pays one unit of the consumption good during the next period with certainty. If $R_f(s)$ denotes the gross return of the risk-free asset in the current state, then $R_f(s)$ solves

$$\frac{1}{R_f(s)} = \beta^{\frac{1-\gamma}{1-\rho}} e^{-\tau\gamma(\alpha_1\bar{d}-\tau\gamma\sigma_a^2)} \left(\sum_{s'\in\mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{\tau\gamma\alpha_2 \widetilde{W}_{n,t+1}} \middle| s'\right) \left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right). \tag{7}$$

Therefore, the equilibrium risk-free rate is also driven by aggregate shocks, the topology of $\mathcal{G}_n$, and the dynamics of ζ_t, as these three features affect the distribution of $\widetilde{W}_{n,t+1}$ and prices of aggregate wealth.

I now study the cross section of expected asset returns. To do so, it is convenient to express $\mathcal{G}_n$ as the union of connected components, which are sets of firms connected via at least one sequence of relationships. If $\mathcal{G}_n^i$ denotes the connected component that firm i belongs to, then $\mathcal{G}_n$ can be written as

$$\mathcal{G}_n \equiv \bigcup_{i \in \mathcal{G}_n} \mathcal{G}_n^i.$$

Define the following averages,

$$\widetilde{W}_{n,t+1}^{i} \equiv \frac{1}{n}\left(\sum_{j \in \mathcal{G}_n^i} \widetilde{\varepsilon}_{j,t+1}\right) \quad \text{and} \quad \widetilde{W}_{n,t+1}^{-i} \equiv \frac{1}{n}\left(\sum_{j \in \mathcal{G}_n \setminus \mathcal{G}_n^i} \widetilde{\varepsilon}_{j,t+1}\right),$$

where $\widetilde{W}_{n,t+1}^{i}$ represents the average number of firms in $\mathcal{G}_n^i$ affected by idiosyncratic shocks at $t+1$, whereas $\widetilde{W}_{n,t+1}^{-i}$ represents the average number of firms in $\mathcal{G}_n \setminus \mathcal{G}_n^i$ (the complement set of $\mathcal{G}_n^i$) affected by idiosyncratic shocks at $t+1$. If $v_i(s)$ denotes the current state-price of firm i, then the expected gross return of firm i is given by

$$\begin{aligned} \mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right) &= \frac{e^{(1/\tau)((1/\tau)\sigma_a^2+\alpha_1 \bar{d})}}{v_i(s)}\left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} v_i(s') \mathbb{E}\left(e^{-(1/\tau)\alpha_2 \widetilde{W}_{n,t+1}} \middle| s'\right)\right) \\ &+ \frac{e^{\sigma_a^2+\alpha_1 d_i}}{v_i(s)}\left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{-\alpha_2 \widetilde{\varepsilon}_{i,t+1}} \middle| s'\right)\right), \end{aligned} \tag{8}$$

where $v_i(s)$ solves

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{((1/\tau)-\gamma)^2\sigma_a^2+\alpha_1((1/\tau)-\gamma)\bar{d}} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'}\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{-((1/\tau)-\gamma)\alpha_2 \widetilde{W}_{n,t+1}} \middle| s'\right) v_i(s')\right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{(1+\gamma^2)\sigma_a^2+\alpha_1(d_i-\gamma\bar{d})}\left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \pi_i(s')\pi_{-i}(s')\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right), \end{aligned}$$

with $\pi_i(s') \equiv \mathbb{E}\left(e^{\alpha_2\gamma(\widetilde{W}_{n,t+1}^{i}-\widetilde{\varepsilon}_{i,t+1})} \middle| s'\right)$ and $\pi_{-i}(s') \equiv \mathbb{E}\left(e^{\alpha_2\gamma \widetilde{W}_{n,t+1}^{-i}} \middle| s'\right)$.

To appreciate how firms' connectivity affects expected returns, suppose all firms have the same number of relationships and relationships have the same propensity to transmit idiosyncratic shocks. In this case, the connectivity of any firm is equal to the connectivity of any other firm. Consequently, state-prices are equal across firms, as $d_i = \bar{d}$, $\mathbb{P}[\widetilde{\varepsilon}_{i,t+1} = 1|s] = p$, $\pi_i = \pi$, and $\pi_{-i} = \pi' \ \forall \ i$. Therefore, cross-sectional differences in state-prices and expected returns arise solely from differences in firms' connectivity.

As shown in (8), firm i's state-price and expected return are altered by how frequently other firms are affected by idiosyncratic shocks. To see this effect more clearly, consider two cases. First, consider firms within the same connected component as firm i. If the average number of firms affected by idiosyncratic shocks in $\mathcal{G}_n^i$ increases, but the likelihood that $\widetilde{\varepsilon}_{i,t+1} = 1$ does not, π_i increases and, thus, v_i increases. v_i increases because firm i is less vulnerable to idiosyncratic shocks affecting firms within the same connected component. Consequently, firm i's expected return decreases as a result of the decrease in exposure to contagion risk. Second, consider firms in connected components that are different from the one that firm i belongs to. The higher the average number of firms affected by idiosyncratic shocks in $\mathcal{G}_n \setminus \mathcal{G}_n^i$, the higher π_{-i} and, thus, the higher v_i. v_i increases because firms in $\mathcal{G}_n^i$ are not vulnerable to idiosyncratic shock affecting firms in $\mathcal{G}_n \setminus \mathcal{G}_n^i$ and, thus, they serve investors to improve their portfolio diversification. Consequently, firm i's expected return decreases as a result of gains in diversification. Therefore, firms' expected returns are affected by (a) firms' vulnerability to idiosyncratic shocks that affect other firms within the same connected component and (b) how frequently firms in other connected components are affected by idiosyncratic shocks.

IV. Calibration

So far, the model illustrates how changes in the propagation of idiosyncratic shocks along a firm network potentially alter equilibrium asset prices and expected returns. I now calibrate the model to match several features of supplier–customer networks in the United States and explore the extent to which the model quantitatively explains asset market phenomena. Section IV.A describes the data. Section IV.B describes the strategy employed to calibrate

$\mathcal{G}_n$ and ζ_t. Section IV.C describes the selection of the rest of the parameters in the model.

A. Data

A.1. Material relationships among U.S. public firms

I use annual data on relationships among U.S. public firms and their major customers to identify material relationships. The Statement of Financial Accounting Standards (SFAS) No.131 requires public firms to report information about customers who represent more than 10% of their annual revenues or sales; firms sometimes report customers below the 10% threshold. Reported customers' information is available on the COMPUSTAT Segment files. However, sometimes customers' names are abbreviated inconsistently over time. For these cases, I use a string-matching algorithm, similar to the one used by Atalay et al. (2011), which generates a list of potential customers in COMPUSTAT.[12] I then select the best match by inspecting a firm's name and industry information.

The dataset spans from 1976 to 2016 and consists of 8,779 different public firms. Similar to Barrot and Sauvagnat (2016), I consider firms i and j to be connected in all years ranging from the first to the last year that i reports j as one of its major customers. This assumption yields 66,355 unique annual supplier–customer relationships. Table I reports the distribution of firms across major industry groups. More than 65% of companies in the sample are classified as either manufacturing or service firms. Table II reports the evolution of the set of most connected firms over the sample period. Large manufactures, such as General Motors and Ford, dominate the early eighties. By the end of the sample, the shift in activity from manufacturing to retail and services is widespread, with Walmart and Cardinal Health being the most connected firms. The distribution of firms' sizes resembles the size distribution of the CRSP universe, but the size distribution of firms' customers is tilted toward large companies, as firms are only required to report customers that represent more than 10% of

[12] I thank Enghin Atalay for sharing the soundex code used in Atalay et al. (2011).

their annual revenues or sales.[13]

A.2. Propensity of relationships

Pivotal for my analysis is identifying the propensity of relationships to transmit idiosyncratic shocks. Unfortunately, propensities of supplier–customer relationships are unobservable. To deal with this issue, I rely on a composite of two measures to proxy for $\widetilde{p}_{ijt}$. The first measure is the percentage of annual sales that customers represent for their suppliers. The higher the percentage, the more likely it is that shocks affecting a customer also affect its supplier, other things being equal. The second measure uses information about the specificity of suppliers, as evidence documented by Barrot and Sauvagnat (2016) suggest that input specificity is a key driver in the propagation of idiosyncratic shocks along production networks. Their idea is simple: if supplier i is highly specific, then it is more likely that i is hard to replace in case of distress and, therefore, the likelihood that shocks affecting i also affect j is higher, all other things being equal. With these measures at hand, I proxy for $\widetilde{p}_{ijt}$ as

$$\widetilde{p}_{ijt} \quad = \quad \text{\% company } i\text{'s sales accounted for by } j \text{ at } t \times \text{ Specificity of } i \text{ at } t. \tag{9}$$

Percentages of annual sales are obtained from COMPUSTAT. To measure the specificity of suppliers, I construct a composite of three measures of input specificity, which I borrow from Barrot and Sauvagnat (2016).[14] Following Barrot and Sauvagnat (2016), I assume that firms are more likely to produce specific goods if they (a) operate in industries producing differentiated goods, (b) have high levels of R&D, or (c) hold a large number of patents. I

[13] Because firms need to be sufficiently large to represent at least 10% of the annual sales of publicly traded companies, many firms and their relationships are overlooked. As a consequence, one may be able to construct, in the most favorable case, a network that resembles a sparse representation of the U.S. economy. To partially ensure that the topology of the benchmark economy provides a fair representation of the U.S. economy, I compare the topology of the benchmark economy with the topology of networks constructed from BEA input–output tables. In unreported results, I show that the network in the benchmark economy does a good job representing some features of the time series of U.S. inter–industry networks and, in doing so, potentially provides a reasonable representation of the aggregate U.S. economy.

[14] I thank Julien Sauvagnat for sharing this dataset.

then compute the specificity of supplier i at t as

$$\text{Specificity of supplier } i \text{ at } t \;=\; \left(\frac{\textit{Rauch(i,t)} + \ \textit{R\&D(i,t-2)} + \ \textit{Patents(i,t)}}{3}\right),$$

where *Rauch(i,t)* $\in [0,1]$ denotes the share of differentiated goods produced in the industry of firm i at t according to Rauch (1999)'s classification of differentiated goods. *R&D(i,t)* $\in [0,1]$ denotes the ratio of R&D expenses to sales of firm i at $t-2$, as innovations may take some time to produce changes in the specificity of good i. *Patents(i,t)* $\in [0,1]$ denotes the ratio of the number of patents issued by firm i from $t-2$ to t to the maximum number of patents issued by any given firm within firm i's industry from $t-2$ to t.[15]

A.3. Firm-Level Financial Data

Monthly returns and annual financial data on firms are obtained from the CRSP/COMPUSTAT Merged Database and COMPUSTAT.[16] All continuous variables are winsorized at the 1st and 99th percentiles of their distributions.

A.4. Summary Statistics

Table III reports summary statistics for the sample. Panel A presents statistics at the annual level. The average and median percentages of sales that customers represent for their suppliers are 19% and 14%, whereas the average and median for suppliers' specificity scores are 34.2% and 34.9%. The main variable of interest is the propensity of relationships to transmit idiosyncratic shocks. The average and median for this variable are 11.4% and 4.4%, respectively. On average, there are eight years between the first and the last year a firm reports another firm as a major customer.

[15] Rauch (1999) classifies inputs into differentiated or homogeneous depending on whether goods are traded on an organized exchange. Each industry is coded as being either sold on an exchange, reference priced, or homogeneous. The ratio of R&D expenses to sales aims to capture the importance of relationship specific investments. The number of patents issued by suppliers aims to capture restrictions on alternative sources of inputs. For more details about the construction of these measures see Barrot and Sauvagnat (2016).

[16] Accessed via Wharton Research Data Service (WRDS).

To examine the persistence of the above variables, Panel B presents statistics regarding autocorrelation coefficients computed at the relationship level. The average first and second autocorrelation coefficients for the percentage of sales that customers represent for their suppliers are 31.5% and 26.8%, and their medians are 27.3% and 25.7%. The average first and second autocorrelation coefficients for suppliers' specificity scores are 29.5% and 23.8%, with medians of 25.1% and 21.4%. The propensities of relationships are also fairly persistent as their average first and second autocorrelation coefficients are 29.9% and 24.8%, with medians of 25.9% and 23%.

B. *Uncovering $\mathcal{G}_n$ and ζ_t*

B.1. Uncovering $\mathcal{G}_n$

To calibrate $\mathcal{G}_n$, I construct firm networks at an annual frequency over the sample period. Nodes represent firms and links represent supplier–customer relationships.

Table IV reports averages and standard deviations for key characteristics of U.S. supplier–customer networks. On average, there are 1,112 firms, 1,109 relationships, and 154 connected components per network. For illustration, Internet Appendix C depicts the time series of such networks. As these figures show, U.S. production networks are highly asymmetric in the sense that only a few firms are connected to many others, while most firms have either one or at most two connections. The degree distributions of these networks, which measure the frequency of firms with a given number of customers and suppliers, are highly skewed to the right. Most importantly, this high asymmetry is fairly persistent.[17]

I use the U.S. supplier–customer network in 2015 (depicted in figure 3) to pin down $\mathcal{G}_n$ as its topology matches several of the averages reported in Table IV. Thus, there are

[17] If a power law distribution is fitted to the degree distribution of each network, one obtains

$$\frac{\text{SD(exponent of power law distribution fitted to degree distribution)}}{\text{Mean(exponent of power law distribution fitted to degree distribution)}} = \frac{0.15}{2.23} = 6\%,$$

which emphasizes the fact that between 1976 and 2016 the level of asymmetry in U.S. production networks has been persistently high.

$n = 1,110$ firms, $1,146$ relationships, and 159 connected components in the benchmark economy. The main results continue to hold if supplier–customer networks of other years are fed into the benchmark economy, as the asymmetric structure of U.S. production networks is fairly persistent.

B.2. Uncovering ζ_t

To calibrate ζ_t, I use the cross-sectional distributions of propensities $\left\{\{\widetilde{p}_{ijt}\}_{(i,j)}\right\}_{t=1976}^{2016}$ constructed following equation (9). Using these values, I fit a Beta distribution to each cross sectional distribution. From this procedure, I obtain a time series of estimates, $\{\zeta_t^*\}_{t=1976}^{2016}$, which are depicted in figure 4. I then fit a vector autoregressive (VAR) process to the time series of estimates. After doing so, I discretize the fitted VAR into a four-state Markov chain using Gospodinov and Lkhagvasuren (2014)'s method and obtain $\zeta_{1\mathrm{L}}^* = 0.67$, $\zeta_{1\mathrm{H}}^* = 0.78$, $\zeta_{2\mathrm{L}}^* = 3.24$, $\zeta_{2\mathrm{H}}^* = 4.01$, and

$$\Omega^* = \begin{bmatrix} 0.57 & 0.27 & 0.06 & 0.05 \\ 0.25 & 0.63 & 0.02 & 0.12 \\ 0.12 & 0.02 & 0.63 & 0.25 \\ 0.05 & 0.06 & 0.27 & 0.57 \end{bmatrix},$$

with a stationary distribution given by $\mathbb{P}(\zeta_{\mathrm{LH}}^*) = \mathbb{P}(\zeta_{\mathrm{HL}}^*) = \mathbb{P}(\zeta_{\mathrm{LL}}^*) = \mathbb{P}(\zeta_{\mathrm{HH}}^*) = 0.25$.

C. Selecting the rest of the parameter values

The rest of the parameters can be separated mainly into two groups. Parameters in the first group define the preferences of the representative investor, which I select in line with Bansal and Yaron (2004). Thus, $\beta = 0.997$, $\gamma = 10$ and $\rho = 0.65$ (IES ≈ 1.5). Parameters in the second group define the dynamics of firms' cash flows, which I proxy with operating income. I restrict my focus to firms in the supplier–customer database, as relationships are known only for such firms. With these restrictions, I use the following regressions to determine α_1. First, I run the following OLS regressions,

$$\log\left(\frac{\text{operating income}_{i,t+1}}{\sum_j \text{operating income}_{j,t}}\right) = Controls + \epsilon^a_{t+1} + \epsilon^{ind}_i + \epsilon^z_{i,t+1} \tag{10}$$

where ϵ^a_{t+1} and ϵ^{ind}_i capture year and industry fixed effects, respectively. *Controls* include lagged values for firms' assets, age, and return on assets, to ensure that variation in the error term $\epsilon^z_{i,t+1}$ is not driven by trends in large, young, or profitable firms. Second, I run the following regressions at the annual frequency,

$$\widehat{\epsilon^z}_{i,t} = \beta_0 + \beta_1 d_i + \epsilon^{\varepsilon}_{i,t},$$

where $\widehat{\epsilon^z}_{i,t+1}$ are the residuals obtained from regression (10). I set $\alpha_1 = 0.3$ so that α_1 equals the average annual estimate of β_1 over the sample. I set $\alpha_2 = 0.3$ and $q = 0.1$ so that the unconditional mean and volatility of consumption growth generated by the model are similar to the ones found in the data. I use annual data on Total Factor Productivity (TFP) growth from the Federal Reserve Bank of San Francisco to determine the volatility of aggregate shocks, σ_a. I set $\sigma_a = 1.7$ so that σ_a equals the annual volatility of TFP growth. Finally, I follow Bansal and Yaron (2004) and set $\tau = 1/3$. Table V summarizes the key parameter values in the calibrated model.

V. Implications of the Calibrated Model

This section quantitatively evaluates the ability of the calibrated model to rationalize features of stock returns. It shows that changes in the propagation of idiosyncratic shocks, within a firm network that captures key characteristics of U.S. supplier–customer networks, are important to understanding variations in stock returns in both the aggregate and the cross section. Section V.A shows that the model generates long-run consumption risks. Section V.B shows that the model also matches cross-sectional patterns of portfolio returns sorted by network centrality. Internet Appendix D describes the methodology used to simulate the model.

A. *Firm Networks and Long-Run Risks*

Table VI exhibits moments generated under the benchmark parameterization. By construction, the benchmark economy delivers annual averages and volatilities of consumption and dividend growth similar to those found in the data. It also delivers an average market return of 12.3%, an annual volatility of the market return of 19.5%, an average risk-free rate of 2.16%, an annual volatility of the risk-free rate of 1.8%, an annual equity premium of 10%, and an average Sharpe ratio of 0.51. With the exception of the volatility of the risk-free rate and Sharpe ratio, all values are aligned with those found in the data.

Besides matching the above moments, the calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those assumed by the long-run risks (LRR) model of Bansal and Yaron (2004). As Bansal and Yaron (2004) and Bansal et al. (2012) show, these two features, together with Epstein-Zin-Weil preferences, help quantitatively explain an array of important asset market phenomena.[18] Table VII reports summary statistics of several similarity measures of time series generated with either the calibrated model or the LRR model. To compute averages and standard deviations, I sample from the calibrated model and the LRR model to construct two distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated economies over 620 periods. The first 100 periods are disregarded to eliminate any bias coming from the initial condition. As table VII suggests, both models generate similar time series for conditional expected consumption growth and conditional

[18] Since Bansal and Yaron (2004), several authors have used the long-run risk framework to explain an array of market phenomena. For instance, Kiku (2006) provides an explanation of the value premium within the long-run risks framework. Drechsler and Yaron (2011) show that a calibrated long-run risks model generates a variance premium with time variation and return predictability that is consistent with the data. Bansal and Shaliastovich (2013) develop a long-run risks model that accounts for bond return predictability and violations of uncovered interest parity in currency markets.

consumption volatility.

It is important to appreciate that the persistent component in expected consumption growth and stochastic consumption volatility are endogenously generated rather than exogenously imposed, as in many asset pricing models. The calibrated model generates these two features for two reasons: (1) relationships are long-lasting, and (2) the four parameter vectors estimated from the data, ζ^*_{LL}, ζ^*_{LH}, ζ^*_{HL}, and ζ^*_{HH}, generate similar cross-sectional distributions of $\{\widetilde{p}_{ijt}\}_{(i,j)\in\mathcal{G}_n}$, as figure 5 shows. Consequently, the connectivity of the firm network is fairly stable over time and, thus, the propagation mechanism of idiosyncratic shocks changes infrequently in the benchmark economy. These infrequent changes generate low-frequency movements in firms' growth prospects which, in turn, generate a persistent component in aggregate output and expected consumption growth.

Changes in the propagation mechanism of idiosyncratic shocks are infrequent because in the data firms tend to engage in enduring and stable relationships with their major customers. For instance, on average, relationships with major customers last more than eight years. The interdependency of such relationships generates long-term interdependencies among firms' cash-flow growth rates, fundamentally driving low-frequency movements in aggregate output growth. These low-frequency movements generate persistent changes in aggregate consumption growth in equilibrium. In such an economy, an idiosyncratic shock to a firm has the potential to affect not only the current cash flow growth of all neighboring firms, but also the long-term growth prospects of all such firms, enhancing the temporal effect of idiosyncratic shocks.

While the model endogeneously generates long-run consumption risks, it does not provide a complete micro-foundation of such risks because of the exogenous determination of the relationship structure. Nonetheless, the model provides a novel link between asset returns and firm networks and suggests that changes in the propagation mechanism of idiosyncratic shocks in fairly sticky production networks are quantitatively relevant to understanding asset market phenomena.

B. *Firms' Centrality and the Cross Section of Risk Premiums*

Besides endogenizing long-run consumption risks, the model helps in understanding the cross section of expected returns as it provides a mapping between firms' quantities of priced risk and firms' importance in the network. To measure the importance of a firm in the network, I define the centrality of firm i at period t as the average number of firms that can be affected by an idiosyncratic shock to firm i at t. This measure captures the relative importance of firm i in propagating idiosyncratic shocks at t. Because the cross-sectional distribution of $\{\widetilde{p}_{ijt}\}_{(i,j)\in\mathcal{G}_n}$ changes over time, firms' centrality scores change over time as well.

To quantitatively assess the effect of a firm's importance in the network on a firm's risk–return trade off, I simulate the benchmark economy at a monthly frequency and construct portfolios based on centrality. Firms are assigned into centrality deciles once per year, and the value-weighted portfolios are not rebalanced for the next 12 months. This exercise reveals that a portfolio that is long the lowest centrality decile portfolio and short the highest centrality decile portfolio generates a statistically significant return of 0.8% per month. Such a return is computed using 200 simulated economies over 1100 monthly observations. I disregard the first 100 observations in each simulation to eliminate any potential bias coming from the initial condition.

