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Employment, Wages and Optimal Monetary Policy

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Abstract

We study optimal monetary policy when the empirical evidence leaves the policymaker uncertain whether the true data-generating process is given by a model with sticky wages or a model with search and matching frictions in the labor market. Unless the policymaker is almost certain about the search and matching model being the correct data-generating process, the policymaker chooses to stabilize wage inflation at the expense of price inflation, a policy resembling the policy that is optimal in the sticky wage model, regardless of the true model. This finding reflects the greater sensitivity of welfare losses to deviations from the model-specific optimal policy in the sticky wage model. Thus, uncertainty about important aspects of the structure of the economy does not necessarily translate into uncertainty about the features of good monetary policy.

JEL classifications: E52
Keywords: optimal monetary policy, optimal targeting rules, search and matching, sticky wages, model uncertainty

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1 Introduction

Macroeconomic models built to capture the cyclical properties of employment and wages often feature either nominal wage rigidities or search and matching frictions in the labor market. We study optimal monetary policy when the policymaker is uncertain which of these two approaches is the correct data-generating process. Unless the policymaker is almost certain about the search and matching model being the correct data-generating process, the policymaker chooses to stabilize wage inflation at the expense of price inflation, a policy resembling the policy that is optimal in the sticky wage model, regardless of the true model. This finding reflects the greater sensitivity of welfare losses to deviations from the model-specific optimal policy in the sticky wage model.

Our analysis features two New Keynesian (NK) models that are identical except for the details of the labor market. Following Calvo (1983), prices are sticky as retail firms sell differentiated goods that are priced using staggered contracts. In the sticky wage model, we assume that in addition wages for differentiated labor varieties are set in a staggered fashion, see Erceg, Henderson, and Levin (2000). The empirical NK literature has largely relied on similar settings to generate empirically plausible labor market dynamics in monetary models.[1]

In sharp contrast to the sticky wage model, nominal wages are flexible in the search and matching model. Although common in other areas of macroeconomics, the models with search and matching frictions in the labor market pioneered by Diamond (1982), Mortensen (1982), and Pissarides (1985) are rarely considered in monetary economics.[2] Wholesale firms post vacancies and workers search for jobs. When a firm and a worker are matched, they (Nash) bargain over the terms of employment (wages and hours worked). The details follow Faia (2009) and Ravenna and Walsh (2011) with the important difference that individual hours worked of an employed worker are elastic in our setup to maintain comparability with the sticky wage model. Furthermore, by modeling explicitly the opportunity costs of employment, we improve the model's ability to capture the empirical patterns of unemployment and vacancies.[3]

[1] Fine illustrations of this approach are Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).

[2] Notable exceptions are Krause and Lubik (2007), Ravenna and Walsh (2008), and Christiano, Eichenbaum, and Trabandt (2013). An approach combining the search and matching framework with staggered multi-period wage contracts is due to Gertler and Trigari (2009).

[3] Shimer (2005) argues that search and matching models cannot generate labor market movements that are in line with the empirical evidence for plausible parameter choices—a view subsequently challenged by other authors. Our approach builds on Hagedorn and Manovskii (2008), but models the opportunity costs of employment explicitly.

At the core of our analysis lies the idea that policymakers can formulate multiple models that provide a good approximation to the true data-generating process given the available empirical evidence against which these models are assessed. We implement this view in our setup by showing that both the sticky wage and the search and matching model can generate dynamics that are in line with evidence from structural vector autoregressions for reasonable parameter values under the assumption that monetary policy follows an estimated interest rate rule.[4] Putting other empirical evidence aside, this exercise leaves the policymaker with two plausible descriptions of the data-generating process and uncertainty about the true data-generating process.

Without being able to settle on a unique model of the economy, conducting optimal monetary policy is complicated by the fact that the two models have vastly different normative implications. For each model, we consider the optimal monetary policy under commitment from the timeless perspective when the policymaker's preferences coincide with those of the representative household as in Woodford (1999). In the model with search and matching frictions, the optimal policy keeps price inflation under tight control while nominal wages display large movements. Although the search and matching process leads to inefficient allocations in our setting, monetary policy cannot correct the underlying distortions in the labor market. Thus, the optimal monetary policy addresses exclusively the dynamic distortions stemming from sticky prices in the product market. Low inflation reduces the differences in relative prices across product varieties and the associated inefficient shifts in relative demand. The degree of inflation stabilization is only constrained by the possible trade-off between inflation and resource utilization (as measured by the output gap). By contrast, in the NK model with sticky nominal wages, the optimal policy needs to strike a balance between price and wage inflation. Similar to the product market, wage inflation distorts relative real wages and labor demand; price inflation supports the adjustment of real wages under staggered nominal wages. The near complete stabilization of wages reflects the high welfare costs associated with even minor relative wage differences in empirical sticky wage models.

Given these normative differences between the two models, how important is it for the policymaker to know which one represents the true data-generating process? Using the concept of optimal targeting rules as in Giannoni and Woodford (2016), we show that transplanting the optimal targeting rule from one model into the other results in welfare

[4] Using the formulation of the search and matching model in Hall and Milgrom (2008), Christiano, Eichenbaum, and Trabandt (2013) report a similar finding.

losses that are orders of magnitudes larger than the welfare costs of business cycles in Lucas (2003).[5] The lack of robustness of the optimal targeting rules is far from symmetric. The optimal targeting rule derived from the search and matching model stabilizes price inflation and induces excessive movements in wage inflation when applied to the sticky wage model; the resulting welfare costs are ten times larger than in the opposite case and reflect the high welfare costs of relative wage differences in empirical sticky wage models. Excessive stabilization of wage inflation in the search and matching model under the targeting rule that is optimal in the sticky wage model induces relatively small welfare losses.

The lack of robustness of the optimal targeting rules makes it unattractive to resolve model uncertainty via standard model selection exercises prior to the evaluation of monetary policy. Instead of opting for a specific model based on inconclusive empirical evidence we allow the policymaker to incorporate model uncertainty as a component in the evaluation of policy as advocated in Brock, Durlauf, and West (2007). We show that in this case, the policymaker selects a policy that resembles the optimal targeting rule *derived in the sticky nominal wage model* unless the policymaker is very certain about the search and matching model being the correct data-generating process.

Building on the ideas developed in Levin, Wieland, and Williams (2003), we arrive at this conclusion under two approaches of deriving the optimal monetary policy under model uncertainty. Under the model averaging approach, the policymaker chooses a policy—implemented through an interest rate or a targeting rule—that minimizes the expected loss for a given probability distribution of the policymaker over the relevant reference models. When the policymaker adopts a minmax strategy, the policy minimizes the maximum expected loss. This approach does not require the policymaker to specify a probability distribution over models. Reflecting the lack of robustness of policies that are (close to) optimal in the search and matching model, the optimal policy under model uncertainty mimics the optimal targeting rule derived in the sticky wage model under both approaches and stabilizes wage inflation at the expense of price inflation, unless the policymaker attaches a low (or, in the case of the minmax strategy, zero) probability on the sticky wage model being the true model. Thus, uncertainty about the true model does not necessarily translate into uncertainty about the features of good monetary policy.

[5] The optimal targeting rule specifies the variables—including the relative importance and the dynamic structure of each variable—in a single target criterion that seeks to implement the optimal monetary policy. In other words, the optimal targeting rule is a commitment to a certain relationship between the model variables.

Throughout our analysis, we assume that the policymaker adopts preferences over economic outcomes that are consistent with those of the households in the reference models. If in departure from the microeconomic foundations of the models, the policymaker's preferences do not reflect those of the households it is possible to derive policies that the policymaker deems (close to) optimal for both models. However, such policies are not robust from the perspective of households. Assigning arbitrary preferences to the policymaker as often the case in the literature is not innocuous.

The remainder of the paper proceeds as follows. Section 2 discusses related literature. We present the NK models with search and matching frictions and sticky nominal wages, respectively, in Section 3. In Section 4, we discuss the details of our empirical strategy to parameterize the two models. Section 5 derives optimal targeting rules for each model and assesses their robustness across models. Optimal policy under model uncertainty is discussed in Section 6. Concluding remarks are offered in Section 7.

2 Related literature

Our approach is closest to Levin and Williams (2003) and Levin, Wieland, and Williams (2003) which also study robust monetary policy with competing reference models. Other related papers include Cogley and Sargent (2005) and Svensson and Williams (2005), but we abstract from the learning dynamics featured in these contributions. In sync with our conclusions, these works recommend policymakers not to tailor policies towards a model with recommendations that are not robust to model misspecification and uncertainty even if the model is considered quite likely to be (close to) the correct data-generating process.[6]

Yet, our analysis differs from all these contributions along important dimensions. First, we restrict attention to microfounded models and exclude macro-econometric models from the set of reference models. Thus, we can consider objective functions of the policymaker that are consistent with the preferences of the economic agents in the underlying reference models and that reflect the policymaker's probability distribution over the models. The aforementioned contributions assume that the policymaker's preferences are independent of the reference models, an approach we show to sometimes falsely suggest the existence

[6] Research on model uncertainty and policy evaluation has taken several directions. One direction is to assume a given baseline model and consider all models within a given distance as in Hansen and Sargent (2007), Tetlow and von zur Muehlen (2001), and Giannoni (2002). The second approach, taken in this paper, does not require the models to be close to each other. Another recent example of this approach is Taylor and Wieland (2012). In addition to model uncertainty, data uncertainty and parameter uncertainty are other areas of concern for policymakers.

of robust policies. Second, we parameterize the models to fit the same empirical evidence under empirical interest rate rules *before* deriving the optimal monetary policy. In Cogley and Sargent (2005) and Svensson and Williams (2005), model parameters are estimated conditional on the policymaker setting policy to maximize a given quadratic objective; no two models fit the data equally well over a given historical episode in their works and the ranking of the models according to the quality of fit switches between episodes. In Levin and Williams (2003) and Levin, Wieland, and Williams (2003) the models are not parameterized using the same empirical evidence. Third, labor market aspects are at the core of our analysis and we stress the importance of smoothing wage inflation at the expense of price inflation as a general principle of robust optimal monetary policy. These considerations are ruled out in the earlier contributions by the choice of models. Sensitivity analysis suggests that our results survive if the policymaker's preferences resemble those of earlier studies and are common across models as long as the policymaker has sufficient dislike for price inflation.

3 Two competing models of the labor market

The two reference models of the policymaker build on the New Keynesian (NK) model with sticky nominal prices; the models differ with regard to the details of the labor market. The first model features search and matching frictions in the labor market as in Diamond (1982), Mortensen (1982), and Pissarides (1985). Each worker negotiates the terms of employment with the matched firm. While the real wage may adjust slowly to shocks, the nominal wage is fully flexible. By contrast, the second model introduces sticky nominal wages as in Erceg, Henderson, and Levin (2000). Unlike NK models with Walrasian labor markets, these two models fit well the impulse responses of labor market variables derived from structural vector auto-regressions (SVAR) for reasonable parameter choices. We provide brief model descriptions in the main text and refer to Appendix A for details.[7]

3.1 NK model with search and matching frictions

Households are modeled as in Andolfatto (1996) and Merz (1995). At any point in time n_t agents of the household are employed (w) and $1 - n_t$ agents are unemployed (u).

[7] We chose not to include the model by Gertler and Trigari (2009) which merges the ideas of the search and matching framework with those of nominal rigidities in wage setting. As shown in Thomas (2008), the optimal policy recommendations derived from this hybrid framework resemble those of the sticky wage model.

As in Walsh (2005) and Christiano, Eichenbaum, and Trabandt (2013), we assume that each household member has the same concave preferences over consumption and that the household provides perfect consumption insurance. The household maximizes the inter-temporal utility of the members

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - \mu c_{t-1})^{1-\sigma}}{1-\sigma} - n_t \phi_0 \frac{(h_t)^{1+\phi}}{1+\phi} \right] \tag{1}$$

subject to the budget constraint

$$c_t + \frac{B_{t+1}}{P_t} \leqslant [w_t h_t n_t + b^u(1-n_t)] + \frac{\mathrm{Pr}_t}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1}B_t}{P_t}. \tag{2}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is the time discount factor. Consumption is denoted by c_t, and the hours worked by the n_t employed household members are measured by h_t. Unemployed household members do not experience disutility from working. The real wage is given by w_t and unemployment benefits are measured by b^u. Bond holdings B_t, taxes and transfers T_t, and profits Pr_t are measured in nominal terms and are converted into real units through division by the price level P_t. R_t is the nominal interest rate on bonds. We denote by λ_t the Lagrange multiplier attached to the budget constraint when solving the household's problem. As in Walsh (2005) we assume that total consumption c_t consists of a manufactured good c_t^m and home production $b^u(1-n_t)$, i.e., $c_t = c_t^m + b^u(1-n_t)$. This assumption guarantees that it is in principle possible under the conditions in Hosios (1990) for the outcomes of the search and matching process to be efficient.[8]

The labor market features search and matching frictions. Firms post vacancies v_t. The share of agents searching for jobs is measured by u_t. New matches m_t between firms and agents are formed according to the matching function

$$m_t = \chi u_t^{\zeta} v_t^{1-\zeta} \tag{3}$$

[8] If unemployment benefits are modeled as tax-financed, imposing the conditions in Hosios (1990) is not sufficient to achieve efficiency for $b^u > 0$. The exact way of modeling unemployment benefits is of little consequence for us as for empirical reasons we are not interested in parameterizations that satisfy the conditions in Hosios (1990). However, the modeling choice matters in our companion paper Bodenstein and Zhao (2016) from which we draw in this paper.

while employment n_t evolves according to

$$n_t = (1-\rho)\, n_{t-1} + m_t \tag{4}$$

where ρ is the exogenous rate at which existing matches break up. The number of job seekers in period t follows

$$u_t = 1 - n_{t-1} + \rho n_{t-1} = 1 - (1-\rho)\, n_{t-1}. \tag{5}$$

Wholesale firms employ labor to produce the good y_t^w which is sold at the competitive market price P_t^w. To hire workers, wholesale firms have to first post a vacancy at the cost κ^v. These firms maximize profits subject to the law of motion for employment and the production technology

$$\begin{aligned} &\max_{\{n_t, y_t^w, v_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(\frac{P_t^w}{P_t} y_t^w - \frac{W_t}{P_t} n_t h_t - \kappa^v v_t \right) \\ &s.t. \quad n_t = (1-\rho)\, n_{t-1} + q_t v_t \\ &\qquad\quad y_t^w = a_t n_t h_t \end{aligned} \tag{6}$$

where firms take the probability of filling an open vacancy $q_t = \frac{m_t}{v_t}$ as given. Total factor productivity a_t follows a standard AR(1) process

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_t^a \tag{7}$$

with normally distributed innovations $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$.

When an agent and a firm are matched, they engage in Nash bargaining over wages and hours worked. The solution to the bargaining problem is obtained from

$$\max_{w_t, h_t} J_t^{1-\xi} H_t^{\xi} \tag{8}$$

where ξ stands for the bargaining power of the worker. The marginal value of employment to the firm J_t is given by the period profit of the additional worker, i.e., the excess of the marginal product over the real wage payment, plus the continuation value if the match survives into the next period

$$J_t = \left(\frac{P_t^w}{P_t} a_t - \frac{W_t}{P_t} \right) h_t + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \tag{9}$$

The marginal value of employment to the household H_t satisfies

$$H_t = \left(\frac{W_t}{P_t} h_t - b^u - \frac{\phi_0}{1+\phi} \frac{h_t^{1+\phi}}{\lambda_t} \right) + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1})\, H_{t+1} \tag{10}$$

and consists of the increase in household income $\frac{W_t}{P_t} h_t - b^u$ of having an additional household member employed over the monetary equivalent to compensate the now employed member for the loss of leisure time $\frac{\phi_0}{1+\phi} \frac{h_t^{1+\phi}}{\lambda_t}$ as well as the continuation value if the match survives into the next period.

