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Nonlinearities in the Phillips Curve for the United States: Evidence Using Metropolitan Data

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With the unemployment rate in the United States currently below estimates of its natural rate we examine if the relationship between inflation and unemployment is nonlinear. Using aggregate data we are unable to reject a linear relationship. However, using metropolitan-level data we find the slope of the Phillips curve is roughly twice as large when unemployment is low compared to when it is high. Nevertheless the simple nonlinear Phillips curves used here suggest a core CPI inflation rate that is only slightly different than the linear version over the next couple of years.

1. Introduction

The relationship between the unemployment rate and inflation is often assumed to be linear. That is, the effect on inflation of an extra percentage point in unemployment is the same regardless of the level of unemployment or inflation.[1] However, this need not be the case. Indeed, Phillips' original paper (1958), as well as many introductory textbooks, show the relationship with a steep slope when the unemployment rate is low and a flatter slope as the unemployment rate increases.

There are a number of reasons that we might expect the Phillips curve to be nonlinear including: changes in trend inflation that can influence the frequency of price changes and alter the slope of the Phillips curve[2], downward nominal wage rigidity and financial frictions that keep firms from reducing prices in deep recessions[3], detachment of the long-term unemployed from the labor market[4], and a bell-curve shaped distribution of the reservation wage across the population that causes the wage needed to attract new workers to increase nonlinearly as unemployment falls.[5]

These various ideas can lead to different forms of nonlinearities in the Phillips curve. In this study, however, we examine only one: that the slope of the consumer price index (CPI) Phillips curve in the United States changes with the level of unemployment. We take an agnostic view and do not specify the source of the nonlinearity. Instead we focus on confirming that a relatively robust nonlinearity exists for US price inflation in recent decades and that it is of a relevant magnitude. We leave other nonlinearities and the cause of the nonlinearity for future research.

Our personal experiences from many years of forecasting inflation at a central bank suggests that a possible nonlinearity in the effect of the unemployment rate on price inflation is usually considerably less important than other specification issues when estimating an equation to forecast inflation. In particular, issues such as the time period of estimation, the choice of the dependent variable, inclusion of additional regressors, like inflation expectations, import and oil prices, and even the lag structures of the dependent and independent variables, usually have much more effect on the fit and forecast of the inflation equation than nonlinearities in an unemployment rate term. However, on those occasions when the unemployment rate becomes extremely high or extremely low the question of nonlinearities takes on additional importance. The next couple of years may be one of those times with the current unemployment rate among the lowest rates of unemployment in the US since the second half of the 1960s.

This paper is closely related to contemporaneous independent work by Murphy (2017) who also uses metropolitan data drawn from the same survey to examine nonlinearities in the Phillips curve and arrives at conclusions roughly similar to our own. Other recent work on nonlinearities in a US price

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[1] For example see Yellen (2015), Gordon (2013), Fuhrer (1995) among numerous others.
[2] See Ball, Mankiw, and Romer (1988), Dotsey, King, and Wolman (1999). Akerlof, Dickens, and Perry (1996) also suggests the inflation-unemployment relationship becomes flatter at lower inflation rates
[3] For downward nominal wage rigidity in the United States see Daly and Hobijn (2014), Lebow, Saks, and Wilson (2003), Fallick, Lettau, and Wascher (2016) among others. For financial frictions Gilchrist, et al (2017).
[4] See Kiley (2015), Smith (2014), Council of Economic Advisors (2014).
[5] This might be what Phillips (1958) implicitly was referring to when he suggested that the "relation between unemployment and the rate of changes of wages is … likely to be highly non-linear."

Phillips curve have been, on balance, somewhat supportive of nonlinearities. Nalewaik (2016) introduces a nonlinearity through a squared term on the unemployment rate when it is low in a two-regime switching model using national core PCE price inflation over the 1961 to 2016 time period, and he finds that this nonlinearity is statistically significant. Santoro, et al (2014) find some support for asymmetries over the 1982 to 2008 period, though only look at overall GDP prices making it unclear if their result is being driven by the large run-up in oil prices near the end of their sample. Albuquerque and Baumann (2017) also suggest a nonlinearity, but do not test it directly and find a linear unemployment rate gap and a nonlinear unemployment rate gap both provide the same level of overall regression fit in a US PCE price Phillips curve over the 1992 to 2015 time period. Some other work examining price Phillips curves for the United States over the past decade and a half have failed to find nonlinearities or suggest a nonlinearity going in the opposite direction (Eliasson (2001), Fendel, Lis, and Rulke (2011)). Additionally, Laxton, et al, (1999) argue that how the NAIRU is estimated greatly affects inferences on asymmetries of the Phillips curve.

For wages in the United States the case for a nonlinearity appears clearer. Fisher and Koenig (2014) noted the appearance of a much steeper wage inflation-unemployment tradeoff when the unemployment rate is low using ECI data. Kumar and Orrenius (2014) examined 1- and 2-kink versions of a wage Phillips curve using state level data and found a single kink fits the data quite well. Both Donayre and Panovska (2016), using a three-regime threshold regression, and Nalewaik (2016), mentioned above, also find nonlinearities in compensation per hour. All find steeper Phillips curve slopes at low levels of unemployment.

In many ways our paper is an extension of Kiley (2015), which found that both short and long-term unemployment had roughly similar effects in inflation. We use an extended version of that dataset with annual observations running from 1985 to 2016 covering 23 metropolitan areas in the United States. Many of the regression forms that we test are inspired by, or directly taken from, Kiley's note. The main difference is that instead of looking at short- and long-term unemployment separately, as Kiley did, we take the overall unemployment rate and give it a nonlinear functional form in order to examine the change in the Phillips curve slope at different levels of unemployment. Inflation is measured as the logarithmic change in the Consumer Price Index (CPI) excluding food and energy from one year to the next, $100*\ln(P_{i,t}/P_{i,t-1})$.[6] The main unemployment rate in the regressions is the standard unemployment rate, U-3, published by the Bureau of Labor Statistics for these metropolitan areas, but, for comparison to Kiley's note, we run some regressions using the short-term and long-term unemployment rates.[7]

While we cannot reject a linear Phillips curve using national data, we can reject it using metropolitan data. We do not find the failure to find a nonlinearity in the national data surprising given the small number of observations at the national level. In the metropolitan data the nonlinearity is fairly robust, with, in many of the specifications, the slope of the Phillips curve more than twice as steep when the unemployment rate is at 4.5 percent than when it is at 7.5 percent. Further, in the metropolitan analysis we find that using a nonlinear form fits the data better than dividing the unemployment rate

[6] The FOMC sets its target in terms of Personal Consumption Expenditure (PCE) price inflation. However, PCE price data is not available by metropolitan region. Appendix tables show results using total CPI inflation that we discuss later in the paper.

[7] Metropolitan level unemployment rate series are constructed using Current Population Survey (CPS) data to roughly match the CPI geographic coverage (see appendix B).

into short- and long-term unemployment. Nonetheless, the economic significance of the nonlinearity is small: inflation projections from the nonlinear Phillips curves are within two-tenths of a percentage point of the projections from the linear version until the unemployment rate falls below 3¾ percent.