The above result is explained by the fact that relationships of peripheral firms in the calibrated model (as in data) tend to exhibit higher propensities than relationships of central firms.[19] Consequently, peripheral firms tend to have higher exposure to idiosyncratic shocks affecting their neighbors. On average, such contagion risk outweighs the potential benefits peripheral firms receive from their few relationships and, thus, peripheral firms command higher risk premiums than central firms. Central firms, however, seem to benefit from diversification of their neighbors as their relationships exhibit, on average, small propensities.

[19] Empirical support for this fact can be found if one plots propensity versus centrality of relationships for each annual U.S. supplier–customer network. These plots are depicted in Internet Appendix E.

As a result, their contagion risk is outweighed by the benefits generated by their many relationships.

Table VIII shows that the calibrated model generates a realistic spread between low and high centrality portfolios as the average monthly return difference between low and high centrality portfolios for firms in the database is 0.82% (with a t-statistic of 4.06). Table VIII reports monthly average raw returns, alphas and loadings from the five–factor model of Fama and French (2015) for two portfolios of stocks sorted by annual centrality as well as the portfolio that is long the lowest centrality decile and short the highest centrality decile.

As table VIII suggests, there is a significant negative relation between firms' centrality and future returns in the data that cannot be captured by standard asset pricing models such as the five-factor model. Firms in the lowest centrality decile command an average monthly return of 2.28%, whereas firms in the highest centrality decile command an average monthly return of 1.45%. The 0.82% monthly difference in returns between these two portfolios is economically and statistically significant and appears naturally in an equilibrium context as a compensation for contagion risk.[20]

VI. Conclusion

This paper studies the asset pricing properties that stem from the propagation of idiosyncratic shocks along firm networks. The fundamental insight of this paper is that extending standard asset pricing models to take into account how idiosyncratic shocks propagate along firm networks can make significant progress toward generating a unifying framework that simultaneously captures dynamics of the aggregate and the cross section of stock returns.

A calibrated model that matches key features of supplier–customer networks in the United States generates long-run consumption risks, high and volatile risk premiums, and a low

[20]If one focuses on manufacturing and service firms—as they jointly represent more than 65% of firms in the dataset—the results tend to be stronger, which is consistent with empirical evidence documented by Wu and Birge (2014). See the tables in Internet Appendix E, which report results on manufacturing and service firms.

and stable risk-free rate. In the model, low-frequency changes in the shock propagation mechanism endogenously generate persistence in firms' growth prospects which, in turn, drives a small but persistent component in expected aggregate consumption growth. With investors with preference for early resolution of uncertainty, sizable risk premiums arise because investors fear that extended periods of low economic growth coincide with low asset prices. Similarly, a small risk-free rate is driven by investors saving for long periods of low economic growth.

Additionally, the model helps in understanding the cross section of expected returns, as it provides a mapping between firms' quantities of priced risk and firms' importance in the network. In the calibrated economy, firms that are more central in the network command lower risk premiums than firms that are less central: Central firms tend to benefit from the diversification of their neighbors and, thus, they mitigate contagion risk better than peripheral firms.

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Appendix A. Mathematical Derivations

This section contains the derivations of formulas in the body of the paper. Let s_t denote the state of the parameter vector ζ_t. Because the firm network is fixed, s_t determines the equilibrium distribution of aggregate consumption growth at t. Because ζ_t follows a Markov process, the distribution of aggregate consumption growth varies over time and the dynamics of its moments satisfy the Markov property.

Price and Expected Return of Aggregate Wealth: I look for an equilibrium such that price-dividend ratios are stationary. I conjecture that if c is the current aggregate consumption and s the current state of ζ_t, then $P_a(c, s) = w_s^a c$, in which P_a is the price of aggregate wealth and w_s^a is a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, the realized gross return at period $t+1$ of the asset that delivers aggregate consumption as its dividend each period, $\widetilde{R}_{a,t+1}$, equals

$$\widetilde{R}_{a,t+1} = \frac{\widetilde{P}_{a,t+1} + \mathcal{C}_{t+1}}{P_{a,t}} = \frac{w_{s'}^a + 1}{w_s^a}\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}.$$

Setting $\widetilde{R}_{i,t+1} = \widetilde{R}_{a,t+1}$ in equation (5) yields

$$\begin{aligned}\mathbb{E}_t\left(\left[\beta\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}}\right) &= 1\\ \Rightarrow \mathbb{E}\left(\left[\beta\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\frac{w_{s'}^a + 1}{w_s^a}\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right]^{\frac{1-\gamma}{1-\rho}}\Big|s\right) &= 1.\end{aligned}$$

Because s_t follows a Markov process, the above equation can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'\in\mathcal{S}}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{1-\gamma}\Big|s'\right)\left(\frac{w_{s'}^a + 1}{w_s^a}\right)^{\frac{1-\gamma}{1-\rho}}\right) = 1.$$

Reordering the above equation yields

$$\begin{aligned}w_s^a &= \beta\left(\sum_{s'\in\mathcal{S}}\omega_{s,s'}\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\Big|s'\right)(w_{s'}^a + 1)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}}\\ &= \beta e^{\tau(1-\rho)(\tau(1-\gamma)\sigma_a^2 + \alpha_1\bar{d})}\left(\sum_{s'\in\mathcal{S}}\omega_{s,s'}\mathbb{E}\left(e^{-\tau(1-\gamma)\alpha_2\widetilde{W}_{n,t+1}}\Big|s'\right)(w_{s'}^a + 1)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}} \qquad \text{(A1)}\end{aligned}$$

Risk-free Asset: Setting $\widetilde{R}_{i,t+1} = R_f$ in equation (5) yields

$$\mathbb{E}\left(\left[\beta\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}\Big|s\right) = \frac{1}{R_f(s)}.$$

Because s_t follows a Markov process and $P_a(c,s) = w_s^a c$, the left-hand side of the above equation can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\gamma} \Big| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right).$$

Therefore,

$$\begin{aligned} \frac{1}{R_f(s)} &= \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \big| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) \\ &= \beta^{\frac{1-\gamma}{1-\rho}} e^{-\tau\gamma(\alpha_1 \bar{d} - \tau\gamma\sigma_a^2)} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{\tau\gamma\alpha_2 \widetilde{W}_{n,t+1}} \big| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right). \end{aligned} \tag{A2}$$

Firm i's Expected Return: Consider $s_t = s$ and $s_{t+1} = s'$. Equation (5) can be rewritten as

$$P_{i,t} = \mathbb{E}_t\left(\widetilde{M}_{t+1} \left(\widetilde{P}_{i,t+1} + y_{i,t+1} \right) \right) \qquad i = 1, \cdots, n \tag{A3}$$

where

$$\widetilde{M}_{t+1} \equiv \left[\beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho} - 1}$$

represents the pricing kernel. Dividing equation (A3) by Y_t yields

$$\frac{P_{i,t}}{Y_t} = \mathbb{E}_t\left(\widetilde{M}_{t+1} \widetilde{X}_{t+1} \frac{\widetilde{P}_{i,t+1}}{Y_{t+1}} \right) + \mathbb{E}_t\left(\widetilde{M}_{t+1} \frac{y_{i,t+1}}{Y_t} \right) \qquad i = 1, \cdots, n$$

which can be rewritten as

$$v_{i,t} = \mathbb{E}_t\left(\widetilde{M}_{t+1} \widetilde{X}_{t+1} v_{i,t+1} \right) + \mathbb{E}_t\left(\widetilde{M}_{t+1} \frac{y_{i,t+1}}{Y_t} \right) \qquad i = 1, \cdots, n \tag{A4}$$

with $v_{i,t} \equiv v_i(s) \equiv \frac{P_{i,t}}{Y_t}$. Because s_t follows a Markov process and $P_a(c,s) = w_s^a c$, the first term in the right-hand side of equation (A4) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1} \widetilde{X}_{t+1} v_{i,t+1} \right) = \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{((1/\tau)-\gamma)\Delta \widetilde{c}_{t+1}} \big| s' \right) v_i(s') \right)$$

whereas the second term in the right hand side of equation (A4) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \quad = \quad e^{\sigma_a^2+\alpha_1 d_i}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\right).$$

The expectation term in the right hand side of the above equation can be written as

$$\begin{aligned}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\right) &= \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'\in\ \mathcal{S}}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\gamma}e^{-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\bigg| s'\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)\\ &= \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'\in\ \mathcal{S}}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\bigg| s'\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right).\end{aligned}$$

As a consequence,

$$\begin{aligned}v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'\in\ \mathcal{S}}\omega_{s,s'}\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\mathbb{E}\left(e^{((1/\tau)-\gamma)\Delta\widetilde{c}_{t+1}}\big| s'\right)v_i(s')\right)\\ &+ \beta^{\frac{1-\gamma}{1-\rho}}e^{\sigma_a^2+\alpha_1 d_i}\left(\sum_{s'\in\ \mathcal{S}}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\bigg| s'\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \qquad i=1,\cdots,n\end{aligned}$$

To solve for the second expectation in the right-hand side of the above equation, it is convenient to express $\mathcal{G}_n$ as a set of connected components. If $\mathcal{G}^i_n$ denotes the connected component that firm i belongs to, then $\mathcal{G}_n$ can be written as

$$\mathcal{G}_n \quad \equiv \quad \bigcup_{i\in\mathcal{G}_n}\mathcal{G}^i_n.$$

Define the following averages,

$$\widetilde{W}^i_{n,t+1}\equiv\frac{1}{n}\left(\sum_{j\in\mathcal{G}^i_n}\widetilde{\varepsilon}_{j,t+1}\right) \quad \text{and} \quad \widetilde{W}^{-i}_{n,t+1}\equiv\frac{1}{n}\left(\sum_{j\in\mathcal{G}_n\setminus\mathcal{G}^i_n}\widetilde{\varepsilon}_{j,t+1}\right),$$

where $\widetilde{W}^i_{n,t+1}$ represents the average number of firms in $\mathcal{G}^i_n$ that face firm-level shocks at $t+1$, whereas $\widetilde{W}^{-i}_{n,t+1}$ represents the average number of firms in $\mathcal{G}_n\setminus\mathcal{G}^i_n$ that face firm-level shocks at $t+1$. Because $\widetilde{W}^i_{n,t+1}$ and $\widetilde{W}^{-i}_{n,t+1}$ are independent,

$$\begin{aligned}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\widetilde{\varepsilon}_{i,t+1}}\bigg| s'\right) &= e^{-\gamma(\alpha_1\bar{d}-\gamma\sigma_a^2)}\underbrace{\mathbb{E}\left(e^{\alpha_2\gamma\widetilde{W}^{-i}_{n,t+1}}\bigg| s'\right)}\underbrace{\mathbb{E}\left(e^{\alpha_2\gamma(\widetilde{W}^i_{n,t+1}-\widetilde{\varepsilon}_{i,t+1})}\bigg| s'\right)}\\ &= e^{-\gamma(\alpha_1\bar{d}-\gamma\sigma_a^2)}\times\qquad\pi_{-i}(s')\qquad\times\qquad\pi_i(s')\end{aligned}$$

Therefore,

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{((1/\tau)-\gamma)\Delta \widetilde{c}_{t+1}} \Big| s' \right) v_i(s') \right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\sigma_a^2(1+\gamma^2)+\alpha_1(d_i-\gamma \bar{d})} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \pi_i(s') \pi_{-i}(s') \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) \qquad i = 1, \cdots, n \end{aligned}$$

Using the above computations, the expected one-period gross return of firm i is given by

$$\begin{aligned} \mathbb{E}\left(\widetilde{R}_{i,t+1} \Big| s \right) &= \mathbb{E}\left(\frac{\widetilde{p}_{i,t+1} + \widetilde{y}_{i,t+1}}{\widetilde{p}_{i,t}} \Bigg| s \right) \\ &= \mathbb{E}\left(\frac{v_{i,t+1} Y_{t+1} + \widetilde{y}_{i,t+1}}{v_{i,t} Y_t} \Bigg| s \right) \\ &= \frac{1}{v_i(s)} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} v_i(s') \mathbb{E}\left(e^{(1/\tau)\Delta \widetilde{c}_{t+1}} \Big| s' \right) \right) + \frac{e^{\sigma_a^2 + \alpha_1 d_i}}{v_i(s)} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{-\alpha_2 \widetilde{\varepsilon}_{i,t+1}} \Big| s' \right) \right) \\ &= \frac{e^{(1/\tau)((1/\tau)\sigma_a^2 + \alpha_1 \bar{d})}}{v_i(s)} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} v_i(s') \mathbb{E}\left(e^{-(1/\tau)\alpha_2 \widetilde{W}_{n,t+1}} \Big| s' \right) \right) + \frac{e^{\sigma_a^2 + \alpha_1 d_i}}{v_i(s)} \left(\sum_{s' \in \mathcal{S}} \omega_{s,s'} \mathbb{E}\left(e^{-\alpha_2 \widetilde{\varepsilon}_{i,t+1}} \Big| s' \right) \right). \end{aligned}$$

Appendix B. Tables and Figures

This section contains tables and figures mentioned in the body of the paper.

Table I
Major Industry Groups

The table reports the distribution of firms across major industry groups in the dataset. Major industry groups are defined by the first two digits of firms' SIC codes.

Industry	**Number of firms**
Agriculture, forestry, and fishing	31
Construction	94
Finance, insurance, and real estate	569
Manufacturing	4275
Mining	588
Retail	345
Service	1598
Transportation, communications, electric, gas, and sanitary	881
Wholesale	290
Nonclassifiable establishments	108
Total	8,779

Table II
Most Connected Firms

The table reports the average number of relationships—considering customers and suppliers—of the five most connected firms in the following five-year intervals: 1976–1980, 1981–1985, 1986–1990, 1991–1995, 1996–2000, 2001–2005, 2006–2010, and 2011–2015.

1976 to 1995							
1976–1980		**1981–1985**		**1986–1990**		**1991–1995**	
Name	**N**	**Name**	**N**	**Name**	**N**	**Name**	**N**
General Motors	290	General Motors	393	General Motors	395	Walmart	446
Ford	157	IBM	226	AT&T	373	AT&T	437
Sears Roebuck	106	AT&T	206	IBM	303	General Motors	377
JC Penney	90	Ford	191	Ford	237	IBM	344
Sears Holdings	76	Sears Roebuck	184	Chrysler	143	Ford	334

1996 to 2015							
1996–2000		**2001–2005**		**2006–2010**		**2011–2015**	
Name	**N**	**Name**	**N**	**Name**	**N**	**Name**	**N**
Walmart	525	Walmart	585	Walmart	570	Walmart	555
General Motors	305	General Motors	259	Cardinal Health	180	Cardinal Health	187
Ford	275	Ford	200	Mckesson	154	Amerisourcebergen	155
AT&T	270	Daimler	185	Amerisourcebergen	139	Mckesson	144
IBM	253	Home Depot	134	AT&T	135	AT&T	144

Table III
Descriptive Statistics

The table reports descriptive statistics for the sample. The sample contains 8,779 different firms and 17,322 supplier–customer relationships among different pairs of firms from 1976 to 2016. These relationships represent 66,355 unique annual linkages. Panel A reports summary statistics at the annual level for (a) the percentage of sales that customers represent for their suppliers, (b) the specificity of suppliers, (c) the Rauch (1999)'s score, (d) the R&D's score, (e) the patent's score, (f) $\widetilde{p}_{ijt}$, and (g) the duration of relationships in years. Panel B present summary statistics at the relationship level. |AC1| and |AC2| report the first and second autocorrelation coefficients of: (a) the percentage of sales that customers represent for their supplier, (b) the specificity of suppliers, and (c) the propensity of relationships. In Panels A and B, column Obs denotes the number of non–missing observations used to compute summary statistics. Summary statistics are in percentages; with the exception of duration. All continuous variables are winsorized at the 1st and 99th percentiles of their distributions.

Panel A: Annual level

	Obs	Mean	25th Per.	Median	75th Per.	Min	Max
% of sales	53,620	19.0	9.8	14.0	22.6	0.6	95.0
Specificity of suppliers	62,447	34.2	0.0	34.9	50.2	0.0	100
Rauch's score	60,904	56.6	0.0	100	100	0.0	100
R&D score	26,897	10.0	1.0	4.0	14.0	0.0	78.1
Patent score	43,967	13.1	0.0	0.0	6.3	0.0	100
$\widetilde{p}_{ijt}$	65,232	11.4	0.0	4.4	12.1	0.0	100
Duration	66,355	8.37	3	6	12	1	39

Panel B: Relationship level

	Obs	Mean	25th Per.	Median	75th Per.	Min	Max
|AC1| % of sales	6,411	31.5	11.9	27.3	49.9	0.0	92.8
|AC2| % of sales	6,336	26.8	12.0	25.7	41.3	0.0	85.2
|AC1| specificity	5,032	29.5	13.3	25.1	44.8	0.0	90.1
|AC2| specificity	5,017	23.8	9.4	21.4	34.3	0.0	77.5
|AC1| $\widetilde{p}_{ijt}$	6,057	29.9	11.4	25.9	46.3	0.0	94.1
|AC2| $\widetilde{p}_{ijt}$	6,061	24.8	10.2	23.0	37.7	0.0	84.9

Table IV
Characteristics of Supplier–Customer Networks

The table reports characteristics of supplier–customer networks generated at an annual frequency from 1976 to 2016. Firms i and j are connected in the network of year t if firm i (j) reports j (i) as a principal customer. The number of connected components per network is computed via a depth-first search algorithm as in Tarjan (1972). The benchmark column reports the characteristics of the network in the benchmark economy.

Characteristic	**Mean**	**Standard Deviation**	**Benchmark**
Number of firms per supplier–customer network	1,112	365	1,110
Number of relationships per supplier–customer network	1,109	393	1,146
Average number of suppliers per firm	0.98	0.06	1.03
Average number of suppliers and customers per firm	1.96	0.13	2.06
Number of connected components per network	154	43	159

Table V
Benchmark Parameterization

The table reports the list of parameter values in the benchmark parameterization. Parameters in the first group define the preferences of the representative investor: β represents the time discount factor, γ represents the coefficient of relative risk aversion for static gambles, and ρ represents the inverse of the inter-temporal elasticity of substitution. Parameters in the second group describe firms' cash flows: σ_a measures the volatility of aggregate shocks, α_1 measures the marginal benefits a firm receives from each relationship, and α_2 measures the decrease in a firm's cash-flow growth if that firm is affected by a negative firm-level shock. Parameters in the third group define the stochastic process that determines the propagation of firm-level shocks. Parameter q measures how frequently firms face negative idiosyncratic shocks. The rest of parameters define the cross sectional distribution from which propensities of relationships are drawn: ζ_{1L}, ζ_{1H}, ζ_{2L}, and ζ_{2H}.

Preferences			**Firms' cash flows**			**Propagation of shocks**				
β	γ	ρ	σ_a	α_1	α_2	ζ_{1L}	ζ_{1H}	q	ζ_{2L}	ζ_{2H}
0.997	10	0.65	1.7	0.03	0.3	0.67	0.78	0.1	3.24	4.01

Table VI
Moments under the Benchmark Parameterization

The table reports the first two moments of consumption and dividend growth as well as a set of key asset pricing moments. Column **Data** reports moments found in the data. Column **Model** reports moments generated under the benchmark parameterization described in Table V. Column **BY2004** reports moments generated under the long-run risks model of Bansal and Yaron (2004). Data on consumption and dividends are obtained from Robert Shiller's website http://www.econ.yale.edu/ shiller/data.htm. Moments on the return on aggregate wealth, risk-free rate, equity premium, and Sharpe ratio are based on data from 1928 to 2014 and obtained from Aswath Damodaran's website: http://pages.stern.nyu.edu/∼adamodar/. The annual return on aggregate wealth is approximated by the annual return of the S&P 500. The return on the risk-free asset is approximated by the yield on three-month T-bills. All values are in percentages with the exception of average Sharpe ratios.

