**Finance and Economics Discussion Series**
**Divisions of Research & Statistics and Monetary Affairs**
**Federal Reserve Board, Washington, D.C.**

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**2017-102**

Please cite this paper as:
Grishchenko, Olesya, Sarah Mouabbi, and Jean-Paul Renne (2017). "Measuring Inflation Anchoring and Uncertainty: A US and Euro Area Comparison," Finance and Economics Discussion Series 2017-102. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2017.102.

# Measuring Inflation Anchoring and Uncertainty: A US and Euro Area Comparison*

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**Abstract**

We use several US and euro-area surveys of professional forecasters to estimate a dynamic factor model of inflation featuring time-varying uncertainty. We obtain survey-consistent distributions of future inflation at any horizon, both in the US and the euro area. Equipped with this model, we propose a novel measure of the anchoring of inflation expectations that accounts for inflation uncertainty. Our results suggest that following the Great Recession, inflation anchoring improved in the US, while mild de-anchoring occurred in the euro area. As of our sample end, both areas appear to be equally anchored.

**JEL codes:** C32, E31, E44
**Keywords:** inflation, surveys of professional forecasters, dynamic factor model with stochastic volatility, term structure of inflation expectations and inflation uncertainty, anchoring of inflation expectations

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# 1 Introduction

In this paper we study survey-based inflation expectations and uncertainty about these expectations. In particular, we propose a novel measure that allows macroeconomists and policymakers to assess the extent to which inflation expectations are anchored.

The Federal Reserve System (the Fed) and the European Central Bank (the ECB) are two of many central banks that have adopted a mandate for price stability devised to foster economic activity. To meet this objective, central banks pay close attention to various measures of inflation expectations implied both by financial market data and by surveys of professional forecasters. Surveys, in particular, have received considerable attention from policymakers and academic researchers. What makes surveys so attractive? First, survey-based measures of inflation expectations — unlike market-based measures — are not affected by inflation risk premiums. Market-based inflation compensation measures, such as inflation swaps and TIPS break-even rates, contain two components: inflation expectations and inflation risk premiums. Several studies show that inflation risk premiums can be large in magnitude and tend to vary a lot, thus distorting the readings of inflation expectations.[1] Second, surveys have been documented to be successful in forecasting inflation relative to various time-series models (e.g. Ang, Bekaert, and Wei, 2007). As a result of the above, surveys are closely monitored and used to assess the anchoring of inflation expectations.

This paper contributes to a growing literature on the anchoring of inflation expectations. There are several proposed measures of anchoring, or stability of inflation expectations. One popular measure is the response of inflation compensation measures (TIPS break-even rates of inflation swaps) or interest rates to incoming macroeconomic news (Gürkaynak, Levin, Marder, and Swanson, 2007; Mishkin,

[1] See Campbell and Viceira (2001); Buraschi and Jiltsov (2005); Ang, Bekaert, and Wei (2008); Hördahl and Tristani (2010); Ajello, Benzoni, and Chyruk (2012); Chernov and Mueller (2012); Haubrich, Pennacchi, and Ritchken (2012); Abrahams, Adrian, Crump, and Moench (2013); Grishchenko and Huang (2013); Fleckenstein, Longstaff, and Lustig (2013); Crump, Eusepi, and Moench (2016); Breach, D'Amico, and Orphanides (2016); D'Amico, Kim, and Wei (2016).

2007; Beechey, Johannsen, and Levin, 2011; De Pooter, Robitaille, Walker, and Zdinak, 2014; Speck, 2016). Other measures include the response of (changes in) long-term inflation expectations to (changes in) short-term ones (Buono and Formai, 2016; Gerlach, Moessner, and Rosenblatt, 2017), the precision around estimates of the level of inflation (Mehrotra and Yetman, 2014), the volatility of shocks to trend inflation (Mertens, 2016), and the closeness of average beliefs to the central bank's inflation target (Kumar, Afrouzi, Coibion, and Gorodnichenko, 2015; Łyziak and Paloviita, 2016).

While the above measures are used to define "anchored" expectations, most of them are mainly related to the stability of the conditional mean of inflation. However, the conditional mean can be stable even if the conditional variance (i.e. uncertainty) is relatively high.[2] To better capture the uncertainty underlying the concept of anchoring, we propose to measure anchoring in terms of probabilities of future inflation being in a certain range that is consistent with inflation targets.

To this end, we propose an approach that takes survey-based inflation forecasts (for various horizons, at varying frequencies and with different definitions) as inputs and produces survey-consistent distributions of inflation at any horizon. We rely both on survey-based consensus inflation forecasts that correspond to an average scenario and on probability distributions of future inflation rates that provide information about uncertainty surrounding this scenario. Specifically, we estimate our model by fitting survey-based first- and second-order moments.

Our dynamic factor model has several noteworthy features. First, common latent factors are allowed to drive the dynamics of inflation rates in both economies, reflecting ever-increasing interconnectedness between developed economies (Monacelli and Sala, 2009; Ciccarelli and Mojon, 2010). Second, our model features stochastic volatility of inflation, hence allowing for time-varying inflation uncertainty.[3] Third,

[2] Consider, for instance, a situation when a specific macroeconomic surprise results in a substantial increase in the long-term conditional variance but has no effect on the conditional mean. That is, suppose we face equal increases in both downside and upside risks. In this situation, while long-term inflation expectations remain stable, the probability of having very high or very low future inflation rates increases substantially, which is at odds with the concept of anchoring.

[3] Engle (1982) was the first who emphasized time-varying inflation uncertainty in the context

our model is highly tractable because it offers closed-form solutions for conditional first and second moments of future inflation rates at *any horizon*. This tractability is due to the fact that the factors in our econometric model follow so-called affine processes. The affine property of our factors implies that the model can be easily cast in state-space form and subsequently estimated using Kalman filtering techniques. These techniques easily handle missing observations, which is particularly useful in our case, because different surveys are released at different points in time.

We apply our methodology to the US and euro-area spanning the period from January 1999 to June 2016. We construct inflation expectations, inflation uncertainty, and inflation anchoring measures for both economies using several prominent surveys of professional forecasters. The measures obtained are directly comparable across the two economies, they are available on a regular (monthly) frequency, and can be computed for any horizon. We find that, in the early 2000s, euro-area long-term inflation expectations are more anchored relative to the US ones. Specifically, the probability of euro-area inflation 5- and 10-years ahead being between 1.5% and 2.5% is larger than 60%, compared to 40% for the United States. However, euro-area inflation expectations show mild signs of de-anchoring in the post crisis period. In contrast, the anchoring of US long-term inflation expectations improves during this period. By the end of our sample (2016Q2), US and euro-area inflation expectations are similarly anchored, according to our measures.

In the remainder of the paper Section 2 summarises the data used in our analysis, Section 3 describes our model and estimation strategy and Section 4 presents empirical results, and Section 5 concludes. Appendix 6 gathers proofs and technical results.

of an econometric model by specifying a new class of stochastic processes called autoregressive conditional heteroscedastic (ARCH) processes. Zarnowitz and Lambros (1987) were the first who emphasized time-varying inflation uncertainty in the context of the second moment of survey-based inflation distributions; the concept that we use in our model to proxy for inflation uncertainty.

# 2 Data

## 2.1 Notation

Let us first define a notation which is flexible enough to account for the breadth of data described in the following Subsection 2.2. We denote by $\pi^{(i)}_{t,t+h}$ the annualized inflation rate in economy $i$ between dates $t$ and dates $t+h$, defined as the log difference in the price index $P^{(i)}_t$:

$$\pi^{(i)}_{t,t+h} = \frac{12}{h} \log\left(\frac{P^{(i)}_{t+h}}{P^{(i)}_t}\right), \tag{1}$$

where $h$ is the forecast horizon measured in months.

Further, because it used in some surveys, we denote by $\tilde{\pi}_{t,t+h}$ the annual-quarter-average over annual-quarter-average percent change in prices, defined as follows:

$$\tilde{\pi}^{(i)}_{t,t+h} = \frac{P^{(i)}_{t+h} + P^{(i)}_{t+h-3} + P^{(i)}_{t+h-6} + P^{(i)}_{t+h-9}}{P^{(i)}_{t+h-12} + P^{(i)}_{t+h-15} + P^{(i)}_{t+h-18} + P^{(i)}_{t+h-21}}. \tag{2}$$

Note that when inflation is relatively small, the inflation target in eq. (2) is well approximated by:[4]

$$\bar{\pi}^{(i)}_{t,t+h} = \frac{1}{4}(\pi^{(i)}_{t+h-21,t+h-9} + \pi^{(i)}_{t+h-18,t+h-6} + \pi^{(i)}_{t+h-15,t+h-3} + \pi^{(i)}_{t+h-12,t+h}). \tag{3}$$

## 2.2 Survey data: sources and content

In our model estimation, we use several surveys of professional forecasters for the United States and for the euro area. Specifically, we obtain inflation forecast data from the following surveys: the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia (US-SPF), the Survey of Primary Dealers conducted by the Federal Reserve Bank of New York (SPD), the Blue Chip Survey of Financial Forecasts and Economic Indicators (Blue Chip, or BCFF and BCEI hereafter), the Survey of Professional Forecasters conducted by the European Cen-

[4] This is illustrated in Figure 1.

tral Bank (ECB-SPF) and the Consensus Economics Survey (CES) conducted by Consensus Economics. The sample period extends from January 1999, which coincides with the onset of the euro area and the start date of the ECB-SPF, until June 2016. Table 1 summarizes the forecast variables extracted from the different surveys and Appendix 6.4 provides specific details about each survey.

**[Insert Table 1 about here.]**

We construct a detailed database of inflation expectation surveys at various horizons. Importantly, surveys target different measures of inflation. To illustrate this, Figure 1 depicts year-over-year inflation $\pi_{t-12,t}$, annual-quarter-average over annual-quarter-average percent change in prices $\tilde{\pi}_{t-12,t}$, and its approximation $\bar{\pi}_{t-12,t}$, for the euro area and the US, respectively. Two points are worth mentioning: (i) the annual-quarter-average over annual-quarter-average percent change in prices $\tilde{\pi}_{t-12,t}$ is well approximated by $\bar{\pi}_{t-12,t}$ and (ii) $\pi_{t-12,t}$ and $\bar{\pi}_{t-12,t}$ are considerably different, with spreads reaching up to 1 percentage point, implying that raw survey data, even if they target the same horizon, are not necessarily comparable.

**[Insert Figure 1 about here.]**

Surveys typically provide point estimates but some surveys provide information on the distribution of inflation. In our analysis, we exploit surveys with density forecasts, which either provide information at an aggregate level (SPD) or at an individual forecasters' level (US-SPF and ECB-SPF). In the latter case, our model assumes the existence of a representative forecaster and, therefore, in our estimation, we use the average of survey outputs (i.e. aggregate densities).

The top and middle panels of Figure 2 illustrate, respectively, the point estimates and aggregate densities (histograms) for the five-year ahead ECB-SPF inflation forecasts and the five-year in five years US-SPD inflation forecasts. Note that the information embedded in the aggregate densities (depicted in the middle panels, in black) amounts to the value of the associated cumulative distribution function (CDF) at a few key values (i.e. 1%, 1.5%, 2%, 2.5% and 3%).

**[Insert Figure 2 about here.]**

## 2.3 Measuring uncertainty

Uncertainty can be measured as the conditional variance of the aggregate probability distribution of survey forecasts.[5] As mentioned in Subsection 2.2, the US-SPF, the SPD, and the ECB-SPF provide us with the CDF at key points. However, this information is not sufficient to obtain the variance. Therefore, we first apply Beta-smoothing techniques to the observed aggregated CDF to obtain an estimate of the full distribution. Note that Beta-smoothing has been widely used in the literature (see Engelberg, Manski, and Williams, 2009; Boero, Smith, and Wallis, 2014) and Appendix 6.5 provides further details about this procedure. Once equipped with the smoothed CDFs, we proceed in computing the associated conditional variances (i.e. uncertainty).[6]

The middle panels of Figure 2 illustrate (in grey) the smoothed CDF at a few key values. We observe that the smoothed series (in grey) fit the raw CDFs (in black) reasonably well. The bottom panels display the uncertainty measures associated with the five-year ahead ECB-SPF inflation forecasts and the five-year in five years US-SPD inflation forecasts.

One important implication is that the conditional variance of the aggregate distribution amounts to the sum of disagreement and the average of individual variances (which results from the application of the law of total variance).[7] Thus, denoting by $\sigma^2_{agg,th}$ the conditional variance of the aggregate distribution, the proxy for

[5]Such uncertainty measures based on the diffuseness of probability distributions have been considered by Zarnowitz and Lambros (1987), Conflitti (2011), Rich, Song, and Tracy (2012), Boero, Smith, and Wallis (2014), and D'Amico and Orphanides (2014), among others.

[6]In Section 3, we fit the conditional variances stemming from the smoothed aggregate densities.

[7]An important strand of the literature studies survey-based measures of disagreement and uncertainty. Similarly to our measure, Lahiri and Sheng (2010) decompose forecast errors into common and idiosyncratic shocks and show that aggregate forecast uncertainty can be expressed as the sum of the disagreement among forecasters and the perceived variability of future aggregate shocks. This finding implies that the reliability of disagreement as a proxy for uncertainty depends primarily on the stability of the forecasting environment.

uncertainty is given by:

$$\sigma^2_{agg,th} = d_{th} + \frac{1}{F}\sum_{f=1}^{F}\sigma^2_{fth}, \tag{4}$$

where $F$ is the number of forecasters $f$, $t$ is the time the forecast is made, $h$ is the forecasting horizon, $d_{th}$ is the disagreement among forecasters and $\sigma^2_{fth}$ is the variance associated with forecaster $f$'s distribution. This measure of uncertainty captures both forecasters' heterogeneity via the cross-sectional variance of individual means (i.e. disagreement) and the average uncertainty of individual forecasters.

# 3 Model and estimation strategy

## 3.1 Inflation and its driving factors

We assume that the annual inflation rate, $\pi^{(i)}_{t-12,t}$, is a linear combination of factors gathered in the $n \times 1$ vector $Y_t = (Y_{1,t}, \ldots, Y_{n,t})'$. As specified below, the dynamics of $Y_t$ is such that the marginal mean of $Y_t$ is zero. Importantly, $Y_{j,t}$ factors, where $j \in \{1, \ldots, n\}$, may be common to different economies:

$$\pi^{(i)}_{t-12,t} = \pi^{(i)} + \delta^{(i)\prime} Y_t. \tag{5}$$

We assume that the distribution of $Y_t$ is Gaussian conditional on its past realization $\underline{Y_{t-1}} = \{Y_{t-1}, Y_{t-2}, \ldots\}$ and on another $q \times 1$ exogenous vector $z_t = (z_{1,t}, \ldots, z_{q,t})'$ that affects the variance of $Y_t$.[8] Specifically, $Y_t$ is given by:

$$Y_t = \Phi_Y Y_{t-1} + diag\left(\sqrt{\Gamma_{Y,0} + \Gamma'_{Y,1} z_t}\right)\varepsilon_{Y,t}, \quad \varepsilon_{Y,t} \sim \mathcal{N}(0, I), \tag{6}$$

where $\Gamma_{Y,0}$ is an $n \times 1$ vector and $\Gamma_{Y,1}$ is a $q \times n$ matrix. According to eq. (6), $z_t$ affects the conditional variance of $Y_t$. Vector $z_t$ is essential for modelling the time-varying variance of inflation so we refer to $z_t$ as the uncertainty vector (and to the

[8] Note that this does not imply that the marginal distribution of $Y_t$ is Gaussian (as it is in GARCH models).

