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Hysteresis via Endogenous Rigidity in Wages and Participation

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Abstract

We model hysteresis in the labor market as resulting from a strategic complementarity in firms' wage setting and workers' job search strategies. Strategic complementarity results in a *continuum* of possible equilibria with higher-wage equilibria welfare dominating lower-wage equilibria. Further, we specify a protocol for revelation of the new equilibria following shocks such that the model exhibits (1) periods of endogenous rigidity in wages and participation, (2) persistent changes in wages, participation, and output in response to transitory movements in labor productivity, (3) sluggish recoveries including both a "jobless" phase and a "wageless" phase. Furthermore, regardless of the history, expansions are insufficiently robust in the sense that misallocation remains even during expansions.

JEL Classification: D83, E24, J42
Keywords: Kinked Labor Supply, Strategic Complementarity, Hysteresis, Real Rigidity, "Jobless" and "Wageless" Recovery

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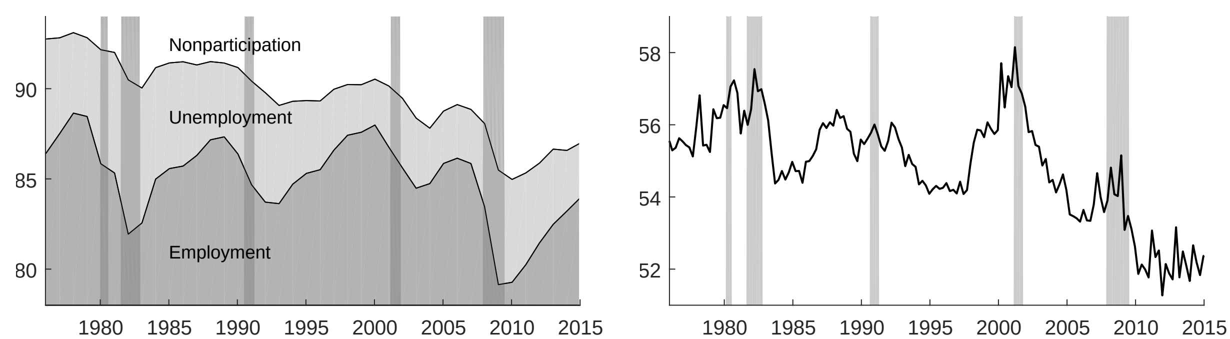
1 Introduction

Employment and compensation exhibit important asymmetries at business cycle frequencies and in the medium run. Figure 1, Panel A shows that employment falls quickly around the onset of a contraction and recovers more slowly, with the "jobless" recoveries following the 1991, 2001, and 2008 contractions being the most severe examples. Panel A also shows that participation in the labor market contributes to this phenomenon. In each contraction since 1975, participation falls and subsequently fails to recover to the pre-contraction level. Panel B shows that compensation of employees over value added spikes during contractions but levels off below pre-contraction levels during subsequent expansions. These "wageless" recoveries are most stark after 2000. The rapid fall in employment and spike of compensation over value added are indicative of sizeable wage rigidities.[1] Meanwhile, the persistent failure to recover pre-shock employment and rent sharing suggests hysteresis: that is, long-lasting scaring such that contractions bleed into a medium-run trend.

Figure 1: Asymmetries in the Labor Market (1975–2015)

Panel A: Participation and Employment | Panel B: Compensation over Value Added



Source: U.S. Census Bureau, Current Population Survey, Annual Social and Economic Supplement [http://doi.org/10.18128/D030.V4.0]. Following methodology of Juhn et al. (1991, 2002); Murphy and Topel (1997) and Elsby and Shapiro (2012), the sample includes healthy, non-institutionalized, civilian males age 25-55 who are not in school in order to abstract from demographic and policy shifts.

Source: U.S. Department of Labor, Bureau of Labor Statistics, Productivity and Costs, [https://www.bls.gov/lpc/]. Following Elsby et al. (2013) we report labor compensation as a share of value added in order to remain agnostic about the distribution of proprietors' income. Note, some of the run up in payroll share during the 2001 dot-com bust is a consequence of exercised stock options.

[1] Daly and Hobijn (2016) show only mild selection on the wages of workers who lose employment.

We model hysteresis as resulting endogenously from a strategic complementarity.[2] Firms' expectations over workers' labor force participation inform their wage setting strategy, while workers' expectations over wages inform their participation strategy. However, unlike the typical model of strategic complementarity, which is aimed mainly at micro-founding hysteresis, our model also features endogenous real rigidities. This arises because, in our case, strategic complementarity results in not just a discrete set of equilibria but rather a *continuum* of equilibria. This implies that there exists an interval of productivity such that fluctuations within this interval are compatible with unchanged participation and wages. As a result, the mechanism we propose can propagate shocks as well as amplify them.

The strategic complementarity can be understood in a stylized two-player representation of the labor market. One worker and one firm can match and produce. The worker has an outside option with a value unknown to the firm but drawn from a known distribution and chooses a search strategy based on her expectation of the wage to be offered by the firm. The worker may decide not to participate and drop out of the game before matching if her expectation for the wage offer is below her reservation wage. Meanwhile, the firm possesses a linear production technology and chooses and commits to a posted wage offer. In posting a take-it or leave-it offer the firm exerts monopsony power. As such, the firm takes into account that a higher wage makes hiring more likely but lowers the profit from production. The firm's choice depends not only on the distribution of possible worker types but also on the worker's participation behavior, which in turn depends on the wage expected by the worker. As a result of the worker's participation strategy, the labor supply curve that the firm expects to face is kinked: the marginal reduction in the probability of hiring from reducing the wage offer below the level expected by the worker discretely exceeds the marginal gain in the probability of hiring from increasing the wage above the expected level since the worker with higher values of leisure may not be searching. Strategic complementarity results since increasing the expected wage leads to higher participation and higher participation lowers the

[2]For the canonical treatment of hysteresis see Blanchard and Summers (1986, 1987).

marginal cost of high wage offers. Thus, many expected wage levels may be self-confirming.

We show the existence of an *interval* of wage and participation pairs which constitute a *continuum* of rational expectations equilibria. The lower boundary is pinned down as the minimum participation threshold such that marginal cost of hiring when approaching the kink in the labor supply curve (induced by the worker's threshold) from below is equal to the marginal revenue. Meanwhile, the upper bound is pinned down by the same calculation taken from above the kink. These equilibria can be welfare ranked, with higher-wage, higher-participation equilibria dominating lower-wage, lower-participation equilibria.[3]

We consider the implications of multiplicity when the model is hit with productivity shocks under the plausible assumption that no player deviates from an existing wage and participation pair unless unilateral deviation is a best response for at least one player. Real rigidity is then simply a corollary of the existence of a continuous set of equilibria. Further, we show that whenever a productivity shock *does* induce a unilateral deviation one can trace out a sequence of simultaneous best responses–a *Cournot tâtonnement*–that converges to a unique equilibrium in the set of equilibria consistent with the new productivity level.[4] Thus, equilibria are learnable in the sense that rational and forward looking agents could deduce the new equilibrium from knowledge of the shock and pre-shock equilibrium.[5]

The kink induced in the labor supply curve by the worker's participation strategy results in upward wage revisions that fall farther from the constrained efficient wage level than

[3] A side result of this observation is that a social planner can potentially "prime the pump" and induce coordination on a welfare dominant equilibrium, for example by an appropriately selected minimum wage.

[4] Specifically, our game shares in common many features of a supermodular game as detailed in Vives (1990, 2005) and Cooper (1994). In particular, the game contains enough structure that a *Cournot tâtonnement* emanating from any point that lies on at least one players best response function will converge to an equilibrium. Further, we will show that the best response of the worker is independent of aggregate productivity shocks. Thus, after all shocks we achieve convergence.

[5] There are alternative approaches to the exceptional element of games with strategic complementarities. One can construct business cycles due to "animal spirits" or "sunspots" in which players spontaneously coordinate expectations on higher or lower output equilibria (Cass and Shell, 1983). Alternatively, one can refine the equilibria prediction via "global games" and perhaps can harness the strategic complementarity to induce amplification (Morris and Shin, 2001). Also alternatively, one may suppose that agents always optimistically coordinate on the highest equilibrium (Krugman, 1991). Since our aim is explicitly to generate hysteresis, we eschew these approaches in favor of the plausible supposition that agents do not deviate from an existing equilibrium unless unilateral deviation is the best response of some player.

do downward revisions. This implies that transient productivity shocks lead to persistent decreases in output, wages, and participation. Further, in order to recover a particular pre-contraction wage and participation level, the subsequent expansion must overshoot the pre-contraction productivity level. Contractions induce hysteresis.

In order to provide microfoundations for the stylized two-player representation, we embed the model in a labor market which features random search and close our model with free entry into vacancy creation and a constant returns to scale matching function.[6] This also provides a mapping between wage, participation, and productivity level to the unemployment rate.

We show that when some workers with high flow values of leisure search for work (but ultimately do not accept the equilibrium wage offer) the effect is congestion in the matching function.[7] Further, changes in participation induced by shocks lead to changes in congestion since the set of workers whose value of leisure exceeds the equilibrium wage swells when shocks induce negative wage revisions. Since, as we have discussed above, negative wage revisions are persistent, the increased congestion persists beyond the duration of the disturbance and can result in "jobless" recoveries. Also, as we have also already noted, recovery of the pre-contraction wage level, and thus the pre-contraction level of congestion, requires that productivity over-shoots the pre-shock level. Thus, unemployment *must* fall below pre-shock levels before pre-shock wages are recovered, resulting also in "wageless" recoveries.

[6]The supposition of a frictional labor market endows the firm with market power in wage setting (monopsony) even for arbitrarily small market frictions (Diamond, 1971). Assuming massless agents and random search renders each worker (resp. firm) unable to affect the average participation threshold (resp. wage) through unilateral deviation. This supports modeling these aggregates as exogenous to each decision maker's problem even when expectations are informed by existing aggregate values as we shall consider. We close the model with the typical free-entry condition into vacancy posting. This provides a map from wages and participation levels to market tightness and unemployment rates. Free entry and the endogenously degenerate wage distribution also support the assumption that players have no continuation values.

[7]On the other hand, as we show in the two-player representation, when a higher proportion of workers with high flow value of leisure search the kink in the labor supply curve is less severe and recovery from contractions is more symmetric.

Related Literature

Our model brings together two branches of the modern Keynesian literature. The first follows Keynes' suggestion that economic fortunes might be governed by "animal spirits." This literature microfounds such economic fluctuations, at least in part, on strategic complementarity, multiplicity, and the potential for coordination on higher or lower output equilibria either spontaneously or under the direction of a social planner. Canonical examples include Heller (1986); Kiyotaki (1988); and Diamond (1982).[8] Recently, Eeckhout and Lindenlaub (2015) have brought these techniques to bear on the issue of jobless recoveries; however, their paper is silent on the issue of wages. Our contribution also generates a degree of cyclical wage rigidity and thus relates to a second branch of the modern Keynesian literature which seeks to harness real rigidities in order to amplify and propagate (nominal) disturbances.