2. National data

We first look at national Phillips curves to see if we can find support for a nonlinearity. For the baseline we follow the simple Phillips curve form of Kiley where inflation is a function of a constant, lagged inflation, and the unemployment rate. These results are shown in table 1 for the 1985 to 2016 period and table 2 for the 1998 to 2016 period. The regressions for the 1985 to 2016 period, which covers a period of generally falling inflation from 1985 to 1998, include inflation expectations as measured by the median 10-year forecast from the Survey of Professional Forecasters (SPF) conducted by the Philadelphia Federal Reserve Bank.[8] Since 1998 inflation expectations have been relatively stable and this variable is dropped from the 1998 to 2016 regressions.[9]

The first column of the two tables shows the baseline regression of a linear Phillips curve. The second column replaces the unemployment rate with the short-term and long term unemployment rate, as in Kiley.[10] In the 1985 to 2016 sample this regression shows a larger difference in the short and long-term unemployment coefficients than Kiley found, but the difference is not statistically significant given the large standard errors. In the 1998 to present sample there is essentially no difference in the coefficients on long and short-run unemployment. Further, the improvement in regression fit from splitting unemployment into short- and long-term unemployment is small and both the adjusted r-squared and AIC suggest a preference for the more parsimonious model without short- and long-term unemployment.

The third and fourth columns use nonlinear forms for the unemployment rate. The third column uses a spline to allow for a single kink, while the fourth column takes the unemployment rate to an exponent.[11] In the 1985 to 2016 sample the nonlinearity suggested by the regression shows a less steep Phillips curve slope when the unemployment rate is low — the opposite of what we would have expected to find. Over the shorter sample in table 2 the nonlinearity reverses with the Phillips curve

[8] Prior to 1990 the 10 Year-Ahead Inflation Forecast is measured as the annual average of the Blue Chip Economic Indicators.

[9] Including the expectations variable in the 1998-2016 regressions does not qualitatively impact the results.

[10] The long-term unemployment rate is measured as the difference in the overall unemployment rate minus the short-term unemployment rate (i.e. the number of people unemployed 26 weeks or less divided by the total size of the labor force).

[11] A grid search was conducted over values of 4.5 to 7.5 by 0.1 increments to determine the best kink point for the unemployment rate. For all regressions in the paper standard errors and goodness of fit statistics have not been adjusted to account for the fitting of the kink points. The exponential regressions were estimated by nonlinear least squares with initial values determined by a grid search using exponent values from -3.0 to 3.0 in 0.1 increments. We also tested the log of the unemployment rate an as option instead of the unemployment rate to an exponent.

steeper when the unemployment rate is low. Either way we cannot reject that the Phillips curve is linear and the improvement in regression fit is small.

The Phillips curves used here are very simple and have few observations, but our experience with more fully specified quarterly national Phillips curves, such as those in Yellen (2015), suggest it is difficult to make a clear case one way or the other for an asymmetric US price inflation Phillips curve using aggregate national data unless the sample is extended to cover the rise in inflation during the second half of the 1960s as is done in Nalewaik (2016).

Table 1. Results from National Data over 1985 to 2016 time period

Equation: $\text{Inflation}_{i,t} = c + \beta_1 \text{ Inflation}_{i,t-1} + \theta \text{ f(Unemployment rate}_{i,t}) + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)
Form of unemployment rate	UR	Short & long term UR	UR with kink at 5.1	UR^{α}
Constant	0.47 (0.36)	0.74 (0.49)	0.61 (0.49)	0.14 (0.75)
Coef on Inflation expectations	0.50^{**} (0.22)	0.51^{**} (0.22)	0.49^{**} (0.22)	0.50^{**} (0.22)
Coef on lagged core inflation	0.51^{***} (0.17)	0.61^{***} (0.21)	0.51^{***} (0.17)	-0.50^{***} (0.14)
Coef on ur	-0.12^{**} (0.05)		-0.14^{**} (0.06)	
Coef on Short-term UR		-0.27 (0.19)		
Coef on Long-term UR		-0.02 (0.14)		
Additional coef on ur below kink 1			0.13 (0.30)	
Coef on u^{α}				-0.01^{**} (0.11)
α (exponent on u)				1.90 (3.75)
Slope at 4.5% U rate	-0.12	NA	-0.01	-0.08
Slope at 7.5% U rate	-0.12	NA	-0.14	-0.13
Nobs	32	32	32	32
SSR	3.73	3.64	3.71	3.72
Adj. R^2	0.880	0.879	0.877	0.880
AIC	0.939	0.977	0.994	1.000

*,**,*** represent difference from 0 is statistically significant at the 90, 95, 99 percent level, except for exponent on UR where it represents the difference from 1.

Table 2. Results from National Data over 1998 to 2016 time period

Equation: $\text{Inflation}_{i,t} = c + \beta_1 \text{ Inflation}_{i,t-1} + \theta \text{ f(Unemployment rate}_{i,t}) + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)
Form of unemployment rate	UR	Short & long term UR	UR with kink at 6.0	UR^{α}
Constant	2.49^{***}	2.46^{***}	2.10^{***}	5.52
	(0.54)	(0.59)	(0.71)	(38.87)
Coef on lagged core inflation	0.20	0.18	0.20	0.19
	(0.18)	(0.26)	(0.18)	(0.19)
Coef on ur	-0.15^{***}		-0.10	
	(0.04)		(0.07)	
Coef on Short-term UR		-0.13		
		(0.18)		
Coef on Long-term UR		-0.16		
		(0.13)		
Additional coef on ur below kink 1			-0.14	
			(0.17)	
Coef on u^{α}				-2.57
				(36.09)
α (exponent on u)				0.24
				(2.33)
Slope at 4.5% U rate	-0.15	NA	-0.24	-0.20
Slope at 7.5% U rate	-0.15	NA	-0.10	-0.13
Nobs	19	19	19	19
SSR	1.36	1.36	1.30	1.36
Adj. R^2	0.496	0.463	0.487	0.464
AIC	0.518	0.622	0.575	0.618

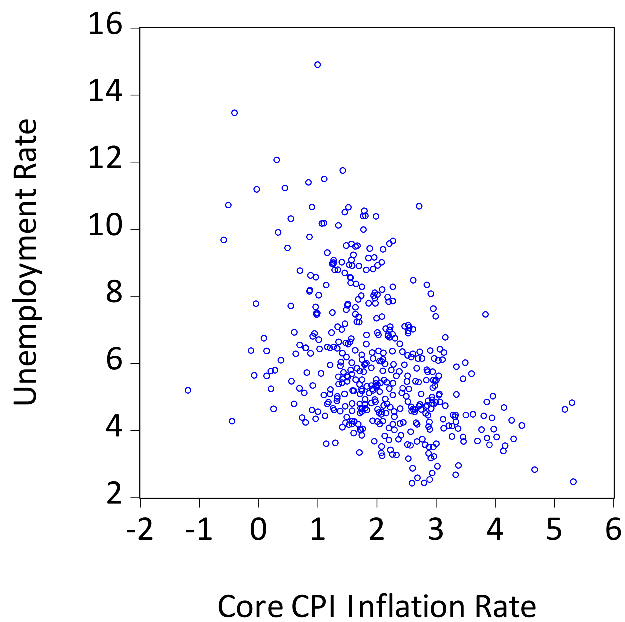
,,*** represent difference from 0 is statistically significant at the 90, 95, 99 percent level, except for exponent on UR where it represents the difference from 1.*

3. Empirical results on metro data

We now turn to examining the metropolitan data. Kiley noted the low power of a single series of U.S. national data to draw statistically significant differences between the effects of short-term and long-term unemployment on inflation. As a result he employed a panel dataset of metropolitan area inflation and unemployment rates. We use the same approach and dataset with a few adjustments as described in Appendix B.

Figure 1 shows a simple plot of the metropolitan inflation and unemployment rate data. The bulk of the observations appear with the unemployment rate between 3 and 8 percent and an inflation rate between 1 and 4 percent. The general shape of the observations appear to be roughly a right triangle, which could be consistent with a steeper Phillips curve at low rates of unemployment.

