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Misallocation Costs of Digging Deeper into the Central Bank Toolkit *

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Abstract

Central bank large-scale asset purchases, particularly the purchase of corporate bonds of nonfinancial firms, can induce a misallocation of resources through their heterogeneous effect on firms cost of capital. First, we analytically demonstrate the mechanism in a static model. We then evaluate the misallocation of resources induced by corporate bond buys and the associated output losses in a calibrated heterogeneous firm New Keynesian DSGE model. The calibrated model suggests misallocation effects from corporate bond buys can be large enough to make them less effective than government bond buys, which is not the case without accounting for misallocation effects.

Keywords: QE, LSAPs, Misallocation
JEL Classification: E22, E51, E52, G21

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1 Introduction

Central banks, such as the Bank of Japan (BOJ), the Bank of England (BOE), the European Central Bank (ECB), and the Federal Reserve (the Fed), have used large-scale asset purchases (LSAPs) as a policy tool once they have reduced the short rate under their control to its effective lower bound.[1] While the BOJ, the ECB, and the Fed purchased government bonds as part of their LSAP programs, as they have chosen to buy other assets, their choices have differed: between them, the different central banks have purchased corporate bonds, exchange-traded funds, mortgage-backed securities, and other assets. Yet there is little theory to guide central banks on whether there are aggregate costs to purchasing some private assets rather than others.

This paper addresses this gap in the literature, focusing specifically on the costs associated with large-scale purchases of nonfinancial corporate bonds. Purchases of corporate bonds may reduce borrowing costs by more for firms whose securities are purchased than those not purchased and potentially create distortions in the cost of capital among firms.[2] In standard models of firm financing and capital choice, differences in the cost of capital induce differences in firm investment decisions and thus the allocation of capital, which has consequences for the efficiency of the allocation and aggregate output. Our work builds a simple theory of LSAPs with this mechanism present to deliver analytical results regarding how large-scale purchases of corporate bonds affect the allocation of capital among firms. We then introduce the key elements of our simple model into a New Keynesian DSGE model. With our calibrated DSGE model, we quantify the potential misallocative effects of large-scale purchases of nonfinancial firm corporate bonds.

To demonstrate the economic mechanism at work within our DSGE model, the first part of the paper takes a static model of firm dynamics with a financial intermediary sector and demonstrates the conditions under which a large-scale purchase of nonfinancial firm corporate bonds by the central bank induces a misallocation of resources. The model allows us to separate two effects of

[1] Such LSAPs are often referred to as quantitative easing (QE) policies.

[2] Krishnamurthy and Vissing-Jorgensen (2011), among others, demonstrate that there are price effects of central bank bond buys that are more pronounced for the securities purchased than other securities.

a shock to interest rates that lowers rates for one set of firms (firms issuing highly rated corporate bonds, which we denote as "large firms") more than for another set of firms (which we denote as "small firms"): (1) the effect on the allocation of capital (2) the effect on the aggregate capital stock. We isolate the key elements of our model that govern the size of each effect.[3]

In our model, following Gertler and Karadi (2011) and Gertler and Karadi (2013) (hereafter, GK11 and GK13), a financially constrained intermediary helps facilitate the financing of capital by firms. Intervention by the central bank in asset markets affects the quantity and distribution of assets which are intermediated and the constraint faced by the financial intermediary. Our simple, static model introduces the two key additional model elements we use to study the effects of misallocation.[4] The first is heterogeneity in production among multiple "groups" of firms. Specifically, we allow there to be two (or more) types of intermediate good firms that have similar production technologies. Their outputs are used in the production of the final good, and are imperfect substitutes.

Second, we introduce a regulatory constraint that allows for richer heterogeneity in spreads than in the model of GK13. The regulatory constraint makes it more costly for banks to hold risky private-sector debt than government bonds.[5] Additionally, the regulatory constraint discourages the concentration of certain classes of private securities.[6] In equilibrium, the regulatory constraints induce asymmetric effects in the credit spread response of purchasing debt of firms relative to that of the government.

We demonstrate that in our model when the central bank buys the securities of one set of firms,

[3] It is possible to use a large-scale purchase of corporate bonds to either *reduce* a misallocation or increase a misallocation. However, we focus on the latter effect in explaining the mechanism, guided by our calibration.

[4] We build on the framework of GK13 which is widely used in studying QE policies across central banks.

[5] Policies such as the Liquidity Coverage Ratio are the realistic counterpart to this model constraint. Details of the Liquidity Coverage Ratio can be found at https://www.occ.gov/news-issuances/bulletins/2014/bulletin-2014-51.html.

[6] This constraint is motivated by guidance on stress testing from bank regulators. For example, in the Guidance for Stress Testing from the Federal Deposit Insurance Corporation, the Office of the Comptroller of the Currency, and the Fed in 2012, found at https://www.federalreserve.gov/supervisionreg/srletters/sr1207a1.pdf, the regulatory authorities state: "Accordingly, stress tests should provide a banking organization with the ability to identify potential concentrations including those that may not be readily observable during benign periods and whose sensitivity to a common set of factors is apparent only during times of stress and to assess the impact of identified concentrations of exposures, activities, and risks within and across portfolios and business lines and on the organization as a whole."

it lowers the cost of capital for that set of firms by more than that for the other sets of firms, all else being equal. When the central bank buys government bonds, if spreads between loans to various types of firms (large and small firms, for instance) are small in steady state, the central bank reduces the cost of capital for all firms approximately evenly. Hence, in equilibrium, there is an additional effect on the allocation of resources from a corporate bond purchase that does not occur to the same degree from a government bond purchase. Such a framework thus endogenizes the heterogeneous effect on borrowing costs from a large-scale purchase of nonfinancial firm corporate bonds.

The second part of this paper quantifies the misallocation effects of LSAPs of bonds issued by one set of firms (large firms) and not another set of firms (small firms) in a calibrated, DSGE model similar to our two-period model but with DSGE elements that we did not include in our static model: sticky prices, endogenous net worth of banks, and a representative household with habits that can hold bonds facing a holding cost. In the DSGE model, the response of credit spreads to an asset purchase is not only heterogeneous, as in our static model, but also time-varying. We calibrate the new parameters we introduce using U.S. data.

In the calibrated model, a QE policy in which the central bank purchases public debt produces a positive, large effect on investment and output after a bad shock. However, a QE policy in which the central bank purchases the debt of large firms—although potentially inducing a similar effect on output—reduces the response of investment for small firms whose debt is not purchased by the central bank and induces a non-negligible misallocation of resources. In fact, away from the zero lower bound (ZLB), our calibration implies that the misallocation effect is large enough to make a government bond purchase *more effective* than a private bond purchase in terms of increasing output, even though without misallocation a government bond purchase is *less effective* in increasing output than a large-scale corporate bond buy. The misallocation effect is non-negligible away from the ZLB relative to movements in output; however, at the ZLB, the effect of LSAPs on output are amplified, while the misallocation effect is not. Therefore, the misallocation effect as a percentage of the potential output gain from large-scale corporate bond buys will be smaller at the ZLB.

The rest of the paper, after the literature review below, follows as such. Section 2 presents results from the simple model. Section 3 describes the DSGE model, its calibration, and assesses the quantitative implications of large-scale purchases of nonfinancial firm corporate bonds. Section 4 discusses the role of the ZLB and concludes.

1.1 Related Literature

This paper is closely related to a literature that embeds QE policies in macroeconomic models to analyze the channels through which such policies affect the economy. GK11 and GK13 study QE policies in a representative firm DSGE model with constrained financial intermediaries, and their framework is a embedded as special case of our own. As financial frictions are key to our channel, it could be grouped, largely, into an "imperfect asset-substitutability" channel of monetary policy.[7] Other papers in this literature, such as He and Krishnamurthy (2013) and Cúrdia and Woodford (2015), also emphasize the role of financial market imperfections in making QE effective.

In the models of GK11 and GK13, the central bank is less efficient at intermediating financial transactions than the private sector. The calibrated models suggest that QE policies by the central bank can reduce credit spreads and increase investment and output nonetheless. A related literature examines other indirect costs and benefits of QE policies. Hall and Reis (2015) asssesses the potential risks to central bank solvency. Reis (2017) points out that central bank liabilities used to fund LSAPs are special in that they are free of default risk, and thus could prove as a useful policy tool in fighting inflation in a fiscal crisis. To keep to the economic point of interest, our paper does not incorporate such additional tradeoffs.

Our paper is also closely related to the large literature on how misallocation affects the macroeconomy, built on the work of Hopenhayn and Rogerson (1993) and Hsieh and Klenow (2009), among many others. The closest papers to ours are Midrigan and Xu (2014) and Gilchrist, Sim, and Zakrajsek (2013), as they study how financial frictions that induce a misallocation (and result

[7] There is a growing literature addressing different channels in which QE policies affect the economy. In Bhattarai, Eggertsson, and Gafarov (2015), the authors model the "signaling channel" of QE policies. In Greenwood and Vayanos (2014), the authors outline the "duration-risk channel" of QE policies. Other channels have been highlighted in the literature on QE, such as the prepayment-risk channel and open-economy channels.

in a wider dispersion in credit spreads) affect the macroeconomy.[8]

Lastly, there is growing empirical evidence of there being heterogeneous effects of LSAPs through various channels.[9] Darmouni and Rodnyansky (2017) and Kurtzman, Luck, and Zimmermann (2017) show the Federal Reserve's purchases of MBS in QE1 and QE3 incentivized more lending and risk-taking, respectively, by banks with more MBS holdings. Di Maggio et al. (2016) identify an effect of QE on the volume of new mortgages originated and show that the type of mortgages originated were more likely to be those that could be securitized and sold to the Federal Reserve. Chakraborty et al. (2016) show banks that are more active in the MBS market reduce commercial lending subsequent to QE by the Fed, inducing the firms borrowing from these banks to reduce investment. Foley-Fisher et al. (2016) present evidence that the Federal Reserve's maturity extension program had a greater effect on the valuation, investment, and employment of firms which were more dependent on long-term debt.

2 Demonstrating the Mechanism in a Simple Model

We begin by highlighting the main mechanism in our paper within a static framework of firm capital choice and financing. The model details how the capital decisions of heterogeneous firms are affected by the financing environment and how central bank LSAPs change the allocation of capital and affect macroeconomic aggregates.

