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Why Rent When You Can Buy?[*]

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Abstract

Using a model with bilateral trades, we explain why agents prefer to rent the goods they can afford to buy. Absent bilateral trading frictions, renting has no role even with uncertainty about future valuations. With pairwise meetings, agents prefer to sell (or buy) durable goods whenever they have little doubt on the future value of the good. As uncertainty grows, renting becomes more prevalent. Pairwise matching alone is sufficient to explain why agents prefer to rent, and there is no need to introduce random matching, information asymmetries, or other market frictions.

Keywords: Rent, repo, security lending, over-the-counter market, bargaining, bilateral matching, directed search

JEL Codes: G11, E44, C78

1. Introduction

Most of us have already experienced the bitter feeling of purchasing an object that we thought we would use but sits forgotten in our basement. For some it is the pair of crampons used to conquer Denali, for others the chainsaw to put down the old cherry tree. Several months

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or even years later, this lone object with the description "barely used" will be the subject of haggling in a spring garage sale. And when the first lowball offer comes, we really wish we had rented it at the time. This bitterness must also be common in the corporate world. Purchasing managers resort more often than not to the leasing of durable goods (see, for example Gavazza 2011, for aircraft leasing). Human resources managers use staffing agencies to increase their workforce temporarily. Even financial traders "rent" financial assets through repurchase agreements (repo), although they could buy the assets they need. The general view is that these managers are financially constrained (see Eisfeldt & Rampini 2009), but it is not necessarily the case: after all, the human resources manager will have to pay the wage bill to the staffing agencies and therefore must have the means to hire the extra personnel. The weakness of this general view is even clearer with a repo, a spot purchase of an asset combined with a forward contract that promises the repurchase of the asset at a later date. The use of repos begs the question: If we can buy the good or asset, why, and in which situations, do we prefer to rent it?

A plausible explanation is that the seller has private information on the quality of the good for sale that makes it less liquid (see, for example Madison 2016). But most goods for rent are either new or of brands that are well known for their reliability. Also, most financial repos involve highly liquid securities, such as Treasury bills, that are unlikely to be subject to private information. So, the question remains: Why rent when you can buy?

In this paper, we build on the fact that most durable goods or securities are traded over-the-counter and not in a centralized frictionless market. We present a parsimonious environment with bilateral trades where the agents' utilities of holding an asset or a good are stochastic. We assume the goods/assets do not depreciate and that there is a fixed stock, thus allowing us to abstract from issues of production. We show that agents trading in frictionless markets would find selling and renting redundant and thus remains the question "Why rent when you can buy?"

We then introduce frictions in the form of pairwise meeting and trading, which prevent the emergence of a Walrasian market outcome.[1] Unlike a Walrasian environment, in pairwise trading the price of the goods is a function of the sellers' identity, who they meet, their asset holdings, and the asset holdings of others. Thus, agents prefer to own some durable goods, even though they know they may not need these goods in the (near) future. Uncertainty about the future implies that agents prefer to own rather than to rent. If agents do not need

[1] More generally, we assume that agents are unable to trade within a group with an aggregate valuation identical to the Walrasian market valuation.

the durables in one period, they just rent them out. While agents could sell the durables, they prefer not to do so, as they recognize the existence of a hold-up problem inherent to bilateral trades: When the time comes to buy some of the durables back, their next trading partner could exploit their urge to consume and charge them a high price. As a result, agents are not indifferent between renting and selling, and they optimally combine both forms of trades. This argument is robust to different fiscal treatments: If there is a tax or subsidy on rentals, they are still useful as long as the tax or the subsidy is not too large.

Although we present a search environment, our results are not driven by random matching. In fact, we consider both random and directed matching, following Corbae, Temzelides & Wright (2003). With directed matching, we impose a matching rule that, in equilibrium, maximizes the possible surplus from trade. Although agents have a good idea of whom they will meet in the future, they still find renting goods more useful than buying them. In addition, under this matching rule we show that, with two valuation types, an invariant distribution for the ownership of durable goods has a two-point support–that is, there is one level of ownership for each valuation types. The difference between these two levels increases with the persistence of the valuation types: As they become more persistent, agents' levels of ownership diverge. Inversely, as switching between the two valuation shocks becomes more likely, agents tend to hold the same amount of the goods and rent more.

Our results are not due to the inability of agents to find a trading partner. In contrast to other studies in the related literature, agents in our model always meet somebody with whom they can trade. The key friction in this model lies in the assumption of bilateral trading with an inefficient distribution of bargaining power.

Still, some degree of randomness is necessary. Using a simple example, we show that renting becomes redundant if an agent can be matched with the right individual on *and* off the equilibrium path. Therefore, renting is essential to achieve the efficient allocation whenever a trader who bought more than the advisable level (relative to the equilibrium) from a seller is unable to trade with the same seller in the future. For example, it could be that this seller changed type, or that he committed to doing another trade in the meantime. These examples show the extent to which there are search frictions in our environment.

Using our simple environment, we can analyze the effect of the rental market on sales. Specifically, one might wonder if asset sales and rentals are two substitute activities, in which case we should expect rentals and sales to co-move negatively. We find that the sales volume is a hump-shaped function of the degree of persistence in agents' valuation of the durable goods. As valuations become more persistent, there are two margins. First, we find an intensive margin: As persistence increases, the support of the distribution of asset holdings also increases. Agents who just switched their valuation from high to low are matched with agents who switched from low to high. Therefore, as the difference in their level of ownership grows, so do the gains from trade, and they trade more goods. Second, there is also an extensive margin: Because shocks are more persistent, fewer agents switch types. Therefore, total sales can either increase or decrease. As the extensive and intensive margins go against each other, sales are hump-shaped. However, we find that rentals are always decreasing with persistence. As switching types becomes more likely, agents are unwilling to alter their positions through asset sales, but they are willing to rent. Therefore, total rentals are decreasing with persistence and are higher than total sales when the uncertainty is high, or persistence is low.

Related literature Leasing by corporate firms has attracted some attention and our paper relates to the literature on leasing capital goods. Gavazza (2011) studies the leasing and secondary markets for aircrafts and empirically confirms that operators facing more volatile productivity shocks are more likely to lease aircrafts than those facing less volatile shocks. Similarly, we find that the rental volume is a decreasing function in the degree of shock persistence. However, our model differs substantially from Gavazza (2011) in which the frictions are monitoring and transaction costs. In contrast, the only friction in our model is that agents meet in pairs. Our model is also related to the literature on leasing by financially constrained firms (for example, Eisfeldt & Rampini 2009). However, agents that are not financially constrained in our model still prefer to rent rather than to buy in some cases.

The empirical literature on the repo market is large, but only a few papers explain the usefulness of repo markets. As Parlatore Siritto (2015) shows, private information on the quality of assets can be a concern, and the very fact that the seller is willing to repurchase the asset is a guarantee that the asset is of good quality. In an environment with no commitment, Mills & Reed (2008) argue that repos are useful to cover counterparty risk. However, it is not clear why agents do not sell the collateral to obtain funding, especially as lenders generally apply haircuts to the collateral.[2] Maurin, Monnet & Gottardi (2016) emphasize the uncertainty in the asset payoff. Tomura (2013) uses the mechanism we propose to analyze investment through dealers.

Our paper builds on several strands of the literature. First and foremost, our paper is

[2]See Lacker (2001) and Kehoe & Levine (2006) for models of collateralized debt. See Ewerhart & Tapking (2008) for a model with repo contracts and endogenous choice of collateral.

related to the literature on over-the-counter markets initiated by Duffie, Gârleanu & Pedersen (2005) and generalized by Lagos & Rocheteau (2009). An important difference is that we do not introduce matching frictions: Our agents meet with certainty, but they only meet in pairs. This setup lets us characterize an equilibrium distribution of asset holdings, even when holdings are arbitrary. We then obtain results on the distribution of assets. Lagos & Rocheteau (2009) find that more severe search frictions are associated with less dispersion in the equilibrium asset distribution. We find that, as it becomes more likely that agents will have to readjust their portfolios, because of increased uncertainty about future valuation, the distribution of asset holdings also becomes less dispersed.

The rest of the paper proceeds as follows. Section 2 describes the environment. Section 3 characterizes the Walrasian allocations in the benchmark case. Section 4 provides two examples to illustrate the importance of pairwise matching and uncertainty about future valuation for the coexistence of sales and rentals. Section 5 describes general allocations attainable under pairwise meeting and bargaining. Section 6 solves for the equilibrium with directed matching and solve for the equilibrium distribution and volumes in general. Section 7 concludes.

2. Environment

Time is discrete and the horizon is infinite. There is a continuum of anonymous agents. In each period, there is a measure $1/2$ of two types of agents, type h and type ℓ. To be concise, we will refer to agents of type h as agents h and to agents of type ℓ as agents ℓ. At the start of each period, the type of an agent may change. It stays the same with probability $\pi \in [1/2, 1]$, and it changes with probability $1 - \pi \in [0, 1/2]$. We appeal to the law of large numbers to guarantee that there is the same measure of each type in each period.

There is a durable good (durable(s), for short) or asset in positive fixed supply A. There is another good (the numeraire, for short) that all agents can produce. Agents $i \in \{h, \ell\}$ derive utility $u_i(a) + x$ from consuming a units of the durable or the asset's fruits and from a (net) amount x of the numeraire – where x is negative if the agent produces.[3] The

[3] The amount a could also be seen as the quality of the good that is consumed. In the case of the asset, we associate it with a Lucas tree: One unit of the asset yields one unit of some fruit, and agents derive utility $u(a)$ from consuming a units of the asset's fruits. See Lagos & Rocheteau (2009) or Duffie et al. (2005) for different interpretations of these preferences.

numeraire is used to settle trades so that settlement does not generate any net utility gains. For simplicity, we impose the following condition,

Assumption 1. $u'_h(a) \geq u'_\ell(a)$ *for all* a.

Therefore, for a given amount of durables, agents h have a higher marginal utility than agents ℓ, so agents h desire to consume more of the durables than agents ℓ.

Agents can agree to trade durables. In this case, the seller trades some of the durables for some of the numeraire, and the buyer gains ownership of these durables for the next period. Or, agents can agree only to rent the durable: Then the lessor transfers some of the durable this period but retains ownership over it next period. The renter gives the durable back to the lessor after enjoying the benefits of holding it this period.[4] We assume that renters can commit to giving back the good they rent, and we analyze the case with limited commitment at the end of section 4.

3. Walrasian benchmark

We first consider the case where the trading stage is a Walrasian market. We denote by p^r the rental price for the durable and by p^s the sales price for the durable. We consider only a stationary equilibrium so that these prices are the same in each period. An agent $i \in \{h, \ell\}$ holding durable a has a value $W_i(a)$, defined recursively as

$$\begin{aligned} W_i(a) \quad &= \quad \max_{q_i^r, q_i^s} u_i(c_i) - d + \beta \left[\pi W_i(a'_i) + (1-\pi) W_{-i}(a'_i) \right], \\ &\text{s.t.} \quad p^r q_i^r + p^s q_i^s \leq d, \\ &\qquad c_i = a + q_i^s + q_i^r, \\ &\qquad a'_i = a + q_i^s, \end{aligned}$$

where d is the total amount of numeraire spent on renting and purchasing the durables. We also use the usual convention that $i \neq -i \in \{h, \ell\}$. The agent decides to rent q_i^r and to buy

[4] Here, trade and settlement takes place the same time. Koeppl, Monnet & Temzelides (2012) model delayed settlement more precisely. Notice that we model the surplus from trade as originating from reallocating the durable from agents with low to high valuation for the asset. Alternatively, the surplus from trade could originate from reallocating the numeraire, in which case the agent with a high value for the numeraire will compensate the other agent. Also, notice that one interpretation of the model is that the lessor agrees to a repurchase agreement in which the buyer buys an asset at date t using a spot contract, keeps the fruit that the asset yields at date t, and commits to selling the asset back to the seller at the start of date $t+1$ using a forward contract.

q_i^s. Therefore, his current consumption c_i may differ from his holding of durables for the next period a_i'. Naturally, the rented amount q_i^r does not enter into the continuation valuation but only in the momentary utility $u_i(.)$. The first-order and envelope conditions yield

$$\begin{aligned} u_i'(c_i) + \beta\left[\pi W_i'(a_i') + (1-\pi)W_{-i}'(a_i')\right] &= p^s, \\ u_i'(c_i) &= p^r, \\ W_i'(a) &= p^s. \end{aligned} \tag{1}$$

Notice from (1) that all agents value an additional unit of the durable in the same way when they enter the Walrasian market, independent of their type and of their holdings of the durable. There are two reasons for this outcome: First, the utility is linear in the numeraire such that there is no wealth effect, and, second, agents are playing against their budget constraint. The equilibrium prices and quantities satisfy

$$\begin{aligned} (1-\beta)p^s &= p^r, \\ u_h'(c_h) = u_\ell'(c_\ell) &= p^r, \\ c_h + c_\ell &= 2A. \end{aligned}$$

The first equation is a no-arbitrage condition: Agents have to be indifferent between renting, in which case they have to pay the price p^r, and buying some durable at price p^s and reselling it in the next period at price βp^s. These two schemes are payoff equivalent and so should be their cost. As a consequence, anything goes for renting, and, in particular, $q_h^r = q_\ell^r = 0$. In other words, in a Walrasian market, there is no difference between renting or buying and selling the asset: Rentals and outright purchases are perfect substitutes. Therefore, absent any additional frictions, the Walrasian benchmark cannot explain why the rental market exists. In the following sections, we depart from the Walrasian benchmark by assuming that agents trade bilaterally (or over-the-counter), in which case they bargain over the allocation.

4. Two-period examples

In the first example, we illustrate the main mechanism at work in the general setup by showing how *future* pairwise matching can explain a mix of renting and sales today, even though (1) agents trade on a Walrasian market *today* and (2) agents know who they will meet tomorrow.

We consider a two-period economy, where agents are endowed with some durables A in the first period. Also in the first period, agents have access to a Walrasian market where they can both sell and rent durables. An agent $i \in \{h, \ell\}$ derives utility from consuming durables in both periods and producing the numeraire according to the utility

$$U_i(c^1, d^1, c^2, d^2) = u_i(c^1) - d^1 + \beta\pi \left[u_i(c^2) - d^2\right] + (1 - \pi) \left[u_{-i}(c^2) - d^2\right],$$

where $u_h(c)$ and $u_\ell(c)$ satisfy Assumption 1. Because the world ends at date 2, there is no difference between rentals and sales at that date, as only the consumption of the durable matters. However, we will show that pairwise matching at date 2 can explain the mix of rentals and sales in the Walrasian market at date 1.

Let us first consider the problem of an agent i with durables A in the first period when the Walrasian sales price is p^s and the rental price p^r. Then, denoting the expected value for the agent i of holding a_i durable at date 2 by $V_i(a_i')$, the problem of the agent i is

$$\begin{aligned}
\max_{q^s, q^r} \quad & u_i\left(c_i^1\right) - d^1 + \beta V_i(a_i'), \\
s.t. \quad & p^r q^r + p^s q^s \leq d^1, \\
& c_i^1 = A + q^s + q^r, \\
& a_i' = A + q^s.
\end{aligned}$$

Using the first-order conditions for the two types, we obtain $c_i^1 = c_i^*$ and $a_i' = a_i^*$ for $i \in \{h, \ell\}$, where

$$u_\ell'(c_\ell^*) = u_h'(c_h^*), \tag{2}$$

$$V_h'(a_h^*) = V_\ell'(a_\ell^*), \tag{3}$$

where the resource constraint is $c_\ell^* + c_h^* = a_h^* + a_\ell^* = 2A$. Prices satisfy $p^s = p^r + \beta V_i'(a_i^*)$ and $p^r = u_i'(c_i^*)$. As the utility functions are concave, there is a unique pair (c_h^*, c_ℓ^*) that satisfies (2) and the resource constraint. Similarly, if V_i is concave, there is a unique pair (a_h^*, a_ℓ^*) that satisfies (3) and the resource constraint. Notice that the rental market is active whenever $a_i^* \neq c_i^*$. This result is what we will show below by solving for the functions $V_i(a)$.

