Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

On Targeting Frameworks and Optimal Monetary Policy

Martin Bodenstein and Junzhu Zhao

2017-098

Please cite this paper as:
Bodenstein, Martin, and Junzhu Zhao (2017). "On Targeting Frameworks and Optimal Monetary Policy," Finance and Economics Discussion Series 2017-098. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2017.098.

NOTE: Staff working papers in the Finance and Economics Discussion Series (FEDS) are preliminary materials circulated to stimulate discussion and critical comment. The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors. References in publications to the Finance and Economics Discussion Series (other than acknowledgement) should be cleared with the author(s) to protect the tentative character of these papers.

On Targeting Frameworks and Optimal Monetary Policy

Martin Bodenstein — Federal Reserve Board
Junzhu Zhao — Nanjing Audit University

September 13, 2017

Abstract

Speed limit policy, a monetary policy strategy that focuses on stabilizing inflation and the change in the output gap, consistently delivers better welfare outcomes than flexible inflation targeting or flexible price level targeting in empirical New Keynesian models when policymakers lack the ability to commit to future policies. Even if the policymaker can commit under an inflation targeting strategy, the discretionary speed limit policy performs better for most empirically plausible model parameterizations from a normative perspective.

JEL classifications: E52, E58
Keywords: inflation targeting, price level targeting, speed limit policy, optimal monetary policy, delegation.

* The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System. We are grateful to Larry Christiano, Chris Gust, Paul Levine, Joseph Pearlman, Ben Johannsen, and Robert Tetlow for helpful comments and suggestions.
** Contact information: Martin Bodenstein (corresponding author), E-mail Martin.R.Bodenstein@frb.gov; Junzhu Zhao, E-mail junzhuzhao@gmail.com

1 Introduction

The optimal delegation problem in monetary policy studies how a central bank can best serve the interests of society when the optimal state-contingent plan derived under the true social objective function is time-inconsistent. Starting with Rogoff (1985), several authors have shown that assigning the central bank an objective that differs from the true social objective can lead to better normative outcomes under discretionary policymaking than otherwise.[1]

One such central bank objective is the speed limit policy under which, according to Walsh (2003), the policymaker focuses on stabilizing inflation and the change in the output gap. We show that in the discretionary Markov equilibrium, the speed limit policy framework consistently outperforms flexible inflation targeting and often performs better than flexible price level targeting in a set of New Keynesian models (NKM) ranging from the purely forward-looking textbook version of the NKM and its extensions to the medium-scale DSGE model in Christiano, Eichenbaum, and Evans (2005) as implemented and estimated in Smets and Wouters (2007) (CEE/SW model).[2]

The speed limit policy performs strongly in the discretionary Markov equilibrium as it captures a robust feature of the optimal monetary policy under commitment (henceforth, optimal commitment policy) in NKMs: The policymaker promises to keep future monetary policy tight in response to shocks that drive up inflation, such as a positive price markup shock, as evidenced by a slow closing of the negative output gap under the optimal commitment policy. The persistent rise in the policy interest rate deters excessive price and wage adjustments by the private sector in the impact period and reduces overall movements in inflation under the optimal commitment policy. Importantly, the price level is not necessarily stationary under the optimal commitment policy. The speed with and the extent to which nominally rigid prices and wages return to their long-run trend paths depend on the degree of price and wage indexation to past inflation.

As the speed limit policy interprets the idea of stabilizing the real economy as preventing large *changes* in the output gap as opposed to *deviations* of the output gap from zero, the policymaker prefers delaying the closing of the negative output gap after the inflationary shock by construction and keeps future monetary policy tight regardless of the policymaker's ability to commit. If the private sector understands this behavior of the central bank, the rise in inflation is kept small while the price level rises permanently by a small amount. The price level targeting framework also incorporates the idea of keeping monetary policy tight after an inflationary shock albeit through a different mechanism. By assumption, the policymaker is determined to drive the price level back to its trend path under this framework and keeps the interest rate elevated to undo earlier changes induced by the shock. Anticipating such a policy move, households and

[1] Important contributions include King (1997), Svensson (1997), Svensson (1999), Clarida, Gali, and Gertler (1999), Walsh (2003), Woodford (2003b), Nessen and Vestin (2005), Vestin (2006), and Bilbiie (2014).

[2] Consistent with the literature, we define that under a flexible targeting framework the central bank minimizes the discounted infinite sum of a period loss function that reflects the central bank's preferences over stabilizing prices and the real economy subject to its model of the economy. Under inflation targeting, the loss function places weight on the squared deviations of inflation from its long-run target and of the output gap from zero as in Svensson (2010). The price level (in deviation from a deterministic trend) takes the place of inflation in the loss function under price level targeting; in addition the loss function places weight on the squarred deviations of the output gap from zero. Finally as in Walsh (2003), the central bank's loss function features an aversion to squarred deviations of inflation from its target and of the *growth rate* of the output gap under the speed limit policy.

firms feel deterred from implementing large changes in prices and wages in the first place.

By contrast, the inflation targeting framework lacks a built-in mechanism that facilitates implementing tight monetary policy after an inflationary shock in the discretionary Markov equilibrium. As the policymaker intends to stabilize inflation and the level of the output gap, the policymaker will not be expected to drive prices back to their trend level or to delay the closing of the output gap under the inflation targeting objective. In line with the "weight-conservative" central banker of Rogoff (1985), placing a high weight on stabilizing inflation helps improving the performance of the inflation targeting framework, but is generally too crude to make inflation targeting attractive relative to the speed limit policy under discretionary policymaking. Only in the simplest NKMs with a high degree of indexation to past inflation can inflation targeting perform best, since in this case the desirability of returning the price level to its previous trend vanishes under the optimal commitment policy. In more complex models featuring habit persistence in consumption or sticky nominal wages (unless highly indexed to inflation as well) or the empirical CEE/SW model inflation targeting is undesirable irrespective of the degree of price indexation when policymakers cannot commit.[3]

Although, we view the case of discretionary policymaking as more realistic, we also report findings for the case that the central bank can commit to future actions.[4] Under commitment, the inflation targeting central bank does drive prices and wages back towards their long-run trends if so desired under the optimal commitment policy and performs reliably best across models from the textbook NKM to the CEE/SW model with the speed limit policy a close second. Since under price level targeting the central bank will never allow for permanent changes in prices and wages, this framework performs worst when prices and wages are highly indexed to past inflation.[5]

Several experiments in the CEE/SW model lend further support to the speed limit policy framework when policymakers can only act under discretion. Beyond parameterizing the model at the mode of the posterior distribution reported in Smets and Wouters (2007), we consider alternative parameter choices drawn from the Laplace approximation to the posterior distribution. When the objective functions are parameterized optimally for each parameter draw, the speed limit policy dominates for almost all 30,000 empirically plausible draws when policymakers act under discretion. Surprisingly, the speed limit policy *under discretion* outperforms the inflation targeting framework *under commitment* for the majority of draws (including our benchmark parameterization). When we compare the targeting frameworks for selected specifications of the objective functions that do not vary across the 30,000 parameterizations of the CEE/SW model, the speed limit policy almost always dominates regardless of the central bank's ability to commit.

Our findings prevail in a version of the CEE/SW model that is estimated with euro area data instead of US data or a version that reduces the importance of wage markup shocks relative to labor supply shocks

[3] Under habit persistence smoothing a quasi-difference of the output gap enters in the true social loss functions as a motive which is well captured by the speed limit policy objective; under sticky nominal wages with a moderate degree or no inflation indexation, the optimal commitment policy pushes the levels of prices and wages back towards their deterministic trends even if prices are highly indexed.

[4] See Bernanke and Mishkin (1997) and King (2004) for further elaborations on this issue.

[5] In the case of commitment, adopting a simple objective function for the central bank can be justified on the grounds of improving transparency, accountability and the pursuit of the central bank's legal mandate.

to address concerns about identification raised in Chari, Kehoe, and McGrattan (2009) and Justiniano, Primiceri, and Tambalotti (2013). Finally, we also account for the limitations of conventional monetary policy imposed by the zero lower bound constraint on the nominal interest rate. Unless long-lasting and frequent zero-bound episodes cannot be eliminated by raising the long-run inflation target, our results go unchallenged.

In terms of scope and focus, our paper is closest to Walsh (2003). In a simple NKM with sticky prices and backward-looking elements in the form of lagged inflation and lagged output gap Walsh (2003) illustrates the potential advantages of the speed limit policy. However, the model in Walsh (2003) is not fully micro-founded and social welfare is measured by an ad hoc loss function that is not derived from the preferences of the representative household. Furthermore, the underlying model is calibrated rather than estimated and lacks many of the features found to be of empirical relevance in works such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2005). In contrast to Walsh (2003), we find that the speed limit policy outperforms inflation and price level targeting under discretion regardless of the degree of backward-looking inflation dynamics in the CEE/SW model. In Walsh (2003) and in simple NKMs, this conclusion applies only for the case of an intermediate degree of backward-looking behavior.

Restricting attention to the case of a fully committed policymaker Debortoli, Kim, Lindé, and Nunes (2015) report strong support in favor of inflation targeting using the CEE/SW model, a result we confirm and extend to a range of other empirically relevant parameterizations of the CEE/SW model. However, as the optimal inflation targeting under commitment is dominated by the optimal speed limit policy under discretion for many empirically plausible parameterizations, our results appear more general.

The remainder of the paper proceeds as follows. In Section 2, we analyze inflation targeting, price level targeting, and speed limit policy in a sequence of simple NKMs. We consider a wide range of parameterizations and variations of the CEE/SW model in Section 3. Concluding remarks are offered in Section 4. A technical appendix provides information on our methodology, details on the models, and additional results.

2 Baseline New Keynesian Model

Throughout this paper, we refer to the NKM presented in Woodford (2003a), Gali (2008) or Walsh (2010) as the textbook NKM. This model features sticky nominal prices as in Calvo (1983) and a production technology that requires only labor as input. Sales subsidies offset the distortions arising from monopolistic competition in the steady state. Finally, the economy experiences technology and markup shocks. One at a time, we consider the role of features commonly present in empirical DSGE models: (i) intrinsic inflation inertia, (ii) steady state distortions, (iii) consumption habits, and (iv) sticky wages. Appendix A offers details on our computational approach. The models are described in Appendix B.

2.1 Simple objective functions and targeting frameworks

Broadly speaking, analysis of monetary policy distinguishes between targeting frameworks and instrument rules. Under a targeting framework, the central bank optimizes an objective function. An inflation targeting central bank, for example, is instructed to keep a selected inflation measure in the neighborhood of a specific target value. The central bank is granted some flexibility in pursuing this goal and can deviate from its target in the short run to buffer the impact of shocks (flexible inflation targeting).[6] Given a specific model of the economy, the policymaker derives a set of optimality conditions for the targeting variables to fulfill under the targeting framework. By contrast, an instrument rule as in Taylor (1993) is a is a formula that specifies directly the functional relationship between the central bank's instrument and a set of variables.

For model-based policy analysis, the central bank's objective function under a targeting framework specifies the variables that characterize the long-run goal(s) of the central bank and the weights assigned to each of these variables as argued in Svensson (2010). In line with the literature, we represent loss functions associated with the targeting frameworks of interest as:

1. inflation targeting (IT)
$$L_t^{IT} = \pi_{p,t}^2 + \lambda_x^{IT} \left(x_t^{gap}\right)^2 \tag{1}$$

2. price level targeting (PLT)
$$L_t^{PLT} = \hat{p}_t^2 + \lambda_x^{PLT} \left(x_t^{gap}\right)^2 \tag{2}$$

3. speed limit policy (SLP)
$$L_t^{SLP} = \pi_{p,t}^2 + \lambda_x^{SLP} \left(\left(x_t^{gap}\right) - \left(x_{t-1}^{gap}\right)\right)^2 \tag{3}$$

where $\pi_{p,t}$ denotes deviations of the inflation measure from its value along the balanced growth path (henceforth the long-run target), $\hat{p}_t$ is the log-deviation of the price level from its value along the balanced growth path (henceforth the long-run trend), and x_t^{gap} measures the (model-specific) output gap. We refer to λ_x^{TF} as the weight on the activity measure under framework TF.

Each objective function implies a long-run commitment to price stability expressed in terms of a long-run inflation target, or equivalently, a deterministic trend in the price level to provide a nominal anchor. The central bank minimizes the discounted sum of losses subject to the equations that describe the behavior of the economy. We consider both the case that in doing so the policymaker can commit to future policy actions and the case that such a commitment is not feasible (discretion). A targeting framework is referred to as optimal, when the objective function associated with this framework is parameterized to minimize the expected welfare loss under this objective relative to the social optimum. The social optimum is defined by the economic outcomes under the optimal commitment policy when the policymaker's preferences

[6] In practice, a targeting framework fullfils a list of formal criteria. State of the art inflation targeting, for example, is commonly characterized as featuring the following elements, see Hammond (2012): (1) price stability as the main goal of monetary policy, (2) public announcement of a quantitative target for inflation, (3) policy based on inflation forecast, (4) mechanisms for transparency and accountability. Suitably adapted, these elements would also be present in other targeting frameworks. By contrast, our discussion of targeting frameworks treats monetary policy as the solution to an optimal control problem under a specific objective function for each framework. Given our broader perspective, the analysis in this paper is also of relevance for central banks that do not adopt a formal targeting framework, but rather search for monetary policy strategies that achieve the central bank's mandate as in the case of the U.S. Federal Reserve.

are consistent with the true social loss function. Following Woodford (1999), we adopt the concept of "optimality from a timeless perspective" to derive commitment policies throughout this paper.

2.2 Targeting frameworks in the textbook NKM

We start our discussion of targeting frameworks using the textbook NKM. At the core of the linear version of this model lies the New Keynesian Phillips Curve (NKPC) which links inflation, $\pi_{p,t}$, to the (welfare-relevant) output gap, x_t,

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p(\sigma_L + \sigma_C)x_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t}. \tag{4}$$

Here and subsequently, all variables are expressed in deviation from their steady state values (relative if carrying a "hat", absolute otherwise). The markup shock, $u_{p,t}$, follows a known stochastic process. The composite parameter $\kappa_p(\sigma_L + \sigma_C)$ measures the slope of the NKPC and the parameter ι_p represents the degree of indexation to past inflation as in Christiano, Eichenbaum, and Evans (2005). The aggregate demand curve

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_C} \left(i_t - E_t \pi_{p,t+1} - g^*_{mu,t} \right) \tag{5}$$

provides the connection between the output gap, inflation, the nominal interest rate, i_t, and the natural rate of interest, $g^*_{mu,t} = \sigma_C \left[E_t \hat{y}^*_{t+1} - \hat{y}^*_t \right]$. The natural level of output in this model

$$\hat{y}^*_t = \frac{1 + \sigma_L}{\sigma_L + \sigma_C} \hat{\xi}_{A,t} \tag{6}$$

is obtained from a counterfactual economy without nominal rigidities and without markup shocks. The natural level of output responds to changes in technology, $\hat{\xi}_{A,t}$; other shocks that could move the natural level of output and thus the natural rate of interest, but from which we abstract for now, are shocks to household preferences or government spending. The output gap is defined as the difference between actual output and the natural level of output, $x_t = \hat{y}_t - \hat{y}^*_t$. As in Woodford (2003a), the preferences of the representative household (or equivalently the social welfare function in this context) are approximated to the second-order as

$$E_{t_0} \left(\frac{1}{2} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \right) \tag{7}$$

with the true (approximate) social loss function L_t satisfying

$$L_t = (\sigma_L + \sigma_C) (x_t)^2 + \frac{1 + \theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2 \tag{8}$$

with σ_L, σ_C, θ_p being known parameters.

To fix ideas, we consider first the performance of each targeting framework in the fully forward-looking

NKM, i.e., $\iota_p = 0$. The policies associated with each framework are obtained by replacing the true social loss function L_t in equation (7) with the loss functions in (1)-(3). Each framework is evaluated for a range of weights on the activity measure, λ_x^{IT}, λ_x^{PLT}, and λ_x^{SLP}, respectively, both under commitment and discretion with $x_t^{gap} = x_t$. Table 1 provides the parameterization of the model (and of all its extensions). For each targeting framework we consider and for the optimal commitment policy, shocks that transmit through the natural real interest rate, such as the technology shock, have no welfare consequences as adjustments in the nominal interest rate prevent movements of inflation and the output gap so as to prevent any welfare consequences. Blanchard and Gali (2007) refer to this feature of the textbook NKM as divine coincidence.[7] In the following, we restrict attention to markup shocks which by contrast cannot be neutralized.

Figure 1 plots the unconditional welfare loss for each framework relative to the optimal commitment policy expressed as consumption equivalent variation (CEV). The weight on the activity measure for which the welfare loss is minimized under a targeting framework is indicated by "$\circ$" for price level targeting (PLT), "$*$" for speed limit policy (SLP), and "$\diamond$" for inflation targeting (IT). The optimal weights on the activity measure are low relative to the weights on the inflation measure (which is normalized to 1) and the welfare losses of not implementing the optimal commitment policy are small both under commitment and discretion for each framework.

Figure 1 reproduces some well-known results. Under inflation targeting, a central bank acting under commitment can replicate the optimal commitment policy; the solid line in the top panel assumes the value of zero for the optimal choice of the weight λ_x^{IT} in the objective function. In the textbook NKM without indexation, the true social loss function (8) is written solely in terms of contemporaneous inflation and the welfare-relevant output gap. The central bank's preferences over inflation and the output gap under inflation targeting coincide with the true social loss function, if $\lambda_x^{IT} = \lambda_x \equiv (\sigma_C + \sigma_L)\kappa_p \frac{\theta_p}{1+\theta_p}$. Thus, the welfare loss under optimal inflation targeting relative to the optimal commitment policy must be zero. Given the modifications in the objective functions for price level targeting ($\hat{p}_t$ instead of $\pi_{p,t}$) and speed limit policy ($x_t^{gap} - x_{t-1}^{gap}$ instead of x_t^{gap}) relative to the true social loss function the outcomes under these two targeting frameworks are suboptimal by construction.

The equivalence between inflation targeting and the optimal commitment policy breaks down for any change in the model environment, most notably if the central bank lacks commitment. For example, in response to a *transitory* markup shock, the optimal commitment policy manages to reduce deviations of inflation and the output gap from their target values in the impact period by allowing these variables to deviate from their target values also in future periods after the shock has ceased. A central bank acting under discretion with the objective in (8) for $\iota_p = 0$, however, will find it optimal to eliminate these deviations from target in future periods to fully stabilize the economy earlier (stabilization bias). As households and firms correctly anticipate this behavior, the discretionary central bank will not be able to reap the benefits of the optimal commitment policy in the impact period thereby causing larger movements

[7] This feature of the model requires that shocks are sufficiently small in order for policy not to be constrained by the zero lower bound on the nominal interest rate.

in inflation and the output gap.[8]

Borrowing the idea of a "(weight-) conservative central banker" from Rogoff (1985), Clarida, Gali, and Gertler (1999) show that the optimal inflation targeting central bank puts lower weight on the activity measure than society does, i.e., $\lambda_x^{IT} < \lambda_x$, which mitigates, but does not eliminate, the negative welfare consequences of the stabilization bias. Thus, the CEV in Figure 1 is positive for optimal inflation targeting under discretion.[9] Changes to the functional form of the policymaker's objective function can induce further welfare improvements: The welfare loss under optimal price level targeting is close to zero and is marginally higher under the optimal speed limit policy in Figure 1.

To understand the strong performance of price level targeting and speed limit policy in the textbook NKM when the policymaker acts under discretion, we revisit the effects of a markup shock under the optimal commitment policy. Let the shock lead initially to an unexpected rise in inflation and a drop in the output gap. Over time the optimal commitment policy drives the price level back to its long-run trend by pushing inflation temporarily below its long-run target. The explanation for the optimality of price level stability (relative to its long-run trend) recognizes the link between price dispersion and inflation: the cross-sectional variation of prices is proportional to the squared value of inflation as shown in Woodford (2003a) and Appendix B.2. By assumption, firms that do not adjust prices optimally in the current period adjust prices by the value of the long-run inflation target instead. Suppose, that the central bank does not plan to return the price level to its long-run trend. Firms that have not adjusted optimally for some time will be far off the new price level and thus contribute to increased dispersion of prices. When such firms are finally called upon to adjust optimally, a sizable price adjustment will contribute to higher inflation. If the central bank does return the price level to its long-run trend, firms that have not adjusted optimally for some time will find their prices to be close to the expected long-run price level; hence prices adjust little when these firms are called upon to do so. In addition, firms that happen to adjust optimally closer in time to the impact of the shock will be deterred from raising prices: if the price level will return to its long-run trend over time, larger price adjustments early on bear the risk of the firms' prices to be far off the price level over time absent future optimal adjustments. As price level targeting under discretion will drive the price level back to its long-run trend by construction, whereas inflation targeting considers past deviations of inflation from its target bygones, the former outperforms the latter.[10]

An equivalent description of the optimal commitment policy focuses on the dynamics of the output gap after an inflationary markup shock: an increase of inflation above its target is subsequently countered by tighter monetary policy resulting in a negative output gap. Anticipating such a policy, forward-looking

[8] In the case of the textbook NKM with an efficient steady state and the central bank's preferences coinciding with those of the representative household the true social loss function is given by equation (8) regardless of the central bank's ability to commit.

[9] Rogoff (1985) formulates the idea of a conservative central bank to overcome the inflation bias that arises under policy discretion in a model with product or labor market distortions akin to Barro and Gordon (1983). A subsidy to offset market distortions also eliminates the inflation bias under discretionary policy in this setting. Yet, in the textbook NKM, even with an efficient steady state due to such subsidies, the optimal commitment policy continues to be time-inconsistent as discussed in the text.

[10] Following Vestin (2006), we prove in Appendix B.3 that for purely *transitory* markup shocks, as opposed to the ARMA(1,1) shock underlying Figure 1, optimal price level targeting under discretion replicates the optimal commitment policy. Even when the markup shock is *persistent*, the response of the economy under the optimal price level targeting and speed limit policy are close to optimal. Bilbiie (2014) shows how to construct a loss function for the central bank that replicates under discretion the optimal commitment policy regardless of the persistence of the markup push shock.

firms restrain their price response in the first place. Rewriting equation (5), we express the output gap as the sum of current and future real interest rates using

$$x_t = -\frac{1}{\sigma_C}(i_t - \pi_{p,t+1}) - \frac{1}{\sigma_C}E_t\left[\sum_{j=1}^{\infty}(i_{t+j} - \pi_{p,t+1+j})\right], \tag{9}$$

where we have set $g^*_{mu,t+j} = 0$ for all j, and we express inflation as the discounted sum of output gaps

$$\pi_{p,t} = \kappa_p(\sigma_L + \sigma_C)x_t + \kappa_p(\sigma_L + \sigma_C)E_t\left[\sum_{j=1}^{\infty}\beta^j x_{t+j}\right] + E_t\left[\sum_{j=0}^{\infty}\beta^j u_{p,t+j}\right]. \tag{10}$$

Following equation (9), tight future monetary policy in terms of higher future real interest rates affects negatively the contemporaneous and expected future values of the output gap. In turn, expectations of a slowly closing output gap reduce the trade off between contemporaneous inflation and the output gap in equation (10) for a given markup shock. As the speed limit policy assigns dislike to changes in the output gap, $x_t^{gap} - x_{t-1}^{gap}$, it replicates the slow closing of the output gap under the optimal commitment policy.

Yet, the speed limit policy cannot replicate the optimal commitment policy as it fails to drive the price level back to its long-run trend. As under inflation targeting the price level changes permanently under the speed limit policy. However, the built-in mechanism of closing the output gap slowly by running tighter monetary policy after an inflationary shock reduces the initial increase in the price level under the discretionary speed limit policy compared to inflation targeting. The problem with inflation targeting is not that deviations of inflation from target are considered bygones, but the lack of a mechanism to commit to tight future monetary policy after an inflationary shock.

The superior performance of price level targeting should not be mistaken as a general result. The speed with and the extent to which the price level returns to its long-run trend under the optimal commitment policy is sensitive to a range of model features, but the need to promise keeping monetary policy tight after inflationary shocks for longer is a general feature of the optimal commitment policy. Whether price level targeting or speed limit policy strikes a better balance between the path of the price level and other policy considerations when the policymaker lacks commitment is the quantitative question explored in this paper.

2.3 Extensions to the textbook NKM

The welfare ordering of the targeting frameworks in the textbook NKM is robust to the addition of other features. Inflation targeting is the preferred framework under commitment; price level targeting and speed limit policy outperform inflation targeting under discretion. Figure 2 explores the performance of the speed limit policy and price level targeting relative to inflation targeting as a function of the degree of price indexation, ι_p, for (i) the textbook NKM, (ii) the textbook NKM with a distorted steady state, (iii) a model with external consumption habit, (iv) and a model with sticky nominal wages. With the inflation targeting framework set to be the point of reference, a negative CEV indicates that the framework under investigation is inferior to inflation targeting and superior otherwise. We turn to a detailed discussion of each model variation.

2.3.1 The role of price indexation in the textbook NKM

The textbook NKM with price indexation is given by equations (4)-(8) with $0 < \iota_p \leq 1$. The lagged inflation rate enters equation (4) through the behavior of those firms that are not selected to reset prices optimally in the current period. Following the literature, we assume that these non-selected firms adjust prices by the geometric average of the steady state inflation rate and the inflation rate that prevailed in the previous period.

The weight ι_p governs the social desirability of undoing earlier changes in the price level. If non-selected firms adjust prices by the steady state inflation rate ($\iota_p = 0$), prices of these firms grow along the long-run trend of the price level. The optimal commitment policy limits welfare-costly price dispersion by promising to drive the price level back to its long-run trend over the medium run.

By contrast, when inflation is fully indexed ($\iota_p = 1$), the prices of non-selected firms reflect the deviations of the price level from its previous trend. The optimal commitment policy contains price dispersion, which is proportional to $(\pi_{p,t} - \pi_{p,t-1})^2$ for $\iota_p = 1$, by considering past deviations of inflation from its long-run target bygones and by allowing the price level to change permanently. If monetary policies attempted to revert the price level to its previous trend, it would cause unnecessary price dispersion in future periods. In analogy to the case without indexation, the optimal commitment policy under full indexation promises to return inflation (rather than prices) back to its long-run trend while it is the change in inflation (rather than the change in prices) that enters the true social loss function. This promise of the central bank deters firms that adjust prices optimally in a given period from choosing a price that is far off the price under the automatic indexation scheme for non-selected firms.

If the degree of price indexation falls strictly between 0 and 1, the price level is stationary under the optimal commitment policy, but the horizon over which the price level returns to its long-run trend lengthens with the degree of indexation. As in the case of the textbook NKM without indexation, a shock that calls for monetary tightening in the current period under the optimal commitment policy also calls for tighter policy in future periods as evidenced by a slow closing of the output gap.[11]

Turning to the evaluation of targeting frameworks, note that in the presence of indexation to past price inflation, the inflation targeting objective cannot be parameterized to match the true social loss function in equation (8). Nevertheless, as shown in the first row of panels in Figure 2, optimal inflation targeting outperforms price level targeting and speed limit policy under commitment for any degree of price indexation, ι_p, owing to the fact that the objective functions for price level targeting and speed limit policy depart even more from the true social loss function. The dominance of inflation targeting is most striking when indexation is high and the price level returns to its long-run trend very slowly, if at all, under the optimal commitment policy. In particular, price level targeting performs poorly in this case given its

[11] Stationarity of the price level (or the lack thereof) under the optimal commitment policy can be shown by writing the first order conditions as $-\frac{\theta_p}{1+\theta_p} x_t = \hat{p}_t - \iota_p \hat{p}_{t-1}$. For $\iota_p < 1$, the price level must return to its long-run trend for the output gap to be closed and inflation to be at its long-run target. For $\iota_p = 1$, the output gap is closed if and only if $\hat{p}_t - \hat{p}_{t-1} = \pi_{p,t} = 0$.

tendency to force the price level back to trend too quickly.

Under the optimal commitment policy, the monetary authority relates acceptable deviations of inflation from target to the change in the output gap and past inflation:

$$\pi_{p,t} = -\frac{\theta_p}{1+\theta_p}\left(x_t - x_{t-1}\right) + \iota_p \pi_{p,t-1}. \tag{11}$$

An inflation targeting policymaker also aspires to set inflation in accordance with the change in the output gap. But such a policymaker responds to expected future changes in the output gap and discards the role of past inflation:

$$\pi_{p,t} = -\frac{\lambda_x^{IT}}{\lambda_x}\frac{\theta_p}{1+\theta_p}\left(\left(x_t - x_{t-1}\right) - \beta \iota_p E_t\left(x_{t+1} - x_t\right)\right). \tag{12}$$

For a markup shock with a strong transitory component as under our parameterization, the optimal commitment policy allows inflation to rise and the output gap to turn negative initially followed by a period of below-target inflation and a gradual closing of the output gap. Under commitment, inflation targeting induces dynamics similar to those under the optimal commitment policy, when the central bank places a higher weight on stabilizing the output gap, $\lambda_x^{IT} > \lambda_x = (\sigma_L + \sigma_C)\frac{\theta_p \kappa_p}{1+\theta_p}$. The higher weight on the activity measure compensates for the fact that the expected (positive) output gap growth term in equation (12) operates in the opposite direction of the lagged inflation term in equation (11). Finally, inflation targeting under commitment performs strongly although it fails to drive the price level back fully to its original trend.

Under discretion, price level targeting and speed limit policy deliver better outcomes than inflation targeting for low and moderate degrees of price indexation ($\iota_p < 0.8$), but not for a high degree as inflation becomes increasingly persistent irrespective of policy. High inflation persistence feeds into higher expected inflation after an inflationary shock; an inflation targeting central bank will thus be expected to keep interest rates high to curb inflation. This feature of the textbook NKM with (high) indexation allows the discretionary central bank to indirectly commit to running tight future monetary policy and to preventing the output gap from closing too quickly thereby containing the initial response of inflation. The higher the degree of indexation, the more powerful is the fact that the inflation targeting objective replaces the quasi-difference in inflation in the true social loss function with inflation. In the limiting case of $\iota_p = 1$, optimal inflation targeting under discretion can even implement the optimal commitment policy under suitable assumptions for the nature of the underlying stochastic shocks—just as price level targeting can implement the optimal commitment policy for the case of $\iota_p = 0$.

More formally, provided that shocks are sufficiently small to prevent the zero lower bound constraint from binding, note that in the model without indexation, $\iota_p = 0$, the price level targeting central bank adopts the objective function $L_t^{PLT} = \hat{p}_t^2 + \lambda_x^{PLT}\left(x_t\right)^2$ and faces the NKPC of the form

$$\left(\hat{p}_t - \hat{p}_{t-1}\right) = \kappa_p(\sigma_L + \sigma_C)x_t + \beta E_t\left(\hat{p}_{t+1} - \hat{p}_t\right) + u_{p,t}. \tag{13}$$

In the case of full indexation, $\iota_p = 1$, the inflation targeting central bank adopts the objective function $L_t^{IT} = \pi_{p,t}^2 + \lambda_x^{IT} (x_t)^2$ and faces the NKPC of the form

$$(\pi_{p,t} - \pi_{p,t-1}) = \kappa_p(\sigma_L + \sigma_C)x_t + \beta E_t (\pi_{p,t+1} - \pi_{p,t}) + u_{p,t}. \tag{14}$$

Substituting $\pi_{p,t}$ with $\hat{p}_t$ reveals that inflation targeting under discretion in the model with $\iota_p = 1$ is isomorphic with price level targeting under discretion in the model with $\iota_p = 0$. As the optimal commitment policy stabilizes the price level absent indexation, but stabilizes the inflation rate under full indexation, inflation targeting performs close to optimal when $\iota_p = 1$ by analogy. Price level targeting and speed limit policy impose too tight monetary policy in future periods when prices are fully indexed.[12]

Finally, this discussion shows that for a high degree of indexation optimal inflation targeting under discretion can outperform inflation targeting under commitment. This observation raises the question under what conditions it is desirable to assign the central bank a (simple) loss function that departs from the true social loss function when policymakers can fully commit to future actions.

2.3.2 Inefficient steady state

Theoretical works building on the New Keynesian paradigm often assume that the steady state of the model is efficient as subsidies/taxes offset the distortions from monopolistic competition. By contrast, works on empirical DSGE models—including the seminal contributions of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007)—tend to abstract from such subsidies and taxes. The (in-)efficiency of the steady state affects the welfare ranking of policies through the definition of the output gap.