Retail prices experience nominal rigidities. Retail firms produce differentiated goods using wholesale goods as the sole input. The optimization problem of retail firm i consists of two parts. The cost minimization problem

$$\begin{aligned} &\min_{y_t^w(i), y_t(i)} P_t^w y_t^w(i) \\ &s.t. \quad y_t(i) = y_t^w(i). \end{aligned} \tag{11}$$

delivers an expression for the retailer's real marginal costs mc_t

$$mc_t = \frac{P_t^w}{P_t}. \tag{12}$$

Retailer i adjusts its price $P_t(i)$ each period with the fixed probability $1 - \xi^p$. For firms that do not re-optimize their price in a given period, prices will be updated as a weighted average of $\Pi_t = \frac{P_t}{P_{t-1}}$, the nominal price inflation in the previous period, and $\bar{\Pi}$, the steady state inflation rate

$$P_{t+1}(i) = \tilde{P}_t(i) \left(\Pi_t^{\iota_p} \bar{\Pi}^{1-\iota_p} \right). \tag{13}$$

Retail firm i sets its price to maximize

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left((1+\bar{\tau}^p) \tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right) - MC_{t+s} \right) \right] y_{t+s}(i)$$

$$s.t. \quad y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right)}{P_{t+s}} \right)^{-\frac{\lambda^p}{\lambda^p - 1}} y_{t+s} \tag{14}$$

where the subsidy $\bar{\tau}^p$ offsets distortions due to monopolistic competition in the steady state. We introduce a markup shock directly into the first order condition of the retailer which under a linear approximation of the model is equivalent to variations in $\bar{\tau}^p$ or λ^p. In choosing its price, the firm takes into account the demand curve for its differentiated good. This demand curve is derived from the profit maximization problem of the producers of the final composite consumption good y_t which is produced from the differentiated goods according to

$$y_t = \left[\int_0^1 y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p} . \tag{15}$$

The term $\frac{\lambda^p}{\lambda^p - 1}$ refers to the elasticity of substitution between the retail varieties. y_t is used for consumption c_t^m and to cover the costs of posting vacancies v_t.

3.2 NK model with sticky nominal wages

The model with sticky nominal wages differs from the search and matching model with regard to the labor market details. In this model, all household members are employed and nominal wages are set in staggered contracts following Calvo (1983). Each household j chooses consumption and asset holdings by maximizing the inter-temporal utility function

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{(c_t(j) - \mu c_{t-1}(j))^{1-\sigma}}{1-\sigma} - \phi_0 \frac{h_t(j)^{1+\phi}}{1+\phi} \right] \tag{16}$$

subject to the budget constraint

$$P_t c_t(j) + B_{t+1}(j) = (1 + \bar{\tau}^w) W_t(j) h_t(j) + R_{t-1} B_t(j) + Pr_t(j) + T_t(j). \tag{17}$$

Labor bundlers package the differentiated labor services supplied by households and sell the aggregate labor service at the nominal wage W_t. The labor bundling technology satisfies

$$h_t = \left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}} \mathrm{d}j \right]^{\lambda^w} \tag{18}$$

where the term $\frac{\lambda^w}{\lambda^w - 1}$ measures the elasticity of substitution between differentiated labor services. The bundler's demand for variety j of labor services is given by

$$h_t(j) = \left[\frac{W_t(j)}{W_t} \right]^{-\frac{\lambda^w}{\lambda^w - 1}} h_t. \tag{19}$$

Each household j supplies differentiated labor services $h_t(j)$ and sets a wage rate under monopolistic competition. The household can readjust its wage with fixed probability $1 - \xi^w$ each period. If the household cannot reoptimize its wages, wages will increase by a weighted average of past inflation and the steady state inflation rate according to

$$W_{t+1}(j) = \tilde{W}_t(j) \left(\Pi_t^{\iota^\omega} \bar{\Pi}^{(1-\iota^\omega)} \right). \tag{20}$$

A reoptimizing household chooses its wage as the solution to the following problem

$$\begin{aligned}
& \max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^w \beta)^s \left[\frac{(c_{t+s} - \mu c_{t-1+s})^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_{t+s}(j)^{1+\phi} \right] \\
s.t. \quad & P_{t+s} c_{t+s} + B_{t+s+1} = (1 + \bar{\tau}^w) W_{t+s}(j) h_{t+s}(j) + R_{t+s-1} B_{t+s} + Pr_{t+s} + T_{t+s} \\
& h_{t+s}(j) = \left(\frac{W_{t+s}(j)}{W_{t+s}} \right)^{-\dfrac{\lambda^w}{\lambda^w - 1}} h_{t+s} \\
& W_{t+s}(j) = \tilde{W}_t(j) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^w} \bar{\Pi}^{1-\iota^w} \right)
\end{aligned} \tag{21}$$

where the subsidy $\bar{\tau}^w$ is set to eliminate the labor supply distortions arising from monopolistic competition in the steady state.

Wholesale firms purchase aggregate labor services h_t from the labor bundler. Retail firms purchase the wholesale good, differentiate it, and set prices using staggered contracts, just as in the NK model with search and matching frictions.

3.3 Linearized models

We briefly turn to the log-linear approximation of the two models around their respective steady states for the baseline specification without indexation of prices and wages ($\iota^p = \iota^\omega = 0$) and consumption habits ($\mu = 0$); we display the core equations only. The details for the search and matching model are provided in Appendix B. For the sticky wage model, we refer the reader to Erceg, Henderson, and Levin (2000).

Our linear search and matching model resembles the one in Ravenna and Walsh (2011) with two important exceptions: (i) the steady state is inefficient as we do not impose the conditions stated in Hosios (1990), (ii) the individual labor supply is elastic. Under the first assumption, the flexible price economy is not efficient which in turn complicates finding the second order approximation to the preferences of the representative household conducted later. The second assumption yields an additional equation not present in the linearized model of Ravenna and Walsh (2011).

The linearized search and matching model reduces to three equations (excluding a description of monetary policy) in four endogenous variables

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^p \left[\left(\phi + \sigma \frac{\varpi^{y_{ss}}}{1-\kappa^c} \right) \hat{y}_t - (1+\phi)\, \hat{a}_t - (\theta_1 \hat{n}_t + \theta_2 \hat{n}_{t-1}) \right] + \hat{\theta}_{p,t} \quad (22)$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1-\kappa^c}{\sigma \varpi^{y_{ss}}} \left(i_t - E_t \pi_{t+1} + (\theta_1 - \phi)\left(E_t \hat{n}_{t+1} - \hat{n}_t\right) + \theta_2 \left(\hat{n}_t - \hat{n}_{t-1}\right) \right) \quad (23)$$

$$\gamma_1 E_t \hat{n}_{t+1} + \gamma_2 \hat{n}_t + \gamma_3 \hat{n}_{t-1} = \left(1 + \phi + \frac{\sigma \varpi^{y_{ss}}}{1-\kappa^c} \right) \hat{y}_t - (1+\phi)\, \hat{a}_t$$
$$- \frac{(1-\rho)\beta}{\nu} \left(1 - \xi q_{ss} \theta_{ss}\right) \left(\frac{\kappa^v}{q_{ss}} \right) \left(i_t - E_t \pi_{t+1} \right). \quad (24)$$

Price inflation π_t, output $\hat{y}_t$, employment $\hat{n}_t$, and the nominal interest rate i_t are expressed in deviations from their steady state values; "hatted" variables are in log-deviations. The exogenous shocks to technology ($\hat{a}_t$) and markups ($\hat{\theta}_{p,t}$) follow standard AR(1) processes. Equations (22) and (23) are the NK Phillips curve and the aggregate demand relationship, respectively. In contrast to standard NK models without search and matching frictions in the labor market, the level of employment $\hat{n}_t$ enters these equations. The third equation, which can be traced back to the Nash bargaining over real wages relates the evolution of employment to the other variables.

This model reduces to the standard NK model if each household member is employed at every point in time which, among other assumptions, requires that vacancy posting costs are set to zero. Absent posting costs, κ^c assumes the value of 0, $\varpi^{y_{ss}} = 1$, and $\hat{n}_t = 0$ for all t. Equation (24) is dropped due to the lack of wage bargaining. Alternatively, the model in Ravenna and Walsh (2011) with inelastic individual labor supply emerges in the limit as ϕ approaches infinity implying $\lim_{\phi\to\infty}\theta_1 = \phi$ and $\lim_{\phi\to\infty}\gamma_2 = \phi$. Equation (24) converges to $\hat{n}_t = \hat{y}_t - \hat{a}_t$ which simply describes the production technology when hours worked are fixed. Using this result, equations (22) and (23) can be written in terms of inflation, the nominal interest rate, and employment. Notice, that the search and matching model with fixed hours worked does not nest the standard NK model.

The sticky nominal wage model features NK Phillips curves for prices and wages

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^p \left(\hat{w}_t - \hat{a}_t\right) + \hat{\theta}_{p,t} \tag{25}$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w \left(\sigma + \phi\right) \left(\hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t\right) - \kappa^w \left(\hat{w}_t - \hat{a}_t\right) \tag{26}$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) \tag{27}$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t \tag{28}$$

Equation (28) describes the evolution of the real wage. If wages are fully flexible, i.e., $\kappa^w \to \infty$, the model reduces to the standard NK model since equation (26) reduces to $\hat{w}_t - \hat{a}_t = (\sigma+\phi)\left(\hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t\right)$ and equation (25) can be written in its standard formulation.

Absent price indexation, inflation is a forward-looking phenomenon in both models and can be expressed as the discounted present value of real marginal costs. Although real wages are not the exclusive determinant of real marginal costs, the real wage dynamics shape the dynamics of real marginal costs importantly. Thus, the dynamics of price inflation depend on the adjustment process for real wages and in turn on the impact that price inflation has on the adjustment of real wages. Monetary policy influences the interplay between these variables.

4 Empirical Strategy

At the core of our analysis lies the idea that the policymaker can formulate multiple models that provide a good approximation to the true data-generating process given the available empirical evidence against which the models are assessed. The large variety of business cycle models found in the academic literature (that all try to explain similar aspects of the data) and the diverse set of models used within central banks in practice support this view.

To arrive at a setting in which the policymaker considers multiple models for the purpose of policymaking, we search for parameterizations of the two models introduced in the previous section that match the same empirical evidence. Given the number of free parameters in most theoretical models, it is basically impossible to reduce the set of candidate models to a single one and obtain model certainty. This section discusses the criteria by which we judge the empirical performance of the sticky wage model and the search and matching model.

4.1 Estimation

We estimate selected parameters of the two models using impulse response function matching introduced by Rotemberg and Woodford (1997). We focus on the case of the neutral technology shock; the empirical impulse responses are taken from Christiano, Eichenbaum, and Trabandt (2013).[9]

For each model, we divide the parameters into two groups: calibrated and estimated parameters. The values assigned to the first group of parameters are taken from the literature. The parameters in the second group shape the dynamics of the model importantly; with clear evidence hard to come by for these parameters, we allow the data to determine their values.

In this part of our analysis, monetary policy is assumed to follow a simple rule for the nominal interest rate as commonly found in the literature and in central bank analysis. In detail, we assume

$$i_t = \rho_R i_{t-1} + \rho_\pi \pi_t + \rho_x x_t \tag{29}$$

where π_t refers to price inflation in deviation from its long-run target value, and i_t denotes the short term nominal interest rate in deviation from steady state. The output gap is x_t. The coefficients ρ_R and ρ_π govern the degree of interest rate smoothing and the reaction of the nominal interest rate to current price inflation, respectively. In what follows, we

[9] Estimating a structural vector autoregressive (SVAR) model, Christiano, Eichenbaum, and Trabandt (2013) identify shocks to monetary policy, as well as neutral and investment-specific technology shocks. Our focus on the neutral technology shock is determined by certain features of our analysis: (i) we abstract from capital (and thus the investment-specific shock) to simplify the derivation of optimal targeting rules in the next section, (ii) monetary policy shocks of the kind identified in SVAR models play no role in our subsequent analysis of optimal monetary policy.

abstract from the output gap by setting $\rho_x = 0$. When we included the output gap in the estimation, our results hardly changed.

For each model, conditional on the calibrated parameters Γ^c, we search over the remaining parameters — collected in the vector Γ — to minimize the distance between the impulse response functions generated from the model, denoted by $G(\Gamma, \Gamma^c)^{model}$, and the empirical impulse response functions from the SVAR in Christiano, Eichenbaum, and Trabandt (2013), denoted by G:

$$\hat{\Gamma} = \arg\min_{\Gamma} \left(G - G(\Gamma, \Gamma^c)^{model}\right)' \left(\Psi^0\right)^{-1} \left(G - G(\Gamma, \Gamma^c)^{model}\right). \tag{30}$$

As customary, the diagonal weighting matrix Ψ^0 is obtained from the empirical variance-covariance matrix of the empirical impulse response functions Ψ by setting all off-diagonal elements in Ψ to zero. The estimate $\hat{\Gamma}$ minimizes the objective function in (30).

Before reporting the results of the estimation, we review the values assigned to some key parameters collected in Γ^c. The parameter values are recorded in Table 1. To the extent appropriate, we assign identical values to the parameters both models have in common. In particular, we set the labor supply elasticity equal to 1/2, implying $\phi = 2$, in line with the results reported by Smets and Wouters (2007). Hours worked are assumed to be 1/3 in the steady state. The parameter ξ^p which governs the degree of nominal price rigidities is fixed at 0.75. The markup for prices is set at 20 percent in the steady state implying λ^p equal to 1.2.

In the sticky wage model, we also need to specify the parameter governing the stickiness of nominal wages ξ^w and the steady state wage markup λ^w. As in Christiano, Eichenbaum, and Trabandt (2013), we set $\xi^w = \xi^p$ and $\lambda^w = \lambda^p$.

Parameters specific to the model with search and matching frictions are chosen as follows. The breakup probability of a match with $\rho = 0.1$ implies a quarterly separation rate of 10 percent which is in line with the estimate in Shimer (2005) of 3.4 percent per month. The parameter ζ in the matching function is set at 0.54, just in the range of plausible values between 0.5 and 0.7 reported in Pissarides and Petrongolo (2001). We target a vacancy filling rate in the steady state q_{ss} of 0.7 following Ramey, den Haan, and Watson (2000). The unemployment rate in the steady state is set at 0.055, the average US unemployment rate over the period from 1951Q1 to 2008Q4 reported by Christiano, Eichenbaum, and Trabandt (2013). We assume that the costs associated with posting and filling vacancies are proportional to the number of posted vacancies. Relative to output these costs amount to $\eta_s = 0.0066$.

We estimate the remaining parameters for each model by matching their impulse responses to a neutral technology shock to the corresponding SVAR estimates in Christiano, Eichenbaum, and Trabandt (2013). We include the first 15 periods after the shock. While we fix the persistence of the technology shock at $\rho_a = 0.9999$, we estimate the standard deviation of the shock σ_a. Furthermore, for each model we estimate the coefficients in the interest rate rule, ρ_R and ρ_π, the degree of internal consumption habits μ, and the degree of price indexation ι^p. In the search and matching model, we also estimate the replacement ratio r^u. Finally, we estimate two versions of the sticky wage model. In the first one we abstract from wage indexation, i.e., ι^ω is fixed at 0, and in the second one we estimate the degree of wage indexation. For the sticky wage model, the estimation includes the impulse response functions for output, inflation, the short-term interest rate as measured by the federal funds rate, hours worked, real wages, and consumption. In the case of the search and matching model, we also include the responses of the unemployment rate, vacancies, and the job finding rate.

Table 2 summarizes the estimates. Figure 1 shows the resulting impulse responses for the two models absent wage indexation in the sticky wage model and Figure 2 plots the responses when the degree of wage indexation is estimated. In both figures, the theoretical models match the empirical responses well. With the exception of hours worked, the model responses lie within the confidence bands of the empirical responses and the responses are reasonably close to the SVAR point estimates and to each other. The sticky wage model with indexation, estimated at $\iota^\omega = 1$, provides a better fit to the data than the model without wage indexation according to the value of our criterion function, see bottom of Table 2. Most of the difference in fit stems from the model's implications for hours worked. However, some authors have expressed skepticism regarding the presence of wage indexation in the data, see Levine, McAdam, and Pearlman (2012) and Christiano, Eichenbaum, and Trabandt (2013).