Figure 1. Core CPI Inflation and Unemployment Rate for metropolitan areas, 1985-2016



Note: Core CPI data sourced from the Bureau of Labor Statistics, *Consumer Price Index for All Urban Consumers*: All items less food and energy for the 23 MSAs listed in Appendix B, retrieved from FRED, Federal Reserve Bank of St. Louis. Data for the Unemployment Rate are authors' calculations derived from the Current Population Survey from the Bureau of Labor Statistics.

Moving to regressions, using the metropolitan data we repeat the regression forms used in tables 1 and 2. The first two columns of table 3 update Kiley's results using the 1985-2016 sample. The results are largely unchanged from that paper—short and long-term unemployment have roughly similar effects on inflation when looking at their coefficient values. Similarly, comparing the sum of squared residuals (SSR), adjusted R-squared, and AIC suggests that overall regression fit is not much improved by breaking the unemployment rate into short and long-term unemployment. These regressions include fixed effects on both the cross section and time dimension, which should absorb constant differences in the natural rate of unemployment across cities (cross-sectional fixed effects) and average changes over time in the natural rate of unemployment across all cities (time fixed effects).

Nonlinear forms that allow the effect of the unemployment rate to change as the level of unemployment changes are shown in regressions (3) through (5) of table 3. Regression (3) uses a linear spline with one kink point at 5.0 percent unemployment, regression (4) uses a linear spline with two kink points at 5.7 and 5.9 percent, and regression (5) tests a functional form where the unemployment rate term is taken to an exponent.[12] Examining the SSR, AIC, and the adjusted r-squareds suggests that all of the nonlinear versions fit somewhat better than either the linear or short-term/long-term unemployment versions, though the overall improvement in the fit of the regression is small.

The nonlinearity term in each of these regressions—the additional coefficient above or below a kink or the exponent on the unemployment rate—is highly significant and the expected sign. The spline with one break point, regression 3, shows a slightly flatter Phillips curve slope than the linear version, regression 1, when the unemployment rate is high, -0.18 versus -0.23, but a much steeper slope when the unemployment rate is low. In the single-kink spline the effective coefficient when the unemployment rate is below the 5.0 percent breakpoint is -.45, suggesting a Phillips curve that is more than twice as steep when the unemployment rate is below 5.0 percent than when it is above it. Similarly, the 2-kink regression, column 4, and the exponential regression, column 5, show a slope of the Phillips curve that is roughly twice as steep when the unemployment rate is at 4½ then when it is at 7½ percent.

For the spline regressions, the steeper Phillips curve when unemployment is low is not a function of the chosen kink point. Using different kink points for regression 3, figure 2 shows the slopes of Phillips curve above and below the kink point. At all kink points the slope is steeper (i.e. more negative) when the unemployment rate is below the kink point than when the unemployment rate is above the kink point. The lower the kink point, the steeper the slope below the kink point, which suggests to us a smooth function—like the exponential form used in column 5 of table 1, or a similar form—is a more natural fit to the data than a spline with a sharp kink..

[12] For the metropolitan regressions the grid search for the kink points that led to the lowest SSR was conducted over values of 3 to 10 by 0.1 increments.

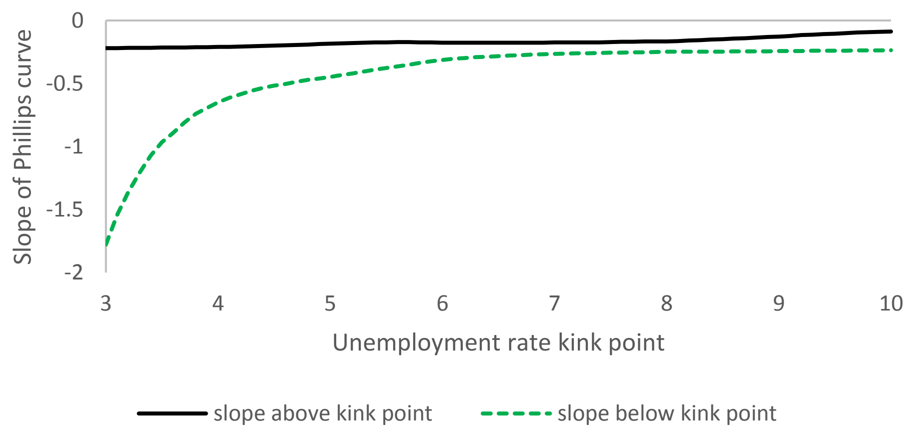
Table 3. Results from Metropolitan Data over 1985 to 2016 time period

Equation: $\text{Inflation}_{i,t} = c + \beta_1 \text{ Inflation}_{i,t-1} + \theta \text{ f(Unemployment rate}_{i,t}) + \varepsilon_{i,t}$

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
Form of unemployment rate	UR	Short-run and total UR	UR with kink at 5.0	UR with kinks at 5.7 and 5.9	UR^{α}
Constant	2.78^{***} (0.26)	2.73^{***} (0.26)	2.52^{***} (0.26)	-5.62^{**} (2.51)	-4.83 (10.17)
Coef on ldcore	0.46^{***} (0.05)	0.46^{***} (0.05)	0.44^{***} (0.04)	0.43^{***} (0.04)	0.44^{***} (0.04)
Coef on unemployment rate	-0.23^{***} (0.03)		-0.18^{***} (0.03)	1.21^{***} (0.43)	
Coef on short-term unemployment rate		-0.20^{***} (0.04)			
Coef on long-term unemployment rate		-0.27^{***} (0.06)			
Additional coef on ur below kink 1			-0.26^{***} (0.08)	-1.65^{***} (0.46)	
Additional coef on ur above kink 2				-1.44^{***} (0.45)	
Coef on ur^{α}					9.36 (9.30)
α (Exponent on ur)					-0.23^{***} (0.36)
Cross sectional dummies and period fixed effects	Yes	Yes	Yes	Yes	Yes
Slope at 4.5% U rate	-0.23	NA	-0.45	-0.44	-0.34
Slope at 7.5% U rate	-0.23	NA	-0.18	-0.22	-0.18
Nobs	736	736	736	736	736
SSR	300.1	299.8	294.4	291.8	295.3
Adj. R^2	0.742	0.742	0.746	0.748	0.746
AIC	2.09	2.09	2.08	2.07	2.08

*,**,*** represent difference from 0 is statistically significant at the 90, 95, 99 percent level, except for exponent on UR where it represents the difference from 1.

Figure 2. Phillips curve slopes above and below single unemployment rate kink point



