

**Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.**

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Incentives**

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2017-040

Please cite this paper as:

Weingarden, Alison (2017). "Employment Dynamics in a Signaling Model with Workers' Incentives," Finance and Economics Discussion Series 2017-040. Washington: Board of Governors of the Federal Reserve System, <https://doi.org/10.17016/FEDS.2017.040>.

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Employment Dynamics in a Signaling Model with Workers' Incentives

Alison Weingarden*

September 28, 2016

Abstract

Many firms adjust employment in a “lumpy” manner – infrequently and in large bursts. This paper shows how workers’ incentives can function as an endogenous cost that leads to lumpy firm-level employment adjustments such as mass layoffs. The signaling model features a firm that has private information about idiosyncratic demand shocks and relies on workers’ effort for production. When workers observe the firm’s employment choice but not the underlying shock, the equilibrium is characterized by partial pooling in which the firm hoards labor to maintain workers’ incentives. In response to small negative shocks, the firm initially underadjusts in order to conceal information from workers; it conducts a mass layoff after business conditions deteriorate past a certain point.

JEL Classification: D82, J21, J63, M51.

Keywords: Labor demand, Layoffs, Downsizing, Displacement, Asymmetric information, Signaling, Moral hazard.

*Federal Reserve Board. The analysis and conclusions set forth are those of the author and do not indicate concurrence by other members of the research staff or the Board of Governors. I would like to thank my PhD advisors James Albrecht and Susan Vroman for extremely helpful guidance and feedback. I am grateful to Ross Doppelt, Jin-Hyuk Kim, Pawel Krolikowski, Amanda Michaud, Andrea Pozzi, Paolo Ramezzana, John Rust, and Francis Vella for insightful suggestions. This paper has also benefited from comments in presentations at Georgetown University, EconCon 2014, SOLE 2014, 2014 Midwest Economics Conference, the Federal Reserve Board, the U.S. Census Bureau, the Consumer Financial Protection Bureau, and Cornell University. However, any errors are my own. Please send comments or questions to alison.e.weingarden@frb.gov.

1 Introduction

Neoclassical models suggest that a profit-maximizing firm should adjust its workforce frequently in response to idiosyncratic demand or productivity shocks. If wages are not fully flexible, a firm might want to increase or reduce its workforce each time its marginal product of labor rises or falls. In reality though, many U.S. firms conduct layoff episodes infrequently and in large bursts.¹ While fixed costs of adjustment can produce lumpy patterns, in a flexible labor market such as the U.S. such explicit fixed costs are not necessarily large enough to explain observed layoff patterns. The theoretical model presented in this paper explores an additional reason that a firm might make lumpy adjustments to employment instead of more gradual changes: the negative effect of layoff episodes on the incentives and effort provision of the firm’s workers.

In the signaling model of this paper, the firm has private information about idiosyncratic shocks to its business condition (i.e. productivity or demand for its product). The firm’s revenue depends on its business condition and on the non-contractible effort exerted by its homogeneous workers. When each worker’s compensation is based on a combination of the worker’s own productivity and the firm’s privately observed business condition, workers use their beliefs about the firm’s profitability to inform their effort decisions. Since the firm’s optimal choice of the size of its labor force depends on its business condition, workers view the firm’s decision of employment (which is observable to every worker) as a signal. Therefore, the firm chooses its employment level strategically considering its impact on workers’ beliefs and effort provision.

Using this framework I show that there generally exists an equilibrium in which firm types with intermediate levels of business conditions pool their actions, leading to labor hoarding in response to small shocks. These firm types have incentives to delay layoffs and bear the costs of being overstaffed for an extended period of time in order to discourage workers from shirking or looking for other jobs. This layoff avoidance strategy leads to “pent-up” adjustments: a layoff becomes optimal only if business conditions deteriorate beyond some threshold. This employment adjustment consolidates all previous periods of falling labor demand into a single layoff episode, and thus the adjustment is much larger than the magnitude of the underlying shock. The setup of this model suggests that lumpy employment patterns are generated by factors other than the frequency of large shocks; they may be more common in industries in which monitoring is costly or infeasible but

¹Davis, Faberman and Haltiwanger (2006) examine several sources of data on job creation and job destruction from 1990–2005 and conclude that, “job flows mainly reflect lumpy employment changes at the establishment level”. Further empirical evidence is presented in Section 2.

workers' effort is important.

A principal-agent problem underlies the contracting environment between the firm and its workers, with the realistic implication that workers' compensation is increasing in their employer's productivity.² Although this microfoundation leads to workers sharing directly in the firm's production, workers may also derive indirect benefits from working at better firms and thus the type of profit-sharing arrangement that arises in the model can encompass any situation in which workers have a stake in the business prospects of their employer. For example, the firm's condition could determine whether the worker will have future promotion opportunities with salary increases or it may impact the worker's future layoff probability. Compensation that is increasing in effort and business conditions is meant to capture workers' incentives to invest in their inside option when they are employed by a better firm. Instead, if workers believe that their current employer is not doing well, they may shirk, start looking for other jobs, and/or invest less in firm-specific human capital.

Although my signaling model is mostly concerned with the effort response to a firm's choice of employment, workers' wage and bonus structures play an important role. Therefore, in an extension to the model I consider the case in which the firm offers workers a wage contract before the signaling game takes place. For a meaningful signaling game, I only consider parameters values for which the firm finds it too costly to propose a contract that implements high effort for all possible business conditions. Instead, some states of the world result in low effort provision from workers.³

The literature about labor adjustments at firms spans many decades. A closely related paper is Bentolila and Bertola (1990), which looks at the optimal labor adjustments of a firm that faces shocks to demand and explicit (per-worker) costs of hiring or firing.⁴ In the equilibrium of their model, a firm makes no adjustment to employment in response to very small shocks; layoffs are optimal only when negative shocks are sufficiently large. However, the explicit per-worker adjustment costs in their model introduce a wedge between a firm's genuine employment choice and its optimal adjustments. Consequently, the magnitude of

²Bargaining between workers and employers typically yields a similar output-sharing rule.

³My principal-agent problem is similar to the one presented in Kim (1997) in which a firm contracts with a risk-neutral worker who has limited liability. In his model the optimal contract implements a unique effort level whereas in my model the contract is set ex-ante and the worker's effort decision is determined in the subsequent signaling game.

⁴While explicit per-worker firing costs are high in many European countries, as discussed in Section 2, empirical evidence indicates that per-worker hiring or firing costs in the U.S. are probably not high enough to explain the lumpiness of firm-level employment adjustments.

any layoff episode in their model is *smaller* than required by a full adjustment, which is the key difference between their result and mine. Another classic paper, Azariadis (1975), looks at the contracting decision of a firm with its workers when the firm has private information about business conditions. Like much of the implicit contracting literature, in his paper labor contracts play an insurance role and the model hinges on the assumption that workers are risk-averse.⁵ In contrast, workers in my model are risk-neutral and labor contracts are designed to elicit workers' effort.

Several recent papers look at the interaction between workforce decisions at firms and workers' incentives to provide effort. Golosov and Menzio (2015) take a macro approach, incorporating effort provision into a general equilibrium search model. Their model shows how state-contingent contracts to induce effort can provide an amplification mechanism for aggregate separations. However, since their model has only single-worker firms, it does not speak to the lumpiness of firm-level layoffs. Jeon and Shapiro (2007) look at a contracting game in which workers do not observe whether the firm has experienced a negative shock; subsequently the firm chooses its employment level and then workers decide whether to provide effort. Their paper has a rich characterization of workers' value functions and outside options but, differently from my model, theirs has only two periods and, more importantly, only two types of firms. Levin (2002) has a setting similar to mine with a key difference that he assumes multilateral relational employment contracts can be implemented between a firm and its employees. The form of such contracts implies a stark result: labor hoarding can be sustained as an equilibrium because any layoff would invalidate the firm's promise to pay bonuses and trigger low effort from workers. In contrast, the bilateral and verifiable nature of contracts in my model leads to some very different conclusions. For example, Levin shows that multilateral contracts (and thus labor hoarding) are more likely when workers' bargaining power is low whereas I show that labor hoarding and mass layoffs are *less* likely when workers share less in the profits of their employers.

Methodologically, some features of my model are closely related to Spence (1973). The signal senders in his paper are students who undertake education, where education is less costly for high types. In my model, firms are the signal senders and a large workforce is less costly for good firms. In the equilibrium of the Spence model, high types choose more education to separate from low types. The equilibrium of my model instead has some intermediate types of firms pooling on employment choices. This partial pooling result is driven by two mechanisms: the binary nature of workers' incentives and the

⁵Critics cite these two features of implicit contract theory as its central weaknesses— see Akerlof and Miyazaki (1980).

non-monotonicity of firm types' preferences over workforce size. Regarding the latter mechanism, since it is costly for a firm to depart up or down from its optimal workforce size, in equilibrium some firm types choose their optimal workforce rather than incur the costs of pooling.⁶

In a survey paper, Hamermesh and Pfann (1996) draw attention to a gap in the current literature on labor and capital adjustment patterns at firms: the source of adjustment costs are not well understood. They say, "Without knowing the source of the costs we have little hope of using estimates of parameters describing the lag structures that we specify, no matter how sophisticated they are, to extrapolate to the likely effects of policies that impose gross or net costs of adjusting inputs". And, "Discovering the size of adjustment costs and how these too vary by industries' and workers' characteristics should be high on anyone's research agenda in the study of factor demand." The signaling model in my paper shows that workers' effort can act as an endogenous cost that resembles a fixed cost of labor adjustment at firms. By building a micro-founded model, it sheds light on the size of the effect and how it may vary with industry and worker characteristics.

2 Evidence of and explanations for lumpy layoffs

This section highlights the lumpy nature of firm-level layoffs in the U.S. and discusses possible explanations. First I present evidence from previous literature that illustrates the costs of downward employment adjustments. Then I use microdata from large U.S. companies to discuss a few stylized facts about the size, frequency, and reasons for mass layoffs. Finally, I evaluate explanations such as indivisibilities, stock price reactions, and legal considerations and conclude that none of these reasons provide a full explanation for the prevalence of lumpy adjustments.

2.1 Related literature

A number of papers in the literature show that infrequent and large employment changes appear to be a general feature of U.S. microdata. Using plant-level output and employment measures for a single firm, Hamermesh (1989) is one of the first to conclude that employment appears to be lumpy due to fixed costs of adjustment. Caballero, Engel and Haltiwanger (1997) also find evidence of infrequent and large adjustments, based on differences between intensive and extensive labor margin adjustments in a large number of establishments.

⁶In this sense, my model resembles Bernheim (1994), which focuses on partial pooling outcomes when signal senders have different underlying preferences but are "conformist" in that they want to be perceived as central types.

Bloom (2009) looks at the joint behavior of firm-level capital and labor adjustments in response to uncertainty shocks. With regard to labor adjustment, he finds “high” fixed costs of labor adjustments in the U.S. (around 2.1% of a firm’s annual revenue) and only “limited” per capita costs of hiring and firing. Cooper, Haltiwanger and Willis (2007), using a search model calibration, also reach a clear conclusion that the lumpy pattern of employment adjustments in the U.S. appears to be due to large fixed costs of hiring and/or firing. However, analysis in Davis, Faberman and Haltiwanger (2013) shows that hiring activities have very minimal returns to scale, which suggests that fixed costs of hiring are not the primary driver of lumpy employment patterns. Davis and von Wachter (2011) emphasize that not only are mass layoffs common in the U.S., they are a particularly important policy issue because of their substantial negative effects on workers.

2.2 Mass layoffs at large U.S. companies

This section uses microdata from the 2007-2012 Challenger Job Cut Reports to show the prevalence of large, lumpy mass layoffs. The reported reason for many of these mass layoffs is weak financial conditions; this section discusses some evidence about the relationship between firm-level financial conditions and mass layoff decisions.

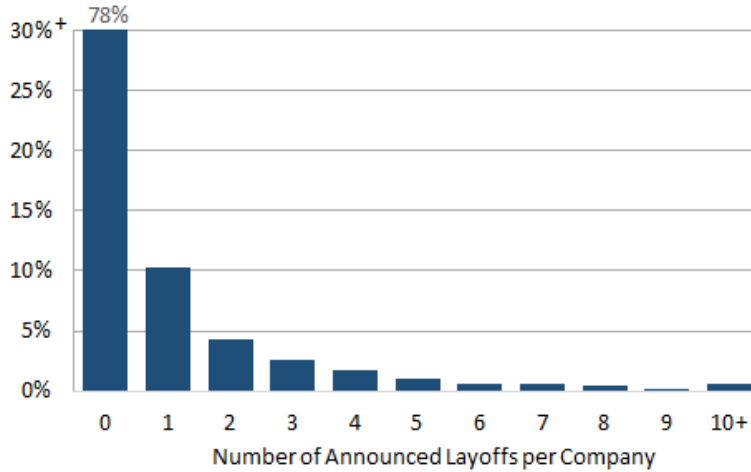
The Challenger Job Cut dataset captures all layoffs by U.S. employers announced in the media, company financial reports, and those reported as part of the Worker Adjustment and Retraining Act (WARN) requirement. The chart and table below limit the sample to companies listed on U.S. stock exchanges, mainly because listed companies are generally larger and more likely to have public reporting of their layoffs.⁷ At these companies, an average reported layoff affects more than 900 workers (or 7% of the workforce) at a given firm in a given quarter. In fact, nearly 85% of these episodes are above the 50 worker threshold that is generally used to define a “mass layoff”.⁸

Even during the years of the Great Recession, mass layoffs appear to be relatively infrequent at a company level. As shown in Figure 1, 78% of publicly-listed companies had no layoff episode reported during the six year sample period. Indeed, more than 10% had only one large layoff and a further 4% had only two large layoffs. A histogram of layoff frequency at nearly four thousand publicly-listed companies is shown below.