In period 2, agents trade bilaterally. The matching rule specifies that each agent h is matched with an agent ℓ and agents who switched types are matched together. Given that the distribution of durables has a two-point support at the end of period 1, this matching rule implies that agent i holding durables a will be matched with an agent $-i \neq i$ holding durables $2A - a$. Therefore, in the equilibrium agents know exactly the type and holdings of the agent they will meet. We assume that the agents h and ℓ bargain over the terms of trade so that the consumption levels c_h^2, c_ℓ^2, and the payment d are the solutions to the following bargaining problem at $t = 2$:

$$\begin{array}{ll} \max_{c_h^2, c_\ell^2, a_h, a_\ell, d} & \left[u_h(c_h^2) - d^2 - u_h(a_h)\right]^\theta \left[u_\ell(c_\ell^2) + d^2 - u_\ell(a_\ell)\right]^{1-\theta}, \\ s.t. & c_h^2 + c_\ell^2 = a_h + a_\ell = 2A, \end{array}$$

where θ denotes the bargaining power of the agent h. Let us emphasize that a_h is not necessarily equal to a_h^*: Indeed, some agents ℓ holding a_ℓ^* may change type so that in period 2 some agents h are holding a_ℓ^* (and some agents ℓ are holding a_h^*). Still, the matching is such that in equilibrium $a_h + a_\ell = 2A$ for all pairs. The first-order conditions give us

$$u_h'(c_h^*) = u_\ell'(c_\ell^*),$$

where $c_h^* + c_\ell^* = 2A$ and d^2 satisfies

$$d^2 = (1-\theta)\left[u_h(c_h^*) - u_h(a_h)\right] + \theta\left[u_\ell(c_\ell^*) - u_\ell(a_\ell)\right].$$

Therefore, the consumption of agents i in period 2 is efficient, as in period 1. Using these first-order conditions, we obtain the usual expression for the payoff of an agent of type i holding a units of the asset, $v_i(a)$ for $i \in \{h, \ell\}$:

$$\begin{aligned} v_h(a) &= u_h(a) + \theta S(a, a_\ell), && (4) \\ v_\ell(a) &= u_\ell(a) + (1-\theta) S(a_h, a), && (5) \end{aligned}$$

so that agents split the surplus $S(a_h, a_\ell) = u_h(c_h^*) + u_\ell(c_\ell^*) - u_h(a_h) - u_\ell(a_\ell)$ according to their bargaining power. Notice that, contrary to the case with a frictionless market, the payoff is not linear in the amount of durables held, but naturally depends on both agents' holdings of durables. In particular, a quick inspection of (4) and (5) and the bargaining problem reveals that the agents' outside options affect the marginal payoffs in period 2. While this effect is well known, it is worth pointing out, because it explains why agents prefer to rent when they could instead buy more durables.

We can now write the expected value for agent i of holding durables a_i at date 2 as,

$$V_i(a_i) = \pi v_i(a_i) + (1-\pi)v_{-i}(a_i).$$

Clearly, the marginal payoffs are sensitive to the holdings of durables because $V_i''(a) < 0$, which confirms our initial guess that (3) was giving a two-point support distribution for durables holdings at the end of the Walrasian market. Using (4) and (5), we can see that $V_i'(a)$ is a scaled weighted average of the marginal utility of each type, that is

$$V_i'(a) = u_i'(c_i^*) + \pi(1-\theta_i)\left[u_i'(a) - u_i'(c_i^*)\right] + (1-\pi)\theta_i\left[u_{-i}'(a) - u_i'(c_i^*)\right], \tag{6}$$

with the natural conventions $\theta_h = \theta$ and $\theta_\ell = 1-\theta$. To derive (6) we assume that an agent i holding an amount of durables that does not belong to the equilibrium support of the holdings distribution is almost surely matched with any agents of type $-i$ who just changed type. If $\pi < 1$, then (1) and (6) evaluated at $a = c_i^*$ imply

$$V_\ell'(c_\ell^*) > u_\ell'(c_\ell^*) = u_h'(c_h^*) > V_h'(c_h^*), \tag{7}$$

so that at (c_h^*, c_ℓ^*) there is a wedge between the value of present and future durables consumption.[5] A quick inspection of (6) reveals why this is so: If an ℓ agent holds c_ℓ^*, he may switch to being a high type with probability $1-\pi$. In this case, he holds too little durables, in which case his marginal utility is higher. A similar intuition holds for agents h.

As the marginal continuation utilities $V_i'(.)$ across the two types are not equated at the optimal consumption levels c_h^* and c_ℓ^*, the current marginal utility $u_i'(.)$ cannot be the same at the optimal holdings a_h^* and a_ℓ^*. Therefore, a_i^* cannot equate c_i^* for $i \in \{h, \ell\}$, and both the rental and the sales markets are active at $t = 1$. If $\pi = 1$, there is no wedge in the sense that $V_i'(c_i^*) = u_i'(c_i^*)$, and in this case $a_i^* = c_i^*$: The rental market is inactive.

Notice that we could also obtain that rentals are not essential, i.e., $V_i'(c_i^*) = u_i'(c_i^*)$, by defining the bargaining power appropriately as a function of both agents' holdings of durables. Therefore, one could reinterpret our result in terms of bargaining power: Agents rent, as they could later be matched with agents with too much bargaining power.

Figure 1 illustrates the choice of c_h^* and c_ℓ^* as well as why the rental market is active whenever the inequalities in (7) hold. On the left, figure 1 shows the intratemporal indifference curves for $u_i(a) - d_i$ for the two types of agents, where in equilibrium $d_\ell = -d_h$.

[5] The wedge equals $(1-\pi)\theta_i[u_j'(c_i^*) - u_i'(c_i^*)]$, and it is negative for agents h and positive for agents ℓ.

These indifference curves are tangential at the equilibrium consumption level (c_h^*, c_ℓ^*), where $c_h^* + c_\ell^* = 2A$ (the axis for d is reversed). On the right, figure 1 superimposes the intertemporal indifference curves $V_i(c) - d_i$. As (7) holds, the intratemporal indifference curve of the agent h is steeper than its intertemporal indifference curve. Inversely, the intertemporal indifference curve of the agent ℓ is steeper than its intratemporal indifference curve. Therefore, and as illustrated in figure 1, the intertemporal indifference curves for both agents will be tangent at a point northwest of (c_h^*, c_ℓ^*). The difference between intertemporal and intratemporal indifference curves explains why renting is useful: Asset sales alone are leaving unexploited gains from trade.

This example shows the importance of an agent's outside option to bargaining: It is this outside option that determines the marginal value of holding durables. Holding too little durables would give a bad outside option, and, as $V_i''(a) < 0$, holding too much may cost too much relative to the additional benefits. This problem explains why agents do not want to take extreme positions where they would rent or sell everything. Also, this example illustrates that it is the hold-up problem from pairwise trading which matters for the result, as well as the random matching off equilibrium. Below, we will show that this result holds true in the more general set-up.

Finally, how does the sales market react to a shock to the rental market? Suppose that the rental market shuts down in the first period so that agents can only buy or sell durables. In appendix 8.1., we show that agents h will purchase more than a_h^*, as they want to get closer to c_h^* today, and because they will not be able to do so by renting, the sales volume will increase. However, agents h will not buy as much as c_h^*, as they may face a hold-up problem tomorrow. Therefore, agents will buy some amount between a_h^* and c_h^*. The spot price with no rental market will then be $p^s = u_h'(a_h) + \beta V_h'(a_h)$ for some $a_h \in [a_h^*, c_h^*]$. It is difficult to say how the sales price will move, however, as there are two countervailing effects: First, more durables is desirable, as agents are able to consume more today and this increased sale volume tends to increase the sales price. At the same time there is a second effect in play. The hold-up problem is more severe than with rentals, as agents have to keep any asset they buy, and this effect tends to decrease the sales price. Which effect dominates depends on the curvature of the utility function. In the case of the CRRA utility function with $u_h(a) = \alpha u_\ell(a)$, we find that the sales price is lower when the rental market is inoperative, for all possible levels of persistence.

Security deposit In the case of small- (and sometimes large-) ticket items, the lessor can require a security deposit that can be substantial and even as large as the sales price. In the context of this two-period example, we want to show how a security deposit can arise. We first need to introduce the notion of security deposit s per unit of durables. When the rental price includes a security deposit, the effective rental price is $p^r + s$ and the lessor rents some durables at price p^r in the first period with the promise to return the security deposit d when the renter returns the durables. To have a notion of default risk, we suppose that a fraction δ of agents h or ℓ (but not both) can choose to keep the durables they are renting. Therefore, the security deposit must guarantee that all agents execute the second leg of their rental agreement.

In the Walrasian market, agents of type i solve the following problem:

$$
\begin{aligned}
\max_{q^r,q^s} \quad & u_i\left(c_i^1\right) - (p^r + s) \cdot q^r - p^s \cdot q^s + +\beta\left\{s \cdot q^r + V_i(a_i')\right\}, \\
s.t. \quad & c_i^1 = A + q^s + q^r, \\
& a_i' = A + q^s.
\end{aligned}
$$

The solutions to both problems imply

$$
p^r + s - \beta s = u_h'(c_h^*) = u_\ell'(c_\ell^*)
$$

and

$$
p^s = u_h'(c_h^*) + \beta V_h'(a_h^*) = u_\ell'(c_\ell^*) + \beta V_\ell'(a_\ell^*),
$$

where $V_i'(a)$ is given in (6). The security deposit is set such that no agent defaults. As the marginal benefit is higher for agents h, they do not default whenever

$$
s_h = v_h'(a_h^*) = (1 - \theta_h) \cdot u_h'(a_h^*) + \theta_h \cdot u_h'(c_h^*).
$$

In the case where the renter can default, the lessor, the agent ℓ, will apply such a high security deposit that the overall transfer in a rental agreement is higher than the sales price. That is, the security deposit satisfies

$$
p^s = u_h'(c_h^*) + \beta V_h'(a_h^*) < u_h'(c_h^*) + \beta v_h'(a_h^*) = (p^r + s_h - \beta \cdot s_h) + \beta s_h = p^r + s_h.
$$

In the second case, where the lessor, the agent ℓ, can default and fail to return the deposit, the security deposit should induce the lessor to reclaim even the very last rented unit, or

$$s_\ell = v'_\ell(a^*_\ell) = \theta_h \cdot u'_\ell(a^*_\ell) + (1-\theta_h) \cdot u'_\ell(c^*_\ell).$$

Then the agent ℓ receives $p^r + s$ units per each unit of durables rented rents to the agent h, and there is a small and even negative security deposit, as

$$p^s = u'_\ell(c^*_\ell) + \beta V'_\ell(a^*_\ell) > u'_\ell(c^*_\ell) + \beta v'_\ell(a^*_\ell) = (p^r + s_\ell - \beta \cdot s_\ell) + \beta s_\ell = p^r + s_\ell.$$

When the security deposit is negative, the lessor is paid when the rented durables are returned.

Note that in both cases, the (positive or negative) security deposit is independent of the probability of possible default δ. But the deposit is a function of the type persistence, π, and the value of durables for different types.[6] In both cases, the security deposit is increasing in our measures of uncertainty: as π decreases to $1/2$, or as agents become more risk averse, the ratio of security deposit to sales price increases, as shown in figure 2. The intuition is that, as $\pi \to 1$, agent i's expected marginal value of holding the good is very close to his marginal value in the second period, that is, $V'_i(a^*_i)$ is close to $v'_i(a^*_i)$ for $i \in \{h, \ell\}$. Therefore, the sales price that accounts for the expected marginal value has to be close to the rental price, which accounts for the marginal value in the second period. Symmetrically, as $\pi \to 1/2$, the difference between the expected and the realized marginal values, and thus the security deposit, increases.

5. Pairwise meeting and bargaining in infinite horizon

In this section we return to the infinite horizon setup for our analysis. We assume that each agent h is matched with exactly one agent ℓ when trading. We describe the matching technology later. We consider allocations that are the solution to a bargaining game between both agents. We will consider a generic meeting between an agent h holding a generic amount of durables a_h and an agent ℓ holding a generic amount of durables a_ℓ. An allocation is a triple $\{q^s(a_h, a_\ell), q^r(a_h, a_\ell), d(a_h, a_\ell)\}$, where q^s denotes the quantity of durables that agent h buys from agent ℓ (sells if negative), q^r is the quantity of durables that agent h rents from agent ℓ, and d is the numeraire transfer that agent h makes to agent ℓ (receives if negative).

[6] For financial assets, this finding is as in Krishnamurthy, Nagel & Orlov (2014), who find that haircuts (akin to security deposit for financial assets) do not depend on the risk characteristics of borrowers.

We focus only on stationary and symmetric allocations. An allocation is feasible if

$$\begin{aligned} q^s(a_h, a_\ell) &\in [-a_h, a_\ell], \\ q^r(a_h, a_\ell) + q^s(a_h, a_\ell) &\in [-a_h, a_\ell]. \end{aligned}$$

Notice that we do not allow short selling. We will denote by $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ the feasible allocations for all possible matches such that $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ defines invariant distributions of durables holdings for agents h and ℓ. We denote these distributions by $\mu_i(a)$ for $i \in \{h, \ell\}$, where we have dropped the reference to the allocation for convenience. If they exist, a property of any pair of invariant distributions is that

$$\frac{1}{2} \int a d\mu_h(a) + \frac{1}{2} \int a d\mu_\ell(a) = A.$$

Then we can define recursively the expected value for agent $i \in \{h, \ell\}$ of holding durables a before trading, $V_i(a)$, as

$$\begin{aligned} V_h(a) &= \pi \int [u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell) \\ &+ (1 - \pi) \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h), \end{aligned} \tag{8}$$

where $c_h(a, a_\ell) = a + q^s(a, a_\ell) + q^r(a, a_\ell)$ is the durables consumption of a type h endowed with a units of durables matched with a type ℓ holding an amount a_ℓ of durables. Similarly, $c_\ell(a_h, a) = a - q^s(a_h, a) - q^r(a_h, a)$ is the durables consumption of a type ℓ endowed with a units of durables matched with a type h holding a_h units of durables. With probability π, an agent h maintains his type. Then he meets an agent ℓ with durables a_ℓ according to the distribution μ_ℓ. Because he remains an agent h, he enjoys instant utility $u_h(.)$ from consuming $a + q^s(a, a_\ell) + q^r(a, a_\ell)$. However, he only carries $a + q^s(a, a_\ell)$ to the next period with a value $\beta V_h(a + q^s(a, a_\ell))$, and he rents the difference. With probability $1 - \pi$, an agent h becomes an agent ℓ. In this case, he meets an agent h according to the distribution μ_h and he enjoys instant utility $u_\ell(.)$ from consuming $a - q^s(a_h, a) - q^r(a_h, a)$. He values his ownership according to $\beta V_\ell(a - q^s(a_h, a))$. Similarly for agents ℓ,

$$
\begin{aligned}
V_\ell(a) &= \pi \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h) \\
&+ (1-\pi) \int [u_h(c_h(a, a_\ell) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell).
\end{aligned} \tag{9}
$$

We assume that agents cannot commit to ex-ante participation, and an allocation $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ is individually rational if all agents prefer the allocation to being in autarky this period. That is, for any endowment a, an agent h matched with an agent ℓ with an endowment a_ℓ prefers the allocation to not trading today, that is,

$$
u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell)) \geq u_h(a) + \beta V_h(a),
$$

and similarly for an agent ℓ matched with an agent h with endowment a_h,

$$
u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a)) \geq u_\ell(a) + \beta V_\ell(a).
$$

With general Nash bargaining, where agents h have bargaining power $\theta \in [0, 1]$, the allocation of an agent h with endowment a_h matched with an agent ℓ with endowment a_ℓ solves the following problem:

$$
\begin{aligned}
&\max_{q^s, q^r, d} [u_h(c_h) - d + \beta V_h(a_h + q^s) - u_h(a_h) - \beta V_h(a_h)]^\theta \\
&\quad \times [u_\ell(c_\ell) + d + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta V_\ell(a_\ell)]^{1-\theta},
\end{aligned}
$$

subject to the allocation being feasible. The first-order conditions for an interior solution are[7]

$$
V_h'(a_h + q^s) = V_\ell'(a_\ell - q^s), \tag{10}
$$

$$
u_h'(c_h) = u_\ell'(c_\ell), \tag{11}
$$

$$
\begin{aligned}
d(a_h, a_\ell) &= (1-\theta)[u_h(c_h) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta V_h(a_h)] \\
&\quad - \theta[u_\ell(c_\ell) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta V_\ell(a_\ell)].
\end{aligned} \tag{12}
$$

Equations (10) and (11) characterize the allocations $q^s(a_h, a_\ell)$ and $q^r(a_h, a_\ell) = c_\ell -$

[7]In the case of a corner solution where $q^s = a_\ell$, (10) becomes $V_h'(a_h + q^s) > V_\ell'(a_\ell - q^s)$, while in the case where $q^s + q^r = a_\ell$, (11) becomes $u_h'(a_h + a_\ell) \geq u_\ell'(0)$.

$[a_\ell + q^s(a_h, a_\ell)]$. Inspecting (10) and (11), we see that agents rent whenever $c_h \neq a_h + q^s$ and $c_\ell \neq a_\ell - q^s$. In addition, notice that (11) together with $c_h + c_\ell = a_h + a_\ell$ uniquely define c_h and c_ℓ. The transfer $d(a_h, a_\ell)$ redistributes the surplus from trade according to the bargaining weights. Finally, notice that the allocation depends on the distributions of durables μ_i for $i \in \{h, \ell\}$, as they affect the value functions V_i. Therefore, to fully characterize the equilibrium with an invariant distribution, we need to specify how agents are matched. In the next section, we assume that agents direct their search, and we study random matching in appendix 8.5.

6. Directed search

We now describe a rather sophisticated matching technology. Following Corbae et al. (2003), we use a directed search: The matching function specifies that agents who switched types are matched together. Therefore a "new" agent h will be matched with a "new" agent ℓ, while an "old" agent h is matched with an "old" agent ℓ.[8] Below we verify that this matching function is an equilibrium matching rule (where such a term is precisely defined). We devote the rest of this section to the following result.