Heterogeneous firms choose their capital to maximize profits, but their capital choice and profits are affected by their (heterogeneous) cost of capital. Firms must finance their capital via constrained financial intermediaries. Purchases of securities by the central bank will loosen the financial intermediary's collateral constraint, lowering spreads and therefore firms' cost of capital. Due to a regulatory constraint that enters the financial intermediary's problem, purchases of corporate

[8] In Gilchrist, Sim, and Zakrajsek (2013), the financial frictions are not explicitly modeled but it is assumed that financial frictions induce the dispersion in borrowing. Further, in the robustness section of Midrigan and Xu (2014), there is an evaluation of how heterogeneity in borrowing rates induce a misallocation, although the focus of the paper is on how a worsening of financial frictions can induce a misallocation. Also, it is important to note that financial frictions can also distort entry and technology adoption in Midrigan and Xu (2014).

[9] This literature has grown out of empirical work by Krishnamurthy and Vissing-Jorgensen (2013) and Chodorow-Reich (2014) who assess the effects of LSAPs on economic outcomes.

securities will lower spreads for purchased securities by more than for non-purchased securities.

2.1 Model and Equilibrium

The model consists of heterogeneous intermediate good firms and a representative final good firm, financial intermediary, capital producer, and household. There are J continuums of intermediate good firms. Intermediate good firms are indexed by i and the continuum they belong to by j. The total mass of firms is normalized to 1 ($\sum_{j=1}^{J}\int_{i\in j} di = 1$). Each intermediate good firm, i, produces a differentiated good using capital, k_i, with technology $y_i = A_i k_i^{\alpha}$.

The final good is produced from intermediate goods with technology $Y = \left(\int_i y_i^{\rho} di\right)^{\frac{1}{\rho}}$. The final good can either be consumed or used to produce capital. Capital can be produced from the final good with technology that requires $\phi^K(K)$ units of the final good to produce K units of capital, where $\phi^K(K)$ is weakly convex. Therefore, the clearing condition for the final good is $Y = C + \phi^K(K)$.

Finally, intermediate good firms, in total, cannot use more of the capital good than is produced: $K \geq \int_i k_i di$.

2.1.1 Final Good Firms, Capital Producers, and Households

The final good sector is competitive, and the final good is the numeraire. Thus, it earns zero profits and maximizes the problem:

$$\max_{y_i} \left(\int y_i^{\rho} di\right)^{\frac{1}{\rho}} - \int_i p_i y_i di. \tag{1}$$

From (1), we can obtain the standard expression for the price of intermediate good firms:

$$p_i = \left(\frac{y_i}{Y}\right)^{\rho-1}.$$

Aggregate capital is chosen to maximize profits of a firm that converts the final good into capital:

$$\max_{K} QK - \phi^{K}(K),$$

where Q is the price of capital, shared across firms. We assume that $\phi^{K} = \frac{h_k K^{1+b_k}}{1+b_k}$, where h_k is a parameter that affects the level of the cost of producing capital, while b_k is a parameter that affects the convexity of the cost of producing capital.

Households are the residual claimants of all profits by firms or intermediaries, and there is a representative household that consumes the final good. Households can also lend or borrow from the financial intermediary at a gross interest rate r. Households make consumption and lending decisions to maximize consumption subject to their budget constraint:

$$C + D_h = D_h r + X,$$

where D_h is the net lending of the household to the financial intermediary and X is the sum of the household endowment and profits of intermediate good firms, financial intermediaries, and capital producing firms. All lending occurs within the period. In equilibrium, the gross return on household lending must be $r = 1$. Otherwise, the household would want to lend or borrow infinite amounts to financial intermediaries.

2.1.2 Intermediate Good Firms

Intermediate good firm i maximizes profits, defined as revenues from production less expenditures on capital:

$$\max_{k_i} \frac{p_i y_i}{\tau_i^k} - r_i Q k_i,$$

where τ_i^k is an exogenous wedge that represents distortions or inefficiencies in capital allocation not included in the model. Capital expenditures by the firm must be financed, and r_i is the gross interest rate at which a firm can borrow (the "cost of capital"). This maximization problem yields first-order condition for capital:

$$k_i = \left(\frac{\alpha \rho Y^{1-\rho} A_i^{\rho}}{\tau_i^k r_i Q} \right)^{\frac{1}{1-\alpha\rho}}. \tag{2}$$

2.1.3 Financial Intermediation

As firm capital must be financed, the capital expenditures of firm i must be equal to the firm i securities held by financial intermediaries and the central bank:

$$Qk_i = S_{b,i} + S_{g,i},$$

where $S_{b,i}$ denotes the representative financial intermediary's holdings of firm i securities and $S_{g,i}$ denotes central bank holdings of firm i securities.

A representative financial intermediary has exogenous financial wealth N and, along with investing in corporate securities, invests in government bonds. The total supply of government bonds, B^S, is net positive, and clearing implies $B^S = B_b + B_g$, where B_g are central bank holdings of government bonds.

If the intermediary's holdings are not equal to its wealth, it either borrows or lends to households at gross interest rate r. The financial intermediary also faces a regulatory constraint limiting its leverage. This regulatory constraint will depend on the assets purchased and their concentration.[10] We define our J continuums of firms so that each continuum represents firms whose securities are treated identically in terms of capital requirements and concentration risk. In our model, the representative financial intermediary faces the following regulatory collateral constraint:

$$V \geq \sum_j \theta \Delta_j \left(\int_{i \in j} S_{b,i} di \right)^{\nu_j} + \theta B_b, \tag{3}$$

[10] This regulatory constraint is motivated by capital requirements placed on banks, which will differ for different asset classes, as well as the required bank stress testing, which penalizes high concentration of similar risky assets (due to greater exposure to common risk factors).

where $\theta\Delta_j$ and θ are constants that reflect capital requirements for corporate bonds of firms $i \in j$ and government bonds (if holding government bonds is associated with lower capital requirements than holding corporate bonds, $\theta < \theta\Delta_j$). Parameters $\nu_j \geq 1$ reflect the penalty for concentration of assets of type j.[11] V is the market value of the financial intermediary's equity. Note that with $\Delta_j = 1 \forall j$, concentration risk, and a representative intermediate good firm, this is similar to the collateral constraint in the model of GK13.

The financial intermediary maximizes its market value:

$$V = \max_{S_{b,i}, B_b} \sum_j \int_{i \in j} S_{b,i} r_i di + B_b r_b + \left(N - \sum_j \int_{i \in j} S_{b,i} di - B_b \right) r, \tag{4}$$

subject to the collateral constraint (3).

The maximization problem (4) yields the following first-order condition for the interest rate on the debt of firm i of type j:

$$(r_i - r) = \frac{\lambda}{(1+\lambda)} \theta\Delta_j \nu_j \left(\int_{s \in j} (Qk_s - S_{g,s})\, ds \right)^{\nu_j - 1}, \tag{5}$$

where λ is the Lagrangian multiplier on the collateral constraint. Note that since $S_{b,i} = Qk_i - S_{g,i}$, central bank purchases of corporate bonds enter this condition. A similar condition results for the interest rate on government bonds:

$$(r_b - r) = \frac{\lambda}{(1+\lambda)} \theta. \tag{6}$$

The collateral constraint of the intermediary can thus be written as:

$$\left(\sum_j (r_j - r) B_j \right) + B_b (r_b - r) + Nr \geq \sum_j \theta\Delta_j S_{b,j}^{\nu_j} + \theta B_b, \tag{7}$$

where r_j is the spread on firms $i \in j$, which must be identical following (5), and $S_{b,j} = \int_{i \in j} S_{b,i} di =$

[11] If $\nu_j = 1$, there is no penalty for concentration of securities of type j firms.

$\int_{i\in j}(Qk_i - S_{g,i})\,di$. As we will detail below, government purchases of corporate securities, $S_{b,i}$, or long-term government bonds, B_b, lower the amount the financial intermediary has to intermediate.

2.1.4 Equilibrium

Given exogenous bank net worth, N, firm-level productivities and wedges, $\left\{A_i, \tau_i^k\right\}\forall i$, central bank purchases of corporate bonds, $\left\{S_{g,i}\right\}\forall i$, and government bonds, B_g, equilibrium in this model is a set of allocations, $\left\{C, Y, K, B_b, D_h\right\}$ and $\left\{k_i, y_i, S_{b,i}\right\}\forall i$, and prices, $\left\{Q, r, r_B\right\}$ and $\left\{r_i, p_i\right\}\forall i$, such that households maximize consumption subject to their budget constraint; intermediate good firms, final good firms, and capital producers maximize profits; financial intermediaries maximize profits subject to their collateral constraint; and clearing conditions hold.

2.2 Effect of Central Bank Bond Buys

Central bank purchases of either long-term government bonds (B_g) or corporate securities of firm i ($S_{g,i}$) will reduce the amount of that particular asset that has to be intermediated by the financial intermediary. LSAPs will directly affect bond spreads by changing: (1) the Lagrange multiplier on the collateral constraint, λ; (2) the first-order condition (5), by reducing the concentration of firms of type j in the intermediary's balance sheet. Additionally, LSAPs will indirectly affect spreads through their effect on firm capital choices, as can be seen in the first-order condition for capital. Changing firm capital choices will also affect the price of capital, Q, and the amount of firm capital that needs to be intermediated, $Qk_i - S_{g,i}$, which enters directly into the collateral constraint. Proposition 1 states that we can analytically demonstrate the effect of LSAPs on bond spreads, holding firm-capital choices fixed.

Proposition 1. *Holding firm capital choices, k_i, fixed, central bank LSAPs have the following effects:*

(i) A purchase of long-term government bonds, B_g,

(a) decreases λ, the Lagrangian multiplier on the collateral constraint.

(b) proportionately decreases firm spreads, that is, $\frac{\Delta(r_i - r)}{(r_i - r)}$ *is constant for all firms.*

(ii) A purchase of firm securities, $S_{g,i}$*, for* $i \in j$

(a) decreases λ*, the Lagrangian multiplier on the collateral constraint, if* $Nr \geq \frac{(\nu_j - 1)}{\nu_j} B_b \theta + \left(\sum_s \int_{i \in s} \left(1 - \frac{\nu_s}{\nu_j}\right) \theta_s S_{b,i}^{\nu_s} \right)$ *where* $s \neq j$*.*

(b) does not lead to a proportionate decrease in firm spreads, that is, $\frac{\Delta(r_i - r)}{(r_i - r)}$ *is greater for firms of type* $i \in j$ *than other types whose debt is not purchased* ($i \notin j$)*.*

Proof. See Appendix A.