Moments	**Data**	**Model**	**BY2004**
Average annual log of consumption growth rate	1.9	1.9	1.8
Annual volatility of log consumption rate	3.5	3.5	2.8
Average annual log dividend growth rate	3.8	3.8	1.8
Annual volatility of the log dividend growth rate	11.63	11.9	12.3
Average annual market return (S&P 500)	11.53	12.3	7.2
Annual volatility of the market return	19	19.5	19.42
Average annual risk-free rate (3-month T-bill)	3.53	2.16	0.86
Annual volatility of risk-free rate	3	1.8	0.97
Average annual equity risk premium	8	10	6.33
Average annual Sharpe ratio	0.4	0.51	0.33

Table VII
Similarities between the calibrated model and the LRR model

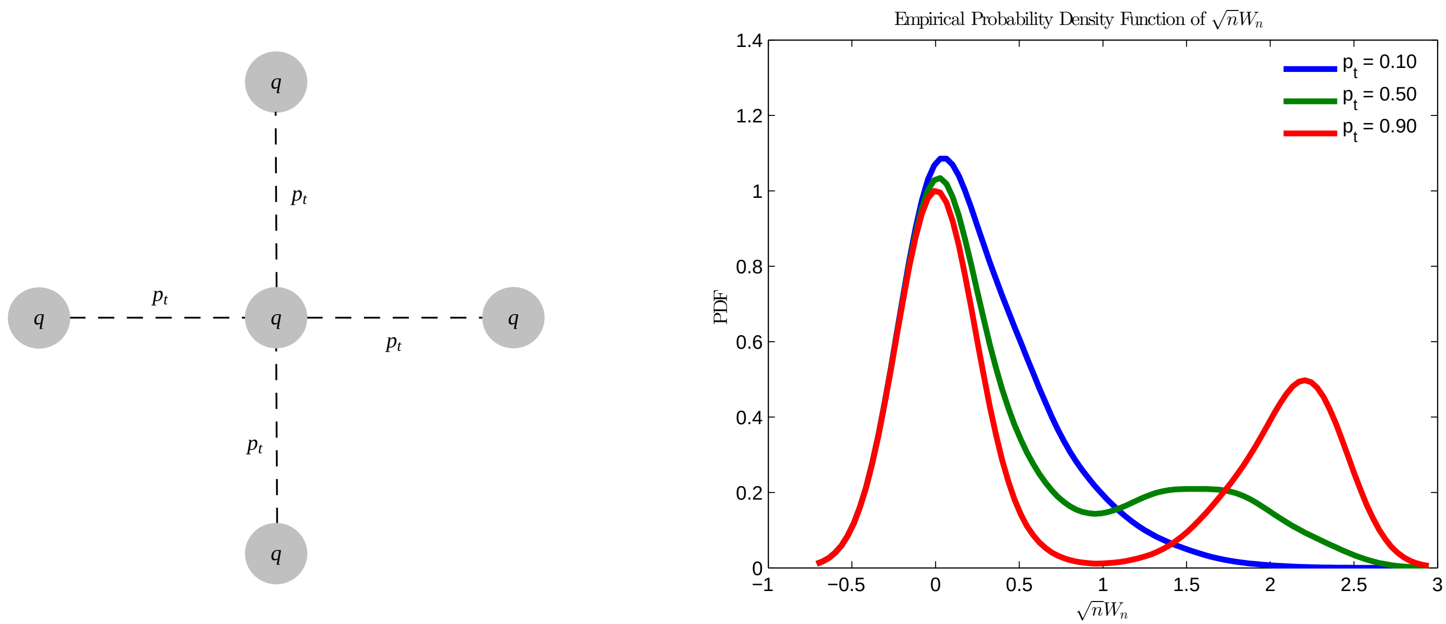
The table reports averages and standard deviations of similarity measures between time series generated with either the calibrated model or the benchmark parameterization in the LRR model of Bansal and Yaron (2004). To compute averages and standard deviations, I sample from the calibrated model and the LRR model to construct two empirical distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated samples over 620 periods. The first 100 periods in each sample are disregarded to eliminate bias coming from the initial condition. All similarity measures report scores computed as $\frac{1}{1+distance}$, where *distance* is defined according to each similarity measure. Let $\mathbf{X}_T = (X_1, \cdots, X_T)$ and $\mathbf{Y}_T = (Y_1, \cdots, Y_T)$ denote realizations from two time series, $X = \{X_t\}$ and $Y = \{Y_t\}$. The first and second similarity measures focus on the proximity between X and Y at specific points of time. The euclidean distance (ED) is defined as $\sqrt{\sum_{t=1}^{T}(X_t - Y_t)^2}$, whereas the dynamic time warping (DTW) distance is defined as $\min_r\left(\sum_{i=1}^{m}|X_{a_i} - Y_{b_i}|\right)$, where $r = ((X_{a_1}, Y_{b_1}), \cdots, (X_{a_m}, Y_{b_m}))$ is a sequence of m pairs that preserves the order of observations, i.e., $a_i < a_j$ and $b_i < b_j$ if $j > i$. DTW seeks to find a mapping such that the distance between X and Y is minimized. This way of computing distance allows two time series that are similar but locally out of phase to align in a nonlinear manner. The third measure focuses on correlation-based distances. It uses the partial autocorrelation function (PACF) to define the distance between time series. In particular, distance is defined as $\sqrt{(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})'\Omega(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})}$, where Ω is a matrix of weights, whereas $\hat{\rho}_{X_t}$ and $\hat{\rho}_{Y_t}$ are the estimated partial autocorrelations of X and Y, respectively. The fourth and fifth measures assume that a specific model generates both time series. The idea is to fit the specific model to each time series and then measure the dissimilarity between the fitted models. The fourth measure computes the distance between two time series as the ED between the truncated AR operators. In this case, distance is defined as $\sqrt{\sum_{j=1}^{k}(e_{j,X_t} - e_{j,Y_t})^2}$, where $e_{X_t} = (e_{1,X_t}, \cdots, e_{k,X_t})$ and $e_{Y_t} = (e_{1,Y_t}, \cdots, e_{k,Y_t})$ denote the vectors of $AR(k)$ parameter estimators for X and Y, respectively. The fifth measure computes dissimilarity between two time series in terms of their linear predictive coding in ARIMA processes, as in Kalpakis et al. (2001). The last measure defines distance based on nonparametric spectral estimators. Let f_{X_T} and f_{Y_T} denote the spectral densities of X_T and Y_T, respectively. The dissimilarity measure is given by a nonparametric statistic that checks the equality of the log-spectra of the two time series. It defines distance as $\sum_{k=1}^{n}\left[Z_k - \hat{\mu}(\lambda_k) - 2\log(1 + e^{Z_k - \hat{\mu}(\lambda_k)})\right] - \sum_{k=1}^{n}\left[Z_k - 2\log(1 + e^{Z_k})\right]$, where $Z_k = \log(I_{X_T}(\lambda_k)) - \log(I_{Y_T}(\lambda_k))$, and $\hat{\mu}(\lambda_k)$ is the local maximum log-likelihood estimator of $\mu(\lambda_k) = \log(f_{X_T}(\lambda_k)) - \log(f_{Y_T}(\lambda_k))$ computed with local lineal smoothers of the periodograms. All similarity measures are computed using the **R** package TSclust (see Montero and Vilar (2014)).

	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$		$\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$	
Similarity Measure	**Mean**	**Standard Deviation**	**Mean**	**Standard Deviation**
ED	0.99	0.02	0.96	0.01
DTW	0.74	0.10	0.75	0.12
PACF	0.80	0.04	0.78	0.05
ED in AR	0.90	0.12	0.93	0.11
Linear predictive in ARIMA	0.77	0.34	0.75	0.33
Spectral distance	1.00	0.00	1.00	0.00

Table VIII
Performance of Centrality Portfolios

The table reports monthly average raw returns, alphas and loadings from the five-factor model of Fama and French (2015) for three portfolios constructed by sorting stocks based on centrality: a portfolio that holds stocks on the lowest decile of centrality (Low), a portfolio that holds stocks on the highest decile of centrality (High), and a portfolio that is long on stocks on the lowest decile and short on stocks on the highest decile of centrality (Low - High). The bottom row provides the t-statistics for the low minus high portfolio. Firms are assigned into deciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1976 to December 2016. Raw returns and alphas are in percent.

	Raw	5-Factor Model					
Decile	Return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.28	1.38	0.98	0.41	-0.27	-0.47	0.06
High	1.45	0.55	0.94	-0.23	-0.07	-0.06	0.10
Low - High	0.82	0.43	0.05	0.64	-0.20	-0.40	-0.02
t-statistic	[4.06]	[2.49]	[1.22]	[10.49]	[-2.58]	[-4.84]	[-0.21]



(a) Network Topology

(b) Empirical density function of $\widetilde{W}_{n,t}$

Figure 2. The figure illustrates how changes in the propensity of inter-firm relationships to transmit shocks at t, $\widetilde{p}_t$, affect the distribution of $\widetilde{W}_{n,t}$. Figure 2(a) depicts an economy with $n = 5$ firms, whereas figure 2(b) depicts estimates of the density function of $\widetilde{W}_{n,t}$ for different values of $\widetilde{p}_t$. These estimates are computed via normal kernel smoothing estimators using function ksdensity$(\cdot)$ in MATLAB.

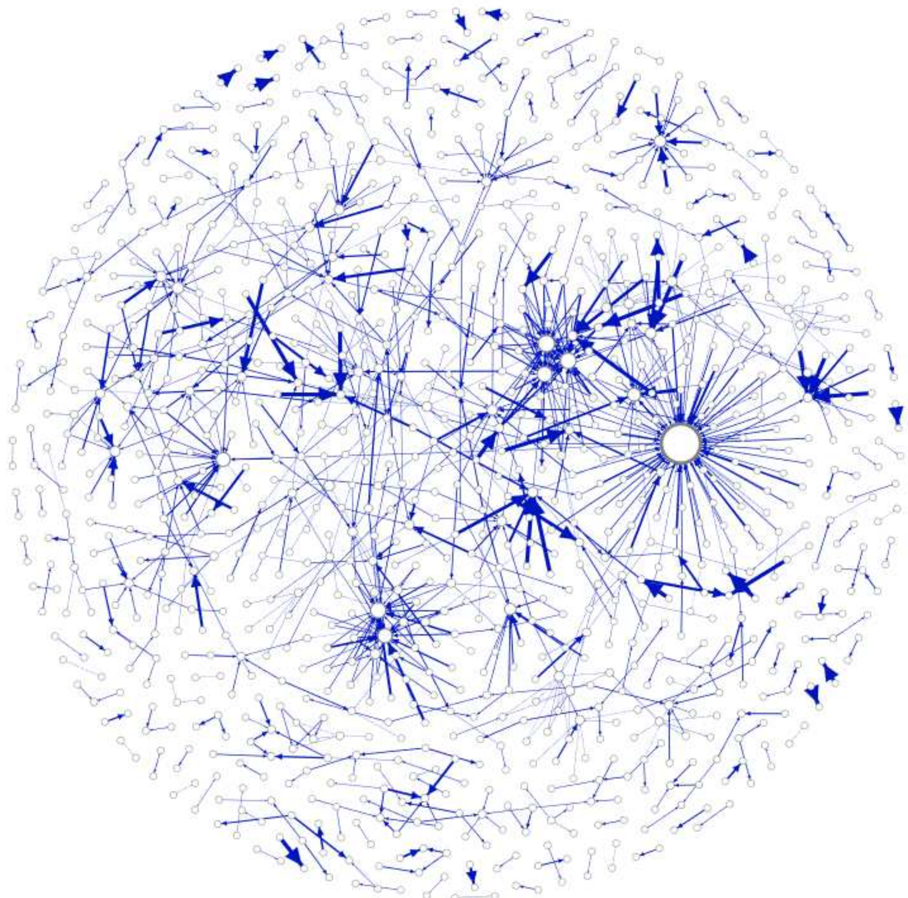


Figure 3. The figure shows the customer-supplier network in the benchmark parameterization.

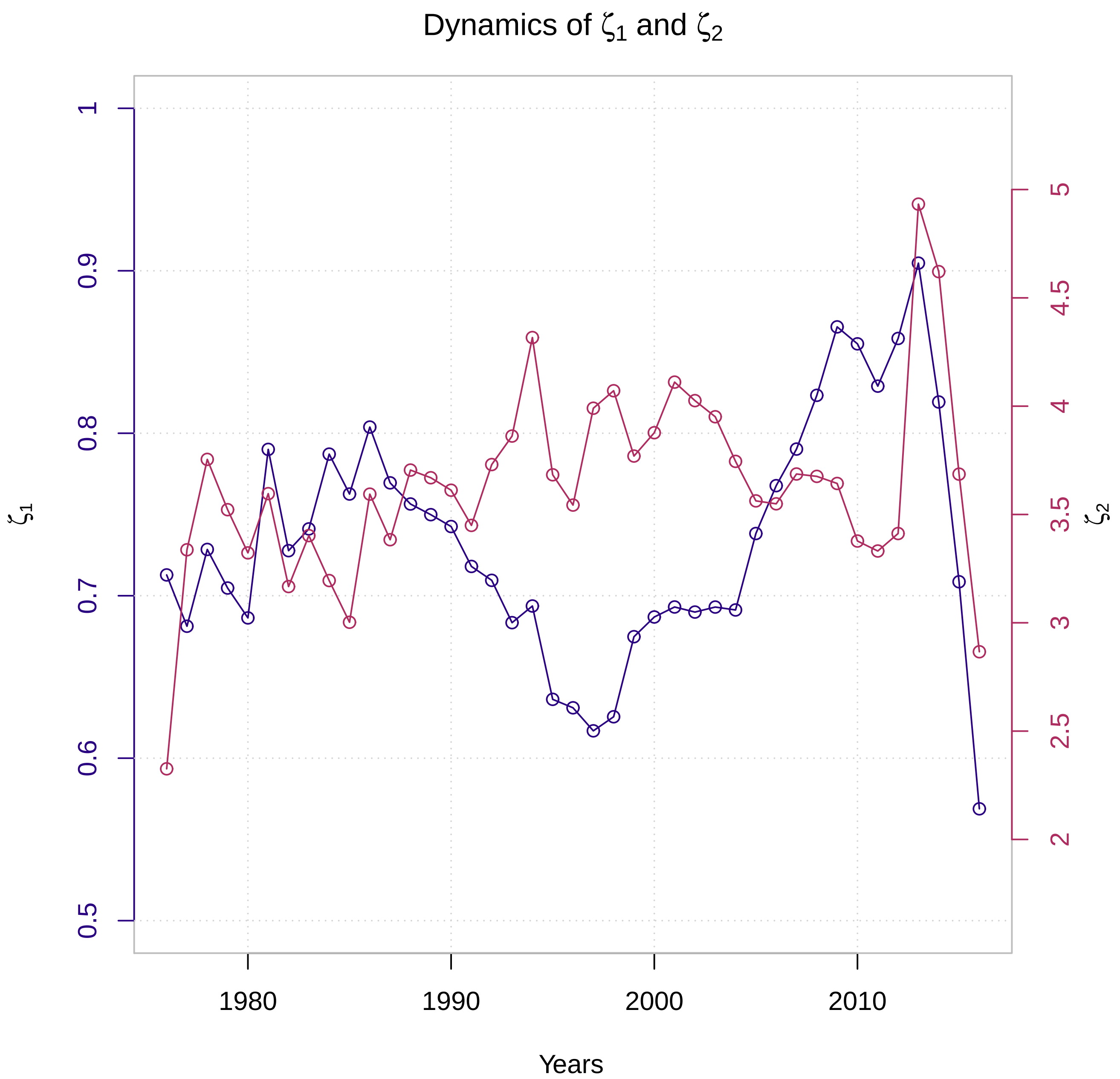


Figure 4. The figure shows annual estimates of parameters ζ_1 and ζ_2. I obtain these estimates by fitting Beta distributions to the annual link weight distributions using maximum likelihood.

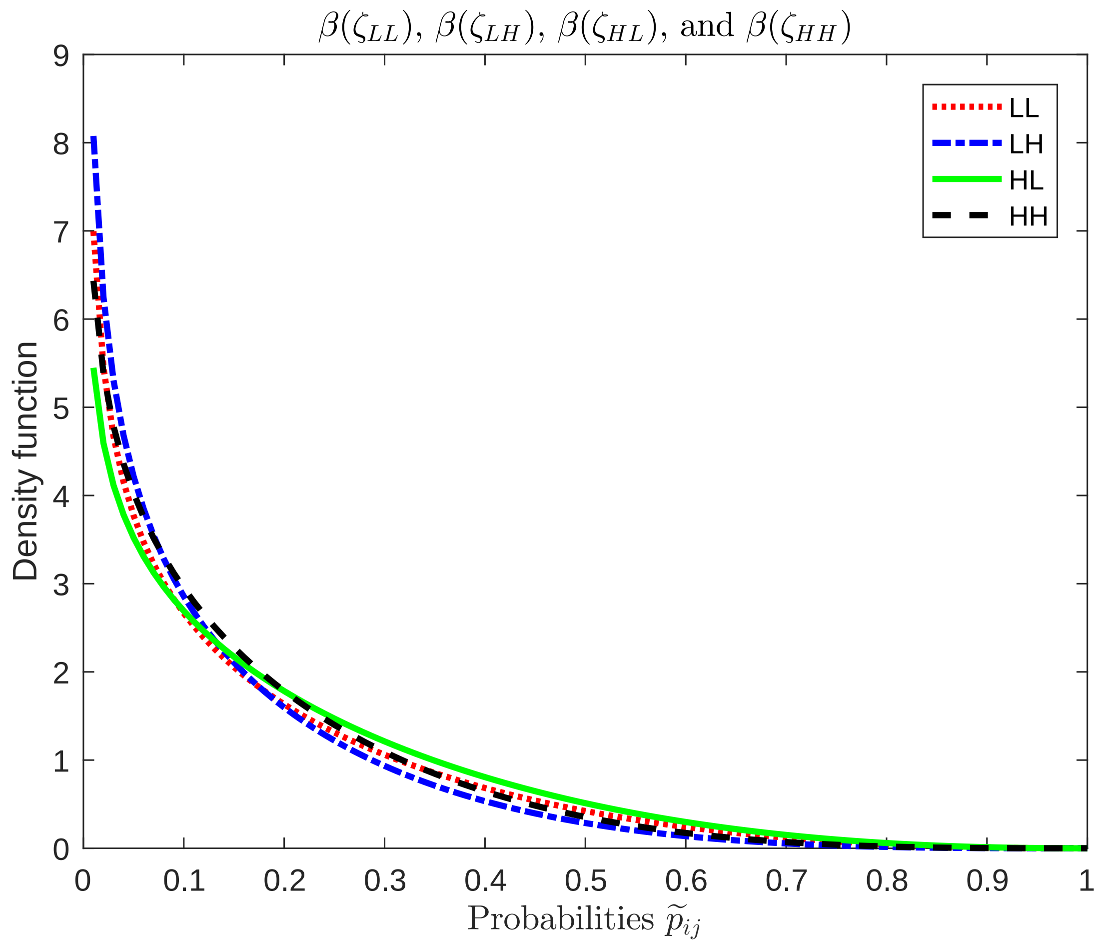


Figure 5. The figure shows probability density functions for Beta distributions with shape parameter vectors ζ_{LL}, ζ_{LH}, ζ_{HL}, and ζ_{HH}.

For Online Publication: Appendix for "Firm Networks and Asset Returns"

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This internet appendix contains supporting results, tables, and figures to supplement the analysis in the paper "Firm Networks and Asset Returns." Section A presents an equilibrium network model where production is explicitly modeled. Section B provides conditions under which $\widetilde{W}_{n,+1}$ follows a Poisson or Normal distribution. Section C depicts the time series of U.S. supplier–customer networks over the sample period. Section D provides a description of the algorithm used to simulate the model. Section E presents figures that depict the propensity versus centrality of relationships in U.S. supplier–customer networks as well as tables that support the cross–sectional results of the paper.

A. A Production-Based Equilibrium Network Model

The model embeds a variant of the multisector models of Long and Plosser (1983) and Acemoglu et al. (2012) into a standard asset pricing model with investors with Epstein-Zin-Weil preferences. Section A.A describes the production side of the economy. Section A.B describes investors preferences. Section A.C defines the equilibrium. Section A.D examines the equilibrium distribution of consumption growth. Using approximate analytical solutions, section A.E analyzes the asset pricing implications of changes in the propagation of idiosyncratic shocks along a firm network. Section A.F presents the results of a simple calibration exercise to check whether the cross-sectional results obtained in the paper can be supported by a calibrated version of the production-based network equilibrium model.

A. Production

Consider an economy with n different perishable goods and an infinite time horizon, with n being potentially large. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. Goods are produced using both labor and intermediate inputs. Each good is potentially used as an input in the production of every other good. There are n different competitive sectors, each populated by a large number of identical, infinitely lived firms that are aggregated into a representative firm. Within each period, representative firm i produces good i, with $i \in \{1, 2, \cdots, n\}$. Representative firms, henceforth referred to as firms, buy inputs and produce at the same time. There is one share per firm.

Firms use Cobb-Douglas technologies with constant returns to scale. Firm i's output at period t, denoted

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by y_{it}, is given by

$$y_{it} \quad \equiv \quad (a_t z_{it} l_{it})^{\chi} \prod_{j=1}^{n} y_{ijt}^{(1-\chi)w_{ij}} \ , \text{ with} \sum_{j=1}^{n} w_{ij} = 1,$$

where $\log(a_t) \xrightarrow{d}$ i.i.d. $\mathcal{N}(0, \sigma_a^2)$ is an aggregate productivity shock at period t, z_{it} is a productivity shock to firm i at period t, l_{it} is the amount of labor hired by firm i at period t, and y_{ijt} is the amount of good j used in the production of good i at period t. Parameter $\chi \in (0,1)$ represents the share of labor. Parameter w_{ij} denotes the share of good j used in the production of good i.

Let p_{it} denote the price of good i at period t. Taking input prices as given, firms choose how much labor and inputs to buy to maximize per-period profits. For simplicity, firms' choice of labor is deliberately normalized to 1.[1] Thus, at period t firm i solves

$$\begin{aligned} \pi_{i,t} \equiv \max_{\{y_{ijt}\}_{j=1}^{n},\ l_{it}} \qquad & p_{it} y_{it} - \sum_{j=1}^{n} p_{jt} y_{ijt} \\ st. \qquad & l_{it} = 1. \end{aligned}$$

where $\pi_{i,t}$ denotes the dividend of firm i at t.

A.1. The firm network and firm-level productivity shocks

If firm-level productivity shocks are independent across firms, as in Acemoglu et al. (2012), they affect downstream production only via changes in production costs.[2] For example, if firm i faces a negative productivity shock at period t, its production decreases and its output price increases, which, in turn, increases production costs in all firms that (directly or indirectly) use good i as an input in period t. However, if firm-level productivity shocks are dependent across firms, they affect downstream production not only via changes in production costs but also via changes in firms' productivity.

I assume that the dependence structure among shocks to firm-level productivity growth is determined by a firm network. In particular, $\Delta z_{i,t+1} \equiv \log\left(\frac{z_{i,t+1}}{z_{i,t}}\right)$ follows

$$\Delta z_{i,t+1} \quad = \quad \alpha_1 d_i - \alpha_2 \widetilde{\varepsilon}_{i,t+1} \ , \quad i \ \in \{1, \cdots, n\} \ ,$$

where parameters α_1 and α_2 are non-negative and equal across firms. Parameter d_i represents the number of relationships of firm i. Uncertainty on $\Delta z_{i,t+1}$ is introduced by $\widetilde{\varepsilon}_{i,t+1}$ which equals one if firm i is affected by a negative firm-level productivity shock at period $t+1$ and zero otherwise.

The distribution of $\widetilde{\varepsilon}_{i,t+1}$ is determined as in the baseline model. Similarly, propensities $\{\widetilde{p}_{ij,t+1}\}_{(i,j)}$ are drawn from a Beta distribution with parameters $\zeta_{1,t+1} > 0$ and $\zeta_{2,t+1} > 0$, which are drawn prior to drawing from the Beta distribution at period $t+1$. The shape parameter vector $\zeta_t \equiv [\zeta_{1t} \ \zeta_{2t}]$ follows a four-state ergodic Markov process with transition matrix Ω.

[1] The main results continue to hold if a competitive labor market is introduced.

[2] In principle, firm-level productivity shocks would also affect upstream demand, as they not only change a firm's output, but also change the amount of input needed to produce any given level of output. Shea (2002) shows that these two effects cancel out with Cobb-Douglas technologies.

B. Representative Investor

The economy is populated by a large number of identical, infinitely lived individuals who are aggregated into a representative investor. The representative investor owns all assets in the economy and is endowed with n units of labor each period. The representative investor does not benefit from leisure and her preferences are defined over the following consumption bundle:

$$\mathcal{C}_t \equiv \prod_{i=1}^{n} c_{i,t}^{1/n},$$

where $c_{i,t}$ denotes her consumption of good i at period t. The representative investor has Epstein-Zin-Weil preferences and, hence,

$$U_t = \left[(1-\beta)\mathcal{C}_t^{1-\rho} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$$

represents her utility at period t. Parameter $\rho > 0$, $\rho \neq 1$, represents the inverse of the inter-temporal elasticity of substitution (IES), $\gamma > 0$ is the coefficient of relative risk aversion for static gambles, and $\beta > 0$ measures the subjective discount factor under certainty.