⁷Listed companies account for 2 million worker displacements, or half of all displacements recorded in the layoff announcement dataset.

⁸The BLS defines a mass layoff episode as one in which 50 or more workers from a given establishment file initial unemployment insurance claims within a five week period.

Figure 1: Distribution of Mass Layoff Episodes per Company, 2007-2012



Source: Challenger layoff data. Mass layoff episodes are defined as the number of quarters in which a company reports layoffs at at least one establishment.

To understand the reasons that companies conduct mass layoffs, the next table summarizes the explanations recorded in the Challenger dataset.⁹

Table 1: Reported Reason for Mass Layoff Episode

Reason	% of episodes	Reason	%
Weak demand	30%	Outsourcing/relocation	3%
Reorganization	26%	Acquisition/merger	3%
Bankruptcy/closing	21%	Loss of contract/funding	2%
Cost reduction	10%	Other reasons	1%

Source: Challenger layoff data.

Table 1 above shows that “weak demand” is the primary reason reported for 30% of mass layoff episodes. This number could actually be a lower bound if companies under-report “weak demand” because of the negative signal it sends to investors.

In a companion paper, Weingarden (2016), I use the Challenger dataset to show that a firm’s propensity to conduct layoffs increases in response to declines in firm-level revenue and profits. Moreover, I find that the response to firm-level financial conditions happens over one year or more. While a fall in financial conditions at a firm triggers an immediate

⁹The primary explanation was sorted into one of eight broad categories and observations with no reported reason (4% of the data) were dropped.

rise in its layoff propensity, past quarterly financial declines also have significant marginal effects. Consistent with the empirical evidence, in this paper layoffs are a pent-up response to the accumulation of negative firm-level shocks.

2.3 Reasons for lumpy layoffs

While it has been shown that generic fixed costs of firing can match the patterns of employment to generate lumpy changes, are the sources of these costs observable? As shown in Table 1, some mass layoffs are motivated by indivisibilities such as bankruptcy/closings or reorganization (perhaps eliminating divisions within a company or replacing a large number of workers with new workers or new technologies). However, at large U.S. companies a third of mass layoffs appear to be conducted in response to deteriorating business conditions; a similar result emerges for the reasons reported by a broader set of companies compiled by BLS Mass Layoff Statistics.

Anecdotally, workers' incentives are cited as a reason that firms hoard labor and conduct lumpy layoffs. In Bewley (1999), interviews with corporate managers reveal some of the interaction between worker effort and mass layoff decisions by firms. In particular, one human resources manager says, "Continued layoffs are demoralizing. It is better to take your hit all at once". Another echoes this sentiment, saying "Layoffs should be done quickly. You should pay those left more and say that you are done". This highlights a common corporate strategy of consolidating layoffs into large, infrequent episodes rather than gradual layoffs that could damage the attachment of remaining workers to the firm.

Although two other reasons are often mentioned as explanations for the mass layoff phenomenon, neither seems to be adequate. First, the anticipated reaction of stock prices to mass layoff announcements probably cannot explain why U.S. firms choose mass layoffs over smaller, more frequent layoff episodes. Farber and Hallock (2009) have found that a company's stock price generally declines in response to a layoff announcement but they found that the effect on stock prices is closer to zero in their more recent data. Accordingly, anticipated stock market reactions may have very little impact on layoff decisions at companies- if the reaction to layoff announcements is negative then companies might even prefer small dismissal episodes to mass layoffs. Second, legal considerations may dissuade companies from laying off workers one-at-a-time, but these considerations are probably not large enough to explain the size and extent of mass layoffs. For example, Oyer and Schaefer (2000) show that companies shifted away from individual dismissals after 1991, when penalties for employment discrimination increased. Nonetheless, this shift was only

evident for workers in protected categories, suggesting that companies do consider the legal implications of dismissals, but the magnitude of the effect is limited.

Not only are there few clear benefits to mass layoffs, but such lumpy adjustments may actually have diseconomies of scale. For example, mass layoffs can trigger advance notification provisions, increase the administrative burden on human resources departments, increase the number of workers that might initiate lawsuits, and lead to a disconnect between a company's optimal and actual labor force size. This reveals an important discrepancy: in a flexible labor market such as the U.S., there are not many costs to conducting frequent layoffs that affect a small number of workers. However, firm-level employment dynamics suggest that U.S. firms behave as if there are large fixed (or non-convex) costs to such layoffs. This paper shows that workers' incentives can generate costs that are large and non-convex, and may therefore help explain the extent of lumpy layoff patterns seen in the data.

Brief outline

This paper presents a model in which the firm chooses its employment strategically in order to elicit effort from workers. Section 3 presents the timing and payoffs of the signaling game; the firm's production and workers' compensation both depend on worker effort and the firm's exogenous business condition realization. Section 4 focuses on the strategic interaction between the firm and its workers in this one period model. The strategies of the firm and workers are outlined in the context of a perfect Bayesian equilibrium, where the firm's choice of employment is a signal to workers about its most recent business condition realization. Section 5 presents a numerical example and provides a more rigorous discussion of the equilibrium properties of the model, including conditions under which partial pooling occurs in equilibrium. A multiperiod model is presented in Section 6 and its dynamics are illustrated with a numerical example. Finally, Section 7 concludes. Proofs and lengthy discussions about assumptions are relegated to the Appendix.

3 The Model

Overview

In the signaling model of this paper, a single firm draws a random "business condition" parameter, which could be a random realization of the firm's productivity or of the demand for its final output. The firm's revenues are determined both by its business condition parameter and the effort exerted by its workers; the firm's costs are determined by the

number of workers and the wage paid to these workers. Workers are homogeneous and their expected contributions to production are increasing in effort and the firm's business condition, with the marginal impact of effort being higher in better business conditions. Effort is non-contractible and therefore the firm offers each of its workers a wage that is based on the worker's individual contribution to the firm.

The signaling model features asymmetric information between the firm and its workers. After the firm has received private information about its business condition, it decides its employment level, which is an observable signal that workers use to update their beliefs. Workers decide effort based on their beliefs about the firm's business condition and provide more effort when they expect business conditions to be good.

In the remainder of this section I describe each of these aspects in more detail. In particular, in Section 3.1 I describe the firm's production technology and workers' effort provision. In Section 3.2 I outline the timing of the game, including the wage contracting stage, while in Section 3.3 I define both players' strategies and workers' beliefs in the signaling game, and finally in Section 3.4 I present a solution to the full information benchmark case.

3.1 Production technology

Workers each contribute y to production, where y is a random variable. Its distribution, and thus ultimately its expected value, depends on the effort exerted by the worker and on the business condition of the firm. I denote worker effort by e and I assume that it can take only two values $e \in \{e_L, e_H\}$, with $e_L < e_H$.¹⁰ Workers have a disutility of effort, $\Psi(e)$ that is strictly increasing in e . The business condition of the firm is denoted by z , such that higher values of z correspond to better business conditions. The firm type z is drawn from density $g(z)$ with full support on $[z_{min}, z_{max}]$, where $z_{min} \geq 0$.

Business condition parameter z can be interpreted as a realization of the firm's (1) productivity and/or (2) external demand. If z is a productivity shock then it affects workers' expected contributions to output, whereas if it is a demand shock then it affects the firm's revenues. To allow for either interpretation of z , I refer to it more generally as the firm's exogenous business condition.

As anticipated above, the expected value of y depends on e and z (it is weakly positive and

¹⁰Binary effort could result from a utility maximization problem with corner solutions or from a worker's decision about whether or not to search for a new employer. Appendix section 10.1 discusses the binary effort assumption in further detail.

strictly increasing in both variables). In particular,

$$q(e, z) = E[y|e, z] \tag{1}$$

Thus $q(e, z)$ represents the expected individual contribution of any worker given effort choice e and business condition parameter z . This function is characterized by increasing differences in z and e ,

Assumption 1 $q(e_H, z) - q(e_L, z)$ is strictly increasing in z

The assumption that $q(e, z)$ has increasing differences in its two arguments means that the firm's business conditions and workers' effort are productive complements. Therefore, a firm with better business conditions is more inclined to encourage high effort from workers. The increasing differences property also leads workers to prefer providing high effort for an employer with a higher value of z , since (as it will be shown later) each worker earns a share of his individual contribution to output.

The firm's production technology aggregates the individual worker contributions of its n workers using a fixed factor of production. This fixed factor of production is also subject to a congestion effect, $X(n)$, leading to higher costs as employment increases. In this setting $X(n)$ is increasing and strictly convex: $X' > 0$, $X'' > 0$, and also satisfies $X(0) = 0$. The firm's expected production with n workers, business condition realization z , and workers' effort choice e is therefore,

$$Q(e, z, n) = q(e, z)n - X(n) \tag{2}$$

The main role of $X(n)$ in this model is to produce decreasing returns to scale, which ensures that the firm's profit function is concave in n . The separability of $q(e, z)$ from $X(n)$ has the additional useful property that the effort of individual workers can be separated from the production technology of the firm. Since effort only enters the first term of the expression, each worker contributes $q(e, z)$ to production and has a constant marginal return to effort that does not depend on the total number of workers at the firm. When $Q(e, z, n)$ is determined as above and y is observable (and contractible) for each worker, team production is not a concern and there are no externalities between workers. Thus the firm can contract with individual workers in a standard principal-agent framework.

The next section discusses the relevant principal-agent contracting problem and the sequence of events in each period of the model.

3.2 Model timing

At the beginning of the period, before receiving private information about z ,¹¹ the firm offers workers a state-contingent wage contract that will depend on the worker's contribution to production. Although effort is non-contractible, workers' individual contributions are observable and verifiable ex-post in *Stage 4*.

Stage 0 (Wage contracts): The firm offers a contract that specifies the wage payment $w(y)$ as a linear share ω^* of the worker's individual contribution y ,

$$w(y) = \omega^* y \tag{3}$$

Workers are risk-neutral and have a limited (zero) liability. For the main body of the paper ω^* is treated as an exogenous parameter, however, the firm's optimal endogenous contract offer is derived in Appendix section 10.2 in terms of the model primitives. The principal-agent problem shows that any optimal contract must have $\omega^* \in [0, 1)$ but I only look at the interesting setting in which ω^* is non-zero (implemented with Assumption 2 in Section 3.4.2).

Stage 1 (Private information): The firm observes private information about its business condition, z . Workers know the prior distribution of business conditions, $g(z)$ but they do not observe the firm's realization of z .

Stage 2 (Employment signal): Based on its realization of z , the firm chooses its number of employees, n according to its optimal strategy laid out in Section 4.1. Workers observe n and know that higher levels of n are more profitable for firms with higher levels of z . Therefore, as detailed in Section 4.2, workers use the signal of n to update their beliefs about the firm's business condition.

Stage 3 (Effort decision): Workers decide effort e based on their posterior beliefs about z . Workers are risk-neutral with additively separable utility (and no income effects). Their utility for a given level of effort e is,

$$W(e) = w(y) - \Psi(e) \tag{4}$$

¹¹If instead the firm observed z before offering the contract, this would result in an informed principal problem in which the contract terms are a signal to the worker about the firm's business condition, in addition to the signaling value of the firm's employment level. An interesting discussion of the informed principal contracting problem with continuous effort and no additional signal can be found in Beaudry (1994).

Since the expected value of y , $q(e, z)$, has increasing differences in e and z , workers obtain relatively greater benefits from providing high effort when the firm's business condition is good. The workers' optimal strategy is given in Section 4.3.

Stage 4 (Production and payments): Workers' contributions are realized. The firm earns revenue from production (as described in Section 3.1) and pays workers as agreed in *Stage 0*. The firm's realized profits are,

In each period of the model described above, the signaling game takes place after wage contracting in *Stage 0*. For expositional ease, the wage parameter ω^* is taken as exogenous in the following section, which focuses on the strategic interaction between the firm and its workers as given by the signaling game in *Stages 1-4*.

3.3 Perfect Bayesian equilibrium

The notation for strategies and beliefs is introduced here and is used again in Sections 4 and 5 to characterize the equilibrium solution.