$z_{j,t}$s as the uncertainty factors) hereinafter.

The specification of the conditional variance in eq. (6) implies that the entries of $\Gamma_{Y,0} + \Gamma'_{Y,1} z_t$ have to be non-negative for all $t$. To that end, we assume that all elements of $\Gamma_Y$ vectors are non-negative and that $z_t$ follows a multivariate auto-regressive gamma process. As shown in Appendix 6.2, the dynamics of $z_t$ admits the following semi-strong VAR representation:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag\left(\sqrt{\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}}\right) \varepsilon_{z,t}, \tag{7}$$

where, conditional on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ has a zero mean and a unit diagonal covariance matrix, and where $\Gamma_{z,0}$ is a $q \times 1$ vector and $\Gamma_{z,1}$ is a $q \times q$ matrix.

Given the dynamics of $Y_t$ and $z_t$, the semi-strong VAR form of the dynamics followed by $X_t = (Y'_t, z'_t)'$ is:

$$X_t = \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \mu_X + \Phi_X \begin{bmatrix} Y_{t-1} \\ z_{t-1} \end{bmatrix} + \Sigma_X(z_{t-1})\varepsilon_{X,t}, \tag{8}$$

where $\varepsilon_{X,t}$ is a $(n+q)$-dimensional unit-variance martingale difference sequence and where:

$$\mu_X = \begin{bmatrix} 0 \\ \mu_z \end{bmatrix}, \quad \Phi_X = \begin{bmatrix} \Phi_Y & 0 \\ 0 & \Phi_z \end{bmatrix}, \quad \Sigma_X(z_{t-1})\Sigma_X(z_{t-1})' = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma'_{12} & \Sigma_{22} \end{bmatrix},$$

$$\text{with} \quad \begin{cases} \Sigma_{11} = diag(\Gamma_{Y,0} + \Gamma'_{Y,1}(\mu_z + \Phi_z z_{t-1})), \\ \Sigma_{22} = diag(\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}), \\ \Sigma_{12} = 0. \end{cases}$$

An important property of $X_t$ is that it is affine (see Appendix 6.1.1). This implies that, conditionally on $\underline{X_t} = \{X_t, X_{t-1}, \dots\}$, the first and second conditional moments of any linear combination of future values of $X_t$ are affine functions of $X_t$. In particular, since the realized log annual growth rate of the price index $\pi^{(i)}_{t-12,t}$ is an affine transformation of $X_t$ (see eq. (5)), its first and second moments can be written as affine functions of the $X_t$ factors as well:

$$\mathbb{E}_t(\pi^{(i)}_{t+h-12,t+h}) = \pi^{(i)} + a^{(i)}_h + b^{(i)\prime}_h X_t \quad (9)$$

$$\mathbb{V}ar_t(\pi^{(i)}_{t+h-12,t+h}) = \alpha^{(i)}_h + \beta^{(i)\prime}_h X_t, \quad (10)$$

where $\mathbb{E}_t(\bullet)$ and $\mathbb{V}ar_t(\bullet)$ respectively denote the expectations and variances conditional on $\underline{X_t}$. As explained in Section 2, we have to consider other measures of inflation because of the nature of the different surveys we fit. In particular, we study the annualized $h$-period ahead inflation rates $\pi^{(i)}_{t,t+h} = (12/h)\log(P^{(i)}_{t+h}/P^{(i)}_t)$, that we can write as:

$$\pi^{(i)}_{t,t+h} = \pi^{(i)} + \frac{1}{k}\delta^{(i)\prime}(Y_{t+12} + Y_{t+24} + \cdots + Y_{t+h}), \quad (11)$$

where $h = 12 \times k$. Because $\pi^{(i)}_{t,t+h}$ is affine in $Y_t$ (and therefore in $X_t$), the first and second conditional moments of $\pi^{(i)}_{t,t+h}$ can also be written as affine functions of $X_t$. The same applies to $\bar{\pi}^{(i)}_{t,t+h}$ (see Subsection 2.1). Appendices 6.1.2 and 6.1.3 detail the recursive algorithms used to compute the loadings defining all these affine relationships.

## 3.2 State-space model and Kalman-filter estimation

### 3.2.1 Objective and strategy

In addition to model parameters, we have to estimate the factors $X_t$ that are not observed by the econometrician. We handle both estimations in a joint manner using Kalman filtering techniques. The affine property of the process $X_t$ is key to the tractability of the estimation. Specifically, we not only have closed-form formulas for conditional expectations and variances (as in eqs. (9) and (10) for $\pi^{(i)}_{t+h-12,t+h}$), but they also are affine. This allows us to easily cast the model into a linear state-space form, which is the required form of the model for the Kalman filter algorithm to be applied. This is a fundamental difference between our approach and alternative inflation models exhibiting stochastic volatility (see, e.g. Stock and Watson, 2007; Mertens, 2016). Indeed, while the latter models entail closed-form expressions for the first two conditional moments of inflation, the second-order moments are non-linear in the unobserved factors, which substantially complicates the model estimation.

A state-space model consists of two types of equations: transition equations and measurement equations. Transition equations describe the dynamics of the latent factors, as in eq. (8). Measurement equations specify the relationship between the observed variables and the latent factors. A by-product of the Kalman filter algorithm is the likelihood function. Parameter estimates can therefore be obtained by maximising this function.

### 3.2.2 Measurement equations

The state-space model involves three types of measurement equations:

(a) The first set of equations states that, for each economy $i$, the realised inflation rate is equal to a linear combination of factors $Y_t$, as stated by eq. (5), with area-specific loadings.

(b) The second set of equations states that, up to the measurement error, survey-based expectations of future inflation rates are equal to the model-implied ones, that is:

$$SPF_t = \boldsymbol{\pi} + \mathbf{a} + \mathbf{b}' X_t + diag(\sigma^{avg})\eta_t^{avg}, \tag{12}$$

where $\eta_t^{avg}$ is a vector of *iid* Gaussian measurement errors, $SPF_t$ gathers all survey-based inflation expectations available at date $t$ and the vector $\boldsymbol{\pi}$, the vector $\mathbf{a}$ and the matrix $\mathbf{b}$ are filled with the appropriate $\pi^{(i)}$s and with the parameters defining the affine relationships between conditional expectations and $X_t$ (such as eq. (9)).

(c) The third set of equations states that, up to the measurement error, survey-based variances are equal to the model-implied ones, i.e.:

$$VSPF_t = \boldsymbol{\alpha} + \boldsymbol{\beta}' X_t + diag(\sigma^{var})\eta_t^{var} \tag{13}$$

where $\eta_t^{var}$ is a vector of *iid* Gaussian measurement errors, $VSPF_t$ gathers all survey-based conditional variances of inflation forecasts available at date $t$, and the vector $\boldsymbol{\alpha}$ and the matrix $\boldsymbol{\beta}$ are filled with the parameters defining the affine relationships between conditional variances and $X_t$ (such as eq. (10)).

Let us denote by $S_t$ the vector of observations used in the state-space model. Since the latter is based on equations of types (a), (b) and (c), we have $S_t = [\pi_t^{(1)}, \pi_t^{(2)}, SPF_t', VSPF_t']'$. Using obvious notations, the measurement equations of the state-space model read:

$$S_t = A + B' X_t + diag(\sigma^S)\eta_t^S, \tag{14}$$

where $\mathbb{V}ar(\eta_t^S) = Id$.

### 3.2.3 Discussion of model estimation

At this stage, three remarks are in order. First, most survey forecasts are not released every month so $SPF_t$ and $VSPF_t$ are not available every month and thus their series contain missing observations when measured at a monthly frequency.[9] Fortunately, it is straightforward to adjust the Kalman filter in order to handle missing observations (for details see Harvey and Pierse, 1984; Harvey, 1989). For the months when no $SPF_t$ and $VSPF_t$ variables are available, the filter can still produce estimates of all latent factors, though with lower precision.

The second remark is about the Kalman filter performance in our case. While the affine form of our transition and measurement equations facilitates the implementation of the filter, the filter we eventually run is not optimal. It would have

[9] An alternative, but equivalent, view would be that the vectors and matrices $\boldsymbol{\pi}$, $\mathbf{a}$, $\mathbf{b}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ have time-varying sizes.

been optimal had the conditional covariance matrix $\Sigma_X \Sigma_X'$ in eq. (8) not been dependent on $X_{t-1}$. However, this is not the case given some entries of $\Gamma_{Y,1}$ are non-null. Therefore, we estimate our model using a quasi-maximum-likelihood (QML) approach based on a modifed version of the Kalman filter.[10]

The third remark pertains to the standard deviations of the measurement equations, i.e. to the components of vectors $\sigma^{avg}$ and $\sigma^{var}$ (see eqs. (12) and (13)). In order to reduce the number of parameters to estimate, these standard deviations are calibrated in a preliminary step. We employ the approach of Green and Silverman (1994) and proceed as follows. We apply a smoothing spline to the raw survey-based expectations and variances ($SPF_t$ and $VSPF_t$). Next, we compute the standard deviations of the differences between the survey-based series and their smoothed counterparts. The standard errors of the measurement equations are set to these values.

# 4 Results

## 4.1 Estimated model

Table 2 presents the fit of the data resulting from three different specifications: $n = 3, 4, 5$ (dimension of $Y_t$) and $q = 2$ (dimension of $z_t$).[11] The fitting performances of the model are reported in terms of root mean squared errors (RMSE) and ratios of mean squared errors (MSE) to variances of the measurement equations (eq. 14). Out of the 22 considered moments, 7 relate to the euro area (4 means and 3

[10]Our filter algorithm makes use of the standard forecasting and updating steps of the Kalman filter except that, at iteration $t$, we replace the unobserved covariance matrix of the $X_t$ innovations ($\Sigma_X(z_{t-1})\Sigma_X(z_{t-1})'$) by $\Sigma_X(z_{t-1|t-1})\Sigma_X(z_{t-1|t-1})'$, where $z_{t-1|t-1}$ denotes our filtered estimate of $z_{t-1}$ (using the information up to date $t-1$). Another adjustment we have to make to the filter pertains to the fact that factors $z_t$ are non-negative. For this purpose, after each updating step of the algorithm, negative entries in the $z_t$ estimate are replaced by 0. Monte Carlo analyses run by Duan and Simonato (1999), Zhou (2001) and Monfort, Pegoraro, Renne, and Roussellet (2017) suggest that in the case of linear but heteroscedastic models, that kind of approximation is of limited importance in practice (see also Duffee and Stanton (2012)).

[11]These results are for the joint model (US and euro area). Individual estimations per country, as well as estimations excluding second conditional moments, have been conducted and their results are qualitatively similar. We opt for the joint specification including second conditional moments as it allows for interesting studies of inflation co-movements and uncertainty. Results of these robustness checks are available upon request.

variances) 15 to the US (8 means and 7 variances). Overall, the fitting performances are substantially better for $n = 5$, especially for long-run conditional moments. Unreported results suggest that, for $q = 1$, second order moments are poorly fitted and that, compared to the case where $q = 2$, they are only slightly improved for $q = 3$. As a result, in what follows, we focus on results obtained for $(n, q) = (5, 2)$.

Table 3 presents the parameter estimates for our joint model estimation. For the sake of identification, the euro-area loadings on latent factors, i.e. the $\delta_j^{(1)}$s in eq. (5), are set to 1. This is also the case for the scale parameters associated with the volatility factor $z_t$.[12] Further, realized inflation are assumed to be measured without error (eq. 5). To facilitate estimation, the marginal means of inflation, i.e. $\pi^{(1)}$ and $\pi^{(2)}$, are set to their sample values. The estimated autoregressive parameters of the first and the second level factors $Y_t$ are very close to one: $\Phi_Y[1,1] = 0.994$ and $\Phi_Y[2,2] = 0.999$, suggesting that these persistent processes are the main long-run drivers of inflation. The three other factors are less persistent. The autoregressive parameter of the first volatility factor $z_{1,t}$ is also very persistent ($\Phi_z[1,1] = 0.994$), while the second volatility factor $z_{2,t}$ is less persistent ($\Phi_z[2,2] = 0.969$).

**[Insert Table 3 about here.]**

Figure 3 displays the factor loadings of the estimated model. The more persistent the considered factor, the flatter the loadings curve. As a result, the loading curve associated with the second factor, which is highly persistent, is almost flat. This factor seems to be particularly important in the euro area and far less so in the US. By contrast, the third factor, which is less persistent, is more important for the US inflation than for the euro-area one. The first factor seems to be of equal importance in both areas. With regard to conditional variances (second row of plots), the first uncertainty factor $z_{1,t}$ plays the biggest role across all horizons in the euro area. In the US, the second factor is more important for short horizons.

**[Insert Figure 3 about here.]**

[12]These scale parameters are the entries of vector $\mu$ in Appendix 6.2. The entries of $\Gamma_{1,Y}$ and of $\mu$ are not simultaneously identified.

Figure 4 illustrates the fit obtained for some selected survey-based moments.[13] While the left-hand side shows the fit of euro area surveys, the right-hand side shows the fit of US surveys. The top panels show realized HICP/CPI inflation. Panels in row 2 (3) show the fit of one-year-ahead conditional expectations (variances) from the ECB-SPF and US-SPF surveys. Panels in row 4 (5) show the fit of the long-term conditional expectations (variances), $\pi_{t+48,t+60}$ for the euro area and $\pi_{t+60,t+120}$ for the US. The euro-area long-term moments are taken from the ECB-SPF survey; US ones are taken from a combination of surveys: the conditional expectations correspond to BCFF/BCEI observations before March 2007, and to SPD observations afterwards.[14] For the US, long-term conditional variance dots correspond to the SPD observations only.[15] It is important to stress that the definition of inflation forecasts,[16] both across areas and surveys, differs and, thus, the outputs are not directly comparable.

**[Insert Figure 4 about here.]**

## 4.2 Model-implied conditional distributions

Figure 5 compares the one-year ahead survey-based histograms (grey and red bars) to the one-year ahead model-implied distributions for February 2005 and April 2016 (euro area) and January 2005 and January 2016 (US).[17] In order to illustrate the effect of Beta-smoothing raw survey data (Appendix 6.5), we also plot survey-based Beta-smoothed distributions. For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect uncertainty associated with the estimation of the latent factors $X_t$ and are obtained

[13] For the sake of readability, this figure does not show the fit of all observed surveys.