Proposition 2. *With a directed search, there is an equilibrium characterized by a degenerate distribution of durables for each type at some level $\bar{a}_i$ for $i \in \{h, \ell\}$ with $q^s(\bar{a}_h, \bar{a}_\ell) = 0$, $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$, and $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$ where q^r solves $u_h'(\bar{a}_h + q^r) \geq u_\ell'(\bar{a}_\ell - q^r)$ (with equality if $q^r < \bar{a}_\ell$).*

In other words, each agent $i \in \{h, \ell\}$ is holding a specific amount of durables, either $\bar{a}_h$ or $\bar{a}_\ell$, for agent h and ℓ, respectively. Agents who did not switch type just rent. Agents who switched type adjust their holdings of durables to their type's level. Then they rent as if they did not change type. Loosely speaking, agents first access the sales market to adjust their holdings of durables and then rent if they want to consume more or less.

Suppose that there is a pair $(\bar{a}_h, \bar{a}_\ell)$ such that the functions q^r and q^s specified in Proposition 2 satisfy conditions (10) and (11). To verify that this pair forms an equilibrium, we first need to verify that an agent would not prefer to be matched with a different agent

[8] Thus, matching only depends on an agent's type and whether an agent just changed type. It turns out that in equilibrium this matching function will maximize the gains from trade in each match, so that if agents could choose, they would actually want to be matched in this way.

than the one he is assigned to and that no other agents would prefer to interact with that agent rather than trading with their assigned agents. In the terminology of Corbae et al. (2003), the proposed matching rule is an equilibrium matching if no coalition consisting of one or two agents can do better (in the sense that the discounted lifetime utility of all agents in the coalition increases) by deviating in the following ways. An individual can deviate by matching with himself (i.e., being in autarky this period) rather than as prescribed by the matching rule, and a pair can deviate by matching with each other rather than as prescribed by the matching rule. It should be clear that the bargaining solution is always weakly better than autarky. Hence, we only have to consider a deviation by two agents. However the matching rule maximizes the gains from trade for those agents who just switched, so these agents would be worse off if they were matched differently. It follows that the matching rule is an equilibrium matching rule.

We now show that given the functions q^r and q^s specified in Proposition 2, there exists an equilibrium support $\{\bar{a}_h, \bar{a}_\ell\}$ for the distribution of assets. We first consider the terms of trade for renting and selling the asset. The rental price is $d(\bar{a}_h, \bar{a}_\ell)$, the transfer between an agent h holding $\bar{a}_h$ and an agent ℓ holding $\bar{a}_\ell$. Indeed, there are no sales in this match, as agents h and ℓ keep the same amount of durables through the next period. The rental-sales price is $d(\bar{a}_\ell, \bar{a}_h)$, as agents h and ℓ now rent but also change their holdings of durables. In appendix 8.3., we show that

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1-\theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)], \\ d(\bar{a}_\ell, \bar{a}_h) &= d(\bar{a}_h, \bar{a}_\ell) + \bar{u} + \beta(1-\theta)[V_h(\bar{a}_h) - V_h(\bar{a}_\ell)] + \beta\theta[V_\ell(\bar{a}_h) - V_\ell(\bar{a}_\ell)], \end{aligned}$$

where

$$\bar{u} = (1-\theta)[u_h(\bar{a}_h) - u_h(\bar{a}_\ell)] + \theta[u_\ell(\bar{a}_h) - u_\ell(\bar{a}_\ell)].$$

As expected, $d(\bar{a}_\ell, \bar{a}_h) > d(\bar{a}_h, \bar{a}_\ell)$, and the combined rental-sales price is composed of the rental price, plus the weighted present and discounted lifetime gains of switching holdings for agents h and ℓ.

The directed matching technology specifies that an agent h with $\bar{a}_\ell$ meets an agent ℓ with $\bar{a}_h$ and an agent h with $\bar{a}_h$ meets an agent ℓ with $\bar{a}_\ell$. Given $q^s(\bar{a}_h, \bar{a}_\ell) = 0$, $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$ and $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$, we obtain the following value functions:

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)], \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)]. \end{aligned}$$

Notice that we need to specify the value of the outside option in order to solve for the bargaining solution in equilibrium, i.e., $V_h(\bar{a}_\ell)$ and $V_\ell(\bar{a}_h)$. Because the matching technology pairs agents who just changed type, an agent h holding $\bar{a}_\ell$ (respectively, $\bar{a}_h$) will be paired with an agent ℓ holding $\bar{a}_h$ (respectively, $\bar{a}_\ell$), irrespective of their trading history.[9] Therefore, we obtain

$$\begin{aligned} V_h(\bar{a}_\ell) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)], \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)]. \end{aligned}$$

Using these value functions, we find

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \frac{\bar{u}}{1-\beta},$$

so that the value of selling durables is just $\bar{u}/(1-\beta)$, the lifetime discounted surplus from adjusting one's holdings. Notice that $\bar{a}_h$ converges to $\bar{a}_\ell$ as $\pi \to 1/2$, so that $\bar{u} \to 0$ and selling durables has no value. In appendix 8.3., we characterize the bargaining solution with directed matching q^r, $\bar{a}_h$ and $\bar{a}_\ell$.

Proposition 3. *The degenerate supports $\bar{a}_h$ and $\bar{a}_\ell$ of the two distributions with bargaining are fully characterized by the following equations:*

$$\begin{aligned} u_h'(\bar{c}_h) &= u_\ell'(\bar{c}_\ell), \\ u_h'(\bar{c}_h) &= \frac{[\pi - (2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)}, \\ \bar{c}_h + \bar{c}_\ell &= \bar{a}_h + \bar{a}_\ell = 2A. \end{aligned}$$

These equations imply that agents h and ℓ hold the same amount of durables, $\bar{a}_\ell = \bar{a}_h = A$, when types are not persistent, $\pi = 1/2$. In this case agents only rent and never conduct any sales, $q^s = 0$ and $q^r > 0$. This result is intuitive: When an agent's type today is unrelated to his type tomorrow, all agents have the same future value of owning durables, irrespective of their current types, and so they equate their holdings. The reverse holds in the case with full persistence, $\pi = 1$. Then agents neither rent, buy, or sell: $q^r = q^s = 0$ and the distribution of durables satisfies $u_\ell'(\bar{a}_\ell) = u_h'(\bar{a}_h)$. Again, this is intuitive: When types are permanent, agents know that they always value durables the same way. Therefore, they just

[9] Notice that in equilibrium, there is always trade unless $\pi = 1$ in which case agents never switch type so this ad-hoc rule is never used.

hold the amount they need and they never trade. We can then vary the degree of persistence to obtain market volumes as a function of persistence, as shown in figure 5.

Corollary 4. $\bar{a}_h - \bar{a}_\ell \geq 0$ *is increasing in* π. *The sales volume is hump-shaped in* π, *while the rentals volume is strictly decreasing in* π.

With $\pi = 1/2$ we know there are no sales, and agents h just rent durables from agents ℓ. Types are more persistent when π increases. An agent h's valuation for durables increases, and starting from $a_h = a_\ell = A$, there are now gains from trade between agents h and ℓ. There is an intensive margin: As π increases, the difference in the equilibrium ownership level of durables between agents h and ℓ increases. Therefore, agents who just changed their type to h buy more from agents ℓ. However, there is also an extensive margin: As π increases, fewer agents switch types and so fewer pairs of agents trade. The combination of the extensive and intensive margins explains why asset sales are hump-shaped in π. In contrast, the rental volume is always decreasing with persistence. As π increases agents adjust their position to the optimal quantity of durables they would like to hold absent any future uncertainty. Therefore, they need to rent less to achieve the efficient level of consumption as π increases. As a result, the total rental volume is decreasing with π and is higher than the total sales volume when $\pi = 1/2$. Finally, when $\pi = 1$, agents know their type for sure. Hence, in equilibrium, all gains from trades (sales and rentals) are extinguished and there are neither sales nor rentals.

Additionally, we can find the rental and sales prices.

Corollary 5. *Let* p^r *be the rental price and* p^s *the sales price. Then*

$$\begin{aligned}
p^r &= \frac{(1-\theta)[u_h(\bar{c}_h) - u_h(\bar{c}_h - q^r)] + \theta[u_\ell(\bar{c}_\ell + q^r) - u_\ell(\bar{c}_\ell)]}{q^r}, \\
p^s &= \frac{(1-\theta)[u_h(\bar{c}_h - q^r) - u_h(\bar{c}_\ell + q^r)] + \theta[u_\ell(\bar{c}_h - q^r) - u_\ell(\bar{c}_\ell + q^r)]}{(1-\beta)(\bar{a}_h - \bar{a}_\ell)}.
\end{aligned}$$

We find these prices by using the definition of the payments $d(a_h, a_\ell)$. Because agents who did not switch types only rent, we infer that $d(\bar{a}_h, \bar{a}_\ell) = p^r q^r$. Therefore,

$$p^r q^r = (1-\theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)].$$

A pair of agents that switched conducts both a sale q^s to adjust their position, and then a rental q^r, so the transfer includes both transactions $d(\bar{a}_\ell, \bar{a}_h) = p^r q^r + p^s q^s$. And, as

$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \bar{u}/(1-\beta)$, we obtain that

$$p^s q^s = \frac{\bar{u}}{1-\beta}.$$

Using the expression for $\bar{u}$, with $q^s = \bar{a}_h - \bar{a}_\ell$, we get the result.

Figure 3 shows how $\bar{a}_h$ (red curve) and $\bar{a}_\ell$ (blue curve) evolve as π varies from 1/2 to 1. The parameters chosen are $\theta = 0.5, \lambda = 0.1, \sigma = 2, \beta = 0.9,$ and $A = 50$. Interestingly, the rate of divergence increases as types become more persistent. Hence, as π becomes large, we should expect some movements in prices and quantities. This intuition is confirmed by figure 4, which shows rental and sales prices.

Similarly, the total rental volume q^r and total sales volume $(1-\pi)q^s$ display very different patterns, as illustrated in figure 5. At $\pi = 0.9$, the total rental volume is approximately 20 percent of the stock of durables, while total sales are only 1 percent. Finally, the coefficient of risk aversion σ has the largest effect on volumes and prices.

7. Conclusion

What should one do with the pair of crampons used to conquer Denali, or the chainsaw used to put down the old cherry tree? One could sell these barely used items, accepting the low-ball offer, or just rent them. The possibility of renting will affect the price at which one is willing to sell or buy these items. In other words, the rental market for some durables can have important consequence for sales. Many shops, for example Home Depot and car dealers, have long recognized the interaction between rentals or leases and sales by offering both options to their customers. In this paper, we analyzed this interaction using an environment with trading frictions where agents can rent and buy. The model is simple and concentrates on the simple idea that trading frictions are enough to explain why agents could prefer to rent a good when they can afford to buy.

Our model has several implications that we list here. (1) Absent the possibility to rent, more severe trading frictions imply a drop in sales volume and an increase in sales price. (2) Adding a rental market to our model predicts a decline in sales and an increase in the sales price, and some agents will now rent. (3) In an increasingly uncertain environment (where valuation becomes less persistent), our theory predicts an increase in rentals. Sales, however, could go both ways (see figure 5), but the theory predicts a decrease in the sales price. (4) Finally, we expect there will be more rentals and fewer sales as the hold-up problem induced by bilateral trading becomes more prevalent: Our theory predicts an increase in rental volume as the bargaining power distribution becomes more skewed.

In appendix II, we offer many extensions to better understand this basic trade-off. First, we analyze the effects of patience. We may expect rentals to become less useful when agents become more patient: As future trades become more important, the current losses from the hold-up problem are less important relative to the future benefits of holding the right amount of durables. Second, we consider whether agents' ability to trade often once they receive a shock affects renting. Again, trading often should reduce the hold-up problem, as agents get closer to a Walrasian market, where they can buy from a large pool of sellers. Therefore, we expect the rental volume to decline with agents' ability to trade often. Nevertheless, rental volumes remain positive, because even with highly frequent bilateral trades, the outcome of these trades still depends on the level of ownership and the bargaining power of the participating agents. Third, we also analyze a different option for future trades: We assume that agents have the option to trade on a Walrasian market from next period on. This assumption weakens the hold-up problem today as tomorrow's option becomes better. Fourth, we study the effect of a wedge between the cost of renting and selling, such as a rental tax. We show that renting is robust to such a tax. Finally, in our framework, agents switch from being borrowers or lenders as they switch types. This assumption may appear unrealistic, as some market participants are always on the same side of the rental market. So we study a slightly modified version of the model that agents have ex-ante heterogeneous valuations for the asset and face preference shocks. Agents still prefer to adjust their holdings temporarily, thus, the model accounts for the fact that some market participants are always a renter or lessor.

Of course, our set-up does not fit a particular market exactly but aims at capturing the important features of markets where agents could buy the item but choose to rent it instead. Some rentals markets, such as housing, do not satisfy this assumption: Ordinary people can hardly buy a home with cash. But our environment makes clear that uncertainty and trading frictions are key to explaining rentals in markets for other (maybe) smaller-ticket items – although what is a small-ticket item for one can be a large-ticket item for another. While our model has some implications, they are necessarily quite general and may not fit one particular case. To understand the rentals market for a specific good, one will need to focus on the specifics of that market (for example, see Gavazza, Lizzeri & Roketskiy (2014)), but this specifity is beyond the scope of this paper, which aimed to highlight a basic trade-off between renting and selling.

References

Corbae, D., Temzelides, T. & Wright, R. (2003), 'Directed matching and monetary exchange', *Econometrica* **71**(3), 731–756.

Duffie, D. (1996), 'Special repo rates', *The Journal of Finance* **51**(2), 493–526.

Duffie, D., Gârleanu, N. & Pedersen, L. H. (2005), 'Over-the-counter markets', *Econometrica* **73**(6), 1815–1847.

Eisfeldt, A. L. & Rampini, A. A. (2009), 'Leasing, ability to repossess, and debt capacity', *Review of Financial Studies* **22**(4), 1621–1657.

Ewerhart, C. & Tapking, J. (2008), 'Repo markets, counterparty risk, and the 2007/2008 liquidity crisis', *European Central Bank Working Paper* .

Gavazza, A. (2011), 'Leasing and secondary markets: Theory and evidence from commercial aircraft', *Journal of Political Economy* **119**, 325–377.

Gavazza, A., Lizzeri, A. & Roketskiy, N. (2014), 'A quantitative analysis of the used-car market', *The American Economic Review* **104**(11), 3668–3700.

Kehoe, T. J. & Levine, D. K. (2006), 'Bankruptcy and collateral in debt constrained markets', *NBER Working Paper* **12656**.

Kiyotaki, N. & Wright, R. (1989), 'On money as a medium of exchange', *The Journal of Political Economy* pp. 927–954.

Koeppl, T., Monnet, C. & Temzelides, T. (2012), 'Optimal clearing arrangements for financial trades', *Journal of Financial Economics* **103**(1), 189–203.

Krishnamurthy, A., Nagel, S. & Orlov, D. (2014), 'Sizing up repo', *The Journal of Finance* **69**(6), 2381–2417.

Lacker, J. (2001), 'Collateralized Debt as the Optimal Contract', *Review of Economic Dynamics* **4**(4), 842–859.

Lagos, R. & Rocheteau, G. (2009), 'Liquidity in asset markets with search frictions', *Econometrica* **77**(2), 403–426.

Lagos, R. & Wright, R. (2005), 'A unified framework for monetary theory and policy analysis', *Journal of Political Economy* **113**, 463–484.

Madison, F. (2016), 'Asymmetric information in frictional markets for liquidity: on the non-equivalence between collateralized credit and immediate settlement', *mimeo, University of Basel* .

Maurin, V., Monnet, C. & Gottardi, P. (2016), 'A theory of repurchase agreement, collateral re-use, and repo intermediation', *mimeo* .

Mills, D. C. & Reed, R. R. (2008), 'Default risk and collateral in the absence of commitment', *mimeo, Board of Governors* .

Parlatore Siritto, C. (2015), 'Collateralizing liquidity', *Working Paper* .

Tomura, H. (2013), 'Repos in over-the-counter markets', *mimeo, Hokkaido University* .

