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GDP Trend-cycle Decompositions Using State-level Data

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Abstract

This paper develops a method for decomposing GDP into trend and cycle exploiting the cross-sectional variation of state-level real GDP and unemployment rate data. The model assumes that there are common output and unemployment rate trend and cycle components, and that each state's output and unemployment rate are subject to idiosyncratic trend and cycle perturbations. The model is estimated with Bayesian methods using quarterly data from 2005:Q1 to 2016:Q1 for the 50 states and the District of Columbia. Results show that the U.S. output gap reached about -8% during the Great Recession and is about 0.6% in 2016:Q1.

Keywords: Unobserved components model, trend-cycle decomposition, state-level GDP data

JEL Classification Numbers: C13, C32, C52

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[*]The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System.

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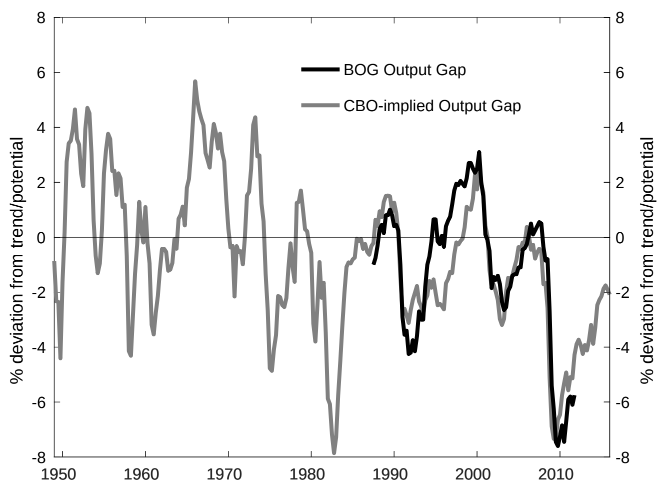
1 Introduction

The output gap—the difference between actual and potential or trend output—is a central measure in policy making. Both monetary and fiscal policy look at the output gap to gauge inflationary pressures or the degree of labor market slack. Because the output gap is a nonobservable variable, several statistical and econometric techniques have been proposed to estimate the cyclical position of the economy. The Congressional Budget Office (CBO), for example, offers a measure of potential output that helps forecast future federal revenue and spending, and from which the output gap can be obtained (see Congressional Budget Office, 2014, for a recent report). Central banks also construct estimates of the output gap that inform their monetary policy decisions (see Federal Reserve Bank of Philadelphia, 2017). Figure 1 plots the CBO-implied and Board of Governors (BOG) of the Federal Reserve System output gaps for the available periods.[1]

Figure 1: CBO-implied and BOG Output Gaps



Among the statistical techniques used to discriminate the permanent component from the transitory or cyclical component is the Beveridge and Nelson (1981) decomposition of integrated time series. Other techniques involve filters of different natures to separate the cycle from the trend or long-run component, like the Hodrick and Prescott (1997) filter, or variations of a band-pass filter, as in Baxter and King (1999) and Christiano and Fitzgerald (2003), or the Kalman filter to estimate models with unobserved components (UC), as in Clark (1987). Another approach suggested by Stock and Watson (1989) uses multiple macroeconomic time series to extract a common cyclical component in the spirit of dynamic factor models.

Even though there are existing methods to estimate the cyclical position of the economy or of a group of economies exploiting the variability of multiple variables, multiple cross-

[1]The BOG output gap are real-time estimates and projections of the output gap used by the staff of the Board of Governors of the Federal Reserve System in constructing its forecast. These estimates are released with a lag of 5 years and published by the Federal Reserve Bank of Philadelphia. Hence, the information ends in the fourth quarter of 2011, as of the date of publication of the current draft.

sectional units, or both, none of these methods has explored the estimation of the output gap—defined as the difference between actual and potential output—using multiple cross-sectional units such as geographical regions or industries. In this paper, I propose a UC model that combines information about output and the unemployment rate from many cross-sectional units—the U.S. states—to estimate the aggregate GDP cycle and its trend. The proposed model exploits the panel structure of the state-level data and adds richness to the conventional UC approach in a way that resembles the dynamic factor models but that allows me to measure the output gap as the difference between actual and potential GDP.

The setup assumes that (the log of) each state's real GDP is the sum of state-specific trend and cycle components. Each of the state-specific trends, in turn, is a linear combination of a trend common across states and an idiosyncratic trend. Likewise, each of the state's cycles is a linear combination of a cycle common to all of the states and an idiosyncratic component. Lastly, an Okun's law at the state level relates the cycle of GDP to the cycle of the unemployment rate. Hence, information from multiple cross sections and variables can be used to estimate the aggregate output cycle.

The model is similar to the one suggested by Stock and Watson (2016) for estimating trend inflation, in which different inflation categories are used as the source of cross-sectional variation. The difference is that the model in this paper incorporates a bivariate analysis within the cross sections, including another dimension of variation that adds information to the estimation. Additionally, in the model of this paper, the cyclical components are assumed to have autocorrelated dynamics, while autocorrelation is absent from the setup of Stock and Watson.

The aim of the paper is similar in spirit to that of Owyang, Rapach and Wall (2009), who extract business cycle factors using information on income and payroll employment at the state level. In the present paper, however, I use real GDP and unemployment rate data, so it is possible to interpret the cyclical component of GDP as a measure of the output gap.

I estimate the model with quarterly information for the 50 states and the District of Columbia for the period from 2005:Q1 to 2016:Q1. The results are obviously influenced and characterized by the features of the Great Recession. In particular, the estimated cycle is very persistent, with a hump-shaped response of the cycle to an innovation in itself.

The estimated smoothed cycle is -7.9% at the trough of the recession in the fourth quarter of 2009, slightly lower than the CBO-implied cycle, which reached about -7.3% two quarters before. The model's estimates imply that the trend of GDP declined about 1.9% between the end of 2008 and the beginning of 2009, whereas the CBO estimate lowers its slope much more slowly. At the end of the sample, the estimated output gap is about 0.6% and trend output would be growing at an average annual pace of ¾ percent, compared to 3¼ percent before the recession.

The proposed model also allows one to examine the cyclical features of the state economies during the recession and at the end of the sample. For example, the states that were hit the hardest by the recession were those in the rust belt, as well as Arizona, California, Nevada, and Florida. At the end of the sample, states like West Virginia and Wyoming were experiencing negative output gaps, probably driven by low commodity prices.

I find that adding cross-sectional information reduces the uncertainty around the estimator of the cycle compared with a model that uses aggregate data only. I also find that the model with many cross sections outperforms two competing models in predicting the growth rate of real GDP. The first is a simple AR(1) process of real GDP growth. The second is a (bivariate) UC model that uses aggregate information on real GDP and the unemployment rate. The reduced uncertainty and the better forecasting ability shed light on the advantage of using cross-sectional information to obtain estimates of the cycle with a UC model.

The paper is structured as follows: Section 2 reviews the existing literature. In Section 3, I set up the UC model with many cross sections. Section 4 presents the estimation technique. Section 5 describes the data at the state level and shows the features of the Great Recession. In Section 6, I present and discuss the results of the estimation. Section 7 evaluates the performance of the model. Section 8 concludes.

2 Contacts with the Literature

This work is related to the literature on UC models of trend-cycle decompositions. It also relates to the literature on common components estimation with dynamic factor models and, indirectly, to the literature on extracting common stochastic trends with panel data.

Introducing multidimensional variability into the UC model's setup can be done in three ways. One is to incorporate many variables belonging to one unit of interest, where these variables provide information to extract common trends and/or cycles. Another possibility is to incorporate many units of interest for which the same variable is used to extract common trends and/or cycles. The third possibility considers a combination of many units of interest and many variables to extract common trends and/or cycles. In this paper, I adopt the third possibility.

2.1 Many Variables and a Single Economic Unit

With respect to the first approach, in which many variables of the same unit of interest provide information about a common cycle, Crone and Clayton-Matthews (2005) describe how to use mixed frequency data from the U.S. states to estimate state-level monthly indexes of economic activity for each state. The setup is the UC model discussed in Stock and Watson (1988, 1989). Observable variables for each state are the first difference of the following: (the log of) nonagricultural employment, the unemployment rate, (the log of) average hours worked in manufacturing, and (the log of) real wage and salary disbursements. A scalar latent stationary series is common to the state-level observable variables and is interpreted as the state's cycle. The observable series load on the state's cycle with leads and/or lags. In this analysis, only the cross-sectional structure of the many variables is exploited, not the variability along the state-level dimension.

Basistha and Startz (2008) and Fleischman and Roberts (2011) also use several variables, but at the aggregate level, to estimate the U.S. NAIRU in the first case and to estimate the U.S. potential output and its associated business cycle in the second. In both cases, the authors emphasize the advantages of using a multivariate approach in the estimation of unobserved components models. Such advantages include gaining precision of the estimates and the ability to assess the trade-offs among competing signals in a coherent way.

2.2 A Single Variable and Many Economic Units

Within the second approach —many units of interest and one variable—Del Negro and Otrok (2008) extend a factor model to incorporate time-varying factor loadings and stochastic volatility in order to extract the international business cycle using a panel of 19 countries. The model is estimated with Bayesian methods and allows one to obtain the common and country-specific cycles for the growth rates of GDP, but it does not consider the common GDP trends.

Mitra and Sinclair (2012) also use many cross sections and one variable. They propose a multivariate UC model to simultaneously decompose real GDP for each of the G-7 countries into its respective trend and cycle components. The setup considers real GDP as the only observable variable and assumes that each country's GDP is driven by specific trend and cycle components. The setup allows for possible correlations between any of the contemporaneous shocks to the unobserved (trend and cycle) components.

Stock and Watson (2016) propose a multivariate model to estimate trend inflation. They use 17 components of the U.S. personal consumption expenditure inflation to construct an index akin to core inflation, but with time-varying distributed lags of weights, where each sectoral weight depends on the time-varying volatility and persistence of the sectoral inflation series, and on the co-movement among sectors. The modeling framework is a dynamic factor model with time-varying coefficients and stochastic volatility estimated with Bayesian methods. This work is the most similar to the present paper, although it only considers one variable in the analysis—the inflation rate—while this paper considers two, GDP and the unemployment rate.

2.3 Many Variables and Many Economic Units

Regarding the third approach—many variables and many cross sections—Gregory, Head and Raynauld (1997) use a dynamic factor model estimated with classical methods to decompose aggregate output, consumption, and investment for the G-7 countries into factors that are common across all countries and aggregates, common across aggregates within a country, and specific to each individual aggregate. Data have to be de-trended to use the dynamic factor models approach because the underlying factors are assumed to be stationary. Similarly, Kose, Otrok and Whiteman (2003) estimate a dynamic latent factor model, but with Bayesian methods, to extract common components from macroeconomic aggregates (output, consumption, and investment) in a 60-country sample covering seven regions of the world. They allow factors common to the world, the regions, and the countries. Here, too, data are de-trended.

For the United States, Owyang, Rapach and Wall (2009) use state-level income and payroll employment data to estimate a dynamic factor model of the 48 contiguous states and the District of Columbia in order to extract business cycle factors. The estimation of the model identifies three common factors underlying the fluctuations in state-level income and employment growth, with the first common factor resembling aggregate fluctuations in real activity at the national level. The factors explain a large proportion of the total variability in state-level variables, although there is still a substantial amount of cross-state heterogeneity.