The estimates for the coefficients in the policy rule, the variance of the technology shock, price indexation, and consumption habits are almost identical across models. The estimated policy rule features a high degree of interest rate smoothing and the implied long-run response of the interest rate to inflation is just strong enough to satisfy the Taylor principle, e.g., for the search and matching model this value is $1.0003 = (1 - 0.8555)/0.1445$. By coincidence, the estimated simple interest rate rules are close to identical across models. When forcing the simple rule to coincide across models the estimates of the remaining parameters stayed stable. For all model specifications, price indexation is estimated to be zero. Overall, our estimates resemble those in Christiano, Eichenbaum, and Trabandt (2013) despite the greater simplicity of our models.[10]

4.2 The elasticity of labor market tightness

We also estimate the replacement ratio r^u in the search and matching model. At a value of 0.5345 our point estimate is well below the implausibly high estimate in Christiano, Eichenbaum, and Trabandt (2013) for the search and matching model with Nash bargaining. The subsequent discussion explains how this difference across models related to our decision of modeling the disutility from labor explicitly.

The responses of unemployment and vacancies are important dimensions to judge the performance of the search and matching model. The unemployment rate (and thus the number of job seekers u_t) drops significantly after rising initially and vacancies v_t increase strongly over the medium term. Both in the data and the model the directions and the magnitudes of these responses imply a strong response of labor market tightness (the ratio of unfilled vacancies to job seekers).

As shown in Appendix B.1, labor market tightness $\hat{\theta}_t$ (expressed in log deviation from steady state) is approximately proportional to (the log-deviations from steady state of) the marginal product of labor, hours worked, and real marginal costs in our model:

$$\begin{aligned} \hat{\theta}_t &= \hat{v}_t - \hat{u}_t \\ &\approx \frac{1}{\Upsilon} \frac{\frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss}}{\left[\left(\frac{\phi}{1+\phi} - r^u\right) mpl_{ss} h_{ss} mc_{ss} + r^u \left(1-(1-\rho)\beta\right) \frac{\kappa^v}{q_{ss}}\right]} \left(\widehat{mpl}_t + \hat{h}_t + \widehat{mc}_t\right) \end{aligned} \tag{31}$$

where

$$\Upsilon = \zeta + \frac{(1-\rho)\beta\xi q_{ss}\theta_{ss}\left(1-\zeta\right)}{\left[1-(1-\rho)\beta\left(1-\xi q_{ss}\theta_{ss}\right)\right]}. \tag{32}$$

[10] Christiano, Eichenbaum, and Trabandt (2013) include investment, capacity utilization, and the relative price of investment in the set of impulse responses and they require their models to also match the empirical responses to monetary policy and investment-specific technology shocks. Our estimate of no consumption habits contrasts with their estimate of μ lying between 0.7 and 0.8, a difference that stems from the inclusion of the monetary policy shock in Christiano et al. Their (recursively identified) monetary policy shock induces a hump-shaped response in consumption, a feature not shared by the neutral technology shock. Habit persistence is key to match the consumption response to the monetary policy shock. When we fixed μ at a strictly positive value the parameters reported in Table 2 changed marginally.

Υ lies in the interval $[\zeta, 1]$, where ζ is often set around 0.5 (in our case 0.54).

Abstracting from the disutility of working for employed workers (i.e., $\phi \rightarrow \infty$), Shimer (2005) argues that standard search and matching models cannot reproduce the strong response of labor market tightness relative to the movements in the marginal product of labor found in the empirical evidence for plausible parameter choices, in particular for the replacement ratio r^u. According to Shimer, a strongly pro-cyclical real wage dampens the responses of vacancies and unemployment resulting in a much muted response of labor market tightness vis-a-vis the data.[11]

Numerous authors have offered approaches to resolve this issue: Hall (2005) and Shimer (2005) propose real wage rigidities; Hagedorn and Manovskii (2008) argue in favor of high opportunity costs of employment; Hall and Milgrom (2008) suggest departures from Nash bargaining over wages; Petrosky-Nadeau and Wasmer (2013) analyze the role of financial frictions. Our framework avoids the criticism in Shimer (2005) by modeling the disutility from working explicitly building on the ideas in Hagedorn and Manovskii (2008). With a labor supply elasticity of 0.5, i.e., $\phi = 2$, the value for r^u required to match the empirical evidence on unemployment, vacancies, and labor market tightness drops from almost 1 to near 0.5.

4.3 Additional shocks

In addition to technology shocks, our model features markup shocks. Unfortunately, we are not aware of a broadly accepted scheme to identify markup shocks using SVAR analysis. We assume that each economy is subject to purely transitory markup shocks. The standard deviation of the markup shock is set at 0.0135 in the sticky wage model. The standard deviation of the markup shock in the search and matching model of 0.0104 minimizes the distance between the impulse responses for the markup shocks in the two models given the remaining parameters in Tables 1 and 2. The smaller value of the standard deviation in the search and matching model reflects the stronger impact of an equal-sized markup shock on output and inflation in the search and matching model compared to the sticky wage model.

An alternative approach to ours would employ full information estimation of each

[11] For our parameterization, the steady state values of the marginal product of labor mpl_{ss} and marginal costs mc_{ss} are 1, and hours worked h_{ss} are 1/3, implying $h_{ss} mpl_{ss} mc_{ss} = 1/3$. With the term $(1-(1-\rho)\beta)\frac{\kappa^v}{q_{ss}}$ assuming the value 0.0024, the elasticity of labor market tightness can be raised to its value in the data by choosing r^u sufficiently close to 1. In a setting similar to ours, Christiano, Eichenbaum, and Trabandt (2013) estimate r^u to be 0.88.

model as in Smets and Wouters (2007). While delivering an empirical specification of each shock, full information estimation comes at the cost of requiring additional modeling features and assumptions about a range of stochastic disturbances that are of varying economic plausibility.

5 Optimal policy and robustness

As both models match the SVAR evidence well for reasonable parameters under the estimated simple interest rate rule, the policymaker has little guidance for choosing one model over the other. In addition, the two models have conflicting normative implications, making it potentially worthwhile for the policymaker to shy away from selecting one model prior to the evaluation of monetary policy: in the search and matching model, the optimal monetary policy seeks to stabilize price inflation at the expense of wage inflation whereas the optimal monetary policy in the sticky wage model seeks the opposite outcome. Thus, if the policymaker formulates policies on the assumption that the search and matching model is the true data-generating process when in fact the sticky wage model constitutes the true process, the policymaker implements a policy that may result in big welfare losses. Instead, it appears suitable to make model uncertainty a component of the policy evaluation.

One way for the policymaker to address the tensions between the normative recommendations of the models for the purpose of finding an optimal policy under model uncertainty is to choose a policy that minimizes an expected loss function under a probability distribution Ω_t over the candidate models. Let the loss function be of the general form

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathcal{L} \left(\tilde{y}_t, \Omega_t \right) \right\} \tag{33}$$

where $\tilde{y}_t$ is a vector defined to include the variables of all reference models including their leads and lags when appropriate. The loss function depends explicitly on the policymaker's probability distribution over models to reflect the fact that the policymaker's preferences over economic outcomes may differ across models. In principle, the policymaker updates Ω_t over time in response to the observed economic outcomes under the policymaker's choices. The modelling of this updating process can take on different degrees of complexity with no learning about the true data-generating process and Bayesian optimal learning defining the range of possible setups.

In this paper, we consider the following scenarios to provide insight into the policymaker's decision problem under model uncertainty:

1. the policymaker knows the correct model and implements a policy that is optimal given the preferences of the representative household (standard assumption as in Woodford (2003)),
2. the policymaker selects policy on the assumption that model 1 is true, when in fact model 2 is true and the other way around (this approach requires the use of optimal targeting rules),
3. the policymaker has a time-invariant probability distribution over models and fixes a policy rule at the beginning of time (as in Levin, Wieland, and Williams (2003), Brock, Durlauf, and West (2007), and Taylor and Williams (2010)) consistent with a weighted-average over the preferences of the representative household in the reference models,
4. the policymaker chooses a policy rule that minimizes the maximum welfare losses of the representative household given the reference models (as in Levin, Wieland, and Williams (2003)).

We discuss the first two scenarios in this section, and the remaining two in Section 6.

5.1 Optimal policy and optimal targeting rule

To implement scenarios 1 and 2, we follow Svensson and Woodford (2004), Giannoni and Woodford (2003, 2016) who advocate for the use of optimal targeting rules to characterize the optimal monetary policy. The optimal targeting rule specifies the variables—including the relative importance and the dynamic structure of each variable—in a single target criterion that seeks to implement the optimal monetary policy.

Optimal targeting rules can be computed for any preferences assigned to the policymaker. However, we adopt the approach in Woodford (2003) and assume the preferences of the policymaker to coincide with those of the representative household in the model. Therefore, obtainaing optimal targeting rules in our settings requires to: (1) derive the objective function of the policymaker as a purely quadratic approximation to the preferences of the representative household;[12] (2) obtain the first order conditions associated with the policymaker's problem of optimizing the (quadratic) objective function subject to the (linear) equations that describe the behavior of the private sector using the Lagrangian approach; (3) combine the first order conditions to obtain a single equation without Lagrange multipliers; this targeting rule describes the relationship between the endogenous and exogenous variables under the optimal policy. Importantly, the evolution of the economy that is consistent with the targeting rule is unique.

The optimal targeting rule implements the optimal monetary policy in the model from which it is derived, in contrast to simple instrument rules with optimally chosen coefficients. However, similar to instrument rules, the optimal targeting rule is expressed in terms of economically relevant model variables only; instrument rules prescribe how to adjust the policy instrument (such as the short term interest rate) in response to variables such as inflation, output, etc., and targeting rules describe how to adjust a target variable (for example, price inflation) to output, wage inflation and other variables. Finally, the evolution of the optimal target criterion does not necessarily involve the policy instrument of the central bank directly. Optimal targeting rules are therefore ideally suited to investigate the robustness of the optimal policy implied by one model in a different model as in our second scenario.

The optimal targeting rule for the NK model with sticky wages is easily obtained from the linear quadratic approximation of the original model. Employing results from Woodford (2003) for the policymaker's objective function and the linear approximation of the model's structural equations, the policymaker's problem is to

$$\min_{\{\pi_t,\pi_t^w,\hat{y}_t,i_t,\hat{w}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sw} \tag{34}$$

s.t. equations (25) to (28) and pre-commitments for period 0 (timeless perspective).

Absent consumption habits, price and wage indexation, the loss function consistent with the second order approximation of household preferences satisfies

$$\mathcal{L}_t^{sw} = \frac{\sigma+\phi}{2}\left(\hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t\right)^2 + \frac{1+\theta^p}{2\theta^p\kappa^p}\pi_t^2 + \frac{1+\theta^w}{2\theta^w\kappa^w}(\pi_t^w)^2. \tag{35}$$

[12] We follow Woodford (1999) and Benigno and Woodford (2012) in adopting the concept of "optimality from the timeless perspective"—a necessary assumption to obtain the correct linear quadratic approximation to our (nonlinear) model.

Giannoni and Woodford (2003) show that the first order conditions associated with the problem in (34) can be recombined to obtain the optimal targeting rule

$$\begin{aligned} 0 \;=\; & \left(\frac{1+\theta^p}{\theta^p}\pi_t + x_t - x_{t-1}\right) + \frac{1+\beta+\kappa^p}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_t + x_t - x_{t-1}\right) \\ & -\frac{\beta}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_{t+1} + x_{t+1} - x_t\right) - \frac{1}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_{t-1} + x_{t-1} - x_{t-2}\right) \end{aligned} \quad (36)$$

with $x_t = \hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t$. See Appendix D for details of the derivation. Absent sticky wages, i.e., $\kappa^w \to \infty$, equation (36) reduces to the optimal targeting rule in the standard NK model with flexible wages

$$\frac{1+\theta^p}{\theta^p}\pi_t + x_t - x_{t-1} = 0 \quad (37)$$

which suggests lowering the target value for price inflation in the current period below its long run value when the growth rate of the output gap is positive. When wages are sticky, the targeting rule also features the position of wage inflation relative to the output gap (over the near past and future). For stickier nominal wages and thus a flatter NK Phillips curve for wages ($\kappa^w \to 0$), the policymaker is less concerned with deviations of price inflation from its long-run target. In the limiting case of flexible prices ,i.e., $\kappa^p \to \infty$, the target value for wage inflation in the current period is set below its long run value when the growth rate of the output gap is positive.

Thus far, derivations of the optimal targeting rules in models with search and matching frictions are absent from the literature. Blanchard and Galí (2010), Thomas (2008), and Ravenna and Walsh (2011) derive purely quadratic objectives for the policymaker from household preferences under the assumption that the search and matching process does not induce inefficiencies as in Hosios (1990). None of these papers derives the implied optimal targeting rule. Furthermore, if the Hosios condition is not imposed, even the first step of obtaining an explicit second order approximation to the preferences of the representative household is missing in the literature.

To derive a purely quadratic objective for the policymaker in the presence of a distorted steady state for the search and matching model, we employ the numerical approach described in Bodenstein, Guerrieri, and LaBriola (2014) which is consistent with the theoretical results in Benigno and Woodford (2012). Appendix C shows that the policymaker's (period) loss function consistent with a second order approximation to the preferences of the representative household can be written as

$$\mathcal{L}_t^{s\&m} = P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t + P_{y,n^-}\hat{y}_t\hat{n}_{t-1} + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t}. \quad (38)$$

This formulation of the loss function is already simplified to include those variables only that enter the linear model in equations (22)-(24). We cannot obtain closed form expressions for the composite coefficients in (38), but our approach provides numerical values based on the underlying deep parameters of the model. The (linear-quadratic) problem of the policymaker is

$$\min_{\{\pi_t, i_t, \hat{n}_t, \hat{y}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{s\&m} \quad (39)$$

s.t. equations (22) to (24) and pre-commitments for period 0 (timeless perspective).

Rearranging the first order conditions associated with the optimization problem in (39) delivers the optimal targeting rule in the search and matching model

$$\varpi_1\hat{n}_t + \varpi_2\hat{n}_{t-1} + \varpi_3\hat{n}_{t+1} + \varpi_4\hat{y}_t + \varpi_5\hat{y}_{t+1} + \varpi_6\hat{a}_t + \varpi_7\hat{\theta}_{p,t} + \varpi_8\pi_t + \varpi_9\pi_{t+1} + \varpi_{10}\hat{P}_{t-1} + \varpi_{11}\hat{y}_t^{WA} + \varpi_{12}\hat{n}_t^{WA} + \varpi_{13}\hat{a}_t^{WA} + \varpi_{14}\hat{\theta}_{p,t}^{WA} + \varpi_{15}\hat{P}_t^{WA} = 0 \quad (40)$$

where we define

$$\pi_t = \hat{P}_t - \hat{P}_{t-1} \quad (41)$$

$$\hat{y}_t^{WA} = \beta_\delta\hat{y}_{t-1}^{WA} + \hat{y}_t \quad (42)$$

$$\hat{n}_t^{WA} = \beta_\delta\hat{n}_{t-1}^{WA} + \hat{n}_t \quad (43)$$

$$\hat{P}_t^{WA} = \beta_\delta\hat{P}_{t-1}^{WA} + \hat{P}_t. \quad (44)$$

$$\hat{a}_t^{WA} = \beta_\delta\hat{a}_{t-1}^{WA} + \hat{a}_t \quad (45)$$

$$\hat{\theta}_{p,t}^{WA} = \beta_\delta\hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p,t} \quad (46)$$

In the steady state, the price level grows with the steady state inflation rate. The term $\hat{P}_t$ denotes deviations of the price level from this growth path. Compared to the sticky wage model, the optimal targeting rule in the search and matching model involves weighted infinite-moving averages of output, employment, the price level, and the shocks. The presence of the markup shock $\hat{\theta}_{p,t}$ in the targeting rule is solely due to our decision not to impose the efficiency condition by Hosios (1990)—a decision that was needed to obtain a good fit of the search and matching model to the data in Section 4.[13]

To be able to exchange the optimal targeting rules between models requires expressing the targeting rules in variables that are common across models. For example, when implementing the rule in equation (40) in the sticky wage model we first substitute out for employment $\hat{n}_t$ in terms of hours worked, output, and technology using the aggregate production function and define the price level in the sticky wage model. Similarly, a definition of wage inflation is added to the search and matching model when solving that model under the optimal targeting rule derived in the sticky wage model, equation (36).