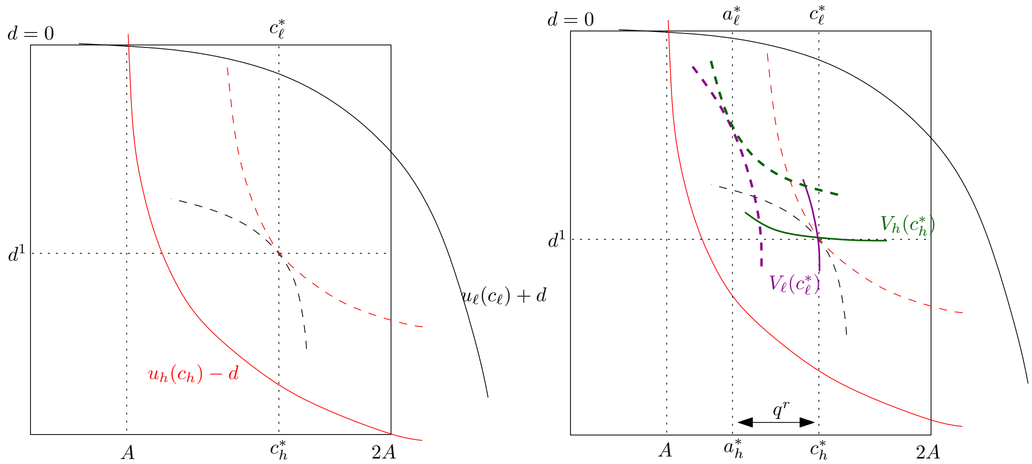


Figure 1: Active repo market

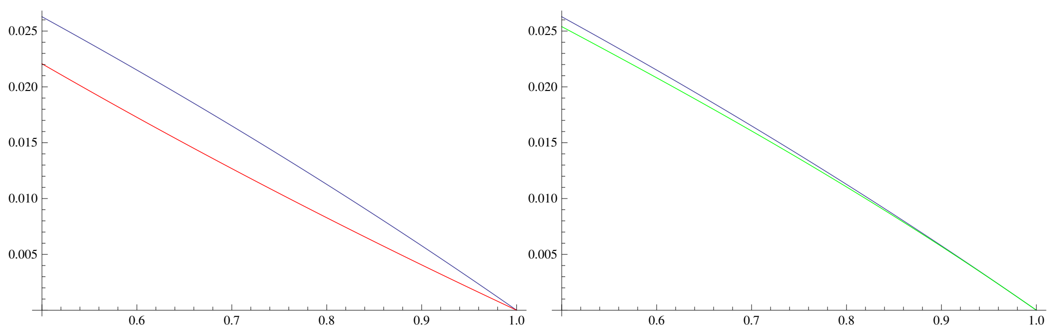


Figure 2: Left, $p_h^r/p^s - 1$ (blue) and $1 - p_\ell^r/p^s$ (red)
Right, $p_h^r/p^s - 1$ for higher risk aversion (blue) and lower risk aversion (green)

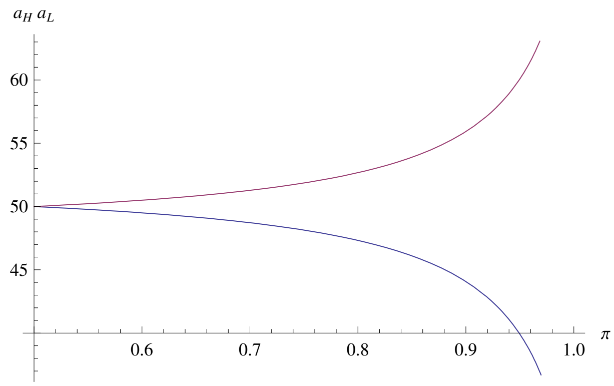
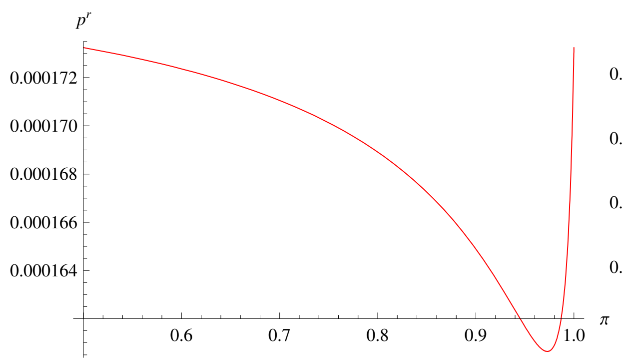


Figure 3: Asset holdings as a function of π.



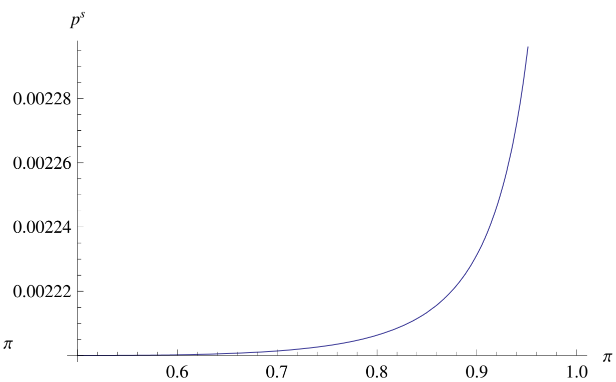
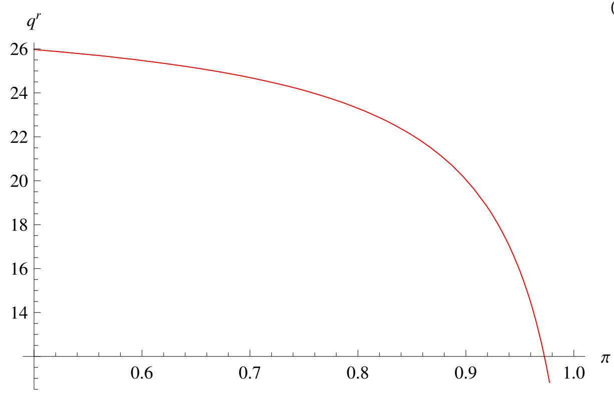


Figure 4: Repo prices (left) and outright purchase price (right)



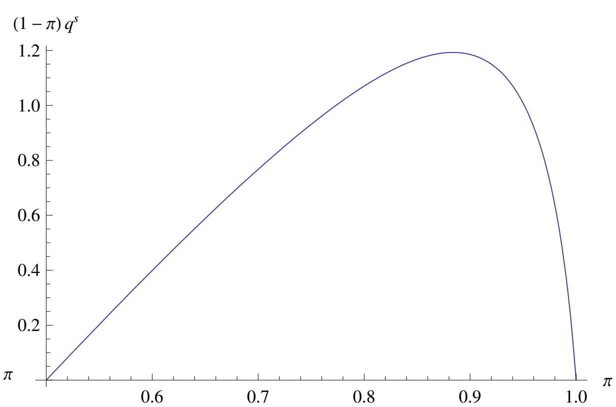


Figure 5: Repo volumes (left) and outright purchase volumes (right)

8. Appendix I

8.1. Shut down of the rental market

In a two period example of an economy, with Walrasian markets in the first period and bilateral trade in the second period, the competitive price for the outright sales market, as a function of persistent π, is given by

$$\begin{aligned} p^s(\pi) &= u^{h\prime}(c_h^*) + \frac{\beta}{2}\left[\pi u^{h\prime}(a_h^*) + (1-\pi)u^{\ell\prime}(a_h^*)\right] \\ &= u^{\ell\prime}(c_\ell^*) + \frac{\beta}{2}\left[\pi u^{\ell\prime}(a_\ell^*) + (1-\pi)u^{h\prime}(a_\ell^*)\right], \end{aligned}$$

where c_i^* and a_i^* are efficient consumption and ownership of durables for type $i \in \{h, \ell\}$.

Now assume that only the sales market is active and agents cannot rent to adjust their consumption in the first period. In this case, $c_i = a_i$, and the sales price will be given by

$$\begin{aligned} \tilde{p}^s(\pi) &= u^{h\prime}(\tilde{a}_h) + \frac{\beta}{2}\left[\pi u^{h\prime}(\tilde{a}_h) + (1-\pi)u^{\ell\prime}(\tilde{a}_h)\right] \\ &= u^{\ell\prime}(\tilde{a}_\ell) + \frac{\beta}{2}\left[\pi u^{\ell\prime}(\tilde{a}_\ell) + (1-\pi)u^{h\prime}(\tilde{a}_\ell)\right], \end{aligned}$$

where $\tilde{a}_i$ is the consumption and ownership of durables of type $i \in \{h, \ell\}$. Note that for $\pi \in (\frac{1}{2}, 1]$ we have

$$c_\ell^* < \tilde{a}_\ell \leq a_\ell^* \quad \leq A \leq \quad a_h^* \leq \tilde{a}_h < c_h^* \tag{13}$$

with the equality holding for $\pi = 1$, in which case $p^s(1) = \tilde{p}^s(1)$.

If we assume that $u^h(c) = \alpha \cdot u^\ell(c) = \alpha \cdot u(c)$, for a given $\alpha > 1$, then we can show that

$$\frac{dp^s}{d\pi} = \frac{\beta}{2}(\alpha - 1)\left\{\frac{-u''(a_\ell^*)\cdot a_\ell^*}{-u'(a_\ell^*)} \cdot \frac{u'(a_h^*)}{a_\ell^*} - \frac{-u''(a_h^*)\cdot a_h^*}{-u'(a_h^*)} \cdot \frac{u'(a_\ell^*)}{a_h^*}\right\} \Big/ \left\{\frac{-u''(a_\ell^*)}{-u'(a_\ell^*)} + \frac{-u''(a_h^*)}{-u'(a_h^*)}\right\}$$

and

$$\frac{d\tilde{p}^s}{d\pi} = \frac{\beta}{2}(\alpha - 1)\left\{\frac{-u''(\tilde{a}_\ell)\cdot \tilde{a}_\ell}{-u'(\tilde{a}_\ell)} \cdot \frac{u'(\tilde{a}_h)}{\tilde{a}_\ell} - \frac{-u''(\tilde{a}_h)\cdot \tilde{a}_h}{-u'(\tilde{a}_h)} \cdot \frac{u'(\tilde{a}_\ell)}{\tilde{a}_h}\right\} \Big/ \left\{\frac{-u''(\tilde{a}_\ell)}{-u'(\tilde{a}_\ell)} + \frac{-u''(\tilde{a}_h)}{-u'(\tilde{a}_h)}\right\}.$$

If $u(\cdot)$ is $CRRA$ and $\sigma = \frac{-u''(c)\cdot c}{u'(c)}$, using the fact that $\tilde{a}_h + \tilde{a}_\ell = a_h^* + a_\ell^* = 2A$, we have

$$\begin{aligned}\frac{dp^s}{d\pi} &= \frac{\beta}{2}(\alpha - 1)\frac{(a_h^*)^{1-\sigma} - (a_\ell^*)^{1-\sigma}}{2A}, \\ \frac{d\tilde{p}^s}{d\pi} &= \frac{\beta}{2}(\alpha - 1)\frac{(\tilde{a}_h)^{1-\sigma} - (\tilde{a}_\ell)^{1-\sigma}}{2A}.\end{aligned}$$

Hence, if $\sigma > 1$ then p^s and $\tilde{p}^s$ are both decreasing in π, and for $\sigma < 1$ they are increasing in π. Moreover, given (13) for both cases of $\sigma > 1$ and < 1, we have $\tilde{p}^s < p^s$.[10] In other words, the possibility of renting increases the sales price of durables.

8.2. Proof of Proposition 2

We show that no two agents benefit from being matched with each other more than their prescribed partner. It should be clear that the bargaining solution is always better than autarky, even though not necessarily in a strict sense. Therefore, we only need to check deviations by a coalition of two agents. An agent ℓ with $\bar{a}_h$ could decide to form a coalition with an agent ℓ with $\bar{a}_\ell$ or an agent h with $\bar{a}_h$. It is a property of the bargaining solution that an agent ℓ will obtain a lower payoff being matched with an agent h with a higher amount of durables. Indeed, the agent ℓ can extract less from a relatively rich agent h, as the marginal utility of obtaining more durables is lower for this agent. Hence, an agent ℓ with $\bar{a}_h$ prefers to be matched with an agent h with $\bar{a}_\ell$. Also, it is a property of the bargaining solution that, given he has to meet an agent holding a, an agent ℓ prefers to be matched with the agent with the highest marginal utility, who will be an agent h.

We now turn to agents h. An agent h with $\bar{a}_\ell$ could decide to form a coalition with an agent h with $\bar{a}_h$ or an agent ℓ with $\bar{a}_\ell$. As above, however, it is a property of the bargaining solution that the payoff of agent h matched with an agent ℓ is higher whenever the agent ℓ is holding more durables. Hence, agent h will not want to be matched with an agent ℓ holding $\bar{a}_\ell$. Also, an agent h with $\bar{a}_h$ prefers to be matched with the agent holding $\bar{a}_\ell$ with the lowest marginal utility, that is, with an agent ℓ. Therefore, there is no coalition of two agents where both agents would do better than under the prescribed matching technology. This result, combined with the distribution over $\{\bar{a}_h, \bar{a}_\ell\}$, shows that we have an equilibrium.

[10] If $\sigma = 1$, i.e., agents have a log utility, then p^s and $\tilde{p}^s$ are equal and independent of π.

8.3. Proof of Proposition 3

The value functions are

$$\begin{aligned}
V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)], \\
V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)].
\end{aligned}$$

Adding the above equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1-\beta}. \tag{14}$$

Also

$$\begin{aligned}
V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)], \\
V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)].
\end{aligned}$$

Adding the above two equations, we obtain

$$V_h(\bar{a}_\ell) + V_\ell(\bar{a}_h) = V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell). \tag{15}$$

From the first order conditions of the bargaining problem, we then can compute $d(\bar{a}_h, \bar{a}_\ell)$ as

$$d(\bar{a}_h, \bar{a}_\ell) = (1-\theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_h)] - \theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell)], \tag{16}$$

where we have used (14) and (15) and the fact that $q^s(\bar{a}_h, \bar{a}_\ell) = 0$. Therefore, using (16) we obtain

$$\begin{aligned}
& u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h) \\
= \; & u_h(\bar{a}_h) + \beta V_h(\bar{a}_h) + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell)].
\end{aligned}$$

Also

$$\begin{aligned}
& u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) \\
= \; & u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) + (1-\theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell)].
\end{aligned}$$

In a similar fashion, using $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$, we can rewrite $d(\bar{a}_\ell, \bar{a}_h)$ as

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \bar{u} + \beta(1-\theta)[V_h(\bar{a}_h) - V_h(\bar{a}_\ell)] + \beta\theta[V_\ell(\bar{a}_h) - V_\ell(\bar{a}_\ell)], \tag{17}$$

where

$$\bar{u} = (1-\theta)[u_h(\bar{a}_h) - u_h(\bar{a}_\ell)] + \theta[u_\ell(\bar{a}_h) - u_\ell(\bar{a}_\ell)].$$

Therefore, using (15) and (17) and simplifying we obtain

$$\begin{aligned} & u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell) \\ = \; & u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h) + (1-\theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]. \end{aligned}$$

Similarly

$$\begin{aligned} & u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) \\ = \; & u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell) + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]. \end{aligned}$$

Hence, combining all these expressions, we obtain

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + \pi\theta S + (1-\pi)(1-\theta)\tilde{S}, \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + \pi(1-\theta)S + (1-\pi)\theta\tilde{S}, \\ V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + (1-\pi)[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)(1-\theta)S + \pi\theta\tilde{S}, \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + (1-\pi)[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)\theta S + \pi(1-\theta)\tilde{S}, \end{aligned}$$

where

$$\begin{aligned} S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell), \\ \tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h). \end{aligned}$$

Solving for $V_h(\bar{a}_h)$, we obtain

$$(1-\beta)V_h(\bar{a}_h) = \frac{(1-\pi)[u_\ell(\bar{a}_h) + (1-\theta)\tilde{S}] + [\pi - (2\pi-1)\beta][u_h(\bar{a}_h) + \theta S]}{1-(2\pi-1)\beta}.$$

And taking the derivative, we have

$$(1-\beta)V_h'(\bar{a}_h) = \frac{u_\ell'(\bar{a}_h)(1-\pi) + [\pi-(2\pi-1)\beta]u_h'(\bar{a}_h) + (1-\pi)(1-\theta)\frac{\partial\tilde{S}}{\partial\bar{a}_h} + \theta\frac{\partial S}{\partial\bar{a}_h}[\pi-(2\pi-1)\beta]}{1-(2\pi-1)\beta}.$$

Using the first order condition for q^r, we obtain after some simplifications

$$\begin{aligned}(1-\beta)(1-(2\pi-1)\beta)V_h'(\bar{a}_h) &= u_\ell'(\bar{a}_h)\theta(1-\pi) + u_h'(\bar{a}_h)(1-\theta)[\pi-(2\pi-1)\beta] \\ &\quad + u_h'(\bar{a}_h+q^r)[1-\pi+(2\pi-1)(1-\beta)\theta].\end{aligned}$$

Since (14) holds, we use the first order condition for q^r and simplify to obtain

$$\begin{aligned}(1-\beta)(1-(2\pi-1)\beta)V_\ell'(\bar{a}_\ell) &= u_\ell'(\bar{a}_\ell-q^r)[\pi-(2\pi-1)(\beta+(1-\beta)\theta)] + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) \\ &\quad + \theta[\pi-(2\pi-1)\beta]u_\ell'(\bar{a}_\ell).\end{aligned}$$

The first order condition for q^s imposes that $V_h'(\bar{a}_h) = V_\ell'(\bar{a}_\ell)$. Using the fact that $u_\ell'(\bar{a}_\ell - q^r) = u_h'(\bar{a}_h+q^r)$ and simplifying, we obtain

$$u_h'(\bar{a}_h+q^r) = \frac{[\pi-(2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell)-(1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h)-(1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)}.$$

Together with the first order condition on asset sales and the feasibility constraint, this completes the proof.