3 Unobserved Components Models for Trend-cycle Decomposition of GDP

There is a relatively long history of formulations of UC models to decompose GDP into trend and cycle components. Clark (1987) proposes a univariate estimation in which the log of aggregate real GDP is the sum of orthogonal trend and cycle components and where the cycle follows a stationary AR(2) process. Clark (1989) adds the aggregate unemployment rate as an additional variable to inform the trend-cycle decomposition and sets up the following model:

$$y_t = \tau_t^y + c_t \tag{1}$$
$$u_t = \tau_t^u + \theta_1 c_t + \theta_2 c_{t-1} \tag{2}$$
$$\tau_t^y = \mu + \tau_{t-1}^y + \eta_t^y \tag{3}$$
$$\tau_t^u = \tau_{t-1}^u + \eta_t^u, \tag{4}$$
$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t, \tag{5}$$

with

$$\left.\begin{bmatrix} \varepsilon_t \\ \eta_t^y \\ \eta_t^u \end{bmatrix}\right| \mathfrak{F}_{t-1} \sim \text{iid } \mathbb{N}\left(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 \\ 0 & \sigma_{\eta^y}^2 & 0 \\ 0 & 0 & \sigma_{\eta^u}^2 \end{bmatrix}\right), \tag{6}$$

and $\mathfrak{F}_{t-1}$ is the sigma-field containing the information up to period $t-1$.

In equations (1)-(6), y_t is the log of aggregate real GDP, τ_t^y is its unobserved trend, assumed to be a random walk with mean growth rate μ, and c_t is the unobserved stationary cycle that follows an AR(2) process with autoregressive coefficients ϕ_1 and ϕ_2. Additionally, u_t is the aggregate unemployment rate and τ_t^u is its unobserved trend, which follows a random walk process, whereas the cyclical component is given by a linear combination of current and lagged values of the cycle of GDP with coefficients θ_1 and θ_2, in a way resembling Okun's law. Finally, ε_t is a disturbance to the cycle, while η_t^y and η_t^u are trend disturbances, all of them independent of each other.

Estimation of this model has been typically made with classical methods using the Kalman filter (see Clark, 1989; Gonzalez-Astudillo and Roberts, 2016, for example). Also, the inflation rate can be introduced in a way resembling a Phillips curve to inform the estimation of the cycle (see Basistha and Nelson, 2007, for example).

3.1 The Unobserved Components Model With Many Cross Sections

In this paper, I introduce a cross-sectional dimension to the UC model in equations (1)-(6). Specifically, the cross-sectional information comes from state-level data. The model specifies (the log of) each state's real GDP, y_{it}, and unemployment rate, u_{it}, for states $i = 1, 2, \ldots, n$ in period $t = 1, 2, \ldots, T$ as follows:

$$y_{it} = \tau_{it}^y + c_{it} \tag{7}$$

$$
\begin{aligned}
u_{it} &= \tau^u_{it} + \theta_{1i} c_{it} + \theta_{2i} c_{i,t-1} && (8)\\
\tau^y_{it} &= \delta^y_i \tau^y_t + \xi^y_{it} && (9)\\
\tau^u_{it} &= \delta^u_i \tau^u_t + \xi^u_{it} && (10)\\
\xi^y_{it} &= \mu_i + \xi^y_{i,t-1} + \eta^y_{it} && (11)\\
\xi^u_{it} &= \xi^u_{i,t-1} + \eta^u_{it} && (12)\\
\tau^y_t &= \mu + \tau^y_{t-1} + \eta^y_t && (13)\\
\tau^u_t &= \tau^u_{t-1} + \eta^u_t && (14)\\
c_{it} &= \alpha_i c_t + \upsilon_{it} && (15)\\
\upsilon_{it} &= \rho_i \upsilon_{i,t-1} + \zeta_{it} && (16)\\
c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t, && (17)
\end{aligned}
$$

where the error terms are assumed to be white noise, uncorrelated with each other and normally distributed, as follows:

$$
\left.\begin{bmatrix} \varepsilon_t \\ \eta^y_t \\ \eta^u_t \\ \eta^y_{it} \\ \eta^u_{it} \\ \zeta_{it} \end{bmatrix}\right| \mathfrak{F}_{t-1} \sim \mathbb{N}\left(0, \begin{bmatrix} \sigma^2_\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta^y} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta^u} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta^y_i} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\eta^u_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\zeta_i} \end{bmatrix}\right), \qquad (18)
$$

and $\mathfrak{F}_{t-1}$ is the sigma-field containing the information up to period $t-1$. Appendix A presents the state-space model in matrix form.

In the model characterized by equations (7)-(18), each state's GDP and unemployment rate are the sum of GDP and unemployment rate trends and cycles. The state's GDP and unemployment rate trends, τ^y_{it} and τ^u_{it}, respectively, are the sum of two components. The first component is a function of the common trends, τ^y_t or τ^u_t, with loadings δ^y_i or δ^u_i, respectively. The common trends, in turn, are unit-root processes with drift μ in the case of GDP and without drift in the case of the unemployment rate. The second component is a state-specific trend, ξ^y_{it} or ξ^u_{it}, which are unit-root processes with drift μ_i in the case of GDP and without drift in the case of the unemployment rate.

The state-specific GDP cycle is c_{it}, whereas in the case of the unemployment rate, the cycle is a linear combination of the cycle of GDP with coefficients θ_{1i} and θ_{2i} in a way resembling an Okun's law, as in Clark (1989). Each state's GDP cycle loads on the common cycle, c_t, with coefficient α_i and is affected by a state-specific disturbance, υ_{it}, which in turn follows a stationary AR(1) process with persistence coefficient ρ_i. Lastly, the common cycle follows a stationary AR(2) process with persistence coefficients ϕ_1 and ϕ_2.

The model outlined in equations (7)-(18) differs from the existing UC models for GDP trend-cycle decompositions in the literature because it incorporates cross-sectional information in the analysis. The common trend and cycle of aggregate GDP are informed by the cross-sectional structure of the data. Although the idiosyncratic disturbances are uncorrelated across states, the common trend and cycle allow for correlated movements in the state-level GDPs and unemployment rates. The model differs from the conventional dynamic factor models approach because it explicitly allows for trends, which are generally removed in the factor analysis from each of the variables to deal with stationary processes. The exception to this de-trending treatment is the setup in Stock and Watson (2016). Their model is similar to the model presented here, except that I do not allow for stochastic volatilities or time-varying loading coefficients, in particular because of the short longitude of the data and the large cross-section. What is allowed in the model considered here, however, is the possibility that the cyclical components, c_t and υ_{it}, follow autocorrelated dynamics, whereas Stock and Watson assume that they are martingale processes. Additionally, the model presented here incorporates two variables for each cross section, namely real GDP and the unemployment rate, to inform the trend and cycle of GDP, whereas in Stock and Watson the only variable of interest was inflation to inform its own trend.

4 Estimation Strategy

This section lays out the identification restrictions and the Bayesian estimation strategy to obtain the parameters that characterize the model in equations (7)-(18), as well as the common and state-specific trends and cycles of output.

4.1 Identification

Regarding the identification of the parameters and the unobserved components, the variance of the error term of the common cycle, σ^2_{ε}, is set to one to be able to identify the scale of the loading coefficients, α_i. The same is done with the variances of the error terms of the common trends, $\sigma^2_{\eta^y}$ and $\sigma^2_{\eta^u}$, in order to identify the loading coefficients δ^y_i and δ^u_i, respectively. The drift of the common output trend, μ, is set to zero to allow the unrestricted estimation of the state-specific drifts, μ_i. To avoid state-level GDP trends with negative slopes, the loading coefficients, δ^y_i, and the state-specific drifts, μ_i, are restricted to be positive for all the states. The variances of the idiosyncratic components of the cycle, $\sigma^2_{\zeta_i}$, are identified from the covariance and autocovariance structure, including the cross autocovariances, of the state-level GDPs and unemployment rates.

Because the common cycle, c_t, is identified only up to its sign, I restrict $\alpha_i \geq 0$ for $i =$ CALIFORNIA. I also restrict the loading coefficients of each state's unemployment rate on the common unemployment trend, δ^u_i, to be nonnegative for all the states in order to prevent unemployment rate trends at the state level that may move opposite to the common trend permanently.

4.2 Bayesian Estimation

The estimation of the UC model with many cross sections in equations (7)-(18) is carried out with Bayesian methods and involves 512 coefficients when the identification restrictions are imposed. I set up the following Metropolis-within-Gibbs procedure.[2] Let $\mathbf{z}_{it}$ be a 2×1 vector containing the rate of real GDP growth and the first difference of the unemployment rate for $i = 1, \ldots, n$ and $t = 1, \ldots, T$. Also, let $\mathbf{x}_{it}$ be a vector including the latent variables

[2] Appendix B describes in detail the sampling procedure.

for $i = 1, \ldots, n$ and $t = 1, \ldots, T$. Additionally, denote as $\mathbf{\Theta}_{\text{mi}}$ the parameters of the state-space model's measurement equation for state i and $\mathbf{\Theta}_{\text{si}}$ the parameters of the state-space model's transition equation for state i. For $\mathbf{\Theta}_{\text{m}} = \bigcup_{\text{i}=1}^{n} \mathbf{\Theta}_{\text{mi}}$ and $\mathbf{\Theta}_{\text{s}} = \bigcup_{\text{i}=1}^{n} \mathbf{\Theta}_{\text{si}}$, the steps are the following:

1. Start with initial values for the model's parameters, $\mathbf{\Theta} = \mathbf{\Theta}_{\text{m}} \bigcup \mathbf{\Theta}_{\text{s}}$.
2. Conditional on the data, $\mathbf{Z}_T = [\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2, \ldots, \tilde{\mathbf{z}}_T]$, and $\mathbf{\Theta}$, generate a draw of the latent variables, $\mathbf{X}_T = [\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_T]$, using the Durbin and Koopman (2002) simulation smoother. Here, $\tilde{\mathbf{z}}_t = [\mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{nt}]$ and $\tilde{\mathbf{x}}_t = [\mathbf{x}_{1t}, \mathbf{x}_{2t}, \ldots, \mathbf{x}_{nt}]$.
3. Conditional on $\mathbf{X}_T, \mathbf{Z}_T$, and $\mathbf{\Theta}_{\text{m}}$, generate a draw of $\mathbf{\Theta}_{\text{s}}$ using the Normal-Inverse-Gamma posterior distribution and the Metropolis-Hastings algorithm.
4. Conditional on $\mathbf{X}_T, \mathbf{Z}_T$, and $\mathbf{\Theta}_{\text{s}}$, generate a draw of $\mathbf{\Theta}_{\text{m}}$ using the Metropolis-Hastings algorithm imposing the identification restrictions.
5. Return to step 2.

5 Data

I consider real GDP in chained 2009 dollars for the 50 states and the District of Columbia at a quarterly frequency from the first quarter of 2005 to the first quarter of 2016. The information is obtained from the U.S. Bureau of Economic Analysis Regional Economic Accounts. The unemployment rate data at the state level are obtained from the Local Area Unemployment Statistics of the U.S. Bureau of Labor Statistics.

Because the aim of the paper is to obtain and characterize the national business cycle from state-level data, this section briefly describes the characteristics of the state economies. I pay special attention to the features of the state economies during the Great Recession. I show the industry contribution to each state's GDP, the states that were most strongly affected by the recession, and the industries that have contributed the most to the growth of GDP in each state between 2005 and 2015.

