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Optimal Bank Regulation In the Presence of Credit and Run Risk*

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Abstract

We modify the Diamond and Dybvig (1983) model of banking to jointly study various regulations in the presence of credit and run risk. Banks choose between liquid and illiquid assets on the asset side, and between deposits and equity on the liability side. The endogenously determined asset portfolio and capital structure interact to support credit extension, as well as to provide liquidity and risk-sharing services to the real economy. Our modifications create wedges in the asset and liability mix between the private equilibrium and a social planner's equilibrium. Correcting these distortions requires the joint implementation of a capital and a liquidity regulation.

Keywords: Bank Runs, Credit Risk, Limited Liability, Regulation, Capital, Liquidity
JEL Classification: E44, G01, G21, G28

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1 Introduction

Financial intermediaries, hereafter banks, perfom various socially useful functions. These include providing liquidity (Diamond and Dybvig, 1983), facilitating credit extension to fund productive investment (Diamond, 1984), and improving risk-sharing (Benston and Smith, 1976; Allen and Gale, 1997, 2004). Banks' asset portfolio composition and liabilities structure interact to allow them to perform these services. However, these same interactions can also be a source of fragility. Transforming illiquid long-term assets into liquid short-term claims, such as demandable deposits, is desirable, but exposes banks to the possibility of a run which can be disastrous for the bank, its borrowers and its depositors. Likewise, funding risky loans through both debt and equity improves risk-sharing (and potentially raises growth), but can lead to socially wasteful bankruptcy costs. Finally, the presence of short-term liabilities can generate better incentives for banks to monitor borrowers and honor their liabilities (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), but creates run risk that long-term funding avoids.

In this paper we expand the Diamond and Dybvig (1983) model of banking to incorporate all of the three aforementioned banking functions and explore whether private decisions result in an efficient level of intermediation. Our analysis is related to the emerging literature exploring optimal macroprudential regulation to address various inefficiencies, such as aggregate demand externalities in the presence of nominal rigidities (Farhi and Werning, 2016), pecuniary externalities operating through collateral constraints (Bianchi and Mendoza, forthcoming) or fire-sales externalities (Stein, 2012). However, our focus is different as we are interested in identifying externalities that pertain to banks choices associated with endogenous credit risk and run risk.

Bankers have a comparative advantage at intermediating funds, but their incentives to monitor their investment can differ from their investors due to private benefits that are available to them. On one hand, a fragile funding structure and run risk can be optimal to discipline the banker and align the incentives for monitoring. Short-term debt is, thus, preferred to long-term funding for its disciplining function even when both are able to provide liquidity services through retrading in capital markets. On the other hand, a fragile funding structure can misalign the incentives between bankers and debt-holders in the presence of credit risk and, hence, result in both distorted asset holdings and a capital structure. Contrary to the case of pure run risk, equity financing has an advantage over short-term debt in dealing with externalities from the management of credit risk. Overall, run and credit risk endogenously interact to determine banks' asset portfolio and capital structure, which, in turn, has implications for the level of intermediation and the allocation of benefits from intermediation.

We make five modifications to the original Diamond-Dybvig model to capture the aforementioned interaction of credit and run risk. First, bank loans are risky. The risk arises because borrowers use the bank funding to invest in a technology whose payoff is uncertain and whose results are private information for the borrower.

Because loans are risky, borrowers can default due to insufficient funds. This can potentially cause the bank to be unable to fully repay depositors who incur additional bankruptcy costs. Our second modification is to assume that both banks and borrowers are subject to limited liability.

Third, the private information about a borrower's success leads banks to have to monitor the borrower. The banks are run by bankers who seek to maximize the value of dividends they receive from the banks. Absent monitoring the bankers enjoy a private benefit, but without monitoring their borrowers will never repay their loans.

Fourth, a full set instruments to insure against all risks in the environment is unavailable. Both loan and deposit contracts cannot be made contingent on the aggregate realization of risks. As a result, credit risk occurs in equilibrium. Moreover, contracts can be incomplete such that not all actions of borrowers are ex-ante contractible. For example, a more comprehensive debt contract would not only specify an interest rate and a debt amount, but also the composition and amount of assets that can be seized if default occurs. Likewise, bankers choose how much equity to contribute to the bank in addition to accepting deposits. Not only the asset portfolio, but also the capital structure of the bank is endogenously determined.

Fifth, we assume that depositors receive signals about the value of loans that the bank can recover before they are due. This interim liquidation value is available to pay depositors who are seeking to withdraw. We suppose that the depositors make a decision whether to run based on these signals. Our assumption about the nature of these signals means that the decision to run depends on the asset and liability structure of the bank, and the value of the signal. There is a unique signal threshold that determines whether there is a run.

There are three important consequences of these modifications. First, they create an environment where the level of credit risk and run risk in the economy are endogenously determined and interact. Second, the bank's choices of both the mix between the level of liquid and illiquid assets and between debt and equity differ from what a social planner would select. The private equilibrium features excessive levels of lending relative to liquid asset holding and more debt financing relative to equity financing. Because of the market incompleteness there is not a unique social planner's allocation. Instead the preferred allocations will depend on the planner's weights on the different actors in the economy. Loosely speaking, when the planner favors the savers, she will choose to limit risk while emphasizing liquidity provision for depositors. Alternatively, if the planner is primarily looking out for borrowers, the allocations are arranged to control run risk while increasing lending.

The third outcome in the model is that regulations akin to those embedded in the new Basel regulations for liquidity and capital can be studied to see if they could align the private asset and liability mix with the social efficient one. In particular, we study two capital regulations, one that ties capital requirements to the riskiness of bank assets and a second leverage requirement that is determined by the total scale of bank assets. We also look at a pair of liquidity regulations. One, akin to the so-called liquidity coverage ratio, makes the bank hold more short-term liquid assets when it uses more runnable funding. The other, like the so-called net stable funding ratio, requires the bank to increase its long term funding to match its longer term assets.

Although regulations can individually reduce the probability that a run occurs and improve welfare, they affect the asset mix and the liability mix in different ways. Capital regulations result in more lending, but in lower liquid asset holdings than it is socially optimal. In contrast, liquidity regulations reduce lending, but leave the level of capital below the social optimum. Because the private allocations diverge from the socially optimal allocations in two ways, no single regulation is sufficient to implement the social optimum; we show that at least two tools are needed. Yet, the optimal regulatory mix cannot arbitrarily include any two tools, because some regulations may be redundant. For example, we find that the two liquidity regulations cannot be jointly binding. Nevertheless, combinations of a capital and a liquidity regulation are feasible and are sufficient to implement the social planner's solution.

A special case arises when bankers have ample wealth to invest in so much bank equity, which pushes their economic surplus down to zero in equilibrium. Therein, planning outcomes are decentralized with only one regulation, which depends on the deadweight losses in bankruptcy. If the latter are low, then only liquidity regulation is needed, while for high ones, capital regulation is used.

The remainder of the paper is separated into four parts. In section 2, we describe the model and show the privately optimal choices for the bank, the savers and the entrepreneurs. In section 3 we study the efficient allocations chosen by a social planner and derive expressions for the wedges between the private and social decisions. In section 4, we explore how regulation can be used to correct the private inefficiencies. The following section analyzes a special case where less regulation is needed. Section 6 concludes by summarizing the main findings, reiterating the intuition for them, and describing a few directions for future research. Additional derivations and model extensions are relegated to an online appendix.

2 Model

The model consists of three periods, $t = \{1,2,3\}$, features a single consumption good and includes three types of (representative) agents; an entrepreneur (E), a saver (S) and a banker (B). The entrepreneur has access to a productive, but illiquid, risky technology. The entrepreneur's primary decision is how much of her own money to allocate to the project and how much to borrow.

Funds invested at date 1 yield an uncertain payoff $A_{3s} \cdot F(\cdot)$ at date 3 depending on the realization of state s, where F is a concave production function and A_{3s} a productivity shock. State $s = \{g,b\}$ occurs with probability ω_{3s} and these states represent a good and a bad realization of the shock, i.e., $A_{3g} > A_{3b}$. The project delivers no output at date 2 but it can be liquidated. The liquidation value, ξ, is uncertain and independent of the productivity shock.[1]

The banker manages an institution which we call a bank that acts as an intermediary between the

[1]The discrete state space for the productivity shock is not important for our results, but it facilitates the computation of the numerical equilibria. As described below, all agents have linear preference at $t = 3$, so that they care only about the expected payoffs and not the state by state payoffs. Moreover, another shock will be realized at $t = 2$, which follows a continuous distribution and is independent of the realization of the productivity shock. Thus, from the perspective of $t = 1$, there is an "infinite" dimensional state space in the future. We are more precise below.

entrepreneurs and savers. The bank is funded partly from the banker's endowment and by raising addtional funds from the saver. The funds raised at date 1 are invested into either a liquid storage asset or in a loan to the entrepreneur, which the bank can recall in the intermediate period. Moreover, the banker decides whether to monitor the entrepreneur's project at $t = 3$ or not. Monitoring is important because the productivity shock is private information to the entrepreneur.

The saver has a large endowment at date 1 that is used to fund initial consumption and savings. The savers have uncertain future consumption needs and, as in Diamond and Dybvig (1983), after date 1 some fraction will need to consume at t=2 and the rest can wait to consume at date 3. The saver invests in bank deposits or holds a liquid storage asset. The deposits are demandable, which is important to provide incentives to the banker to monitor as we describe in detail later.[2]

The liquidation value, ξ, follows a uniform distribution $U \sim \left[\underline{\xi}, \overline{\xi}\right]$ with $0 \leq \underline{\xi} < 1 < \overline{\xi}$ and $\Delta_{\xi} = \overline{\xi} - \underline{\xi}$. The fact that $\overline{\xi}$ can exceed 1 will be important in what follows. We assume that long-term loans are callable in which case the entrepreneur forfeits the portion of the project that is funded by the loan. Moreover, when a project is liquidated it yields an immediate gross return ξ.

The liquidation value can be justified in several ways. For instance the incomplete project could have a secondary use in the interim period because it can be used in conjunction with alternative short-term technology. Or we could assume that it can be sold to some outside investors as in Shleifer and Vishny (1992). In other words, ξ does not strictly represent the salvage value of the long-term investment, as for example in Cooper and Ross (1998), but rather the liquidation/resale value of long-term investment. $\overline{\xi}$ has to be high enough that the bank can always withstand a panic for some realizations. Yet, $\underline{\xi}$ has to be low enough that the bank may run out of liquidity even if a panic does not occur. We describe the importance of these bounds in section 2.4.[3]

Sections 2.1-2.4 describe in detail the agents' optimization problems in the private equilibrium. As we introduce the agents' problems, we emphasize the reasons why individual agents will make choices that would differ from a social planner. Section 2.5 discusses the key modeling assumptions.

[2] We assume that savers cannot buy equity in the bank in order to simplify the exposition of our baseline model. In the online appendix, we present a more complicated model where where the bank raise both inside equity from bankers and outside equity from savers. Therein, the bank shares purchased by savers are tradable in a frictionless market in the intermediate period and, thus, also provide liquidity services. Although bank equity can also provide liquidity services, because it can be traded in a secondary market similar to Jacklin (1987), an all-equity funding structure would not be optimal in even in this richer setup due to the disciplinary role of runnable debt. Overall, the main results from the model in the body of the paper continue to hold.

[3] Our model can easily be adjusted to make the liquidation value depend on the expected value of the loans, i.e, $\xi \cdot \mathbb{E}_s V^I_{3s}(1+r^I)$, where V^I_{3s} is the percentage repayment on the loan given by (15) later and r^I is the loan rate. Then, ξ would capture the fraction (between 0 and 1) of the expected value that can be obtained at liquidation. The expected value is computed over the possible realizations of state s in the last period for known probabilities ω_{3s}. The liquidation value, $\xi \cdot \mathbb{E}_s V^I_{3s}(1+r^I)$, would vary because ξ varies. Given that the expected value of loans is higher than one, the two approaches would yield qualitatively similar results. Alternatively, we could have assumed that ξ does not vary, but the probability distribution $\tilde{\omega}_{3s}$ varies as in Goldstein and Pauzner (2005). Then, the liquidation value would continuously vary with the realization of the true probability distribution ω_{3s} because $\mathbb{E}_s V^I_{3s}(1+r^I)$ varies. The upper and lower dominance regions in the incomplete information game would still be endogenously determined in these cases.

2.1 Savers

The savers are endowed with e_1^S and e_2^S at $t = 1$ and $t = 2$, respectively. In the initial period, they invest in bank deposits, D, and can additionally save by investing in the liquid storage technology, LIQ_1^S. A portion of savers, δ, receive a preference shock to consume in the intermediate period, while the rest, $1-\delta$, want to consume at $t = 3$. The preference shock is private information, i.i.d. and is not contractible ex-ante.

Deposits are demandable, early withdrawals are serviced sequentially and the interest rates, r_2^D and r_3^D, for withdrawals at $t = 2$ or $t = 3$ respectively, are uncontingent. This contract structure creates the possibility of a run, since patient savers may choose to demand their deposits early depending on their own information and their expectations about the actions of other patient savers; every (patient) depositor receives a noisy signal at $t = 2$ about the liquidation value, ξ, of the bank's loans and there is a threshold, ξ^*, determining whether a patient saver decides to run or keep her deposits in the bank. The probability of a run will be unique and depend on fundamentals similarly to Goldstein and Pauzner (2005).[4]

In order to facilitate the exposition of the model, while retaining precision, we denote all variables that are not (pre-)determined at $t = 1$ as functions of the liquidation value, ξ, and the portion of savers who decide to withdraw at $t = 2$, $\lambda \in [\delta, 1]$. In equilibrium, either all savers choose to withdraw, $\lambda = 1$, or only the impatient savers withdraw, $\lambda = \delta$. However, the out-of-equilibrium beliefs, which play an important role in the determination of the run probability derived below in section 2.4, depend on the conjectured portion of savers withdrawing. This conjecture can be anywhere between δ and 1.

It is instructive to review the different possible scenarios separately. If there is no run, only impatient depositors withdraw and they receive the full amount of promised payment, $D\left(1+r_2^D\right)$. Patient depositors' repayments are determined as a function of the technology shock in the next period. In a run, all depositors attempt to withdraw and there is probability $\theta(\xi, 1)$ that a depositor is served.[5] Conditional on the bank surviving to $t = 3$ patient depositors receive their promised payment in full or in part if the bank defaults, $V_{3s}^D(\xi, \delta) D\left(1+r_3^D\right)$. The percentage repayment on

[4]Bank-runs in our model can also be panic based rather than purely information based as in Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), Uhlig (2010), Angeloni and Faia (2013), Boissay, Collard and Smets (2016). In other words, a bank-run can occur due to a coordination problem among depositors even if the bank is solvent in the long-run. Similarly, a bank-run can also occur because the information about fundamentals is very bad. In determining the optimal ex-ante decisions, it is important to know what determines panics. In the Diamond-Dybvig model panics are a multiple equilibrium outcome. Cooper and Ross (1998), Peck and Shell (2003) and Keister (2015) suppose instead that the probability of a bank-run is driven by sunspots. In our earlier working paper Kashyap, Tsomocos and Vardoulakis (2014), in Gertler and Kiyotaki (2015) and in Choi, Eisenbach and Yorulmazer (2016) the probability of a run is determined by an exogenous function of key fundamentals. Ennis and Keister (2005) take an axiomatic approach to equilibrium selection and link the probability of a particular equilibrium being played to appropriately defined incentives of agents. Instead, we use the global games approach developed by Morris and Shin (1998) and applied to banks runs by Goldstein and Pauzner (2005) to derive a unique probability of run which depends on fundamentals. Although we maintain they key assumption in Goldstein and Pauzner (2005) to obtain a unique equilibrium, we amend their approach by introducing noisy signals on a different variable so that the upper dominance region is endogenously derived rather than assumed. Rochet and Vives (2004) and Vives (2014) also take a global game approach, but delegate the withdrawal decision to a (deposit) fund manager with a simpler payoff function.

[5]This probability is determined by equation (12), derived in section 2.2. In a run all savers attempt to withdraw, $\lambda = 1$.

period 3 deposit withdrawals, $V_{3s}^D(\xi,\delta)$, is given by equation (16) that is derived below. Depositors have to pay an additional cost, c_D, per unit of promised payments to receive a payment when the bank defaults.

Thus, the net repayment on deposit is $\left(V_{3s}^D(\xi,\delta) - c_D \cdot \mathbb{I}_d\right) D\left(1+r_3^D\right)$, where $\mathbb{I}_d$ is an indicator function that takes the value of 1 when the bank defaults.

We proceed by formally presenting savers' problem. Savers' consumption at $t=1$ is given by

$$c_1 = e_1^S - D - LIQ_1^S. \tag{1}$$

Given the various cases that occur in the latter periods when the savers may or may not be patient, choose to withdraw or run, and be paid or not in a run, it is helpful to introduce some further notation. Denote by $j = i, p$ the saver's type, which is realized at $t = 2$. They will be impatient $(j = i)$ with probability δ and patient $(j = p)$ with probability $1-\delta$. A run occurs when $\xi \in \left[\underline{\xi}, \xi^*\right]$. In this case, all savers attempt to withdraw, but only a fraction of them are repaid. We denote by $\mathbb{I}_\theta$ an indicator which takes the value of 1 if an individual saver is repaid, where the (endogenous) probability of repayment is $\theta(\xi,1)$. Then, the consumption of a saver of type j is given by

$$c_{ts}\left(j, \mathbb{I}_\theta\right) = \mathbb{I}_\theta \cdot D\left(1+r_2^D\right) + LIQ_1^S + e_2^S, \tag{2}$$

where $ts = 2$ for $j = i$ and $ts = 3s$ for $j = p$, because patient savers still only consume at $t = 3$ and will transfer their resources from period 2 to 3 using the storage technology.

We also define an indicator $\mathbb{I}_w$ which takes the value of 1 if an agent of type j withdraws when a run does not occur, i.e., when $\xi \in \left[\xi^*, \overline{\xi}\right]$, and is 0 otherwise. Her consumption is denoted by $c_{ts}\left(j, \mathbb{I}_w\right)$. Although in equilibrium patient savers will truthfully report their type and only impatient savers will withdraw, we need to contemplate deviations where a patient saver could opt to withdraw and show that such deviations never are optimal (see section 2.4). The consumption of an impatient saver is, then, given by

$$c_2\left(i, \mathbb{I}_w = 1\right) = D\left(1+r_2^D\right) + LIQ_1^S + e_2^S. \tag{3}$$

The consumption at $t = 3$ of a patient saver who chooses to wait or withdraw are, respectively, given by

$$c_{3s}\left(p, \mathbb{I}_w = 0\right) = \left(V_{3s}^D(\xi,\lambda) - c_D \cdot \mathbb{I}_d\right) D\left(1+r_3^D\right) + LIQ_1^S + e_2^S, \tag{4}$$

$$\text{or} \qquad c_{3s}\left(p, \mathbb{I}_w = 1\right) = D\left(1+r_2^D\right) + LIQ_1^S + e_2^S. \tag{5}$$

Note that in equilibrium, $\lambda = \delta$ in (4).

Finally, short-selling of deposits and the liquid asset is not allowed. So in solving the model we add the following constraints (with the associated Lagrange multipliers indicated in parentheses): $D \geq 0$ (ν_{D}); and $LIQ_1^S \geq 0$ $(\nu_{\mathrm{LIQ_1^S}})$.

The savers choose the level of deposits and their holdings of the liquid asset to maximize their utility subject to constraints (1)-(5). The expected utility of a representative saver is given by

$$\mathbb{U}^S = U_1(c_1) + \sum_{t=2,3} \left\{ \overbrace{\int_{\underline{\xi}}^{\xi^*} \mathbb{E}_{j,\theta} U_t\left(c_{ts}\left(j, \mathbb{I}_\theta\right); j\right) \frac{d\xi}{\Delta_\xi}}^{\text{run}} + \overbrace{\int_{\xi^*}^{\overline{\xi}} \mathbb{E}_{j,s} U_t\left(c_{ts}\left(j, \mathbb{I}_w\right); j\right) \frac{d\xi}{\Delta_\xi}}^{\text{no run}} \right\}. \tag{6}$$

Conditional on a run occurring, patient savers compute the expected utility from remaining patient or attempting to withdraw and probabilistically receiving payment on their deposits ($\mathbb{E}_{j,\theta}$). If a run does not occur, patient savers compute the expected utility from withdrawing early, and from receiving the state contingent payment on deposits ($\mathbb{E}_{j,s}$). Note that we have indexed the utility function by the time t and agent type j. Impatient savers receive utility only at $t = 2$ which is discounted to the present by $\beta < 1$, while patient savers receive utility only at $t = 3$ which is discounted to the present by β^2. Moreover, we assume that savers have quasi-linear preferences; at t=1 and at $t = 2$ (for $j = i$) savers have concave utility U, while savers have linear preferences at $t = 3$ (for $j = p$).[6]

An individual saver takes the probability of being repaid in a run, $\theta(\xi, 1)$ and the percentage repayment, $V^D_{3s}(\xi, \delta)$, as given. These objects depend on the aggregate bank portfolio and we suppose that the individual saver is sufficiently small so as to not account for her impact on them. A social planner would internalize the effect of the choices.

The optimal supply of deposits by savers is given by:

$$DS: -U_1'(c_1) + \left(1 + r_2^D\right) \left[\overbrace{\sum_{t=2,3} \left\{ \int_{\underline{\xi}}^{\xi^*} \theta(\xi, 1) \cdot \mathbb{E}_j U_t'\left(c_{ts}(j, 1); j\right) \frac{d\xi}{\Delta_\xi} \right\}}^{\text{run}} + \overbrace{\delta \int_{\xi^*}^{\overline{\xi}} U_2'\left(c_2(i, 1); i\right) \frac{d\xi}{\Delta_\xi}}^{\text{no run, impatient}} \right]$$
$$\underbrace{+ (1 - \delta) \int_{\xi^*}^{\overline{\xi}} \sum_s \omega_{3s} U_3'\left(c_{3s}(p, 0); p\right) \cdot \left(V^D_{3s}(\xi, \delta) - c_D \cdot \mathbb{I}_d\right) \cdot (1 + r_3^D) \frac{d\xi}{\Delta_\xi}}_{\text{no run, patient}} + \nu_D = 0. \tag{7}$$

Condition (7) says that savers equate the marginal utility of forgone consumption at $t = 1$ to the expected marginal utility gain from holding deposits in the future. In a run, all savers withdraw; their marginal utility depends on their type, j, and the probability that they are repaid, $\theta(\xi, 1)$. If a run does not occur, impatient savers are fully repaid at the promised rate, $1 + r_2^D$, while patient savers do not withdraw and receive the uncertain deposit payoff, $V^D_{3s}(\xi, \delta) \cdot \left(1 + r_3^D\right)$, minus any marginal bankruptcy costs.

Savers may want to self-insure and hold the liquid asset. The optimal liquid holdings, LIQ_1^S, are

[6] The linearity of utilities in the final period is not important for our results and we have assumed it for simplicity of exposition. We discuss further this assumption in section 2.5.

given by:

$$-U_1'(c_1) + \sum_{t=2,3} \left\{ \overbrace{\int_{\underline{\xi}}^{\xi^*} \mathbb{E}_{j,\theta} U_t'\left(c_{ts}\left(j, \mathbb{I}_\theta\right); j\right) \frac{d\xi}{\Delta_\xi}}^{\text{run}} + \overbrace{\int_{\xi^*}^{\bar{\xi}} \mathbb{E}_{j,s} U_t'\left(c_{ts}\left(j, \mathbb{I}_w\right); j\right) \frac{d\xi}{\Delta_\xi}}^{\text{no run}} \right\} + \nu_{LIQ_1^S} = 0. \quad (8)$$

Condition (8) says that holding the liquid asset allows the saver to self-insure against states that she is not repaid in a run, i.e., compared to using a deposit as in (7) the liquid asset always delivers future consumption, but does so by forgoing the higher uncertain return that deposits promise.