This proposition formalizes the direct effects of LSAPs on spreads. Directly buying the securities of only firms of type $i \in j$ will, holding capital choices constant, lower their spreads by more than of firms with $i \notin j$. This can be contrasted with the effect of purchases of long-term government debt, which will lower spreads proportionately. However, purchases of both long-term government bonds and corporate securities will, under some reasonable conditions, decrease spreads by loosening the collateral constraint faced by financial intermediaries (implying a lower multiplier on the constraint, λ).[12]

Therefore, central bank LSAPs of corporate securities will induce asymmetric changes in spreads and therefore firm *cost of capital*. Through the first-order condition for capital, (2), these spreads will induce changes in firm capital choices and therefore both the allocation of capital and aggregate capital supply. Subsection 2.2.1 demonstrates how such differences in spreads affect the allocation of resources.

2.2.1 Allocative Efficiency

Proposition 1 tells us how government and corporate bond buys directly affect borrowing rates of firms and the government, holding fixed the indirect effect of firm decisions changing in response. Although we cannot directly map bond buys to aggregates analytically, to develop intuition

[12] Holding capital decisions fixed, this is always true for purchases of government debt. For purchases of corporate securities, the condition in Proposition 1 (ii)(a) must hold, which can be understood as implying that firms must not hold too much government debt relative to wealth, with an adjustment for heterogeneous ν_j.

as to how bond buys affect allocative efficiency it is useful to walk through how spreads affect the allocation and aggregates.[13] In equilibrium, we can express the relative holdings of firm capital as

$$\frac{k_i}{K} = \frac{\left(\frac{A_i^\rho}{r_i \tau_i^k}\right)^{\frac{1}{1-\gamma-\alpha\rho}}}{\sum_j \int_{i \in j} \left(\frac{A_i^\rho}{r_i \tau_i^k}\right)^{\frac{1}{1-\gamma-\alpha\rho}} di}.$$

Therefore, the relative levels of firm cost-of-capital, r_i, have implications for the *relative* allocation of capital. Additionally, firm cost-of-capital affects the aggregate demand for capital:

$$K = \frac{\left(\int \left(\frac{A_i^\rho}{\left(r_i \tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho(1-\alpha+b_k)}} \left(\int \left(\frac{A_i^\rho}{\left(r_i \tau_i^k\right)}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1-\alpha}{1-\alpha+b_k}}}{\left(\frac{h_k}{\alpha\rho}\right)^{\frac{1}{1-\alpha+b_k}}}. \tag{8}$$

In equilibrium, our model yields macroeconomic aggregates as an analytical function of firm interest rates, r_i, and exogenous variables ($A_i, \tau_i^k \, \forall i$). Building on (8), we can express aggregate output as

$$Y = \underbrace{K^\alpha}_{\text{Capital}} \underbrace{\frac{\left(\int \left(\frac{A_i^\rho}{\left(r_i \tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1}{\rho}}}{\left(\int \left(\frac{A_i^\rho}{r_i \tau_i^k}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\alpha}}}_{\text{Allocation}}. \tag{9}$$

Note that (9) shows that output can be expressed as a function of aggregate capital, K^α, modified by a term that captures both the productivity of intermediate good firm production functions and the efficiency of the allocation of capital. For example, if there is no heterogeneity ($A_i = A, r_i = r_A, \tau_i^k = \tau^k$), then (9) reduces to $Y = AK^\alpha$.

To more clearly demonstrate the effect of spreads on output and allocative efficiency, we define r_A, the *weighted-average interest rate faced by firms*, such that aggregate capital depends only on

[13] This is due to the nonlinearity in the equation linking firm credit spreads to central bank bond purchases (due to firm-capital choices reacting to the change in spreads, a second-order effect).

r_A (and does not depend on heterogeneity in interest rates):

$$\frac{1}{r_A} = \frac{\left(\int \left(\frac{A_i^\rho}{\left(\tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}} \left(\frac{1}{r_i}\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int \left(\frac{A_i^\rho}{\tau_i^k}\right)^{\frac{1}{1-\alpha\rho}} \left(\frac{1}{r_i}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}}{\left(\int \left(\frac{A_i^\rho}{\left(\tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int \left(\frac{A_i^\rho}{\left(\tau_i^k\right)}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}}.$$

Note that if all firms have the same interest rate, $r_i = r_A$, we can then define *interest rate wedges*, $r_{\tau,i}$, between firm interest rates and the weighted-average firm interest rate as

$$r_{\tau,i} = \frac{r_A}{r_i}.$$

These expressions allow us to derive an expression for output as a function of aggregate capital, which depends only on the weighted-average interest rate, r_A, and for productivity and allocative efficiency, which depends only on firm productivities, A_i, interest rate wedges, $r_{\tau,i}$, and other exogenous distortions, τ_i^k:

$$Y = \underbrace{K^\alpha}_{\text{Capital}} \underbrace{\frac{\left(\int \left(A_i^\rho \left(\frac{r_{\tau,i}}{\tau_i^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1}{\rho}\left(\frac{1-\alpha\rho}{1-\alpha}\right)}}{\left(\int \left(\frac{A_i^\rho}{\left(\tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{(1-\rho)\alpha}{\rho(1-\alpha)}} \left(\int \left(\frac{A_i^\rho}{\left(\tau_i^k\right)}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\alpha}}}_{\text{Allocation}}. \tag{10}$$

From (10), and building on proposition 1, there are two first-order consequences of central bank purchases of corporate securities that lower bond spreads heterogeneously. First, they will lower the weighted-average interest rate, r_A, leading to greater aggregate investment and capital. However, they can also generate interest rate wedges, $r_{\tau,i}$, which have consequences for the allocation of capital relative to the efficient level. The size (and direction) of these effects depend on how far the baseline allocation is from its efficient level and whether the interest rate wedge changes induced by bond buys exacerbate or undo distortions. The latter point can be further formalized by deriving output-maximizing interest rate wedges, as we do in Proposition 2 below.

Proposition 2. *The output-maximizing allocation of firm interest rate wedges satisfies* $r^*_{\tau,i} \propto \tau^k_i$ *Equivalently,* $r^*_i \propto r_A \frac{1}{\tau^k_i}$

Proof. See Appendix A.

Proposition 2 shows that the optimal interest rate wedges are such that they exactly offset exogenous distortions τ^k_i. If there are no exogenous distortions then the optimal allocation arises when firms all have identical costs of capital $r_A = r_i$.

When there are only two types of firms ($j \in 1, 2$), where r_i and τ^k_i are symmetric for all $i \in j$, we can characterize all interest rate wedges using just a single interest rate wedge, $r_{\tau,1}$, and the weighted average cost of capital, r_A. The following corollary shows that in this case, output is monotonically decreasing as the interest rate wedges move further from their output-maximizing values:

Corollary 2.1. *In the case with only two types of firms,* $\frac{\partial Y}{\partial |r_{\tau,1} - r^*_{\tau,1}|} < 0$.

Thus, given that large-scale corporate bond buys can induce heterogeneous movements in spreads, they can cause (reverse) a misallocation by moving interest rate wedges away from (toward) the efficient allocation.[14] For example, if interest rate wedges of type 1 (large) firms are greater (therefore interest rates are lower) than those of type 2 (small) firms in steady state, central bank purchases of large firm assets increase large firm interest rate wedges relatively further. In this setting, central bank bond buys of large firm assets will distort the allocation further from its efficient level. Looking ahead, our New Keynesian DSGE model will be calibrated such that interest rate wedges of large firms are greater than those of small firms in steady state, and central bank bond buys of large firm assets will distort the allocation further from its efficient level. Thus, Corollary 2.1 provides the key intuition behind the results we will present in the calibrated New Keynesian DSGE model.

[14] A version of Corollary 2.1 can be derived for a case with more than two types of firms, but the measure of 'distance' from the efficient allocation will be a more complicated function of firm interest rate wedges, exogenous distortions, and productivities.

3 New Keynesian DSGE Model

We evaluate the impact of the misallocative effect of LSAPs by the central bank in a richer environment where the effects of central bank asset purchases on firm borrowing rates are endogenized. We embed our simple model in a standard New Keynesian DSGE model following GK13; along with explicitly modeling banks, the model has the key elements of a New Keynesian model: households, nonfinancial firms, capital goods producers, retail good firms with sticky prices, a central bank, and a government. It is useful to start by outlining the changes we make to the nonfinancial firm sector and then discussing the changes made to households and the financial sector. Afterwards, we outline the remainder of the model and define an equilibrium.[15]

3.1 Model Description

Nonfinancial and Capital Good Firms There are two continuums, indexed by $j \in (1,2)$, of nonfinancial intermediate good firms. Each firm i in continuum j produces output with technology:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{(1-\alpha)},$$

where $Y_{i,t}$ is the intermediate good output of firm i, $K_{i,t}$ its capital stock, $L_{i,t}$ its employment, $\alpha \in (0,1)$ governs capital's share in production. Total intermediate good firm output, $Y_{m,t}$, is then computed using a CES aggregator:

$$Y_{m,t} = \left(\sum_j \omega_j \int_{i \in j} Y_{j,t}^{\rho} di \right)^{\frac{1}{\rho}},$$

where ω_j is a parameter greater than or equal to 0 that is a factor affecting the extent to which the output of intermediate good firms of type j enters total output, and ρ is the CES parameter. Note if $\rho = 1$, then intermediate goods are perfectly substitutable.

[15] Besides the production and financing environment, all of the remaining model elements are exactly as in GK13 to allow for easy comparison of quantitative results. Their model can be thought of as a special parameterization of our model in which firm heterogeneity and concentration risk are unimportant.