Following Benigno and Woodford (2005) the true social loss function in the model with an inefficient steady state satisfies

$$L_t = (\sigma_L + \sigma_C) (\tilde{x}_t)^2 + \frac{1 + \theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2 \tag{15}$$

where $\tilde{x}_t$ denotes the welfare-relevant output gap. The structural equations are given by

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p(\sigma_L + \sigma_C)\tilde{x}_t + \beta E_t(\pi_{p,t+1} - \iota_p \pi_{p,t}) + \frac{\sigma_L + \sigma_C}{\sigma_L + \sigma_C + (\Phi - 1)(1 + \sigma_L)} u_{p,t} \tag{16}$$

$$\tilde{x}_t = E_t \tilde{x}_{t+1} - \frac{1}{\sigma_C} (i_t - E_t \pi_{p,t+1} - \tilde{g}_{mu,t}) \tag{17}$$

$$\tilde{y}_t = \frac{1 + \sigma_L}{\sigma_L + \sigma_C} \hat{\xi}_{A,t} - \frac{(\Phi - 1)\frac{1+\sigma_L}{\sigma_L+\sigma_C}}{\sigma_L + \sigma_C + (\Phi - 1)(1 + \sigma_L)} u_{p,t} \tag{18}$$

with $\tilde{g}_{mu,t} = \sigma_C [E_t \tilde{y}_{t+1} - \tilde{y}_t]$. At first glance, it appears that we have merely replaced the output gap term "x_t" with "$\tilde{x}_t$" and rescaled the impact of the markup shock. However, the two definitions of the output gap respond differently to the markup shock. Under the definition $x_t \equiv \hat{y}_t - \hat{y}_t^*$, the target output

[12] For an intermediate degree of indexation, $0 < \iota_p < 1$, hybrid price level targeting with the objective function $L_t^{hPLT} = (\hat{p}_t - \iota_p \hat{p}_{t-1})^2 + \lambda_{hPLT} (x_t^{gap})^2$ can be shown to perform at least as well as inflation or price level targeting. See Roisland (2005) and Gaspar, Smets, and Vestin (2007) for additional discussion.

level $\hat{y}_t^*$ defined in equation (6) does not respond to the markup shock; all else equal under the definition $\tilde{x}_t \equiv \hat{y}_t - \tilde{y}_t$, the output gap will respond by less to a markup shock since the relevant output level $\tilde{y}_t$ defined in equation (18) moves in the same direction as actual output. Absent steady state distortions, i.e., $\Phi = 1$, the two definitions of the output gap coincide. Furthermore, in response to a technology shock, the divine coincidence continues to apply under the optimal commitment policy regardless of steady state distortions.

Applying this change in the definition of the relevant output gap to the three targeting frameworks, i.e. $x_t^{gap} = \tilde{x}_t$, the second row of panels in Figure 2 plots the results for the case of a distorted steady state with the sales subsidy set equal to zero. Both under commitment and discretion, price level targeting and speed limit policy appear closer to inflation targeting than in the case of an efficient steady state. The reason for this finding is the reduced impact of the markup shock in the model with an inefficient steady state ($\Phi > 1$): in the NKPC the markup shock is scaled by a term smaller than unity and movements in the output gap are curtailed by the adjustments in $\tilde{y}_t$. With the effective magnitude of the markup shock reduced the welfare losses under each targeting framework relative to the optimal commitment policy shrink.

The behavior of the output gap, and thus the ranking of targeting frameworks, is sensitive to the definition of potential output. If $x_t^{gap} = x_t$ despite the distorted steady state the measured output gap is larger after a markup shock all else equal, and calls for a larger adjustment in policy than under the output gap definition of $\tilde{x}_t$. When using x_t as the output gap measure despite the presence of steady state distortions, inflation targeting improves its perfomance and dominates price level targeting and speed limit policy already for the moderate degree of price indexation of $\iota_p = 0.4$.

2.3.3 Habit persistence

When the household's utility function depends on a quasi-difference in consumption (habit persistence), the implied output gap enters with its quasi-difference into the (approximate) true social loss function. Under external consumption habits as in Smets and Wouters (2007), the linear-quadratic form of the model is given by the loss function

$$L_t = \sigma_L (x_t)^2 + \frac{\sigma_C}{(1-h)(1-h\beta)} (x_t - hx_{t-1})^2 + \frac{1+\theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2 \tag{19}$$

and the structural equations

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p \widehat{mc}_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t} \tag{20}$$

$$\widehat{mc}_t = \sigma_L x_t + \frac{\sigma_C}{1-h} (x_t - hx_{t-1}) + \frac{h\beta}{1-h\beta} g^*_{mu,t} \tag{21}$$

$$(x_t - hx_{t-1}) = E_t (x_{t+1} - hx_t) - \frac{1-h}{\sigma_C} \left(i_t - E_t \pi_{p,t+1} - g^*_{mu,t}\right) \tag{22}$$

where $g^*_{mu,t}$ is defined as $g^*_{mu,t} = \frac{\sigma_C}{1-h}\left[E_t\left(\hat{y}^*_{t+1} - h\hat{y}^*_t\right) - \left(\hat{y}^*_t - h\hat{y}^*_{t-1}\right)\right]$. The efficient output level satisfies the difference equation

$$\sigma_L \hat{y}^*_t + \frac{\sigma_C}{(1-h)(1-h\beta)}\left(\hat{y}^*_t - h\hat{y}^*_{t-1}\right) - h\beta\frac{\sigma_C}{(1-h)(1-h\beta)}E_t\left(\hat{y}^*_{t+1} - h\hat{y}^*_t\right) = (1+\sigma_L)\,\hat{\xi}_{A,t}. \tag{23}$$

The degree of habit persistence is measured by the parameter $h \in [0,1)$. The model with habit persistence features endogenous persistence, since the lagged value of the output gap enters into the NKPC and the aggregate demand curve, which in turn affects the dynamics of inflation.[13] The presence of the lagged output gap term in the true social loss function (19) strengthens the motive for smoothing the evolution of the output gap under the optimal commitment policy.

As shown in the third row of panels in Figure 2, the speed limit policy can outperform inflation targeting under commitment for a moderate degree of habit persistence ($h = 0.7$) and low inflation inertia due to little or no price indexation. Abstracting from price indexation, the true social loss function resembles the objective function of the speed limit policy framework: A reasonably high degree of habit persistence implies that most of the weight is placed on the term $(x_t - hx_{t-1})^2$ in the true social loss function and the optimal speed limit policy under commitment mimics the optimal commitment policy. Overall, under commitment, the differences between speed limit policy and inflation targeting are much reduced for any degree of price indexation. Price level targeting performs relatively poorly under commitment for a high degree of price indexation as in the previous two model variations.

When policy is conducted under discretion, inflation targeting never outperforms the other two frameworks regardless of the degree of inflation indexation. Compared to the textbook NKM the differences between frameworks are of much larger magnitude. The advantage of speed limit policy and price level targeting over inflation targeting narrows considerably as the degree of price indexation ι_p approaches 1. However, the isomorphism of inflation targeting for $\iota_p = 1$ with price level targeting for $\iota_p = 0$ under discretion no longer applies in the presence of consumption habits. Higher inflation persistence as a result of indexation allows the discretionary inflation targeting central bank to commit indirectly to tighter monetary policy in the future after an inflationary shock. Yet, the expected future policy under inflation targeting is not tight enough. When consumption experiences habit persistence, the optimal commitment policy engages in more smoothing of the output gap which strengthens the motive of keeping monetary policy tight after an inflationary shock. The inflation targeting objective does not capture this additional motive and provides less stabilization of the economy.

[13] When habits are external, the decisions taken by the household members are not efficient under flexible prices even if a sales subsidy removes the distortions from monopolistic competition in the goods market. To render the steady state of the model efficient, we introduce a consumption tax; yet, the dynamics remain inefficient even for technology shocks. With the term $\frac{h\beta}{1-h\beta}g^*_{mu,t}$ entering equation (20) through the definition of the marginal cost term, $\widehat{mc}_t$, the central bank is unable to perfectly stabilize inflation and the welfare-relevant output gap in response to technology shocks. As discussed in Leith, Moldovan, and Rossi (2012) and Woodford (2003a), consumption habits have to be specified as internal in order for the divine coincidence to re-emerge.

2.3.4 Sticky wages

Sticky nominal wages as in Erceg, Henderson, and Levin (2000) are the final feature that we consider in isolation. In detail, the loss function can be shown to satisfy

$$L_t = (\sigma_L + \sigma_C)(x_t)^2 + \frac{1+\theta_p}{\theta_p \kappa_p}(\pi_{p,t} - \iota_p \pi_{p,t-1})^2 + \frac{1+\theta_w}{\theta_w \kappa_w}(\pi_{w,t} - \iota_w \pi_{p,t-1})^2 \tag{24}$$

while the structural equations are summarized by

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p \widehat{mc}_t + \beta E_t(\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t} \tag{25}$$

$$\widehat{mc}_t = \hat{\omega}_t - \hat{\xi}_{A,t} \tag{26}$$

$$(\pi_{w,t} - \iota_w \pi_{p,t-1}) = \kappa_w(\widehat{mrs}_t - \hat{\omega}_t) + \beta E_t(\pi_{w,t+1} - \iota_w \pi_{p,t}) + u_{w,t} \tag{27}$$

$$\widehat{mrs}_t - \hat{\omega}_t = (\sigma_L + \sigma_C) x_t - (\hat{\omega}_t - \hat{\omega}_t^*) \tag{28}$$

$$(\hat{\omega}_t - \hat{\omega}_t^*) = (\hat{\omega}_{t-1} - \hat{\omega}_{t-1}^*) + \pi_{w,t} - \pi_{p,t} - (\hat{\omega}_t^* - \hat{\omega}_{t-1}^*) \tag{29}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_C}(i_t - E_t \pi_{p,t+1} - g_{mu,t}^*). \tag{30}$$

The NKPC for wages, equation (27), links wage inflation, $\pi_{w,t}$, to the gap between the marginal rate of substitution (between consumption and leisure), $\widehat{mrs}_t$, and the real wage, $\hat{\omega}_t$. The policymaker places weight on stabilizing price and wage inflation with the weights inversely related to the slopes of the respective NKPCs. To maintain comparability with the previous models we focus on price markup shocks.

As in the model with flexible wages, a policy that promises to be tight in the future—summarized by the discounted sum of future (negative) output gaps in equation (31)—acts towards stabilizing the output gap, and (a weighted average of price and wage) inflation in the impact period:

$$\pi_{p,t} + \frac{\kappa_p}{\kappa_w}\pi_{w,t} = \kappa_p(\sigma_L + \sigma_C)x_t + \kappa_p(\sigma_L + \sigma_C)E_t\left[\sum_{j=1}^{\infty}\beta^j x_{t+j}\right] + E_t\left[\sum_{j=0}^{\infty}\beta^j u_{p,t+j}\right]. \tag{31}$$

The optimal split between movements in wage and price inflation depends on the relative stickiness between prices and wages as captured by the slope coefficients κ_p and κ_w and the evolution of the real wage. According to equation (25),

$$\pi_{p,t} = \kappa_p E_t\left[\sum_{j=0}^{\infty}\beta^j \omega_{t+j}\right] + E_t\left[\sum_{j=0}^{\infty}\beta^j u_{p,t+j}\right]. \tag{32}$$

If the central bank allows the real wage to fall persistently, it can lean against the initial rise in inflation. However, a decline in the future real wage also requires that prices rise faster than wages. If the policymaker places a high weight on stabilizing price inflation, the adjustment process has to operate more through wage inflation. Under the optimal commitment policy, tight monetary policy in the periods following an inflationary shock undoes almost all of the earlier changes in the price and wage level, but prices and wages are not stationary unless there is no inflation indexation, i.e., $\iota_p = \iota_w = 0$. The speed with which price and wage changes are undone depends on the degree of indexation. Unless both prices and wages are highly indexed, this process is rather fast. When prices and wages are fully indexed ($\iota_p = \iota_w = 1$), there is no partial undoing of earlier changes in prices and wages at all.

With these features of the optimal commitment policy in mind, we return to Figure 2. The fourth row of the figure shows that inflation targeting outperforms the other frameworks, when the policymaker can commit. To induce outcomes that are close to the optimal commitment policy, inflation targeting under commitment must place a sufficiently low weight on price inflation to prevent wages from carrying too much of the burden of the real wage adjustment. Overall, when the central bank implements its objective under commitment, the welfare differences across targeting frameworks are small and comparable to those in the previous models.

When the targeting frameworks are implemented under discretion, speed limit policy and price level targeting dominate inflation targeting—and for the case of no wage indexation depicted in Figure 2—this finding does not depend on the degree of price indexation. Given the features of the optimal commitment policy, price level targeting is best suited to stabilize the economy although it pushes prices and wages back to their long-run trends. Discretionary inflation targeting views all changes to prices and wages as permanent; promising to revert price inflation to its long-run target is not a sufficient deterrent against changes in prices and wages. Finally, the speed limit policy keeps the initial response of prices and wages in check as the private sector expects changes in the output gap to be smooth reflecting once again the idea to keeping future monetary policy tight after an inflationary shock. Overall, the welfare outcomes under the speed limit policy are close to those under price level targeting.

In contrast to the previous models, the relative performance of discretionary inflation targeting worsens when prices are increasingly indexed while keeping the degree of wage indexation unchanged. More price indexation implies more persistent price inflation after a markup shock, which leads the inflation targeting central bank wanting to stabilize price inflation more aggressively and thereby to put more burden on wage inflation in the adjustment process. The performance of inflation targeting improves for a higher degree of price indexation, when wage indexation is also high—in this case the optimal commitment policy ends up stabilizing inflation rates and does little to push prices and wages back towards their previous trends.[14]

Finally, if we keep the degree of price indexation constant and low, a higher degree of wage indexation implies a better relative performance of the optimal inflation targeting under discretion. An increase in wage indexation has little impact on the persistence of price inflation and on the optimal parameterization of the inflation targeting objective. Furthermore, changes in prices and wages are quickly pushed back under the optimal commitment policy. However, the welfare losses under each framework relative to the optimal commitment policy shrink since wage dispersion, measured by $\pi_{w,t} - \iota_w \pi_{p,t}$, drops for higher values of ι_w. While the welfare differences become smaller, the ranking of targeting frameworks is preserved.s

[14] If wage indexation is kept fixed at a high value, the advantage of speed limit policy and price level targeting over inflation targeting first increases as the degree of price indexation rises from 0 before eventually falling (and possibly turning negative) as the degree of price indexation approaches 1.

2.3.5 Comparison with Walsh (2003)

Walsh (2003) concludes that a high degree of price indexation is necessary in order for inflation targeting to outperform speed limit policy and price level targeting when policymakers cannot commit to future policy paths.[15] Our analysis generalizes this insight to the case of sticky wages: all prices and wages that experience nominal rigidities must be highly indexed for inflation targeting to perform strongly under discretion. Furthermore, our findings point to the role of consumption habits as increasing the central bank's motive for keeping future monetary policy tight after an inflationary shock to curb the dispersion of prices and wages. This feature is not captured in Walsh (2003) who assumes a model-invariant social loss function of the form $\pi_{p,t}^2 + \lambda\,(x_t^{gap})^2$ in departure from the linear-quadratic approximation of the preferences of the representative household.

3 Empirical models of the business cycle

Moving beyond the textbook NKM, we extend our analysis to the medium scale CEE/SW model which features sticky nominal prices and wages both with partial indexation to past inflation, physical capital and investment with capital utilization and investment adjustment costs, habit persistence in consumption, a variable elasticity of substitution between intermediate goods as in Kimball (1995) and the same for labor types, a distorted steady state, and shocks to technology, the risk premium, government spending, investment, price and wage markups, and monetary policy as detailed in Appendix D.

An important step in extending our analysis is to obtain a second-order accurate approximation to the preferences of the representative household. We follow a numerical approach. Let the $N \times 1$ vector of endogenous variables in the CEE/SW model be denoted by x_t, with the partition $x_t = (\tilde{x}_t', i_t)'$. The variable i_t is the policy instrument of the central bank. The vector ζ_t refers to the set of exogenous variables. Given the central bank's choice of the policy instrument for all periods $t \geq t_0$, $\{i_t\}_{t=t_0}^{\infty}$, the remaining $N-1$ endogenous variables satisfy the $N-1$ structural model equations

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \tag{33}$$

in equilibrium.

With the intertemporal preferences of society given by $\mathcal{U} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t)$, the optimal commitment policy is derived from the maximization program

$$\begin{aligned}
&\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \\
&g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}.
\end{aligned} \tag{34}$$

[15] See Appendix C for model details. Figure 14 replicates our analysis for the model in Walsh (2003) for both the case of discretion and commitment with the latter one not being included in Walsh (2003).

The constraint $g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}$ captures the policymaker's ability to pre-commit before the beginning of time in $t = t_0$ to embed the idea of *optimality from a timeless perspective* as in Woodford (2003a).[16]

Using the toolbox developed in Bodenstein, Guerrieri, and LaBriola (2014), the first-order conditions associated with the program in (34) can be used to obtain the purely quadratic approximation to the intertemporal preferences of society. The true social loss function

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} \hat{x}_t' A(L) \hat{x}_t + \hat{x}_t' B(L) \zeta_{t+1} \right] + \beta^{-1} \hat{\varphi}_{t_0-1}^{*\prime} C(0) \hat{x}_{t_0} \tag{35}$$

correctly ranks (the first-order accurate) outcomes $\{\hat{x}_t\}_{t=t_0}^{\infty}$ obtained under any monetary policy from the perspective of the optimal commitment policy (from a timeless perspective). The matrices $A(L)$ and $B(L)$ represent the approximation of the preferences with "L" denoting the lag-operator. As discussed in Benigno and Woodford (2012), the term $\beta^{-1} \hat{\varphi}_{t_0-1}^{*\prime} C(0) \hat{x}_{t_0}$ punishes violations of the pre-commitment constraint under the assessed policy in the case of discretion.[17] Appendix A provides the details of obtaining and evaluating the welfare criterion (35) and of solving for the decision rules under discretion and commitment.

As in Section 2, we compare the welfare implications under inflation targeting, speed limit policy, and price level targeting both under commitment and discretion. At times, we also report results from two nominal income targeting frameworks included in Walsh (2003):

1. nominal income targeting 1 (NIT)

$$L_t^{NIT} = \pi_{p,t}^2 + \lambda_x^{NIT} \left(\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1} \right)^2 \tag{36}$$

2. nominal income targeting 2 (NIT-II)

$$L_t^{NIT\text{-II}} = (x_t^{gap})^2 + \lambda_x^{NIT\text{-II}} \left(\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1} \right)^2 . \tag{37}$$

The optimal parameterization of a targeting framework, i.e., the optimal choice of λ_x^{TF}, minimizes the welfare distance between the targeting framework and the optimal commitment policy as measured by the welfare criterion in equation (35). In this section, we follow Smets and Wouters (2007) in measuring the output gap as the difference between actual output and the potential output defined as the output level that would have prevailed absent nominal rigidities and inefficient markup shocks to prices and wages.

Our analysis of targeting frameworks in the CEE/SW model proceeds as follows. First, we fix the parameters of the model at their posterior mode estimated in Smets and Wouters (2007). We then explore alternative parameterization of the CEE/SW model obtained by drawning from the Laplace approximation

[16] Benigno and Woodford (2012) and Debortoli and Nunes (2006) show that assuming policy to be conducted under suitable pre-commitments is generally needed to obtain a purely quadratic approximation to the preferences of the representative household. For the models in Section 2, the assumption of the timeless perspective is key for deriving the true social loss function when the steady state is not efficient; see also Appendix B.

[17] In practice, the correction term tends to be small. Although we did not emphasize this term in Section 2, we did include it in our computations when needed.

to the posterior distribution in Smets and Wouters (2007).

We close by assessing robustness of our findings along three dimensions. First, we compute optimal targeting frameworks for the CEE/SW model when the model is estimated with data for the euro area instead of the United States. Second, we investigate how our findings are affected by the difficulties of distinguishing between wage markup shocks and preference shocks that shift the marginal utility of labor. And finally, we explore the implications resulting from the zero lower bound on nominal interest rates.

3.1 Targeting frameworks in the CEE/SW model

Figure 3 summarizes our findings for the CEE/SW model. As before, we consider variations in the degrees of price and wage indexation. The top row of panels shows how the degree of price indexation ι_p impacts the relative ordering of the five targeting frameworks in the CEE/SW model. A vertical line marks the posterior mode of $\iota_p = 0.22$. The results nicely relate to our earlier findings. With consumption habits at 0.71 and sticky nominal wages, the optimal speed limit policy is a close second to inflation targeting when the policymaker acts under commitment. As price level targeting places too much importance on price stability and disregards the need to smooth the evolution of the output gap, the welfare outcomes are somewhat inferior. The two nominal income targeting frameworks are strictly outperformed by the speed limit policy and the price level targeting framework. The overall magnitude of the welfare differences is significantly larger in the CEE/SW model than in the simple NKMs, reflecting the presence of additional model features and shocks that introduce welfare-relevant policy trade-offs.

Under discretion, the speed limit policy framework strictly outperforms all other frameworks irrespective of the degree of price indexation. At the posterior mode parameterization of the model, the optimal speed limit policy exceeds welfare under inflation targeting by more than 0.30% of steady state consumption, whereas the advantage of the price level targeting framework over inflation targeting is a bit smaller with 0.25%. As in the textbook NKM with sticky wages, the advantage of the optimal speed limit policy over inflation targeting is larger when the degree of price indexation is higher while keeping the degree of wage indexation constant. Even the two nominal income targeting frameworks strongly outperform inflation targeting in the discretionary Markov equilibrium.

As shown in Figure 4, discretionary speed limit policy and price level targeting capture key features of the optimal commitment policy in the CEE/SW model in response to price markup and wage markup shocks. Given the estimated moderate degree of indexation ($\iota_p = 0.22$ and $\iota_w = 0.59$), price and wage dispersion are closely related to price and wage inflation, which are kept low by the promise of tight future monetary policy after an inflationary shock under the optimal commitment policy. As a result, the price and wage levels return slowly towards their pre-shock trends, although not completely. Noticeably, the speed limit policy considers deviations of price and wage inflation from their long-run target values bygones. However, given the built-in promise of keeping future policy tight after an inflationary shock this policy reduces overall inflation and the rise in the price and wage levels. Price level targeting as a monetary policy strategy signals tight monetary policy in response to inflationary shocks through explicitly promising to return prices and wages to their earlier trends. For a moderate degree of indexation, the resulting stabilization of price and wage inflation is close to optimal. By contrast, the inflation targeting objective does not include built-in features that would allow the central bank to promise tight future monetary policy in an environment with low to moderate inflation indexation under discretion. Thus, the inflation targeting central bank is less effective at stabilizing the economy: Inflation is persistently higher and the output gap drops by more on impact compared to the optimal commitment policy and the other targeting frameworks in Figure 4.

The CEE/SW model abstracts from taxes/subsidies that could correct the distortions associated with monopolistic competition in the production of intermediate goods and the labor market. The second row of panels in Figure 3 reveals that if these distortions are removed, inflation targeting improves its relative performance slightly.

As for the textbook NKM with sticky wages, we vary the degree of wage indexation in the bottom row of panels. Varying the degree of wage indexation away from its posterior mode of $\iota_w = 0.59$ while keeping the degree of price indexation at its posterior mode of $\iota_p = 0.22$ reveals that a lower degree of wage indexation goes along with a relatively poorer performance of inflation targeting under discretion as in the previous section. Under commitment, changing the degree of wage indexation impacts the relative performance of the frameworks in a manner similar to changes in price indexation.

3.2 Deconstructing the results

While the outcomes in the CEE/SW model resemble those in Section 2, we also consider one of the many sequences of expanding the textbook NKM step-by-step to the CEE/SW model. We present results for the case of discretion. Figure 5 plots the CEV values for each framework relative to the inflation targeting framework under discretion. Starting from the textbook NKM with preferences being specified as in Smets and Wouters (2007)—titled SW–Woodford—and using the parameters estimated by Smets and Wouters (2007) where applicable we introduce the following changes step-by-step:

- remove taxes/subsidies for intermediate goods,
- government spending, physical capital and investment, including capital utilization and investment adjustment costs, and related shocks,
- sticky wages (with a wage subsidy to offset distortions in the labor market and no wage markup shock),
- a wage markup shock,
- remove the wage subsidy,
- habit persistence,
- a higher degree of nominal rigidities measured by the probabilities of not adjusting prices or wages optimally from $\xi_p = 0.65$ and $\xi_w = 0.73$ to $\xi_p = 0.85$ and $\xi_w = 0.88$, respectively, in order to match the slopes of the NKPC between a model with and without a variable elasticity of substitution (Kimball aggregator),

- a variable elasticity of substitution as in Kimball (1995).

The figure confirms the importance of indexation, sticky wages, and habit persistence in determining the ranking of targeting frameworks under discretion. Absent sticky wages, a higher degree of price indexation plays out in favor of inflation targeting under discretion. In the presence of sticky nominal wages this finding is overturned. Furthermore, the magnitude of welfare differences increases with sticky wages and the associated wage markup shocks. Habit persistence in consumption raises the overall welfare costs of not implementing the optimal commitment policy and thus the advantage of speed limit policy and price level targeting over inflation targeting. With the true social loss function featuring an explicit motive for smoothing the quasi-difference in the output gap, the speed limit policy gets even closer to the price level targeting framework. The role of capital accumulation and investment adjustment costs on the quantitative differences between targeting frameworks is relatively minor.

In addition to the features discussed in Section 2, the variable elasticity of substitution is the other feature of quantitative importance as it increases the strategic complementarity in price setting. The Kimball aggregator impacts our outcomes mostly through changing the slope of the NKPCs. Moving from the bottom left panel in the figure (constant elasticity of substitution and $\xi_p = 0.65$ and $\xi_w = 0.73$) to the bottom right panel (variable elasticity of substitution and $\xi_p = 0.65$ and $\xi_w = 0.73$) directly, the welfare differences between price level targeting (or speed limit policy) and inflation targeting triple. Yet, considering the intermediate step of the middle panel (constant elasticity of substitution and $\xi_p = 0.85$ and $\xi_w = 0.88$) reveals that this increase could also be obtained by raising the degree of nominal rigidities while keeping the slopes of the NKPCs the same between the last two panels. Similar conclusions regarding the importance of the various model features emerge when we change the sequence of introducing them or when policymakers act under commitment.

3.3 Robustness to alternative parameterizations

To explore the sensitivity of our findings to alternative, yet empirically plausible, parameter choices. We draw 30000 parameter specifications from the Laplace approximation to the posterior distribution Smets and Wouters (2007) and we

1. compute the optimal weights on the activity measure in the objective functions, λ_x^{TF}, associated with inflation targeting, speed limit policy, and price level targeting for each parameter draw and compare welfare for each parameter draw under these optimal weights,

2. compare welfare across targeting frameworks for each parameter draw when the weights on the activity measure in the objective function are fixed at specific values.

We exclude the NIT and NIT-II framework from this exercise as they were strictly dominated by price level targeting and speed limit policy.

The first experiment, referred to as the "optimal weights case," confirms that the ordering of targeting frameworks is robust to alternative empirically plausible parameterizations of the CEE/SW model. Figure 6 plots the distribution of welfare losses relative to the optimal commitment policy (expressed in CEV) for each draw of parameters and targeting framework. Under commitment (the top row of panels), the distribution of welfare losses is similar across targeting frameworks, although the losses tend to be slightly smaller under inflation targeting. The median loss under inflation targeting is -0.0288, whereas it reaches -0.0538 under price level targeting and -0.0454 under the speed limit policy. Large losses are rare for all frameworks. Table 2 Panel (a) reports the frequency with which each of the frameworks performs better than the remaining two. The optimal inflation targeting framework emerges as the winner for 97% of the parameter draws. Table 2 Panel (d) is designed to shed light on the magnitude of the welfare differences. For each draw of parameters we compute the welfare difference between a given targeting framework and the best performing framework of the remaining two and report the percentiles of the resulting distribution of welfare differences in increasing order. Since inflation targeting almost always performs best, when policymakers can commit, the differences reported in columns 3 and 4 basically coincide with the differences between price level targeting and inflation targeting and between the speed limit policy and inflation targeting, respectively. Only for 5% of the parameter draws does the difference between the price level targeting and the inflation targeting framework exceed -0.0493; for the speed limit policy framework, the value is even smaller with -0.0280. For the inflation targeting framework, the advantage over the next best targeting framework is smaller than 0.0280 for about 95% of the draws. The values at the nth percentile for column 2 (IT) and the $(100-n)$th percentile for column 4 (SLP) indicate that the speed limit policy framework is the second-best performing framework for most parameter draws.

Under discretion, the distributions of welfare losses induced by the three targeting frameworks look much less alike. In Figure 6 (the middle row of panel), the distribution of welfare losses relative to the optimal commitment policy is noticeably more dispersed for price level targeting and, in particular, for inflation targeting than under commitment. By contrast, the distribution under the speed limit policy is more concentrated, an observation leading us to speculate whether the optimal speed limit policy under discretion may deliver better welfare outcomes (1) than the optimal speed limit policy under commitment, and (2) than optimal inflation targeting under commitment. The first claim is true for any parameterization we consider; the second claim is true for more than 50% of the parameter draws and in particular it is true when the parameters in the CEE/SW model are fixed at their posterior mode. Table 2, Panel (a) further reveals the superiority of speed limit policy under discretion. It is found to perform better than inflation targeting and price level targeting for most parameter draws (around 98%). As shown in Panel (d), the advantage of the speed limit policy framework over the inflation targeting framework can be sizeable (column 5). Although price level targeting performs consistently better than inflation targeting under discretion, it rarely performs best (column 6).

The final row of Figure 6 plots the cumulative distribution functions of the optimal weights on the activity measure. For each framework, the optimal weights tend to be larger and the distributions of weights are more dispersed under commitment than under discretion. For example, the median weight under the speed limit policy framework is $\lambda_x^{SLP} = 11.86$ for commitment, but only $\lambda_x^{SLP} = 3.39$ for discretion.

The robust performance of the speed limit policy framework across commitment and discretion not only applies to a wide range of empirically plausible parameterizations of the CEE/SW model when the weights on the activity measure are set optimally for each draw and framework. Our second set of experiments finds that the performance of the speed limit policy framework is also less sensitive to the exact choice of the weight on the activity measure: (1) We fix the weight on the activity measure for each targeting framework at the value found to be optimal when the parameters in the CEE/SW model are fixed at their posterior mode (under commitment and discretion, respectively) and compute the welfare losses relative to the optimal commitment policy for each of the 30000 parameter draws. (2) We fix the weight on the activity measure for each targeting framework at the value found to be optimal under commitment (discretion) when the parameters in the CEE/SW model are fixed at their posterior mode and compute the welfare losses relative to the optimal commitment policy for each of the 30000 parameter draws, but solve the model under the assumption that the policymaker acts under discretion (commitment). Subsequently, we refer to (1) as the "fixed weights case" and to (2) as the "exchanged weights case."

As reported in Table 2, Panel (b), in the fixed weights case, the speed limit policy performs best for 16.5% of the parameter draws under commitment—up from 2.7% in the original experiment—and it maintains its superior performance under discretion by outperforming the other frameworks for 98% of the draws. Figure 7 plots the distribution of welfare losses under the fixed weights case relative to the optimal weights case. The welfare losses that are caused by the policymaker using the optimal weights for a given parameter draw are small under commitment across regimes, but are often sizeable under discretion for both price level targeting and, in particular, inflation targeting.

The exchanged weights case explores the sensitivity of the targeting frameworks to both parameter uncertainty and uncertainty about the ability of the policymaker to commit. As shown in Table 2, Panel (c) when policy is conducted under commitment, but the policymaker uses the weights found to be optimal under discretion for the posterior mode parameterization of the CEE/SW model, the speed limit policy framework performs best for 99% of the parameter draws. Under discretion, the speed limit policy framework performs best for 98% of the draws. Figure 8 also plots the distribution of welfare losses under the exchanged weights case relative to the optimal weights case for each framework. The inflation targeting framework is very sensitive to getting the weight on the activity measure right as evidenced by the high share of large welfare losses exceeding 1% (measured as CEV) for more than 50% of the parameter draws. Under the speed limit policy framework such large losses are never observed.

The speed limit policy framework emerges as the most desirable setting in our analysis of the CEE/SW model. Across parameterizations, the optimal speed limit policy consistently outperforms the inflation targeting and the price level targeting framework under discretion; under commitment the speed limit policy framework is a very close second to the inflation targeting framework; the optimal speed limit policy framework implemented under discretion delivers higher social welfare than optimal inflation targeting under commitment. Finally, the performance of the economy under a speed limit policy is much less sensitive to the exact parameterization of the objective function which is of relevance if the policymaker faces uncertainty about the correct specification of the economy.

3.4 Additional robustness checks

We conclude our analysis with robustness checks regarding the data used to estimate the model, the role of the relative importance of labor supply shocks versus wage markup shocks, and the limitations of monetary policy imposed by the zero lower bound constraint on the nominal interest rate.

3.4.1 Robustness to alternative data

Smets and Wouters (2007) estimated the CEE/SW model using U.S. data. Figure 9 compares the performance of all five targeting frameworks when the CEE/SW model is estimated using data for the euro area instead.[18]

Qualitatively, the results for the euro area are similar to those derived from U.S. data. From a quantitative perspective, the case for price level targeting and speed limit policy is even stronger. Their advantage over inflation targeting measured in terms of steady state consumption doubles under discretion. Under commitment the inflation targeting framework maintains a small advantage over speed limit policy and price level targeting.