Moreover, note that the equilibrium outcomes will be incentive compatible, i.e., a patient saver will not have an incentive to misrepresent her type and withdraw for $\xi > \xi^*$. This is guaranteed by the way the run threshold is determined, which we describe in detail in section 2.4.

Finally, we need to specify what would happen if the savers chose to avoid using the bank. One possibility is that they save using only the liquid asset. We refer to this as autarky. The liquidity choice in this case is the solution to $U_1'(e_1^S - LIQ_1^{S,aut}) + \sum_{t=2,3} E_j U_t'(e_2^S + LIQ_1^{S,aut}; j) = 0$. These holdings imply a utility in autarky $U^{S,aut}$, which is a useful benchmark for gauging the benefits that intermediation delivers through liquidity provision. A second alternative is that the savers could attempt to directly lend to entrepreneurs (assuming that they would also have to monitor after doing so). We denote the resulting level of utility by by $U^{S,dl}$. The participation constraint of savers is, then, given by:

$$\mathbb{U}^S \geq \max\left(U^{S,aut}, U^{S,dl}\right). \quad (9)$$

Given that this constraint will mostly not bind for the results we present, we report the detailed problem when savers lend directly to entrepreneurs in the online appendix. We will be explicit about the occasions that the constraint binds.

2.2 Bankers and Banks

The banker makes all investment and funding decisions to maximize her own utility. At $t = 1$, she is endowed with e^B and decides how much equity, E, to put into the bank. Her utility is given by:

$$\mathbb{U}^B = \gamma \cdot U\left(e^B - E\right) + \overbrace{\int_{\xi^*}^{\bar{\xi}} \sum_s \omega_{3s} DIV_{3s}(\xi, \delta) \frac{d\xi}{\Delta_\xi}}^{\text{no run}}, \quad (10)$$

where DIV_{3s} are the dividends in state s at $t = 3$.

The banker trades off foregoing current consumption to investing in equity and receiving dividends in the future. The banker has also quasi-linear preferences and the same utility function at $t = 1$ as the savers, but unlike the saver never needs to consume in the interim period. We have maintained the linearity of preferences at $t = 3$, though we restrict parameters so that savers never insure the banker against future uncertainty.[7]

Additionally, the banker chooses how many deposits to raise, D, and using the total funds, the banker invests in the liquid assets, LIQ_1, and illiquid loans, I. The loan contract is uncontingent and requires a payment of $1+r^I$ per dollar of lending at $t=3$. As already mentioned, loans are callable at any time before maturity at which case the entrepreneur surrenders the projects funded by these loans and does not have an obligation to repay them at $t=3$.

The balance sheet constraint at $t=1$ is given by:

$$BS: I + LIQ_1 = D + E. \tag{11}$$

We define ψ_{BS} to be the multiplier on the balance sheet constraint (11), which represents the shadow value of funding, i.e., the endogenous cost of expanding assets by raising a unit of funds.

The balance sheet and profits after t=2 depend on the realization of ξ and the number of people withdrawing, λ. If a bank-run occurs then the bank is liquidated and the proceeds are distributed according to sequential service constraint. Thus, the probability that any saver is served is equal to

$$\theta(\xi,\lambda) = \frac{LIQ_1 + \xi \cdot I}{\lambda \cdot D \cdot (1+r_2^D)}. \tag{12}$$

If the bank survives the run, it will have to recall and liquidate a portion $y(\xi,\lambda)$ of its loan portfolio to serve the early withdrawals given by

$$y(\xi,\lambda) = \frac{\lambda \cdot D \cdot (1+r_2^D) - LIQ_1 + LIQ_2(\xi,\lambda)}{\xi \cdot I}, \tag{13}$$

where $LIQ_2(\xi,\lambda) \geq 0$ are the liquid holdings carried over to the third period. Our assumptions regarding the distribution of ξ lead bank to hold insufficient liquid assets to service all early deposit withdrawals even when only the impatient savers withdraw. So the bank is always planning to call some loans. In principle, the bank could want to liquidate its whole loan portfolio and carry the proceeds forward using the storage technology, but this would only be the case if the realization of ξ is higher than the expected return from holding the loan to maturity, which we have excluded by assumption. As a result, $y(\xi,\lambda)$ will take interior values between zero and one, and it will be decreasing in ξ and increasing in λ for a pre-determined bank portfolio.

Conditional on the bank surviving, the dividends depend on the portion of the portfolio liqui-

[7]The difference between the banker and the savers expected utility is that the former values future consumption more than current or in other words $\gamma < 1$. Assigning to the banker the same utility function requires high enough e^B or low enough γ such that she would be willing to invest enough of her own wealth in equity to provide risk-sharing benefits to savers. We do the second because we want the banker endowment to represent only a small part of the total endowment in the economy, with the vast majority accruing to the savers. For $\gamma = 1/\beta^2$, such that savers and the banker discount the future the same way, and for logarithmic utility, we can obtain the same equilibrium for banker's wealth $\breve{e}^B = E + (e^B - E)/(\beta^2\gamma)$, where E is the equilibrium value of contributed equity. Finally, given that bankers are protected by limited liability and that future endowments are not contractible, quasi-linear preferences allow us to exclude final period endowments from our analysis by setting them to zero.

dated $y(\xi, m)$ and are given by

$$DIV_{3s}(\xi,\lambda) = (1 - y(\xi,\lambda)) \cdot V^I_{3s}(\xi,\lambda) \cdot I \cdot \left(1 + r^I\right) + LIQ_2(\xi,\lambda) - V^D_{3s}(\xi,\lambda) \cdot (1-\lambda) \cdot D \cdot \left(1 + r^D_3\right), \tag{14}$$

where $V^I_{3s}(\xi,\lambda)$ is the percentage repayment on the remaining risky loans. It is given by

$$V^I_{3s}(\xi,\lambda) = \min\left[1, \frac{A_{3s}F\left(I^E + (1 - y(\xi,\lambda)\cdot I)\right)}{(1 - y(\xi,\lambda)) \cdot I \cdot (1 + r^I)}\right] \tag{15}$$

and $V^D_{3s}(\xi,\lambda)$ is the repayment rate on deposits. It is given by

$$V^D_{3s}(\xi,\lambda) = \left[1, \frac{(1 - y(\xi,\lambda)) \cdot V^I_{3s}(\xi,\lambda) \cdot I \cdot \left(1 + r^I\right) + LIQ_2(\xi,\lambda)}{(1-\lambda) \cdot D \cdot \left(1 + r^D_3\right)}\right]. \tag{16}$$

In other words, bank profits in (14) are equal to the revenue received from the repayment on the outstanding loans plus any liquid assets carried forward minus the repayment on the deposits that were not withdrawn early. Equation (15) says that the loan is fully repaid when the revenue available to the entrepreneur, which is derived from the own funds invested by the entrepreneurs, I^E, and bank loans, is higher than the outstanding loan obligation; otherwise the entrepreneur defaults and the bank seizes everything that is available. Equation (16) says that late depositors are repaid in full when the value of bank assets is higher than the promised deposit payments; otherwise the bank defaults and depositors divide the assets in a pro-rata fashion.

After the run uncertainty has been resolved and the true value of ξ is learned, the banker can choose to monitor the borrower to learn the true value of the productivity shock at $t = 3$, which is private information to the entrepreneur. Alternatively, the banker can forgo the monitoring and enjoy a private benefit from running the bank. We follow the long tradition in the literature assuming that monitoring is costly for the banker because she would have to give up a private benefit she would otherwise receive from managing the bank (see, for example, Holmström and Tirole, 1997). This assumption creates an ex-post moral hazard problem in which the banker will choose to monitor only if the expected dividends are higher than the private benefit. If the banker opts not to monitor, the entrepreneur would always report the lowest realization of the productivity shock and default on the loan.[8] The banker will choose to monitor for all $\xi \geq \xi^*$ if the following incentive compatibility constraint is satisfied:

$$IC : \sum_s \omega_{3s} DIV_{3s}(\xi^*, \delta) - PB \geq 0, \tag{17}$$

where PB is the private benefit.

The first term in the IC constraint is the expected payoff to the banker if she monitors when $\xi = \xi^*$. We take the expectation because the banker has to decide whether to monitor before she

[8] The productivity level is common across projects. Therefore, as in Diamond (1984), monitoring costs are conserved by having a bank monitor all borrowers, relative to having individual lenders monitor individual borrowers. Thus, the bank monitoring expands the supply of credit.

learns the true value of A_{3s}. The second term is the private benefit. If the banker does not monitor, then the entrepreneur reports the lowest realization for A_{3s}, defaults on the loan repayment and forces the bank to default on its deposits (so that bank equity is worthless). It suffices that the *IC* constraint is satisfied for $\xi = \xi^*$, because expected dividends are increasing in ξ, thus the banker will always have an incentive to monitor if there is no run.

The bank and the depositors may want to write a deposit contract not only on the deposits rate(s) and the amount of deposits, but also over all the factors affecting the riskiness of the deposits. These risks are governed by all aspects of the bank's balance sheet, in particular, its choice of leverage (or equivalently a capital ratio), its asset allocation between loans and liquid assets (i.e., a liquidity ratio) and its maturity mismatch (i.e., a net stable funding ratio). However, such comprehensive contracts may not be possible for a number of reasons and do not resemble observed deposit or unsecured funding arrangements in reality.[9] As a result, the bank would be tempted to deviate in the way it chooses its leverage, liquidity and maturity mismatch after it has entered into a deposit contract and received the deposits. Technically, this *lack of commitment* means that the bank will optimize only over states of the world in which it is solvent because it is protected by limited liability. Likewise, it will only internalize how it affects the supply of deposits when it chooses the contract terms $\left(D, r_2^D, r_3^D\right)$. The bank does understand that taking more risk increases the cost of raising deposits, and would ideally want to promise depositors that it will behave prudently. But, after the deposit contract has been signed, the bank has an incentive to deviate towards lending more, holding fewer liquid assets and raising less equity.

Depositors have rational expectations and ex-ante require that the bank offers higher deposit rates to compensate for the anticipated risk-taking due to the lack of commitment. In contrast, a social planner would recognize that the bank's insolvency adversely impacts savers, and would account for this in making allocations. We believe that incomplete contracting is an important feature of reality when financial institutions have a rich balance sheet and their activities expose savers to credit risk. Nevertheless, we also examine the case where comprehensive contracts, specifying the full set of choices made by the banker, can be written. We denote by $\mathbb{I}_c$ an indicator function, which takes value one if deposit contracts are comprehensive and zero if they are incomplete. Our conclusions regarding the need for bank regulation hold for both cases.

One force in the model that partially disciplines the banker is the possibility of a bank run. The banker will internalize how her investment and funding choices affect the probability of a run via condition (32) (that is derived below), and hence the probability that she will make profits. Similarly, the banker understands that her ex-post incentives to monitor need to be consistent with condition (17); otherwise depositors would anticipate that the banker will not have an incentive to monitor and would always run at $t = 2$ driving the banker's rents to zero. In this respect, the run risk creates

[9] For example, Dewatripont and Tirole (1994) argue that individual depositors are sufficiently small and diverse to enforce comprehensive contracts which discipline all banking choices. See also Stiglitz and Weiss (1981), Matutes and Vives (2000), Boyd and De Nicoló (2005) among others for models with risk-taking incentives when loan contracts are not comprehensive. Contrary to these papers, which maintain the price-taking assumption for the borrowing rate, we allow borrower to optimally choose all the terms specified in the contract.

an incentive for the banker to monitor its borrowers and to prudently choose its capital structure and amount of lending at $t = 1$ (see Calomiris and Kahn, 1991, Diamond and Rajan, 2000, 2001).

Overall, the banker will understand how the investment and funding decision matter for future behavior by savers and will take equations (32) and (17) as additional constraints in her optimization problem, but she neglects the other effects of her decisions on savers and entrepreneurs utilities given by (6) and (26).

In solving for bank's optimal choices, we will focus on equilibria such that the bank is always solvent in state g and always defaults in state b for all realizations of $\xi \geq \xi^*$.[10] Substituting into (10) equations (13), (14), (15), (16), the banker optimizes over the risky loan, I, the liquid asset holdings, LIQ_1 and $LIQ_2(\xi, \delta)$ for each ξ, the equity contributed, E, the run threshold, ξ^*, the level of deposits, D, and the deposit rates, r_2^D and r_3^D. She takes (11), (17) and (32) as constraints in her problem. The last constraint is the global game condition, GG, which determines the run threshold is derived in section 2.4 below. Due to limited liability the banker will only consider the states in which she is solvent.

The optimality condition for loans, I, is:

$$\frac{d\mathbb{U}^B}{dI} - \psi_{BS} + \psi_{IC}\frac{dIC}{dI} + \psi_{GG}\frac{dGG}{dI} + \psi_{DS}\frac{dDS}{dI} \cdot \mathbb{I}_c = 0, \tag{18}$$

where $d\mathbb{U}^B/dI = \int_{\xi^*}^{\bar{\xi}} \left\{\omega_{3g}\left(1+r^I\right)\right\} d\xi/\Delta_\xi$ is the marginal effect of investment on banker's share of profits and ψ_{BS}, ψ_{IC}, ψ_{GG} and ψ_{DS} are the multipliers on constraints (11), (17), (32) and (7) respectively.

The expression (18) says that optimal level of lending is determined by having the banker trade off the marginal return accruing to her against the shadow cost of funding additional lending and the way it affects the incentive compatibility and the run threshold determination constraints. As already mentioned, the banker only internalizes states where she is solvent due to limited liability. Finally, the banker considers how her investment decisions affects the deposit supply only when the level of investment is a contractual deposits term at which she can commit to. Equation (18) corresponds to the loan supply schedule, denoted by LS, offered to entrepreneurs.

The optimality condition for first period liquid assets, LIQ_1, is:

$$\frac{d\mathbb{U}^B}{dLIQ_1} - \psi_{BS} + \psi_{IC}\frac{dIC}{dLIQ_1} + \psi_{GG}\frac{dGG}{dLIQ_1} + \psi_{DS}\frac{dDS}{dLIQ_1} \cdot \mathbb{I}_c = 0, \tag{19}$$

where $d\mathbb{U}^B/dLIQ_1 = \int_{\xi^*}^{\bar{\xi}} \left\{\omega_{3g}\left(1+r^I\right)/\xi\right\} d\xi/\Delta_\xi$ is the marginal effect of liquidity on banker's share of profits. The optimal choice of liquid assets is governed by the same considerations to determine optimal lending. The only difference is that the marginal return on the liquid assets is scaled by the liquidation value ξ, because the bank needs to liquidate $1/\xi$ fewer loans to serve early

[10] Equivalently, entrepreneurs default on their loan in state b and deliver fully in state g. We have also solved the model with more states for the realization of the productivity shock, such that entrepreneurs' default does not need to coincide with banks' default. Given that our result continue to hold, we have chosen to present the model with two level of productivity to simplify the analysis and present the more complicated case in an online appendix.

withdrawals for each additional unit of the liquid asset.

The banker will optimally choose the run threshold, ξ^*, which yields:

$$\frac{d\mathbb{U}^B}{d\xi^*} + \psi_{IC}\frac{dIC}{d\xi^*} + \psi_{GG}\frac{dGG}{d\xi^*} + \psi_{DS}\frac{dDS}{d\xi^*}\cdot \mathbb{I}_c = 0, \tag{20}$$

where $d\mathbb{U}^B/d\xi^* = -\sum_s \omega_{3s}DIV_{3s}(\xi^*,\delta)/\Delta_\xi$. In making this choice, (20) says that the banker balances the reduction in dividends because of a marginally higher ξ^* against the effect from relaxing the *IC* and *GG* constraints (and *DS* if the run threshold is a deposit contract term).

The optimal choice of liquidity holdings, $LIQ_2(\xi,\delta)$ is made after the run uncertainty is resolved and depends on the realization of ξ. As a result, the banker will only consider the effect on (her) profits, but not the effect on the run threshold due to the inability to commit. Banks may want patient investors to think that they will hold liquid assets from $t = 2$ to $t = 3$ to reduce the probability of a run, but if the bank survives, then banks may not have an incentive to hold liquid assets because they only care about states in which they are solvent (unless the deposit contract specifies the level of second period liquid assets for every realization of ξ). Under incomplete contracts, the banker will carry liquidity in period 3 only if the liquidation value is higher than the expected loan return in the states that the bank is solvent, i.e., if $\xi > \omega_{3g}(1+r^I)$. In the equilibria we examine this is never the case, because $\overline{\xi} < 1+r^I$, so it is optimal for the bank to recall loans only to serve early withdrawals and not to hoard liquidity.

The optimality condition with respect to contributed equity, E, is:

$$\frac{d\mathbb{U}^B}{dE} + \psi_{BS} = 0, \tag{21}$$

where $d\mathbb{U}^B/dE = -\gamma\cdot U'\left(e^B - E\right)$. Condition (21) says that injecting more equity requires the banker to give up consumption in the initial period in exchange for increasing the funds of the bank. Note the condition does not include a term for the effect of additional equity on constraints *GG* or *IC* (as well as *DS* for comprehensive contracts). This is true because, E does not appear directly in (32), (17) or (7), but this doesn't mean that equity is irrelevant for their determination. On the contrary, equity issuance can affect the run probability, the incentives to monitor and the deposit supply through its joint determination with the other equilibrium variables.

Condition (21) governs the shadow cost of funds ψ_{BS}, which is inversely related to the amount of equity the banker puts in the bank. In banking models without endogenous credit or run risk, the higher funding costs of injecting more equity would feed in higher loan rates and lower investment. This does not need to be true when equity changes the level of credit and run risk as in our model; higher equity and cost of funding can be compatible with lower loan rates and more investment.

Finally, the banker chooses the deposit contract $\left(D, r_2^D, r_3^D\right)$ which needs to lie on the deposit supply curve (7). The optimal deposit contract satisfies the following first-order conditions:

$$\frac{d\mathbb{U}^B}{dD} + \psi_{BS} + \psi_{IC}\frac{dIC}{dD} + \psi_{GG}\frac{dGG}{dD} + \psi_{DS}\frac{dDS}{dD} = 0 \tag{22}$$

$$\frac{d\mathbb{U}^B}{dr_2^D} + \psi_{IC}\frac{dIC}{dr_2^D} + \psi_{GG}\frac{dGG}{dr_2^D} + \psi_{DS}\frac{dDS}{dr_2^D} + \nu_{r_2^D} = 0, \tag{23}$$

$$\frac{d\mathbb{U}^B}{dr_3^D} + \psi_{IC}\frac{dIC}{dr_3^D} + \psi_{GG}\frac{dGG}{dr_3^D} + \psi_{DS}\frac{dDS}{dr_3^D} = 0, \tag{24}$$

where $d\mathbb{U}^B/dD = -\int_{\xi^*}^{\overline{\xi}} \left\{\omega_{3g}\left(1+r^I\right)\left(\delta\left(1+r_2^D\right)\right)/\xi + (1-\delta)\left(1+r_3^D\right)\right\} d\xi/\Delta_\xi$ captures the effect of deposits, $d\mathbb{U}^B/dr_2^D = -\int_{\xi^*}^{\overline{\xi}} \left\{\omega_{3g}\left(1+r^I\right)(\delta \cdot D)/\xi\right\} d\xi/\Delta_\xi$ the effect of the early deposit rate and $d\mathbb{U}^B/dr_3^D = -\int_{\xi^*}^{\overline{\xi}} \{\omega_{3g}(1-\delta)\cdot D\} d\xi/\Delta_\xi$ the effect of the late deposit rate—the three deposit contract terms— on banker's profits, respectively. Finally, $\nu_{r_2^D}$ is the multiplier on non-negativity constraint $r_2^D \geq 0$, which we discuss below.

Condition (22) can be easily interpreted. ψ_{BS} is the shadow benefit of raising an additional unit of deposits. When a deposit is accepted, it entails paying the interest rate on late withdrawals to $1-\delta$ depositors and liquidating the long term asset to service the early withdrawals of δ depositors, and it alters the run threshold and the incentive compatibility constraint. Similarly to the other decisions, the banker considers the effect of repaying deposits on her profits only in states that she expects to be solvent. The last term captures the effect on the privately optimal supply of deposits and is present even if deposit contracts are not comprehensive. Conditions (23) and (24) can be similarly interpreted with the difference that deposit rates do not entail a direct balance sheet cost and, thus, ψ_{BS} does not appear in the respective optimality conditions.

We restrict deposit rates to be positive, which can be particularly important for the choice of r_2^D in (23). Absent constraints, the banker may want to offer an early deposit rate that is negative, since this would allow her to reduce the probability of a run. Such run-preventing deposit contracts have been studied for example in Cooper and Ross (1998). In our model, however, runnable deposits are important to discipline the banker and there are limits to how low the early deposit rate can be set both because of the disciplinary role and because savers can stop using the bank if the rates become too low. In the numerical examples we present, r_2^D hits the non-negativity constraint both in the private and planning equilibria, but we have also solved for cases where it is allowed to take negative values. The implications of our model for the distortions between the private and planning equilibria as well as the effects and desirability of regulation continue to hold under a negative deposit rate for early withdrawals. We present these results in the online appendix.[11]

The banker is willing to intermediate funds between savers and entrepreneurs if the utility she

[11] See also Keister (2015) for a model with flexible deposit contracts, i.e., the payment that a depositor receives is determined by the bank as a best response to realized withdrawals in the intermediate period. Runs in his framework are partial in the sense that the bank can alter payments to stop withdrawals by patient depositors and avoid liquidation once the run state is revealed.

obtains is higher that the utility is autarky, i.e., if the following participation constraints is satisfied:

$$\mathbb{U}^B \geq U^{B,aut}, \tag{25}$$

where $U^{B,aut} = \gamma \cdot U\left(e^B - LIQ_1^B\right) + LIQ_1^B$. In autarky, the consumption of the banker at $t = 1$ is equal to her endowment, e^B, minus any holding of the liquid asset, LIQ_1^B, carried forward to $t = 3$. LIQ_1^B is the solution to equation $\gamma \cdot U'\left(e^B - LIQ_1^B\right) = 1$ if positive and zero otherwise.[12]

2.3 Entrepreneurs

Entrepreneurs have the rights to real projects that are in elastic supply, require a unit of funding at $t = 1$, are infinitely divisible when liquidated, and mature at $t = 3$. Entrepreneurs have endowment e^E in the initial period and borrow I from the bank at interest rate r^I. Denote by I^E the own funds put into the real projects at $t = 1$. Then, E consumes $e^E - I^E$ and takes a bank loan which depends on both the loan amount (or equivalently the loan-to-value ratio, $LTV = I/\left(I + I^E\right)$), and a loan rate, r^I. For simplicity, we will assume that the entrepreneur is risk-neutral and that she derives utility only from consumption at $t = 3$. Hence, she will invest all her endowment in the risky project as long as the return is higher that the return on the storage technology which has zero yield. Finally, E is protected by limited liability when projects mature and loans are due. If a run does not occur, she will repay the outstanding loans, $1 - y(\xi, \delta)$, not recalled at $t = 2$ only if the investment payoff is higher than the contractual loan obligation. In a run, all projects funded by bank loans are liquidated ($y(\xi, \delta) = 1$ for all $\xi < \xi^*$) and the entrepreneur can only produce using her own capital committed at $t = 1$.