5.1 Economic Activities by State

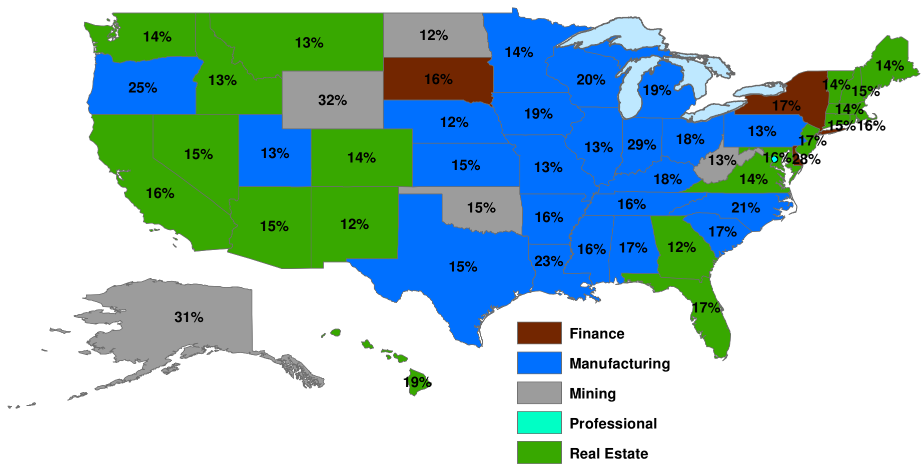
This section describes the most relevant industries by state. Using annual information by industries at the state level from 2005 to 2015, Figure 2 presents two maps of the United States with the two most important industries in terms of GDP participation, on average, for each state.[3]

The industries that contribute the most to the states GDPs, using the North American Industry Classification System, are the following: (i) Manufacturing, (ii) Real Estate and Rental and Leasing, (iii) Mining, (iv) Professional, Scientific, and Technical services, (v) Health Care and Social Assistance, (vi) Finance and Insurance, (vii) Accommodation and

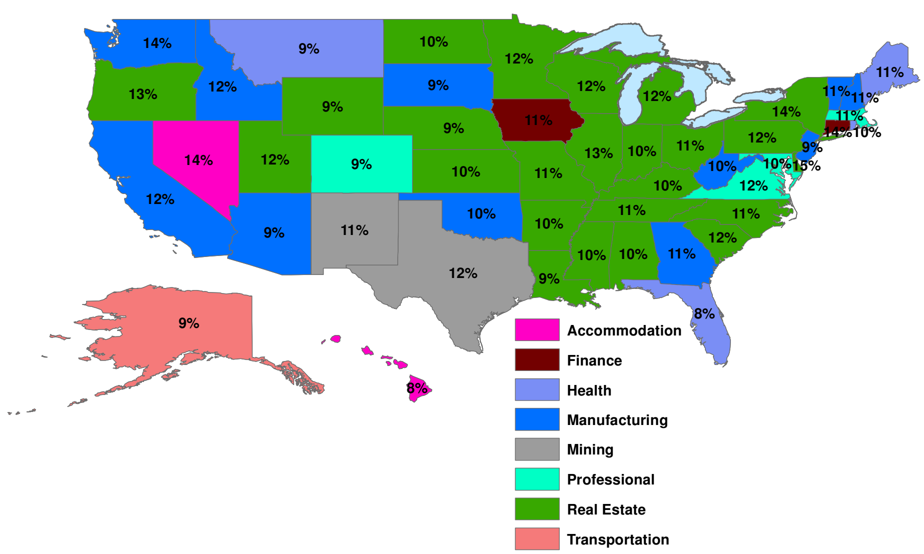
[3] This analysis excludes the importance of the government sector in each state, which can be large for some of the state economies.

Figure 2: Most Important Industries by State 2005-2015

(a) First Industry

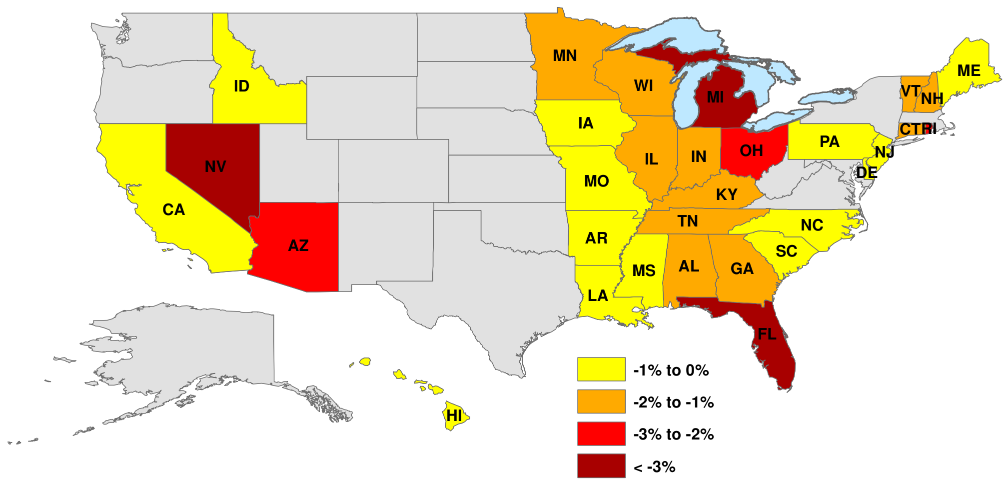


(b) Second Industry



Note: Percentages are the average contribution of the industry to the state's GDP between 2005 and 2015. Source: U.S. Bureau of Economic Analysis (BEA), *Regional Economic Accounts*.

Figure 3: States with Negative Average Real GDP Annual Growth in 2007-2009



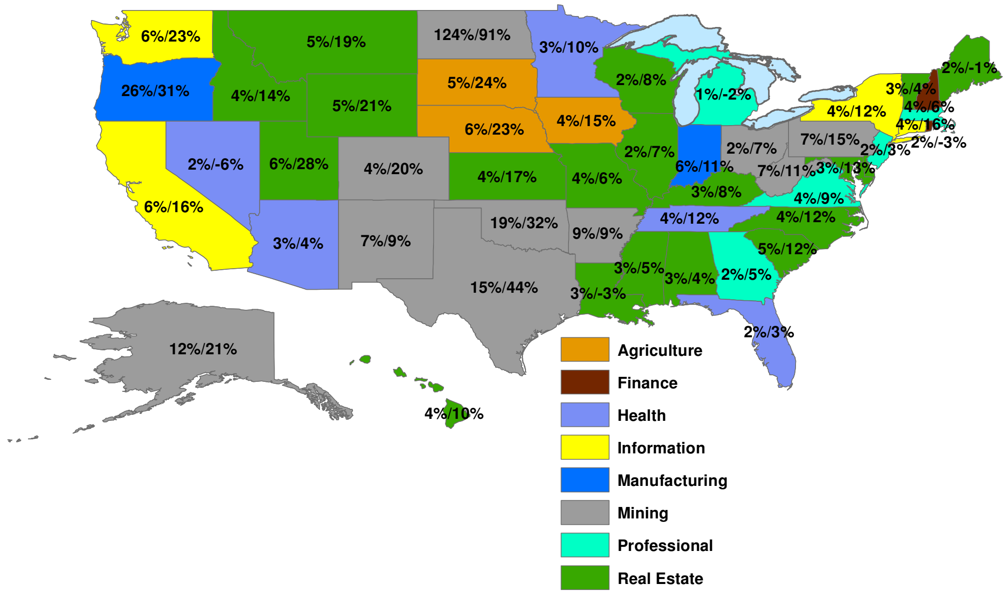
Note: States in light gray experienced positive average growth rates.
Source: U.S. Bureau of Economic Analysis (BEA), *Regional Economic Accounts.*

Food Services, and (viii) Transportation and Warehousing. By a large margin, the two most important industries for the vast majority of states are manufacturing and real estate. However, some other states have other sectors that contribute importantly to their GDP. For example, mining is an important industry in Alaska, New Mexico, North Dakota, Oklahoma, Texas, West Virginia, and Wyoming. Colorado, the District of Columbia, Maryland, Massachusetts, and Virginia have an important contribution from the professional services sector. Financial services contribute importantly to the GDP of Connecticut, Delaware, Iowa, New York and South Dakota. Health-related services are an important sector in Florida, Maine, Montana, and Rhode Island, while Hawaii and Nevada both have large contributions from accommodation and food services.

5.2 States Most Strongly Affected by the Great Recession

The states that were most strongly affected by the Great Recession in terms of real GDP growth rates in the 2007–09 period appear in Figure 3. The states that experienced the lowest average growth rates of real GDP were Florida, Michigan, and Nevada, with rates lower than -3%. Following this group are Arizona and Ohio, with growth rates between -3% and -2%. Having real estate as the first or the second most important economic activity was most likely associated with GDP growth declines across states. This situation is obvious, given the nature of the Great Recession. By contrast, having mining as one of the two most important economic activities—like in Alaska, New Mexico, North Dakota, Oklahoma, Texas, West Virginia, and Wyoming—was associated with positive average growth rates in that period. All told, 30 of the 48 states experienced zero or negative average growth rates in the 2007–09 period.

Figure 4: Most Important Contributing Industries to GDP Growth 2005-2015



Note: The first percentage is the average contribution of the most important industry to GDP growth. The second percentage is the rate of real GDP growth between 2005 and 2015.
Source: U.S. Bureau of Economic Analysis (BEA), *Regional Economic Accounts*.

5.3 Most Important Contributing Industries to the States' Real GDP Growth

Now that we have seen the states that were affected the most by the Great Recession, Figure 4 presents the real GDP growth rates between 2005 and 2015 by state and the industries that contributed the most to the growth of real GDP in each state. The five states with the highest growth rates are North Dakota (91%), Texas (44%), Oklahoma (32%), Oregon (31%), and Utah (28%). The five states with the lowest growth rates are Nevada (-6%), Louisiana (-3%), Connecticut (-3%), Michigan (-2%), and Rhode Island (-1%). Mining has been the most important contributor to growth in the three fastest growing states. In Oregon, manufacturing contributed the most, while the real estate sector has affected growth the most in Utah. In the majority of states, the construction and manufacturing industries have been the biggest drags with respect to GDP growth. In fact, construction is the sector with the largest negative contribution to the growth of real GDP in Nevada, Michigan, and Rhode Island, whereas the manufacturing sector has the largest negative contribution in Louisiana and Connecticut.

In the next section, I present the results of the estimation and link these results to the narrative of this section.

6 Estimation Results

This section presents the results of the estimation of the UC model of equations (7)-(18). It describes the results of the estimation focusing on the aggregate GDP trend and cycle. It

Table 1: Prior Distributions of the Parameters - State Level Data

Parameter	Distribution	Mean	Standard Deviation
ϕ_1	Truncated Normal	1.5	1
ϕ_2	Truncated Normal	-0.6	1
μ_i	Truncated Normal	0.8	0.8
α_i	Normal	1	1
δ_i^y	Truncated Normal	1	1
δ_i^u	Truncated Normal	1	1
θ_{1i}	Normal	-0.25	0.25
θ_{2i}	Normal	-0.25	0.25
ρ_i	Truncated Normal	0	1
$\sigma^2_{\eta_i^y}$	Inverse Gamma	—	—
$\sigma^2_{\eta_i^u}$	Inverse Gamma	—	—
$\sigma^2_{\zeta_i}$	Inverse Gamma	—	—

also discusses the features of each state's estimated trend and cycle and the magnitudes of the Okun's law coefficients.

6.1 Prior Distributions

The prior distributions of the parameters of the UC model with many cross sections appear in Table 1.

The prior means of the parameters of the common cycle, ϕ_1 and ϕ_2, are similar to those found in the literature of trend-cycle decompositions of output. For example, Morley, Nelson and Zivot (2003) find that the estimation of an unobserved components model with aggregate data on real GDP in absence of correlation between trend and cycle innovations yields estimated coefficients $\hat{\phi}_1 = 1.53$ and $\hat{\phi}_2 = -0.61$. Also with aggregate data, but including the unemployment rate along with real GDP, Gonzalez-Astudillo and Roberts (2016) find estimates around 1.6 for ϕ_1 and -0.65 for ϕ_2 either with or without correlation between output innovations. The joint prior distribution of these two parameters is truncated to satisfy the weak stationarity feature of the common cycle. The mean growth rate of each state's GDP has a truncated normal prior distribution with mean 0.8, which yields an annualized growth rate of real GDP around 3%, roughly the historical average. The distributions of the parameters that load on the common cycle, α_i, are normal with means and standard deviations equal to one.[4] The coefficients that load on the common trends, δ_i^y, and δ_i^u, have truncated normal distributions with means and standard deviations equal to one. The Okun's law coefficients, θ_{1i} and θ_{2i}, are normally distributed with prior means equal to -0.25 each, such that the long-run Okun's law coefficient for each state is centered at -0.5 under the prior distribution. This is the usual Okun's law coefficient used in the literature (see Abel, Bernanke and Croushore, 2013). The prior distribution of the parameter of the idiosyncratic cycle, ρ_i, is a truncated standard normal. In general, the standard deviations

[4] In the case of California, the prior distribution of α_i is truncated normal with mean and standard deviation equal to one.