A perfect Bayesian equilibrium consists of (i) a strategy for the firm, which is a choice of employment n for each (privately observed) z ; (ii) a strategy for workers, which is a choice of effort e given the signal n ; and (iii) workers' beliefs given n such that:

1. The firm's strategy $a : Z \rightarrow N$ maximizes expected profits, given its private information about business condition z and given the workers' strategy for choosing effort;
2. Workers' beliefs $\mu : N \rightarrow \Delta_Z$ assign a probability distribution of Z based on the signal n . The probability that the firm is type z conditional on the worker observing action n is denoted $\mu(z|n)$ such that $\int_Z \mu(z|n) dz = 1$ for each $n \in N$. Beliefs are determined by Bayes' rule where possible and otherwise, if workers observe an n off the equilibrium path, beliefs are based on the equilibrium dominance refinement;
3. The workers' strategy $b : N \rightarrow E$ selects an effort level that maximizes each worker's expected utility, given the firm's strategy and the signal n that is used to inform beliefs;
4. The firm's strategy $a(z) = n$ and workers' strategy $b(n) = e$ are sequentially rational best responses and workers' beliefs $\mu(z|n)$ are consistent given the strategy profiles.

The optimal strategies and the off-path refinement for workers' beliefs are detailed in Section 4. First, however, it is helpful to define the full information benchmark.

3.4 Full information benchmark

This section characterizes the solution to a counterfactual model in which there is no private information (i.e. if both the firm and its workers observe z).

3.4.1 Firm's profit maximization problem

In this benchmark case in which workers have full information about z , the firm's choice of n does not serve as a signal. The firm's expected profits are denoted $\pi(e, z, n)$, where e can be any anticipated effort level by workers.

$$\pi(e, z, n) = (1 - \omega^*)q(e, z)n - X(n) \tag{6}$$

The firm's optimal choice of n in the full information case is based on $\pi(e, z, n)$ as defined above. Although this represents the firm's choice of employment under full information, the notation introduced below is used in proofs and propositions in later sections.

Notation: $N^*(e, z)$ is the firm's "genuine" labor force choice that would be optimal in each period of the model, under full information (if the level of employment had no signaling value to workers), where e can be any anticipated level of effort from workers.

$$N^*(e, z) = \operatorname{argmax}_{n \geq 0} \pi(e, z, n) \tag{7}$$

Since $X(0) = 0$ and $\omega^* < 1$, there always exists some employment level $n \geq 0$ for which $\pi \geq 0$. Based on the profit function, $N^*(e, z)$ is increasing in z (for a given level of effort e), which means that firms with better business conditions prefer to employ more workers, all else equal.

Next, in Section 3.4.2, I discuss the workers' utility maximization problem in the full information case and I introduce an additional assumption for the exogenous value of ω^* . In Section 3.4.3 I conclude the full information benchmark model by characterizing the equilibrium strategies of the firm and its workers.

3.4.2 Workers' utility maximization problem

When there is full information and workers observe z directly, they choose effort to maximize expected utility as given in *Stage 3* of the signaling game. Workers have a trade-off when choosing e : higher effort leads to higher expected wages (based on expected output $q(e, z)$) but it also imposes a utility cost, $\Psi(e)$. Workers' expected utility is described by,

$$U(e, z) = \omega^* q(e, z) - \Psi(e) \quad (8)$$

when the wage contract is treated as exogenous or pre-determined with parameter ω^* . In this full information case, workers choose e depending on the observed level of z .

$$e_F^*(z) = \operatorname{argmax}_e U(e, z) \quad (9)$$

Since $q(e, z)$ has increasing differences, workers generally prefer $e = e_L$ for lower levels of z and $e = e_H$ for higher levels of z . The workers' indifference point in terms of z is used frequently in later analysis and thus is defined formally below.

Notation: *The worker's indifference point between providing low and high effort is denoted by \tilde{z} in the full information case in which z is observed by both the firm and the workers. It is assumed, for convenience, that each worker chooses high effort at $z = \tilde{z}$.*

Before proceeding, I make an assumption about \tilde{z} to ensure that a worker prefers to provide low effort for some values of z and high effort for other values of z : the worker's indifference point \tilde{z} should lie in the domain of z .

Assumption 2 *The exogenous share ω^* is such that $\exists \tilde{z} \in (z_{min}, z_{max})$, which requires:*

$$\frac{\Psi(e_H) - \Psi(e_L)}{q(e_H, z_{max}) - q(e_L, z_{max})} < \omega^* < \frac{\Psi(e_H) - \Psi(e_L)}{q(e_H, z_{min}) - q(e_L, z_{min})}.$$

This assumption is essentially a restriction on the magnitude of $\Psi(e_L)$ and $\Psi(e_H)$ relative to share ω^* that is specified by the wage contract. The left-hand side of the inequality establishes that the expected wage payment must be large enough that it outweighs the marginal cost of providing high effort, at least for the best possible state (z_{max}). The right-hand side ensures that the share is not so high that the worker finds it optimal to provide high effort even in the worst state (z_{min}). This rules out the uninteresting cases in which the worker chooses only e_L for all possible z or only e_H for all possible z . As mentioned earlier, the optimal value of ω^* is given in Appendix section 10.2 and a numerical example satisfying Assumption 2 is provided in Section 11. Regardless of whether ω is endogenous

or exogenous, \tilde{z} is inversely related to it. With a higher sharing parameter, workers are willing to provide high effort for lower levels of z .

3.4.3 Equilibrium of full information benchmark

Using the notation introduced above, I conclude the discussion of the full information case by stating the equilibrium strategies of the firm and its workers. Since workers observe z directly in this case, their beliefs about z are obviously consistent with its true value and their strategy depends only on the realized value of z .

When workers observe z directly, their strategy is simple: they choose e_L for values of z below \tilde{z} and e_H for higher z realizations. The workers' strategy in the counterfactual full information case is,

$$b_F(z) = \begin{cases} e_L & \text{for } z \in [z_{min}, \tilde{z}) \\ e_H & \text{for } z \in [\tilde{z}, z_{max}] \end{cases} \quad (10)$$

The firm's strategy when workers observe z is to choose its genuine employment level based on e_L where z is low, and based on e_H where z is high. In particular, the firm's strategy in the counterfactual full information case is,

$$a_F(z) = \begin{cases} N^*(e_L, z) & \text{for } z \in [z_{min}, \tilde{z}) \\ N^*(e_H, z) & \text{for } z \in [\tilde{z}, z_{max}] \end{cases} \quad (11)$$

Therefore, the firm's strategy is to choose the genuine employment level (based on the realization of z) that is a best response to the workers' effort provision strategy.

The following property of $a_F(z)$ is invoked in Section 4, where it is used to show that even with asymmetric information, the worker can generally determine z when the firm chooses its genuine level of n .

Claim 1 $a_F(z)$ is invertible in z .

Proof: See Appendix section 9.

In other words, the strategy of the firm entails a unique choice of $n = a_F(z)$ for each possible realization of z .

Now that I have characterized an equilibrium of the full information benchmark case, I will return to the signaling model in which the firm has private information about z and the

firm's choice of n is a signal to workers about the true underlying value of z . The next section presents a candidate equilibrium of the signaling model that includes pooling for a range of z values.

4 Strategies and Beliefs

In the following discussion, the notation from Section 3.3 is used to expand upon the optimal strategies and equilibrium beliefs in the signaling model. Here I lay out the structure and notation for the candidate equilibrium in which firms pool for intermediate values of z . Throughout, I restrict attention to pure (non-mixed) strategies.

4.1 Firm's optimal strategy with private information

In the full model, the firm faces a profit maximization problem that includes signaling considerations. The firm privately observes the realized level of z in *Stage 1* and then chooses its labor force size n in *Stage 2*. Under asymmetric information, instead of choosing its genuine labor force size for each value of z , the firm's optimal strategy anticipates the workers' subsequent effort strategy in *Stage 3*.

The firm's profit maximization problem with signaling considerations is to choose the optimal n for each z subject to the workers' strategy for effort provision.

$$\begin{aligned} \max_{n \geq 0} \quad & \pi(e^*, z, n) \\ \text{s.t.} \quad & e^* = b(n) \end{aligned} \tag{12}$$

In general, the firm may want to pursue a strategy in which it chooses its genuine level of n for every realization of z or, alternatively, some firm types may benefit from a pooling strategy in which they choose the same level of n for multiple realizations of z . Although pooling is costly since it generally requires a firm to deviate from its optimal genuine level of employment, it may be attractive to some firm types. For example, when the firm draws z just below \tilde{z} (i.e. close to the workers' effort indifference point), pooling with a firm type above \tilde{z} provides an increase in worker effort from e_L to e_H and the costs of pooling are relatively small.¹² However, for firms with very low z realizations, pooling may be prohibitively costly because it would require significant distortions to n .

With asymmetric information, an equilibrium may be characterized by partial or complete

¹²See Proposition 4 for a full argument.

pooling. In the equilibrium proposed below, the firm's pooling actions are confined to an unique, compact interval $z \in [\underline{z}, \bar{z}]$ around \tilde{z} .¹³ Firm types in this pooling interval choose the same pooling level of employment, N_P , and outside of the pooling interval firm types choose their genuine full information level of employment. In the signaling model with asymmetric information between the firm and its workers, the firm's partial pooling strategy is,

$$a(z) = \begin{cases} N^*(e_L, z) & \text{for } z \in [z_{min}, \underline{z}) \\ N_P & \text{for } z \in [\underline{z}, \bar{z}] \\ N^*(e_H, z) & \text{for } z \in (\bar{z}, z_{max}] \end{cases} \quad (13)$$

When z is either lower or higher than the pooling interval, the optimal strategy in the candidate equilibrium corresponds to the firm's strategy $a_F(z)$ in the full information benchmark. In the signaling game, the non-pooling component of the firm's strategy can be defined as $N_S(z)$, where $N_S(z) \equiv a(z) \equiv a_F(z)$ for all $z \notin [\underline{z}, \bar{z}]$.

Although here the pooling interval location, employment level, and resulting beliefs were taken as given, they will be established formally in Section 5.

4.2 Workers' beliefs about z

Based on the standard common knowledge and rationality assumptions in strategic games, workers know the firm's strategy even though they do not observe the true value of z . Therefore, when workers observe n they form beliefs about z . Workers' beliefs about z , denoted as $\mu(z|n)$, are probability distributions that depend on whether the firm's action was a separating or pooling action.

Given the firm's strategy in the signaling model, there are two different situations that can arise with regard to workers' beliefs: (i) workers observe a value of n that is part of the firm's strategy (either pooling or non-pooling); or (ii) workers observe a value of n that is off-path (i.e. not part of the firm's strategy). In Section 4.2.1 I discuss workers' beliefs in case (i) in which they observe an on-path value of n and then in Section 4.2.2 I discuss case (ii) in which they apply an equilibrium refinement to update their beliefs after observing an off-path value of n .

¹³In other words, the interval bounds are: $\underline{z} < \tilde{z} < \bar{z}$. Section 5 establishes the existence and likely uniqueness of this proposed equilibrium.

4.2.1 On-path beliefs

Pooling action observed: A pooling level of employment is one type of on-path action that comes from the firm's optimal strategy, $a(z)$, presented above. If the firm chooses a pooling action, workers cannot work out the true value of z from the firm's strategy because any $z \in [\underline{z}, \bar{z}]$ would lead to the same choice of employment. Therefore, the workers' belief function places positive probability on all possible z realizations for which the firm optimally chooses the pooling action ($n = N_P$).

When pooling action N_P is observed, workers' beliefs are:

$$\mu(z|n = N_P) = \begin{cases} \frac{g(z)}{G(\bar{z}) - G(\underline{z})} & \text{for } z \in [\underline{z}, \bar{z}] \\ 0 & \text{for all other } z \end{cases} \quad (14)$$

After workers observe $n = N_P$, Bayesian updating of their prior $g(z)$ normalizes the distribution over the possible z realizations that could have led to the pooling action.

Non-pooling action observed:

Instead, the firm might choose a value of n that is consistent with a value of $N_S(z)$, the non-pooling component of the firm's equilibrium strategy. This choice of n would also constitute an on-path action for the firm. In this case (unlike with pooling), the worker can determine the exact value of z given the firm's choice of n and thus the belief function is degenerate with probability mass of one at the true z . More formally,

When any action $N_S(z)$ is observed, workers' beliefs are:

$$\mu(z|n = N_S(z)) = \begin{cases} 1 & \text{for } z = N_S^{-1}(z) \\ 0 & \text{for all other } z \end{cases} \quad (15)$$

Since $a_F(z)$ is invertible in z (as established by Claim 1), $N_S(z)$ is also invertible. Thus when workers see a particular $n \in N_S(z)$ they are able to tell exactly which z was realized.

Finally, workers' beliefs assign a zero probability to off-path actions by the firm. Consistent beliefs contain no information about the firm type when workers observe a value of n that does not correspond to the firm's equilibrium strategy (i.e. an off-path action). However, certain firm types might prefer the equilibrium outcome to the potential deviation; a belief refinement is included in the equilibrium definition to reflect the fact that certain deviations are attractive to certain firm types and not to others. In the next section I present a general

introduction to the belief refinement used for off-path actions in this model.