3.4.2 Shocks to labor supply and wage markups

Chari, Kehoe, and McGrattan (2009) point to an identification problem in the CEE/SW model that preference shocks shifting the marginal disutility of labor cannot be easily distinguished from wage markup shocks. Gali, Smets, and Wouters (2011) and Justiniano, Primiceri, and Tambalotti (2013) impose assumptions to overcome this identification problem.[19] While in comparison to the CEE/SW model wage markup shocks play a less important role in both these papers, wage markup shocks continue to contribute significantly to the fluctuations in inflation in Gali, Smets, and Wouters (2011). Given the different welfare implications of the inefficient wage markup shocks, which creates a monetary policy trade off, and the efficient labor supply shocks, the relative importance of these two shocks may influence the ranking of targeting frameworks.

Figure 10 provides a preliminary inquiry into the importance of the issues raised by Chari, Kehoe, and McGrattan (2009) for the ranking of frameworks. We compute the welfare differences between targeting frameworks by changing the relative importance of wage markup and labor supply shocks. Following Gali, Smets, and Wouters (2011) and Justiniano, Primiceri, and Tambalotti (2013), we model the labor supply shock as a shock to the marginal disutility of labor. The labor supply shock is specified to match the unconditional variance of the wage markup shock and to induce responses similar in magnitude to those

[18] Smets and Wouters (2005) estimate a medium-scale DSGE model for the euro area, but the details of the model differ from those in Smets and Wouters (2007). To maintain comparability of results, we estimate the model specified in Smets and Wouters (2007) using data for the euro area from the Area Wide Model database (see Fagan, Henry, and Mestre (2005)). Data on consumption, investment, GDP, hours and wages are expressed in 100 times the log. Inflation is the first difference of the log GDP deflator. The interest rate is the short-term interest in the AWM database. As stated in Smets and Wouters (2005), total employment data is used in place of hours worked due to the absence of hours worked data for the euro area.

[19] Gali, Smets, and Wouters (2011) obtain identification by embeding a theory of unemployment and by including data on unemployment. Justiniano, Primiceri, and Tambalotti (2013) do not exploit the connection between unemployment and wage markups and assume instead a particular stochastic structure for the latter (white noise) to obtain identification.

induced by the wage markup shock. The relative weight on the labor supply shock depicted along the horizontal axis governs the relative importance of the two shocks.

Both for the commitment and the discretion case, the ranking of targeting frameworks is independent of the relative importance of wage markup and labor supply shocks with the exception of the NIT and the NIT-II framework for the case of discretion and a high importance of the labor supply shock. As the importance of the inefficient wage markup shock is reduced, the welfare differences between targeting frameworks shrink by construction. Monetary policy can mostly offset the welfare consequences of the labor supply shock; when wage markup shocks are absent from the model, price markup shocks are the only remaining source of inefficient fluctuations.

As long as one believes wage markup shocks to play some role in driving business cycle fluctuation as in Gali, Smets, and Wouters (2011), the speed limit policy framework under discretion strongly outperforms all other frameworks under discretion (as well as the inflation targeting framework under commitment). But even for the assessment in Justiniano, Primiceri, and Tambalotti (2013), which assigns little importance to wage markup shocks, the speed limit policy framework performs well. Absent certainty about the true data-generating process, adopting the speed limit policy framework may turn out to be a prudent choice.

3.4.3 Zero lower bound on nominal interest rates

Following earlier work on optimal policy design, we have abstracted from the implications for monetary policy imposed by the zero lower bound on the nominal interest rate. This way of preceding allows us to include larger models and to consider aspects of parameter uncertainty. Furthermore, the probability of the policy rate reaching zero (and staying at zero for several periods) is low in the CEE/SW model. As long as the time that the economy spends at the zero bound is short, economic outcomes when the zero bound is enforced barely differ from the outcomes when the policy rate is allowed to violate the zero bound. Thus, the optimal parameterization of each targeting framework is expected to change by little if we were to impose the zero bound in our analysis. Nevertheless, we want to touch on the challenges for monetary policy design presented by the zero bound at least in closing.

Figure 11 plots the impulse responses of selected variables to a combination of contractionary demand shocks under the optimal commitment policy. The figure also plots the responses under inflation targeting, price level targeting, and the speed limit policy: the policymaker acts under discretion, the model parameters are fixed at the posterior mode, and the objective functions are parameterized as found to be optimal absent the zero bound constraint.[20] In response to the shock, the optimal commitment policy lowers the short-term interest rate to zero, although not for long, and allows for mild deflation of prices and wages. The output gap turns negative and closes slowly. Further out, the optimal commitment policy allows for only very minor overshooting of price and wage inflation above their long-run target values and the output

[20] Initially, the economy is assumed to be growing along the balanced growth path. In period 1 the economy experiences a negative one-standard deviation risk-premium shock together with a negative 10-standard deviation shock to government spending. In addition, we lowered the value of the nominal interest rate along the balanced growth path to 4%. The problem is solved using the piece-wise linear approach in Eggertsson and Woodford (2003), Coibion, Gorodnichenko, and Wieland (2012), and Guerrieri and Iacoviello (2015); we abstract from modifications of the social loss function that could result from the zero bound constraint.

gap hardly rises above zero.

Although operated under discretion, all three targeting frameworks perform closely to the optimal commitment policy. The inflation and price level targeting central banks are more aggressive at stabilizing price and wage inflation and the output gap. As the shock pushes price and wage inflation, and the output gap in the same direction, the high relative weight on price inflation in the objective function of the inflation targeting central bank allows the inflation targeting central bank to mimic the behavior of the price level targeting central bank.[21]

The optimal speed limit policy computed in Section 3.1 allows for larger deviations of inflation and the output gap than the optimal commitment policy. Under the speed limit policy, the policymaker seeks to adjust the output gap gradually. While such gradualism is of advantage in response to price and wage markup shocks—keeping the output gap negative after an inflationary shocks signals tight future monetary policy and reduces the initial rise in inflation—it is of potential disadvantage after large demand shocks that push the policy interest rate to zero. The slow closing of the output gap under the speed limit policy prevents price and wage inflation from a fast return to their long-run targets. Shocks that are more contractionary than the ones underlying Figure 11 can exacerbate this feature of the speed limit policy.

This potential drawback of the speed limit policy can be ameliorated by reducing the weight on the activity measure in the objective function. To convey this idea, Figure 11 also plots the impulse responses under a speed limit policy with a reduced weight on the output gap under the label Alt. SLP (that is one tenth of the weight found to be optimal in Section 3.1). Under the reduced weight, the speed limit policy closely resembles the optimal commitment policy. While the dramatic reduction in weight on the activity measure worsens the performance of the speed limit policy to price and wage markup shocks in particular, this specific parameterization of the speed limit policy still outperforms the optimal inflation and the optimal price level targeting framework under discretion computed in Section 3.1 for the posterior mode parameterization of the model.[22]

The optimal parameterization let alone the ranking of targeting regimes in the CEE/SW model may hardly be affected if we enforced the zero bound constraint on nominal interest rates. If shocks that call for lowering the policy interest rates to zero are more frequent than in the CEE/SW model, price level targeting might be preferred to the speed limit policy under discretionary policymaking given a low value of the long-run inflation target. However, raising the long-run inflation target may constitute a viable alternative: the monetary authority can adopt a speed limit policy which is effective in ameliorating the time inconsistency problem associated with price and wage markup shocks while significantly reducing the likelihood of zero bound events. Whether these benefits outweigh the costs of achieving a long-run inflation

[21] This result is not at odds with Adam and Billi (2007) or Bodenstein, Hebden, and Nunes (2012) who point out the importance of commitment at the zero lower bound when the central bank maximizes the discounted utility of the representative household. In our application, the discretionary central bank places a higher weight on stabilizing price inflation than under the true social loss function and is therefore much better positioned to stabilize the economy through accommodative monetary policy than in those papers for the case of discretion.

[22] Abstracting from the zero lower bound, the optimal parameterization of each framework is primarily determined by the price and wage markup shocks. Ironically, the optimal weight on the activity measure under the speed limit policy is higher when these markup shocks are more important which in turn impedes the central bank's ability to stabilize the economy in the face of large negative demand shocks and zero interest rates.

target is an empirical question beyond the scope of this section.[23]

4 Conclusion

The debate on targeting frameworks has often focused on the differences between inflation and price level targeting. In models that follow the New Keynesian paradigm, the optimal commitment policy tends to undo most, if not all, changes of price and wage inflation from their long-run targets over time to realign prices and wages with their long-run trends. Given this insight, price level targeting appears to be a natural contender to inflation targeting when policymakers act under discretion.

However, we argue that speed limit policy is a clear alternative to both the inflation targeting and the price level targeting framework. The objective function underlying the speed limit policy framework with its long-run commitment to stable inflation and its short-run focus on inflation and smooth changes in the output gap leads to better outcomes than all other frameworks when policymakers act under discretion in many circumstances. When policymakers act under commitment, the differences between the three targeting frameworks are negligible. Most importantly, the speed limit policy under discretion outperforms inflation targeting under commitment in numerous cases. We show the relative superiority of the speed limit policy framework in a sequence of simple NK models, that introduce inflation indexation, habit persistence in consumption, and sticky wages, and in the CEE/SW model. The optimal speed limit policy is more robust to empirically-relevant alternative parameterizations of the CEE/SW model and to unclarity about the ability of the central bank to commit. Unless the economy can experience large and persistent negative (demand) shocks and the costs of raising the long-run inflation target are high, the speed limit policy will also outperform inflation and price level targeting under discretion when the zero lower bound constraint on nominal interest rates is enforced in the model.

Since speed limit policies have not yet been as thoroughly examined as inflation and price level targeting, a range of open questions remain to be addressed. How would a speed limit policy perform under model settings that included informational rigidities, or financial frictions? How does a central bank's ability to measure the output gap accurately in real-time—an issue explored in Orphanides (2003)— influence the relative performance of targeting frameworks? What about central bank communication of current and future policy goals? Given the promising performance of speed limit policies shown in this paper, it appears worth to continue exploring the implications of this policy and find answers to the preceding questions.

[23] Pursuing higher inflation targets has captured the imagination of policymakers in the aftermath of the Great Recession, see Williams (2016). Coibion, Gorodnichenko, and Wieland (2012) compute the optimal inflation target for a discretionary central bank to fall just below 3%; Billi (2011) reports significantly higher numbers.

References

Adam, K. and R. M. Billi (2007). Discretionary monetary policy and the zero lower bound on nominal interest rates. *Journal of Monetary Economics 54*(3), 728–752. [26]

Anderson, E. W., E. R. McGrattan, L. P. Hansen, and T. J. Sargent (1996). Mechanics of forming and estimating dynamic linear economies. In H. M. Amman, D. A. Kendrick, and J. Rust (Eds.), *Handbook of Computational Economics*, Volume 1 of *Handbook of Computational Economics*, Chapter 4, pp. 171–252. Elsevier. [49]

Barro, R. J. and D. B. Gordon (1983). A Positive Theory of Monetary Policy in a Natural Rate Model. *Journal of Political Economy 91*(4), 589–610. [8]

Benigno, P. and M. Woodford (2005). Inflation Stabilization And Welfare: The Case Of A Distorted Steady State. *Journal of the European Economic Association 3*(6), 1185–1236. [12, 45, 63, 64]

Benigno, P. and M. Woodford (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory 147*(1), 1–42. [18, 44, 45, 47, 48]

Bernanke, B. S. and F. S. Mishkin (1997). Inflation Targeting: A New Framework for Monetary Policy? *Journal of Economic Perspectives 11*(2), 97–116. [3]

Bilbiie, F. O. (2014). Delegating optimal monetary policy inertia. *Journal of Economic Dynamics and Control 48*, 63–78. [2, 8]

Billi, R. M. (2011). Optimal Inflation for the US Economy. *American Economic Journal: Macroeconomics 3*(3), 29–52. [27]

Blanchard, O. and J. Gali (2007). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking 39*(s1), 35–65. [7, 63]

Bodenstein, M., L. Guerrieri, and J. LaBriola (2014). Macroeconomic Policy Games. Finance and Economics Discussion Series 2014-87, Board of Governors of the Federal Reserve System (U.S.). [18, 46, 47, 48]

Bodenstein, M., J. Hebden, and R. Nunes (2012). Imperfect credibility and the zero lower bound. *Journal of Monetary Economics 59*(2), 135–149. [26]

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398. [4, 56, 57, 80, 81]

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2009). New Keynesian Models: Not Yet Useful for Policy Analysis. *American Economic Journal: Macroeconomics 1*(1), 242–66. [4, 24, 66]

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy 113*(1), 1–45. [2, 4, 6, 12]

Clarida, R., J. Gali, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature 37*(4), 1661–1707. [2, 8]

Coibion, O., Y. Gorodnichenko, and J. Wieland (2012). The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?

Review of Economic Studies 79(4), 1371–1406. [25, 27]

Debortoli, D., J. Kim, J. Lindé, and R. Nunes (2015). Designing a Simple Loss Function for the Fed: Does the Dual Mandate Make Sense? Cepr discussion papers, C.E.P.R. Discussion Papers. [4]

Debortoli, D. and R. Nunes (2006). On Linear Quadratic Approximations. MPRA Paper 544, University Library of Munich, Germany. [18, 45]

Dennis, R. (2007). Optimal Policy In Rational Expectations Models: New Solution Algorithms. *Macroeconomic Dynamics 11*(01), 31–55. [51]

Eggertsson, G. B. and M. Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity 34*(1), 139–235. [25]

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics 46*(2), 281–313. [15, 55, 66]

Fagan, G., J. Henry, and R. Mestre (2005). An area-wide model for the euro area. *Economic Modelling 22*(1), 39–59. [24]

Gali, J. (Ed.) (2008). *Monetary Policy, inflation, and the Business Cycle*. Princeton University Press. [4, 55, 59, 66]

Gali, J., F. Smets, and R. Wouters (2011). Unemployment in an Estimated New Keynesian Model. In *NBER Macroeconomics Annual 2011, Volume 26*, NBER Chapters, pp. 329–360. National Bureau of Economic Research, Inc. [24, 25]

Gaspar, V., F. Smets, and D. Vestin (2007). Is time ripe for price level path stability? Working Paper Series 0818, European Central Bank. [12]

Guerrieri, L. and M. Iacoviello (2015). OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily. *Journal of Monetary Economics 70*(C), 22–38. [25]

Hammond, G. (2012). *State of the art of inflation targeting*. Number 29 in Handbooks. Centre for Central Banking Studies, Bank of England. [5]

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013). Is There a Trade-Off between Inflation and Output Stabilization? *American Economic Journal: Macroeconomics 5*(2), 1–31. [4, 24, 25]

Kimball, M. S. (1995). The Quantitative Analytics of the Basic Neomonetarist Model. *Journal of Money, Credit and Banking 27*(4), 1241–77. [17, 21, 37, 80]

King, M. (1997). Changes in UK monetary policy: Rules and discretion in practice. *Journal of Monetary Economics 39*(1), 81–97. [2]

King, M. (2004). The Institutions of Monetary Policy. NBER Working Papers 10400, National Bureau of Economic Research, Inc. [3]

Leith, C., I. Moldovan, and R. Rossi (2012). Optimal Monetary Policy in a New Keynesian Model with Habits in Consumption. *Review of Economic Dynamics 15*(3), 416–435. [14, 59, 63]

Nessen, M. and D. Vestin (2005). Average Inflation Targeting. *Journal of Money, Credit and Banking 37*(5), 837–63. [2]

Orphanides, A. (2003). The quest for prosperity without inflation. *Journal of Monetary Economics 50*(3), 633–663. [27]

Otrok, C. (2001). On measuring the welfare cost of business cycles. *Journal of Monetary Economics 47*(1), 61–92. [54]

Rogoff, K. (1985). The Optimal Degree of Commitment to an Intermediate Monetary Target. *The Quarterly Journal of Economics 100*(4), 1169–89. [2, 3, 8]

Roisland, O. (2005). Inflation inertia and the optimal hybrid inflation/price-level target. Working Paper 2005/4, Norges Bank. [12]

Smets, F. and R. Wouters (2005). Comparing shocks and frictions in US and euro area business cycles: a Bayesian DSGE Approach. *Journal of Applied Econometrics 20*(2), 161–183. [4, 24]

Smets, F. and R. Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review 97*(3), 586–606. [2, 3, 12, 13, 18, 19, 20, 21, 24, 32, 35, 37, 38, 39, 40, 42, 50, 54, 65, 79]

Svensson, L. E. (2010). Inflation Targeting. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3 of *Handbook of Monetary Economics*, Chapter 22, pp. 1237–1302. Elsevier. [2, 5]

Svensson, L. E. O. (1997). Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts. *American Economic Review 87*(1), 98–114. [2]

Svensson, L. E. O. (1999). How should monetary policy be conducted in an era of price stability? *Proceedings - Economic Policy Symposium - Jackson Hole*, 195–259. [2]

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*(1), 195–214. [5]

Vestin, D. (2006). Price-level versus inflation targeting. *Journal of Monetary Economics 53*(7), 1361–1376. [2, 8, 72]

Walsh, C. (2003). Speed Limit Policies: The Output Gap and Optimal Monetary Policy. *American Economic Review 93*(1), 265–278. [2, 4, 17, 18, 72, 76, 77]

Walsh, C. (Ed.) (2010). *Monetary Theory and Policy*. MKT Press. [4]

Williams, J. C. (2016). Monetary Policy in a Low R-star World. *FRBSF Economic Letter*. [27]

Woodford, M. (1999). Commentary: how should monetary policy be conducted in an era of price stability? *Proceedings - Economic Policy Symposium - Jackson Hole*, 277–316. [6]

Woodford, M. (Ed.) (2003a). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press. [4, 6, 8, 14, 18, 45, 53, 59, 61, 63, 66]

Woodford, M. (2003b). Optimal Interest-Rate Smoothing. *Review of Economic Studies 70*(4), 861–886. [2]

Table 1: Parameter values for the textbook NKM and its extensions

		Parameters governing											
		sticky prices				sticky wages				other			
Model	description	ξ_p	ι_p	θ_p	$\bar{\tau}_p$	ξ_w	ι_w	θ_w	$\bar{\tau}_w$	h	σ_C	σ_L	β
1	textbook model	0.8	0	0.61	0.61	–	–	–	–	0	1.39	1.92	0.9984
2	price indexation	0.8	[0; 1]	0.61	0.61	–	–	–	–	0	1.39	1.92	0.9984
3	inefficient steady state	0.8	[0; 1]	0.61	0	–	–	–	–	0	1.39	1.92	0.9984
4	consumption habits	0.8	[0; 1]	0.61	0.61	–	–	–	–	0.7	1.39	1.92	0.9984
5	sticky wages	0.8	[0; 1]	0.61	0.61	0.8	0	0.5	0.5	0	1.39	1.92	0.9984

Note: The table documents the parameter values of the textbook NKM and its extensions underlying Figures 1 and 2. Model 1 is the textbook NKM without indexation. In Model 2 we augment the textbook NKM to allow for price indexation. Model 3 features distortions in the steady state. Habit persistence in consumption is introduced in Model 4. Finally, Model 5 allows for sticky nominal prices and wage. In all models, an ARMA(1,1) price markup shock is the sole source of fluctuations with the autocorrelation coefficient $\rho_u = 0.9$, the moving average coefficient $\rho_{u\epsilon} = 0.74$, and the standard deviation for innovations $\sigma_u = 0.0014$.

Table 2: Performance of targeting frameworks under parameter uncertainty

a: Frequency of being the best framework: optimal weights case

	IT	PLT	SLP
Commitment	0.9723	0.0004	0.0273
Discretion	0.0000	0.0167	0.9833

b: Frequency of being the best framework: fixed weights case

	IT	PLT	SLP
Commitment	0.8281	0.0073	0.1646
Discretion	0.0000	0.0168	0.9832

c: Frequency of being the best framework: exchanged weights case

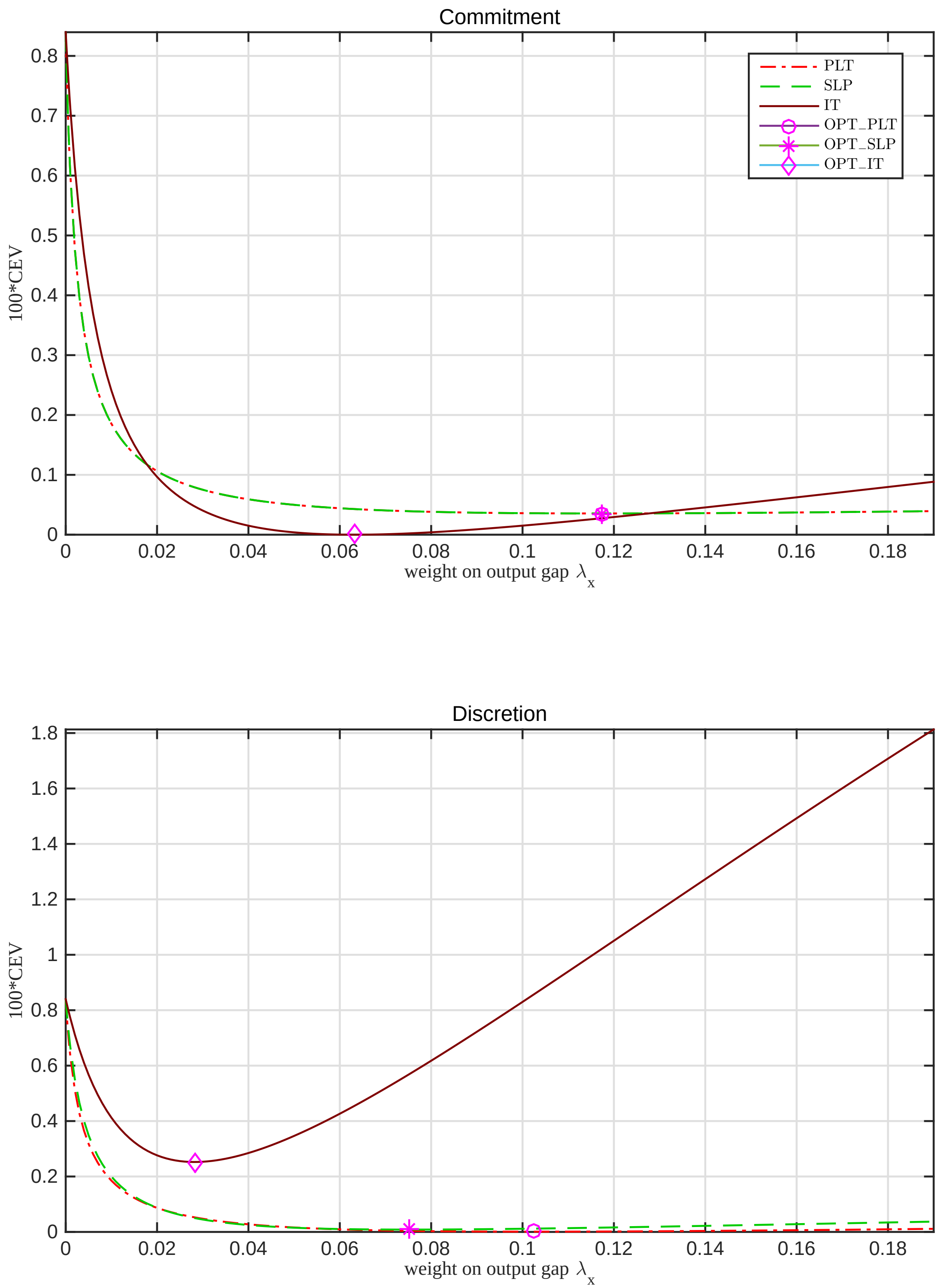
	IT	PLT	SLP
Commitment	0.0036	0.0056	0.9908
Discretion	0.0000	0.0162	0.9838

d: Percentiles of CEV differences

	Commitment			Discretion		
Quantile	IT	PLT	SLP	IT	PLT	SLP
5%	0.0022	-0.0493	-0.0280	-0.7717	-0.2169	0.0114
10%	0.0055	-0.0413	-0.0244	-0.6086	-0.1621	0.0193
15%	0.0081	-0.0368	-0.0222	-0.5291	-0.1350	0.0258
20%	0.0098	-0.0338	-0.0207	-0.4772	-0.1172	0.0310
25%	0.0110	-0.0316	-0.0194	-0.4349	-0.1035	0.0362
30%	0.0119	-0.0297	-0.0184	-0.4013	-0.0927	0.0409
35%	0.0128	-0.0280	-0.0174	-0.3728	-0.0837	0.0458
40%	0.0135	-0.0264	-0.0166	-0.3475	-0.0757	0.0510
45%	0.0142	-0.0250	-0.0158	-0.3253	-0.0685	0.0565
50%	0.0150	-0.0237	-0.0150	-0.3053	-0.0624	0.0624
55%	0.0158	-0.0226	-0.0143	-0.2856	-0.0565	0.0685
60%	0.0165	-0.0213	-0.0136	-0.2674	-0.0510	0.0757
65%	0.0174	-0.0202	-0.0128	-0.2483	-0.0458	0.0837
70%	0.0183	-0.0191	-0.0120	-0.2294	-0.0409	0.0927
75%	0.0193	-0.0178	-0.0110	-0.2105	-0.0362	0.1035
80%	0.0206	-0.0165	-0.0099	-0.1910	-0.0310	0.1172
85%	0.0222	-0.0149	-0.0082	-0.1675	-0.0258	0.1350
90%	0.0243	-0.0130	-0.0056	-0.1388	-0.0193	0.1621
95%	0.0279	-0.0094	-0.0023	-0.0970	-0.0114	0.2169

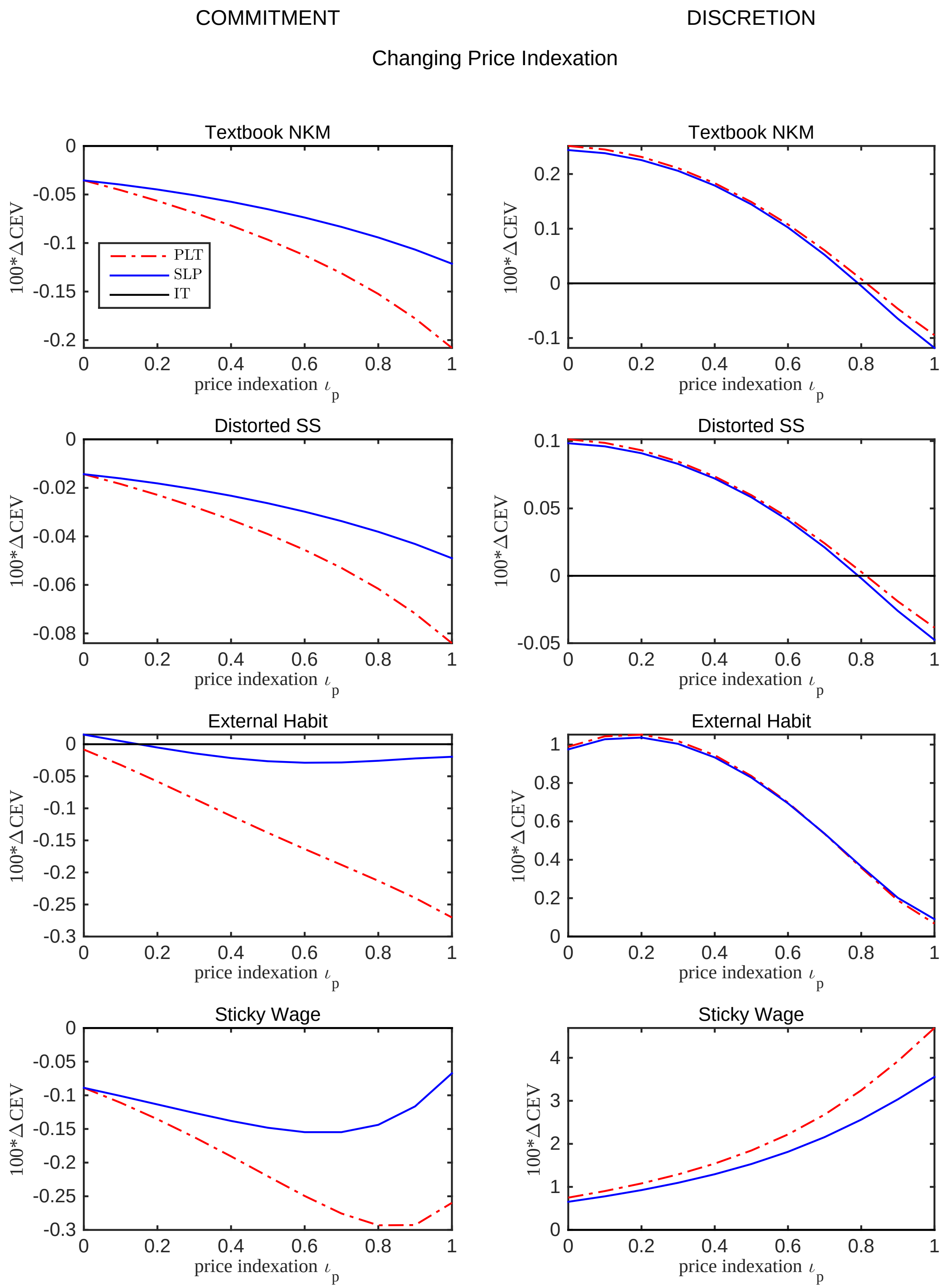
Note: The table summarizes the performance of inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the parameters of the CEE/SW model are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). Panel (a) states the frequency of each targeting regime being the best performing one for both the case of commitment and discretion. The weight on the activity measure λ_x^{TF} is chosen optimally for each framework and each parameter draw. In Panel (b) the weight on the activity measure λ_x^{TF} is fixed for each framework at the value that is optimal when the model is parameterized at the posterior mode. All other parameters are drawn from the Laplace approximation to the posterior distribution. In Panel (c) when policy is conducted under commitment (discretion) the weight on the activity measure λ_x^{TF} is fixed for each framework at the value that is optimal under discretion (commitment) for the posterior mode parameterization of the model. All other parameters are drawn from the Laplace approximation to the posterior distribution. In Panel (d), we first compute the CEV difference between the best performing and the second best performing framework for each parameterization; we then rank the differences by size for each framework and compute percentiles.

Figure 1: Targeting frameworks in the textbook NKM



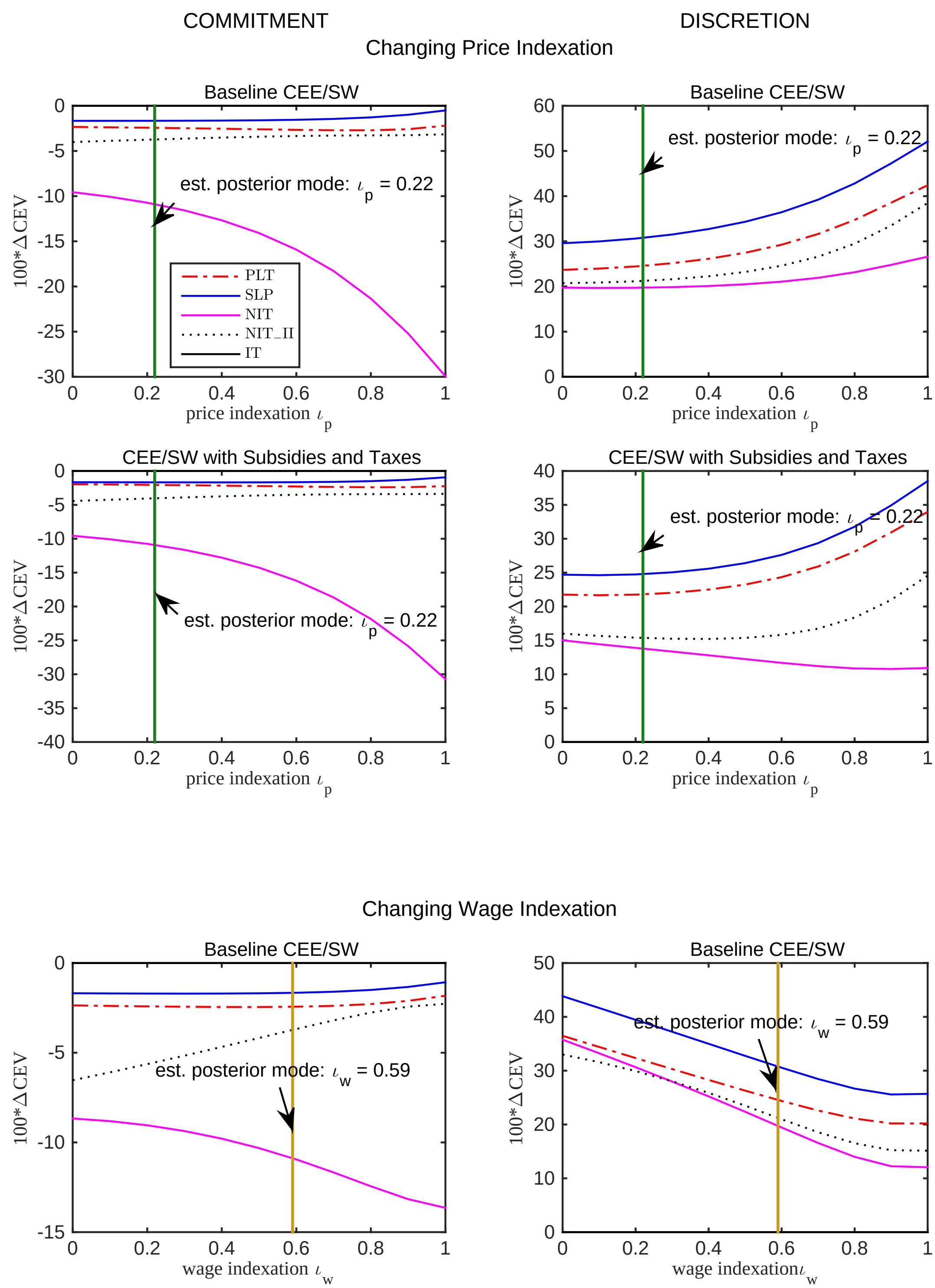
Note: The figure plots the welfare loss for each targeting framework against the optimal commitment policy under different values for λ_x^{TF}. The only source of fluctuations is an ARMA(1,1) markup shock. Welfare is reported in terms of consumption equivalent variation multiplied by 100. The weight λ_x^{TF} for which the welfare loss is minimized is indicated by "$\circ$" under price level targeting (PLT), "$*$" under speed limit policy (SLP), and "$\diamond$" under inflation targeting (IT), respectively.