Table 2: Parameter Estimates of the Common Trend and Cycle

Parameter	Posterior Mean	90% Confidence Set
ϕ_1	1.97	[1.95, 1.98]
ϕ_2	-0.98	[-0.99, -0.96]
σ_ε	1	—
σ_{η_y}	1	—
σ_{η_u}	1	—

of the parameters imply that the distributions are neither too tight nor too narrow.

In the current setup, because of the lack of previous estimates of these coefficients in the literature, it may be difficult to characterize the prior distributions of the variances of the idiosyncratic components of the trend and the cycle, $\sigma^2_{\eta^y_i}$, $\sigma^2_{\eta^u_i}$, and $\sigma^2_{\zeta_i}$. Hence, the estimation of these parameters relies exclusively on the likelihood function.

6.2 Estimate of the U.S. Output Gap

The estimation produced 60,000 draws from the posterior distribution. The last 30,000 draws were kept. The acceptance rate for the coefficients drawn with the Metropolis-Hastings algorithm was between 30% and 40%, on average, across states. Results of the estimation for the parameters that characterize the common cycle appear in Table 2. The posterior mean of the parameters ϕ_1 and ϕ_2 imply a very persistent hump-shaped response of the cycle to a shock, in line with the findings of Fleischman and Roberts (2011). Under the posterior distribution of these coefficients, the lag polynomial has complex roots that imply a 90% confidence set for the period of the cycle between approximately 13½ and 19 years, with a mean of approximately 15¾ years.

Using the derivation detailed in Appendix C to obtain the cycle of aggregate GDP from the estimation with state-level data, I obtain the following:

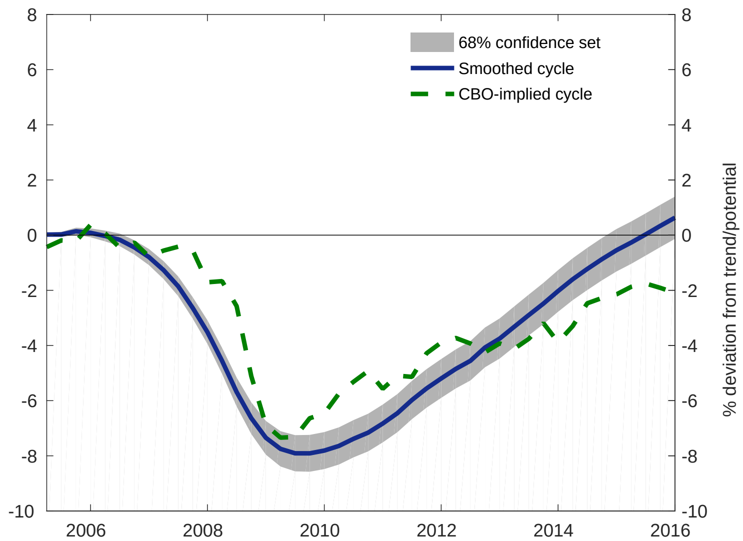
$$\text{Aggregate GDP cycle}_t \approx \bar{\alpha} c_t + \bar{\upsilon}_t, \tag{19}$$

with $\bar{\alpha} = \sum_{i=1}^{n} w_i \alpha_i$ and $\bar{\upsilon}_t = \sum_{i=1}^{n} w_i \upsilon_{it}$, where the smoothed estimates of $c_{t|T}$ and $\upsilon_{it|T}$ are used to obtain the estimated cycle and w_i is the average contribution of state i's GDP to aggregate GDP in the sample period.

The smoothed estimate of the cycle of aggregate GDP for the United States, along with the implied cycle from the CBO estimate of potential output, appears in Figure 5. After reaching a peak of about 0.14% in the fourth quarter of 2005, the estimated cycle starts to decline and becomes negative in the second quarter of 2006. The trough occurs in the fourth quarter of 2009, when the cycle reaches about -7.9%. The estimated cycle differs with respect to the CBO-implied cycle in that during the recession the latter reached -7.3% in the second quarter of 2009, about ½ percentage point higher than and two quarters before the former. However, the trough of the estimated cycle coincides with that of the BOG estimate, which reached -7.6% in the fourth quarter of 2009.

The estimations results regarding the output gap at the trough of the recessions can

Figure 5: Smoothed Estimate of the Aggregate GDP Cycle from State-level Data



Source: Congressional Budget Office (CBO) and author's calculations.

also illustrate those states that experienced the largest deviations from trend. Figure 6 shows these results. In concordance with the results on the average real GDP annual growth rates in Figure 3, Nevada, Michigan, and Florida are the most affected states. Arizona, California, Delaware, Rhode Island, and Ohio also have negative output gaps larger than 10%. Eleven states—including, for example, Illinois and Indiana—experienced output gaps between 7% and 10%. Eighteen states—including, for example, Massachusetts and Texas—reached output gaps between -5% and -7%. The least affected states that still had negative output gaps were the District of Columbia and states like South Dakota, West Virginia, and Louisiana. Finally, there is a group of states that, according to the estimation, did not go through episodes of negative output gaps during the 2006:Q2-2010:Q1 period. These states are Alaska, New Mexico, Oregon, and Wyoming, three of which are states where the oil and gas sectors, as well as mining, are important contributors.

Figure 5 also shows that the aggregate GDP cycle has almost constantly increased after the recession to end at about 0.6% in the first quarter of 2016, well above the CBO-implied estimate of -2.1%. In any case, the 68% confidence set of the estimated cycle contains zero, indicating that it is not unreasonable to predict that the output gap is closed by the first quarter of 2016. At the state level, the results in Figure 7 classify the states according to the estimated output gap at the end of the sample period.

Most of the states lie in the region of a positive estimated output gap. The exceptions are Alaska, Louisiana, New Mexico, Oregon, West Virginia, and Wyoming, several of which have relevant oil, gas, and mining industries. In fact, there are reports that West Virginia and Wyoming have recently been suffering from a recession that is most likely due to lower commodity prices (see Matthews, 2016). On the other side of the spectrum, with the largest

Figure 6: Largest Negative Output Gaps between 2006:Q2 and 2010:Q1

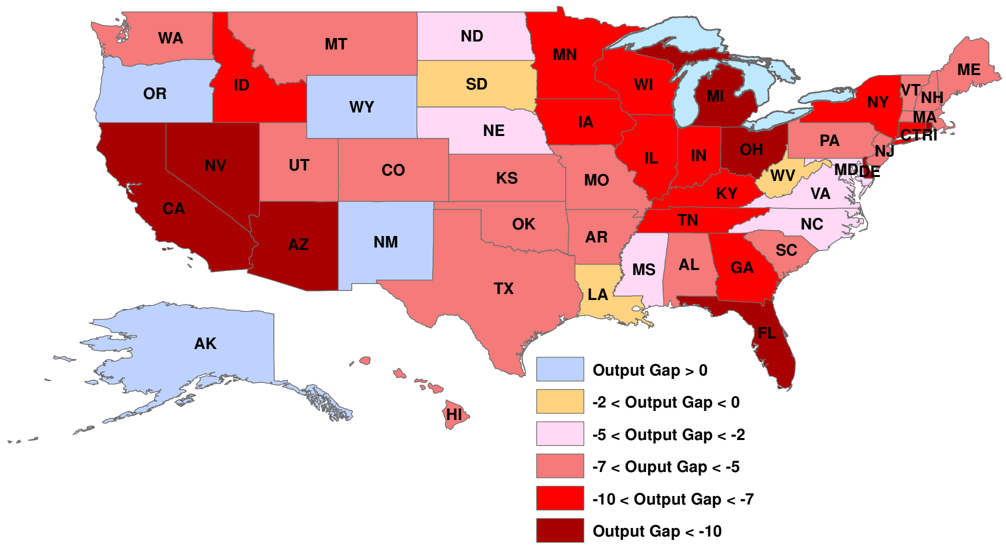


Figure 7: State Output Gaps in 2016:Q1

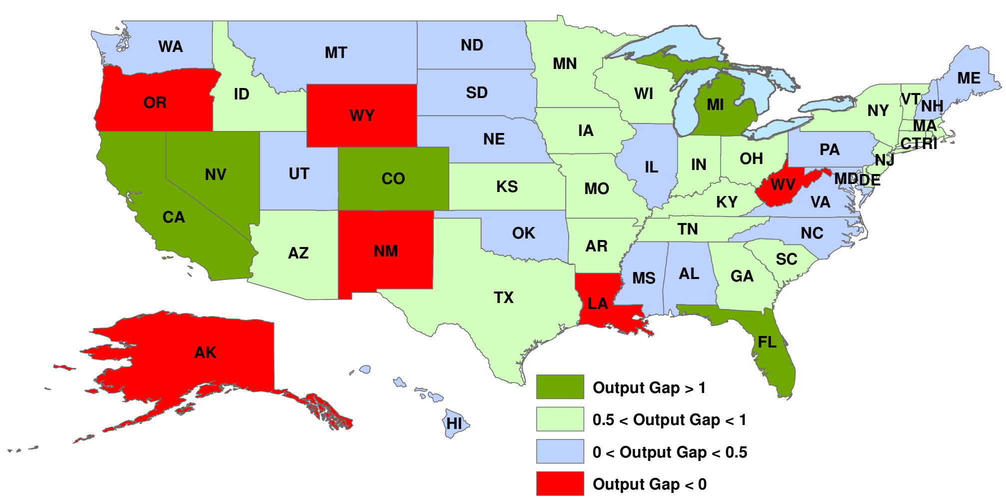
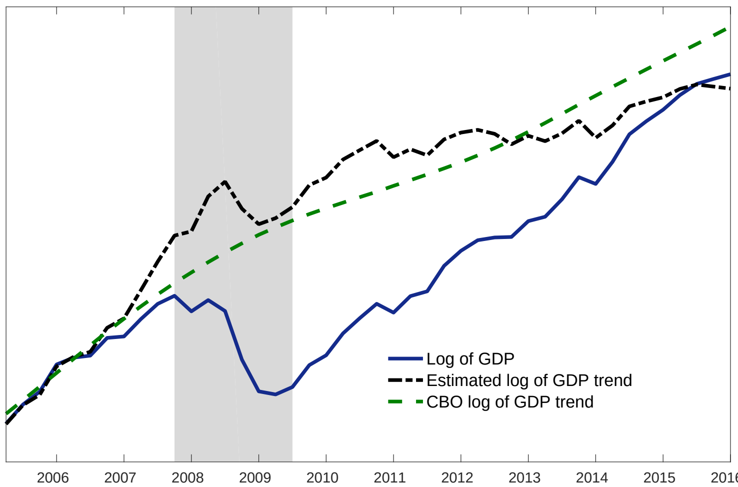


Figure 8: Estimate of the aggregate GDP trend



Note: The shaded area indicates an NBER recession.
Source: Congressional Budget Office (CBO), Federal Reserve Economic Data (FRED), and author's calculations

positive output gaps, are California, Colorado, Florida, Nevada, and Michigan. It might be counterintuitive that the states with the most-negative output gaps during the recession, as illustrated in Figure 6, are also the states with the highest output gaps at the end of the sample. One possible explanation is that the GDP trends of these states have been significantly affected by the recession, hence the level of output necessary to achieve a positive output gap is relatively lower than for other states that did not see their trends adjusted downward.