4.2.2 Refinement for off-path beliefs

The refinement invoked here for workers' beliefs about a firm's off-path behavior is "equilibrium dominance", as discussed in Cho and Kreps (1987). Equilibrium dominance requires that any firm choosing a particular off-path action has done so because the action could have improved its payoff for at least one configuration of workers' beliefs, relative to the payoff that the firm expects to receive in equilibrium. If workers were to observe an action that is not part of the firm's equilibrium strategy (in other words, any off-path employment level $N_O \notin a(z)$), they use the equilibrium dominance refinement to update beliefs. As I show in Section 5, this refinement limits the types of firms from which a deviation could come. This has important implications for the existence of a pooling interval in the equilibrium of the model.

4.3 Workers' optimal strategy with asymmetric information

This section outlines the worker's optimal strategy for effort provision in the signaling model in which workers observe n and use it to inform their posterior beliefs about z . To simplify the model exposition, from now on I will consider the case in which workers' contributions are linear in z such that $y = zf(e)$, where $0 \leq f(e_L)$ and $E[f(e_L)] < E[f(e_H)]$ and therefore the expected value of y exhibits increasing differences in e and z as required by Assumption 1.

With a slight abuse of notation, let $E_\mu[z|n] = \int_{z_{min}}^{z_{max}} z\mu(z|n)dz$. This expression represents a worker's posterior expectation of z after observing the firm's choice of n . When each worker's expected contribution (and therefore also expected compensation) is linear in z , his or her strategy in the signaling model depends only on his or her posterior beliefs about z . The strategy of workers is therefore,

$$b(n) = \begin{cases} e_L & \text{for } n \text{ such that } E_\mu[z|n] \in [z_{min}, \tilde{z}) \\ e_H & \text{for } n \text{ such that } E_\mu[z|n] \in [\tilde{z}, z_{max}] \end{cases} \quad (16)$$

where $E_\mu[z|n]$ is the expected value of z based on workers' posterior beliefs. The workers' optimal strategy is to provide low effort when they believe z is low and provide high effort when they believe z is at or above their indifference point \tilde{z} .

This formulation, however, does not fully satisfy the perfect Bayesian equilibrium criterion

that beliefs be consistent and strategies be (sequentially rational) mutual best responses. The analysis presented here instead lays the groundwork for the next section, in which the equilibrium properties are discussed more rigorously.

5 Equilibrium Characterization

This section shows that a perfect Bayesian equilibrium of the signaling model can be characterized by partial pooling in the firm’s employment decision across type-space z in each period. I first present an example for illustrative purposes and then derive a series of lemmas about the properties of any pooling interval in the signaling model. At the end of Section 5.2 I look at conditions for the existence of a pooling interval in equilibrium.

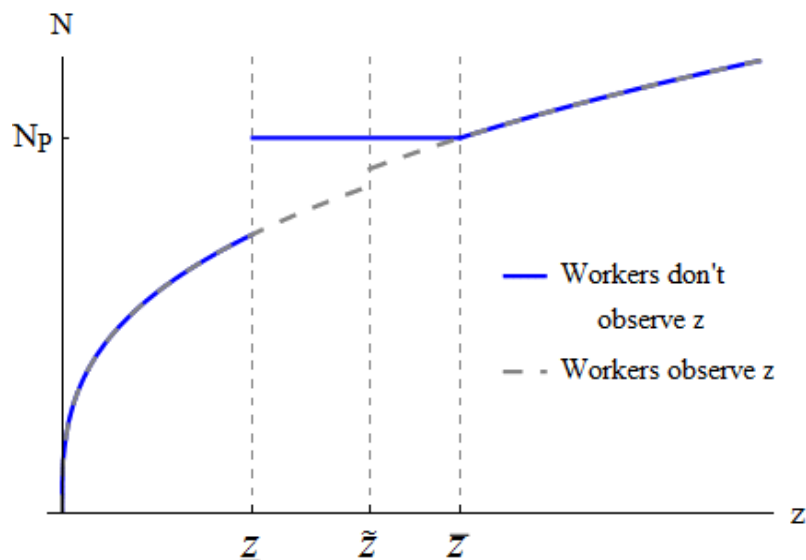
5.1 Numerical example with partial pooling equilibrium

Figure 2 shows an example of the firm’s optimal employment strategy in each period of the signaling model. In particular, $g(z)$ is normal and $X(n)$ has constant elasticity – see Appendix section 11 for specific parameter values.

The dashed line is the firm’s counterfactual genuine choice of n ; this is the level of employment that a firm would choose under full information (if workers could observe z directly). The solid line is the firm’s optimal strategy in the signaling model (when workers cannot observe z). This line has a flat region in an interval around \tilde{z} , which is the indifference point where workers switch from e_L to e_H . This flat portion is the model’s predicted pooling equilibrium. Note that, when a pooling interval exists, the interval bounds are always around \tilde{z} , the only discontinuity in the graph is at \underline{z} , and firm types within the pooling interval always have higher employment in the signaling model than in the full information benchmark. Although the binary nature of workers’ effort leads to a discontinuity in the firm’s labor demand even in the counterfactual case, the model of asymmetric information shows both pooling and a larger discontinuity than the counterfactual model. In the multiperiod model equilibrium, pooling will produce underadjustment; the large discontinuity will produce a lumpy overadjustment that was discussed in the introduction.

This graph also highlights the implications of asymmetric information. In the signaling model, firm types below \underline{z} (and those above \bar{z}) choose their genuine employment levels and get low (high) effort from workers, just as in the full information case. In the pooling interval however, firms choose a higher level of employment in the signaling model equilibrium

Figure 2: Model Equilibrium



compared with their genuine choices under full information. For the lower-type firms in the interval, those below \tilde{z} , asymmetric information allows them to pool with better type firms to get high effort from workers. However, the incentives for lower-type firms to pool imposes an externality on the higher-type firms in the pooling interval. In particular, the firm types above \tilde{z} would have been better-off in the full information equilibrium in which they chose genuine levels of employment and received high effort from workers. Asymmetric information leads to an inefficient outcome: some firm types below \tilde{z} have an incentive to mimic those above \tilde{z} , which leads to labor hoarding by all firms in the pooling interval (since the higher types do not want to be perceived as low types).

The next section discusses the general properties of any pooling interval in this model. In particular, I show the existence of an equilibrium with partial pooling on a central interval by first describing the interval, characterized by a pooling employment level (Section 5.2.1) and lower/upper interval bounds (Section 5.2.2). Next, I show that this proposed equilibrium has no profitable deviation. Finally, in Section 5.2.3 I state sufficient conditions for the existence of an equilibrium with partial pooling.

5.2 Pooling in equilibrium

As a first step in characterizing the equilibrium with partial pooling in Section 5.2.1, I assume that this equilibrium exists and I discuss the equilibrium beliefs that workers hold when pooling is observed (Lemma 1), the pooling level of employment in relation to firms' genuine preferences (Lemma 2), and workers' beliefs regarding possible off-path deviations

(Lemma 3).

5.2.1 Pooling level of employment and workers' beliefs

In order for a pooling interval to exist in equilibrium, it must be profitable for at least some firm type relative to the alternative of choosing a genuine level of employment. The first lemma asserts that pooling can only be sustained if it induces workers to choose high effort.

Lemma 1 (Pooling beliefs) *Pooling on employment level N_P is profitable for some firm types only if it leads to workers' beliefs such that $E_\mu[z|n = N_P] \geq \tilde{z}$ and the workers' equilibrium strategy upon observing the pooling action leads to effort choice e_H ,*

$$b(n = N_P) = e_H. \tag{17}$$

Proof: If workers' posterior beliefs lead to $E_\mu[z|n = N'_P] < \tilde{z}$ upon observing employment level N'_P , then from the workers' strategy defined in eq.(16), firms that pool on N'_P would get low effort from workers. However, holding worker effort constant at e_L , $N^*(z, e_L)$ is the genuine choice of employment for these firm types. The genuine choice of employment is at least weakly more profitable than the pooling action N'_P when $E_\mu[z|n = N'_P] < \tilde{z}$, so firms would have a profitable deviation from N'_P . Therefore, the pooling interval must be characterized by $E_\mu[z|n = N_P] \geq \tilde{z}$.

The lemma above shows that the pooling interval has no profitable deviation only if workers provide high effort after observing the pooling level of employment. This is the case only if their expectation of z for the pooling interval is at least \tilde{z} , where \tilde{z} denotes their indifference point between providing low or high effort.

In the proposed equilibrium with partial pooling, a firm with type \bar{z} is the best-type firm that pools. As the next lemma shows, this firm type has an incentive (and the capacity) to deviate from any pooling level of employment, unless the pooling level coincides with its genuine choice of employment.

Lemma 2 (Pooling level) *When a pooling interval exists in equilibrium and $\bar{z} < z_{max}$ is the highest-type firm involved in pooling, the pooling level of employment is given by the genuine employment choice of type \bar{z} ,*

$$N_P = N^*(e_H, \bar{z}) \tag{18}$$

Proof: See Appendix section 9.

Therefore, eq.(18) defines the level of pooling employment in terms of the interval upper bound. The firm type \bar{z} is included in the pooling interval by virtue of choosing its genuine level of employment and it has no reason to try to move out of the pooling interval, since as established by Lemma 1, pooling induces high effort provision from workers. Aside from the firm at \bar{z} , which is choosing its genuine level of employment even with pooling, the pooling action involves over-employment for other firm types in the interval. Their genuine choices for n are lower than the pooling level (in Figure 1, note that firms' genuine employment choices when workers observe z lie below the pooling level of employment). Although these firms prefer lower employment levels, their ability to deviate from the pooling equilibrium and still receive high effort from workers depends crucially on workers' beliefs about off-path actions.

Based on Lemma 2 above, the pooling level of employment N_P entails over-employment for all firm types in the pooling interval (except the firm type at upper bound \bar{z}). Therefore, the potentially attractive deviations for firms in the pooling interval would be off-path actions with lower levels of employment than the pooling level, $N_O < N_P$.¹⁴

As discussed in Section 4.2.2, when workers see a deviation to an off-path action N_O , their posterior beliefs are updated with the equilibrium dominance refinement. With this refinement, workers' beliefs about z when observing off-path actions N_O depend upon the types of firms that might have reason to deviate from the equilibrium strategy. The lemma below characterizes workers' beliefs for the most plausible deviation to a lower level of employment,

Lemma 3 (Off-path refinement) *The equilibrium dominance refinement implies that $E_\mu[z|n = N_O] < E_\mu[z|n = N_P]$ when they observe off-path action $N_O < N_P$.*

Proof: See Appendix section 9.

Since workers use their posterior beliefs about z to decide effort, workers' beliefs about off-path actions help to pin down the firm's on-path equilibrium strategy. As shown in the next section, although N_O might appear to be an attractive deviation for some firm types in the pooling interval, the resulting impact on workers' posterior beliefs imposes a

¹⁴Since workers' beliefs for on-path actions are already pinned down, N_O is a relevant off-path action only if it lies above $N^*(e_L, \underline{z})$.

constraint on the deviations that would actually be profitable.

In the next section, I first show that the only equilibrium pooling level without a profitable deviation is located such that workers provide high effort for $n = N_P$ (Lemma 4) but low effort for any lower n , including any off-path action (Corollary 4a). Therefore, firms face a trade-off: pool to get high effort or choose a genuine level of employment but get low effort from workers. I show in Lemma 5 that the incentives for lower-type firms to pool is monotonically increasing in z . This result is used to derive a cutoff for the pooling interval (Corollary 5a); firms below this cutoff value find pooling too costly relative to their genuine choice of employment even though their genuine employment choice leads to low effort from workers. The section concludes with Proposition 1 in which I provide a complete definition of the pooling interval, including its bounds and the relevant firm and workers' strategies for pooling.

5.2.2 Pooling interval bounds

This section introduces two equations, both in terms of interval lower bound \underline{z} and upper bound \bar{z} , that determine the pooling interval location. The first equation is summarized in the lemma below.

Lemma 4 (Pooling action) *In an equilibrium with partial pooling, workers hold beliefs such that $E_\mu[z|n = N_P] = \tilde{z}$, consistent with the pooling interval location defined by,*

$$\int_{\underline{z}}^{\bar{z}} z \frac{g(z)}{G(\bar{z}) - G(\underline{z})} dz = \tilde{z} \quad (19)$$

Proof: From Lemma 1, the pooling action must have $E_\mu[z|n = N_P] \geq \tilde{z}$. However, if pooling action N'_P resulted in $E_\mu[z|n = N'_P] \gg \tilde{z}$, then some firm types in the pooling interval would have a profitable deviation to a lower level of employment. A deviation to $N'_P - \epsilon$ would still lead to beliefs $E_\mu[z|n = N'_P - \epsilon] > \tilde{z}$ and worker effort choice e_H . Since this logic can be repeated for any N'_P with $E_\mu[z|n = N'_P] > \tilde{z}$, in equilibrium the only pooling action without a profitable deviation must satisfy $E_\mu[z|n = N_P] = \tilde{z}$.

This lemma shows that the equilibrium pooling action is one that minimizes the over-employment associated with pooling but still satisfies the belief requirement of Lemma 1. The corollary below discusses the implications of workers' beliefs about on-path pooling actions for their beliefs about off-path actions. Specifically, the pooling interval is located such that any level of employment lower than the pooling level induces workers to provide

low effort.