5.2 Robustness of optimal targeting rules

We assess the robustness of the optimal targeting rules across the two models starting with the search and matching model. Figure 3 depicts the case of the technology shock $\hat{a}_t$, and Figure 4 shows the case of the price markup shock $\hat{\theta}_{p,t}$. Under the technology shock, the optimal monetary policy as implemented by the targeting rule in equation (40) for the search and matching model calls for almost full stabilization of price inflation. No meaningful trade-offs arise as the welfare-relevant gaps move in the same direction: the technology shock exerts downward pressure to prices, and upward pressure on output and employment with the expansions being held back by sticky nominal prices. An interest rate cut reduces the downward pressure on prices and speeds up the expansion in output and employment. As a result, the real variables follow closely their paths in a real economy without sticky prices. These findings are reminiscent of the standard NK model with flexible wages (and no search and matching frictions in the labor market) in which the optimal monetary policy fully stabilizes inflation and closes the output gap. In fact, for the parameters in Table 1 and 2, the weights ϖ_j with $j = 1, ..., 15$ in equation (40) are such that the optimal targeting rule from the standard NK model (without search and matching frictions) displayed in equation (37) is a close approximation to the optimal targeting rule derived from the search and matching model.[14]

Notably, the labor market adjusts almost instantly to the shock in sharp contrast to

[13] In the sticky wage model the markup shock does not enter equation (36) since the steady state is assumed to be efficient; otherwise the markup shock would appear in the targeting rule as well. See also Benigno and Woodford (2005).

[14] See Appendix C and the discussion in Bodenstein and Zhao (2016).

the empirical responses in Figure 1. Under the optimal policy the real wage adjusts swiftly facilitated by a pronounced spurt in wage inflation. The movements in wages reflect the persistent jump in the marginal value of employment to the firm that gives rise to the front-loaded response in vacancies and the fall in unemployment.

In contrast to the rule in equation (40), the optimal targeting rule derived from the sticky wage model given in equation (36) places greater emphasis on stabilizing wage inflation and less emphasis on price inflation. Applied to the search and matching model, this second rule keeps nominal wages basically constant. Yet, the persistent rise in technology pressures real wages to rise and thus price inflation must fall below its target value to facilitate at least gradual adjustment in the real wage. As firms and households cannot reap all the benefits of improved technology and higher real wages immediately, vacancy postings, employment, and unemployment display inertia relative to the optimal responses. Adjustments in output and consumption are consequently delayed, as well. As is apparent from Figure 3, the targeting rule that is optimal in the sticky wage model does not induce the optimal responses in the search and matching model after a technology shock.

In the case of the markup shock, similar differences emerge between the two policy rules in the search and matching model. With the exception of price inflation, all other variables react more strongly to the shock under the optimal policy, equation (40). As the markup shock induces a trade-off between variables, price inflation is not fully stabilized under the optimal policy to temper the fluctuations in the other variables. Again, when the targeting rule derived from the sticky wage model is imposed instead, i.e., the rule in equation (36), wage inflation is almost fully stabilized at the expense of higher price inflation. The responses of all other variables are greatly muted compared to the optimal policy when the policymaker follows equation (40).

The lack of robustness of the targeting rules across models also applies to the model with sticky wages. Figures 5 and 6 plot the responses in the sticky wage model to technology and markup shocks, respectively, for the two targeting rules displayed in equations (36) and (40). The optimal policy under sticky nominal wages, implemented through equation (36), stabilizes wage inflation in response to both shocks. This policy avoids welfare-costly wage dispersion in the sticky wage model, whereas price inflation induces movements in the real wage that in turn facilitate the adjustment process for all other variables under the optimal policy. By overly stabilizing price inflation, the targeting rule derived in the search and matching model (40) allows more wage inflation than is optimal in the sticky wage model and in turn causes hours worked, output, and consumption, in particular, to exceed the optimal responses.

To sum up, the targeting rule (40), which is optimal in the search and matching model, favours stabilizing prices over stabilizing wages irrespective of the model in which the rule is implemented. The targeting rule (36), which is optimal in the sticky wage model, favours stabilizing wages over stabilizing prices irrespective of the model under consideration. Exchanging targeting rules between the models induces welfare losses that are orders of magnitudes larger than the welfare costs of business cycles in Lucas (2003). For the sticky wage model the welfare loss (measured in CEV) is considerably higher than for the search and matching model (1.3033 versus 0.1133) reflecting the high welfare costs associated with even minor relative wage differences in the sticky wage model. The lack of robustness of the optimal targeting rules also applies when nominal wages are indexed to past inflation in the sticky wage model as in the final column of Table 2.[15]

5.3 Sensitivity

Thus far, we have derived optimal targeting rules for each model assuming that the policymaker adopts the preferences of the representative household. To distinguish the role of preferences from the remaining model features for our results, we consider the case that the policymaker's preferences are identical across models and are given by the widely-used simple quadratic loss function $\mathcal{L}_t^{sql} = \pi_t^2 + \lambda_x x_t^2$. The parameter λ_x governs the relative importance of stabilizing price inflation versus the output gap. We consider the case of $\lambda_x = 0.0429$ under which the policymaker places high emphasis on price inflation similar to $\mathcal{L}_t^{s\&m}$, and the case of $\lambda_x = 1$ which implies a low emphasis on price inflation as under $\mathcal{L}_t^{sw}$.[16] Figure 7 shows the impulse responses under the targeting rules derived from the simple loss functions.

In the search and matching model, the optimal policy consistent with preferences $\mathcal{L}_t^{sql}$

[15] For the case of full indexation, the focus of optimal monetary policy in the sticky wage model shifts from smoothing wage inflation to smoothing the difference between wage inflation and lagged price inflation, i.e., $\pi_t^w - \pi_{t-1}$. The welfare loss of implementing in the sticky wage model with full indexation the (unchanged) targeting rule that is optimal in the search and matching model rises to 1.9728 when measured as CEV. The overall welfare loss in the search and matching model rises to 0.1680 when policy follows the optimal targeting rule derived in the sticky wage model. Appendix E provides more details on this case.

[16] The choice $\lambda_x = 0.0429$ is consistent with the weight on the output gap in the loss function derived for the standard NK model with flexible wages under the parameters in Tables 1 and 2. The alternative specification of $\lambda_x = 1$ is popular in the literature.

for $\lambda_x = 0.0429$ resembles the optimal policy derived under preferences $\mathcal{L}_t^{s\&m}$—the top row of panels in the figure. However, with the exception of price inflation all variables react by less to the markup shock than in Figure 4, indicating that under this parameterization of $\mathcal{L}_t^{sql}$ the policymaker prefers price inflation to bear more of the burden of adjustment than in our original case. When imposing onto the search and matching model the optimal targeting rule derived in the sticky wage model under preferences $\mathcal{L}_t^{sql}$, the same qualitative differences emerge as in Figure 4 despite the fact that the policymaker's preferences are now constant across models. Similarly, in the sticky wage model the gaps between the impulse responses under the two targeting rules derived for preferences $\mathcal{L}_t^{sql}$ remain large albeit smaller than in Figure 6. Even when the policymaker's preferences are held constant across models, the optimal targeting rules are not necessarily robust.

However, if the policymaker assigns even lower relative importance to price inflation, the optimal targeting rules *are* robust. The lower two rows of panels in Figure 7 show the impulse responses for $\lambda_x = 1$. In both the search and matching model and the sticky wage model, the gaps between the impulse responses generated by the optimal targeting rules derived for $\lambda_x = 1$ are minor. The robustness of optimal targeting rules is therefore sensitive to the preferences assigned to the policymaker.[17] It is important to realise, though, that the robustness of the optimal targeting rules under $\lambda_x = 1$ only applies from the viewpoint of the policymaker with preferences $\mathcal{L}_t^{sql} = \pi_t^2 + x_t^2$. For the representative households in the respective models these policies are suboptimal as household preferences continue to be given by $\mathcal{L}_t^{s\&m}$ and $\mathcal{L}_t^{sw}$.

6 Robust policy

Having established, that the reference models differ with respect to their normative recommendations and that the welfare costs of applying the wrong policy to a given model can be sizeable, we move on to scenarios 3 and 4 outlined in Section 5 and make model uncertainty a direct component of the evaluation of monetary policies.

[17] Using a very different set of models, Levin and Williams (2003) also find that optimal targeting rules are more likely to be robust if the policymaker focuses less on price stability.

6.1 Methodology

To obtain optimal policies under model uncertainty, we need to specify the policymaker's preferences and possibly a probability distribution over models. Furthermore, we need to make assumptions about the implementation of monetary policy.

6.1.1 Objective function of the policymaker

We begin with specifying the preferences of the policymaker. Previous works on model uncertainty separate the policymaker's preferences from the underlying models by assuming a simple quadratic loss function that punishes deviations of inflation and the output gap from their respective long-run target values. However, as discussed in the previous section, the model-consistent preferences over outcomes are given by the function $\mathcal{L}_t^{s\&m}$ in the search and matching model and by the function $\mathcal{L}_t^{sw}$ in the sticky wage model, respectively. Thus, we propose to better align the preferences of the policymaker with those of the representative household. We present results for a model averaging approach and a minmax approach.

Under model averaging, the policymaker's preferences over economic outcomes are a weighted average over the preferences of the representative household in the reference models. The weights are taken to coincide with the policymaker's probability distribution over models. We assume the probability distribution over models to be time-invariant. With these assumptions in place, the policymaker's preferences are

$$L^{av}(\Theta) = \omega \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{s\&m}(\Theta) \right) + (1-\omega) \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sw}(\Theta) \right) \tag{47}$$

where Θ indicates the monetary policy common to both models as specified below. Levin and Williams (2003) and Taylor and Williams (2010) refer to this approach as Bayesian strategy.

The model averaging approach can be interpreted literally as the case of a single policymaker assigning a probability distribution over the reference models based on statistical analysis. For example, Levine, McAdam, and Pearlman (2012) translate posterior odds ratios of models estimated with full information Bayesian techniques into the policymaker's probability distribution over models. The policymaker then engages in model averaging. In this sense, the policymaker's model is a weighted average across the reference models. An alternative interpretation is related to decision-making in committees. Each member of the committee selects a single model that reflects her/his views over the economy. The optimal policy under uncertainty is not optimal from any individual member's point of view, but it produces outcomes that might be acceptable to all members.

When the policymaker pursues the minmax approach, the policymaker's loss under policy Θ is the maximum welfare loss across the two reference models

$$L^{minmax}(\Theta) = \max \left\{ \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{s\&m}(\Theta) \right), \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sw}(\Theta) \right) \right\}. \tag{48}$$

This policymaker is concerned with avoiding the worst case scenario of setting a policy that could result in large welfare losses under any circumstances. In comparison to the model averaging approach, the minmax approach allows the optimal policy under model uncertainty to be greatly influenced by the model which displays greater sensitivity of welfare losses to deviations from its model-specific optimal policy even if that model is considered unlikely. The results under the minmax approach are independent from the policymaker's probability distribution over models as long as non-zero probability is attached to each model.

6.1.2 Formulating policy

We assume that monetary policy follows a simple instrument rule of the type

$$i_t = \rho_R i_{t-1} + \rho_\pi \pi_t + \rho_\pi^w \pi_t^w + \rho_x x_t. \tag{49}$$

The vector $\Theta = \left\{ \rho_R, \rho_\pi, \rho_\pi^w, \rho_x \right\}$ collects the coefficients of the rule. According to the rule, the nominal interest rate is adjusted in response to movements in price and wage inflation, as well as the output gap. Furthermore, the rule allows for interest rate inertia. We search for the (non-negative) values of the parameters in Θ that minimize the welfare loss of the policymaker under the given objective as in Levin and Williams (2003) and Levin, Wieland, and Williams (2003).[18]

Using simple rules limits the number of coefficients to be determined and thus facilitates computations and transparency. Despite the small number of variables included in

[18] For a given parameterization of the rule, we compute the unconditional welfare loss for each model to approximate the conditional welfare as the discount factor is close to 1 and to eliminate the potential impact of arbitrary initial conditions. Following Benigno and Woodford (2012) we include a correction term to account for the fact that the rule may violate the pre-commitment conditions imposed in deriving the loss functions. Rules that lead to equilibrium indeterminacy are discarded.

the simple rule, these rules can provide high-quality approximations to the optimal policy. We confirm that absent model uncertainty the simple rule (49) can indeed approximate the optimal policy with a high degree of accuracy for each reference model.

In addition to simple instrument rules with optimized coeffients, we also consider a simple targeting rule of the form

$$\begin{aligned} 0 &= \rho_{tr}\left(\frac{1+\theta^p}{\theta^p}\pi_t + x_t - x_{t-1}\right) + \frac{1+\beta+\kappa^p}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_t + x_t - x_{t-1}\right) \\ &\quad -\frac{\beta}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_{t+1} + x_{t+1} - x_t\right) - \frac{1}{\kappa^w}\left(\frac{1+\theta^w}{\theta^w}\pi^w_{t-1} + x_{t-1} - x_{t-2}\right) \qquad (50) \end{aligned}$$

with the policymaker choosing the parameter ρ_{tr} given the values for the remaining parameters. This simple targeting rule can approximate well the optimal targeting rule from the search and matching model for large values of ρ_{tr} under our parameterization as discussed earlier and it replicates the optimal targeting rule from the sticky wage model for $\rho_{tr} = 1$ regardless of the parameterization. In implementing each of these two rules we define the output gap as the difference between actual output and the output that would have prevailed absent nominal rigidities.[19]

6.2 Monetary policy rules under model uncertainty

Table 3 reports in Panel (a) the optimal simple rules under the benchmark parameterization of the search and matching model and the sticky wage model. For the model averaging approach, we consider multiple specifications of the policymaker's probability distribution with ω, the probability that the policymaker assigns to the search and matching model being the true data-generating process, ranging from 0 to 1. We refer to the optimal simple rule associated with a given probability distribution as the "ω-optimal simple rule." Under the minmax strategy, the policymaker's probability distribution over models is irrelevant. Welfare is reported in terms of consumption equivalent variations (CEV). In Panel (b), we report the findings when the policymaker follows the simple targeting rule. Finally, for comparison, the table repeats in Panel (c) the welfare implications of implementing the optimal targeting rules derived in the previous section across models.

In Panel (a), we distinguish three regions for the probability ω under model averaging:

[19] This implementation differs from Section 5 where we abstain from defining output gaps in the implementation. The differences in outcomes under the two approaches are minor for targeting rules.

low ($\omega \leq 0.2$), intermediate ($0.3 \leq \omega \leq 0.8$), and high ($\omega \geq 0.9$). The ω-optimal simple rule varies distinctly across these regions. In the first region with little probability weight on the search and matching model, the nominal interest rate responds primarily to wage inflation in line with optimal policy prescriptions of the sticky wage model. In the second region, the rule responds to wage and price inflation with the coefficients assigned to the two variables being of similar magnitude. It is only in the third region that the ω-optimal simple rule displays significant interest rate inertia. The coefficient on wage inflation basically drops to zero whereas the nominal interest rate responds to price inflation. With the policymaker assigning a high probability to the search and matching model, the importance of wage inflation stabilization fades. Consequently, in the sticky wage model the welfare loss (relative to the optimal monetary policy in that model) under the ω-optimal simple rule is larger for higher values of ω and the welfare loss in the search and matching model is reduced.