[14] Because SPD are released more frequently than BCFF/BCEI surveys, we favour the former once these become available (in March 2007).

[15] BCFF/BCEI surveys are about conditional expectations only. SPD started asking respondents about long-term inflation densities in March 2007, so these series are not available before this date.

[16] We mean here differences between $\mathbb{E}_t(\pi_{t+h-12,t+h})$, $\mathbb{E}_t(\pi_{t,t+h})$ and $\mathbb{E}_t(\bar{\pi}_{t,t+h})$ (see Subsection 2.1).

[17] We cannot use the same months for both areas because surveys that feature histograms are not released on the same month in the US and in the euro area. The first selected dates – namely February 2005 for the euro area and January 2005 for the US – are chosen because they correspond to periods when inflation uncertainty was somewhat low relative to the end-of-sample dates.

by applying the delta method on the function relating $X_t$ factors to the conditional cumulative distribution function (CDF) of future inflation.[18] Again, we stress that the raw survey data are not comparable across areas unless the inflation measure is the same, which is not the case here.[19] By contrast, as will be exploited below, we can deduce from the estimated model moments that are adequately comparable across economies.

In both economies, conditional inflation distributions have shifted noticeably to the left from 2005 to 2016, suggesting a decline in inflation expectations. The euro-area inflation distribution widened, indicating an increase in the variance of inflation expectations and, thus, greater inflation uncertainty.[20] By contrast, the US conditional distribution of the one-year ahead inflation became less dispersed, which indicates diminished short-term uncertainty about future inflation, possibly reflecting the announced inflation target in January 2012 by the Fed.[21]

**[Insert Figure 5 about here.]**

Figure 6 displays the model-implied term structure of conditional inflation expectations ($\pi_{t,t+h}$; top charts) and one-year forward inflation expectations ($\pi_{t+h-12,t+h}$; bottom charts) for two dates: February 2005 and April 2016 for the euro area and January 2005 and January 2016 for the US.[22] The figure also displays the 5th and 95th quantiles associated with the conditional distributions.[23] The top charts demonstrate that survey-based inflation expectations declined over the last decade, the decline being more marked for shorter horizons.

**[Insert Figure 6 about here.]**

[18]The covariance matrix of the smoothed values of $X_t$ (i.e. $\mathbb{E}(X_t|\underline{S_T})$, where $T$ is the length of our sample) stems from the Kalman smoothing algorithm (Harvey, 1989). Appendix 6.3 details the computation of the CDF of future inflation rates.

[19]While the inflation targeted by the SPF is $\pi_{t,t+12}$ in the euro area, it is $\bar{\pi}_{t,t+12}$ in the US (see Subsection 2.1).

[20]This is also documented in Rich, Song, and Tracy (2012), who find that uncertainty measures stemming from the ECB-SPF display countercyclical behavior and find evidence of increased inflation uncertainty since 2007.

[21]See `https://www.federalreserve.gov/newsevents/press/monetary/20120125c.htm`.

[22]These dates are the same as for Figure 5.

[23]The quantiles are derived from closed-form formulas (exploiting the affine property of the model) given in Appendix 6.3.

Figure 7 displays the term structure of model-implied variances of inflation forecasts ($\pi_{t,t+h}$; top charts) and one-year forward inflation forecasts ($\pi_{t+h-12,t+h}$; bottom charts) for the same two dates as in Figure 6.[24] As discussed in Subsection 2.3, we interpret these variances as inflation uncertainty measures. According to the top charts, the uncertainty associated with average future inflation, i.e. between $t$ and $t + h$, has been higher in the United States than in the euro area at all horizons $h$ in earlier parts of our sample, in particular in early 2005. Towards the end of our sample, the term structure of US inflation uncertainties is close to the euro-area one. By contrast, the euro-area area uncertainty increased over the same period, for all horizons. The lower row of charts focuses on shorter future periods (between $t+h-12$ and $t+h$). To a certain extent, the inflation measure targeted by the first row of charts is the average, over future horizons, of the inflation targeted by the lower plots. With this in mind, the second row of charts suggests that the previously mentioned decrease in average US uncertainty is essentially accounted for by a substantial decrease in short-term inflation uncertainty. This substantial decrease in short-term uncertainty more than compensates the increase in long-run uncertainty associated with far-ahead inflation rates between 2005 and 2016.

**[Insert Figure 7 about here.]**

Figure 8 shows the time series of date-$t$ model-implied conditional means (left-hand side) and variances (right-hand side) for the following selected inflation rates: $\pi_{t+48,t+60}$ (1Y4F), $\pi_{t+108,t+120}$ (1Y9F), and $\pi_{t+60,t+120}$ (5Y5F).[25] US Inflation forecasts kept declining almost steadily throughout our sample, more drastically for 1Y4F and 5Y5F than for 1Y9F. Overall, while inflation forecasts are around 2.5 percent in the beginning of our sample, they are close to 2 percent at the end of our sample. Euro-area inflation increases from 1.8 percent at the beginning of the sample to 2.1 percent around 2008, and then declines to about 1.8 percent at the end of our

[24] Note that these conditional variances were partially reflected by the width of the 90% confidence bands displayed in Figure 6.

[25] The notation xYzF is by analogy with the notation used for forward interest rates.

sample.[26] One prominent feature is that the gap between inflation forecasts in the two areas narrowed considerably towards the end of our sample, pointing to more commonalities in the macroeconomic conditions in the euro area and US. The right-hand side charts show that conditional variances are higher in the US than in the euro area, reflecting higher US inflation uncertainty. The US future inflation measures encompassing medium-run horizons (i.e. 1Y4F and 5Y5F) feature more volatile conditional variances than the far-ahead inflation rate (1Y9F). In particular, the former variances spiked during the 2007-2008 financial crisis, which is not the case for the uncertainty associated with the far-ahead inflation rate. For both the euro area and the US, the uncertainty associated with far-ahead inflation is higher at the end of the sample than at the beginning. However, while the uncertainty levels associated with 1Y4F and 5Y5F inflations have also increased over the estimation period for the euro area, their US equivalent return to their pre-crisis level at the end of our sample.[27]

Our finding that, overall, inflation uncertainty is higher in the US than in the euro area echoes the results of Beechey, Johannsen, and Levin (2011) who show that (a) US expectations show much greater dispersion across survey respondents' long-term forecasts and that (b) euro-area daily changes of the market-based inflation compensation measures (inflation swaps and/or breakeven inflation) are less responsive to macroeconomic news than their US counterparts.

**[Insert Figure 8 about here.]**

## 4.3 Anchoring of the inflation expectations

This section proposes a new measure of the degree to which inflation expectations are anchored. We define this measure in terms of the conditional probability of

[26]Market-based measures of inflation compensation also suggest declining inflation expectations in the post-crisis period relative to the 2004 pre-crisis period. See, e.g. D'Amico, Kim, and Wei (2016) and Chen, Engstrom, and Grishchenko (2016). However, these measures are affected by inflation risk premiums and/or liquidity issues and, therefore, may not reflect inflation expectations accurately.

[27]Using ECB-SPF data, Abel, Rich, Song, and Tracy (2016) also find higher inflation uncertainty (along with GDP and unemployment uncertainties) at all horizons since 2007.

inflation forecasts falling in a certain range:

$$\mathbb{P}(\pi_{t+h-m,t+h} \in I|\underline{X_t}), \tag{15}$$

where $h$ is the forecast horizon (in months), $m$ is the tenor of the future inflation rate, and the interval of inflation outcomes $I$ can be defined according to the economic conditions/inflation target of a specific economy. We suggest that these conditional probabilities of future inflation rates, which can be computed for any horizon, capture the spirit of the anchoring of inflation expectations while accounting for uncertainty around inflation expectations. We thus propose using these probabilities to gauge the anchoring of inflation expectations. Figure 9 reports time-series of conditional probabilities of future inflation rates being in a certain range, $I = [1.5\%, 2.5\%]$.[28] Our chosen range $I$ is consistent with the inflation targets specified by the ECB and the Fed. In particular, the ECB clarified *the medium-term* inflation target in May 2003, formulated as follows: *"The primary objective of the ECB's monetary policy is to maintain price stability. The ECB aims at inflation rates of below, but close to, 2 percent over the medium term."*[29] The Fed specified *the longer-run* inflation target in January 2012, stated as follows: *"The Committee judges that inflation at the rate of 2 percent, as measured by the annual change in the price index for personal consumption expenditures, is most consistent over the longer run with the Federal Reserve's statutory mandate."*[30] Thus, the ECB and the Fed target inflation over somewhat different periods. We interpret the "medium-term" as the one-year four years ahead inflation (1Y4F) and the "long-term" as the average annual inflation five-year five years ahead (5Y5F). We plot conditional probabilities eq. (15) for 1Y4F, 1Y9F, and 5Y5F inflation rates.[31]

[28] Since an econometrician does not perfectly observe $X_t$, Figure 9 actually displays an estimate of $\mathbb{P}(\pi^{(i)}_{t+h-m,t+h} \in I|\underline{X_t})$. To make it clear, let us denote by $f(X_t)$ the function that is such that $f(X_t) = \mathbb{P}(\pi^{(i)}_{t+h-m,t+h} \in I|\underline{X_t})$; then Figure 9 displays $f(\mathbb{E}(X_t|\underline{S_T}))$ where $T$ is the length of our sample and $\mathbb{E}(X_t|\underline{S_T})$ therefore is the Kalman-smoothed estimate of $X_t$ (Harvey, 1989).

[29] See https://www.ecb.europa.eu/mopo/html/index.en.html.

[30] See https://www.federalreserve.gov/newsevents/press/monetary/20120125c.htm.

[31] The latter horizon was notably used by Beechey, Johannsen, and Levin (2011) to study the anchoring of inflation expectations as the sensitivity of interest rates and inflation compensation

Figure 9 conveys a few interesting findings. First, the probability of inflation expectations being in the [1.5%, 2.5%] range has been higher in the euro area than in the US throughout our sample, suggesting that inflation expectations are better anchored in the euro area. Second, the euro-area probability declined by up to 10 percent since the end of 2008, more so for medium horizon (1Y4F) than for longer horizons (1Y9F and 5Y5F). This points to a mild de-anchoring of inflation expectations in the euro area over the recent period.[32] Third, US anchoring measures are on average higher at the end of the sample than during its first half: for the 1Y4Y and 1Y9Y measures, the measures have increased by about 20 percent (from about 40 to 60 percent), and the 5Y5F probabilities have gained 30 percent (from about 50 to 80 percent). As the grey lines on the three charts show, the difference in our anchoring measures between the two areas has been diminishing almost monotonically since the early 2000s, though in a more volatile manner for the 5Y5F inflation. At the end of the sample, the difference is small and not significant for the 5Y5F measure. Interestingly, the 1Y4F and 5Y5F US probabilities have steadily increased since the Fed's specification of the inflation target in 2012 (vertical red solid line in Figure 9).

**[Insert Figure 9 about here.]**

Figure 10 essentially reproduces Figure 9 but adds single-area model-based anchoring probabilities.[33] This figure serves as a robustness check to show that our conclusions about the anchoring of inflation expectations in each area are not convoluted by the joint use of surveys. Indeed two-area-based probabilities (solid lines) and single-area-based probabilities (dotted lines) are relatively close to each other. There are some differences for the US probabilities depending on the model used, which is likely due to the sample difference: the sample period for the US single-area estimation starts from 2007 due to the unavailability of long-term density projections

to various macroeconomic news releases.

[32] To some extent, this finding of a decrease in euro-area anchoring is in line with Łyziak and Paloviita (2016), who find that euro-area long-term inflation forecasts became more sensitive to realized HICP inflation and to shorter-term forecasts in the post-crisis period.

[33] By "single-area" we mean that either euro-area or US surveys have only been used in estimation.

(via SPD), prior to that date.[34] Note that when single-area models are estimated, we cannot compute confidence intervals around differences in anchoring measures.

**[Insert Figure 10 about here.]**

# 5 Conclusion

In this paper, we construct inflation expectations, inflation uncertainty, and inflation anchoring measures for the US and the euro area using several prominent surveys of professional forecasters. To that end, we build a dynamic latent factor model with stochastic volatility for the joint estimation of inflation expectations and inflation uncertainty in the US and the euro area. We estimate our model using Kalman filtering techniques, using different types of survey-based inflation forecasts to fit the first and second moments of distributions of future inflation rates. The measures we obtain are directly comparable across the two economies, they are available on a regular (monthly) frequency, and can be computed for any horizon.

Our results suggest that following the Great Recession, inflation anchoring improved in the US, while mild de-anchoring occurred in the euro-area. As of our sample end, both areas appear to be equally anchored.

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[34] In the absence of survey-based density data, the estimation dataset contains no information about long-term uncertainty. This translates into a large uncertainty surrounding filtered (long-term) conditional variances during the half of the sample when the single US model is estimated from 1999 onwards.

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# 6 Appendix

## 6.1 Conditional means and variances of $X_t$

In this appendix we compute conditional expectations and variances of linear combinations of future $X_t$s. Formally, we consider the first two moments of the random variable $\Sigma_{i=1}^{h}\gamma_i' X_{t+i}$ conditionally on the information available as of date $t$ (i.e. $\underline{X_t}$).

Appendix 6.1.1 shows that $X_t$ is an affine process. This property implies that the first two conditional moments of $X_t$ are affine in $X_t$. That is, there exist functions $a_h$, $b_h$, $\alpha_h$ and $\beta_h$ such that, for any set of $\gamma_i$s:

$$\begin{aligned}\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= a_h(\gamma_1,\dots,\gamma_h) + b_h(\gamma_1,\dots,\gamma_h)' X_t \\ \mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \alpha_h(\gamma_1,\dots,\gamma_h) + \beta_h(\gamma_1,\dots,\gamma_h)' X_t.\end{aligned}$$

Appendix 6.1.2 (Appendix 6.1.3) provides the recursive formulas that can be used to compute $a_h$ and $b_h$ ($\alpha_h$ and $\beta_h$).

### 6.1.1 Affine property of $X_t$

Showing that $X_t$ has an affine dynamics amounts to showing that the Laplace transform of $X_{t+1}$, conditional on $\underline{X_t}$, is exponential affine in $X_t$.