Figure 2: Welfare evaluation of targeting frameworks in extensions of the textbook NKM



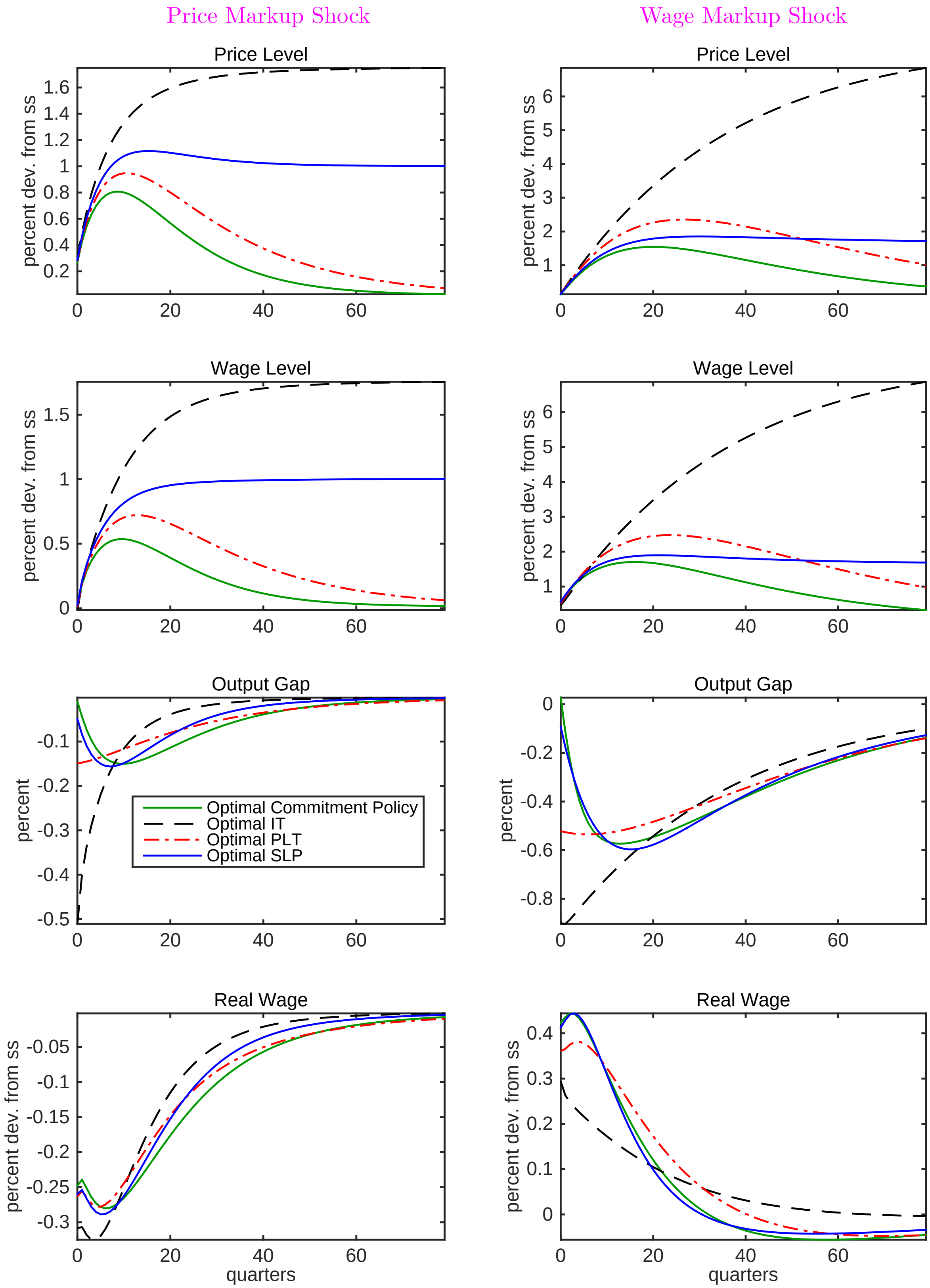
Note: Welfare performance of price level targeting and speed limit policy relative to inflation targeting in the textbook NKM and its extensions with varying degree of price indexation ι_p under commitment and discretion. The only source of fluctuations is an ARMA(1,1) markup shock. Welfare is reported in terms of consumption equivalent variation multiplied by 100. The top row depicts the results in the textbook NKM with an efficient steady state and price indexation. Each of the following rows differs from the textbook NKM by a single feature: distorted steady state (second row), external consumption habits (third row), and sticky nominal wages (last row).

Figure 3: Welfare evaluation of targeting frameworks in the CEE/SW model



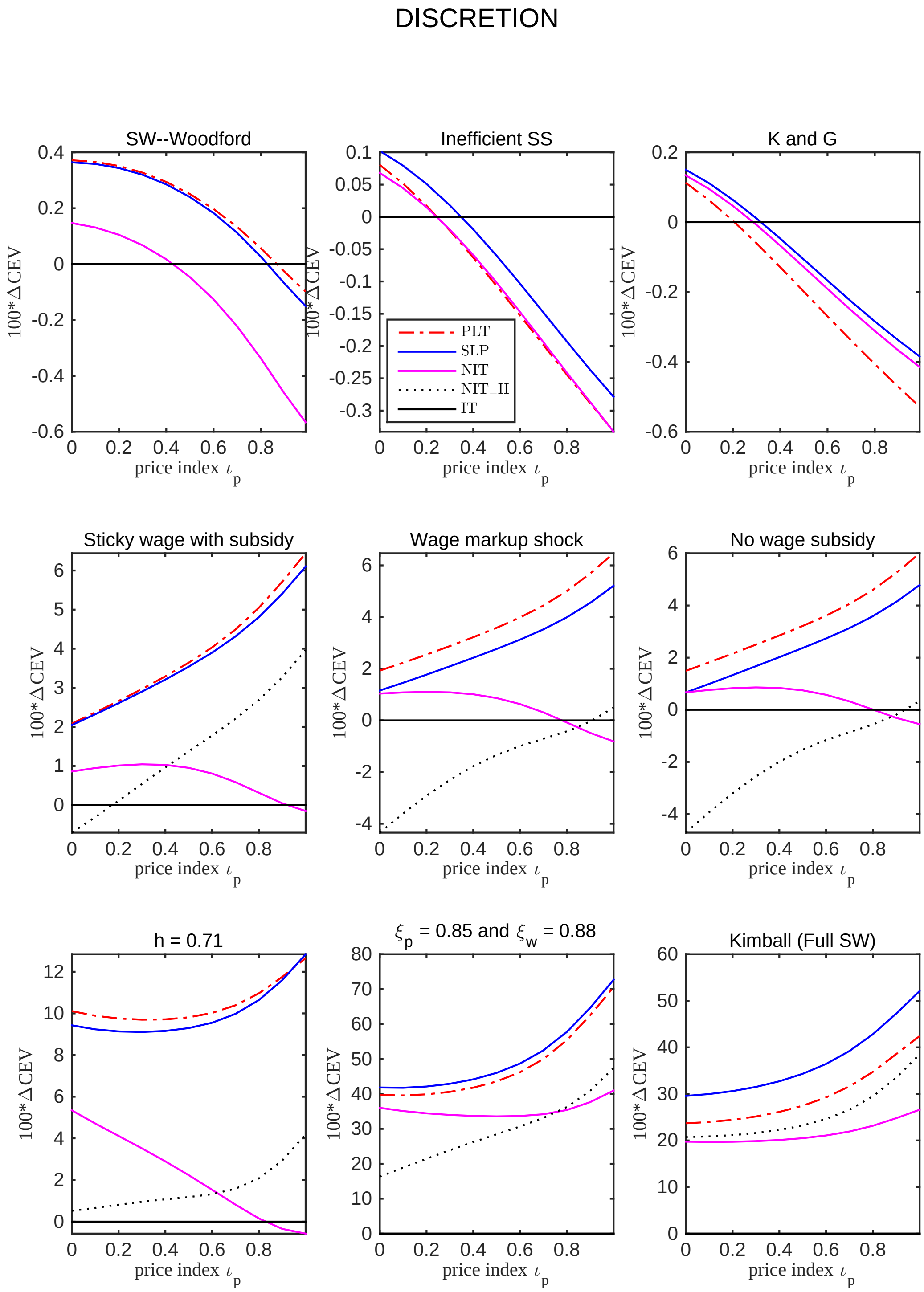
Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting (IT) in the CEE/SW model under commitment and discretion. Parameters are set at the mode of the posterior distribution reported in Smets and Wouters (2007). Welfare is measured in terms of consumption equivalent variation multiplied by 100. In the first two rows of panels, we vary the degree of price indexation. The second row deviates from Smets and Wouters (2007) by correcting steady state inefficiencies due to external habits and monopolistic competition. The third row considers variations in the degree of wage indexation.

Figure 4: Impulse responses in the CEE/SW model to price and wage markup shocks



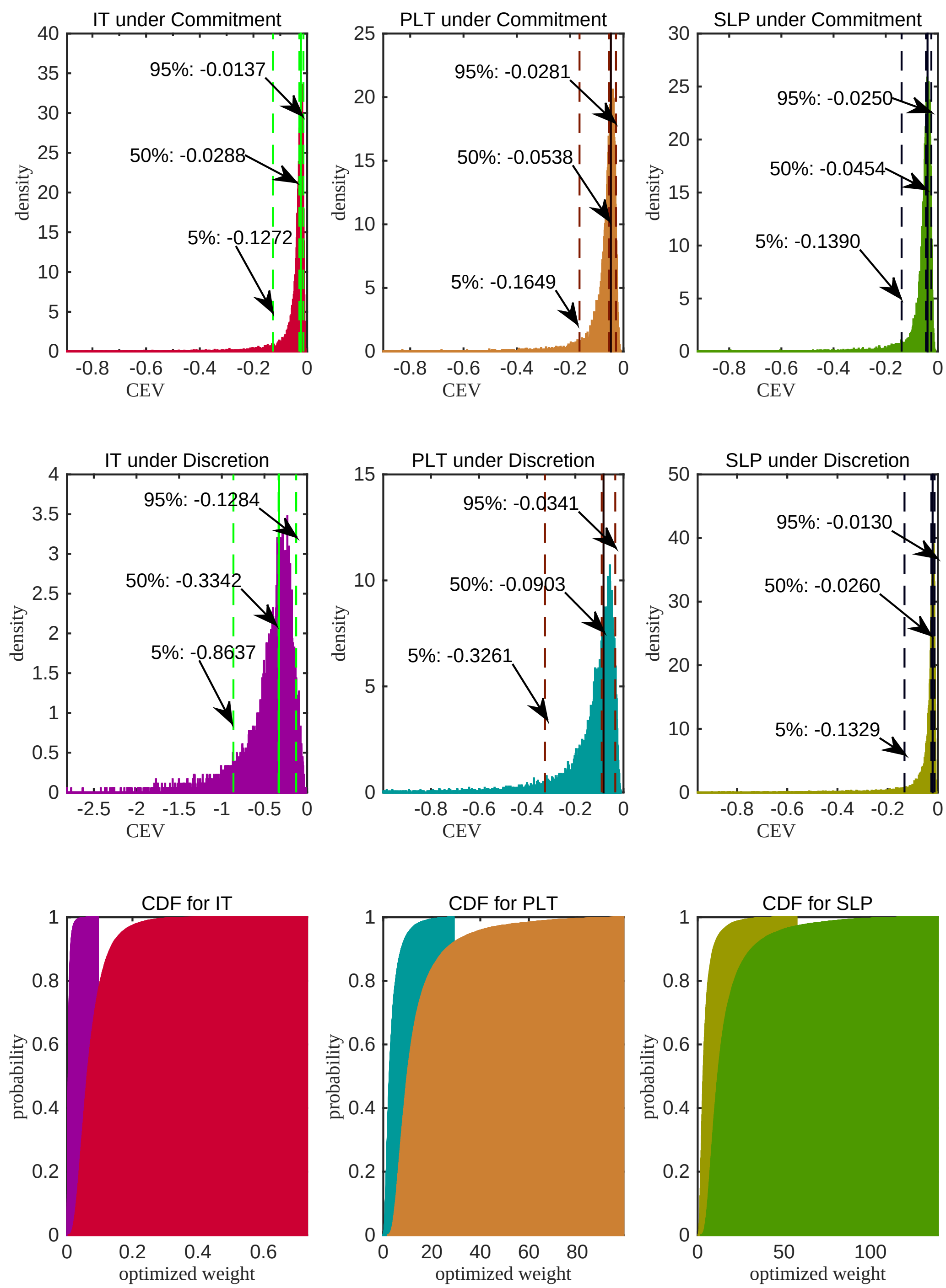
Note: The figure compares the impulse responses to a price and wage markup shock under the optimal commitment policy, inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP). The two markup shocks follow ARMA(1,1) processes. See also Appendix D.

Figure 5: Understanding the welfare rankings in the CEE/SW model under discretion: introducing features sequentially



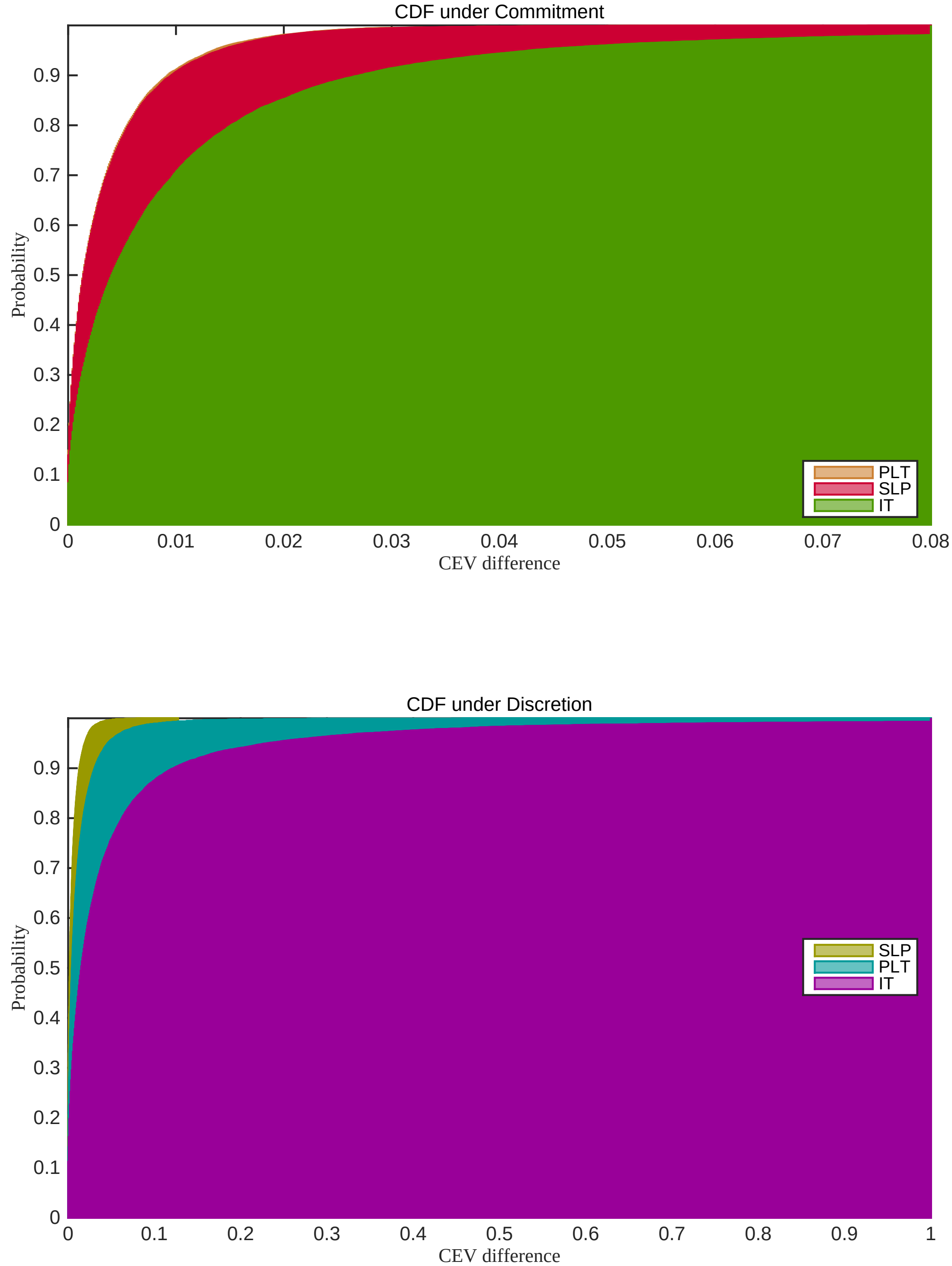
Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting (IT) in the CEE/SW model under discretion. From top left to bottom right we augment the textbook NKM step-by-step by the features in Smets and Wouters (2007): Goods subsidies are removed to render the steady state inefficient, capital and government spending are added in top right panel. In the second row, sticky wages with a wage subsidy to remove distortions in the labor market are introduced, a wage markup shock is added, and finally, the wage subsidy is removed. In the final row, we introduce external consumption habits, increase the nominal rigidities to obtain the same slopes in the NKPCs in the model without variable elasticity of substitution as in the full CEE/SW model with a Kimball (1995) aggregator in the bottom right panel.

Figure 6: Targeting frameworks in the CEE/SW model for alternative parameterizations: optimal weights case



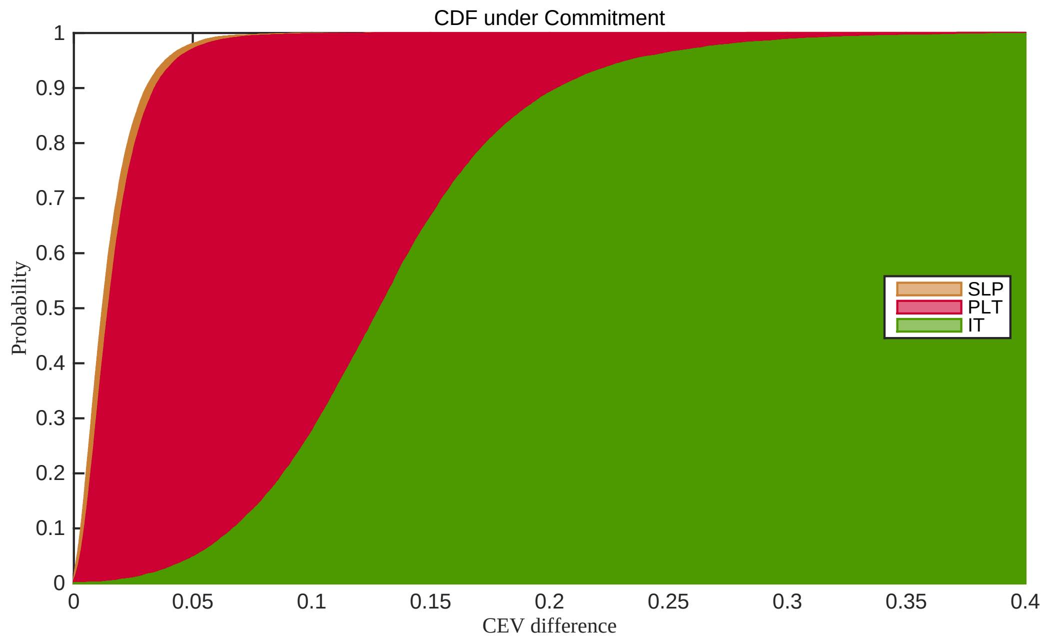
Note: The figure plots the distribution of welfare and the optimized weights λ_x^{TF} for inflation targeting (IT), price level targeting (PLT) and speed limit policy (SLP) under commitment and discretion when the parameters of the CEE/SW model are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The top row shows the density distribution of the consumption equivalent variation (CEV) under commitment, the middle row shows the results under discretion. The bottom row of panels depicts the cumulative distribution function (CDF) of the optimal weights under discretion and commitment for each framework in a single panel.

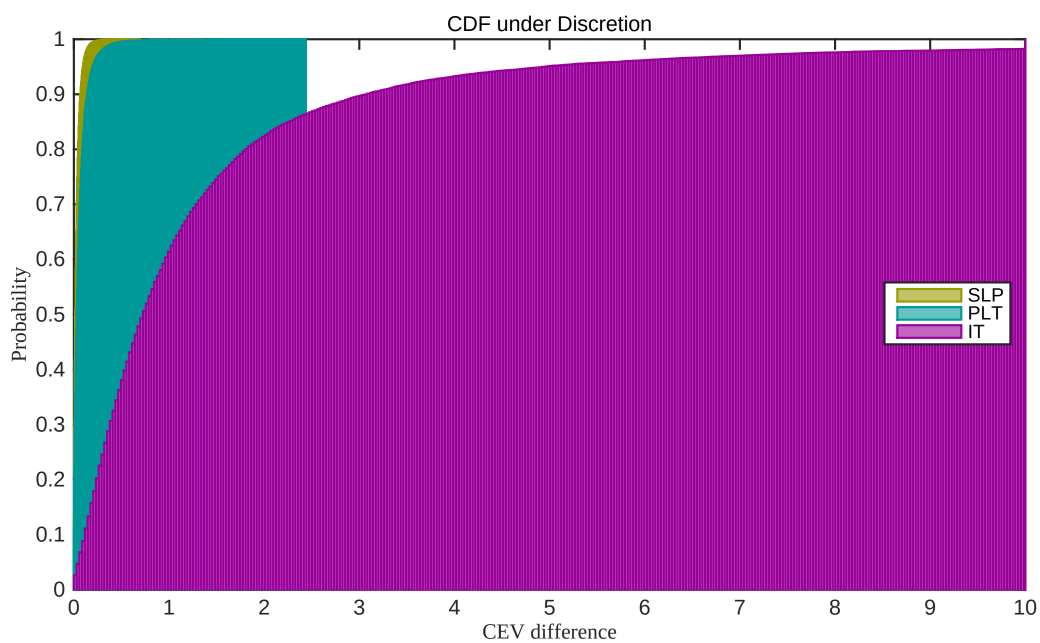
Figure 7: Targeting frameworks in the CEE/SW model for alternative parameterizations: fixed weights case



Note: The figure plots the cumulative welfare distribution under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the weights on the activity measure are fixed at the values that are optimal under the posterior mode parameterization of the CEE/SW model relative to the case when the weights on the activity measure are set optimally for each parameter draw and targeting framework. All other parameters are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The upper panel plots the cumulative distribution function (CDF) under commitment; the bottom panel plots the cumulative distribution function (CDF) under discretion.

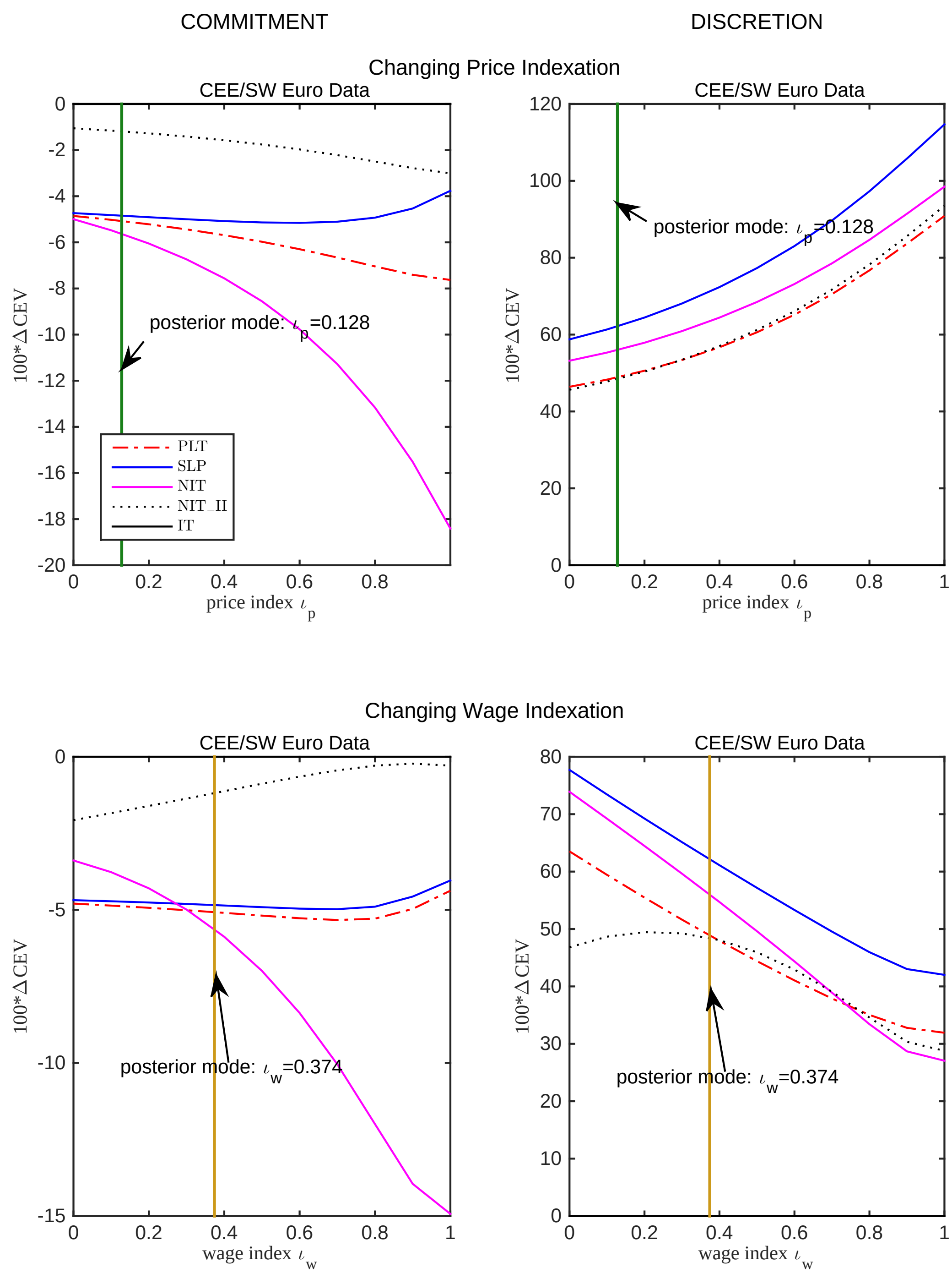
Figure 8: Targeting frameworks in the CEE/SW model for alternative parameterizations: exchanged weights case





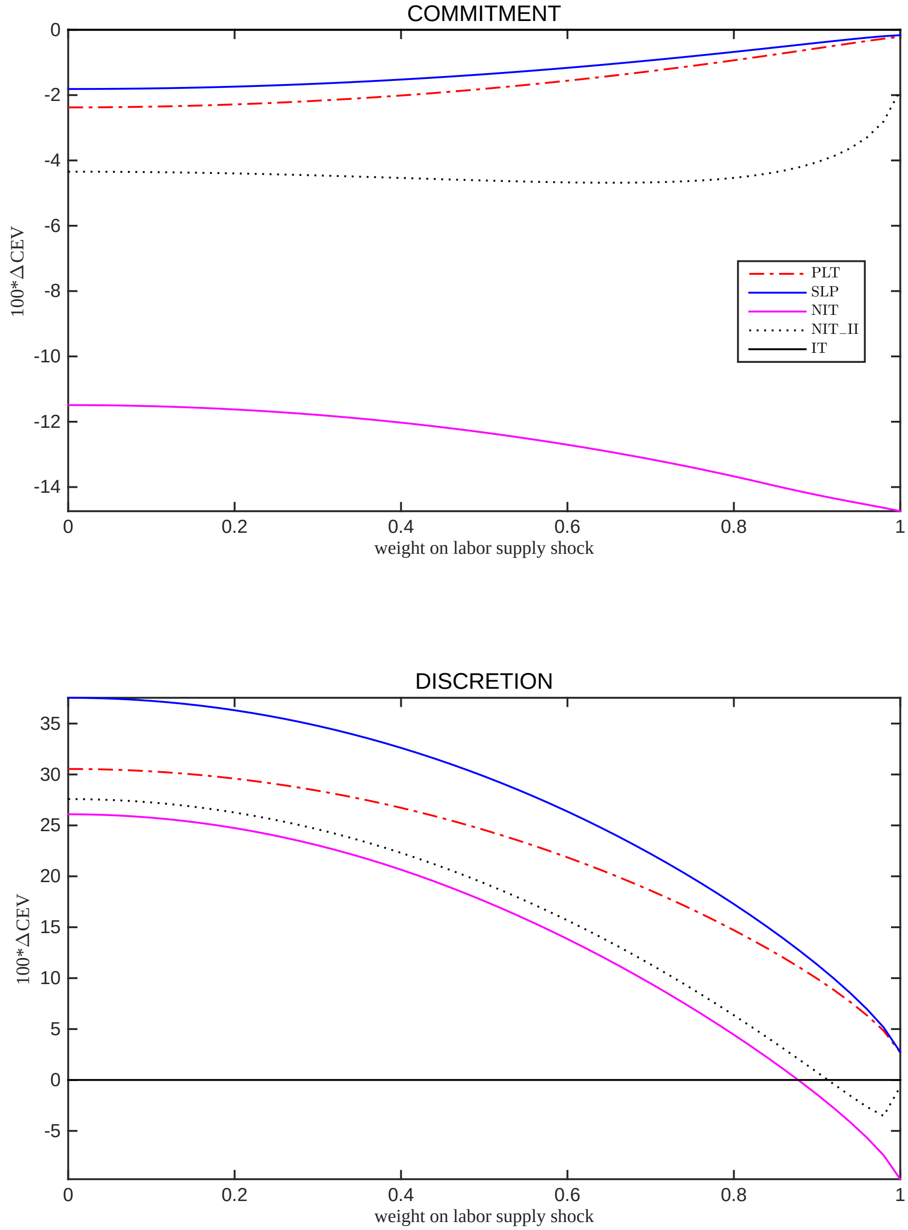
Note: The figure plots the cumulative welfare distribution under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the weights on the activity measure under commitment (discretion) are fixed at the values that are optimal under the posterior mode parameterization of the CEE/SW model with discretion (commitment) relative to the case when the weights on the activity measure are set optimally for each parameter draw and targeting framework. All other parameters are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The upper panel plots the cumulative distribution function (CDF) under commitment; the bottom panel plots the cumulative distribution function (CDF) under discretion.