To illustrate the dynamic implications of the various optimal simple rules, we plot in Figure 8 the impulse responses of output, price and wage inflation in both models to the technology shock and the markup shock for $\omega = 0, 0.2, 0.3, 0.8, 0.9, 1$. In the sticky wage model, the ω-optimal simple rules with $\omega < 0.9$ induce impulse responses (bottom two rows of panels) that are reasonably close to those under $\omega = 0$ (the optimal simple rule if the policymaker is certain about the sticky wage model being the true data-generating process). For the search and matching model (top two rows of panels), ω-optimal simple rules with $\omega < 0.9$ induce responses that differ noticeably from those under $\omega = 1$ (the optimal simple rule if the policymaker is certain about the search and matching model being the true data-generating process).[20] Thus, the policymaker effectively biases policies towards the optimal policy in the sticky wage model for the low and the intermediate region of ω even though the optimally chosen parameters differ across the two regions.

The welfare losses induced by the ω-optimal simple rules reported in Table 3 confirm this conclusion from a normative perspective. When moving from the 0.8-optimal simple rule to the 0.9-optimal simple rule the CEV value for the sticky wage model goes from negligible to 0.2. While the welfare losses in the search and matching model are generally small, the CEV value is practically zero under the 0.9-optimal simple rule.

[20] For the search and matching model, the Euclidean distance between the impulse responses of price inflation, wage inflation and output to the price markup shock for the rules $\omega = 0.8$ and $\omega = 0.9$ measured against the case of $\omega = 1$, respectively, drops from 0.0347 to 0.019. For the sticky wage model, by contrast, the Euclidean distance between the impulse responses of price inflation, wage inflation and output to the price markup shock for the rules $\omega = 0.8$ and $\omega = 0.9$ measured against the case of $\omega = 0$, respectively, more than doubles from 0.0116 to 0.0245.

The reason for the apparent bias of the optimal policy under model uncertainty towards the sticky wage model lies in the high welfare costs associated with even minor relative wage differences in the sticky wage model. The desire to avoid bad economic outcomes caused by bad monetary policy is even more explicit when the policymaker adopts a minmax strategy. In this case, the optimal simple rule coincides with the 0-optimal simple rule, which in turn mimics the optimal targeting rule derived in the sticky wage model.[21]

To complement the findings for the optimal simple rules, Panel (b) reports the optimal parameterization of the simple targeting rule proposed in equation (50). In the case of model averaging, when the policymaker holds the sticky wage model reasonably likely, the coefficient ρ_{tr} is set near 1 and the rule allows for wage inflation to be a primary concern of monetary policy. Only for ω close to 1 does the policymaker switch to stabilizing price inflation aggressively: ρ_{tr} exceeds 6e+05 for $\omega = 1$, but it assumes a value around 15 for $\omega = 0.9$. It is in the interval $\omega \in [0.9, 1]$ that the welfare loss in the search and matching model under the simple targeting rule drops to almost zero, whereas the welfare loss in the sticky wage model soars.[22] Under the minmax approach, the policymaker chooses $\rho_{tr} = 1$ and implements the optimal targeting rule of the sticky wage.

6.3 Sensitivity of results

6.3.1 Functional form of the policy rule

In principle, the simple rule in equation (49) allows the policymaker to respond to the lagged value of the nominal interest rate, price and wage inflation, and the output gap. However, the response coefficient on each variable is assigned the value of zero for some ω; the patterns of zeroes define the three distinct regions of the ω-optimal simple rules in Table 3 for the model averaging approach. To assess the sensitivity of our findings to the functional form of the simple rule, Table 4 reports optimal simple rules that, in comparison to (49), are restricted not to respond to either the lagged interest rate, the output gap, price inflation, or wage inflation, respectively.[23]

[21] The 0-optimal simple rule is close to but not identical to the optimal targeting rule derived in the sticky wage model, as the functional form of the simple rule in equation (49) is not quite flexible enough.

[22] By construction, the simple targeting rule is biased in favor of the sticky wage model and somewhat less flexible than the optimal simple rule. The value of the objective function assumes larger values than in the case of optimal simple rules for high values of ω.

[23] The presence of three distinct parameter regions in Table 3 Panel (a) under model averaging suggests the existence of multiple local optima. In computing restricted optimal simple rules we can also confirm that the ω-optimal simple

Absent interest rate smoothing (Case I in Table 4), the optimal simple rule changes only for $\omega \geq 0.9$ compared to Table 3. The response coefficient for price inflation becomes very large to compensate for the lack of interest rate smoothing in the rule, but overall welfare and welfare in the search and matching model deteriorate nevertheless. In its eagerness to fight price inflation, the rule for $\omega = 1$ and $\rho_R = 0$ is particularly unattractive, as it induces welfare losses in the sticky wage model that by far exceed the corresponding loss in Table 3 Panel(a).

Eliminating the output gap from the list of response variables (Case II) affects the computations of the optimal simple rules only for $\omega \leq 0.2$. These restricted rules respond to wage inflation by more than in Table 3—the optimizer reaches the upper bound of 100—where the ω-optimal simple rule responded importantly to the output gap for $\omega \leq 0.2$. The overall welfare loss is higher mostly because the restricted rules perform worse in the sticky wage model.

More dramatic changes in the optimal simple rules appear if the rules are restricted not to respond to price inflation or wage inflation (Case III). Setting $\rho_\pi = 0$ leads to higher response coefficients for wage inflation and, depending on the value of ω, the output gap or interest rate smoothing. The deterioration in overall welfare is borne by the search and matching model; welfare in the sticky wage model improves for most values of ω and never declines.

Finally, when eliminating the policymaker's ability to respond to wage inflation directly, welfare losses increase in both the sticky wage and the search and matching model for most values of ω (Case IV). The form of the simple rule in this final case coincides with the specification adopted in our estimation. Even more so, under $\omega = 0.6$ and $\omega = 0.7$, the restricted optimal simple rules feature parameter values that are close to the values retrieved in our estimation: the interest rate smoothing coefficient lies around 0.8 and the short-run coefficient assigned to price inflation lies between 0.1 and 0.2. If we interpret the estimated simple rules obtained in Section 4 (which basically coincide for the two models) as arising from optimal policy considerations under model uncertainty—where the policymaker intentionally excludes a direct response to wage inflation—U.S. policymakers assign probability 0.6 to 0.7 to the search and matching model being the true data-generating process.

rules are indeed globally optimal.

6.3.2 Shock persistence and consumption habits

Thus far, we have assumed that the markup shock is transitory and that households do not experience habit persistence in consumption. As our estimation strategy is silent on the parameterization of the markup shock, we also explore the possibility of a mildly persistent markup shock ($\rho_u = 0.2$). In a second alternative, we investigate the impact of habit persistence in consumption ($\mu = 0.6$) on our results.[24]

Table 5, summarizes in Panel (a) the results for the case of mildly persistent markup shocks. Overall, the results are similar to those in Table 3, if not stronger. Under model averaging, the ω-optimal simple rule is biased towards improving the outcomes in the sticky wage model: the welfare loss (measured in CEV) in the sticky wage model is smaller than in the search and matching model as long as $\omega \leq 0.8$ and negligible for $\omega \leq 0.4$ (compared to $\omega \leq 0.2$ in Table 3). The minmax strategy continues to pick the ω-optimal simple rule for $\omega = 0$. Our results also withstand the introduction of habit persistence as shown in Panel (b) of Table 5. Yet, in the presence of this real rigidity the bias of the optimal policy under model uncertainty towards the sticky wage model is slightly less pronounced.

6.3.3 Simple loss function

Lastly, we return to the role of the policy objective function in obtaining robust monetary policy under model uncertainty. We replace the preferences of the policymaker under model averaging in equation (47) with

$$L^{sql}(\Theta) = \omega \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sql,s\&m}(\Theta) \right) + (1-\omega) \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sql,sw}(\Theta) \right) \tag{51}$$

where $\mathcal{L}_t^{sql,i}(\Theta) = (\pi_t^i)^2 + (x_t^i)^2$ is the value of the simple loss function for model $i = \{s\&m, sw\}$. As in Section 5.3, but in contrast to our baseline case, the preferences of the policymaker are thus assumed to be independent of the reference models.

Table 6 summarizes the optimal simple rules derived under the simple loss function. Although the parameterization of the ω-optimal simple rule varies strongly with ω, the induced dynamics in the two economies hardly differ across the rules (not shown)—similar

[24] The impulse response function matching in Section 4 finds no support for consumption habits in contrast to Christiano, Eichenbaum, and Trabandt (2013). In part, this result emerges as we exclude monetary policy shocks from the empirical analysis. Monetary policy shocks induce a pronounced hump-shaped response of consumption and output in the SVAR. One way to capture this feature is to introduce habit persistence in consumption.

to Figure 7 when we analyzed the robustness of the optimal targeting rules under this simple loss function ($\lambda_x = 1$). Further evidence along these lines stems from the observation that the welfare losses in each model under the model-specific true loss functions (as opposed to the simple loss function) are highly stable across values of ω; the low CEVs computed for the sticky wage model indicate that all the rules resemble closely the optimal monetary policy in the sticky wage model (under the true loss function of the sticky wage model).

The results in Table 6 could be viewed as evidence for the existence of a monetary policy rule that is (more or less) robust to the extent of model uncertainty. Yet, this conclusion is true only from the perspective of the policymaker whose preferences are described by the simple loss function. From the perspective of the representative household with preferences $\mathcal{L}_t^{s\&m}$ and $\mathcal{L}_t^{sw}$ the results are suboptimal: the simple rules are unnecessarily biased towards the sticky wage model when ω is close to 1 compared to Table 3. To the extent that the welfare implications of mircofounded models are of interest, this result discourages the use of arbitrary loss functions for policy analysis.

7 Conclusion

We contrast the optimal monetary policy recommendations in two models that the policymaker views as good approximations of the true data-generating process. The models differ with regard to the details of the labor market. The first model follows the search and matching literature in assuming that workers have to search for jobs and firms post vacancies. For a worker to become employed and for the production of goods to occur, the worker has to be matched with a firm. The two parties then negotiate over the real wage which, as a result, may experience substantial inertia. In the second model, nominal wages are rigid as the result of staggered (nominal) wage contracts. We apply impulse response function matching to estimate key parameters of the models.

While the two models produce very similar impulse responses for common variables under the estimated policy rules, the responses differ importantly when monetary policy is chosen optimally. Under sticky wages without indexation, the optimal policy induces little variation in nominal wages; the dynamics of the real wage are determined by the adjustment in prices.[25] In the search and matching model, it is optimal to stabilize prices

[25] When wages are fully indexed, the optimal policy smoothes the difference between wage and past price inflation.

and to allow for substantial real wage adjustment brought about by changes in nominal wages. We fill a gap in the literature by deriving the optimal targeting rule for a search and matching model—a single target criterion that seeks to implement the optimal monetary policy. We investigate the performance of each economy under the optimal targeting rule derived in the other model. In particular, the optimal targeting rule derived for the search and matching model is not robust, in the sense that it induces large welfare losses in the sticky wage model. While the optimal targeting rule derived for the sticky wage model also alters the dynamics in the search and matching model relative to the optimal monetary policy in that model, the welfare consequences are less dramatic.

Given the models' sensitivity to the optimal targeting rules, we compute optimal simple rules and simple targeting rules when the policymaker considers both the sticky wage and the search and matching model to be good candidates for the true data-generating process. Applying a model averaging and a minmax approach to obtain the optimal parameterization of each rule, we find that unless the policymaker places high probability weight on the search and matching model the resulting policies are biased towards stabilizing wage inflation, the key feature of the optimal monetary policy in the sticky wage model.

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Table 1: Calibrated Parameters

Description	Parameter	Search and Matching	Sticky Wage
discount factor	β	0.99	0.99
exogenous seperation rate	ρ	0.1	-
matching function share of unemployment	ζ	0.54	-
steady state inflation rate	$\bar{\pi}$	1	1
Calvo price stickiness	ξ^p	0.75	0.75
steady state price markup	λ^p	1.2	1.2
Calvo wage stickiness	ξ^w	-	0.75
steady state wage markup	λ^w	-	1.2
invers consumption elasticity	σ	1	1
inverse labor supply elasticity	ϕ	2	2
hiring flow cost / output	η_s	0.0066	-
steady state unemployment rate	$\tilde{u}_{ss}$	0.055	-
steady state vacancy filling rate	q_{ss}	0.7	-
steady state working hour	h_{ss}	1/3	1/3
Shock Process			
technology shock: AR	ρ_a	0.9999	0.9999
markup shock: AR	ρ_u	0	0
markup shock: Std	σ_u	0.0104	0.0135
Implied Deep Parameter Value			
hiring fixed cost	$\bar{\kappa}$	0	-
hiring flow cost	κ^v	0.0154	-
unemployment benefit	b^u	0.1769	-
worker's share of surplus	ξ	0.7438	-
matching efficiency	χ	0.6625	-
scaling of working hour disutility	ϕ_0	27.8940	27

Note: Table 1 summarizes the parameters and calibration targets for the NK model with search and matching frictions and the NK model with sticky wages.

Table 2: Estimated Parameters

Description	Estimated Parameter	Search	Sticky Wage	Sticky Wage with Indexation
interest rate smoothing	ρ_R	0.8555	0.8379	0.8895
		[0.0294]	[0.0450]	[0.0260]
weights on inflation	ρ_π	0.1445	0.1622	0.1105
		[1.5e-05]	[3.12e-05]	[2.4e-05]
std technology shock	σ_a	0.0031	0.0033	[0.0031]
		[0.0002]	[0.0002]	[0.0002]
habit persistence	μ	0	0	0
		[0.5148]	[0.4394]	[0.7734]
replacement ratio	r^u	0.5345	-	-
		[0.0185]	-	-
price indexation	ι^p	0	0	0
		[0.3123]	[0.3204]	[0.2714]
wage indexation	ι^w	-	-	1
		-	-	[0.1656]

Minimum Distance Estimator

Description	Search	Sticky Wage	Sticky Wage with Indexation
criterion value (9 variables)	124.8128	-	-
criterion value (6 variables)	99.6490	136.0783	77.2143

Note: The top panel of Table 2 summarizes the estimated parameters for the NK model with search and matching frictions and the NK model with and without wage indexation. The parameters are estimated using impulse response function matching under neutral technology shocks. The empirical impulse responses against which the performance of the theoretical models is assessed are taken from the SVAR estimation in Christiano, Eichenbaum, and Trabandt (2013). The numbers in the square bracket are the standard deviations of the estimates. The lower panel provides the value of the criterion function (30) at the minimum.

