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Inferring the Shadow Rate from Real Activity †

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Abstract

We estimate a shadow rate consistent with the paths of time series capturing real activity. This allows us to quantify the real effects of unconventional monetary policy in terms of equivalent short-term interest rate movements. We find that large-scale asset purchases and forward guidance had significant real effects equivalent of up to a four percent reduction in the federal funds rate.

JEL Classification: E43, E47, E52

Keywords: effective lower bound, external instrument VAR, Kalman filter, shadow rate, unconventional monetary policy

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1 Introduction

From December 2008 to December 2015, the federal funds rate was held near zero. Because its traditional policy instrument could no longer be used to provide further monetary accommodation, the Federal Reserve turned to forward guidance and large-scale asset purchases (LSAPs). These tools were meant to loosen financial conditions by decreasing longer-term interest rates, thereby increasing employment and inflation.

There is general agreement that forward guidance and LSAPs helped to reduce longer-term interest rates.[1] However, as central banks have limited experience with these policies, their overall effects on the economy remain uncertain. Further understanding these effects is important. In recent years, evidence suggests that the natural rate of interest- the real interest rate compatible with low inflation- has fallen to historic lows (Laubach and Williams, 2016; Holston et al., 2017). In turn, current expectations of longer-run nominal interest rates are low, and are at levels below the magnitude of typical federal funds rate easings during recessions. It is thus likely that in the event of a negative economic shock, the Federal Reserve will again be constrained by the effective lower bound (ELB), and will again use unconventional policy.

Standard approaches to quantify the macroeconomic effects of unconventional policy commonly proceed in two steps: First, a shadow interest rate is derived using a dynamic term structure model, measuring an unbounded short-term interest rate consistent with the remainder of the yield curve; second, this shadow rate is used in a vector autoregression (VAR) or dynamic stochastic general equilibrium (DSGE) model to evaluate the macroeconomic effects of unconventional policy (Wu and Xia, 2016; Mouabbi and Sahuc, 2016). A similar approach is to use a term structure model to measure longer-term interest rate effects of unconventional policy, and enter those into a macroeconomic model

[1] See, for example, Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2011), Gilchrist et al. (2015) and Swanson (2017).

(Engen et al., 2015)[2].

In this paper, we present a new approach to measuring the macroeconomic effects of unconventional policy. Rather than first estimate a shadow rate from financial markets using term structure models, we directly infer a shadow rate from macroeconomic time series using VARs. Typically, a monetary VAR is estimated using the federal funds rate, and the resulting impulse response functions are used to measure the effects of a monetary shock. Our exercise instead inverts this process: Given estimated effects of federal funds rate changes, which interest rate path is most consistent with macroeconomic dynamics since reaching the ELB? Our approach thus quantifies the net effects of policy in terms of the Federal Reserve's traditional policy instrument.

Unlike previous literature, we do not assume that changes in a shadow rate measured from financial markets, as determined by dynamic term structure models, map one-for-one to real effects from conventional interest rate changes. However, we still find that unconventional policy created a negative interest rate environment, in that policy increased the growth of macroeconomic aggregates as further federal funds rate decreases would have. Thus, our paper confirms that the accommodative financial market effects of non-traditional policy resulted in improvements in real economic outcomes.

As our baseline, we infer the shadow rate from a monetary VAR identified recursively (Christiano et al., 1999). With this type of VAR identification, the most traditional in the literature, monetary shocks are identified with the restriction that shocks to the federal funds rate do not affect macroeconomic aggregates contemporaneously. However, as we wish to credibly estimate the dynamic causal effects of monetary policy, we also present results where we identify exogenous monetary policy shocks using unanticipated changes in interest rates around FOMC (Federal Open Market Committee) announcements as instruments (Stock and Watson, 2012; Mertens and Ravn, 2013; Gertler and Karadi, 2015). We find that the external instrument approach results in a similar estimated

[2] These authors also incorporate surveys to measure the effects of forward guidance.

shadow rate to our baseline results using recursive identification.

As we use pre-ELB data to estimate the VAR parameters, we perform a host of checks testing for structural change, parameter instability, and forecast accuracy at the ELB. The results show that our empirical model accurately captures dynamics following the incidence of the ELB. This finding mirrors other results in the time series literature (Stock and Watson, 2012) suggesting that the Great Recession is better explained as the result of large shocks rather than underlying changes to the structure of the economy. We provide further evidence of a lack of structural change over the ELB period through variations of our shadow rate that are estimated with ELB data, which yield similar results to our baseline specification.

Finally, we cross-validate our technique by showing that it correctly recovers the federal funds rate out-of-sample. To our knowledge, we are the first to show that the federal funds rate can be recovered from the evolution of real variables, which reflects the rate's importance in macroeconomic fluctuations. We can only cross-validate out-of-sample using data before the ELB is reached, during which time the federal funds rate as a measure of policy is an observed variable. However, given our affirmative tests of parameter stability, the fact that our methodology recovers monetary shocks correctly before the ELB implies that it should be valid during the ELB period as well.

We continue our paper as follows: We further remark upon relevant literature in Section 2, we explain the data and methodology in Section 3, we present our baseline and instrumental variable identified results in Section 4, we provide evidence that the results are valid in Section 5, we explore the implications of the results for policy effectiveness in Section 6, and we conclude in Section 7.

2 Relevant Literature

Our approach departs from the existing literature using dynamic term-structure models (Krippner, 2013; Wu and Xia, 2016; Christensen and Rudebusch, 2016), in that our methodology does not rely on post-2008 financial variables or no-arbitrage restrictions to identify the shadow rate. In a related paper outside of the no-arbitrage framework, Lombardi and Zhu (2014) estimate a shadow rate with a dynamic factor model derived from interest rate series, monetary aggregates, and balance sheet data. In contrast, our underlying data includes standard series that capture real activity such as gross domestic product (GDP) and investment, and we explicitly measure the shadow rate from its causal effects on our underlying data. Interestingly, we find that the stance of policy instruments, as measured by Lombardi and Zhu (2014), corresponds closely to our measure of policy effects, as measured by our shadow rate.