Hence, the utility of an individual entrepreneur is:

$$\mathbb{U}^E = \sum_s \omega_{3s} \left\{ \overbrace{\int_{\xi^*}^{\bar{\xi}} \left[A_{3s} F\left(I^E + (1 - y(\xi, \delta))I\right) - (1 - y(\xi, \delta))I(1 + r^I)\right]^+ \frac{d\xi}{\Delta_\xi}}^{\text{no run}} + \overbrace{\int_{\underline{\xi}}^{\xi^*} A_{3s} F\left(I^E\right) \frac{d\xi}{\Delta_\xi}}^{\text{run}} \right\}. \tag{26}$$

The loan contract is comprehensive and E optimally chooses a combination of I and r^I that lie on the loan supply curve (18) to maximize (26). In addition to the loan contract terms, E's utility and the loan supply curve offered by the bank to each individual entrepreneur depend on a set of aggregate bank variables that the entrepreneur takes as given. These aggregate variables include

[12]The outside option is important because the planner will drive the banker to her participation constraint. Offering the most competitive lending terms to entrepreneurs would require the intermediation of deposits from savers. Thus, assuming that entrepreneurs can freely choose the banker that offers the best terms, the outside option for the banker is her utility in autarky. Alternatively, we could have assumed that entrepreneurs are captive of bankers and, hence, the outside option is equal to the utility they would obtain by lending to entrepreneurs using only their own capital. We derive the conditions for this case in an online appendix. We should note that for the equilibrium we examine, entrepreneurs would not borrow from bankers, unless the latter raise deposits to reduce funding costs. The reason is that entrepreneurs would obtain a higher utility investing only out of their own funds. Thus, the autarkic utility is the relevant outside option for bankers under either assumption.

the probability of bank run, which depends on ξ^*, $y(\xi,\delta)$, and the shadow values ψ_{BS}, ψ_{IC}, and ψ_{GG}. Although the individual loan characteristics will matter for the aggregate bank variables in equilibrium, each individual entrepreneur is small compared to the aggregate bank portfolio such that she neglects the effect of the loan terms on them.

Combining the optimality conditions with respect to the loan terms I and r^I, we obtain the optimal loan demand, LD, of the entrepreneur:

$$LD : \int_{\xi^*}^{\bar{\xi}} (1-y(\xi,\delta)) \left[A_{3g}F'\left(I^E + (1-y(\xi,\delta))\cdot I\right) - (1+r^I) + I \cdot \frac{\partial LS}{\partial I}\left(\frac{\partial LS}{\partial r^I}\right)^{-1} \right] \frac{d\xi}{\Delta_\xi} = 0. \quad (27)$$

The first two terms in the entrepreneur's loan demand (27) schedule correspond to the profit margin to the entrepreneur, given by the difference between the marginal product of investment and the gross loan rate. Limited liability means that the entrepreneur only cares about the states in which she does not default, i.e., she considers the profit margin only in state g. The third term captures the dependence of the loan rate on the loan level I. Although the entrepreneur cares only about the states in which she is solvent, her loan demand is influenced by the states in which she defaults because the bank cares about this when the interest rate is determined. Because entrepreneurial default and bank default occur at the same time, $\partial LS/\partial I = 0$ from the perspective of an individual entrepreneur who takes the aggregate portion of loans recalled, $y(\xi,\delta)$, and the other aggregate bank variables as given.[13] A social planner would instead take full account of how a default by the entrepreneur influences the other two agents.

Finally, the entrepreneur is willing to borrow from the bank if her utility is higher than the utility from just investing her own funds in the project, i.e., if the following participation constraints is satisfied:

$$\mathbb{U}^E \geq U^{E,aut}, \quad (28)$$

where $U^{E,aut} = \sum_s \omega_{3s} A_{3s} F\left(I^E\right)$. Constraint (28) implies that there is at least one state that the entrepreneur does not default on her loan.

2.4 Global Game and Bank-run Threshold

We conclude our description of the model by examining the incentives of patient savers to run or not. As already mentioned, we take all variables that are not predetermined at t=2 to be functions of the realization of ξ and the number of people that choose to withdraw, $\lambda \in [\delta, 1]$. Patient savers receive at t=2 private signals $x_i = \xi + \varepsilon_i$, where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, \varepsilon]$. Focusing on threshold strategies, an individual patient saver will run if the private signal realization is lower than a threshold, $x_i \leq x^*$, and will not run otherwise. The threshold for the strategies implies a threshold for fundamentals ξ^*.

The number of savers that withdraw under threshold strategy x^* at a given level of fundamentals

[13] This is not generally true if there are additional states such that the bank remains solvent even if entrepreneurs default on their loans. As already mentioned, expanding our model to account for such outcomes is not important for our results.

ξ is

$$\lambda(\xi, x^*) = \begin{cases} 1 & \text{if } \xi < x^* - \varepsilon \\ \delta + (1-\delta) Prob\,(x_i \leq x^*) & \text{if } x^* - \varepsilon \leq \xi \leq x^* + \varepsilon\,, \\ \delta & \text{if } \xi > x^* + \varepsilon \end{cases} \tag{29}$$

where $Prob\,(x_i \leq x^*) = (x^* - \xi + \varepsilon)\,/2\varepsilon$. The number of savers withdrawing is decreasing in ξ, so the bank is liquidated in a run only if $\xi \leq \xi^*$ where ξ^* is the unique solution to $\theta\left(\xi^*, \lambda\left(\xi^*, x^*\right)\right) = 1$ (see equation (12)):

$$\begin{aligned} \lambda\left(\xi^*, x^*\right) D(1+r_2^D) &= LIQ_1 + \xi^* I \\ \Rightarrow \xi^* &= \frac{\varepsilon\left[(1+\delta)D(1+r_2^D) - 2\cdot LIQ_1\right] + x^*(1-\delta)D(1+r_2^D)}{2\varepsilon I + (1-\delta)D(1+r_2^D)} \end{aligned} \tag{30}$$

Next consider the decision of an individual patient saver to withdraw given her expectation about the total number of people withdrawing and the signal she receives.

For any λ and ξ such that the bank survives the run, i.e., $\lambda \leq (LIQ_1 + \xi \cdot I)/\left(D\left(1+r_2^D\right)\right)$, equations (4) and (5) give the period 3 consumption of a patient saver who waits, $c_{3s}\,(p, \mathbb{I}_w = 0)$, and withdraws, $c_{3s}\,(p, \mathbb{I}_w = 1)$. The difference between (4) and (5) arises because the person who waits will receive a late deposit payment, while the other person will get her deposits early and transfer them to period 3 using the liquid asset. The expected utility differential between waiting and withdrawing conditional on the bank surviving the run is $\sum_s \{\omega_{3s}\left(V_{3s}^D(\xi,\lambda) - c_D \cdot \mathbb{I}_d\right) \cdot D \cdot \left(1+r_3^D\right)\} - D \cdot \left(1+r_2^D\right)$.

On the other hand, in a run, i.e., for $\lambda \geq (LIQ_1 + \xi \cdot I)/\left(D\left(1+r_2^D\right)\right)$, a patient saver who waits consumes $LIQ_1^S + e_2^S$, while a patient saver who attempts to withdraw consumes $D\left(1+r_2^D\right) + LIQ_1^S + e_2^S$ with probability $\theta(\xi,\lambda)$ and $LIQ_1^S + e_2^S$, otherwise. The expected utility differential between waiting and withdrawing is $LIQ_1^S + e_2^S - \theta(\xi,\lambda)\cdot(D\left(1+r_2^D\right) + LIQ_1^S + e_2^S) - (1-\theta(\xi,\lambda))\cdot(LIQ_1^S + e_2^S) = -\theta(\xi,\lambda) D\left(1+r_2^D\right)$, where $\theta(\xi,\lambda) = (LIQ_1 + \xi \cdot I)/(\lambda \cdot D\left(1+r_2^D\right))$.

Overall, the utility differential between waiting and withdrawing when fundamentals are ξ and λ savers withdraw is given by the following piecewise function:

$$\nu(\xi,\lambda) = \begin{cases} \sum_s \{\omega_{3s}\left(V_{3s}^D(\xi,\lambda) - c_D \cdot \mathbb{I}_d\right) \cdot D \cdot \left(1+r_3^D\right)\} - D \cdot \left(1+r_2^D\right) & \text{if} \quad \dfrac{LIQ_1 + \xi \cdot I}{D \cdot (1+r_2^D)} \geq \lambda \geq \delta \\ -\dfrac{LIQ_1 + \xi \cdot I}{\lambda \cdot D \cdot (1+r_2^D)} \cdot D \cdot \left(1+r_2^D\right) & \text{if} \quad 1 \geq \lambda \geq \dfrac{LIQ_1 + \xi \cdot I}{D \cdot (1+r_2^D)} \end{cases}. \tag{31}$$

To understand the decision to run, consider an individual patient saver who receives signal x_i. The agent will use the signal to update her beliefs about the realization of ξ. Given the distributional assumptions we make (both ξ and ε_i are uniformly distributed), the posterior distribution of ξ given x_i is $\xi\,|x_i \sim U[x_i - \varepsilon, x_i + \varepsilon]$. This implies that the utility differential between waiting and withdrawing for a patient saver who receives signal x_i as a function of the cutoff value for running is

$$\Delta(x_i, x^*) = \frac{1}{2\varepsilon}\int_{x_i-\varepsilon}^{x_i+\varepsilon} \nu(\xi, \lambda(\xi, x^*))d\xi.$$

Consider next an agent who receives a signal equal to the threshold x^*. This agent by definition is indifferent between waiting and withdrawing, i.e., $\Delta(x^*, x^*) = 0$. The posterior distribution of $\lambda(\xi, x^*)$ for this agent is uniform over $[\delta, 1]$.[14] As ξ decreases from $x_i + \varepsilon$ to $x_i - \varepsilon$, λ increases from δ to 1. Changing variables and taking the limit $\varepsilon \to 0$, which implies that $x^* \to \xi^*$, provides the indifference condition that determines the unique value for ξ^* in the global game:

$$\begin{aligned} GG: &\int_{\delta}^{\theta^*}\left[\sum_s \left\{\omega_{3s}\cdot\left(V_{3s}^D(\xi^*,\lambda) - c_D\cdot\mathbb{I}_d\right)\cdot D\cdot\left(1+r_3^D\right)\right\} - D\cdot\left(1+r_2^D\right)\right]d\lambda \\ &- \int_{\theta^*}^{1}\frac{LIQ_1+\xi^* I}{\lambda\cdot D\cdot(1+r_2^D)}\cdot D\cdot\left(1+r_2^D\right)d\lambda = 0 \end{aligned} \tag{32}$$

where $\theta^* = \left(LIQ_1 + \xi^* I\right)/\left(D\cdot(1+r_2^D)\right)$.[15]

As in Goldstein and Pauzner (2005), our model exhibits one-sided strategic complementarities, i.e., ν in (31) is monotonically decreasing in λ whenever it is positive. We refer the reader to Goldstein-Pauzner for a detailed proof of existence and uniqueness of the equilibrium run threshold. In contrast to their setup, we obtain well-defined upper and lower dominance regions under our assumptions for the liquidation value ξ, with each patient agent's best action being independent of her belief concerning other patient agents' behavior. The existence of these regions is critical for obtaining a run threshold.

The lower dominance region is defined by a threshold ξ^{LD} for fundamentals such the every individual patient depositor will run on the bank irrespective of what other patient depositors do when $\xi < \xi^{LD}$. This threshold is given by $\xi^{LD} = \left(\delta\cdot D\cdot(1+r_2^D) - LIQ_1\right)/I$. In other words, when the liquidation value turns out to be so low that the impatient depositors cannot be fully repaid, then the patient depositors will always run.

The upper dominance region is defined by a threshold ξ^{UD} for fundamentals such that every individual patient depositor will not run on the bank when $\xi > \xi^{UD}$, irrespective of what other patient depositors do. This threshold is given by $\xi^{UD} = \left(D\cdot(1+r_2^D) - LIQ_1\right)/I$. This condition says that the liquidation value is so high that even if everyone were to run the bank would be able to pay them. In that case, running makes no sense.

In the equilibria we consider we verify that $\underline{\xi} < \xi^{LD} < \xi^* < \xi^{UD} < \overline{\xi}$. The conditions that are needed to establish the two regions are not very restrictive. Because there is aggregate uncertainty about the liquidation value and the loans may be worth more than their face value if liquidated, the bank will hold fewer liquid assets than the predicted withdrawals by impatient depositors. This

[14]This is true because $Prob\left(\lambda(\xi, x^*) \le N\right) = 1 - Prob\left(\xi \le \xi^* + \varepsilon - (N-\delta)/(1-\delta)2\varepsilon\right) = 1 - (\xi^* + \varepsilon - (N-\delta)/(1-\delta)2\varepsilon - \xi^* + \varepsilon)/(2\varepsilon) = (N-\delta)/(1-\delta)$, hence $\lambda(\xi, x^*) \sim U[\delta, 1]$.

[15]Equation (32) is sufficient to guarantee that a patient saver will not withdraw if a run does not occur; only impatient savers withdraw in equilibrium. In other words, her incentive compatibility constraint $\sum_s \left\{\omega_{3s}\cdot\left(V_{3s}^D(\xi,\delta) - c_D\right)\cdot D\cdot\left(1+r_3^D\right)\right\} - D\cdot\left(1+r_2^D\right) \ge 0$ is always satisfied as it is positive for ξ^* and increasing in ξ.

establishes the lower dominance region. Moreover, if the liquidation value is high enough and/or if the bank has sufficient equity, then it would be able to repay all depositors early without running out of funds. This will guarantee the upper dominance region.

2.5 Discussion of Modeling Assumptions

Before analyzing the model's properties, it is helpful to clarify the role that the various modifications we have made to the standard Diamond-Dybvig model play in our analysis. There are three important changes that are essential for our results and several lesser alterations that are made to simplify the analysis and exposition.

Private Benefit and Monitoring. One critical change is the assumption that the bankers have an outside option which depositors must take into account in providing funding. We have introduced this consideration by assuming that the realized productivity of entrepreneurial projects is private information, hence there is a need for monitoring. However, bankers are willing to monitor only when the profits accruing to them are higher than their private benefits. Demandable deposits exert discipline because depositors would run if they expected that the incentive compatibility constraint of bankers to be violated ex-post. These adjustments are important in generating endogenous run risk.

Incomplete Markets for Aggregate Risk. The second fundamental adaptation is the assumption that real economic activity is subject to aggregate productivity risk and agents cannot write contingent contracts on the realization of the productive shock in state s. The uncontingent debt contracts could be set so that they would be riskless by restricting the loan amount so that the borrower could repay in all states of the world. This is not profit-maximizing and instead the bank is willing to take some credit risk. In addition to the aggregate risk, the liquidation value of long-term investment is uncertain (and uninsurable and uncontractible). The combination of these modifications creates endogenous risk of a run. The technical assumptions about the signals regarding the liquidation value means that the probability of a run is a uniquely defined as a function of fundamentals. This assumption is also important to generate endogenous credit risk.

Banker as an Agent and Banker's Wealth. Our third important modification is the assumption that the intermediaries are run by bankers who want to maximize their own utility rather than the utility of depositors and also enjoy private benefits from operating the bank. This assumption is important to justify short-term funding as a discipline device and generate divergent incentives due to credit risk. We suppose that in our baseline case that bankers can earn profits. As mentioned already, it is possible that the bankers are so wealthy that they desire to lend so much that profits are driven to zero. This reduces one of the distortions in the model, but, as we show in section 5, there is still scope for banking regulation.

There are several other modifications that we make to the Diamond-Dybvig set up that are for convenience and are not essential for the results.

Quasi-linear Preferences and Equity Financing. We have assumed quasi-linear preferences for savers and bankers to simplify the exposition of the model. Our results would also hold under concave utilities in period 3 and, arguably, they would be stronger given that the stability of the banking sector would interact positively with risk-aversion. Our results go through provided that bankers are not more risk-averse than savers, so that bankers are willing to inject equity and are not insured by savers. Nevertheless, quasi-linear preferences are important to simplify the solution of the incomplete information game when savers are allowed to also invest in bank equity as explained in the online appendix. To facilitate a comparison to that case we have maintained the assumption in our baseline model.

Bankruptcy Costs. The introduction of bankruptcy costs essentially gives banker's an advantage at investing in entrepreneurs' projects and introduces a risk-sharing role of equity.[16] Although assuming zero bankruptcy costs would be inconsequential for most of our analysis, the level of these costs matters when bankers have ample wealth and planning equilibria can be implemented with one regulation as we explain in section 5.

Finally, it is not necessary that the value of liquidity provision arises only for the reasons emphasized by Diamond and Dybvig. We could change agents' preferences to reduce the complexity of our model. The first drawback of doing so is that we would need to introduce another source of outflows that the bank experiences in the intermediate period such that the lower dominance region in (27) is well defined. Such outflows could result, for example, from tax obligations; yet, certain assumptions about the seniority of these outflows and short-term debt would have to be made. The second drawback is that we would not be able to study the effect of regulation on liquidity provision, which would matter for the welfare implications of our model.

3 Efficient Allocations

Bankers internalize how their investment and capital decisions change the probability of a run and choose the deposit terms optimally given the supply schedule offered by savers. However, bankers may still have an incentive to take risk to exploit their limited liability and choose banking allocations that maximize their own utility at the expense of the other agents. Savers and entrepreneurs are sufficiently small to internalize how their own decisions matter for aggregate bank allocations driving run risk and credit risk. In order to examine how these externalities distort the efficient allocations we consider a social planner who internalizes the effects of investment and capital decisions on all agents, but still is constrained by the market structure of the economy. We will show that there are two major distorted margins in banker's private decisions.[17] Section 3.1 sets the planner's problem and derives the socially efficient optimization margins. Section 3.2 derives expressions for the distortions between the private and social optimization margins. Section 3.3 presents a numerical

[16] See also Allen, Carletti and Marquez (2015) who also introduce bankruptcy costs to endogenize the cost of equity and deposit finance for banks.

[17] In a model with Diamond-Dybvig preferences and complete asset markets for aggregate risk, Allen and Gale (2004) show that equilibrium allocations under financial intermediation are constrained efficient. In our framework, the presence of incomplete markets, incomplete contracts and limited liability makes the asset and capital structure of banks matter for equilibrium outcomes and bank risk. The optimality conditions of a social planner will differ from those in the private equilibrium and welfare improvements are possible.

solution to the model and describes how the privately and socially allocation differ.

3.1 Social Planner

The social planner chooses banking assets, $\{I, LIQ_1, LIQ_2\}$, banking liabilities, $\{D, E_1^B\}$, the run threshold, ξ^*, savers' liquidity holdings, $\{LIQ_1^S\}$, and interest rates, $\{r^I, r_2^D, r_3^D\}$, to maximize the following social welfare function:

$$\mathbb{U}^{sp} = w_E \mathbb{U}^E + w_S \mathbb{U}^S + w_B \mathbb{U}^B, \tag{33}$$

where w_E, w_S and w_B are the weights assigned to the three agents, which are positive and sum up to 1. Agents' utilities are given by (26), (6) and (10). It will be useful in what follows to introduce some additional notation. Define the set of the aforementioned optimizing variables as $\mathbb{X}$. The planner will optimally choose variables $X \in \mathbb{X}$ subject to a set of constraints $\mathbb{B}(X)$, which are described below, i.e., the planner's problem is

$$\begin{aligned} &\max_{\mathbb{X}} \quad \mathbb{U}^{\text{sp}} \\ &s.t. \;\; \mathbb{B}(X) \geq 0. \end{aligned} \tag{34}$$

The planner is constrained by the market structure of the economy, i.e., she cannot use lump-sum transfers to allocate resources across agents,[18] and needs to respect: the individual budget constraints (1), (2), (3), (4), (5); the balance sheet constraints (11), (12), (13), (14); the private incentives to default, i.e., constraints (15) and (16); the banker's incentive compatibility constraint (17); the global game constraint (32); and the fact that liquidity, deposits, equity and interest rates cannot be negative. Moreover, the planner takes the deposit supply and loan demand schedules (7) and (27) as additional constraints. Yet, in principle, she doesn't need to respect them, which means that (7) and (27) do not need to hold with equality in the planner solution. For example, the planner could choose deposit or loan rates that do not necessarily satisfy all these conditions with equality and implement the resulting allocations by choosing instruments, such as Pigouvian taxes on interest income/expenses, that distorts (7) or (27).[19] If these conditions do not hold with equality, then the Lagrange multipliers associated with them are zero. Given that our focus is on banking regulation, we will impose the private deposit supply and loan demand schedules as equalities in the planner's problem. Hence, in our baseline analysis the planner is essentially choosing a set of allocations that need to satisfy the pricing equations given by the deposit supply and loan demand schedules. We relax this assumption in the online appendix and show that our conclusions on the need for

[18] Given the absence of lump-sum transfers, we cannot unambiguously construct a welfare criterium to maximize the total surplus. Thus, we assign weights for different agents in a social welfare function and study different constellations of these weights. Although we remain agnostic about the origin of such weights, we discuss the potential political economy considerations of regulation.

[19] Farhi and Werning, 2016, and Bianchi and Mendoza, forthcoming, consider such taxes to implement the constrained efficient allocations. Note that the taxes can also take negative values, in which case they are interpreted as subsidies.

banking regulation continue to hold. To summarize, the set $\mathbb{B}(X)$ includes constraints (1)-(5), (7), (9), (11)-(17), (25), (27), (28), (32).

We report the planner's first order conditions in a compact form, because the detailed expressions are long and not particularly enlightening. The first-order condition with respect to a variable $X \in \mathbb{X}$ will, in general, take the following form:

$$\sum_{h=\{E,R,B\}} \overline{w}_h \frac{d\mathbb{U}^h}{dX} + \zeta_{BS}\frac{dBS}{dX} + \zeta_{IC}\frac{dIC}{dX} + \zeta_{GG}\frac{dGG}{dX} + \zeta_{LD}\frac{dLD}{dX} + \zeta_{DS}\frac{dDS}{dX} = 0, \tag{35}$$

where ζ_{BS}, ζ_{IC}, ζ_{GG}, ζ_{LD}, ζ_{DS} and ζ_{ES} are the multipliers on (11), (17), (32), (27) and (7), respectively.

The first term in (35) captures how variable X matters for the weighted utilities of agents where $\bar{w}_h = w_h + \zeta_{PC,h}$ and $\zeta_{PC,h}$, $h = \{E,S,B\}$ are the multiplier on E's, S's and B's participation constraints given by (28), (9) and (25), respectively. The second, third and fourth terms capture the effect of variable X on the balance sheet, the banker's incentive compatibility and the global game constraints, while the last two terms capture how variable X changes the loans demand and deposit supply schedules.[20]

3.2 Private versus Social decisions

In this section, we compare the allocations chosen by private agents to the efficient allocations chosen by the social planner above. We identify two distorted margins of optimization in the banker's private decisions; first, a distorted asset mix, and, second, a distorted liabilities mix. The former captures the way the banker and the planner choose between investing in the risky loans or the liquid asset. The latter captures the choice of funding used for investment.[21]

To see why there are two independent intermediation margins, observe that the banker's optimizing behavior in the previous section yields seven optimizing condition for $\{I, LIQ_1, E, D, r_2^D, r_3^D, \xi^*\}$. Moreover, there are four Lagrange multipliers $\{\psi_{BS}, \psi_{IC}, \psi_{GG}, \psi_{DS}\}$ associated with four constraints. Given that the constraints bind, as is the case, one can use four of the optimizing condition to determine the multipliers. In particular, but not exclusively, use (18) to determine ψ_{IC}, (20) to determine ψ_{GG}, (24) to determine ψ_{DS}, and (21) to determine ψ_{BS}. Moreover, use the four constraints to pin down I, r_3^D, E and ξ^*, and (23) to pin down r_2^D, as function of LIQ_1 and D. The latter two variables are determined by (19) and (22). Alternatively, we could have expressed everything in terms of functions I and E – or in fact in terms of any other combination of one of the liabilities and one of the assets. So a natural way to think of the two "free" banking choices is that the proportions of liquid to illiquid assets and deposits to equity are the critical endogenous objects in the model. The same logic applies to solve for the free variables in the planner's problem.[22]

[20]Note that the multipliers in the planner's solution are denoted by ζ rather than ψ in the private equilibrium, because the two will be determined differently.