The representative investor's budget constraint is given by

$$\sum_{i=1}^{n} p_{it} c_{it} + \sum_{i=1}^{n} (v_{it} - \pi_{it}) \phi_{i,t+1} = \sum_{i=1}^{n} v_{it} \phi_{i,t}$$

where $v_{i,t}$ denotes the cum-dividend value of firm i at t, and $\phi_{i,t}$ denotes the number of shares of firm i owned by the representative investor at the beginning of t (determined at $t-1$).

C. Competitive Equilibrium

DEFINITION 1: *A competitive equilibrium of the economy at period t consists of spot prices $(p_{1t}^*, \cdots, p_{nt}^*)$, a consumption bundle $\mathcal{C}_t^* = \left(c_{1,t}^*, \cdots, c_{n,t}^*\right)$, and quantities $\left(\phi_{i,t}^*, l_{it}^*, y_{it}^*, \left\{y_{ijt}^*\right\}_j\right)_{i=1\cdots n}$ such that: (a) each firm maximizes per-period profits, (b) the representative investor maximizes utility, and (c) labor and good markets clear; that is,*

$$\begin{aligned} y_{it}^* &= c_{i,t}^* + \sum_{j=1}^{n} y_{jit}^*, \quad \forall i, \\ \sum_{i=1}^{n} l_{it}^* &= n, \\ \phi_{i,t}^* &= 1, \quad \forall i. \end{aligned}$$

D. Equilibrium Consumption Growth

The first-order conditions of firm i imply

$$y_{ijt}^* = \left(\frac{p_{it}}{p_{jt}}\right)(1-\chi) w_{ij} y_{it}^*.$$

Substituting these values into the market clearing condition yields

$$\begin{aligned} y_{it}^* &= c_{i,t}^* + \sum_{j=1}^n \left(\frac{p_{jt}^*}{p_{it}^*}\right)(1-\chi) w_{ji} y_{jt}^* \\ &= c_{i,t}^* \left(1 + q_{it}^*\right) \end{aligned}$$

where $q_{it}^* \equiv (1-\chi)\frac{\sum_{j=1}^n p_{jt}^* w_{ji} y_{jt}^*}{p_{it}^* c_{i,t}^*}$. Thus,

$$\begin{aligned} \log\left(y_{it}^*\right) &= \log\left(c_{i,t}^*\right) + \log\left(1 + q_{it}^*\right) \\ &= \log\left(c_{i,t}^*\right) + q_{it}^* - \frac{1}{2}\left(q_{it}^*\right)^2 + \frac{1}{3}\left(q_{it}^*\right)^3 + \cdots \end{aligned}$$

Consequently,

$$\begin{aligned} \log\left(\frac{y_{it+1}^*}{y_{it}^*}\right) &= \log\left(\frac{c_{i,t+1}^*}{c_{i,t}^*}\right) + (q_{it+1}^* - q_{it}^*) - \frac{1}{2}((q_{it+1}^*)^2 - (q_{it}^*)^2) + \frac{1}{3}((q_{it+1}^*)^3 - (q_{it}^*)^3) + \cdots \\ &= \log\left(\frac{c_{i,t+1}^*}{c_{i,t}^*}\right) + (q_{it+1}^* - q_{it}^*)\left\{1 - \frac{1}{2}(q_{it+1}^* + q_{it}^*) + \frac{1}{3}((q_{it+1}^*)^2 + q_{it+1}^* q_{it}^* + (q_{it}^*)^2) + \cdots\right\}. \end{aligned}$$

Because $\phi_{i,t}^* = 1$ at equilibrium, the representative investor's budget constraint implies $c_{i,t}^* = \chi y_{i,t}^*$ when equilibrium prices are different from zero. As a consequence, the second term in the right-hand side of the previous equation is zero. Hence,

$$\frac{1}{n}\sum_{i=1}^n \log\left(\frac{y_{it+1}^*}{y_{it}^*}\right) = \frac{1}{n}\sum_{i=1}^n \log\left(\frac{c_{i,t+1}^*}{c_{i,t}^*}\right)$$

at equilibrium.

Now, I express $\frac{1}{n}\sum_{i=1}^n \log\left(\frac{y_{it+1}^*}{y_{it}^*}\right)$ as a function of the propagation of idiosyncratic shocks along the network. Taking the logarithm of $y_{i,t}^*$ and using the first-order conditions of firm i into firm i production function yields

$$\log(y_{i,t}^*) = \chi \log(a_t z_{i,t}) + (1-\chi)\sum_{j=1}^n w_{ij}\left\{\log\left(\frac{p_{i,t}^*}{p_{j,t}^*}\right) + \log(1-\chi) + \log(w_{ij}) + \log(y_{i,t}^*)\right\}.$$

Provided that $\sum_{j=1}^n w_{ij} = 1$, the above expression can be reduced to

$$\begin{aligned} \log(y_{i,t}^*) &= \log(a_t z_{i,t}) + \left(\frac{1-\chi}{\chi}\right)\left(\sum_{j=1}^n w_{ij}\log\left(\frac{p_{i,t}^*}{p_{j,t}^*}\right)\right) + \left(\frac{1-\chi}{\chi}\right)\log(1-\chi) \\ &+ \left(\frac{1-\chi}{\chi}\right)\sum_{j=1} w_{ij}\log(w_{ij}). \end{aligned}$$

Therefore, $\log\left(\frac{y_{i,t+1}^*}{y_{i,t}^*}\right)$ can be rewritten as

$$\log\left(\frac{y_{i,t+1}^*}{y_{i,t}^*}\right) = \log\left(\frac{a_{t+1}}{a_t}\right) + \log\left(\frac{z_{i,t+1}}{z_{i,t}}\right) + \left(\frac{1-\chi}{\chi}\right)\left(\sum_{j=1}^n w_{ij}\left\{\log\left(\frac{p_{i,t+1}^*}{p_{j,t+1}^*}\right) - \log\left(\frac{p_{i,t}^*}{p_{j,t}^*}\right)\right\}\right).$$

To simplify the above expression, I use the following normalization on spot prices

$$\prod_{i=1}^{n}\prod_{j=1}^{n}\left(\frac{p^*_{i,t}}{p^*_{j,t}}\right)^{w_{ij}} = 1, \ \forall t.$$

Using the above price normalization and summing over all firms yields

$$\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{y^*_{i,t+1}}{y^*_{i,t}}\right) = \log\left(\frac{a_{t+1}}{a_t}\right)+\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{z_{i,t+1}}{z_{i,t}}\right).$$

As a result, $\Delta\widetilde{c}_{t+1}\equiv\log\left(\frac{\mathcal{C}^*_{t+1}}{\mathcal{C}^*_t}\right)$ equals

$$\begin{aligned}
\Delta\widetilde{c}_{t+1} &= \log\left(\prod_{i=1}^{n}\left(\frac{c^*_{i,t+1}}{c^*_{i,t}}\right)^{1/n}\right)\\
&= \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{y^*_{it+1}}{y^*_{it}}\right)\\
&= \log\left(\frac{a_{t+1}}{a_t}\right)+\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{z_{i,t+1}}{z_{i,t}}\right)\\
&= \underbrace{\log\left(\frac{a_{t+1}}{a_t}\right)}_{\Delta a_{t+1}}+\alpha_1\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}d_i\right)}_{\bar{d}}-\alpha_2\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}\widetilde{\varepsilon}_{i,t+1}\right)}_{\widetilde{W}_{n,t+1}}\\
&= \Delta a_{t+1}+\alpha_1 \bar{d} - \alpha_2 \widetilde{W}_{n,t+1},
\end{aligned}$$

where Δa_{t+1} denotes innovations to aggregate TFP and $\bar{d}$ denotes the average number of relationships per firm, and $\widetilde{W}_{n,t+1}$ denotes the average number of firms affected by negative shocks to firm-level productivity growth at period $t+1$. Consequently, innovations to consumption growth are driven by either innovations in aggregate productivity or innovations in $\widetilde{W}_{n,t+1}$ as in the baseline model; see equation (4).

E. Equilibrium Asset Pricing

Although the model is solved numerically, the mechanisms working at equilibrium are shown via approximate analytical solutions. I first derive approximate equalities among variables of interest using log linearizations. I then use those equalities to explore the equilibrium asset pricing implication of changes in the propagation of idiosyncratic shocks along the network.

E.1. Approximate Equalities

Let $Y_t\equiv\prod_{i=1}^{n}y_{i,t}^{1/n}$. Define

$$g_{i,t+1}\equiv\log\left(\frac{y_{i,t+1}}{Y_t}\right),\ \widehat{y}_{i,t+1}\equiv\log\left(\frac{y_{i,t+1}}{Y_{t+1}}\right),\ \widehat{p}_{i,t+1}\equiv\log\left(\frac{p_{i,t+1}}{Y_{t+1}}\right),\ \widehat{p}_{a,t+1}\equiv\log\left(\frac{p_{a,t+1}}{Y_{t+1}}\right),$$

where $p_{a,t+1}$ denotes the price of aggregate wealth at $t+1$.

At equilibrium, the following two conditions must hold:

$$p_{a,t+1} = \sum_{i=1}^{n} p_{i,t+1},$$

and

$$\begin{aligned} \Delta\widetilde{c}_{t+1} &= \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{y^*_{it+1}}{y^*_{it}}\right) \\ &= \frac{1}{n}\sum_{i=1}^{n} g_{i,t+1} + \underbrace{\frac{1}{n}\sum_{i=1}^{n}\widehat{y}_{i,t}}_{=0 \text{ (provided } Y_t \text{ definition).}} \end{aligned} \tag{IA.1}$$

Relationship between $\widehat{p}_{a,t+1}$ and $\widehat{p}_{i,t+1}$: Using a first-order Taylor approximations yields

$$\widehat{p}_{a,t+1} \approx \varphi_0 + \sum_{i=1}^{n}\varphi_i \widehat{p}_{i,t+1}, \tag{IA.2}$$

where $\varphi_i = \mathbb{E}\left(\frac{\widehat{p}_{i,t}}{\sum_{j=1}^{n}\widehat{p}_{j,t}}\right)$, and $\sum_{i=1}^{n}\varphi_i = 1$. The term φ_0 is selected to ensure that first order approximations hold in levels as well.

Conditional distribution of $g_{i,t+1}$: At equilibrium,

$$g_{i,t+1} = \Delta a_{t+1} + \Delta z_{i,t+1} + \widehat{y}_{i,t}.$$

Then, the conditional distribution of $g_{i,t+1}$ at t is given by

$$g_{i,t+1} \xrightarrow{d} \mathcal{N}\left(\mathbb{E}_t[\Delta z_{i,t+1}] + \widehat{y}_{i,t}, 2\sigma_a^2 + \mathrm{Var}_t[\Delta z_{i,t+1}]\right).$$

Consequently, $g_{i,t+1}$ can be approximated by

$$g_{i,t+1} \approx x_{i,t} + \sigma_{i,t}\eta_{i,t+1},$$

where $\eta_{i,t+1} \xrightarrow{d} \mathcal{N}(0,1)$ and

$$\begin{aligned} x_{i,t} &\equiv \alpha_1 d_i - \alpha_2\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right] + \widehat{y}_{i,t}, \\ \sigma_{i,t}^2 &\equiv 2\sigma_a^2 + \alpha_2^2\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\left(1 - \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\right). \end{aligned}$$

Given the information at time t, $x_{i,t}$ determines $\mathbb{E}_t[g_{i,t+1}]$ while $\sigma_{i,t}$ determines the conditional volatility of $g_{i,t+1}$. Because the propensity matrix $\widetilde{p}_t$ follows an ergodic Markov process, $x_{i,t}$ and $\sigma_{i,t}^2$ can be approximated by the following autoregressive processes:

$$\begin{aligned} x_{i,t+1} &\approx \mu_0 + \mu_1 x_{i,t} + \mu_2\sigma_{i,t}\zeta_{p,t+1}, \\ \sigma_{i,t+1}^2 &\approx \nu_0 + \nu_1\sigma_{i,t}^2 + \nu_2\sigma_p\zeta_{p,t+1}, \end{aligned}$$

where $0 < \mu_1 < 1$, $\mu_2 > 0$, $0 < \nu_1 < 1$ and $\nu_2 > 0$. Variable $\zeta_{p,t+1} \xrightarrow{d} \mathcal{N}(0,1)$ represents the uncertainty coming from unexpected changes in $\widetilde{p}_{t+1}$ (network connectivity). Variable $\eta_{i,t+1}$ represents the uncertainty coming from the unexpected changes in exposure of firm i to idiosyncratic productivity shocks affecting other firms in the economy.

Approximate equalities for equilibrium asset returns: Define the continuous return of firm i at $t+1$ as

$$r_{i,t+1} \equiv \log\left(\frac{p_{i,t+1}+y_{i,t+1}}{p_{i,t}}\right),$$

and the continuous return on the market portfolio at $t+1$ as

$$r_{a,t+1} \equiv \log\left(\frac{p_{a,t+1}+\mathcal{C}_{t+1}}{p_{a,t}}\right) \approx \log\left(\frac{p_{a,t+1}+Y_{t+1}}{p_{a,t}}\right).$$

Using first-order Taylor approximations yields[3]

$$r_{i,t+1} \approx k_i + \rho_i \widehat{p}_{i,t+1} - \widehat{p}_{i,t} + \rho_i \Delta\widetilde{c}_{t+1} + (1-\rho_i) g_{i,t+1}, \tag{IA.3}$$

$$r_{a,t+1} \approx k_m - \widehat{p}_{a,t} + \rho_m \widehat{p}_{a,t+1} + \Delta\widetilde{c}_{t+1}, \tag{IA.4}$$

where $\{k_i\}_{i=1}^n$ and k_m ensure that first-order approximations hold in levels.

E.2. Equilibrium Asset Returns

With the above definitions and approximations at hand, I study the asset pricing implication of changes in the propagation of idiosyncratic shocks along the network.

The pricing kernel is given by

$$m_{t+1} \equiv \theta \log(\beta) - \frac{\theta}{\psi}\Delta\widetilde{c}_{t+1} + (\theta-1) r_{a,t+1}.$$

with $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $\psi = \frac{1}{\rho}$.

The stock price and return of firm i can be determined using the pricing kernel and the representative investor's first-order condition

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1. \tag{IA.5}$$

I first solve for the return of the market portfolio $r_{a,t+1}$ substituting $r_{i,t+1}$ for $r_{a,t+1}$ in (IA.5). I then solve for the risk-free rate. Finally, I solve for the risk premium of firm i, $\forall\ i \in \{1,\cdots,n\}$.

Return of the Market Portfolio: I conjecture that firm i's log price-output ratio follows

$$\widehat{p}_{i,t} = a_0 + a_1 x_{i,t} + a_2 \sigma_{i,t}^2. \tag{IA.6}$$

[3] Approximation (IA.4) follows directly from the dividend-ratio model of Campbell and Shiller (1989). Approximation (IA.3) follows from Campbell and Shiller (1989) once noting that

$$\begin{aligned} r_{i,t+1} &\approx k_i + \log\left(\frac{y_{i,t}}{p_{i,t}}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{p_{i,t+1}}\right) + \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right), \\ &= k_i + \log\left(\frac{y_{i,t}}{Y_{t-1}}\frac{Y_t}{p_{i,t}}\frac{Y_{t-1}}{Y_t}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{Y_t}\frac{Y_{t+1}}{p_{i,t+1}}\frac{Y_t}{Y_{t+1}}\right) + \log\left(\frac{y_{i,t+1}}{Y_t}\frac{Y_t}{Y_{t-1}}\frac{Y_{t-1}}{y_{i,t}}\right), \\ &= k_i + \rho_i \widehat{p}_{i,t+1} - \widehat{p}_{i,t} + \rho_i \Delta\widetilde{c}_{t+1} + (1-\rho_i) g_{i,t+1} \end{aligned}$$

To solve for constants a_0, a_1 and a_2 I substitute (IA.1), (IA.2) and (IA.4) into the Euler equation (IA.5). As $\eta_{i,t+1}$ and $\zeta_{p,t+1}$ are conditionally normal, $r_{a,t+1}$ and m_{t+1} are also normal. Exploiting this normality, I write down the Euler equation in terms of the state variables $\{x_{i,t}, \sigma_{i,t}\}_{i=1}^n$. As the Euler equation must hold for all values of the states variables, the terms involving $x_{i,t}$ must satisfy

$$\varphi_i \left\{ \left(1 - \frac{1}{\psi}\right) - a_1 + \rho_m a_1 \mu_1 \right\} - \left(\varphi_i - \frac{1}{n}\right)\left(1 - \frac{1}{\psi}\right) \quad = \quad 0$$

ASSUMPTION 1: *To a first order approximation, $\varphi_i \approx \frac{1}{n}$.*

Assumption 1 is satisfied if $\mathcal{G}_n$ is regular (i.e. all firms have the same degree). If $\mathcal{G}_n$ exhibits power-law degree distributions, assumption 1 is also satisfied for a large fraction of firms.

If assumption 1 is satisfied, then

$$a_1 \quad \approx \quad \frac{\left(1 - \frac{1}{\psi}\right)}{1 - \mu_1 \rho_m}.$$

Assume n is large. Collecting all terms that involve $\sigma_{i,t}^2$ yields

$$a_2 \quad \approx \quad \frac{\frac{\theta}{2}\left(\left(1 - \frac{1}{\psi}\right)^2 + \rho_m^2 a_1^2 \mu_2^2\right)}{1 - \nu_1 \rho_m + \frac{\theta}{2}\rho_m^2 a_1 \mu_2 \nu_2 \sigma_p}.$$

Using (IA.6), the innovation to the return of the market portfolio can be written as

$$\begin{aligned} r_{a,t+1} - \mathbb{E}_t[r_{a,t+1}] \quad &\approx \quad \rho_m \left(a_1 \mu_2 \left(\sum_{i=1}^n \varphi_i \sigma_{i,t}\right) + a_2 \nu_2 \sigma_p\right) \zeta_{p,t+1} + \frac{1}{n}\sum_{i=1}^n \sigma_{i,t}\eta_{i,t+1} \\ &\approx \quad \rho_m \left(a_1 \mu_2 \left(\sum_{i=1}^n \varphi_i \sigma_{i,t}\right) + a_2 \nu_2 \sigma_p\right) \zeta_{p,t+1} + \sum_{i=1}^n \varphi_i \sigma_{i,t}\eta_{i,t+1} \\ &= \quad \rho_m \Delta_{p,t}\zeta_{p,t+1} + \sum_{i=1}^n \varphi_i \sigma_{i,t}\eta_{i,t+1} \end{aligned} \tag{IA.7}$$

where $\Delta_{p,t} \equiv a_1 \mu_2 \left(\sum_{i=1}^n \varphi_i \sigma_{i,t}\right) + a_2 \nu_2 \sigma_p$. The conditional variance of the market portfolio is given by

$$\mathrm{Var}_t[r_{a,t+1}] \quad \approx \quad \rho_m^2 \Delta_{p,t}^2 + \mathrm{Var}_t\left(\sum_{i=1}^n \varphi_i \sigma_{i,t}\eta_{i,t+1}\right) + 2\rho_m \Delta_{p,t}\sum_{i=1}^n \varphi_i \sigma_{i,t}\mathrm{Cov}_t\left(\zeta_{p,t+1}, \eta_{i,t+1}\right)$$

NOTATION 1: *Given two sequences $\{a_n\}_n$ and $\{b_n\}_n$, I write $a_n = o(b_n)$ if $\frac{a_n}{b_n} \to 0$ as $n \to \infty$, and $a_n = O(b_n)$ if $\left|\frac{a_n}{b_n}\right|$ is bounded from above as $n \to \infty$.*

REMARK 1: *If assumption 1 is satisfied and*

$$Var_t\left(\sum_{i=1}^n \sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \text{and} \quad \sum_{i=1}^n \sigma_{i,t}\, Cov_t\left(\zeta_{p,t+1}, \eta_{i,t+1}\right) = o(n),$$

then

$$\lim_{n\to\infty} Var_t[r_{a,t+1}] \quad = \quad a_2^2 \nu_2^2 \rho_m^2 \sigma_p^2.$$

Hence, the volatility of the market portfolio is only driven by changes in network connectivity.

Pricing Kernel: Using (IA.1) and (IA.4), I rewrite the pricing kernel in terms of the state variables,

$$
\begin{aligned}
m_{t+1} &\equiv \theta \log(\beta) - \frac{\theta}{\psi}\Delta\widetilde{c}_{t+1} + (\theta-1)r_{a,t+1} \\
&\approx \theta \log(\beta) - \frac{\theta}{\psi}\left(\sum_{i=1}^{n}\varphi_i(x_{i,t}+\sigma_{i,t}\eta_{i,t+1})\right) \\
&+ (\theta-1)\left(k_m - \varphi_0 - \sum_{i=1}^{n}\varphi_i\left(a_0 + a_1 x_{i,t} + a_2\sigma_{i,t}^2\right)\right) \\
&+ (\theta-1)\rho_m\left(\varphi_0 + \sum_{i=1}^{n}\varphi_i\left(a_0 + a_1\mu_0 + a_1\mu_2 x_{i,t} + a_1\mu_2\sigma_{i,t}\zeta_{p,t+1}\right)\right) \\
&+ (\theta-1)\rho_m\left(\sum_{i=1}^{n}\varphi_i\left(a_2\nu_0 + a_2\nu_1\sigma_{i,t}^2 + a_2\nu_2\sigma_p\zeta_{p,t+1}\right)\right) \\
&+ (\theta-1)\left(\sum_{i=1}^{n}\varphi_i(x_{i,t}+\sigma_{i,t}\sigma_{i,t+1})\right).
\end{aligned}
$$

Innovations to the pricing kernel are then given by

$$
m_{t+1} - \mathbb{E}_t[m_{t+1}] \approx \lambda_{m,q}\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\eta_{i,t+1}\right) + \lambda_{m,p}\Delta_{p,t}\zeta_{p,t+1}, \tag{IA.8}
$$

where λ represents the aggregate market price of risk for each source of risk, namely $\{\eta_{i,t+1}\}_{i=1}^{n}$ and $\zeta_{p,t+1}$, which are defined as

$$
\begin{aligned}
\lambda_{m,q} &\equiv \theta\left(1-\frac{1}{\psi}\right) - 1, \\
\lambda_{m,p} &\equiv (\theta-1)\rho_m.
\end{aligned}
$$

It follows from (IA.8) that the conditional variance of the pricing kernel is given by

$$
\mathrm{Var}_t[m_{t+1}] \approx \lambda_{m,q}^2\mathrm{Var}_t\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\eta_{i,t+1}\right) + \lambda_{m,p}^2\Delta_{p,t}^2 + 2\lambda_{m,q}\lambda_{m,p}\Delta_{p,t}\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\mathrm{Cov}_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right) \tag{IA.9}
$$

REMARK 2: *If assumption 1 is satisfied and*

$$
Var_t\left(\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \text{and} \quad \sum_{i=1}^{n}\sigma_{i,t}Cov_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) = o(n),
$$

then

$$
\lim_{n\to\infty} Var_t[m_{t+1}] = \lambda_{m,p}^2 a_2^2\nu_2^2\sigma_p^2.
$$

Hence, the volatility of the pricing kernel is only driven by changes in network connectivity.