Table 3: Optimal Simple Rules and Optimal Simple Targeting Rules

Panel a: Optimal Simple Rules

Approach	Prior	Coefficients				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Model Averaging	$(0,1)$	0	0	66.6844	2.3852	3.1047	2.1568	0.1094	3.1047	0.0010
	$(0.1,0.9)$	0	0	61.5860	2.0019	3.0099	2.1566	0.1092	3.1048	0.0010
	$(0.2,0.8)$	0	0	56.4038	1.6763	2.9151	2.1565	0.1091	3.1048	0.0011
	$(0.3,0.7)$	0	0.6240	0.5226	0	2.8139	2.1028	0.0554	3.1186	0.0149
	$(0.4,0.6)$	0	0.6368	0.5160	0	2.7123	2.1025	0.0551	3.1188	0.0151
	$(0.5,0.5)$	0	0.6558	0.5131	0	2.6106	2.1022	0.0548	3.1190	0.0153
	$(0.6,0.4)$	0	0.7005	0.5158	0	2.5088	2.1014	0.0540	3.1199	0.0162
	$(0.7,0.3)$	0	0.8135	0.5231	0	2.4067	2.0994	0.0520	3.1240	0.0202
	$(0.8,0.2)$	0	1.1725	0.5245	0	2.3031	2.0920	0.0446	3.1475	0.0438
	$(0.9,0.1)$	0.8177	0.8860	0	0	2.1870	2.0623	0.0149	3.3098	0.2061
	$(1,0)$	0.9366	2.1197	0	0	2.0477	2.0477	0.0003	4.0851	0.9814
Minmax	N.A.	0	0	66.6844	2.3852	3.1047	2.1568	0.1094	3.1047	0.0010

Panel b: Optimal Simple Targeting Rules

Approach	Prior	Coefficients	Welfare Loss				
		ρ_{tr}	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Model Averaging	$(0,1)$	1.0000	3.1037	2.1602	0.1128	3.1037	0.0000
	$(0.1,0.9)$	1.0847	3.0094	2.1601	0.1127	3.1037	0.0000
	$(0.2,0.8)$	1.2319	2.9150	2.1600	0.1126	3.1038	0.0000
	$(0.3,0.7)$	1.4166	2.8206	2.1598	0.1124	3.1038	0.0001
	$(0.4,0.6)$	1.6773	2.7262	2.1595	0.1121	3.1040	0.0003
	$(0.5,0.5)$	2.0417	2.6317	2.1591	0.1117	3.1043	0.0006
	$(0.6,0.4)$	2.6065	2.5371	2.1585	0.1111	3.1050	0.0013
	$(0.7,0.3)$	3.5951	2.4423	2.1576	0.1102	3.1068	0.0031
	$(0.8,0.2)$	5.7812	2.3471	2.1557	0.1083	3.1129	0.0091
	$(0.9,0.1)$	15.0510	2.2499	2.1495	0.1021	3.1534	0.0497
	$(1,0)$	6.1979e+05	2.0488	2.0488	0.0014	4.3001	1.1964
Minmax	N.A.	1.0000	3.1037	2.1607	0.1133	3.1037	0.0000

Panel c: Optimal Targeting Rules

Optimal Targeting Rule	Welfare Loss			
	$\mathcal{L}_t^{s\&m}$	$CEV^{s\&m}$	$\mathcal{L}_t^{sw}$	CEV^{sw}
s&m	2.0474	0.0000	4.4070	1.3033
sw	2.1607	0.1133	3.1037	0.0000

Note: Table 3 reports the optimal parameterizations of the simple rule in (49) in Panel (a) and simple targeting rule (50) in Panel (b) when the policymaker has two reference model, the NK model with search and matching frictions (s&m) and the NK model with sticky wages and no indexation (sw). The model is parameterized as in Tables 1 and 2. Under model averaging, the policymaker minimizes the expected loss given a probability distribution (prior). Under the minmax strategy, the policymaker searches for a policy rule that minimizes the maximum loss. "Objective" measures the value of the policymaker's objective function at the optimum. The columns $\mathcal{L}_t^{s\&m}(\Theta^*)$ and $\mathcal{L}_t^{sw}(\Theta^*)$ give the value of the expected loss in each model, the columns $CEV^{s\&m}(\Theta^*)$ and $CEV^{sw}(\Theta^*)$ translate these losses into consumption equivalent variations. Panel (c) displays the welfare costs of implementing the optimal targeting rules in each model.

Table 4: Restricted Optimal Simple Rules

Model Averaging	Prior	Restricted Optimal Rule				Welfare Loss		
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$
Case I: No interest rate smoothing	$(0, 1)$	0	0	65.8388	2.3084	3.1047	2.1567	3.1047
	$(0.1, 0.9)$	0	0	60.6858	1.9798	3.0099	2.1566	3.1048
	$(0.2, 0.8)$	0	0	55.4967	1.6522	2.9151	2.1565	3.1048
	$(0.3, 0.7)$	0	0.6230	0.5235	0	2.8139	2.1028	3.1186
	$(0.4, 0.6)$	0	0.6369	0.5160	0	2.7123	2.1025	3.1188
	$(0.5, 0.5)$	0	0.6561	0.5133	0	2.6106	2.1022	3.1190
	$(0.6, 0.4)$	0	0.7006	0.5159	0	2.5088	2.1014	3.1199
	$(0.7, 0.3)$	0	0.8136	0.5231	0	2.4067	2.0994	3.1240
	$(0.8, 0.2)$	0	1.1724	0.5245	0	2.3031	2.0920	3.1475
	$(0.9, 0.1)$	0	2.2488	0.4734	0	2.1922	2.0730	3.2651
	$(1, 0)$	0	51.5995	5.2078	0.5170	2.0485	2.0485	20.0584
Case II: No output gap	$(0, 1)$	0	0	100	0	3.1059	2.1556	3.1059
	$(0.1, 0.9)$	0	0	100	0	3.0108	2.1556	3.1059
	$(0.2, 0.8)$	0	0	100	0	2.9158	2.1556	3.1059
	$(0.3, 0.7)$	0	0.6242	0.5224	0	2.8139	2.1028	3.1186
	$(0.4, 0.6)$	0	0.6389	0.5167	0	2.7123	2.1025	3.1188
	$(0.5, 0.5)$	0	0.6564	0.5130	0	2.6106	2.1022	3.1190
	$(0.6, 0.4)$	0	0.7012	0.5160	0	2.5088	2.1014	3.1200
	$(0.7, 0.3)$	0	0.8179	0.5238	0	2.4067	2.0993	3.1242
	$(0.8, 0.2)$	0	1.1725	0.5245	0	2.3031	2.0920	3.1475
	$(0.9, 0.1)$	0.8177	0.8860	0	0	2.1870	2.0623	3.3098
	$(1, 0)$	0.9366	2.1197	0	0	2.0477	2.0477	4.0851
Case III: No price inflation	$(0, 1)$	0	0	66.0161	2.3594	3.1047	2.1567	3.1047
	$(0.1, 0.9)$	0	0	61.0171	1.9932	3.0099	2.1566	3.1048
	$(0.2, 0.8)$	0	0	56.0134	1.6655	2.9151	2.1565	3.1048
	$(0.3, 0.7)$	0	0	93.0510	2.1243	2.8203	2.1563	3.1048
	$(0.4, 0.6)$	0	0	93.5958	2.3938	2.7254	2.1564	3.1048
	$(0.5, 0.5)$	0.5709	0	5.0689	0.2221	2.6309	2.1554	3.1063
	$(0.6, 0.4)$	0.3093	0	20.0193	0.4160	2.5357	2.1559	3.1055
	$(0.7, 0.3)$	0.4090	0	20.0150	0.2744	2.4407	2.1556	3.1060
	$(0.8, 0.2)$	0.5257	0	20.0071	0.1165	2.3457	2.1554	3.1067
	$(0.9, 0.1)$	0.6127	0	19.9914	0	2.2505	2.1553	3.1075
	$(1, 0)$	0.9269	0	19.3246	0	2.1552	2.1552	3.1076
Case IV: No wage inflation	$(0, 1)$	0	1.0001	0	12.0434	3.1107	2.2814	3.1107
	$(0.1, 0.9)$	0.0587	0.9414	0	2.3607	3.0267	2.2598	3.1119
	$(0.2, 0.8)$	0.9900	0.0113	0	0.0011	2.9370	2.1308	3.1386
	$(0.3, 0.7)$	0.9900	0.0112	0	7.5200e04	2.8357	2.1232	3.1410
	$(0.4, 0.6)$	0.9900	0.0111	0	5.1800e04	2.7335	2.1178	3.1439
	$(0.5, 0.5)$	0.9900	0.0110	0	3.1900e-04	2.6304	2.1130	3.1478
	$(0.6, 0.4)$	0.9079	0.1172	0	0	2.5243	2.1024	3.1572
	$(0.7, 0.3)$	0.8947	0.2260	0	0	2.4176	2.0912	3.1790
	$(0.8, 0.2)$	0.8669	0.4487	0	0	2.3061	2.0769	3.2226
	$(0.9, 0.1)$	0.8177	0.8861	0	0	2.1870	2.0623	3.3099
	$(1, 0)$	0.9366	2.1204	0	0	2.0477	2.0477	4.0856

Note: Table 4 reports the optimal simple rules under the model averaging approach similar to Table 3 when restricting the rule not to respond to one of the variables in (49) at the time. See also footnote Table 3.

Table 5: Sensitivity of Optimal Simple Rules

Panel a: persistent markup shock $\rho_u = 0.2$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Model Averaging	$(0, 1)$	0	0	66.6812	1.6100	3.2524	2.2305	0.1520	3.2524	0.0014
	$(0.1, 0.9)$	0	0	62.1310	1.3100	3.1501	2.2302	0.1518	3.2524	0.0014
	$(0.2, 0.8)$	0	0	57.0042	1.0663	3.0479	2.2300	0.1516	3.2524	0.0015
	$(0.3, 0.7)$	0	0	51.9820	0.8663	2.9457	2.2299	0.1514	3.2524	0.0015
	$(0.4, 0.6)$	0	0	47.4445	0.6725	2.8434	2.2297	0.1513	3.2525	0.0016
	$(0.5, 0.5)$	0	0.6436	0.5084	0	2.7087	2.1448	0.0663	3.2726	0.0217
	$(0.6, 0.4)$	0	0.6743	0.5110	0	2.5958	2.1441	0.0656	3.2735	0.0226
	$(0.7, 0.3)$	0	0.7419	0.5165	0	2.4827	2.1424	0.0639	3.2768	0.0259
	$(0.8, 0.2)$	0	0.9590	0.5250	0	2.3683	2.1361	0.0576	3.2973	0.0463
	$(0.9, 0.1)$	0.8744	0.5682	0	0	2.2419	2.0993	0.0208	3.5253	0.2744
	$(1, 0)$	0.9789	1.4644	0	0	2.0785	2.0785	0.0000	4.7228	1.4719
Minmax	N.A.	0	0	66.6812	1.6100	3.2524	2.2305	0.1520	3.2524	0.0014

Panel b: habit persistence $\mu = 0.6$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Model Averaging	$(0, 1)$	0	0	68.1424	1.4345	3.1519	2.1796	0.1347	3.1519	0.0007
	$(0.1, 0.9)$	0	0.6175	1.3949	0	3.0546	2.1689	0.1240	3.1530	0.0018
	$(0.2, 0.8)$	0	0.6561	1.3497	0	2.9561	2.1678	0.1229	3.1532	0.0020
	$(0.3, 0.7)$	0	0.7064	1.3276	0	2.8575	2.1664	0.1215	3.1537	0.0025
	$(0.4, 0.6)$	0	0.7754	1.3139	0	2.7586	2.1646	0.1197	3.1547	0.0035
	$(0.5, 0.5)$	0	0.8786	1.3099	0	2.6594	2.1621	0.1172	3.1568	0.0056
	$(0.6, 0.4)$	0.9882	1.4557	0	0.2369	2.5576	2.1205	0.0756	3.2132	0.0620
	$(0.7, 0.3)$	0.9198	2.4368	0	0.3228	2.4461	2.1042	0.0593	3.2438	0.0926
	$(0.8, 0.2)$	0.8178	4.0965	0	0.4114	2.3289	2.0867	0.0418	3.2976	0.1464
	$(0.9, 0.1)$	0.6763	6.9621	0	0.4346	2.2020	2.0677	0.0228	3.4110	0.2598
	$(1, 0)$	0	41.4453	1.6503	0	2.0465	2.0465	0.0016	5.0518	1.9006
Minmax	N.A.	0	0	68.1424	1.4345	3.1519	2.1796	0.1347	3.1519	0.0007

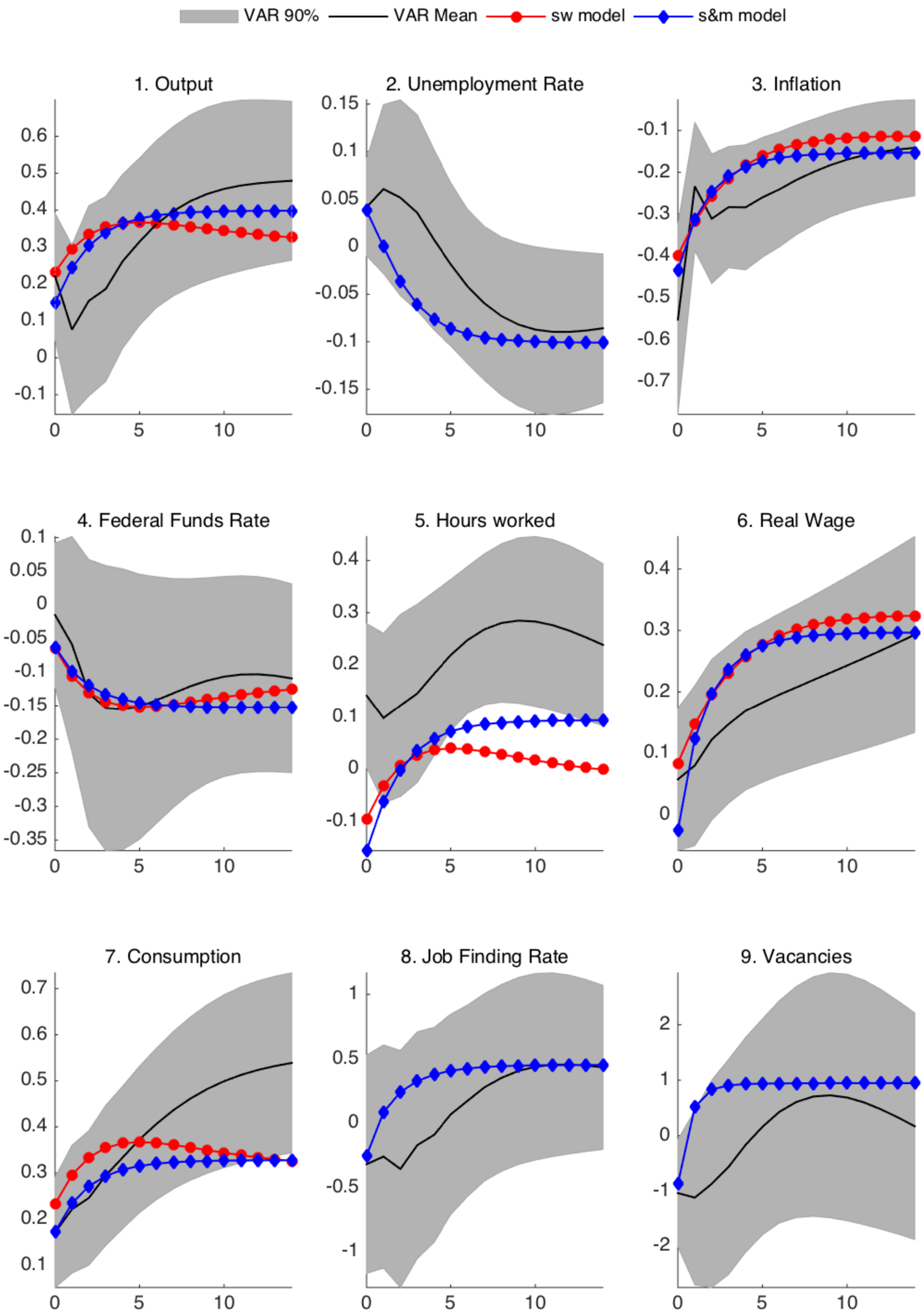
Note: Table 5 reports the optimal parameterizations of the simple rule in (49) when the policymaker has two reference model, the NK model with search and matching frictions (s&m) and the NK model with sticky wages and no indexation (sw) for two alternative specifications of the model. The model is parameterized as in Tables 1 and 2, with the exception that we raise the persistence of the price markup shock from zero to $\rho_u = 0.2$ in Panel (a), and we raise the degree of habit persistence from zero to $\mu = 0.6$ (Panel b). Under model averaging, the policymaker minimizes the expected loss given a probability distribution (prior). Under the minmax strategy, the policymaker searches for a policy rule that minimizes the maximum loss. "Objective" measures the value of the policymaker's objective function at the optimum. The columns $\mathcal{L}_t^{s\&m}(\Theta^*)$ and $\mathcal{L}_t^{sw}(\Theta^*)$ give the value of the expected loss in each model, the columns $CEV^{s\&m}(\Theta^*)$ and $CEV^{sw}(\Theta^*)$ translate these losses into consumption equivalent variations.