8.4. Proof of Corollary 4

The equilibrium allocation is given by

$$\begin{aligned} u_h'(\bar{c}_h) &= u_\ell'(\bar{c}_\ell), \\ u_h'(\bar{c}_h) &= \frac{[\pi-(2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell)-(1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h)-(1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)}, \\ \bar{c}_h+\bar{c}_\ell &= \bar{a}_h+\bar{a}_\ell = 2A. \end{aligned} \qquad (18)$$

Let

$$\alpha_1(\pi) = \frac{\pi-(2\pi-1)\beta}{2\pi-1} = \frac{\pi}{2\pi-1} - \beta,$$

and

$$\alpha_2(\pi) = \frac{1-\pi}{2\pi-1},$$

then $\alpha_1'(\pi) = \alpha_2'(\pi) = \frac{-1}{(2\pi-1)^2} < 0$. And we can rewrite (18) as

$$u_h'(\bar{c}_h)(1-\beta)(2\theta-1) = \alpha_1(\pi)[\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - \alpha_2(\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)].$$

Notice that $\bar{c}_h$ is not a function of π, so that using $\bar{a}_h + \bar{a}_\ell = A$ and the implicit function theorem, we have

$$\begin{aligned} 0 \;=\; & \alpha_1'(\pi)[\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)]d\pi + \alpha_1(\pi)[-\theta u_\ell''(A-\bar{a}_h) - (1-\theta)u_h''(\bar{a}_h)]d\bar{a}_h \\ & -\alpha_2'(\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]d\pi - \alpha_2(\pi)[\theta u_\ell''(\bar{a}_h) + (1-\theta)u_h''(A-\bar{a}_h)]d\bar{a}_\ell, \end{aligned}$$

which we can simplify as

$$\frac{d\bar{a}_h}{d\pi} = \frac{\alpha_1'(\pi)\left\{\theta\left[u_\ell'(\bar{a}_\ell) - u_\ell'(\bar{a}_h)\right] + (1-\theta)\left[u_h'(\bar{a}_\ell) - u_h'(\bar{a}_h)\right]\right\}}{\alpha_1(\pi)[\theta u_\ell''(A-\bar{a}_h) + (1-\theta)u_h''(\bar{a}_h)] + \alpha_2(\pi)[\theta u_\ell''(\bar{a}_h) + (1-\theta)u_h''(A-\bar{a}_h)]}.$$

Since $u_i'(\bar{a}_\ell) > u_i'(\bar{a}_h)$ for both i and $\alpha_i'(\pi) < 0$, the numerator is negative. Concavity of the utility function implies that the denominator is also negative. Therefore we have $d\bar{a}_h/d\pi > 0$.

Given π, the rental volume in this economy is given by q^r, because all agents rent, while the volume of asset sales is given by $(1-\pi)q^s = (1-\pi)(\bar{a}_h - \bar{a}_\ell)$. Clearly, the sales volume is hump-shaped as when $\pi = 1/2$, we have $\bar{a}_h = \bar{a}_\ell$ so that $q^s = 0$, while when $\pi = 1$, $q^s = 0$ as well. However, $(1-\pi)q^s > 0$ for all other values of π. Because, the problem is continuous, sales volume is hump-shaped. Also, the fact that $\bar{c}_h = \bar{a}_h + q^r$ is a constant implies that total rental volume (i.e., q^r because all agents rent) is declining in π. Agents do not rents when $\pi = 1$, therefore, the rental volume is declining to zero with π.

8.5. Random matching: special cases

Here we study two extreme cases with either $\pi = 1/2$ or $\pi = 1$ when an agent h is randomly matched with an agent ℓ. In the case with $\pi = 1/2$, preference shocks have no persistence and current preferences do not give any information on future preferences. In the case with $\pi = 1$, preference shocks are fully persistent as they are fixed forever.

With no persistence and random matching, we obtain the following result.

Proposition 6. *With random matching and $\pi = 1/2$, there is a unique invariant equilibrium* *characterized by a distribution of durables ownership levels for each type that are degenerate at some level $\bar{a} = A$ with $q^s(\bar{a}, \bar{a}) = 0$, and $q^r(\bar{a}, \bar{a}) > 0$.*

In the case without persistence, (8) and (9) imply that $V_h(a) = V_\ell(a)$ for all a, such that agents h and ℓ enjoy the same value of holding durables. In this case, (10) implies that $a_h + q^s(a_h, a_\ell) = a_\ell - q^s(a_h, a_\ell)$ with $q^s(a_h, a_\ell) > 0$, if and only if $a_\ell > a_h$, and $q^s(a_h, a_\ell) < 0$ otherwise. That is, agents leave the match holding the same quantity of durables. Hence, the unique invariant equilibrium is one where the distribution of durables holdings is degenerate at $\bar{a} = A$ and $q^s(\bar{a}, \bar{a}) = 0$. This is very intuitive: Because all agents give the same value to future returns, they extinguish all surplus from trading durables by averaging their holdings (i.e., once an agent holding a_h trade with an agent holding a_ℓ, they both end up with $(a_h + a_\ell)/2$) and in equilibrium they hold the same amount of durables. Then (11) together with Assumption 1 imply that $q^r(\bar{a}, \bar{a}) > 0$: While agents value future durable consumption the same way, they differ in their valuation of current consumption. Therefore, there is a benefit from renting.

With full persistence, however, there is an equilibrium with neither sales nor rentals in equilibrium.

Proposition 7. *With random matching and $\pi = 1$, there is an equilibrium with a degenerate distribution of durables holdings for each type at some level $\bar{a}_h$ and $\bar{a}_\ell$ with $\bar{a}_h > \bar{a}_\ell$, where $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ and $q^r(\bar{a}_h, \bar{a}_\ell) = 0$.*

We will first verify that the proposed allocation is an equilibrium. Because $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ and $q^r(\bar{a}_h, \bar{a}_\ell) = 0$, equation (12) implies that $d(\bar{a}_h, \bar{a}_\ell) = 0$. Using (8) and (9), we then have for $i \in \{h, \ell\}$

$$V_i(\bar{a}_i) = \frac{u_i(\bar{a}_i)}{1-\beta}, \tag{19}$$

and (10) and (11) imply that $\bar{a}_h$ and $\bar{a}_\ell$ are uniquely given by

$$u_h'(\bar{a}_h) = u_\ell'(\bar{a}_\ell),$$

with $\bar{a}_h = 2A - \bar{a}_\ell$. This verifies that there is no sales or rentals in the equilibrium. Also, combining the last equation with Assumption 1, we can verify that $\bar{a}_h > \bar{a}_\ell$. This equilibrium is unique whenever endowments are symmetric across all agents (and no constraint binds – which may happen if some agents ℓ are endowed with too much durables in the first place) so that all agents ℓ hold the same amount a_ℓ and all agents h holds a_h. To see this point note that if an agent h endowed with a_h meets an agent ℓ endowed with a_ℓ, then the bargaining solution imposes that they trade so that (11) holds. But the unique solution is $a_h + q^s(a_h, a_\ell) = \bar{a}_h$ and $a_\ell - q^s(a_h, a_\ell) = \bar{a}_\ell$. Because $a_h + a_\ell = 2A$, such a q^s exists and takes the agents directly to the equilibrium distribution of durables holdings.

For general levels of persistence $\pi \in (0, 1)$ and random matching, we are unable to determine analytically the total sales and rentals volumes, as we cannot solve analytically for the invariant equilibrium distribution of durables holdings.[11] We suspect that as an agent ℓ is better endowed, he will sell more to agent h, and as agent h is less endowed, he will buy more from agent ℓ. This would hint to more trade as agents valuations differ and we would expect that the distributions of asset holdings become more spiked around their respective mean $\bar{a}_h$ and $\bar{a}_\ell$ as π increases, where the means are diverging as π increases. However, since agents can switch randomly from one type to the other, it is difficult to fully characterize the equilibrium without resorting to numerical simulations.

[11]This is a usual problem in models with pairwise trade and arbitrary holdings. Agents in Kiyotaki & Wright (1989) or Duffie et al. (2005) trade an indivisible asset with a unit upper bound. Lagos & Wright (2005) introduces a Walrasian market with quasi-linear preferences so that agents can level their asset holdings, thus giving a degenerate distribution of assets. In a Lagos & Wright (2005) environment, there is no role for rentals.

9. Appendix II - Some variations on the model

In this section we consider several extensions to the basic framework.

Renting is useful because of a hold-up problem, thus, it is natural to ask about the consequences of weakening (or strengthening) the hold-up problem on rentals. We consider several ways to affect the severity of the hold-up problem. First, we analyze the effects of patience. We may expect renting to become less useful when agents become more patient: as future trades become more important, the current losses from the hold-up problem are less important relative to the future benefits of holding the right amount of durables.

Second, we consider whether agents' ability to trade often once they receive a shock affects the existence of renting. Again, trading often should reduce the hold-up problem, as agents gets closer to a Walrasian market where they can buy from a large pool of sellers. So we expect repos to decline with agents' ability to trade often.

Third, we also analyze a different option for future trades: we assume that agents have the option to trade on a Walrasian market from next period on. This weakens the hold-up problem today as tomorrow's option becomes better.

Fourth, we study the effect of a wedge between the cost of renting and the one for an outright purchase, such as a tax on rentals. We show that renting is robust to such a tax.

Finally, in our framework, agents switch from being borrowers or lenders as they switch types. This may appear unrealistic as some market participants are always on the same side of the rental market. So we study a slightly modified version of the model which accounts for the fact that some market participants are always renter or lessor.

9.1. Patience

In general prices in Corollary 5 are different from their Walrasian equivalents, and in particular that $(1-\beta)p^s$ is different from p^r. However, an interesting case to consider is when agents become very patient. Then it is legitimate to guess that the allocation will converge to the Walrasian one, as it is in some sense equivalent to agents trading with each other very frequently. However, it is also as if agents were also changing type very often and although we have the illusion that they can trade very fast when β converges to one, they are also bargaining a lot to readjust their ownership of durables so this friction remains. Indeed, as β tends to one, the solution to the bargaining problem is characterized by $\bar{a}_h, \bar{a}_\ell \to A$, so that sales converge to zero. Hence, we obtain

$$\lim_{\beta \to 1}(1-\beta)p^s = (1-\theta)u'_h(A) + \theta u'_\ell(A).$$

However, in the limit q^r satisfies $u'_h(A+q^r) = u'_\ell(A-q^r)$ and Assumption 1 guarantees that $q^r > 0$ is bounded away from zero. Because $q^r > 0$ and $u_i(.)$ is concave

$$\begin{aligned} q^r u'_h(A+q^r) &< u_h(A+q^r) - u_h(A) < q^r u'_h(A), \\ q^r u'_\ell(A) &< u_\ell(A) - u_\ell(A-q^r) < q^r u'_\ell(A-q^r), \end{aligned}$$

and in general $p^r \neq (1-\beta)p^s$. For illustration, we use the following utility function: $u_h(a) = \frac{a^{1-\sigma}}{1-\sigma}$ and $u_\ell(a) = \lambda u_h(a)$, where $\lambda \in (0,1)$. Then we obtain

$$q^r = \frac{\lambda^{-\frac{1}{\sigma}}\bar{a}_\ell - \bar{a}_h}{1+\lambda^{-\frac{1}{\sigma}}},$$

so that

$$\bar{a}_h + q^r = \lambda^{-\frac{1}{\sigma}}\frac{2A}{1+\lambda^{-\frac{1}{\sigma}}} \text{ and } \bar{a}_\ell - q^r = \frac{2A}{1+\lambda^{-\frac{1}{\sigma}}}.$$

Figure 6 shows how rentals volume moves along the (β, π)-dimensions.

As we have argued above, a_ℓ and a_h tends to A, whenever $\beta \to 1$. Therefore, in this case

$$\lim_{\beta \to 1}(1-\beta)p^s = (1-\theta+\lambda\theta)A^{-\sigma},$$

while

$$\lim_{\beta \to 1} p^r = \frac{A^{-\sigma}}{1-\sigma}\frac{\left(\lambda^{\frac{1}{\sigma}}+1\right)^{\sigma}}{\left(1-\lambda^{\frac{1}{\sigma}}\right)}\left[2^{1-\sigma} - \left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma}\right]\left\{1-\theta+\lambda\theta\frac{\left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma} - \left(2\lambda^{\frac{1}{\sigma}}\right)^{1-\sigma}}{\lambda 2^{1-\sigma} - \lambda\left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma}}\right\}.$$

With $\lambda = 0.1$ and $\sigma = 2$, we plot the ratio $\lim_{\beta\to1}(1-\beta)p^s/\lim_{\beta\to1}p^r$ as a function of $\theta \in [0, 0.4]$. As figure 7 shows, $\lim_{\beta\to1}(1-\beta)p^s > \lim_{\beta\to1}p^r$ for low values of θ, and the inequality is reversed otherwise. In the next subsection we correct for the frequency with which agents change type as they can trade more often.

9.2. Frequent trades

In this subsection we study the consequences of agents meeting more frequently. More specifically, what are the consequences of reducing the time until the next meeting from one to $\Delta < 1$? And, what happens when $\Delta \to 0$? If β denotes discounting over a period of unit length, and π denotes the probability of maintaining the same type over a period of unit length, we assume agents discount future at rate $\beta_\Delta = 1 - \Delta(1 - \beta)$ and the probability of maintaining the same type is $\pi_\Delta = 1 - \Delta(1 - \pi)$ over a period of $\Delta < 1$ length. Clearly the level of consumption by agents h and agents ℓ will remain the same, however, the share of renting and sales changes. We denote the repo level when $\Delta < 1$ units of time elapses until the next meeting by q^r_Δ, and we show in section 9.7.:

Proposition 8. *For any $\Delta < 1$, we have $q^r_\Delta < q^r$. Also, as $\Delta \to 0$, q^r_Δ decreases to $q^r_0 > 0$.*

That is even with very frequent trades agents rent. Although agents can trade more often they still face the friction that trading has to be bilateral and this gives a role for renting some of the durables.

9.3. Outside option

An intuitive explanation for our results, reminiscent of the intuition from our earlier example, is that agents may prefer renting over buying, because they do not want to lock in a position that may be difficult to undo later at an agreeable price. When they rent, agents are not locked into a position. To make this intuition more precise, we modify the environment slightly and assume that agents' *outside option* is to access a Walrasian market from next period onward. Then the outside option for an agent i endowed with durables a is $u_i(a) + \beta\tilde{W}_i(a)$, where $\tilde{W}_i(a) = \pi W_i(a) + (1 - \pi)W_{-i}(a)$ and $W(a)$ has been defined in section 3. The possibility to trade on a Walrasian market would make the lock-in problem a little less severe, as agents could sell (some of) their durables on the Walrasian market next period. Therefore, we would expect a decrease in rentals relative to the economy where agents do not have the option to unload their durables on a Walrasian market. Still, agents are locked-in for one period and we would still expect renting to have a role. Indeed, let $\bar{q}^r$ be the rent amount in equilibrium when agents have the option to trade at Walrasian prices in the next period, and let q^r be the rent amount when they do not have this option. Then in the section 9.7. we show:

Proposition 9. *With directed search and bargaining, there is an equilibrium where* $q^r > \bar{q}^r > 0$.

The equilibrium with the option to trade on a Walrasian market displays the same features as the one in our original set-up. That is, whether agents switch types or not, they always rent $\bar{q}^r$. In addition, those agents who have switched types trade $\bar{q}^s = \bar{a}_h - \bar{a}_\ell$, and zero otherwise, where $\bar{a}_h$ and $\bar{a}_\ell$ are given by some equilibrium conditions. The important result is that, although agent's outside option is the Walrasian price (in the next period), agents will still rent, but less so than if they did not have the option to trade at the Walrasian price in the following period. Therefore, the volume of rentals declines as agents have better outside options, thus, alleviating the hold-up problem.

9.4. Transaction wedge

For financial repos, Duffie (1996) introduces transaction costs to model the need for special repo. It is indeed fair to argue that renting may be preferred if they are treated differently from, for example, a fiscal perspective. Therefore, in this subsection, we study the robustness of our results when there is a wedge – positive or negative – between the transactions costs of rentals and sales.

More specifically, we assume that agents are matched bilaterally in two markets: a sales and a rentals market. They first meet in the sales market and trade their assets in bilateral matches. In the rentals market, agents rent/lend durables bilaterally for one period. In both markets, agents pay with the numeraire. However, there is a wedge between the production cost and consumption benefit when settling the terms of a sale, while there is no wedge when agents settle a rental agreement. When an agent h owning a_h is matched in the sales market with an agent ℓ owning a_ℓ, they solve the following problem.

$$\max_{d^s,q^s} \quad \left[W_h^r\left(a_h+q^s\right)-\omega\cdot d^s-W_h^r\left(a_h\right)\right]^{\theta_s}\left[W_\ell^r\left(a_\ell-q^s\right)+d^s-W_\ell^r\left(a_\ell\right)\right]^{1-\theta_s}, \qquad (20)$$

where $W_i^r(a)$ is the value of going to the rentals market with a durables for an agent $i \in \{h,\ell\}$. In the rentals market, an agent h owning a_h' meets an agent ℓ owning a_ℓ', and they solve the following problem.

$$\max_{d^r,q^r} \quad \left[u_h\left(a_h'+q^r\right)-d^r-u_h\left(a_h'\right)\right]^{\theta_r}\left[u_\ell\left(a_\ell'-q^r\right)+d^r-u_\ell\left(a_\ell'\right)\right]^{1-\theta_r}. \qquad (21)$$

Note that we are also allowing for different bargaining powers in the rentals and sales markets. The reason is that we are not necessarily assuming that agents are trading with the same counterparty in both markets. When agents rent, we have

$$\begin{aligned} W_h^r(a'_h) &= u_h(a'_h+q^r)-d^r+\beta\left\{\pi\cdot W_h^s(a'_h)+(1-\pi)\cdot W_\ell^s(a'_h)\right\}, \\ W_\ell^r(a'_\ell) &= u_\ell(a'_\ell-q^r)+d^r+\beta\left\{\pi\cdot W_\ell^s(a'_\ell)+(1-\pi)\cdot W_h^s(a'_\ell)\right\}, \end{aligned}$$

where

$$\begin{aligned} W_h^s(a_h) &= W_h^r(a_h+q^s)-\omega\cdot d^s, \\ W_\ell^s(a_\ell) &= W_\ell^r(a_\ell-q^s)+d^s. \end{aligned}$$

In the equilibrium we have $W_h^{r\prime}(a_h^*)=\omega\cdot W_\ell^{r\prime}(a_\ell^*)$. In section 9.7. we use this equation as well as $a_\ell^*=2A-a_h^*$ to solve for a_h^* and a_ℓ^*. We find that an increase in the wedge ω decreases a_h^* and increases a_ℓ^*. In other words, when adjusting their consumption of durables via sales becomes more costly, agents rent more. Figure 8 shows a numerical example of how the share of rentals changes with respect to changes in the sales cost wedge ω and persistence π.