Equity Premium: The risk premium of the market return is determined by the conditional covariance between the market portfolio and the pricing kernel. It follows that

$$\mathbb{E}_t[r_{a,t+1} - r_{f,t}] \quad = \quad -\mathrm{Cov}_t\left(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{a,t+1} - \mathbb{E}_t[r_{a,t+1}]\right) - \frac{1}{2}\mathrm{Var}_t(r_{a,t+1})$$

Substituting (IA.7) and (IA.8) in the above equation yields

$$\begin{aligned}
\mathbb{E}_t[r_{a,t+1} - r_{f,t}] \quad &\approx \quad -\left(\lambda_{m,q} + \frac{1}{2}\right)\mathrm{Var}_t\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\eta_{i,t+1}\right) - \rho_m\left(\lambda_{m,p} + \frac{\rho_m}{2}\right)\Delta_{p,t}^2 \\
&- \quad \Delta_{p,t}\left(\lambda_{m,q}\rho_m + \lambda_{m,p} + \rho_m\right)\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\mathrm{Cov}_t\left(\eta_{i,t+1}, \zeta_{p,t+1}\right)\right) \qquad \text{(IA.10)}
\end{aligned}$$

REMARK 3: *If assumption 1 is satisfied and*

$$Var_t\left(\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \textit{and} \quad \sum_{i=1}^{n}\sigma_{i,t}\,Cov_t\left(\zeta_{p,t+1}, \eta_{i,t+1}\right) = o(n),$$

then

$$\lim_{n\to\infty}\mathbb{E}_t[r_{a,t+1} - r_{f,t}] \quad = \quad -\rho_m\left(\lambda_{m,p} + \frac{\rho_m}{2}\right)a_2^2\nu_2^2\sigma_p^2.$$

Hence, the equity premium is determined by temporal changes in network connectivity.

Risk-free Rate: The risk-free rate satisfies

$$r_{f,t} \quad = \quad -\log(\beta) + \frac{1}{\psi}\mathbb{E}_t[\Delta\widetilde{c}_{t+1}] + \frac{1-\theta}{\theta}\mathbb{E}_t[r_{a,t+1} - r_{f,t}] - \frac{1}{2\theta}\mathrm{Var}_t[m_{t+1}]$$

Substituting (IA.9) and (IA.10) in the above equation yields

$$\begin{aligned}
r_{f,t} \quad \approx \quad & -\log(\beta) + \frac{1}{\psi}\left(\sum_{i=1}^{n}\varphi_i x_{i,t}\right) \\
- \quad & \frac{1-\theta}{\theta}\left(\left(\lambda_{m,q} + \frac{1}{2}\right)\mathrm{Var}_t\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\eta_{i,t+1}\right) + \rho_m\left(\lambda_{m,p} + \frac{\rho_m}{2}\right)\Delta_{p,t}^2\right) \\
- \quad & \frac{1-\theta}{\theta}\Delta_{p,t}\left(\lambda_{m,q}\rho_m + \lambda_{m,p} + \rho_m\right)\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\mathrm{Cov}_t\left(\eta_{i,t+1}, \zeta_{p,t+1}\right)\right) \\
- \quad & \frac{1}{2\theta}\left(\lambda_{m,q}^2\mathrm{Var}_t\left(\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\eta_{i,t+1}\right) + \lambda_{m,p}^2\Delta_{p,t}^2 + 2\lambda_{m,q}\lambda_{m,p}\Delta_{p,t}\sum_{i=1}^{n}\varphi_i\sigma_{i,t}\mathrm{Cov}_t\left(\eta_{i,t+1}, \zeta_{p,t+1}\right)\right)
\end{aligned}$$

Firms' Risk Premiums: As with the market portfolio, the risk premium of firm i is determined by the conditional covariance between firm i's return and the pricing kernel. Therefore,

$$\mathbb{E}_t[r_{i,t+1} - r_{f,t}] \quad = \quad -\mathrm{Cov}_t\left(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]\right) - \frac{1}{2}\mathrm{Var}_t(r_{i,t+1}) \qquad \text{(IA.11)}$$

To simplify the above expression, it becomes convenient to compute the innovations on firm i's return,

$$\begin{aligned} r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}] &\approx \rho_i \left(a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} \\ &+ \rho_i \left(\sum_{j\neq i}^{n} \varphi_j \sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i(1-\varphi_i))\sigma_{i,t}\eta_{i,t+1} \\ &= \rho_i \nabla_{i,t}\zeta_{p,t+1} + \rho_i \left(\sum_{j=1}^{n} \varphi_j \sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i)\sigma_{i,t}\eta_{i,t+1} \end{aligned} \tag{IA.12}$$

where $\nabla_{i,t} \equiv a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p$. Then, it follows from (IA.12) that

$$\begin{aligned} \mathrm{Var}_t(r_{i,t+1}) &\approx \rho_i^2\nabla_{i,t}^2 + \rho_i^2 \mathrm{Var}_t\left(\sum_{j=1}^{n} \varphi_j \sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i)^2\sigma_{i,t}^2 \\ &+ \rho_i(1-\rho_i)\sigma_{i,t}\left(\mathrm{Cov}_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) + \sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right) \\ &+ \rho_i^2\nabla_{i,t}\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\zeta_{p,t+1}\right) \end{aligned}$$

REMARK 4: *Suppose assumption 1 is satisfied and*

$$Var_t\left(\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \text{and} \quad \sum_{i=1}^{n}\sigma_{i,t}Cov_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) = o(n),$$

then

$$\lim_{n\to\infty} Var_t(r_{i,t+1}) = \rho_i^2\nabla_{i,t}^2 + (1-\rho_i)^2\sigma_{i,t}^2 + \rho_i(1-\rho_i)\sigma_{i,t}\left(Cov_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) + \sum_{j=1}^{n}\varphi_j\sigma_{j,t}Cov_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right).$$

Hence, firm i's return volatility depends on $\sigma_{i,t}$ and the covariance of $\eta_{i,t+1}$ with innovations to network connectivity and idiosyncratic productivity shocks to other firms. Additionally, if firm i is such that

$$\sum_{j=1}\sigma_{j,t}Cov_t\left(\eta_{j,t+1},\eta_{i,t+1}\right) = o(n),$$

then

$$\lim_{n\to\infty} Var_t(r_{i,t+1}) = \rho_i^2\nabla_{i,t}^2 + (1-\rho_i)^2\sigma_{i,t}^2 + \rho_i(1-\rho_i)\sigma_{i,t}Cov_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right).$$

Substituting (IA.8) and (IA.12) in (IA.11) yields

$$\begin{aligned}
\mathbb{E}_t[r_{i,t+1} - r_{f,t}] &\approx -\rho_i \left(\lambda_{m,q}\nabla_{i,t} + \lambda_{m,p}\Delta_{p,t}\right)\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\zeta_{p,t+1}\right)\right) - \lambda_{m,q}\rho_i\mathrm{Var}_t\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\eta_{j,t+1}\right)\\
&- \lambda_{m,q}(1-\rho_i)\sigma_{i,t}\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\eta_{i,t+1}\right) - \lambda_{m,p}\Delta_{p,t}\rho_i\nabla_{i,t}\\
&- \lambda_{m,p}\Delta_{p,t}(1-\rho_i)\sigma_{i,t}\mathrm{Cov}_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right) - \frac{1}{2}\mathrm{Var}_t(r_{i,t+1}). \qquad \text{(IA.13)}
\end{aligned}$$

REMARK 5: *Suppose assumption 1 is satisfied and*

$$Var_t\left(\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \text{and} \quad \sum_{i=1}^{n}\sigma_{i,t}Cov_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) = o(n).$$

Define

$$\vartheta_{i,t} \equiv -\lambda_{m,p}\Delta_{p,t}\rho_i\nabla_{i,t} - \frac{1}{2}\left(\rho_i^2\nabla_{i,t}^2 + (1-\rho_i)^2\sigma_{i,t}^2\right).$$

Then,

$$\begin{aligned}
\lim_{n\to\infty}\mathbb{E}_t[r_{i,t+1} - r_{f,t}] &= \vartheta_{i,t} - (1-\rho_i)\sigma_{i,t}\left(\lambda_{m,p}\Delta_{p,t} + \frac{1}{2}\rho_i\right)Cov_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right)\\
&- (1-\rho_i)\sigma_{i,t}(\lambda_{m,q}+\rho_i)\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}Cov_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right).
\end{aligned}$$

F. Calibration exercise

This section analyzes whether the main results obtained in the paper can be supported by a calibrated version of the production-based network equilibrium model.

F.1. Values for model parameters

I follow Bansal and Yaron (2004) and set the preferences parameters to $\beta = 0.997$, $\gamma = 10$, and $\rho = 0.65$. I follow Acemoglu et al. (2012) and set the share of labor to $\chi = 0.55$. As in the paper, I set $\sigma_a = 1.7$ so that σ_a equals the annual volatility of aggregate TFP growth. To compute the share of good j used in good i, $\{w_{ij}\}_{(i,j)}$, I use industry-level data from BEA Input-Output (IO) tables from 1997 to 2015. IO tables are at the annual frequency. I compute the percentage of industry j's sales purchased by industry i at year t following Ahern and Harford (2014). I then set w_{ij} equal to the average percentage of industry j's sales purchased by industry i over the sample period. Firms in the same industry are assumed to share the same values for $\{w_{ij}\}_{(i,j)}$. To calibrate the benchmark topology, I use the U.S. supplier–customer network of 2015 as in the paper. Thus, $n = 1,100$ and the network exhibits a power-law degree distribution. The shape parameter vector ζ_t and its dynamics are calibrated as in the paper. I set $\alpha_1 = 0.1$ and $\alpha_2 = 1$ so that the unconditional mean and volatility of consumption growth generated by the calibrated model are similar to the ones found in data.

F.2. Implications of the calibrated model

Because equilibrium consumption growth can be decomposed as in the baseline model, long-run consumption risks endogenously arise as long as $\widetilde{W}_{n,t+1}$ exhibits a long-run predictable component. This is the case because the shape parameter vector ζ_t follows a persistent process, and thus, changes in network connectivity are infrequent.

To analyze whether cross-sectional results in the paper can be supported by a production-based equilibrium framework, I study how the risk premium of a firm changes in the presence of a small increase in its centrality. Let κ_{it} denote the centrality of firm i at t as defined in the paper. If assumption 1 is satisfied and the following two equalities hold:

$$\mathrm{Var}_t\left(\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1}\right) = o(n^2) \quad \text{and} \quad \sum_{i=1}^{n}\sigma_{i,t}\mathrm{Cov}_t\left(\zeta_{p,t+1},\eta_{i,t+1}\right) = o(n).$$

then the following approximation is accurate for a large fraction of firms in the economy:

$$\begin{aligned}\frac{\partial \mathbb{E}_t[r_{i,t+1}-r_{f,t}]}{\partial \kappa_{i,t}} &\approx \frac{\partial \vartheta_{i,t}}{\partial \kappa_{i,t}} - (1-\rho_i)\frac{\partial}{\partial \kappa_{i,t}}\left(\sigma_{i,t}\left(\lambda_{m,p}\Delta_{p,t}+\frac{1}{2}\rho_i\right)\mathrm{Cov}_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right)\right)\\ &- (1-\rho_i)(\lambda_{m,q}+\rho_i)\frac{\partial}{\partial \kappa_{i,t}}\left(\sigma_{i,t}\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right)\right),\end{aligned}$$

because n is large and the network exhibits a power-law degree distribution.

Note that

$$\frac{\partial \vartheta_{i,t}}{\partial \kappa_{i,t}} = -a_1\mu_2\left(\rho_i\lambda_{m,p}(\varphi_i\nabla_{i,t}+\Delta_{p,t})+\rho_i^2\nabla_{i,t}+(1-\rho_i)^2\frac{\sigma_{i,t}}{a_1\mu_2}\right)\frac{\partial \sigma_{i,t}}{\partial \kappa_{i,t}}.$$

Additionally, the following first-order approximations hold

$$\begin{aligned}\frac{\partial}{\partial \kappa_{i,t}}\left(\sigma_{i,t}\left(\lambda_{m,p}\Delta_{p,t}+\frac{1}{2}\rho_i\right)\mathrm{Cov}_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right)\right) &\approx \left(\frac{\partial \sigma_{i,t}}{\partial \kappa_{i,t}}\right)\left(\lambda_{m,p}\Delta_{p,t}+\frac{1}{2}\rho_i+\lambda_{m,p}a_1\mu_2\varphi_i\sigma_{i,t}\right)\mathrm{Cov}_t\left(\eta_{i,t+1},\zeta_{p,t+1}\right),\\ \frac{\partial}{\partial \kappa_{i,t}}\left(\sigma_{i,t}\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right)\right) &\approx \frac{\partial \sigma_{i,t}}{\partial \kappa_{i,t}}\left(\sum_{j=1}^{n}\varphi_j\sigma_{j,t}\mathrm{Cov}_t\left(\eta_{j,t+1},\eta_{i,t+1}\right)\right).\end{aligned}$$

Under the benchmark parameterization, $a_1 > 0$, and thus, $\frac{\partial \vartheta_{i,t}}{\partial \kappa_{i,t}} < 0$. In addition, the previous two derivatives are non-negative in the benchmark parameterization. As a result, $\frac{\partial \mathbb{E}_t[r_{i,t+1}-r_{f,t}]}{\partial \kappa_{i,t}} \leq 0$. This result shows that central firms command lower risk premiums than peripheral firms within a calibrated version of the production-based equilibrium framework. A realistic spread between firms in the highest and lowest centrality decile can be generated by fine tuning the values of μ_0 and ν_0.

B. *Distribution of* $\widetilde{W}_{n,t+1}$ *and* $\Delta\widetilde{c}_{t+1}$

Under specific dependence assumptions, explicit bounds between the distribution of $\widetilde{W}_{n,t+1}$ and well-known distributions can be found. For instance, effective bounds between the distribution of $\widetilde{W}_{n,t+1}$ and the Poisson and Binomial distributions can be found if the dependence structure among variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ decreases as the distance between them increases. Chen (1975) finds bounds between the distribution of the sum of dependent Bernoulli trials and the Poisson distribution. Soon (1996) finds bounds between the distribution of the sum of dependent Bernoulli trials and the Binomial distribution.

A. *Distribution of $\widetilde{W}_{n,t+1}$ for finite n.*

Given how idiosyncratic shocks propagate along the network, variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ are not independent. The following definition becomes handy for analyzing the distribution of $\widetilde{W}_{n,t+1}$ as it reconciles the notion of dependence between two random variables $\widetilde{\varepsilon}_{i,t+1}$ and $\widetilde{\varepsilon}_{j,t+1}$, and the position of firms i and j in $\mathcal{G}_n$.

DEFINITION 2 (Dependency Graphs): *A graph $\mathcal{G}$ is said to be a dependency graph for a family of random variables if the following two conditions are satisfied:*

(i) The set of random variables can be indexed by the nodes of $\mathcal{G}$.

(ii) If S_1 and S_2 are two disjoint set of nodes in $\mathcal{G}$ such that no edge in $\mathcal{G}$ has one endpoint in S_1 and the other in S_2, then the corresponding sets of random variables are independent.

Note that the above definition does not define a unique dependency graph for every family of random variables. For instance, adding one edge to a dependency graph yields a new dependency graph for the same family of random variables.

It is worth noting that $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ has $\mathcal{G}_n$ as its dependency graph. Using the previous definition, the following proposition says that if every firm has a sufficiently small probability of being affected by an idiosyncratic shock, then $\widetilde{W}_{n,t+1}$ follows approximately a Poisson distribution despite the fact that $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ are not independent.

PROPOSITION 1 (Poisson Approximation of $\widetilde{W}_{n,t+1}$): *Define*

$$\begin{aligned}
\widetilde{\pi}_{i,t+1} &\equiv \mathbb{E}\big[\widetilde{\varepsilon}_{i,t+1}\big|\widetilde{p}_{t+1}\big] \\
\lambda_{t+1} &\equiv \mathbb{E}\big[\widetilde{W}_{n,t+1}\big|\widetilde{p}_{t+1}\big] \\
\sigma^2_{t+1} &\equiv Var\big[\widetilde{W}_{n,t+1}\big|\widetilde{p}_{t+1}\big]
\end{aligned}$$

Provided that $\mathcal{G}_n$ is the dependency graph of the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ then

$$d_{TV}\left(\widetilde{W}_{n,t+1}, Po\big(\lambda_{t+1}\big)\right) \leq \min\left\{1, \frac{1}{\lambda_{t+1}}\right\}\left(\sigma^2_{t+1} - \lambda_{t+1} + 2\sum_{i=1}^{n}\widetilde{\pi}^2_{i,t+1} + \sum_{(i,j)\,\in\,\mathcal{R}_n}\widetilde{\pi}_{i,t+1}\widetilde{\pi}_{j,t+1}\right)\ \forall\ t$$

where $d_{TV}\left(\widetilde{W}_{n,t+1}, Po\big(\lambda_{t+1}\big)\right)$ denotes the total variation distance between the distribution of $\widetilde{W}_{n,t+1}$ and a random variable with Poisson distribution of parameter λ_{t+1}, denoted by $Po\big(\lambda_{t+1}\big)$.[4]

Proof of Proposition 1. The result follows directly from Janson et al. (2000, Theorem 6.23) □

[4]The total variation distance between the distribution of two random variables X and Y is defined as

$$d_{TV}\big(X, Y\big) \equiv \sup_{A}\big|\mathbb{P}[X \in A] - \mathbb{P}[Y \in A]\big|$$

taking the supremum over all borel sets A. If X and Y are integer valued, then

$$d_{TV}\big(X, Y\big) \equiv \frac{1}{2}\sum_{k}\big|\mathbb{P}[X = k] - \mathbb{P}[Y = k]\big|$$

B. Distribution of $\widetilde{W}_{n,t+1}$ as n grows large

If variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ are independent, the central limit theorem implies that $\sqrt{n}(\widetilde{W}_{n,t+1} - q)$ is normally distributed as n grows large. Unfortunately, variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ are not independent. The following proposition says that if the number of relationships of the most connected firm is not too large, then $\widetilde{W}_{n,t+1}$ follows a normal distribution as n grows large.[5]

PROPOSITION 2 (Asymptotic Normality of $\widetilde{W}_{n,t+1}$): *Let D_n denote the highest number of relationships per firm in the economy. If there are no relationships in $\mathcal{G}_n$ define $D_n = 1$. For each $t+1$, define*

$$\mu_{t+1} \equiv \lim_{n\to\infty} \mathbb{E}\left[\sum_{i=1}^{n} \widetilde{\varepsilon}_{i,t+1} \middle| \widetilde{p}_{t+1}\right] \quad \textit{and} \quad \sigma^2_{t+1} \equiv \lim_{n\to\infty} \mathit{Var}\left[\sum_{i=1}^{n} \widetilde{\varepsilon}_{i,t+1} \middle| \widetilde{p}_{t+1}\right]$$

If there exists an integer $m \geq 3$ such that

$$\lim_{n\to\infty} \left(\frac{n}{D_n}\right)^{\frac{1}{m}} \left(\frac{D_n}{n\sigma_t}\right) = 0 \tag{IA.14}$$

then, $\widetilde{W}_{n,t+1} \xrightarrow{d} \mathcal{N}\left(\mu_{t+1}, \sigma^2_{t+1}\right)$ as $n \to \infty$.[6]

Proof of Proposition 2. Provided that $\widetilde{\varepsilon}_{i,t}$ are Bernoulli random variables, $\left|\widetilde{\varepsilon}_{i,t}\right| \leq 1$, $\forall\ i$. As $\mathcal{G}_n$ is a dependency graph for the family $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$, $\forall\ t$, it follows from Janson (1988, Theorem 2) that

$$\sum_{i=1}^{n} \widetilde{\varepsilon}_{i,t} \xrightarrow{d} \mathcal{N}\left(\mu_n(t), \sigma^2_n(t)\right)$$

$\square$

The following corollary provides a more detailed characterization of the types of economies wherein $\widetilde{W}_{n,t+1}$ follows a normal distribution as n gets large.

COROLLARY 1: *Suppose $\sigma_t < \infty$. If $D_n = o(n)$ then log consumption growth is approximately normally distributed as n gets large.*

Proof of Corollary 1. If $\sigma_t < \infty$ and $D_n = o(n)$ then (IA.14) is satisfied. Hence $\widetilde{W}_{n,t+1}$ and $\Delta\widetilde{c}_{t+1}$ are normally distributed as n grows large. $\square$

Therefore, as long as the number of relationships of the most connected firm grows less than linearly with n, $\widetilde{W}_{n,t+1}$ and thus, $\Delta\widetilde{c}_{t+1}$, follow a normal distribution as n grows large.

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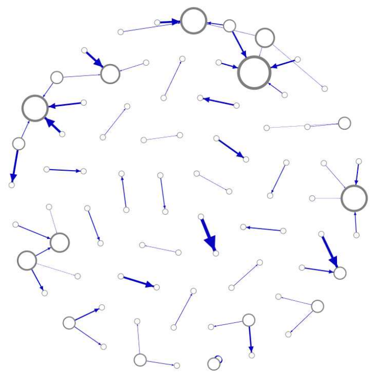
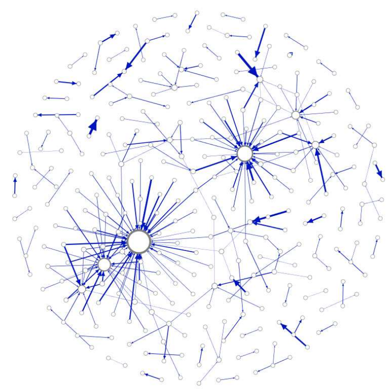
[5] See Baldi and Rinott (1989, Corollary 2) for a similar result to that in Janson (1988, Theorem 2).