[21]A third distorted intermediation margin arises if we allow savers to also buy bank equity (see the online appendix).

[22]In the expressions below the Lagrange multipliers are considered to be at their equilibrium values and are not sub-

The asset mix distortion is derived by combining the investment and liquid asset optimality conditions of the banker, (18) and (19), and of the planner, equation (35) for $X = I$ and $X = LIQ_1$, respectively. The banker's *investment-liquidity* margin, $ILIQ_B$, is, then, given by

$$\frac{d\mathbb{U}^B}{dI} - \frac{d\mathbb{U}^B}{dLIQ_1} + \psi_{IC}\left[\frac{dIC}{dI} - \frac{dIC}{dLIQ_1}\right] + \psi_{GG}\left[\frac{dGG}{dI} - \frac{dGG}{dLIQ_1}\right] + \psi_{DS}\left[\frac{dDS}{dI} - \frac{dDS}{dLIQ_1}\right] \cdot \mathbb{I}_c = 0. \tag{36}$$

In contrast the socially optimal investment-liquidity margin, $ILIQ_{sp}$, will include additional terms capturing how banking decisions also affect savers and entrepreneurs:

$$\sum_{h=\{E,R,B\}} \overline{w}_h \left[\frac{d\mathbb{U}^h}{dI} - \frac{d\mathbb{U}^h}{dLIQ_1}\right] + \zeta_{IC}\left[\frac{dIC}{dI} - \frac{dIC}{dLIQ_1}\right] + \zeta_{GG}\left[\frac{dGG}{dI} - \frac{dGG}{dLIQ_1}\right]$$
$$+ \zeta_{DS}\left[\frac{dDS}{dI} - \frac{dDS}{dLIQ_1}\right] + \zeta_{LD}\left[\frac{dLD}{dI} - \frac{dLD}{dLIQ_1}\right] = 0. \tag{37}$$

We can group the differences between the banker's and the planner's margin in three categories. First, the planner considers the direct effect of a portfolio shift from liquid asset to risky loans on weighted social welfare rather than on only the welfare of the banker. This can be seen by comparing the first term in (37), $\sum_h \bar{w}_h \left[d\mathbb{U}_h/dI - d\mathbb{U}_h/dLIQ_1\right]$, to the first term in (36), $d\mathbb{U}_B/dI - d\mathbb{U}_B/dLIQ_1$. Second, the planner considers how the welfare of all agents, not only of the banker, matters for the level of multipliers on constraints GG, IC and DS.[23] In other words, the planner internalizes how picking the run threshold, how relaxing the banker's incentive compatibility constraint and how moving along the deposit supply curve also affects entrepreneurs' and savers' welfare. Third, the planner internalizes how the choice of investment and liquidity affects the deposit supply and loan demand schedules. The banker only prices the effect of investment and liquidity on deposit supply when deposit contracts are comprehensive, i.e., $\mathbb{I}_c = 1$.

For further reference, denote the sum of these distortions in the investment-liquidity margin by $ILIQ_{wedge}$, such that

$$ILIQ_{sp} = ILIQ_B + ILIQ_{wedge}. \tag{38}$$

The liabilities mix distortion can be derived similarly by combining (21) and (22) for the private margin, and equations (35) for $X = E$ and $X = D$ for the planner's margin. We call this the *equity-deposits* margin, ED. The differences between the private margin, ED_B, and the social margin, ED_{sp}, fall in the same categories described above for the investment-liquidity margin. A subtle distinction is that the banker internalizes the effect of deposit taking on deposit supply even when

stituted out following the strategy outlined above. We report in an online appendix the intermediation margins expressed only in terms of allocations such that the multipliers are substituted. These expression are convoluted and do not provide additional intuition. Thus, we have opted to present the intermediation margins without substituting the multipliers herein.

[23]For example, the multiplier ψ_{GG} in (36) only depends on $d\mathbb{U}^B/d\xi^*$, while the multiplier ζ_{GG} in (37) depends on $\sum_h \bar{w}_h d\mathbb{U}_h/d\xi^*$. This can be seen from the optimality conditions for the run threshold, (20) and (35) for $X = \xi^*$. Similarly for the other two multipliers.

deposit contracts are incomplete. For further reference, denote the sum of these distortions in the equity-deposits margin by ED_{wedge}, such that

$$ED_{sp} = ED_B + ED_{wedge}. \tag{39}$$

In the next section we present a numerical example of the private and planning equilibria and discuss how the allocations differ in reference to the aforementioned intermediation margins.

3.3 Numerical example

The full set of parameters we used to solve the model is shown in Table 1. The parameterization should be taken more as an illustrative example to highlight the mechanisms in the model rather than as a realistic calibration of the economy attempting to make quantitative statements about the absolute optimal level of banking regulations. We have experimented with various other parameter choices and the findings that we emphasize are quite robust.

Our model would require some obvious modifications to use it for quantitative policy analysis. For example, all liabilities in our model are unsecured, while in practice certain types of deposits are insured. Deposit insurance, even partial, would reduce the market discipline exerted by depositors and hence the credit risk premia in deposit rates bringing them closer to what is observed in reality. Moreover, it is not clear whether the various capital regulations in practice (Basel requirements, stress tests, restrictions on dividend payouts) are indeed binding and whether one should be calibrating to match a regulated economy rather than an unregulated private equilibrium. Finally, the assumption of linearity of utilities in third period consumption, which simplifies the computation of the run threshold significantly, as well as the finite horizon of the model make depositors willing to accept a higher probability of a run if they were risk-averse or if there was a continuation value for the bank. One could add convex bankruptcy costs to mimic a higher degree of risk-aversion as well as model the continuation value, but we have not done so because it is not important to make our fundamental analytic points.

With these caveats in mind, let us call attention to some of the considerations that we took into account while choosing the model parameters.

First, the probabilities of default and losses given default will determine the amount of default risk that the bank is facing. We opt to have entrepreneurs and banks default in the bad state irrespective of the realization of the liquidation value in the intermediate period.

Second, the bank is profitable enough, and the initial equity of the banker and her preference for current consumption are such that she voluntarily uses some of her endowment to buy more equity in the bank. In our baseline equilibrium, the banker enjoys a positive economic surplus from intermediating. In section 5 we examine a case that the banker has sufficient initial wealth so that she invests in the bank up to the point that the economic surplus accruing to her is driven to zero, i.e., she enjoys the same utility as in autarky. We believe that this is not realistic, but describing this case is still useful to highlight that the justification for banking regulation is not to capture the economic surplus of bankers, but rather to improve allocative and productive efficiency.

Third, the liquidity provision by the bank leads savers not choose to additionally self-insure by holding the liquid asset. When savers self-insure, the banking sector is under-performing as a provider of liquidity and, hence, intermediation, and regulations that make banks more stable would have an additional positive effect. In our baseline parameterization we want to mute this channel and make it harder for regulation to improve economic outcomes. However, our results hold even when savers self-insure in the private equilibrium.

Fourth, we have chosen the parameters, among them, most importantly, the liquidity preference shock, the distribution of the liquidation value and risky technology payoffs, such that the bank holds a portfolio of both liquid assets and risky loans, and also liquidates part of its risky holdings to serve early withdrawals.

Fifth, we have chosen logarithmic utility for period 1 and period 2 consumption, while we specialize the production function to be $F = \left(I + I^E\right)^{\alpha} \ell^{1-\alpha} = \left(I + I^E\right)^{\alpha}$, with $\alpha < 1$ and entrepreneurial skills' supply normalized to $\ell = 1$.[24]

Before presenting the model solution and explaining how the private and social equilibria differ, we briefly describe some regulatory ratios and risk metrics that we have constructed to facilitate the analysis.

The capital adequacy ratio (*CR*) is equal to the value of equity divided by the level of risky loans. We have normalized the risk-weight on loans to one, while liquid assets receive a risk-weight of zero:

$$CR = \frac{E}{I}. \tag{40}$$

The leverage ratio (*LevR*) includes both the risky and liquid holdings and is given by:

$$LevR = \frac{E}{I + LIQ_1}. \tag{41}$$

The liquidity coverage ratio takes the (lowest) liquidation value of the bank's portfolio in a run relative to runnable liabilities:[25]

$$LCR = \frac{LIQ_1 + \underline{\xi} \cdot I}{D \cdot \left(1 + r_2^D\right)}. \tag{42}$$

Finally, we compute a net stable funding ratio which is computed as the fraction illiquid assets

[24] The original Diamond-Dybvig framework requires the relative risk-aversion coefficient to be higher than one. This is not necessary when the liquidation value of long-term investment can be lower than one as pointed out by Cooper and Ross (1998). In addition, the share of income for the risky technology accruing to entrepreneurial human capital (set to 0.25) is chosen to reflect estimates from the literature. Gollin (2005) finds that the share of profits in entrepreneurial activities is 0.10. The rest is the share of labor and capital. In our setting, labor from workers is not modeled, and we are interested in the share of the remaining output which is distributed to entrepreneurs and suppliers of capital. Setting the share of capital relatively to labor to 0.30, which is standard in the literature, give a relative share for entrepreneurial and capital profits of $0.1/(0.1 + 0.9 \cdot 0.3) = 0.28$ and $(0.9 \cdot 0.3)/(0.1 + 0.9 \cdot 0.3) = 0.72$, respectively.

[25] It is not obvious whether to count the portion of the loans that are always available as being liquid or not. Our results are very similar if we exclude them from the numerator of this regulation.

funded by relatively stable sources:

$$NSFR = \frac{E + (1-\delta)\cdot D}{I}. \tag{43}$$

Moreover, the probability of a run is computed as $q = \left(\xi^* - \underline{\xi}\right)/\Delta_\xi$ and can be further disaggragated into a fundamental-driven and a panic-driven component. The probability that depositors run only because fundamentals turn out to be bad is $q^f = \left(\xi^{LD} - \underline{\xi}\right)/\Delta_\xi$.

Finally, we compute a measure of the liquidity provision delivered by the bank. As already mentioned, savers expected utility must be higher than in autarky. However, the bank can make this happen in different ways. For example, it could offer higher compensation for patient savers in exchange for lower liquidity provision to impatient ones. We, thus, separately compute the expected utility of impatient savers when the bank intermediates relative to their utility in autarky as a measure of liquidity provision:

$$Liq.Prov. = \frac{\int_{\underline{\xi}}^{\xi^*} \mathbb{E}_\theta U_2\left(c_2(i,\mathbb{I}_\theta);i\right)\frac{1}{\Delta_\xi}d\xi + \int_{\xi^*}^{\overline{\xi}} U_2\left(c_2(i,1);i\right)\frac{1}{\Delta_\xi}d\xi}{U_2\left(e_2^S + LIQ_1^{S,aut};i\right)}. \tag{44}$$

Table 2 reports the equilibrium values of some main variables of interest along with the computed metrics for the private equilibrium (PE), and the social planner's solution for different weights on E, S, and B. We have set banker's weight to 0.2 and also set a lower value of 0.2 for the weights of the other two agents. As we will explain in more detail later, this choice is not important for the generality of the results. We have normalized the utility of all agents in the private equilibrium to one when we compute the welfare change. $\%\Delta\ \mathbb{U}^{\text{sp}}$ and $\%\Delta\ \mathbb{S}^{\text{sp}}$ are the percentage change in social welfare given the weights and the change in total (unweighted) utility from the private equilibrium, respectively. Note that we report two types of private equilibria in Table 2: one where the funding contracts with depositors are incomplete, and another where the banker can write comprehensive contracts. We start by discussing the first case, and examine the second at the end of this section.

We focus the analysis around the two intermediation margins derived in section 3.2. First, the planner corrects the distortion in the asset mix between risky loans and liquid assets. Due to limited liability and incomplete contracting the banker has an incentive to tilt her portfolio towards risky loans, which have a higher payoff in the states where B is solvent, while fully internalizing the effect of the asset mix on the run probability and the stability of banking profits accruing to her. The planner also internalizes the effect of the bank's choices on savers and entrepreneurs and chooses a more liquid asset mix. This can be seen from the big increase in liquid assets in the planner's solution for all weights and the higher liquidity coverage and net stable funding ratios.

The funding mix is also distorted. The banker prefers to fund herself with deposit rather than equity because by levering up she can exploit her limited liability. Also, deposits carry a liquidity premium, in that the deposit rate reflects more than credit risk and time-preference. The banker herself has no preference for liquidity given that she doesn't want to consume in the intermediate period and would like to extract any liquidity premium by using deposit funding. The planner instead prefers more equity funding and a more stable capital structure. Hence, the planner chooses higher common equity and higher capital adequacy. By doing so, the planner's allocations allow the bank to operate with a larger balance sheet. Despite a higher capital ratio, the bank operates with a lower leverage ratio and delivers its intermediation services more efficiently.

The more stable asset and funding choices of the planner come from the desire to reduce run risk. The probability that a run occurs drops in the planner's solution, as does the probability of fundamental runs. Overall, this improves the stability of the banking sector and enhances the stability of real economic activity as the entrepreneur sees her funding being withdrawn less frequently.[26]

Moreover, the planner reduces maturity transformation in relative terms, which can be seen by the higher risk weighted capital and net stable funding ratios. However, the reduction in maturity transformation is not necessarily accompanied by a drop in the *level* of credit extension. For sufficiently high w_E, the planner chooses higher investment than in the private equilibrium. Indeed, we show that increasing the equity in the bank, without requiring more liquidity, leads to more credit extension through a combination of channels described in detail in sections 4.1 and 4.2 where we study capital and leverage requirements in isolation. On the contrary, credit extension is lower when the saver is favored and the planner forces the bank to hold more liquid assets. Overall, private allocations can exhibit both under- and over-investment indicating that the source of inefficiency is not the total level of lending, but its relation to the holding of liquid assets and the funding structure of the bank.

Liquidity provision is also higher in the planner's solution. In fact, liquidity provision as measured in (44) is below one in the private equilibrium, which suggests that the banker offers attractive enough returns to patient depositors to induce them to accept lower utility when they turn out to be impatient – the deposit terms still deliver higher overall utility than in autarky. This may not be surprising given that the banker favors lending over holding liquidity, so the bank finds it more difficult to provide insurance to impatient savers. Instead the planner delivers more liquidity provision to impatient depositors without sacrificing long-term returns to patient savers.[27] This is possible because the planner's choices lead to a larger overall level of intermediation.

The enhanced stability of both the asset portfolio and capital structure of the bank are beneficial

[26]The fact that panic-driven runs occur with non-negligible probability suggests that government guarantees, such as deposit insurance or implicit bailout subsidies, may be useful policy interventions. We have abstracted from introducing government guarantees in the model for two reasons. First, it would not unambiguously improve outcomes as in the original Diamond-Dybvig set-up because of risk-taking incentives (see, for example, Kareken and Wallace, 1978, Cooper and Ross, 2002, Admati et al., 2012). Second, designing deposit insurance when runs have both a fundamental and panic risk component is far from straightforward. Such an exercise is not trivial and is beyond the scope of the current paper which aims to identify the banking externalities arising from incomplete contracting for credit and run risk (Allen et al., 2015, study government guarantees within a global games framework and a simpler banking sector that the one in our paper). In the same token, we do not study emergency liquidity assistance from a Lender of Last Resort (Rochet and Vives, 2004) or suspension of convertibility (Ennis and Keister, 2009), which would also require non-trivial modifications in the model we present. See also Keister (2015) for an analysis of efficient bailouts, which should be complemented with prudential regulation. We believe that these are important avenues for future research in models that feature an elaborate banking sector subject to both credit and run risk like ours.

[27]Though not shown in the table, the utility of the patient savers is higher than in the private equilibrium.

to savers and entrepreneurs. However, the banker is worse-off; in our model the banker internalizes all the effects that matter for her welfare and optimally chooses more risk to maximize her own utility. The planner cares about the externalities to the other agents as well and weakens the ability of the banker to take advantage of her limited liability. In fact, the banker's welfare always drops in the planner's solutions and drives the banker to her participation constraint, unless the weight on the banker is (unreasonably) high. We do not explore such planning equilibria because they cannot be implemented with banking regulation, which always drives bankers utility down, as it will become more clear in section 4. Nevertheless, the planner not only increases social welfare ($\mathbb{U}^{sp}$), which depends on Pareto weights, but also the overall surplus in the economy, which is captured by the change in $\mathbb{S}^{sp}$. The planner could improve the welfare all agents if she had access to a re-distributive, non-distortionary (lump-sum), tax system to transfer resources across agents. In section 5, we examine equilibria where a very wealthy banker injects enough equity into the bank to drive the economic profits to zero. In this case, we show that a Pareto improvement over the private equilibrium is possible.

Finally, the planner chooses different levels of capital and liquidity depending on which agents she favors most. When the weight on the entrepreneur is higher, the planner chooses more capital to support higher lending. This is beneficial for the entrepreneur, but results in lower liquidity provision, which is relatively bad for savers. On the contrary, when the weight on savers is higher, the planner chooses a more liquid asset mix.

We should note that neither the banker nor the planner hold excess liquidity, i.e., $LIQ_2(\xi,\delta) = 0$ for all ξ. Holding excess liquidity could be desirable in order to eliminate the probability of a run altogether. If the liquidation value of the bank for the lowest possible realization of ξ was higher that the total runnable deposit obligations, i.e., $LIQ_1 + \underline{\xi} \cdot I \geq D(1+r_2^D)$, then only impatient depositors would withdraw. The excess liquidity carried over to period 3 would then be $LIQ_2 = LIQ_1 - \delta \cdot D(1+r_2^D) \geq (1-\delta)D(1+r_2^D) - \underline{\xi} \cdot I$. Such run-proof equilibria may not be desirable when the lowest liquidation value of long-term investment is small or when savers are not very risk-averse. A fair amount of literature has focused on run-proof equilibria, which naturally restrict credit intermediation (see Cooper and Ross, 1998, Ennis and Keister, 2006, Diamond and Kashyap. 2016). Run-proof contracts require certain assumptions to be optimal and our work has, instead, focused on optimal policy in the presence of both run risk and credit risk.

Before turning to the implementation of the planning outcomes, we discuss the private equilibrium when deposit contracts are comprehensive, i.e., $\mathbb{I}_c = 1$ in the banker's optimization conditions. The results are reported in the third column in Table 2. Comprehensive deposit contracts allow the banker to commit, but she still chooses the levels of risk that improves her own utility and does not internalize all the effects on other agents. Acting in her own interests, the banker still chooses a riskier asset and liabilities structure than a planner would. The social planner instead internalizes the effects of all decisions on all agents utility, and choose a less risky asset mix and liabilities structure. The gap between the social and private equilibrium grows with commitment, because in this case the banker offers a contract that is even more skewed towards her interests at the expense of the other agents. Indeed, as the table shows the banker's choices result in lower welfare for entrepreneurs and savers compared to the private equilibrium with incomplete deposit contract, and the participation constraint of savers starts binding.

4 Regulation

We now explore how the planner's solution can be decentralizing via various regulatory interventions, which tighten the regulatory ratios (40)-(43). Sections 4.1-4.4 discuss the effects when the tools are used in isolation. Section 4.5 discusses how the regulations can be optimally combined to implement the planner's solution as a private equilibrium. Table 3 reports the results for the various regulations.

4.1 Capital Requirements

Capital regulation requires the bank to hold a certain percentage of equity for every unit of risky loans extended and it formally amounts to increasing CR in equation (40).

Mandating higher capital requirements reduces the ability of the banker to take risk through deposit funding. In models where the bank cannot raise additional equity, stricter capital requirements (mechanically) result in a drop in credit extension (see, for example, Corbae and D'Erasmo, 2014, Clerc et al. 2015 and the references therein). More generally, one could allow banks to raise both equity and deposits. Then, capital regulation has an effect if the Modigliani-Miller theorem is violated. Despite abstracting from any tax advantages of debt, which is the most common violation put forward, our environment breaks Modigliani-Miller in various ways, even holding the probability that a run occurs constant. First, deposits carry a liquidity premium.[28] Second, the available equity capital is not perfectly elastically supplied and, from (21), the banker requires a higher return if she contributes more equity. Third, default is costly and there are positive bankruptcy costs. The first two frictions push for higher cost of funding, while the last for lower, when capital requirements increase. Although the overall partial equilibrium effect, fixing the probability of a run and the liquidity premium, seems ambiguous, it is plausible to suppose that the three forces result in higher funding costs and lower lending for low enough bankruptcy costs.

But our model features additional channels which push up lending. The first important channel is that higher capital reduces the probability of a run. This makes savers more willing to make deposits and the entrepreneurs more inclined to borrow.[29] Second, substituting equity financing for deposit financing on the margin allows the bank to hold less liquidity to serve the impatient households. This would incrementally free up resources to be invested in risky loans. Finally, the

[28] See Van den Heuvel (2008) and Hanson, Kashyap and Stein (2011) for estimates of the liquidity premium for bank deposits.

[29] In a model where the bank is funded only with deposits, Ennis and Keister (2006) show that shifting the asset mix towards more illiquid loans would result in a lower probability of being repaid given that a run occurs, which counterbalances the increase in credit extension when the probability of a run decreases. This does not need to be true in our model, because the increase in the credit extension is funded by more capital.

reduced demand for deposits suppresses incrementally the deposit rate, other things equal, due to impatient savers' liquidity demand.[30]

Accounting for all these considerations, lending rises when capital requirements increase.[31] There are other noteworthy general equilibrium effects that also arise. For example, the cost of funding decreases which also allows for lower loan rates.[32] Moreover, the lower probability of a run allows for more deposit taking, which pushes the deposit rate up; the banker continues to try to take advantage of her limited liability and funding higher investment exclusively with equity is expensive. Finally, capital requirements do not necessarily need to result in lower liquidity holdings. The opportunity cost of liquidating risky loans increases because the loans are funded with a more expensive source of financing on the margin. Hence, the bank chooses to liquidate a smaller portion of its loan portfolio in the intermediate period for any realization of the liquidation value (though this is not shown in Table 3). This result is why the holdings of liquid assets also increase with higher capital requirements, but the bank still holds less liquidity that the planner would. This implies that capital and liquidity regulation could be optimally combined as we show in section 4.5.

Although credit extension goes up, the regulation has two important implications. First, the higher credit extension needs to be funded exclusively with more equity. Second, liquid asset holdings also rise. As we will discuss in the following section, this is not the case under the risk-insensitive leverage regulation, because that type of rule allows the banker to offset lending increases with reductions in liquid assets.

On net, run risk falls because the bank uses a lower percentage of deposit funding. The reduction in run risk is beneficial for savers and entrepreneurs. The level of deposits need not fall, because the bank is safer overall and hence its total balance sheet grows. This mean that liquidity provision can be maintained. However, the banker is made worse-off with higher capital requirements, which is not surprising given that she could have voluntarily chosen more capital if it was beneficial for her.

Of note, the level of capital requirements that maximizes social gains without violating bankers' participation constraints, is higher than the capital ratio in the planner's solution. As we discuss in section 4.5, the planner uses multiple tools to implement the socially optimal allocations. If a regulator is limited to one tool, that tool must be used more aggressively than if several can be deployed, so it's value will "over-shoot" the level that a planner will pick. As we see below, this is true for all the tools.

[30] Begenau (2015) also shows in a real business cycle framework that the fall in the deposit rate, when capital requirements increase and savers value the liquidity services of deposits, can push the overall cost of funding down and result in higher credit extension. The strength of this mechanism is mitigated when savers can also purchase bank equity (see the extended model in the online appendix).

