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A Model of Endogenous Debt Maturity with Heterogeneous Beliefs*

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Abstract

This paper studies optimal debt maturity in an economy with repayment enforcement frictions and investors disagree about repayment probabilities. The optimal debt maturity choice is a mix of long- and short-term debt securities. Spreading risky debt claims on cash flows over time allows debt to be priced by investors most willing to hold risk at each point in time, thereby increasing investment and output. By contrast, a single maturity, either all long- or short-term, will be priced by investors less willing to hold risk, which reduces investment and output. The model provides a novel explanation for the stylized fact that large and mature companies almost always issue debt with multiple maturities rather than a single maturity, and is broadly consistent with empirical debt maturity results. Lastly, we show that non-financial covenants that prevent debt dilution only serve as substitutes for short-term debt and do not affect real outcomes as they do not allow the firm to create additional collateral against which to borrow.

Keywords: Debt maturity, investment, cost of capital, covenants, debt dilution

JEL Codes: D92, G11, G12, G31, G32, E22

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1 Introduction

Many large and mature corporations typically raise capital by issuing debt of various maturities. For example, IBM tapped the bond market 5 times in the first 6 months of 2017 with a different maturity offering each time. In July 2017, AT&T raised $22.5 billion issuing maturities ranging from 5.5 years to 41 years. In 2013, Verizon raised $49 billion issuing debt through 6 different maturities. Microsoft offered 7 different maturities when it raised $19.75 billion in 2016. [1] In addition, Kalemli-Ozcan, Laeven, and Moreno (2018) show that both large firms and SMEs in Europe utilize both long- and short-term debt.

Theoretically explaining why the largest, most mature firms with low default and rollover risk issue a mix of debt maturity is a challenge. For example, the scant existing explanations for why any firm would issue multiple maturities rely on balancing inefficient liquidation risk from short-term debt and maintaining control rents or optimal continuation policies from long-term debt (Diamond (1991), (1993), and Houston and Venkataraman (1994)). Yet, empirical and survey evidence suggests that liquidation and information asymmetries are not likely to have significant effects on large, mature corporations with abundant public information and analyst coverage.[2]

This paper presents a novel theory to explain why issuing multiple debt maturities is cost minimizing and value maximizing for firms in the absence of agency conflicts or liquidity risk. Firms use debt maturity to inter-temporally cater risky claims on cash-flows to investors most willing to hold them. For any given investment, a firm has three ways to structure its debt maturity: 1) use long-term debt that matches the timing of its assets and liabilities. Long-term debt requires paying positive credit spreads due to default risk, but it insulates the firm from changes in the cost of issuing debt in the future; 2) issue short-term

[1] In fact, virtually every recent major public debt offering involves multiple maturities.

[2] Graham and Harvey (2001) find that listed firms are typically not concerned with information asymmetries and the mis-pricing of securities due to agency problems. Johnson (2003) finds that the liquidity risk effects of short-term debt matter almost exclusively for unrated firms, and are virtually non-existent for rated firms. Billet, King, and Mauer (2007) do not find evidence that liquidity risk drives short-term debt use at all for rated firms.

debt that needs to be rolled over. Short-term debt is initially safe without liquidity risk, but completely exposes the firm to price fluctuations in the future; or 3) issue a combination of the two maturities in which only the short-term component is subject to price changes, which generates a dilution cost for long-term debt.[3]

Debt maturity choice is analyzed as a tradeoff between the cost of risky debt over time due to changes in expected firm cash-flows and heterogeneity in the price investors are willing to pay to hold risky debt claims on those cash flows. For a given investment, issuing more long-term debt requires offering higher compensation to less optimistic investors. At the same time, more long-term debt also reduces the amount of short-term debt that needs to be rolled over, which results in more optimistic investors determining the market value of short-term debt and reduces dilution costs. Therefore, changes in the aggregate supply of debt issued across time lead to changes in the relative cost of debt financing firms face during those times. The main result is that issuing a mix of debt maturities allows firms to issue the optimal amount of risky debt in each time period and firms capitalize on investors' different willingness to hold risk. By contrast, issuing a single debt maturity only allows the optimal amount of risky debt to be issued in a single time period and forces the firm to raise a larger portion of its capital from investors who seek higher compensation. Issuing a mix of debt maturities allows firms to raise additional capital for investment on better terms and increase overall firm value.

Our model is an incomplete markets economy with binomial states and three-periods: 1) an initial state, 2) intermediate states, and 3) terminal states. The key frictions are investor heterogeneity and repayment enforcement problems.[4] Creditors cannot coerce

[3]Recent studies of Hugonnier, Malamud, and Morellec (2014 and 2015) have used search frictions to highlight the point that capital supply frictions can generate new predictions for firm financial policy. A simple collateral constraints and investor heterogeneity in our model produces a similar environment in which one can easily study how debt maturity interacts with investment decisions.

[4]Our focus is on limited enforcement frictions as in Rampini and Vishwanathan (2010) and Fostel and Geanakoplos (2015, 2016). Distinct from these papers, we ask how a firm should optimally structure its debt maturity in an economy with collateral constraints and heterogeneous investors. Rampini and Vishwanathan (2010) focus on how a firm should allocate scarce collateral between investment and risk management. Fostel and Geanakoplos (2016) focus on how collateral constraints and financial innovation affect investment

debtors into repayment, and collateral is used as enforcement. When a debtor fails to honor its debts, the creditor has the right to seize collateral to be made whole, but no more.[5] We consider risk neutral creditors with heterogeneous beliefs over the expected value of repayment. Investors are willing to pay different prices to hold risky debt, and these prices change in intermediate states as uncertainty is either resolved in good states or grows in bad states.

We begin with simple examples that show how investor heterogeneity permits debt maturity to affect firm value, even without intermediate liquidity risk or bankruptcy costs. Compared to a common belief in a homogeneous investor economy, investor heterogeneity allows the firm to utilize the debt maturity choice in a way that reduces financing costs and increases firm value. Firm value is increased in the heterogeneous agent economy when the marginal investor pricing debt in equilibrium has a higher expectation of being repaid than the common belief. But this need not always be the case if the common belief is sufficiently high. However, allowing the firm to utilize both long and short-term debt in the heterogeneous agent economy spreads the debt issuance over time which actually consolidates risk to investors with higher repayment expectations. Therefore, a mix of long- and short-term debt will always be the optimal choice in the heterogeneous agent economy and will generally increase the value of the firm.

The intuition is the following: Consider *all* long-term debt funding. There are two benefits from substituting a portion of the long-term debt for short-term debt. First, short-term debt is risk free in the initial period. Second, because less long-term debt will be issued, a more optimistic investor will price it in equilibrium, raising long-term bond prices. The costs of partially substituting into short-term debt are the following: The first is the expected cost to rollover short-term debt. The second is the dilution cost to long-term debt

efficiency.

[5]Our choice to highlight a collateral friction is supported by recent empirical evidence suggesting that collateral plays an important role in the design of debt contracts and the provision of credit (Cerqueiro, Ongena, and Roszbach (2016)).

due to the expected increase in short-term debt face value needed to rollover expiring claims. In general, the substitution benefits always outweigh the costs for two reasons. Substituting risky for risk-free debt is always cheaper, and short-term refinancing costs are paid in expectation rather than with certainty as with long-term debt. Moreover, the dilution effect on long-term debt prices is mitigated by the fact that increasingly optimistic investors finance long-term debt as long-term debt is substituted for short-term debt. This means that the marginal investor cares less about the dilution costs the more the firm substitutes away from long-term debt.

By similar reasoning, multiple debt maturities generally dominate *all* short-term debt financing. The benefits of substituting a portion of debt financing into long-term debt are the following: Raising one dollar long-term is cheaper than raising one dollar short-term conditional on bad news because short-term debt is information sensitive. In addition, issuing less short-term debt lowers the dilution cost to long-term debt, which further increases long-term debt prices. The only cost of substituting into long-term debt is paying a positive credit spread with certainty and giving up the opportunity to finance short-term debt risk free.

An interesting implication of a multiple debt maturity equilibrium is that debt dilution can actually *increase* the asset value of the firm. The intuition is that a more optimistic marginal bond buyer ends up pricing long-term debt as the firm substitutes long- for short-term. Therefore, even though the value of long-term debt is being diluted, the marginal investor cares less about the diluted recovery value of her debt. In other words, the substitution effect of reducing the face value of long-term debt in leiu of some short-term debt outweighs the dilution cost.

We then ask how non-financial debt covenants that prevent debt dilution from occurring may impact our maturity results. Specifically, we allow long-term debt to be secured by specific firm assets and show that the recovery value of long-term debt in a mixed debt maturity equilibrium is the same as the recovery value in a long-term only equilibrium i.e.

there is no dilution effect.[6] We show that utilizing a mix of long- and short-term debt remains the optimal debt maturity strategy even in the presence of protected long-term debt. The intuition is the following: *Ceteris paribus*, the price any investor is willing to pay for a protected long-term bond rises when their claims cannot be diluted. This causes the firm to raise more long-term debt resulting in a more pessimistic marginal long-term bond buyer completely undoing the initial price increase. In equilibrium, the price of long-term debt remains the same and there is no relative cost advantage for the firm. The only effect of the covenant is that relatively more long-term debt is issued in equilibrium, and issuing multiple debt maturities remains the optimal debt issuance strategy.

More broadly, debt maturity at issuance in emerging markets has lengthened as firms have increase their reliance on bond finance (Fuertes and Serena (2014) and Shin (2014)) A recent IMF report on emerging market corporate leverage suggests that global factors explain most of the increase in debt maturity (IMF 2015). Our model suggests that global factors may be a reflection of the differences among investors to hold risky claims on firm cash flows and hence firms cater the maturity of their debt issuance to these investors and increase leverage.

The organization of the paper is as follows: related literature is below. Section 2 introduces the model, agents, the different debt contracts considered, and works through 3 simple examples. Section 3 characterizes the equilibrium debt liability structure and comparative static results. Section 4 introduces the covenant and provides a numerical example highlighting its effects and the comparative static result. Section 5 concludes. All proofs that are not obvious from the text are contained in the appendix.

Related literature

In a series of papers, Hart and Moore (1994, 1995, 1998) show that debt is an optimal contract to resolve agency problems and discipline management to payout cash flows and

[6] In section 4 and the appendix, we discuss that our secured covenant can be interpretation as a negative pledge covenant, one of most common covenants found in long-term corporate debt indentures.

undertake efficient investment projects. These models either only examine long-term debt (Hart and Moore (1995)), or characterize repayment paths as either the fastest or slowest (Hart and Moore (1994, 1998)), but never a combination of the two. Zwiebel (1996) shows that multiple repayment paths will constrain empire building and prevent control takeovers, but only for the *most risky firms* for whom debt is a possible financing source, but is at odds with empirical observations. While agency frictions certainly explain why management may use debt in its capital structure, they do not appear to adequately describe why multiple maturities are simultaneously used to raise capital (see also Jensen and Meckling (1986), Bolton and Scharfstein (1990, 1998)).

Private information can affect the types of debt securities firms issue. When firms have inside information, Flannery (1986) shows that firms will use short-term debt to signal quality. Diamond (1991, 1993) and Houston and Venkataraman (1994) show that liquidity risk breaks the reliance on short-term debt and generates different debt maturity choices in the cross section based on credit ratings.[7] We show that multiple maturity debt is optimal without liquidation risk and firms do not use private information in any way.

There are many models in which debt affects firm value. For example, debt maturity can improve investment incentives due to debt overhang (Myers (1977), He and Diamond (2014)), optimal default timing (He and Milbradt (2016)), and information asymmetries (Flannery (1986), Kale and Noe (1990)). Debt maturity affects corporate financial policy in our model because the same risky debt claim will be priced differently in different periods when investors disagree about repayment probabilities.[8] Another distinction in our model are non-exclusive relationships between debtors and creditors. In practice, non-

[7]Proposition 2 of Diamond (1991) shows that short-term debt is the unique funding outcome in the model absent liquidation risk or loss of control rents. The agency problem is therefore necessary in his model to obtain an equilibrium with multiple maturities. Firm borrowing is fixed in Diamond (1991) and firms can borrow up to the fixed amount at the same interest rate.