Other papers outside the dynamic term-structure literature (Johannsen and Mertens, 2016) also use time series models to better understand the effective short-term interest rate. A key difference in our approach is that while Johannsen and Mertens (2016) measure the nominal interest rate that would prevail in the absence of the ELB, our results are interpreted as the likeliest effective monetary policy stance, measured in terms of the equivalent interest rate during normal times. Other papers close to Johannsen and Mertens (2016) include Iwata and Wu (2006), Nakajima (2011), and Chan and Strachan (2014). A hybrid approach, which bridges dynamic term-structure and time series models, can be found in Jackson Young (2014).

Finally, our paper relates to state-of-the-art non-linear DSGE models created to characterize the ELB period (Gust et al., 2017). While our approach is a linear approximation of macroeconomic dynamics, we find evidence that a simple VAR model accurately forecasts at the ELB, as has been found in previous literature.[3] Of note, other shadow rate

[3] Aastveit et al. (2016) and Ferrara et al. (2015) find that linear constant parameter VARs are difficult to beat in out-of-sample forecasting during and following the crisis. Note that even if the crisis data-

estimates also assume a linear structure.[4] In practice, evidence of non-linearities using the quarterly aggregate data that we use has been hard to find (Ng and Wright, 2013), in part because non-linearities that are present at the household and firm-level may not survive aggregation.[5]

3 Data and Methodology

3.1 Data

In order to adequately capture the relationship between interest rates and the rest of the economy, we utilize a broad set of macroeconomic variables. As in Smets and Wouters (2007) we consider output, private consumption, private investment, real wages, hours, inflation (the GDP deflator), and the federal funds rate. The first four variables are per capita and used in log levels so as to contain memory in the system and allow for cointegrating relationships. Real wages and hours are from the non-farm business sector (Bureau of Labor Statistics). In additional estimations, we also include log per capita government consumption and investment from the national accounts, and Moody's BAA corporate bond to 10-year Treasury constant maturity spread.

We choose this set of variables for the baseline specification as we find them to be adequate on three levels. First, they are broad enough to encompass several dimensions of economic activity, but few enough to avoid causing identification problems by significantly reducing the degrees of freedom. Second, they are at a sufficiently high level of aggregation such that we are less concerned with non-linearities present in more granular data, such as

generating process is non-linear, the error in estimation of non-linear models could be much larger than the bias due to a linear approximation.

[4] Wu and Xia (2016), for example, assume that their shadow rate is an affine function of factors following a linear VAR(1) process.

[5] Financial market responses to monetary policy at the ELB also do not show signs of non-linearities. Swanson (2017) finds that responses to unconventional monetary policy are largely similar to those of conventional monetary policy, and that the factors capturing those responses are essentially unchanged when estimating over the ELB period separately.

at the firm and household-levels. Third, the variables have been identified as economically significant in the Smets and Wouters (2007) model and subsequent variations.

We show that our baseline specification provides a good system from which to measure the federal funds rate as an unobserved variable, both before and after the incidence of the ELB. Although other alternative specifications for the VAR are equally plausible, if our shadow rate methodology were to be applied to a different underlying VAR system it would be necessary to comprehensively validate the system as we do in this paper with our chosen specification.

3.2 Methodology

As in Skaperdas (2016), our methodology follows from the observation that VARs can be used to measure the federal funds rate as an unobserved variable. Given a set of observed economic variables and previously estimated inter-relationships between these variables and the federal funds rate, we ask the following question: which interest rate path has the highest likelihood of being consistent with observed economic variables?

More explicitly, consider a bivariate VAR(1) system in structural form, where $\mathbf{Y_t} = (x_t \;\; z_t)'$ as follows:

$$A_0\mathbf{Y_t} = A_1\mathbf{Y_{t-1}} + \epsilon_\mathbf{t} \tag{1}$$

Rewrite Equation 1 as explicit equations for the two variables with a recursive ordering, letting (i, j) denote the ith row and jth column of the preceding matrix as follows:

$$x_t = A_1(1,1)x_{t-1} + A_1(1,2)z_{t-1} + \epsilon_t^x \tag{2}$$

$$z_t = -A_0(2,1)x_t + A_1(2,2)x_{t-1} + A_1(2,1)z_{t-1} + \epsilon_t^z \tag{3}$$

The structural shocks are by construction independent, zero mean, and normally distributed. Assume that the VAR parameters have been solved for using a sample of observed values. Then, in state-space, form, the measurement equation is,

$$x_t = \underbrace{\hat{A}_1(1,1)x_{t-1}}_{\text{deterministic}} + \hat{A}_1(1,2)z_{t-1} + \epsilon_t^x \tag{4}$$

while the state equation is

$$z_t = \underbrace{-\hat{A}_0(2,1)x_t + \hat{A}_1(2,2)x_{t-1}}_{\text{deterministic}} + \hat{A}_1(2,1)z_{t-1} + \epsilon_t^z \tag{5}$$

Using an observed series of values, x_t, we can solve for the optimal z_t using the Kalman filter for the unobserved state, both in- or out-of-sample, using pre-estimated VAR parameters $\hat{A}$'s. Because the shocks are orthogonalized, the Kalman filter provides an optimal linear estimate of the state. The filter minimizes one-step-ahead forecast errors subject to the coefficient estimates of the model. We treat deterministic terms as exogenous forcing variables. In practice, our measurement and state equations are set up as functions of the variables as follows:

$$\mathbf{X_t} = \begin{bmatrix} Output \\ Consumption \\ Investment \\ RealWage \\ Hours \\ Inflation \end{bmatrix}$$

$$Z_t = \begin{bmatrix} Federalfundsrate \end{bmatrix}$$

As shown in the two variable example, we identify the VAR using the Cholesky decomposition.[6] Because our data consists of non-financial variables, we consider the timing restrictions of Cholesky identification to be plausible. However, we also show that the shadow rate is estimated to be similar while instrumenting for monetary shocks using

[6]As is common in the VAR literature, we encounter a prize puzzle where a monetary shock is followed by a rise in inflation. In our estimations, the puzzle is not significant at the 66 percent confidence level, and inflation is not an influential variable in measuring our shadow rate. This is consistent with recent findings of a flattened Phillips curve (Leduc and Wilson, 2017).

high frequency changes in interest rates around FOMC announcements as in Gertler and Karadi (2015).[7] In all cases, we initialize the Kalman filter with the diffuse method of De Jong (1988).