All firms within each continuum face the same financing environment and production technology, as in Section 2. We can therefore represent each continuum of firms with a representative firm of type j. We can thus write the production technologies for representative firms as follows:

$$Y_{j,t} = A_{j,t} K_{j,t}^{\alpha} L_{j,t}^{(1-\alpha)},$$

where $Y_{j,t}$ is the intermediate good output of type j firms, $A_{j,t}$ an index of type j TFP, $K_{j,t}$ is the type j firm capital stock, $L_{j,t}$ is the type j total employment.[16] Similarly, total intermediate good output is thus combined from type j intermediate good outputs with production function:

$$Y_{m,t} = \left(\sum_j \omega_j Y_{j,t}^{\rho} \right)^{\frac{1}{\rho}}.$$

We set $A_{j,t} = A_t$, so firms receive identical productivity shocks.[17]

Following the usual arguments from cost-minimization, the price of intermediate good j can be written as

$$P_{j,t} = \omega_j P_{m,t} Y_{m,t}^{1-\rho} Y_{j,t}^{\rho-1},$$

where $P_{m,t}$ is the relative price of intermediate goods. Firms choose labor to maximize revenues less labor expense, where W_t is the wage rate which is constant across firms. Then we can recover firm j's demand for labor from

$$W_t = P_{m,t} \frac{Y_{m,t}^{1-\rho} Y_{j,t}^{\rho}}{L_{j,t}} \omega_j \rho (1-\alpha),$$

which holds for each type j. Remaining revenues accrue to capital, so gross profits per unit of

[16] If we define $Y_{j,t} = \left(\int_{i \in j} Y_{i,t}^{\rho} di \right)^{\frac{1}{\rho}}$, $A_{j,t} = \left(\int_{i \in j} A_{i,t}^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}}$, $K_{j,t} = \int_{i \in j} K_{i,t} di$, $L_{j,t} = \int_{i \in j} L_{i,t} di$, it can be verified that all equilibrium conditions will be identical for representative firm j and all firms $i \in j$.

[17] We use ω_j to calibrate the output shares of each type of firms in steady-state.

capital for firm j, $Z_{j,t}$, are

$$Z_{j,t} = \omega_j \left(1 - \rho(1-\alpha)\right) \frac{P_{m,t} Y_{m,t}^{1-\rho} Y_{j,t}^{\rho}}{\xi_t K_{j,t-1}},$$

where ξ_t is the capital quality shock, and this equation holds for each type j.[18]

We assume that capital is transferable between firms: thus, we have the capital accumulation equation:

$$K_{t+1} = \xi_{t+1}[I_t + (1-\delta)K_t]. \tag{11}$$

The capital good producer solves the following maximization problem:

$$\max E_t \sum_{t=\tau}^{\infty} \Lambda_{t,\tau} \{Q_\tau I_\tau - [1 + f(\frac{I_\tau}{I_{\tau-1}})] I_\tau\},$$

where Λ_t is a discount factor that will be obtained from the household's problem, in equilibrium.

Thus, the price of capital goods can be determined from profit maximization as

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - E_t \Lambda_{t,t+1} (\frac{I_{t+1}}{I_t})^2 f'(\frac{I_{t+1}}{I_t}). \tag{12}$$

Imposing the functional form for f considered by GK13 in (12), we get

$$Q_t = 1 + \frac{\eta_i}{2}(\frac{I_t}{I_{t-1}} - 1)^2 + \eta_i(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}} - E_t \Lambda_{t,t+1}(\frac{I_{t+1}}{I_t})^2 \eta_i (\frac{I_{t+1}}{I_t} - 1),$$

where η_i is the inverse elasticity of net investment to the price of capital.

Firms require financing of their capital stocks, and they do so by issuing state-contingent claims that are perfectly monitored and enforced and, thus, perfectly state-contingent. We assume that

[18] In this setup, capital owners receive firm income less wages paid, which is equal to the marginal product of capital if firms produce with constant returns to scale ($\rho = 1$).

only part of firm capital expenditures must be financed using external sources:

$$K_{j,t} = K_{j,I} + S_{j,t},$$

where $K_{j,I}$ is the amount of capital firms of type j do not need to finance externally. We assume that $K_{j,I}$ is low enough that firms will always use some external financing ($S_{j,t} > 0$) in equilibrium, and that firm profits are split proportionally between capital financed externally ($S_{j,t}$) and not ($K_{j,I}$), so the gross profits per unit of capital are $Z_{j,t+1}$ for every unit of capital of type j.[19] To keep $K_{j,I}$ constant, we impose that intermediate good firms make net transfers to the household of $K_{j,I}\,(Z_{j,t+1} - P_{k,t+1}\delta)$ each period, paying out earnings beyond those required to purchase $K_{j,I}$ units of capital.[20]

If ξ_{t+1} is the capital quality shock, the period $t+1$ payoff of the security of firm j is $(Z_{j,t+1} + (1-\delta)Q_{t+1})\xi_{t+1}$. Thus, the security of firm j has a rate of return of

$$R_{k,j,t+1} = \frac{Z_{j,t+1} + (1-\delta)Q_{t+1}}{Q_t}\xi_{t+1}. \tag{13}$$

The rate of return therefore has a relationship with $Z_{j,t}$, profits per unit of capital, which depends on $K_{j,t}$. Since Q_t is common to the two types of firms, difference in the rates of return, $R_{k,j,t+1}$, imply differences in per-capital profit rates and allocation of capital between the two types.

Retail Good Firm Problem The final good, Y_t, is produced using a mass one continuum of differentiated retail goods using CES production:

$$Y_t = [\int_0^1 Y_{ft}^{\frac{\epsilon-1}{\epsilon}} df]^{\frac{\epsilon}{\epsilon-1}}.$$

Retail good firms, however, just take intermediate output and repackage it. Thus, the marginal

[19] $K_{j,I}$ is incorporated to match the fact that firms in the U.S can finance much of their investment in the aggregate with internal funds.

[20] While positive in steady-state, in the case of a particularly bad shock, this transfer can be negative.

cost of production is P_{mt}, the price of the output of intermediate good firms. The retail good firm faces Calvo pricing. It can adjust its price with probability $1-\gamma$. The firms choose the same reset price P_t^*. Following the usual arguments, we can obtain the first-order condition:

$$\sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i}[\frac{P_t^*}{P_{t+i}} - \mu P_{mt+i}]Y_{ft+i} = 0,$$

with $\mu = \frac{1}{1-1/\epsilon}$. We can thus recover the law of motion for prices:

$$P_t = [(1-\gamma)(P_t^*) + \gamma(P_{t-1}^{1-\epsilon})]^{\frac{1}{1-\epsilon}}.$$

Households There is a measure one continuum of households (all identical), each of which consumes the final good, saves by lending funds to banks and potentially the central bank and supplies labor.[21]

Each household is composed of a fraction $1-f$ workers and f bankers and has perfect consumption insurance. Workers are the members who supply labor to earn real wage, W_t, which the household shares among itself. Bankers also share any earnings with the household as a whole. In effect, the household owns the bank that its bankers manage. Define the overall transfers to households from firms and banks as Π_t. Households pay taxes, T_t. The household deposits funds in banks but only in banks the household's bankers do not manage. Workers can become bankers and vice versa over time. With probability σ, bankers stay bankers, and with probability $1-\sigma$, bankers become workers. Bankers face a finite horizon problem; in effect, they cannot retain earnings beyond the point at which they can fund all investment from their own capital. Workers are randomly selected to replace the bankers who switch to workers and receive a startup fund of $\frac{X}{(1-\sigma)f}$.

The household consumes C_t units of the final good. L_t is family labor supply. The household

[21] The economy we consider is the cashless limit.

has habits in consumption, and the household's utility, u_t, is defined as follows:

$$u_t = E_t \sum_t^{\infty} \beta^i [\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+1+i}^{1+\varphi}], \tag{14}$$

where $0 < \beta < 1$, $0 < h < 1$, and $\chi, \phi > 0$.

Households are indifferent between deposits and government debt, as they both pay rate of return between periods $t-1$ and t of R_t, in equilibrium. Thus, we make this assumption throughout, calling both short-term debt, $D_{h,t}$. We can thus define the household's budget constraint to be

$$C_t = W_t L_t + \Pi_t - X + T_t + R_t D_{h,t-1} - D_{h,t}. \tag{15}$$

The household thus solves (14) subject to (15) choosing C_t, L_t, and D_{ht}. Define u_{Ct} to be the marginal utility of consumption. We then have labor supply condition:

$$u_{Ct} W_t = \chi L_t^{\varphi},$$

and consumption-savings optimality condition:

$$E_t \beta \frac{u_{C,t+1}}{u_{C,t}} R_{t+1} = 1.$$

It is also useful to define

$$\Lambda_{t,t+1} = E_t \beta \frac{u_{C,t+1}}{u_{C,t}}, \tag{16}$$

as it enters the discount factor of firms and intermediaries.

Holding Costs We also allow households to directly hold securities in the face of holding costs. Define $S_{h,j,t}$ as the securities of firm j held by the household at time t and $B_{h,j,t}$ as securities of the government held by the household at time t with price q_t. Holding costs for type j firm securities are $\frac{\kappa_j}{2}\frac{\left(S_{h,j}-\bar{S_{h,j}}\right)^2}{S_{h,j}}$, where parameters κ_j and $\bar{S_{h,j}}$ are positive and $S_{h,j} \geq \bar{S_{h,j}}$. Holding costs for government securities are $\frac{\kappa_j}{2}\frac{\left(B_h-\bar{B_h}\right)^2}{B_h}$, where parameters κ_j and $\bar{B}_h$ are positive and $B_h \geq \bar{B}_h$.

With holding costs, we rewrite budget constraint of the household:

$$
\begin{aligned}
C_t + D_{h,t} + \sum_{j=1}^{j=2} Q_{j,t}(S_{h,j,t} + \frac{1}{2}\kappa(S_{h,j,t} - \bar{S_{h,j}})^2) + q_t(B_{h,t} + \frac{1}{2}\kappa(B_{h,t} - \bar{B_h})^2) \\
= W_t L_t + \Pi_t + T_t - X + R_t D_{h,t-1} + \sum_{j=1}^{j=2} R_{k,j,t} S_{h,j,t-1} + R_{b,t} B_{h,t-1},
\end{aligned}
$$

where $R_{b,t}$ is the return on government bonds.