Note that as the sales cost wedge ω rises, the shares of rentals in the consumption of durables increases up to a level where all change in consumption is done through renting durables. The opposite holds as well. If selling become much cheaper than renting, then all changes in consumption would be conducted via sales. For the intermediate levels of the wedge ω, sales coexist with rentals.

9.5. Ex-ante Asymmetric Agents

It is common for agents renting to be always on one side of the rentals market, participating only as lessor or as renters. In this subsection we study a case of ex-ante asymmetric agents to show that renting is essential to achieve the efficient allocation even if agents always participate on one side of the rentals market. More specifically, assume there is an equal measure of two types of agents denoted by B and L. Each type receives a non-persistent i.i.d preference shock h or ℓ, such that in every period a fraction π of type B agents have h preference shock and a fraction π of type L agents have ℓ preference shock. Denoting the utility from durables consumption of type B agents with h and ℓ preferences by $u_{B,h}(\cdot)$ and $u_{B,\ell}(\cdot)$, and for type L agents with h and ℓ preferences by $u_{L,h}(\cdot)$ and $u_{L,\ell}(\cdot)$, we assume that $u'_{B,h}(a)>u'_{B,\ell}(a)>u'_{L,h}(a)>u'_{L,\ell}(a)$. Moreover, the efficient allocation is described by the consumption levels $c^*_{B,h}, c^*_{B,\ell}, c^*_{L,h}$, and $c^*_{L,\ell}$ that satisfy

$$u'_{B,h}(c^*_{B,h}) = u'_{B,\ell}(c^*_{B,\ell}) \quad = \quad u'_{L,h}(c^*_{L,h}) = u'_{L,\ell}(c^*_{L,\ell}), \tag{22}$$

for

$$\begin{aligned} 2A &= c^*_{B,h} + c^*_{L,\ell}, \\ &= c^*_{B,\ell} + c^*_{L,h}, \end{aligned}$$

where A is the average endowment of durables in the economy. We assume that agents with opposite types meet bilaterally via directed search after realization of their shocks in every period. Terms of trade are set with bargaining power θ and $1-\theta$ for type B and L. In section 9.7., we show that asset sales alone cannot achieve the efficient allocation.

Proposition 10. *With two types of agents who are subject to preference shocks, only asset sales in bilateral trade cannot implement the efficient allocation, even as the preference shock becomes unlikely.*

Finally, it is straightforward to show that given (22), there exists an efficient equilibrium where agents rent. In particular, if all type B agents hold $c^*_{B,\ell}$ units and type L agents hold $c^*_{L,h}$ units of asset, we attain the efficient allocation when type B agents with h shock rent $c^*_{B,h} - c^*_{B,\ell} = c^*_{L,\ell} - c^*_{L,h}$ from type L agents with ℓ shock. Note that in this equilibrium type B agents always rent, while type L agents always lend.

9.6. More on directed search: the case with no rentals

In the context of a lending relationship, we usually understand counterparty risk as the risk that a borrower fails to repay his debt. In the paper, we illustrate another type of counterparty risk: The risk that the next counterparty does not hold the right amount of durables. To show this, we re-consider our simple two-period example. In this section, we assume that, in addition to being matched with the right type (h or ℓ), an agent endowed with a is always being matched with an agent holding $2A - a$. To make sure that this is always feasible, we have to modify the environment slightly by adding noise traders. These traders do not hold the equilibrium amount of durables and an agent who deviates from the equilibrium strategy and holds $a \neq a^*$ can always be matched with a noise trader holding $2A - a$. This is how this example differs from our earlier one.

Under this assumption the first order conditions on the Walrasian market at $t = 1$ are still given by (2) and (3). Prices still satisfy $p^s = p^r + \beta V_i'(a_i^*)$ and $p^r = u_i'(c_i^*)$. Moreover, the value of endowment a for a type i at the start of the bargaining game is still given by $v_i(a)$ for $i \in \{h, \ell\}$ as described in (4) and (5) with the added requirement that $a_{-i} = 2A - a_i$, or

$$
\begin{aligned}
v_h(a) &= u_h(a) + \theta S(a, 2A - a), && (23)\\
v_\ell(a) &= u_\ell(a) + (1-\theta) S(2A - a, a), && (24)
\end{aligned}
$$

where $S(a_h, a_\ell) = u_h(c_h^*) + u_\ell(c_\ell^*) - u_h(a_h) - u_\ell(a_\ell)$ is again the match surplus from trade. Notice that contrary to our earlier example, an agent's payoff only depends on his durables ownership, as in a Walrasian market. Loosely speaking, agents now have better control over the optimum amount of durables. With the new matching technology, $c_h + c_\ell = 2A$ so that $dc_h/da_h = -dc_\ell/da_h$. We can then compute the marginal expected value for agent i of holding a_i at date 2 as

$$
V_i'(a) \;=\; \pi \left\{(1-\theta_i)u_i'(a) + \theta_i u_{-i}'(2A - a)\right\} + (1-\pi)\left\{(1-\theta_{-i})u_{-i}'(a) + \theta_{-i} u_i'(2A - a)\right\},
$$

where $\theta_h = \theta$ and $\theta_\ell = 1 - \theta$. Hence, evaluating at $a = c_h^*$ and c_ℓ^* for V_h' and V_ℓ' respectively, we obtain

$$
V_h'(c_h^*) = \pi \left\{(1-\theta)u_h'(c_h^*) + \theta u_\ell'(c_\ell^*)\right\} + (1-\pi)\left\{\theta u_\ell'(c_h^*) + (1-\theta)u_h'(c_\ell^*)\right\}
$$

and

$$
V_\ell'(c_\ell^*) = \pi \left\{\theta u_\ell'(c_\ell^*) + (1-\theta)u_h'(c_h^*)\right\} + (1-\pi)\left\{(1-\theta)u_h'(c_\ell^*) + \theta u_\ell'(c_h^*)\right\},
$$

so that $V_\ell'(c_\ell^*) = V_h'(c_h^*)$. Because $u_h'(c_h^*) = u_\ell'(c_\ell^*)$, there is no wedge at (c_h^*, c_ℓ^*) between the value of present and future consumption. As a consequence agents do not rent and only the sales market is active at $t = 1$.

9.7. Proofs

Proof of Proposition 8

Given the time to the next meeting is $\Delta < 1$, we denote the ownership of durables and the rental amount by $\bar{a}_{\ell,\Delta}$, $\bar{a}_{h,\Delta}$ and q_Δ^r. Using this notation, we have $c_\ell^* = \bar{a}_\ell - q^r = \bar{a}_{\ell,\Delta} - q_\Delta^r$

and $c_h^* = \bar{a}_h + q^r = \bar{a}_{h,\Delta} + q^r_\Delta$. Using the equilibrium condition of Proposition 3, we get

$$\begin{aligned}
& (2\theta-1)u_h'(c_h^*) \\
= \; & \frac{[\pi-(2\pi-1)\beta][\theta u_\ell'(c_\ell^*+q^r)-(1-\theta)u_h'(c_h^*-q^r)]-(1-\pi)[\theta u_\ell'(c_h^*-q^r)-(1-\theta)u_h'(c_\ell^*+q^r)]}{(2\pi-1)(1-\beta)} \\
= \; & \frac{[\pi_\Delta-(2\pi_\Delta-1)\beta_\Delta][\theta u_\ell'(c_\ell^*+q^r_\Delta))-(1-\theta)u_h'(c_h^*-q^r_\Delta)]}{(2\pi_\Delta-1)(1-\beta_\Delta)} \\
& -\frac{(1-\pi_\Delta)[\theta u_\ell'(c_h^*-q^r_\Delta)-(1-\theta)u_h'(c_\ell^*+q^r_\Delta))]}{(2\pi_\Delta-1)(1-\beta_\Delta)},
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
& LH(q^r)+\left(\frac{(1-\pi)}{(1-2(1-\pi))(1-\beta)}\right)(LH(q^r)-RH(q^r)) \\
= \; & LH(q^r_\Delta)+\left(\frac{(1-\pi_\Delta)}{(1-2(1-\pi_\Delta))(1-\beta_\Delta)}\right)(LH(q^r_\Delta)-RH(q^r_\Delta)), \qquad (25)
\end{aligned}$$

where

$$LH(q^r) = \theta u_\ell'(c_\ell^*+q^r)-(1-\theta)u_h'(c_h^*-q^r)$$

and

$$RH(q^r) = \theta u_\ell'(c_h^*-q^r)-(1-\theta)u_h'(c_\ell^*+q^r).$$

For $q \leq ({c_h}^* - {c_\ell}^*)/2$, we have

$$u_\ell'(c_h^*-q) < u_\ell'(c_\ell^*+q) < u_\ell'(c_\ell^*) = u_h'(c_h^*) < u_h'(c_h^*-q) < u_h'(c_\ell^*+q).$$

Therefore, we have $LH(q) > RH(q)$. Moreover, concavity of u_h and u_ℓ implies

$$\frac{d}{dq}LH(q) \quad < 0 < \quad \frac{d}{dq}RH(q).$$

Notice that for $\Delta < 1$

$$\begin{aligned}
\frac{(1-\pi_\Delta)}{(1-2(1-\pi_\Delta))(1-\beta_\Delta)} & = \frac{(1-\pi)}{(1-2\Delta(1-\pi))(1-\beta)} \\
& < \frac{(1-\pi)}{(1-2(1-\pi))(1-\beta)}.
\end{aligned}$$

Thus, (25) implies $q^r_\Delta < q^r$ for $\Delta < 1$. Notice that, as $\Delta \to 0$, the share of change in consumption due to renting decreases to $q^r_0 > 0$, which is determined by

$$(2\theta - 1)u'_h(c^*_h) \quad = \quad LH(q^r_0) + \frac{(1-\pi)}{(1-\beta)}\left(LH(q^r_0) - RH(q^r_0)\right).$$

This completes the proof.

Proof of Proposition 9

We still assume that agents who did not switch are matched together, while those agents who just switched are matched with each other. With Nash bargaining, the allocation of an agent h with endowment a_h matched with an agent ℓ with endowment a_ℓ solves the following problem.

$$\begin{aligned}\max_{q^s,q^r,d}&[u_h(a_h + q^s + q^r) - d + \beta V_h(a_h + q^s) - u_h(a_h) - \beta\tilde{W}_h(a_h)]^\theta \\ &\times[u_\ell(a_\ell - q^s - q^r) + d + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta\tilde{W}_\ell(a_\ell)]^{1-\theta},\end{aligned}$$

which has the first order conditions

$$\begin{aligned}V'_h(a_h + q^s) &= V'_\ell(a_\ell - q^s), \\ u'_h(a_h + q^s + q^r) &= u'_\ell(a_h - q^s - q^r), \\ d(a_h, a_\ell) &= (1-\theta)[u_h(a_h + q^s + q^r) + \beta V_h(a_h + q^s) - u_h(a_h) - \beta\tilde{W}_h(a_h)] \\ &\quad -\theta[u_\ell(a_\ell - q^s - q^r) + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta\tilde{W}_\ell(a_\ell)].\end{aligned}$$

We still assume that agents who did not switch types are matched together while those agents who just switched are matched together. We first solve for $\tilde{W}_i(a)$. By definition, $\tilde{W}_i(a) = \pi W_i(a) + (1-\pi)W_j(a)$, with $i \neq j \in \{h, \ell\}$, and where W_i denotes the value of participating in the Walrasian market as a type i. From the problem of agents in the Walrasian market, it follows that $W_i(a) = pa + W_i(0)$, where $W_i(0)$ is given by

$$\begin{aligned}W_i(0) &= u_i(a^w_i) - pa^w_i + \beta E_{k|i}W_k(a^w_i) \\ &= u_i(a^w_i) - p^r a^w_i + \beta E_{k|i}W_k(0),\end{aligned}$$

where a_i^w is the solution to $u_i'(a_i^w) = p^r$ and $u_h'(a_h^w) = u_\ell'(a_\ell^w)$ with $a_\ell^w + a_h^w = 2A$. Solving for $W_i(0)$ we have

$$\begin{aligned} W_h(0) &= u_h(a_h^w) - p^r a_h^w + \beta\pi W_h(0) + \beta(1-\pi)W_\ell(0), \\ W_\ell(0) &= u_\ell(a_\ell^w) - p^r a_\ell^w + \beta\pi W_\ell(0) + \beta(1-\pi)W_h(0). \end{aligned}$$

Thus

$$(1-\beta)W_h(0) = \alpha[u_h(a_h^w) - p^r a_h^w] + (1-\alpha)[u_\ell(a_\ell^w) - p^r a_\ell^w],$$

where $\alpha = \frac{1-\beta\pi}{1+\beta-2\beta\pi} \in [0,1]$. Similarly,

$$(1-\beta)W_\ell(0) = \alpha[u_\ell(a_\ell^w) - p^r a_\ell^w] + (1-\alpha)[u_h(a_h^w) - p^r a_h^w].$$

Therefore,

$$\begin{aligned} (1-\beta)\tilde{W}_h(0) &= \pi(1-\beta)W_h(0) + (1-\pi)(1-\beta)W_\ell(0) \\ &= u(a_h^w) - p^r a_h^w + [\pi + \alpha - 2\pi\alpha][u_\ell(a_\ell^w) - p^r a_\ell^w - u_h(a_h^w) + p^r a_h^w], \end{aligned}$$

and

$$\begin{aligned} (1-\beta)\tilde{W}_\ell(0) &= (1-\pi)(1-\beta)W_h(0) + \pi(1-\beta)W_\ell(0) \\ &= u(a_\ell^w) - p^r a_\ell^w + [\pi + \alpha - 2\pi\alpha][u_h(a_h^w) - p^r a_h^w - u_\ell(a_\ell^w) + p^r a_\ell^w]. \end{aligned}$$

Notice that

$$\tilde{W}_h(0) + \tilde{W}_\ell(0) = \frac{u(a_h^w) - p^r a_h^w + u(a_\ell^w) - p^r a_\ell^w}{1-\beta}.$$

In this environment, the first order condition of the bargaining problem gives us

$$u_h'(\bar{a}_h + q^r) = u_\ell'(\bar{a}_\ell - q^r).$$

Thus

$$\begin{aligned} \bar{a}_h + q^r &= a_h^w, \\ \bar{a}_\ell - q^r &= a_\ell^w. \end{aligned}$$

The value functions are

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)], \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)]. \end{aligned}$$

Adding both equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1-\beta} = \tilde{W}_h(a_h^w) + \tilde{W}_\ell(a_\ell^w). \tag{26}$$

From the bargaining first order condition, we obtain

$$\begin{aligned} d(a_h, a_\ell) &= (1-\theta)[u_h(a_h + q^s + q^r) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta \tilde{W}_h(a_h)] \\ &\quad -\theta[u_\ell(a_\ell - q^s - q^r) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta \tilde{W}_\ell(a_\ell)]. \end{aligned}$$

Thus, using the fact that $q^s(\bar{a}_h, \bar{a}_\ell) = 0$, the transfer $d(\bar{a}_h, \bar{a}_\ell)$ is given by

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1-\theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_h) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_h)] \\ &\quad -\theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_\ell)]. \end{aligned}$$

Therefore, using the relation between $\bar{a}_i$ and a_i^w as well as equation (26), we obtain:

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_h) + \beta \tilde{W}_h(\bar{a}_h) \\ &\quad +\theta \left\{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\right\}. \end{aligned}$$

Also,

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_\ell) + \beta \tilde{W}_\ell(\bar{a}_\ell) \\ &\quad +(1-\theta) \left\{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\right\}. \end{aligned}$$

In a similar fashion, using $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$, we obtain

$$\begin{aligned} d(\bar{a}_\ell, \bar{a}_h) &= (1-\theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_\ell)] \\ &\quad -\theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_h)]. \end{aligned}$$

Therefore,

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_h) + \beta \tilde{W}_\ell(\bar{a}_h) \\ &\quad + (1-\theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)], \end{aligned}$$

and similarly

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_\ell) + \beta \tilde{W}_h(\bar{a}_\ell) \\ &\quad + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]. \end{aligned}$$