Mechanically, our firm's problem resembles the kinked demand curve theory of real price rigidity. This theory originated in the Industrial Organization literature with Sweezy (1939) and Hall and Hitch (1939) and has received attention in macroeconomics, due in large part to Kimball (1995), in the context of goods market rigidities. The innovations contained in this work are twofold. First, the empirically plausible size of real rigidities in the goods market provide insufficient amplification and propagation to match business cycle facts (Ball and Romer, 1990; Klenow and Willis, 2016). In contrast, plausible rigidities in the labor market are large. By switching the focus from product demand curves to labor supply curves, we open the possibility to harness the larger rigidity. Second, while the kinked demand curve is heuristically appealing, it is difficult to build a consistent equilibrium microfoundation. To these authors' knowledge, no satisfactory equilibrium microfoundation has yet been published.[9] Strategic labor force participation provides perhaps the first satisfactory equilibrium microfoundation of a kink.

Although it is not a focus of our paper, endogenous wage stickiness due to our coordi-

[8] See Cooper and John (1988) and Solow (1998) for discussions.

[9] The often cited work, Woglom (1982), does not formally model the consumers' problem. Dupraz (2016) provides the most complete microfouncation of kinked product demand of which these authors are aware.

nation failure also speaks to the Shimer (2005) puzzle: the observation that the Diamond-Mortensen-Pissaredes (DMP) model of a frictional labor market fails to produces sufficiently volatile unemployment. As such, it relates to a larger literature that seeks to induce amplification through wage rigidity, for example Hall (2005), Gertler and Trigari (2009), and Kennan (2010). We differ from this literature in our focus on producing hysteresis as opposed to only amplification. A strict focus on amplification will typically yield rebounds of the unemployment rate following severe shocks that are *steeper* than the contractions that precede them. Consider a transient shock severe enough to yield a downward wage revision. Since this new wage is sticky, the value of a vacancy after the shock has dissipated must be *larger* than it was initially. This implies an greater abundance of vacancies and a lower unemployment rate post-shock than pre-shock. Without cyclical variation in congestion, a rigid wage theory therefore implies "jobfull" recoveries. Instead, the data show slow recoveries of unemployment and *outward* shifts in the Beveridge curve. Our model can generate both "jobless" recoveries, in which unemployment remains persistently high even after labor productivity has recovered, and "wageless" recoveries, in which unemployment falls to unprecedented lows before wages recover.

2 A Stylized Two Player Game

In this section, we consider a stylized single-shot, one-worker, one-firm labor market and show that there exist a continuum of rational expectations equilibria. Computing player's best response functions requires positing a fixed expectation for the firm's posted wage on the worker's behalf and for the worker's participation strategy on the firm's behalf. We call the worker's expectation of the wage w_0. We will see that we can summarize the worker's strategy as a participation threshold and we call the firm's expectation of the threshold r_0. Thus, w_0 (resp. r_0) is the *belief* that the worker (resp. firm) holds about the firm's (resp. worker's) wage offer (resp. threshold). Worker and firm maximize income and profit.

2.1 Worker

For the worker, the game proceeds in two stages. In the first stage, the worker draws a value of leisure, b, from a known distribution, $H(b)$, with density $h(b)$, defined on $[\underline{b}, \bar{b}]$.[10] In the second stage, the worker may costlessly seek to contact the firm.[11] If she does not seek to contact the firm, we call this nonparticipation. If she does seek to contact the firm, she has a $(1-u)$ probability of making contact and being made a wage offer. If the worker accepts this wage offer we call her employed. If she rejects we call her voluntarily unemployed. Finally, in the case that there is no contact between worker and firm we call her involuntarily unemployed. We will see that the decision to participate will depend on her expectation of the wage offer, w_0.

As is typical for such problems, the worker's strategy takes the form of threshold rules: a reservation wage and a threshold for labor force participation. Since search is costless, the reservation wage is equal to the value of leisure. Let $V^W(r, w_0)$ be the expected payoff to a worker of choosing threshold r when the expected wage choice of the firm is w_0. Observe that there are two cases to characterize this value function:

$$V^W(r, w_0) = \begin{cases} \overbrace{(1-u)w_0 H(r)}^{\textit{employment}} + \overbrace{u \int_{\underline{b}}^{r} bh(b)db}^{\substack{\text{involuntary} \\ \text{unemployment}}} + \overbrace{\int_{r}^{\bar{b}} bh(b)db}^{\substack{\text{non-} \\ \text{participation}}}. & \text{if } r < w_0 \\ \underbrace{(1-u)w_0 H(w_0)}_{\textit{employment}} + \underbrace{u \int_{\underline{b}}^{w_0} bh(b)db}_{\substack{\text{involuntary} \\ \text{unemployment}}} + \underbrace{\int_{w_0}^{r} bh(b)db}_{\substack{\text{voluntary} \\ \text{unemployment}}} + \underbrace{\int_{r}^{\bar{b}} bh(b)db}_{\substack{\text{non-} \\ \text{participation}}}. & \text{if } r \geq w_0 \end{cases}$$

[10] We do not rule out $\underline{b} = -\infty$ or $\bar{b} = \infty$.

[11] Under costless search the reservation wage and threshold participation level coincide. We will see when we turn to the firm's problem that this coincidence is essential for multiplicity. Costly search results in a unique equilibrium with less participation than the minimal participation consistent with the set of equilibria we recover under costless search. Costly non-participation results in a unique equilibrium with participation exceeding the maximal participation consistent with the set of equilibria we recover under costless search. We note that evidence on job search behavior does not support substantial cost–see Mukoyama et al. (2016).

Differentiating, it follows that $\frac{dV^W(r,w_0)}{dr} > 0$ whenever $r < w_0$. In this case, the worker can improve her payoff by increasing her participation threshold. That is, a low threshold causes the worker not to seek work in some cases when there is a positive probability of receiving an acceptable employment offer. One can also see that $\frac{dV^W(r,w_0)}{dr} = 0$ whenever $r \geq w_0$: in this case, the payoffs from increasing the participation threshold are nil. In the marginal case, the worker obtains the same value–the value of leisure–in voluntary unemployment as she does in nonparticipation .

In the region of indifference ($r \geq w_0$) we posit a mixed strategy over the pure strategies "voluntary unemployment" and "nonparticipation":

Assumption 1. *When the worker expects to be indifferent between voluntary unemployment and nonparticipation she randomises between the two states with probability i placed on voluntary unemployment.*

Under Assumption 1, the workers best response, $r^*(w_0)$, is to set a threshold:

$$r^*(w_0) = w_0, \tag{1}$$

such that she participates with probability one if $b < r^*(w_0)$ and with probability $i < 1$ if $b > r^*(w_0)$.[12]

Lemma 1. *This game exhibits* ***positive spillovers*** *and* ***strategic complementarities*** *for the worker.*

Proof. An increase in the posted wage–which we will see is the strategy of the firm–increases the payoff for the worker regardless of worker's participation threshold strategy (positive spillovers). Also, an increase in the posted wage–which we will see is the strategy of the

[12]It is also possible that the worker adopts a more complex randomization strategy. In particular one might posit that the weight placed on "voluntary unemployment" might depend on the realization of b. However, any randomization such that the probability of nonparticipation exceeds zero as the realized value of leisure approaches the expected wage level from above will produce a kink in the labor supply curve and thus will yield qualitatively similar results regarding rigidity and propagation. Further, if $i \to 1$ as $b \searrow w_0$ then we have a "smoothed" kink, similar to Kimball (1995), and we will obtain results regarding prorogation.

firm–increases the optimal participation threshold strategy of the worker (strategic complementarity).[13] □

2.2 Firm

The firm posts a wage offer, w, ex-ante and may meet the worker with probability $(1-v)$. If the firm successfully hires the worker it will earn rent $p-w$, where p is the output of the match. If the firm and the worker do not meet or if the worker rejects the wage offer then the payoff to the firm is zero.

Let $V^F(w, r_0)$ be the payoff to the firm of posting wage w when the firm's expectation for the worker's threshold for participation is r_0. We can write the firm's value function:

$$V^F(w,r_0) = \underbrace{(1-v)}_{\substack{\text{match}\\ \text{probability}}} \underbrace{(p-w)}_{\substack{\text{revenue}\\ \text{from}\\ \text{successful}\\ \text{hiring}}} \times \underbrace{\begin{cases} \frac{H(w)}{H(r_0)+i[1-H(r_0)]} & \text{if } w < r_0 \\ \frac{H(r_0)+i[H(w)-H(r_0)]}{H(r_0)+i[1-H(r_0)]} & \text{if } w \geq r_0 \end{cases}}_{\text{expected labor supply schedule}}$$

The first term, $(1-v)$, and the second term, $(p-w)$, are the probability of meeting the worker and the payoff from successfully hiring. The third term contains the core of the problem, encoding the firm's expectation of the labor supply schedule it faces. We can interpret the probability that a wage offer of w will be accepted as the labor supply curve faced by the firm. Notice that the participation threshold introduces *a kink in the expected labor supply* at the expected threshold. To the left of the threshold, where $w < r_0$, the expected probability that the worker participates is one. Thus, posting a wage less than or equal to the expected labor force participation threshold results in hiring a worker with probability $\frac{H(w)}{H(r_0)+i(1-H(r_0))}$. To the right of the threshold, the expected probability that the worker participates is only

[13] Formally, define *positive spillovers* for player j as the case in which an increase in the other player's strategy increases the payoff to player j (Cooper and John (1988)). Now simply note that $\frac{dV^W(r,w)}{dw} = (1-u)H(r) > 0$ if $r < w$ and $\frac{dV^W(r,w)}{dw} = (1-u)H(w) + uwh(w) > 0$ if $r > w$. Also, define *strategic complementarities* for player j as the case when an increase in the other player's strategy increases the best response of player j (Cooper and John (1988)). This trivially follows from expression (1), that is $\frac{dr^*(w)}{dw} = 1 > 0$.

$i < 1$. Thus, posting a wage greater than the expected labor force participation threshold results in hiring a worker with probability $\frac{H(r_0)+i[H(w)-H(r_0)]}{H(r_0)+i(1-H(r_0))}$.

The firm's best response given a particular expectation for the worker's strategy is to post a wage that satisfies:

$$w^*(r_0) = \arg\max_w \left\{ V^F(w, r_0) \right\}.$$

The problem takes the form of monopsony wage setting. The firm's expectation over the worker's participation threshold introduces a kink in the expected labor supply schedule. As a result the best response function–the optimal wage posting strategy of the firm–is defined piecewise on intervals of the expectation for the worker's participation threshold. Our task is to determine these intervals and the optimal posted wage schedule within each of them. We distinguish three cases.