Figure 9: Welfare evaluation of targeting frameworks in the CEE/SW model estimated with euro area data



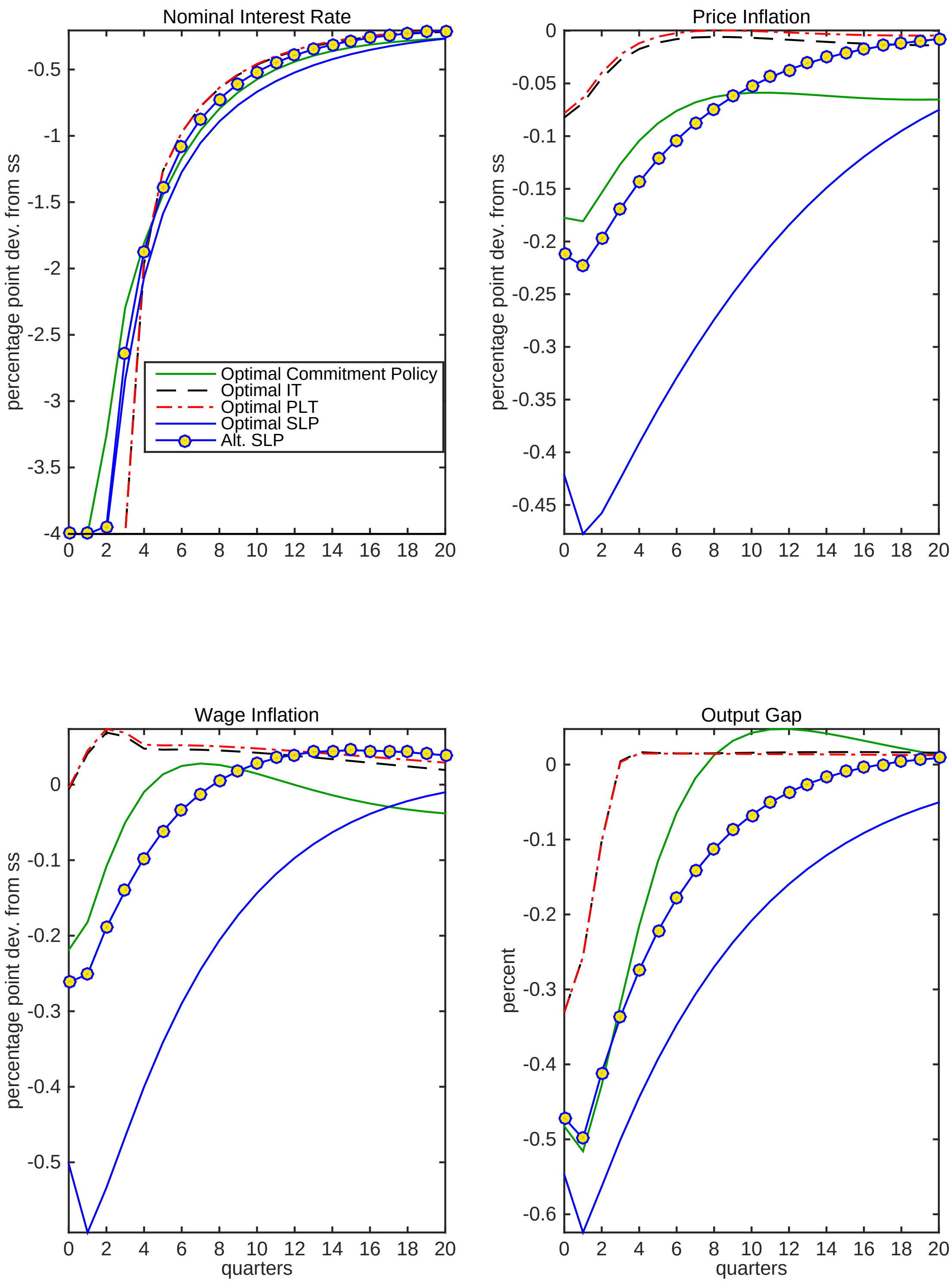
Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting in the CEE/SW model estimated with euro area data (1975Q4 to 2008Q3) under commitment and discretion. In the first row of panels the degree of price indexation is varied. The second row considers variations in the degree of wage indexation. The degree of indexation at the posterior mode is indicated with $\iota_p = 0.128$ for prices and $\iota_w = 0.374$ for wages, respectively.

Figure 10: Welfare evaluation of targeting frameworks: relative importance of wage markup shocks and labor supply shocks



Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting in the modified CEE/SW model when allowing for labor supply and wage markup shocks. This version of the model features preferences that are separable in consumption and leisure. The relative importance of the two shocks is controlled by the weight parameter indicated on the x-axis. “0” indicates the absence of the labor supply shock and “1” indicates the absence of the wage markup shock. The wage markup shock follows an ARMA(1,1) process as in Smets and Wouters (2007), whereas the labor supply shock is assumed to be an AR(1) process. The labor supply shock is scaled to ensure similar magnitudes of the shock as the ARMA(1,1) wage markup shock when comparing the unconditional variances of the shocks.

Figure 11: Welfare evaluation of targeting frameworks under the zero lower bound constraint



Note: The figure compares the impulse responses to a negative one-standard deviation risk premium shock and a negative 10-standard deviation shock to government spending under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) each under discretion and the under the optimal commitment policy. The shocks are large enough for the policy interest rate to be constrained by the zero lower bound.

A Methodology

This Appendix discusses the computational details of our analysis. We describe how to:

1. obtain a valid second-order accurate welfare criterion for any nonlinear model
2. evaluate this welfare criterion given the (linear) decision rules under each monetary policy framework
3. compute the linear decision rules under each targeting framework for the case of
   (a) commitment
   (b) discretion
4. translate the welfare differences that arise between the monetary policy frameworks into consumption units.

Given a fully-specified model, our analysis proceeds as follows. First, we obtain a purely quadratic approximation of the welfare function that describes the preferences of society (in standard applications, the utility function of the representative household)—the true social loss function. This approximation is summarized by the matrices $(A(L), B(L))$. Next, we assume that the central bank optimizes a given, yet arbitrary, quadratic objective subject to the linearized structural equations of the underlying model of the economy. The linearized economy is summarized by the matrices $(C(L), D(L))$. Solving the system of first order conditions delivers linear decision rules that describe the behaviour of the economy under the given objective function for monetary policy. Finally, we use the matrices $(A(L), B(L))$ and the linear decision rules to measure the welfare implications of each policy objective. Within each class of policy objectives we search for its loss-minimizing parameterization. While we restrict attention to linear-quadratic frameworks, i.e., quadratic objective functions and linear constraints, for comparability with the existing literature, our approach can be implemented at higher orders of approximation without restrictions.

A.1 Welfare criterion

For a given model, let the $N \times 1$ vector of endogenous variables be denoted by x_t with the partition $x_t = (\tilde{x}_t', i_t)'$. The variable i_t is the policy instrument of the central bank, typically a short-term interest rate. The vector ζ_t refers to the complete set of exogenous variables. Given the central bank's choice of the policy instrument for all periods $t \geq t_0$, $\{i_t\}_{t=t_0}^{\infty}$, the remaining $N-1$ endogenous variables satisfy the $N-1$ structural model equations

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \tag{38}$$

in equilibrium. The system of equations in (38) is assumed to be differentiable up to the desired order of approximation. We refrain from splitting $g(\cdot)$ into equations that contain no forward-looking variables and equations that do contain forward-looking variables for ease of notation and proceed as if each equation in $g(\cdot)$ contains at least one forward-looking variable.[24] The intertemporal preferences of society are described

[24] When implementing our numerical procedure, however, we carefully separate the equations into those with and without forward-looking variables as in Benigno and Woodford (2012).

by $\mathcal{U} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t)$ with the utility function $U(x_{t-1}, x_t, \zeta_t)$. Within this setting, the optimal monetary policy under full commitment is derived from the maximization program

$$\begin{aligned}
&\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0. \qquad (39)
\end{aligned}$$

As is well understood, the problem stated in (39) does not deliver time-invariant decision rules. Following a large body of the literature, we opt for the optimal monetary policy under commitment from the timeless perspective as the reference point to evaluate the performance of different policies, henceforth referred to as the *optimal commitment policy*; see Woodford (2003a). Optimality from a timeless perspective assumes that the policymaker can "pre-commit" to a policy before period t_0 of the form

$$g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0} \qquad (40)$$

to yield the new optimization program

$$\begin{aligned}
&\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \\
&g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}. \qquad (41)
\end{aligned}$$

If not all equations in $g(\cdot)$ contain forward-looking variables, pre-commitments only need to be specified for those equations with forward-looking variables.

As stressed in Benigno and Woodford (2012) and Debortoli and Nunes (2006) assuming that policy is conducted under suitable pre-commitments is generally key in order to obtain a purely quadratic approximation of the welfare function.[25] Two important remarks are in order:

1. Including the pre-commitment constraints in (40) into problem (39) changes the original optimization problem.

2. Policies that violate the initial pre-commitments (40) are penalized with regard to welfare in accordance with the severity of the violation. In particular, the path of the endogenous variables derived from the original problem (39) may no longer be deemed optimal under the new program (41).

There are two equivalent approaches to obtain the correct linear-quadratic approximation of the optimization problem stated in (41). The first approach (LQ problem), described in Benigno and Woodford (2012), is often followed to obtain a compact characterization of the policy problem in small-scale models. Starting from a second-order Taylor-series expansion of the utility function $U(x_{t-1}, x_t, \zeta_t)$, second-order Taylor-series expansions of the structural equations, $E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0$, and of the pre-commitment

[25] Most prominently, assuming optimality from a timeless perspective is necessary if the deterministic steady state of the model is inefficient. Compare Benigno and Woodford (2005) for details.

constraint, $g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}$, are used to substitute out the linear terms in the approximation to the utility function. As a result, the approximation to the welfare function involves quadratic terms only and it can be maximized subject to the linear approximation of the constraints in (38) and (40) to get a first-order accurate approximation to the problem in (41). The alternative approach computes the first order conditions of the problem in (41) and then seeks the approximation of the resulting system of equations to the desired order. Both approaches can be implemented numerically. We utilize the toolbox developed in Bodenstein, Guerrieri, and LaBriola (2014) which follows the second approach.

The first order conditions associated with the program (41)

$$\begin{aligned}
&\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \\
&g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}
\end{aligned}$$

imply that under the assumptions of full commitment and optimality from a timeless perspective the equilibrium process $\{x_t, \varphi_t\}_{t=t_0}^{\infty}$ satisfies

$$\begin{aligned}
&D_x U(x_{t-1}, x_t, \zeta_t) + \beta E_t D_{x^-} U(x_t, x_{t+1}, \zeta_{t+1}) \\
&+\beta E_t \left\{\varphi'_{t+1} D_{x^-} g(x_t, x_{t+1}, x_{t+2}, \zeta_{t+1})\right\} + E_t \left\{\varphi'_t D_x g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right\} \\
&+\beta^{-1} \varphi'_{t-1} D_{x^+} g(x_{t-2}, x_{t-1}, x_t, \zeta_{t-1}) = 0
\end{aligned} \tag{42}$$

and the structural equations

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \tag{43}$$

at each date $t \geq t_0$. The notation D_x denotes the vector of partial derivatives of any functions with respect to the elements of x_t; likewise do D_{x^-} and D_{x^+} for derivatives with respect to x_{t-1} and x_{t+1}, respectively.

Taking a first order approximation of the equations in (42) around the deterministic steady state of the model delivers

$$\begin{aligned}
&D^2_{xx^-}\bar{U}\hat{x}_{t-1} + \left[D^2_{xx}\bar{U} + \beta D^2_{x^-x^-}\bar{U}\right]\hat{x}_t + \beta D^2_{x^-x}\bar{U}E_t\hat{x}_{t+1} + D^2_{x\zeta}\bar{U}\zeta_t + \beta D^2_{x^-\zeta}\bar{U}E_t\zeta_{t+1} \\
&+\beta\bar{\varphi}\left\{D^2_{x^-x^-}\bar{g}\hat{x}_t + D^2_{x^-x}\bar{g}E_t\hat{x}_{t+1} + D^2_{x^-x^+}\bar{g}E_t\hat{x}_{t+2} + D^2_{x^-\zeta}\bar{g}E_t\zeta_{t+1}\right\} \\
&+\bar{\varphi}\left\{D^2_{xx^-}\bar{g}\hat{x}_{t-1} + D^2_{xx}\bar{g}\hat{x}_t + D^2_{xx^+}\bar{g}E_t\hat{x}_{t+1} + D^2_{x\zeta}\bar{g}\zeta_t\right\} \\
&+\beta^{-1}\bar{\varphi}\left\{D^2_{x^+x^-}\bar{g}\hat{x}_{t-2} + D^2_{x^+x}\bar{g}\hat{x}_{t-1} + D^2_{x^+x^+}\bar{g}\hat{x}_t + D^2_{x^+\zeta}\bar{g}\zeta_{t-1}\right\} \\
&+\beta E_t D_{x^-}\bar{g}'\hat{\varphi}_{t+1} + D_x\bar{g}'\hat{\varphi}_t + \beta^{-1}D_{x^+}\bar{g}'\hat{\varphi}_{t-1} = 0.
\end{aligned} \tag{44}$$

The notation $D^2_{xx^-}$ marks the matrix of second derivatives of a function with respect to x and x^-. $\bar{U}$ and $\bar{g}$ are used as short-hand to indicate that a function (or its partial derivatives) is evaluated at the steady-state values $\{\bar{x}, \bar{\varphi}\}$. "Hatted" variables refer to the deviation of the original variable from its steady-state value.

Regrouping terms delivers

$$
\begin{aligned}
&\bar{\varphi}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right]\hat{x}_{t-2}+\left\{D^2_{xx^-}\bar{U}+\bar{\varphi}\left[D^2_{xx^-}\bar{g}+\beta^{-1}D^2_{x^+x}\bar{g}\right]\right\}\hat{x}_{t-1}\\
&+\left\{\left[D^2_{xx}\bar{U}+\beta D^2_{x^-x^-}\bar{U}\right]+\bar{\varphi}\left[D^2_{xx}\bar{g}+\beta D^2_{x^-x^-}\bar{g}+\beta^{-1}D^2_{x^+x^+}\bar{g}\right]\right\}\hat{x}_t\\
&+\left\{\beta D^2_{xx^-}\bar{U}+\beta\bar{\varphi}\left[D^2_{xx^-}\bar{g}+\beta^{-1}D^2_{x^+x}\bar{g}\right]\right\}' E_t\hat{x}_{t+1}\\
&+\beta^2\bar{\varphi}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right]' E_t\hat{x}_{t+2}+\left\{\beta D^2_{x^-\zeta}\bar{U}+\beta\bar{\varphi}D^2_{x^-\zeta}\bar{g}\right\}E_t\zeta_{t+1}\\
&+\left\{D^2_{x\zeta}\bar{U}+\bar{\varphi}D^2_{x\zeta}\bar{g}\right\}\zeta_t+\beta^{-1}\bar{\varphi}D^2_{x^+\zeta}\bar{g}\zeta_{t-1}\\
&+\beta E_t D_{x^-}\bar{g}'\hat{\varphi}_{t+1}+D_x\bar{g}'\hat{\varphi}_t+\beta^{-1}D_{x^+}\bar{g}'\hat{\varphi}_{t-1}=0 \qquad (45)
\end{aligned}
$$

which coincides with the first order conditions of the following LQ problem, where we have turned the maximization problem of the utility function into a minimization problem of the (approximated) true social loss function

$$
\begin{aligned}
&\min_{\{\hat{x}_t\}_{t=t_0}^{\infty}} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\,\hat{x}_t' A(L)\hat{x}_t+\hat{x}_t' B(L)\zeta_{t+1}\right]\\
&s.t.\\
&E_t C(L)\hat{x}_{t+1}+D(L)\zeta_t=0\\
&C(L)\hat{x}_{t_0}=d_{t_0}\\
&\zeta_t=\Gamma\zeta_{t-1}+\Upsilon\xi_t \qquad (46)
\end{aligned}
$$

where

$$
\begin{aligned}
A_2 &= -2\bar{\varphi}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right]\\
A_1 &= -2\left(D^2_{xx^-}\bar{U}+\bar{\varphi}\left[D^2_{xx^-}\bar{g}+\beta^{-1}D^2_{x^+x}\bar{g}\right]\right)\\
A_0 &= -\left[D^2_{xx}\bar{U}+\beta D^2_{x^-x^-}\bar{U}\right]-\bar{\varphi}\left[D^2_{xx}\bar{g}+\beta D^2_{x^-x^-}\bar{g}+\beta^{-1}D^2_{x^+x^+}\bar{g}\right]\\
A(L) &= A_0+A_1L+A_2L^2\\
B(L) &= -\left\{\beta D^2_{x^-\zeta}\bar{U}+\beta\bar{\varphi}D^2_{x^-\zeta}\bar{g}\right\}-\left\{D^2_{x\zeta}\bar{U}+\bar{\varphi}D^2_{x\zeta}\bar{g}\right\}L-\beta^{-1}\bar{\varphi}D^2_{x^+\zeta}L^2\\
C(L) &= D_{x^+}\bar{g}+D_x\bar{g}L+D_{x^-}\bar{g}L^2\\
D(L) &= D_\zeta\bar{g}.
\end{aligned}
$$

where $\hat{x}_t$ measures the (log-) deviation of variable "x" from its value assumed in the deterministic steady state. The matrices $(A(L), B(L))$ capture the second-order approximation of the welfare function, where "L" denotes the lag-operator. The matrices $C(L)$ and $D(L)$ capture the linear approximation of the constraints. The linear constraints $C(L)\hat{x}_{t_0} = d_{t_0}$ implement the timeless perspective through the appropriate choice of d_{t_0}. The model description is completed by the evolution of the exogenous variables, the last equation in (46). The innovations ξ_t follow *iid* standard normal distributions. To a first-order approximation, the output of the toolbox in Bodenstein, Guerrieri, and LaBriola (2014) is equivalent to that of the LQ approach studied in Benigno and Woodford (2012) and using the above definitions, it is easy to compute the matrices for the LQ problem from the numerical output produced by the toolbox described in Bodenstein, Guerrieri, and LaBriola (2014).

The criterion $E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2}\hat{x}_t' A(L)\hat{x}_t + \hat{x}_t' B(L)\zeta_{t+1}\right]$ ranks outcomes $\{x_t\}_{t=t_0}^{\infty}$ obtained from policies that satisfy the initial pre-commitment constraints $C(L)\hat{x}_{t_0} = d_{t_0}$ correctly by their welfare implications. However, if the policies considered do not respect the initial pre-commitment constraints, the criterion needs to be augmented to include a penalty for violations of the initial pre-commitment. As discussed in detail in Benigno and Woodford (2012), the correct criterion that allows for meaningful welfare comparisons of arbitrary policies against the optimal commitment policy is given by

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2}\hat{x}_t' A(L)\hat{x}_t + \hat{x}_t' B(L)\zeta_{t+1}\right] + \beta^{-1}\hat{\varphi}_{t_0-1}^{*\prime} C(0)\hat{x}_{t_0}. \tag{47}$$

$\hat{\varphi}_{t_0-1}^{*\prime}$ denotes the values of the Lagrange multipliers associated with the pre-commitment constraints under the optimal commitment policy. $C(0)$ is the coefficient matrix going along with the forward-looking variables in the first order approximation of the equations in $g(\cdot)$. Finally, $\hat{x}_{t_0}$ contains the values of the endogenous variables at time t_0 under the policy that is actually implemented. Intuitively, insuring that the optimal commitment policy is the best policy among all feasible policies requires a change in preferences. Rather than viewing preferences as being described by $E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t)$, preferences need to be viewed as

$$E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) + \beta^{-1}\varphi_{t_0-1}' \left(g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) - \bar{g}_{t_0}\right). \tag{48}$$

The optimal policy problem is then given by

$$\begin{aligned} &\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t) + \beta^{-1}\varphi_{t_0-1}' \left(g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) - \bar{g}_{t_0}\right) \\ &s.t. \\ &E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0. \end{aligned} \tag{49}$$

Approximating this problem following the same steps as above yields the criterion function in (47). By construction, the problem in (49) implies the same first-order conditions as the optimization program in (41). In finding a second-order approximation of the augmented utility function one only needs to include the second-order expansion of the penalty term, which after eliminating first-order terms, is simply given by $\beta^{-1}\hat{\varphi}_{t_0-1}' D_{x^+}\bar{g}'\hat{x}_{t_0} = \beta^{-1}\hat{\varphi}_{t_0-1}' C(0)\hat{x}_{t_0}$.

A.2 Applying the welfare criterion

We focus on unconditional welfare, but similar steps apply for computing conditional welfare. In doing so, we integrate out initial conditions with the help of the invariant unconditional distribution over possible initial conditions — including the pre-commitments.

Consider an arbitrary policy regime, indexed by TF, and suppose that the (linear) equilibrium decision rules can be summarized by

$$z_t^{TF} = P^{TF} z_{t-1}^{TF} + Q^{TF}\xi_t. \tag{50}$$

If policy is conducted under commitment (from a timeless perspective), the vector z_t^{TF} contains the endogenous variables $\hat{x}_t^{TF}$, the exogenous shocks ζ_t and ζ_{t-1}, and a set of Lagrange multipliers $\hat{\varphi}_t^{TF}$. Under discretion, Lagrange multipliers are not part of the state space and will be omitted from the vector z_t^{TF}. The same applies if one were to include instrument rules in the analysis. We denote the decision rules under the optimal commitment policy by a star, "*", instead of TF.

The unconditional variance-covariance matrix $Cov_{z^{TF},z^{TF}}$ satisfies

$$Cov_{z^{TF},z^{TF}} = P^{TF}\left[Cov_{z^{TF},z^{TF}}\right]P^{TF'} + Q^{TF}Q^{TF'} \tag{51}$$

which can be computed efficiently using the doubling algorithm suggested in Anderson, McGrattan, Hansen, and Sargent (1996). The (first) auto-covariance term is obtained by recognizing that $Cov_{z^{TF},z^{TF}_{-1}} = P^{TF}Cov_{z^{TF},z^{TF}}$.

To compute the unconditional welfare implied by the policy TF we simplify the two terms in equation (47) as follows. The first term of the welfare criterion can be written in terms of the unconditional covariances and auto-covariances between the endogenous variables, $\hat{x}_t^{TF}$, and exogenous variables, ζ_t, since

$$\begin{aligned}&\frac{1}{1-\beta}tr\left\{E\left[\frac{1}{2}\hat{x}_t^{TF'}A(L)\hat{x}_t^{TF} + \hat{x}_t^{TF'}B(L)\zeta_{t+1}\right]\right\}\\ &= \frac{1}{1-\beta}tr\left\{\frac{1}{2}\sum_{i=0}^{2}A(i)Cov_{\hat{x}^{TF}_{-i},\hat{x}^{TF}} + \sum_{i=0}^{2}B(i)Cov_{\zeta_{-i+1},\hat{x}^{TF}}\right\}\end{aligned} \tag{52}$$

where $tr(M)$ denotes the trace of the matrix M.

Because the second term in (47) involves the Lagrange multipliers associated with the optimal commitment policy, evaluation of the term requires knowledge of the variance-covariance matrix of the endogenous variables under the optimal commitment policy, $\hat{\varphi}^*_{t_0-1}$. The pre-commitments are drawn from the invariant distribution of the endogenous variables under the optimal policy.

If the policy TF is conducted under commitment, the second term can be written as

$$E\left[\beta^{-1}\hat{\varphi}^{*\prime}_{t_0-1}C(0)\hat{x}^{TF}_{t_0}\right] = \beta^{-1}tr\left\{C(0)S_xP^*Cov_{z^*,\hat{\varphi}^*} + C(0)S_xQ^*Cov_{\xi,\hat{\varphi}^*_{-1}}\right\} \tag{53}$$

from the unconditional perspective. The matrix S_x selects the elements in z_t that coincide with those in the vector $\hat{x}_t$. If the policymaker respects the pre-commitments consistent with the optimal commitment policy, it must be that $C(0)\hat{x}^{TF}_{t_0} = C(0)\hat{x}_{t_0}$. Thus, the term (53) does not depend on the decision rules of the policy regime under consideration as long as the policymaker respects pre-commitments.

When pre-commitments are not honoured, in particular under discretion or an instrument rule, the second term does depend on the decision rules of the policy implemented by the central bank and therefore the correction term satisfies

$$E\left[\beta^{-1}\hat{\varphi}^{*\prime}_{t_0-1}C(0)\hat{x}^{TF}_{t_0}\right] = \beta^{-1}tr\left\{C(0)S_xP^{TF}Cov_{z^*,\hat{\varphi}^*} + C(0)S_xQ^{TF}Cov_{\xi,\hat{\varphi}^*_{-1}}\right\} \tag{54}$$

with S_x defined appropriately to select the elements in z_t that coincide with those in the vector $\hat{x}_t$ under discretion.

A.3 Decision rules under commitment and discretion

For each targeting framework, we consider the case of the central bank optimizing its assigned objective under full commitment from a timeless perspective and the case of optimization under discretion. We assume that the central bank is committed to an explicit long-run inflation target. Thus, our analysis abstracts from the inflationary bias under discretion; our work focuses purely on the stabilization bias.

Each targeting framework is represented by a quadratic loss function:

1. inflation targeting (IT)
$$L_t^{IT} = \pi_{p,t}^2 + \lambda_x^{IT} \left(x_t^{gap}\right)^2 \tag{55}$$

2. price level targeting (PLT)
$$L_t^{PLT} = \hat{p}_t^2 + \lambda_x^{PLT} \left(x_t^{gap}\right)^2 \tag{56}$$

3. speed limit policy (SLP)
$$L_t^{SLP} = \pi_{p,t}^2 + \lambda_x^{SLP} \left(\left(x_t^{gap}\right) - \left(x_{t-1}^{gap}\right)\right)^2 \tag{57}$$

4. nominal income targeting (NIT)
$$L_t^{NIT} = \pi_{p,t}^2 + \lambda_x^{NIT} \left(\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1}\right)^2 \tag{58}$$

5. nominal income targeting II (NIT-II)
$$L_t^{NIT\text{-II}} = \left(x_t^{gap}\right)^2 + \lambda_x^{NIT\text{-II}} \left(\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1}\right)^2 \tag{59}$$

where $\pi_{p,t}$ denotes deviations of the inflation measure from its value along the balanced growth path, $\hat{p}_t$ is the log-deviation of the price level from its value along the balanced growth path, and x_t^{gap} measures the output gap. We follow Smets and Wouters (2007) and define the output gap as the difference between actual output (in deviations from the balanced growth path), $\hat{y}_t$, and the output level that would prevail absent nominal rigidities and markup shocks.

A.3.1 Targeting frameworks under commitment

For a given parameterization of a targeting framework, a central bank, that formulates policy under commitment and respects the same pre-commitments as the optimal commitment policy, solves the optimization problem

$$\begin{aligned}
&\min_{\{\hat{x}_t^{TF}\}_{t=t_0}^{\infty}} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\, \hat{x}_t^{TF\prime} A^{TF}(L) \hat{x}_t^{TF} \\
&s.t. \\
&E_t C(L) \hat{x}_{t+1}^{TF} + D(L)\zeta_t = 0 \\
&C(L)\hat{x}_{t_0}^{TF} = d_{t_0} \\
&\zeta_t = \Gamma \zeta_{t-1} + \Upsilon \xi_t.
\end{aligned} \tag{60}$$

The matrix $A^{TF}(L)$ is parameterized to reflect the loss function that characterizes the targeting framework under consideration with $TF = \{IT, PLT, SLP, NIT, NIT\text{-II}\}$. The entries into $A^{TF}(L)$ are zero except for those diagonal elements that correspond to the positions of the targeting variables in the vector $\hat{x}_t$ for the targeting regime TF. Thus, the problem resembles the one of obtaining the optimal commitment policy in (46) with $(A(L), B(L))$ being replace by $A^{TF}(L)$.

The first-order conditions associated with this linear quadratic program can be solved using standard algorithms to obtain the decision rules of the endogenous variables and the Lagrange multipliers. These decision rules are then used to compute the relevant variance-covariance matrices to evaluate the welfare criterion (47).

A.3.2 Targeting frameworks under discretion

To find the (Markov equilibrium) decision rule of a central bank acting under discretion we follow the methodology suggested in Dennis (2007). Today's central bank is viewed as the Stackelberg leader; households and firms as well as future policymakers are the Stackelberg followers. Define $\tilde{z}_t$

$$\tilde{z}_t = \begin{pmatrix} \hat{x}_t^{TF,\setminus i} \\ \zeta_t \end{pmatrix} \tag{61}$$

to be the vector that contains the endogenous variables, $\hat{x}_t^{TF}$, except for the vector of policy instruments, $i_t = \hat{x}_t^{TF,i}$, and the exogenous shocks. We start by writing the linearized equilibrium conditions $E_t C(L)\hat{x}_{t+1}^{TF} + D(L)\zeta_t = 0$ as

$$M_0\tilde{z}_t = M_1\tilde{z}_{t-1} + M_2 E_t\tilde{z}_{t+1} + M_3 i_t + M_4 E_t i_{t+1} + M_5\xi_t \tag{62}$$

with

$$M_0 = -\begin{bmatrix} C^{\setminus i}(1) & 0 \end{bmatrix} \tag{63}$$
$$M_1 = \begin{bmatrix} C^{\setminus i}(2) & D(0)\Gamma + D(1) \end{bmatrix} \tag{64}$$
$$M_2 = \begin{bmatrix} C^{\setminus i}(0) & 0 \end{bmatrix} \tag{65}$$
$$M_3 = c(1) \tag{66}$$
$$M_4 = c(0) \tag{67}$$
$$M_5 = D(0)\Upsilon. \tag{68}$$

The matrix $C^{\setminus i}(1)$ is derived from $C(1)$ by eliminating from $C(1)$ the column $c(1)$ which is associated with the policy instrument and similarly for $C(0)$ and $C(2)$. We assume $c(2)$ to be a vector of zeros.

Similarly, we write the objective function of the central bank—originally characterized by $A^{TF}(L)$—to conform with the inclusion of the exogenous variables into the vector $\tilde{z}_t$ and the separating out of the policy instrument

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left[\tilde{z}_t' W^{TF}\tilde{z}_t + i_t' K^{TF} i_t\right] \tag{69}$$

where

$$
\begin{aligned}
W^{TF} &= \begin{bmatrix} A^{TF,\backslash i} & 0 \\ 0 & 0 \end{bmatrix} && (70)\\
K^{TF} &= a^{TF}. && (71)
\end{aligned}
$$

We proceed under the conjecture that the solution will be of the form

$$\tilde{z}_t = H_1 \tilde{z}_{t-1} + H_2 \xi_t \tag{72}$$

$$i_t = F_1 \tilde{z}_{t-1} + F_2 \xi_t. \tag{73}$$

Substituting this conjecture into equation (62) we obtain

$$[M_0 - M_2 H_1 - M_4 F_1]\, \tilde{z}_t = M_1 \tilde{z}_{t-1} + M_3 i_t + M_5 \xi_t. \tag{74}$$

Similarly, the objective function (69) can be written as

$$
\begin{aligned}
& E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left[\tilde{z}_t' W^{TF} \tilde{z}_t + i_t' K^{TF} i_t \right] \\
= \; & \tilde{z}_t' N^{TF} \tilde{z}_t + i_t' K^{TF} i_t + \frac{\beta}{1-\beta} tr \left(H_2' N^{TF} H_2 + F_2' K^{TF} F_2 \right) && (75)
\end{aligned}
$$

since

$$
\begin{aligned}
E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\tilde{z}_t' W^{TF} \tilde{z}_t \right] &= \tilde{z}_t' \left(\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(H_1'^{t-t_0} \right) W^{TF} \left(H_1^{t-t_0} \right) \right) \tilde{z}_t \\
& \quad + \beta \sum_{t=t_0}^{\infty} \sum_{\tilde{t}=t_0}^{\infty} \beta^{(t-t_0)+(\tilde{t}-t_0)} tr \left(H_2' \left(H_1'^{t-t_0} \right) W^{TF} \left(H_1^{t-t_0} \right) H_2 \right) \\
&= \tilde{z}_t' S \tilde{z}_t + \frac{\beta}{1-\beta} tr \left(H_2' S H_2 \right) && (76)
\end{aligned}
$$

and

$$
\begin{aligned}
E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[i_t' K^{TF} i_t \right] &= i_t' K^{TF} i_t + \beta \tilde{z}_t' \left(\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(H_1'^{t-t_0} \right) F_1' K^{TF} F_1 \left(H_1^{t-t_0} \right) \right) \tilde{z}_t \\
& \quad + \frac{\beta}{1-\beta} tr \left(F_2' K^{TF} F_2 \right) \\
& \quad + \beta \sum_{t=t_0}^{\infty} \sum_{\tilde{t}=t_0}^{\infty} \beta^{(t-t_0)+(\tilde{t}-t_0)} tr \left(H_2' \left(H_1'^{t-t_0} \right) F_1' K^{TF} F_1 \left(H_1^{t-t_0} \right) H_2 \right) \\
&= i_t' K^{TF} i_t + \beta \tilde{z}_t' R \tilde{z}_t + \frac{\beta}{1-\beta} tr \left(F_2' K^{TF} F_2 \right) + \frac{\beta^2}{1-\beta} tr \left(H_2' R H_2 \right). && (77)
\end{aligned}
$$

The matrices S, R and N^{TF} are defined implicitly as

$$
\begin{aligned}
S &= W^{TF} + \beta H_1' S H_1 && (78)\\
R &= F_1' K^{TF} F1 + \beta H_1' R H_1 && (79)\\
N^{TF} &= S + \beta R. && (80)
\end{aligned}
$$

S and R are fixed points provided that the spectral radius of H_1 is less than one. In our application, $K^{TF} = 0$ and the second term of the objective function drops out.