[8]Heterogeneity is at the heart of Jung and Subramanian (2014), but an agency problem gives maturity a role in their model. Specifically, heterogeneous beliefs between managers and equity holders leads to a tradeoff between manager optimism and long-term debt issuance. Our model also has a flavor of this effect, but heterogeneity between the firm and investors is only material for determining what portion long-term debt constitutes of total debt issuance.

exclusive relationships are common for large corporations. For example, Dass and Massa (2014), using Lipper eMAXX data, highlight that the average corporation has 17 institutional investors acting as creditors at any point in time (see also Detragiache, Garella, and Guiso (2000)).[9]

Diamond and He (2014) highlight the subtle effects of debt maturity on debt overhang and investment incentives. The optimal debt maturity balances the symptoms of short- and long-term debt overhang; respectively, earlier default versus reduced investment incentives. However, they consider different debt maturities with equivalent market values and a fixed asset. We do not consider overhang effects because how debt maturities are structured in our model affects the *ex ante* value of the asset/project the firm undertakes.

Dynamic debt maturity models in continuous time focus on refinancing policies, optimal leverage ratios, and target average debt maturity (see Leland (1994, 1998)). He and Milbradt (2016) bring debt maturity to the forefront of these models. They emphasize the joint determination of default and maturity by showing that a firm actively manages maturing debt depending on the firm's distance to default. The firm issues short- or long-term debt and commits to a constant book leverage policy, *i.e.* maintains a constant aggregate face value of outstanding debt. Rollover losses arise as equity holders must absorb any cash flow shortfall when maturing bonds are refinanced when credit conditions deteriorate. The rollover losses feed back to the default decision by equity holders, leading to earlier default. Our model characterizes the firm's optimal debt financing strategy with multiple maturity issuances. Though we do not focus on the timing of default, our model has a similar feedback mechanism in which the anticipated short-term debt rollover losses induce the firm to substitute toward more initial long-term debt. We view our paper as complementary.

[9]Large firms typically raise capital from a syndicate of creditors rather than a single creditor even when considering private loan markets. Using supervisory data on bank holding companies, Caglio, Darst, and Parolin (2016) show that larger corporates borrow from, on average, 8 banks compared to small firms that tend to borrow from one.

Brunnermeier and Oehmke (2013) show that financial firms' inability to commit to a maturity structure leads short-term debt to dilute long-term debt. Their model has a fixed supply of assets where the firm increases aggregate debt liabilities when new debt is issued to repay expiring claims, diluting the per-claim value of existing debt. Equity holders cannot absorb losses in their model as they can in our model and He and Milbradt (2016). He and Xiong (2012b), with a fixed maturity structure, show how short-term debt can amplify default risks when liquidity risk is present because equity holders will default at earlier valuation thresholds. Default timing is fixed in our model, but maturity is allowed to adjust.

Geanakoplos (2009) and He and Xiong (2012a) study debt financing in incomplete asset markets with heterogeneous agents. In their models, short-term debt is the unique equilibrium because a sequence of short-term claims allows agents to take maximum leverage. The difference in their models is that all agents own risky assets, some of whom have higher valuations than others, and agents can borrow against the assets by issuing safe promises to obtain leverage. Issuing consecutive short-term claims allows optimists to borrow against the lowest value of the asset one period in the future, while a long-term claim only allows an agent to borrow against the final period worst case outcome. We adopt the same uncertainty structure to highlight that our result is not a special case of what one assumes about uncertainty, but that introducing an optimizing agent with production into a heterogeneous agent framework delivers the multiple maturity phenomenon that we commonly see in practice.

2 Model

2.1 Time and uncertainty

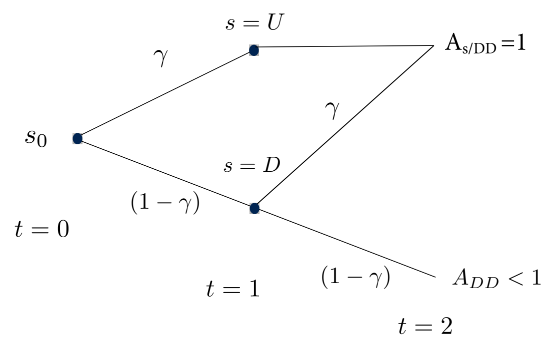
The model is a dynamic three-period production economy with incomplete asset markets. Time is denoted $t = \{0,1,2\}$. Uncertainty is given by a tree of state events $s \in S$ with root s_0, intermediate states $s \in S$ that take values $\{U,D\}$, and a set of terminal nodes denoted $S_T = \{UU,UD,DU,UU\} \subset S$. Let state realization U be up or a "good" state and D be down or a "bad" state.

The only uncertainty in the model is an aggregate shock that affects output at $t = 2$. The parameter A_{s_T} captures the effect of the shock to production. The expected value of the shock is conditional on the information revealed at $t = 1$. We assume for simplicity that good news at $t = 1$ resolves uncertainty at $t = 2$ and there is no shock: $A_{UU} = A_{UD} = 1$. Bad news at $t = 1$ raises uncertainty at $t = 2$ about the ability of the firm to repay debts, akin to "scary bad news" in Geanakoplos (2009). Specifically, there is no shock at terminal node $s = DU$, but there is a shock at terminal node $s = DD$, $A_{DD} < A_{DU} = 1$. Note that this uncertainty structure is the same as the simplification made in the continuous time version of Diamond and He (2014).[10] Figure 1 depicts the economy's state tree.

The assumptions about time and uncertainty are made for simplicity. We show in the appendix that equilibrium is *qualitatively* unaffected under different assumptions. For example, one may assume that good news is more likely to follow good news rather than bad news, which alleviates the concern that bad news is effectively not as bad. This would change only *quantitatively* how much long- versus short-term debt the firm would issue. Alternatively, one could allow for uncertainty conditional on $s = U$ and that $A_{UD} \neq A_{DU}$. This alternative would also only affect the relative amounts of long- and short-term debt issued, and would not change the result that issuing both maturities is optimal. The lat-

[10]In example 2 of Diamond and He (2014), they assume that asset volatility is state-contingent. Specifically, $\sigma_H = \varepsilon > 0 = \sigma_L$ where σ_i is asset volatility conditional on state i. Clearly uncertainly is resolved when $\sigma_L = 0$.

Figure 1: Economy State Tree



Tree3.pdf

ter alternative would equate the model's uncertainty structure with what He and Xiong (2012a) consider. Moreover, Fostel and Geanakoplos (2010) prove that agents have the incentive to produce projects that become more volatile in bad times. The reason is simple: uncertainty following bad news is not very informative, which implies that price declines in bad intermediate periods are relatively small. Alternatively, if uncertainty was completely resolved after bad news, then prices in bad intermediate periods would fall much further and reflect the certain bad outcome in the final period. Lower intermediate prices would limit *ex ante* how much agents could leverage and borrow against their projects.

2.2 Debt contracts

There is a single durable consumption good available in the economy at $t = 0$, which is the numeraire. There are two types of debt contracts that can be made, each with different maturity. Short-term debt matures after one period and long-term debt matures after two periods. All debt contracts are non-contingent and pay zero-coupons. For simplicity, we normalize the repayment value of each contract to 1.

Let the quantity of debt issued at any state and time be q_{s/S_T}. The quantity of long-term debt issued at $t = 0$ is denoted q^{ℓ} and the market price denoted by p^{ℓ}. Short-term debt may be issued at $t = 0, 1$. The quantity of short-term debt issued at $t = 0$ is given by q_0^{ς} and the quantity of short-term debt issued at $t = 1$ is given by q_s^{ς}, $s = U, D$. The prices of short-term debt at $t = 0, 1$ are respectively $p_0^{\varsigma}, p_U^{\varsigma}$, and p_D^{ς}. Following much of the literature, we assume equal seniority between short- and long-term debt. All market prices of debt securities will be determined through equilibrium market clearing conditions.

The key friction in our model is that agents cannot be coerced to repay debts. As in Rampini and Vishwanathan (2010) and Fostel and Geanakoplos (2016), collateral serves as the payment enforcement mechanism. Specifically, creditors have the right to confiscate debtor collateral up to the value of the promise but nothing more. "Collateral" in our economy will be the firm itself, and can be thought of as the physical assets it produces from its investment decision. The collateral value of a debt contract is given by a state-contingent delivery function, $d_{S_T}(\cdot)$. Implicitly we are assuming there are no collateral cash flow problems in that all agents anticipate the state-contingent value of collateral (see Fostel and Geanakoplos (2015)). We return to the debt delivery functions in section 2.4.

2.3 Agents

We first describe the firm and its objective, followed by the investors' problem.

2.3.1 Firm

We assume a representative firm is owned and operated by a manager (equity claimant) with access to a two-period decreasing returns to scale production technology.[11] The production function is denoted by $f(I; \alpha, A_s) = A_s I^{\alpha}$, $\alpha < 1$ where I is the amount of capital the manager raises and puts into production. We assume the firm has no cash

[11] All agency problems are resolved with the assumption that the firm is owned and operated by the same agent. This is done to contrast our maturity results from extant agency-based models.

endowment, does not generate cash flow at $t=1$, and that new promises issued at $t=1$ do not scale the project's original size.[12]

The firm's objective is to maximize expected profits by choosing how much capital to raise and the maturity of the debt contracts it issues.[13] Let ρ denote the portion of debt that is raised long-term, $\rho=\frac{p^{\ell}q^{\ell}}{I}$, and let γ denote the probability of good news.[14] Formally, the firm maximizes the following problem:

$$\begin{cases} \max\limits_{I,\rho}\prod = & \max\left\{\gamma\left(I^{\alpha}-q^{\ell}-q_{U}^{\varsigma}\right)+(1-\gamma)\,\gamma\left(I^{\alpha}-q^{\ell}-q_{D}^{\varsigma}\right),0\right\} \\ \text{s.t.} & I=p^{\ell}q^{\ell}+p_{0}^{\varsigma}q_{0}^{\varsigma} \\ & p_{s}^{\varsigma}q_{s}^{\varsigma}=q_{0}^{\varsigma}, s=U,D \\ & 0\leq\rho=\frac{p^{\ell}q^{\ell}}{I}\leq 1 \end{cases} \tag{1}$$

Conditional on $s=U$, γ fully characterizes the firm's decision for both terminal states, because firm always repays. At $t=1$ the firm must decide whether it is beneficial to roll-over the short-term component of its debt portfolio. The firm repays short-term debt holders by raising $p_{s}^{\varsigma}q_{s}^{\varsigma}=q_{0}^{\varsigma}, s=\{U,D\}$. At $s=U$, the firm can always repay debts and $p_{0}^{\varsigma}=p_{U}^{\varsigma}=1$. The firm owes $q_{U}^{\varsigma}+q^{\ell}$ at $t=2$. At $s=D$, there is uncertainty regarding whether the firm can repay debt at $t=2$. In this case, $p_{D}^{\varsigma}<p_{0}^{\varsigma}=1$ and the firm owes $q_{D}^{\varsigma}+q^{\ell}$

[12]Alternatively, one could assume that there is an extreme form of limited commitment at the interim date in which no cash flows can be verified at a reasonable cost so debt repayments cannot come from cash flow. Under this alternative, cash-flow is independent of how the project is financed. The debt maturity mix will affect the investment cost that generates the cash-flow. Even if management could abscond with all intermediate cash, they would still issue the types of debt securities that would reduce costs to make higher profits in the final period.

[13]We restrict the analysis to debt issuance and do not allow for equity financing. This allows us to focus the analysis entirely on the endogenous composition of debt issuance in terms of the debt liability structure. Incorporating equity is a natural extension to the model.