Table 6: Optimal Simple Rules under a Simple Loss Function $\mathcal{L}_t^{sql}$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}_t^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}_t^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Model Averaging	$(0,1)$	0	0	0	41.8350	0.0120	2.2892	0.2418	3.1106	0.0069
	$(0.1,0.9)$	0.4376	0	4.0245	20.1245	0.0118	2.2708	0.2234	3.1104	0.0067
	$(0.2,0.8)$	0.9269	0	4.1139	20.1418	0.0117	2.2703	0.2229	3.1107	0.0070
	$(0.3,0.7)$	0.9999	0	4.0938	19.9256	0.0116	2.2702	0.2228	3.1107	0.0070
	$(0.4,0.6)$	0.9999	0	2.1613	10.2326	0.0114	2.2697	0.2223	3.1108	0.0071
	$(0.5,0.5)$	0.9999	0	1.3662	6.2492	0.0113	2.2691	0.2217	3.1109	0.0072
	$(0.6,0.4)$	0.9999	0	0.8275	3.5735	0.0112	2.2681	0.2207	3.1111	0.0074
	$(0.7,0.3)$	0.9999	0	0.4872	1.9215	0.0110	2.2665	0.2191	3.1114	0.0077
	$(0.8,0.2)$	0.9999	0	0.2646	0.9015	0.0109	2.2636	0.2162	3.1123	0.0086
	$(0.9,0.1)$	0.9999	0	0.1128	0.3014	0.0107	2.2582	0.2108	3.1148	0.0111
	$(1,0)$	0.9999	0.05	0	0.1003	0.0104	2.2501	0.2027	3.1236	0.0199
Minmax	N.A.	0	0	0	41.8350	0.0120	2.2892	0.2418	3.1106	0.0069

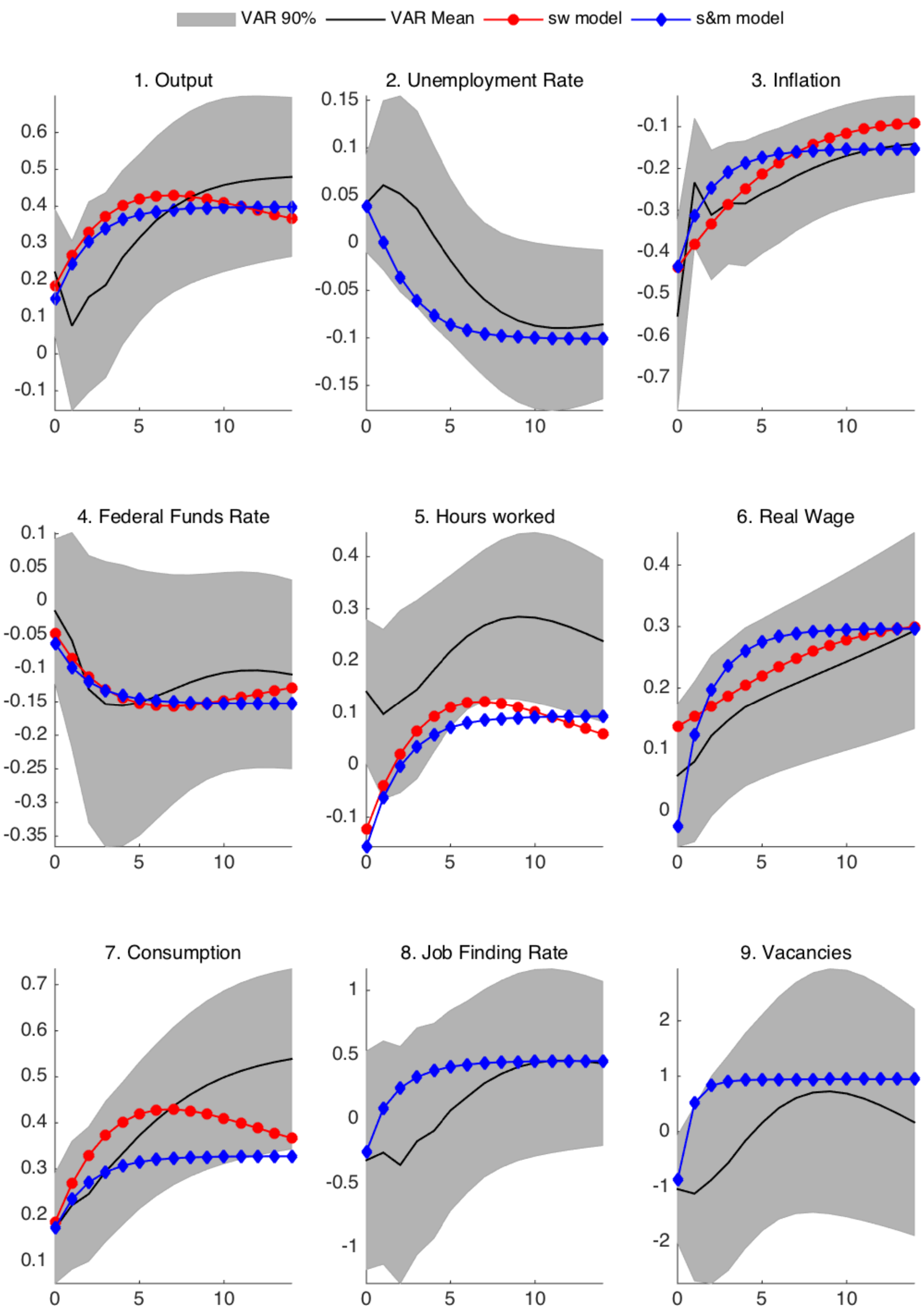
Note: Table 6 reports the optimal parameterizations of the simple rule in (49) when the policymaker has two reference model, the NK model with search and matching frictions (s&m) and the NK model with sticky wages and no indexation (sw). In contrast to Table 3, the policymaker's preferences are described by the simple loss function of the form $\mathcal{L}_t = \pi_t^2 + x_t^2$ in both models. The model is parameterized as in Tables 1 and 2. Under model averaging, the policymaker minimizes the expected loss given a probability distribution (prior). Under the minmax strategy, the policymaker searches for a policy rule that minimizes the maximum loss. "Objective" measures the value of the policymaker's objective function at the optimum, i.e., the simple loss function. The columns $\mathcal{L}_t^{s\&m}(\Theta^*)$ and $\mathcal{L}_t^{sw}(\Theta^*)$ give the values of the expected loss in each model from the perspective of the representative household, the columns $CEV^{s\&m}(\Theta^*)$ and $CEV^{sw}(\Theta^*)$ translate these losses into consumption equivalent variations.

Figure 1: Impulse response function matching under neutral technology shock



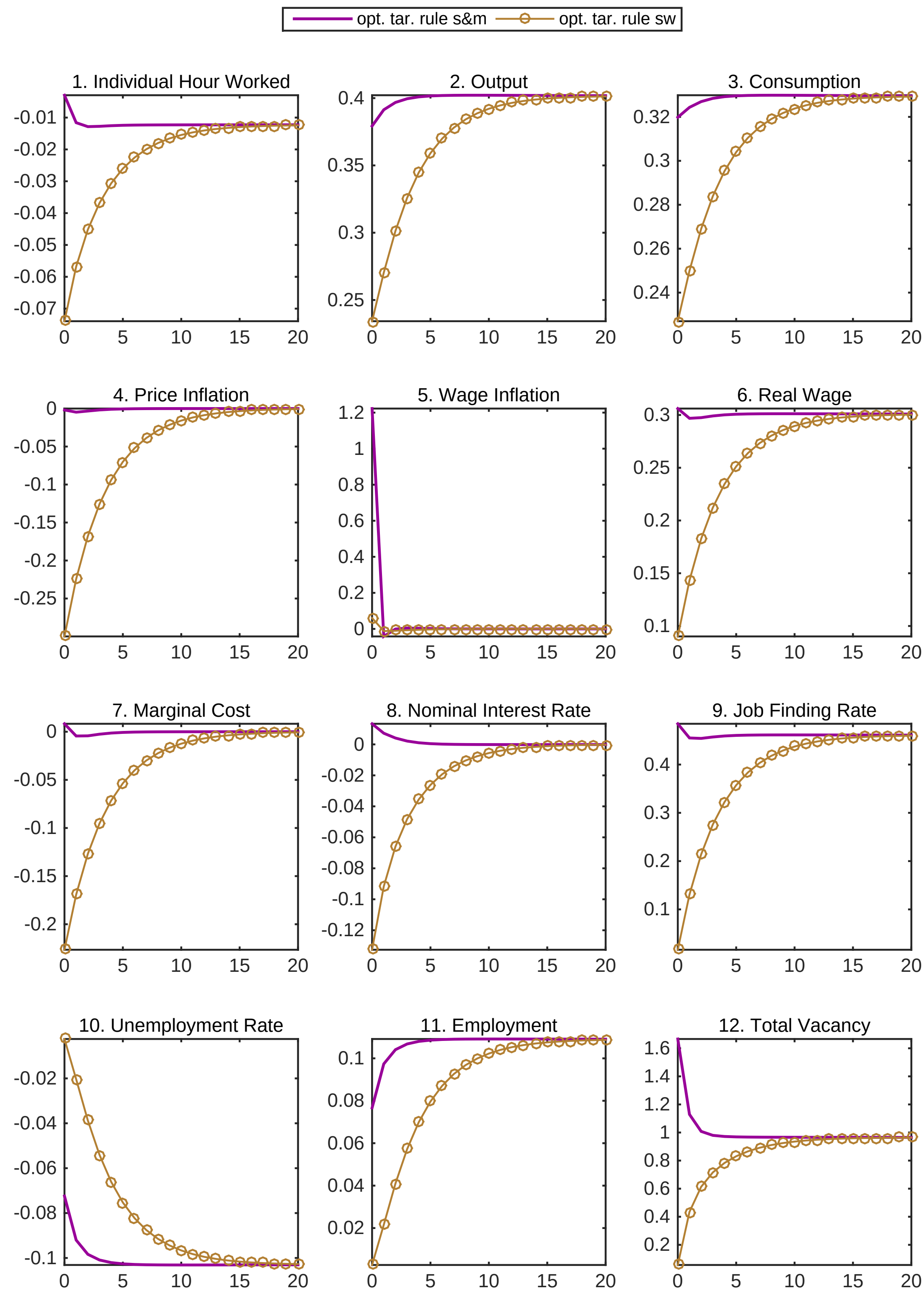
Note: Figure 1 depicts the impulse responses to a neutral technology shock in the search and matching model (blue) and the sticky wage model (red). The solid black lines show the point estimates of the empirical impulse responses along with the 90% confidence interval, the grey shaded area. Inflation rates and the federal fund rate are annualized.

Figure 2: Impulse response function matching under neutral technology shock with wage indexation in the sticky wage model



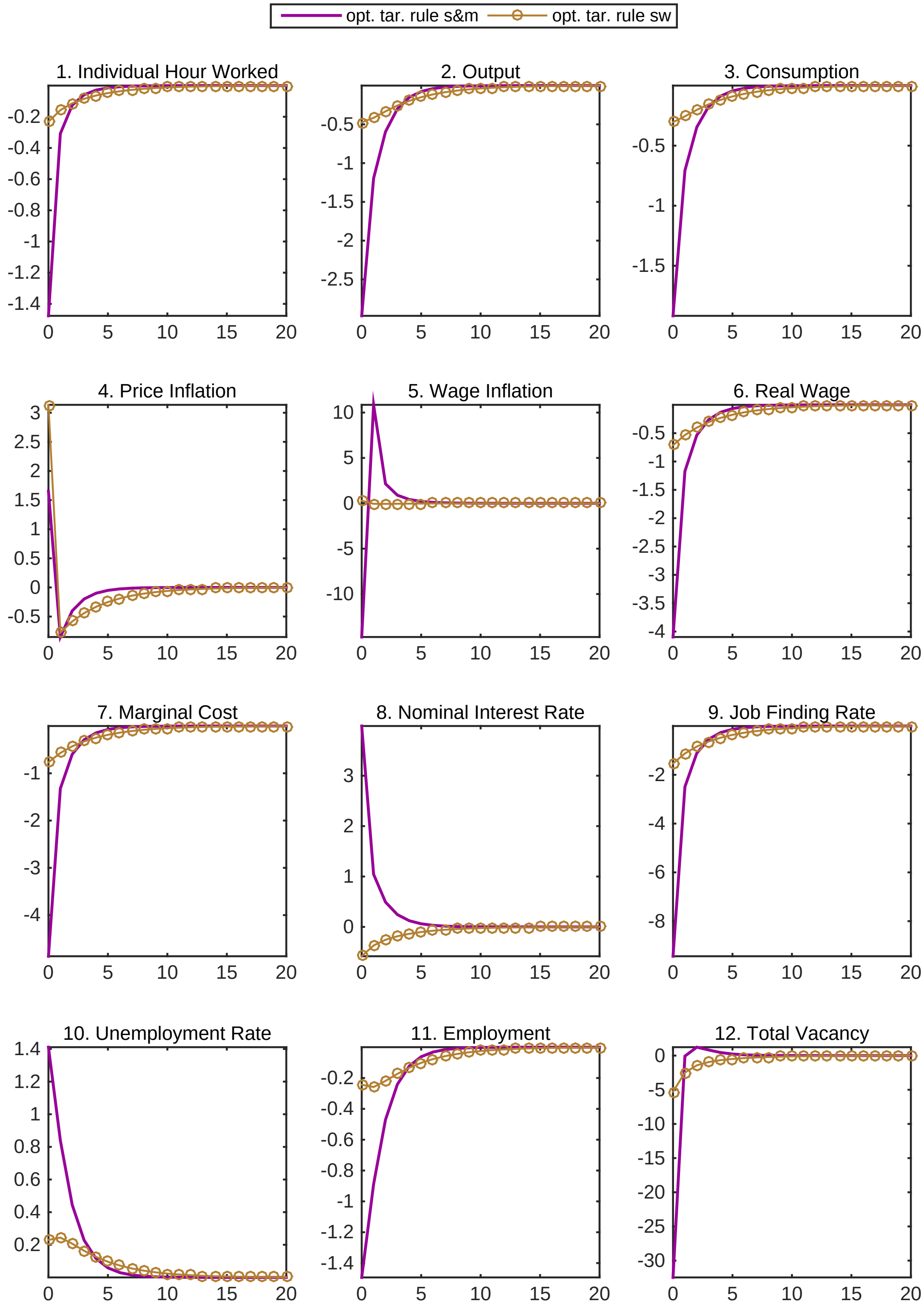
Note: Figure 2 depicts the impulse responses to a neutral technology shock in the search and matching model (blue) and the sticky wage model (red). The solid black lines show the point estimates of the empirical impulse responses along with the 90% confidence interval, the grey shaded area. Inflation rates and the federal fund rate are annualized.