Under discretion, the policymaker optimizes the objective function (75) subject to the conditions in (74). Taking first-order conditions and applying the method of undetermined coefficients yields

$$\bar{M} \equiv M_0 - M_2 H_1 - M_4 F_1 \tag{81}$$

$$N^{TF} \equiv W^{TF} + \beta F_1' K^{TF} F_1 + \beta H_1' N^{TF} H_1 \tag{82}$$

$$F_1 = -(K^{TF} + M_3' \bar{M}'^{-1} N^{TF} \bar{M}^{-1} M_3)^{-1} M_3' \bar{M}'^{-1} N^{TF} \bar{M}^{-1} M_1 \tag{83}$$

$$F_2 = -(K^{TF} + M_3' \bar{M}'^{-1} N^{TF} \bar{M}^{-1} M_3)^{-1} M_3' \bar{M}'^{-1} N^{TF} \bar{M}^{-1} M_5 \tag{84}$$

$$H_1 = \bar{M}^{-1}(M_1 + M_3 F_1) \tag{85}$$

$$H_2 = \bar{M}^{-1}(M_5 + M_3 F_2). \tag{86}$$

Equations (72) and (73) can be combined to deliver the law of motion to the full vector z_t under discretionary policies as in equation (50).

In order to evaluate the five targeting frameworks under discretionary policymaking, we do not need to characterize the optimal policy under discretion when the central bank's objective is derived from the utility function of the representative household. Each targeting framework can be evaluated by applying the criterion stated in (47) to assess the welfare implications of the policy paths under discretion—the true social loss function. The reason for condition (47) to suffice for welfare evaluations lies in the fact that absent shocks, the central bank chooses the same policy path under each objective regardless of policy being conducted under commitment or discretion. In particular, an inflationary bias cannot arise even if the steady state is not efficient.[26]

A.4 Welfare comparison

We compute welfare under the targeting regime W^{TF} and the optimal commitment policy W^* and convert the difference into consumption units. More concretely, the difference is expressed in terms of the consumption equivalent variation (CEV). The CEV is defined as the amount of (steady state) consumption that the representative household—with preferences over consumption and leisure $U(C, N)$—would need to give up to be indifferent between the optimal commitment policy and the targeting framework being implemented. Algebraically, the CEV is defined as

$$\begin{aligned} W^{TF} - W^* &= U((1+CEV)\bar{C}, \bar{N}) - U(\bar{C}, \bar{N}) \\ &= \left.\frac{\partial U}{\partial C}\right|_{C=\bar{C}} [(1+CEV)\bar{C} - \bar{C}] \\ &= \left.\frac{\partial U}{\partial C}\right|_{C=\bar{C}} \bar{C} CEV \end{aligned}$$

[26] As pointed out in Woodford (2003a), Chapter 7, page 470, footnote 4, characterizing the optimal policy under discretion is a complicated task, in particular when the steady state is distorted. Assigning to a central bank acting under discretion the objective in (47) does not yield the optimal policy under discretion as the derivations underlying expression (47) *assume* that policy is conducted under commitment from a timeless perspective.

or solved for the CEV

$$CEV = \frac{W^{TF} - W^*}{\left.\frac{\partial U}{\partial C}\right|_{C=\bar{C}} \bar{C}}. \tag{87}$$

When households experience habit persistence in consumption—here of the form $U(C_t, C_{t-1}, L_t) = U(C_t - hC_{t-1}, N_t)$—we follow the approach in Otrok (2001). In this case, we obtain

$$\begin{aligned} W^{TF} - W^* &= U((1+CEV)\bar{C}, (1+CEV)\bar{C}, \bar{N}) - U(\bar{C}, \bar{C}, \bar{N}) \\ &= \left.\frac{\partial U}{\partial C}\right|_{C=\bar{C}} [((1+CEV)\bar{C} - (1+CEV)h\bar{C}) - (\bar{C} - h\bar{C})] \\ &= \left.\frac{\partial U}{\partial C}\right|_{C=\bar{C}} (1-h)\bar{C}CEV. \end{aligned}$$

Under additive separable preferences, as conventionally assumed in the textbook NKM, it is

$$CEV = \frac{W^{TF} - W^*}{((1-h)\bar{C})^{1-\sigma_C}}.$$

Under the preferences assumed in Smets and Wouters (2007) the CEV is given by

$$CEV = \frac{W^{TF} - W^*}{((1-h)\bar{C})^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L} \bar{N}^{1+\sigma_L}\right)}.$$

B Baseline New Keynesian model

B.1 Model description

For completeness, we lay out the assumptions of the textbook NK model and its variations discussed in the text. We then derive the linear-quadratic framework for versions of the NK model with price indexation, external habits, inefficient steady state, and sticky wages.

B.1.1 Household

Household j chooses consumption $C_t(j)$, labor supply $N_t(j)$, bond holdings to maximize expected discounted lifetime utility taking prices, wages, taxes, and transfers as given. The household's preferences over consumption and leisure are given by

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{\left(C_t(j)-hC_{t-1}^{A}\right)^{1-\sigma_C}}{1-\sigma_C}-\frac{N_t(j)^{(1+\sigma_L)}}{1+\sigma_L}\right\}. \tag{88}$$

Consumption habits are external; C_{t-1}^{A} refers to the aggregate level of consumption in the previous period and the degree of habit persistence is governed by the parameter h. The inverse of the intertemporal elasticity of substitution of consumption is denoted by σ_C, and the parameter σ_L is the inverse of the Frisch elasticity of labor supply. We assume that financial markets are complete due to a set of Arrow securities. As a result consumption is equalized across households in equilibrium. In addition, each household can invest in a simple bond without state-contingent payoffs.

The budget constraint of household j satisfies

$$P_tC_t(j)+\frac{B_t(j)}{R_t}=W_tN_t(j)+B_{t-1}(j)+Profits_t+Transfers_t. \tag{89}$$

The household earns income by supplying labor services $N_t(j)$ for the nominal wage W_t, receives payments from holding bonds $B_t(j)$, receives an aliquot share of profits $Profits_t$ and government transfer $Transfers_t$. This income is used to purchase the consumption good and bonds. The notation abstracts from the household's transactions in Arrow securities.

B.1.2 Labor market

We consider the case with and without flexible wages. If wages are flexible, workers receive the same nominal wage W_t in period t. The household chooses the labor supply optimally.

In modeling nominal sticky wages we follow in general Erceg, Henderson, and Levin (2000), but the details of the implementation are as in Gali (2008). Households supply their homogenous labor to labor unions. The labor union differentiates the labor services, and resells them to a labor bundler. These aggregated labor services are then hired out to firms.

The labor bundlers aggregate the labor services provided by the labor unions according to

$$L_t=\left[\int_0^1 L_t(j)^{\frac{1}{1+\theta_w}}\,\mathrm{d}j\right]^{1+\theta_w}. \tag{90}$$

Labor bundlers buy labor services $L_t(j)$ from labor union j, combine the differentiated services into L_t, and then resell the aggregate labor service to intermediate goods producers. The Labor bundlers maximize profits under perfect competition. The first order conditions associated with this maximization problem can be combined to obtain the labor demand functions for the labor services $L_t(j)$ offered by labor union j

$$L_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\frac{1+\theta_w}{\theta_w}} L_t \tag{91}$$

and the aggregate wage index

$$W_t = \left[\int_0^1 W_t(j)^{-\frac{1}{\theta_w}} \mathrm{d}j\right]^{-\theta_w}. \tag{92}$$

Labor unions take the household's marginal rate of substitution between consumption and leisure as the costs of labor services. The labor unions act under monopolistic competition and wages are set using staggered contracts as in Calvo (1983). Each period, the union faces a constant probability $1-\xi_w$ to re-optimize its wage $\tilde{W}_t(j)$. This probability is independent across unions and time. Unions that cannot adjust their wage optimally in the current period will increase their wage by the weighted average of (gross) inflation $\Pi_t = \frac{P_t}{P_{t-1}}$ in the previous period and steady state inflation $\bar{\Pi}$ with weights ι_w and $1-\iota_w$, respectively. Let $\tilde{W}_t(j)$ be the optimal wage set by union j in period t. The union charges

$$W_{t+1}(j) = \tilde{W}_t(j)\left(\Pi_t^{\iota_w}\bar{\Pi}^{(1-\iota_w)}\right) \tag{93}$$

in period $t+1$, if it is not allowed to adjust the wage optimally in period $t+1$. When the union can choose its wage optimally, the union solves the following optimization problem

$$\begin{aligned}
&\max_{\tilde{W}_t(j)} E_t \sum_{s=0}^{\infty} (\xi_w)^s \frac{\beta^s \lambda_{t+s}}{\lambda_t}\left[(1+\bar{\tau}_w)W_{t+s}(j) - W_{t+s}^h\right] L_{t+s}(j)\\
&s.t. \quad \frac{L_{t+s}(j)}{L_{t+s}} = \left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\frac{1+\theta_w}{\theta_w}}\\
&\qquad W_{t+s}(j) = \tilde{W}_t(j) X_{t,s}^W\\
&X_{t,s}^W = \begin{cases} 1 & for \quad s=0\\ \prod_{l=1}^{s}\left(\Pi_{t+l-1}^{\iota_w}\bar{\Pi}^{1-\iota_w}\right) & for \quad s=1,...,\infty. \end{cases}
\end{aligned} \tag{94}$$

B.1.3 Intermediate goods producer

Each intermediate goods producer employs labor to produce a variety. The cost minimization problem of the producer is

$$\begin{aligned}
&\min_{L_t(i)} W_t L_t(i)\\
&s.t. \quad Y_t(i) = \xi_{A,t} L_t(i).
\end{aligned} \tag{95}$$

$\xi_{A,t}$ denotes a shock to total factor productivity which follows an exogenous stochastic process

$$\log\left(\xi_{A,t}\right) = \rho_A \log\left(\xi_{A,t-1}\right) + \varepsilon_{A,t} \tag{96}$$

$\varepsilon_{A,t}$ is white noise following $N(0, \sigma_A^2)$.

Prices are set using staggered contracts as in Calvo (1983). Each period, a firm faces a constant probability $1 - \xi_p$ to re-optimize its price $\tilde{P}_t(i)$. This probability is independent across firms and time. A firm that does not re-optimize its price in period t, the price increases by the weighted average of (gross) inflation $\Pi_t = \frac{P_t}{P_{t-1}}$ in the previous period and steady state inflation $\bar{\Pi}$ with weights ι_p and $1 - \iota_p$, respectively. Finally, firms engage in monopolistic competition.

Thus, the price setting problem of intermediate goods producer i can be stated as

$$\begin{aligned}
&\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \left(\beta\xi_p\right)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(1+\bar{\tau}_p)P_{t+s}(i) - MC_{t+s}\right] Y_{t+s}(i) \\
&s.t. \quad Y_{t+s}(i) = \left(\frac{P_{t+s}(i)}{P_{t+s}}\right)^{-\frac{1+\theta_p}{\theta_p}} Y_{t+s} \\
&\qquad P_{t+s}(i) = \tilde{P}_t(i) X_{t,s}^P \\
&X_{t,s}^P = \begin{cases} 1 & for \quad s = 0 \\ \prod_{l=1}^{s} \left(\Pi_{t+l-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}\right) & for \quad s = 1, ..., \infty. \end{cases}
\end{aligned} \tag{97}$$

In the following, we will assume the presence of a price markup shock, often also referred to as markup shock. In the literature, several ways have been suggested to motivate this shock: (i) a shock to the subsidy $\bar{\tau}_p$, (ii) a shock to the elasticity of substitution θ_p, or (iii) a shock in the first order condition associated with the maximization problem of the intermediate goods producers. While all three approaches lead to the same set of equations when the model is approximated to the first order, this is not true, when the model is approximated to the second order. We offer a short discussion on this topic later in this appendix.

B.1.4 Final good bundlers

Intermediate goods are combined into a composite final good by a continuum of representative bundlers acting under perfect competition. The standard Dixit-Stiglitz aggregator implies

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta_p}} \mathrm{d}i\right]^{1+\theta_p} \tag{98}$$

where $\frac{1+\theta_p}{\theta_p}$ denotes the elasticity of substitution between the intermediate goods.
Each bundler maximizes profits by choosing the amount of each intermadiate good to obtain the final good

$$\begin{aligned}
&\max_{Y_t(i), Y_t} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\
&s.t. \quad Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta_p}} \mathrm{d}i\right]^{1+\theta_p}.
\end{aligned} \tag{99}$$

The first order conditions to this problem provide the demand function for each intermediate goods and an expression for the aggregate price level

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} Y_t \tag{100}$$

and

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\theta_p}} \mathrm{d}i \right]^{-\theta_p}, \tag{101}$$

respectively.

B.1.5 Resource constraint

Market clearing in the market for the final good requires that

$$Y_t = C_t. \tag{102}$$

When wages are flexible, the supply of the final good is given by

$$\Omega_t^y Y_t = \xi_{A,t} N_t \tag{103}$$

where Ω_t^y is the measure of price dispersion

$$\Omega_t^y = \frac{\left[\int_0^1 Y_t(i) \mathrm{d}i \right]}{\left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta_p}} \mathrm{d}i \right]^{1+\theta_p}}. \tag{104}$$

We have made use of the fact that under flexible wages the labor market clears when

$$N_t = L_t. \tag{105}$$

Under sticky wages an additional term that captures wage dispersion arises in equation (103). Note that

$$N_t = \int_0^1 L_t(j) dj = \left[\int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\theta_w}{\theta_w}} dj \right] L_t = \Omega_t^l L_t \tag{106}$$

where j is the index of a labor union. Since the labor supplied by the households is homogeneous, $N_t(i) = N_t$. Similarly, aggregate output and manufactured varieties satisfy the relationship

$$\int_0^1 Y_t(i) di = \left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\theta_p}{\theta_p}} di \right] Y_t = \Omega_t^y Y_t. \tag{107}$$

Market clearing implies

$$\int_0^1 Y_t(i) di = \xi_{A,t} L_t \tag{108}$$

or making use of the above relationships

$$\Omega_t^l \Omega_t^y Y_t = \xi_{A,t} N_t. \tag{109}$$

B.2 Linear-quadratic frameworks

We derive the linear-quadratic framework consistent with the NK model laid out in the preceding section. We begin with a version of the model that features flexible wages and an efficient steady state. Then we discuss the derivations of the linear-quadratic framework for the case of a distorted steady state and flexible wages. Finally, we move on to the case of sticky wages.

B.2.1 NKM with external consumption habits

Our model with external consumption habits and inflation persistence resembles Leith, Moldovan, and Rossi (2012).[27] Following the steps outlined in Woodford (2003a) and Gali (2008), the second-order approximation of the household preferences around the efficient steady state can be shown to be of the form

$$L_t = \sigma_L \left(x_t\right)^2 + \frac{\sigma_C}{\delta}\left(x_t - h x_{t-1}\right)^2 + \frac{1+\theta_p}{\theta_p \kappa_p}\left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2. \tag{110}$$

To arrive at this results, we first approximate each of the utility contributions from consumption and labor in equation (88). In the private sector equilibrium, the utility from consumption can be written as

$$\begin{aligned} & \frac{\left(C_t - hC_{t-1}\right)^{1-\sigma_C}}{1-\sigma_C} \\ = \quad & U_c \bar{C}\left\{\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) - h\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right) - \frac{\sigma_C}{2(1-h)}\left(\hat{c}_t - h\hat{c}_{t-1}\right)^2\right\} + t.i.p. + O(2) \end{aligned} \tag{111}$$

with $U_c = \left((1-h)\bar{C}\right)^{-\sigma_C}$. Summing over all periods leads to the expression

$$\begin{aligned} & E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{\left(C_t - hC_{t-1}\right)^{1-\sigma_C}}{1-\sigma_C} \\ & = U_c\bar{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{(1-h\beta)\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) - \frac{\sigma_C}{2(1-h)}\left(\hat{c}_t - h\hat{c}_{t-1}\right)^2\right\} + t.i.p. + O(2). \end{aligned} \tag{112}$$

Given the linearity of production in labor, the disutility from labor can be written as

$$\frac{N_t^{1+\sigma_L}}{1+\sigma_L} = U_n \bar{N}\left\{\hat{n}_t + \frac{1+\sigma_L}{2}\hat{n}_t^2\right\} + t.i.p. + O(2) \tag{113}$$

where $U_n = \bar{N}^{\sigma_L}$. Applying the following result from Woodford (2003a)

$$\hat{n}_t \quad = \quad \hat{y}_t - \hat{\xi}_{A,t} + \frac{1+\theta_p}{2\theta_p} var_i\left(p_t(i)\right) \tag{114}$$

in equation (113), the disutility from labor can be expressed as

$$\begin{aligned} & E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{N_t^{1+\sigma_L}}{1+\sigma_L} \\ = \quad & \bar{N}^{1+\sigma_L}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\hat{y}_t + \frac{1+\sigma_L}{2}\hat{y}_t^2 - 2\frac{1+\sigma_L}{2}\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{2\theta_p}var_i\left(p_t(i)\right)\right\} \\ & + t.i.p. + O(2). \end{aligned} \tag{115}$$

Before re-combining the expressions for the utility from consumption and the disutility from labor, we

[27] Leith, Moldovan, and Rossi (2012) abstract from inflation persistence and focus on the conceptually more challenging derivations under various formulations of consumption habits (internal versus external, deep versus superficial habits).

turn to three relationships that allow us to simplify the approximation. The market clearing condition implies

$$\hat{c}_t = \hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \frac{1}{2}\hat{c}_t^2 + O(2) \tag{116}$$

and the price dispersion term can be expressed as

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} var_i\left(p_t(i)\right) = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2. \tag{117}$$

The third relationship is

$$(1-h\beta)U_c\bar{C} = U_n\bar{N} \tag{118}$$

which is derived as follows. The deterministic steady state of the market economy is not necessarily efficient under external habits as agents do not internalize the impact of today's consumption choice on tomorrow's marginal utility. To render the steady state efficient, we introduce a tax on consumption, which satisfies $1+\bar{\tau}_c = \frac{1}{1-h\beta}$. With this tax in place, the first order conditions for consumption and labor imply that in the steady state

$$\frac{\left((1-h)\bar{C}\right)^{-\sigma_C}}{\bar{N}^{\sigma_L}} = (1+\bar{\tau}_c)\frac{\bar{N}}{\bar{C}} \tag{119}$$

or

$$(1-h\beta)U_c\bar{C} = U_n\bar{N}. \tag{120}$$

Combining the utility from consumption and the disutility from labor using these three relationships, we obtain the second-order approximation to household preferences as

$$\begin{aligned}
& U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right) - \frac{\sigma_C}{2(1-h)(1-h\beta)}\left(\hat{c}_t - h\hat{c}_{t-1}\right)^2\right\} \\
& -U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right) + \frac{\sigma_L}{2}\hat{y}_t^2 - 2\frac{1+\sigma_L}{2}\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{2\theta_p}var_i\left(p_t(i)\right)\right\} \\
& +t.i.p. + O(2) \\
= \; & -\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\sigma_L\hat{y}_t^2 + \frac{\sigma_C}{\delta}\left(\hat{y}_t - h\hat{y}_{t-1}\right)^2 - 2(1+\sigma_L)\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t} - \iota_p\pi_{p,t-1}\right)^2\right\} \\
& +t.i.p. + O(2)
\end{aligned} \tag{121}$$

with

$$\begin{aligned}
\kappa_p &= \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} \\
\delta &= (1-h)(1-h\beta).
\end{aligned}$$

Our baseline model with sticky prices and external consumption habits can be summarized in linear-quadratic form by the (hybrid) New Keynesian Phillips curve

$$(\pi_{p,t} - \iota_p\pi_{p,t-1}) = \kappa_p\widehat{mc}_t + u_{p,t} + \beta E_t\left(\pi_{p,t+1} - \iota_p\pi_{p,t}\right) \tag{122}$$

where u_t denotes a stationary markup shock.[28] Marginal costs follow

$$\widehat{mc}_t = \sigma_L \hat{y}_t + \frac{\sigma_C}{1-h}\left(\hat{y}_t - h\hat{y}_{t-1}\right) - \left(1+\sigma_L\right)\hat{\xi}_{A,t} \tag{123}$$

and the aggregate demand curve satisfies

$$\left(\hat{y}_t - h\hat{y}_{t-1}\right) = E_t\left(\hat{y}_{t+1} - h\hat{y}_t\right) - \frac{1-h}{\sigma_C}\left(i_t - E_t\pi_{p,t+1}\right) \tag{124}$$

where i_t denotes the nominal interest rate. The social loss function satisfies

$$E_{t_0}\left(\frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right) \tag{125}$$

with

$$L_t = \sigma_L \hat{y}_t^2 + \frac{\sigma_C}{\delta}\left(\hat{y}_t - h\hat{y}_{t-1}\right)^2 - 2(1+\sigma_L)\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t} - \iota_p\pi_{p,t-1}\right)^2. \tag{126}$$

To express the model consisting of equations (122) to (126) in terms of "gaps," we adopt the notion of the welfare-relevant output gap as in Woodford (2003a). The welfare-relevant output gap is computed by defining potential output as the output level that would prevail absent nominal rigidities and markup shocks, but the internalization of consumption habits . Although the consumption tax $1+\bar{\tau}_c$ removes all static distortions arising from external habits, the dynamics remain distorted. To obtain the (linear) equilibrium dynamics of this efficient output level, we solve the model under internal habits, as a social planner would do, to deliver

$$\begin{aligned}
\widehat{mc}_t^* &= \left[\widehat{\frac{W}{P}}\right]_t^* - \left(\hat{y}_t^* - \hat{n}_t^*\right) \\
&= \sigma_L \hat{n}_t^* + \frac{\sigma_C}{\delta}\left(\hat{c}_t^* - h\hat{c}_{t-1}^*\right) - h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{c}_{t+1}^* - h\hat{c}_t^*\right) - \left(\hat{y}_t^* - \hat{n}_t^*\right) \\
&= \sigma_L\left(\hat{y}_t^* - \hat{\xi}_{A,t}\right) + \frac{\sigma_C}{\delta}\left(\hat{y}_t^* - h\hat{y}_{t-1}^*\right) - h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}_{t+1}^* - h\hat{y}_t^*\right) - \hat{\xi}_{A,t} \\
&= \sigma_L \hat{y}_t^* + \frac{\sigma_C}{\delta}\left(\hat{y}_t^* - h\hat{y}_{t-1}^*\right) - h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}_{t+1}^* - h\hat{y}_t^*\right) - \left(1+\sigma_L\right)\hat{\xi}_{A,t}.
\end{aligned} \tag{127}$$

In the efficient economy (flexible prices, no markup shocks) real marginal costs are constant and therefore efficient output evolves according to

$$\sigma_L \hat{y}_t^* + \frac{\sigma_C}{\delta}\left(\hat{y}_t^* - h\hat{y}_{t-1}^*\right) - h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}_{t+1}^* - h\hat{y}_t^*\right) = \left(1+\sigma_L\right)\hat{\xi}_{A,t}. \tag{128}$$

Equation (128) can be used to rewrite the model in terms of the welfare-relevant output gap. Applied to equation (121), we obtain

$$-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\sigma_L\hat{y}_t^2 - 2\sigma_L\hat{y}_t\hat{y}_t^* + \frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t} - \iota_p\pi_{p,t-1}\right)^2\right\}$$

[28] The literature offers several ways of modelling markup shocks such as variations in desired markups in the price setting rules of firms, exogenous variations in wage markups, shocks to the price subsidy paid to producers, or even shocks to the elasticity of substitution between varieties. To a first order approximation all these models imply the same dynamic responses of the economy to the markup shock. As discussed below, the second-order approximation of the household preferences, however, is not identical across approaches if the steady state is inefficient.

$$
\begin{aligned}
& -\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{\sigma_C}{\delta}\left\{\left(\hat{y}_t-h\hat{y}_{t-1}\right)^2-2\left[\hat{y}_t\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-\beta h\hat{y}_t\left(\hat{y}_{t+1}^*-h\hat{y}_t^*\right)\right]\right\} \\
& +t.i.p.+O(2) \\
= & -\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\sigma_L\hat{y}_t^2-2\sigma_L\hat{y}_t\hat{y}_t^*+\frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2\right\} \\
& -\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{\sigma_C}{\delta}\left\{\left(\hat{y}_t-h\hat{y}_{t-1}\right)^2-2\left(\hat{y}_t-h\hat{y}_{t-1}\right)\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)]\right\} \\
& +t.i.p.+O(2) \qquad (129)
\end{aligned}
$$

using

$$
E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{y}_t\left\{\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-h\beta\left(\hat{y}_{t+1}^*+h\hat{y}_t^*\right)\right\}=E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\hat{y}_t-h\hat{y}_{t-1}\right)\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right). \qquad (130)
$$

Let the welfare-relevant output gap be denoted by $x_t=\hat{y}_t-\hat{y}_t^*$. Equations (122) to (126) can be stated as

$$
\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)=\kappa_p\widehat{mc}_t+u_{p,t}+\beta E_t\left(\pi_{p,t+1}-\iota_p\pi_{p,t}\right) \qquad (131)
$$

with marginal costs following

$$
\widehat{mc}_t = \sigma_L x_t+\frac{\sigma_C}{1-h}\left(x_t-hx_{t-1}\right)+\frac{h\beta}{1-h\beta}g_{mu,t}^* \qquad (132)
$$

and the aggregate demand curve

$$
\left(x_t-hx_{t-1}\right)=E_t\left(x_{t+1}-hx_t\right)-\frac{1-h}{\sigma_C}\left(i_t-E_t\pi_{p,t+1}-g_{mu,t}^*\right). \qquad (133)
$$

$g_{mu,t}^*$ is defined as

$$
g_{mu,t}^*=\frac{\sigma_C}{1-h}\left[E_t\left(\hat{y}_{t+1}^*-h\hat{y}_t^*\right)-\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)\right]. \qquad (134)
$$

The social loss function is now written as

$$
E_{t_0}\left(\frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right) \qquad (135)
$$

with

$$
L_t=\sigma_L\left(x_t\right)^2+\frac{\sigma_C}{\delta}\left(x_t-hx_{t-1}\right)^2+\frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2. \qquad (136)
$$

The equilibrium efficient output follows

$$
\hat{y}_t^*=\Gamma_{\hat{y}^*}\hat{y}_{t-1}^*+\Gamma_{\hat{\xi}^A}\hat{\xi}_{A,t} \qquad (137)
$$

as derived from equation (128) with $\Gamma_{\hat{y}^*}$ being the solution to

$$
-h\beta\frac{\sigma_C}{\delta}\Gamma_{\hat{y}^*}^2+\left(\sigma_L+\frac{\sigma_C}{\delta}(1+h^2\beta)\right)\Gamma_{\hat{y}^*}^2-h\frac{\sigma_C}{\delta}=0 \qquad (138)
$$

and $\Gamma_{\hat{\xi}^A}$ being given by

$$\Gamma_{\hat{\xi}^A} = \frac{1+\sigma_L}{\sigma_L + \frac{\sigma_C}{\delta}(1+h^2\beta) - h\beta\frac{\sigma_C}{\delta}\left(\Gamma_{\hat{y}^*} + \rho_A\right)}. \tag{139}$$

As the term $\frac{h\beta}{1-h\beta} g^*_{mu,t}$ appears in equation (132), the central bank is unable to perfectly stabilize inflation and the welfare-relevant output gap under external consumption habits in response to technology shocks. As discussed in Leith, Moldovan, and Rossi (2012) and Woodford (2003a), consumption habits have to be specified as internal in order for the "divine coincidence" to re-emeerge; also compare to Blanchard and Gali (2007).

B.2.2 Linear quadratic framework with distorted steady state

In our discussion of the case of a distorted steady state, we return to the simple New Keynesian Model with flexible wages and no consumption habits (h=0) as in Benigno and Woodford (2005). In our derivations, we allow for two sources that could justify the presence of a shock in the NKPC. The first one is an ad hoc markup shock $\mu_{p,t}$ that is introduced into the first order condition of price setting firms. The second one is a shock to the sales subsidy $\tau_{p,t}$. If the subsidies to prices do not fully offset the monopolistic distortions in the product market, the steady state relationship between consumption and labor is determined by

$$\bar{C}^{1-\sigma_C} = \bar{N}^{1+\sigma_L}\Phi \tag{140}$$

with the steady state markup satisfying $\frac{1}{\bar{mc}} = \frac{1+\theta_p}{1+\bar{\tau}_p} = \Phi$. In combining the utility from consumption, equation (112), and the disutility from labor, equation (115), the linear term $\hat{y}_t$ does not drop out

$$\begin{aligned}
& \Phi U_n \bar{N} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 \right) - \frac{\sigma_C}{2}\hat{y}_t^2 \right\} \\
& -U_n \bar{N} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 \right) + \frac{\sigma_L}{2}\hat{y}_t^2 - 2\frac{1+\sigma_L}{2}\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{2\theta_p} var_i\left(p_t(i)\right) \right\} \\
& +t.i.p. + O(2) \\
= \quad & -\frac{1}{2} U_n \bar{N} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ -2\left(\Phi - 1\right)\hat{y}_t + \left[(\sigma_L + \sigma_C) - (1-\sigma_C)(\Phi - 1)\right]\hat{y}_t^2 \right\} \\
& -\frac{1}{2} U_n \bar{N} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ -2(1+\sigma_L)\hat{y}_t\hat{\xi}_{A,t} + \frac{1+\theta_p}{\theta_p \kappa_p}\left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2 \right\} + t.i.p. + O(3).
\end{aligned} \tag{141}$$

Absent distortions, $\Phi = 1$, and the linear term $(\Phi - 1)\hat{y}_t$ in equation (141) cancels out. With distortions, we employ the second-order approximation to the nonlinear New Keynesian Phillips curve as in Benigno and Woodford (2005) to substitute out for this linear term. The first order condition for price setting is given by

$$P_t^{opt} = \frac{H_t}{G_t} \tag{142}$$

where

$$H_{t_0} = \frac{1}{C_{t_0}^{-\sigma_C}} E_{t_0} \sum_{t=t_0}^{\infty} (\beta\xi_p)^{t-t_0} (1+\mu_{p,t}) N_t^{\sigma_L} \left(\frac{\Pi(t) P_{t_0}}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} Y_t \tag{143}$$

$$G_{t_0} = \frac{1}{C_{t_0}^{-\sigma_C}} E_{t_0} \sum_{t=t_0}^{\infty} (\beta\xi_p)^{t-t_0} (1+\tau_{p,t}) C_t^{-\sigma_C} \left(\frac{\Pi(t) P_{t_0}}{P_t}\right)^{1-\frac{1+\theta_p}{\theta_p}} Y_t \tag{144}$$

and $\Pi(t) = \prod_{l=1}^{t} \left(\pi_{t_0+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p}\right)$. Following the steps outlined in Benigno and Woodford (2005), this relationship can be shown to be approximated by

$$\begin{aligned} \frac{V_t}{\sigma_L+\sigma_C} &= \kappa_p E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{\hat{h}_t - \hat{g}_t}{\sigma_L+\sigma_C} + \frac{\left(\hat{h}_t - \hat{g}_t\right)\left(\hat{h}_t + \hat{g}_t\right)}{2\left(\sigma_L+\sigma_C\right)} \right] \\ &\quad + \kappa_p E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1+\theta_p}{2(\sigma_L+\sigma_C)\kappa_p\theta_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2 \right] + t.i.p. + O(3) \end{aligned} \tag{145}$$

where the terms $\hat{g}_t$ and $\hat{h}_t$ are given by

$$\begin{aligned} \hat{g}_t &= \frac{\bar{\tau}_p}{1+\bar{\tau}_p} \hat{\tau}_{p,t} - \left(\sigma_C - 1\right) \hat{y}_t \\ \hat{h}_t &= \frac{\bar{\mu}_p}{1+\bar{\mu}_p} \hat{\mu}_{p,t} + \left(1+\sigma_L\right) \hat{y}_t - \left(1+\sigma_L\right) \hat{\xi}_{A,t} + \sigma_L \frac{1+\theta_p}{2\kappa_p\theta_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2. \end{aligned}$$

Substituting the definitions $\hat{g}_t$ and $\hat{h}_t$ into equation (145),

$$\begin{aligned} & \frac{\hat{h}_t - \hat{g}_t}{\sigma_L+\sigma_C} + \frac{\left(\hat{h}_t - \hat{g}_t\right)\left(\hat{h}_t + \hat{g}_t\right)}{2\left(\sigma_L+\sigma_C\right)} + \frac{1+\theta_p}{2(\sigma_L+\sigma_C)\kappa_p\theta_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2 \\ = \;& \hat{y}_t + \frac{2+\sigma_L-\sigma_C}{2} \hat{y}_t^2 + \frac{1+\sigma_L}{\sigma_L+\sigma_C} \frac{1+\theta_p}{2\kappa_p\theta_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2 \\ & - \frac{(1+\sigma_L)^2}{\sigma_C+\sigma_L} \hat{\xi}_{A,t} \hat{y}_t + \frac{\bar{\mu}_p}{1+\bar{\mu}_p} \frac{1+\sigma_L}{\sigma_C+\sigma_L} \hat{\mu}_{p,t} \hat{y}_t - \frac{\bar{\tau}_p}{1+\bar{\tau}_p} \frac{1-\sigma_C}{\sigma_C+\sigma_L} \hat{\tau}_{p,t} \hat{y}_t + t.i.p. \end{aligned} \tag{146}$$

multiplying with $-\frac{1}{\kappa_p} U_N \bar{N}(\Phi - 1)$ and adding into equation (141) we obtain the approximation

$$\begin{aligned} & -\frac{1}{2} U_n \bar{N} \left\{ 1 + (\Phi-1) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \right\} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\sigma_L+\sigma_C) \hat{y}_t^2 \\ & -\frac{1}{2} U_n \bar{N} \left\{ 1 + (\Phi-1) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \right\} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} - 2(\sigma_L+\sigma_C) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \hat{y}_t \hat{\xi}_{A,t} \\ & -\frac{1}{2} U_n \bar{N} \left\{ 1 + (\Phi-1) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \right\} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1+\theta_p}{\kappa_p\theta_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2 \\ & -\frac{1}{2} U_n \bar{N} \left\{ 1 + (\Phi-1) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \right\} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{2\frac{\bar{\mu}_p}{1+\bar{\mu}_p}(\Phi-1)(1+\sigma_L)}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)} \hat{\mu}_{p,t} \hat{y}_t \\ & -\frac{1}{2} U_n \bar{N} \left\{ 1 + (\Phi-1) \frac{1+\sigma_L}{\sigma_L+\sigma_C} \right\} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{-2\frac{\bar{\tau}_p}{1+\bar{\tau}_p}(\Phi-1)(1-\sigma_C)}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)} \hat{\tau}_{p,t} \hat{y}_t \\ & + t.i.p. + O(3). \end{aligned} \tag{147}$$

Therefore, the model with a distortionary steady state can be written as

$$E_{t_0}\left(\frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right) \tag{148}$$

with

$$L_t = (\sigma_L+\sigma_C)\left(\hat{y}_t-\tilde{y}_t\right)^2+\frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2 \tag{149}$$

and the target output level $\tilde{y}_t$

$$\begin{aligned}\tilde{y}_t &= \frac{1+\sigma_L}{\sigma_L+\sigma_C}\hat{\xi}_{A,t}\\ &-\frac{(\Phi-1)\frac{1+\sigma_L}{\sigma_L+\sigma_C}}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)}\frac{\bar{\mu}_p}{1+\bar{\mu}_p}\hat{\mu}_{p,t}\\ &+\frac{(\Phi-1)\frac{1-\sigma_C}{\sigma_L+\sigma_C}}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)}\frac{\bar{\tau}_p}{1+\bar{\tau}_p}\hat{\tau}_{p,t}.\end{aligned}$$

The linear New Keynesian Phillips curve is given by

$$\begin{aligned}(\pi_{p,t}-\iota_p\pi_{p,t-1}) &= \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left((\sigma_L+\sigma_C)\hat{y}_t+\frac{\bar{\mu}_p}{1+\bar{\mu}_p}\hat{\mu}_{p,t}-\frac{\bar{\tau}_p}{1+\bar{\tau}_p}\hat{\tau}_{p,t}-(1+\sigma_L)\hat{\xi}_{A,t}\right)\\ &+\beta E_t(\pi_{p,t+1}-\iota_p\pi_{p,t})\end{aligned} \tag{150}$$

or written in terms of the welfare relevant output gap $\hat{y}_t-\tilde{y}_t$

$$\begin{aligned}(\pi_{p,t}-\iota_p\pi_{p,t-1}) &= \kappa_p(\sigma_L+\sigma_C)\left(\hat{y}_t-\tilde{y}_t\right)\\ &+\kappa_p\left(1-\frac{(\Phi-1)(1+\sigma_L)}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)}\right)\frac{\bar{\mu}_p}{1+\bar{\mu}_p}\hat{\mu}_{p,t}\\ &-\kappa_p\left(1-\frac{(\Phi-1)(1-\sigma_C)}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)}\right)\frac{\bar{\tau}_p}{1+\bar{\tau}_p}\hat{\tau}_{p,t}\\ &+\beta E_t(\pi_{p,t+1}-\iota_p\pi_{p,t}).\end{aligned} \tag{151}$$

In contrast to the model with an efficient steady state, the target output level $\tilde{y}_t$ responds to the price markup shock $\hat{\mu}_{p,t}$ and the shock to the subsidy $\hat{\tau}_{p,t}$. Only when $\Phi = 1$ does the target level remain unchanged after such shocks. While under an undistorted steady state the two shocks have the same impact under the optimal policy, this is no longer true if the steady state is distorted. This can easily be seen if $\sigma_C = 1$. In this case, the shock $\hat{\tau}_{p,t}$ does not impact the target output level at all.