**Lemma 6.1** *The Laplace transform of $X_{t+1}$, conditional on $\underline{X_t}$, is given by:*

$$\begin{aligned}& \mathbb{E}(\exp(u'X_{t+1})|\underline{X_t}) \\ = \; & \exp(u_Y'\Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2)' z_t + \\ & a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2) - u_Y'\Theta\bar{z} + 0.5\Gamma_{Y,0}'u_Y^2),\end{aligned} \tag{16}$$

*where $u = (u_Y', u_z')'$, $u_Y^2 = u_Y \odot u_Y$ (by abuse of notation), $\Gamma_Y$ is a $q \times n$ matrix and where the functions $a_z$ and $b_z$ define the conditional Laplace transform of $z_t$ (see Appendix 6.2, eq. (19) and (20)).*

**Proof** We have:

$$
\begin{aligned}
& \mathbb{E}(\exp(u'X_{t+1})|\underline{X_t}) \\
= \ & \mathbb{E}(\exp(u'_Y Y_{t+1} + u'_z z_{t+1})|\underline{X_t}) \\
= \ & \mathbb{E}(\mathbb{E}[\exp(u'_Y Y_{t+1} + u'_z z_{t+1})|\underline{X_t}, z_{t+1}]|\underline{X_t}) \\
= \ & \exp(u'_Y\{\Phi_Y Y_t - \Theta\bar{z}\})\mathbb{E}(\exp((u_z + \Theta' u_Y)' z_{t+1} + 0.5 u'_Y diag(\Gamma_{Y,0} + \Gamma'_{Y,1} z_{t+1}) u_Y)|\underline{X_t}) \\
= \ & \exp(u'_Y \Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)' z_t + \\
& a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u'_Y \Theta \bar{z} + 0.5\Gamma'_{Y,0} u_Y^2),
\end{aligned}
$$

which leads to the result. ■

The fact that $X_t$ follows an affine process implies the following result.

**Lemma 6.2** *The multi-horizon Laplace transforms of $X_t$, conditional on $\underline{X_t}$, are exponential affine in $X_t$. Specifically, for any set of vectors $u_i$, $i \in [1, h]$, we have:*

$$
\mathbb{E}(\exp(u'_1 X_{t+1} + \cdots + u'_h X_{t+h})|\underline{X_t}) = \exp(A_h(u_1, \ldots, u_h) + B_h(u_1, \ldots, u_h)' X_t),
$$

*where the functions $A_i$ and $B_i$ are given by:*

$$
\left\{
\begin{array}{l}
A_h([u'_Y, u'_z]') = a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u'_Y \Theta \bar{z} + 0.5\Gamma'_{Y,0} u_Y^2 \\
B_h([u'_Y, u'_z]') = [u'_Y \Phi_Y, b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)']'
\end{array}
\right. \quad \textit{if } h = 1,
$$

*and*

$$
\left\{
\begin{array}{l}
A_h(u_1, \ldots, u_h) = A_{h-1}(u_2, \ldots, u_h) + A_1(u_1 + B_{h-1}(u_2, \ldots, u_h)) \\
B_h(u_1, \ldots, u_h) = B_1(u_1 + B_{h-1}(u_2, \ldots, u_h))
\end{array}
\right. \quad \textit{otherwise.}
$$

**Proof** eq. (16) proves that Lemma 6.2 is valid for $h = 1$. Assume Lemma 6.2 is valid for a given $h \geq 1$, we have:

$$
\begin{aligned}
& \mathbb{E}(\exp(u'_1 X_{t+1} + \cdots + u'_{h+1} X_{t+h+1})|\underline{X_t}) \\
= \ & \mathbb{E}\{\exp(u'_1 X_{t+1})\mathbb{E}[\exp(u'_2 X_{t+2} + \cdots + u'_{h+1} X_{t+h+1})|\underline{X_{t+1}}]|\underline{X_t}\} \\
= \ & \mathbb{E}\{\exp(u'_1 X_{t+1}) \exp(A_h(u_2, \ldots, u_{h+1}) + B_h(u_2, \ldots, u_{h+1})' X_{t+1})|\underline{X_t}\} \\
= \ & \exp(A_h(u_2, \ldots, u_{h+1}) + A_1(u_1 + B_h(u_2, \ldots, u_{h+1})) + B_1(u_1 + B_h(u_2, \ldots, u_{h+1})' X_t)),
\end{aligned}
$$

which leads to the result. ■

### 6.1.2 Computation of $a_h$ and $b_h$

We have:

$$\begin{aligned}
\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{E}_t\left(\mathbb{E}_{t+1}\sum_{i=1}^{h}\gamma_i' X_{t+i}\right)\\
&= \mathbb{E}_t\left(\gamma_1' X_{t+1} + a_{h-1}(\gamma_2,\dots,\gamma_h) + b_{h-1}(\gamma_2,\dots,\gamma_h)' X_{t+1}\right)\\
&= a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)) +\\
&\quad b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h))' X_t,
\end{aligned}$$

which implies that:

$$\left\{\begin{array}{l}
a_h(\gamma_1,\dots,\gamma_h) = a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h))\\
b_h(\gamma_1,\dots,\gamma_h) = b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)),
\end{array}\right. \tag{17}$$

with $a_1(\gamma) := \gamma'\mu_X$ and $b_1(\gamma) := \Phi_X'\gamma$.

### 6.1.3 Computation of $\alpha_h$ and $\beta_h$

We have:

$$\begin{aligned}
\mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{V}_t\left(\mathbb{E}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \mathbb{V}_t\left(\gamma_1' X_{t+1} + \mathbb{E}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1)' X_t +\\
&\quad \alpha_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h))' X_t.
\end{aligned}$$

Therefore:

$$\left\{\begin{array}{l}
\alpha_h(\gamma_1,\dots,\gamma_h) = \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \alpha_{h-1}(\gamma_2,\dots,\gamma_h) +\\
\quad a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h))\\
\beta_h(\gamma_1,\dots,\gamma_h) = \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)),
\end{array}\right. \tag{18}$$

where, with $\quad S_p = \sum_{i=1}^{p}[e_i^{(p)} \otimes e_i^{(p)}]e_i^{(p)\prime}$ :

$$\left\{\begin{array}{l}
\alpha_1(\gamma) = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta)S_q\Gamma_{z,0} + S_n\Gamma_{Y,0} + S_n\Gamma_{Y,1}'\mu_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma_{z,0}\\
\quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta)S_q\Gamma_{z,0},\\
\beta_1(\gamma)' = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta)S_q\Gamma_{z,1}' + S_n\Gamma_{Y,1}'\Phi_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma_{z,1}'\\
\quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta)S_q\Gamma_{z,1}'.
\end{array}\right.$$

## 6.2 Auto-regressive Gamma processes

The vector $z_t$ follows a multivariate ARG$_\nu(\varphi,\mu)$ process. This process, introduced by Gouriéroux and Jasiak (2006), is the time-discretized Cox, Ingersoll, and Ross (1985) process (see also Monfort, Pegoraro, Renne, and Roussellet (2017)).

Conditionally on $\underline{z_{t-1}} = \{z_{t-1}, z_{t-2}, \dots\}$, the different components of $z_t$, denoted by $z_{i,t}$, are independent and drawn from non-centered Gamma distributions, i.e.:

$$z_{i,t}|\underline{z_{t-1}} \sim \gamma_{\nu_i}(\varphi_i' z_{t-1}, \mu_i),$$

where $\nu$, $\mu$, $\varphi_1$, ..., $\varphi_{q-1}$ and $\varphi_q$ are $q$-dimensional vectors. (Recall that $W$ is drawn from a non-centered Gamma distribution $\gamma_\nu(\varphi,\mu)$, iif there exists an exogenous $\mathcal{P}(\varphi)$-distributed variable $Z$ such that $W|Z \sim \gamma(\nu + Z, \mu)$ where $\nu + Z$ and $\mu$ are, respectively, the shape and scale parameters of the gamma distribution.)

Importantly, it can be shown that this process is affine, in the sense that its conditional Laplace transform is exponential affine. Formally, the conditional log-Laplace transform of $z_{t+1}$, denoted by $\psi_t$, is given by:

$$\psi_t(w) := \log(E_t[\exp(w' z_{t+1})]) = a_z(w) + b_z(w)' z_t,$$

with

$$\begin{aligned} a_z(w) &= -\nu' \log(1 - \mu \odot w) && (19)\\ b_z(w) &= \varphi \left( \frac{w \odot \mu}{1 - w \odot \mu} \right), && (20) \end{aligned}$$

where $\varphi$ is the $q \times q$ matrix equal to $[\varphi_1, \dots, \varphi_q]$, where $\odot$ is the element-by-element (Hadamard) product and where, by abuse of notations, the log and division operator are applied element-by-element wise.

The semi-strong vector auto-regressive (VAR) form of process $z_t$ is given by:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag\left(\sqrt{\Gamma_{z,0} + \Gamma_{z,1}' z_{t-1}}\right) \varepsilon_{z,t},$$

where, conditionally on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ is of mean zero and has a covariance matrix equal to the identity matrix and where:

$$\mu_z = \mu \odot \nu, \quad \Phi_z = (\mu \mathbf{1}_{q\times 1}') \odot (\varphi'), \quad \Gamma_{z,0} = \mu \odot \mu \odot \nu \quad \text{and} \quad \Gamma_{z,1}' = 2[(\mu \odot \mu)\mathbf{1}_{q\times 1}'] \odot (\varphi').$$

Assuming that the eigenvalues of $\Phi_z$ lie (strictly) within the unit circle, this last formula notably implies that the unconditional mean of $z_t$ is equal to $(I_q - \Phi_z)^{-1}\mu_z$ whilst $z_t$'s unconditional variance is equal to $(I_{q^2} - \Phi_z \otimes \Phi_z)^{-1} S_q(\Gamma_{z,0} + \Gamma_{z,1}' \bar{z})$.

## 6.3 Computation of model-implied conditional distributions

In the model, inflation rates of different areas are equal to the linear combinations of the affine process $X_t$. This implies the existence of closed-form formulas to derive the conditional distribution functions of future inflation rates for any maturity (see Duffie, Pan, and Singleton (2000)). Specifically, we have:

$$\mathbb{P}(\gamma_1' X_{t+1} + \cdots + \gamma_h' X_{t+h} < y | \underline{X_t}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\Psi_h(iv\boldsymbol{\gamma}, X_t)]e^{-ivy}}{v} dv,$$

where $Im(c)$ denotes the imaginary part of $c \in \mathbb{C}$ and where $\Psi_h$ is the multi-horizon Laplace transform of $X_t$, defined by:

$$\Psi_h(\mathbf{u}, X_t) = \mathbb{E}(\exp(u_1' X_{t+1} + \cdots + u_h' X_{t+h}) | \underline{X_t}), .$$

with $\mathbf{u} = [u_1, \ldots, u_h]$. A simple computation of $X_t$'s Laplace transform is provided by Lemma 6.2 in Appendix 6.1.1.

## 6.4 Survey data

### 6.4.1 US surveys

US surveys used in our study include the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (US-SPF), Blue Chip Financial Forecasts (BCFF) and Blue Chip Economic Indicators (BCEI) surveys, the Survey of Primary Dealers (SPD) published by the Federal Reserve Bank of New York and the Consensus Economics Survey (CES). Panel A of Table 1 summarizes the data set described below.

The US-SPF survey is conducted quarterly and provides forecasts on a wide range of macroeconomic and financial variables since 1968:Q4.[35] For the purpose of this study, we use several inflation forecasts from the US-SPF.

First, we use density forecasts – available in the form of histograms – for the price change in the GDP price deflator (survey variable PRPGDP) for the current and the following calendar year.[36] The density functions are available on an individual forecaster basis and we aggregate this information by using the averaged forecast density functions. The US-SPF defines a price change as the annual-average over

[35] The US-SPF survey was formerly conducted by the American Statistical Association and the National Bureau of Economic Research and was taken over by the Philadelphia Fed in 1990:Q2.

[36] US-SPF started providing density projections of the core Consumer Price Index (survey variable PRCCPI) and of the core Personal Consumer Expenditures Index (survey variable PRCPCE) only in 2007:Q1. Therefore we concentrate on the density projections of the GDP price deflator (despite small level differences with the headline CPI index) in order to have information about the second moments of the future US inflation rates starting from the beginning of our sample, 1999:Q1. The US-SPF does not provide any density projections about headline CPI inflation.

annual-average percent change in the level of the GDP price index that is available quarterly (see eq. (2)). Note that forecast density functions are *fixed event* forecasts (they target the current and the next calendar years), therefore, the forecast horizon changes with the survey's timing. Our sample for the density functions is from 1999:Q1 to 2016:Q2.[37]

Second, we use the US-SPF five-year average headline CPI inflation consensus forecasts (survey variable CPI5YR) in order to identify more distant-horizon inflation forecasts. This projection is defined as the annual average inflation rate over the next five years. The "next five years" includes the year in which the survey is conducted and the following four years. Our sample for this variable spans from 2005:Q3 (its starting point in the US-SPF) to 2016:Q2.

The BCFF and BCEI surveys are published monthly. Both Blue Chip surveys provide individual point estimates of inflation forecasts, from which consensus and disagreement measures can be obtained. Monthly surveys provide inflation forecasts up to six quarters out. In addition to those, BCFF and BCEI surveys publish long-range forecasts twice a year. These long-range forecasts contain average annual forecasts usually five years out from the survey publication year and the average five-year forecast five years ahead. We use five-year five years ahead consensus inflation forecasts in our model estimation.

The SPD survey started in 2004 and, although recent, nicely complements information from the US-SPF that only provides density inflation forecasts for shorter horizons by providing densities for longer horizons. Prior to each FOMC meeting, the survey asks primary dealers (currently 22) a number of questions including inflation density forecasts. The survey questions sometimes vary depending on the economic environment.[38] Nonetheless, certain questions such as the density forecasts for headline CPI inflation are routinely asked. In particular, since the FOMC meeting of March 2007, survey participants are asked to provide a percent chance attached to the five-year average annual CPI inflation five years ahead falling in pre-determined bins. Since the FOMC meeting of December 2014, primary dealers are also asked to provide the same inflation density forecasts over the next five years.

The CES survey provides inflation forecasts for a range of developed countries, on a monthly basis. Survey participants provide point estimates for the average annual percent change of the headline CPI index relative to the previous calendar year. These projections are available for the current and the next calendar year, allowing us, via weighted averages, to obtain $\mathbb{E}_t(\pi_{t,t+12})$.

[37] The beginning of our sample is motivated by the onset of the euro-zone and availability of the euro-area surveys.

[38] See posted questions on the website of the Federal Reserve Bank of New York: https://www.newyorkfed.org/markets/primarydealer_survey_questions.html.

### 6.4.2 Euro-area surveys

Euro-area surveys include the Survey of Professional Forecasters (ECB-SPF) published by the European Central Bank and the Consensus Economics Survey (CES). Panel B of Table 1 summarizes the data set described below.

The ECB-SPF survey was launched in the first quarter of 1999 and provides GDP forecasts, inflation expectations and unemployment forecasts on a quarterly frequency, at a forecaster level. It also provides assumptions made by different forecasters. For the purpose of our analysis we only focus on inflation expectations. Specifically, we use probability distribution forecasts – available in the form of histograms – for rolling horizons (one and two years ahead year-on-year forecasts) and longer-term inflation expectations (five years ahead) defined as changes in the Harmonized Index of Consumer Prices (HICP).