[14]We will show that γ does not determine the general existence of multiple maturities as an equilibrium outcome. The fact that γ is known to the firm and may not be equivalent to the marginal investor's expectation of good news is not completely without loss of generality. γ will determine the relative amount of long- versus short-term claims, $0<\rho<1$, that makeup the optimal debt liability structure. However, one can solve the model by restricting γ to almost surely equal the marginal buyer's expectation so that there is a "true" state probability. This approach will pin down a unique ρ for all A_{DD} rather than have a state-space consisting of (A_{DD},γ)-pairs.

at $t = 2$. In the event of default, the firm makes no profits and all assets are distributed to creditors pro rata. The maximization problem is subject to the following constraints: the amount of capital the firm can use for production has to be raised by issuing bonds at $t = 0$. Conditional on rolling over short-term debt at $t = 1$, the firm issues new short-term debt, $q_0^{\varsigma} = p_s^{\varsigma} q_s^{\varsigma}, s = \{U, D\}$. Lastly, ρ, is bound between 0 and 1. To derive the firm's first order conditions for a maximum, first use the definition of $\rho = \frac{p^{\ell} q^{\ell}}{I}$ to write the problem in terms of choice variables I and ρ :

$$\max_{I,\rho} \prod = \max \left\{ \gamma \left(I^{\alpha} - \frac{\rho I}{p^{\ell}} - \frac{(1-\rho) I}{p_0^{\varsigma}} \right) + (1-\gamma)\gamma \left(I^{\alpha} - \frac{\rho I}{p^{\ell}} - \frac{(1-\rho) I}{p_D^{\varsigma}} \right), 0 \right\}.$$

If an interior maximum for ρ exists, the first order necessary conditions with respect to I and ρ respectively, are

$$\alpha I^{\alpha-1} \left[1-(1-\gamma)^2\right] = \frac{\rho \left[1-(1-\gamma)^2\right]}{p^{\ell}} + \frac{(1-\rho)}{p_0^{\varsigma}} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right] \tag{2}$$

$$\frac{\left[1-(1-\gamma)^2\right]}{p^{\ell}} = \frac{1}{p_0^{\varsigma}} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]. \tag{3}$$

The necessary conditions for the corner solutions are easily obtained by plugging either $\rho = 0 \text{ or } 1$ into the firm's maximization problem–there will be no first order condition with respect to ρ. Equation (2) says that the marginal product of capital in states where the firm makes profits–which occurs with probability $1-(1-\gamma)^2$–must equal the maturity-weighted expected marginal cost of debt. The marginal cost of long-term debt is given by $\frac{\left[1-(1-\gamma)^2\right]}{p^{\ell}}$ and the marginal cost of a sequence of short-term bonds is given by $\frac{1}{p_U^{\varsigma}} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]$. With probability γ, the firm will issue a sequence of risk-free bonds ($p_U^{\varsigma} = 1$). With probability $\gamma(1-\gamma)$, the firm pays a higher short-term borrowing cost $p_D^{\varsigma} < p_U^{\varsigma} = p_0^{\varsigma} = 1$ per bond to roll over existing claims. Equation (3) says that in an interior debt maturity optimum, the marginal cost of a long-term bond must equal the marginal cost of a sequence of short-term bonds. Intuitively, if marginal cost of one maturity is lower than the other, the firm will find it optimal to always issue the less expensive maturity, raise more capital, and make higher profits. But since the marginal costs must be equivalent in any interior optimum, we can combine equations (2) and (3) into a simplified version of either long- or short-term debt respectively:

$$\alpha I^{\alpha-1} = \frac{1}{p^{\ell}}, \tag{4}$$

$$\alpha I^{\alpha-1}\left(1-(1-\gamma)^2\right) = \frac{1}{p_0^{\varsigma}}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]. \tag{5}$$

The equilibrium debt maturity the firm chooses will be determined by the relative market prices of risky debt securities, $\left(p^{\ell}, p_D^{\varsigma}\right)$. In order to find debt prices and characterize equilibrium, we must solve the investors' problem with the debt delivery functions for the different maturity strategies.

2.3.2 Investors

There exists at $t = 0$ a continuum of uniformly distributed investors with unit mass, $h \in H \sim U[0,1]$, each of whom is endowed with a unit of the durable consumption good in all non-terminal states, $e^h, e^h_s, s \neq S_T$. The uniform distribution allows one to rank investors according to the likelihood each places on the subsequent state being good, denoted by h. Investors are risk-neutral, expected utility maximizers that consume at $t = 2$, and do not discount the future. Without loss of generality, we assume investors have different priors (see Fostel and Geanakoplos (2015)).[15]

Investors also have access to a riskless storage technology and form portfolios consisting of cash and bonds purchased from the firm. The von-Neumann-Morgenstern prefer-

[15]One could assume investors differ in a measure of risk aversion; have different endowments across states, which produces different marginal utilities across states; or have different degrees of "patience." The critical assumption is the heterogeneity of marginal utilities across investors. We choose to think about beliefs because it is most familiar in these models.

ences are given by:

$$U^h(x_{UU}, x_{UD}, x_{DU}, x_{DD}) = h^2 x_{UU} + h(1-h)x_{UD} + (1-h)hx_{DU} + (1-h)^2 x_{DD}. \quad (6)$$

We now characterize the investors' budget sets. Given debt prices, $\left(p^\ell, p_0^\varsigma, p_U^\varsigma, p_D^\varsigma\right)$, each investor, $h \in H$, chooses cash holdings, $\left\{x_0^h, x_D^h, x_U^h\right\}$, debt holdings, $\left\{q^{\ell,h}, q_0^{\varsigma,h}, q_U^{\varsigma,h}, q_D^{\varsigma,h}\right\}$, and final period consumption decisions, $\left\{x_s^h\right\}, s \in S_T$, to maximize utility given by (6) subject to the budget set defined by:

$$\begin{aligned} B^h\left(p^\ell, p_0^\varsigma, p_U^\varsigma, p_D^\varsigma\right) = & \Big\{\left(x_0, x_D, x_U, q^\ell, q_0^\varsigma, q_U^\varsigma, q_D^\varsigma, x_s\right)_{h \in H} \\ & x_0^h + p^\ell q^{\ell,h} + p_0^\varsigma q_0^{\varsigma,h} = e_0^h, \\ & x_U^h + p_U^\varsigma q_U^\varsigma = e_1^h + q_U^\varsigma d_U\left(q_0^\varsigma\right) + x_0^h \\ & x_D^h + p_D^\varsigma q_D^\varsigma = e_1^h + q_D^\varsigma d_D\left(q_0^\varsigma\right) + x_0^h \\ & x_s^h = x_0^h + x_{U,D}^{h_1} + q^\ell d_s\left(q^\ell\right) + q_s^\varsigma d_s\left(q_s^\varsigma\right), s \in S_T\Big\}. \quad (7) \end{aligned}$$

Each investor may use their initial cash endowment to purchase either type of debt at $t = 0$. The endowment received at $t = 1$ plus any returns from short-term debt holdings and cash carried forward are used to either purchase short-term debt at $t = 1$ or held for final consumption. All cash that is not used to purchase debt is carried forward to consume at $t = 2$. All final period consumption comes from debt purchases and cash holdings.

It is clear that each investor will choose the debt maturity that delivers repayment in the state that the investor finds most likely. And because the contracts that investors purchase are debt contracts that pay out 1 in repayment states, the most optimistic investors simply purchase the debt security that they can purchase in the largest quantity. Put differently, the optimists purchase the cheapest debt securities with the highest expected yield.

Consider a conjectured equilibrium with both long- and short-term debt: at $t = 0$, optimists will use all of their endowment to purchase long-term bonds, while relative pessimists will hold a mix of short-term debt and cash. If the firm issues short-term debt that it rolls over at $t = 1$, the optimists will use their $t = 1$ endowment to purchase risky short-term debt at $s = D$ as well. Alternatively, consider a conjectured short-term only debt equilibrium: all investors at $t = 0$ will hold a portfolio of safe short-term debt and cash, and optimists will use all of their $t = 1$ endowment to purchase risky short-term debt at $s = D$. Lastly, consider a conjectured long-term only debt equilibrium: optimistic investors at $t = 0$ will purchase long-term debt and all other investors will remain in cash. All investors will use their $t = 1$ endowment for consumption because no further bonds are issued.

In order to close the model and characterize which type of debt securities will trade, we must determine exactly how investors price long- and short-term debt based on the recovery value of debt given default.

2.4 Debt repayment

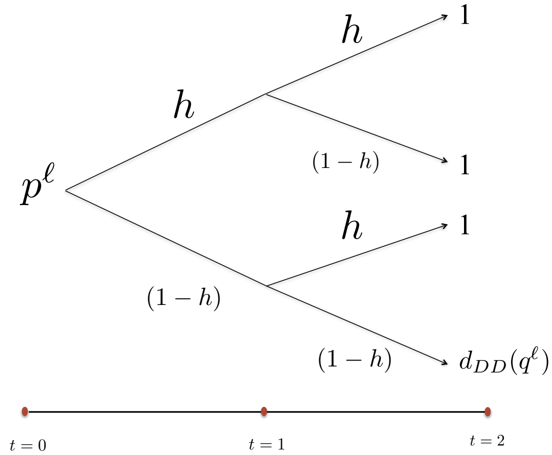
Due to the repayment enforcement friction, debt is effectively collateralized by future output as in Rampini and Vishwanathan (2010) and Fostel and Geanakoplos (2016). The implicit assumption underlying these models is that there are no collateral "cash-flow problems."[16]

Short-term Debt

Let $d_{U,D}\left(q_0^{\varsigma}\right)$ describe short-term debt delivery at time 1. Short-term debt repayment is conditional on whether the firm rolls over debt at $t = 1$. Specifically, short-term debt will be "safe" if it is always rolled over, and both recovery and prices are equal to 1; otherwise, short-term debt will be risky due to potential liquidation. To highlight the novelty of investor heterogeneity rather than liquidation risk ala Diamond (1991), we assume there is

[16] Traditional macro/finance models such as Kiyotaki and Moore (1993) assume that creditors can confiscate land, but not the fruit produced by the land. Corporate finance models following Holmstrom and Tirole (1997) assume an information asymmetry between borrowers and lenders. Borrowing too much in these models reduces cash-flow and reduces incentives to work hard to produce good cash flows.

Figure 2: Long Term Financing



no inefficiency from default in any state. Risky debt, regardless of its maturity, is always fairly priced. As such, liquidation does not provide any additional insights, and we proceed by assuming debt is always rolled over. We later derive that rolling over short-term debt is indeed optimal.[17]

The firm must issue q_s^ς one-period debt contracts to rollover expiring claims. Short-term debt delivery in the final period is state-contingent with default only at $s = DD$.[18]

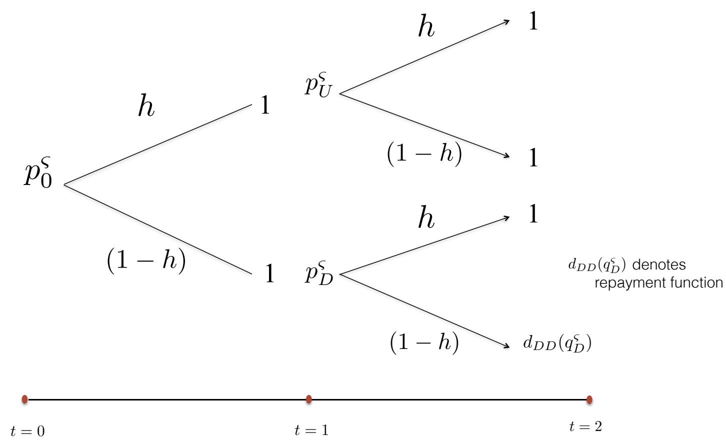
$$d_{DD}(q^\varsigma) = \begin{cases} 1, & s \neq DD \\ \frac{A_{DD} I^\alpha}{q_D^\varsigma + q^\ell}, & s = DD \end{cases}. \tag{8}$$

Equation (8) says that all debts are honored as long as two periods of bad news do not occur. Firms default after two periods of bad news due to the technology shock and all

[17]Flannery (1986) considers only safe short-term debt at $t = 0$ as well. His analysis highlights the importance on asymmetric information in determining debt maturity.

[18]This is simply restating absolute priority ala Merton (1974) via a collateral constraint. Equity receives nothing when debt holders are not repaid *ex post*, but collateral delivery is required to obtain debt *ex ante*. We will show that $A_{DD} < \alpha$ is sufficient for $d_{DD}(\cdot) < 1$, and consider this parametrization throughout the paper to focus on risky debt.

Figure 3: Short-Term Financing



firm assets are divided pro rata to debt holders. The sequence of short-term debt contract payouts is depicted in figure 3.