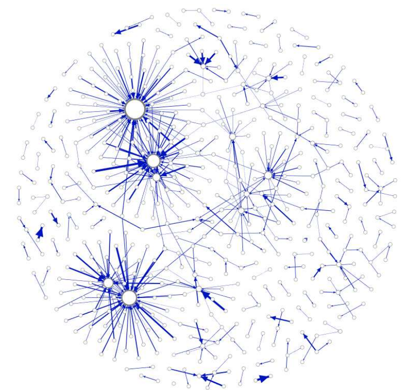
[6] If condition (IA.14) is satisfied, then $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$ can be interpreted as a m-dependent sequence of random variables. Namely, if the distance between variable j and k is greater than m then $\widetilde{\varepsilon}_{j,t}$ is independent of $\widetilde{\varepsilon}_{k,t}$, for all t. To do so, however, a notion of distance between $\widetilde{\varepsilon}_{j,t}$ and $\widetilde{\varepsilon}_{k,t}$ needs to be properly defined to reconcile the position of firms i and j in $\mathcal{G}_n$ with their position in the sequence $\{\widetilde{\varepsilon}_{l,t}\}_{l=1}^{n}$.



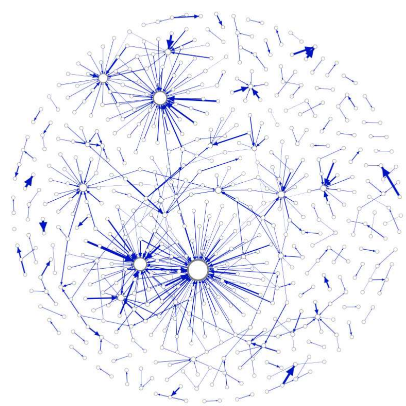
(a) 1976



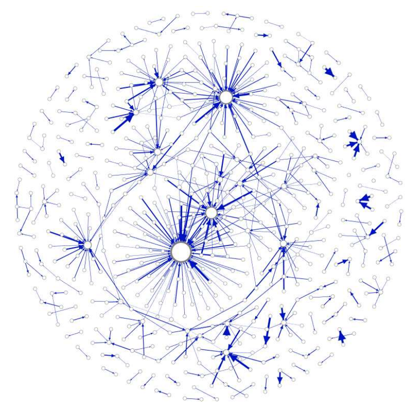
(b) 1977



(c) 1978

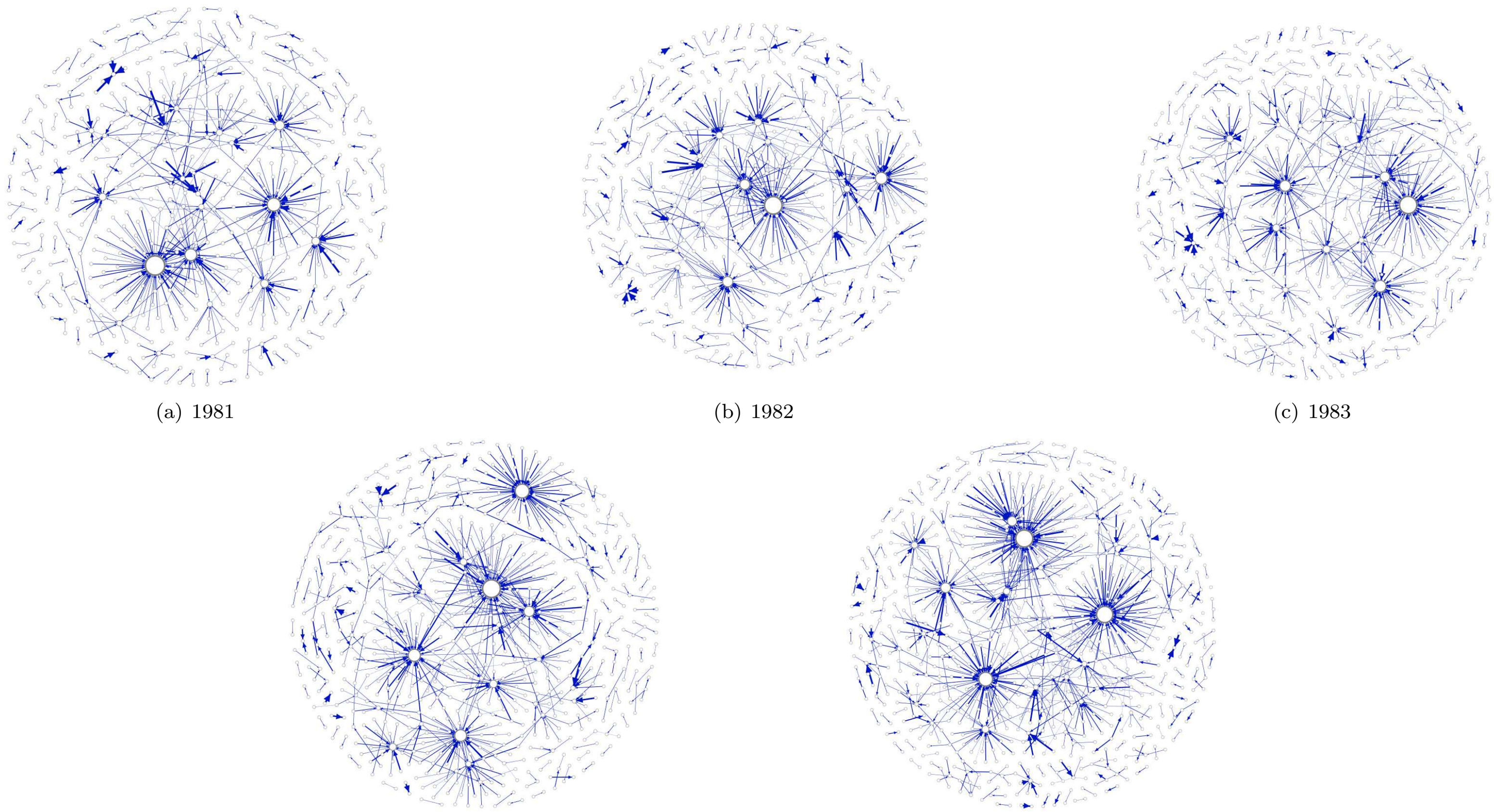


(d) 1979



(e) 1980

Figure IA.1. The figure shows supplier–customer networks from 1976 to 1980.



(a) 1981

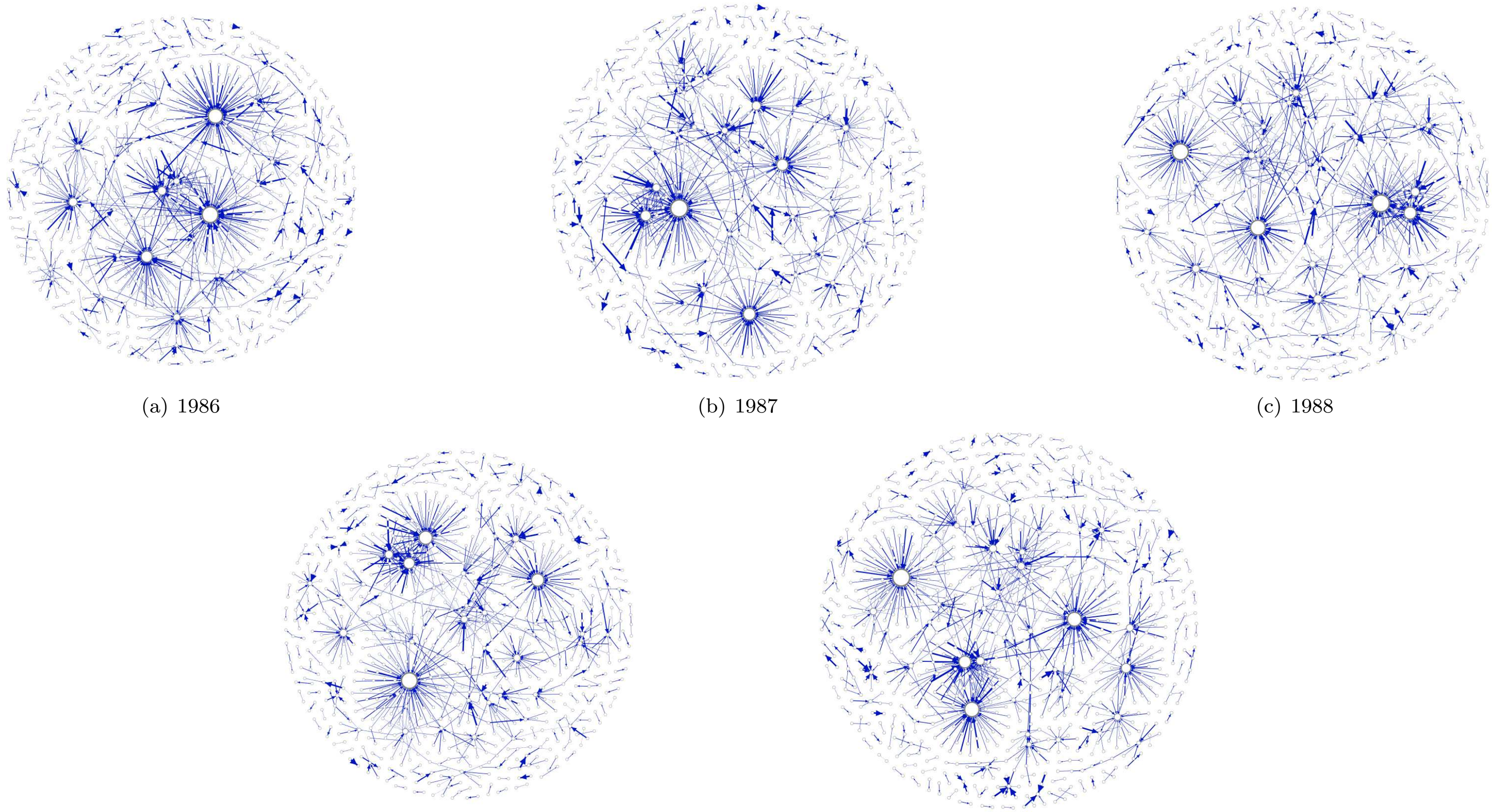
(b) 1982

(c) 1983

(d) 1984

(e) 1985

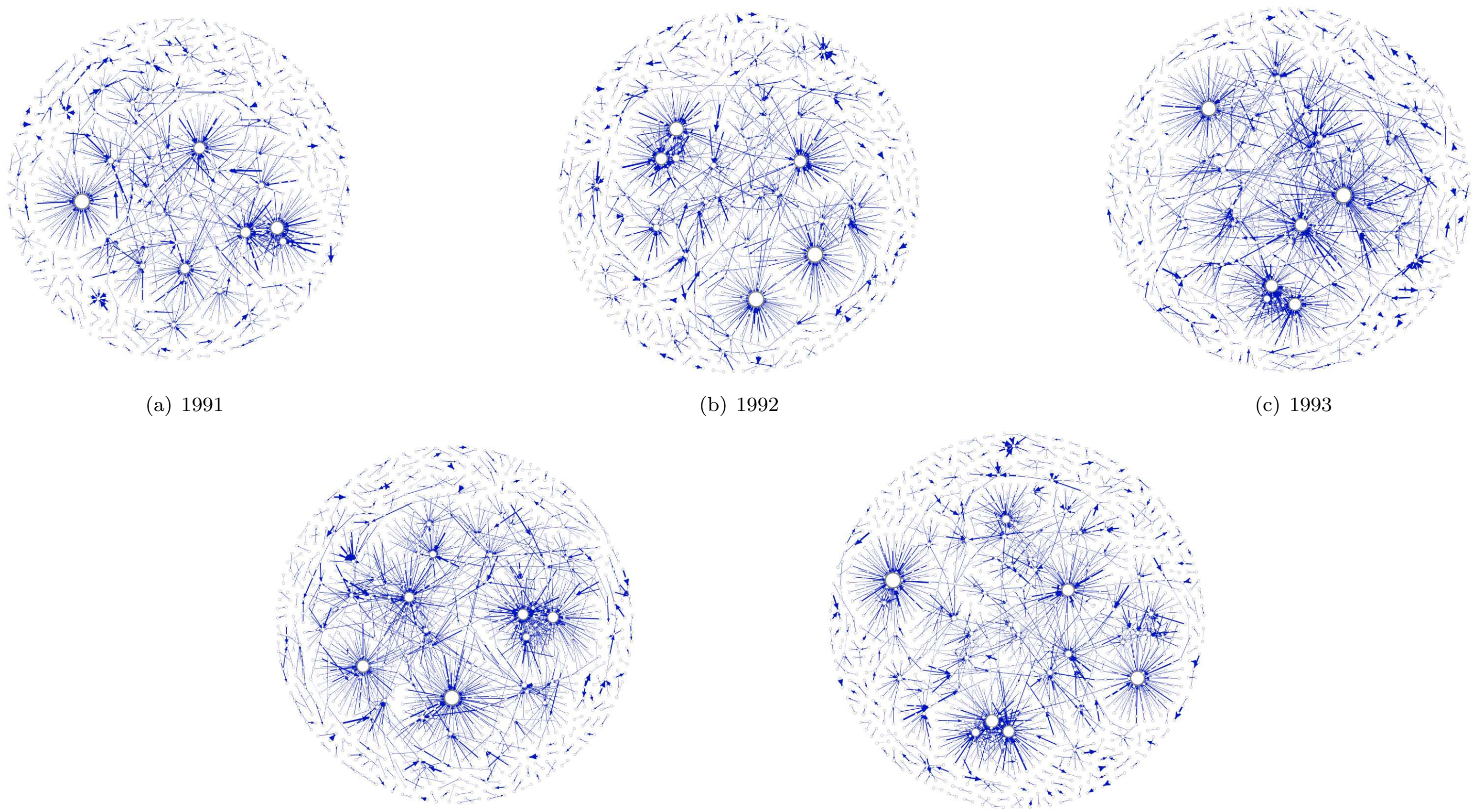
Figure IA.2. The figure shows supplier–customer networks from 1981 to 1985.



(a) 1986 (b) 1987 (c) 1988

(d) 1989 (e) 1990

Figure IA.3. The figure shows supplier–customer networks from 1986 to 1990.



(a) 1991

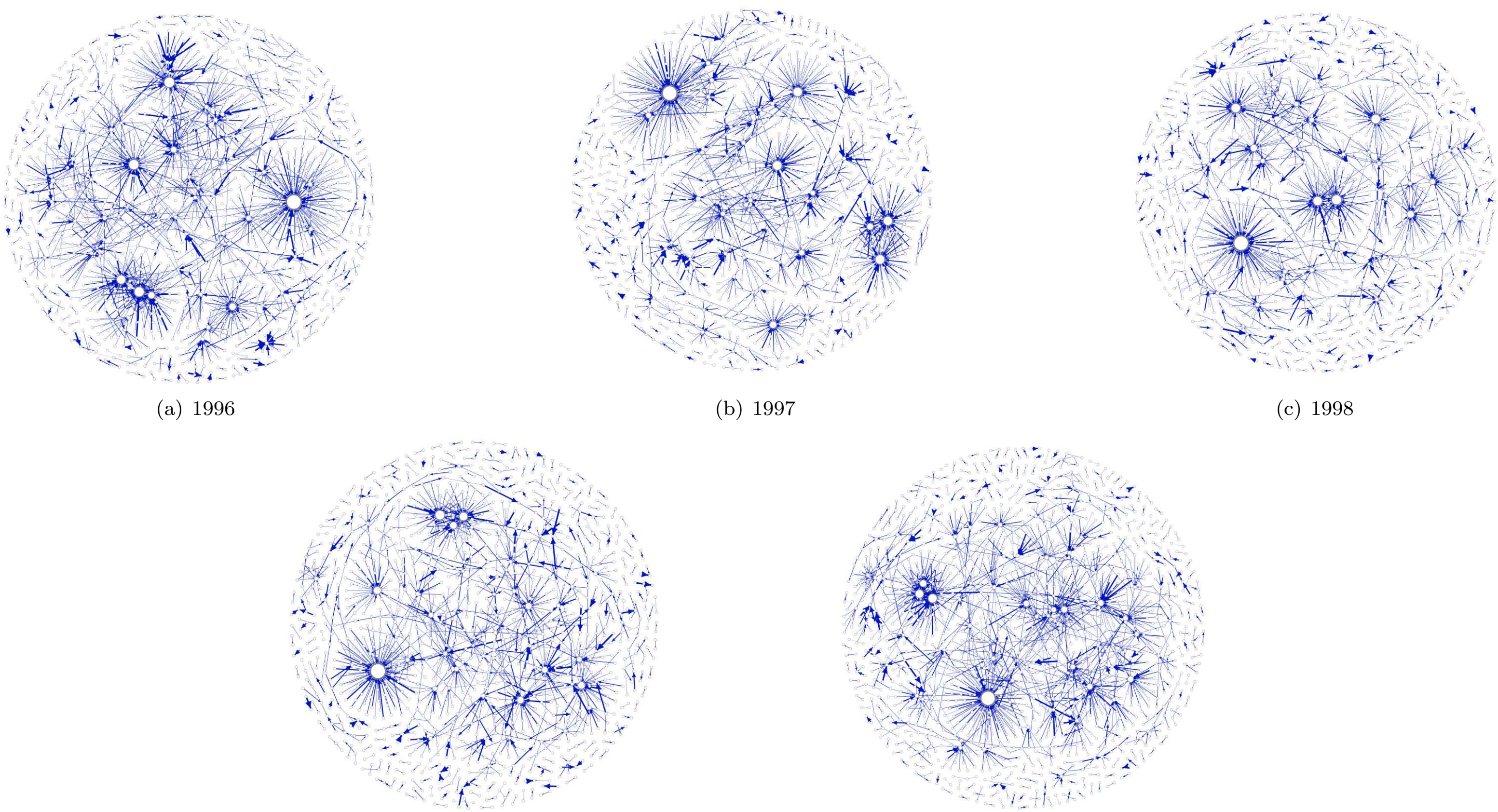
(b) 1992

(c) 1993

(d) 1994

(e) 1995

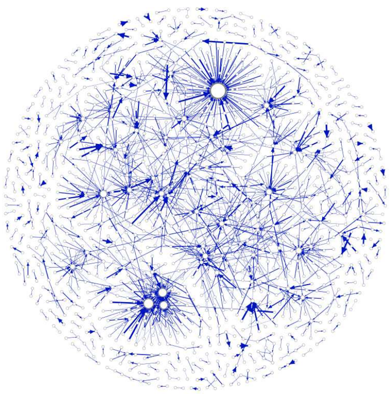
Figure IA.4. The figure shows supplier–customer networks from 1991 to 1995.



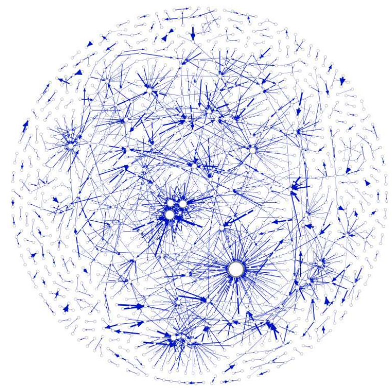
(a) 1996 (b) 1997 (c) 1998

(d) 1999 (e) 2000

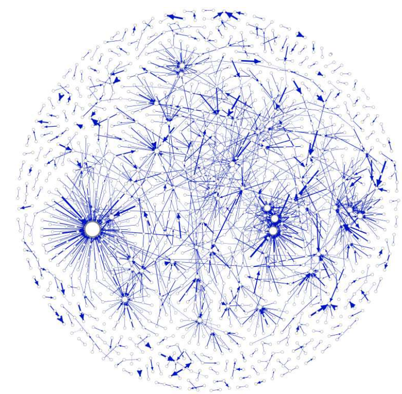
Figure IA.5. The figure shows supplier–customer networks from 1996 to 2000.