The CES survey publishes long-term forecasts on a semi-annual basis (in April and October), in which five-year five years ahead inflation projections are available. These forecasts are available since 1999 (in the case of the euro area). We use these long-term forecasts to complement the ECB-SPF survey information.

## 6.5 Beta-smoothing methodology

### 6.5.1 Overview

This appendix presents the methodology used to smoothen forecasters' views about the probabilities of future inflation outcomes. These views are available in the form of histograms in the ECB-SPF, the US-SPF and the SPD (see Appendix 6.4 for details).

The spirit of the smoothing methodology broadly builds on Engelberg, Manski, and Williams (2009) (see also Boero, Smith, and Wallis, 2014; Clements, 2014). We consider the data associated with a specific inflation distribution, as defined by: (a) one area, (b) one horizon ($h$) and (c) one measure of inflation ($\pi_{t+h-12,t+h}$, $\pi_{t,t+h}$ or $\bar{\pi}_{t+h-12,t+h}$). We then look for the parametrisation of a generalized Beta distribution that provides the closest fit to the considered data (by minimising the sum of weighted squared deviations between the data and its "theoretical" counterpart).

The data consists of survey-based probabilities of future inflation outcomes falling within given ranges and provides evaluations of the cumulative distribution function (CDF) of the associated distribution at the right-hand bounds of the bins (excluding the last bin, which usually is of the form $[\gamma, \infty[$, where $\gamma$ is the right-hand bound of the penultimate bin). It is important to note that the resulting smoothed distribution is fundamentally different from the model-implied distribution obtained by the approach developed in the present paper. Indeed, the latter are coherent across time and horizons, which is not the case of the former. Heuristically, the smoothing approach presented in this appendix constitutes a preliminary processing of the data before using them in the model estimation.

### 6.5.2 Generalised Beta distribution

$X$ is distributed as a generalised Beta distribution of parameters $(a, b, c, d)$ if $(X - c)/(d - c)$ is distributed as $B(a, b)$. In that case, we use the following notation: $X \sim \mathcal{B}(a, b, c, d)$. If $X \sim \mathcal{B}(a, b, c, d)$, we have $\mathbb{P}(X < x) = \mathbb{P}(Y < (x - c)/(d - c))$, where $Y$ is distributed as $B(a, b)$. Therefore, the CDF of $X$ is:

$$F(x) = \frac{Beta((x - c)/(d - c); a, b)}{B(a, b)},$$

where $Beta(x; a, b)$ is the incomplete Beta function, defined by:

$$Beta(x; a, b) := \int_0^x t^{a-1}(1 - t)^{b-1} dt.$$

The distribution function of $X$ then is:

$$f(x; a, b, c, d) := \mathbb{I}_{\{x \in [c,d]\}} \frac{1}{(d - c)B(a, b)} \left(\frac{x - c}{d - c}\right)^{a-1} \left(\frac{d - x}{d - c}\right)^{b-1}.$$

### Table 1: Summary of the survey data

| Survey | Horizon | Description | Index | Inflation Rate Definition | Frequency | Sample |
|---|---|---|---|---|---|---|
| **Panel A: US surveys of inflation forecasts** | | | | | | |
| US SPF | $\bar{\pi}_{t,t+12}$ | Density | GDP | Annual average over annual average | Q | 1999:Q1 - 2016:Q2 |
| US SPF | $\bar{\pi}_{t,t+15}$ | Density | GDP | Annual average over annual average | Q | 1999:Q1 - 2016:Q2 |
| US SPF | $\bar{\pi}_{t,t+18}$ | Density | GDP | Annual average over annual average | Q | 1999:Q1 - 2016:Q2 |
| US SPF | $\bar{\pi}_{t,t+21}$ | Density | GDP | Annual average over annual average | Q | 1999:Q1 - 2016:Q2 |
| US SPF | $\bar{\pi}_{t,t+24}$ | Density | GDP | Annual average over annual average | Q | 1999:Q1 - 2016:Q2 |
| US SPF | $\pi_{t,t+60}$ | Point estimates | CPI | Average of Q4-over-Q4 forecasts | Q | 2005:Q3 - 2016:Q2 |
| BCFF & BCEI | $\pi_{t+60,t+120}$ | Point estimates | CPI | Annual average | 4/year | 3/1999 - 6/2016 |
| SPD | $\pi_{t,t+60}$ | Density | CPI | Annual average | 8/year | 12/2014 - 6/2016 |
| SPD | $\pi_{t+60,t+120}$ | Density | CPI | Annual average | 8/year | 3/2007 - 6/2016 |
| CES | $\pi_{t,t+12}$ | Point estimates | CPI | Annual average (current and next calendar year) | M | 1/1999 - 6/2016 |
| **Panel B: Euro-area surveys of inflation forecasts** | | | | | | |
| ECB SPF | $\pi_{t,t+12}$ | Density | HICP | Annual inflation | Q | 1/1999 - 6/2016 |
| ECB SPF | $\pi_{t+12,t+24}$ | Density | HICP | Annual inflation | Q | 1/1999 - 6/2016 |
| ECB SPF | $\pi_{t+48,t+60}$ | Density | HICP | Annual inflation | Q | 1/1999 - 6/2016 |
| CES | $\pi_{t+60,t+120}$ | Point estimates | HICP | Annual average | SA | 4/2003 - 4/2016 |

This table summarizes inflation forecast variables from the US and euro-area surveys used in the study. We denote each survey as follows: US-SPF (Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia), BCFF and BCEI (Blue Chip Financial Forecasts and Economic Indicators surveys), SPD (Survey of Primary Dealers conducted by the Federal Reserve Bank of New York), CES (Consensus Economics Survey, conducted by Consensus Economics), and ECB-SPF (Survey of Professional Forecasters conducted by the European Central Bank).

**Table 2: Model fit**

| | $n=3$ and $q=2$ | | $n=4$ and $q=2$ | | $n=5$ and $q=2$ | | |
|---|---|---|---|---|---|---|---|
| Variable | $RMSE$ $(\times 100)$ | $\frac{MSE}{Var}$ | $RMSE$ $(\times 100)$ | $\frac{MSE}{Var}$ | $RMSE$ $(\times 100)$ | $\frac{MSE}{Var}$ | $Nb.$ |
| EA CF $\mathbb{E}_t(\pi_{t+60,t+120})$ | 6.43 | 0.576 | 4.16 | 0.241 | 5.85 | 0.478 | 27 |
| EA SPF $\mathbb{E}_t(\pi_{t,t+12})$ | 22.34 | 0.446 | 17.49 | 0.273 | 15.53 | 0.215 | 71 |
| EA SPF $\mathbb{E}_t(\pi_{t+12,t+24})$ | 11.75 | 0.452 | 8.19 | 0.219 | 8.57 | 0.240 | 71 |
| EA SPF $\mathbb{E}_t(\pi_{t+48,t+60})$ | 5.42 | 0.651 | 1.98 | 0.087 | 1.66 | 0.061 | 65 |
| EA SPF $\mathbb{V}ar_t(\pi_{t,t+12})$ | 5.18 | 0.180 | 5.39 | 0.195 | 5.33 | 0.191 | 71 |
| EA SPF $\mathbb{V}ar_t(\pi_{t+12,t+24})$ | 3.58 | 0.088 | 3.81 | 0.100 | 3.63 | 0.091 | 71 |
| EA SPF $\mathbb{V}ar_t(\pi_{t+48,t+60})$ | 2.87 | 0.082 | 2.71 | 0.073 | 2.71 | 0.073 | 65 |
| US SPF $\mathbb{E}_t(\pi_{t,t+60})$ | 17.33 | 0.482 | 19.95 | 0.639 | 18.01 | 0.520 | 44 |
| US BC+SPD $\mathbb{E}_t(\pi_{t+60,t+120})$ | 11.33 | 0.446 | 6.96 | 0.168 | 3.70 | 0.047 | 106 |
| US SPD $\mathbb{V}ar_t(\pi_{t,t+60})$ | 0.04 | 0.000 | 0.02 | 0.000 | 0.79 | 0.113 | 13 |
| US SPD $\mathbb{V}ar_t(\pi_{t+60,t+120})$ | 3.73 | 0.297 | 3.00 | 0.192 | 3.52 | 0.265 | 74 |
| US CES $\mathbb{E}_t(\pi_{t,t+12})$ | 38.67 | 0.292 | 36.17 | 0.255 | 27.21 | 0.144 | 211 |
| US SPF $\mathbb{E}_t(\bar{\pi}_{t,t+12})$ | 47.23 | 1.171 | 40.01 | 0.841 | 22.08 | 0.256 | 18 |
| US SPF $\mathbb{E}_t(\bar{\pi}_{t,t+15})$ | 41.63 | 1.285 | 33.91 | 0.853 | 11.49 | 0.097 | 17 |
| US SPF $\mathbb{E}_t(\bar{\pi}_{t,t+18})$ | 33.08 | 0.873 | 24.74 | 0.488 | 18.76 | 0.281 | 18 |
| US SPF $\mathbb{E}_t(\bar{\pi}_{t,t+21})$ | 21.63 | 0.361 | 18.81 | 0.273 | 18.64 | 0.268 | 18 |
| US SPF $\mathbb{E}_t(\bar{\pi}_{t,t+24})$ | 22.58 | 0.617 | 15.06 | 0.274 | 13.08 | 0.207 | 18 |
| US SPF $\mathbb{V}ar_t(\bar{\pi}_{t,t+12})$ | 3.29 | 0.044 | 4.03 | 0.067 | 3.83 | 0.060 | 18 |
| US SPF $\mathbb{V}ar_t(\bar{\pi}_{t,t+15})$ | 4.07 | 0.055 | 2.31 | 0.018 | 2.51 | 0.021 | 17 |
| US SPF $\mathbb{V}ar_t(\bar{\pi}_{t,t+18})$ | 5.64 | 0.080 | 3.98 | 0.040 | 4.77 | 0.057 | 18 |
| US SPF $\mathbb{V}ar_t(\bar{\pi}_{t,t+21})$ | 9.03 | 0.139 | 7.10 | 0.086 | 6.79 | 0.079 | 18 |
| US SPF $\mathbb{V}ar_t(\bar{\pi}_{t,t+24})$ | 10.05 | 0.155 | 11.56 | 0.205 | 7.72 | 0.091 | 18 |

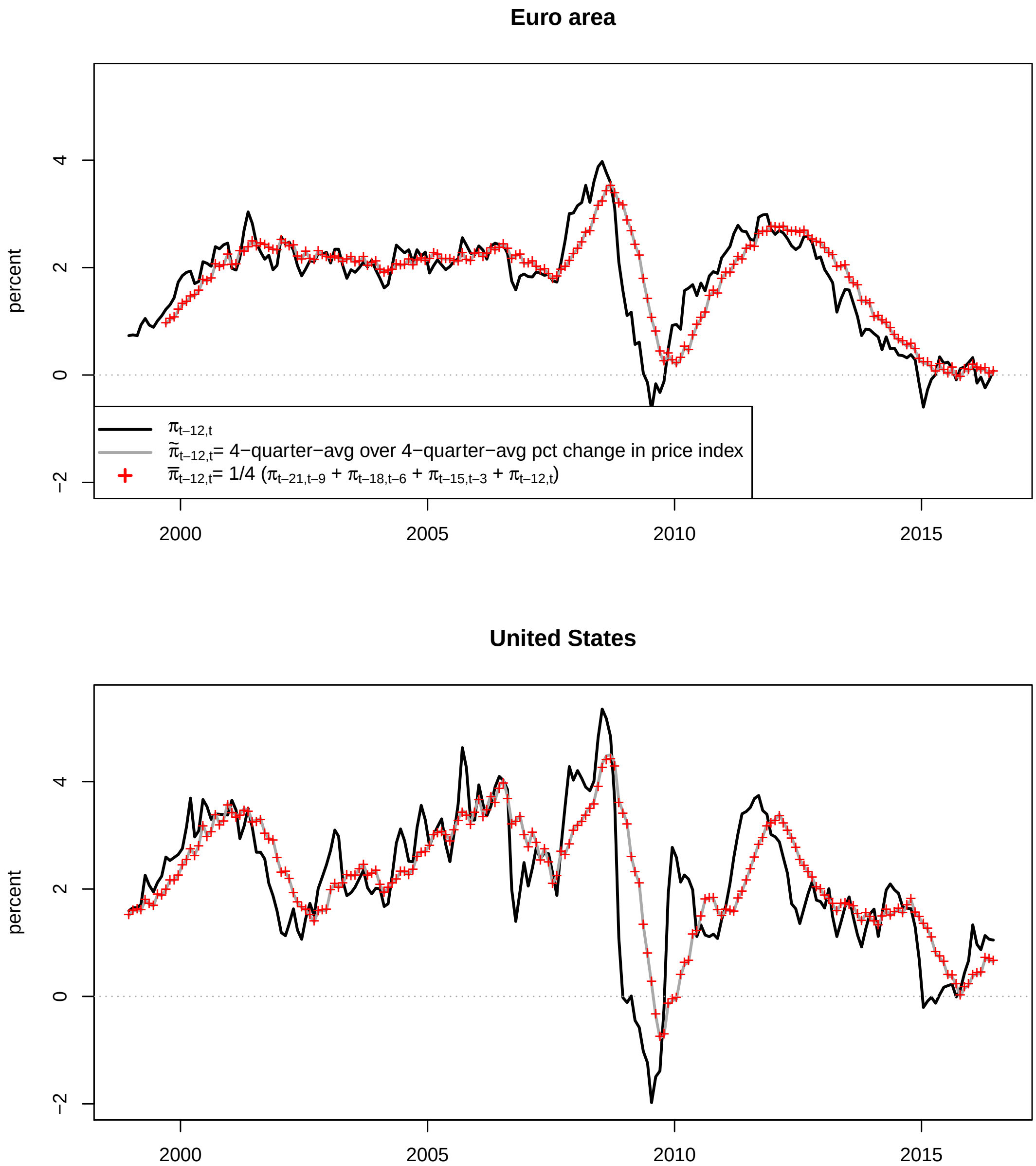
The table reports our model fit results for three cases: $(n,q) = \{(3,2),\ (4,2),\ (5,2)\}$. Columns $RMSE$ report the root mean-squared errors of the measurement equations, expressed in percentage points. Columns $\frac{MSE}{Var}$ report the ratio of the mean squared error to the variance of associated survey-based series. Column $Nb.$ gives the number of observations of each survey-based series.