Long-term Debt

Long-term debt is very simple to describe as it matches the maturity of assets with liabilities. Let $d_s\left(q^{\ell}\right)$ denote the long-term debt delivery function. Given our assumptions on short-term debt being rolled over, both long- and short-term deliveries are given by (8), $d_s\left(q^{\ell}\right) = d_s\left(q^{\varsigma}\right)$, or generically $d_s\left(\cdot\right), s \in S_T$.[19] Equation (8) gives the recovery value of firm assets for any of the possible debt maturity equilibria by simply setting either $q_D^{\varsigma} = 0$ for a long-term only equilibrium or $q^{\ell} = 0$ for a short-term only equilibrium. We now characterize equilibrium.

2.5 Equilibrium

Definition 1 *Equilibrium is a collection of the following non-negative items: debt prices, firm investment decision, investor cash holdings, debt holdings, and final consumption*

[19]We are implicitly assuming that the collateral value of the firm is not being split as explicit collateral pieces for long- versus short-term debt. We return to this assumption in section 4 of the paper when we consider restrictive negative covenants.

decisions that maximize firm profits given by (1)*, investor utility given by* (6) *subject to their budget constraints in* (7)*, and both the goods and debt markets clear in all periods.*

To highlight the importance of heterogeneity and catering debt securities to investors, we begin by solving the special case of the model with homogeneous investors. We then show how moving to heterogeneous investors delivers both similar predictions as Diamond (1991) and the new prediction that issuing both a combination of debt maturities is generally the least costly financing option, even absent liquidity risk.

2.5.1 Examples of investor beliefs and debt maturity choice

Example 1 *Homogeneous investors and debt maturity irrelevance.*

Assume a unit mass of competitive investors all share the common prior that the up state occurs with probability, γ. The only change a common belief changes is how investors value debt. We will use the following parameters throughout the examples: $A_{DD} = .5$, $\alpha = 0.8$, $\gamma = 0.7$. There is no risk of liquidation at $t = 1$ so that all short-term debt issued at $t = 0$ is risk free.

Long-term–If long-term debt is the only security traded, $\rho = 1$. A competitive equilibrium requires that investors make zero profits in expectation. If γ is the probability of $s = U$, then long-term debt is valued by all investors according to

$$\left(1-(1-\gamma)^2\right)1+(1-\gamma)^2 d_{DD}\left(q^\ell\right) = p^\ell. \tag{9}$$

One needs to solve for the debt delivery function, (8), with $\rho = 1$ in order to price the debt security. From the firm's funding condition, $\frac{1}{q^\ell} = \frac{p^\ell}{I}$. Lastly, use (4) to solve for $I^{\alpha-1}$ to arrive at the following:

$$d_{DD}\left(q^\ell\right) = \frac{A_{DD}}{\alpha}. \tag{10}$$

Table 1: Long-term homogeneous equilibrium

	MC_{γ}^{ℓ}	p_{γ}^{ℓ}	I	V_{γ}^{ℓ}	Π_{γ}^{ℓ}	$d\left(q^{\ell}\right)$
$(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$	1.034	.9663	.2760	.3570	.0650	.625

Intuitively, the less production is affected by the technology shock (high A_{DD}), the more assets are available for investors to recover. Likewise, the more output a firm generates per unit of investment capital (low α), the more investors recover on a per claim basis. We define the following objects for comparison across economies: the marginal cost of issuing long-term debt, $\frac{1}{p^{\ell}}$, investment, $I = \left(\alpha p^{\ell}\right)^{\frac{1}{1-\alpha}}$, the value of the firm output, $V_{\gamma}^{\ell} = I^{\alpha}$, and expected profits are $\Pi_{\gamma}^{\ell} = \left(1 - [1-\gamma]^2\right)\left(V_{\gamma}^{\ell} - q^{\ell}\right)$, where the superscript ℓ denotes the long-term debt regime and the subscript γ denotes all agents' homogeneous belief. Table 1 contains the value of the key objects.

Short-term–If short-term debt is the only security traded, $\rho = 0$. All risky short-term debt purchased at $s = D$ must yield zero profit to investors in expectation:

$$\gamma 1 + (1-\gamma)\, d_{DD}\left(q_D^{\varsigma}\right) = p_D^{\varsigma}. \tag{11}$$

Following the same proceedure laid out in the long-term debt regime, the debt delivery function for short-term debt funding, (8), becomes

$$d_{DD}\left(q_D^{\varsigma}\right) = \frac{A_{DD}}{\alpha} \underbrace{\left(\frac{p_D^{\varsigma}\gamma + \gamma(1-\gamma)}{1-(1-\gamma)^2}\right)}_{\text{dilution factor}}. \tag{12}$$

Debt delivery with short-term funding is the product of two components. The first component is the same fundamental recovery value as long-term debt delivery, $\frac{A_{DD}}{\alpha}$. The second component represents the dilution effect of issuing more short-term debt in expectation at $t = 1$ to honor expiring claims. The dilution effect is given by, $\frac{p_D^{\varsigma}\gamma + \gamma(1-\gamma)}{1-(1-\gamma)^2} < 1$. The marginal cost of short-term debt is given by $MC_{\gamma}^{\varsigma} = \frac{1}{1-(1-\gamma)^2}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]$. Solving (11)

Table 2: Short-term homogeneous equilibrium

	MC_γ^ς	p_D^ς	I	V_γ^ς	Π_γ^ς	$d\left(q_D^\varsigma\right)$
$(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$	1.034	.8685	.2760	.3570	.0650	.524

and (12) simultaneously, gives $p_D^\varsigma = .8685$, and a recovery rate of .5242. Table 2 provides the values of the key objects for the short-term debt economy.

The examples show that the economies are equivalent from the firm's perspective. Intuitively, the firm's efficient investment scale is given by the expected marginal cost of issuing debt. The expected marginal costs are equivalent across the two economies ($MC_\gamma^\varsigma = MC_\gamma^\ell = 1.034$) because all agents have the same common information. The dilution effect of short-term debt is correctly priced in expectation and the firm is indifferent between issuing long- or short-term debt. However, the firm is not indifferent between issuing a combination of long- *and* short-term debt and issuing *either* long- *or* short-term debt. The reason is that as soon as the firm issues any *combination* of long- and short-term debt, the dilution effect of short-term debt reduces the price of long-term debt as investors will not pay the same price for a security whose value falls in expectation. Moreover, the more short-term debt the firm issues, the more long-term bond prices must fall due to the dilution effect. Put differently, risky long- and short-term debt prices positively co-move with homogeneous investors because of debt dilution. In order for the firm to issue both long- and short-term debt, the expected costs of the two debt securities must be equal.[20] But the only way that the expected costs can be equivalent is if the prices are lower than what they are in either corner solution, because the prices positively co-move. Therefore, the firm cannot operate at the same efficient scale by simultaneously issuing both long- and short-term debt.

Example 2 *Debt maturity with heterogeneous investors*

Now consider the heterogeneous investor case described in section 2.3.2. All of the

[20] See equation (3).

Table 3: Long-term heterogeneous equilibrium

	MC_h^ℓ	p^ℓ	h_0	I	V_h^ℓ	Π_h^ℓ	$d\left(q^\ell\right)$
$(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$	1.030	.9702	.7182	.2817	.3629	.0665	.625

firm's first order conditions for an optimal are the same. The only thing that changes are the debt pricing equations. We ask the following question: Is the firm indifferent between short- and long-term debt when investors are heterogeneous, and if not, what will be the equilibrium financing structure?

Long-term–The marginal long-term bond buyer at $t = 0$ must be indifferent between buying the bond and cash. The break even condition for the marginal long-term bond buyer is:

$$1 - (1 - h_0)^2 + (1 - h_0)^2 d_{DD}\left(q^\ell\right) = p^\ell. \tag{13}$$

In equilibrium, long-term bond prices are determined by market clearing in the bond market. Specifically, investor demand for risky bonds must equal the supply of bonds the firm issues:

$$\frac{1 - h_0}{p^\ell} = q^\ell, \text{ Long-term debt market clearing} \tag{14}$$

There are four unknowns in this economy: $\left(I, p^\ell, q^\ell, h_0\right)$, and four equations: (4), (13), (14), and $I = p^\ell q^\ell$. The value of the endogenous variable in this economy are contained in table 3.

Notice the value of firm output and profits are higher under the heterogeneous regime: $\Pi_h^\ell = .0660 > \Pi_\gamma^\ell = .0650$ and $V_h^\ell = .3629 > V_\gamma^\ell = .3570$. This is because the marginal investor's prior is higher than the common belief $h = .7182 > \gamma = .70$, which leads to lower credit spreads and more investment. The results show that the firm benefits when issuing debt in a heterogeneous agent economy compared to the homogeneous investor economy.

Table 4: Short-term heterogeneous equilibrium

	MC_h^{ς}	p_D^{ς}	h_1	I	V_h^{ς}	Π_h^{ς}	$d\left(q_D^{\varsigma}\right)$
$(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$	1.0318	.8786	.7199	.2800	.3612	.0657	.5286

Short-term–The marginal short-term debt holder at $t = 1$ must be indifferent between risky short-term debt and cash:

$$h_1 + (1 - h_1)\, d_{DD}\left(q_D^{\varsigma}\right) = p_D^{\varsigma}. \tag{15}$$

The short-term debt market must clear at both $t = 0, 1$. Without liquidity risk, all short-term debt is initially risk free, $p_0^{\varsigma} = 1$. Risk-free short-term debt implies that all investors at $t = 0$ hold a combination of short-term debt and cash. The firm must issue $q_U^{\varsigma} = q_0^{\varsigma}$ in the good state and $q_D^{\varsigma} = \frac{q_0^{\varsigma}}{p_D^{\varsigma}}$ in the bad state to ensure short-term debt is risk free. This reflects the fact that, conditional on $s = U$, all short-term debt is risk free and repaid, but conditional on $s = D$ the face value of short-term debt must rise to clear the market. Conditional on $s = D$, investor demand for risky bonds must equal supply:

$$\frac{1 - h_1}{p_D^{\varsigma}} = q_D^{\varsigma}, \text{ Short-term debt market clearing.} \tag{16}$$

Market clearing, (16) along with the pricing equation, (15), the debt delivery function, (12), and the firm's funding condition, $I = p_0^{\varsigma} q_0^{\varsigma}$ can be simultaneously solved for the equilibrium endogenous variables in the short-term debt economy, $\left(I, p_D^{\varsigma}, q_D^{\varsigma}, h_1\right)$. The solution to this economy is given in table 4.

Unlike the homogeneous investor case, the value of firm output is lower under the short-term debt regime than the long-term debt regime for the exact same set of parameters. To understand why debt maturity affects the value of firm output, note that, for a given investment, I, the expected marginal product of capital across the two economies will be the same:

$$E_0\left[MP\right] = \alpha I^{(\alpha-1)}\left(1-(1-\gamma)^2\right).$$

However, in general, the expected marginal costs across the economies will not be the same. The equilibrium price of long-term debt is determined by market clearing in equation (14). Likewise, market clearing for short-term debt is given by (16). It must be the case that $h_0 = h_1$ if the firm operates at the same scale if debt maturity is irrelevant. However, there is more uncertainty at $s = D$ than at $s = 0$, which means that the same investor will never price risky short- and long-term debt equivalently. In fact, the same investor will always price risky short-term debt at $s = D$ lower than long-term debt at $s = 0$; otherwise, short-term debt will always dominate long-term debt. The firm's financial policy influences the value of firm output and the efficient investment scale when investors are heterogeneous and the price of risk is time varying.

The following example asks whether or not debt liabilities can be more efficiently structured by spreading risk across time through a combination of long- and short-term debt.