Interior solution such that $w < r_0$

Suppose that the firm expects that the participation threshold is high enough that it is non-binding. In this case $V^F(w, r_0)$ simplifies to $(1-v)(p-w)\frac{H(w)}{H(r_0))+i(1-H(r_0))}$ and the firm's best response satisfies:

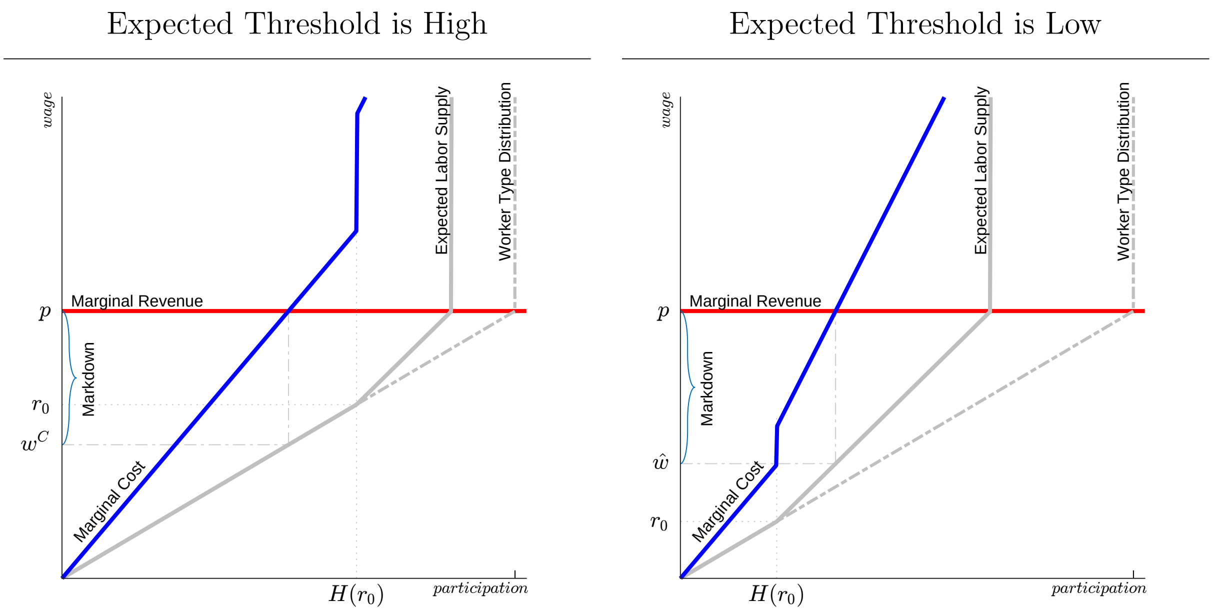
$$w^C = \arg\max_w \left\{ H(w)(p-w) \right\},$$

noting that $(1-v)$ and $(H(r_0) + i[1 - H(r_0)])$ are exogenous to the firm. The first order condition can be written as follows:[14]

$$\underbrace{p}_{\substack{\text{marginal}\\ \text{revenue}}} = \underbrace{w^C \left[1 + \frac{H(w^C)}{w^C h(w^C)} \right]}_{\substack{\text{marginal}\\ \text{cost}}} \tag{2}$$

[14]A sufficient condition such that w^C is the unique maximizer is given by: $\frac{1}{p-b} > \frac{d^2H(b)}{db^2}\frac{1}{2h(b)}$. This states that the distribution $H(b)$ is not more convex than the hyperbola defined by the firms iso-profit curves: $\frac{1}{p-b}$. This is trivially satisfied whenever the density is weakly decreasing – for example, the uniform and Pareto distributions as well as by the normal and logistic distributions when at least half of workers participate in the labor force. The remainder of this paper focuses on the case where the second order condition holds.

Figure 2: Optimal Wage Choice at Interior Solutions.



Note: The left panel illustrates an expected threshold such that the firm finds an unconstrained interior solution. The right panel illustrates an expected threshold such that the firm finds a constrained interior solution. The location of the kink in the expected labor supply curve depends on the firm's expectation for the worker's participation threshold, r_0. The angle of rotation and the associated jump in the marginal cost curve depend on the probability that the worker searches when her value of leisure exceeds this threshold, i. Fore ease of illustration we take $H(b)$ to be uniformly distributed on $[\underline{b}, \bar{b}]$.

The left hand side is the marginal revenue from an employee. The right hand side is the marginal cost of hiring the worker in all the cases in which she will accept wages no less than w^C.

The wage choice, w_C, is illustrated in the left panel of Figure 2. The x-axis plots the probability that a worker will accept a wage offer of w. From the firm's perspective, this is the labor supply schedule. The y-axis plots wages. We plot both the underlying distribution of worker types and the labor supply schedule that the firm expects to face given the worker's participation threshold in gray hashed and solid respectively. Note that the expected labor supply schedule is rotated counterclockwise relative to the underlying distribution of worker types around the probability that the worker is type r_0 or less, $H(r_0)$, creating a kink in the labor supply curve. The angle of rotation depends on the probability, i, that the worker searches when her value of leisure exceeds this threshold, i.

We also plot the marginal revenue and marginal cost faced by the firm. As in the typical monopsony problem, we find the optimal quantity of labor demanded, $H(w^C)$, at the intersection between marginal revenue and marginal cost. Since we have assumed that the firm is unconstrained by the worker's expected participation choice, the kink in the expected labor supply curve falls to the right of the intersection of marginal cost and marginal revenue. The wage and markdown are determined as in a normal monopsony diagram.

Interior solution such that $r_0 < w$

Suppose that the firm expects that the participation threshold is low enough that it is binding. In this case:

$$\hat{w} = \arg\max_{w} \left\{ [H(r_0) + i[H(w) - H(r_0)]](p - w) \right\},$$

and the first order condition is given by:

$$\underbrace{p}_{\substack{\text{marginal}\\\text{revenue}}} = \underbrace{\hat{w}\left[1 + \frac{H(r_0) + i[H(\hat{w}) - H(r_0)]}{ih(\hat{w})\hat{w}}\right]}_{\substack{\text{marginal}\\\text{cost}}} \tag{3}$$

Again, the left hand side is the marginal revenue from an employee. The right hand side is the marginal cost of hiring the worker in all the cases in which she will accept wages no less than $\hat{w}$.

The wage choice, $\hat{w}$, is illustrated in the right panel of Figure 2. Now we are considering that the firm's labor demand is greater than $H(r_0)$ and thus that the kink induced by the rotation of the expected labor supply schedule relative to the underlying distribution of worker types is binding. We again plot the marginal revenue and marginal cost faced by the firm. At the quantity of labor supplied at the expected participation threshold, $H(r_0)$, the marginal cost jumps due to the kink in the expected labor supply curve. Again, we find the optimal quantity of labor demanded, $H(r_0) + i[H(\hat{w}) - H(r_0)]$, at the intersection between marginal revenue and marginal cost.

Corner solution

Suppose that the constraint imposed by the expected participation threshold is binding and it induces a corner solution (i.e., $w = r_0$). This occurs if the marginal revenue strictly exceeds the marginal cost when approaching the expected kink from below while the marginal cost strictly exceeds the marginal revenue when approaching from above:[15]

$$p > r_0 + \frac{H(r_0)}{h(r_0)} \quad \text{and} \quad p < r_0 + \frac{H(r_0)}{ih(r_0)}.$$

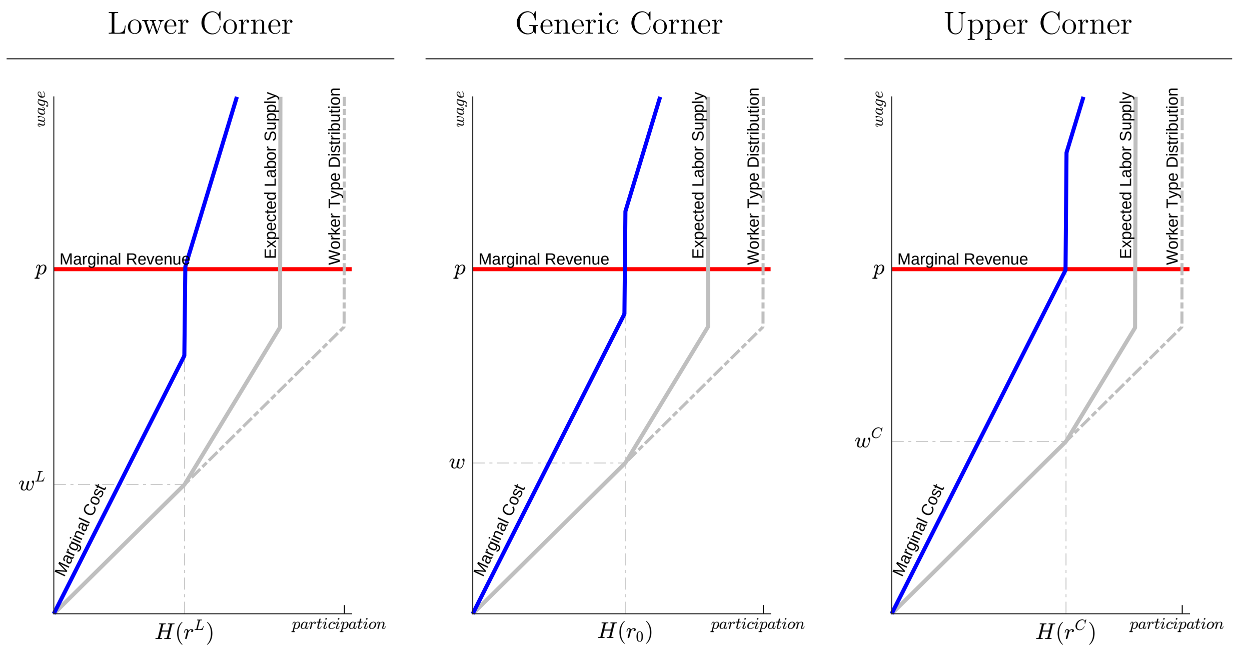
What remains is to find the lowest expected participation threshold such that the firm prefers to hire weakly more than $H(r_0)$. In other words, the lowest participation threshold, r^L, for which the corner solution is consistent must satisfy:

$$p = r^L + \frac{H(r^L)}{ih(r^L)}.$$

Figure 3 plots the range of participation thresholds for which the corner solution is optimal for the firm. The left panel plots the *smallest* threshold such that quantity of labor demanded coincides with the quantity supplied at the expected participation threshold:

[15] Formally, taking the derivative from the left, $\lim_{w \nearrow r_0} \frac{dV^F(w,r_0)}{dw}$, and from the right, $\lim_{w \searrow r_0} \frac{dV^F(w,r_0)}{dw}$, respectively. A particularly useful form of representing the firm's best response is that the wage is a markdown from the marginal cost of labor, the markdown is given by the elasticity of the expected labor supply faced by the firm. That is, expression (2) can be written as $\frac{p-w^C}{w^C} = \frac{H(w^C)}{w^C h(w^C)} = \frac{1}{\eta^1}$, where η^1 is the elasticity of the expected labor supply curve in the region below the firm's expectation for the worker's participation threshold. Notice that, $\eta^1 = \eta$, where $\eta = \frac{dH}{dw}\frac{w}{H}$ denotes the elasticity of labor supply with respect to wages would pertain if the worker participates for all values of b. Manipulating equation (3) one can show that, hiring in this side of the market, the monopsony markdown is equal to $\frac{p-\hat{w}}{\hat{w}} = \frac{H(r_0)+i[H(\hat{w})-H(r_0)]}{ih(\hat{w})\hat{w}} = \frac{1}{\eta^i}$, where η^i is the elasticity of the expected labor supply curve above the kink. Note that $\eta^i = \eta \frac{H(w)i}{H(w)i+H(r_0)(1-i)}$. In other words, the markdown just above the kink market exceeds the markdown that the firm would choose if it were not constrained by the expected participation threshold. Notice also that, when $i < 1$, the elasticity of the expected labor supply with respect to the wage at the expected threshold is discontinuous: $\eta^i(r_0^+) < \eta^1(r_0^-)$ and there is a jump in the marginal costs at the kink stemming from that discontinuity. For a given r_0 the size of the jump in the marginal cost depends on i, fraction of the time that the worker draws $b \geq w_0$ and searches for work: when the worker always searches then the marginal cost is smooth; and when all workers with $b \geq w_0$ non-participate, the marginal cost of hiring greater than $H(r_0)$ is infinite.