[31] We have experimented with several versions of the model and parameterizations and this conclusion is very robust.

[32] The first order condition with respect to E, (21), becomes $\psi_{BS} = \gamma \cdot U'(e^B - E) - \psi_{CR}$. Although more equity pushes the cost of funding up, as measured by the bankers marginal utility, the Lagrange multiplier on the capital requirement, ψ_{CR}, operates in the opposite direction and in equilibrium it dominates.

4.2 Leverage Requirements

Leverage regulation ties the level of capital to the overall size of the bank's balance sheet and it formally amounts to decreasing *LevR* in equation (41).

Leverage requirements operate through the same channels as capital requirements, i.e., they reduce the ability of the banker to take risk through deposit funding. Overall, they push credit extension up, but there is a critical difference. Risk-weighted capital regulation requires banks to hold more capital only against risky loans, while leverage regulation mandates more capital against all assets and, hence, it does not directly affect the marginal choice between investment and liquidity. Although the banker is required to operate with a safer liability structure, she can tilt the asset mix to reduce liquid assets and raise lending. As a result, credit extension should increase relatively more from tightening the leverage requirements than from raising risk-weighted capital requirements.

The increase in asset illiquidity increases fundamental run risk. The drop in liquid holdings and lower demand for deposits makes savers worse-off, who are pushed to their participation constraint for small changes in the regulation. Entrepreneurs are marginally better-off due to the higher credit extension, but the drop in social welfare suggests that leverage requirements would not be used in isolation in this economy. This conclusion is not robust to some reasonable modifications of the environment – see for example the extension in the online appendix. However, even in the model as it stands, below we show that leverage regulation can be combined with other regulations to improve economic outcomes. As with all the regulations we consider, the banker is worse-off.

4.3 Liquidity Requirements

A liquidity-coverage-ratio regulation requires that the immediately available funding for the bank is at least a certain percentage of runnable debt (deposits) and, in our model, it formally amounts to increasing *LCR* in equation (42).[33]

Mandating that the bank holds more liquidity changes the trade-off between investing in risky loans and liquid assets, since liquid assets count fully towards this regulation and loans do not. Looked at in isolation, this regulation reduces the incentive to fund loans through deposits.

Liquidity requirements are good tools for raising liquidity and reducing credit extension. However, they erode bank profitability and make it harder to raise equity. The amount of equity falls as does the capital adequacy ratio (despite the decrease in credit extension). The bank switches to more deposit financing to compensate for lower equity financing and leverage is higher than both the PE and SP outcomes. Overall this regulation has the ability to reduce risk on the asset side, but it results in higher risk on the liability side.[34]

[33] We focus on a form of liquidity coverage regulation, but the results in this section hold more generally for other types of liquidity regulation, such simple restrictions on the ratio of liquid to illiquid assets (LIQ_1/I) or reserve ratio requirements (LIQ_1/D).

[34] The literature has studied additional market failures that justify the regulation of banks' liquidity. In Allen and Gale (2004) and Diamond and Rajan (2011) the need for policy intervention stems from the presence of fire-sales, in Farhi, Golosov and Tsyvinski (2009) liquidity regulation tackles inefficient risk-sharing due to hidden trades, while Diamond and Kashyap (2016) show that liquidity requirements are important to deter run risk when depositors have incomplete

However, the lower probability of runs is beneficial to both savers and entrepreneurs, and the high level of deposits improves liquidity provision. Yet, the decrease in credit extension results in higher benefits for savers than for entrepreneurs compared to the planner's solution, while the opposite hold for capital and leverage regulations. Finally, the banker sees her welfare going down for the same reason described before.

4.4 Net Stable Funding Requirements

This type of regulation requires that the bank funds a certain percentage of illiquid assets with long-term, stable sources of financing, which in our model are equity and the portion of deposits that will not be withdrawn. Formally, the regulation sets a higher value for $NSFR$ in equation (43).

The effects of NSFR regulation are parallel to the LCR. Investment goes down, liquidity improves, and the bank relies more on deposit funding rather than equity capital. Similarly, the probability of bank runs goes down.

A generalized version of the NSFR could be calibrated to look more like capital or liquidity regulation. Suppose that the relative weights on equity capital and stable deposits that appear in the numerator of the NSFR could vary. Figure 1 shows the change in credit extension for this generalized version of the net stable funding ratio where equity and long-term deposit funding are weighted differently, i.e., $NSFR = (E + w \cdot (1-\delta) \cdot D)/I$ with $0 < w \leq 1$. Depending on the weight on deposits, an increase in the NSFR would resemble more closely the effects of capital versus liquidity regulations. In particular for a low weight on deposits, the NSFR results in higher credit extension similarly to capital requirements, while for higher weights credit extension decreases similarly to liquidity requirements.

4.5 Optimal Regulatory Mix

This section examines whether and how regulation can be combined to implement the social planner's solution as a private equilibrium. The social planner solves for allocations without taking into consideration how the optimal behavior of the bank will change, or in other words the first-order conditions of the banker (adjusted for regulatory interventions) are not taken as additional constraints in (34) – though the banker's participation constraint must be satisfied. Hence, the planner's allocations are computed without tying the planner to specific tools. This way we have been able to clearly identify the distorted margins between the privately and socially optimal decisions in section 3.2. The rest of the section shows how the regulatory tools studied above can be combined to correct for the distorted banking decisions derived as the wedges in conditions (38) and (39).

For that purpose we set-up an augmented planner endowed with certain regulatory tools. Let $\mathbb{T}$ be the available set of regulatory tools which will include at least the four regulations studied above and possibly others. For each $T \in \mathbb{T}$ there is a regulatory constraint $RC(T,X) \geq 0$, which ties the tool with the endogenous variables $X \in \mathbb{X}$ (for example, constraints (40)-(43)). It is important to

information about banks' asset portfolios. Finally, Calomiris, Heider and Hoerova (2015) discuss how liquidity regulation can provide the right incentives for managing risks under deposit insurance guarantees.

note that the regulatory constraints are defined as inequalities, i.e., the planner can tighten them, but not loosen them. Let ψ_T be the multipliers that the banker in the private equilibrium assigns to constraint $RC(T,X) \geq 0$.

Under regulation, the optimization margins change to:

$$ILIQ_{\mathbb{T}} : ILIQ_B + \sum_T \psi_T \left[\frac{dRC(T,X)}{dI} - \frac{dRC(T,X)}{dLIQ_1} A_{ILIQ} + \frac{dRC(T,X)}{dE} (1 - A_{ILIQ}) \right] = 0, \tag{45}$$

$$ED_{\mathbb{T}} : ED_B + \sum_T \psi_T \left[\frac{dRC(T,X)}{dE} (1 + A_{ED}) - \frac{dRC(T,X)}{dD} + \frac{dRC(T,X)}{dI} A_{ED} \right] = 0, \tag{46}$$

where A_{ILIQ} and A_{ED} are given by (A.9) and (A.11) in the online appendix.

The tools-augmented planner's problem, akin to a Ramsey planner's problem in the public finance literature (see, for example, Lucas and Stokey, 1983), is derived in an online appendix.[35] To implement the equilibrium allocations of the social planner, denoted by X^{sp}, the available tools, T, have to be chosen such that, first, X^{sp} satisfy the regulatory constraints $RC(T,X^{sp}) = 0$, and, second, the intermediation margins in the associated equilibrium are the same as the intermediation margins of the planner. Essentially, this means that the additional terms in (45) and (46) need to equal the wedges derived in (38) and (39). In matrix form, this can be written as:

$$\Delta RC \cdot \Psi = WD_{sp}, \tag{47}$$

where Ψ is the Tx1 vector of the multiplier on the T regulatory constraints, WD_{sp} is the 2x1 vector of the wedges in the two intermediation margins evaluated at the planner's equilibrium values, and ΔRC is the 2xT matrix of the partial derivatives of the relevant variables for each intermediation margin on the T regulatory constraints. These derivatives are also evaluated at the equibrium values for the variables X^{sp} and for the levels of the tools T^{sp}, which implicitly solve $RC(T^{sp},X^{sp}) = 0$.

Hence, it suffices to find two regulatory tools such, first, the matrix ΔRC is invertible, and, second, all elements in Ψ are positive. We will now explain these conditions in more detail and provide the underlying economic intuition.

Given that the banks' asset and liability mix are each distorted, two tools are generally needed to implement the planner's allocations. The exception would be if the distortions turn out to alter both mixes in identical ways. In this, measure zero, case the wedges would be "equal" to each other in equilibrium. Moreover, the optimization variables should not load on the regulatory constraints in a collinear way, or, in other words, the matrix $\Delta \mathrm{RC}^{sp}$ should not be singular. This means that the choice of one tool should not determine the level of another tool and, hence, there are enough degrees of freedom to correct both of the distortions.

[35] The problem in the Ramsey literature is to maximize a social welfare function subject to all the constraints constituting a competitive equilibrium for the purpose of financing government expenditure with distortionary taxation. Although the purpose of our augmented planner is different, the methodology to optimally choose the level of instruments that she is endowed with is the same.

Finally, the regulatory tools should be jointly binding, which means that the multipliers ψ_T should be strictly positive. The reason is that quantity regulations as in (40)-(43) mandate a minimum level of capital, liquidity or combinations of assets and liabilities, and the bank cannot be forced to operate at a lower level. This becomes important when tools, which are binding by themselves in the private equilibrium, are combined.

In order to examine which regulations can be jointly binding, we set the level of tools to the regulatory ratios in the planner's solution and compute the vector of multipliers Ψ for all possible combinations. It turns out that many pairs of regulations can be combined to deliver the planner's allocations. As long as one of the pair is a capital tool (CR or LevR) and the other a liquidity tool (LCR or NSFR) then the regulations will replicate the planner's allocations. This result is intuitive. The planner wants to hold more liquidity and more capital than in the private equilibrium. Liquidity requirements can force more liquidity in the bank, but at the cost of reducing capital ratios or, equivalently, increasing leverage. Hence adding a capital or leverage requirement can correct for the (unintended) consequences of liquidity regulation. Yet, two liquidity tools cannot be jointly binding as they reinforce each other and move the key variables in the same direction.[36] The same problem arises if only the two capital regulations are deployed. Otherwise, the palnner's outcome shown in the last column of Table 3 can be delivered by any of the four combinations of that involve a single capital regulation along with a single liquidity regulation.

5 Social outcomes, regulation and banker's wealth abundance

Until now, our analysis has focused on equilibria where banking is sufficiently profitable that the banker's utility is above her reservation value. We have shown that regulation depletes this surplus, but improves economic efficiency and increases the total surplus created. This raises the question of whether regulation is needed just because the banker, acting in her own interests, maximizes the surplus accruing to her. Although regulation has redistributive aspects, we show in this section that it is beneficial even when the banker is receives zero economic surplus in the private equilibrium.

One way to clarify these issues is to consider a banker who is wealthy enough that the marginal value of consumption at t=1 (or the relevant outside option in a richer model) is sufficiently low. In this case, the banker will invest in so much bank equity that the banker's participation constraint is binding in the private equilibrium. Table 4 reports the private and planning outcomes for two levels of the bankruptcy cost. As we discuss below, the level of the bankruptcy cost is not important to obtain divergent private and social outcomes, but matters for the kind of regulation that can

[36]Our results are consistent with the analysis in Checchetti and Kashyap (2016), who show that *LCR* and *NSFR* regulations almost surely will never bind at the same time. However, the collinearity of the CR and LevR regulations may be specific to our model. If the bank that could choose between more types of assets with different levels of risk, or to hold off-balance sheet assets, this result may no have obtained – though this would not likely deliver the planner's allocations. Although we can only speculate at this point, we believe that such modifications are important avenues for future research. Other papers that study the use of capital and liquidity requirements include Walther (2016) and Kara and Ozsoy (2016) in the presence of fire sale externalities, Boissay and Collard (2016) when the interbank market cannot efficiently allocate resources, and Van den Heuvel (2017) who quantifies the welfare costs of capital and liquidity requirements in a neoclassical growth model.

decentralize the planner's solution.

As in the general case when the banker's utility exceeds her reservation value, the planner continues to favor a lower run probability. Social welfare improves and the total surplus is higher than in the private equilibrium. Moreover, Pareto improvements are possible, whereby both savers and entrepreneurs are better-off compared to the private equilibrium. Nevertheless, the choice of the asset mix and of the liabilities structure depends on the agent who is favored more and the level of deadweight losses in bankruptcy. The tensions arise because in this equilibrium the banker cannot simultaneously increase both capital and liquidity. The banker is already at her participation constraint in the private equilibrium, asking her to contribute more equity requires a reduction in liquidity so the lower return to equity is counterbalanced by the positive effect of holding fewer liquid assets (and vice versa). The planner cannot require both higher capital and liquidity without violating the banker's participation constraint.

If bankruptcy costs are low, the risk-sharing effect of higher capital is less strong and the planner chooses more liquid assets. As a result, investment goes down and savers enjoy most of the gains from the intervention, while entrepreneurs are slightly better-off for most weights and worse-off for low w_E. In this case, planning outcomes can be fully decentralized with just one liquidity regulation.

If bankruptcy costs are high, raising capital requirements is a more efficient way to improve social welfare and investment goes up. This outcome is equally beneficial for entrepreneurs and savers. Finally, planning outcomes can be fully decentralized with just one capital tool.

For all of the prior results, where the banker enjoys positive economic surplus in the private equilibrium, both a capital tool and a liquidity tool are needed for decentralization irrespective of the level of the bankruptcy cost.

6 Conclusions

Banks perform important services for the real economy using both sides of their balance sheet. However, the private banking equilibria may not be socially optimal and regulating banking activities can improve social welfare. We have examined how many regulations that are often discussed in policy discussions perform in a relatively familiar model of banking. We started from the Diamond and Dybvig (1983) benchmark precisely because it is so thoroughly studied. The modifications that we made trade-off tractability to keep the model relatively simple, against our preference for additional realistic forces that the baseline model excludes.

Our modifications generate endogenous credit risk in banks' portfolios as well as the risk of an endogenous funding run. This simple pair of features interact in interesting and unexpected ways. We draw several general lessons from the model that we believe will carry over to many other models.

First, we identify two general intermediation margins that are distorted, i.e., the relative amounts of liquid and illiquid assets and the mix of deposits and equity. The way that banks privately set these margins diverges from what a social planner would choose, because bankers do not fully internalize the effects of their choices on savers and entrepreneurs. In particular, a social planner chooses relatively more liquidity and equity than the banker. As a result, the planner reduces run risk, improves the provision of liquidity, and guarantees a more stable extension of credit and real production compared to the private equilibrium. These two distortions will be present if we expand the set of assets that banks can invest in or the types of funding sources, but additional ones may also arise.

Second, the two wedges between the private and social choices are not collinear. Thus, more that one regulatory tool is needed to implement the socially optimal allocations. Optimal policy in models without both distortions can be misleading. For example, if the liability structure is constrained, say because deposit levels are exogenously determined and equity is fixed, studying asset allocations and distortions becomes much easier. But, regulation, if any is needed, will amount to fixing liquidity ratios. Similarly, shutting down the liquidity demand and liquidity risk makes it easier to focus on the optimal capital structure and level of investment. But, regulation, if again any is needed, would amount to fixing capital ratios. Instead, when both sides of the bank's balance sheet are endogenously determined the distortions from each side interact and a combination of both capital and liquidity requirements emerge in the optimal regulatory mix.

Third, the political economy aspects of regulation deserve attention. Our bankers internalize how their decisions matter for run risk and choose funding contracts optimally to maximize their own welfare. Their distorted choices, from a social point of view, have real macroeconomic consequences. Regulation improves aggregate welfare, but reduces the rents accruing to bankers. If possible, therefore, banks' incentives to engage in regulatory arbitrage would be strong. The lack of regulatory arbitrage in the model we have studied is one of its main shortcomings. Moreover, regulating capital and/or liquidity is beneficial for both savers and entrepreneurs, but the relative benefits of the type of regulation differ. Savers gain more with liquidity regulation given that it has a bigger effect of liquidity provision, while entrepreneurs gain more with capital regulation given that it allows for more credit extension.

There are other interesting avenues to extend our model, some of which we have already been mentioned and are analyzed in the online appendix. One further direction would be to allow banks to issue long-term debt together with demandable deposits and equity. Including loss-absorbing debt instruments in the regulatory mix could introduce additional ways to tackle with run risk and credit risk. But it would not constitute a full remedy by itself due to the disciplinary role that demandable liabilities play. Moreover, our model is flexible enough to incorporate fire-sale dynamics by endogenizing the liquidation value of long-term investment. Although this would introduce pecuniary externalities as an additional reason why private allocations are inefficient, it would not qualitatively overturn our main conclusions; the asset and liability side distortions would be similar. Finally, one could enrich the set of risky investments from which a banker could choose and, thus, increase the scope for asset substitution. Setting the (relative) risk-weights in capital requirements to capture social risks would be, then, highly important.

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Tables and Figures

$e_1^S = 2.50$	$A_{3g} = 3.40$	$\omega_{3g} = 60\%$	$\alpha = 0.77$
$e_2^S = 0.80$	$A_{3b} = 0.80$	$PB = 0.20$	$\gamma = 0.10$
$e^E = 0.22$	$\overline{\xi} = 1.20$	$\delta = 0.50$	$\rho = 1.00$
$e^B = 0.30$	$\underline{\xi} = 0.01$	$\beta = 0.70$	$c_D = 1\%$

Table 1: Parameterization.

	PE Incomplete Contracts	PE Compr/ve Contracts	SP for weights (w_E, w_S) (0.2,0.6)	(0.3,0.5)	(0.4,0.4)	(0.5,0.3)	(0.6,0.2)
I	0.745	0.745	0.734	0.736	0.739	0.743	0.747
LIQ_1	0.166	0.164	0.276	0.275	0.273	0.271	0.268
D	0.679	0.676	0.778	0.778	0.778	0.777	0.776
E	0.233	0.232	0.232	0.233	0.235	0.237	0.239
CR	0.312	0.312	0.316	0.316	0.317	0.319	0.320
$LevR$	0.255	0.256	0.229	0.230	0.232	0.234	0.236
LCR	0.256	0.253	0.364	0.363	0.361	0.358	0.355
$NSFR$	0.768	0.766	0.846	0.845	0.843	0.842	0.839
r^I	1.796	1.797	1.750	1.749	1.748	1.747	1.746
r_3^D	1.278	1.272	1.549	1.548	1.547	1.545	1.541
q	0.408	0.409	0.369	0.369	0.369	0.369	0.370
q_f	0.187	0.188	0.121	0.122	0.123	0.125	0.127
$Liq.Prov.$	0.949	0.943	1.219	1.218	1.217	1.214	1.210
$\%\Delta\mathbb{U}^E$	-	-0.02%	1.04%	1.05%	1.07%	1.09%	1.11%
$\%\Delta\mathbb{U}^S$	-	-0.07%	3.43%	3.42%	3.41%	3.38%	3.34%
$\%\Delta\mathbb{U}^B$	-	0.02%	-1.45%	-1.45%	-1.45%	-1.45%	-1.45%
$\%\Delta\mathbb{U}^{sp}$	-	-0.03%	1.97%	1.74%	1.50%	1.27%	1.04%
$\%\Delta\mathbb{S}^{sp}$	-	-0.02%	1.00%	1.01%	1.01%	1.01%	1.00%

Table 2: Privately versus Socially Optimal Solutions. The table reports private equilbria under incomplete and comprehensive contracts. The welfare changes are computed over the level of welfare in the private equilibrium with incomplete contracts, which is normalized to one for each agent.

	PE	CR	LevR	LCR	NSFR	SP
I	0.745	0.769	0.751	0.701	0.717	0.739
LIQ_1	0.166	0.246	0.161	0.283	0.282	0.273
D	0.679	0.764	0.676	0.772	0.777	0.778
E	0.233	0.251	0.236	0.212	0.222	0.235
CR	0.312	0.326	0.314	0.303	0.309	0.317
$LevR$	0.255	0.247	0.259	0.216	0.222	0.232
LCR	0.256	0.332	0.249	0.376	0.372	0.361
$NSFR$	0.768	0.823	0.764	0.854	0.851	0.843
r^I	1.796	1.745	1.795	1.765	1.757	1.748
r_3^D	1.278	1.504	1.272	1.536	1.547	1.547
q	0.408	0.374	0.408	0.373	0.370	0.369
q_f	0.187	0.140	0.190	0.115	0.117	0.123
$Liq.Prov.$	0.949	1.173	0.941	1.209	1.219	1.217
$\%\Delta\mathbb{U}^E$	-	1.12%	0.02%	0.70%	0.89%	1.07%
$\%\Delta\mathbb{U}^S$	-	2.87%	-0.07%	3.21%	3.39%	3.41%
$\%\Delta\mathbb{U}^B$	-	-1.45%	-0.01%	-1.45%	-1.45%	-1.45%
$\%\Delta\mathbb{U}^{sp}$	-	1.31%	-0.02%	1.27%	1.42%	1.50%
$\%\Delta\mathbb{S}^{sp}$	-	0.85%	-0.02%	0.82%	0.94%	1.01%

Table 3: Single regulations versus planner's solution for $(w_E, w_S) = (0.4, 0.4)$. Regulation is set at its highest level such that there are gains in social welfare, while agents' participation constraints are satisfied.

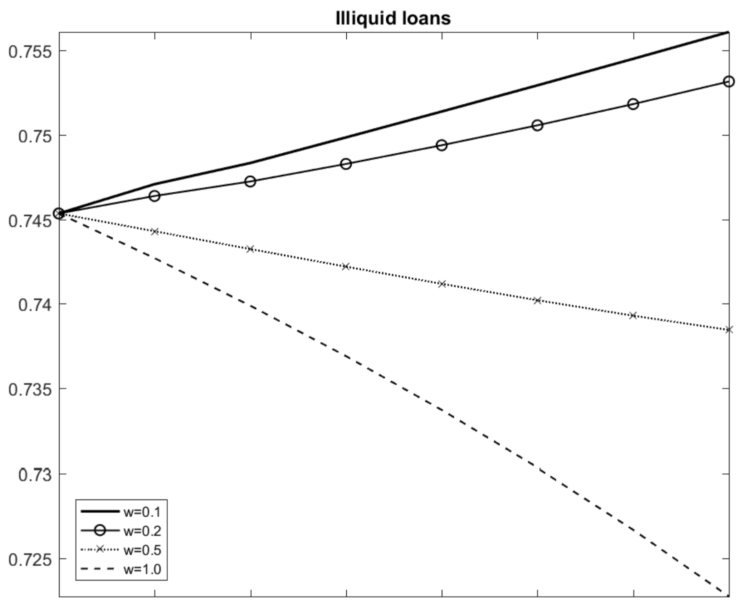

Figure 1: The figure shows the response of credit extension for different levels of the deposit weight in the NSFR. The horizontal axis represents the number of successive times the NSFR is tightened. The first iteration corresponds to the competitive equilibrium level where the tool is not binding.