In order for our methodology to capture the overall effects of unconventional monetary policy, it must be the case that unconventional policy affects macro variables in a manner than is comparable to the way a change in the federal funds rate would have. Our estimates provide a benchmark estimate of the effects of unconventional policy through this lens.[8] Other researchers have made the case that LSAPs affect the macroeconomy in a similar way to conventional interest rate changes (Gagnon, 2016), while forward guidance was used both during and before the ELB period (Swanson, 2017). An alternative to the baseline estimation approach examined in this paper would be to estimate all parameters in the state-space model at once, and treat the federal funds rate as an unobserved variable at the ELB. In Section 4.5.2 we show that using a similar methodology yields results that are comparable to our baseline specification.

[7] Papers using high frequency identification have found insignificant contemporaneous effects on real variables, validating the use of Cholesky identification (Gertler and Karadi, 2015; Caldara and Herbst, 2018).

[8] Note that if unconventional policy affects macroeconomic aggregates through additional channels not occurring through conventional federal funds rate changes, we would understate its positive effects.

4 Results

4.1 Baseline

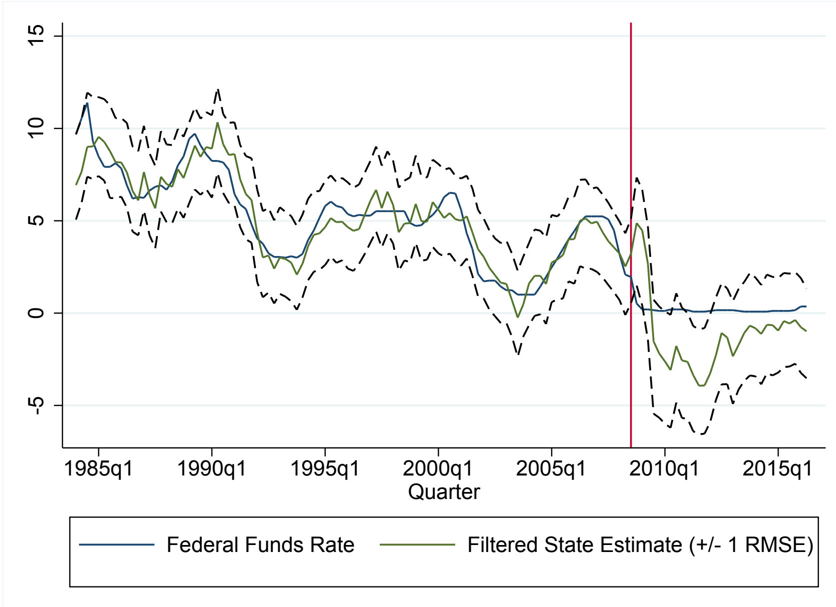
Figure 1 presents our baseline estimation results.[9] We use one lag, as suggested by both the Hannan-Quinn and Schwarz information criteria. In-sample, the state estimates closely follows the federal funds rate. Out-of-sample, the estimates directly infer the equivalent federal funds rate following the incidence of the ELB. These results provide a summary statistic of the net effect of Federal Reserve policies at each quarter, to the extent that they affect other variables in the model in the way traditional monetary policy would have. Of note, the estimates are negative shortly after the financial crisis. Thus, unconventional policy was able to affect macroeconomic aggregates in the same way that a strong and persistent reduction of the federal funds rate would have.

The fact that the shadow rate initially increases following 2008:Q3 may at first appear unintuitive; however, the shadow rate estimated in this paper is the most likely interest rate given observed variables. Although the Federal Reserve decreased the policy rate close to its ELB in 2008:Q4, it is reasonable to believe that the tightening of financial markets following the crisis caused macroeconomic aggregates to grow as if a contractionary monetary policy had been in place. Thus, the increase in the shadow rate reflects the severity of the financial shock to the real economy.[10]

[9]The confidence bands on the estimates are calculated so as to correct for the generated regressors in the first state estimation of the VAR parameters. In all figures, standard error bands are calculated using a bootstrap. Steps are as follows: an artificial time series is created, equal in length to the sample, using random samples of residuals drawn with replacement from the VAR residuals, initiated with 1984:Q1 values. The VAR model is estimated over the artificial time series, yielding a distribution of the VAR parameters. This process is repeated 100 times. We then estimate the shadow rate over the original 1984:Q1-2016:Q2 data with each iteration of the VAR parameters, and report the 16th and 84th percentile intervals of the resulting distribution of state estimate root mean squared errors.

[10]To the extent that financial frictions have impaired policy, our methodology would estimate a shadow rate more restrictive than the actual policy stance. In the appendix, we present evidence of the financial shock. In it, one can see that though the federal funds rate was set to near zero, interbank rates spiked heavily, which would influence broader financing conditions and thus the shadow rate we estimate. We control for spreads in Section 4.2, which nullifies the increase in our shadow rate following the financial crisis.

Figure 1: Implied Monetary Stance, Baseline Estimate



VAR coefficients estimated using 1984Q1-2008:Q3 data, one lag.

4.2 Identification through External Instruments

Our baseline estimation strategy features a VAR with standard timing restrictions on the contemporaneous impacts of monetary shocks. In this VAR model, structural shocks are estimated from the reduced form errors ($\epsilon_\mathbf{t} = A_0\mathbf{u_t}$) under the assumption that interest rate shocks have null effects during the quarter of impact on all other VAR variables. However, these restrictions have been criticized as implausible when financial variables are included in the VAR because it is thought that financial markets are efficient in quickly incorporating any new information into market prices, including any changes in the monetary authority's policy instrument. Since this paper focuses on the effects of monetary policy on real variables, the timing restrictions are less controversial. Nonetheless, it is reassuring to see that the shadow rate estimates are similar while using credible external instruments to estimate the dynamic causal effects of a monetary shock.