Example 3 *Optimal debt maturity mix with heterogeneous investors*

In this example, the firm can choose $0 < \rho < 1$ in addition to either corner solution. This economy has eight unknowns $\left(I, \rho, p^{\ell}, q^{\ell}, p^{\varsigma}_D, q^{\varsigma}_D, h_0, h_1\right)$ and eight equations, seven of which are a collection of equations from the long- and short-term debt funding regimes:

$$\alpha I^{\alpha-1} = \frac{1}{p^{\ell}}, \text{ combined first order}$$
$$1-(1-h_0)^2+(1-h_0)^2 d_{DD}\left(q^{\ell}\right) = p^{\ell}, \text{ long-term debt pricing}$$
$$h_D+(1-h_D)\, d_{DD}\left(q_D^{\varsigma}\right) = p_D^{\varsigma}, \text{ short-term debt pricing}$$
$$\frac{1-h_0}{p^{\ell}} = q^{\ell}, \text{ long-term debt market clearing}$$
$$\frac{1-h_D}{p_D^{\varsigma}} = q_D^{\varsigma}, \text{ short-term debt market clearing}$$
$$I = p^{\ell}q^{\ell}+p_D^{\varsigma}q_D^{\varsigma}, \text{ firm funding condition}$$
$$\rho = \frac{p^{\ell}q^{\ell}}{I}, \text{ long-term debt portion.}$$

The only new equation is the definition of ρ–the long-term portion of total debt. Notice the first order conditions for an interior optimum can be collapsed into a single equation, because in expectation, the marginal costs of long- and short-term debt must be equivalent.[21] The eighth equation is the debt delivery equation. Through the same procedure as the long- and short-term debt economies, the recovery value of all debt in the mixed maturity economy is given by:

$$d_{DD}(\cdot) = \frac{A_{DD}}{\alpha}\underbrace{\left(\frac{p_D^{\varsigma}}{(1-\rho)\,p^{\ell}+\rho\, p_D^{\varsigma}}\right)}_{\text{dilution factor}}. \tag{17}$$

The dilution factor is a function of risky debt prices and the debt maturity mix determined by ρ.

The solution to this system is found in table 5. First, all bond pricing is significantly higher under the maturity mix regime than either of the respective corner solutions for the same parameters. The two marginal buyers in the multiple debt maturity regime are $h_0 = .8227$ and $h_1 = .8690$. Both marginal buyers are significantly more optimistic about firm cash flows than either of their counterparts in the corner solution regimes. Firms cater

[21] Collapsing into the long-term first order condition eases the exposition.

Table 5: Maturity mix heterogeneous equilibrium

	p_D^ς	p^ℓ	MC	I	ρ	h_0	h_1	V	Π	$d(\cdot)$
Maturity mix	.9495	.9879	1.012	.3083	.5752	.8227	.8690	.3900	.0710	.6159
Long-term	-	.9702	1.030	.2817	1	.7182	-	.3629	.0665	.625
Short-term	.8786	-	1.032	.2800	0	-	.7199	.3612	.0657	.5286

risky debt securities to investors across time and receive better prices in the heterogeneous investor economy. Second, this has real positive effects on the value of firm output and investment. Firms are able to finance more investment and production, which increases the firm's scale. Third, the recovery rate in the maturity-mix regime is slightly lower than the fundamental value of the recovery rate in the long-term regime. Interestingly, debt dilution can actually raise the value of firm ouput. The intution for this result is that, on the one hand, short-term debt dilutes long-term debt under equal seniority, which lowers long-term debt prices. On the other hand, short-term debt substitutes for long-term debt and concentrates the placement of long-term debt to investors with higher marginal valuations, which raises long-term debt prices. In this example, the substitution effect dominates the dilution effect and all prices rise compared to issuing only long- or short-term debt. We show in the following section that the substitution effect always dominates the dilution effect and a maturity mix is generally the optimal funding regime for firms facing heterogeneous investors.

3 The general debt maturity solution with heterogeneous investors

The examples in the previous section show that heterogeneous investors alone lead to equilibrium outcomes where firm debt maturity choice matters in a novel way. There is no liquidation risk in the examples making the model similar to Flannery (1996) and the special case of Diamond (1991), but the optimal debt maturity choice is very different. In those models, short-term debt is the unique maturity choice. A combination of long- and short-term debt emerges in our model. In this section we show that no liquidation is an endogenous outcome and that a mix of long- and short-term debt is in general the optimal debt maturity choice.

Short-term debt is "safe" at $t = 0$ if and only if it is unconditionally rolled over at $t = 1$. The rollover condition states that profits must be greater than or equal to zero after repaying both long- and short-term debts:

$$I^{\alpha} \geq q^{\ell} + q_D^{\varsigma}. \tag{18}$$

We can focus only on the down-state without loss of generality because the firm is always better off conditional on $s = U$ than $s = D$. Note that state probabilities, γ, do not factor in this decision because the firm only retains profits when it fully repays all debts, both of which occur with probability $1-(1-\gamma)^2$. The price of short-term debt at $t = 0$ must be $p_0^{\varsigma} = 1$ if the firm continues and produces at $t = 2$. Lemma 1 states the condition when short-term debt is always successfully repaid and there is no liquidation in equilibrium.

Lemma 1 ***Short-term Debt Rollover:*** *Suppose* $Q = \left(q^{\ell}, q_0^{\varsigma}, q_s^{\varsigma}\right) > 0$, $s = \{U, D\}$. *Short-term debt at* $t = 0$ *is safe if and only if*

$$\varepsilon \equiv \alpha \frac{1-\rho}{(1-\alpha\rho)} \leq \frac{p_D^{\varsigma}}{p^{\ell}} < 1. \tag{19}$$

Safe short-term debt is possible in equilibrium as long as there is a balance between the price of risky long-term debt today versus risky short-term debt conditional on bad news tomorrow, $\frac{p_D^{\varsigma}}{p^{\ell}}$. The firm will always choose short-term debt if there is no difference between long-term and risky short-term debt because short-term debt is financed risk free conditional on $s = U$. Therefore, in expectation, short-term debt will dominate long-term debt if the risky component of short-term debt is the same as long-term debt. Alternatively, risky short-term debt prices increase as the portion of financing through short-term debt rises. This drives a wedge between long- and risky short-term debt prices, and short-term debt will be relatively expensive in expectation compared to long-term debt.

To gain some intuition for when equation (19) holds, let ρ vary in the limit between 0 and 1. The relative price of risky debt, $\frac{p_D^{\varsigma}}{p^{\ell}}$, at $t=0,1$ is determined by investor expectations at each point in time, h_0 and h_D. For any given investment, I, ρ approaches 1 as more long-term debt is chosen. Substitution into long-term debt does two things: 1) it lowers long-term debt prices at $t=0$ as more pessimistic investors finance investment; and 2) it reduces short-term debt issuance, which lowers the dilution cost to long-term debt holders. Thus (19) holds trivially. Alternatively, let ρ approach 0, which increases short-term debt issuance. Substitution into short-term debt has the opposite effect on relative debt prices, p_D^{ς} falls and p^{ℓ} rises. As long as the ratio of risky debt prices, $\frac{p_D^{\varsigma}}{p^{\ell}}$, is greater than a measure of firm production and collateral, α, equation (19) holds, which becomes trivial as α falls. Intuitively, α measures the return to a unit of capital input (firm marginal productivity rises as α falls, $0<\alpha<1$). Higher marginal returns to production make it very easy to repay small amounts of debt. This reasoning implies that there is an upper boundary on $\bar{\alpha} \lessapprox 1$ for which equation (19) holds with equality from below. While we cannot give a precise mathematical expression for $\bar{\alpha}$, numerical simulations suggest $\alpha \lessapprox 0.9$ satisfies the condition.

Proposition 1 ***Multiple debt maturity structure:*** *When there is no liquidity risk and investors are heterogeneous, a mix of long- and short-term debt is optimal,* $Q \equiv \left(q^{\ell}, q_0^{\varsigma}, q_s^{\varsigma}\right) > 0$, $s=\{U,D\}$.

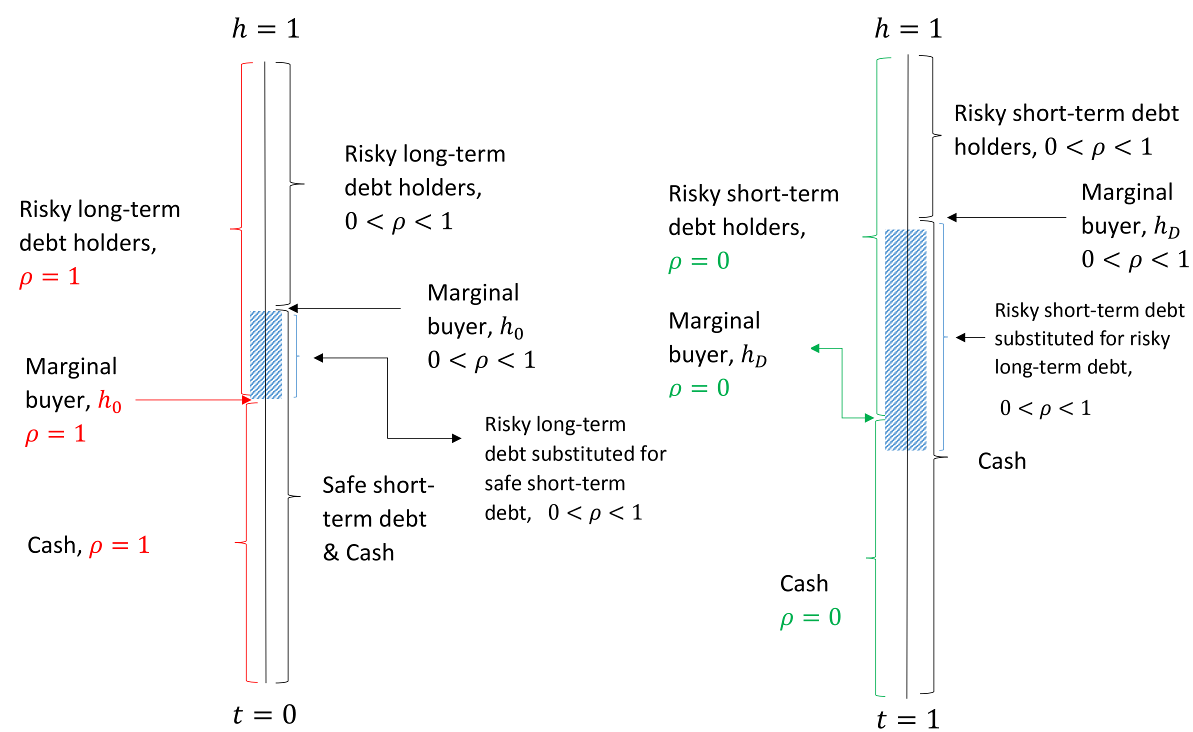
For any given investment, I, moving from $\rho=1$ to $\rho<1$ has two benefits. First, issuing a mix a debt maturities allows a more optimistic buyer to price long-term debt at $t=0$, which lowers the cost of long-term capital. Second, the portion of long-term debt that is substituted for short-term debt allows the firm to borrow risk free at $t=0$ and gives the firm the opportunity to borrow risk free at $t = 1$, both of which lower overall financing costs. The costs of substituting into short-term debt are: 1) borrowing costs rise as more short-term debt is issued, and 2) the introduction of the dilution effect on long-term debt. The dilution effect is increasing in the amount of short-term debt issued and helps temper the increase in long-term bond prices.

For the same investment, I, moving from $\rho = 0$ to $\rho > 0$ lowers short-term rollover costs and reduces the dilution effect, both of which represent the substitution benefit of moving into long-term debt.[22] The substitution cost is that the firm must pay a positive credit spread on long-term debt rather than the risk free rate on short-term debt at $t = 0$. However, long-term debt buyers are generally optimists meaning that long-term credit spreads are relatively low and close to the risk-free rate. Thus, on the margin, the cost of substituting into long-term debt is very small compared to the benefit of reducing both short-term credit spreads and the dilution effect.

Figure 4 captures the essense of the benefits of substituting from a single debt maturity to a combination of debt maturities. Using a combination of debt maturities concentrates fewer total long-term bonds to investors most willing to hold risk at $t = 0$ than a maturity with no short-term debt, $q_0^{\varsigma} = 0$. Concurrently, the debt needed to ensure short-term debt is rolled over at $t = 1$ is also more concentrated to investors with higher willingness to hold risk than if the firm only issued long-term debt, $q_0^{\varsigma} = 0$. A combination of debt maturities reallocates risky debt away from investors today, who require more collateral to borrow at a given credit spread, to investors tomorrow who require less collateral to borrow at the same rate, and vice versa. Therefore, the firm is able to optimize the amount of risky debt it issues at each point in time. For a given amount of pledgable collateral, issuing a mix of debt a maturities allows the firm to invest and produce more than if it issued a single debt maturity.