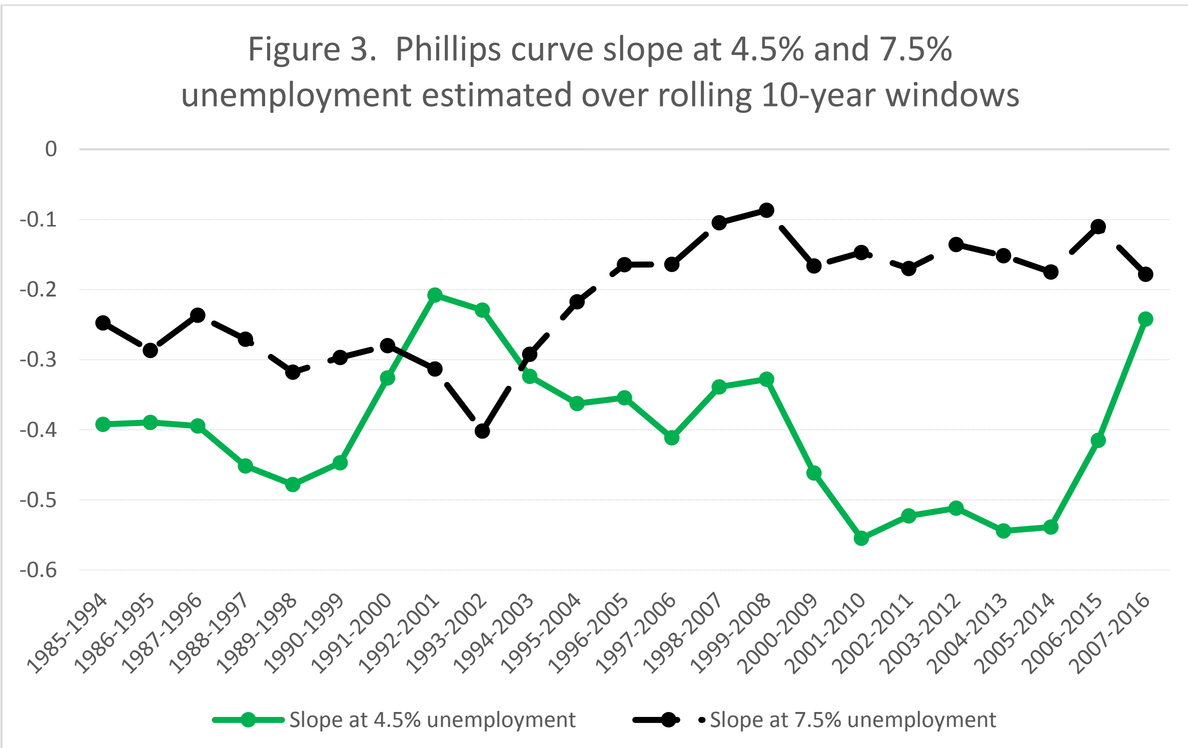
Kiley also looked at the 1998 to present period. This shorter sample roughly aligns with the period of time where national core inflation has been relatively low and stable, and for that reason the 1998 to present sample is probably more relevant for the current period than the longer sample results. The results over this shorter time period are shown in table 4. Here we see that the results from table 3 largely hold through to the shorter time period: a steeper Phillips curve when the unemployment rate is low and a small improvement in fit when using a nonlinear unemployment rate. Compared to the results from the longer time period, the 1998 to 2016 results display lower kink points, though the degree of curvature in the exponential version, column 5, is similar in the two time periods. In the exponential and single kink forms the nonlinearity terms are again highly significant, but they are not in the two-kink regression, regression 4.

Table 4. Results from Metropolitan Data over 1998 to 2016 time period
(Standard errors in parentheses)
Equation: $\text{Inflation}_{i,t} = c + \beta_1 \text{Inflation}_{i,t-1} + \theta f(\text{Unemployment rate}_{i,t}) + \varepsilon_{i,t}$

Form of unemployment rate	(1) UR	(2) SR & LR UR	(3) UR with kink at 4.2	(4) UR with kinks at 3.2 and 5.2	(5) UR^{α}
Constant	2.33^{***} (0.38)	2.17^{***} (0.39)	2.16^{***} (0.37)	2.85^{***} (0.58)	-2.49 (5.12)
Coef on ldcore	0.44^{***} (0.07)	0.44^{***} (0.07)	0.42^{***} (0.07)	0.43^{***} (0.07)	0.43^{***} (0.07)
Coef on ur	-0.20^{***} (0.05)		-0.17^{***} (0.05)	-0.31^{***} (0.10)	
Coef on short-term UR		-0.12 (0.08)			
Coef on long-term UR		-0.29^{***} (0.08)			
Additional coef on ur below kink 1			-0.40^{***} (0.15)	-0.72 (0.51)	
Additional coef on ur above kink 2				0.16 (0.11)	
Coef on u^{α}					6.64 (4.23)
α (Exponent on ur)					-0.36^{***} (0.49)
Cross sectional dummies and period fixed effects	Yes	Yes	Yes	Yes	Yes
Slope at 4.5% U rate	-0.20	NA	-0.17	-0.31	-0.31
Slope at 7.5% U rate	-0.20	NA	-0.17	-0.16	-0.15
Nobs	437	437	437	437	437
SSR	174.2	173.3	170.5	169.9	170.7
Adj. R^2	0.508	0.509	0.517	0.517	0.516
AIC	2.11	2.11	2.10	2.10	2.10

*,**,*** represent difference from 0 is statistically significant at the 90, 95, 99 percent level, except for exponent on UR where it represents the difference from 1.

Re-running the exponential Phillips curve used in regression 5 of tables 3 and 4 on rolling 10-year windows of the data suggests that the nonlinearity has been present in most time periods. Figure 3 displays the Phillips curve slope at a 4½ percent unemployment rate and also at a 7½ percent unemployment rate from these rolling 10-year window regressions. In almost every 10-year sample the best fitting exponent implies a slope at 4½ percent unemployment, the green solid line, that is steeper (more negative) than at a 7½ percent unemployment rate, the dashed black line, though the difference has narrowed considerably in the most recent couple of 10-year windows.



The Phillips curves used in tables 3 and 4 are fairly simplistic, but the basic finding of a steeper Phillips curve when the unemployment rate is low is quite robust across a number of specification choices. In particular, appendix tables 1 through 4 show a number of robustness regressions based on a single kink point, and appendix tables 1a through 4a use the exponential form. These tables show that the finding in the metropolitan data of a steeper Phillips curve when the unemployment rate is low is robustness to:

1. *Fixed effects*: The first four regressions in each of the appendix tables show different choices of the cross sectional and time fixed effects. In all cases the Phillips curve is steeper at low levels of unemployment regardless of the inclusion or exclusion of fixed effects in the regressions, though the amount, and significance, of the difference in the slope is sensitive to inclusion or exclusion of fixed, time, and location effects (as is the location of the kink point). Including both fixed effects makes sense if there are persistent differences over time (such as energy and import price effects or drifts in inflation expectations) or location (such as different natural rates of unemployment) that are not captured in this simple model.

2. *Inclusion of food and energy price changes, national import price changes, lagged national inflation, and/or national inflation expectations*: In the fifth through eighth regressions in the appendix tables the time fixed effects are replaced with various combinations of national import price changes, lagged inflation rates and lagged inflation expectations as proxied by the average of expected CPI inflation over the next 10 years from the Survey of Professional Forecasters lagged by a year. Also the effect of food and energy prices at the metropolitan level, measured as the difference between core and total inflation, is included in some of the forms. The eighth regression also includes lagged forms of the unemployment rate. In general these various forms do not fit as well as having period fixed effects, but they have little effect on the nonlinearity, which continues to show a steeper Phillips curve when the unemployment rate is low.

3. *Total instead of core inflation*: Appendix tables 3 and 4 repeat appendix tables 1 and 2 but substituting total CPI inflation for core CPI inflation. Again, in all regressions the slope of the Phillips curve is higher when the unemployment rate is lower.[13]

4. How important is the asymmetry in practice?

Despite the fairly strong statistical case for a steeper Phillips curve slope at low unemployment rates, in a practical sense the effect found in this paper is small: Using the 2016-2015 annual average core CPI inflation rate of 2.2 percent, and assuming an unemployment rate of 4.5 percent, the linear Phillips curve of table 4 suggests a 2017-2016 annual average core CPI inflation rate of 2.4 percent. The 2-kink version would also suggest a 2.4 percent inflation rate, while both the 1-kink and exponential versions would suggest inflation of 2.3 percent.[14] Given the standard errors on Phillips curve forecasts these are quite small differences.