As stated previously, there are differences between the estimate of the aggregate output gap and the CBOs, both in terms of magnitude and timing. This situation has implications for the estimate of the trend. Specifically, between the third quarter of 2008—when the estimated trend peaks during the recession—and the first quarter of 2009, the estimated decline of the aggregate trend in the model used in this paper is about 1.9%. Figure 8 shows the sudden drop in the estimated trend of GDP during the recession, which occurs only gradually in the CBO counterpart. The figure also shows that the estimated trend has experienced a marked deceleration in recent years compared with the years before the recession. In fact, between 2006 and 2008 the trend is estimated to have grown at an average annual rate of about 3¼ percent, whereas the average growth rate has been about ¾ percent since 2012. Additionally, there is a substantial difference compared with the growth rate of the CBO potential in recent years. The CBO estimates that potential has been growing at an annual rate of about 1½ percent since 2012, twice the estimate in this paper.

At the state level, Figure 9 shows selected states whose trends have increased significantly above or below the estimated aggregate trend. The first panel illustrates how the states that benefited the most from high commodity prices (oil, in particular)—North Dakota, Texas, Utah, and Oklahoma—significantly increased their trend output during the past decade. The middle panel shows the states in the rust belt that suffered during the Great Recession.

Michigan was not only affected in terms of its cyclical situation, but its trend GDP has also declined after the recession. As mentioned previously, this decline in the trend slope for Michigan can be associated with its positive estimated output gap at the end of the sample. The other states—namely, Indiana, Kentucky, Illinois, and Ohio—have recovered, but at a slower pace than the aggregate. Finally, the third panel shows states that are unusual in their characteristics such that they cannot be grouped with the states in the first two panels. Oregon, for example, has seen its manufacturing industry grow significantly in the past decade. The important increase in trend output for Oregon and a deceleration of output toward the end of the sample can explain the negative output gap discussed previously for this state. South Dakota's trend may have been driven by industries related mainly to agriculture and financial services. States whose trend output growth has been among the lowest of the country are Louisiana, Maine, and Nevada, according to the model used in this paper.

6.3 State Trend Intercepts and Cycle Loading Coefficients

To illustrate the features of the state economies, in Figure 10 I present the results related to the average output trend growth rates, μ_i, and the common cycle loadings, α_i. Figure 10a shows that the states that experienced higher average growth rates of trend output are North and South Dakota, followed by Alaska, Nebraska, Oregon, Utah, Texas, Washington, and Wyoming. In most of these states, the mining sector strongly influences output growth, as Figure 4 shows. Some of these states with the highest average output trend growth rates also appear in Figure 9 as the states with the fastest growing trends. Those states that do not appear in the state trends figure despite having high estimated values of μ_i may have experienced negative idiosyncratic trend shocks. On the other side of the spectrum, with average trend annualized growth rates lower than 0.4%, are Maine and Michigan. As indicated in Section 5.3, the construction industry was the most negative contributor to growth in Michigan, and the same is true for Maine.

With regard to the loading coefficients on the common cycle, Figure 10b shows the estimation results for the states ordered by quartiles. Among the states with the highest loadings are Arizona, Florida, Michigan, and Nevada, which are some of the states that experienced the most negative output gaps during the recession years. One can interpret this result as these states being the most sensitive to the national business cycle conditions during the sample period. With respect to the states with the lowest loadings, it turns out that some of them actually have negative posterior mean estimates. This set includes Alaska, Louisiana, New Mexico, Oregon, and Wyoming, although in all of the cases, the confidence sets of the loading coefficients include zero. These states seem to be among the less sensitive to the national business cycle conditions.

6.4 Variance Decomposition of States' Cycles

Another way to look at the sensitivity of the states economies cycles to the common cycle is to compute the variance decomposition of the state-specific cycle. From equation (16), the cycle for each state $i = 1, 2, \ldots, n$ is specified as $c_{it} = \alpha_i c_t + v_{it}$. Hence, one can obtain the fraction of the variance of the cycle that is due to the common cycle, $\alpha_i^2 \operatorname{var}(c_t)/ \operatorname{var}(c_{it})$,

Figure 9: Estimate of the GDP Trend for Selected States

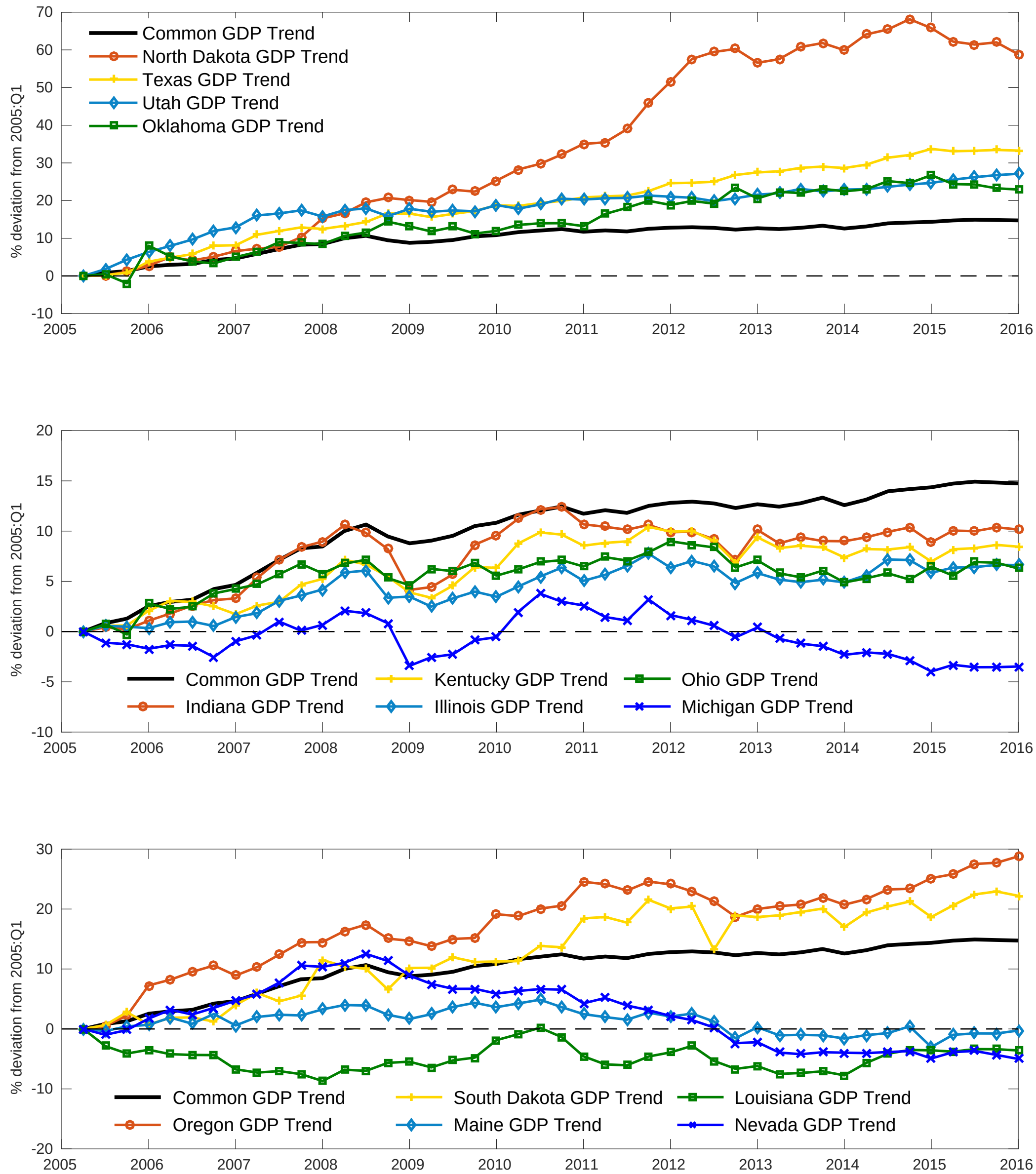
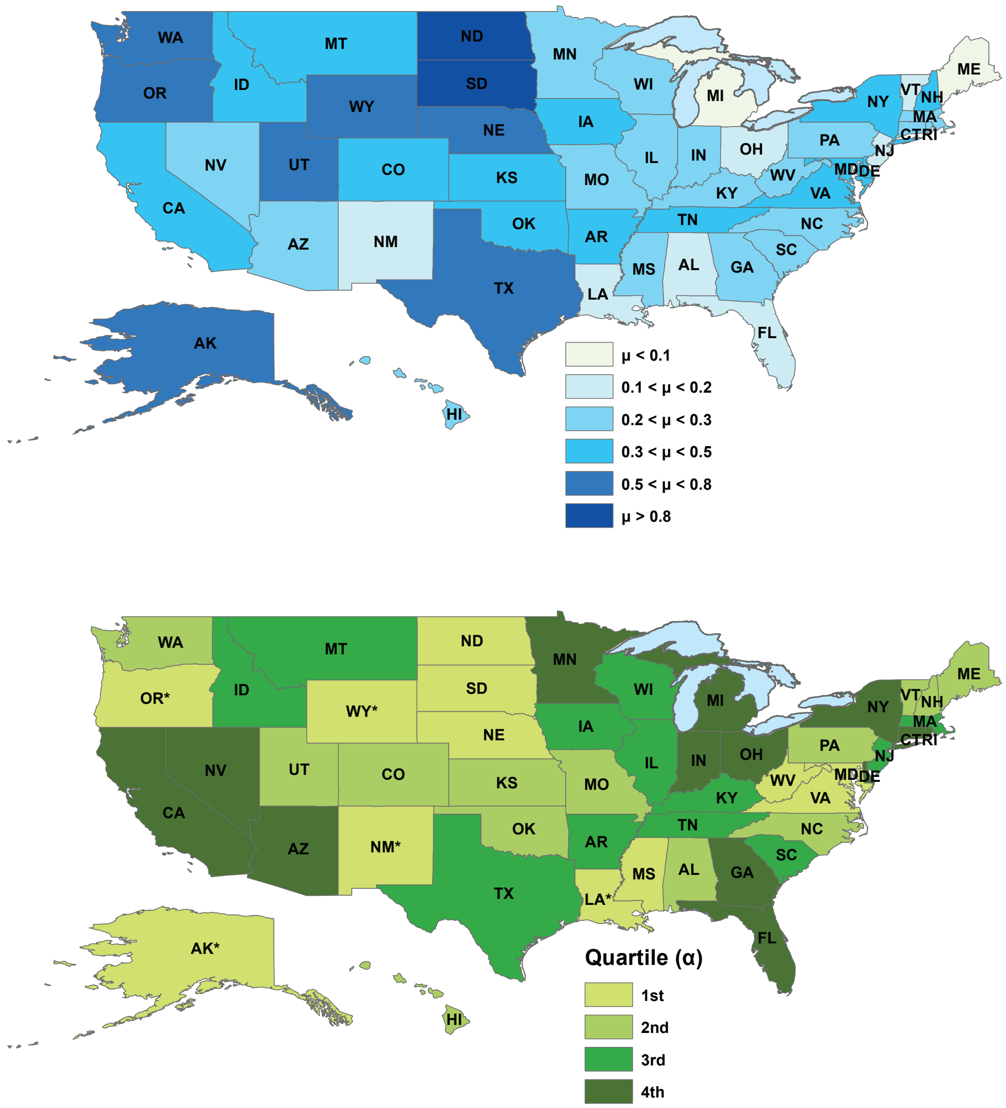
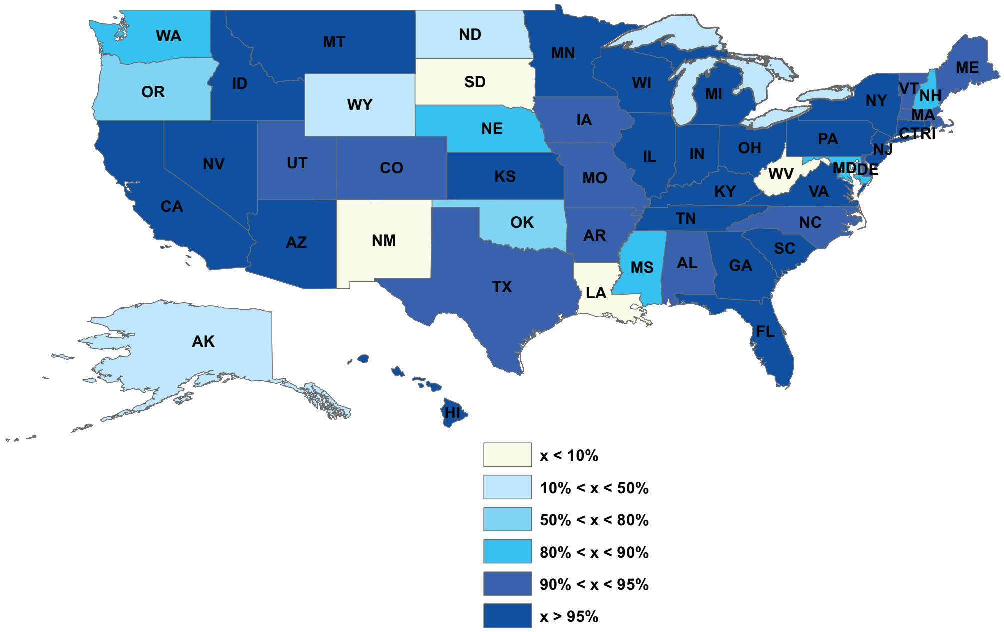