Banks There is a single bank which makes long-term loans to nonfinancial firms and the government, which are funded by the bank's liabilities (short-term deposits of households). The bank is jointly owned by all of the bankers, and when bankers become workers they bring back to the household their fraction of the net worth of the bank. The rate of return on a loan will be equal to the return on the security defined in (13). There are government bonds, b_t, that are available to households and banks, which are perpetuities and pay one dollar per period. If q_t is the price of the bond and P_t is the price level, the real rate of return on the bond $R_{b,t+1}$ is

$$
R_{b,t+1} = \frac{\frac{1}{P_t} + q_{t+1}}{q_t}.
$$

The balance sheet of the bank is

$$
Q_t S_{b,1,t} + Q_t S_{b,2,t} + q_t B_{b,t} = N_t + d_t, \tag{17}
$$

where N_t is bank net worth, d_t is deposits held, $B_{b,t}$ is government bonds held, and $S_{b,j,t}$ for $j \in \{1, 2\}$ is the securities holdings by the bank of firms 1 and 2, respectively. Net worth is the difference between the gross return on assets and the cost of deposits:

$$
N_t = R_{k,1,t} Q_{t-1} S_{b,1,t-1} + R_{k,2,t} Q_{t-1} S_{b,2,t-1} + R_{b,t} q_{t-1} B_{b,t-1} - R_t d_{t-1}. \tag{18}
$$

The bank will maximize its expected discounted value of net worth:

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+1} N_{t+1}. \tag{19}$$

The bank faces an incentive constraint due to an imperfect monitoring problem of the bank by depositors wherein the government regulations on asset holdings also enter:

$$V_t \geq \theta Q_t S_{b,1,t}^{\nu_{s,1}} + \theta \Delta_s Q_t S_{b,2,t}^{\nu_{s,2}} + \Delta \theta q_t B_{b,t}^{\nu_b}, \tag{20}$$

where $\nu_{s,1}$, $\nu_{s,2}$, and ν_b are parameters all greater than or equal to one and govern the extent to which it is costly to hold a given asset $s_{1,t}$, $s_{2,t}$, and $B_{b,t}$, respectively. Also, parameters θ, $\theta\Delta_s$, and $\theta\Delta$ are the respective amounts of the bank's portfolios of $S_{b,1,t}$, $S_{b,2,t}$, and $B_{b,t}$ the bank can divert, where $0 \geq \{\Delta, \Delta_s\}$. This constraint can also be interpreted as a collateral/regulatory constraint, where θ and $\{\Delta, \Delta_s\}$ are parameters that govern how tightly the collateral constraint binds on different assets, while $\nu_{s,1}$, $\nu_{s,2}$, and ν_b act as either holding costs or tighter regulatory constraints due to concentration in particular asset classes.

The bank chooses $S_{b,1,t}$, $S_{b,2,t}$, and $B_{b,t}$ to maximize (19) subject to (17), (18), and (20). In addition, banks are price takers, taking interest rates and spreads as given. We describe the solution to the problem of the bank in Appendix B.

Central Bank and Government Policy The central bank can purchase either government bonds (short or long term) or private securities. We only allow the central bank to purchase the securities of type 1 (large) firms.

The central bank can issue riskless short-term debt $D_{g,t}$ which pay R_{t+1}.

Thus, the central bank has balance sheet

$$Q_t S_{g,1,t} + q_t B_{g,t} = D_{g,t},$$

where $S_{g,1,t}$ is central bank holdings of type 1 securities, and $B_{g,t}$ is central bank holdings of government bonds.[22] The central bank costlessly transfers any profits to, or recovers any losses from, the government. We assume the central bank is less efficient in intermediation than banks and thus pays $\tau_{s,j}$ per unit of type j bonds intermediated and τ_b per unit of government bonds.

The central bank determines monetary policy using a Taylor rule. Define i_t as the net nominal interest rate, i as the steady-state nominal rate, π_t as the inflation rate P_{t+1}/P_t, and Y_t^* as the flexible-price equilibrium level of output. Then

$$i_t = i + \kappa_\pi \pi_t + \kappa_y(\log(Y_t) - \log(Y_t^*)) + \epsilon_t,$$

where ϵ_t is an exogenous shock. When we allow for a ZLB on interest rates:

$$i_t = \max\left\{0, i + \kappa_\pi \pi_t + \kappa_y(\log(Y_t) - \log(Y_t^*)) + \epsilon_t\right\}.$$

We can then determine the real interest rate with the standard Fisher relation:

$$1 + i_t = R_{t+1}\frac{P_{t+1}}{P_t}.$$

Clearing for each type j securities implies

$$S_{j,t} = S_{b,j,t} + S_{h,j,t} + S_{g,j,t},$$

where $S_{j,t}$ is total holdings of type j securities.

Also, we have clearing for government bonds, which implies

$$B_t = B_{b,t} + B_{h,t} + B_{g,t}.$$

Government consumption, G, and the net interest payments from fixed amount of long-term

[22] $S_{g,2,t}$ or central bank holdings of type 2 assets is restricted to be zero and, thus, we do not write it into the constraint.

bonds $\bar{B}$ are fixed. Revenues will include central bank earnings net costs plus collected taxes.

We thus have the consolidated government budget constraint:

$$G + (R_{b,t} - 1)\bar{B} \;\;=\;\; T_t + \sum_{j=1}^{j=2}(R_{k,j,t} - R_t - \tau_{s,j})Q_{j,t-1}S_{g,j,t-1} + (R_{b,t} - R_t - \tau_b)q_{t-1}B_{g,t-1}.$$

Central bank LSAPs involve purchasing a fraction, $\varphi_{s,1,t}$ and $\varphi_{b,t}$, of outstanding type 1 private-sector securities or long-term government securities, respectively. To be precise, these policies are respectively modeled as

$$S_{g,1,t} \;\;=\;\; \varphi_{s,1,t}S_{1,t-1},$$

and

$$B_{g,t} \;\;=\;\; \varphi_{b,t}B_{t-1},$$

where $\varphi_{s,1,t}$ and $\varphi_{b,t}$ are modeled as second-order regressive policies.

When Δ_s >1, limits to arbitrage are weaker for private securities of type 2 than for private securities of type 1 firms. Thus, all else being equal, if there is an asset purchase, private securities of type 1 should move by more than private securities of type 2. A similar result is true for government bonds when $\Delta < 1$. Further, $\nu_{s,1}$ and $\nu_{s,2}$, when greater than one, affect the desired stock of holdings, further altering the extent to which excess returns adjust after a bond purchase.

Resource Constraint, Further Clearing Conditions, and Equilibrium We have the resource constraint:

$$Y_t = C_t + [1 + f(\frac{I_t}{I_{t-1}})]I_t + G + \sum_{j=1}^{j=2}\tau_{s,j}Q_{t-1}S_{g,j,t-1} + \tau_g q_{t-1}B_{g,t-1}.$$

We then require that supply equals demand in our different markets. In the market for labor:

$$\omega_1(1-\alpha)\rho\frac{Y_{1,t}^{\rho}Y_{m,t}^{1-\rho}}{L_{1,t}}E_t u_{C,t} = \frac{1}{P_{m,t}}\chi L_t^{\phi},$$

and

$$\omega_2(1-\alpha)\rho\frac{Y_{2,t}^{\rho}Y_{m,t}^{1-\rho}}{L_{2,t}}E_t u_{C,t} = \frac{1}{P_{m,t}}\chi L_t^{\phi},$$

where $L_t = L_{1,t} + L_{2,t}$.

In the market for capital, we have

$$K_{1,t+1} + K_{2,t+1} = I_t + (1-\delta)K_t,$$

where $K_t = K_{1,t} + K_{2,t}$.

Notice that with clearing in the markets for goods, labor, and all securities, by Walras' Law the market for riskless short-term debt also clears.

3.2 Misallocation

We can construct a measure of misallocation by first constructing a counterfactual measure of output: the maximum output, $\hat{Y}$, which can be produced with a fixed amount of labor and capital. In our production environment, $\hat{Y}$ can be expressed as

$$\hat{Y}_t = A_t K_t^{\alpha} L_t^{(1-\alpha)} \left(\sum_{j=1}^{j=2} \omega_j^{\frac{1}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}.$$

We therefore can define the losses from misallocation as $\hat{Y}_t - Y_t$.

3.3 Calibration

We present the parameters used in our quantitative exercise in Table 1. In our calibration exercise, we follow the calibration strategy of GK13 for their parameters and calibrate the new parameters we introduced.[23] The new parameters, listed at the bottom of Table 1, concern firm heterogeneity and the regulatory constraint. Our calibration of the parameters that govern the extent of misallocation in steady state is meant to be conservative.

For the regulatory constraint parameters, we motivate the calibration with the following two points: (1) it is generally less costly for a bank to hold government debt than corporate debt due to differences in liquidity in these assets, and (2) government debt is considered Level 1 capital as a High Quality Liquid Asset (HQLA) in computing the Liquidity Coverage Ratio for Basel III, while nonbank investment-grade corporate debt is considered a Level 2B asset, while CDOs of other corporate loans do not count as HQLA. To be conservative with our calibration of the extent of misallocation in steady state, we choose parameters $\nu_b = 1.0$, $\nu_{s,1} = \nu_{s,2} = 1.2$, which imply only a modest amount of convexity (which can be interpreted as low holding costs) and where the convexity does not differ between private sector assets. We parameterize $\Delta_{s,2}$ so that the difference in spreads between the two types of firms is 0.9% in steady-state, which is well below that implied by the dispersion in credit spreads in Gilchrist, Sim, and Zakrajsek (2013), consistent with our goal of being conservative as to the extent of misallocation that exists in steady state. We set $\bar{S_{h,1}}$ and $\kappa_{s,1}$ to the values of private securities in GK13.

For intermediate good firms, we set the standard, CES parameter, ρ, to 0.9, implying a great deal of substitutability between the two types of firms. Consistent with our goal of being conservative with regard to the extent of misallocation that exists in steady state, our calibration is above of Atkeson and Burstein (2010), a representative value in the literature. We set the share of labor and output in production of type 1 firms to 0.5.

We calibrate our financing parameters to account for the following fact: Shourideh and Zetlin-

[23] The only parameter we change is σ, which we lower to 0.92. At the previous value of σ, intermediaries could save out of their constraint, because we have introduced a new higher yielding asset, that is, type 2 firms.

Jones (2012) show that about 80% of investment by private firms is financed externally, compared to 20% for publicly traded firms.[24] We set the proportion of steady-state capital that firms of type 1 finance internally at 80% and the amount that they finance externally directly from households at 10% (therefore 10% of their capital is financed via intermediaries in steady-state, which is consistent with half of their external financing being met by households). Firms of type 2 finance 20% of their capital internally, finance 10% directly from households (the same proportion as type 1 firms), and must rely on financial intermediaries to finance the remainder of their capital stock.

3.4 Quantitative Results

Figure 1 presents results from the impulse responses from type 1 (large) firm bond purchases and government bond purchases. In our case with firm heterogeneity and potential misallocative effects, government bond purchases are *more effective* in boosting output than corporate bond purchases (of type 1 firm debt).[25] This is the reverse of the result in the work GK13, which does not consider heterogeneity. The result of GK13 occurs in our model when the output of the two types of firms is perfectly substitutable ($\rho = 1$).