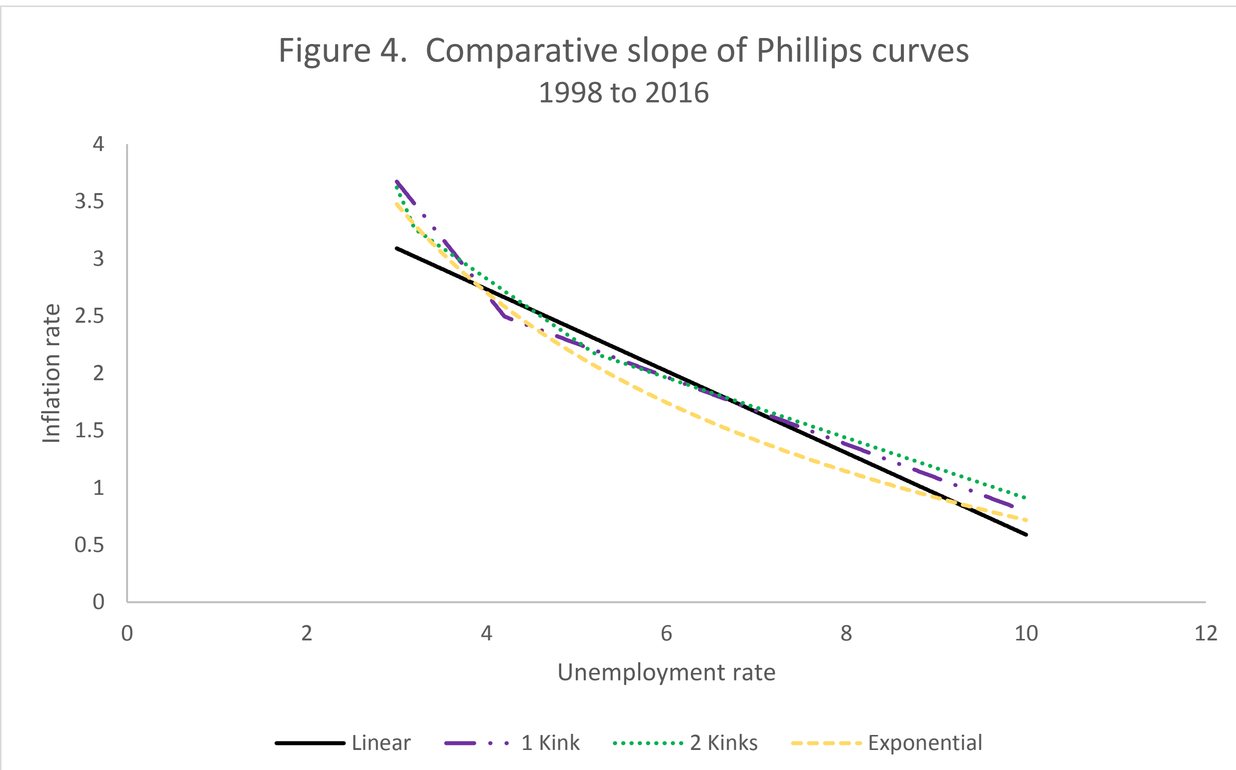
Forecasting further ahead the differences across the models grow, but not by much. Assuming the unemployment rate stays at some specified level for a considerable number of years and also assuming inflation expectations and the natural rate of unemployment, which are embedded in the constant in

[13] We were also able to obtain CPI data for all areas for the CPI components of housing services, services, and services excluding rent of shelter. Over both the 1985 to 2016 and 1998 to 2016 time periods the exponent of these regressions is more negative than -0.25, with the services less rent of shelter showing the exponents of -1.1 and -2.25. This suggests the asymmetries are at least as large in these services components as in the overall core CPI.

[14] The projected inflation rate for the 1-kink and exponential forms being *lower* than the linear version may surprise some readers. It is a result we have also found with many other Phillips curve variants including quarterly variants that control for changes in the natural rate of unemployment, import and energy prices. One possible explanation is that the effect occurs from the 10 percent unemployment rate during the Great Recession and an actual inflation rate that remained well above zero. Whereas the linear form takes this as evidence for a strongly positive trend in inflation, the high unemployment rate pushes down inflation less in the nonlinear forms, hence they have a somewhat lower inflation trend than in the linear model (i.e. the constant is lower in the nonlinear models than the linear one).

these regressions, remain unchanged, we can back out a steady-state unemployment-inflation rate trade off.[15] Figure 4 compares that trade-off for the linear version (regression 1) to the nonlinear versions of regressions 3, 4, and 5 for the 1998 to 2016 sample. All the nonlinear forms display a steeper slope than the linear version when the unemployment rate is low, and a slightly flatter slope than when it is high. Nonetheless, the difference between the linear and nonlinear Phillips curves is small. At 4½ percent unemployment rate the linear and 2-kink versions of this simple Phillips curve show a steady-state core CPI inflation rate of just above 2½ percent with the one-kink and exponential versions being about 15 basis points lower. The nonlinear versions do not suggest inflation more than a tenth higher than the linear version until the unemployment rate falls below 3¾ percent—a low rate of unemployment that has not been seen nationally since the late 1960s.

These small differences between the linear and nonlinear forms suggest that as a practical matter accounting for the possibility of a steeper Phillips curve slope at low rates of unemployment is unlikely to improve forecasts from Phillips curve models by very much.[16]



[15] The assumptions needed for this exercise are fairly strong, so while we use it as an example, we would not suggest that this gives a reasonable practical Phillips curve for use in policy or analysis.

[16] Additionally, the metropolitan results may not be applicable to the national level because either the sample of cities that we have is not representative of the national data or the nonlinearities are only in the cross-sectional dimension. We do not think either of these are the case (averaging the cities by population weights or unweighted leads to unemployment rates and inflation rates that are relatively close to the national average, and running Phillips curves on each city separately over the 1998-2016 period finds that 18 of the 23 cities show a steeper Phillips curve when unemployment is low). Nonetheless we cannot rule out that the metropolitan results are not applicable to the national level.

5. Conclusions

Results from US metropolitan areas since the mid-1980s show a fairly robust presence of a steeper slope of the Phillips curve when the unemployment rate is low and a flatter slope when the unemployment rate is high. Nevertheless, the degree of asymmetry is small and, compared to a linear model, inflation projections from the nonlinear models only suggest a noticeably higher inflation rate once unemployment falls well below its current level.

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Appendix Table 1. Metropolitan Data, 1985 to 2016 time period, 1-Kink, various specifications

Equation: Core Inflation$_{i,t}$ = c + β Core Inflation$_{i,t-1}$ + θ * (Unemployment rate$_{i,t}$) + θ$_2$ *(if Unemployment rate$_{i,t}$<kink)*(Unemp rate$_{i,t}$-kink)+ Additional terms +ε$_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	2.52*** (0.26)	1.96*** (0.22)	1.39*** (0.31)	1.25*** (0.27)	0.77** (0.34)	1.62*** (0.29)	0.76** (0.34)	-0.14 (0.48)
Coef on lagged core inflation	0.44*** (0.04)	0.53*** (0.04)	0.72*** (0.05)	0.73*** (0.05)	0.47*** (0.05)	0.67*** (0.05)	0.46*** (0.05)	0.49*** (0.05)
Coef on unemployment rate	-0.18*** (0.03)	-0.12*** (0.03)	-0.12*** (0.04)	-0.10*** (0.03)	-0.15*** (0.03)	-0.14*** (0.04)	-0.14*** (0.03)	-0.05 (0.05)
Additional coef on unemp rate below kink 1	-0.26*** (0.08)	-0.79** (0.35)	-0.92** (0.41)	-0.75* (0.39)	-1.19*** (0.39)	-0.74*** (0.25)	-0.32*** (0.10)	0.09 (0.08)
Coef on lagged nat'l inflation expectations					0.44** (0.21)		0.37* (0.20)	0.49* (0.25)
Coef on lagged nat'l core price inflation					0.05 (0.17)		0.09 (0.16)	0.06 (0.18)
Coef on metro total-core inflation						-0.21*** (0.08)	-0.17** (0.07)	-0.19*** (0.07)
Coef on lagged metro total-core inflation						0.12 (0.08)	0.14* (0.07)	0.12* (0.07)
Coef on nat'l import price inflation						0.06** (0.03)	0.05** (0.02)	0.04 (0.02)
Coef on lagged nat'l import price inflation						0.05* (0.03)	0.04* (0.02)	0.04* (0.02)
Coef of change in Unemployment rate								-0.19*** (0.06)
Coef of lagged u rate below kink								-0.37*** (0.08)
Coef of second lag of core inflation								-0.08 (0.05)
Kink point	5.0	3.3	3.3	3.3	3.3	3.6	3.7	6.6
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	736	736	736	736	736	736	736	713
SSR	294.4	316.7	454.4	463.18	392.99	396.88	351.96	329.25
Adj. R^2	0.746	0.736	0.626	0.629	0.675	0.671	0.708	0.706
AIC	2.08	2.09	2.43	2.39	2.29	2.30	2.19	2.16