Corollary 4a (Off-path beliefs) *Off-path employment level $N_O < N_P$ induces workers' beliefs $E_\mu[z|n = N_O] < \tilde{z}$ and workers' effort choice $e(n = N_O) = e_L$.*

Proof: Lemma 3 concludes that $E_\mu[z|n = N_O] < E_\mu[z|n = N_P]$. Since Lemma 4 established that N_P results in beliefs $E_\mu[z|n = N_P] = \tilde{z}$, it follows that $E_\mu[z|n = N_O] < \tilde{z}$ when $N_O < N_P$.

In other words, firms cannot choose a lower employment level than the pooling level and still get high effort from workers. Therefore, any firm considering pooling faces the choice between its genuine employment choice or the pooling level of employment, which is higher than the genuine level for low-type firms.

The following notation is used to denote the change in profits that a firm of type z could expect in a deviation from the pooling level of employment, where workers' on-path beliefs lead to e_H for the pooling level of employment (Lemma 1) and off-path beliefs lead to e_L for any deviation to a lower level of employment (Corollary 4a).

Notation: *When the pooling level upper bound is \hat{z} and the pooling employment level is $N_P(\hat{z}) = N^*(e_H, \hat{z})$ based on the result of Lemma 2, $\Delta\Pi_P(z, \hat{z})$ is the difference in profits between pooling and not pooling for firm type z ,*

$$\Delta\Pi_P(z, \hat{z}) = \pi(e_H, z, N_P(\hat{z})) - \pi(e_L, z, N^*(e_L, z)) \text{ where } z < \hat{z}$$

$\Delta\Pi_P(z, \hat{z})$ therefore represents the difference between the firm's profits with the pooling level of employment and high effort from workers, compared to the firm's profits with its genuine level of employment but low effort from workers.

As established below, this difference is increasing in firm type; firms with lower levels of z are more sensitive to the costs of pooling. Firms with intermediate levels of z (those close to \tilde{z}) benefit more from pooling in order to get high effort. Of course, firms with very high levels of z can choose N^* and still get e_H . The next lemma establishes that, for firms with low to intermediate values of z , the incremental profits from pooling to get high effort are increasing in firm type.

Lemma 5 (Partial sorting condition)

If pooling is weakly profitable for a certain firm type \underline{z} then it is also profitable for all firm types $z \in [\underline{z}, \hat{z}]$ compared to the alternative of N^* with e_L . Furthermore, $\forall \hat{z} \in [\tilde{z}, z_{max}]$, there exists a unique $\underline{z} \in (0, \hat{z})$ where $\Delta\Pi_P(\underline{z}, \hat{z}) = 0$.

Proof: See Appendix section 9.

This result is used in the next corollary to define the lower bound of the pooling interval in terms of its upper bound (and later, in Proposition 2, the sorting condition is used to show that an equilibrium with partial pooling has no profitable deviation). Although the sorting condition is satisfied for any $\hat{z} \geq \tilde{z}$ (all plausible values of \bar{z}), the upper bound of pooling is also pinned down by the beliefs condition of eq.(19), which is the first equation that defines the pooling interval bounds.

The second such condition, eq.(20) below, defines \underline{z} as the type of firm that is indifferent between choosing its genuine employment level and getting low effort from workers or choosing the pooling action, which entails over-employment but results in high effort.

Corollary 5a (Zero profit from pooling at lower bound)

In an equilibrium with partial pooling, if

$$\Delta\Pi_P(\underline{z}, \bar{z}) = \pi(e_H, \underline{z}, N^*(e_H, \bar{z})) - \pi(e_L, \underline{z}, N^*(e_L, \underline{z})) = 0 \quad (20)$$

then firms have no profitable deviation from pooling action $a(z) = N_P$ when $z \in [\underline{z}, \bar{z}]$.

Proof: Lemma 2 established that a deviation to a higher level of employment is not attractive to these firm types $z \in [\underline{z}, \bar{z}]$. Of all deviations to a lower level of employment, N^* is the most attractive, however, Corollary 4a established that any employment level less than N_P results in low effort from workers. When $\Delta\Pi_P(\underline{z}, \bar{z}) = 0$, Lemma 5 established that for all $z \in [\underline{z}, \bar{z}]$, pooling on $N_P = N^*(e_H, \bar{z})$ is more profitable than a genuine level of employment with low effort from workers. Therefore, firms in the pooling interval defined above have no profitable deviation from N_P .

The lemma above uses the sorting condition to show that no firm type in the pooling interval would have a profitable deviation away from pooling when $\Delta\Pi_P(\underline{z}, \bar{z}) = 0$. However, the reverse is also true: when Corollary 5a holds, no firm type outside of the pooling interval would have an incentive to deviate by choosing the pooling level of employment (this is shown formally in Proposition 2). Since pooling in this model requires over-employment

for low firm types, types below \underline{z} find pooling for high effort too costly relative to their alternative genuine employment choice with low effort.

To conclude this section, I use Lemmas 1-5 and their corollaries to state Proposition 1. This proposition focuses on the pooling interval; it characterizes the bounds of the interval, the firm's strategy when $z \in [\underline{z}, \bar{z}]$, and the workers' strategy when N_P is observed.

Proposition 1 (Pooling interval) *A pooling interval of the signaling model is defined by $[\underline{z}, \bar{z}]$ where the upper and lower bounds of the interval and the pooling level of employment are determined by the following,*

Iff $\exists \{z, \bar{z}\}$, with $z_{min} < \underline{z} < \bar{z} < z_{max}$ s.t. eq.(19) and eq.(20);

then in equilibrium, the firm's optimal action on the pooling interval is

$a(z) = N_P \quad \forall z \in [\underline{z}, \bar{z}]$ where N_P is given by eq.(18); workers' beliefs are consistent,

and their optimal action when pooling occurs is given by eq.(17).

The proposition above characterizes a pooling interval in which firms pool on an employment level that is at least weakly higher than their genuine choice. Once pooling arises, because of the beliefs that would result from an off-path deviation, higher-type firms in the interval (those above \tilde{z}) cannot "opt-out" lest they be perceived as a lower-type firm. Lemma 5 tells us that all of the firms in the pooling interval earn higher profits by pooling than they would earn from choosing a genuine employment level when workers perceive them as a lower-type firm. Therefore, in this model, pooling necessarily occurs on a compact interval.

In the next section, I show that the equilibrium with partial pooling that was characterized here is robust to potential deviations (Proposition 2). In Proposition 3, I present a sufficient condition for existence. I address the issue of uniqueness by establishing that the most natural candidate, a separating equilibrium, does not exist in the signaling model (Proposition 4).

5.2.3 Equilibrium existence

As the next proposition establishes, when the pooling interval bounds can be defined based on Proposition 1, a perfect Bayesian equilibrium of the signaling model includes a pooling interval. The firm's strategy is to pool for levels of z in the pooling interval but choose

its genuine level of employment for other values of z . In equilibrium, workers provide high effort to firms that pool or choose even higher employment levels than N_P ; workers provide low effort to firms with lower levels of employment.

Proposition 2 (Equilibrium with partial pooling) *When a pooling interval exists for $[\underline{z}, \bar{z}]$ defined in Proposition 1, the firm's strategy is $a(z)$ as already specified in eq.(13); workers' beliefs are defined by eq.(14) and eq.(15); their strategy is,*

$$b(n) = \begin{cases} e_L & \text{for } n < N_P \\ e_H & \text{for } n \geq N_P \end{cases} \quad (21)$$

and there is no profitable deviation from this equilibrium.

Proof: See Appendix section 9.

When the conditions in Proposition 1 are satisfied, Proposition 2 shows that a partial pooling equilibrium has no profitable deviation. Firm types below \underline{z} find pooling too costly; firm types above \bar{z} can get high effort from workers by choosing their genuine employment level. For reasons discussed earlier, firm types within the pooling interval have no profitable deviation from the pooling level of employment.

In fact, the partial pooling equilibrium stated above exists under fairly general conditions. To anticipate the results of Proposition 3, both \underline{z} and \bar{z} are well-defined values in the domain of z whenever $z_{min} = 0$ and $z_{max} = \infty$.

Proposition 3 (Sufficient condition for existence) *The partial pooling equilibrium given in Proposition 2 exists when $z_{min} = 0$ and $z_{max} = \infty$ (with $\tilde{z} \ll z_{max}$).*

Proof: See Appendix section 9.

The proof uses Lemma 5 to show that a solution for \underline{z} in terms of \bar{z} based on eq.(19) yields $0 < \underline{z} < \bar{z}$ and therefore \underline{z} is well-defined whenever $z_{min} = 0$. For \bar{z} to satisfy eq.(20), it is enough that $z_{max} = \infty$ and \tilde{z} is not excessively large. When these conditions are met, there exists an equilibrium of the signaling model with partial pooling on an interval that lies within the domain of z .

The next proposition establishes that the signaling model has no equilibrium in which all firm types choose their genuine levels of employment, which argues for the uniqueness of the proposed equilibrium with partial pooling.

Proposition 4 (Distortions from signaling) *When workers do not observe z there is always a profitable deviation from the firm's full information strategy defined by eq.(11).*

Proof: Let $\tilde{z}^- = \tilde{z} - \epsilon$. In a fully separating equilibrium, a firm of type \tilde{z}^- would get low effort from its genuine employment choice $N^*(e_L, \tilde{z}^-)$ and a firm of type \tilde{z} would get high effort from its genuine employment choice $N^*(e_H, \tilde{z})$. However, if a firm of \tilde{z}^- type were to get high effort, its optimal genuine labor choice would be $N^*(\tilde{z}^-, e_H)$. By an envelope theorem argument, the firm at \tilde{z}^- has a profitable deviation to $N^*(e_H, \tilde{z})$, since workers observing $n = N^*(e_H, \tilde{z})$ provide e_H in the fully separating candidate equilibrium. The higher employment choice imposes a second-order loss in profits, while the benefit of workers switching from low to high effort provision provides a first-order gain. Therefore, the proposed fully separating equilibrium always has a profitable deviation for some firm type just below \tilde{z} .

The proposition above shows that the signaling model does not have an equilibrium that corresponds to the full information benchmark in which every firm chooses its genuine employment level.

6 Employment Dynamics

Having presented the main mechanisms at work in the one period signaling model above, this section sheds some light on the main question of this paper, namely the implications of signaling for the path of employment adjustments.

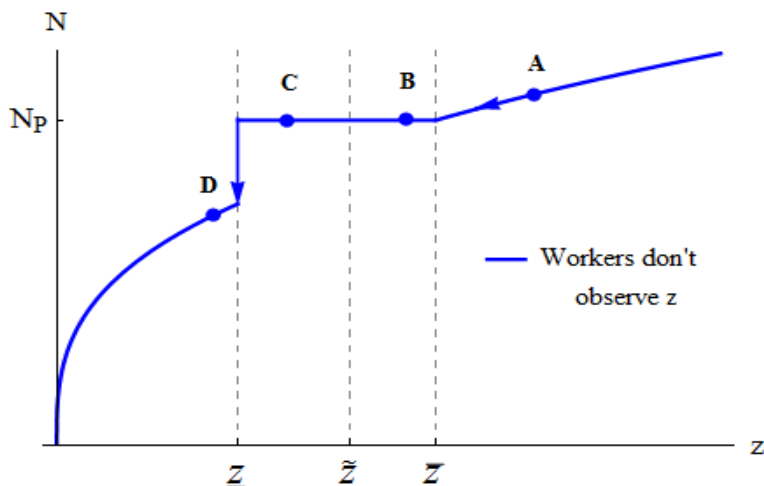
Throughout this discussion I focus on the negative shocks that a firm experiences over time.¹⁵ In particular, I focus on shocks for which z_{t-1} and/or z_t are in the pooling interval. For other values of z below \underline{z} and above \bar{z} , the signaling model is uninteresting because the pooling equilibrium looks the same as the full information benchmark.

The intuition of the model dynamics can be grasped by looking at the graph below. In this stylized example, the firm has been doing well for a long period of time. Assume it starts from point A, where it is choosing a genuine level of employment and workers can infer the exact value of z . At some point in time, the firm experiences a negative shock to point B (that lies in the pooling interval). The firm reduces employment, but by less than prescribed by the full information benchmark. If the firm gets another shock to point C, it

¹⁵An interesting question arises if there is an underlying positive trend in the distribution of z such that workers expect the firm to hire in each period, however, extending the model in that manner would add considerable complication.

makes almost no adjustment to employment.¹⁶ Only a further shock to a level of z below the pooling interval will prompt a layoff (for example, the shock from Point C to Point D). At this point, the firm conducts a lumpy, mass layoff. Any negative shock that takes the firm below Point D will lead it to make the same employment adjustments as in the case of full information.

Figure 3: Employment Changes with Negative Shocks



While this graph shows an example of the model dynamics, the next section develops a fully fledged multiperiod model to study the magnitude of employment adjustments as a function of the previous level of z and the size of the most recent shock.