**Table 3: Parameter estimates**

| | Adjust. | Value | St.dev. | | Adjust. | Value | St.dev. |
|---|---|---|---|---|---|---|---|
| $\pi^{(1)}$ | | 1.731 | – | $\Gamma_{Y,0[1]}$ | $(\times 10^3)$ | 0.652 | *0.931* |
| $\pi^{(2)}$ | | 2.356 | – | $\Gamma_{Y,0[2]}$ | $(\times 10^3)$ | 0.451 | *0.578* |
| | | | | $\Gamma_{Y,0[3]}$ | $(\times 10^3)$ | 0.000 | *0.555* |
| $\delta_1^{(1)}$ | | 1.000 | – | $\Gamma_{Y,0[4]}$ | $(\times 10^3)$ | 12.383 | *5.005* |
| $\delta_2^{(1)}$ | | 1.000 | – | $\Gamma_{Y,0[5]}$ | $(\times 10^3)$ | 5.533 | *6.685* |
| $\delta_3^{(1)}$ | | 1.000 | – | | | | |
| $\delta_4^{(1)}$ | | 1.000 | – | $\Gamma_{Y,1[1,1]}$ | $(\times 10^5)$ | 0.000 | *0.775* |
| $\delta_5^{(1)}$ | | 1.000 | – | $\Gamma_{Y,1[2,1]}$ | $(\times 10^5)$ | 0.098 | *1.156* |
| $\delta_1^{(2)}$ | | 0.788 | *0.255* | $\Gamma_{Y,1[3,1]}$ | $(\times 10^5)$ | 0.000 | *0.594* |
| $\delta_2^{(2)}$ | | −0.078 | *0.129* | $\Gamma_{Y,1[4,1]}$ | $(\times 10^5)$ | 0.000 | *0.495* |
| $\delta_3^{(2)}$ | | 3.867 | *0.445* | $\Gamma_{Y,1[5,1]}$ | $(\times 10^5)$ | 1.975 | *0.931* |
| $\delta_4^{(2)}$ | | 0.411 | *0.096* | $\Gamma_{Y,1[1,2]}$ | $(\times 10^3)$ | 0.074 | *0.031* |
| $\delta_5^{(2)}$ | | 3.072 | *0.223* | $\Gamma_{Y,1[2,2]}$ | $(\times 10^3)$ | 0.550 | *0.200* |
| | | | | $\Gamma_{Y,1[3,2]}$ | $(\times 10^3)$ | 0.127 | *0.070* |
| $\Phi_{Y[1,1]}$ | | 0.994 | *0.001* | $\Gamma_{Y,1[4,2]}$ | $(\times 10^3)$ | 0.001 | *0.056* |
| $\Phi_{Y[2,2]}$ | | 0.999 | *0.000* | $\Gamma_{Y,1[5,2]}$ | $(\times 10^3)$ | 0.243 | *0.099* |
| $\Phi_{Y[3,3]}$ | | 0.962 | *0.002* | | | | |
| $\Phi_{Y[4,4]}$ | | 0.915 | *0.003* | $\mu_{z[1]}$ | | 0.273 | *0.277* |
| $\Phi_{Y[5,5]}$ | | 0.694 | *0.035* | $\mu_{z[2]}$ | | 0.166 | *0.123* |
| | | | | $\Phi_{z[1,1]}$ | | 0.994 | *0.002* |
| | | | | $\Phi_{z[2,2]}$ | | 0.969 | *0.003* |

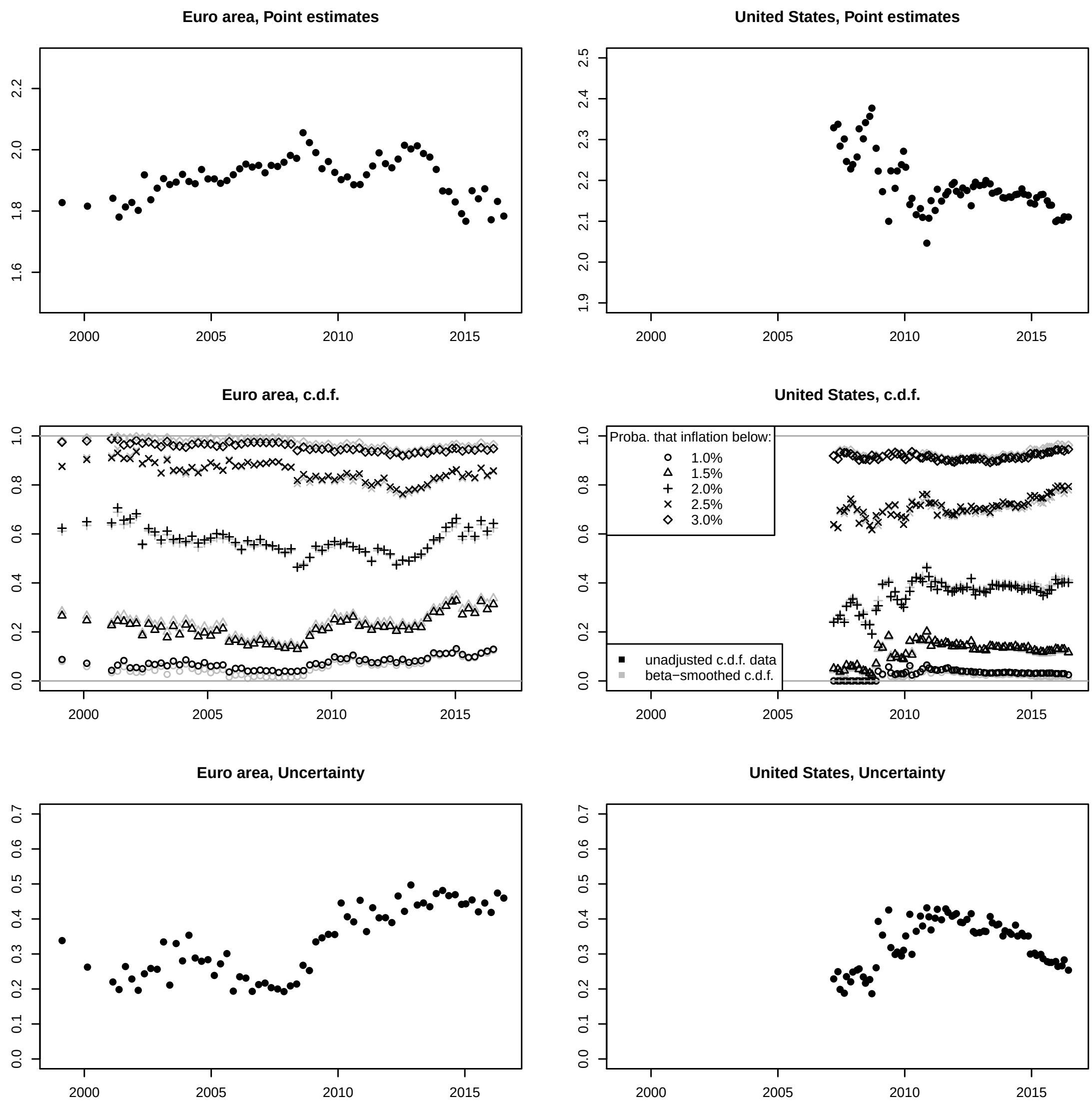
The model is estimated by maximizing the quasi-likelihood stemming from a modified Kalman filter. Standard deviations (in italics) are calculated from the outer product of the log-likelihood gradient, evaluated at the estimated parameter values. For the sake of identification, different elements of $\delta$ are set to 1. Superscripts in parentheses indicate the currency areas: 1 for the euro area and 2 for the US.

## Figure 1: Differences in inflation definitions



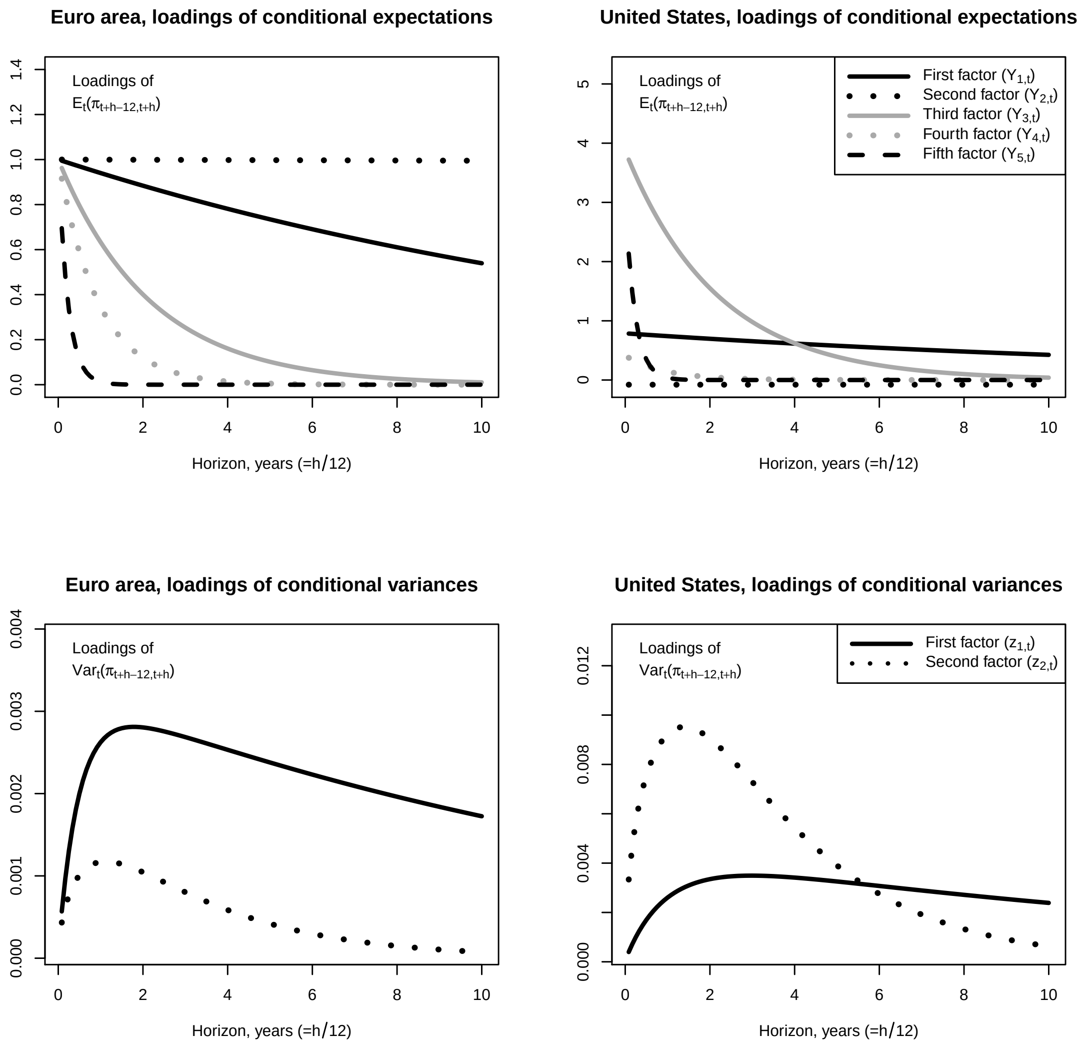

This figure depicts the definitions of inflation forecasts used in various surveys. Black lines on both charts show the realizations of the year-over-year inflation targeted by the ECB-SPF, defined as $\pi_{t-12,t} = \ln \frac{P_t}{P_{t-12}}$, where $P_t$ is the price index of the corresponding economy at time $t$. Grey lines show the realizations of the 4-quarter-average over 4-quarter-average inflation targeted by the US-SPF, defined as $\tilde{\pi}_{t-12,t} = \frac{P_t+P_{t-3}+P_{t-6}+P_{t-9}}{P_{t-12}+P_{t-15}+P_{t-18}+P_{t-21}}$. Red crosses indicate the approximated version of the grey lines: $\bar{\pi}_{t-12,t} = 1/4(\pi_{t-21,t-9} + \pi_{t-18,t-6} + \pi_{t-15,t-3} + \pi_{t-12,t})$.

## Figure 2: Original survey data, ECB-SPF and US-SPD surveys



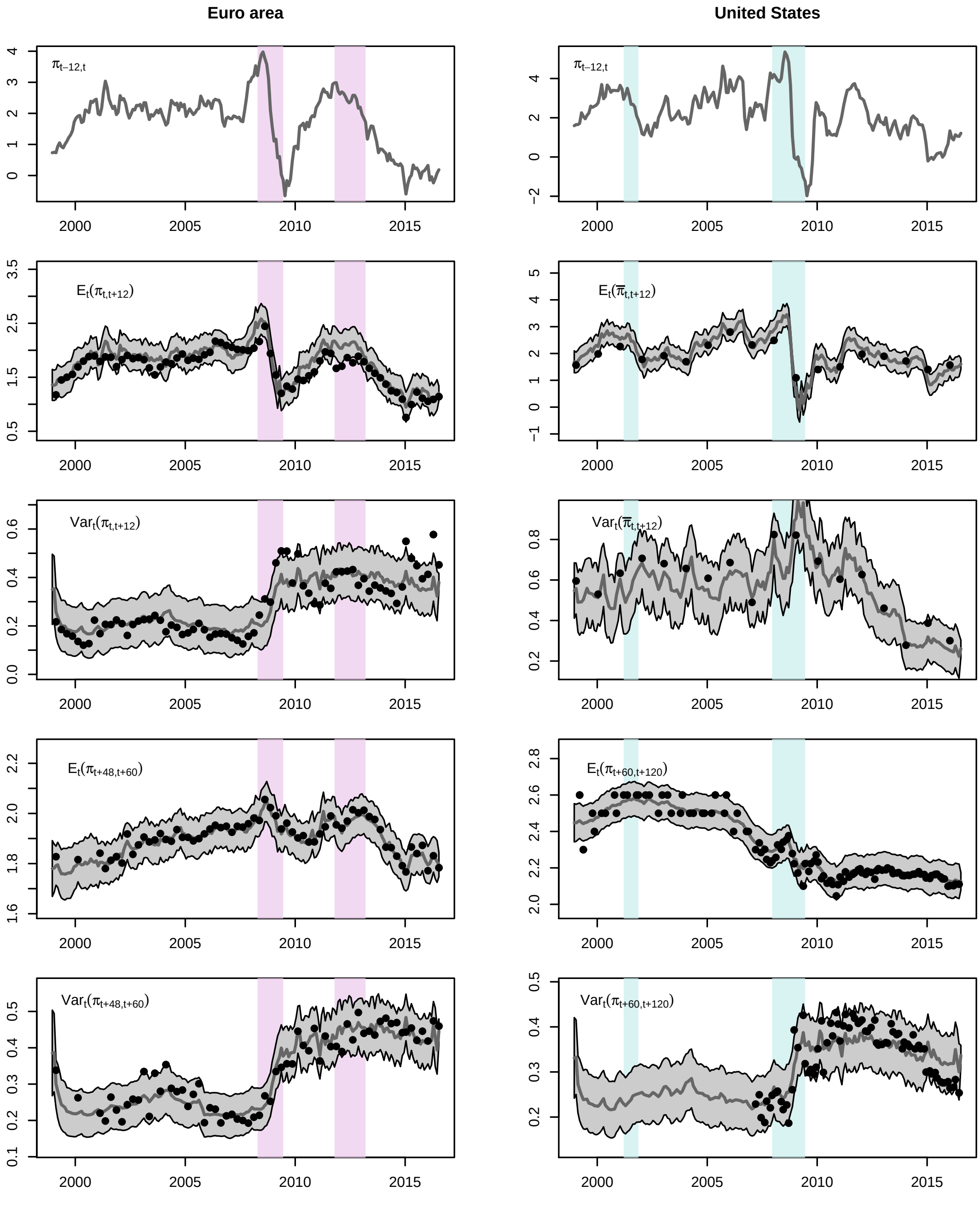

This figure shows the original survey data from the ECB-SPF and the US-SPD surveys. The left-hand side charts show the point estimates of the 1-year 4 years ahead ($\pi_{t+48,t+60}$) inflation forecasts (top panel), original CDF survey results for $\pi_{t+48,t+60}$ inflation rate forecasts along with their beta-smoothed distribution (middle panel), and a derived uncertainty measure about $\pi_{t+48,t+60}$ inflation rate forecasts (bottom figure). The ECB-SPF sample is from January 1999 to June 2016. The right-hand side charts show the point estimates of the 5-year 5 years ahead ($\pi_{t+60,t+120}$) inflation forecasts (top panel), raw CDF survey results for $\pi_{t+60,t+120}$ inflation rate forecasts along with their beta-smoothed distribution (middle panel), and a derived uncertainty measure about $\pi_{t+60,t+120}$ inflation rate forecasts (bottom figure). The US-SPD sample is from March 2007 to June 2016.