	$c_D = 1\%$ & $e^B = 0.33$				$c_D = 5\%$ & $e^B = 0.33$			
	PE	SP for weights (w_E, w_S)			PE	SP for weights (w_E, w_S)		
		(0.2,0.6)	(0.4,0.4)	(0.6,0.2)		(0.2,0.6)	(0.4,0.4)	(0.6,0.2)
I	0.827	0.787	0.792	0.798	0.757	0.786	0.789	0.795
LIQ_1	0.154	0.198	0.195	0.191	0.150	0.147	0.145	0.141
D	0.701	0.726	0.726	0.724	0.666	0.675	0.675	0.674
E	0.279	0.259	0.261	0.265	0.241	0.257	0.259	0.263
CR	0.338	0.329	0.330	0.332	0.319	0.328	0.329	0.330
$LevR$	0.285	0.263	0.265	0.268	0.266	0.276	0.278	0.280
LCR	0.231	0.283	0.280	0.275	0.237	0.230	0.227	0.221
$NSFR$	0.762	0.790	0.788	0.786	0.759	0.758	0.756	0.754
r^I	1.755	1.757	1.756	1.754	1.795	1.780	1.779	1.777
r_3^D	1.331	1.400	1.399	1.394	1.295	1.319	1.317	1.313
q	0.396	0.388	0.388	0.389	0.413	0.408	0.408	0.409
q_f	0.192	0.168	0.169	0.172	0.194	0.195	0.196	0.198
Liq.Prov.	0.992	1.069	1.067	1.061	0.914	0.933	0.931	0.926
$\%\Delta\mathbb{U}^E$	-	-0.01%	0.02%	0.05%	-	0.30%	0.32%	0.34%
$\%\Delta\mathbb{U}^S$	-	0.85%	0.83%	0.78%	-	0.28%	0.26%	0.22%
$\%\Delta\mathbb{U}^B$	-	0.00%	0.00%	0.00%	-	0.00%	0.00%	0.00%
$\%\Delta\mathbb{U}^{sp}$	-	0.51%	0.34%	0.19%	-	0.22%	0.23%	0.25%
$\%\Delta\mathbb{S}^{sp}$	-	0.28%	0.28%	0.28%	-	0.19%	0.19%	0.19%

Table 4: Privately versus Socially Optimal Solutions: Zero economic surplus to bankers.

Optimal Bank Regulation
In the Presence of Credit and Run Risk

Anil K Kashyap Dimitrios P. Tsomocos Alexandros P. Vardoulakis

Online Appendix

Appendix A reports derivations and results that were omitted in the main body of the paper. Subsection A.1 provides additional details about the computation of the run threshold in the incomplete information game described in section 2.4. Section A.2 reports the detailed expressions for the intermediation margins in section 3.2. Section A.3 derives the problem of the tools-augmented Ramsey planner described in section 4.5. Section A.4 derives the equilibrium conditions when the bank funds lending with her own funds. Section A.5 presents the problem when savers lend directly to entrepreneurs. Section A.6 reports the planning outcomes when the planner can use other tools to distort the deposit supply and loan demand schedules. Section A.7 presents the privately and socially optimal solutions when interest rates are allowed to take negative values.

Appendix B presents an extension of the model where savers can also purchase equity in the bank and the probability of bankruptcy in the final period conditional of the bank surviving the run is endogenous.

A Additional derivations and computations

A.1 Run threshold

This section provides details about the calculation of the run threshold in equation (32). The utility differential between waiting and withdrawing depends on the expected repayment on deposits, which in turn is a function of the expected repayment on bank loans. Moreover, they both vary as the portion of depositors, λ, varies from δ to θ^*. The entrepreneur always delivers in the good state of the world and the bank is solvent when the portion of depositors withdrawing is δ. However, for given $\xi = \xi^*$, bank profits fall as λ increases and there is a $\tilde{\lambda} \in (\delta, \theta^*)$ such that the bank is insolvent, i.e., $V_{3g}^D(\xi^*, \lambda) < 1$, for $\lambda > \tilde{\lambda}$. The threshold $\tilde{\lambda}$ is calculated as the solution to equation

$$\left(1 - y(\xi^*, \tilde{\lambda})\right) \cdot I \cdot \left(1 + r^I\right) + LIQ_2(\xi^*, \tilde{\lambda}) - \left(1 - \tilde{\lambda}\right) \cdot D \cdot \left(1 + r_3^D\right) = 0. \tag{A.1}$$

The bank is always insolvent in the bad state of the world, but the entrepreneur may not be. The reason is that the entrepreneur's loan obligation decreases as λ increases and the bank recalls

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more loans. As a result, there is a threshold $\hat{\lambda}$ such that the entrepreneur repays fully her remaining loans in the bad state, i.e., $V^I_{3b}(\xi^*,\lambda) = 1$, for $\lambda > \hat{\lambda}$. The threshold $\hat{\lambda}$ is calculated as the solution to equation

$$A_{3b} \cdot F\left[\left(1 - y(\xi^*, \hat{\lambda})\right)\right] - \left(1 - y(\xi^*, \hat{\lambda})\right) \cdot I \cdot \left(1 + r^I\right) = 0. \tag{A.2}$$

Taking into consideration these two thresholds, condition (32) can be written as:

$$\begin{aligned}
&\int_{\delta}^{\tilde{\lambda}} \omega_{3g} D \cdot \left(1 + r_3^D\right) d\lambda + \int_{\tilde{\lambda}}^{\theta^*} \omega_{3g} \frac{(1 - y(\xi^*, \lambda)) \cdot I \cdot \left(1 + r^I\right)}{1 - \lambda} d\lambda \\
&\int_{\delta}^{\hat{\lambda}} \omega_{3b} \frac{A_{3b} \cdot F\left[(1 - y(\xi^*, \lambda)) \cdot I + I^E\right]}{1 - \lambda} d\lambda + \int_{\hat{\lambda}}^{\theta^*} \omega_{3b} \frac{(1 - y(\xi^*, \lambda)) \cdot I \cdot \left(1 + r^I\right)}{1 - \lambda} d\lambda \\
&- \left[\int_{\tilde{\lambda}}^{\theta^*} \omega_{3g} d\lambda + \int_{\delta}^{\theta^*} \omega_{3b} d\lambda\right] \cdot c_D \cdot D \cdot \left(1 + r_3^D\right) - \int_{\delta}^{\theta^*} D \cdot \left(1 + r_2^D\right) d\lambda - \int_{\theta^*}^{1} \frac{\theta^*}{\lambda} D \cdot (1 + r_2^D) d\lambda = 0.
\end{aligned} \tag{A.3}$$

When computing the derivatives of (A.3) with respect to the choice variables, the banker and the planner explicitly consider how they affect the two thresholds $\tilde{\lambda}$ and $\hat{\lambda}$. The respective derivatives are computed by totally differentiating (A.1) and (A.2).

A.2 Intermediation margins

This section presents the detailed expressions for the intermediation margins derived in section 3.2. The approach proceeds by using first-order conditions to solve for and substitute out the Lagrange multipliers, such that the final remaining first-order conditions are only expressed in terms of allocations.

First, use (20) and (24) to express ψ_{GG} and ψ_{DS} in terms of allocations and ψ_{IC} such that:

$$\psi_{GG} = A_{GG} + \Gamma_{GG}\psi_{IC} \tag{A.4}$$

$$\psi_{DS} = A_{DS} + \Gamma_{DS}\psi_{IC} \tag{A.5}$$

where

$$A_{GG} = -\frac{\frac{d\mathbb{U}^B}{dr_3^D}\frac{dDS}{d\xi^*}\mathbb{I}_c - \frac{d\mathbb{U}^B}{d\xi^*}\frac{dDS}{dr_3^D}}{\frac{dGG}{dr_3^D}\frac{dDS}{d\xi^*}\mathbb{I}_c - \frac{dGG}{d\xi^*}\frac{dDS}{dr_3^D}} \quad \text{and} \quad \Gamma_{GG} = -\frac{\frac{dIC}{dr_3^D}\frac{dDS}{d\xi^*}\mathbb{I}_c - \frac{dIC}{d\xi^*}\frac{dDS}{dr_3^D}}{\frac{dGG}{dr_3^D}\frac{dDS}{d\xi^*}\mathbb{I}_c - \frac{dGG}{d\xi^*}\frac{dDS}{dr_3^D}} \tag{A.6}$$

$$A_{DS} = -\frac{\frac{d\mathbb{U}^B}{dr_3^D} + A_{GG}\frac{dGG}{dr_3^D}}{\frac{dDS}{dr_3^D}} \quad \text{and} \quad \Gamma_{DS} = -\frac{\frac{dIC}{dr_3^D} + \Gamma_{GG}\frac{dGG}{dr_3^D}}{\frac{dDS}{dr_3^D}} \tag{A.7}$$

Substitute in (18) and (19) the values for ψ_{BS}, ψ_{GG} and ψ_{DS} from (21), (A.4) and (A.5), respectively, to get the *investment-liquidity* margin in the private equilibrium:

$$\frac{d\mathbb{U}^B}{dI}-\frac{d\mathbb{U}^B}{dLIQ_1}+(1-A_{ILIQ})\left(\frac{d\mathbb{U}^B}{dLIQ_1}+\frac{d\mathbb{U}^B}{dE}\right)$$
$$+A_{GG}\left(\frac{dGG}{dI}-\frac{dGG}{dLIQ_1}A_{ILIQ}\right)+A_{DS}\left(\frac{dDS}{dI}-\frac{dDS}{dLIQ_1}A_{ILIQ}\right)\mathbb{I}_c=0, \tag{A.8}$$

where

$$A_{ILIQ}=\frac{\frac{dIC}{dI}+\frac{dGG}{dI}\Gamma_{GG}+\frac{dDS}{dI}\Gamma_{DS}\mathbb{I}_c}{\frac{dIC}{dLIQ_1}+\frac{dGG}{dLIQ_1}\Gamma_{GG}+\frac{dDS}{dLIQ_1}\Gamma_{DS}\mathbb{I}_c}. \tag{A.9}$$

Similarly, combine (21), (22), (18), (A.4) and (A.5) to get the *equity-deposit* margin in the private equilibrium:

$$\frac{d\mathbb{U}^B}{dE}-\frac{d\mathbb{U}^B}{dD}+A_{ED}\left(\frac{d\mathbb{U}^B}{dI}+\frac{d\mathbb{U}^B}{dE}\right)$$
$$-A_{GG}\left(\frac{dGG}{dD}-\frac{dGG}{dI}A_{ED}\right)-A_{DS}\left(\frac{dDS}{dD}-\frac{dDS}{dI}A_{ED}\mathbb{I}_c\right)=0, \tag{A.10}$$

where

$$A_{ED}=\frac{\frac{dIC}{dD}+\frac{dGG}{dD}\Gamma_{GG}+\frac{dDS}{dD}\Gamma_{DS}}{\frac{dIC}{dI}+\frac{dGG}{dI}\Gamma_{GG}+\frac{dDS}{dI}\Gamma_{DS}\mathbb{I}_c}. \tag{A.11}$$

The same process is followed to derive the investment-liquidity and equity-deposit margins in the planner's solution, which are, respectively given by:

$$\sum_h \overline{w}_h\left(\frac{d\mathbb{U}^h}{dI}-\frac{d\mathbb{U}^h}{dLIQ_1}\right)+(1-\Delta_{ILIQ})\sum_h \overline{w}_h\left(\frac{d\mathbb{U}^h}{dLIQ_1}+\frac{d\mathbb{U}^h}{dE}\right)$$
$$+\Delta_{GG}\left(\frac{dGG}{dI}-\frac{dGG}{dLIQ_1}\Delta_{ILIQ}\right)+\Delta_{DS}\left(\frac{dDS}{dI}-\frac{dDS}{dLIQ_1}\Delta_{ILIQ}\right)+\Delta_{LD}\left(\frac{dLD}{dI}-\frac{dLD}{dLIQ_1}\Delta_{ILIQ}\right)=0, \tag{A.12}$$

and

$$\sum_h \overline{w}_h\left(\frac{d\mathbb{U}^h}{dE}-\frac{d\mathbb{U}^h}{dD}\right)+\Delta_{ED}\sum_h \overline{w}_h\left(\frac{d\mathbb{U}^h}{dI}+\frac{d\mathbb{U}^h}{dE}\right)$$
$$-\Delta_{GG}\left(\frac{dGG}{dD}-\frac{dGG}{dI}\Delta_{ED}\right)-\Delta_{DS}\left(\frac{dDS}{dD}-\frac{dDS}{dI}\Delta_{ED}\right)-\Delta_{LD}\left(\frac{dLD}{dD}-\frac{dLD}{dI}\Delta_{ED}\right)=0, \tag{A.13}$$

where

$$\Delta_{ILIQ}=\frac{\frac{dIC}{dI}+\frac{dGG}{dI}Z_{GG}+\frac{dDS}{dI}Z_{DS}+\frac{dLD}{dI}Z_{LD}}{\frac{dIC}{dLIQ_1}+\frac{dGG}{dLIQ_1}Z_{GG}+\frac{dDS}{dLIQ_1}Z_{DS}+\frac{dLD}{dLIQ_1}Z_{LD}}, \tag{A.14}$$

$$\Delta_{ED} = \frac{\frac{dIC}{dD} + \frac{dGG}{dD} Z_{GG} + \frac{dDS}{dD} Z_{DS} + \frac{dLD}{dD} Z_{LD}}{\frac{dIC}{dI} + \frac{dGG}{dI} Z_{GG} + \frac{dDS}{dI} Z_{DS} + \frac{dLD}{dI} Z_{LD}}, \tag{A.15}$$

$$\Delta_{GG} = -\frac{\sum_h \overline{w}_h \left[\frac{dU^h}{dr_3^D} \frac{dDS}{d\xi^*} - \frac{dU^h}{d\xi^*} \frac{dDS}{dr_3^D} + \left(\frac{dLD}{dr^I}\right)^{-1} \frac{dU^h}{dr^I} \frac{dLD}{d\xi^*} \frac{dDS}{dr_3^D} \right]}{\frac{dGG}{dr_3^D} \frac{dDS}{d\xi^*} - \frac{dGG}{d\xi^*} \frac{dDS}{dr_3^D} + \left(\frac{dLD}{dr^I}\right)^{-1} \frac{dLD}{d\xi^*} \frac{dGG}{dr^I} \frac{dDS}{dr_3^D}}, \tag{A.16}$$

$$Z_{GG} = -\frac{\frac{dIC}{dr_3^D} \frac{dDS}{d\xi^*} - \frac{dIC}{d\xi^*} \frac{dDS}{dr_3^D} + \left(\frac{dLD}{dr^I}\right)^{-1} \frac{dLD}{d\xi^*} \frac{dIC}{dr^I} \frac{DS}{dr_3^D}}{\frac{dGG}{dr_3^D} \frac{dDS}{d\xi^*} - \frac{dGG}{d\xi^*} \frac{dDS}{dr_3^D} + \left(\frac{dLD}{dr^I}\right)^{-1} \frac{dLD}{d\xi^*} \frac{dGG}{dr^I} \frac{dDS}{dr_3^D}}, \tag{A.17}$$

$$\Delta_{DS} = -\frac{\sum_h \overline{w}_h \frac{dU^h}{dr_3^D} + \Delta_{GG} \frac{dGG}{dr_3^D}}{\frac{dDS}{dr_3^D}} \qquad \text{and} \qquad Z_{DS} = -\frac{\frac{dIC}{dr_3^D} + Z_{GG} \frac{dGG}{dr_3^D}}{\frac{dDS}{dr_3^D}}, \tag{A.18}$$

$$\Delta_{LD} = -\frac{\sum_h \overline{w}_h \frac{dU^h}{dr^I} + \Delta_{GG} \frac{dGG}{dr^I}}{\frac{dLD}{dr^I}} \qquad \text{and} \qquad Z_{LD} = -\frac{\frac{dIC}{dr^I} + Z_{GG} \frac{dGG}{dr^I}}{\frac{dLD}{dr^I}}. \tag{A.19}$$

As discussed in the paper, the private intermediation margins differ from the planner's in a number of ways. Most importantly, the banker does not care how her choices directly affect the utility of savers and entrepreneurs. Thus, additional terms enter into the planner's solution, which capture the direct total effect of the banking choices governing credit and run risk on savers' and entrepereneurs' welfare. For example, the derivatives $dU^j/d\xi^*$, $j = S, E$, are present in (A.12) and (A.13), but not in (A.8) and (A.10). These derivatives introduce a wedge between the private and social intermediation margins. Additionally, the banker is protected by limited liability and will not internalize all effects when contracts are incomplete, i.e., $\mathbb{I}_c = 0$ in (A.12) and (A.13) contrary to the planner's intermediation margins where the relevant terms are always present.

A.3 Tools-augmented planner

In this section we specify the problem of the tools-augmented planner and show that (47) is (generically) a necessary and sufficient condition such that the social planner's solution described in section 3.1 can be decentralized as a private equilibrium by using regulatory tools $T \in \mathbb{T}$. The tools-augmented planner not only chooses optimally allocations and prices $X \in \mathbb{X}$, but also the level of tools $T \in \mathbb{T}$ and the multipliers ψ_T, which are the shadow values that the bank assigns to constraints $RC(T,X) \geq 0$ in the new equilibrium. Her problem is:

$$\max_{\mathbb{X},\mathbb{T},\psi_T} \quad \mathbb{U}^{sp}$$

$$s.t. \;\; \mathbb{B}(X) = 0, \;\; RC(T,X) \geq 0, \;\; ILIQ_{\mathbb{T}}(T,X,\Psi_T) = 0, ID_{\mathbb{T}}(T,X,\Psi_T) = 0, \tag{A.20}$$

where $ILIQ_{\mathbb{T}}$ and $ID_{\mathbb{T}}$ are the regulation-distorted margins given by (45) and (46).

The first-order condition with respect to X (similar to first-order condition (35)) are:

$$\sum_{h=\{E,R,B\}} \overline{w}_h \frac{d\mathbb{U}^h}{dX} + \zeta_{BS}\frac{dBS}{dX} + \zeta_{IC}\frac{dIC}{dX} + \zeta_{GG}\frac{dGG}{dX} + \zeta_{LD}\frac{dLD}{dX} + \zeta_{DS}\frac{dDS}{dX}$$
$$+ \sum_T \zeta_T \frac{dRC}{dX} + \zeta_{ILIQ}\frac{dILIQ_{\mathbb{T}}}{dX} + \zeta_{ID}\frac{dID_{\mathbb{T}}}{dX} = 0, \tag{A.21}$$

where ζ_T, ζ_{ILIQ} and ζ_{ID} are the multipliers the tool-augmented planner assigns to regulatory constraints $RC(T,X)$ and the three regulation-distorted intermediation margins.

The first-order conditions with respect to the level of tools T are:

$$\zeta_T \frac{dRC}{dT} + \zeta_{ILIQ}\frac{dILIQ_{\mathbb{T}}}{dT} + \zeta_{ID}\frac{dID_{\mathbb{T}}}{dT} = 0, \tag{A.22}$$

and choosing optimally the multipliers ψ_T yields:

$$\zeta_{ILIQ}\frac{dILIQ_{\mathbb{T}}}{d\psi_T} + \zeta_{ID}\frac{dID_{\mathbb{T}}}{d\psi_T} = 0. \tag{A.23}$$

To prove sufficiency, (47) implies that there need to be two regulatory tools such that a solution to multipliers ψ_T can be obtained. In turn, this means that there are two first-order conditions of the form in (A.22) and two of the form in (A.23). Conditions (A.23) can be written in matrix form as $transpose(\Delta RC) \cdot transpose([\zeta_{ILIQ} \ \ \zeta_{\mathrm{ID}}]) = 0$. Given that ΔRC is invertible, $\zeta_{ILIQ} = \zeta_{ID} = 0$. Thus, the only solution is one where all ζ_T, ζ_{ILIQ} and ζ_{ED} are zero and the first-order conditions (A.21) coincide with the first-order conditions (35) of the social planner.

To prove necessity, suppose that (47) does not hold or in other words the span of ΔRC is less than two. Using conditions (A.21) we can derive intermediation margins $ILIQ_{tap} = ILIQ_{sp} + ILIQ_{tap}^{wedge}$ and $ID_{tap} = ID_{sp} + ID_{tap}^{wedge}$ for the tool-augmented planner, where the wedges are linear combination of one multiplier ζ_T, ζ_{ILIQ} and ζ_{ID}. The social planner's and tools-augmented planner's solutions coincide if both wedges are zero, which in principle is possible because there are three multipliers, hence three degrees of freedom. However, equations (A.22) and (A.23) remove two degrees of freedom. Hence, it is not possible to replicate the social planner's solution with fewer that two independent tools, which leads to a contradiction.

A.4 Equilibrium without deposit intermediation

The participation constraint (25) of the banker supposes that utility in autarky is the outside option. As already mentioned, we could consider the utility that the banker obtains by lending to entrepreneurs using only her own funds as an outside option. This section derives the conditions for the alternative outside option and shows that for the equilibrium considered in section 3.3 the autarkic utility is the relevant outside option.

The utility of the banker who lends to the entrepreneur only using her own capital and not taking deposits is

$$\mathbb{U}^{B,n} = \gamma \cdot U\left(e^B - I^n\right) + \sum_s \omega_{3s} V_{3s}^{I,n} I^n \left(1 + r^{I,n}\right), \tag{A.24}$$

where I^n is the loan to E, $r^{I,n}$ the loan rate, and $V_{3s}^{I,n} = \min\left(1, A_{3s}F(I^E + I^n)/(I^n(1 + r^{I,n}))\right)$ the percentage repayment on the loan. The optimal choice of I^n yields:

$$-\gamma \cdot U'\left(e^B - I^n\right) + \omega_{3g}\left(1 + r^{I,n}\right) + \omega_{3b} F'\left(I^E + I^n\right) = 0, \tag{A.25}$$

considering that E defaults in the bad state of the world.

An individual entrepreneur chooses a loan rate and loan amount that satisfy the loan supply by the banker (A.25) to maximize her utility given by:

$$\mathbb{U}^{E,n} = \sum_s \omega_{3s}\left[A_{3g}F\left(I^E + I^n\right) - I^n\left(1 + r^{I,n}\right)\right]^+. \tag{A.26}$$

Given that an individual entrepreneur does not internalize how her loan demand affects the shadow cost of funds for the banker, i.e., the first term in (A.25), but does internalize how it affects the repayment in default, $E's$ loan demand schedule is given by:

$$\omega_{3g}\left[A_{3g}F'\left(I^E + I^n\right) - \left(1 + r^{I,n}\right)\right] + \omega_{3b} I^n A_{3b} F''\left(I^E + I^n\right) = 0. \tag{A.27}$$

Conditions (A.25) and (A.27) yield a solution for the loan amount and the loan rate. Table A.1 below compares the private equilibria when banks intermediate deposits and when they do not. The percentage change in welfare for E and B is calculated over the utility level in autarky (normalized to one). The participation constraint of entrepreneurs is violated when the banker do not raise deposits to lower lending rates.

	Loan rate	Loan amount	$\mathbb{E}V_{3b}^I$	$\%\Delta\mathbb{U}^E$
Intermediation	1.796	0.745	0.454	0.55%
No intermediation	2.014	0.276	0.560	-4.57%

Table A.1: Private equilibrium solutions under deposit and no deposit intermediation.

A.5 Direct lending

This section derives the conditions for direct lending to entrepreneurs by savers and computes the equilibrium outcomes for the parameterization in section 3.3.

Direct lending requires the individual savers to be able to monitor the entrepreneur. Denote by MC the monitoring cost to an individual saver, which can be higher or equal to the cost for the banker, i.e., her private benefit. At $t = 1$, an individual saver can invest in the liquid asset, LIQ^{dl}, or lend to the entrepreneur, I^{dl}, at interest rate $r^{I,dl}$. In the intermediate period, she would liquidate all her loans if she turns out to be impatient. Otherwise, the saver waits until the final period and receives the percentage repayment on the loans she made. Her utility under direct lending is given by:

$$U^{S,dl} = U_1\left(c_1^{dl}\right) + \delta \int_{\underline{\xi}}^{\overline{\xi}} U_2\left(c_2^{dl}; i\right) \frac{d\xi}{\Delta_\xi} + (1-\delta)\sum_s \omega_{3s} U_3\left(c_3^{dl}; p\right),$$

where $c_1^{dl} = e_1^S - I^{dl} - LIQ^{dl}$, $c_2^{dl} = e_2^S + LIQ^{dl} + \xi \cdot I^{dl}$, and $c_3^{dl} = e_2^S + LIQ^{dl} + (V_{3s}^{I,dl} - c_D \cdot \mathbb{I}_{dl}) \cdot I^{dl} \cdot (1 + r^{I,dl}) - MC$. Moreover, $V_{3s}^{I,dl} = min[1, A_{3s} \cdot F(I^E + I^{dl})/(I^{dl} \cdot (1 + r^{I,dl}))]$ is the percentage repayment on the loan and $\mathbb{I}_{dl}$ is the indicator function for default.