Figure 3: Optimal Wage Choice at Corner Solutions.



Note: The left panel illustrates the *smallest* threshold such that the firm prefers to hire $H(r_0)$. The right panel illustrates the *largest* threshold for which the firm weakly prefers to hire $H(r_0)$. The center panel illustrates a generic corner solution.

$r_0 = r^L = w^L$. The right panel plots the *largest* threshold such that quantity of labor demanded coincides with the quantity supplied at the expected participation threshold: $r_0 = r^C = w^C$. The center panel plots a generic corner solution when $r^L < r_0 < r^H$ and strictly prefers to more than $H(r_0 - \varepsilon)$ and strictly less than $H(r_0 + \varepsilon)$ for arbitrarily small ε.

We can now write the firm's wage best response (i.e., wage posting strategy) as follows:

$$w^*(w_0) = \begin{cases} \hat{w} & \text{if } r_0 < r^L \\ r_0 & \text{if } r_0 \in [r^L, r^C] \\ w^C & \text{if } r_0 > r^C. \end{cases} \tag{4}$$

Lemma 2. *For all expected participation thresholds, r_0, in the interval $[r^L, r^C]$ the game exhibits **positive spillovers** and **strategic complementarities** for the firm.*

Proof. Whenever the marginal searching worker would accept the posted wage offer an increase in participation increases the pay off to the firm (positive spillovers). Moreover, whenever $r \in [r^L, r^C]$ an increase in the participation threshold increases the optimal posted wage strategy of the firm (strategic complementarity).[16] □

2.3 Rational Expectations Equilibria

Definition 1. *A **rational expectation equilibrium** of the two-player game is a pair–wage and participation threshold–such that wage and participation threshold are mutual best responses.*

In other words, in any rational expectations equilibrium both worker's and firm's expectations must be self-confirming: $r^* = r_0$ and $w^* = w_0$.

Proposition 1. *For $i < 1$, a **continuum of equilibria** exists on the interval $[w^L, w^C]$, with higher welfare for higher wage levels.*

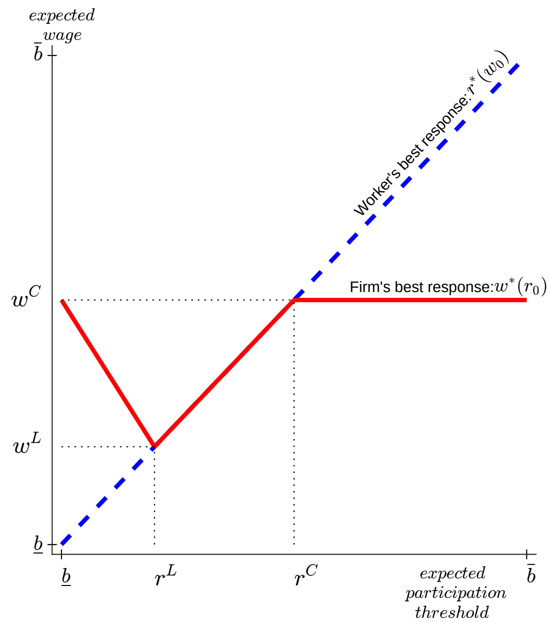
Proof. Suppose the worker expects wages to be w_0. Her best response is to set a participation threshold such that $r^* = w_0$. If this expectation is consistent with an equilibrium then it must be the case that the firm's best response is to confirm the expectation by setting a low wage such that $w^* = r^* = w_0$. This will be consistent with the firm's strategy as long as $w_0 \in [w^L, w^C]$.[17] Since we have already shown that both the firm and worker face positive spillovers and strategic complementarities on this interval the welfare ranking result is a straightforward application of Lemmas 1 and 2 and Cooper and John (1988) Proposition 5. □

Figure 4 illustrates the best response function of the worker (hashed) and the best response of the firm (solid). These are mutual best responses on the 45 degree line from

[16]Formally, $\frac{dV^F(w,r)}{dr} = (1-v)(p-w)\frac{h(r)(1-i)(i(1-H(w)])}{[H(r)+i(1-H(r)]^2} > 0$ whenever $w \geq r_0$ and $\frac{dw^*(r)}{dr} = 1 > 0$ whenever $r \in [r^L, r^C]$.

[17]It is easy to construct an analogous example illustrating that (off equilibrium) expectations above w^C or below w^L are not confirmed.

Figure 4: Mutual Best Responses.



Note: The Firm's and Worker's strategies are mutual best responses for an interval of wage and participation thresholds on the on the 45 degree line from (r^L, w^L) to (r^C, w^C).

(r^L, w^L) to (r^C, w^C). Note that as i approaches 1, w^L approaches w^C and the equilibrium set collapses to a unique equilibrium. This equilibrium is constrained efficient and coincides with the equilibrium found in Diamond (1971).

Robustness

Note that if there is non-zero probability that wage offer exceeds the worker's expectation then the worker's threshold must be raised to exceed the support of the distribution of possible wage offers. At this point, one might be tempted to impose an equilibrium refinement with a "trembling hand" flavor as in Selten (1975). If there is any chance that the firm's hand "trembles" and it may make a mistake in some off equilibrium path and post a high wage, such a consideration could be used to rule out all equilibria other than the constrained efficient. This consideration is a valid criticism of our two-player game. However, we have devised this simply to illustrate clearly the minimal assumptions that drive results in our model. In the full labor market that we present in Section 4 one can see that such a refinement is less plausible as it would require not just that a single firm's hand "trembles" but that a discrete mass of firms all simultaneously mistakenly play an off-equilibrium wage. We suggest that such a deviation from equilibrium is too implausible to impose as a refinement.

Animal Spirits

The stylized two player representation of the labor market can be used to generate endogenous business cycles. Consider that the worker and firm may experience a spontaneously and coordinated change in expectations–for instance due to a sunspot–such that the equilibrium shifts from a low wage to a high wage. From Proposition 1, we can infer that this shift will be attended by an increase in employment and an increase in output. Thus we can generate business cycles motivated purely by the "animal spirits" of market participants. However, we find such cycles unsatisfying. In particular, such spontaneity has no dependence on history and thus can not explain why contraction is quick relative to expansion. We prefer instead to investigate the potential that there may be a degree of rigidity in wages and participation with respect to productivity shocks and that productivity shocks that are large enough to induce wage and participation revisions may lead to long-lasting changes in expectations.[18]

Priming the Pump

A straightforward implication of Proposition 1 is that a policy maker empowered with either the power to set a minimum wage or require participation could increase welfare by appropriately levering her instrument. While welfare improving policies are interesting to consider, we focus our paper instead on the positive implications of the existence of these multiple equilibria. We turn now to demonstrating that the two player game exhibits rigidity and hysteresis.

[18] Further, when we embed the model in a two-sided frictional labor market in Section 4 we will see that without further assumptions cycles driven by sunspots may produce unemployment dynamics that are contrary to the data. Meanwhile, expectation shifts driven by realized productivity shocks are consistent with both the "jobless" and "wageless" phases of recovery observed in the data.

3 Productivity Shocks, Rigidity, and Hysteresis

We have established the existence of a continuum of equilibria. We now consider the implications in the context of productivity shocks. In particular we compare equilibria before and after the arrival of shocks. Since the two player game considered both before and after the shock is a single shot game there is no need to distinguish between expected and unexpected shocks.[19]

3.1 Endogenous Rigidity

Assumption 2. *No player deviates from an existing wage and participation threshold pair unless unilateral deviation is the best response.*

Under this plausible assumption, the model predicts that there is a range of values both for labor productivity over which wages and participation are endogenously ridged.

Proposition 2. *Given an initial wage and participation threshold, $\{w_0, r_0\}$, these values are* ***endogenously rigid*** *for a range of labor productivity, $[p^L, p^H]$, where*

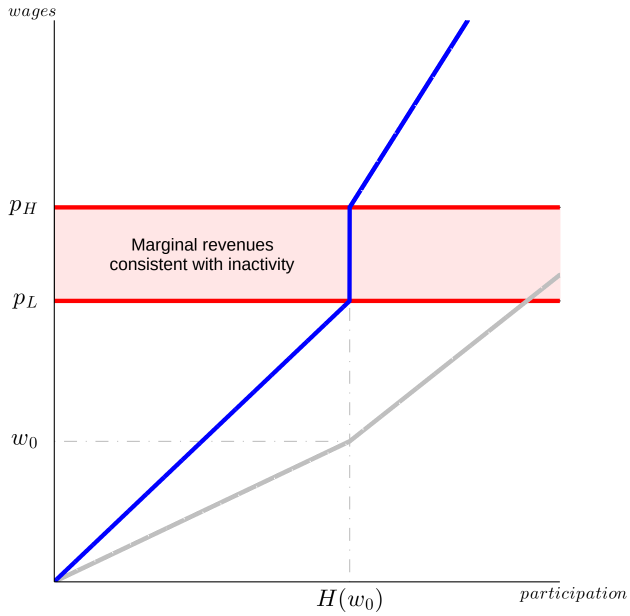
$$p^L(w_0) = w_0 + \frac{H(w_0)}{h(w_0)} \qquad\qquad p^H(w_0) = w_0 + \frac{H(w_0)}{ih(w_0)}.$$

Proof. The proof follows from considering the first order conditions of the problem of a firm and worker. In this interval, no player has an incentive to unilaterally deviate. This is essentially a corollary to the existence of the interval of equilibria on $[w^L, w^C]$ for any given productivity level. See Proposition 1. $\square$

For $i \in (0, 1)$ this range of inactivity is depicted in Figure 5. The figure presents firm's monopsony problem constrained by the participation threshold as in Figures 2 and 3. For productivity in $[p^L, p^H]$, we see that the intersection between marginal cost and marginal revenue is at $H(w_0)$ and consistent with wage w_0 and participation threshold r_0.

[19]When we consider a frictional labor market in Section 4 we will discuss the extent to which the expected distribution of shocks is orthogonal to player's strategies.

Figure 5: Endogenous Rigidity.



Note: The firm's monopsony problem constrained by the participation threshold as in Figures 2 and 3. The red shaded region highlights the range of productivities for which w_0 is in the set of equilibria.

Endogenous Rigidity and the Barro (1977) Critique

Barro (1977) suggests that any price rigidity that rules out a rent-producing trade is implausible. In addressing this criticising, it is useful to dissect rigidity in our model into ex-ante rigidity (before meeting the worker) and ex-post rigidity (after having met the worker). The firm in our model is free to choose any wage ex-ante (there are no menu costs). In this sense our rigidity is impervious to the Barro (1977) critique. Still, in equilibrium ex-ante posted wages are at times non-responsive to fundamentals due to coordination failure and, from Proportion 1, we know this has a welfare effect.