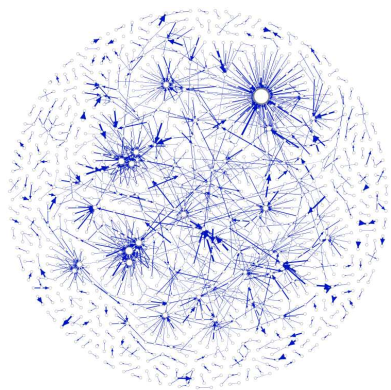
(a) 2001



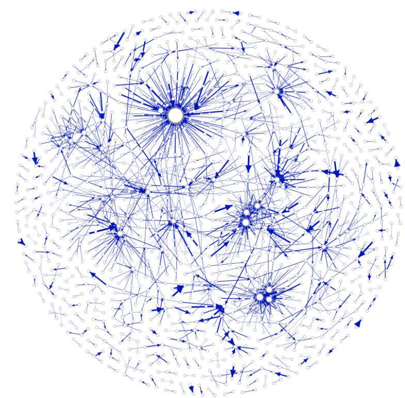
(b) 2002



(c) 2003

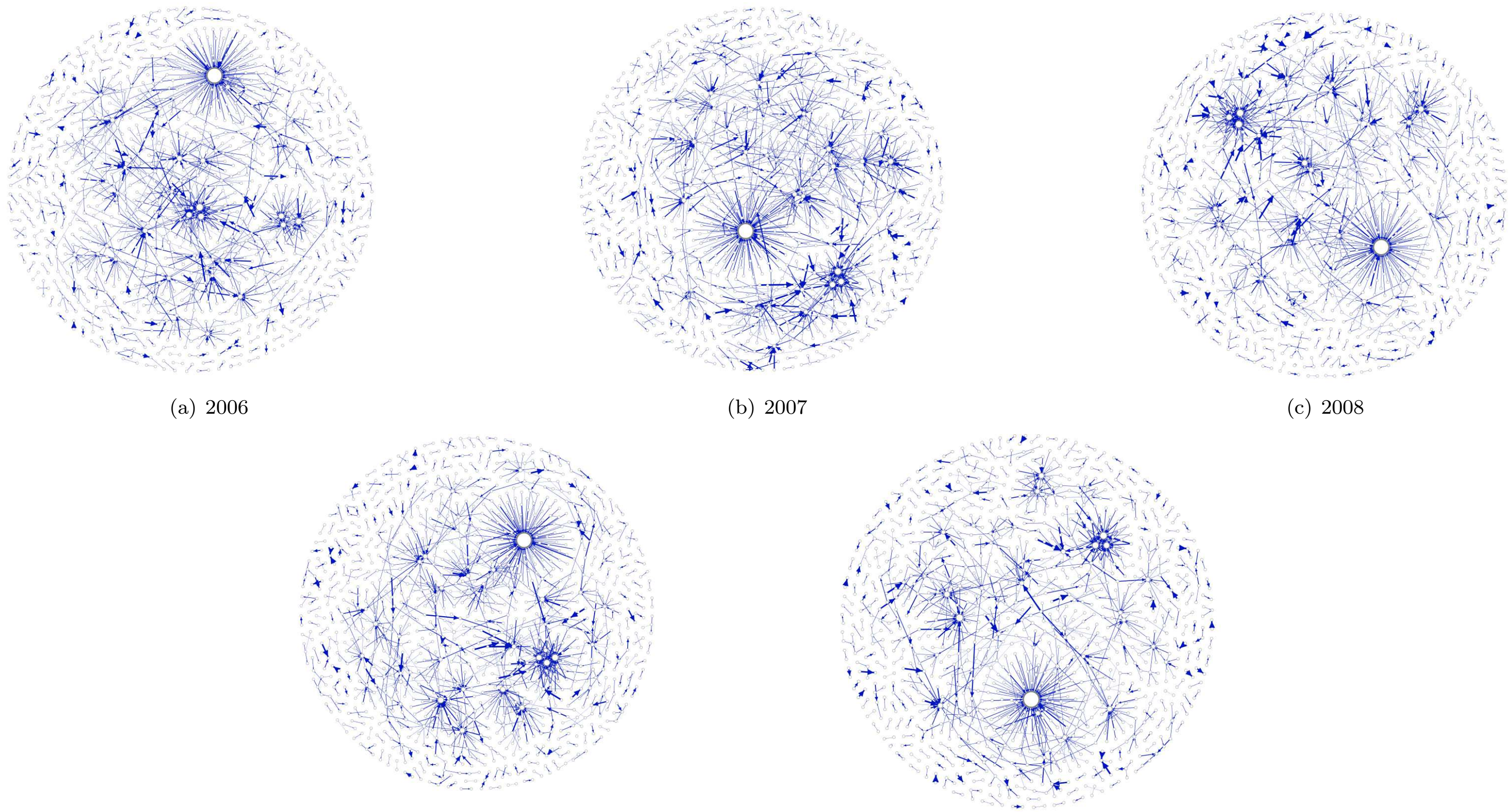


(d) 2004



(e) 2005

Figure IA.6. The figure shows supplier–customer networks from 2001 to 2005.



(a) 2006

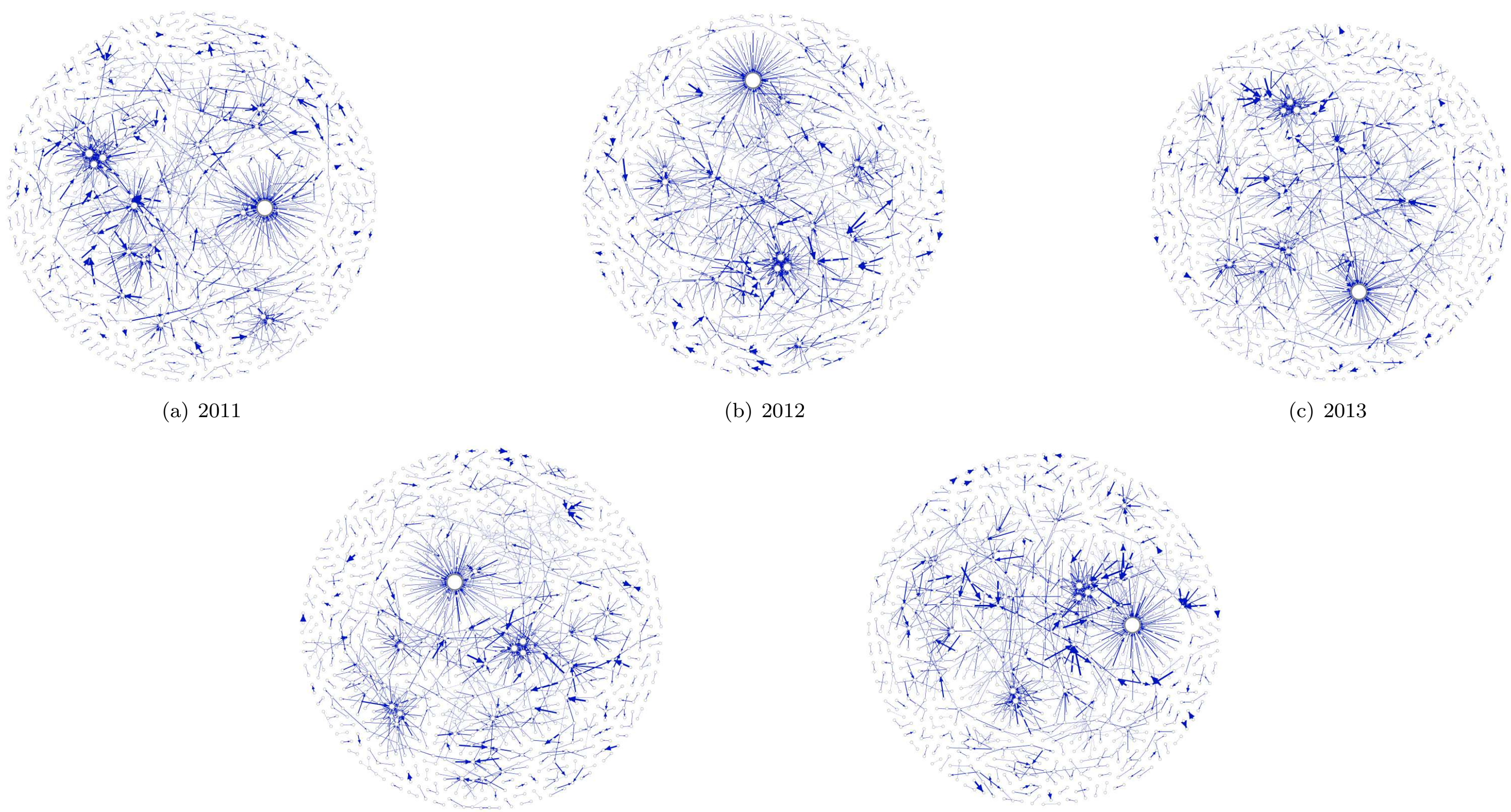
(b) 2007

(c) 2008

(d) 2009

(e) 2010

Figure IA.7. The figure shows supplier–customer networks from 2006 to 2010.



(a) 2011 (b) 2012 (c) 2013

(d) 2014 (e) 2015

Figure IA.8. The figure shows supplier–customer networks from 2011 to 2015.

D. Simulation of the Model

This section describes the algorithm used to simulate the model. Let

$$Z_{t+1} \equiv f\left(\widetilde{W}_{n,t+1}\right),\ T_{i,t+1} \equiv g\left(\widetilde{W}^{i}_{n,t+1}\right), U_{i,t+1} \equiv h\left(\widetilde{W}^{-i}_{n,t+1}\right),\ \text{and}\ e_{i,t+1} \equiv m\left(\widetilde{\varepsilon}_{i,t+1}\right)$$

denote four random variables which are functions of $\widetilde{W}_{n,t+1}$, $\widetilde{W}^{i}_{n,t+1}$, $\widetilde{W}^{-i}_{n,t+1}$, and $\widetilde{\varepsilon}_{i,t+1}$, respectively. Given ζ_{t+1} and q, I use the following procedure to compute the conditional expectation of Z_{t+1}, $T_{i,t+1}$, $U_{i,t+1}$, and $e_{i,t+1}$:

(a) Determine the set of firms that initially face idiosyncratic shocks by drawing a Bernoulli random variable (with success probability q) per each firm.

(b) Determine the set of relationships that transmit shocks by drawing a Bernoulli random variable per each relationship. The success probability of relationship (i, j) at $t+1$ is given by $\widetilde{p}_{ij,t+1}$. Probabilities $\{\widetilde{p}_{ij,t+1}\}_{(i,j)\in\mathcal{G}_n}$ are drawn from a Beta distribution of parameter ζ_{t+1}. Higher values of $\widetilde{p}_{ij,t+1}$ are assigned to relationships with lower betweenness scores to capture features of the data.

(c) Compute $\widetilde{W}_{n,t+1}$ by adding all firms affected by idiosyncratic shock at $t+1$. Compute $\widetilde{W}^{i}_{n,t+1}$ by adding all firms in $\mathcal{G}^{i}_{n}$ affected by idiosyncratic shock at $t+1$. Compute $\widetilde{W}^{-i}_{n,t+1}$ by adding all firms in the complement set of $\mathcal{G}^{i}_{n}$ affected by idiosyncratic shock at $t+1$. Firms are considered to be affected by idiosyncratic shocks according to the propagation mechanism described in the paper. Using $\widetilde{W}_{n,t+1}$, $\widetilde{W}^{i}_{n,t+1}$, and $\widetilde{W}^{-i}_{n,t+1}$, I compute Z_{t+1} as $Z_{t+1} = f\left(\widetilde{W}_{n,t+1}\right)$, $T_{i,t+1}$ as $T_{i,t+1} = g\left(\widetilde{W}^{i}_{n,t+1}\right)$, and $U_{i,t+1}$ as $U_{i,t+1} = h\left(\widetilde{W}^{-i}_{n,t+1}\right)$.

(c) Repeat steps (a), (b), and (c) 10,000 times. I set $\mathbb{E}[Z_{t+1}|\zeta_{t+1}]$, $\mathbb{E}[T_{i,t+1}|\zeta_{t+1}]$, and $\mathbb{E}[U_{i,t+1}|\zeta_{t+1}]$ equal to their corresponding sample averages. To compute $\mathbb{E}[e_{i,t+1}|\zeta_{t+1}]$, I only repeat steps (a) and (b). I set $\mathbb{E}[e_{i,t+1}|\zeta_{t+1}]$ equals to the sample average over the 10,000 simulations.

When needed, one needs to repeat the above procedure for each firm. This turns out to be computationally intensive. To reduce time, one can take advantage of the topology of $\mathcal{G}^{i}_{n}$. In particular, if $\mathcal{G}^{i}_{n}$ is a tree, the following algorithm can be use to compute the probability that firm i is affected by an idiosyncratic shock. This algorithm exploits the fact that computing such probabilities can be framed as a recursive problem.

Algorithm *Firm i's Probability (G^{i}_{n}, s_t, q)*
($*$ Description: Algorithm that computes firm i's probability of facing shocks if G^{i}_{n} is a tree $*$)
Input: G^{i}_{n} (a tree), s_t (state of the economy), q.
Output: Firm i's probability of facing shocks at time t, $\pi^{*}_{i}(s_t)$
1. **if** firm i has a no connections
2. **return** $\pi^{*}_{i}(s_t) = q$
3. **else** **return** Prob(i,G^{i}_{n},s_t,q)

where Prob(i,G^{i}_{n},s_t,q) corresponds to the following recursive program:

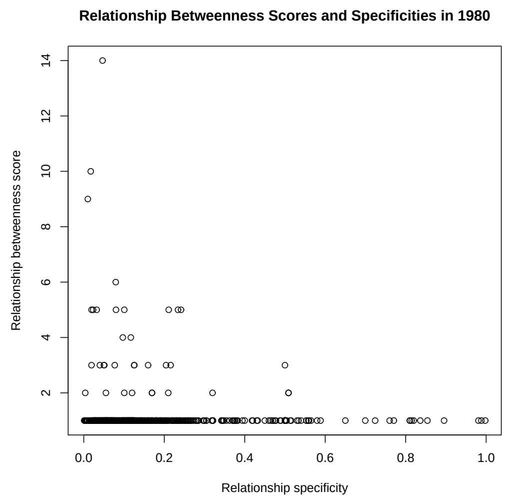
Algorithm *Prob(i,G^{i}_{n},s_t,q)*
($*$ Description: Recursive algorithm that computes firm i's probability of facing a shock $*$)
Input: Tree G^{i}_{n} wherein node i is the root, s_t, and q.
Output: $\pi^{*}_{i}(s_t)$

1. Determine the set of children of node i in G_n^i, say $\mathcal{C}_i$.[7]
2. **if** $\mathcal{C}_i = \emptyset$
3. **return** $\pi_i^*(s_t) = q$
4. **else if** every node in $\mathcal{C}_i$ has no children
5. **return** $\pi_i^*(s_t) = q + (1-q)\left(1 - \mathbb{E}\left[\prod_{k \in \mathcal{C}_i}(1 - q\widetilde{p}_{ikt})\middle| s_t\right]\right)$
6. **else return** $\pi_i^*(s_t) = q + (1-q)\left(1 - \mathbb{E}\left[\prod_{k \in \mathcal{C}_i}\left(1 - \widetilde{p}_{ikt}\mathrm{Prob}(k, T_{i,k}, s_t, q)\right)\right) \middle| s_t\right]$

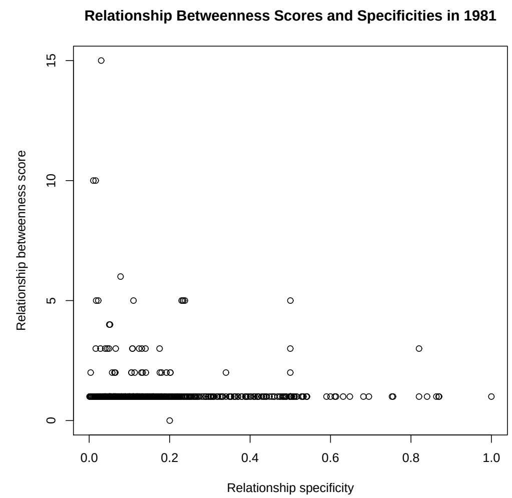
where $T_{i,k}$ denotes the branch of tree G_n^i that starts at node k.

E. Figures and Tables

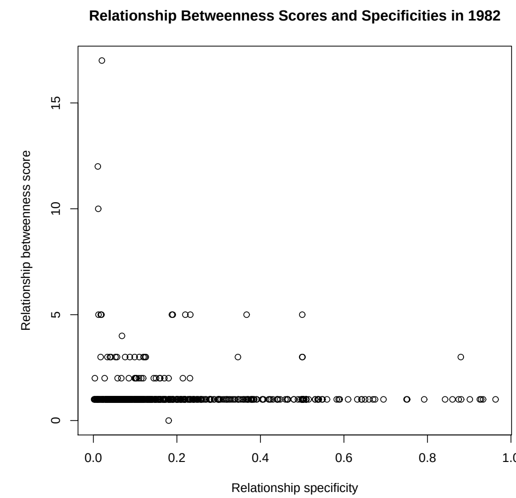
[7] In a rooted tree, the parent of a node is the node connected to it on the path to the root. Every node except the root has a unique parent. A child of a node v is a node of which v is the parent.



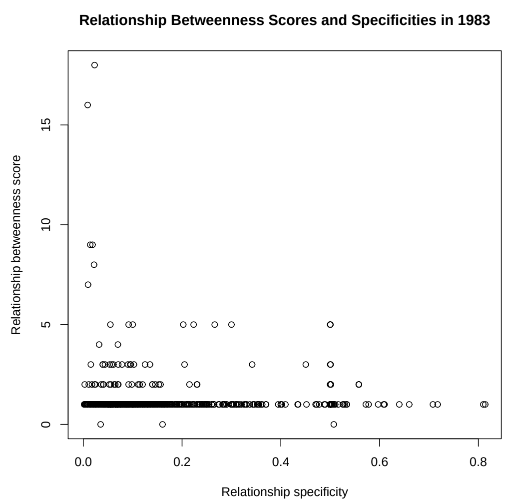
(a) 1980



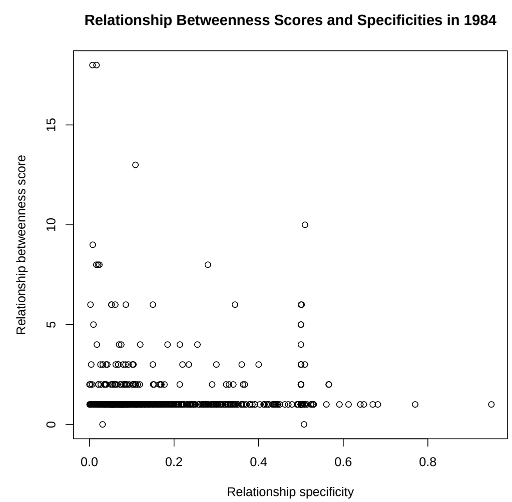
(b) 1981



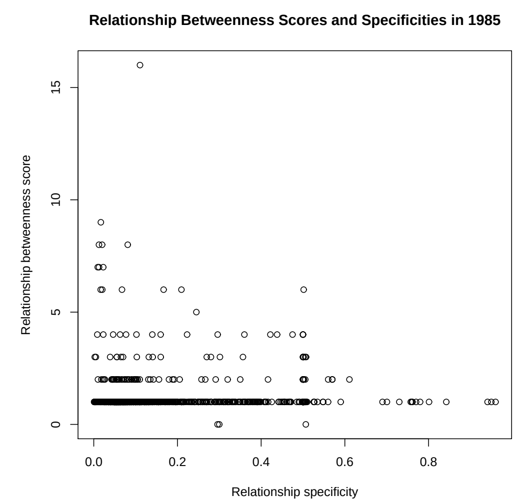
(c) 1982



(d) 1983

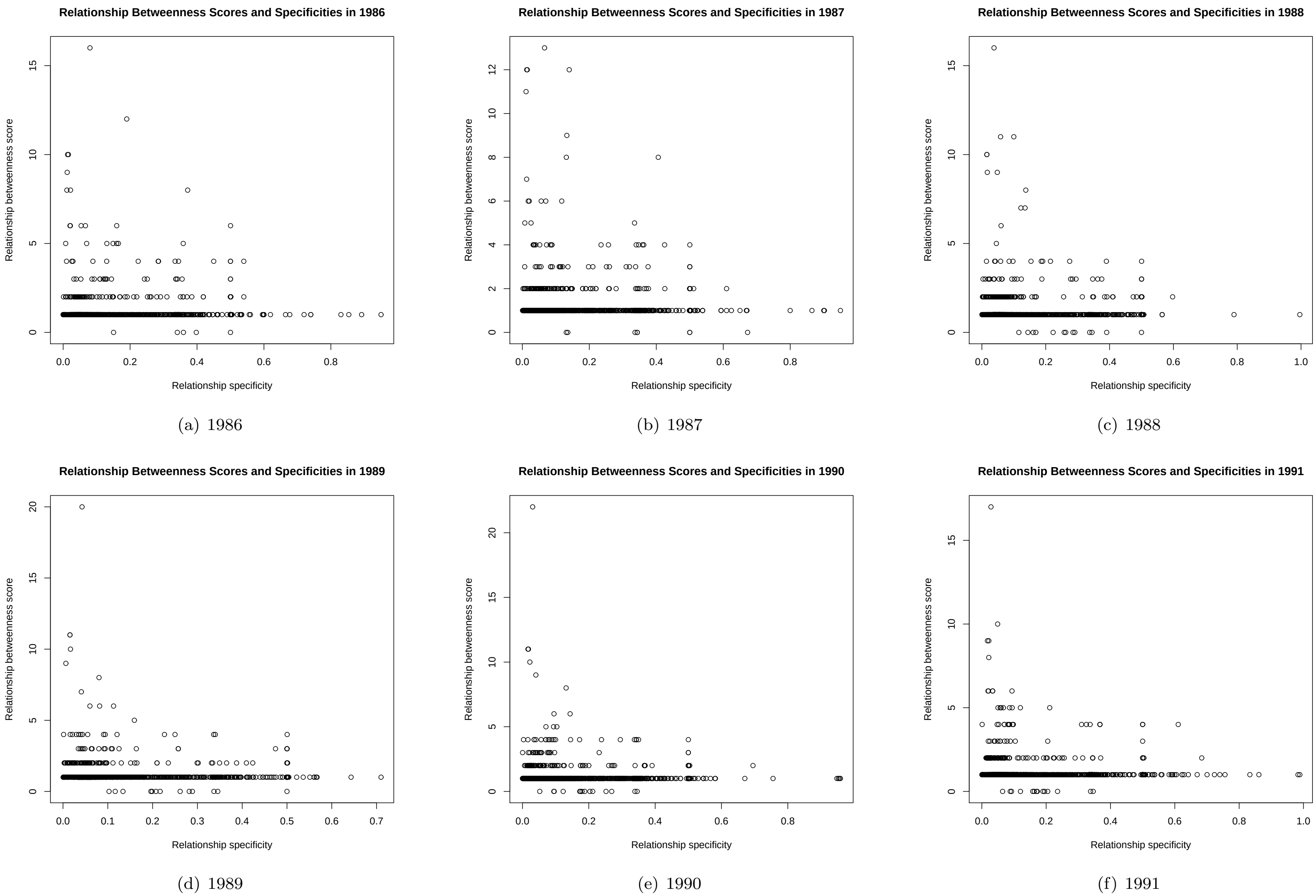


(e) 1984



(f) 1985

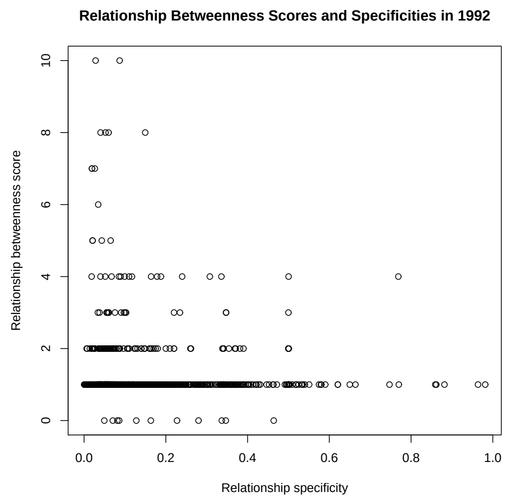
Figure IA.9. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 1980 to 1985.