Under the assumption that an individual saver lends to an individual entrepreneur, the former will internalize how her loan extension affects the expected delivery in default (much like the banker does). Hence, the optimal choice of lending, I^{dl}, yields:

$$-U_1'\left(c_1^{dl}\right) + \delta \int_{\underline{\xi}}^{\overline{\xi}} \xi \cdot U_2'\left(c_2^{dl}; i\right) \frac{d\xi}{\Delta_\xi} + \beta^2(1-\delta) \cdot \left[\omega_{3g}\left(1 + r^{dl}\right) + \omega_{3b}\left(A_{3b}F'\left(I^E + I^{dl}\right) - c_D\right)\right] = 0 \tag{A.28}$$

where we have used the facts that $U_3'(\cdot; p) = \beta^2$ and that the entrepreneur would default, in equilibrium, in the bad state.

Similarly, the optimal choice of liquid holdings, LIQ^{dl}, yields:

$$-U_1'\left(c_1^{dl}\right) + \delta \int_{\underline{\xi}}^{\overline{\xi}} U_2'\left(c_2^{dl}; i\right) \frac{d\xi}{\Delta_\xi} + \beta^2(1-\delta) + \nu_{LIQ^{DL}} = 0, \tag{A.29}$$

where $\nu_{LIQ^{DL}}$ is the Lagrange multiplier on the constraint $LIQ^{DL} \geq 0$.

The utility of an individual entrepreneur

$$U^{E,dl} = \delta \cdot U^{E,aut} + (1-\delta) \cdot \omega_{3g}\left[A_{3g}F\left(e^E + I^{dl}\right) - I^{dl} \cdot \left(1 + r^{dl}\right)\right], \tag{A.30}$$

given that E invests all of her wealth in the project, i.e., $I^E = e^E$. With probability δ an individual entrepreneur has her project liquidated and continues to produce only with her own capital. As a result, she enjoys the same utility as in autarky. With probability $1-\delta$, the saver does not liquidate the project and the entrepreneur defaults in that bad state. The entrepreneur chooses the loan amount, I^{dl}, and the loan rate, r^{dl}, that satisfy (A.28) to maximize (A.30). Consistent with our analysis in the rest of the paper, the entrepreneur internalizes her effect on the marginal payoff accruing to the saver, but takes the other forces determining saver's costs of funds (marginal utilities at $t = 1$ and $t = 2$) as given. Thus, the optimal loan demand by the entrepreneur is:

$$\omega_{3g}\left[A_{3g}F'\left(e^E + I^{dl}\right) - \left(1 + r^{dl}\right)\right] + \omega_{3b}A_{3b}F''\left(e^E + I^{dl}\right)I^{dl} = 0. \tag{A.31}$$

Conditions (A.28), (A.29) and (A.31) jointly determine I^{dl}, LIQ^{dl} and r^{dl} in equilibrium. Using the parameterization in section 3.3 and setting MC equal to PB, the utility of savers is 0.07% higher under bank intermediation compared to direct lending which, in turn, is higher than the utility in autarky. By increasing MC we can obtain equilbria where direct lending delivers lower utility to savers and eventually is dominated by autarky. In addition, the utility of entrepreneurs is higher than in autarky, thus they are willing to borrow directly from savers.

A.6 Additional distortionary tools

This section extends the analysis in section 3.3 by allowing the planner to use tools to distort the deposit supply and loan demand schedules of savers and entrepreneurs. We consider generic tools, τ_{DS} for the deposit supply schedule, and τ_{LD} for the loan demand schedule, and discuss how they can be implemented in practice.

The deposit supply schedule (7) that the planner faces becomes:

$$-U_1'(c_1)+\left(1+r_2^D\right)\left[\sum_{t=2,3}\left\{\int_{\underline{\xi}}^{\xi^*}\theta(\xi,1)\cdot\mathbb{E}_jU_t'(c_{ts}(j,1);j)\frac{d\xi}{\Delta_\xi}\right\}+\delta\int_{\xi^*}^{\overline{\xi}}U_2'(c_2(i,1);i)\frac{d\xi}{\Delta_\xi}\right]$$
$$+(1-\delta)\int_{\xi^*}^{\overline{\xi}}\sum_s\omega_{3s}U_3'(c_{3s}(p,0);p)\cdot\left(V_{3s}^D(\xi,\delta)-c_D\cdot\mathbb{I}_d\right)\cdot(1+r_3^D)\frac{d\xi}{\Delta_\xi}+\nu_D+\tau_{DS}=0. \quad \text{(A.32)}$$

To the extent that savers supply deposits, i.e., $\nu_D=0$, the planner can distort their willingness to hold deposits at given deposit rates by varying the level of the distortionary tool τ_{DS}. In other words, the planner can set $\tau_{DS}\neq 0$, which implies that (A.32) stops being a constraint in her optimization problem (33) and $\zeta_{DS}=0$ in (35). The intervention can be implemented, for example, either as a tax on the supply of deposits at $t=1$ or as a tax on the interest income accruing to late depositors at $t=3$ when the bank is solvent. In the first case, the tax can be computed as $-\tau_{DS}/U_1'(c_1)$, while in the second as $-\tau_{DS}/\left(\omega_{3g}(1-q)\cdot(1-\delta)\cdot U_3'(c_{3g}(p,0);p)\cdot(1+r_3^D)\right)$. If $\tau_{DS}<0$, then a tax is levied, while $\tau_{DS}>0$ implies a subsidy. We assume that the planner rebates the tax proceeds back to the same agents in the same period in a lump-sum fashion in order to neutralize any income effects.

Similarly, the loan demand schedule (27) becomes:

$$\int_{\xi^*}^{\overline{\xi}}(1-y(\xi,\delta))\left[A_{3g}F'\left(I^E+(1-y(\xi,\delta))\cdot I\right)-(1+r^I)+I\cdot\frac{\partial LS}{\partial I}\left(\frac{\partial LS}{\partial r^I}\right)^{-1}\right]\frac{d\xi}{\Delta_\xi}+\tau_{LD}=0. \quad \text{(A.33)}$$

The planner can distort the willingness of entrepreneurs to borrow by varying the level of the distortionary tool τ_{LD}. such that if $\tau_{LD}\neq 0$, then $\zeta_{LD}=0$ in (35). The intervention can be implemented with a tax on loan repayment in the good state of the world, which can be computed as $-\tau_{LD}/\left(\int_{\xi^*}^{\overline{\xi}}(1-y(\xi,\delta))(1+r^I)d\xi/\Delta_\xi\right)$. If $\tau_{LS}<0$, then a tax is levied, while $\tau_{LS}>0$ implies a subsidy. A tax can also be implemented with restrictions on the maximum loan-to-value ratio for entrepreneurial loans, i.e., $I \leq \overline{LTV}/(1-\overline{LTV}) \cdot I^E$ where $\overline{LTV}$ is the loan-to-value limit. Then, τ_{LD} is the value of the Lagrange multiplier on the LTV constraint. Note that this limit is imposed on the entrepreneur rather than the banker, because the objective is to distort the loan demand schedule.

Table A.2 below reports the planning equilibria under two sets of weights when distortionary tools are available (using the parameterization discussed in section 3.3). Comparing the planning outcomes with and without distortionary tools, we can observe that the planner can improve social gains if she is endowed with more tools. The reason is that both savers and entrepreneurs do not internalize how their behavior affects the aggregate bank variables, and most importantly the probability of a run.

Nevertheless, there are three important observations about this extension of the model. First, banking regulation is still needed to implement socially optimal outcomes. The additional distortionary tools affect the deposit supply and loan demand schedules, but do not correct for the distortions in the banker's optimization condition. Capital and liquidity regulation are required for the latter. Second, the use of the distortionary tools has implications for the allocation of social gains. While the banker remains at her participation constraint, either the saver or the entrepreneur can be made better-off when these tools are used compared to the social planning outcomes without them. Third, a tax that restricts the supply of deposits can be beneficial for savers. In particular, the bank has to offer higher deposit rates to attact deposits and the smaller reliance on deposits in combination with capital and liquidity regulations improves the bank's stability. Liquidity provision is lower, but this does not hurt savers overall because they are able to self-insure by holding the liquid asset. The benefits are smaller when savers are not allowed to self-insure. Moreover, entrepreneurs are worse-off because the level of funds channeled through the bank goes down and they are driven to their participation constraint. As a result, τ_{LD} cannot be combined in this example with τ_{LD} because there are no additional social gains to be made.

A.7 Negative interest rates

This section relaxes the assumption about the non-negativity of the early deposit rate, r_2^D, and shows that our conclusions about the necessity for capital and liquidity regulations carry over. Table A.3 reports the private and socially optimal outcomes for negative r_2^D.

Negative early deposit rates reduce the probability of a run, since both the savers' incentive to run and the bank's liquidity needs are lower. The banker will weigh the reduction in the run probability to the potential increase in late deposit rates when choosing to set early deposit rates negative. However, the banker is not able to decrease r_2^D all the way to the level that the probability of a run is zero, because she would either need to offer very high r_3^D, which eliminates her own profits, or violate the participation constraint of savers. In the private equilibrium in Table A.3 savers are driven to their participation contraint (9). So merely allowing for negative rates does not allow the private sector to deliver run-free banking.

The planner can reduce the early deposit rate all the way to the point that runs are ruled out.

	PE	SP for (w_E, w_S)=(0.4,0.4)			SP for (w_E, w_S)=(0.6,0.2)		
		No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$	No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$
I	0.745	0.739	0.550	0.744	0.747	0.550	0.724
LIQ_1	0.166	0.273	0.221	0.286	0.268	0.221	0.192
D	0.679	0.778	0.510	0.794	0.776	0.510	0.685
E	0.233	0.235	0.261	0.236	0.239	0.261	0.231
CR	0.312	0.317	0.474	0.317	0.320	0.474	0.320
$LevR$	0.255	0.232	0.338	0.229	0.236	0.338	0.253
LCR	0.256	0.361	0.444	0.369	0.355	0.444	0.291
$NSFR$	0.768	0.843	0.938	0.851	0.839	0.938	0.793
r^I	1.796	1.748	1.828	1.774	1.746	1.828	1.638
r_3^D	1.278	1.547	2.321	1.609	1.541	2.319	1.215
q	0.408	0.369	0.207	0.364	0.370	0.207	0.399
q_f	0.187	0.123	0.043	0.117	0.127	0.044	0.166
$Liq.Prov.$	0.949	1.217	0.933	1.254	1.210	0.933	0.987
$\%\Delta\mathbb{U}^E$	-	1.07%	-0.55%	0.49%	1.11%	-0.54%	3.07%
$\%\Delta\mathbb{U}^S$	-	3.41%	13.73%	4.01%	3.34%	13.72%	0.25%
$\%\Delta\mathbb{U}^B$	-	-1.45%	-1.45%	-1.45%	-1.45%	-1.45%	-1.45%
$\%\Delta\mathbb{U}^{sp}$	-	1.50%	4.99%	1.51%	1.04%	2.13%	1.60%
$\%\Delta\mathbb{S}^{sp}$	-	1.01%	3.91%	1.02%	1.00%	3.91%	0.62%
τ_{DS}	-	-	-0.321	-	-	-0.321	-
τ_{LD}	-	-	-	0.017	-	-	-0.068

Table A.2: Privately versus Socially Optimal Solution when additional distortionary tools are available.

Doing so requires the liquidation value of the bank's assets to exceed the total value of runnable liabilities for any realization of the liquidation value, i.e., $(LIQ_1 + \underline{\xi} \cdot I)/(D(1+r_2^D)) \geq 1$. This is exactly the condition that LCR must equal 1 in (42). Any excess liquidity on top of what is needed to serve early withdrawals would then be carried over to the final period using the storage technology, i.e., $LIQ_2 = LIQ_1 - \delta \cdot D(1+r_2^D)$. For the planning equilibrium reported in the last column in Table A.3 the planner does not carry over excess liquidity, because she is able to eliminate runs by driving the early deposit rate very negative. As a result, the liquidity the planner needs to hold is small, yet the LCR goes to its highest level. Although liquidity provision is lower, the saver gains further from the reduction in the run probability. And most of the gains accrue to the entrepreneur, since the lower amount of liquidity needed to control run risk allows for more investment. The further increase in the late deposit rate and the decrease in the loan rate, makes it more difficult to raise equity from the banker without violating her participation constraint.[1] The social planner's allocations force the banker to invest in more equity than she would do voluntarily. Hence, to decentralize this allocation, capital regulation would also be needed. Therefore just like in the baseline model in the body of the paper, the private equilibrium is inefficient and one capital and one liquidity regulation

[1] Keep in mind that all the utility levels in the table are normalized to one in the private equilibrium. The banker's utility skyrockets when negative rates are allowed. So the large drop for the social planner's allocations come because the starting point for the banker is so favorable.

is required to match the social planner's allocations.

	PE	SP
I	0.870	1.492
LIQ_1	0.123	0.015
D	0.742	1.343
E	0.251	0.164
CR	0.288	0.110
$LevR$	0.253	0.109
LCR	0.282	1.000
$NSFR$	0.715	0.560
r^I	1.675	1.313
r_2^D	-0.370	-0.976
r_3^D	1.030	3.644
q	0.193	0.000
q_f	0.099	0.000
Liq.Prov.	0.946	0.724
$\%\Delta\mathbb{U}^E$	-	23.74%
$\%\Delta\mathbb{U}^S$	-	29.14%
$\%\Delta\mathbb{U}^B$	-	-27.01%
$\%\Delta\mathbb{U}^{sp}$	-	15.75%
$\%\Delta\mathbb{S}^{sp}$	-	8.63%

Table A.3: Privately versus Socially Optimal Solutions for $r_2^D < 0$. The planning outcomes are for weights $(w_E, w_S) = (0.4, 0.4)$. We have added a fixed number (equal to 1) to the utility of impatient depositors, because it takes negative values for $q = 0$ as early consumption, $c_2(i)$, is less than 1. This does not affect marginal decisions and equilibrium outcomes, but it allows the easy comparison of the *Liq.Prov.* ratio across equilibria, which would otherwise have a negative value for $q = 0$.

B Extended model

This section extends the baseline model so that savers can also purchase equity in the bank and the probability of bankruptcy in the final period conditional on the bank surviving the run is endogenous.

The first modification implies that the banker and the planner have an alternative source of funding apart from the equity contributed by the banker and deposits offered by savers. We will refer to equity contributed by bankers and savers as “inside” and “outside” equity, respectively. The introduction of an additional source of funding adds another intermediation margin for banking decisions. We show that this margin is also distorted and that a planner would need an additional tool on top of a capital and a liquidity regulation to fully implement a solution with positive outside equity.

The second modification allows us to examine how regulation differentially affects run risk and credit risk. To do so, we introduce a third "medium" state for the realization of the productivity shock in the final period, which is between the level in the good and the bad state. Thus, the state space at $t = 3$ is $s \in \{g, m, b\}$ and the productivity realization satisfy $A_{3g} > A_{3m} > A_{3b}$. We focus on cases in which entrepreneurs default in states m and b, while they fully repay in state g. The bank is solvent is state g and defaults in state b, while the bankruptcy decision depends on the realization of ξ in state m. Hence, there is a threshold $\hat{\xi} \in \left(\xi^*, \overline{\xi}\right)$ such that the bank is solvent in state m only if the realization of the liquidation value is higher than $\hat{\xi}$. The threshold is endogenous and depends on the balance sheet of the bank. Thus, it plays a critical role in the expected probability of bank default and the benefit of raising equity to reduce expected bankruptcy costs.[2] We also consider a general specification for bankruptcy costs and introduce investment adjustment costs for entrepreneurs when their loans are recalled and investment liquidated.

These modifications allow us to study equilibria where the planner chooses positive outside equity and there is room for redistributive effects of regulation. To avoid repeating ourselves, we only present the equations where these modifications enter.

B.1 Modified savers’ problem

As in the baseline model, savers invest in bank deposits and the liquid asset at $t = 1$ to maximize their lifetime expected utility (6). But, they can additionally buy bank (outside) equity shares, E^S, in a primary market at a price P per share. Equity is valuable because of the dividends paid on each share, $DPS_{3s}(\xi, \lambda)$, at $t = 3$. Recall that in the baseline model we did not distinguish between bank profits and dividends per share given that the banker is the sole equity-holder. We will be precise

[2] Having only three levels of productivity does not change the fundamental economic outcomes in the extended model. In particular, even with many more outcomes for technology there are fundamentally only three different types of outcomes. For some realizations of productivity the resources are sufficient so that the loans are fully paid and in this case the deposits are also fully paid. Conversely, it is possible that the investment outcome is so poor that the loan repayment is so low that the depositors can never be fully paid. Finally, there are interim cases where the loans may not be completely paid, but the bank still can fully pay deposits. Nothing in the analysis would change if we had many more states.

about how the dividends per share are determined in the modified banker's problem below. As a result, the budget constraint at $t = 1$ –equation (1)– becomes:

$$c_1 = e_1^S - D - P \cdot E^S - LIQ_1^S. \tag{B.1}$$

Each share can be re-traded in a secondary market as a price $P_{sec}(\xi,\lambda)$. In a run, equity is worthless, i.e., $P_{sec}(\xi,1) = 0$ and $DPS_{3s}(\xi,1) = 0$ because the bank is liquidated. Patient savers will enter the secondary market to buy equity from impatient savers. The patient savers' total funds are the sum of their new endowment, e_2^S, and their liquid holdings carried over from the first period, LIQ_1^S. The patient savers total equity holdings after trading are $E_{sec}^S(\xi,\lambda)$. Thus, the net purchase is $P_{sec}(\xi,\delta)\cdot(E_{sec}^S(\xi,\lambda) - E^S)$ and the remaining resources are transferred to $t = 3$ using the storage technology. Conditional on a run ocurring, the consumption of a saver of type j is still given by (2). However, the consumption of an impatient saver when a run does not occur –equation (3)– is now given by:

$$c_2(i,\mathbb{I}_w = 1) = D\left(1 + r_2^D\right) + LIQ_1^S + P_{sec}(\xi,\lambda) + e_2^S. \tag{B.2}$$

Similarly, the consumption at $t = 3$ of a patient saver who chooses to wait or withdraw –equations (4) and (5) respectively– will be given by[3]

$$\begin{aligned} c_{3s}(p,\mathbb{I}_w = 0) = {} & E_{sec}^S(\xi,\lambda,\mathbb{I}_w = 0)\, DPS_{3s}(\xi,\lambda) + P_{sec}(\xi,\lambda)\left(E^S - E_{sec}^S(\xi,\lambda,\mathbb{I}_w = 0)\right) \\ & + \left(V_{3s}^D(\xi,\lambda) - c_D(D)\cdot\mathbb{I}_d\right) D\left(1 + r_3^D\right) + LIQ_1^S + e_2^S, \end{aligned} \tag{B.3}$$

or

$$\begin{aligned} c_{3s}(p,\mathbb{I}_w = 1) = {} & E_{sec}^S(\xi,\lambda,\mathbb{I}_w = 1)\, DPS_{3s}(\xi,\lambda) + P_{sec}(\xi,\lambda)\left(E^S - E_{sec}^S(\xi,\lambda,\mathbb{I}_w = 1)\right) \\ & + D\left(1 + r_2^D\right) + LIQ_1^S + e_2^S. \end{aligned} \tag{B.4}$$

Since the individual saver takes the bank dividends as given, the optimal decision to purchase equity, E^S, in the primary market is chosen so that

$$ES : -P\cdot U_1'(c_1) + \overbrace{\sum_{t=2,3}\left\{\int_{\xi^*}^{\bar{\xi}} \mathbb{E}_j U_t'(c_{ts}(j,\mathbb{I}_w);j)\cdot P_{sec}(\xi,\delta)\frac{1}{\Delta_\xi}d\xi\right\}}^{\text{no run}} + \nu_{E^S} = 0, \tag{B.5}$$

[3] In the extended model we consider a more general function for the bankruptcy costs given by $c_D(D) = c_D \cdot D^{\phi_D}$, $\phi_D \geq 0$. The more general specification can enhance the risk-sharing role of equity because different allocations imply different marginal bankruptcy costs. Savers' take $c_D(D)$ as given since it is a function of the total deposits in the bank. Hence, the deposit supply equation (7) has the same functional form in the extended model with the difference that the marginal cost depends on D in equilibrium. The banker and the planner account for this dependance.

where ES stands for *equity supply* and ν_{ES} is the multiplier on the no short-sale constraint $E^S \geq 0$. Condition (B.5) says that savers equate the marginal utility of lost consumption from buying one bank share at price P to the expected marginal utility gain from the value of the share in the future, $P_{sec}(\xi, \delta)$. The share only has any value if the bank survives a run, since otherwise equity is worthless.

The value of equity that emerges from the secondary market trading satisfies

$$P_{sec}(\xi, \delta) = \sum_s \omega_{3s} DPS_{3s}(\xi, \delta), \tag{B.6}$$

i.e., the secondary equity price is equal to the expected value of future dividends because patient savers have linear utility at $t = 3$ and their outside option pays zero interest. If a run does not occur, impatient savers sell their bank shares to patient savers. Market clearing requires the equity holdings of each individual patient saver at $t = 3$ be such that $E^S_{sec}(\xi, \delta) = E^S/(1-\delta)$.