Figure 3: Targeting rules in the search and matching model: neutral technology shock



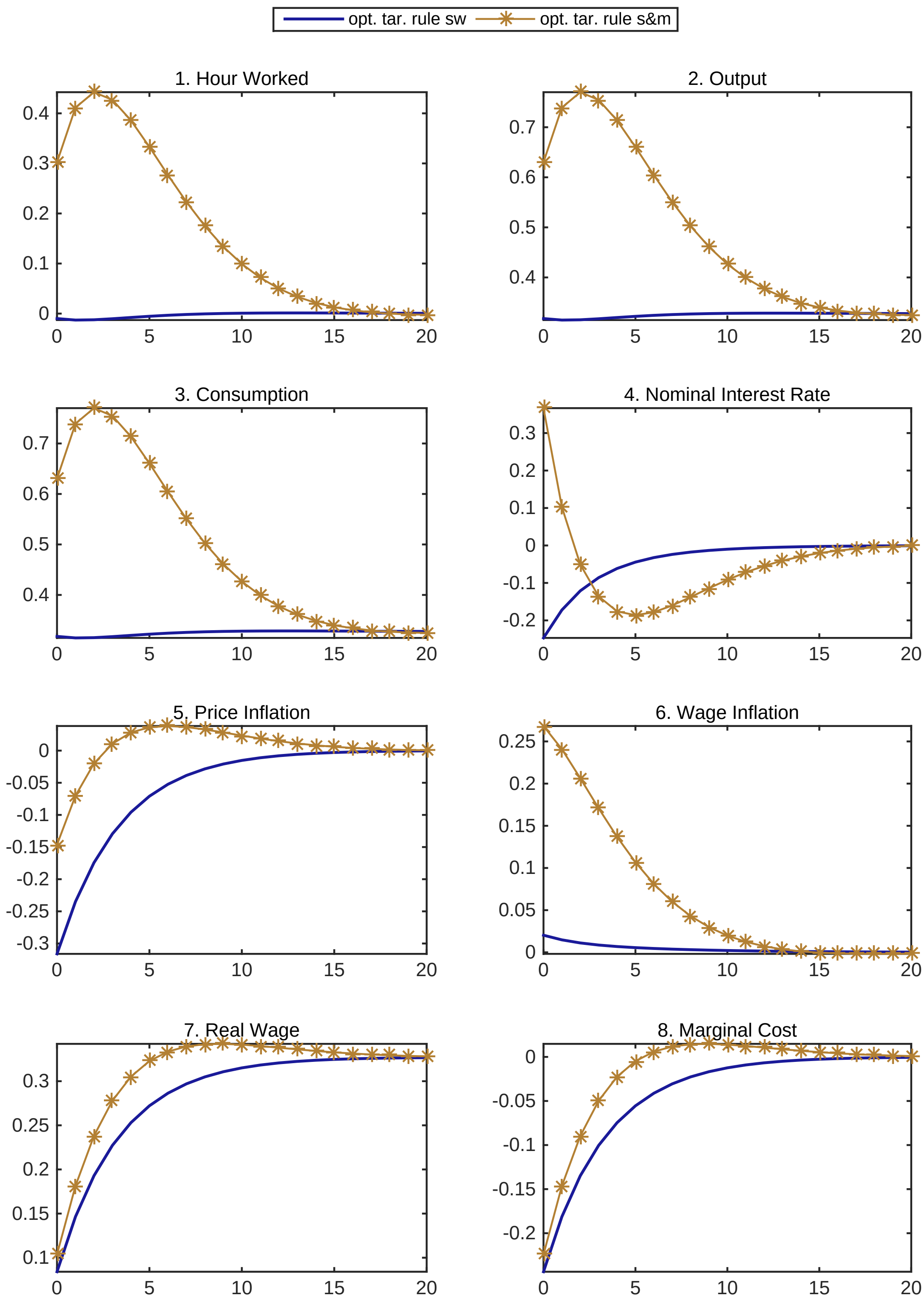
Note: Figure 3 plots the impulse responses in the search and matching model to a neutral technology shock when policy follows the optimal targeting rule from the search and matching model (purple) and the sticky wage model (yellow).

Figure 4: Targeting rules in the search and matching model: price markup shock



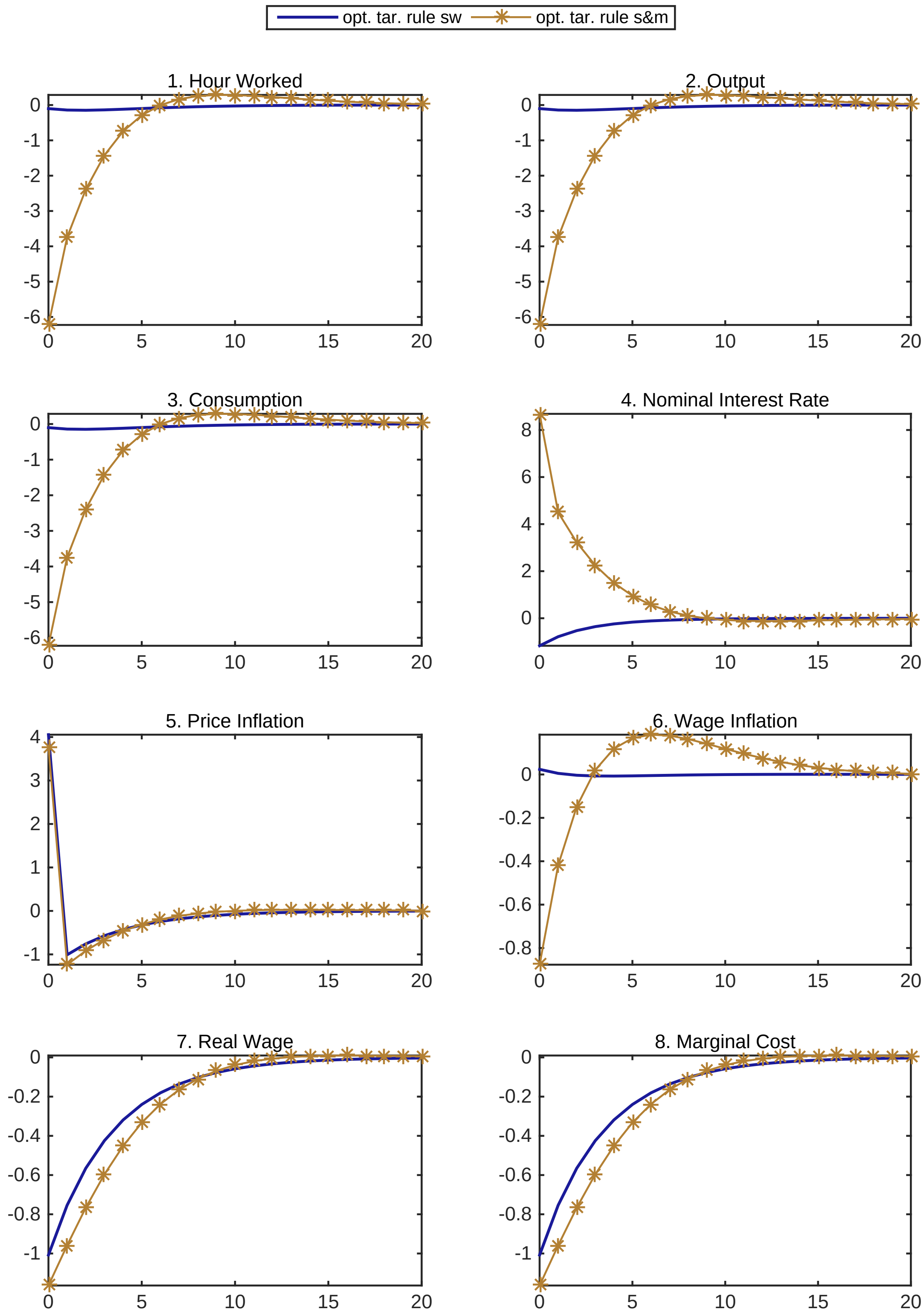
Note: Figure 4 plots the impulse responses in the search and matching model to a price markup shock when policy follows the optimal targeting rule from the search and matching model (purple) and the sticky wage model (yellow).

Figure 5: Targeting rules in the sticky wage model: neutral technology shock



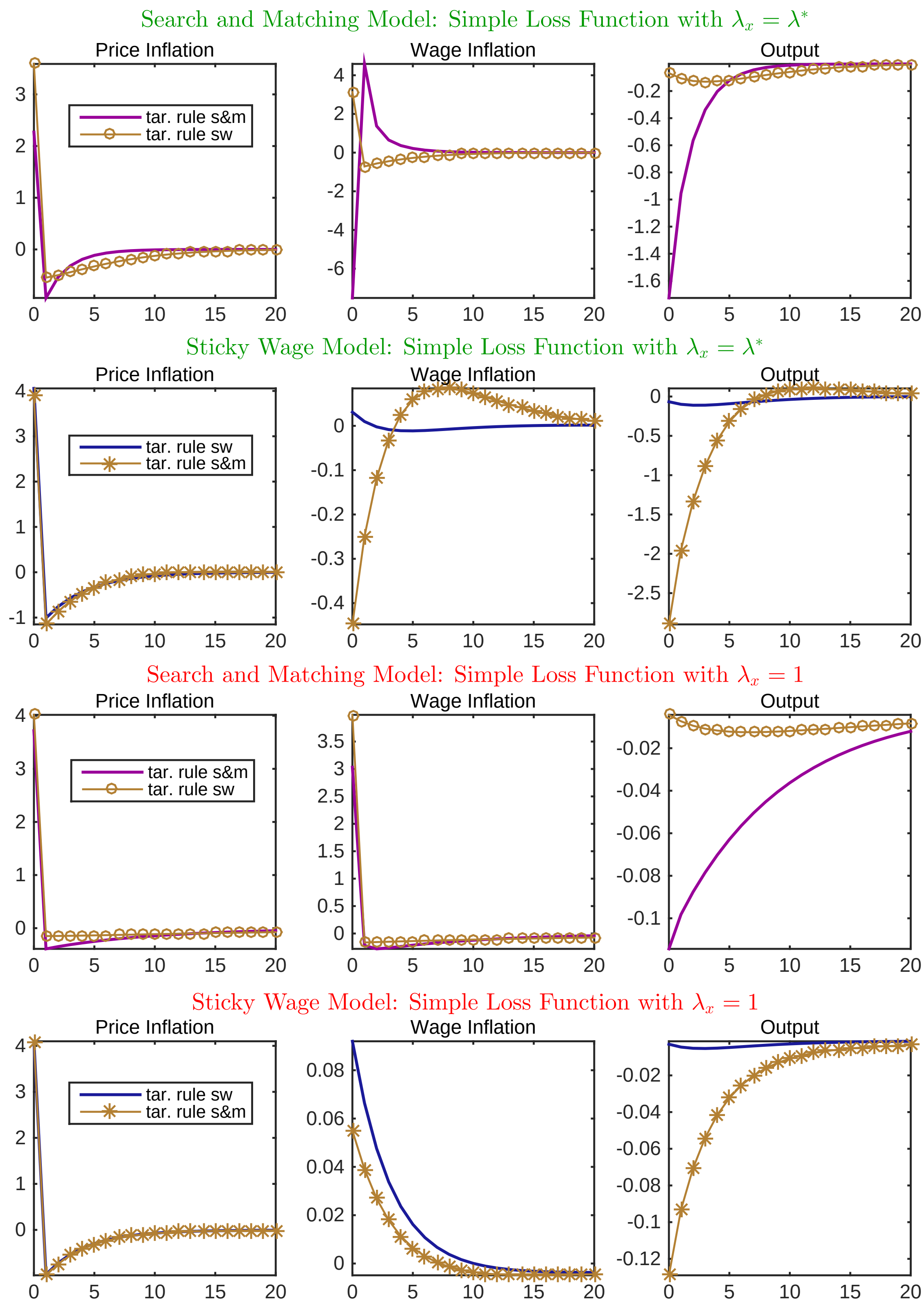
Note: Figure 5 plots the impulse responses in the sticky wage model to a neutral technology shock when policy follows the optimal targeting rule from the sticky wage model (blue) and the search and matching model (yellow).

Figure 6: Targeting rules in the sticky wage model: price markup shock



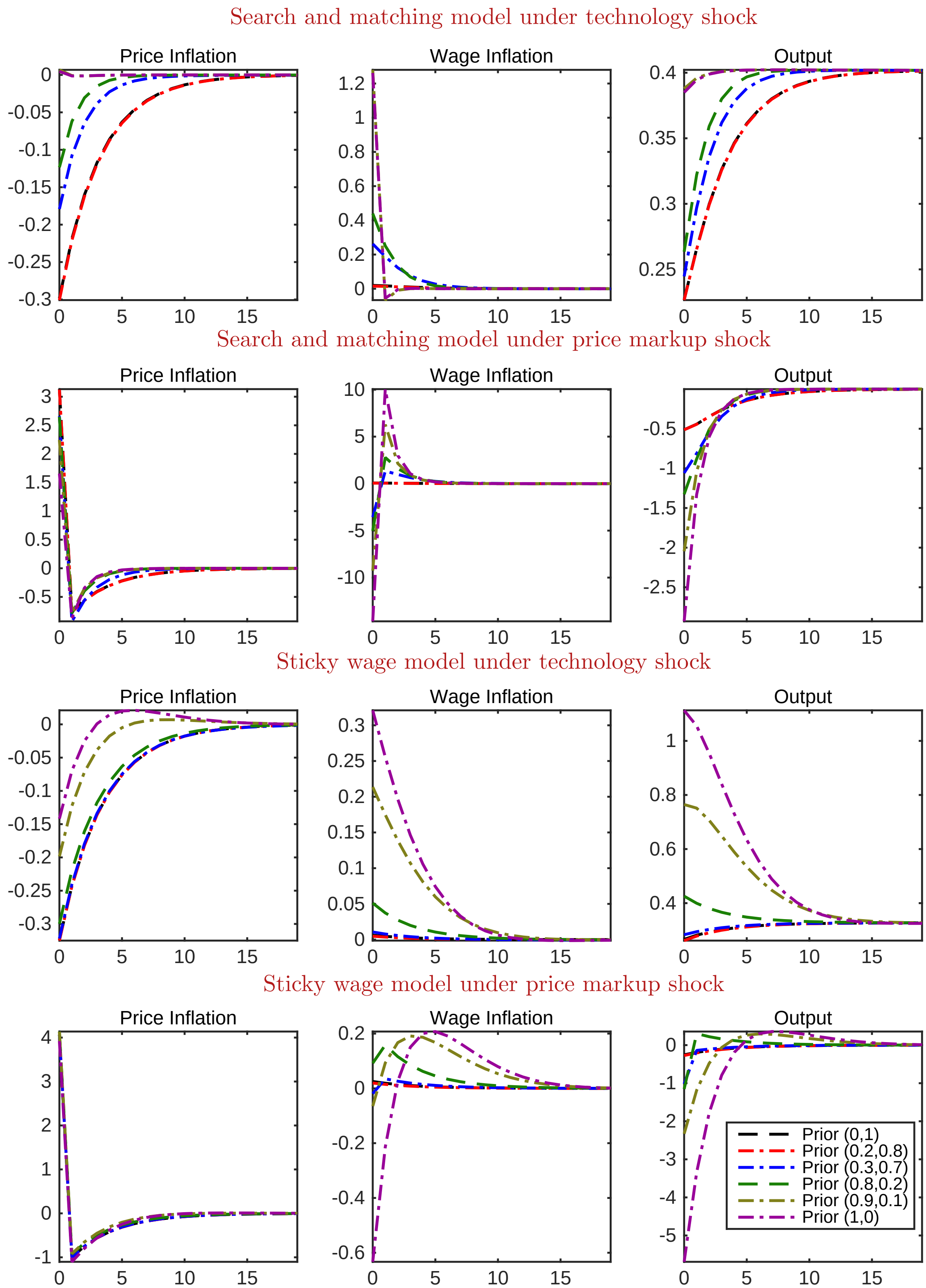
Note: Figure 6 plots the impulse responses in the sticky wage model to a price markup shock when policy follows the optimal targeting rule from the sticky wage model (blue) and the search and matching model (yellow).

Figure 7: Targeting rules with simple loss function: price markup shock



Note: Figure 7 compares the performance of optimal targeting rules derived from the loss function $\left(\pi_t^2 + \lambda_x x_t^2\right)$ for both the search and matching model and the sticky wage model in response to a *price markup shock*. In the upper six panels, it is $\lambda_x = \lambda^* = 0.0429$; in the lower six panels it is $\lambda_x = 1$.

Figure 8: Impulse responses under optimal simple rules



Note: Figure 8 compares the performance of the search and matching and the sticky wage model under ω-optimal simple rules $(0,1)$, $(0.2,0.8)$, $(0.3,0.7)$, $(0.8,0.2)$, $(0.9,0.1)$, and $(1,0)$ for the neutral technology shock and the price markup shock.