Using the above calculations, we obtain

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta \tilde{W}_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_h) + \beta \tilde{W}_\ell(\bar{a}_h)] + \theta\pi S + (1-\theta)(1-\pi)\tilde{S}, \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta \tilde{W}_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_\ell) + \beta \tilde{W}_h(\bar{a}_\ell)] + \pi(1-\theta)S + (1-\pi)\theta\tilde{S}, \end{aligned}$$

where

$$\begin{aligned} S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell), \\ \tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h). \end{aligned}$$

And taking the derivatives, we have

$$V_h'(\bar{a}_h) = \pi[u_h'(\bar{a}_h) + \beta p] + (1-\pi)[u_\ell'(\bar{a}_h) + \beta p] + (1-\pi)(1-\theta)\frac{\partial \tilde{S}}{\partial \bar{a}_h} + \theta\pi\frac{\partial S}{\partial \bar{a}_h},$$

and, using the first order condition for q^r, we obtain

$$\begin{aligned} V_h'(\bar{a}_h) &= \beta p + (1-\pi)u_\ell'(\bar{a}_h) + \pi u_h'(\bar{a}_h) \\ &\quad + (1-\pi)(1-\theta)[u_h'(\bar{a}_h + q^r) - u_\ell'(\bar{a}_h)] \\ &\quad + \theta\pi[u_h'(\bar{a}_h + q^r) - u_h'(\bar{a}_h)]. \end{aligned}$$

Also, using the first order condition for q^r, we obtain

$$\begin{aligned} V_\ell'(\bar{a}_\ell) &= \beta p + \pi u_\ell'(\bar{a}_\ell) + (1-\pi)u_h'(\bar{a}_\ell) \\ &\quad + \pi(1-\theta)[u_\ell'(\bar{a}_\ell - q^r) - u_\ell'(\bar{a}_\ell)] + (1-\pi)\theta[u_\ell'(\bar{a}_\ell - q^r) - u_h'(\bar{a}_\ell)]. \end{aligned}$$

The first condition for q^s imposes that $V_h'(\bar{a}_h) = V_\ell'(\bar{a}_\ell)$. Using the fact that $u_\ell'(\bar{a}_\ell - q^r) = u_h'(\bar{a}_h + q^r)$ and simplifying, we obtain

$$u_h'(\bar{a}_h + q^r) = \frac{\pi\theta u_\ell'(\bar{a}_\ell) + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) - (1-\pi)\theta u_\ell'(\bar{a}_h) - \pi(1-\theta)u_h'(\bar{a}_h)}{(1-2\pi)(1-2\theta)}.$$

Therefore, the equilibrium is given by

$$\begin{aligned} u_h'(\bar{a}_h + q^r) &= u_\ell'(\bar{a}_\ell - q^r), \\ \bar{a}_h + \bar{a}_\ell &= 2A, \\ u_h'(\bar{a}_h + q^r) &= \frac{\pi\theta u_\ell'(\bar{a}_\ell) + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) - (1-\pi)\theta u_\ell'(\bar{a}_h) - \pi(1-\theta)u_h'(\bar{a}_h)}{(1-2\pi)(1-2\theta)}. \end{aligned}$$

Notice that β does not impact the equilibrium allocation.

Also, suppose that $q^r = 0$ is an equilibrium. Then $u_h'(\bar{a}_h) = u_\ell'(\bar{a}_\ell)$ and

$$u_h'(\bar{a}_h)(1-2\theta) = (1-\theta)u_h'(\bar{a}_\ell) - \theta u_\ell'(\bar{a}_h),$$

or

$$\theta\left[u_\ell'(\bar{a}_h) - u_h'(\bar{a}_h)\right] = (1-\theta)\left[u_h'(\bar{a}_\ell) - u_h'(\bar{a}_h)\right].$$

However, because $\bar{a}_\ell \leq \bar{a}_h$, the RHS is positive, while the LHS is negative by assumption. Therefore, $q^r = 0$ cannot be an equilibrium.

Now we show that the rented amount is actually lower under this arrangement. The equilibrium allocations under the benchmark and the Walrasian outside-option can be summarized by

$$\begin{aligned} & (1-2\pi)(1-\beta)(1-2\theta)u_h'(c_h^*) \\ = \; & \pi(1-\beta)\left[\theta u_\ell'(c_\ell^* + \bar{q}^r) - (1-\theta)u_h'(c_h^* - \bar{q}^r)\right] \\ & -(1-\pi)(1-\beta)\left[\theta u_\ell'(c_h^* - \bar{q}^r) - (1-\theta)u_h'(c_\ell^* + \bar{q}^r)\right], \end{aligned}$$

where $u_h'(c_h^*) = u_\ell'(c_\ell^*)$ with $c_h^* + c_\ell^* = 2A$ and q^{r*} and $\bar{q}^*$ being the rented amounts under the benchmark and the Walrasian outside-option respectively. Define

$$LH(q^r) = \theta u_\ell'(c_\ell^* + q^r) - (1-\theta)u_h'(c_h^* - q^r),$$

and

$$RH(q^r) \;=\; \theta u_\ell'(c_h^* - q^r) - (1-\theta)u_h'(c_\ell^* + q^r).$$

Note that for $q^r \leq (c_h{}^* - c_\ell{}^*)/2$, we have

$$u_\ell'(c_h^* - q^r) < u_\ell'(c_\ell^* + q^r) < u_\ell'(c_\ell^*) = u_h'(c_h^*) < u_h'(c_h^* - q^r) < u_h'(c_\ell^* + q^r).$$

Therefore, we have $LH(q^r) > RH(q^r)$, which implies for all q^r

$$[\pi(1-\beta) + (1-\pi)\beta]\, LH(q^r) - (1-\pi)RH(q^r) > \pi(1-\beta)LH(q^r) - (1-\pi)(1-\beta)RH(q^r).$$

Finally, concavity of u_h and u_ℓ implies

$$\frac{d}{dq^r}LH(q^r) \;\; < 0 < \;\; \frac{d}{dq^r}RH(q^r).$$

Hence, $[\pi(1-\beta) + (1-\pi)\beta]\, LH(q^r) - (1-\pi)RH(q^r)$ and $\pi(1-\beta)LH(q^r) - (1-\pi)(1-\beta)RH(q^r)$ are both decreasing in q^r. This means we have

$$\begin{aligned}
& (1-2\pi)(1-\beta)(1-2\theta)u_h'(c_h^*) \\
= \;& [\pi(1-\beta) + (1-\pi)\beta]\, [\theta u_\ell'(c_\ell^* + q^{r*}) - (1-\theta)u_h'(c_h^* - q^{r*})] \\
& -(1-\pi)\, [\theta u_\ell'(c_h^* - q^{r*}) - (1-\theta)u_h'(c_\ell^* + q^{r*})] \\
> \;& \pi(1-\beta)\, [\theta u_\ell'(c_\ell^* + q^{r*}) - (1-\theta)u_h'(c_h^* - q^{r*})] \\
& -(1-\pi)(1-\beta)\, [\theta u_\ell'(c_h^* - q^{r*}) - (1-\theta)u_h'(c_\ell^* + q^{r*})]\,.
\end{aligned}$$

Therefore, $q^{r*} > \bar{q}^r$. Note that the change in β does not affect $\bar{q}^r$, but as β approaches zero, then $q^{r*} \to \bar{q}^r$.

Wedge

The solution for (21) implies

$$\begin{aligned}
u_h'\left(a_h' + q^r\right) \;&=\; u_\ell'\left(a_\ell' - q^r\right) \\
d^r \;&=\; (1-\theta_r)\left[u_h\left(a_h' + q^r\right) - u_h\left(a_h'\right)\right] + \theta_r\left[u_\ell\left(a_\ell'\right) - u_\ell\left(a_\ell' - q^r\right)\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
W_h^r\left(a_h'\right) &= \theta_r\left[u_h\left(a_h'+q^r\right)+u_\ell\left(a_\ell'-q^r\right)\right]+\left[(1-\theta_r)u_h\left(a_h'\right)-\theta_r u_\ell\left(a_\ell'\right)\right] \\
&\quad +\beta\left\{\pi\cdot W_h^s\left(a_h'\right)+(1-\pi)\cdot W_\ell^s\left(a_h'\right)\right\}, \\
W_\ell^r\left(a_\ell'\right) &= (1-\theta_r)\left[u_h\left(a_h'+q^r\right)+u_\ell\left(a_\ell'-q^r\right)\right]-\left[(1-\theta_r)u_h\left(a_h'\right)-\theta_r u_\ell\left(a_\ell'\right)\right] \\
&\quad +\beta\left\{\pi\cdot W_\ell^s\left(a_\ell'\right)+(1-\pi)\cdot W_h^s\left(a_\ell'\right)\right\}.
\end{aligned}$$

The solution for (20) implies

$$\begin{aligned}
W_h^{r\prime}\left(a_h+q^s\right) &= \omega\cdot W_\ell^{r\prime}\left(a_\ell-q^s\right) \\
d^s &= \frac{1}{\omega}\left\{(1-\theta_s)\left[W_h^r\left(a_h+q^s\right)-W_h^r\left(a_h\right)\right]-\omega\theta_s\left[W_\ell^r\left(a_\ell-q^s\right)-W_\ell^r\left(a_\ell\right)\right]\right\}.
\end{aligned}$$

Hence,

$$\begin{aligned}
W_h^s\left(a_h\right) &= \theta_s\left[W_h^r\left(a_h+q^s\right)+\omega W_\ell^r\left(a_\ell-q^s\right)\right]+\left[(1-\theta_s)W_h^r\left(a_h\right)-\omega\theta_s W_\ell^r\left(a_\ell\right)\right], \\
W_\ell^s\left(a_\ell\right) &= \frac{1}{\omega}(1-\theta_s)\left[W_h^r\left(a_h+q^s\right)+\omega W_\ell^r\left(a_\ell-q^s\right)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r\left(a_h\right)-\omega\theta_s W_\ell^r\left(a_\ell\right)\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
W_h^r\left(a_h'\right) &= \theta_r\left[u_h\left(a_h'+q^r\right)+u_\ell\left(a_\ell'-q^r\right)\right]+\left[(1-\theta_r)u_h\left(a_h'\right)-\theta_r u_\ell\left(a_\ell'\right)\right] \\
&\quad +\beta\pi\left\{\theta_s\left[W_h^r\left(a_h'+q^s\right)+\omega W_\ell^r\left(a_\ell'-q^s\right)\right]+\left[(1-\theta_s)W_h^r\left(a_h'\right)-\omega\theta_s W_\ell^r\left(a_\ell'\right)\right]\right\} \\
&\quad +\beta(1-\pi)\left\{\frac{1}{\omega}(1-\theta_s)\left[W_h^r\left(a_h''+q^s\right)+\omega W_\ell^r\left(a_h'-q^s\right)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r\left(a_h''\right)-\omega\theta_s W_\ell^r\left(a_h'\right)\right]\right\}, \\
W_\ell^r\left(a_\ell'\right) &= (1-\theta_r)\left[u_h\left(a_h'+q^r\right)+u_\ell\left(a_\ell'-q^r\right)\right]-\left[(1-\theta_r)u_h\left(a_h'\right)-\theta_r u_\ell\left(a_\ell'\right)\right] \\
&\quad +\beta\pi\left\{\frac{1}{\omega}(1-\theta_s)\left[W_h^r\left(a_h'+q^s\right)+\omega W_\ell^r\left(a_\ell'-q^s\right)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r\left(a_h'\right)-\omega\theta_s W_\ell^r\left(a_\ell'\right)\right]\right\} \\
&\quad +\beta(1-\pi)\left\{\theta_s\left[W_h^r\left(a_\ell'+q^s\right)+\omega W_\ell^r\left(a_\ell''-q^s\right)\right]+\left[(1-\theta_s)W_h^r\left(a_\ell'\right)-\omega\theta_s W_\ell^r\left(a_\ell''\right)\right]\right\},
\end{aligned}$$

where in the equilibrium sales market an agent h holding a_h' is matched with an agent ℓ with a_ℓ', an agent h holding a_ℓ' is matched with an agent ℓ holding a_ℓ'', and an agent ℓ holding a_h' is matched with an agent h with a_h''.

Assuming that by a marginal change in durables the counterparty's holding of durables remains constant, and also applying of the Envelope theorem for the effects on q^r and q^s, we obtain

$$
\begin{aligned}
W_h^{r\prime}(a_h') &= \theta_r u_h'(a_h' + q^r) + (1-\theta_r) u_h'(a_h') \\
&\quad + \beta\pi \left\{ \theta_s W_h^{r\prime}(a_h' + q^s) + (1-\theta_s) W_h^{r\prime}(a_h') \right\} \\
&\quad + \beta(1-\pi) \left\{ (1-\theta_s) W_\ell^{r\prime}(a_h' - q^s) + \theta_s W_\ell^{r\prime}(a_h') \right\}, \\
W_\ell^{r\prime}(a_\ell') &= (1-\theta_r) u_\ell'(a_\ell' - q^r) + \theta_r u_\ell'(a_\ell') \\
&\quad + \beta\pi \left\{ (1-\theta_s) W_\ell^{r\prime}(a_\ell' - q^s) + \theta_s W_\ell^{r\prime}(a_\ell') \right\} \\
&\quad + \beta(1-\pi) \left\{ \theta_s W_h^{r\prime}(a_\ell' + q^s) + (1-\theta_s) W_h^{r\prime}(a_\ell') \right\}.
\end{aligned}
$$

In the equilibrium agents h would adjust their durables level to a_h^* and agents ℓ would adjust theirs to a_ℓ^* after realizing their types, where $a_h^* + a_\ell^* = 2A$ and $W_h^{r\prime}(a_h^*) = \omega \cdot W_\ell^{r\prime}(a_\ell^*)$. Hence, $q^s(a_h^*, a_\ell^*) = 0$ and $q^s(a_\ell^*, a_h^*) = a_h^* - a_\ell^*$. Moreover, we have $q^r(a_h^*, a_\ell^*) = c_h^* - a_h^*$, where $u_h'(c_h^*) = u_\ell'(2A - c_h^*) = u_\ell'(c_\ell^*)$. Therefore, we have

$$
\begin{aligned}
W_h^{r\prime}(a_h^*) &= \theta_r u_h'(c_h^*) + (1-\theta_r) u_h'(a_h^*) \\
&\quad + \beta\pi \left\{ \theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_h^*) \right\} \\
&\quad + \beta(1-\pi) \left\{ (1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*) \right\}, \\
W_\ell^{r\prime}(a_\ell^*) &= (1-\theta_r) u_\ell'(c_\ell^*) + \theta_r u_\ell'(a_\ell^*) \\
&\quad + \beta\pi \left\{ (1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_\ell^*) \right\} \\
&\quad + \beta(1-\pi) \left\{ \theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_\ell^*) \right\} \\
W_h^{r\prime}(a_\ell^*) &= \theta_r u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r) u_h'(a_\ell^*) \\
&\quad + \beta\pi \left\{ \theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_\ell^*) \right\} \\
&\quad + \beta(1-\pi) \left\{ (1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_\ell^*) \right\}, \\
W_\ell^{r\prime}(a_h^*) &= (1-\theta_r) u_\ell'(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u_\ell'(a_h^*) \\
&\quad + \beta\pi \left\{ (1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*) \right\} \\
&\quad + \beta(1-\pi) \left\{ \theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_h^*) \right\},
\end{aligned}
$$

where $q^r(a_\ell^*, a_\ell^*)$ and $q^r(a_h^*, a_h^*)$ are set such that $u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) = u_\ell'(a_\ell^* - q^r(a_\ell^*, a_\ell^*))$ and $u_h'(a_h^* + q^r(a_h^*, a_h^*)) = u_\ell'(a_h^* - q^r(a_h^*, a_h^*))$. Note that in the equilibrium type i agents would enter the rentals market holding a_i^* for $i \in \{h, \ell\}$, therefore, when a type i agent shows up in the rentals market holding a_{-i}^*, she will still be matched with a $-i$ agent with asset holding a_{-i}^*.