Figure 10: Intercept and Loading Coefficients Estimates



Note: States with an asterisk (*) have a negative estimated posterior mean coefficient α_i.

Figure 11: Variance Decomposition of the State's Cycles



Note: x = $100 \times \frac{\alpha_i^2 \operatorname{var}(c_t)}{\operatorname{var}(c_{it})}$ is the proportion of the variance of the state's cycle that is explained by the variance of the common cycle.

and the fraction that corresponds to the idiosyncratic component, $\operatorname{var}(\upsilon_{it})/\operatorname{var}(c_{it})$. Figure 11 shows the percent of the variance of the cycle of each of the states that is due to the common cycle.

Connecticut, California, Georgia, and Nevada are among the states with cycle variability that is explained the most by the variance of the common cycle, whereas Louisiana, New Mexico, South Dakota, and West Virginia have the smallest variation of their cycle attributed to the common cycle. In other words, most of the variability of the cycle in these states comes primarily from features inherent to those states. For example, New Mexico and West Virginia are states where mining can have very particular cyclical effects which are not necessarily related to the aggregate economic cycle.

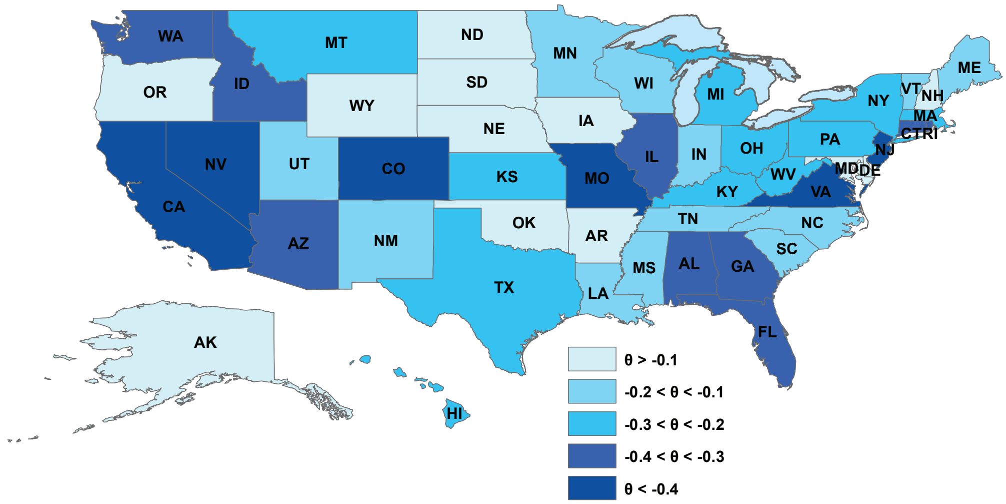
6.5 Variance Decomposition of Aggregate GDP

As described in Section 3.1, the common trend and cycle components imply co-movement between the state economies that, in turn, influence the variability of the aggregate GDP. Hence, one can compute the variance decomposition of aggregate GDP growth, as follows:[5]

$$\operatorname{var}(\Delta\% \text{ Aggregate GDP}) = \operatorname{var}(\Delta \text{ Aggregate GDP trend}) + \operatorname{var}(\Delta \text{ Aggregate GDP cycle}).$$

[5]The full derivation appears in Appendix D.

Figure 12: Okun's Law Coefficients ($\theta_{1i} + \theta_{2i}$)



Note:θ is the sum $\theta_1 + \theta_2$ for each state.

Therefore, the percent of the variance of GDP growth that is due to the variance of the change in the cycle is given by the following:

$$\frac{\text{var}(\Delta \text{ Aggregate GDP cycle})}{\text{var}(\Delta \text{ Aggregate GDP trend}) + \text{var}(\Delta \text{ Aggregate GDP cycle})} = 47\%.$$

The contribution of the variability of the cycle to the variability of GDP growth of 47% is lower than estimates obtained with longer samples. For example, Gonzalez-Astudillo and Roberts (2016) find that the contribution is around 60% or 65%, depending on the assumption about the correlation between trend and cycle components. One possible explanation for the lower contribution obtained in this paper is that the sample exclusively contains the Great Recession, which the results indicate affected the trend of GDP more strongly than what estimations with longer samples would obtain.

6.6 State Okun's Law Coefficients

Another dimension along which the results can be analyzed are the cyclical features of the labor markets, in particular the sensitivity of the unemployment gap to the output gap in each state. This feature can be measured by the sum of the Okuns law coefficients, $\theta_{1i} + \theta_{1i}$. The higher this sum in absolute value, the more responsive is the unemployment rate to the cyclical fluctuations of output. Figure 12 presents the results grouped by values of the posterior mean estimates.

The states with more-cyclical labor markets would be California, Colorado, Missouri, Nevada, New Jersey, and Virginia. The states with less cyclical labor markets would be Alaska, New Hampshire, North Dakota, and South Dakota. Three of these states are mining states, a sector that does not require significant labor force inputs. The average of the posterior mean of the Okuns law coefficients across states is about -0.2, which is smaller in

Table 3: Prior Distributions of the Parameters - Aggregate Data

Parameter	Distribution	Mean	Standard Deviation
ϕ_1	Truncated Normal	1.5	1
ϕ_2	Truncated Normal	-0.6	1
μ	Truncated Normal	0.8	0.8
θ_1	Normal	-0.25	0.25
θ_2	Normal	-0.25	0.25
σ_ε^2	Inverse Gamma	—	—
$\sigma_{\eta^y}^2$	Inverse Gamma	—	—
$\sigma_{\eta^u}^2$	Inverse Gamma	—	—

absolute value than the usual coefficient of -0.5.

7 Evaluation of Results

This section evaluates the performance of the proposed model in comparison to a UC model of aggregate real GDP and unemployment rate. The aim is to offer insights about the gain of including cross-sectional information to estimate the cycle of output. First, I present the results of the estimation of such an aggregate model and compare them to the results of the model with state-level data, in particular with regard to the efficiency of the estimator of the GDP cycle. Then, I evaluate the forecast performance of the UC model with many cross sections proposed in this paper. Finally, I discuss extensions to the model and the limitations of the sample size.

7.1 Unobserved Components Model with Aggregate Data

In this section, I compare the results obtained from the estimation of the UC model with many cross sections to the results obtained from the estimation of the model with aggregate data given in equations (1)-(6) over the same sample period.

The prior distributions of the coefficients appear in Table 3 and are similar to those in the specification with data at the state level; in this case, however, it is possible to estimate the variances of the aggregate cycle and trend shocks, σ_ε^2, $\sigma_{\eta^y}^2$, and $\sigma_{\eta^u}^2$. I use quarterly GDP and unemployment rate data from the St. Louis FRED database. Results appear in Table 4.

The mean annual growth rate of real GDP is estimated to be about 1.4%, very similar to the estimate with state-level data obtained from weight-averaging the estimated state-level mean growth rates. With aggregate data, the estimation yields complex roots in the lag polynomial 88% of the times, and the 90% confidence set for the period of the cycle is between approximately 5¾ and 26¼ years, a set substantially wider than with state-level data. The estimated mean of the period is approximately 12½ years, 3¼ years shorter than with state-level data. Finally, the variance decomposition of real GDP growth indicates that about 46% of its variation is due to variations in the cycle, 1% below the result with state-level data.

Table 4: Parameter Estimates with Aggregate GDP and Unemployment Rate Data

Parameter	Posterior Mean	90% Confidence Set
μ	0.34	[0.18,0.50]
ϕ_1	1.81	[1.63,1.95]
ϕ_2	-0.85	[-0.98,-0.67]
σ_ε	0.27	[0.15,0.40]
σ_{η_y}	0.56	[0.47,0.67]
θ_1	0.07	[-0.27,0.48]
θ_2	-0.70	[-1.21,-0.35]
σ_{η_u}	0.20	[0.19,0.30]

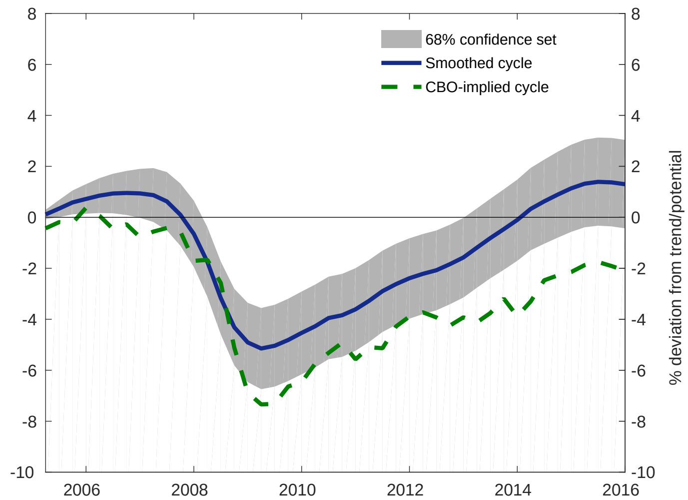
With respect to the Okun's law coefficients, the long-run sensitivity of the unemployment gap to the output gap is about -0.64. Owyang, Vermann and Sekhposyan (2013) investigate the Okun's law coefficients across states, although using the growth rates version of the law, and find that there can be discrepancies between the estimates with state-level data (like those in Figure 12) and the estimate obtained from aggregate data.

The smoothed cycle of this estimation along with the CBO-implied output gap appear in Figure 13. The trough of the cycle reaches about -5.1% in the second quarter of 2009; this value is about 2¾ percentage points higher than the estimate with state-level data. Also, the estimated cycle recovers faster than the estimate with state-level data, reaching about 1.2% in the first quarter of 2016, compared with 0.6% with state-level data. These results are consistent with an estimated output trend growing, at the end of the sample, at an average annual rate of a little above 1 percent with aggregate data, compared with ¾ percent with state-level data.

Stock and Watson (2016) argue that the width of the confidence set of the cycle, in this case, reflects two sources of uncertainty: the signal extraction uncertainty conditional on the values of the parameters of the model and the parameter uncertainty. The model with state-level data contains more information than the model with aggregate data, and hence the signal extraction uncertainty is smaller in the former. But the model with aggregate data contains many fewer parameters than the model with state-level data, which implies greater parameter uncertainty in the latter. Therefore, there is no a priori ranking of the width of the posterior intervals between the two models.