Before turning to the modified problems of the banker and the entrepreneur, it is easy to show that the global game analysis in section 2.4 remains intact (once we account for the additional state m). The reason is that because of quasi-linear utilities the expected utility differential between waiting and withdrawing conditional on the bank surviving the run –upper part in (31)– is not affected by the decision to purchase outside equity. To be more precise, the expected utility differential is the difference in expected consumption in (B.3) and expected consumption in (B.4), which differ in two ways. One arises from the different equity holding after secondary trading for a saver that waits and a saver that withdraws, and the other comes because the person who waits will receive a late deposit payment, while the other person will get her deposits early and transfer them to period 3 using the liquid asset. However, the demand for equity in the secondary market determines the secondary equity price $P_{sec}(\xi, \lambda) = \sum_s \omega_{3s} DPS_{3s}(\xi, \lambda)$. Substituting the secondary price in (B.3) and (B.4), the expected utility differential between waiting and withdrawing conditional on the bank surviving the run is $\sum_s \{\omega_{3s}(V^D_{3s}(\xi, \lambda) - c_D(D) \cdot \mathbb{I}_d) D\left(1 + r^D_3\right)\} - D\left(1 + r^D_2\right)$.[4]

B.2 Modified banker's problem

The banker makes the same decisions as in the baseline model, but additionally needs to decide how much outside equity to raise from savers. The equity shares in the bank will be split between the banker and savers and the respective holdings are denoted by E^B and E^S. At this point we distinguish between the initial equity, E^B_0, that the banker holds, and the additional equity, E^B_1, that

[4]The linearity of utility from consumption at $t = 3$ simplifies the run decision substantially since the expected utility differential between waiting and withdrawing, given by equation (31), depends only on predetermined variables and not on actions taken after the run decision, such as trading in the secondary equity market. The terms $\sum_s \omega_{3s} E^S_{sec}(\xi, \delta, \mathbb{I}_w) \left(DPS_{3s}(\xi, \delta) - P_{sec}(\xi, \delta)\right)$, for both $\mathbb{I}_w = 0$ and $\mathbb{I}_w = 1$, in patient agents' period 3 expected utility drop out. This is an outcome of the linear preference at $t = 3$ and is true for any portion of savers λ deciding to withdraw. As a result, the computation of the run threshold in the global game is largely simplified, because the distribution of equity holdings between patient savers that choose to withdraw and those that choose to wait does not matter for the utility differential.

she decides to put into the bank at price P by participating in the primary equity market at $t = 1$. This distinction was inconsequential in the baseline model that the banker is the sole owner of the bank. Hence, the total share holdings of the banker are $E^B = E_0^B + E_1^B$. Pinning down the share of ownership is important because the profits accruing to the banker depend on her relative holdings, $E^B/(E^B + E^S)$, or in other words she will receive a dividend per share for each of the E^B she holds. The banker's utility –equation (10)– changes to:

$$\mathbb{U}^B = \gamma \cdot U\left(e^B - P \cdot E_1^B\right) + \overbrace{E^B \int_{\xi^*}^{\bar{\xi}} \sum_s \omega_{3s} DPS_{3s}(\xi, \delta) \frac{1}{\Delta_\xi} d\xi}^{\text{no run}}. \tag{B.7}$$

The banker trades off foregoing current consumption to investing in equity and receiving dividends in the future. Note that the banker gets to consume her share of dividends only if the bank survives the run. The banker "buys" additional equity at the same price at which she issues equity to savers. The balance sheet constraint at $t = 1$ –equation (11)– becomes:

$$BS: \;\; \mathrm{I} + \mathrm{LIQ}_1 = \mathrm{D} + \mathrm{CEQ}, \tag{B.8}$$

where $CEQ = P \cdot \left(E^S + E_1^B\right) + E_0^B$ is the total common equity.

Raising outside equity does not affect the balance sheet constraints at $t = 2$, thus the probability that a depositors is served, $\theta(\xi, \lambda)$, in a run are given by (12) and the fraction of loans recalled, $y(\xi, \lambda)$, when a run does not occur are given by (13). The dividends per share are the total dividends divided by the total number of shares, i.e.,

$$DPS_{3s}(\xi, \lambda) = \frac{DIV_{3s}(\xi, \lambda)}{E^B + E^S}, \tag{B.9}$$

where $DIV_{3s}(\xi, \lambda)$ are given by (14).

Moreover, the banker will choose to monitor if her share of the dividends rather than total dividend are higher than the private benefit. Thus, the incentive compatibility constraint –equation (17)– becomes:

$$IC: \;\; \mathrm{E^B} \sum_{\mathrm{s}} \omega_{3\mathrm{s}} \mathrm{DPS}_{3\mathrm{s}}(\xi^*, \delta) - \mathrm{PB} \geq 0. \tag{B.10}$$

Finally, the second modification to the baseline model implies an endogenous bankruptcy threshold, $\hat{\xi}$, in state m is determined by the following equation:

$$\left(1 - y\left(\hat{\xi}, \delta\right)\right) V_{3m}^I \left(\hat{\xi}, \delta\right) I\left(1 + r^I\right) + LIQ_2\left(\hat{\xi}, \delta\right) - (1 - \delta) D\left(1 + r_3^D\right) = 0. \tag{B.11}$$

We now turn into describing how the optimality conditions are altered and what are the new optimality conditions with respect to outside equity and the equity price. We will focus attention to incomplete funding contracts, i.e., deposit contracts specify the tuple (D, r_2^D, r_3^D) and equity contracts specify the tuple (E_1^B, E^S, P)

The marginal effect of investment on banker's utility in the optimality condition for loans, I –equation (18)– becomes:

$$\frac{d\mathbb{U}^B}{dI} = \frac{E^B}{E^B+E^S}\left[\overbrace{\int_{\xi^*}^{\bar{\xi}}\left\{\omega_{3g}\left(1+r^I\right)\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } g}+\overbrace{\int_{\hat{\xi}}^{\bar{\xi}}\left\{\omega_{3m}A_{3m}F'\left(I^E+(1-y(\xi,\delta))I\right)\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } m}\right]. \tag{B.12}$$

As shown in (B.12), limited liability means that the banker still only internalizes states where she is solvent.

Similarly, the marginal effect of investment on banker's utility in the optimality condition for first period liquid assets, LIQ_1 –equation (19)– becomes:

$$\frac{d\mathbb{U}^B}{dLIQ_1} = \frac{E^B}{E^B+E^S}\left[\overbrace{\int_{\xi^*}^{\bar{\xi}}\left\{\omega_{3g}\left(1+r^I\right)\frac{1}{\xi}\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } g}+\overbrace{\int_{\hat{\xi}}^{\bar{\xi}}\left\{\omega_{3m}A_{3m}F'\left(I^E+(1-y(\xi,\delta))I\right)\frac{1}{\xi}\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } m}\right]. \tag{B.13}$$

The optimal choice of the run threshold –equation (20)– becomes

$$-E^B\sum_s \omega_{3s}DPS_{3s}(\xi^*,\delta)\frac{1}{\Delta_\xi}+\psi_{IC}\frac{dIC}{d\xi^*}+\psi_{GG}\frac{dGG}{d\xi^*}=0. \tag{B.14}$$

The optimal choice of liquidity holdings, $LIQ_2(\xi,\delta)$, at $t=2$ after the run uncertainty is resolved is given by:

$$\frac{E^B}{E^B+E^S}\left(\omega_{3g}\left[1-\frac{V_{3g}^I\left(1+r^I\right)}{\xi}\right]+\omega_{3m}\left[1-\frac{V_{3m}^I\left(1+r^I\right)}{\xi}\right]\cdot(1-\mathbb{I}_d)\right)+\nu^{LIQ_2(\xi,\delta)}=0, \quad \forall \quad \xi\geq\xi^*, \tag{B.15}$$

where $\nu^{LIQ_2(\xi,\delta)}$ is the multiplier on the short-sale constraint $LIQ_2(\xi,\delta)\geq 0$.

Turning to the deposit contract, the marginal effects of the deposit contract terms on banker's utility in the optimality conditions (22) to (24)–become:

$$\frac{d\mathbb{U}^B}{dD} = -\frac{E^B}{E^B+E^S}\left[\overbrace{\int_{\xi^*}^{\bar{\xi}}\left\{\omega_{3g}\left(1+r^I\right)\frac{\delta\left(1+r_2^D\right)}{\xi}+(1-\delta)\left(1+r_3^D\right)\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } g}\right.$$
$$\left.\overbrace{+\int_{\hat{\xi}}^{\bar{\xi}}\left\{\omega_{3m}A_{3m}F'\left(I^E+(1-y(\xi,\delta))I\right)\frac{\delta\left(1+r_2^D\right)}{\xi}+(1-\delta)\left(1+r_3^D\right)\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } m}\right], \tag{B.16}$$

$$\frac{d\mathbb{U}^B}{dr_2^D}=-\frac{E^B}{E^B+E^S}\left[\overbrace{\int_{\xi^*}^{\bar{\xi}}\left\{\omega_{3g}\left(1+r^I\right)\frac{\delta\cdot D}{\xi}\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } g}+\overbrace{\int_{\hat{\xi}}^{\bar{\xi}}\left\{\omega_{3m}A_{3m}F'\left(I^E+(1-y(\xi,\delta))I\right)\frac{\delta\cdot D}{\xi}\right\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } m}\right], \tag{B.17}$$

$$\frac{d\mathbb{U}^B}{dr_3^D}=-\frac{E^B}{E^B+E^S}\left[\overbrace{\int_{\xi^*}^{\bar{\xi}}\{\omega_{3g}(1-\delta)\cdot D\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } g}+\overbrace{\int_{\hat{\xi}}^{\bar{\xi}}\{\omega_{3m}(1-\delta)\cdot D\}\frac{1}{\Delta_\xi}d\xi}^{\text{no default, state } m}\right]=0. \tag{B.18}$$

We now turn to the decisions in the primary equity market.

Buying more equity not only requires the banker to give up consumption in the initial period in exchange for a higher share of future dividends, but it also changes the mix of inside and outside equity which matters for the incentives to monitor through equation (B.10), In addition, the banker understands how putting more of her own equity changes the incentives of savers to buy equity and to hold deposits. The latter will be priced by the banker to the extent that contracts are comprehensive. Overall, the optimality condition with respect to inside equity –equation (21)– becomes:

$$-\gamma\cdot P\cdot U'\left(e^B-P\cdot E_1^B\right)+\frac{E^S}{E^B+E^S}\int_{\xi^*}^{\bar{\xi}}\sum_s\omega_{3s}DPS_{3s}(\xi,\delta)\frac{1}{\Delta_\xi}d\xi+\psi_{BS}\cdot P+\psi_{IC}\frac{dIC}{dE_1^B}+\psi_{ES}\frac{dES}{dE_1^B}=0. \tag{B.19}$$

where ψ_{ES} is the multiplier on the equity supply schedule (B.5) offered by savers, satifying the complementarity slackness condition $\psi_{ES}\cdot\nu_{E^S}=0$

Finally, the banker also chooses how much outside equity to raise from savers, E^S, and the price at which the bank will issue equity in the primary market, P. As was the case for inside equity, these choices will matter for the incentive of savers to buy equity as described in the equity supply schedule (B.5).

The optimality conditions for E^S and P, which do not have a counterpart in the baseline model, are:

$$-\frac{E^B}{E^B+E^S}\int_{\xi^*}^{\bar{\xi}}\sum_s\omega_{3s}DPS_{3s}(\xi,\delta)\frac{d\xi}{\Delta_\xi}+\psi^{BS}\cdot P+\psi_{IC}\frac{dIC}{dE^S}+\psi_{ES}\frac{dES}{dE^S}+\psi_{DS}\frac{dDS}{dE^S}\mathbb{I}_c=0, \tag{B.20}$$

$$-\gamma\cdot E_1^B\cdot U'\left(e^B-P\cdot E_1^B\right)+\psi_{BS}\cdot\left(E_1^B+E^S\right)+\psi_{ES}\frac{dES}{dP}+\psi_{DS}\frac{dDS}{dP}\mathbb{I}_c=0. \tag{B.21}$$

Conditions (B.20) and (B.21) can be easily interpreted. Selling equity to the savers delivers the shadow benefit of more equity but reduces the banker's share of future dividends, thus changing the incentive to monitor. This combination moves the banker to a different point in the equity supply schedule of the savers. Finally, a higher equity issuance price affects the banker's current consumption negatively, because she has to pay this price, but has a positive balance sheet effect and allows the banker to move at a different point on the savers' equity supply schedule.

B.3 Modified entrepreneurs' problem

As in the baseline model, the entrepreneur uses her own capital and borrows from the bank to invest in the project. The loan contract specifies the loan amount and the loan rate. As already mentioned, the entrepreneur repays her loan only if state g realizes. We, additionally, introduce adjustment costs when part of entrepreneur's initial investment is liquidated. These costs are paid by the entrepreneur in the intermediate period and are a function of the required adjustment, $c_I\left(y(\xi,\delta)\cdot I\right) = c_I\left(y(\xi,\delta)\cdot I\right)^{\phi_I}$, where $c_I > 0$, $\phi_I \geq 0$. For simplicity, we assume that E pays these costs out of new endowment, e_2^E, she receives at $t = 2$. Note that E cannot invest in more long-term projects at $t = 2$, thus she consumes at $t = 3$ what is left of the period 2 endowment after paying the adjustment costs.

Hence, the utility of an individual entrepreneur –equation (26)– becomes:

$$\mathbb{U}^E = \sum_s \omega_{3s}\left\{\overbrace{\int_{\xi^*}^{\overline{\xi}}\left[A_{3s}F\left(I^E+(1-y(\xi,\delta))I\right)-(1-y(\xi,\delta))I(1+r^I)\right]^+\frac{d\xi}{\Delta_\xi}}^{\text{no run}}+\overbrace{\int_{\underline{\xi}}^{\xi^*}A_{3s}F\left(I^E\right)\frac{d\xi}{\Delta_\xi}}^{\text{run}}\right\}$$
$$+e_2^E-\underbrace{\int_{\underline{\xi}}^{\overline{\xi}}c_I\left(y(\xi,\delta)\cdot I\right)\frac{d\xi}{\Delta_\xi}}_{\text{adjustment costs}}. \tag{B.22}$$

The optimal loan demand of an individual entrepreneur –equation (27)– becomes:

$$LD:\quad \omega_{3g}\int_{\xi^*}^{\overline{\xi}}(1-y(\xi,\delta))\left[A_{3g}F'\left(I^E+(1-y(\xi,\delta))\cdot I\right)-(1+r^I)+I\cdot\frac{\partial LS}{\partial I}\left(\frac{\partial LS}{\partial r^I}\right)^{-1}\right]\frac{d\xi}{\Delta_\xi}$$
$$-\int_{\underline{\xi}}^{\overline{\xi}}y(\xi,\delta)\cdot c_I'\left(y(\xi,\delta)\cdot I\right)\frac{d\xi}{\Delta_\xi}=0. \tag{B.23}$$

The second line in (B.23) shows the impact of investment on the marginal adjustment costs. The first line has the same terms as in the baseline model but there is a subtle difference. Entrepreneurial and bank default do not necessarily occur at the same time given that the bank is solvent in state m for $\xi > \hat{\xi}$, while the entrepreneur always default. Hence, $\partial LS/\partial I \neq 0$ as the entrepreneur prices the recovery value of her investment in state m. The partial derivatives of the loan supply curve with respect to the loan characteristics, taking all aggregate variables as given, are:

$$\frac{\partial LS}{\partial I}=\int_{\hat{\xi}}^{\overline{\xi}}\left\{\omega_{3m}A_{3m}\left(1-y(\xi,\delta)\right)F''\left(I^E+\left(1-y(\xi,\delta)\right)I\right)\right\}\frac{1}{\Delta_\xi}d\xi, \tag{B.24}$$

$$\frac{\partial LS}{\partial r^I} = \int_{\xi^*}^{\overline{\xi}} \{\omega_{3g}\} \frac{1}{\Delta_\xi} d\xi. \tag{B.25}$$

B.4 Modified planner's problem and intermediation margins

The planner's problem is similar to the one described in section 3.1. The only difference is that the planner will also internalize the effect of her decisions on the equity supply schedule and will also have two additional optimality conditions for E^S and P (the secondary equity price is substituted out in the budget sets). Thus, the generic first-order condition for the planner is:

$$\sum_{h=\{E,R,B\}} \overline{w}_h \frac{d\mathbb{U}^h}{dX} + \zeta_{BS}\frac{dBS}{dX} + \zeta_{IC}\frac{dIC}{dX} + \zeta_{GG}\frac{dGG}{dX} + \zeta_{LD}\frac{dLD}{dX} + \zeta_{DS}\frac{dDS}{dX} + \zeta_{ES}\frac{dES}{dX} = 0,$$

where ζ_{ES} is the multipliers on the equity supply schedule (B.5), satifying the complementarity slackness condition $\zeta_{ES} \cdot \nu_{E^S} = 0$.

The ability to choose the level of outside equity introduces an additional intermediation margin. To see this fix the assets mix, i.e., the investment-liquidity margin, and also fix the liabilities mix, i.e., the equity-deposits margin. Then, one can additionally use the balance sheet and incentive compatibility constraints to express all variables in terms of the amount of outside equity issued. In other words, the banker can scale up or down the level of credit extension and bank size by choosing different levels of outside equity even if the marginal relationship between liquid and illiquid assets and between equity and deposits is fixed. One way to express this margin is to combine the optimality conditions for outside equity and inside equity. We will refer to it as the *equity-mix* margin, denoted by EE. The banker and the planner will have different incentives when choosing between inside and outside equity. Hence, there is an additional wedge between the private and planning solution on top of $ILIQ_{wedge}$ and ED_{wedge} in equations (38) and (39):

$$EE_{sp} = EE_B + EE_{wedge}. \tag{B.26}$$

The wedge in (B.26) represents a distortion in the equity mix or, as discussed above, in the scale of credit intermediation chosen by the banker versus the planner. Indeed, we show in the next section that usual prudential tools are insufficient to correct this third margin, though the $ILIQ$ and ED margins can be corrected. The EE margin it can be addressed with targeted corrective taxes.

B.5 Numerical example

This section presents a numerical example for the equilibrium in the extended model. Table B.4 shows the parameterization of the exogenous variables, which have been chosen such that it is optimal for the private economy and the planner, at least for some weights, to invest in outside equity. Table B.5 reports the private equilibrium as well as the planner's solutions for different weights in the social welfare function. Table B.6 reports the effects of individual regulations.

The planner chooses to raise outside equity as long as the weight on entrepreneurs is high enough ($w_E \geq 0.5$ in this example). The reason is that raising outside equity reduces the reliance on deposits, which reduces the need for holding liquidity and allows for more credit extension. In addition, the lower demand for deposits suppresses deposit rates and allows the planner to set lower loan rates given the intermediation spread required to satisfy the banker's incentive compatibility constraint. These effects are beneficial for entrepreneurs, but reduce savers' utility. Hence, the planner will choose to raise outside equity when w_E is high enough. For lower w_E, the planner will not choose to raise outside equity and the analysis is the same as in the baseline model.

The rest of the conclusions derived in sections 3.3 and 4 continue to hold in the extended model. To summarize a few, the planner chooses both higher liquidity and capital ratios to address the distorted investment-liquidity and equity-deposits margins.[5] The run probability goes down and liquidity provision is higher in the planner's solution. As in the baseline model, the welfare of savers and entrepreneurs improves, while the banker is driven to her participation constraint. The total surplus created by the planner is positive. Moreover, the planner chooses higher common equity capital, fewer liquid asset holdings and higher investment when the weight on entrepreneurs is higher, and vice versa. Finally, the impact of individual regulations is similar to that in the baseline model.

Extending the analysis in section 4.5 to three intermediation margins, three independent tools are, in principle, needed to replicate the planner's solution when $E^S > 0$. However, the tools need to be jointly binding, which is not the case for any of the combinations of the four capital and liquidity regulations discussed in section 4.5. Instead, corrective (Pigouvian) taxes can be used in combination with a capital and a liquidity tool to replicate the planner's solution. These taxes can affect marginal decisions, but the tax proceeds are assumed to be fully rebated to agents in a lump-sum fashion in order to eliminate any income implications. Despite the fact that such taxes may seem unrealistic from the lens of actual policy implementation, they can point to the direction that the additional distortion operates. For example, a capital requirement and a liquidity tool can be combined with a corrective tax levied on inside equity to push relatively more outside equity into the bank and bring the scale of credit intermediation down to desirable levels. Alternatively, a leverage requirement and a liquidity tool can be used in combination to a corrective tax on the total size of the bank (or just deposits) to push the scale of credit intermediation down. Overall, the third intermediation margin determines the scale of credit intermediation, because the banker can decide on the level of equity issued to scale up their balance sheet. Given regulations that pin down the other two margins, a targeted tool is needed to control the size of the bank.

[5]For $w_E \in [0.2, 0.4]$, where the planner sets $E^S = 0$, the capital ratio in the planner's solution is lower than in the private equilibrum. This does not mean that the planner chooses a lower capital ratio compared to an *LCR* regulated economy, since the drop in capital is due to the big increase in liquidity.

Tables

e_1^S	2.95	ρ	1.00	ω_{3g}	65%
e_2^S	1.10	γ	0.10	ω_{3m}	30%
e_1^E	0.05	A_{3g}	3.30	c_D	2.5%
e_2^E	0.01	A_{3m}	1.15	ϕ_D	0.50
e^B	0.20	A_{3b}	0.70	c_I	2.5%
E_0^B	0.13	α	0.75	ϕ_I	3.0
δ	0.50	$\overline{\xi}$	1.20	PB	0.14
β	0.70	$\underline{\xi}$	0.10		

Table B.4: Parameterization.

	PE	SP for weights (w_E, w_S)				
		(0.2,0.6)	(0.3,0.5)	(0.4,0.4)	(0.5,0.3)	(0.6,0.2)
I	0.895	0.831	0.838	0.845	0.847	0.854
LIQ_1	0.085	0.243	0.239	0.233	0.175	0.172
D	0.789	0.907	0.906	0.904	0.812	0.815
CEQ	0.191	0.167	0.170	0.174	0.209	0.211
$E^B/(E^S+E^B)$	0.997	1.000	1.000	1.000	0.671	0.685
CR	0.213	0.201	0.203	0.206	0.247	0.247
$LevR$	0.194	0.156	0.158	0161	0.205	0.206
LCR	0.221	0.360	0.356	0.351	0.320	0.315
$NSFR$	0.654	0.747	0.744	0.741	0.727	0.724
P	0.958	1.000	1.000	1.000	1.018	0.998
r^I	1.650	1.672	1.668	1.664	1.665	1.662
r_3^D	1.161	1.342	1.339	1.335	1.127	1.136
q	0.482	0.450	0.450	0.450	0.431	0.433
q_f	0.224	0.139	0.142	0.145	0.157	0.160
$Liq.Prov.$	0.884	1.011	1.009	1.007	1.013	1.010
$\%\Delta\mathbb{U}^E$	-	0.84%	0.91%	0.98%	1.36%	1.38%
$\%\Delta\mathbb{U}^S$	-	3.19%	3.16%	3.10%	2.70%	2.66%
$\%\Delta\mathbb{U}^B$	-	-0.22%	-0.22%	-0.22%	-0.22%	-0.22%
$\%\Delta\mathbb{U}^{sp}$	-	2.04%	1.81%	1.59%	1.45%	1.32%
$\%\Delta\mathbb{S}^{sp}$	-	1.27%	1.28%	1.29%	1.28%	1.27%

Table B.5: Privately versus Socially Optimal Solutions.

	PE	CR	LevR	LCR	NSFR	SP
I	0.895	0.902	0.913	0.861	0.862	0.847
LIQ_1	0.085	0.108	0.111	0.217	0.216	0.175
D	0.789	0.797	0.816	0.897	0.897	0.812
CEQ	0.191	0.213	0.209	0.181	0.181	0.209
$E^B/(E^S+E^B)$	0.997	0.844	0.912	1.000	1.000	0.671
CR	0.213	0.236	0.229	0.210	0.210	0.247
$LevR$	0.194	0.211	0.204	0.168	0.168	0.205
LCR	0.221	0.249	0.249	0.338	0.337	0.320
$NSFR$	0.654	0.678	0.676	0.731	0.730	0.727
P	0.958	0.918	0.889	1.000	1.000	1.018
r^I	1.650	1.623	1.635	1.655	1.654	1.665
r_3^D	1.161	1.134	1.181	1.320	1.319	1.127
q	0.482	0.459	0.463	0.452	0.452	0.431
q_f	0.224	0.202	0.204	0.154	0.154	0.157
$Liq.Prov.$	0.884	0.937	0.933	0.998	0.998	1.013
$\%\Delta\mathbb{U}^E$	-	1.27%	0.91%	1.10%	1.10%	1.36%
$\%\Delta\mathbb{U}^S$	-	1.08%	1.13%	2.89%	2.87%	2.70%
$\%\Delta\mathbb{U}^B$	-	-0.22%	-0.22%	-0.22%	-0.22%	-0.22%
$\%\Delta\mathbb{U}^{sp}$	-	0.92%	0.75%	1.37%	1.37%	1.45%
$\%\Delta\mathbb{S}^{sp}$	-	0.71%	0.60%	1.25%	1.25%	1.28%

Table B.6: Single regulations versus planner's solution for $(w_E, w_S) = (0.5, 0.3)$. Regulation is set at its maximum level such that there are gains in social welfare, while the banker's participation constraint is satisfied.