[22]Equation (17) collapses to $\frac{A_{DD}}{\alpha}$ as ρ goes to 1.

Figure 4: Marginal buyer regimes



Regime Image.pdf

3.1 Debt maturity optimization and comparative statics

This section briefly discusses the model's comparative static results related to how the maturity profile is optimized toward long- or short-term debt depending on model parameters.

Let $0 < \rho^*(\gamma, A_{DD}) < 1$ denote the equilibrium amount of long-term debt issued for any given set of parameters. Specifically, γ is the likelihood that good news arrives in the following period, from the firm's perspective. A_{DD} determines the amount of collateral the firm can pledge at $s = DD$ and is a measure of down risk, while α is the returns to scale parameter.

More short-term debt is issued the more likely good news arrives in $t = 1$, $\frac{\partial \rho}{\partial \gamma} < 0$. The reason is that the likelihood of rolling over short-term debt at the risk-free rate increases, which lowers expected rollover costs relative to long-term financing.

More short-term debt is issued the more collateral the firm can pledge at $s = DD$, $\frac{\partial \rho}{\partial A_{DD}} < 0$. The reason is that risky short-term debt prices at $t = 1$ are more responsive to movements in A_{DD} than risky long-term debt prices at $t = 0$. To see why, consider any given investor, h. This investor puts more weight on $s = DD$ at time $t = 1$ than she does at $t = 0$, $(1-h) > (1-h)^2$. The value of an investor's claim at $s = DD$, irrespective of maturity, is the delivery rate given by (17). Investor h values the recoverable claim more at $t = 1$ than at $t = 0$.

Proposition 2 *With no liquidity risk and heterogeneous investors, debt maturity is optimized more long-term:*

- *the lower the likelihood of good states, low γ, $\frac{\partial \rho}{\partial \gamma} < 0$;*
- *the lower are expected cash-flows or the higher is down risk, low A_{DD}, $\frac{\partial \rho}{\partial A_{DD}} < 0$;*

3.2 Discussion and interpretation

In this section we discuss the predictions of the model relative to the existing heterogeneous agent models on which it is based, the general consistency of the model's comparitive statics results with the broad empirical literature, and the new insights that reconcile differences between empirical debt maturity studies.

Short-term debt is the unique funding outcome in the class of heterogeneous agent models of Geanakoplos (2003, 2009), Fostel and Geanakoplos (2008, 2010), and He and Xiong (2012a). In these models, all agents are endowed with both a risk-less and risky asset. Optimists want to hold more risky assets than pessimists. Optimists purchase all the risky assets by issuing a riskless claim backed by the maximum value of the asset in each period. Issuing a sequence of short-term claims allows optimists to borrow against the asset's worst intermediate-state and terminal-state values. By contrast, long-term claims only allow agents to borrow against the value of the asset in the worst terminal state . Thus, for optimists who price the asset in equilibrium, short-term debt always dominates long-term debt. These models are best suited to describe debt financing of financial assets for which the use of leverage is paramount. Banks, hedge funds, and institutional investors typically use leverage to make their asset purchases.

By contrast, the "firm" in our model is endowed with a risky production technology and issues debt backed by its technology–similar to Fostel and Geanakoplos (2016) and Rampini Vishwanathan (2010)–while investors have riskless assets that they use to purchase firm debt. Optimists, use their riskless asset to buy risky debt. The firm maximizes the expected value of equity value by concentrating risky claims across time because that is how it most efficiently accesses optimistic investor capital.

Our model is particularly relevant for large corporations where liquidity risk and information asymmetries are likely second order concerns. Lemma 1 and proposition 1 imply that safe short-term debt should be used *in conjunction* with risky-long term debt because it will help lower aggregate risky financing costs. This intuition rationalizes the existence of corporate commercial paper (CP) programs for large safe corporations. In our model, short-term CP is safe short-term debt issued at $t = 0$. The CP issuance must be refinanced at $t = 1$. This interpretation is consistent with the "bridge financing" findings of Kahl, Shivdasani, and Wang (2015). Moreover, safe-debt is *sufficient* for a debt maturity mix, which contrasts the liquidation risk stories underpinning Diamond (1991) and Houston and Venkataraman (1994).

The predictions of our model are broadly consistent with existing empirical studies. For example, we can interpret γ as a measure of management "optimism." Landier and Thesmar (2008) and Graham et. al (2013) find that management optimism leads to more short-term debt issuance, controlling for firm risk factors and leverage. Choi, Hackbarth, and Zechner (2016) show that corporations typically issue debt into, on average, more than 3 distinct maturity bins, and that large and mature corporations are more likely to issue multiple debt maturities. Norden, Rooenboom, and Wang (2016) show that borrowing costs are lower and leverage is positively associated with debt granularity *i.e.* a mix of debt maturities rather than a single debt maturity.

Empirical studies measure growth options as the market-to-book value of assets. In our model, the market value of the assets is the amount the firm produces because there is only one asset whose price is normalized to 1. The book value of the firm's asset is the amount of capital it raises to produce, or the book value of its liabilities. The market-to-book value of the firm is given by [23]

$$\text{market-to-book} = \frac{I^{\alpha}}{I} = I^{(\alpha-1)} = \frac{1}{\alpha p^{\ell}\left(\alpha, A_{DD}, \gamma\right)}. \tag{20}$$

Notice that the growth option of the firm is inextricably linked to the exogenous parameters

[23] We use the first order conditions (2) and (3) to derive the market-to-book in terms of the long-term bond price, p^{ℓ}. It can also be expressed in terms of short-term bond prices since the expected costs across maturities must be the same in an interior maturity equilibrium.

of the model through the market price of debt. Therefore, growth options are endogenously determined along with the firm's maturity choice and leverage through an asset's fundamental collateral value via α. This suggests that empirical studies should not treat growth options as exogenous to leverage and maturity choices, which is typical. The endogeneity of growth options with maturity choice may help explain why empirical debt maturity results are often mixed. Barclay and Smith (1995) and Guedes and Opler (1996) find that growth options and maturity are negatively related. Stohs and Mauer (1996) and Johnson (2003) find a positive relationship, while Billet et. al. (2007) find no relationship when controlling for covenants.

Lastly, investor heterogeneity generates differences in cost of capital through time and helps reconcile puzzling survey evidence that firms try to time the market when choosing debt financing. For example, Graham and Harvey (2001) and Servaes and Tufano (2006) find that global CFO respondents largely issue debt maturity to time market interest rates and limit the amount of debt that needs to be refinanced at any point in time. Economists typically view a market timing response with circumspect. Our model suggests that debt maturity choice is not about market timing *per se*. Rather, debt maturity is used to smooth financing costs by limiting the amount of risky debt that firms issue at any point in time. Lastly, both surveys find very little support for information asymmetries (Flannery (1986)) and debt overhang (Myers (1977)) as the main factors driving maturity choice, while credit ratings are important insofar as they affect the terms of borrowing, but expectations about credit rating changes are second order at best (Diamond (1991) and Houston and Venkataraman (1994)).

4 Protected debt with endogenous maturity

In this section we ask what happens when investors structure debt contracts with covenents that prevent debt dilution. Afterall, a combination of debt maturities without any covenents does in fact lead short-term debt to dilute long-term debt. If one does not allow long-term debt to be diluted, does using a combination of long- and short-term debt have any benefit relative to only long-term debt. The answer is yes. Protecting long-term debt raises the value any individual investor is willing to pay for the debt contract. The firm responds by re-optimizing its maturity more toward the relatively cheap long-term debt and away from the relatively more expensive short-term debt. In equilibrium, the marginal long-term bond buyer will be more pessimistic but the recovery value is higher. These two effects cancel one another and the relative prices between long- and short-term debt remain the same.

We begin by showing how to protect long-term debt from dilution and what the effect is on the equilibrium amount of long- versus short-term debt. We then show that firm value is unaffected by a non-financial covenant.

To see this, assume that a covenant ensures ρ portion of the firm assets are used as exclusive collateral for long-term debt, irrespective of short-term debt financing at $t = 1$. These assets cannot be used as collateral for short-term debt without violating the covenant and opening the firm up to costly litigation. The remainder of the assets, $(1-\rho)$, are used as collateral to secure short-term debt. A natural interpretation of this covenant is a negative pledge covenant. Negative pledges are among the most common non-financial covenants found in public long-term debt indentures. A description of the covenant and its relevance can be found in the appendix. With the covenant, the recovery values given by (8) become

$$\begin{cases} d_{DD}\left(q^{\ell}\right) = & \frac{\rho A_{DD} I^{\alpha}}{q^{\ell}}, \text{ long-term recovery} \\ d_{DD}\left(q_D^{\varsigma}\right) = & \frac{(1-\rho) A_{DD} I^{\alpha}}{q_D^{\varsigma}}, \text{ short-term recovery} \end{cases}.$$

Firm output in default is split between protected long-term creditors and short-term creditors who fund the short-term debt rollover at $t = 1$. Following the same procedure outlined in the example section, one can show that the debt delivery functions become:

$$\begin{cases} \hat{d_{DD}}\left(\hat{q}^{\ell}\right) = & \frac{A_{DD}}{\alpha} \\ \hat{d_{DD}}\left(\hat{q}^{\varsigma}_{D}\right) = & \frac{A_{DD}}{\alpha}\left(\frac{\hat{p}^{\varsigma}_{D}}{\hat{p}^{\ell}}\right) \end{cases}. \tag{21}$$

The hats represent variables in an economy with the covenant. The maturity specific recovery values in (21) behave as if $\rho = 1$ for long-term debt and $\rho = 0$ for short-term debt in (17), even though $0 < \hat{\rho} < 1$. All long-term debt holders are protected from debt dilution.

Proposition 3 *Consider any* $\hat{d_{DD}}\left(\hat{q}^{\ell}\right)$ *defined by* (21) *with a secured covenant and corresponding* $d^{*}_{DD}(\cdot)$ *defined by* (17) *without a covenant. In any debt financing strategy for which* $Q \equiv \left(q^{\ell}, q^{\varsigma}_{0}, q^{\varsigma}_{s}\right) > 0$, $s = \{U, D\}$ *,the following hold*

- $\hat{d_{DD}}\left(\hat{q}^{\ell}\right) > d^{*}_{DD}(\cdot)$. *Moreover,* $\hat{d_{DD}}\left(\hat{q}^{\ell}\right) = d_{DD}\left(q^{\ell}\right)\big|_{q^{\varsigma}_{0}=0}$*–long-term debt with secured covenants are protected from short-term debt dilution.*
- $\frac{1-(1-h_0)^2+(1-h_0)^2\left[\hat{d}_{DD}\left(q^{\ell}\right)\right]}{p^{\ell}} > \frac{1-(1-h_0)^2+(1-h_0)^2\left[d_{DD}(\cdot)\right]}{p^{\ell}}$*–any given investor is willing to pay more for long-term debt with a secured covenant than without.*

An immediate implication of proposition 3 is that debt maturity will be optimized more long-term.

Corollary 1 *Let* $\hat{\rho}$ *be the equilibrium portion of long-term debt when collateral values are determined by* (21)*. Let* ρ^{*} *be the equilibrium portion of debt issued long-term debt when collateral values are determined by* (17)*. It follows that* $\hat{\rho} > \rho^{*}$.

Corollary 1 says that protective covenants act as substitutes for risky short-term debt. This prediction is consistent with the empirical findings of Billet et. al. (2007). Do these non-financial covenants affect firm value? The answer is no. The firm increases the supply of risky long-term bonds it issues but reduces the supply of risky short-term bonds. In equilibrium, the relative prices of the two debt maturities must be equivalent in expectation (see equation (2)).