We follow Gertler and Karadi (2015) in the use of high frequency changes in futures markets around FOMC announcements to instrument for unanticipated and exogenous movements in the federal funds rate. Denote z_t as the instrument, $\epsilon_t^{p'}$ as the policy shocks of interest, and $\epsilon_t^{q'}$ another structural shock in the model. In order for z_t to be a valid instrument, the following two conditions must hold

$$E[z_t \epsilon_t^{p'}] = \beta \tag{6}$$

$$E[z_t \epsilon_t^{q'}] = 0 \tag{7}$$

The instrument z_t must be correlated with the structural policy shock, $\epsilon_t^{p'}$, but uncorrelated with every other structural shock in the model. Following Kuttner (2001), Gürkaynak et al. (2005), and Gertler and Karadi (2015), we use the three-month-ahead monthly federal funds futures (FF4), and the six-month, nine-month, and one-year-ahead futures on three-month Eurodollar deposits (ED2, ED3, and ED4) as instruments.[11]

Armed with our instruments, we estimate the following regression to isolate exogenous variation in the policy residuals:

$$u_t^p = \beta z_t + \zeta_t \tag{8}$$

Table 1 presents the first stage estimates of β for each instrument using the baseline VAR reduced form residuals. As shown, the instruments are statistically correlated with the reduced form shocks, satisfying condition 6. In particular, the first stage regressions are all well above the one instrument F-statistic cutoff of 10 for weak instruments (Stock et al., 2002).

We then use the predicted structural shocks to estimate the contemporaneous effects,

[11] Since our data are quarterly, we aggregate from the monthly level by taking three month averages of Gertler and Karadi (2015)'s data, which is measured in 30 minute windows around FOMC announcements to ensure no other news drives the changes. We use the instruments through 2008:Q3, as it is until this quarter that we estimate the underlying VAR, after which the federal funds rate reaches its ELB. We do not use the current month federal funds future, as Gertler and Karadi (2015) find it to be a poor instrument for financial variables affecting real activity (such as corporate spreads and household mortgage rates).

Table 1: Effects of High Frequency Instruments on First Stage Residuals of Federal Funds Rate Equation

Instrument	(1)	(2)	(3)	(4)
FF4	1.702***			
	(0.257)			
ED2		2.150***		
		(0.322)		
ED3			1.943***	
			(0.307)	
ED4				1.773***
				(0.341)
Constant	0.062	0.079**	0.059	0.053
	(0.038)	(0.039)	(0.039)	(0.041)
Observations	75	99	99	99
Adjusted R-squared	0.261	0.297	0.273	0.223
F-statistic	44.01	44.46	40.15	27.08

*** p<0.01, ** p<0.05, * p<0.1. Robust standard errors in parentheses.

$\boldsymbol{\phi}$, of the structural policy shocks on the reduced form non-policy residuals as follows.

$$\mathbf{u_t^q} = \boldsymbol{\phi}\hat{u}_t{}^p + \boldsymbol{\xi_t} \tag{9}$$

The vector $\boldsymbol{\phi}$ corresponds to identifying the column in $\boldsymbol{A_0}^{-1}$, where $\boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t} = \mathbf{u_t}$, governing the effects of ϵ_t^p on $\mathbf{u_t^q}$. We then use the variance-covariance matrix of the reduced form VAR to estimate the corresponding row of $\boldsymbol{A_0}$.[12] To identify the remainder of $\boldsymbol{A_0}$, we impose the recursive ordering of the non-policy variables as used in our baseline estimation and maximize the log likelihood function. Finally, to recover the shadow rate from the estimated VAR system, we impose random walks on the state equation.[13]

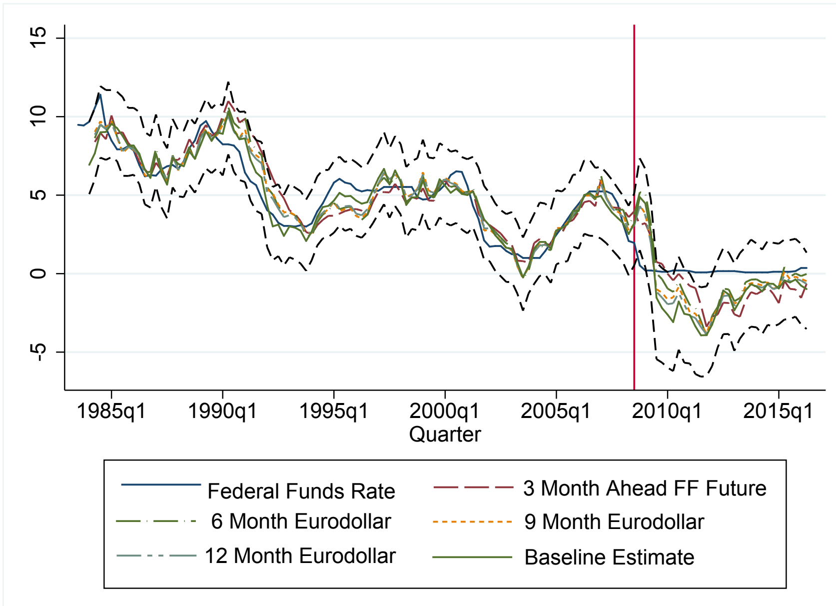
Figure 2 presents the shadow rate estimates with our baseline variables from VARs

[12] Further details are available in the appendix.

[13] We present the random walk results because the instruments result in roughly the same shadow rates with the endogenously estimated state equation, leading us to believe that the systematic policy rule is driving the results. This reformulation is explained at greater length in Section 5.1. In any case, we present the specifications with endogenously estimated state equations in the appendix, which suggest somewhat looser policy.

identified using each of the monetary surprise instruments. As shown, the estimates validate those using recursive identification, and all lie within the baseline identified shadow rate.[14]

Figure 2: Implied Monetary Stance, VAR Identified with External Instruments



Each filtered estimate uses a VAR identified with the noted instrument, estimated with VAR coefficients from 1984Q1-2008:Q3. Dashed black lines: 1984Q1-2008:Q3 baseline state estimate one standard error band. Federal funds rate state equations replaced by random walks.