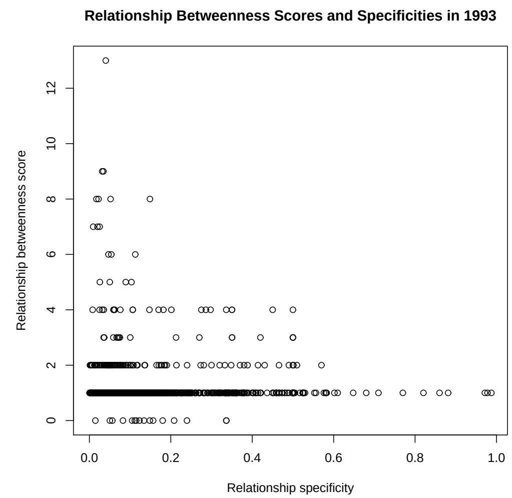
(a) 1986 (b) 1987 (c) 1988

(d) 1989 (e) 1990 (f) 1991

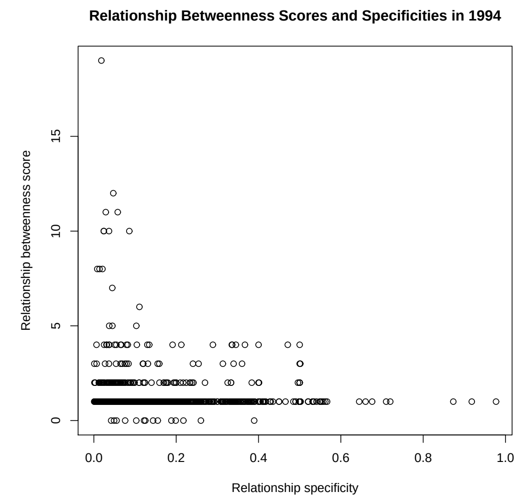
Figure IA.10. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 1986 to 1991.



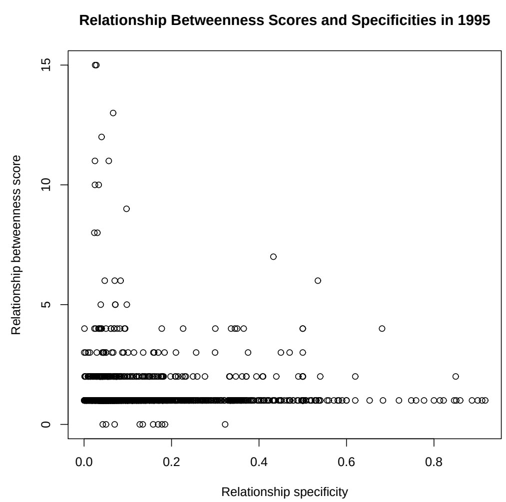
(a) 1992



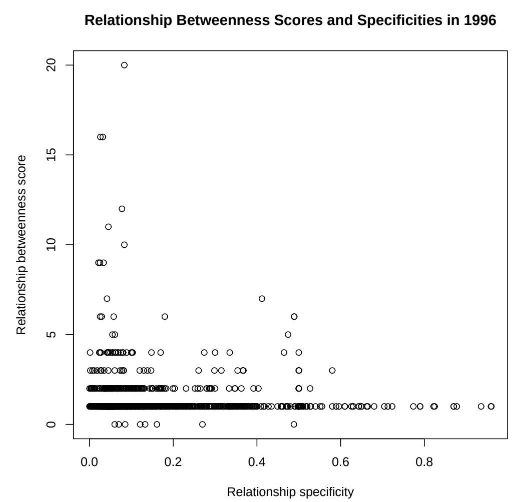
(b) 1993



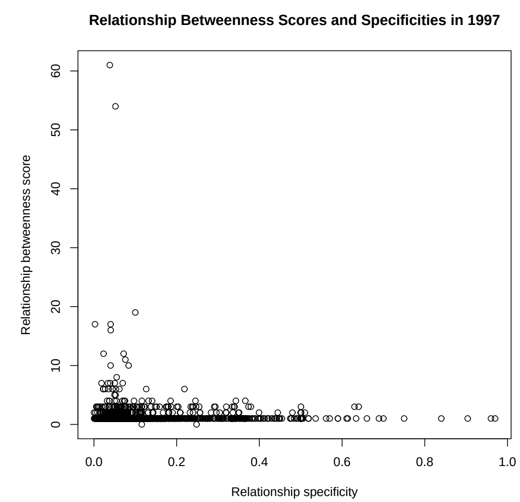
(c) 1994



(d) 1995

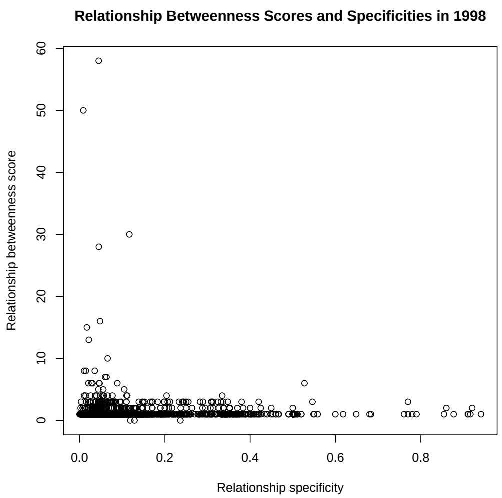


(e) 1996

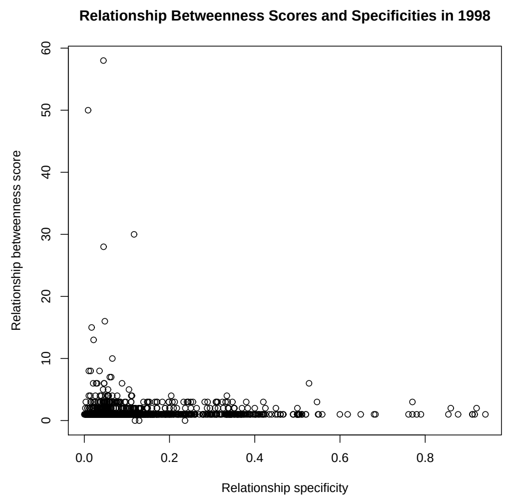


(f) 1997

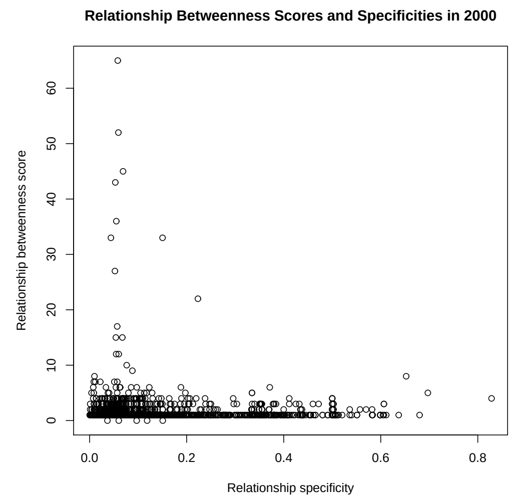
Figure IA.11. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 1992 to 1997.



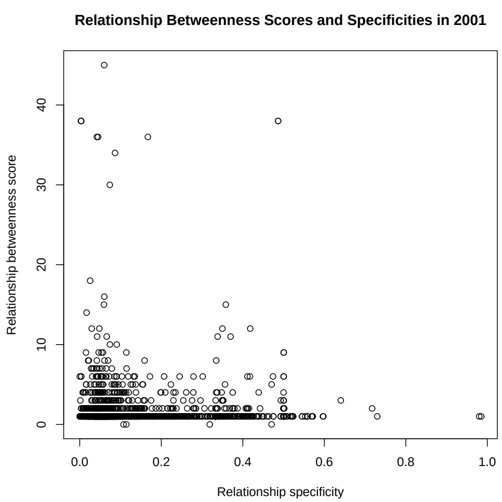
(a) 1998



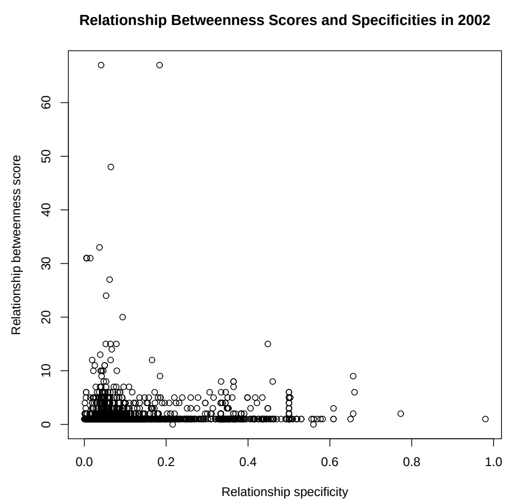
(b) 1999



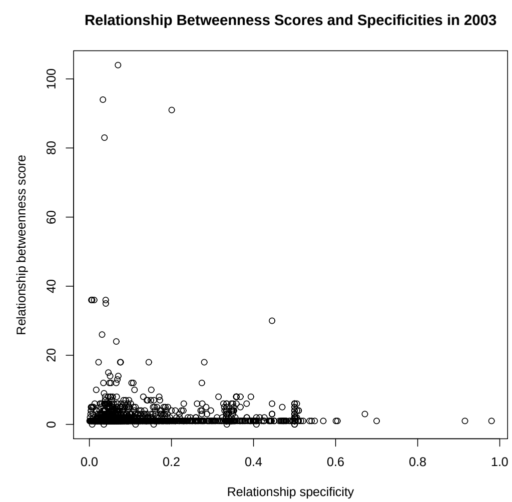
(c) 2000



(d) 2001

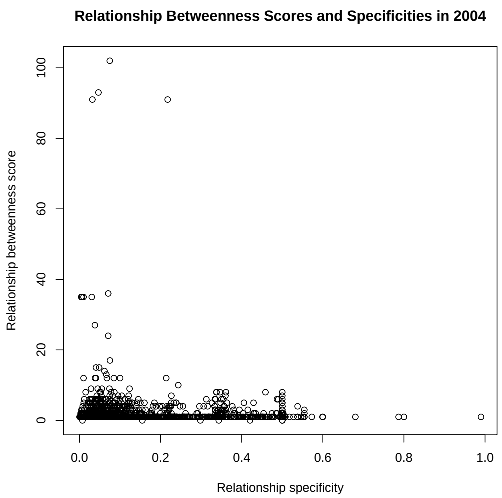


(e) 2002

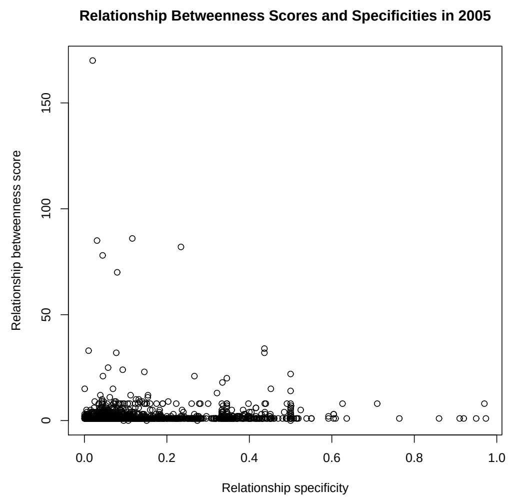


(f) 2003

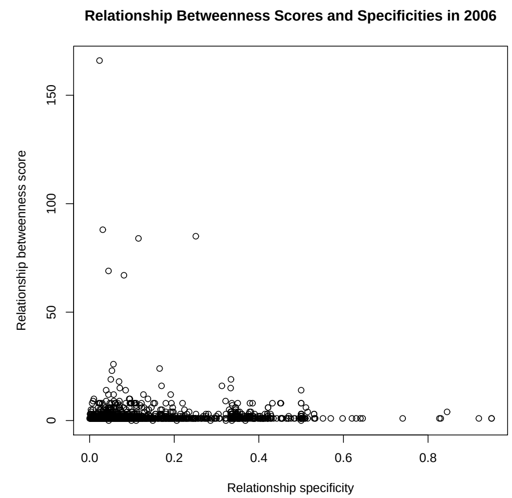
Figure IA.12. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 1998 to 2003.



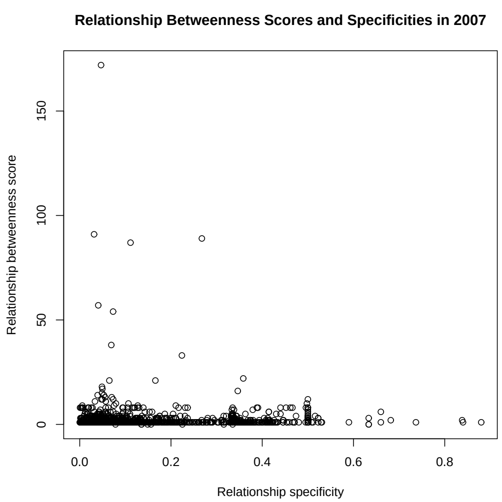
(a) 2004



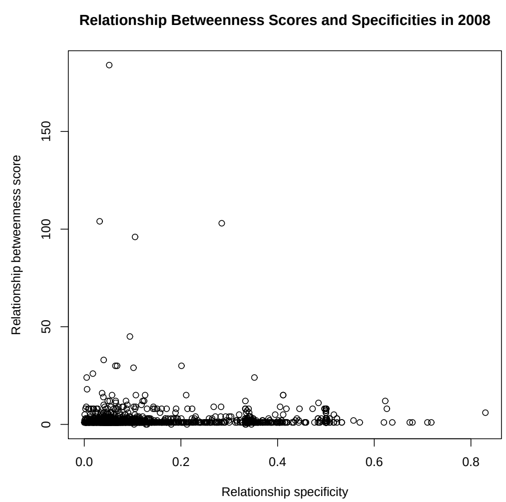
(b) 2005



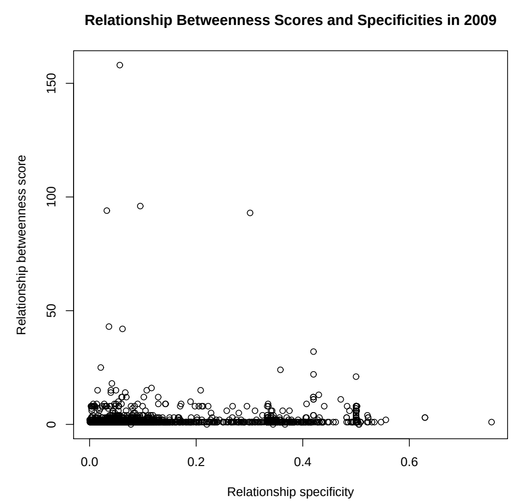
(c) 2006



(d) 2007

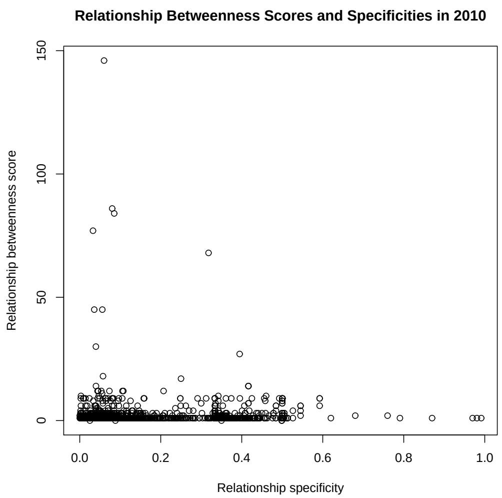


(e) 2008

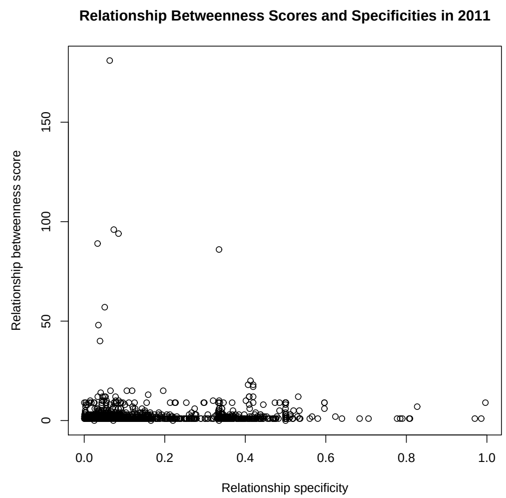


(f) 2009

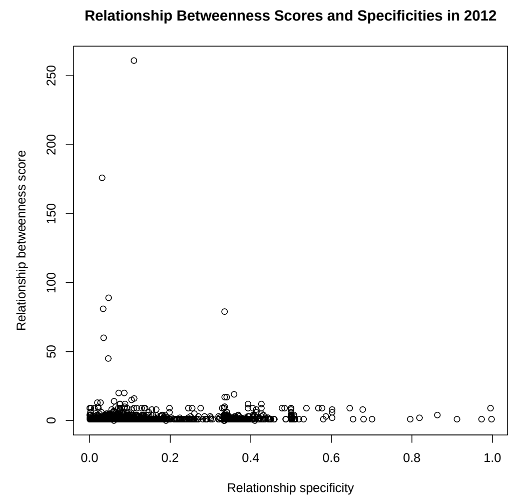
Figure IA.13. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 2004 to 2009.



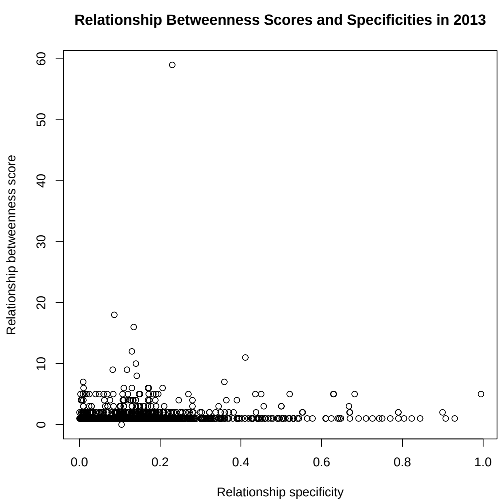
(a) 2010



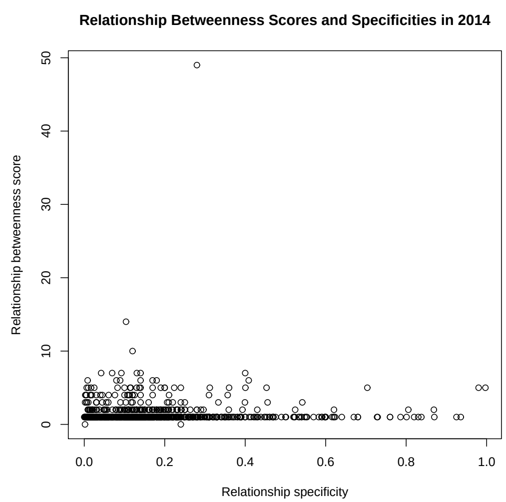
(b) 2011



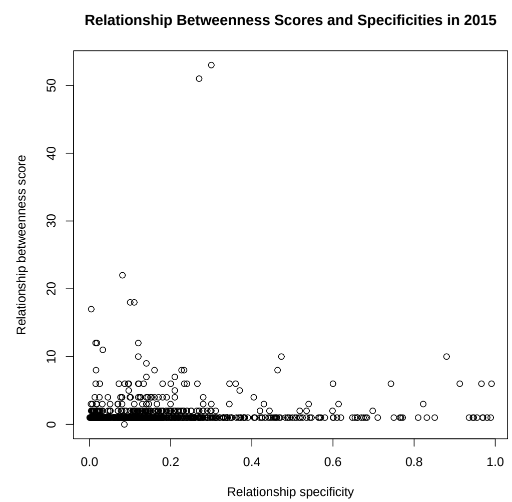
(c) 2012



(d) 2013



(e) 2014



(f) 2015

Figure IA.14. The figure shows the relation between betweenness and specificity scores of relationships in supplier–customer networks from 2010 to 2015.

Table IA.1
Performance of Centrality Portfolios in Manufacturing

The table reports monthly average raw returns, alphas and loadings from the five-factor model of Fama and French (2015) for three portfolios constructed by sorting manufacturing stocks based on centrality: a portfolio that holds stocks on the lowest decile of centrality (Low), a portfolio that holds stocks on the highest decile of centrality (High), and a portfolio that is long on stocks on the lowest decile and short on stocks on the highest decile of centrality (Low - High). The bottom row provides the t-statistics for the low minus high portfolio. Manufacturing firms are assigned into deciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1976 to December 2016. Raw returns and alphas are in percent.

	Raw	5-Factor Model					
Decile	Return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.21	1.24	1.02	0.53	-0.40	-0.34	0.06
High	1.42	0.46	0.94	-0.19	-0.14	-0.02	0.25
Low - High	0.78	0.38	0.08	0.73	-0.27	-0.32	-0.18
t-statistic	[4.37]	[3.23]	[3.03]	[17.36]	[-4.91]	[-5.62]	[-2.19]

Table IA.2
Performance of Centrality Portfolios in Service

The table reports monthly average raw returns, alphas and loadings from the five-factor model of Fama and French (2015) for three portfolios constructed by sorting service stocks based on centrality: a portfolio that holds stocks on the lowest decile of centrality (Low), a portfolio that holds stocks on the highest decile of centrality (High), and a portfolio that is long on stocks on the lowest decile and short on stocks on the highest decile of centrality (Low - High). The bottom row provides the t-statistics for the low minus high portfolio. Service firms are assigned into deciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1976 to December 2016. Raw returns and alphas are in percent.

	Raw	5-Factor Model					
Decile	Return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.56	1.70	1.11	0.47	-0.56	-0.41	-0.05
High	1.18	0.61	0.84	-0.25	-0.14	-0.24	-0.56
Low - High	1.41	0.67	0.29	0.75	-0.39	-0.17	0.52
t-statistic	[4.72]	[2.45]	[4.37]	[7.60]	[-3.03]	[-1.28]	[2.66]

Table IA.3
Performance of Centrality Portfolios in Manufacturing and Service

The table reports monthly average raw returns, alphas and loadings from the five-factor model of Fama and French (2015) for three portfolios constructed by sorting manufacturing and service stocks based on centrality: a portfolio that holds stocks on the lowest decile of centrality (Low), a portfolio that holds stocks on the highest decile of centrality (High), and a portfolio that is long on stocks on the lowest decile and short on stocks on the highest decile of centrality (Low - High). The bottom row provides the t-statistics for the low minus high portfolio. Manufacturing and service firms are assigned into deciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1976 to December 2016. Raw returns and alphas are in percent.

	Raw	5-Factor Model					
Decile	Return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.31	1.36	1.04	0.52	-0.45	-0.36	0.04
High	1.41	0.51	0.93	-0.20	-0.14	-0.04	0.12
Low - High	0.89	0.45	0.11	0.72	-0.31	-0.32	-0.07
t-statistic	[5.05]	[4.08]	[4.39]	[18.27]	[-6.06]	[-5.95]	[-0.91]

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