Proposition 4 *Let* $Q^* \equiv \left(q^{\ell^*}, q_0^{\varsigma^*}, q_s^{\varsigma^*}\right), s = \{U,D\}, \forall q \in Q^* > 0$ *be a set of equilibrium bond quantities with corresponding investment and price functions,* $\left(I^*, p^{\ell^*}, p_0^{\varsigma^*}, p_s^{\varsigma^*}\right)$*, as the solution to program* (1) *with debt deliveries given by* (17)*. Covenants that secure long-term debt and prevent dilution alter debt deliveries via* (21)*. The resulting equilibrium with the secured covenant has the following properties:*

1. *The optimal debt financing strategy is given by* $\hat{Q} \equiv \left(\hat{q}^{\ell}, \hat{q}_0^{\varsigma}, \hat{q}_s^{\varsigma}\right), s = \{U,D\}, \forall q \in \hat{Q} > 0$ *such that* $q^{\ell^*} > \hat{q}^{\ell}$ *and* $q_s^{\varsigma^*} < \hat{q}_s^{\varsigma}, s = \{0,U,D\}$*; and*

2. $\left(I^*, p^{\ell^*}, p_0^{\varsigma^*}, p_s^{\varsigma^*}\right)$ *is unchanged.*

The intuition is the following. Collateral is required due to the payment enforcement friction. Making debt even more "secure" by explicitly preventing dilution has no real impact because it does not allow the firm to create additional value given its collateral constraint. The covenant simply reallocates collateral to the protected debt instrument and the firm substitutes unprotected for protected debt due to changes in the relative equilibrium prices investors require. Our model provides a simple rationalization for the substitution effects of covenants and short-term debt empirically documented by Billet et. al (2007).

The notion that the covenant prevents dilution is equivalent to what Hart and Moore (1995) consider to be a *hard claim* on firm cash flows. They show that hard claims on the value of assets in place prevent managers with empire building motives from undertaking negative net present value projects. The resources needed to fund such intermediate investment projects are encumbered by existing long-term debt claims and cannot be diluted. Secured debt improves investment incentives in models with agency concerns. Secured debt does not improve firm value with repayment enforcement frictions and multiple debt maturities.

The following numerical example highlights the major comparative static results from proposition 2 and covenant results of corollary 1 and proposition 4.

4.1 Numerical example

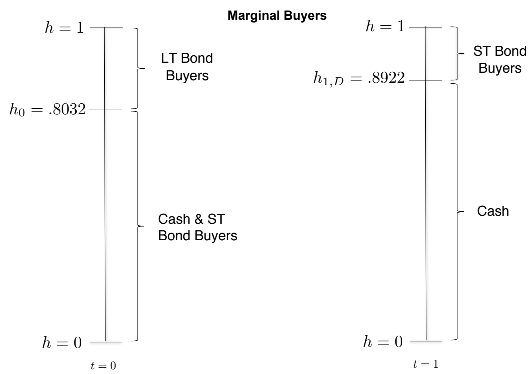
We keep α the same as in the previous example and show how different values the technology shock, A_{DD}, the good state probability, γ, and the covenant change the relative amount of long- versus short-term debt given by ρ. Figure (5) shows the equilibrium marginal investor regime for $A_{DD} = 0.5$ and $\gamma = 0.8$. Table 6 highlights the major effects of the secured debt covenant for various $(A_{DD,}\gamma)$-pairs. The top (bottom) panel contains the endogenous variables for the economy with (without) the covenant. The numbers in red highlight the key changes. First, note that all debt prices, investment levels and profits are unchanged across the two panels. More (Less) long-term (short-term) debt is issued in the economy with the covenant. The covenant simply tilts the maturity in favor of long-term debt, $\rho \uparrow$, and the firm substitutes away from short-term debt.

The comparative static results are contained by comparing rows within each panel. The first two rows of either panel show how variables change as A_{DD} decreases, while the third row shows changes in γ for the same A_{DD} as the first row. More down risk at $t = 2$ lowers all risky debt prices, resulting in lower investment and profits. The firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$. Second, consider a decrease in γ for the same A_{DD} as in the first row. The bottom row shows that the firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$, resulting in lower long-term debt prices, but higher risky short-term debt prices. The firm also invests less and is less profitable.

Table 6: Endogenous Variables

Covenant	p_0^ς	p_D^ς	p^ℓ	q_0^ς	q_D^ς	q^ℓ	I	ρ	Π
(A_{DD},γ)=(.5,.8)	1	.941	.989	.145	.154	.167	.311	.533	.075
(A_{DD},γ)=(.2,.8)	1	.894	.980	.136	.153	.163	.297	.539	.072
(A_{DD},γ)=(.5,.5)	1	.957	.985	.107	.112	.199	.304	.646	.057
No Covenant	p_0^ς	p_D^ς	p^ℓ	q_0^ς	q_D^ς	q^ℓ	I	ρ	Π
(A_{DD},γ)=(.5,.8)	1	.941	.989	.149	.158	.163	.311	.519	.075
(A_{DD},γ)=(.2,.8)	1	.894	.980	.138	.154	.162	.297	.534	.072
(A_{DD},γ)=(.5,.5)	1	.957	.985	.113	.115	.197	.304	.638	.057

Figure 5: Regime: Portfolio - Rollover



5 Conclusion

This paper characterizes optimal debt maturity in an economy with payment enforcement frictions and heterogeneous lenders. A debt financing strategy with both long- and short-term debt is generally the least costly way for the firm to obtain financing. Issuing multiple debt maturities to heterogeneous investors allows firms to cater risky debt securities to investors most willing to hold risk, which facilitates a reduction in borrowing costs and an increase in firm value. Moreover, a combination of debt maturities arises naturally when firms issue "safe" short-term debt and rationalizes why large corporates use commercial paper as bridge financing to finance long-term investment projects. Further, the model predicts that firms will use more short-term debt when managers operating in shareholders' best interest are optimistic about investment returns, or when expected cash flows are high, or down-side risk is low. We also show how growth options and leverage are endogenous to the firm's debt maturity choice because the price of the securities issued are affected by maturity, which in turn affects investment. Finally, we show that protective non-financial debt covenants prevent dilution, leads to more long-term financing and a substitution away from short-term financing. However, protective covenants do not affect real outcomes because they simply reallocate collateral claims among long- and short-term debt holders.

Our model also rationalizes one of Myers' (1993) most striking findings. Myers notes that almost all leverage increasing actions are good news and leverage decreasing activities are bad news. In our model, the firm uses debt maturity to reduce financing costs and issue more debt for any fundamental collateral value it can pledge to lenders. This raises the value of the firm because it invests and produces more. We have abstracted away from agency concerns to highlight that the mechanism operates through investor heterogeneity rather than liquidation risk or asymmetric information and signaling. In so doing, our model suggests that leverage, debt maturity and proxies for growth options are all jointly determined, which may help explain the different findings of various empirical studies.

An important assumption of the model is that there are no collateral cash flow problems, which means that the future value of the firm that serves as collateral can be rationally anticipated. One outcome of this assumption is that investors will always demand to hold risky claims on firm cash flows even following bad news. Altering the model to allow for investor coordination failure and self-fulfilling debt runs with collateral may produce new and interesting interactions between debt maturity, liquidity risk and the design of corporate securities that internalize such outcomes.

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A Appendix Omitted Proofs

Proof of Lemma 1: Combining (2) and (3) and plugging into (18) immediately gives (19). Note that $\varepsilon(\rho;\alpha) \in (\alpha,0)$, $0<\rho<1$ and clearly decreases in the arguments that increase ρ. Proposition 2 shows that $\frac{\partial \rho}{\partial \gamma} < 0$ and $\frac{\partial \rho}{\partial A_{DD}} < 0$, meaning that $\left.\frac{\partial \varepsilon(\rho;A_{DD},\gamma)}{\partial A_{DD}}\right|_{\alpha} < 0$ and $\left.\frac{\partial \varepsilon(\rho;A_{DD},\gamma)}{\partial \gamma}\right|_{\alpha} < 0$. Therefore, $A_{DD} \downarrow 0$ and $\gamma \downarrow 0 \implies \varepsilon \overset{\lim}{\to} 0$. Any risky bond price ratio $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} > 0$ will satisfy (19) for small values of A_{DD} and γ because $\rho \overset{\lim}{\to} 1$. It is less obvious that (19) is always satisfied when $\rho \overset{\lim}{\to} 0$ because $\varepsilon \overset{\lim}{\to} \alpha$. The reason is that moving from all short- to an interior solution involves reducing the safe short-term debt issued at $t=0$ in favor or risky long-term debt which is always costly at $t=0$. By contrast, moving from all long to an interior involves issuing less risky long-term for safe short-term at $t=0$, for which the cost benefits are always clear. $\varepsilon \overset{\lim}{\to} \alpha$ as $\gamma \uparrow 1$ because $\rho \to 0$. As long as $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \geq \alpha$ as $\gamma \uparrow 1$, condition (19) will hold for all γ because $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \uparrow$ as $\gamma \downarrow 0$ and $\varepsilon \uparrow$ .Similarly, if $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \geq \alpha$ holds for $A_{DD} \to 0$, then it will hold for all $A_{DD} \to \alpha$ because $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \uparrow$ as $A_{DD} \downarrow$. For the numerical example in Table 1 of appendix B, $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \approxeq 0.95$, with $\alpha = 0.7$, $\gamma = 0.8$, and $A_{DD} = 0.5$. We can show numerically that (19) does indeed hold $\forall (A_{DD},\gamma)-$pairs. *Q.E.D.*

Proof of Proposition 1: We show for any investment amount I_0, issuing $q_0^{\ell} > 0$ and $q_D^{\varsigma} > 0$ is cost reducing relative to either $q_0^{\ell} = 0$ or $q_0^{\varsigma} = 0$. First, note that (2) and (3) can be combined to express the firm's marginal product equal to either only the marginal cost of long-term debt or the marginal cost of short-term debt. This relationship simply reflects the fact that an interior maximum must be characterized by maturity cost equivalence at the margin. Suppose all short-term debt is rolled over so that $p_0^{\varsigma} = 1$ always. Next, suppose maturity is irrelevant, and the firm can obtain the same terms of financing all long-term or via interior solution. Let I_0^* be the optimal investment amount for some parameter set $\Gamma(\alpha, A_{DD}, \gamma)$. If maturity is irrelevant, the firm must be indifferent to raising I_0^* by issuing all long-term debt, $Q = \tilde{q}_0^{\ell}$, at price $\tilde{p}_0^{\ell}$ or to issuing both long- and short-term debt,

$Q = \hat{q}_0^\ell + \hat{q}_0^\varsigma$, at prices $\hat{p}_0^\ell$ and $\hat{p}_0^\varsigma = 1$. Clearly it must be the case that $\tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^\varsigma > 0$, and since the firm takes prices as given, it must be the case that $\tilde{p}_0^\ell > \hat{p}_0^\ell$. Market clearing implies the supply of financing equals the firm's demand for financing. For only long-term debt, market clearing is given by $\left(1-\tilde{h}_0\right) = \tilde{p}_0^\ell \tilde{q}_0^\ell = I_0^*$ and for both long- and short-term debt by $\left(1-\hat{h}_0\right) + \left(1-\hat{h}_D\right) = \hat{p}_0^\ell \hat{q}_0^\ell + \hat{p}_D^\varsigma \hat{q}_D^\varsigma = I_0^*$. Equating the two market clearing conditions for the same I_0^* gives $\left(1-\tilde{h}_0\right) = \left(1-\hat{h}_0\right) + \left(1-\hat{h}_D\right)$. This can only hold if $\hat{h}_D = 1$ meaning that $\hat{q}_D = 0$–no short-term debt is issued–or if $\tilde{h}_0 < \hat{h}_0$–the marginal long-term bond buyer in an interior solution is more optimistic than the marginal bond buyer in the corner solution. But, the more optimistic the investor, the higher the price she is willing to pay $\implies \tilde{p}_0^\ell < \hat{p}_0^\ell$, which contradicts $\tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^\varsigma > 0$. The same logic will also show that the firm can never be indifferent between all short-term financing and a combination of short- and long-term debt. *Q.E.D.*

Proof of Proposition 2

$\frac{\partial \rho}{\partial \gamma} < 0$: From (2) and a given set of risky debt prices $\left(p_0^{\ell^*}, p_D^{\varsigma^*}\right)$, $\uparrow \gamma$ increases the l.h.s more than the right. If the firm issues more long-term debt, $\rho \uparrow$, long-term debt prices fall and short-term debt prices rise, causing further deviation from the necessary equality. Thus, the firm must issue more short-term debt, $\frac{\partial \rho}{\partial \gamma} < 0$.