4.2.1 Controlling for Credit Spreads

The use of credit spreads also allows us to use the full-sample, including post-ELB data, for estimation purposes.[15] We use an approach inspired by Johannsen and Mertens (2016) for missing data according to the following steps:

- Estimate the instrumented SVAR, ending the sample at time $t = 2008$:Q3

[14] We present our baseline confidence intervals because of computational burden, and because our purpose in the paper is to show that all changes to our baseline VAR lie within this interval. There is also no consensus in the literature regarding what the appropriate frequentist confidence intervals are for SVARs identified using external instruments.

[15] We cannot use the following procedure for the VARs without credit spreads. The positive increase in the shadow rate beginning in 2008:Q4 biases the VAR/Kalman filter iteration process such that the shadow rate does not converge to reasonable values.

- Run the Kalman filter through time $t+1$, and input the shadow rate from time $t+1$ as the federal funds rate in time $t+1$

- Reestimate the SVAR through time $t+1$, and input the estimated shadow rate in time $t+2$ as the federal funds rate

- Repeat until the end of the sample (2016:Q2)

In this way, estimation of the shadow rate and VAR parameters includes data through the ELB.[16] In order to ensure that our estimates are not guided by the systematic monetary policy response of the state equation, we reformulate it as a random walk. For high frequency identification of the federal funds rate equation in A_0, we use the estimated coefficients from pre-ELB data as in the previous section in order to avoid the turbulence in financial markets that occurred during the crisis.[17]

Figure 3 presents the resulting shadow rate estimate.[18] As shown, including the BAA spread in the VAR results in an important change from the baseline estimates. The filter no longer identifies the financial crisis as a contractionary monetary shock, as the shadow rate estimate no longer increases following the crisis.[19] In addition, the shadow rate estimate is somewhat more accommodative than in the baseline estimation. This is not surprising given the findings of Caldara and Herbst (2018). If there is a systematic (negative) response of monetary policy to financial spreads, not accounting for this could bias the estimated response of the real sector to the federal funds rate, and thus the estimated shadow rate, confounding the effects of higher than usual spreads with contractive monetary policy.

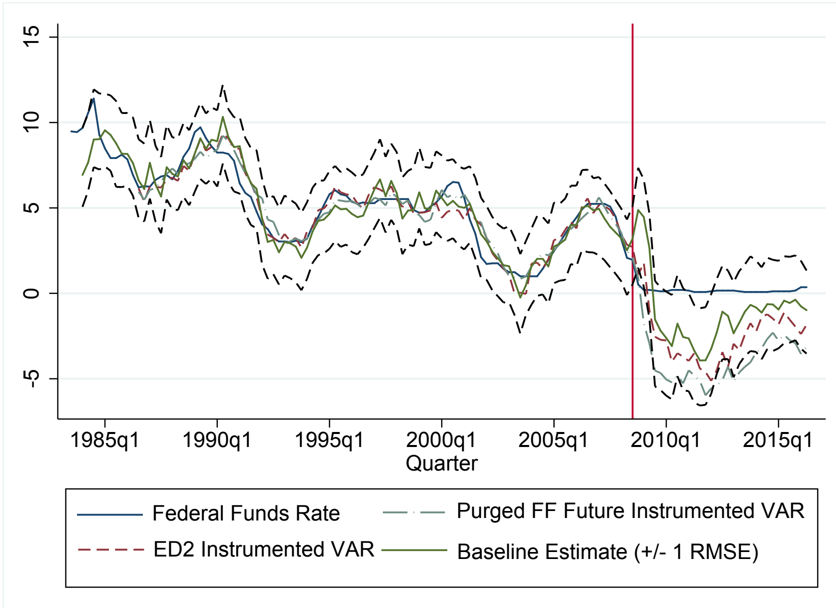
[16]We obtain very similar results when using only pre-ELB data as in previous estimations.

[17]In practice, we find that using financial market responses during the crisis period of the ELB for high frequency identification is problematic, as in many case the SVARs fail to converge.

[18]From the previous section's instruments, we only show the VAR shadow rate instrumented from ED2, the nine-month-ahead Eurodollar future surprises. The other instruments fail to converge using a random walk state equation, though they do converge to similar results using the endogenously estimated state equation. The first-stage F statistic for ED2 is 24.53, with an R^2 of 0.22.

[19]We find that the shadow rate still increases using the excess bond premium of Gilchrist and Zakrajšek (2012) in lieu of the BAA spread. Because the excess bond premium is cleaned of default risk, this suggests that the crisis was more closely associated with increased default risk rather than other factors measured by the excess bond premium.

Figure 3: Implied Monetary Stance, VAR Identified with External Instruments and Controlling for BAA Spread



ED2 indicates a VAR instrumented with surprise 6-month-ahead Eurodollar futures movements on FOMC announcements. The Purged FF uses the purged 3-month-ahead federal funds future shock of Miranda-Agrippino (2016). VARs other than the baseline VAR are estimated over 1986:Q1-2016:Q4, using the missing data procedure described in text, and with the federal funds rate state equations replaced by random walks.

4.2.2 Controlling for Information Content of Announcements

Nakamura and Steinsson (2018), Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2017) show that the use of high frequency monetary surprises as an instrument for exogenous monetary policy shocks can also be confounded by the presence of information asymmetries between the Federal Reserve and market participants.[20] In particular, if the Federal Reserve has a larger or different information set than the public, monetary policy announcements may contain new information for participants about the future state of the economy. Therefore, the estimated responses to monetary surprises can be thought

[20] Miranda-Agrippino and Ricco (2017) also provide evidence that VAR impulse response functions to monetary shocks can be misspecified at longer horizons, as errors are compounded after iteration. As a solution, they proposes a new method bridging local projections with VARs. Within the context of this paper this is not a concern, as our shadow rate methodology uses only one-step-ahead forecasts of VARs, which by construction yield equal parameters to local projections.

of as the aggregate responses of participants to both policy changes and to the central bank's information. Without controlling for this information content, high frequency monetary surprises may not be truly orthogonal to prices and other variables that the Federal Reserve has private knowledge of.

Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2017) deal with this problem by constructing monetary shock series from three-month-ahead Federal Funds rate futures (FF4) purged of central bank information, past information, and public data. Figure 3 presents the shadow rate using Miranda-Agrippino (2016)'s FF4 purged monetary shock measure.[21] The use of this measure results in a similar, though again somewhat more accommodative estimated shadow rate. Thus, our shadow rate is not a result of the public's information revelation.

4.2.3 Fiscal Policy

The use of external instruments also allows us to easily include other variables in the VAR without making timing assumptions on their contemporaneous responses. Within the context of our estimations, it does not seem to be the case that government spending is an important omitted variable, despite the fact that the ELB period was also characterized by unprecedented fiscal contraction. In a variety of specifications, we do not find that including government spending increases the forecast accuracy of our methodology either in- or out-of-sample.

Figure 4 presents specifications where we add government consumption and investment, include credit spreads, and omit the real wage in the underlying SVAR using the procedure for missing data from the previous sections.[22] For brevity, we report the results using the purged FF4 instrumented. We exclude government spending from the measure-

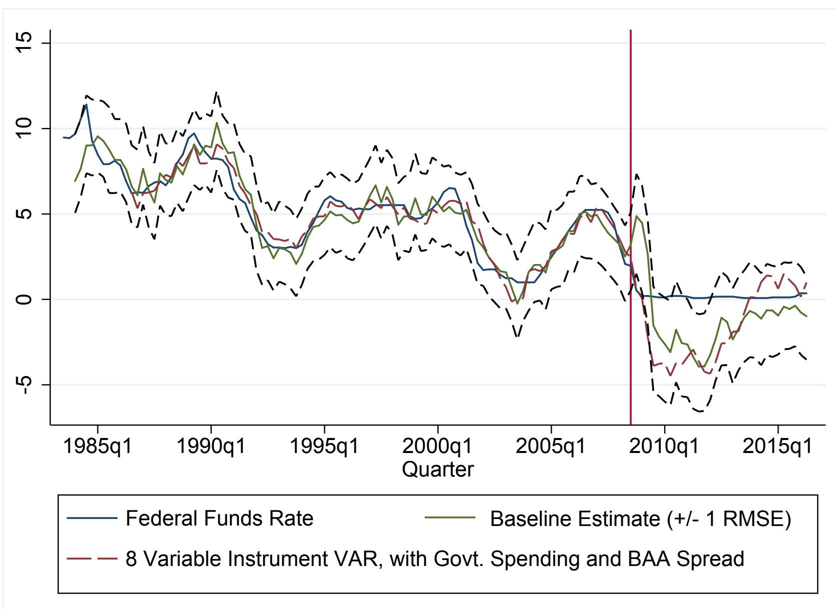
[21] We thank Silvia Miranda-Agrippino for kindly providing her shock series. The first-stage F statistic for this instrument is 15.93, with an R^2 of 0.18. The first stage is estimated using 1990:Q2-2008:Q3 VAR residuals, as the instrument is available beginning in 1990:Q2.

[22] The first-stage F statistic for the instrument is 19.55, with an R^2 of 0.20. We obtain identical results when we include real wages, which we omitted for parsimony.

ment equations, as we are not interested in measuring the shadow rate from its effect on government spending.[23]

As evidenced by the similar state estimates, government spending shocks seem to be associated with shocks other than monetary in the model.[24] The results imply slightly more accommodation before 2011 and slightly less beginning in late 2014. Both monetary and fiscal shocks are expected to have positive effects on inflation and GDP; however, whereas a monetary shock typically increases private investment and consumption, a government spending shock would do the opposite because of crowding out (Coenen et al., 2013; Coenen and Straub, 2005). The identification comes from the heterogeneity in the effects of the two policy shocks.

Figure 4: Implied Monetary Stance, VAR Identified with External Instruments and Controlling for Government Spending



VAR instrumented with the purged three-month-ahead Fed funds future shock series of Miranda-Agrippino (2016), estimated over 1986:Q1-2016:Q4, using the missing data procedure described in text, and replacing federal funds rate state equation with a random walk.

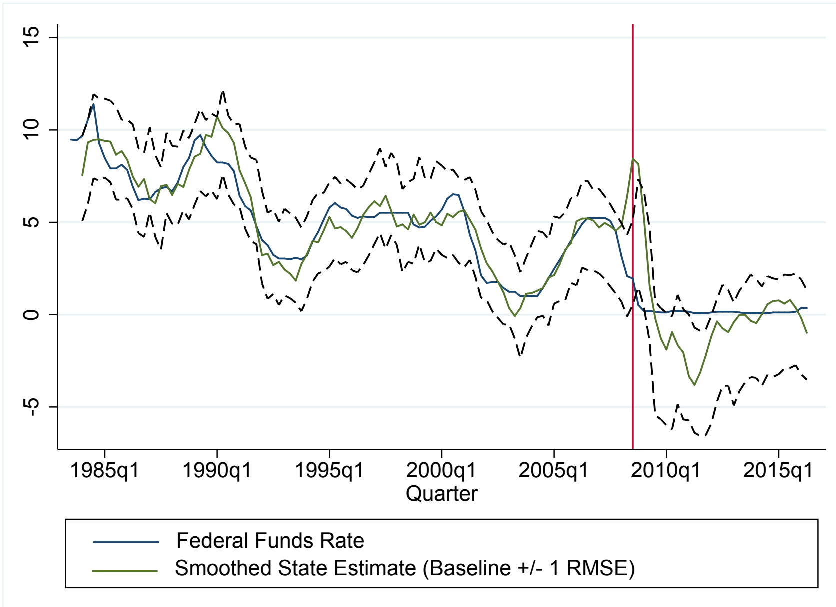
[23]While our modified VAR does not include taxes, previous studies examining fiscal policy have found similar VAR results when omitting taxation (Rossi and Zubairy, 2011).

[24]We obtain almost identical results (not reported) using the Kalman smoother.