Appendix Table 1a. Metropolitan Data, 1985 to 2016 time period, exponential, various specifications

Equation: Core Inflation$_{i,t}$ = c + β Core Inflation$_{i,t-1}$ + θ * (Unemployment rate$_{i,t}$)$^{\alpha}$ + Additional terms +$\varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	-4.83	-0.50	2.52	1.69	-1.99	-1.97	-1.50	-1.32*
	(10.17)	(2.36)	(4.30)	(1.76)	(2.13)	(5.16)	(1.05)	(0.73)
Coef on lagged core inflation	0.44***	0.53***	0.72***	0.73***	0.47***	0.68***	0.46***	0.49***
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)
Coef on unemployment rate to exponent	9.36	4.00***	-0.87	-0.33	4.91***	5.14	5.86***	5.40***
	(9.30)	(1.34)	(3.54)	(1.19)	(1.08)	(3.94)	(1.22)	(2.03)
Coef on lagged nat'l inflation expectations					0.43**		0.36*	0.55**
					(0.22)		(0.20)	(0.27)
Coef on lagged nat'l core price inflation					0.08		0.09	0.03
					(0.17)		(0.16)	(0.18)
Coef on metro total-core inflation						-0.21***	-0.17**	-0.16**
						(0.08)	(0.07)	(0.07)
Coef on lagged metro total-core inflation						0.12	0.14**	0.12
						(0.08)	(0.07)	(0.07)
Coef on nat'l import price inflation						0.06**	0.05*	0.05*
						(0.03)	(0.02)	(0.03)
Coef on lagged nat'l import price inflation						0.05*	0.04	0.03
						(0.03)	(0.02)	(0.02)
Coef on lagged u rate to exponent								1.98
								(1.72)
Coef of second lag of core inflation								-0.10*
								(0.05)
Exponent on U rate	-0.22***	-0.48***	0.42	0.64	-0.56***	-0.37**	-0.79***	-1.09***
	(0.36)	(0.63)	(0.98)	(1.07)	(0.60)	(0.66)	(0.48)	(0.49)
Slope at 4.5 percent	-0.33	-0.21	-0.16	-0.12	-0.26	-0.24	-0.31	-0.25
Slope at 7.5 percent	-0.18	-0.10	-0.11	-0.10	-0.12	-0.12	-0.13	-0.09
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	736	736	736	736	736	736	736	713
SSR	295.3	317.39	458.57	466.38	398.9	400.1	352.86	338.38
Adj. R^2	0.746	0.735	0.622	0.627	0.673	0.669	0.707	0.699
AIC	2.08	2.09	2.44	2.39	2.28	2.31	2.19	2.19

Appendix Table 2. Metropolitan Data, 1998 to 2016 time period, 1-Kink, various specifications

Equation: $\text{Core Inflation}_{i,t} = c + \beta\ \text{Core Inflation}_{i,t-1} + \theta * (\text{Unemployment rate}_{i,t}) + \theta_2 *(\text{if Unemployment rate}_{i,t}<\text{kink})*(\text{Unemp rate}_{i,t}-\text{kink}) + \text{Additional terms} + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	2.15*** (0.37)	1.56*** (0.27)	1.85*** (0.36)	1.62*** (0.33)	3.17** (1.26)	1.81*** (0.35)	3.19** (1.30)	3.24** (1.36)
Coef on lagged core inflation	0.42*** (0.07)	0.53*** (0.06)	0.39*** (0.08)	0.46*** (0.07)	0.44*** (0.07)	0.40*** (0.07)	0.47*** (0.08)	0.54*** (0.08)
Coef on unemployment rate	-0.17*** (0.05)	-0.10*** (0.03)	-0.12*** (0.04)	-0.10** (0.04)	-0.13*** (0.04)	-0.12*** (0.04)	-0.13*** (0.04)	-0.08* (0.04)
Additional coef on unemp rate below kink 1	-0.40*** (0.15)	-0.82** (0.43)	-0.26** (0.13)	-0.20* (0.11)	-0.26** (0.12)	-0.36*** (0.11)	-0.33*** (0.11)	0.04 (0.09)
Coef on lagged nat'l inflation expectations					-0.37 (0.55)		-0.41 (0.58)	-0.03 (0.63)
Coef on lagged nat'l core price inflation					-0.21 (0.20)		-0.25 (0.18)	-0.51*** (0.17)
Coef on metro total-core inflation						-0.40*** (0.11)	-0.37*** (0.10)	-0.42*** (0.09)
Coef on lagged metro total-core inflation						0.20* (0.11)	0.31** (0.12)	0.28** (0.11)
Coef on nat'l import price inflation						0.15*** (0.05)	0.13*** (0.05)	0.14*** (0.04)
Coef on lagged nat'l import price inflation						-0.01 (0.05)	-0.05 (0.05)	-0.06 (0.04)
Coef of change in Unemployment rate								-0.05 (0.06)
Coef of lagged u rate below kink								-0.38*** (0.09)
Coef of second lag of core inflation								-0.20*** (0.07)
Kink point	4.2	3.2	5.1	5.2	5.1	5.1	5.1	6.5
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	437	437	437	437	437	437	437	414
SSR	170.5	185.1	214.0	230.6	209.2	185.8	181.8	158.6
Adj. R^2	0.517	0.503	0.420	0.407	0.430	0.492	0.500	0.541
AIC	2.10	2.08	2.24	2.21	2.23	2.12	2.11	2.05

Appendix Table 2a. Metropolitan Data, 1998 to 2016 time period, exponential, various specifications