$\frac{\partial \rho}{\partial A_{DD}} < 0$: From (17) we know that long-term debt holders and risky short-term debt holders expect the same delivery at $s = DD$. An increase in A_{DD} raises expected delivery for bond holders of all maturities. However, long-term debt holders at $t = 0$ place $(1-h_0)^2$ weight on $s = DD$ while short-term debt holders at $t = 1$ place $\left(1-h_D^\varsigma\right)$ weight on $s = DD$. Therefore, unless $h_0 \ll h_D$, short-term debt holders place more weight on recovery for which A_{DD} has an ultimate affect. This implies that an increase in A_{DD} tilts debt maturity towards short-term funding and ρ falls. Alternatively, suppose to the contrary that $(1-h_0)^2 > (1-h_D) \Leftrightarrow h_0 \ll h_D$ so that long-term marginal buyer places more weight on the down state than the short-term marginal buyer. This condition can be re-written as

$h_0(2-h_0) > h_D$. It then follows that $1-(1-h_0)^2 > h_D \Leftrightarrow h_0(2-h_0) > h_D$. The long-term marginal buyer is also more optimistic about the up-state than the short-term marginal buyer. The only way the long-term price is more responsive to changes in A_{DD} is if the long-term buyer is simultaneously more optimistic and pessimistic than the short-term marginal buyer. A contradiction.

Q.E.D

Proof of Proposition 3: A necessary condition for any $0 < \rho < 1$ in any collateral economy with or without the covenant is $p_0^{\ell} > p_D^{\varsigma}$ from $\frac{d\Pi}{d\rho} = 0$. Thus, the necessary condition also ensures $\hat{d_{DD}}\left(\hat{q}_0^{\ell}\right) > d_{DD}^{*}(\cdot)$. Let $\rho^* = 1$ in which case $q_0^{\varsigma} = 0$. From (17), $d_{DD}^{*}(\cdot) = \frac{A_{DD}}{\alpha} = \hat{d_{DD}}\left(\hat{q}_0^{\ell}\right)$ in (21). For the second item in the proof, it is clear from (13) and (15) that any given buyer with the same marginal utilities across states must pay a higher price for securities with higher deliveries. *Q.E.D.*

Proof of Corollary 1: From Proposition 5 and (21) we know that $d_{DD}\left(q_0^{\ell}\right) > d_{DD}\left(q_D^{\varsigma}\right)$ when long-term indentures include the covenant for a given $\left(I_0^*, \rho^*\right)$. Suppose the firm does not alter its debt structure and $\rho^*$is unchanged. Then, long-term debt prices must rise to a new level reflecting greater marginal valuations, $p_0^{\ell c} > p_0^{\ell *}$, where the superscript c denotes prices with the covenant. But if long-term debt is now cheaper in equilibrium, then the maturity structure for a given $\left(I_0^*, \rho^*\right)$cannot be optimizing and the firm must adjust. Thus the firm issues more long-term debt and reduces its short-term debt, leaving I_0^*unchanged and $\rho^c > \rho^*$so lowering $\downarrow p_0^{\ell c} = p_0^{\ell *}$. *Q.E.D.*

Proof of Proposition 4: Follows immediately from the proof of Corollary 1 and investment optimality in (2) and (3). *Q.E.D.*

B Appendix

B.1 Multiple debt maturity funding

The ten endogenous variables are $\left(p_0^{\varsigma}, p_0^{\ell}, p_D^{\varsigma}, q_0^{\varsigma}, q_0^{\ell}, q_D^{\varsigma}, I_0, \rho, h_0, h_D\right)$. The system of equations, along with (2) and (3) is:

$$p_0^{\varsigma} = 1 \tag{22}$$

$$1 = \frac{1-(1-h_0)^2+(1-h_0)^2 d_{DD}\left(q_0^{\ell}\right)}{p_0^{\ell}} \tag{23}$$

$$1 = \frac{h_D+(1-h_D)\, d_{DD}\left(q_D^{\varsigma}\right)}{p_D^{\varsigma}} \tag{24}$$

$$I_0 = p_0^{\ell} q_0^{\ell} + p_0^{\varsigma} q_0^{\varsigma} \tag{25}$$

$$\rho = \frac{p_0^{\ell} q_0^{\ell}}{I_0} \tag{26}$$

$$q_0^{\varsigma} = p_s^{\varsigma} q_s^{\varsigma} \tag{27}$$

$$1-h_0 = p_0^{\ell} q_0^{\ell} \tag{28}$$

$$1-h_D = p_D^{\varsigma} q_D^{\varsigma} \tag{29}$$

The first three equations are bond pricing equations. Equation (22) shows that short-term bonds issued at time 0 are risk free because all short-term debt is rolled over at time 1. Equation (23) states that long-term bonds are priced based on the time 0 marginal investor's expectations because he is indifferent between buying the bond and holding a cash equivalent asset. Similarly, equation (24) states that time 1 short-term bonds are priced based on the time 1 marginal investor's expectations because cash is the only other alternative asset. Equation (25) says that the amount of capital the firm raises in the bond market is equal to the investment it puts into its production technology. Equations (2) and (3) are the first order conditions *w.r.t.* the portfolio allocation ρ and investment level I_0, respectively. The necessary condition for the firm to issue a portfolio of both long and short-term bonds in (2) says that on the margin the expected cost of issuing either type of bond must be the same. The left hand side of (3) is the expected marginal product of capital irrespective of whether or not it is issued via long-term or short-term bonds. The right hand side is the expected-weighted marginal cost of capital. Equation (26) sets ρ equal to the portion of the firm's investment that is raised via long-term debt. Equation (27) shows that the firm will issue as many short-term bonds at time 1 as it takes to fully repay its time 0 short-term creditors. Equations (28) and (29) are, respectively, the long-term and time 1 short-term bond market clearing conditions.

C Appendix

Here we show that changing the uncertainty structure of the economy does not materially alter the optimal choice to issue both long- and short-term debt. Instead of the structure given by figure 1 where $\gamma = \gamma|_{s=D}$ let $\gamma_1 = \Pr(s = U) > \gamma_2 = \Pr(s = DU|_{s=D})$ so that the likelihood of receive a good state following a bad state is less that receiving an unconditional good state. Breaking the firm's problem given by (1) into its constituent pieces, we can write profits as

$$\max_{I_0,\rho} \prod = \left\{ \gamma_1 \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1-\rho) \frac{I_0}{1} \right] + (1-\gamma_1)\gamma_2 \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1-\rho) \frac{I_0}{p_D^\varsigma} \right] \right\}.$$

This profit expression simply states that conditional on good news at $t = 1$, both long- and short-term debt is repaid, and conditional on bad news at $t = 1$ long- and short-term debts are repaid only if good news arrives at $t = 2$. Notice that the only difference between this problem and the one presented in the main body of the paper is that $\gamma_2 < \gamma_1 = \gamma$. The first order conditions for a maximum simply become

$$\begin{aligned}\frac{[\gamma_1+\gamma_2(1-\gamma_1)]}{p_0^{\ell}} &= \frac{1}{p_0^{\varsigma}}\left[\gamma_1+\frac{\gamma_2(1-\gamma_1)}{p_D^{\varsigma}}\right]\\ \alpha I_0^{\alpha-1}[\gamma_1+\gamma_2(1-\gamma_1)] &= \frac{\rho[\gamma_1+\gamma_2(1-\gamma_1)]}{p_0^{\ell}}+\frac{(1-\rho)}{p_0^{\varsigma}}\left[\gamma_1+\frac{\gamma_2(1-\gamma_1)}{p_D^{\varsigma}}\right].\end{aligned}$$

Plugging into the other we obtain $\alpha I_0^{\alpha-1}=\frac{1}{p_0^{\ell}}$ which of course arises because in equilibrium the marginal cost of a long-term bond must equal the marginal cost of a short-term bond for $0<\rho<1$ allowing us to express the first order condition for a maximum as a function of either long- or short-term debt. Let $A\equiv\gamma+\gamma(1-\gamma)$ when $\gamma=\gamma|_{s=D}$ and $B\equiv\gamma_1+\gamma_2(1-\gamma_1)$ from the restated problem above and $A>B$. Then, $\forall(I_0,\rho)$: $\alpha I_0^{(\alpha-1)}A>\alpha I_0^{(\alpha-1)}B$. This implies that $\frac{1}{p_0^{\ell}}\big|_A>\frac{1}{p_0^{\ell}}\big|_B\Rightarrow p_0^{\ell}\big|_B>p_0^{\ell}\big|_A$ at the optimum. In other words, for a given ρ, the firm will only raise the same amount of capital across the two economies if long-term bond prices are higher in the economy with more uncertainty at $s=D$, which is a contradiction because the firm is less likely to repay debt at $s=DU$ with in the more uncertainty case. Alternatively, the firm can raise less long-term debt and more short-term debt in the economy with more uncertainty at $s=D$, leaving total I_0 unchanged and tilting ρ more toward short-term debt. This results in lower short-term bond prices and higher long-term bond prices. And by proposition 3, starting from a corner solution, it will always be less costly to balance long- and short-term borrowing costs against one another rather than issuing all long- or short-term debt. The only thing that will change is the relative maturity tilt.

The same logic applies if we were to allow for uncertainty at $s=U$ and default at $s=UD$. For this, assume that firm deliver at $s=UD$ is higher than $s=DD$, where generically $d_{UD}(Q)=d_{DD}(Q)+\varepsilon<1$. This simply reflects the fact that the ultimate shock to collateral is worse in two consecutive bad states than in an up state followed by a down state. The firm's maximization problem can be split and written as follows:

$$\max_{I_0,\rho}\prod=\left\{\gamma^2\left[I_0^\alpha-\rho\frac{I_0}{p_0^\ell}-(1-\rho)\frac{I_0}{p_U^\varsigma}\right]+(1-\gamma)\gamma\left[I_0^\alpha-\rho\frac{I_0}{p_0^\ell}-(1-\rho)\frac{I_0}{p_D^\varsigma}\right]\right\}.$$

Only two things change in the problem. 1) Debts are no longer repaid conditional on $s=U$ so that now the first set of repayment states are given by γ^2 rather than γ. 2) $p_U^\varsigma\neq 1$ as it does with full repayment. Taking first order conditions for an interior maximum and plugging in, one can express the same marginal product equals marginal cost as $\alpha I_0^{\alpha-1}=\frac{1}{p_0^\ell}$. And by proposition 3, we know that for any given I_0 and a candidate corner solution, it is always be cheaper to fund a portion of the investment outlay by substituting into either long or short-term debt rather so that both debt maturities are utilized. *QED*.

D Negative pledge covenant

Our treatment of protected long-term debt can be thought either as an explicit collateral pledge or earmark, or the inclusion of a negative pledge covenant that explicitly spells out how long-term debt is secured from short-term debt dilution. The benefit of thinking about negative pledge covenants, as detailed below, is two fold: 1) negative pledges are among the most common covenants found in public debt indentures, 2) given their prominence, surprisingly little is known in the academic literature of their impact. We thus attempt to fill this void with the support of strong practical relevance.

Negative pledges are widely recognized by the law and economics profession (see Bjerre (1999), Wood (2007, 2008)). The covenant stipulates that the firm cannot issue secured debt in the future without securing the current debt issue. For example, Billet et. al. (2007) classify negative pledge covenants as "Secured Debt Restrictions" because they restrict the security of future debt issues. Table III in their paper shows that negative pledges are typically the 3rd or 4th most common covenant, behind cross default or accel-

Table 7: Negative pledge covenant

	Negative pledge covenant	
	Yes	No
Non-financial	14,783	11,424
Financial	3,117	4,825
< 5yr	2,244	2,376
5yr - 30 yr	15,284	13,401
Total	17,900	16,249

eration, asset sale, and merger clauses. Negative pledges are more common than leverage, dividend, and share repurchase restrictions. Table 7 gives a general sense for the basic statistics on types of bonds that contain a negative pledge covenant. They are more prone in medium-to-long-term non-financial corporate indentures.