6.1 Set-up of a dynamic model

This section illustrates a firm's optimal employment choices over multiple periods where the firm and its workers maximize expected profits/utility and employment choices reflect signaling considerations. In the following analysis, z_t is privately observed by the firm and evolves according to a first-order Markov process. Although the firm experiences an idiosyncratic shock at the beginning of every period, this model will show that firm types in the pooling interval may hoard labor across multiple periods. Layoffs, when they happen in the model, are often pent-up and disproportionate relative to the most recent change in business conditions.

The dynamic model consists of multiple periods of the signaling game, given by *Stages 1-4* from the one period model. Wage contracts are treated as state-dependent but prede-

¹⁶As workers learn z_{t-1} , this information defines their prior about z_t in the next period. Since N_P depends on z_{t-1} , the level on which pooling occurs slightly increases after each negative shock.

terminated and it is assumed that workers observe the firm's business condition when they receive their wages in *Stage 4*.¹⁷

The level of the firm's business conditions evolves as a first-order Markov process (i.e. the realization of z_t depends on z_{t-1} in the previous period but does not depend explicitly on any earlier history of z realizations). The process evolves as $g : \Delta_{z_{t-1}} \rightarrow \Delta_{z_t}$ and workers have rational expectations about the firm's business condition from one period to the next. Therefore, $g(z_t|z_{t-1})$ summarizes their prior at the beginning of period t .

6.1.1 Value functions

In this section I present the multiperiod profit maximization problem of the firm and multiperiod utility maximization problem of the worker, both of which are additively separable across periods with constant discount rates β . Although these are formulated as recursive problems, Claim 2 will show that equilibrium beliefs and strategies can depend on the state variable z_{t-1} but are otherwise stationary.

The firm's dynamic profit maximization problem is,

$$V_t^\pi(z_t) = \max_{n_t} \pi_t(e_t^*, z_t, n_t) + \beta E_t[V_{t+1}^\pi(z_{t+1})|z_t] \quad (22)$$

where $V_t^\pi(z_t)$ is the firm's value function with current business condition z_t and $\pi(\cdot)$ denotes the firm's one period expected profit function from eq.(6). These functions are indexed by t for the case of asymmetric information in which workers' beliefs depend on z_{t-1} and their beliefs determine the firm's pooling strategy.

The workers' dynamic utility maximization problem is,

$$V_t^E(z_t) = U(e_t^*, z_t) + \beta E_t[(1 - s_{t+1}(z_{t+1}))V_{t+1}^E(z_{t+1})|z_t] \quad (23)$$

$$\text{s.t. } e_t^* = \operatorname{argmax}_{e_t} E_\mu[V_t^E(z_t)|n_t, z_{t-1}]$$

where $V_t^E(z_t)$ is the value to a worker of being employed by a firm of type z_t and $U(\cdot)$ is the workers' one period expected utility. A worker's probability of layoff from a firm of type z_{t+1} is given by $s_{t+1}(z_{t+1})$, which is indexed by time to reflect its dependence on the firm's previous employment choice.

¹⁷The model abstracts from firms' exit decisions; since $\rho(e, z, n = 0) = 0$ the firm can always choose zero employment and earn zero profits.

To summarize eq.(23) above, a worker's value of employment at firm of type z_t is the worker's expected utility at time t plus the expected future value of employment at the firm. That future value is discounted by the probability of being laid off between time t and time $t + 1$; the value of being unemployed is set to zero (without loss of generality).

In Claim 2 below I show that, for both the firm and the workers, each period of the dynamic problem can be reduced to the same decision as in the one period model.

Claim 2 (Separability) *Both the workers' and the firm's optimal strategies are stationary when conditioning on the information from time $t - 1$.*

Proof (Full information case): For both the firm and the workers, the process by which z evolves is exogenous. The workers' effort decisions in each period have no dynamic implications. Effort does not affect the probability of future layoffs and therefore the second term on the right-hand side of eq.(23) is constant with regard to workers' actions. Similarly, the firm incurs no explicit cost to changing n from one period to another. Since its choice of n only impacts workers' effort choices in the current period, the firm's choice of n in each period of eq.(22) has no dynamic implications.

Proof (Signaling model): The claim above holds even when there is asymmetric information between the firm and its workers. In this case, n_t has signaling value and workers use their beliefs to choose effort. However, despite the differences from the full information benchmark, the timing of the multiperiod model dictates that workers observe the true value of z in *Stage 4* when their wages are paid. Since the worker knows the exact value of z_{t-1} at time t , the firm would not be able to deceive the worker about the longer history of z through its choice of n_t . For the same reasons as stated in the proof above, the firm's optimal strategy in the dynamic signaling model is stationary and analogous to eq.(13) in the static model: any deviation from its optimal static strategy in a given period of the dynamic model would entail lower profits in that period and no increase in profits in the following periods. The workers' optimal strategy in the dynamic signaling model is analogous to eq.(16) in the one period model. Of course, unlike the one period model, at each time t of the multiperiod model all equilibrium strategies (and workers' beliefs) depend on z_{t-1} .

To provide a more formal definition based on the discussion above, a Markov perfect Bayesian equilibrium for the multiperiod signaling game consists of (i) the firm's optimal strategy, $a(z_t|z_{t-1}) = n_t$; (ii) the workers' optimal strategy, $b(z_t|n_t, z_{t-1}) = e_t$; and (iii)

workers' beliefs, $\mu(z_t|n_t, z_{t-1})$. In equilibrium, workers' beliefs must be consistent and strategies must be sequentially rational best responses.

Obviously there are infinitely many possible sequences of shocks and adjustment paths that could arise in the equilibrium of a dynamic model. Although the equilibrium of the multiperiod model is well-defined, an additional complication arises: the pooling bounds and pooling level of employment change as z_{t-1} changes.¹⁸ Therefore, in order to characterize the changing employment levels, the next section uses three dimensional graphs to show how adjustments to employment depend on past business conditions z_{t-1} and realized shocks $\Delta z = z_t - z_{t-1}$.

6.1.2 Illustration of model dynamics

The signaling model in this paper is not a steady state model but rather one of adjustments over the short- to medium-term. In line with the main research question, I focus primarily on negative shocks for which the signaling model dynamics differ from the full information benchmark. These involve either z_{t+1} and/or z_t in the pooling interval.

When z is initially high and is then affected by a negative shock that brings it into the pooling region, this leads to inaction and labor hoarding – an initial underadjustment. When another negative shock (or series of shocks) arrives and is large enough to result in z below the pooling interval, this leads to a lumpy downward change in employment – an overadjustment relative to the counterfactual model. The next two sections highlight the signaling model's underadjustment and overadjustment dynamics, respectively.

Initial underadjustment

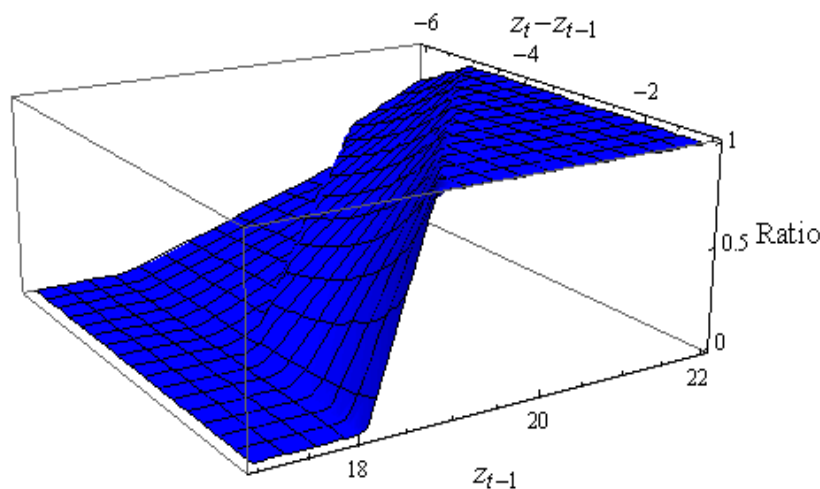
This section shows the change in employment in the signaling model when the firm experiences a negative shock into the pooling interval. This can be understood as either a shock from Point A to B or from Point B to C in Graph 3.

The graph below compares adjustments in the signaling model to the adjustments in a model with full information. This comparison is reported as the ratio of adjustments; the ratio is equal to one when the signaling model adjusts by the same number of workers as the counterfactual model. A ratio of less than one indicates that the signaling model shows

¹⁸Since higher levels of z_{t-1} give the workers a higher prior about z_t , N_P is always decreasing in the level of z_{t-1} and \bar{z} and \underline{z} are also decreasing in the level of z_{t-1} . In other words, pooling at time t becomes more costly and probably less attainable for firm types with low z_{t-1} . These types would need to counteract workers' priors about z_t in order to get high effort from pooling.

labor hoarding relative to the counterfactual model.

Figure 4: Labor Hoarding for Initial Negative Shocks



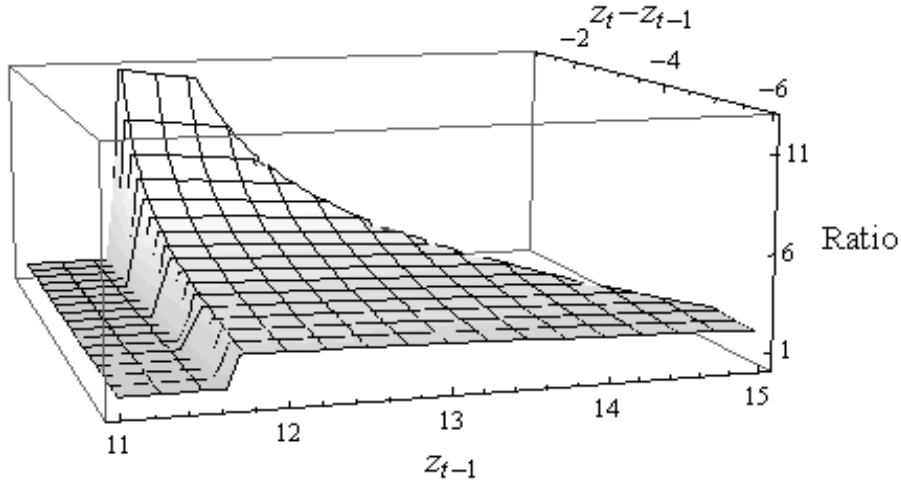
The counterfactual model always has negative adjustments in response to negative shocks, whereas the signaling model has smaller adjustments in many instances and sometimes even shows small positive adjustments to negative shocks. In all of the cases graphed above, once the level of z falls into the pooling interval, the signaling model shows underadjustment relative to the full information benchmark (i.e. a ratio of < 1 in the graph).

Subsequent overadjustment

Here I look at the next possible type of shock, from an intermediate value of z to a value below the pooling interval, such as the change from Point C to Point D in Graph 4.

The three dimensional graph below shows that changes in employment in the signaling model can be larger, in some cases much larger, than employment changes in the counterfactual full information setting. While the adjustment ratio is around 2 for many combinations of initial levels and shocks (indicating that these adjustments are twice as large as those in the benchmark model), in this example the ratio is well above 7 when z starts near the pooling lower bound. In the signaling model, these lumpy patterns come from one-time overadjustments that occur when the firm switches from pooling to choosing its genuine level of employment in response to a negative shock. This shock does not need to be large for an overadjustment to take place: with a small shock, the counterfactual model predicts a small adjustment whereas the signaling model shows a very large adjustment. Therefore, the ratio in the graph is often highest for the smaller-sized shocks.

Figure 5: Mass Layoffs after Additional Negative Shocks



Summary– The simulations presented in this section illustrate the role of private information and workers’ incentives. In the signaling model, negative shocks that lead to pooling result in underadjustment. Firms experiencing shocks that are not too large hoard labor while in the pooling interval and can appear inactive. However, a subsequent negative shock can lead to a layoff that is considerably larger than the adjustment predicted by the counterfactual full information model. The inaction and lumpy layoff pattern in the multiperiod signaling model matches many empirical findings from other papers that were mentioned in the introduction.

6.2 Aggregate shocks: the case of recessions

As emphasized by Lazear, Shaw and Stanton (2013), lower outside options might have positive effects on workers’ effort provision. The most straightforward extension is one in which workers must shirk in order to search for a new job.¹⁹ When a recession lowers the value of workers’ outside options, it could therefore decrease their indifference point between providing low/high effort from \tilde{z} to $\tilde{z}' < \tilde{z}$. When their outside option is weak, workers are willing to stay at a firm with values of $z \in [\tilde{z}', \tilde{z}]$, whereas before the recession began they would have wanted to leave these firm types. The next section provides a summary of this and other findings from the signaling model with workers’ effort decisions.

¹⁹Note, however, that the threat of worker quits cannot replace the effort mechanism in the model. Specifically, if a fraction of workers would quit rather than work at an employer of type $z < \tilde{z}$, this changes the incentives for pooling. Furthermore, lumpy adjustments in employment would be characterized as “mass quits” rather than “mass layoffs”.