Ignoring the movements in the output target level induced by markup/subsidy shocks when formulating policies leads to inefficiencies. Although the optimal commitment policy can be described as an inflation targeting framework, the definition of the output gap is key. If the output gap measure applied by the policymaker rests on a definition of potential output as $\bar{y}_t = \frac{1+\sigma_L}{\sigma_L+\sigma_C}\hat{\xi}_{A,t}$—as would be the case under the definition applied in Smets and Wouters (2007)—instead of $\tilde{y}_t$ the central bank's response will not be optimal.

B.2.3 Linear quadratic framework with sticky wages

When prices and wages are sticky, we follow the steps outlined in Gali (2008) and Erceg, Henderson, and Levin (2000) to approximate the utility function of the household to the second-order. Our derivations include a shock to the marginal disutility of labor to illustrate the discussion in Chari, Kehoe, and McGrattan (2009).

In comparison to the previous section, the approximations of the utility from consumption shown in equation (112) and the disutility from labor given in equation (113) remain unchanged with the small qualifier that the latter expression is augmented by a term to capture the labor supply shock

$$U_c\bar{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{(1-h\beta)\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right)-\frac{\sigma_C}{2(1-h)}\left(\hat{c}_t-h\hat{c}_{t-1}\right)^2\right\}+t.i.p.+O(2). \tag{152}$$

and

$$U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\hat{n}_t+\frac{1+\sigma_L}{2}\hat{n}_t^2+\hat{\xi}_{L,t}\hat{n}_t\right\}+t.i.p.+O(2). \tag{153}$$

Absent sticky wages aggregate labor supply N_t is related to final output Y_t and the level of technology via a term that measures price dispersion. Under sticky wages an additional term that captures wage dispersion arises in this relationship. Note that

$$N_t=\int_0^1 L_t(j)dj=\left[\int_0^1\left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\theta_w}{\theta_w}}dj\right]L_t=\Omega_t^l L_t \tag{154}$$

where j is the index of a labor union. Similarly, aggregate output and manufactured varieties satisfy the relationship

$$\int_0^1 Y_t(i)di=\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}di\right]Y_t=\Omega_t^y Y_t. \tag{155}$$

Market clearing implies

$$\int_0^1 Y_t(i)di=\xi_{A,t}L_t \tag{156}$$

or making use of the above relationships

$$\Omega_t^l\Omega_t^y Y_t=\xi_{A,t}N_t. \tag{157}$$

Applying results from Woodford (2003a) and Gali (2008) regarding the second-order approximations of Ω_t^y and Ω_t^l we obtain

$$\hat{n}_t \quad = \quad \hat{y}_t-\hat{\xi}_{A,t}+\frac{1+\theta_p}{2\theta_p}var_i\left(p_t(i)\right)+\frac{1+\theta_w}{2\theta_w}var_f\left(w_t(f)\right). \tag{158}$$

Thus, the disutility from labor can be approximated by

$$\xi_{L,t}\frac{N_t^{1+\sigma_L}}{1+\sigma_L} = U_n\bar{N}\left\{\hat{y}_t-\hat{\xi}_{A,t}+\frac{1+\theta_p}{2\theta_p}var_i\left(p_t(i)\right)+\frac{1+\theta_w}{2\theta_w}var_f\left(w_t(f)\right)\right\} + U_n\bar{N}\left\{\frac{1+\sigma_L}{2}\left(\hat{y}_t-\hat{\xi}_{A,t}\right)^2+\hat{\xi}_{L,t}\hat{y}_t\right\}+t.i.p.+O(2). \quad (159)$$

Assuming that the steady state is efficient due to appropriately chosen subsidies, i.e., $(1-h\beta)U_c\bar{C} = U_n\bar{N}$, the utility function of the representative household can be approximated as

$$\begin{aligned}&U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\hat{y}_t+\frac{1}{2}\hat{y}_t^2-\frac{\sigma_C}{2(1-h)(1-h\beta)}\left(\hat{y}_t-h\hat{y}_{t-1}\right)^2\right\}\\&-U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\hat{y}_t-\hat{\xi}_{A,t}+\frac{1+\theta_p}{2\theta_p}var_i\left(p_t(i)\right)+\frac{1+\theta_w}{2\theta_w}var_f\left(w_t(f)\right)\right\}\\&-U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1+\sigma_L}{2}\left(\hat{y}_t-\hat{\xi}_{A,t}\right)^2+\hat{\xi}_{L,t}\hat{y}_t\right\}+t.i.p.+O(2)\end{aligned} \quad (160)$$

or after simplifying

$$\begin{aligned}&-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\sigma_L\hat{y}_t^2+\frac{\sigma_C}{\delta}\left(\hat{y}_t-h\hat{y}_{t-1}\right)^2-2\left(1+\sigma_L\right)\hat{y}_t\hat{\xi}_{A,t}+\frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2\right\}\\&-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{2\hat{y}_t\hat{\xi}_{L,t}+\frac{1+\theta_w}{\theta_w\kappa_w}\left(\pi_{w,t}-\iota_w\pi_{p,t-1}\right)^2\right\}+t.i.p.+O(2)\end{aligned} \quad (161)$$

where

$$\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$$
$$\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$$
$$\delta = (1-h)(1-h\beta).$$

To obtain the (linear) equilibrium dynamics of the efficient output level in the model with labor supply shocks, note that

$$\begin{aligned}\widehat{mc}_t^* &= \hat{\xi}_{L,t}+\sigma_L\hat{n}_t^*+\frac{\sigma_C}{\delta}\left(\hat{c}_t^*-h\hat{c}_{t-1}^*\right)-h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{c}_{t+1}^*-h\hat{c}_t^*\right)-\left(\hat{y}_t^*-\hat{n}_t^*\right)\\&=\sigma_L\hat{y}_t^*+\frac{\sigma_C}{\delta}\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}_{t+1}^*-h\hat{y}_t^*\right)-\left(1+\sigma_L\right)\hat{\xi}_{A,t}+\hat{\xi}_{L,t}.\end{aligned} \quad (162)$$

and

$$\sigma_L\hat{y}_t^*+\frac{\sigma_C}{\delta}\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}_{t+1}^*-h\hat{y}_t^*\right)=\left(1+\sigma_L\right)\hat{\xi}_{A,t}-\hat{\xi}_{L,t}. \quad (163)$$

Efficient output is therefore a function of lagged efficient output, technology, and the labor supply shock.

By substituting this last expression back into equation (161), we can approximate the utility function in terms of the welfare-relevant output gap, price and wage inflation, and the labor supply shock

$$-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\sigma_L\hat{y}_t^2-2\sigma_L\hat{y}_t\hat{y}_t^*\right\}$$

$$
\begin{aligned}
&-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2+\frac{1+\theta_w}{\theta_w\kappa_w}\left(\pi_{w,t}-\iota_w\pi_{p,t-1}\right)^2\right\}\\
&-\frac{1}{2}U_n\bar{N}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{\sigma_C}{\delta}\left\{\left(\hat{y}_t-h\hat{y}_{t-1}\right)^2-2\left[\hat{y}_t\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-\beta h\hat{y}_t\left(\hat{y}_{t+1}^*-h\hat{y}_t^*\right)\right]\right\}\\
&+t.i.p.+O(2). \qquad (164)
\end{aligned}
$$

Applying equation (130) once more allows us to define the social loss function for the model with sticky wages and prices, indexation, labor supply shocks, and external habits as

$$
E_{t_0}\left(\frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right) \qquad (165)
$$

with

$$
\begin{aligned}
L_t &= \sigma_L\left(x_t\right)^2+\frac{\sigma_C}{\delta}\left(x_t-hx_{t-1}\right)^2\\
&+ \frac{1+\theta_p}{\theta_p\kappa_p}\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)^2+\frac{1+\theta_w}{\theta_w\kappa_w}\left(\pi_{w,t}-\iota_w\pi_{p,t-1}\right)^2 \qquad (166)
\end{aligned}
$$

$x_t=\hat{y}_t-\hat{y}_t^*$ denotes the welfare-relevant output gap. The structural equations of the model are given by the New Keynesian Phillips curve for prices

$$
\left(\pi_{p,t}-\iota_p\pi_{p,t-1}\right)=\kappa_p\widehat{mc}_t+u_{p,t}+\beta E_t\left(\pi_{p,t+1}-\iota_p\pi_{p,t}\right) \qquad (167)
$$

with

$$
\widehat{mc}_t=\hat{\omega}_t-\hat{\xi}_{A,t}=\hat{\omega}_t-\hat{\omega}_t^* \qquad (168)
$$

and the price markup shock $u_{p,t}$, the New Keynesian Phillips curve for wages

$$
\left(\pi_{w,t}-\iota_w\pi_{p,t-1}\right)=\kappa_w\left(\widehat{mrs}_t-\hat{\omega}_t\right)+u_{w,t}+\beta E_t\left(\pi_{w,t+1}-\iota_w\pi_{p,t}\right) \qquad (169)
$$

with

$$
\begin{aligned}
\widehat{mrs}_t-\hat{\omega}_t &= \sigma_L\hat{y}_t+\frac{\sigma_C}{1-h}\left(\hat{y}_t-h\hat{y}_{t-1}\right)-\sigma_L\hat{\xi}_{A,t}+\hat{\xi}_{L,t}-\hat{\omega}_t\\
&= \sigma_L\hat{y}_t+\frac{\sigma_C}{1-h}\left(\hat{y}_t-h\hat{y}_{t-1}\right)-\left(1+\sigma_L\right)\hat{\xi}_{A,t}+\hat{\xi}_{L,t}-\left(\hat{\omega}_t-\hat{\omega}_t^*\right)\\
&= \sigma_Lx_t+\frac{\sigma_C}{1-h}\left(x_t-hx_{t-1}\right)-\left(\hat{\omega}_t-\hat{\omega}_t^*\right)\\
&\quad+\sigma_L\hat{y}_t^*+\frac{\sigma_C}{1-h}\left(\hat{y}_t^*-h\hat{y}_{t-1}^*\right)-\left(1+\sigma_L\right)\hat{\xi}_{A,t}+\hat{\xi}_{L,t}\\
&= \sigma_Lx_t+\frac{\sigma_C}{1-h}\left(x_t-hx_{t-1}\right)-\left(\hat{\omega}_t-\hat{\omega}_t^*\right)+\frac{h\beta}{1-h\beta}g_{mu,t}^* \qquad (170)
\end{aligned}
$$

and the wage markup shock $u_{w,t}$, the evolution of real wages

$$
\left(\hat{\omega}_t-\hat{\omega}_t^*\right)=\left(\hat{\omega}_{t-1}-\hat{\omega}_{t-1}^*\right)+\pi_{w,t}-\pi_{p,t}-\left(\hat{\omega}_t^*-\hat{\omega}_{t-1}^*\right) \qquad (171)
$$

and the aggregate demand curve

$$(x_t - hx_{t-1}) = E_t\left(x_{t+1} - hx_t\right) - \frac{1-h}{\sigma_C}\left(i_t - E_t\pi_{p,t+1} - g^*_{mu,t}\right). \tag{172}$$

$g^*_{mu,t}$ is defined as

$$g^*_{mu,t} = \frac{\sigma_C}{1-h}\left[E_t\left(\hat{y}^*_{t+1} - h\hat{y}^*_t\right) - \left(\hat{y}^*_t - h\hat{y}^*_{t-1}\right)\right]. \tag{173}$$

The efficient equilibrium output follows

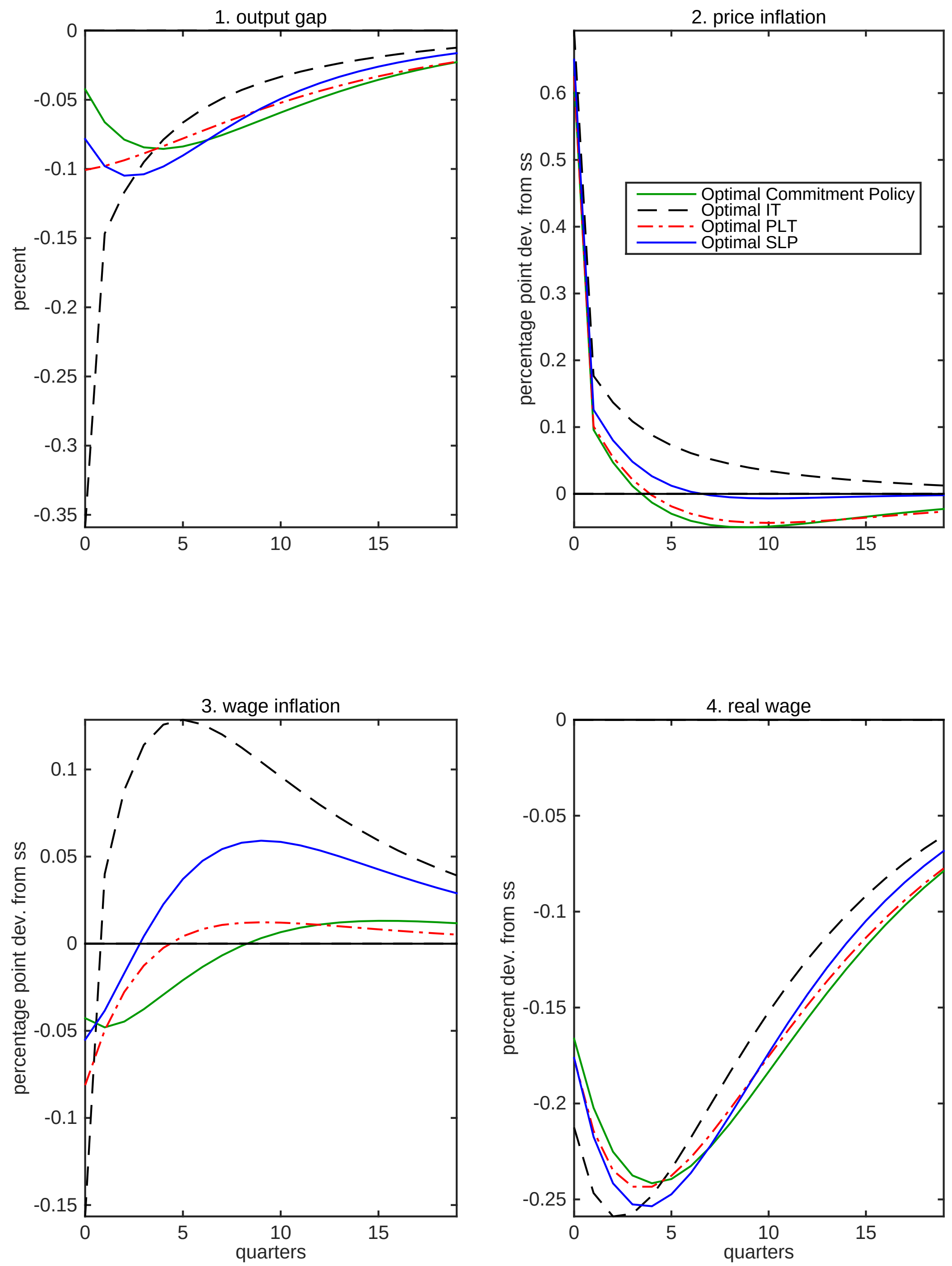
$$\sigma_L \hat{y}^*_t + \frac{\sigma_C}{\delta}\left(\hat{y}^*_t - h\hat{y}^*_{t-1}\right) - h\beta\frac{\sigma_C}{\delta}E_t\left(\hat{y}^*_{t+1} - h\hat{y}^*_t\right) = \left(1+\sigma_L\right)\hat{\xi}_{A,t} - \hat{\xi}_{L,t} \tag{174}$$

and the efficient real wage is determined by

$$\hat{\omega}^*_t = \hat{\xi}_{A,t}. \tag{175}$$

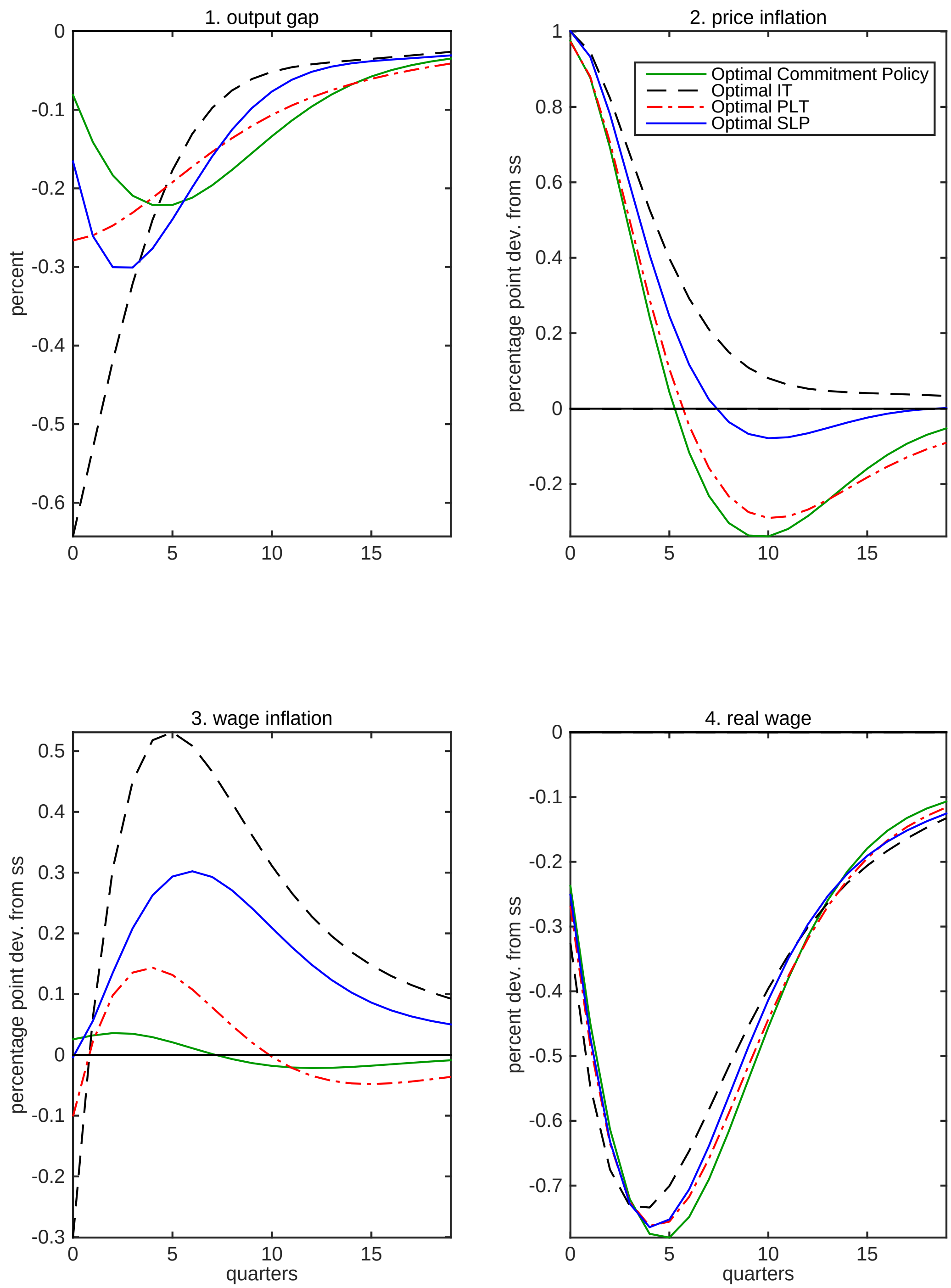
Figures 12 and 13 depict the impulse responses after a price markup shock in the model with sticky wages and no price indexation and with full price indexation, respectively.

Figure 12: Impulse responses to a price markup shock in a model with sticky wages and no price indexation



Note: The figure shows the impulse responses of selected variables in the sticky wage model with no price or wage indexation ($\iota_p = \iota_w = 0$) after a price markup shock. The results for four policies are shown: the optimal commitment policy (Ramsey), optimal inflation targeting framework (IT), the optimal price level targeting (PLT), and the optimal speed limit policy (SLP).

Figure 13: Impulse responses to a price markup shock in a model with sticky wages and full price indexation



Note: The figure shows the impulse responses of selected variables in the sticky wage model with full price indexation ($\iota_p = 1$) but no wage indexation ($\iota_w = 0$) after a price markup shock. The results for four policies are shown: the optimal commitment policy (Ramsey), optimal inflation targeting framework (IT), the optimal price level targeting (PLT), and the optimal speed limit policy (SLP).

B.3 Inertia under price level targeting and speed limit policy in the textbook NKM

When policymakers act under discretion, price level targeting and speed limit policy perform strongly in comparison to the optimal commitment policy as discussed in Walsh (2003) and Vestin (2006). This appendix reproduces the key steps to show how inertia in the output gap and inflation arise in the textbook NKM without price indexation or habit persistence under these two targeting frameworks.

In the following, we assume that the markup shock is transitory, i.e., $E_t(u_{p,t+1}) = 0$, and we abstract from the zero lower bound on the nominal interest rate, which allows us to ignore the aggregate demand curve of the model. Under these assumptions, the optimal commitment policy implies the following dynamics for the output gap and inflation

$$x_t = \Upsilon_x x_{t-1} + \Upsilon_u u_{p,t} \tag{176}$$

$$\pi_{p,t} = \frac{\lambda}{\tilde{\kappa}_p}(1 - \Upsilon_x) x_{t-1} - \frac{\lambda}{\tilde{\kappa}_p}\Upsilon_u u_{p,t} \tag{177}$$

with Υ_x being the solution to

$$\left(1 + \beta + \frac{(\tilde{\kappa}_p)^2}{\lambda}\right)\Upsilon_x - \beta\Upsilon_x^2 - 1 = 0 \tag{178}$$

that satisfies $\Upsilon_x < 1$. The value for Υ_u is

$$\Upsilon_u = \frac{-\tilde{\kappa}_p}{\lambda\{1 + \beta(1 - \Upsilon_x)\} + (\tilde{\kappa}_p)^2}. \tag{179}$$

λ is the weight on the output gap in the true social loss function when the weight on inflation is normalized to unity in the textbook NKM without price indexation, i.e., $\lambda = \frac{\tilde{\kappa}_p\theta_p}{1+\theta_p}$. Recall, that under the assumptions of the textbook NKM the functional form of the true social loss function is identical to the objective function under the inflation targeting framework.

In equilibrium, both inflation and the output gap depend on the pervious realization of the output gap. This feature of the optimal commitment policy is not found in the optimal discretion policy.

B.3.1 Price level targeting

To solve for the equilibrium under discretion we conjecture that the value function of the policymaker is quadratic and depends on the price level of the previous period under price level targeting. We start with the assumption that $u_{p,t} = \rho_u u_{p,t-1} + \sigma_u \varepsilon_{u,t}$. When comparing the solution under the price level targeting framework to the solution under the optimal commitment policy, however, we will set $\rho_u = 0$.

The value function of the policymaker satisfies

$$V(\hat{p}_{t-1}, u_{p,t}) = \min_{\hat{p}_t, x_t} \frac{1}{2}\left(\hat{p}_t^2 + \lambda^{PLT}(x_t)^2\right) + \beta E_t V(\hat{p}_t, u_{p,t+1}) \tag{180}$$

$s.t.$

$$(\hat{p}_t - \hat{p}_{t-1}) = \tilde{\kappa}_p x_t + \beta E_t^{private}(\hat{p}_{t+1} - \hat{p}_t) + u_{p,t}. \tag{181}$$

We conjecture that the value function is quadratic of the form

$$V_p(\hat{p}_{t-1}, u_{p,t}) = a_1 u_{p,t} + \frac{1}{2} a_2 (u_{p,t})^2 + a_3 u_{p,t} \hat{p}_{t-1} + a_4 \hat{p}_{t-1} + \frac{1}{2} a_5 (\hat{p}_{t-1})^2 \tag{182}$$

implying the derivative with respect to $\hat{p}_{t-1}$ to be of the form

$$V_p(\hat{p}_{t-1}, u_{p,t}) = a_3 u_{p,t} + a_4 + a_5 \hat{p}_{t-1} \tag{183}$$

and that in equilibrium the price level evolves according to

$$\hat{p}_t = \Theta_p \hat{p}_{t-1} + \Theta_u u_{p,t}. \tag{184}$$

$$(\hat{p}_t - \hat{p}_{t-1}) = \tilde{\kappa}_p x_t + \beta E_t^{private} (\hat{p}_{t+1} - \hat{p}_t) + u_{p,t} \tag{185}$$

$$(\hat{p}_t - \hat{p}_{t-1}) = \tilde{\kappa}_p x_t + \beta (\Theta_p - 1) \hat{p}_t + (\beta \Theta_u \rho_u + 1) u_{p,t} \tag{186}$$

Combining equations (181) and (184) to eliminate $\hat{p}_{t+1}$ and imposing expectations to be rational delivers

$$x_t = \frac{\omega}{\tilde{\kappa}_p} \hat{p}_t - \frac{1}{\tilde{\kappa}_p} \hat{p}_{t-1} - \frac{1 + \beta \rho_u \Theta_u}{\tilde{\kappa}_p} u_{p,t}. \tag{187}$$

with $\omega = 1 + \beta (1 - \Theta_p)$. Replacing the term x_t in the value function by the expression in equation (187), the Envelop condition associated with the policymaker's optimization problem implies

$$a_3 u_{p,t} + a_4 + a_5 \hat{p}_{t-1} = -\frac{\lambda^{PLT}}{\tilde{\kappa}_p} x_t \tag{188}$$

and the first order condition with respect to $\hat{p}_t$ delivers

$$\hat{p}_t + \omega \frac{\lambda^{PLT}}{\tilde{\kappa}_p} x_t + \beta [a_4 + a_5 \hat{p}_t] = 0. \tag{189}$$

Combining the last two equations to eliminate x_t and applying equation (184) delivers the parameter restrictions

$$\Theta_p = \frac{\omega a_5}{1 + \beta a_5} \tag{190}$$

$$\Theta_u = \frac{\omega - \beta \rho_u}{1 + \beta a_5} a_3 \tag{191}$$

and $a_4 = 0$. Using this information in the Envelop condition

$$a_3 u_{p,t} + a_5 \hat{p}_{t-1} = \frac{\lambda^{PLT}}{\tilde{\kappa}_p} \frac{1}{\tilde{\kappa}_p} (1 - \omega \Theta_p) \hat{p}_{t-1} + \frac{\lambda^{PLT}}{\tilde{\kappa}_p} \frac{1}{\tilde{\kappa}_p} (1 + \beta \rho_u \Theta_u - \omega \Theta_u) u_{p,t} \tag{192}$$

we obtain the remaining two conditions

$$a_3 = \frac{\lambda^{PLT}}{(\tilde{\kappa}_p)^2} (1 + \beta \rho_u \Theta_u - \omega \Theta_u) \tag{193}$$

$$a_5 = \frac{\lambda^{PLT}}{(\tilde{\kappa}_p)^2}(1 - \omega\Theta_p). \tag{194}$$

By combining equations (190), (191), (193), and (194), we obtain the implicit definition of Θ_p and Θ_u

$$\Theta_p = \frac{\lambda^{PLT}\omega}{(\tilde{\kappa}_p)^2 + \beta\lambda^{PLT}(1 - \omega\Theta_p) + \lambda^{PLT}\omega^2} \tag{195}$$

$$\Theta_u = \frac{\lambda^{PLT}(\omega - \beta\rho_u)}{(\tilde{\kappa}_p)^2 + \beta\lambda^{PLT}(1 - \omega\Theta_p) + \lambda^{PLT}(\omega - \beta\rho_u)^2} \tag{196}$$

noting that ω is a function of Θ_p.

Equipped with the law of motion for prices and the output gap under price level targeting with discretion, we can show that inflation and the output gap follow the same path as under the optimal commitment policy, when $\rho_u = 0$ and therefore $\Theta_u = \Theta_p$. Note that equation (187) can be rewritten as

$$x_t = \frac{1 + \beta(1 - \Theta_p)}{\tilde{\kappa}_p}\hat{p}_t - \frac{1}{\tilde{\kappa}_p}\hat{p}_{t-1} - \frac{1}{\tilde{\kappa}_p}u_{p,t} = \frac{\omega - \frac{1}{\Theta_p}}{\tilde{\kappa}_p}\hat{p}_t \tag{197}$$

and therefore $\hat{p}_t = \frac{\tilde{\kappa}_p}{\omega - \frac{1}{\Theta_p}}x_t$. The price level is proportional to the output gap, just as it is the case under the optimal commitment policy (compare to the optimal targeting rule expressed as $\hat{p}_t = -\frac{\lambda}{\tilde{\kappa}_p}x_t$). Hence, we obtain the law of motion for x_t as

$$x_t = \Theta_p x_{t-1} + \frac{\Theta_p\omega - 1}{\tilde{\kappa}_p}u_{p,t}. \tag{198}$$

Thus, for the price level targeting framework to implement the optimal commitment policy, λ^{PLT} must be chosen to satisfy

$$\Upsilon_x = \frac{\lambda^{PLT}\omega}{(\tilde{\kappa}_p)^2 + \beta\lambda^{PLT}(1 - \omega\Upsilon_x) + \lambda^{PLT}\omega^2} \tag{199}$$

with Υ_x being the solution to equation (178) and $\omega = 1 + \beta(1 - \Upsilon_x)$. Furthermore, it is $\frac{\Theta_p\omega - 1}{\tilde{\kappa}_p} = \Upsilon_u$ given the conditions imposed on Υ_x.