On the other hand, the firm in our model experiences ex-post total wage rigidity. We model wages this way in order to capture an informational asymmetry between worker and firm: the firm does not know and cannot learn the worker's value of leisure. A typical critique of wage posting, in the vein of the Barro (1977) critiques, is that in the sub-game in which a rent-producing match is ruled out the informed party should reveal her type and bargaining should commence.[20]

However, in the limit as i falls to nil, this suspect sub-game never occurs since the worker only searches if she will accept posted wage and all matches are consummated. Although ex-post wage rigidity never binds and ex-ante wages are flexible, wages are endogenously rigid on a vast region of values for labor productivity: for all realizations of productivity above p_L wages remain unchanged and for shocks that return p in this region wages are endogenously rigid. In other words, this limiting case eliminates the sub-game that is the source of criticism but leads to the *largest* possible range of the fundamental for which wages are rigid in our model.

This limiting case, however, provides results that are too strong for our purposes: subjected to shocks, an economy in which the worker always chooses non-participation when her value of leisure exceeds the expected wage will eventually converge to zero participation, zero employment, and zero output. We take an intermediate value of i, allowing $i \in (0, 1)$, in order to admit the possibility of macroeconomic recovery following contractions. When we have $i > 0$ we are *partially* subject to Barro's criticism in the sense that ex-post some rent producing matches are ruled out.[21] We turn now to analyzing equilibrium revisions for intermediate values of i.

3.2 Hysteresis

Assumption 2 imposes that the equilibrium wage-labor force participation pair changes only when productivity evolves in such a way that the existing pair is no longer in the set of possible equilibria. At such a point the pre-exisiting wage and participation pair constitute

[20]This criticism is valid for any model of monopolistic pricing: given the flexibility the monopolist would always prefer to price discriminate and price discrimination is more efficient than monopoly pricing.

[21]Of course, if negotiation is allowed in this sub-game, there will be cases when a mutually beneficial deal can be struck. Further, if such negotiation is permitted this will be known by workers and, as a result, beliefs consistent with inactivity will be difficult to support (at least at the threshold for participation predicted in the baseline model). One possible solution to this sub-game that preserves our results is to assume that workers can trigger bargaining if they wish. If they do their bargaining power is zero. The result is that workers with $b < r_0 = w_0$ will never trigger bargaining. Workers with $b \in (w_0, p]$ may, and if they do they will receive wages equal to their value of leisure: so, they won't care if they do or don't bargain.

a disequilibrium. Thus we require a protocol for equilibrium revelation given the observation of some disequilibrium.

We consider *Cournot tâtonnement* as a plausible equilibrium revelation protocol.

Definition 2. *A* Cournot tâtonnement *is a sequence indexed by* $k = 1, 2, ...$ *such that* $\{w_0, r_0\} \in [\underline{b}, \bar{b}]^2$*, and* $\{w_k, r_k\} = \{w^*(r_{k-1}), r^*(w_{k-1})\}$ *is the simultaneous best-response of the firm and worker at iteration* k *to the* $k-1$ *value of the wage and participation threshold respectively.*

The *Cournot tâtonnement* reveals the new equilibrium if there exists some k for which w_k and r_k are mutual best responses. Note, while the *Cournot tâtonnement* is iterative it is indexed by layers of rationality–e.g. how many best-responses to best-responses each player must compute before convergence. We are agnostic as to the relation between number of iterations required to archive convergence and the calendar time elapsed and consider instead serial equilibria in the following results regarding comparative statics. As already noted, shocks that result in $p \in (p^L(w_0), p^H(w_0))$ trigger no player to unilaterally deviate and the *Cournot tâtonnement* converges in the first iteration.

Lemma 3. *Given any shock to labor productivity,* p*, such that* $p > \underline{b}$*, the ensuing* Cournot tâtonnement *converges to an equilibrium. Further, positive (resp. negative) shocks lead to* weakly *positive (resp. negative) wage and threshold innovations.*

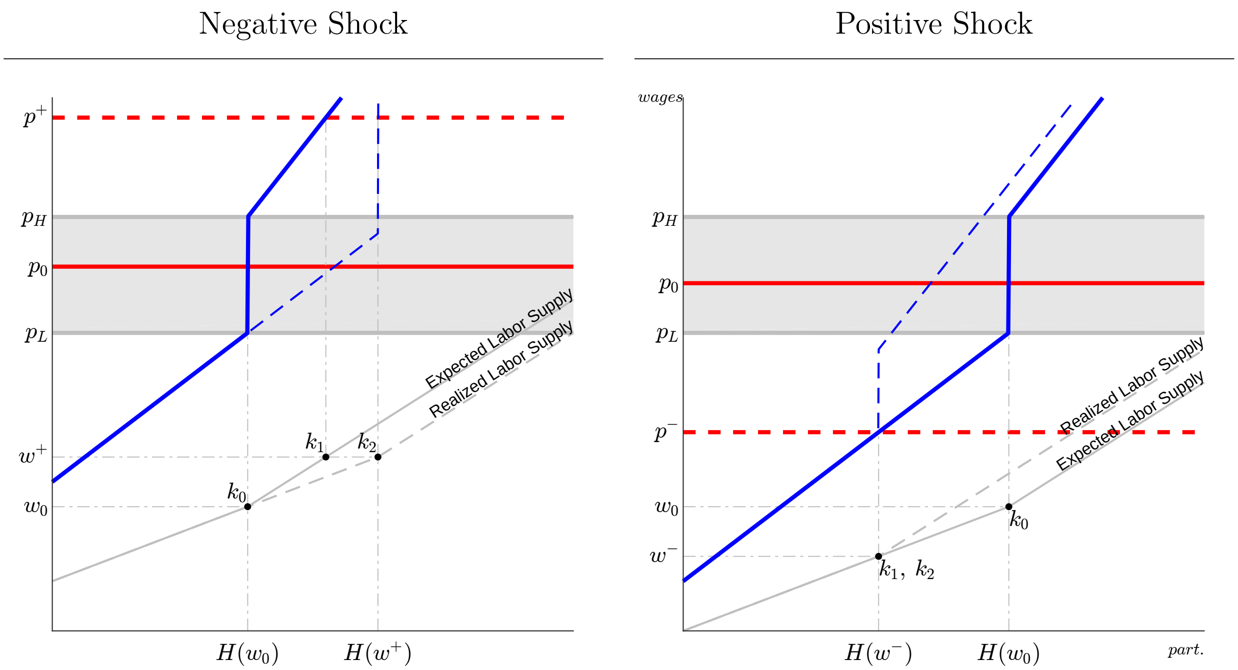
Proof. 1) The optimal posted wage conditional on a fixed expected participation threshold is weakly positively dependent on productivity, $\frac{dw^*(r_0,p)}{dp} \geq 0$. This enables the initial deviation from an existing equilibrium. 2) The optimal threshold conditional on a fixed expected wage is independent of productivity, $\frac{dr^*(w_0,p)}{dp} = 0$. Thus, following any shock even iterations fall on the worker's best response function while odd iterations fall on the firm's. 3) Posted wages exceed the threshold everywhere to the left of equilibria, $\hat{w}(r_0) > r_0$, posted wages fall short of the threshold everywhere to the right of equilibria, $w^C(r_0) < r_0$, and $r^*(w_k)$ is positive monotone. Thus, each iteration raises $r^*(w_k)$ if $w_k < w^L$ and lowers $r^*(w_k)$ if

$w_k > w^C$. 4) Finally, $w^*(r_k)$ is positive monotone for $r_k > r^C$. Thus, whenever $w_0 > w^C$ we have monotone convergence from above to w^C and whenever $w_0 < w^L$ we have either monotone convergence from below or there exists an iteration in which $w^*(r_k) > w^C$ after which we have monotone convergence from above. 5) Whenever the second order condition, $\frac{1}{p-b} > \frac{d^2H(b)}{db^2}\frac{1}{2h(b)}$, holds we have monotone convergence from below to $w^* \in [w^L, w^C]$ where w^C is only attainable if $r_0 = \underline{b}$. In other words efficiency is only attainable if the pre-shock market was in collapse. Note that this second order condition is identical to that required for the unconstrained monopsony's problem to have a unique solution. $\square$

The left panel of Figure 6 illustrates the firm's constrained monopsony problem in the case of a large negative productivity shock that renders $p^- < p^L$. The gray region indicates the interval of labor productivities consistent with a pre-shock equilibrium at wage w_0. The red line indicates a productivity preceding the shock, p_0. The red dashed line represents the productivity following the shock, p^-. The gray solid and dashed lines indicate the expected labor supply schedule on impact and the labor supply schedule that arises in the equilibrium following the shock, respectively. The blue solid and dashed lines plot the marginal cost curve on impact and in the post-shock equilibrium respectively. Prior to the shock the economy is in equilibrium at wage level w_0. We label this as k_0 indicating the initial conditions from whence a *Cournot tâtonnement* will commence. Notice that on impact of the shock the worker best responds to w_0 by maintaining $r^*(w_0) = w_0 = r_0$. Meanwhile, the firm is unconstrained by the existing expected participation threshold, r_0, and selects a new wage such that the first order condition of an unconstrained monopsonist, equation 2, is satisfied. The first iteration of the *Cournot tâtonnement* suggests k_1 as a candidate equilibrium. A second iteration confirms, now the worker best responds with $r^*(w^-) = w^-$ and the labor supply curve shifts to the realized labor supply curve. The firm best responds with $w^*(r_0) = w^-$. At this point the two are mutual best response and the economy has converged to a new equilibrium, we label this k_2.

The right panel of Figure 6 illustrates the firm's constrained monopsony problem the

Figure 6: Equilibrium Revelation Following Large Productivity Shocks.

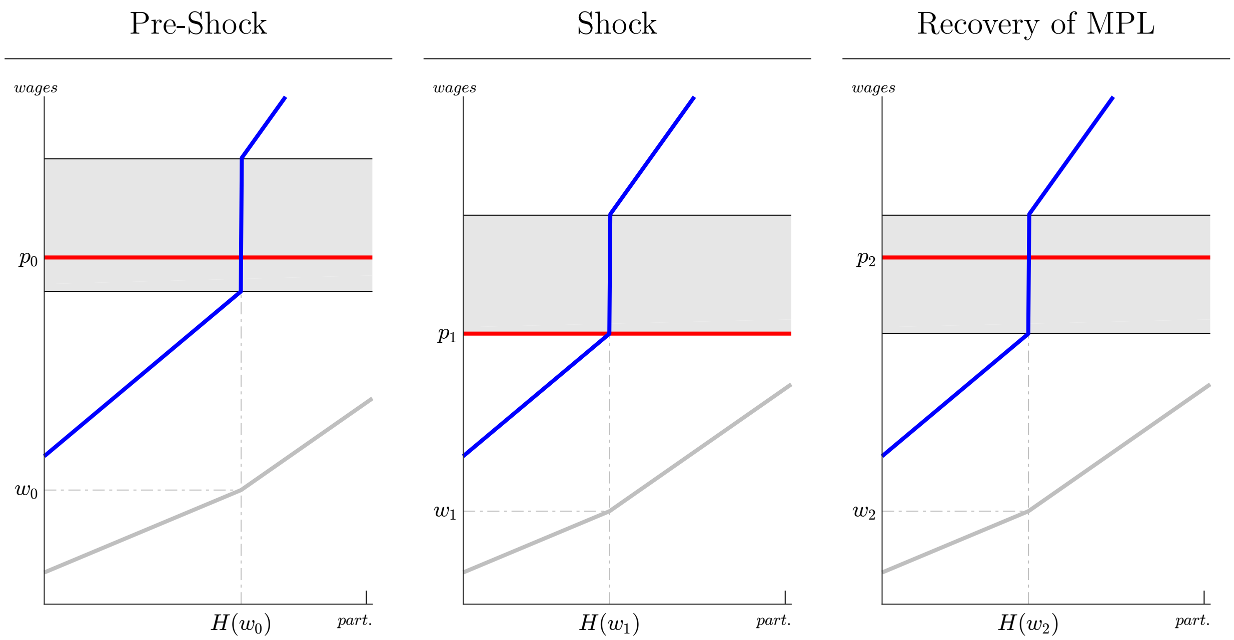


Note: The left panel illustrates a negative productivity shock and the right panel a positive productivity shock. Hashed lines indicate post-shock schedules. Iterations of *Cournot tâtonnement* induced by the productivity shock are indicated as the points labeled k_0, k_1, and k_2. As before, fore ease of illustration we take $H(b)$ to be uniformly distributed on $[\underline{b}, \bar{b}]$.