Equation: Core Inflation$_{i,t}$ = c + β Core Inflation$_{i,t-1}$ + θ * (Unemployment rate$_{i,t}$)$^{\alpha}$ + Additional terms +$\varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	-2.39	-0.01	-0.38	-0.86	0.10	-0.16	1.03	1.22
	(5.12)	(1.20)	(1.73)	(3.39)	(3.22)	(0.90)	(1.76)	(1.35)
Coef on lagged core inflation	0.43***	0.53***	0.39***	0.47***	0.45***	0.41***	0.48***	0.55***
	(0.07)	(0.06)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)
Coef on unemployment rate to exponent	6.64	3.56***	5.08***	4.35**	5.52***	6.54***	6.16***	2.79**
	(4.24)	(1.11)	(1.23)	(1.98)	(1.73)	(1.95)	(1.47)	(1.39)
Coef on lagged nat'l inflation expectations					-0.36		-0.41	-0.22
					(0.55)		(0.58)	(0.48)
Coef on lagged nat'l core price inflation					-0.22		-0.26	-0.53***
					(0.20)		(0.18)	(0.17)
Coef on metro total-core inflation						-0.41***	-0.37***	-0.44***
						(0.11)	(0.10)	(0.09)
Coef on lagged metro total-core inflation						0.21*	0.32***	0.34***
						(0.11)	(0.12)	(0.11)
Coef on nat'l import price inflation						0.16***	0.13***	0.16***
						(0.05)	(0.05)	(0.04)
Coef on lagged nat'l import price inflation						-0.01	-0.05	-0.09**
						(0.05)	(0.05)	(0.04)
Coef on lagged u rate to exponent								5.33***
								(1.70)
Coef of second lag of core inflation								-0.23***
								(0.07)
Exponent on U rate (α)	-0.36***	-0.77**	-0.67**	-0.47*	-0.50**	-0.93***	-0.82***	-0.91***
	(0.49)	(0.86)	(0.69)	(0.79)	(0.63)	(0.55)	(0.52)	(0.40)
Slope at 4.5 percent	-0.31	-0.19	-0.28	-0.22	-0.29	-0.33	-0.33	-0.14
Slope at 7.5 percent	-0.15	-0.08	-0.12	-0.11	-0.13	-0.12	-0.13	-0.05
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	437	437	437	437	437	437	437	437
SSR	170.73	184.97	214.65	231.02	209.65	186.0	181.6	164.85
Adj. R^2	0.516	0.504	0.418	0.406	0.429	0.491	0.501	0.544
AIC	2.10	2.08	2.25	2.22	2.23	2.12	2.11	2.02

Appendix Table 3. Metropolitan Total CPI Data, 1985 to 2016 time period, 1-Kink, various specifications

Equation: $Inflation_{i,t} = c + \beta\ Inflation_{i,t-1} + \theta * (Unemployment\ rate_{i,t}) + \theta_2 *(if\ Unemployment\ rate_{i,t} < kink)*(Unemp\ rate_{i,t} - kink) + Additional\ terms + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	2.48*** (0.24)	1.94*** (0.22)	2.51*** (0.67)	2.31*** (0.60)	1.40* (0.83)	1.62*** (0.29)	0.76** (0.34)	0.14 (0.47)
Coef on lagged inflation	0.39*** (0.05)	0.48*** (0.05)	0.44*** (0.11)	0.46*** (0.11)	0.30** (0.12)	0.67*** (0.05)	0.46*** (0.05)	0.51*** (0.05)
Coef on unemployment rate	-0.16*** (0.03)	-0.11*** (0.02)	-0.18** (0.08)	-0.16** (0.07)	-0.22*** (0.08)	-0.14*** (0.04)	-0.14*** (0.03)	-0.04 (0.05)
Additional coef on unemp rate below kink 1	-0.34*** (0.08)	-0.84*** (0.29)	-0.88* (0.47)	-0.80 (0.50)	-1.16** (0.50)	-0.74*** (0.25)	-0.32*** (0.10)	0.08 (0.08)
Coef on lagged nat'l inflation expectations					0.77 (0.52)		0.37* (0.20)	0.26 (0.24)
Coef on lagged nat'l core price inflation					-0.25 (0.42)		0.09 (0.16)	0.22 (0.18)
Coef on metro total-core inflation						0.79*** (0.08)	0.83*** (0.07)	0.82*** (0.06)
Coef on lagged metro total-core inflation						-0.55*** (0.08)	-0.32*** (0.07)	-0.39*** (0.07)
Coef on nat'l import price inflation						0.06** (0.03)	0.05** (0.02)	0.04* (0.02)
Coef on lagged nat'l import price inflation						0.05* (0.03)	0.04* (0.02)	0.03 (0.02)
Coef of change in Unemployment rate								-0.18*** (0.06)
Coef of lagged u rate below kink								-0.35*** (0.08)
Coef of second lag of inflation								-0.13** (0.05)
Kink point	4.6	3.3	3.4	3.3	3.3	3.6	4.9	6.6
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	736	736	736	736	736	736	736	713
SSR	246.5	265.0	914.6	935.3	811.0	396.8	352.0	323.8
Adj. R^2	0.800	0.792	0.291	0.297	0.37	0.69	0.72	0.74
AIC	1.90	1.91	3.13	3.09	3.01	2.30	2.19	2.15

Appendix Table 3a. Metropolitan Data, 1985 to 2016 time period, exponential, various specifications

Equation: $\text{Inflation}_{i,t} = c + \beta\ \text{Inflation}_{i,t-1} + \theta * (\text{Unemployment rate}_{i,t})^{\alpha} + \text{Additional terms} + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	-0.63	0.56	3.31	2.66	8.01	-1.97	-1.50	-0.77
	(1.48)	(0.56)	(3.69)	(2.13)	(39.3)	(5.16)	(1.05)	(0.65)
Coef on lagged core inflation	0.39***	0.48***	0.44***	0.46***	0.30**	0.68***	0.46***	0.51***
	(0.05)	(0.05)	(0.11)	(0.11)	(0.12)	(0.05)	(0.05)	(0.06)
Coef on unemployment rate to exponent	5.84***	3.99***	-0.62	-0.31	-5.76	5.14	5.86***	5.35**
	(0.84)	(1.40)	(2.49)	(1.20)	(37.8)	(3.94)	(1.22)	(2.26)
Coef on lagged nat'l inflation expectations					0.76		0.36*	0.28
					(0.52)		(0.20)	(0.26)
Coef on lagged nat'l core price inflation					-0.23		0.09	0.22
					(0.42)		(0.16)	(0.19)
Coef on metro total-core inflation						0.79***	-0.83***	0.85***
						(0.08)	(0.07)	(0.07)
Coef on lagged metro total-core inflation						-0.56***	-0.32***	-0.39***
						(0.08)	(0.07)	(0.07)
Coef on nat'l import price inflation						0.06**	0.05**	0.05*
						(0.03)	(0.02)	(0.03)
Coef on lagged nat'l import price inflation						0.05*	0.04	0.03*
						(0.03)	(0.02)	(0.02)
Coef on lagged u rate to exponent								2.03
								(1.79)
Coef of second lag of core inflation								-0.15***
								(0.05)
Exponent on U rate	-0.57***	-0.97***	0.63	0.79	0.18	-0.37**	-0.79***	-1.20***
	(0.36)	(0.62)	(1.17)	(1.24)	(0.90)	(0.66)	(0.48)	(0.52)
Slope at 4.5 percent	-0.31	-0.20	-0.22	-0.18	-0.30	-0.24	-0.31	-0.23
Slope at 7.5 percent	-0.14	-0.07	-0.19	-0.16	-0.20	-0.12	-0.13	-0.08
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	736	736	736	736	736	736	736	713
SSR	246.5	264.5	919.7	938.9	815.6	400.1	352.9	331.3
Adj. R^2	0.800	0.792	0.287	0.294	0.366	0.688	0.724	0.734
AIC	1.90	1.91	3.13	3.09	3.02	2.31	2.19	2.17

Appendix Table 4. Metropolitan Total CPI Data, 1998 to 2016 time period, 1-Kink, various specifications