B.3.2 Speed limit policy

To solve for the equilibrium under discretion in the speed limit policy framework we conjecture that the value function of the policymaker is quadratic and depends on the output gap of the previous period. The value function of the policymaker satisfies

$$V(x_{t-1}, u_{p,t}) = \min_{\pi_{p,t}, x_t} \frac{1}{2}\left(\pi_{p,t}^2 + \lambda^{SLP}(x_t - x_{t-1})^2\right) + \beta E_t V(x_t, u_{p,t+1}) \tag{200}$$

$s.t.$

$$\pi_{p,t} = \tilde{\kappa}_p x_t + \beta E_t^{private}\pi_{p,t+1} + u_{p,t}. \tag{201}$$

We conjecture that the value function is quadratic implying the derivative with respect to x_{t-1} to be of the form

$$V_x(x_{t-1}, u_{p,t}) = a_3 u_{p,t} + a_4 + a_5 x_{t-1} \tag{202}$$

and that in equilibrium inflation evolves according to

$$\pi_{p,t} = \Omega_\pi x_{t-1} + \Omega_u u_{p,t}. \tag{203}$$

Combining equations (201) and (203) and imposing rational expectations delivers

$$\pi_{p,t} = (\tilde{\kappa}_p + \beta\Omega_\pi)\, x_t + (\beta\Omega_u\rho_u + 1)\, u_{p,t}. \tag{204}$$

Thus, the Envelop condition associated with the policymaker's optimization problem implies

$$V_x(x_{t-1}, u_{p,t}) = a_3 u_{p,t} + a_4 + a_5 x_{t-1} = -\lambda^{SLP}(x_t - x_{t-1}). \tag{205}$$

If in equilibrium x_t evolves according to

$$x_t = \Theta_x x_{t-1} + \Theta_u u_{p,t} \tag{206}$$

we obtain the conditions $a_3 = -\lambda^{SLP}\Theta_u$, $a_4 = 0$, and $a_5 = \lambda^{SLP}(1 - \Theta_x)$.

From the first order condition of the value function, we obtain

$$\pi_{p,t}(\tilde{\kappa}_p + \beta\Omega_\pi) + \lambda^{SLP}(x_t - x_{t-1}) + \beta a_3 \rho_u u_{p,t} + \beta a_5 x_t = 0 \tag{207}$$

or after substituting out for $\pi_{p,t}$ and x_t

$$\begin{aligned} &\left[\left((\tilde{\kappa}_p + \beta\Omega_\pi)^2 + \beta a_5\right)\Theta_x + \lambda^{SLP}(\Theta_x - 1)\right] x_{t-1} \\ &+ \left[\left((\tilde{\kappa}_p + \beta\Omega_\pi)^2 + \beta a_5\right)\Theta_u + \left((\beta\Omega_u\rho_u + 1)(\tilde{\kappa}_p + \beta\Omega_\pi) + \lambda^{SLP}\Theta_u + \beta a_3\rho_u\right)\right] u_{p,t} = 0. \end{aligned} \tag{208}$$

Using the fact that $a_5 = \lambda^{SLP}(1 - \Theta_x)$ and $\Omega_\pi = \frac{\tilde{\kappa}_p\Theta_x}{1-\beta\Theta_x}$

$$(\tilde{\kappa}_p)^2 \left(\frac{1}{1 - \beta\Theta_x}\right)^2 \Theta_x - \lambda^{SLP}(1 - \Theta_x)(1 - \beta\Theta_x) = 0 \tag{209}$$

and finally we obtain a relationship between Θ_x and λ^{SLP}

$$\frac{(\tilde{\kappa}_p)^2}{\lambda^{SLP}} = (1 - \Theta_x)\frac{(1 - \beta\Theta_x)^3}{\Theta_x}. \tag{210}$$

Similarly, we have

$$\left((\tilde{\kappa}_p)^2\left(\frac{1}{1 - \beta\Theta_x}\right)^2 + \beta\lambda^{SLP}(1 - \Theta_x)\right)\Theta_u + \left((1 + \beta\Omega_u\rho_u)\frac{\tilde{\kappa}_p}{1 - \beta\Theta_x} + \lambda^{SLP}\Theta_u(1 - \beta\rho_u)\right) = 0 \tag{211}$$

where

$$\Omega_u = \frac{(\tilde{\kappa}_p + \beta\Omega_\pi)\Theta_u + 1}{1 - \beta\rho_u} \tag{212}$$

to determine Θ_u.

We are now in a position to compare the solution under the discretionary speed limit policy to the optimal commitment policy. Rewrite condition (178) as

$$\frac{(\tilde{\kappa}_p)^2}{\lambda} = (1 - \Upsilon_x) \frac{(1 - \beta\Upsilon_x)}{\Upsilon_x}. \tag{213}$$

and compare to condition (210). For $\lambda^{SLP} = \lambda$, the speed limit policy imparts some, but less persistence to the output gap than the optimal commitment policy.

If $\lambda^{SLP} = \lambda / (1 - \beta\Upsilon_x)^2$, the speed limit policy impart the same persistence on the output gap. However, the optimal commitment policy is not replicated for this value of λ^{SLP} since $\Theta_u \neq \Upsilon_u$: setting $\rho_u = 0$ for simplicity condition (211) reduces to

$$\Theta_u = (1 - \beta\Upsilon_x) \frac{-\tilde{\kappa}_p}{\lambda \{1 + \beta(1 - \Upsilon_x)\} + (\tilde{\kappa}_p)^2} = (1 - \beta\Upsilon_x)\Upsilon_u. \tag{214}$$

C Model in Walsh (2003)

Walsh (2003) uses the following linear model which resembles our NK model with price indexation and consumption habits. Backward-looking behavior in the hybrid New Keynesian Phillips curve is measured by the parameter ϕ

$$\pi_{p,t} = (1-\phi)\beta E_t \pi_{p,t+1} + \phi \pi_{p,t-1} + \kappa x_t + e_t \tag{215}$$

where $\pi_{p,t}$ denotes inflation, x_t the output gap and e_t a markup shock. The aggregate demand curve includes a lagged term of the output gap

$$x_t = \theta x_{t-1} + (1-\theta) E_t x_{t+1} - \sigma(R_t - E_t \pi_{p,t+1}) + \mu_t \tag{216}$$

where R_t is the nominal interest rate. The variable μ_t summarizes shocks to the natural rate of interest

$$\mu_t = u_t - [1 - (1-\theta)\bar{\gamma}]\bar{y}_t + \theta \bar{y}_{t-1} \tag{217}$$

where potential output $\bar{y}_t$ and the demand shock u_t follow AR(1) processes

$$\bar{y}_t = \bar{\gamma}\bar{y}_{t-1} + \xi_t \tag{218}$$

$$u_t = \gamma_u u_{t-1} + \eta_t. \tag{219}$$

Finally, the markup shock is given by

$$e_t = \gamma_e e_{t-1} + \varepsilon_t. \tag{220}$$

The welfare criterion in Walsh (2003) is not derived from the preferences of households, but it is simply stated to be of the form

$$\pi_{p,t}^2 + \lambda x_t^2. \tag{221}$$

The parameterization of the model is summarized in Table 3.

Table 3: Parameter Values for Walsh(2003)

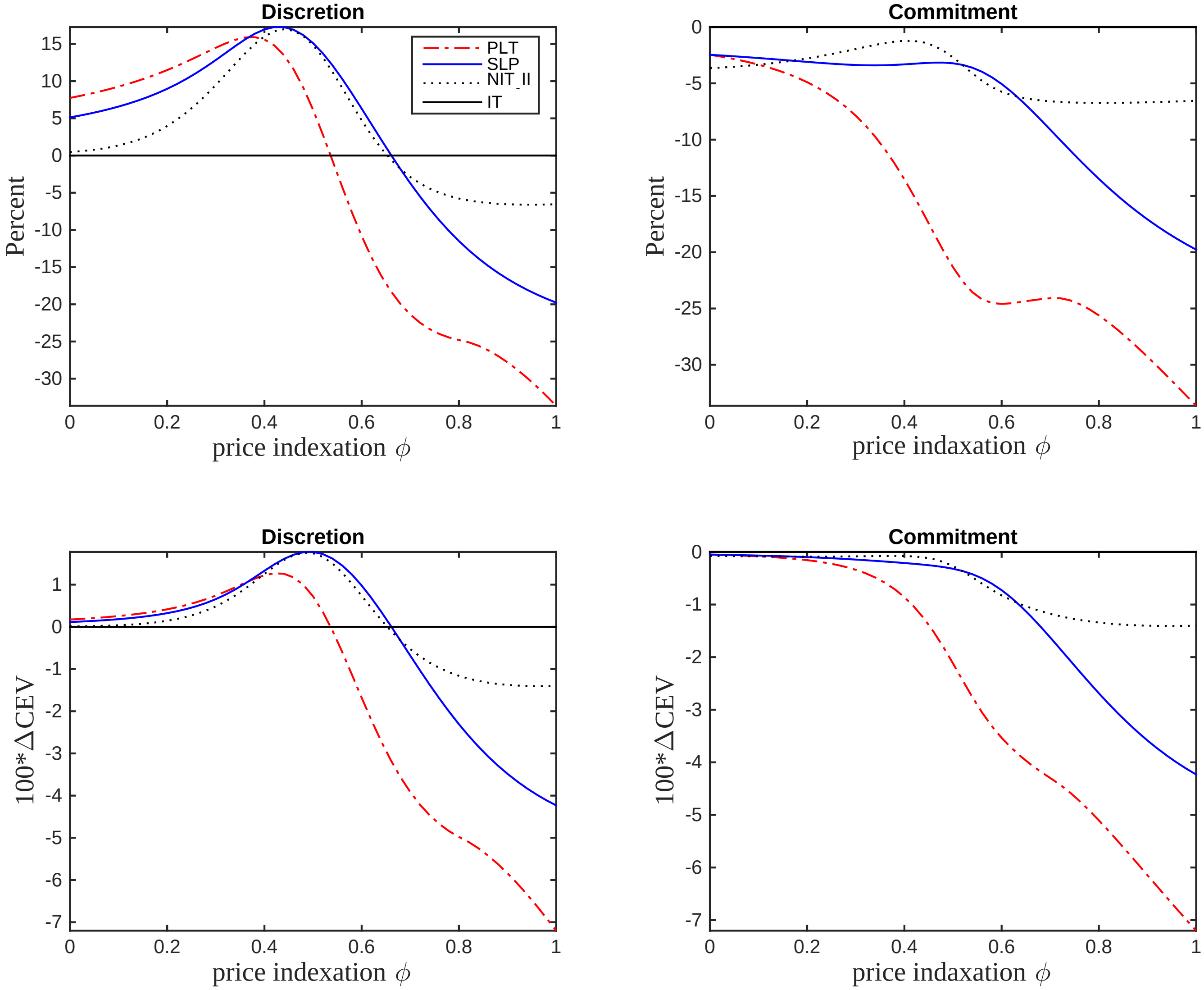
Parameter	Description	Value
β	discount factor	0.99
κ	slope of NKPC	0.05
λ	weight on output gap	0.25
σ	inverse of elast. subs.	1.5
ϕ	lagged inflation in NKPC	0.5
θ	lagged consumption in AD	0.5

Shock	Description	Value
σ_ε	autocorr. markup	0.015
γ_e	std. markup	0
σ_u	autocorr. demand shock	0.015
γ_u	std. demand shock	0.3
σ_ξ	autocorr. natural output	0.005
$\bar{\gamma}$	std. natural output	0.97

We solve the model for inflation targeting, price level targeting, speed limit policy, and the second nominal income targeting framework under commitment and discretion. Walsh (2003) only reports results under discretion. Figure (14) shows the welfare outcomes for each framework relative to the IT framework as a function of the degree of price indexation. The top two panels report welfare differences in percent deviations from the IT framework as in Walsh (2003) while the bottom two panels report the welfare differences in terms of consumption equivalent variation (CEV).

Similar to our findings, the price level targeting and speed limit policy frameworks perform worse than the IT framework when policymakers act under commitment. Given that Walsh (2003) evaluates welfare using a loss function that has the same functional form as the objective function under IT, this result holds by assumption. Under discretion, the price level targeting and the speed limit policy perform much better than the IT framework for moderate degrees of price indexation. This contrasts with our finding that price level targeting and the speed limit policy outperform the IT framework for all degrees of price indexation in the model with moderate consumption habits.

Figure 14: Welfare evaluation of targeting frameworks in Walsh (2003)



Note: The figure shows the welfare performance of price level targeting (PLT), speed limit policy (SLP), and nominal income targeting (NIT-II) relative to inflation targeting (IT) under discretion and commitment in the model of Walsh (2003). In the upper panels, we express the welfare differences of each targeting framework as the percent deviation from the inflation targeting framework as in Walsh (2003), while in the lower panels we express the differences between frameworks in terms of the consumption equivalent variation (CEV).

D The CEE/SW model

This section lays out the nonlinear version of the CEE/SW model as implemented in our paper following Smets and Wouters (2007).

D.1 Households

D.1.1 Household Agent

Each period t, household j chooses consumption $C_t(j)$, labor supply $N_t(j)$, investment $I_t(j)$, the capital stock $K_t(j)$, capital utilization $Z_t(j)$, and domestic bond holdings to maximize expected discounted lifetime utility. In doing so the household takes prices, wages and transfers as given.

Household j's preferences over consumption and leisure are given by

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{1-\sigma_C} \left(C_t(j) - hC_{t-1}^{A} \right)^{1-\sigma_C} exp\left(\frac{\sigma_C - 1}{1+\sigma_L} N_t(j)^{(1+\sigma_L)} \right) \right\} \tag{222}$$

C_{t-1}^{A} refers to the level of aggregate consumption in the previous period; the parameter h captures the degree of external consumption habits.

The budget constraint of the household is given by

$$P_t C_t(j) + P_t I_t(j) + \frac{B_t(j)}{\xi_t^R R_t} = W_t^f N_t(j) + R_t^k K_{t-1}(j) Z_t(j) - a(Z_t(j)) K_{t-1}(j) P_t + Profits_t + T_t \tag{223}$$

The household earns income by supplying homogeneous labor services to labor union $N_t(j)$ and earns the wage rate W_t^f. Furthermore, the household derives income from renting out its capital stock, $R_t^k K_{t-1(j)} Z_t(j)$ net of capital utilization cost $a(Z_t(j))K_{t-1}(j)P_t$. Finally, the household receives payments from holding financial assets, $B_t(j)$, $Profits_t$ and government transfers T_t. This income is spent on consumption, $P_t C_t(j)$, investment, $P_t I_t(j)$, and financial assets.

Capital accumulates following

$$K_t(j) = (1-\delta) K_{t-1}(j) + \xi_t^I I_t(j) \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] \tag{224}$$

with the investment adjustment cost function

$$S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) = \frac{\kappa}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - \gamma \right)^2 \tag{225}$$

where $S(\gamma) = 0$, $S'(\gamma) = 0$, $S''(.) = \kappa > 0$. Capital utilization costs are governed by

$$a(Z_t(j)) = \frac{(R^k)^2}{z} \left[exp\left(\frac{z}{R^k} \left(Z_t(j) - 1 \right) \right) - 1 \right] \tag{226}$$

δ is the depreciation rate. The utilization function satisfies $a(1) = 0$, $a^{'}(1) = R^k$, and $a^{''}(1) = z$.

D.1.2 Labor unions and bundlers

Households supply their homogeneous labor to intermediate labor unions. These unions differentiate the labor services, and resell them to labor bundlers. The union acts under monopolistic competition and sets its wage rate using staggered contracts as in Calvo (1983). The labor bundlers combine the differentiated labor services into an aggregate labor service that is sold to the intermediate goods producers in a competitive market.

Labor bundling takes the form

$$\int_0^1 G\left(\frac{L_t(j)}{L_t}\right) di = 1 \tag{227}$$

following Kimball (1995). G is assumed to be a strictly concave and increasing function

$$G\left(\frac{L_t(j)}{L_t}\right) = \frac{1+\theta_w}{1-\theta_w\epsilon_w}\Big[\Big(\frac{1+\theta_w-\theta_w\epsilon_w}{1+\theta_w}\Big)\frac{L_t(j)}{L_t} + \frac{\theta_w\epsilon_w}{1+\theta_w}\Big]^{\left(\frac{1-\theta_w\epsilon_w}{1+\theta_w-\theta_w\epsilon_w}\right)} - \frac{\theta_w+\theta_w\epsilon_w}{1-\theta_w\epsilon_w} \tag{228}$$

where $\frac{1+\theta_w}{\theta_w}$ refers to the elasticity of substitution among labor varieties, and ϵ_w is referred to as the Kimball elasticity. For $\epsilon_w = 0$, the function G reduces to the standard Dixit-Stiglitz aggregator with a constant elasticity of substitution between varieties.

Each labor bundler buys differentiated labor services from all unions and packages the differentiated services into an aggregate labor service L_t. In doing so, a bundler solves the profit maximization problem

$$\max_{L_t(i),L_t} W_t L_t - \int_0^1 W_t(j)L_t(j)dj \tag{229}$$

$$s.t. \int_0^1 G\left(\frac{L_t(j)}{L_t}\right) dj L_t = L_t \quad (\lambda_t^L). \tag{230}$$

The first order conditions imply the bundlers' demand function for labor of type j

$$\frac{L_t(j)}{L_t} = \frac{1+\theta_w}{1+\theta_w-\theta_w\epsilon_w}\Big(\frac{W_t(j)}{W_t}\frac{W_t}{\lambda_t^L}\Big)^{-\frac{1+\theta_w-\theta_w\epsilon_w}{\theta_w}} - \frac{\theta_w\epsilon_w}{1+\theta_w-\theta_w\epsilon_w} \tag{231}$$

and wage costs charged to an intermediate goods produced satisfies

$$\frac{\lambda_t^L}{W_t} = \Big[\int_0^1 \Big(\frac{W_t(j)}{W_t}\frac{W_t}{\lambda_t^L}\Big)^{-\frac{1-\theta_w\epsilon_w}{\theta_w}} dj\Big]^{-\frac{\theta_w}{1-\theta_w\epsilon_w}}. \tag{232}$$

Each labor union measures the costs of the labor services it differentiates in terms of the marginal rate of substitution of the supplying households. The unions are subject to nominal rigidities as in Calvo (1983). A union can readjust its nominal wage with probability $1-\xi_w$ in each period. For those that cannot adjust wages optimally in the current period, wages increase as the weighted average of inflation in the previous period $\Pi_t = \frac{P_t}{P_{t-1}}$ and inflation rate along the balance growth path $\bar{\Pi}$ taking into account the labor-augment technological progress γ, i.e.,

$$W_{t+1}(j) = \tilde{W}_t(j)\left(\Pi_t^{\iota_w}\bar{\Pi}^{(1-\iota_w)}\gamma\right) \tag{233}$$

For those that can adjust, the problem is to choose a wage $\tilde{W}_t(j)$ that maximizes the wage income in all states of nature where union has to maintain that wage in the future: Wage setting behavior for labor variety j

$$\begin{aligned}
&\max_{\tilde{W}_t(j)} E_t \sum_{s=0}^{\infty} (\xi_w)^s \frac{\beta^s \lambda_{t+s}}{\lambda_t} \left[W_{t+s}(j) - W^h_{t+s} \right] L_{t+s}(j) \\
&s.t. \quad \frac{L_{t+s}(j)}{L_{t+s}} = \frac{1+\theta_w}{1+\theta_w-\theta_w\epsilon_w} \left(\frac{W_{t+s}(j)}{W_{t+s}} \frac{W_{t+s}}{\lambda_{t+s}} \right)^{-\frac{1+\theta_w-\theta_w\epsilon_w}{\theta_w}} - \frac{\theta_w\epsilon_w}{1+\theta_w-\theta_w\epsilon_w} \\
&\qquad W_{t+s}(j) = \tilde{W}_t(j) X^W_{t,s} \\
&X^W_{t,s} = \begin{cases} 1 & for \quad s=0 \\ \prod_{l=1}^{s} (\Pi_{t+l-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} \gamma) & for \quad s = 1, ..., \infty \end{cases}
\end{aligned} \tag{234}$$

A wage markup shock is modeled by allowing θ_w to vary over time. This shock is assumed to follow an ARMA(1,1) process. Accordingly, θ_w is replaced by $\theta_{w,t}$ with

$$\log(\theta_{w,t}) = (1-\rho_w)\, log(\theta_w) + \rho_w \log(\theta_{w,t-1}) + \varepsilon_{w,t} - \rho_{w,\epsilon}\epsilon_{w,t-1} \tag{235}$$

$\varepsilon_{w,t}$ is white noise following $N(0, \sigma_w^2)$.

D.2 Firms

There are two types of firms: intermediate goods producers and final good producers.

D.2.1 Intermediate Goods Producer

Intermediate goods producers choose capital and labor to minimize the cost of producing an intermediate goods variety using a Cobb-Douglas technology. In doing so they take the capital rental rate R_t^k and the aggregate wage rate W_t as given. The cost minimization problem is then given by

$$\begin{aligned}
&\min_{K_t(i), L_t(i)} R_t^k K_t(i) + W_t L_t(i) \\
&s.t. \quad Y_t(i) = \xi_{A,t} K_t(i)^{\omega^k} (\gamma^t L_t(i))^{\omega^l} - \gamma^t \Phi
\end{aligned} \tag{236}$$

where Φ is a fixed cost that is chosen to set the producer's profits equal to zero in the steady state. Marginal costs are equalized across firms as firms share the same technology and factor markets are frictionless. $\hat{\xi}_{A,t}$ denotes a shock to total factor productivity

$$\log(\xi_{A,t}) = (1-\rho_A)\log(\gamma) + \rho_A \log(\xi_{A,t}) + \varepsilon_{A,t} \tag{237}$$

$\varepsilon_{A,t}$ is white noise following $N(0, \sigma_A^2)$. γ refers to steady sate labor-augment technology progress.

Intermediate goods producers set prices using staggered contracts as in Calvo (1983). Each period, a firm can reset its price optimally with a constant probability $1-\xi_p$. This probability is independent across producers and time. Producers that cannot optimally adjust their price in the current period adjust by a weighted average of Π_t the nominal price inflation in the previous period and $\bar{\Pi}$ the steady state inflation rate.

$$P_{t+1}(i) = \tilde{P}_t(i)\left(\Pi_t^{\iota_p}\bar{\Pi}^{1-\iota_p}\right). \tag{238}$$

The intermediate goods producer i solves the profit maximization problem

$$\begin{aligned}
&\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty}(\xi_p)^s \psi_{t,t+s}\left[(P_{t+s}(i) - MC_{t+s})\right] Y_{t+s}(i) \\
&s.t. \\
&\frac{Y_{t+s}(i)}{Y_{t+s}} = \frac{1+\theta_p}{1+\theta_p-\theta_p\epsilon_p}\left(\frac{P_{t+s}(i)}{P_{t+s}}\frac{P_{t+s}}{\lambda_{t+s}^Y}\right)^{-\frac{1+\theta_p-\theta_p\epsilon_p}{\theta_p}} - \frac{\theta_p\epsilon_p}{1+\theta_p-\theta_p\epsilon_p} \\
&\qquad P_{t+s}(i) = \tilde{P}_t(i)X_{t,s}^P \\
&X_{t,s}^P = \begin{cases} 1 & for \quad s=0 \\ \prod_{l=1}^{s}(\Pi_{t+l-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}) & for \quad s=1,...,\infty \end{cases}
\end{aligned} \tag{239}$$

by fixing the price in the current period.

D.2.2 Final Good Producer

Differentiated intermediated products are combined to form the composite good by a continuum of bundlers in a perfectly competitive environment. Using a technology of the form in Kimball (1990), it is

$$\int_0^1 G\left(\frac{Y_t(i)}{Y_t}\right) di = 1 \tag{240}$$

and

$$G\left(\frac{Y_t(i)}{Y_t}\right) = \frac{1+\theta_p}{1-\theta_p\epsilon_p}\left[\left(\frac{1+\theta_p-\theta_p\epsilon_p}{1+\theta_p}\right)\frac{Y_t(i)}{Y_t} + \frac{\theta_p\epsilon_p}{1+\theta_p}\right]^{\left(\frac{1-\theta_p\epsilon_p}{1+\theta_p-\theta_p\epsilon_p}\right)} - \frac{\theta_p+\theta_p\epsilon_p}{1-\theta_p\epsilon_p} \tag{241}$$

where $\frac{1+\theta_p}{\theta_p}$ refers to the elasticity of substitution between intermediate varieties, and ϵ_p stands for the Kimball elasticity. If $\epsilon_p = 0$, the Kimball aggregator reduces to the standard Dixit-Stiglitz aggregator with

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta_p}}\,\mathrm{d}i\right]^{1+\theta_p} \tag{242}$$

Profit maximization for intermediate producer i is defined as:

$$\begin{aligned}
&\max_{Y_t(i),Y_t} P_tY_t - \int_0^1 P_t(i)Y_t(i)di \\
&s.t. \\
&\int_0^1 G\left(\frac{Y_t(i)}{Y_t}\right) diY_t = Y_t \quad (\lambda_t^Y).
\end{aligned} \tag{243}$$

The first order conditions deliver the demand function for each intermediate good and an expression for the aggregate price index

$$\frac{Y_t(i)}{Y_t} = \frac{1+\theta_p}{1+\theta_p-\theta_p\epsilon_p}\left(\frac{P_t(i)}{P_t}\frac{P_t}{\lambda_t^Y}\right)^{-\frac{1+\theta_p-\theta_p\epsilon_p}{\theta_p}} - \frac{\theta_p\epsilon_p}{1+\theta_p-\theta_p\epsilon_p} \tag{244}$$

and

$$\frac{\lambda_t^Y}{P_t} = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\frac{P_t}{\lambda_t^Y}\right)^{-\frac{1-\theta_p\epsilon_p}{\theta_p}} di\right]^{-\frac{\theta_p}{1-\theta_p\epsilon_p}}. \tag{245}$$

Again, if $\epsilon_p = 0$, the demand of each differentiate becomes

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} Y_t \tag{246}$$

and the aggregate price index is

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\theta_p}} \mathrm{d}i\right]^{-\theta_p}. \tag{247}$$

Time variation in the markup can be introduced by replacing θ_p with $\theta_{p,t}$, where $\theta_{p,t}$ follows an ARMA(1,1) process

$$\log(\theta_{p,t}) = \left(1-\rho_p\right)\log(\theta_p) + \rho_p \log(\theta_{p,t-1}) + \varepsilon_{p,t} - \rho_p\varepsilon_{p,t-1} \tag{248}$$

$\varepsilon_{p,t}$ is white noise following $N(0,\sigma_p^2)$.

D.2.3 Fiscal and Monetary Policy

Government budget is balanced with

$$P_tG_t + B_{t-1} = T_t + \frac{B_t}{R_t} \tag{249}$$

and

$$G_t = \xi_{G,t}Y_{ss} \tag{250}$$

The government spending shock $\xi_{G,t}$ follows the stochastic process

$$\log\left(\xi_{G,t}\right) = (1-\rho_G)\log(g_y) + \rho_G\log\left(\xi_{G,t-1}\right) + \rho_{AG}\log(\xi_{A,t}) - \rho_{AG}\log(\xi_{A,t-1}) + \varepsilon_{G,t}. \tag{251}$$

$\varepsilon_{G,t}$ is white noise following $N(0,\sigma_G^2)$. where g_y is the government spending to GDP ratio in the steady state.

D.2.4 Resources Constraint

Capital market clearing The market for capital clears if the total amount demanded by firms $\int_0^1 K_t(i)di$ equals the amount supplied by the households

$$\int_0^1 K_t(i)di = Z_t \int_0^1 K_{t-1}(j)dj. \tag{252}$$

Labor market clearing The relationship between labor supply and aggregate labor demand can be stated as

$$N_t = \Omega_t^l L_t. \tag{253}$$

It can be shown that $\Omega_t^l \geqslant 1$ due to the concavity of the Kimball aggregator. Ω_t^l is defined implicitly by the above equation; see also Appendix B.1.

Final product market clearing Demand for the final product is

$$Y_t = C_t + I_t + G_t + a(Z_t)K_{t-1}. \tag{254}$$

The final product is purchased by households for consumption and investment and capital utilization, and by the government.

Supply of the final product is given by

$$\Omega_t^y Y_t = \xi_{L,t} K_t^\alpha (\gamma^t L_t)^{1-\alpha} - \frac{\theta_p}{1+\theta_p}\gamma^t K_{ss} L_{ss}. \tag{255}$$

It can be shown that $\Omega_t^y \geqslant 1$ due to the concavity of the Kimball aggregator. Ω_t^y is defined implicitly by the above equation; see also Appendix B.1.

Table 4 summarizes the parameters estimated for the CEE/SW model. Figure 4 plots the impulse responses of selected variables to a price and a wage markup shock under the optimal commitment policy, and inflation targeting, price level targeting and speed limit policy under discretion.

Table 4: Parameter values for CEE/SW model estimated with US data

a:Calibrated and Estimated Parameters

Parameter	Description	Value	Parameter	Description	Value
δ	depreciation rate	0.025	κ	invest. adjust. cost	5.48
ϵ_p	Kimball elas. goods	10	ϵ_w	Kimball elas. labor	10
g_y	s.s. G/Y	0.18	β	discount factor	0.9984
γ	tech. progress	1.0043	$\bar{\pi}$	s.s. inflation rate	1.0081
σ_C	inverse cons. elastic.	1.39	σ_L	inverse labor. elastic.	1.92
θ_w	s.s. net wage markup	0.5	θ_p	s.s. net price markup	0.61
h	habit persistence	0.71	ψ	capital util. cost	0.54
ξ_p	price stickiness	0.65	ξ_w	wage stickiness	0.73
ι_p	price indexation	0.22	ι_w	wage indexation	0.59
ω^k	capital share	0.19	ω^l	labor share	0.81
$\bar{\tau}_p$	price subsidies	0	$\bar{\tau}_w$	wage subsidies	0

b:Parameters for Shock Process

Shock	AR(1)		MA(1)		Standard deviation (%)	Value
technology	ρ_A	0.95	-	-	σ_A	0.45
risk premium	ρ_R	0.18	-	-	σ_R	0.24
gov. spending	ρ_G	0.97	ρ_{AG}	0.52	σ_G	0.52
invest. specific	ρ_I	0.71	-	-	σ_I	0.45
price markup	ρ_p	0.90	$\rho_{p\varepsilon}$	0.74	σ_p	0.14
wage markup	ρ_w	0.97	$\rho_{w\varepsilon}$	0.88	σ_w	0.24

Table 5: Parameter values for CEE/SW model estimated with euro area data

a:Calibrated and Estimated Parameters

Parameter	Description	Value	Parameter	Description	Value
δ	depreciation rate	0.025	κ	invest. adjust. cost	5.68
ϵ_p	Kimball elas. goods	10	ϵ_w	Kimball elas. labor	10
g_y	s.s. G/Y	0.18	β	discount factor	0.9977
γ	tech. progress	1.0039	$\bar{\pi}$	s.s. inflation rate	1.0068
σ_C	inverse cons. elastic.	1.32	σ_L	inverse labor. elastic.	2.60
θ_w	s.s net wage markup	0.5	θ_p	s.s. net price markup	0.77
h	habit persistence	0.72	ψ	capital util. cost	0.24
ξ_p	price stickiness	0.64	ξ_w	wage stickiness	0.75
ι_p	price indexation	0.128	ι_w	wage indexation	0.374
ω^k	capital share	0.16	ω^l	labor share	0.84
$\bar{\tau}_p$	price subsidies	0	$\bar{\tau}_w$	wage subsidies	0

b:Parameters for Shock Process

Shock	AR(1)		MA(1)		Standard deviation (%)	Value
technology	ρ_A	0.99	-	-	σ_A	0.27
risk premium	ρ_R	0.69	-	-	σ_R	0.10
gov. spending	ρ_G	0.99	ρ_{AG}	0.39	σ_G	0.29
invest. specific	ρ_I	0.12	-	-	σ_I	0.55
price markup	ρ_p	0.99	$\rho_{p\varepsilon}$	0.92	σ_p	0.16
wage markup	ρ_w	0.98	$\rho_{w\varepsilon}$	0.84	σ_w	0.15