However, the results clearly indicate that the incorporation of many cross sections reduces greatly the signal extraction uncertainty despite the large number of parameters that could increase the parameter uncertainty. When one compares the width of the 68% confidence sets between Figure 5, which is the estimate of the cycle with state-level data, and Figure 13, with aggregate data, it is evident the much narrower confidence set is in the case with many cross sections.

Figure 13: Smoothed Estimate of the GDP Cycle from Aggregate Data



Source: Congressional Budget Office (CBO) and author's calculations.

7.2 Forecasting Performance

In this section, I perform an out-of-sample forecast evaluation of the proposed model against two models of aggregate real GDP. The first model is a simple AR(1) process for the growth rate of real GDP. The second is the UC model of aggregate real GDP and the unemployment rate, whose estimation is described in the previous section.

Given the short span of the sample, the out-of-sample forecast evaluation starts with the sample covering the 2005:Q2 - 2012:Q3 period—that is, 30 quarters—and then predicts real GDP growth one and four quarters ahead. The forecasts are constructed using the prediction step of the Kalman filter directly for the model with aggregate data, and, in the case of the model with state-level data, obtaining the growth rate of aggregate real GDP from the weighted average of the states' real GDP growth forecasts obtained from the prediction step of the Kalman filter for each of them. The forecast of the AR(1) process is trivial. The forecasts are obtained from 10,000 draws each period.

Table 5 presents the average and standard deviation of the mean squared forecast errors of the one- and four-quarter-ahead forecast errors for the three models. A few results stand out from this exercise. First, incorporating richer autoregressive dynamics improves the four-quarter-ahead forecast compared with the simple AR(1) model. Second, incorporating state-level data to the UC model to decompose GDP into trend and cycle more accurately forecasts real GDP growth out of sample. Third, the model with state-level data has a lower variability within the mean square error than the model with aggregate data or the AR(1) model.

Table 5: Mean Squared Forecast Errors for Three Competing Models

Model	One quarter ahead	Four quarters ahead
AR(1)	0.53 (0.04)	0.57 (0.09)
Aggregate	0.57 (0.05)	0.56 (0.08)
State-level	0.49 (0.02)	0.48 (0.06)

Note: In parenthesis appears the standard deviation of the mean square forecast error.

7.3 Extensions and Sample Size Limitation

Given the importance of inflation in defining potential output, it is natural to incorporate this variable in the system I have considered in this paper. Examples of UC models that include inflation to estimate potential output in the literature include Kuttner (1994); Roberts (2001); Basistha and Nelson (2007); Basistha (2007). Unfortunately, there is not information on price inflation at the state level. However, it is possible to introduce an aggregate measure of inflation in the system in equations (7)-(18) to incorporate the information this variable provides on potential output. Hence, I augment the aforementioned system with the following Phillips curve equation:

$$\pi_t = \beta \pi_{t-1} + (1 - \beta)\, \pi_t^e + \kappa \times \text{Aggregate GDP cycle}_t + \eta_t^\pi,$$

with $\beta \in [0, 1]$, where π_t denotes price inflation in period t, π_t^e denotes expected price inflation in period t, Aggregate GDP cycle_t is the aggregate cycle as defined in equation (19), and η_t^π is an i.i.d. disturbance (see Basistha and Nelson, 2007, for more details). To estimate the model, I incorporate information on the core personal consumption expenditures price deflator inflation and the corresponding mean of the four-quarter ahead predictions from the survey of professional forecasters.

Results adding inflation look very similar to the results without inflation. In particular, the Phillips curve coefficient, κ, results in a value too close to zero for inflation to provide information about the aggregate cycle. Blanchard, Cerutti and Summers (2015), for example, find that the slope of the Phillips curve has substantially declined in recent years for the United States. Hence, it does not seem that inflation could make a significant difference to the results obtained with real GDP and unemployment rate data at the state level.

Regarding the short sample size of the state-level data along the time dimension, it could be a concern that the trend of real GDP is estimated with just about 50 quarters, or about 12 years. In particular, the estimates are probably not characterizing the full time series properties of the output trend or cycle because of the particular features of the Great Recession, which is fully included in the sample. Nevertheless, the aim of the paper is to illustrate a novel way to decompose output into trend and cycle exploiting the cross-sectional variation of the data. In the previous two sections, I have shown the advantages of such an approach compared with estimates obtained using aggregate data only and the same time span. Hence, estimating the model proposed with a longer sample could potentially provide more precise estimates, but it would still keep the appeal compared with models that use aggregate data.

8 Conclusions

In this paper, I propose a model to obtain a trend-cycle decomposition of GDP using state-level real GDP and unemployment rate data covering the 2005:Q1–2016:Q1 period. The estimation shows that the economy reached a trough in the fourth quarter of 2009 in which the output gap was about -7.9%, whereas the output gap is estimated to be about 0.6% at the end of the sample in the first quarter of 2016. Across states, the recession hit states in the rust belt especially hard, as well as Arizona, California, Florida, and Nevada. In the first quarter of 2016, states such as Alaska, West Virginia, and Wyoming are experiencing negative output gaps likely because of low commodity prices.

An evaluation of the results shows that incorporating cross-sectional information into the estimation of a UC model of trend-cycle decomposition of GDP improves the precision of the estimator of the cycle. The results also show that the model with state-level data performs better in terms of forecasting compared with a model that only uses aggregate data.

Although introducing cross-sectional variation from many economic units can help reduce the uncertainty around the output gap estimate, having a longer sample along the time dimension would better help identify the output trend and, hence, the cycle. Adopting a mixed-frequency approach can help achieve that goal using yearly state-level data. However, the computational cost can be high as a result of the increase of latent states to accommodate yearly along with quarterly frequencies.

One could modify the approach described in this paper to include a time-varying drift in the trend component of GDP to a different cross-sectional system with a longer time span, such as data on GDP at the industry level, to obtain another measure of the output gap. One could also use information on inflation at the country level, along with real GDP and the unemployment rate, to obtain a measure of the common cycle across those countries. These are topics for future research.

Appendix

A State-space Model in Matrix Form

The model in Equations (7)-(17) can be written in matrix form as the following:

$$\mathbf{z}_{it} = \mathrm{C}(\mathbf{\Theta}_{\mathrm{mi}}) + \mathrm{H}(\mathbf{\Theta}_{\mathrm{mi}})\mathbf{x}_{it} + \mathbf{w}_{it}, \qquad \mathbf{w}_{it}|\mathfrak{F}_{t-1} \sim \text{ iid } \mathbb{N}(0, \mathrm{R}(\mathbf{\Theta}_{\mathrm{mi}})) \tag{20}$$

$$\mathbf{x}_{it} = \mathrm{F}(\mathbf{\Theta}_{\mathrm{si}})\mathbf{x}_{i,t-1} + \mathbf{G}\mathbf{v}_{it}, \qquad \mathbf{v}_{it}|\mathfrak{F}_{t-1} \sim \text{ iid } \mathbb{N}(0, \mathrm{Q}(\mathbf{\Theta}_{\mathrm{si}})), \tag{21}$$

where

$$\mathbf{z}_{it} = \begin{bmatrix} \Delta y_{it} \\ \Delta u_{it} \end{bmatrix}, \qquad \mathbf{x}_{it} = \begin{bmatrix} c_t \\ c_{t-1} \\ c_{t-2} \\ \eta_t^y \\ \eta_t^u \\ \upsilon_{it} \\ \upsilon_{i,t-1} \\ \upsilon_{i,t-2} \end{bmatrix}, \qquad \mathbf{w}_{it} = \begin{bmatrix} \eta_{it}^y \\ \eta_{it}^u \end{bmatrix}, \qquad \mathbf{v}_{it} = \begin{bmatrix} \varepsilon_t \\ \eta_t^y \\ \eta_t^u \\ \zeta_{it} \end{bmatrix},$$

$$\mathrm{C}(\mathbf{\Theta}_{\mathrm{mi}}) = \begin{bmatrix} \mu_i + \delta_i^y \mu \\ 0 \end{bmatrix},$$

$$\mathrm{H}(\mathbf{\Theta}_{\mathrm{mi}}) = \begin{bmatrix} \alpha_i & -\alpha_i & 0 & \delta_i^y & 0 & 1 & -1 & 0 \\ \alpha_i\theta_{1i} & \alpha_i(\theta_{2i} - \theta_{1i}) & -\alpha_i\theta_{2i} & 0 & \delta_i^u & \theta_{1i} & \theta_{2i} - \theta_{1i} & -\theta_{2i} \end{bmatrix},$$

$$\mathrm{R}(\mathbf{\Theta}_{\mathrm{vi}})) = \begin{bmatrix} \sigma^2_{\eta_i^y} & 0 \\ 0 & \sigma^2_{\eta_i^u} \end{bmatrix},$$

$$\mathrm{F}(\mathbf{\Theta}_{\mathrm{si}}) = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$
\mathrm{Q}(\boldsymbol{\Theta}_{\mathrm{vi}}) = \begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 & 0 \\ 0 & \sigma_{\eta^y}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\eta^u}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\zeta_i}^2 \end{bmatrix},
$$

and

$$
\begin{aligned}
\boldsymbol{\Theta}_{\mathrm{mi}} &= \{\mu, \mu_i, \alpha_i, \theta_{1i}, \theta_{2i}, \delta_i^y, \delta_i^u, \sigma_{\eta_i^y}^2, \sigma_{\eta_i^u}^2\} \\
\boldsymbol{\Theta}_{\mathrm{si}} &= \{\phi_1, \phi_2, \rho_i, \sigma_e^2, \sigma_{\eta^y}^2, \sigma_{\eta^u}^2, \sigma_{\zeta_i}^2\},
\end{aligned}
$$

for $i = 1, \ldots, n$.

B Details on the Metropolis-within-Gibbs Algorithm

Let $\mathbf{z}_{it}$, $\mathbf{x}_{it}$, $\boldsymbol{\Theta}_{\mathrm{mi}}$, and $\boldsymbol{\Theta}_{\mathrm{si}}$ for $i = 1, 2, \ldots, n$, be defined as in Appendix A. Let $\mathbf{Z}_T = \{\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2, \ldots, \tilde{\mathbf{z}}_T\}$ denote the observed data and let $\mathbf{X}_T = \{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_T\}$. Here, $\tilde{\mathbf{z}}_t = \{\mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{nt}\}$ and $\tilde{\mathbf{x}}_t = \{\mathbf{x}_{1t}, \mathbf{x}_{2t}, \ldots, \mathbf{x}_{nt}\}$. Denote $\boldsymbol{\Theta}_{\mathrm{m}} = \bigcup_{\mathrm{i}=1}^{n} \boldsymbol{\Theta}_{\mathrm{mi}}$ and $\boldsymbol{\Theta}_{\mathrm{s}} = \bigcup_{\mathrm{i}=1}^{n} \boldsymbol{\Theta}_{\mathrm{si}}$.

Partition $\boldsymbol{\Theta}_{\mathrm{si}} = \boldsymbol{\Theta}_{\mathrm{si}}^1 \bigcup \boldsymbol{\Theta}_{\mathrm{si}}^2 \bigcup \boldsymbol{\Theta}_{\mathrm{si}}^3$, where

$$
\begin{aligned}
\boldsymbol{\Theta}_{\mathrm{si}}^1 &= \{\phi_1, \phi_2\}, \\
\boldsymbol{\Theta}_{\mathrm{si}}^2 &= \{\rho_i, \sigma_{\zeta_i}^2\}, \\
\boldsymbol{\Theta}_{\mathrm{si}}^3 &= \{\sigma_e^2, \sigma_{\eta^y}^2, \sigma_{\eta^u}^2\}.
\end{aligned}
$$

Notice that the identification conditions imply that $\boldsymbol{\Theta}_{\mathrm{si}}^3$ is not random.