We see that following a large-scale type 1 corporate bond purchase, the amount of type 1 firm capital that has to be intermediated, $S_{b,1}$, falls, reducing spreads on firms of type 1 and leading to a larger difference in spreads, $E[R_{K,2}] - E[R_{K,1}]$. This leads to a marked increase in the capital of type 1 firms, K_1. Due to GE price effects, there is a concomitant change in the capital of type 2 firms, K_2, as firms must finance part of their capital stocks. In our calibration, this change in the relative allocation of capital is inefficient, so there is a misallocation cost of corporate bond purchases that reduces their effectiveness. There is still a positive effect on output, as lower average spreads lead to greater capital demand.

After a government bond purchase, we see different dynamics in terms of capital and spreads.

[24] Chari (2013) notes that the fact that GK13 is not calibrated to be consistent with such aggregate facts should be resolved in future work.

[25] We only allow for type 1 bond purchases by the central bank, as representative large-scale nonfinancial corporate sector bond purchases, such as the ECB's Corporate Sector Purchase Program, are typically targeted at purchasing only investment-grade or higher quality corporate debt. This could easily be relaxed.

A government bond purchase loosens the collateral constraint of the financial intermediary, which (all else equal) reduces spreads for all firms. Reducing the spreads for all firms reduces the extent of misallocation as compared to the steady state (arising from the steady-state difference in spreads), and increases capital of type 2 firms (small firms) relatively more than type 1 firms. In this case, the effectiveness of government bond purchases on increasing output is slightly amplified by its effect on the allocation of capital between firms.

The misallocation induced by large-scale corporate bond buys can be important when looking at the difference between the effectiveness of different types of LSAPs. The blue line in Figure 2 is the difference between the impulse response of output during the government bond buy and the impulse response of output during the large firm bond buy. The dashed red line is the output losses *directly* due to misallocation in the corporate bond purchase. The direct misallocation effect is measured as the difference between the maximum output that could be produced with a given amount of capital and labor and what is actually produced.[26]

The losses due to the direct effects of misallocation in large firm corporate bond buys account for the majority of the difference in the effect of corporate versus government bonds buys. In other words, when weighing different options for the types of debt to buy as part of LSAPs, the misallocation effect of corporate bond buys should potentially be weighed as part of the trade-offs involved, as it can be quantitatively meaningful. Notice, this is a different result from that of GK13 who show that government bond buys induce smaller movements in output than private-sector bond buys for a similarly sized bond purchase. Our model generates a similar result to GK13 when intermediate good firm products are perfect substitutes, i.e. when the CES parameter $\rho = 1$.[27] We show this result in panel (c) of Figure 3.

Overall, the calibrated impulse responses suggest that a large-scale corporate bond buy induces a greater misallocation of resources than a large-scale government bond buy and the misallocation

[26] There are also indirect effects of misallocation that our misallocation measure ignores. Capital and labor are taken as given in our misallocation measure, but, in fact, they are endogenously affected by misallocation as well.

[27] This result is due to purchases of private-sector debt having a larger effect on excess returns of private bonds than purchases of government bonds have on excess returns of private bonds, and this effect not being offset by a misallocation effect.

effect is a quantitatively significant fraction of the output gains from a large-scale corporate bond buy.

4 ZLB Discussion

In our model, when at the ZLB, output losses from exogenous shocks, as well as the effectiveness of QE, are amplified.[28] To demonstrate this, we feed in capital quality shocks that force the economy to the ZLB. We then have the central bank perform similarly sized bond purchases to our baseline case when the economy is at the ZLB. We show in panel (d) of Figure 3 that in this case, output gains from a QE program are indeed amplified relative to the baseline case where the ZLB does not bind.[29]

We also compute our misallocation measure in response to bond purchases at the ZLB. We see from Figure 4 that our misallocation measure does not drastically change in response to corporate bond buys when we allow for a binding ZLB. This is because, accounting for level effects on excess returns, there is little change in the relative borrowing costs of type 1 and type 2 firms at the ZLB as compared to the baseline case. Hence, misallocation matters much more relative to movements in real output when the ZLB is not binding. There are arguments for the central bank to make QE part of its toolkit even away from the ZLB (for examples, see Quint and Rabanal (2017) or Gagnon (2016)). Our exercise sheds light on a potential counterargument to be considered when making such a claim, at least for large-scale corporate bond buys, as large-scale corporate bond buys can induce a quantitatively significant misallocation of resources.

[28] To incorporate the ZLB in our model, we follow the work of Guerrieri and Iacoviello (2015).

[29] Our baseline calibration results in corporate bonds buys having a greater stimulative effect on output at the ZLB than government bond buys. However, a small reduction in the CES parameter, ρ, to 0.875 (which would still be a more conservative estimate than that typically considered in the literature) leads to government bond buys remaining more effective than corporate bond buys, even at the ZLB.

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A Proofs to Propositions

Here we present proofs to our propositions.

Proposition 1

We can express the Lagrangian multiplier term in spreads as

$$\frac{\lambda}{1+\lambda} = \frac{\sum_j \theta \Delta_j \left(QK_j - S_{g,j}\right)^{\nu_j} + \left(B_b^T - B_g\right)\theta - N * r}{\sum_j \theta \Delta_j \nu_j \left(QK_j - S_{g,j}\right)^{\nu_j} + \left(B_b^T - B_g\right)\theta}. \tag{21}$$

This derivative of (21) with respect to B_g, holding capital choices (thus, K_j and Q) constant, yields:

$$-\frac{\theta\left(\sum_j \theta\Delta_j\left(\nu_j-1\right)\left(QK_j-S_{g,j}\right)^{\nu_j}+N*r\right)}{\left(\sum_j \theta\Delta_j\nu_j\left(QK_j-S_{g,j}\right)^{\nu_j}+\left(B_b^T-B_g\right)\theta\right)^2}<0,$$

which implies (i)(a) (since $\frac{\lambda}{1+\lambda}$ is increasing in λ). (i)(b) follows immediately from (5).

Similarly, (ii)(a) follows from taking the derivative of $\frac{\lambda}{1+\lambda}$ with respect to $S_{g,i}$ for $i \in j$, holding capital choices constant, and (ii)(b) immediately follows from (5).

Proposition 2

Note that output, defined in (10), is increasing in the term $\int\left(A_i^\rho\left(\frac{r_{\tau,i}}{\tau_i^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}di$. The output-maximizing value for interest rate wedges then can be found by solving the maximization problem:

$$\max_{r_{\tau,i}}\int_i\left(A_i^\rho\left(\frac{r_{\tau,i}}{\tau_i^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}di,$$

such that

$$\frac{\left(\int\left(\frac{A_i^\rho}{\left(\tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,i}\right)^{\frac{\alpha\rho}{1-\alpha\rho}}di\right)^{\frac{1-\rho}{\rho}}\left(\int\left(\frac{A_i^\rho}{\tau_i^k}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,i}\right)^{\frac{1}{1-\alpha\rho}}di\right)^{1-\alpha}}{\left(\int\left(\frac{A_i^\rho}{\left(\tau_i^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}}di\right)^{\frac{1-\rho}{\rho}}\left(\int\left(\frac{A_i^\rho}{\left(\tau_i^k\right)}\right)^{\frac{1}{1-\alpha\rho}}di\right)^{1-\alpha}}=1.$$

This yields a first-order condition that can be simplified as $\frac{r_{\tau,i}}{\tau_i^k}=\Xi$, where Ξ is a constant across firms. Proposition 2 follows.

Corollary 2.1

With only two groups of firms, holding the weighted-average rate of interest fixed, output is only affected by heterogeneous changes in spreads through the term:

$$\sum_{j=1,2}\left(A_j^{\rho}\left(\frac{r_{\tau,j}}{\tau_j^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}, \tag{22}$$

where we define A_j such that $\left(A_j^{\rho}\right)^{\frac{1}{1-\alpha\rho}} = \int_{i\in j}\left(A_i^{\rho}\right)^{\frac{1}{1-\alpha\rho}} di$. Given interest rate wedges, $r_{\tau,j}$, are defined between the interest rates facing firms and the weighted average interest rate, we have clearing condition:

$$\frac{\left(\sum_{j=1,2}\left(\frac{A_j^{\rho}}{\left(\tau_j^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,j}\right)^{\frac{\alpha\rho}{1-\alpha\rho}}\right)^{\frac{1-\rho}{\rho}}\left(\sum_{j=1,2}\left(\frac{A_j^{\rho}}{\tau_j^k}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,j}\right)^{\frac{1}{1-\alpha\rho}}\right)^{1-\alpha}}{\left(\sum_{j=1,2}\left(\frac{A_j^{\rho}}{\left(\tau_j^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}}\right)^{\frac{1-\rho}{\rho}}\left(\sum_{j=1,2}\left(\frac{A_j^{\rho}}{\left(\tau_j^k\right)}\right)^{\frac{1}{1-\alpha\rho}}\right)^{1-\alpha}} = 0. \tag{23}$$

From (23), a shock that increases $r_{\tau,1}$ thus decreases $r_{\tau,2}$. Taking the derivative of (22) with respect to $r_{\tau,1}$, $\frac{\partial \sum_{j=1,2}\left(A_j^{\rho}\left(\frac{r_{\tau,j}}{\tau_j^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}}{\partial r_{\tau,1}}$, yields:

$$\frac{\left(\frac{r_{\tau,2}}{\tau_2^k}-\frac{r_{\tau,1}}{\tau_1^k}\right)}{\left(\left(\frac{\alpha(1-\rho)}{1-\alpha}\right)\frac{\left(\sum_j\left(\frac{A_j^{\rho}}{\tau_j^k}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,j}\right)^{\frac{1}{1-\alpha\rho}}\right)}{\sum_j\left(\frac{A_j^{\rho}}{\left(\tau_j^k\right)^{\alpha\rho}}\right)^{\frac{1}{1-\alpha\rho}}\left(r_{\tau,j}\right)^{\frac{\alpha\rho}{1-\alpha\rho}}}+\left(\frac{r_{\tau,2}}{\tau_2^k}\right)\right)}. \tag{24}$$

The denominator of (24) is always positive, thus the sign of (24) is controlled by the numerator. If the level of the interest rate wedge facing firms of type 1 is above (below) its optimal value $r^{\star}_{\tau,1}$, then Proposition 2 together with (23) imply that $\left(\frac{r_{\tau,2}}{\tau_2^k}-\frac{r_{\tau,1}}{\tau_1^k}\right)<0$ (>0). Corollary 2.1 follows.