case of a large positive productivity shock that renders $p^+ > p^H$. Again, on impact of the shock the worker best responds to w_0 by maintaining $r^*(w_0) = w_0 = r_0$. Meanwhile, the firm *is constrained* by the existing expected participation threshold, r_0, and selects a new wage such that the first order condition of a constrained monopsonist, Equation 3, is satisfied. The first iteration of the *Cournot tâtonnement* suggests k_1 as a candidate equilibrium. In a second iteration the worker responds to the new proposed wage level with $r^*(w^+) = w^+$ and the labor supply curve shifts to the realized labor supply curve. The firm best responds with $w^*(r_0) = w^+$ since is wage level is now in $\{w^L(p^+), w^C(p^+)\}$. At this point the two are mutual best response and the economy has converged to a new equilibrium, we label this k_2.

Proposition 3. *A transient productivity shock leads to* ***persistent*** *changes in wages, participation.*

Figure 7: Persistent Response to Transitory Productivity Shock.

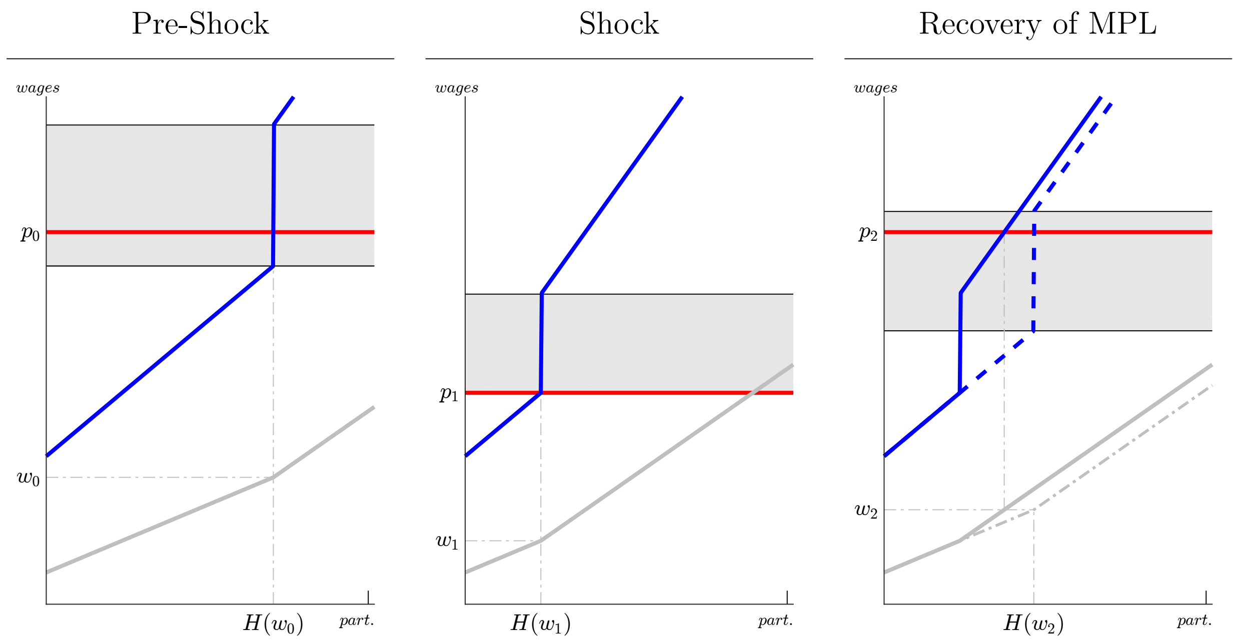


Note: At time 0 the economy is at steady state at wage level w_0. At time 1 a negative productivity shock hits rendering $p_1 < p_L < p_0$ and inducting a negative revision in the wage to $w_1 < w_0$ as firm's restrict hiring. The shock is temporary and at time 2 the pre-shock productivity level returns: $p_2 = p_0$. Despite recovery in the fundamental, however, wages remain depressed: $w_0 > w_2 = w_1$.

Proof. We construct proof by example and illustrate in Figure 7. The left panel depicts an equilibrium at time 0 in which productivity is within some inaction range. Wages and participation are at levels w_0 and $H(w_0)$ respectively. At time 1 a shock arrives such that productivity at 1 is outside and below the time 0 inaction rage. Lemma 3 guarantees that wages and participation fall to $w_1 < w_0$ and $H(w_1) < H(w_0)$ respectively. A new inaction range is established following the shock. Figure 7 illustrates that it is possible that the two inaction ranges overlap and, indeed, that p_0 may fall *inside* the new inaction range. At time 2 productivity recovers to the pre-shock level: $p_2 = p_0$. However, when recovering from the shock, the firm is constrained by the shock-level participation threshold and associated inaction range. Due to Assumption 2, wages and participation *do not* rebound when productivity rebounds. $\square$

Proposition 4. *Transient productivity shocks may lead to* ***scaring****. That is wages, partic-*

Figure 8: Scaring Effects of Contractions



Note: At time 0 the economy is at steady state at wage level w_0. At time 1 a negative productivity shock hits rendering $p_1 < p_L < p_0$ and inducting a negative revision in the wage to $w_1 < w_0$ as firm's restrict hiring. The shock is temporary and at time 2 the pre-shock productivity level returns: $p_2 = p_0$. Despite recovery in the fundamental, however, wages remain depressed: $w_0 > w_2 > w_1$ since firms are constrained by the lower participation threshold induced by the shock.

ipation, and output pruning below pre-shock levels.

Proof. The proof follows from noting that during the contraction the firm's optimization problem coincides with an unconstrained monopsony, equation 2, while during expansion the firm's optimization *is constrained* and follows, equation 3. Further $\hat{w}(r_0) < w^C$ whenever $r_0 \in (\underline{b}, w^C)$ whenever the second order condition $\frac{1}{p-b} > \frac{d^2H(b)}{db^2}\frac{1}{2h(b)}$ holds. This condition coincides with the condition required for a unique w^C. Note that Lemma 3 guarantees convergence to an equilibrium following each shock and that the conditions noted are those required for convergence to $w^* \in [w^L, w^C)$. Further, from Proposition 1 we can establish that, since wages and participation are larger in the pre-shocks equilibrium than the post-shocks equilibrium output must also be larger. □

The scaring effects on wage and participation can be clearly seen in Figure 8. The left panel depicts an equilibrium at time 0 in which productivity is within some inaction range.

Wages and participation are at levels w_0 and $H(w_0)$ respectively. At time 1 a shock arrives such that productivity at 1 is outside and below the time 0 inaction rage. As a result wages and participation fall to $w_1 < w_0$ and $H(w_1) < H(w_0)$ respectively. A new inaction range is established following the shock. As Figure 7 illustrates, it is possible that the two inaction ranges overlap and, indeed, that p_0 may fall *inside* the new inaction range. Figure 8 illustrates that it is possible that p_0 may also fall *outside* and above the new inaction range. At time 2 productivity recovers to the pre-shock level: $p_2 = p_0$. However, when recovering from the shock, the firm is constrained by the shock-level participation threshold and associated inaction range. As a result wages and participation *do not fully* rebound when productivity rebounds. Further, from Proposition 1 we can see that output and welfare are larger in the pre-shocks equilibrium than the post-shocks equilibrium.

4 A Two Sided Frictional Labor Market

We embed this stylized game in a frictional labor market that is endowed with appropriate features to justify the assumptions of our two player game. In the two-player game we assumed that the probability of a worker (firm) meeting a firm (worker) is exogenous to that player's strategy. Here, this assumption is micro-founded by the assumption that every agent is *atomistic*. Thus, the equilibrium job finding and filling hazards are exogenous to each worker's and each firm's strategy. In the two-player game we also assumed that the firm holds monopsony power and posts wages ex-ante. Here, as Diamond (1971) shows, *sequential random matching* endows the firm with this monopsony power and the monopsony wage level prevails even in the limit as search friction fades. Finally, in the two player game we assumed no continuation payoffs. This is justified in the two sided game by the observation that Diamond (1971) guarantees a degenerate wage distribution and thus no option value of search for the worker and *free entry* drives the value of a vacancy to zero for the firm.

In the Appendix, we present the Bellman equations associated with each worker's and each firm's decision problem. These yield participation and wage posting strategies that are identical to the two-player game up to the scaling of the fraction of unemployed who will not accept the going wage offer i. The best response of workers is a participation threshold strategy as in equation (1) and the best response of firms is a wage posting strategy as in equation (4), where i is replaced by $\iota = i/\hat{u}$ and $\hat{u}$ is the equilibrium unemployment rate of worker who *will* accept the equilibrium wage offer.

We close the model by positing a standard matching function as in the DMP model. Firms post vacancies at flow cost, c and workers engage in search at zero cost. Both discount the future at rate ρ. As in the baseline DMP, the flow of new matches is determined by the matching function, denoted as $m(U, V)$, where U is the mass of unemployed workers and V is the mass of vacancies. Imposing Inada conditions and constant returns to scale, the job-finding rate of unemployed workers, $f(\theta) \equiv \frac{m}{U} = m(1, \theta)$, is increasing and concave in the market tightness defined as the ratio of vacancies to the unemployed, $\theta = \frac{V}{U}$. Analogously, the rate at which vacancies meet unemployed workers, $q(\theta) \equiv \frac{m}{V} = \frac{f(\theta)}{\theta}$, is a positive and decreasing function of market tightness.[22]

Note, that since we have assumed random matching, the matching rate of a worker with a high flow value of leisure and a low flow value of leisure are the same whenever they participate in the labor market. Thus, the mass of workers who search for work but will reject wage offers of w^*, $i(1 - H(w^*))$, are just as likely to meet a firm as the $\hat{u}H(w^*)$ mass of workers who are unemployed and will accept wage offer w^*. Since the $i(1 - H(w^*))$ mass of workers reject the wage offer whenever they meet a firm the vacancy filling rate is $g(\theta) = \frac{q(\theta)}{\Lambda(w^*)}$, where $\Lambda(w^*) \equiv \frac{H(w^*)+\iota(1-H(w^*))}{H(w^*)}$ is a measure of the severity of the congestion imposed by workers with leisure value above the equilibrium wage level searching for work

[22] It is important to note here that our model differs in an important way from the classic model of upward sloping labor demand: Diamond (1982). In that model multiplicity derives from a thick market externality generated by increasing returns to scale in the matching function. In our model multiplicity derives from a pair of externalities, a thick market externality derived from workers' participation decision and a pecuniary externality derived from firms' wage posting decision. We follow the main stream DMP literature and impose constant returns to scale on our matching function. We appeal to the empirical results summarized in Pissarides and Petrongolo (2001) to justify this assumption.

while at the same time rejecting all offers. $\Lambda = 1$ occurs when workers with flow value of leisure above the expected wage offer never search and is the case when congestion is minimized. For $\iota > 0$ we have $\Lambda > 1$ and congestion drives a wedge between the rate at which vacancies meet employees and the rate at which jobs fill. For $\iota < \hat{u}$ we have a kink in the labor supply curve as in the two-player representation.