Also, partition $\boldsymbol{\Theta}_{\mathrm{mi}} = \boldsymbol{\Theta}_{\mathrm{mi}}^1 \bigcup \boldsymbol{\Theta}_{\mathrm{mi}}^2$, where

$$
\begin{aligned}
\boldsymbol{\Theta}_{\mathrm{mi}}^1 &= \{\mu_i, \alpha_i, \theta_{1i}, \theta_{2i}, \delta_i^y, \delta_i^u\}, \\
\boldsymbol{\Theta}_{\mathrm{mi}}^2 &= \{\sigma_{\eta_i^y}^2, \sigma_{\eta_i^u}^2\},
\end{aligned}
$$

where μ has been excluded because it is fixed under the identification conditions.

The Metropolis-within-Gibbs procedure is as follows:

1. Draw $\mathbf{X}_T$ from $p(\mathbf{X}_T|\mathbf{Z}_T, \boldsymbol{\Theta}_{\mathrm{m}}, \boldsymbol{\Theta}_{\mathrm{s}})$ using the Durbin and Koopman (2002) simulation smoother.

2. Draw $\boldsymbol{\Theta}_{\mathrm{si}}^1$ from $p(\boldsymbol{\Theta}_{\mathrm{si}}^1|\mathbf{Z}_T, \mathbf{X}_T, \boldsymbol{\Theta}_{\mathrm{m}}, \boldsymbol{\Theta}_{\mathrm{si}}^2, \boldsymbol{\Theta}_{\mathrm{si}}^3)$ using the normal-inverse-gamma distribution.

3. Draw $\boldsymbol{\Theta}_{\mathrm{mi}}^1$ from $p(\boldsymbol{\Theta}_{\mathrm{mi}}^1|\mathbf{Z}_T, \mathbf{X}_T, \boldsymbol{\Theta}_{\mathrm{mi}}^2, \boldsymbol{\Theta}_{\mathrm{m(/i)}}, \boldsymbol{\Theta}_{\mathrm{s}})$ for $i = 1, 2, \ldots, n$, using the Metropolis-Hastings algorithm, where the proposal density is given by the likelihood function of the following nonlinear bivariate model multiplied by the prior distribution of $\boldsymbol{\Theta}_{\mathrm{mi}}^1$:

$$
\mathbf{z}_{it} = \mathrm{C}(\boldsymbol{\Theta}_{\mathrm{mi}}^1) + \mathrm{H}(\boldsymbol{\Theta}_{\mathrm{mi}}^1)\mathbf{x}_{it} + \mathbf{w}_{it}, \quad \mathbf{w}_{it}|\mathfrak{F}_{t-1} \sim \text{ iid } \mathbb{N}(0, \mathrm{R}(\boldsymbol{\Theta}_{\mathrm{mi}}^2)), \quad t = 1, 2, \ldots, T.
$$

4. Draw $\mathbf{\Theta}^2_{\text{mi}}$ from $p(\mathbf{\Theta}^2_{\text{mi}}|\mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}^1_{\text{mi}}, \mathbf{\Theta}_{\text{m(/i)}}, \mathbf{\Theta}_{\text{s}})$ for $i = 1, 2, \ldots, n$ using the inverse gamma distribution.

5. Draw $\mathbf{\Theta}^2_{\text{si}}$ from $p(\mathbf{\Theta}^2_{\text{si}}|\mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}_{\text{mi}}, \mathbf{\Theta}^1_{\text{si}}, \mathbf{\Theta}^3_{\text{si}})$ for $i = 1, 2, \ldots, n$, using the Metropolis-Hastings algorithm, where the proposal density is given by the likelihood function of the following nonlinear bivariate model multiplied by the prior distribution of $\mathbf{\Theta}^2_{\text{si}}$:

$$\mathbf{z}_{it} = \mathrm{C}(\mathbf{\Theta}^1_{\text{mi}}) + \mathrm{H}^-(\mathbf{\Theta}^1_{\text{mi}})\mathbf{x}^-_{it} + \mathbf{e}_{it},$$

where

$$\begin{aligned}
\mathrm{H}^-(\mathbf{\Theta}^1_{\text{mi}}) &= \begin{bmatrix} \alpha_i & -\alpha_i & 0 & \delta^y_i & 0 \\ \alpha_i\theta_{1i} & \alpha_i(\theta_{2i} - \theta_{1i}) & -\alpha_i\theta_{2i} & 0 & \delta^u_i \end{bmatrix}, \\
\mathbf{x}^-_{it} &= \begin{bmatrix} c_t \\ c_{t-1} \\ c_{t-2} \\ \eta^y_t \\ \eta^u_t \end{bmatrix}, \\
\mathbf{e}_{it} &= \begin{bmatrix} \Delta\upsilon_{it} + \eta^y_{it} \\ \theta_{1i}\Delta\upsilon_{it} + \theta_{2i}\Delta\upsilon_{i,t-1} + \eta^u_{it} \end{bmatrix} \equiv \begin{bmatrix} e^y_{it} \\ e^u_{it} \end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{var}\left(e^y_{it}\right) &= \sigma^2_{\Delta\upsilon_{it}} + \sigma^2_{\eta^y_i}, \\
\mathrm{cov}\left(e^y_{it}, e^y_{i,t-j}\right) &= -\frac{1}{2}\rho_i^{j-1}(1-\rho_i)\sigma^2_{\Delta\upsilon_{it}}, \quad \forall j \neq 0, \\
\mathrm{var}\left(e^u_{it}\right) &= \theta^2_{1i}\sigma^2_{\Delta\upsilon_{it}} + \sigma^2_{\eta^u_i}, \\
\mathrm{cov}\left(e^u_{it}, e^u_{i,t-j}\right) &= \begin{cases} \sigma^2_{\Delta\upsilon_{it}}\left(\theta_{1i}\theta_{2i} - \frac{1-\rho_i}{2}\left(\theta^2_{1i} + \rho_i\theta_{1i}\theta_{2i} + \theta^2_{2i}\right)\right) & \text{if } j = 1 \\ -\frac{1}{2}\rho_i^{j-2}(1-\rho_i)\sigma^2_{\Delta\upsilon_{it}}\left(\rho_i(\theta^2_{1i} + \theta^2_{2i}) + \theta_{1i}\theta_{2i}(1+\rho_i^2)\right) & \text{if } j \geq 2 \end{cases}, \\
\mathrm{cov}\left(e^y_{it}, e^u_{it}\right) &= \frac{1}{2}\sigma^2_{\Delta\upsilon_{it}}\left(2\theta_{1i} - \frac{1-\rho_i}{2}\theta_{2i}\right), \\
\mathrm{cov}\left(e^y_{it}, e^u_{i,t-j}\right) &= -\frac{1}{2}\rho_i^{j-1}(1-\rho_i)\sigma^2_{\Delta\upsilon_{it}}\left(\theta_{1i} + \rho_i\theta_{2i}\right), \quad \forall j \neq 0, \\
\mathrm{cov}\left(e^y_{i,t-j}, e^u_{it}\right) &= \begin{cases} \sigma^2_{\Delta\upsilon_{it}}\left(\theta_{2i} - \frac{1-\rho_i}{2}\theta_{1i}\right) & \text{if } j = 1 \\ -\frac{1}{2}\rho_i^{j-2}(1-\rho_i)\sigma^2_{\Delta\upsilon_{it}}\left(\theta_{2i} + \rho_i\theta_{1i}\right) & \text{if } j \geq 2, \end{cases}
\end{aligned}$$

with $\sigma^2_{\Delta\upsilon_{it}} = 2\frac{\sigma^2_{\zeta_i}}{1+\rho_i}$.

C Obtaining the Aggregate Trend and Cycle

The objective of the estimation is to obtain the trend-cycle decomposition of aggregate GDP, exploiting the cross-sectional variability of state-level data. Because of the nonlinearity implicit in the aggregation of the variables and the fact that state-level GDP appears in logs in the specification (7)-(18), an approximation is needed. The quarterly growth rate of aggregate real GDP is given by the following:

$$
\begin{aligned}
\Delta\%Y_t &= \sum_{i=1}^{n} w_{it}\Delta\%Y_{it} \\
&\approx \sum_{i=1}^{n} w_{it}\Delta y_{it} \\
&= \sum_{i=1}^{n} w_{it}\left(\Delta\tau_{it}^{y} + \Delta c_{it}\right) \\
&= \sum_{i=1}^{n} w_{it}\left(\delta_i^y(\mu + \eta_t^y) + \mu_i + \eta_{it}^y + \alpha_i \Delta c_t + \Delta\upsilon_{it}\right) \\
&\approx \underbrace{\bar{\delta^y}(\mu + \eta_t^y) + \bar{\mu} + \bar{\eta^y}_t}_{\Delta \text{ GDP Trend}} + \underbrace{\bar{\alpha}\Delta c_t + \bar{\upsilon}_t^y}_{\Delta \text{ GDP Cycle}},
\end{aligned}
$$

where Y_{it} is real GDP of state i, Y_t is aggregate real GDP in period t, and the contribution of of state's i GDP to aggregate GDP is denoted by w_{it}.

Hence, I can express the trend and cycle components of the aggregate GDP as

$$
\begin{aligned}
\text{GDP Trend} &\approx \bar{\delta^y}\tau_t^y + \bar{\xi}_t^y, \\
\text{GDP Cycle} &\approx \bar{\alpha}c_t + \bar{\upsilon}_t^y,
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\mu}_t &= \sum_{i=1}^{n} w_i\mu_i, \\
\bar{\delta^y} &= \sum_{i=1}^{n} w_i\delta_i^y, \\
\bar{\eta^y}_t &= \sum_{i=1}^{n} w_i\eta_{it}^y, \\
\bar{\xi}_t^y &= \sum_{i=1}^{n} w_i\xi_{it}^y, \\
\bar{\alpha} &= \sum_{i=1}^{n} w_i\alpha_i, \\
\bar{\upsilon}_t^y &= \sum_{i=1}^{n} w_i\upsilon_{it},
\end{aligned}
$$

where $w_i = \sum_{t=1}^{T} w_{it}/T$ is the sample average of each state's contributions to aggregate GDP. I use the smoothed estimates $c_{t|T}$ and $\upsilon_{it|T}$ to obtain the estimate of the cycle and, by residual, the trend.

D Derivation of the Variance Decomposition of Aggregate GDP

Given that (the log of) aggregate GDP can be specified as the sum of its aggregate trend and cycle, one can write the following:

$$\begin{aligned}
\mathrm{var}(\Delta\%Y_t) &= \mathrm{var}(\Delta \text{ Aggregate GDP trend}) + \mathrm{var}(\Delta \text{ Aggregate GDP cycle}) \\
&\approx \mathrm{var}(\bar{\delta^y}\Delta\tau_t + \Delta\bar{\xi}^y_t) + \mathrm{var}(\bar{\alpha}\Delta c_t + \Delta\bar{\upsilon}^y_t) \\
&= \bar{\delta^y}^2\,\mathrm{var}(\eta^y_t + \bar{\eta^y}_t) + \bar{\alpha}^2\,\mathrm{var}(\Delta c_t + \Delta\bar{\upsilon}^y_t) \\
&\approx \bar{\delta^y}^2\left(1 + \sum_{i=1}^{n} w_i^2\sigma^2_{\eta^y_i}\right) + \bar{\alpha}^2\left(2(\gamma_0 - \gamma_1) + \sum_{i=1}^{n} w_i^2\sigma^2_{\Delta\upsilon_{it}}\right),
\end{aligned}$$

where

$$\begin{aligned}
\gamma_0 &= \frac{\frac{1-\phi_2}{1+\phi_2}}{(1-\phi_2^2)-\phi_1^2}, \\
\gamma_1 &= \frac{\phi_1}{1-\phi_2}\gamma_0.
\end{aligned}$$

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