B Solution to the Problem of the Bank

If the bank chooses $S_{b,1,t}$, $S_{b,2,t}$, and $B_{b,t}$ to maximize (19) subject to (17), (18), and (20), the Lagrangian is

$$\begin{aligned}\mathcal{L} &= E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)N_{t+1}+\sigma V_{t+1}\right)\right]+\lambda_t\left(E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)N_{t+1}+\sigma V_{t+1}\right)\right]\right.\\ &\quad \left.-\theta Q_t S_{b,1,t}^{\nu_{s,1}}-\theta\Delta_s Q_t S_{b,2,t}^{\nu_{s,2}}-\Delta\theta q_t B_{b,t}^{\nu_b}\right),\end{aligned}$$

where

$$\begin{aligned}N_t &= R_{k,1,t}Q_{t-1}S_{b,1,t-1}+R_{k,2,t}Q_{t-1}S_{b,2,t-1}+\\ &\quad R_{b,t}q_{t-1}B_{b,t-1}-R_t\left(Q_{t-1}S_{b,1,t-1}+Q_{t-1}S_{b,2,t-1}+q_{t-1}B_{b,t-1}-N_{t-1}\right).\end{aligned}$$

Let λ_t be the Lagrange multiplier associated with the incentive constraint (20).

The first-order conditions here yield

$$E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)+\sigma\frac{\partial V_{t+1}}{\partial N_{t+1}}\right)\left(R_{k,1,t+1}-R_{t+1}\right)\right]=\frac{\lambda_t}{\left(1+\lambda_t\right)}\theta\nu_{s,1}S_{b,1,t}^{\nu_{s,1}-1},$$

$$E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)+\sigma\frac{\partial V_{t+1}}{\partial N_{t+1}}\right)\left(R_{k,2,t+1}-R_{t+1}\right)\right]=\frac{\lambda_t}{\left(1+\lambda_t\right)}\Delta_s\theta\nu_{s,2}S_{b,2,t}^{\nu_{s,2}-1},$$

and

$$E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)+\sigma\frac{\partial V_{t+1}}{\partial N_{t+1}}\right)\left(R_{b,t+1}-R_{t+1}\right)\right]=\frac{\lambda_t}{\left(1+\lambda_t\right)}\Delta\theta\nu_b B_{b,t}^{\nu_b-1},$$

noting that

$$\frac{\partial V_t}{\partial N_t} = E_t \tilde{\Lambda}_{t,t+1} \bigg((R_{k,t+1} - R_{t+1})\phi_t + R_{t+1} \bigg), \tag{25}$$

where

$$\phi_t = \frac{E_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1} \right]}{\theta \nu_{s,1} S_{b,1,t}^{\nu_{s,1}-1} - E_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{k,1,t+1} - R_{t+1} \right) \right]}.$$

Let $\tilde{\Lambda}_{t,t+1}$ be the bank's augmented stochastic discount factor, equal to the product of $\Lambda_{t,t+i}$, that is, the discount factor from the household's problem as defined in (16) and the multiplier $\bigg(\bigg(1 - \sigma \bigg) + \sigma \frac{\partial V_{t+1}}{\partial N_{t+1}} \bigg)$.

Thus, we have the following arbitrage conditions:

$$E_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{b,t+1} - R_{t+1} \right) \right] = \Delta \frac{\nu_b B_{b,t}^{\nu_b - 1}}{\nu_{s,1} S_{b,1,t}^{\nu_{s,1}-1}} E_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{k,1,t+1} - R_{t+1} \right) \right],$$

and

$$E_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{k,2,t+1} - R_{t+1} \right) \right] = \Delta_s \frac{\nu_{s,2} S_{b,2,t}^{\nu_{s,2}-1}}{\nu_{s,1} S_{b,1,t}^{\nu_{s,1}-1}} E_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{k,1,t+1} - R_{t+1} \right) \right].$$

From combining (20) and (25), we obtain the following leverage restriction:

$$N_t \phi_t \geq \frac{Q_t S_{b,1,t}}{\nu_{s,1}} + \Delta_s Q_{2,t} \frac{S_{b,2,t}^{\nu_{s,2}}}{\nu_{s,1} S_{b,1,t}^{\nu_{s,1}-1}} + \Delta q_t \frac{B_{b,t}^{\nu_b}}{\nu_{s,1} S_{b,1,t}^{\nu_{s,1}-1}},$$

which is an inequality when $\lambda_t = 0$ and binds when $\lambda_t > 0$. We can also derive the law of motion for total net worth of all bankers as

$$N_t = \sigma \bigg(\sum_{j=1}^{j=2} ((R_{k,j,t} - R_t) \frac{Q_{j,t-1} S_{b,j,t-1}}{N_{t-1}}) + (R_{b,t} - R_t) \frac{q_{t-1} B_{b,t-1}}{N_{t-1}} \bigg) N_{t-1} + N_e,$$

where N_e is the wealth of entering bankers.

A Tables and Figures

Parameters	Value
From Gertler and Karadi (2013)	
Households	
Discount rate, β	0.995
Habit parameter, h	0.815
Relative utility weight of labor, χ	3.482
Steady-state Treasury supply, B/Y	0.450
Proportion of long-term Treasury holdings of the households, $\bar{B}^h/B$	0.750
Portfolio adjustment cost, κ	1.000
Inverse Frisch elasticity of labor supply, φ	0.276
Financial Intermediaries and Households	
Fraction of capital that can be diverted, θ	0.345
Proportional advantage in seizure rate of government debt, Δ	0.500
Transfer to the entering bankers, X	0.0062
Survival rate of the bankers, σ	0.92
Intermediate Good Firms	
Capital share, α	0.330
Depreciation rate, δ	0.025
Capital-Producing Firms	
Inverse elasticity of net investment to the price of capital, η_i	1.728
Retail Firms	
Elasticity of substitution, ϵ	4.167
Probability of keeping the price constant, γ	0.779
Government	
Steady-state proportion of government expenditures, G/Y	0.200
Inflation coefficient in the Taylor rule, κ_π	1.500
Markup coefficient in the Taylor rule, κ_X	-0.125
New Parameters	
Financial Intermediaries and Households	
Regulatory constraint parameter on government debt, ν_b	1.000
Regulatory constraint parameter on type 1 securities, $\nu_{s,1}$	1.200
Regulatory constraint parameter on type 2 securities, $\nu_{s,2}$	1.200
$\Delta_{s,2}$,	1.0531
$K_{1,I}/K_1$,	0.8
$K_{2,I}/K_2$,	0.2
$\bar{K_{h,1}}/K_1$,	0.1
$\bar{K_{h,2}}/K_2$,	0.1
$\kappa_{s,1}$,	1
$\kappa_{s,2}$,	1
Intermediate Good Firms	
CES parameter, ρ	0.9
Type 1 labor share in production	0.5
Type 2 labor share in production	0.5