This arrangement implies

$$
\begin{aligned}
(1-\beta\pi)\cdot W_h^{r\prime}(a_h^*) &= \theta_r u_h'(c_h^*) + (1-\theta_r)u_h'(a_h^*) \\
&\quad + \beta(1-\pi)\left\{(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*)\right\}, \\
(1-\beta\pi)\cdot W_\ell^{r\prime}(a_\ell^*) &= (1-\theta_r)u_\ell'(c_\ell^*) + \theta_r u_\ell'(a_\ell^*) \\
&\quad + \beta(1-\pi)\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s)W_h^{r\prime}(a_\ell^*)\right\}, \\
(1-\beta\pi(1-\theta_s))\cdot W_h^{r\prime}(a_\ell^*) &= \theta_r u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r)u_h'(a_\ell^*) \\
&\quad + \beta\pi\theta_s W_h^{r\prime}(a_h^*) + \beta(1-\pi)W_\ell^{r\prime}(a_\ell^*), \\
(1-\beta\pi\theta_s)\cdot W_\ell^{r\prime}(a_h^*) &= (1-\theta_r)u_\ell'(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u_\ell'(a_h^*) \\
&\quad + \beta\pi(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \beta(1-\pi)W_h^{r\prime}(a_h^*).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
(1-\beta\pi)\cdot W_h^{r\prime}(a_h^*) &= \left\{\theta_r u_h'(c_h^*) + (1-\theta_r)u_h'(a_h^*)\right\} \\
&\quad + \beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) \\
&\quad + \frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\left\{(1-\theta_r)u_\ell'(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u_\ell'(a_h^*)\right\} \\
&\quad + \frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\left\{\beta\pi(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \beta(1-\pi)W_h^{r\prime}(a_h^*)\right\}, \\
(1-\beta\pi)\cdot W_\ell^{r\prime}(a_\ell^*) &= \left\{(1-\theta_r)u_\ell'(c_\ell^*) + \theta_r u_\ell'(a_\ell^*)\right\} \\
&\quad + \beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) \\
&\quad + \frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\left\{\theta_r u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r)u_h'(a_\ell^*)\right\} \\
&\quad + \frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\left\{\beta\pi\theta_s W_h^{r\prime}(a_h^*) + \beta(1-\pi)W_\ell^{r\prime}(a_\ell^*)\right\},
\end{aligned}
$$

which means

$$\begin{aligned}
&\left\{(1-\beta\pi)-\frac{\beta(1-\pi)\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\right\}\cdot W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)(1-\theta_s)\left\{1+\frac{\beta\pi\theta_s}{(1-\beta\pi\theta_s)}\right\}W_\ell^{r\prime}(a_\ell^*) = \left\{\theta_r u_h'(c_h^*)+(1-\theta_r)u_h'(a_h^*)\right\} \\
&\qquad +\frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}(1-\theta_r)u_\ell'(a_h^*-q^r(a_h^*,a_h^*)) \\
&\qquad +\frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\theta_r u_\ell'(a_h^*),
\end{aligned}$$

$$\begin{aligned}
&\left\{(1-\beta\pi)-\frac{\beta(1-\pi)\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\right\}\cdot W_\ell^{r\prime}(a_\ell^*) \\
&-\beta(1-\pi)\theta_s\left\{1+\frac{\beta\pi(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\right\}W_h^{r\prime}(a_h^*) = \left\{(1-\theta_r)u_\ell'(c_\ell^*)+\theta_r u_\ell'(a_\ell^*)\right\} \\
&\qquad +\frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\theta_r u_h'(a_\ell^*+q^r(a_\ell^*,a_\ell^*)) \\
&\qquad +\frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}(1-\theta_r)u_h'(a_\ell^*).
\end{aligned}$$

In turn

$$\begin{aligned}
&(1-\beta\pi)(1-\beta\pi\theta_s)W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)\beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) = (1-\beta\pi\theta_s)\left\{\theta_r u_h'(c_h^*)+(1-\theta_r)u_h'(a_h^*)\right\} \\
&\qquad +\beta(1-\pi)\theta_s\left\{(1-\theta_r)u_\ell'(a_h^*-q^r(a_h^*,a_h^*))+\theta_r u_\ell'(a_h^*)\right\},
\end{aligned}$$

and

$$\begin{aligned}
&(1-\beta\pi)(1-\beta\pi(1-\theta_s))W_\ell^{r\prime}(a_\ell^*) \\
&-\beta(1-\pi)\beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*), \\
&-\beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) = (1-\beta\pi(1-\theta_s))\left\{(1-\theta_r)u_\ell'(c_\ell^*)+\theta_r u_\ell'(a_\ell^*)\right\} \\
&\qquad +\beta(1-\pi)(1-\theta_s)\left\{\theta_r u_h'(a_\ell^*+q^r(a_\ell^*,a_\ell^*))+(1-\theta_r)u_h'(a_\ell^*)\right\}.
\end{aligned}$$

In the equilibrium, we have $W_h^{r\prime}(a_h^*)=\omega\cdot W_\ell^{r\prime}(a_\ell^*)$, a_h^* and a_ℓ^* are given by $a_h^*=2A-a_\ell^*$ and

$$
\frac{\begin{array}{c}(1-\beta\pi(1-\theta_s))\left\{(1-\theta_r)u_\ell'\left(c_\ell^*\right)+\theta_r u_\ell'\left(a_\ell^*\right)\right\}\\ +\beta(1-\pi)(1-\theta_s)\left\{\theta_r u_h'\left(a_\ell^*+q^r(a_\ell^*,a_\ell^*)\right)+(1-\theta_r)u_h'\left(a_\ell^*\right)\right\}\end{array}}{\left\{(1-\beta\pi)(1-\beta\pi(1-\theta_s))-\beta(1-\pi)\beta(1-\pi)(1-\theta_s)\right\}-\beta(1-\pi)\theta_s\cdot\omega}
$$

$$
= \frac{\begin{array}{c}(1-\beta\pi\theta_s)\left\{\theta_r u_h'\left(c_h^*\right)+(1-\theta_r)u_h'\left(a_h^*\right)\right\}\\ +\beta(1-\pi)\theta_s\left\{(1-\theta_r)u_\ell'\left(a_h^*-q^r(a_h^*,a_h^*)\right)+\theta_r u_\ell'\left(a_h^*\right)\right\}\end{array}}{\left\{(1-\beta\pi)(1-\beta\pi\theta_s)-\beta(1-\pi)\beta(1-\pi)\theta_s\right\}\cdot\omega-\beta(1-\pi)(1-\theta_s)}. \tag{27}
$$

As ω increases, the denominator of the left side of equation (27) decreases and the denominator of the right side increases. The equality is maintained by the increase in the numerator of the left side and decrease in the numerator of the right side. Hence, by the concavity of $u(\cdot)$, an increase in the wedge ω decreases a_h^* and increases a_ℓ^*.

Proof of Proposition 10

When agents cannot rent, they will start the next period with the amount of durables that they consumed in the current period. Hence, the value of holding a durables for a type B agent with shock h is given by

$$
V_{B,h}(a) \quad = \quad u_{B,h}(a+q^s)-d+\beta\cdot V_B(a+q^s),
$$

where $V_B(a)=\pi\cdot V_{B,h}(a)+(1-\pi)\cdot V_{B,\ell}(a)$ and the terms of trade, q^s and d, are set as the solution for the following bargaining problem.

$$
\begin{aligned}
\max_{q^s,d} \quad & \left[u_{B,h}(a_{B,h}+q^s)-d+\beta\cdot V_B(a_{B,h}+q^s)-u_{B,h}(a_{B,h})-\beta\cdot V_B(a_{B,h})\right]^\theta \qquad (28)\\
& \times\left[u_{L,\ell}(a_{L,\ell}-q^s)+d+\beta\cdot V_L(a_{L,\ell}-q^s)-u_{L,\ell}(a_{L,\ell})-\beta\cdot V_L(a_{L,\ell})\right]^{1-\theta}\\
\text{s.t.} \quad & q^s\in\left[-a_{B,h},a_{L,\ell}\right].
\end{aligned}
$$

The value functions $V_{B,\ell}(\cdot)$, $V_{L,h}(\cdot)$, $V_{L,\ell}(\cdot)$, and $V_L(\cdot)$ are defined similarly.

The solution for the bargaining problem (28) and the similar problem for the bargaining between (B,ℓ) and (L,h) agents imply that

$$\begin{aligned} u'_{B,h}\left(a_{B,h}+q^s_{h,\ell}\right)+\beta\cdot V'_B\left(a_{B,h}+q^s_{h,\ell}\right) &= u'_{L,\ell}\left(a_{L,\ell}-q^s_{h,\ell}\right)+\beta\cdot V'_L\left(a_{L,\ell}-q^s_{h,\ell}\right), \quad (29)\\ u'_{B,\ell}\left(a_{B,\ell}+q^s_{\ell,h}\right)+\beta\cdot V'_B\left(a_{B,\ell}+q^s_{\ell,h}\right) &= u'_{L,h}\left(a_{L,h}-q^s_{\ell,h}\right)+\beta\cdot V'_L\left(a_{L,h}-q^s_{\ell,h}\right). \end{aligned}$$

If the equilibrium allocation is efficient then the distribution of type B agents' holdings of durables should have two mass points on $c^*_{B,h}$ and $c^*_{B,\ell}$ with probabilities π and $1-\pi$, and the distribution of type L agents' holdings of durables should have two mass points on $c^*_{L,h}$ and $c^*_{L,\ell}$ with probabilities $1-\pi$ and π, where $c^*_{B,h}$, $c^*_{B,\ell}$, $c^*_{L,h}$, and $c^*_{L,\ell}$ are defined by (22). In this efficient equilibrium, (B,h) agents with durables $c^*_{B,h}$ do not trade. But a (B,h) agent holding durables $c^*_{B,\ell}$ search and match with a (L,ℓ) agent holding durables $c^*_{L,h}$ and they adjust their holdings to $c^*_{B,h}$ and $c^*_{L,\ell}$. Hence, in the equilibrium we have

$$V_{B,h}(c_{B,\ell}) = u(c_{B,\ell}+q)+\beta V(c_{B,\ell}+q)-d(c_{B,\ell},c_{L,h}),$$

where

$$\begin{aligned} d &= (1-\theta)S_{B,h}(c_{B,\ell},c_{L,h})-\theta S_{L,\ell}(c_{B,\ell},c_{L,h})\\ &= (1-\theta)[u_{B,h}(c_{B,\ell}+q)+\beta V_B(c_{B,\ell}+q)-u_{B,h}(c_{B,\ell})-\beta V_B(c_{B,\ell})]\\ &\quad -\theta[u_{L,\ell}(c_{L,h}-q)+\beta V_L(c_{L,h}-q)-u_{L,\ell}(c_{L,h})-\beta V_L(c_{L,h})]. \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} V'_{B,h}(c^*_{B,h}) &= u'_{B,h}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h}),\\ V'_{B,h}(c^*_{B,\ell}) &= \theta\left[u'_{B,h}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h})\right]+(1-\theta)\left[u'_{B,h}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell})\right],\\ V'_{B,\ell}(c^*_{B,\ell}) &= u'_{B,\ell}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell}),\\ V'_{B,\ell}(c^*_{B,h}) &= \theta\left[u'_{B,\ell}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell})\right]+(1-\theta)\left[u'_{B,\ell}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h})\right], \end{aligned}$$

which implies

$$\begin{aligned} V'_B(c^*_{B,h}) &= \pi\cdot u'_{B,h}(c^*_{B,h})+(1-\pi)\theta\cdot u'_{B,\ell}(c^*_{B,\ell})+(1-\pi)(1-\theta)\cdot u'_{B,\ell}(c^*_{B,h})\\ &\quad +\beta\left(\pi+(1-\pi)(1-\theta)\right)\cdot V'_B(c^*_{B,h})+\beta(1-\pi)\theta\cdot V'_B(c^*_{B,\ell}). \quad (30) \end{aligned}$$

Similarly, for type L we have

$$\begin{aligned} V_L'(c_{L,\ell}^*) &= \pi \cdot u_{L,\ell}'(c_{L,\ell}^*) + (1-\pi)(1-\theta) \cdot u_{L,h}'(c_{L,h}^*) + (1-\pi)\theta \cdot u_{L,h}'(c_{L,\ell}^*) \\ &\quad + \beta\left(\pi + (1-\pi)\theta\right) \cdot V_L'(c_{L,\ell}^*) + \beta(1-\pi)(1-\theta) \cdot V_L'(c_{L,h}^*). \end{aligned} \tag{31}$$

Using (22) and the bargaining solutions (29) in the equilibrium we have

$$\begin{aligned} V_B'(c_{B,h}^*) &= V_L'(c_{L,\ell}^*), \\ V_B'(c_{B,\ell}^*) &= V_L'(c_{L,h}^*). \end{aligned}$$

Therefore, subtracting (30) from (31) we should have

$$\begin{aligned} &\theta\left[u_{L,h}'(c_{L,\ell}^*) + u_{B,\ell}'(c_{B,h}^*)\right] \\ = \quad &(2\theta-1)\left[u_{B,\ell}'(c_{B,\ell}^*) + u_{B,\ell}'(c_{B,h}^*) + \beta \cdot \left\{V_B'(c_{B,\ell}^*) - V_B'(c_{B,h}^*)\right\}\right]. \end{aligned}$$

Note that for $\theta \leq \frac{1}{2}$ the left hand of the above equality is positive, while the right hand of it is not. Hence, it cannot hold. This result holds for any $\pi \in [0,1]$.

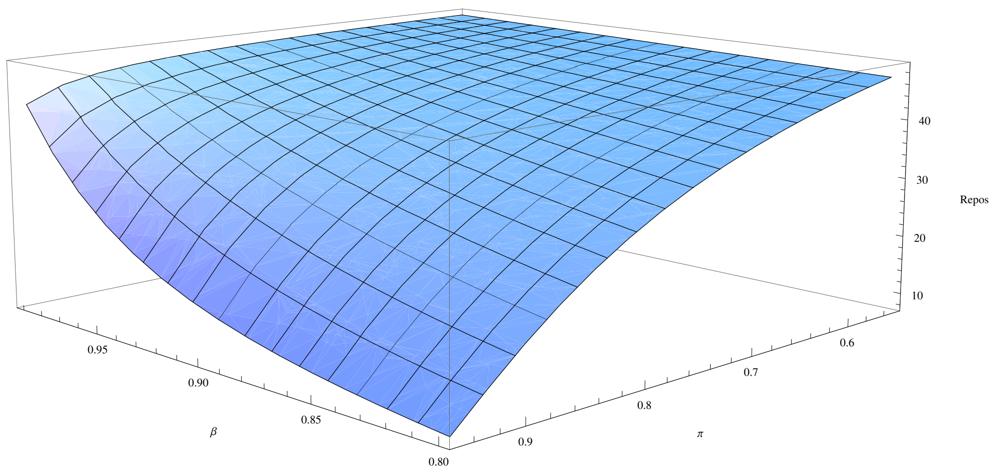


Figure 6: Repo volume in the (β, π)-space.

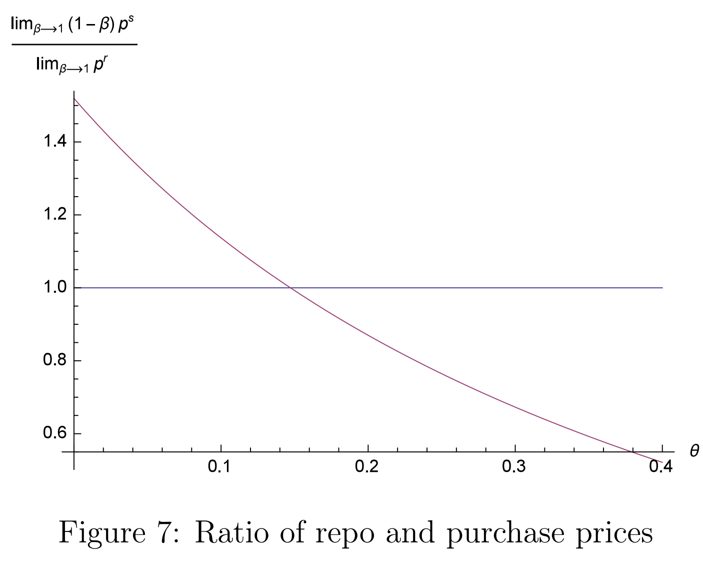


Figure 7: Ratio of repo and purchase prices

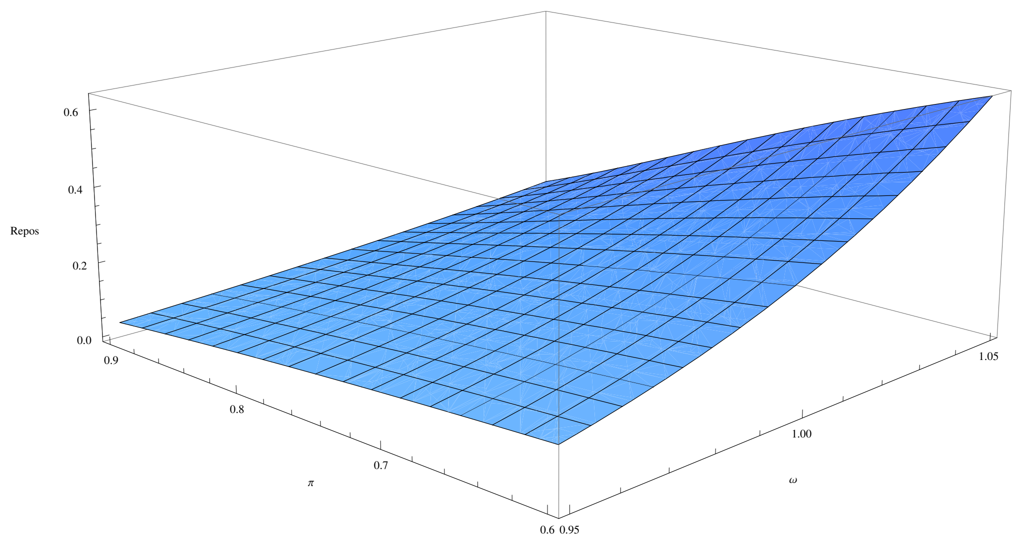


Figure 8: The effect of sale cost wedge ω on repos volume