6.3 Summary of equilibrium dynamics

The equilibrium strategies described in the multiperiod model above have the following implications for the dynamics of employment adjustments:

1. When both z_{t-1} and $z_t \in [\underline{z}_t, \bar{z}_t]$, the model shows labor hoarding (i.e. over-employment) relative to the full information genuine N^* benchmark and the firm underadjusts in response to negative shocks.
2. When z_{t-1} and z_t are both either very low or very high, the magnitude of adjustments are the same as in the model with full information.
3. When $z_{t-1} \in [\underline{z}_t, \bar{z}_t]$, a negative shock that pushes z_t below \underline{z}_t leads to a mass layoff episode. These adjustments move the firm from labor hoarding to a genuine choice of employment. The mass layoff is a pent-up “overadjustment” that reflects changes in labor demand from all previous labor hoarding periods.
4. Workers’ incentives for effort may be affected by the value of their outside option. If workers must shirk in order to search on the job, \tilde{z} defines the point at which workers are indifferent between staying at their employer or not. In the signaling model, if the expected value of workers’ outside options falls during a recession then workers would be willing to provide high effort for firm types that would have otherwise inspired shirking. In this manner, the signaling model may be consistent with evidence in Lazear, Shaw and Stanton (2013) that worker effort increased during the 2008 Recession.

Although it is difficult to summarize the full set of possible scenarios, some of the dynamics described above are consistent with the stylized facts discussed in Section 2. Many firms adjust their employment infrequently, yet there are firms that make disproportionate changes to employment in a short period of time. The mechanisms presented here might help to explain why U.S. employment adjustments might be lumpy even in the absence of large non-convex statutory or technological firing costs.

7 Discussion

Mass layoffs are common in practice, but why do firms choose lumpy patterns over more gradual employment adjustments? This model suggests that the signaling mechanism may contribute to the lumpiness of firm-level employment adjustments when the firm has private information about its business condition, workers’ incentives matter for production, workers’ contracts have a profit-sharing structure, and the terms of their contracts are

predetermined.²⁰

While information asymmetry is a key ingredient for the signaling model, the gap between the firm and the workers' information does not need to be large. One may think that some executives have better information than production workers, and perhaps early departures by key executives also reveal negative information to less informed workers. This additional source of information would not invalidate the model. Even if the firm's only informational advantage is a more accurate one-period ahead forecast (perhaps because it has better knowledge of its own production technology), this still produces enough information asymmetry to generate lumpy employment dynamics.

Several features of workers' contracts are also important for the model's results. As in a standard principal-agent problem, workers' incentives must matter for production and effort must be non-contractible. Although workers' remuneration cannot depend directly on effort, the optimal agreement is essentially a piece rate-style contract in which workers are paid according to what they produce. An additional assumption is that workers produce more when working for a better employer. This is akin to a profit-sharing contract, which could actually represent any of the monetary or non-monetary benefits that workers derive from providing high effort when working for a better employer.²¹ A final assumption is that wages are not negotiated in each period (contracts are determined ex-ante but specify payments that are state-contingent). These three assumptions can be summarized as (1) non-contractible effort, (2) the importance of firm quality, and (3) quasi-rigid wages. Consequently, this model is probably most applicable to industries in which workers have long-term relationships with their employers and effort matters but there is no effective monitoring technology.

A model of endogenous labor adjustment costs has important implications for both positive and normative policy conclusions. First, looking at the model's key assumptions could help policy makers better predict the industries that are more prone to mass layoffs. This could be used to allocate unemployment insurance budgets or to create targeted job-finding resources for the workers most at-risk. Second, labor hoarding may be an important driver of aggregate productivity dynamics— Burnside, Eichenbaum and Rebelo (1993) find, "... a

²⁰The extent of layoff lumpiness (i.e. the degree to which layoffs are large) also depends on the variance of shocks and the mass of workers with similar contracts and effort preferences (because if workers are too heterogeneous then there is no point at which a large number of workers switch from low to high effort provision).

²¹In fact, there may be complex reasons beyond wages that workers derive utility from working for better employers – Brown and Matsa (2012) present an interesting empirical finding in this vein.

significant proportion of movements in the Solow residual are artifacts of labor hoarding type behavior". A better understanding of labor hoarding dynamics could potentially lead to new conclusions about the propagation of aggregate and sectoral shocks. Third, since mass layoffs appear to be particularly detrimental to workers' health and careers, policies encouraging more gradual adjustments may improve workers' welfare.²² The model suggests that the prevalence of large mass layoffs could potentially be reduced by decoupling workers' incentives from their employer's profitability. Finally, the signaling model with worker effort shows that endogenous adjustment costs can have large effects. This suggests that exogenous costs (for example, statutory firing costs, mandated severance payments, and layoff prenotification policies) may have less of an impact on employment frictions than previously thought.

8 Conclusion

This paper shows that a signaling model in which workers reduce their effort in response to layoffs can generate lumpy employment adjustments that seem prevalent in U.S. microdata. In this model, workers' incentives depend on the quality of their employer but workers only observe the firm's employment decisions. Although labor hoarding is costly, some firm types will postpone layoffs in the face of small shocks to conceal negative information and continue receiving high effort from their workers. The equilibrium employment strategy of firms leads to pent-up adjustments: when a firm's business conditions get substantially worse it makes a full adjustment. This adjustment entails laying off all workers that were not needed in the previous periods of decline, which brings the firm to its genuinely optimal (lower) level of employment. The microfoundation in this paper provides some predictions about the relationship between labor contracts, firms' financial conditions, and employment dynamics and therefore can inform policies that try to anticipate and mitigate the negative effects of mass layoffs.

9 Appendix: Additional Proofs

Proof of Claim 1:

Since the profit function is strictly concave in n , for each e the firm's profit maximization problem has a unique maximum for each z . When the set of N is convex, the Maximum

²²Note, however, that this paper did not model firms' entry and exit decisions nor any general equilibrium effects. A full analysis of workers' welfare would have to consider other effects, especially the impact of gradual adjustments on aggregate employment.

Theorem leads to the further conclusion that the optimal employment correspondence for each level of effort e , $N^*(e, z)$, is a single-valued (continuous) function in z . Therefore, for each e , $N^*(e, z)$ is invertible in z . The composite function $a_F(z)$ is given by $N^*(e_L, z)$ for $z < \tilde{z}$ and $N^*(e_H, z)$ for $z \geq \tilde{z}$. Since $N^*(e, z)$ is increasing in both z and e and $e_L < e_H$, $N^*(e_L, \tilde{z}) < N^*(e_H, \tilde{z})$ and the function $a_F(z)$ is also invertible.

Proof of Lemma 2 (Pooling level):

The first step, based on eq.(13), is to notice that the pooling level of employment can only lie in the interval defined by $N_P \in [N^*(e_L, \underline{z}), N^*(e_H, \bar{z})]$. If N_P were lower or higher than that range, it would encroach on the level of employment that another firm type chooses in the equilibrium strategy. This would contradict the definition of the pooling interval bounds as $[\underline{z}, \bar{z}]$. The next step is based on Lemma 1, which concluded that workers' beliefs about N_P are $E_\mu[z|n = N_P] \geq \tilde{z}$ and workers' equilibrium effort choice is $b(n = N_P) = e_H$. Upon observing an employment level greater than the pooling level, workers' beliefs would be slightly higher ($E_\mu[z|n > N_P] > \tilde{z}$) and thus workers would still provide high effort; see Lemma 3 for a related proof based on the equilibrium dominance refinement. For a firm of type \bar{z} , if the pooling employment level were lower than its genuine choice such that $N_P < N^*(e_H, \bar{z})$, it would have a profitable deviation to $n = N^*(e_H, \bar{z})$. Combining these results shows that $N_P = N^*(e_H, \bar{z})$ is the unique pooling employment level.

Proof of Lemma 3 (Off-path refinement):

Equilibrium dominance leads workers to refine their beliefs; only firm types that might stand to improve their equilibrium payoff about the firm type would have chosen a given action. Therefore, when off-path action $N_O < N_P$ is observed and $N_P = N^*(e_H, \bar{z})$, workers look at the types of firms that would want to choose an employment level lower than N_P . Workers' beliefs are refined to:

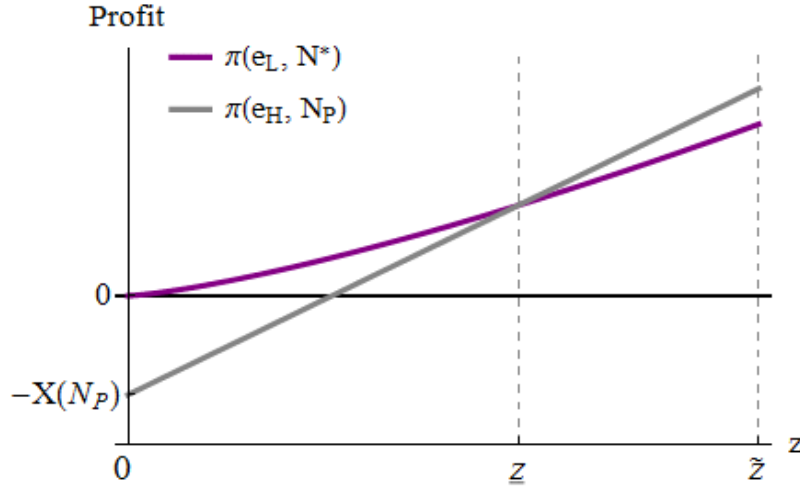
$$\mu(z|n = N_O) = \begin{cases} \frac{g(z)}{G(\bar{z}) - G(\underline{z})} & \text{for } z \in [\underline{z}, \bar{z}) \\ 0 & \text{for all other } z \end{cases} \quad (24)$$

Thus, when an off-path action is observed, workers' beliefs put no weight on the possibility that $z = \bar{z}$ and put more weight on the lower realizations of z relative to when they observe N_P . This leads to the result that $E_\mu[z|n = N_O] < E_\mu[z|n = N_P]$ when $N_O < N_P$.

Proof of Lemma 5 (Partial sorting condition):

When getting low effort from workers, the firm can do no better than choosing its optimal genuine employment level N^* . However, the profits that it gets may be smaller than the profits it would get from pooling on employment to get high effort from workers. To illustrate the firm's trade-off between choosing N^* with e_L or the pooling level N_P with e_H , Figure 6 shows the relevant profit calculations for the numerical example. Although the graph shows one example of the partial sorting condition, the main properties (derived below) hold more generally for any parameter configurations of the model.

Figure 6: Profits Comparison



Changes in profits with respect to z ($\Delta\Pi_P(z, \hat{z})$):

- At $z = 0$, the firm's profits from pooling with $N_P = N^*(e_H, \hat{z})$ are strictly less than its profits from choosing its genuine level of employment, N^* . The firm's profits at $z = 0$ are: $\pi(e_H, 0, N_P) = -X(N_P)$ and $\pi(e_L, 0, N^*(e_L, z)) = 0$ from pooling and not pooling, respectively. Thus, $\Delta\Pi_P(z = 0, \hat{z}) < 0$. Note that $\Delta\Pi_P(z, \hat{z})$ is defined as the difference between the profits that low-type firms would get from pooling, $\pi(e_H, z, N_P)$, and the profits that they would get from not pooling, $\pi(e_L, z, N^*(e_L, z))$.
- At $z = \hat{z}$, the pooling employment level with high effort is strictly more profitable than the firm's genuine employment level with low effort,

$$\pi(e_L, \hat{z}, N^*(\hat{z}, e_L)) < \pi(e_H, \hat{z}, N^*(e_H, \hat{z})) \Rightarrow \Delta\Pi_P(\hat{z}, \hat{z}) > 0. \quad (25)$$

Sorting properties for $z \in [0, \hat{z}]$: The firm's profits with a pooling level of employment or with a genuine employment level are increasing in z . For $z < \hat{z}$, the firm's profits from

pooling increase more steeply than their profits from not pooling with low effort. Applying the envelope theorem, the derivative (with respect to a change in z) of the firm's profit with its genuine employment choice and e_L is,

$$\frac{\partial \pi(e_L, z, N^*(e_L, z))}{\partial z} = (1 - \omega^*)q_z(e_L, z)N^*(e_L, z) \quad (26)$$

The derivative (with respect to a change in z) of the firm's profit with N_P and e_H is,

$$\frac{\partial \pi(e_H, z, N_P)}{\partial z} = (1 - \omega^*)q_z(e_H, z)N_P \quad (27)$$

Since $0 < q_z(e_L, z) < q_z(e_H, z)$ and $0 \leq N^*(e_L, z) < N_P$ for all $z \in [0, \hat{z}]$,

$$\frac{\partial \pi(e_L, z, N^*(e_L, z))}{\partial z} < \frac{\partial \pi(e_H, z, N_P)}{\partial z} \quad \forall z \in [0, \hat{z}] \quad (28)$$

- From the properties detailed above, I can conclude that $\Delta \Pi_P(z, \hat{z})$ is increasing in z for all $z \in [0, \hat{z}]$. The value of the firm's profits from pooling with high effort versus not pooling with low effort cross only once and thus for any \hat{z} there is a unique value of $\underline{z} \in (0, \hat{z})$ for which $\Delta \Pi_P(\underline{z}, \hat{z}) = 0$.