Given the wage, free entry into vacancy creation pins down the labor market tightness as the value of a vacancy is driven to zero. Thus equilibrium tightness satisfies:

$$\frac{c\Lambda(w^*)}{q(\theta)} = \frac{p - w^*}{\rho + \delta}. \tag{5}$$

As in the standard DMP model the job creation condition is downward sloping.[23] Since the wage schedule is flat this guarantees a unique equilibrium tightness for every productivity and wage pair.

Definition 3. *A symmetric* ***rational expectations equilibrium*** *of the frictional labor market is a triple – wage, participation threshold, labor market tightness – such that:*

1. *the wage and participation threshold are mutual best responses.*

2. *labor market tightness satisfies the free entry condition, equation (5).*

Note that all firms (resp. workers) face the same objective function and we have restricted our attention to equilibria that are symmetric in the sense that every firm (resp. worker) plays the same strategy as every other firm (resp. worker). Thus our equilibria, rigidity, and convergence results from the two player game naturally extend to the two sided frictional labor market.

Finally, we pin down the unemployment rate. In steady state, the flow into and out of unemployment for the subset of workers who will accept offer w^* must be equal. Since job destruction is an exogenous shock that arrives with Poisson arrival δ we have $\hat{u} = \frac{\delta}{\delta + f(\theta(w^*))}$.

[23] To see this note that $\frac{d\Lambda}{dw} < 0$ and $\frac{d\Lambda}{d\theta} > 0$.

Also note that a mass equal to $i[1 - H(w^*)]$ are perpetually unemployed. Finally, the total mass of labor force participants is $H(w^*) + i[1 - H(w^*)]$. Thus we have the steady state unemployed rate $u = \frac{\hat{u}H(w^*)+i[1-H(w^*)]}{H(w^*)+i[1-H(w^*)]}$.

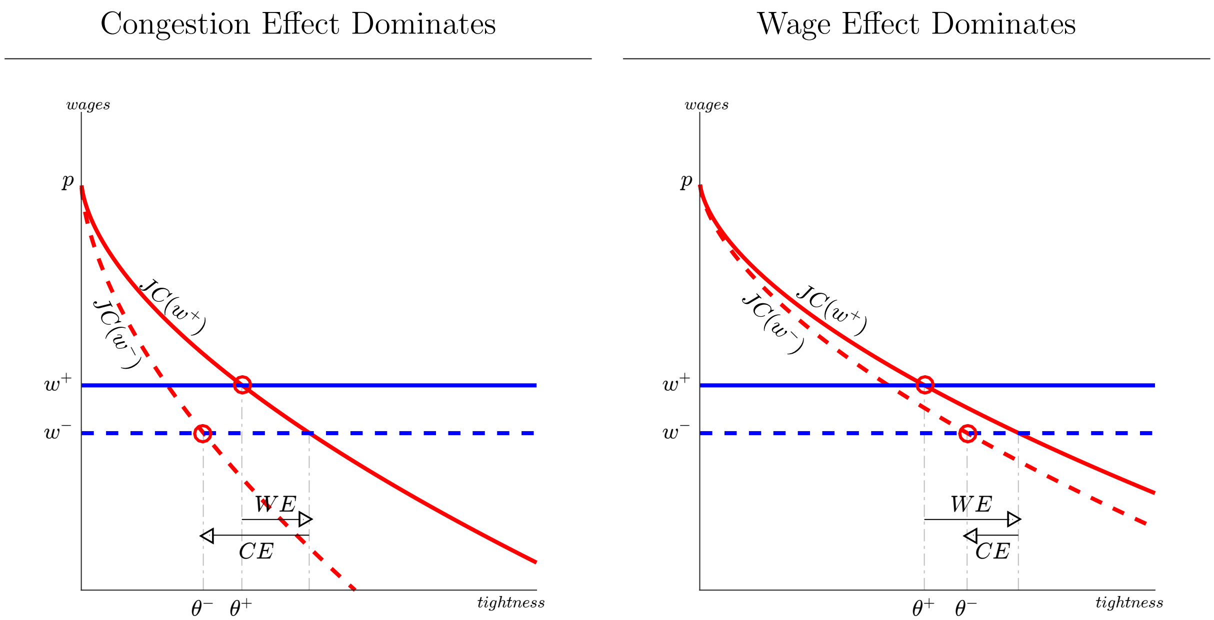
Notice that the slope of the job creation condition, equation (5) depends on the congestion effect. For a given wage, more congestion implies a steeper job creation condition and therefore a looser labor market. This follows from the decrease in the expected value of a vacancy when there is an increase in the probability of meeting a worker who will reject the wage offer. Since congestion lowers the value of a vacancy, fewer are produced.

5 "Job-less" and "Wage-less" Recoveries

The effect of congestion on market tightness is illustrated in Figure 9. In the left panel we plot a market in which the fraction of workers who search even when the expected wage, i, falls short of the value of leisure is higher than in the right panel. Consequently, for any wage level congestion is higher in the left panel than the right, this is reflected in the steeper slope of the job creation conditions in the left panel than in the right. Comparing θ^+ (resp. θ^-) across panels we can see that higher congestion implies lower labor market tightness conditional on the wage level. Higher i increases the steady state unemployment rate. This can be decomposed into two effects. The direct effect is that the mass of perpetually unemployed, $i[1 - H(w^*)]$, increases. The indirect effect is that the mass of unemployed who are willing to work at the equilibrium wage, $\hat{u}H(w^*)$, which increases as congestion reduces the value of a vacancy and loosens the labor market.

Decreasing the wage from w^+ to w^- also increases congestion since the mass of searching workers who will not accept the wage offer rises from $i[1 - H(w^+)]$ to $i[1 - H(w^-)]$. This mechanically increases the fraction of workers are perpetually unemployed. However, the effect on the unemployment rate of workers who *will* accept the wage offer w^- and on the overall unemployment rate is ambiguous. If the increase in congestion triggered by moving

Figure 9: Wage Effect (WE) and Congestion Effect (CE).



Note: Job creation is a downward sloping curve pinned down by the free entry condition, equation (5), while the wage is a horizontal line pinned down by the posted wage. Thus we are guaranteed a unique labor market tightness for every wage and productivity pair. An increase in congestion rotates the job creation condition clockwise. The left panel illustrates equilibrium labor market tightness at a high wage (solid) and low wage (hashed) equilibrium and when the congestion effect is large relative to the wage rigidity effect. The right panel illustrates the converse.

from the high wage to the low wage equilibrium is large enough, then unemployment will be higher in the low-output equilibrium. This is illustrated in the left panel: the congestion effect (labeled CE) is larger than the wage effect (labeled WE). If the increase in congestion is mild, unemployment among workers who will accept w^0 *falls* and the drop may be sufficient to offset the rise in unemployment among the perpetually unemployed, with the total effect being a drop in overall unemployment. This is illustrated in the right panel: the congestion effect is smaller than the wage effect.[24]

Referring to Proposition 1, we note that output under w^+ and w^- can be ranked with output under w^+ exceeding that under w^-.[25] We see that unemployment and output move in opposite directions only when the congestion effect is severe enough. Thus, without

[24] To see that this case exists for every parameterizations of $H(b)$, $m(U,V)$, p, and c for which there are equilibria, consider the limit as $i \searrow 0$ or the limit as $w^- \nearrow w+$ for fixed i.

[25] Whenever both are candidate equilibrium: $w^+ \in [w^L, w^C]$ and $w^- \in [w^L, w^C]$.

knowledge of the magnitude of the congestion effect the relation between fluctuations in unemployment and output is ambiguous, and we fail to consistently generate satisfying business cycle regularities, such as counter-cyclical unemployment, simply from "animal spirits."[26]

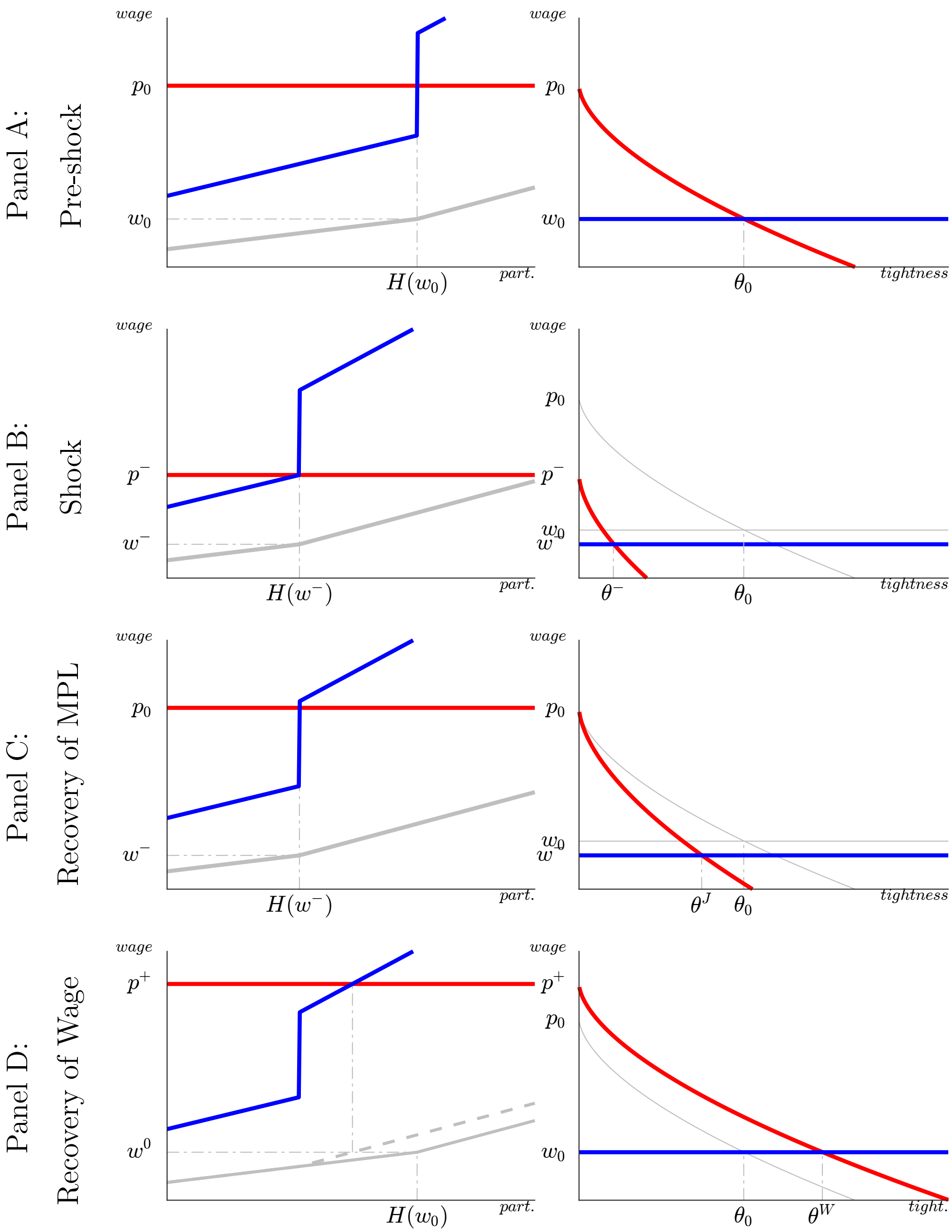
This, seemingly undesirable ambiguity actually works to our benefit and enables us to generate instances of both "job-less" and "wage-less" phases of recovery from productivity contractions–that is a period during which unemployment remains elevated even after productivity has recovered and a phase during which unemployment falls below pre-contraction levels while wages remain depressed. Job-lessness operates through the increase in congestion induced by the fall in the participation threshold during a severe contraction. Wage-lessness operates through an increase in the wage effect induced by persistence of the lower severe-contraction wage level beyond the duration of the disturbance.[27]