5 Proof that the Estimates are Valid

5.1 Further Robustness

Figure 5: Implied Monetary Stance, Smoothed Estimates

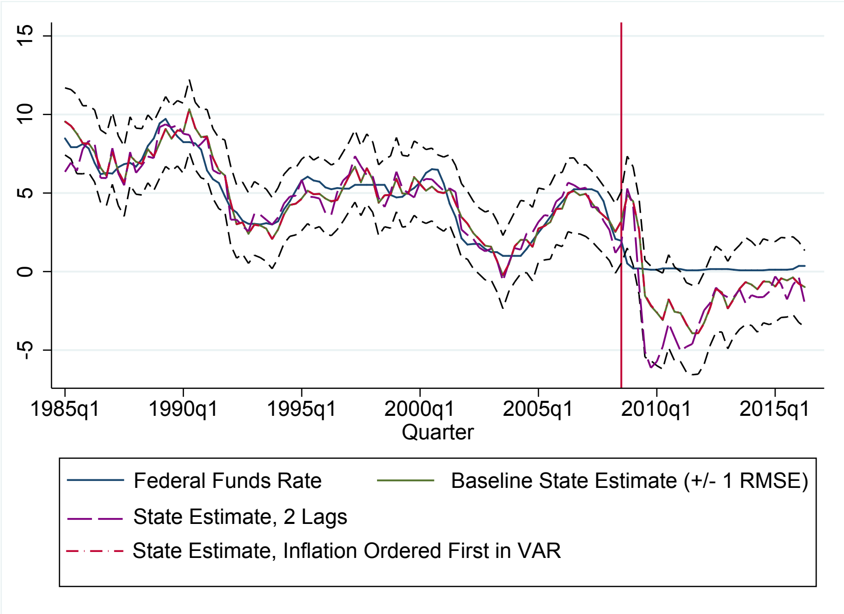


VAR coefficients estimated using 1984Q1-2008:Q3 data, one lag.

In this section, we further elaborate on the robustness of our baseline, (non-IV) estimates. Figure 5 presents the baseline estimation results from the (two-sided) Kalman smoother. In each quarter t, this estimation process uses the entire sample of observations, rather than observations up to quarter t. The smoother results in state estimates that are fairly similar to the filter. This provides evidence that the model is accurately capturing dynamics at the ELB, as the state estimates, and thus the variances of the measurement equations, are not significantly changed by the addition of ELB data. Nonetheless, the filter (one-sided) estimates are our preferred specification, as they allow for some real-time fluctuation in the variances of the measurement equations.

The state estimates are robust to multiple other variations on the VARs. Figure 6 charts several variations of the shadow rate and shows that the state estimates are almost exactly equal with an alternative VAR ordering, where inflation is ordered before all other variables.[25] Likewise, selecting a lag length of two results in similar estimates and dynamics, though a VAR(2) is likely over-fit given our short VAR estimation sample (1984:Q1-2008:Q3).[26]

Figure 6: Implied Monetary Stance, Various Alternative VARs



VAR coefficients estimated using 1984Q1-2008:Q3 data, one lag except for the two lag estimate as indicated.

A possible issue with the the use of the state-space model is that the estimates are driven by the state equation, rather than the measurement equations. If the measurement equations do not provide a good model during the ELB period, it could be the case that the state estimate of the federal funds rate is simply tracing out the pre-ELB reaction function of the Federal Reserve. In order to address this concern, we reformulate the state equation to be a random walk as follows:

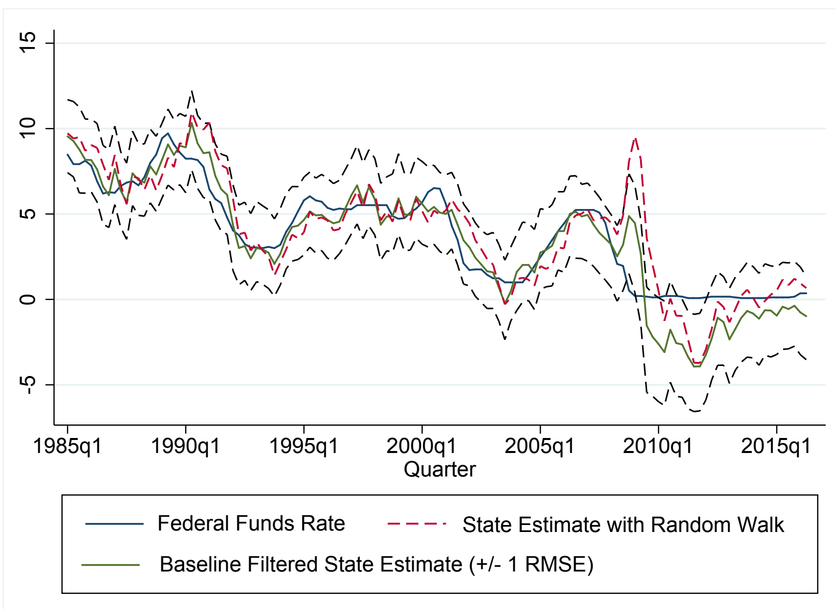
[25] In practice, we do not find the order of the variables before the interest rate to be important.

[26] Since we have a short sample our VAR does not have many lags. Because monetary policy is typically thought to have lagged effects, this could be problematic; Note, however that the interest rate is measured off of its level at each period, not just shocks occurring in each quarter, as would be the case for a VAR estimated in differences.

$$z_t = z_{t-1} + \epsilon_{z,t} \tag{10}$$

The state equation now contains no estimated parameters, and agnostically treats an increase or decrease in the federal funds rate as equally likely in each quarter. The measurement equations remain the same as in the baseline estimation. Figure 7 reveals that the state estimate dynamics are largely unchanged, meaning that the precise structure of the state equation is not crucial for estimation of the shadow rate. However, the random walk formulation does imply somewhat less accommodation overall, and especially in level terms in 2009 and 2010.