Proof of Proposition 2 (Equilibrium with partial pooling):

The workers' equilibrium strategy indicates that workers provide high effort iff they observe $n \geq N_P$. Otherwise, if they observe a lower level of employment they provide low effort. This means that the relevant comparison for a profitable deviation to an employment level below N_P is $\Delta \Pi_P(z, \bar{z})$, which calculates the difference in profits from pooling with high effort compared to separating with low effort.

- When $\Delta \Pi_P(\underline{z}, \bar{z}) = 0$, a firm of type \underline{z} is indifferent between N_P and $N^*(e_L, \underline{z})$. Based on the Partial sorting condition of Lemma 5, for $z \in [z_{min}, \underline{z}]$ (i.e. firm types below the pooling interval) the firm's profits would be lower from pooling; firms with $z \in [z_{min}, \underline{z}]$ have no incentive to deviate from their optimal non-pooling (genuine) employment level, with the resulting low effort.
- As established by the Partial sorting condition above, the difference in profits between pooling and not pooling is greater than zero for all firm types in the pooling interval,

$$\pi(e_H, z, n) - \pi(e_L, z, N^*(e_L, z)) > 0 \quad \forall z \in (\underline{z}, \bar{z}) \quad (29)$$

A firm with type $z \in (\underline{z}, \bar{z})$ within the pooling interval would have no incentive to deviate from the equilibrium because of the impact on workers' beliefs (and their

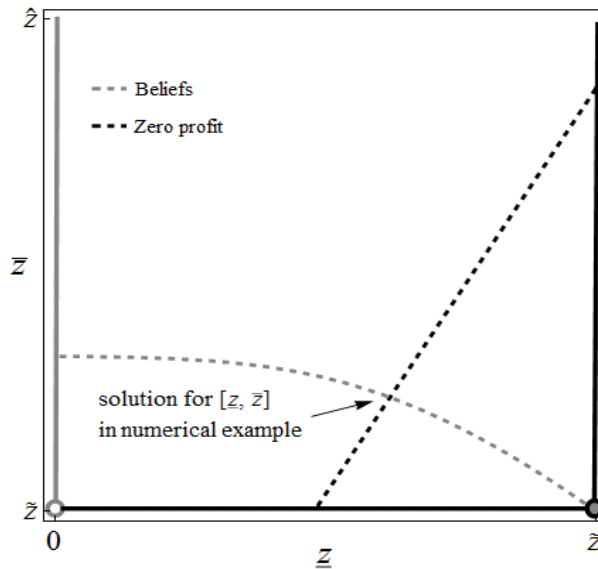
resulting effort decision).

- A firm with type $z = \bar{z}$ is choosing its genuine optimal employment level in the pooling equilibrium and getting high effort from workers, so it has no incentive to deviate (either up or down) from $N_P = N^*(e_H, \bar{z})$.
- Finally, there is no incentive for a firm with $z \in (\bar{z}, z_{max}]$ to deviate from its genuine choice of employment in equilibrium. Since the firm is already getting high effort from workers, its choice of $N^*(e_H, z)$ is better than any profit it could earn from choosing a different n from its optimal genuine employment level.

Proof of Proposition 3 (Sufficient condition for existence):

The bounds of the pooling interval are pinned down by two equations: eq.(19) defines “Beliefs” of workers and eq.(20) defines a “Zero profit” condition for firms. First note that the implicit function for $\underline{z}(\bar{z})$ from eq.(19) is continuous and decreasing in \bar{z} . The implicit function for $\underline{z}(\bar{z})$ from eq.(20) is continuous and increasing in \bar{z} . For eq.(19), $\underline{z} = \tilde{z}$ when $\bar{z} = \tilde{z}$. Instead, at $\bar{z} = \tilde{z}$ eq.(20) has a lower \underline{z} , specifically $\underline{z} \in (0, \tilde{z})$. At the other extreme, $\underline{z} = \tilde{z}$ corresponds to $\bar{z} > \tilde{z}$ in eq.(20).

Figure 7: Illustration of Existence Proof



Existence is established by the fact that both equations define continuous functions. Given the bounds described here, the functions must intersect at some point. Since both functions are monotonic they will cross only once: the solution to these two equations is unique. Finally, to conclude the proof, it should also be clear from the graph that since $\{\underline{z} = \tilde{z}, \bar{z} = \tilde{z}\}$

is not a solution to eq.(20), the pooling interval bounds are given by $z_{min} < \underline{z} < \bar{z} < z_{max}$.

As stated in Proposition 1, if $\tilde{z} \ll z_{max}$ then the system of equations has a solution whenever the domain of z is given by $z_{min} = 0$ and $z_{max} = \infty$. This additional assumption, $\tilde{z} \ll z_{max}$, comes from the requirement that $2G(\tilde{z}) < G(z_{max})$, which means that there exists a point $\underline{z} = 0$ and $\bar{z} > \tilde{z}$ satisfying eq.(19). The last statement establishes that a solution for eq.(19) exists even when $\underline{z} = 0$, whereas \underline{z} is always greater than zero in eq.(20). This range of values for eq.(19) is depicted as the leftmost vertical line in Figure 7. This figure also shows the three other conditions described above and illustrates the intuition of the solution.

10 Appendix: Microfoundations of Assumptions

In the baseline model, two important assumptions were introduced to make the model more tractable. The first important assumption in this model is that effort can take only two values, e_H or e_L . This assumption can be micro-founded from a variety of utility configurations, as discussed in Section 10.1.

The second assumption is that contracts between workers and firms consist of a wage with expected value strictly increasing in z . This feature is microfounded in the context of a principal-agent problem in which the firm makes a take-it-or-leave-it contract offer to the worker in *Stage 0*; the details of such a contract are discussed Section 10.2 below. Alternative models of workers' increasing compensation could involve either worker-firm bargaining or career concerns in which workers' expected tenure and success with a firm depends on the firm's business condition. The principal-agent framework is the most quantifiable of the three options mentioned above, since a bargaining model in which the firm negotiates with each worker over the expected surplus is sensitive to assumptions about the firm's outside options and a career concerns model necessarily spans multiple periods.

10.1 Effort provision

Throughout the model I assume that effort is a binary choice that leads to discontinuous outcomes. A binary effort decision rule by workers could actually stem from a utility maximization problem with corner solutions: once the compensation for effort becomes sufficiently high, the worker may find it optimal to switch from the minimum effort to the maximum effort level. This would typically only arise if the disutility from effort is not convex. The restriction to binary effort provision by workers is common in many models,

such as Shapiro and Stiglitz (1984); it is also common in the principal-agent mechanism design literature. In this signaling model, the effort choice can stem from the workers' decision problem of whether or not to shirk in order to search on the job for a new employer.

Despite the suggestion that workers have a binary decision about whether to search for a new job, their effort decision remains crucial in this model. If the only disciplining force on the firm's layoffs is the threat of an immediate quit, the model's partial pooling equilibrium would not result.

10.2 Derivation of ω^* from an ex-ante principal agent contract

The firm offers each worker a wage contract ex-ante before either the firm or the worker has any knowledge of z beyond the prior $g(z)$. Effort and business conditions are not verifiable, but contracts can be specified in terms of the worker's individual contribution y . In the principal-agent problem below, the worker is subject to limited liability (normalized to zero). Therefore, a feasible contract requires weakly positive payments to the worker for any realized value of y . A general formulation of the problem is presented here and a numerical example is solved for in the table of Appendix section 11.

It is assumed that contracts can only be written in terms of output share ω , which affects the firm's expected profits (π), the workers' expected wages, and their indifference point between providing low and high effort (\tilde{z}).

Assumption 3 *The values of z and e are non-verifiable. Contracts can only specify ω , a constant share of the prospective realized output y .*

The assumption above restricts contracts to those linear in the worker's contribution. Without this restriction, an optimal contract would still be weakly increasing in the expected value of y but it would generally contain non-linearities based on threshold quantities of y .²³ However, a contract that is linear in $q(e, z)$ could arise naturally if output is restricted to two levels $\{0, q_H\}$ and effort and business conditions determine the *probability* that q_H is realized. In that context, a contract to elicit high effort for higher values of z would offer payment $\omega > 0$ when q_H results, which would mean that the worker's expected payment is linear in $q(e, z)$, the expected level of individual output.

In this setting, the firm offers a contract to each worker that maximizes its own expected profit (as given in *Stage 4*). The firm can anticipate the workers' effort provision strategy

²³See Oyer (2000) for a more complete discussion of non-linear bonus contracts.

and thus it designs the ex-ante contract taking the workers' ex-post effort decision as given. For each worker, the firm chooses an optimal sharing parameter to solve its uninformed principal problem,

$$\begin{aligned}
\omega^* &= \operatorname{argmax}_{\omega} \quad \pi(e^*, z, n = a(z), \omega) \\
\text{s.t.} \quad e^* &= b(n) && \text{Incentive compatibility} \\
0 &\leq \int_{z_{\min}}^{z_{\max}} \omega q(e^*, z) dG(z) && \text{Participation constraint with } \bar{U} = 0 \\
0 &\leq \omega q(e, z) \quad \forall e \text{ and } z && \text{Limited liability; minimum of } -l = 0
\end{aligned}$$

In this case, the limited liability constraint is more stringent than the participation constraint and, since $q(e, z) \geq 0$, both can be reduced to the condition $\omega \geq 0$. As shown by Proposition 4, when the firm has private information about z an equilibrium with full separation does not exist.

Here, in addition to determining the bounds of the pooling interval (analogous to those in Proposition 1), the value of ω is an endogenous ex-ante choice for the firm. In choosing ω^* , the firm anticipates the pooling actions that will arise in the equilibrium of the signaling game in *Stages 1-4* of the model. For non-pooling values of z , $a(z) = N^*(e^*, z, \omega)$. For pooling values of z , however, the profit function depends on N_P and the bounds of the pooling interval described by $[\underline{z}, \bar{z}]$.

$$\pi(\cdot) = \int_{z_{\min}}^{\underline{z}} \pi(e_L, z, N^*(e_L, z, \omega), \omega) dz + \int_{\underline{z}}^{\bar{z}} \pi(e_L, z, N_P(\omega), \omega) dz + \int_{\bar{z}}^{z_{\max}} \pi(e_H, z, N^*(e_H, z, \omega), \omega) dz$$

The bounds of the interval and the level of pooling employment are defined in the proposition below.

Proposition 5 (Pooling interval with firm's ex-ante contract offer) *A pooling interval of the signaling model with an optimal ex-ante contract offered by the firm is characterized by, Iff $\exists \{\omega^*, \underline{z}, \bar{z}\}$, with $z_{\min} < \underline{z} < \bar{z} < z_{\max}$ such that*

$$\begin{aligned}
\omega^* &= \operatorname{argmax}_{\omega \geq 0} \quad \pi(e^*, z, n = a(z), \omega) \text{ s.t.} \\
&\int_{\underline{z}}^{\bar{z}} z \frac{g(z)}{G(\bar{z}) - G(\underline{z})} dz = \tilde{z}(\omega) \\
&\pi(e_H, \underline{z}, N^*(\bar{z}, e_H, \omega), \omega) - \pi(e_L, \underline{z}, N^*(e_L, \underline{z}, \omega), \omega) = 0
\end{aligned}$$

$$e^* = b(n)$$

then in equilibrium, the firm's optimal strategy on the pooling interval is:

$$a(z) = N_P \quad \forall z \in [z, \bar{z}], \text{ where } N_P \text{ is given by } N_P = N^*(\bar{z}, e_H, \omega)$$

workers' beliefs are consistent, and their optimal strategy when pooling occurs is:

$$b(n = N_P) = e_H$$

The problem above is most interesting when the endogenous value of ω^* is such that the workers' indifference point $\tilde{z}(\omega^*)$, implicitly defined in terms of ω^* , lies in the interior of the domain of z . This is equivalent to Assumption 2 that was discussed in Section 3.4.2 when ω^* was treated as exogenous. When that condition is satisfied, the firm's equilibrium strategy is characterized by partial pooling as established in Propositions 2 and 3. For the ex-ante principal-agent contract, one such example in which Assumption 2 is satisfied endogenously is provided in the second column of the table in Appendix section 11 below. The optimal wage contract implements low effort for values of z below \underline{z} (i.e. below the pooling interval lower bound) and implements high effort for all other z .

11 Appendix: Example Parameters

The numerical examples are based on the following parameters and functional forms:

Example Parameters

Graphs	Principal-Agent Problem
$X(n) = \frac{1}{4}n^4$ $\Psi_L = 0, \Psi_H = 0.5$ $f(e_L) = 0.6, f(e_H) = 0.7$ (deterministic) $\omega^* = 0.33$ (exogenous) $\tilde{z} = 15$ $\underline{z} = 8.5$ and $\bar{z} = 17.8$	$X(n) = \frac{1}{2}n^2$ $\Psi_L = 0, \Psi_H = 0.01$ $f(e_L) = 0.3, f(e_H) = 0.5$ (deterministic) $\omega^* = 0.089$ (endogenous) $\tilde{z}(\omega^*) = 0.56$ $\underline{z} = 0.50$ and $\bar{z} = 0.62$
$g(z_t z_{t-1}) \sim N(z_{t-1}, 5)$ (with $z_0 = 16$) on domain $[0, 35]$	$g(z)$ uniform on $[0, 1]$

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