Figure 10 illustrates. The first two panels illustrate a steady state at time 0. The firm's wage setting problem is illustrated in the left panel and the job creation condition in the right. At this pre-shock steady state, wages, participation, and labor market tightness are w_0, H_0, and θ_0, respectively. The economy is then hit by a shock that lowers productivity to p^-, a value below the pre-shock inaction range. This is illustrated in the second set of panels. As a result of the shock, wages and participation fall to w^- and H^-. Labor market tightness also falls. The drop in labor market tightness is the the result of three forces: the drop in productivity, an increase in congestion due to the drop in the participation threshold, and a drop in wages. The first two clearly push tightness down. That these dominate the third is a result of firms' optimization over wages. Note that a fall in labor market tightness implies an increase in unemployment even among searchers who will still accept the new, lower, wage:

[26]The model could be used to create economic fluctuations *via* coordinated changes in expectations (i.e., animal spirits). For such fluctuations to match a basic business cycles fact–countercyclical unemployment–we would need to impose that congestion is large enough.

[27]When the increase in congestion is not particularly severe, wage-lessness may dominate throughout a recovery. However, whenever the drop in the participation threshold is severe enough to trigger a job-less recovery, job-lessness will occur during the early part of the recovery and will be followed by a period during which unemployment falls to unprecedented lows while wages continue to lag: a "wage-less" phase.

Figure 10: "Job-less" and "Wage-less" Recoveries.



Note: (Panel A) The economy begins at steady state at wage level w_0. (Panel B) A negative productivity shock hits rendering $p^- < p_0$ and inducting a negative revision in the wage to $w^- < w_0$ and in labor market tightness to $\theta^- < \theta_0$. The shock induces a clockwise rotation in the job creation condition as the decrease in the wage offer increases congestion. (Panel C) Productivity recovers to p_0 but wages remain depressed at $w^- < w_0$. The congestion effect dominates the wage effect and market tightness also remains depressed at $\theta^J < \theta_0$. (Panel D) Productivity overshoots, $p^+ > p_0$ and wages recover the initial level, w_0. Congestion dissipates when wages recover, isolating the wage effect, and we have $\theta^W > \theta_0$.

match efficiency has fallen and the Beveridge curve has shifted outward. Unemployment increases further due to an increase in the share of workers who search but will not accept employment–$i[1 - H(w_0)]$ increases to $i[1 - H(w^-)]$.

The third set of panels illustrates wages, participation, and labor market tightness if the economy is subsequently hit by a productivity shock that returns productivity to the pre-shock level. From Proposition 3 and Proportion 4 we see that wages and participation may not fully recover despite the recovery in productivity. As a result, labor market tightness rises. Now tightness is a result of two of the three forces: the increased in congestion due to the lower participation threshold and the lower wage level. As was illustrated in Figure 9, it is possible that the congestion effect dominates the wage effect. In this case, both the mass of unemployed who will accept and who will not accept the equilibrium wage offer remain elevated. In particular, congestion from the searching workers who will not accept the equilibrium wage offer increases the unemployment rate of those who will: match efficiency remains low and the Beveridge curve shifts outward. We have a job-less recovery.

The final set of panels illustrates the productivity shock required to return the economy to the pre-shock wage level following the contraction. When wages and participation recover to the pre-shock level so too does the magnitude of the congestion effect and the mass of workers who search but will not accept wage offer w_0. However, due to the asymmetry induced by the inaction range, the productivity level required to return the pre-shock wage level exceeds the pre-shock productivity level. Thus, labor market tightness in the period in which wages and participation recover *must* exceed the tightness experienced just before the impact of the first shock, since all that remains from the contraction is the wage effect. Thus we have that at least the end phase of each recovery from any contraction severe enough to yield a downward revision in the wage must be wage-less. Thus, every job-less recovery must be followed by a wage-less recovery.

6 Conclusion

We have considered the possibility that when workers are faced with a labor market in which the anticipated wage falls short of their outside option they may choose to not search. This may induce a kink in the labor supply curve faced by firms. From this consideration we have shown that a strategic complementarity between workers job search and firms wage posting exists and leads to an *continuum* of welfare ranked equilibria. We show that a correlate of the existence of a continuum of equilibria is intervals of endogenous rigidity with respect to wages and labor force participation.

We consider this framework in the context of productivity shocks and posit that players do not deviate from an existing wage and participation threshold pair unless unilateral deviation is a best response. We then show that in the event of deviations players can forecast opponents best response and learn the new equilibrium. Further, the kink in the labor supply curve induces asymmetry such that wages and participation may exhibit persistent responses to temporary shocks and that upward revisions in wages and participation fall farther from efficiency than do downward revisions. These are features of hysteresis.

We embed these intuitions in a model of a frictional labor market and show that such a market can exhibit both "job-less" and "wage-less" phases of recovery from productivity contractions–that is a period during which unemployment remains elevated even after productivity has recovered and a phase during which unemployment falls below pre-contraction levels while wages remain depressed. Joblessness stems from a congesting effect of search on the part of workers whose outside option exceeds the equilibrium wage offer. Wagelessness stems from asymmetric responses to shocks. In order to recover a wage level the rebound must overshoot the pre-shock productivity level and during such a rebound unemployment consequently undershoots the pre-shock unemployment rate.

Quantitative questions are left open by this analysis. In particular, to what extent can the real rigidities modeled here account for the subdued recovery from, for example, the financial crisis? While our analysis shows that the typical recession will feature asymmetry, the degree of asymmetry will depend on the relative efficiency of the pre-shock state of the economy. Thus, without further refinement we are unable to undertake a quantitative exercise. This framework could also be used to design fiscal policies which may mitigate some of the lasting and pernicious effects of hysteresis but may entail a greater degree of congestion during normal times. Similar to the quantitative challenges, this analysis would require further structure in order to isolate the most plausible equilibrium in normal times.

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A Value Functions in the Two Sided Labor Market

We can write out the asset value equations faced by a generic worker:

$$
\begin{aligned}
\rho\mathcal{I}(b, w_0) &= b \\
\rho\mathcal{U}(b, w_0) &= b + f(\theta)max\{0, [\mathcal{W}(b, w_0) - \mathcal{U}(b, w_0)]\} \\
\rho\mathcal{W}(b, w_0) &= w_0 + \delta[\mathcal{U}(b, w_0) - \mathcal{W}(b, w_0)]
\end{aligned}
$$

The first, $\mathcal{I}(b, w_0)$, captures the asset equation for an inactive worker with flow value of leisure b. This worker simply consumes b.[28] The second, $\mathcal{U}(b, w_0)$, captures the asset equation for a searching workers. While searching the worker consumes flow value b and at hazard $f(\theta)$ receive job offers which yield option value $max\{0, [\mathcal{W}(b, w_0) - \mathcal{U}(b, w_0)]\}$. The third, $\mathcal{W}(b, w_0)$ captures the asset equation for employed workers, these consume flow value w_0 and at hazard δ are separated to unemployment.

We begin by solving for the generic worker's reservation wage conditional on participating. This sets the value of unemployment equal to the value of employment: $\mathcal{W}(b, w_0) = \mathcal{U}(b, w_0)$. Thus we have a reservation wage equal to this generic workers realization of the flow value of leisure: b. In the event that w_0 exceeds b this worker anticipates higher value from the employed state than the unemployed state: given the opportunity she accepts all wage offers.[29] If w_0 falls short of b then the worker anticipates lower value from the employed state than the unemployed state: given the opportunity she rejects all wage offers.

Now we turn to considering the threshold participation decision. We can observe that if $b < w_0$ then the worker strictly prefers unemployment to inactivity and employment to unemployment: $I(b, w_0) < U(b, w_0) < W(b, w_0)$. Conversely if $b > w_0$ then the worker is indifferent between inactivity and unemployment and strictly prefers either of these to

[28] Note that in steady state this is independent of the wage level. When adding shocks we note that as long as there is no barrier to reentering the unemployment pool following a shock the independence is preserved.

[29] As we have seen in the two player game and will see again here, information frictions prevent the firm from observing b and thus from setting $w = b$ for each worker and extracting the full rent.

employment: $I(b, w_0) = U(b, w_0) > W(b, w_0)$. Thus we have that whenever $b < w_0$ the worker always prefers to participate and accepts employment whenever matched. Meanwhile if $b > w_0$ the worker is indifferent between participation and non-participation and strictly prefers not to accept employment whenever matched. As in the two player game we break the worker's indifference with Assumption 1.

We can also write out the asset value equations faced by a generic firm:

$$\begin{aligned}
\rho\mathcal{V}(w, r_0) &= -c + q(\theta)\Big[\mathbb{I}_{\{w \leq r_0\}} \tfrac{uH(w)}{uH(r_0)+i(1-H(r_0))} \\
&\quad + (1 - \mathbb{I}_{\{w \leq r_0\}}) \tfrac{uH(r_0)+i[H(w)-H(r_0)]}{uH(r_0)+i(1-H(r_0))}\Big][\mathcal{J}(w, r_0) - \mathcal{V}(w, r_0)] \\
\rho\mathcal{J}(w, r_0) &= p - w + \delta[\mathcal{V}(w, r_0) - \mathcal{J}(w, r_0)]
\end{aligned}$$

The first asset equation, $\mathcal{V}(w, r_0)$, captures the value of a open vacancy. The vacancy costs the firm a flow of c and with hazard $q(\theta)$ the firm meets and makes the posted wage offer w_0 to a worker. The bracketed term follows the logic of the firm's problem in the two player game: higher wage offers are accepted by a larger fraction of the workers that the firm might meet with the return to increasing the wage offer being discontinuous at the expected participation threshold. The final term is the option value of forming a match. The second asset equation, $\mathcal{J}(w, r_0)$ captures the value of a filled job. This is the flow rent $(p - w)$ and the hazard of separation times the option value of separation.

We posit free entry into vacancy creation. This drives the value of a vacancy to zero. Thus the firm's objective function in the two-sided game is isomorphic to the objective function of the firm in the two-player game.