## Figure 3: Factor loadings of expectations and variances of future inflation rates



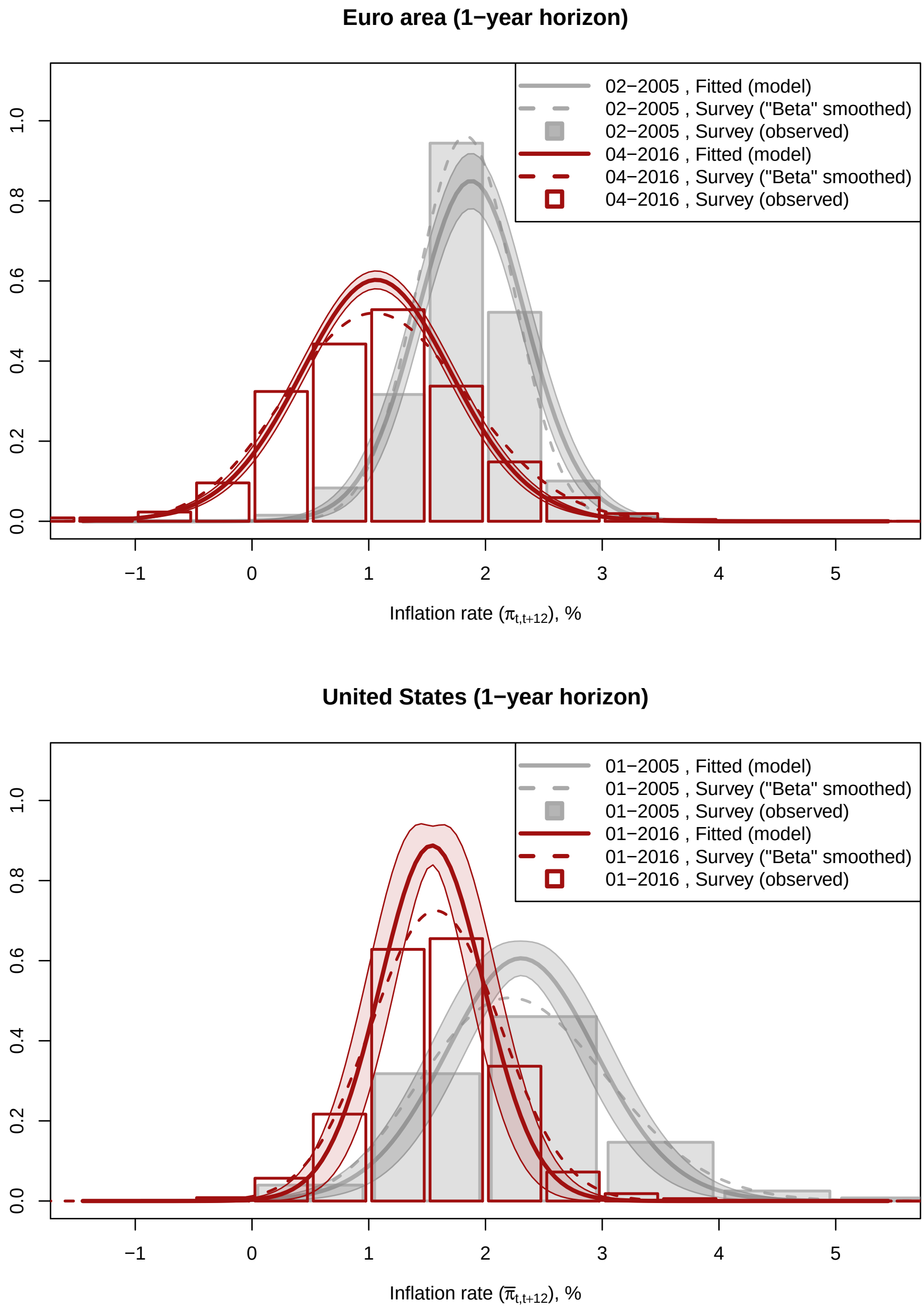

This figure displays, for different horizons $h/12$ (where $h$ is measured in months), the factor loadings $b_h^{(i)}$ and $\beta_h^{(i)}$ for $\mathbb{E}_t(\pi^{(i)}_{t+h-12,t+h})$ and $\mathbb{V}ar_t(\pi^{(i)}_{t+h-12,t+h})$, see eqs. (9) and (10).

## Figure 4: Fit of conditional moments of survey forecasts



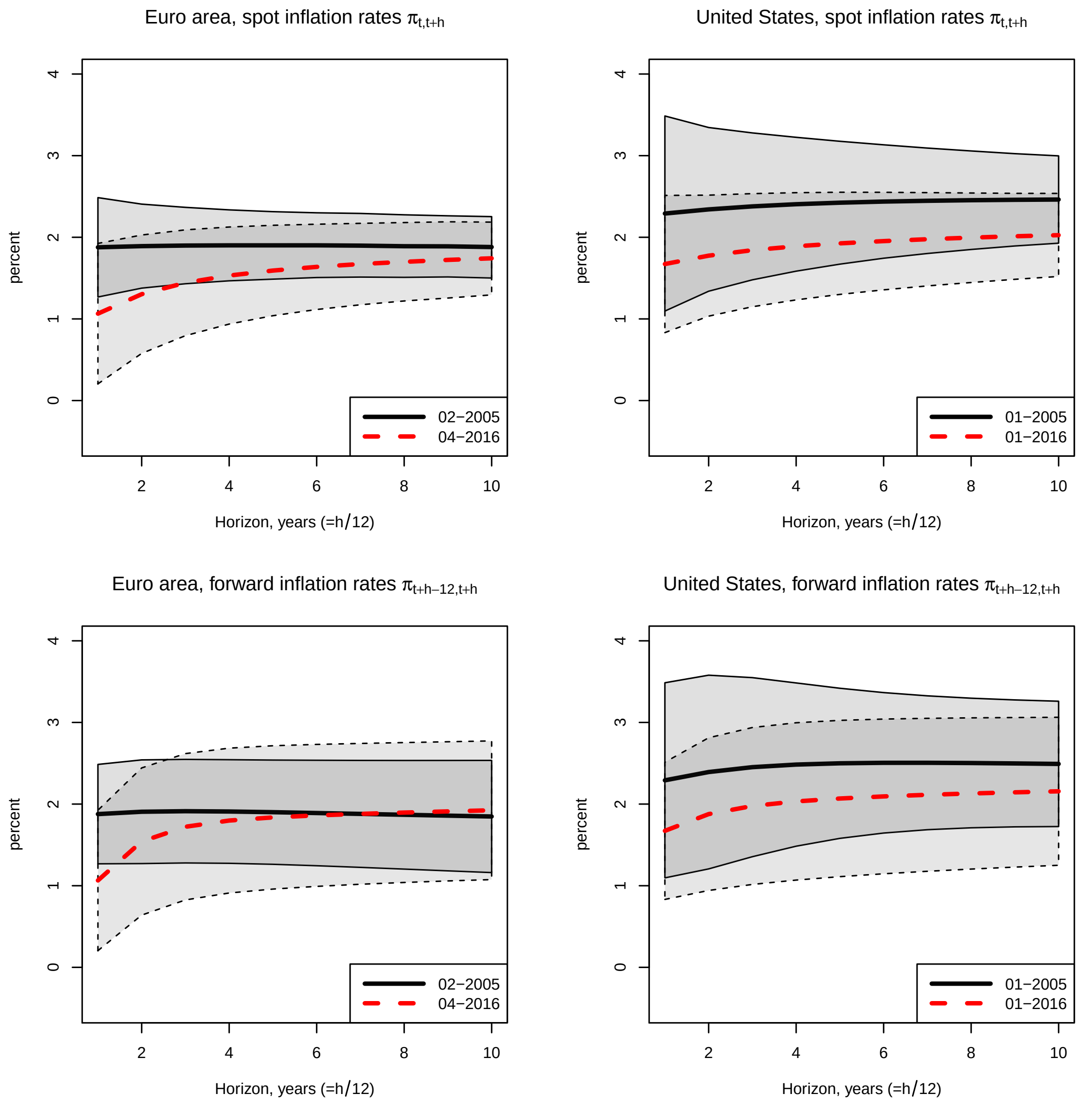

This figure illustrates the fitting properties of the model. The top charts plot realized inflation rates, based on HICP (headline CPI), for the euro area (US). Black dots correspond to survey observations while grey lines correspond to the model-implied quantities. The left-hand (right-hand) sides present euro-area (US) results. The euro-area survey observations correspond to ECB-SPF surveys for both 1-year ahead and 1-year 4 years ahead horizons. The US survey observations correspond to several surveys: observations for the 1-year ahead conditional expectations and variances ($E_t(\bar{\pi}_{t,t+12})$ and $Var_t(\bar{\pi}_{t,t+12})$, panels in rows 2 and 3) correspond to the US-SPF survey; survey observations for $E_t(\pi_{t+60,t+120})$ (panel in row 4) correspond to BCFF/BCEI before March 2007 and to the SPD afterwards; survey observations for $Var_t(\pi_{t+60,t+120})$ (panel in row 5) correspond to the SPD survey. The grey-shaded areas represent 2-standard-deviation confidence intervals. Pink bars indicate euro-area CEPR recessions and blue bars indicate US NBER recessions (here and on the following figures).

## Figure 5: Fit of survey forecast distributions



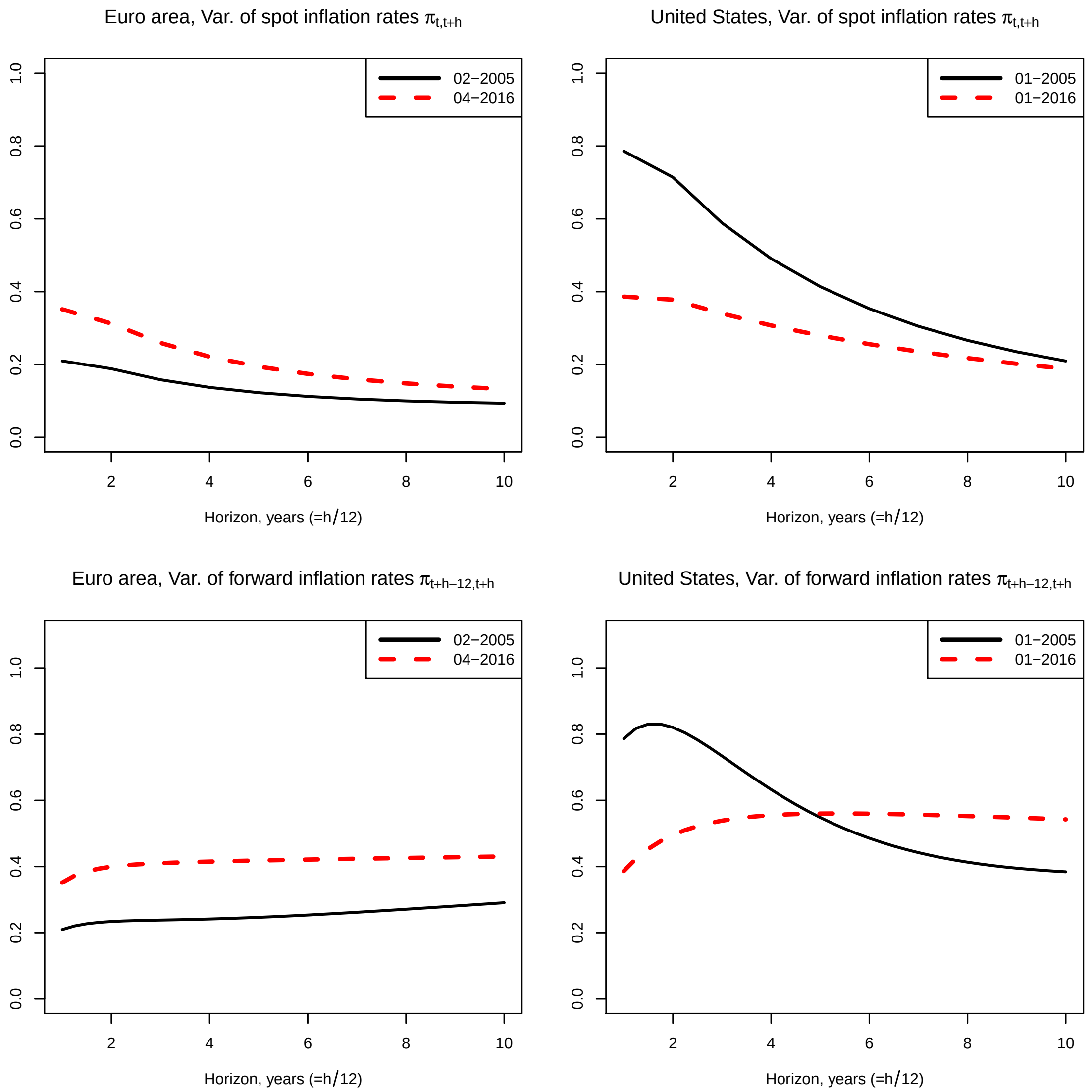

This figure plots the one-year ahead ECB-SPF and US-SPF survey-based histograms, their Beta-smoothed counterparts (dashed lines) (see Appendix 6.5), and the one-year ahead model-implied distributions (solid lines). The grey lines reflect an early date in the sample (February 2005 and January 2005 for the euro area and the US), the red lines reflect a late date in the sample (April 2016 and January 2016 for the euro area and the US). For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect uncertainty associated with the estimation of the latent factors $X_t$ (using the Kalman smoothing algorithm, see e.g. Harvey (1989)). They are obtained by applying the delta method on the function relating factors $X_t$ to the conditional cumulative distribution function (c.d.f.) of future inflation (see Appendix 6.3).