Equation: $\text{Inflation}_{i,t} = c + \beta\ \text{Inflation}_{i,t-1} + \theta * (\text{Unemployment rate}_{i,t}) + \theta_2 *(\text{if Unemployment rate}_{i,t}<\text{kink})*(\text{Unemp rate}_{i,t}-\text{kink}) + \text{Additional terms} + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	2.00***	1.50***	2.59***	2.42***	2.80	1.81***	3.19**	2.00
	(0.36)	(0.28)	(0.95)	(0.92)	(3.77)	(0.35)	(1.30)	(1.64)
Coef on lagged inflation	0.36***	0.46***	0.18	0.22	0.26*	0.40***	0.47***	0.48***
	(0.08)	(0.07)	(0.16)	(0.15)	(0.16)	(0.07)	(0.08)	(0.08)
Coef on unemployment rate	-0.11***	-0.06**	-0.16	-0.14	-0.22*	-0.12***	-0.13***	-0.09**
	(0.04)	(0.03)	(0.12)	(0.12)	(0.12)	(0.04)	(0.04)	(0.04)
Additional coef on unemp rate below kink 1	-0.45***	-0.33**	-0.22	-0.17	-0.16	-0.36***	-0.33***	-0.01
	(0.13)	(0.13)	(0.27)	(0.23)	(0.24)	(0.11)	(0.11)	(0.11)
Coef on lagged nat'l inflation expectations					0.86		-0.41	-0.04
					(1.66)		(0.58)	(0.71)
Coef on lagged nat'l core price inflation					-1.03		-0.25	-0.03
					(0.63)		(0.18)	(0.22)
Coef on metro total-core inflation						0.60***	0.63***	0.53***
						(0.11)	(0.10)	(0.10)
Coef on lagged metro total-core inflation						-0.20*	-0.16	-0.30***
						(0.11)	(0.10)	(0.11)
Coef on nat'l import price inflation						0.15***	0.13***	0.17***
						(0.05)	(0.05)	(0.04)
Coef on lagged nat'l import price inflation						-0.01	-0.05	-0.02
						(0.05)	(0.05)	(0.05)
Coef of change in Unemployment rate								-0.04
								(0.06)
Coef of lagged u rate below kink								-0.41***
								(0.12)
Coef of second lag of inflation								-0.18***
								(0.06)
Kink point	4.2	4.2	5.3	5.5	5.5	5.1	5.1	5.4
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	437	437	437	437	437	437	437	414
SSR	149.0	160.6	545.0	565.5	495.8	185.8	181.8	161.0
Adj. R^2	0.756	0.751	0.148	0.161	0.221	0.707	0.712	0.735
AIC	1.96	1.94	3.18	3.11	3.09	2.12	2.11	2.06

Appendix Table 4a. Metropolitan Data, 1998 to 2016 time period, exponential, various specifications

Equation: $\text{Inflation}_{i,t} = c + \beta \text{ Inflation}_{i,t-1} + \theta * (\text{Unemployment rate}_{i,t})^{\alpha} + \text{Additional terms} + \varepsilon_{i,t}$

	(1)	(2)	(3)	(4)[17]	(5)	(6)	(7)	(8)
Constant	0.37	0.71*	-8.39	3.71***	7.27	-0.16	1.03	0.59
	(0.74)	(0.36)	(104.5)	(1.15)	(19.5)	(0.90)	(1.76)	(1.39)
Coef on lagged core inflation	0.37***	0.46***	0.19	0.22	0.27*	0.41***	0.48***	0.51***
	(0.07)	(0.07)	(0.16)	(0.15)	(0.16)	(0.07)	(0.08)	(0.08)
Coef on unemployment rate to exponent	5.26***	4.52	12.7	-1.17**	-3.44	6.54***	6.16***	3.99**
	(1.74)	(3.06)	(101.9)	(0.57)	(17.4)	(1.95)	(1.47)	(2.06)
Coef on lagged nat'l inflation expectations					0.86		-0.41	-0.05
					(1.66)		(0.58)	(0.54)
Coef on lagged nat'l core price inflation					-1.04		-0.26	-0.11
					(0.63)		(0.18)	(0.23)
Coef on metro total-core inflation						0.59***	0.63***	0.52***
						(0.11)	(0.10)	(0.10)
Coef on lagged metro total-core inflation						-0.20*	-0.16	-0.26**
						(0.11)	(0.10)	(0.11)
Coef on nat'l import price inflation						0.16***	0.13***	0.19***
						(0.05)	(0.05)	(0.05)
Coef on lagged nat'l import price inflation						-0.01	-0.05	-0.06
						(0.05)	(0.05)	(0.05)
Coef on lagged u rate to exponent								4.15*
								(2.18)
Coef of second lag of core inflation								-0.19***
								(0.06)
Exponent on U rate	-1.00***	-1.40***	-0.12		0.29	-0.93***	-0.82***	-1.15***
	(0.61)	(0.90)	(1.32)		(0.97)	(0.55)	(0.52)	(0.53)
Slope at 4.5 percent	-0.26	-0.17	-0.28		-0.34	-0.33	-0.33	-0.18
Slope at 7.5 percent	-0.09	-0.05	-0.16		-0.24	-0.12	-0.13	-0.06
Cross sectional dummies	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	No	No	No	No	No	No
Nobs	437	437	437	437	437	437	437	437
SSR	149.2	160.2	546.0	566.3	495.9	186.0	181.7	167.7
Adj. R^2	0.756	0.752	0.146	0.162	0.221	0.706	0.712	0.732
AIC	1.96	1.94	3.18	3.11	3.09	2.12	2.11	2.04

[17] Regression (4) omits the exponent on the unemployment rate as a variable. Additionally, the coefficient expressed for *unemployment rate to an exponent* in the table refers instead to the coefficient for the *log of the unemployment rate* (not shown in table)

Appendix B: Data Sources and Methodology

National Data

We use the following data for the United Sources (sources in parentheses)

The Consumer Price Index (total), and the Consumer Price Index excluding food and energy (Bureau of Labor Statistics)

The Civilian Unemployment Rate (Bureau of Labor Statistics)

The Imports of Nonpetroleum Goods price index (Bureau of Economic Analysis)

The data above is annual and is thus an average of the underlying monthly or quarterly data

Metropolitan Data

Following Kiley's (2015) methods we take an updated version of the original dataset by moving the end date forward from 2013 to 2016 and a few other changes.

The PCE is the preferred series for measuring inflation, however this data does not exist at the Metropolitan Statistical Area (MSA) level. We use the same Consumer Price Index data for the MSAs as we did at the national level. The monthly index levels are averaged over the year to create annual observations.

We use 23 MSAs. The set is the same as that used in Kiley (2015), except we drop the Washington-Baltimore MSA for which CPI data only starts in 2000. The MSAs used here are: New York-Northern New Jersey-Long Island, Philadelphia-Wilmington-Atlantic City, Boston-Brockton-Nashua, Pittsburgh, Chicago-Gary-Kenosha, Detroit-Ann Arbor-Flint, St. Louis, Cleveland-Akron, Minneapolis-St. Paul, Milwaukee-Racine, Cincinnati-Hamilton, Kansas City, Dallas-Ft. Worth, Houston-Galveston-Brazoria, Atlanta, Miami-Ft. Lauderdale, Los Angeles-Riverside-Orange County, San Francisco-Oakland-San Jose, Seattle-Tacoma-Bremerton, San Diego, Portland-Salem, Honolulu, and Denver-Boulder-Greeley.[18] By using 2010-2015 population counts we note that on average, our block of 23 MSAs accounts for roughly 35% of the total United States population.

We use Kiley's unique dataset for unemployment data prior to 2005. For the years following 2005 (inclusive) we borrow Kiley's methodology and construct our own estimates for both the total unemployment rate and the short-term unemployment rate (where short-term is defined as less than 27 weeks). We use the Current Population Survey taken from the Bureau of Labor Statistics. Our own estimates approximate those found in Kiley's 2015 dataset.

[18] These are the CPI definitions of the MSA; the constructed unemployment series names reference the CBSA code in the CPS dataset which aggregates slightly different areas based off of OMB definitions. In 2014 the codes for Boston, Los Angeles, and Honolulu were changed.