Table 1: Parameters

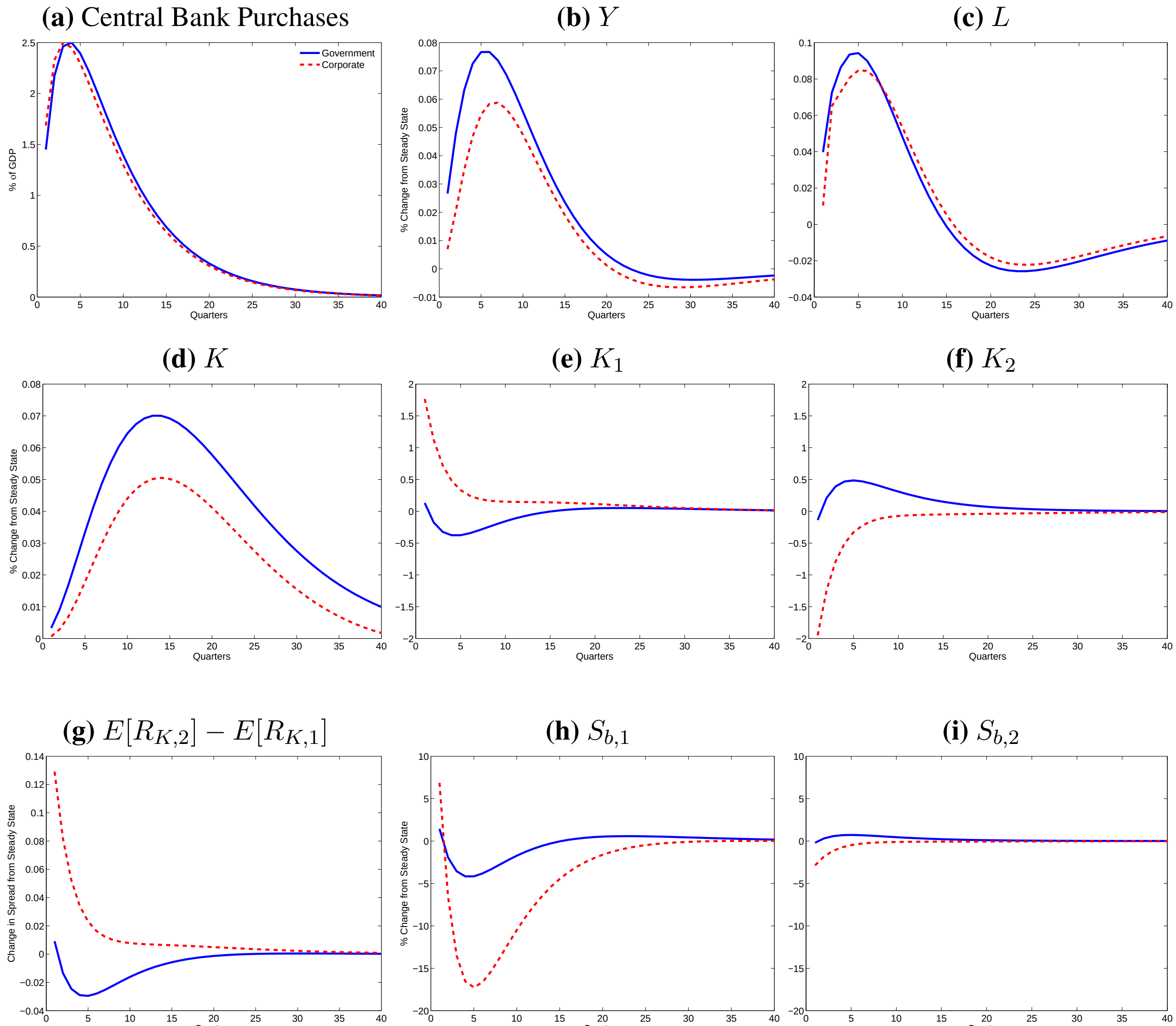

Figure 1: Government and Private Sector Asset Purchase Shocks

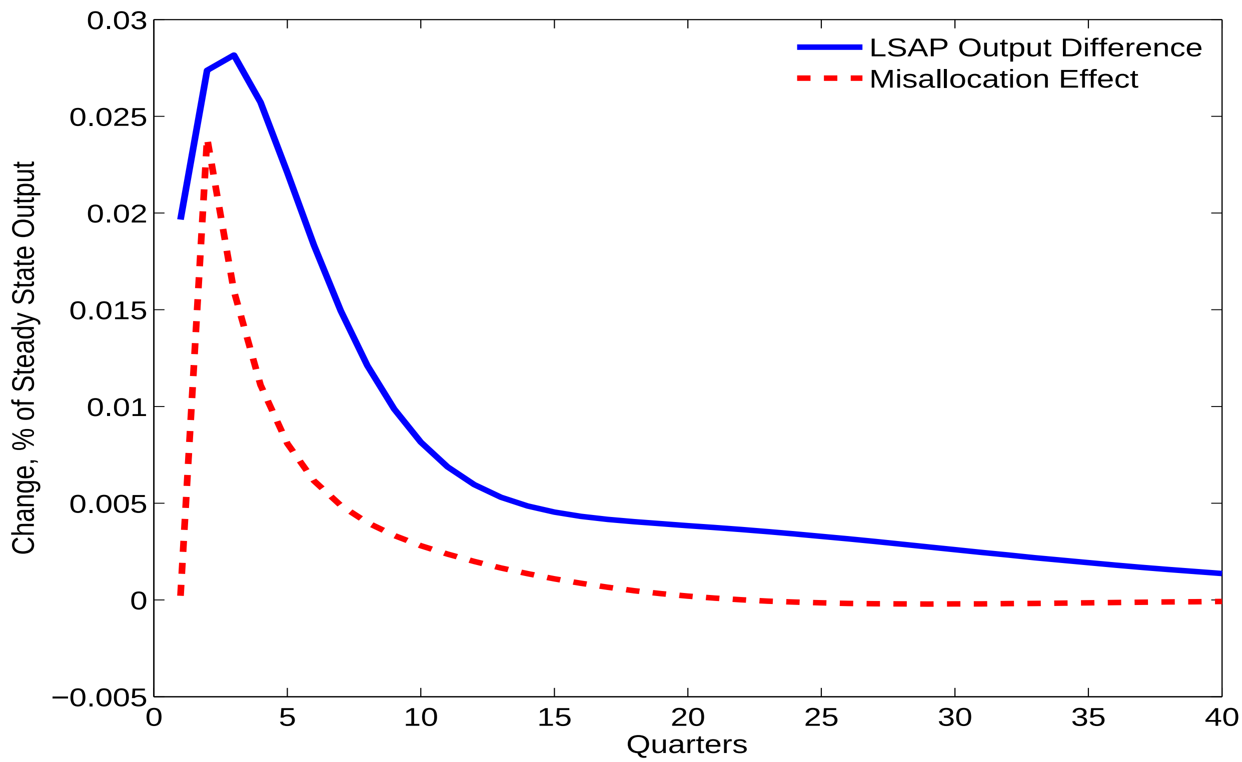

Figure 2: Misallocation Effect and Difference in LSAP Effectiveness

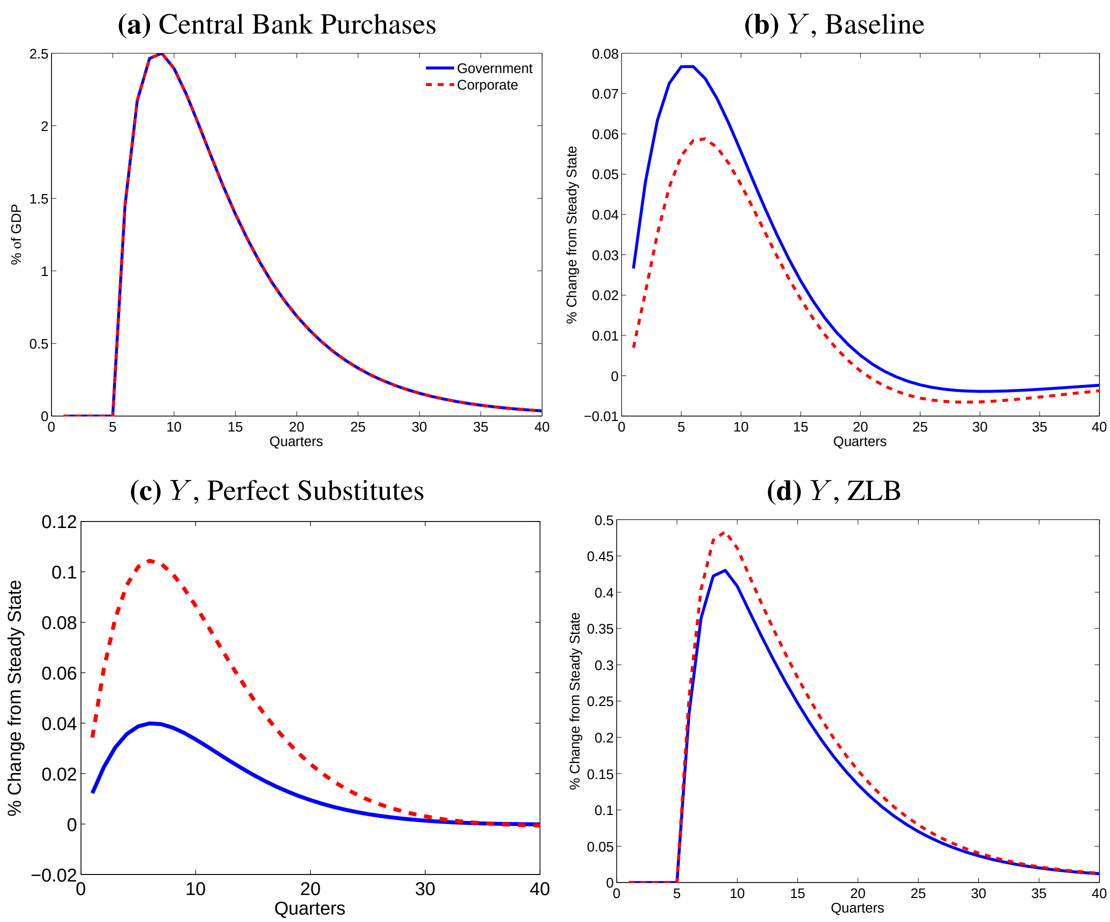

Figure 3: Effect of LSAPs on Output with Perfect Substitutes or ZLB

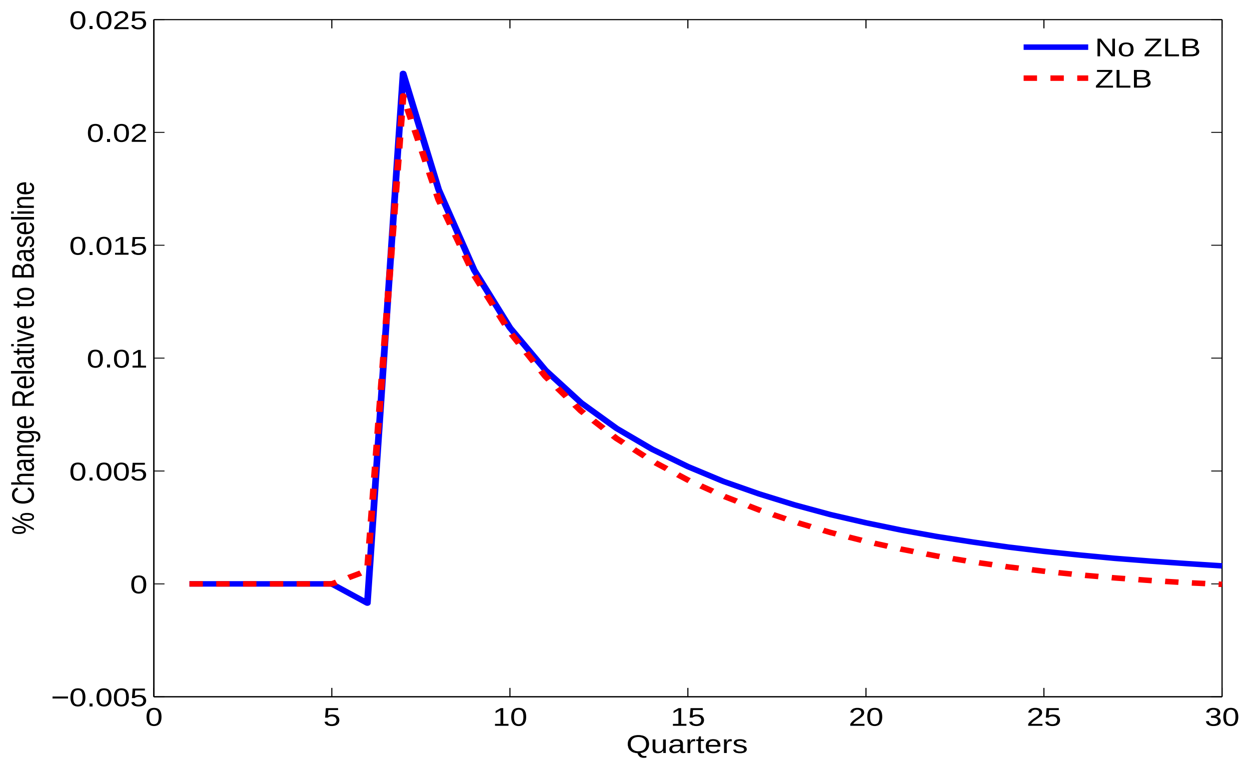

Figure 4: Misallocation Effect of Large-Scale Corporate Bond Buy with and without the ZLB