## Figure 6: Term structure of inflation expectations



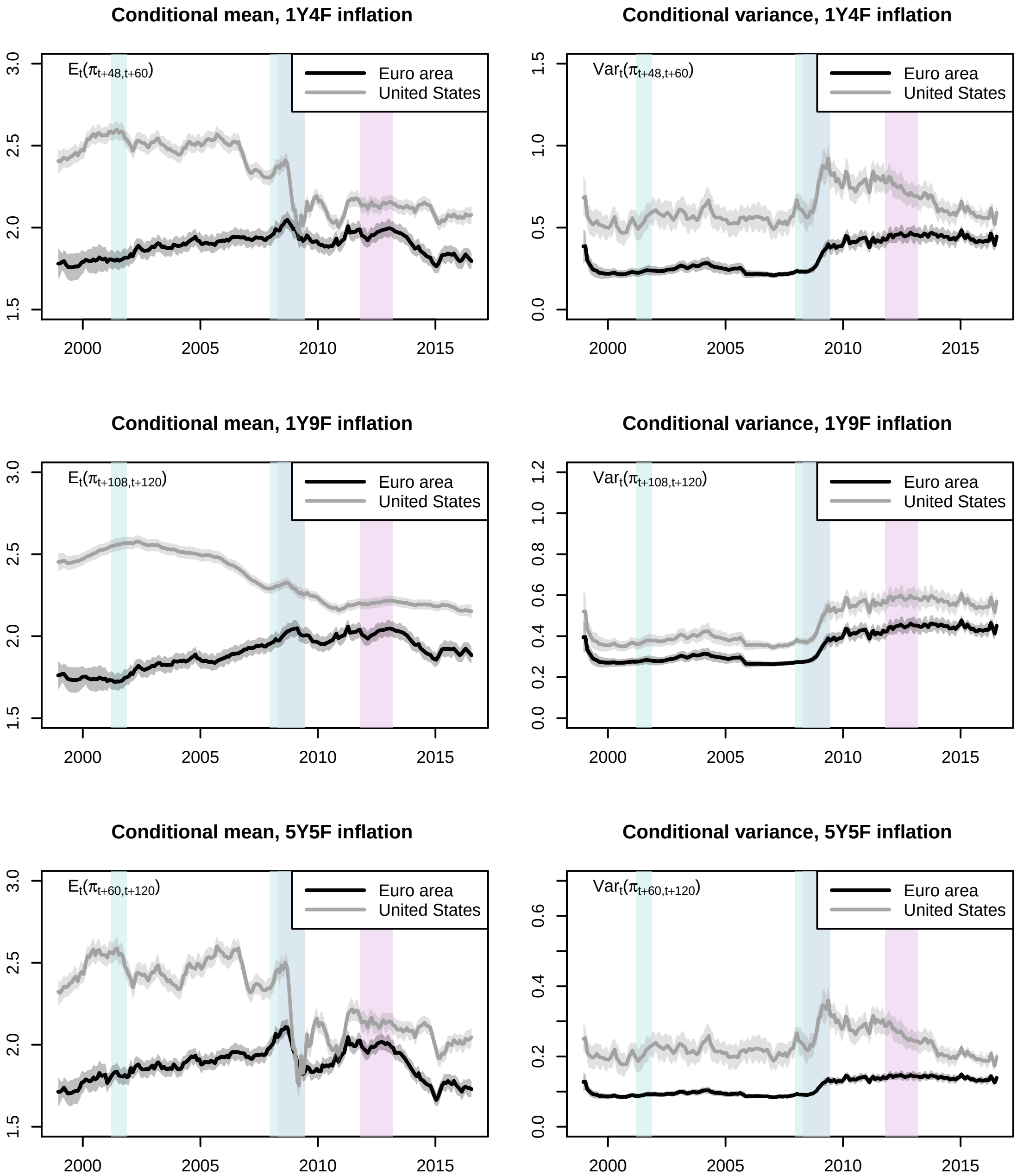

This figure displays the term structure of model-implied expected inflation rates. The top two (bottom two) panels display spot inflation (one-year ahead forward inflation) rates up to a horizon of 10 years in the euro area and the Untied States. The black lines show the term structure for February 2005 (euro area) and January 2005 (US); the red lines show the term structure for April 2016 (euro area) and January 2016 (US). The grey areas with solid (dashed) borders represent the 5th and 95th quantiles associated with the respective conditional distributions for early (late) date in the sample. The quantiles are derived from the closed-form formulas given in Appendix 6.3.

## Figure 7: Term structure of inflation uncertainty



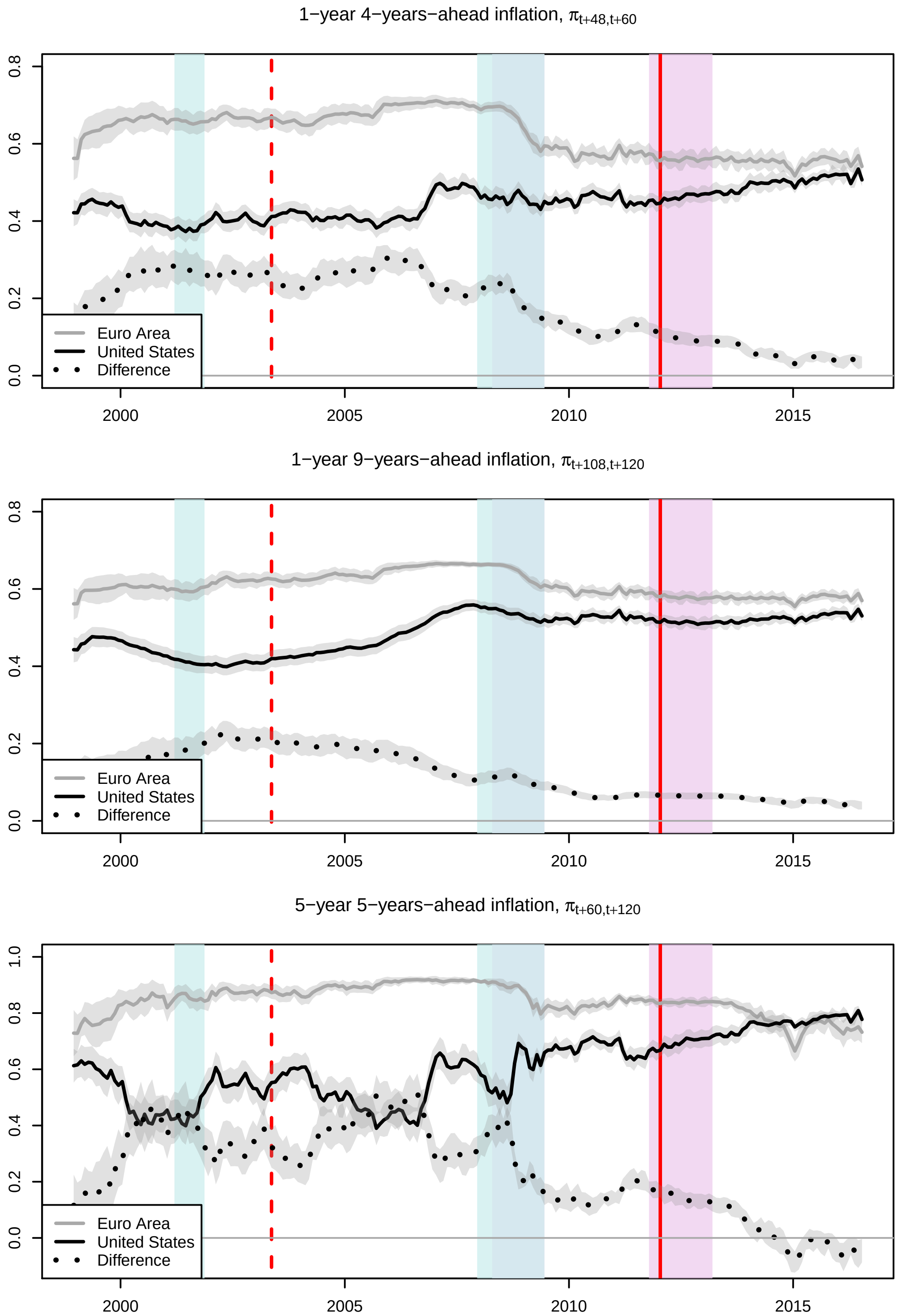

This figure displays the term structure of model-implied conditional variances. The top two (bottom two) panels display variances of spot inflation (one-year ahead forward inflation) rates up to a horizon of 10 years in the euro area and the US. The bottom two panels display the variances of the one-year forward rates $h$ periods ahead. The black lines show the term structure for February 2005 (euro area) and January 2005 (US); the red lines show the term structure for April 2016 (euro area) and January 2016 (US).

**Figure 8: Time series of conditional expectations and variances**



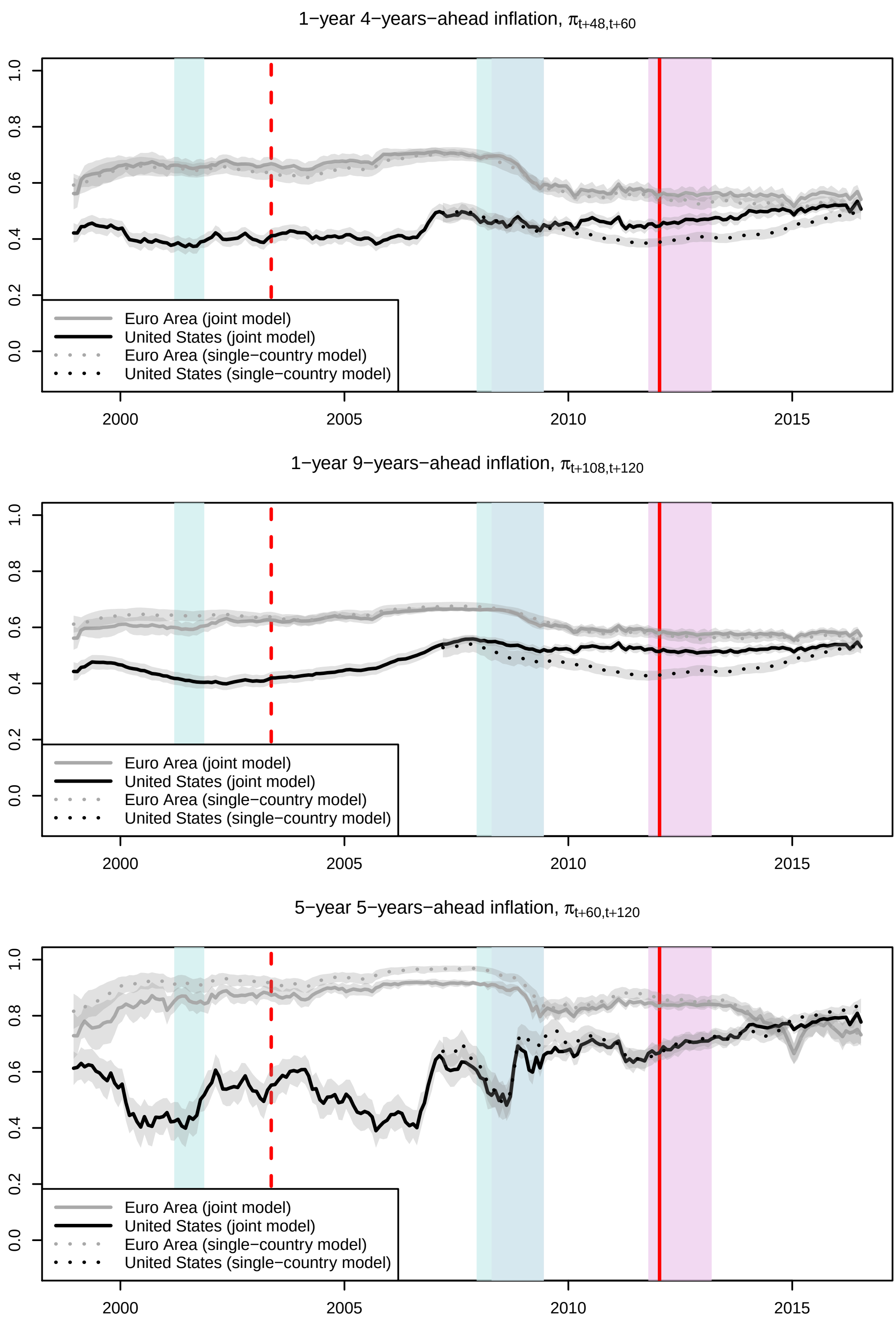

This figure shows the time series of the model-implied conditional means (left-hand side) and conditional variances (right-hand side) for $\pi_{t+48,t+60}$ (top charts), $\pi_{t+108,t+120}$ (middle charts), and $\pi_{t+60,t+120}$ (bottom charts) inflation rates. Black (grey) lines correspond to euro area (US) measures, along with their respective $\pm 2$ standard-deviation bands reflecting the uncertainty associated with factors $X_t$.

**Figure 9: Measure of the anchoring of inflation expectations**




This figure displays probabilities that future inflation rates will fall in the interval $I = [1.5\%, 2.5\%]$. The panels show the time series of the conditional probabilities $\mathbb{P}(\pi^{(i)}_{t+h-m,t+h} \in I|\underline{X_t})$ ($h$ denotes the horizon, $m$ is the tenor, both measured in months, $i$ corresponds to the euro-area or US). Three panels correspond to $\{h = 60, m = 12\}$, $\{h = 120, m = 12\}$ and $\{h = 120, m = 60\}$, respectively. The dashed (solid) red vertical line indicates the timing of the ECB (Fed) clarification (specification) of their inflation objectives, in May 2003 (January 2012). The black line represents US probabilities, the grey line represents euro-area probabilities, and the dotted line – the difference in the two probabilities (euro area − US). Grey-shaded areas are $\pm 2$ standard-deviation bands reflecting the uncertainty associated with factors $X_t$.

**Figure 10: Measure of the anchoring of inflation expectations, comparison between joint and single-area models**




This figure presents both joint-area (grey (euro-area) and black (US) solid lines) and single-area model anchoring probabilities (grey (euro-area) and black (US) dotted lines). Single-area models are the models estimated using the data of one area only (US or euro area). For the US, the estimation of the single-area model is based on the data spanning the period from 2007 to 2016. The dashed (solid) red vertical line indicates the timing of the ECB (Fed) clarification (specification) of their inflation objectives, in May 2003 (January 2012). Grey-shaded areas are $\pm 2$ standard-deviation bands reflecting the uncertainty associated with factors $X_t$.