Figure 7: Implied Monetary Stance, Other Checks

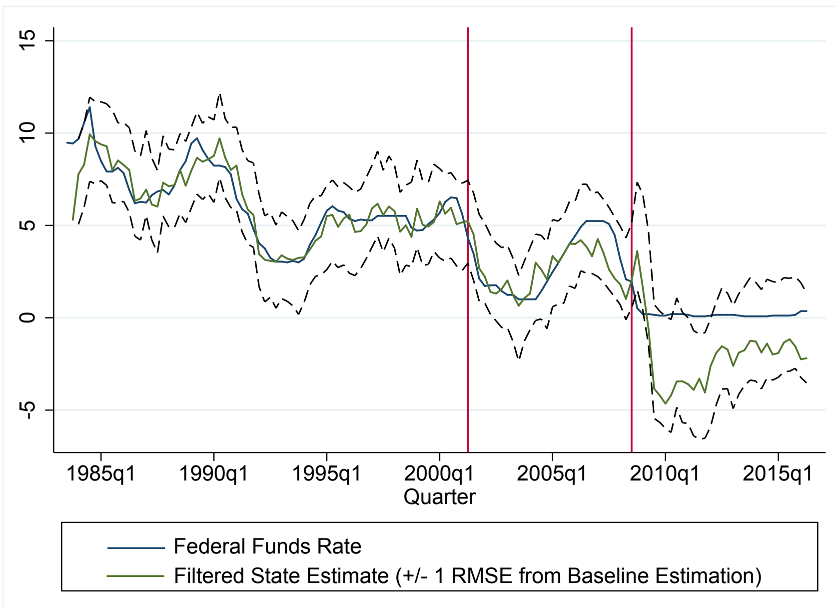


VAR coefficients estimated using 1984Q1-2008:Q3 data, one lag. State estimate with random walk is estimated with the federal funds rate equation replaced by $FFR_t = FFR_{t-1} + \epsilon_t^{FFR}$.

5.2 Model Cross-Validation

If the VAR models are accurately capturing the dynamics of the system, one should be able to estimate them with a shorter time series, and show that the federal funds rate is correctly predicted out-of-sample. Figure 8 presents the shadow rate estimation process using a VAR estimated from 1984Q1-2002:Q1. In-sample, the estimation once again closely recovers the federal funds rate, and lies within the bounds of the baseline estimation standard error bands. More importantly, the out-of-sample results also recover the federal funds rate during which it is observed, and in addition, the ELB period estimate is characterized by similar dynamics.[27]

Figure 8: Implied Monetary Stance, Out-of-Sample Results, VAR coefficients 1984Q1-2002:Q1



Out-of-sample beginning at first red line (one lag). ELB reached at second red line.

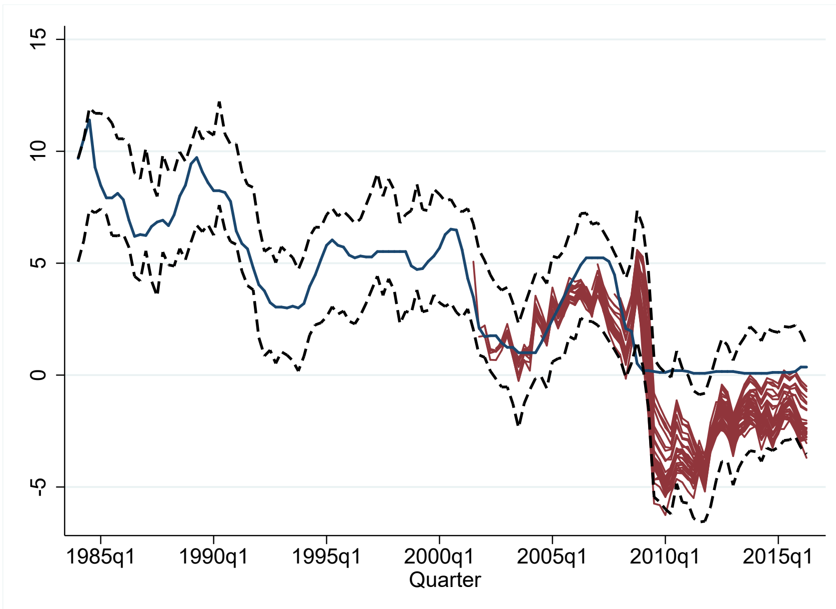
While Figure 8 uses VAR parameters estimated through 2002:Q1 only, we can repeat this process for many endpoints. Figure 9 presents the out-of-sample states estimated with VARs from 1984:Q1 to 2001:Q2, sequentially adding one quarter at a time to the end date, until the baseline VAR estimated through 2008:Q3.[28] In all cases, the methodology

[27]In practice, we have found some modifications to our baseline VAR to perform slightly better in out-of-sample testing before the crisis. However, we use out-of-sample testing to verify our approach, not for model selection, which can be problematic because of over-fitting in the out-of-sample data (Hirano and Wright, 2017). We have also run Monte Carlo simulations to verify that the Kalman filter adequately recovers simulated shocks.

[28]We plot the two-sided estimates in the appendix.

recovers the federal funds rate closely both in- and out-of-sample. Likewise, the state estimates during the effective bound period indicate that unconventional policy gave the economy "under zero" properties.[29]

Figure 9: Implied Monetary Stance, Filtered Out-of-Sample Forecasts Iterated Forward



Blue line: federal funds rate. Red lines: out-of-sample state estimates. Dashed black lines: 1984Q1-2008:Q3 baseline state estimate one standard error band.

5.3 Tests of Parameter Stability

A concern regarding our methodology is that structural change occurred following 2008:Q3. We have assumed that a parsimonious VAR model measured at least partly from before the ELB period can accurately describe dynamics during the effective bound. This could be problematic if, for example, the persistence of GDP decreased following the crisis. We would then find a sequence of contractionary shocks to GDP, as GDP realizations would be lower than the VAR predictions. The filter could then erroneously attribute these

[29] In the appendix, we present similar exercises where we accurately measure a shadow rate out-of-sample in the 1980s using VARs estimated from post-1990 data.

contractionary GDP shocks to contractionary monetary policy, and then estimate a more positive shadow rate.

5.3.1 Explicit Tests

Aastveit et al. (2016) find in a 4-variable VAR that the GDP and inflation equations do not suffer from much instability through the crisis, while the unemployment rate and interest rate parameters do seem to have shifted. The interest rate equation would be expected to have time variation at the ELB because of the ELB constraint. As an explanation for the unemployment parameters shifting, it is known that the unemployment rate has overstated the strength of the labor market during the recovery, as the labor force participation rate has been low by historical standards (Kroft et al., 2016). Since we use total hours as a measure of employment dynamics, we do not have this issue.

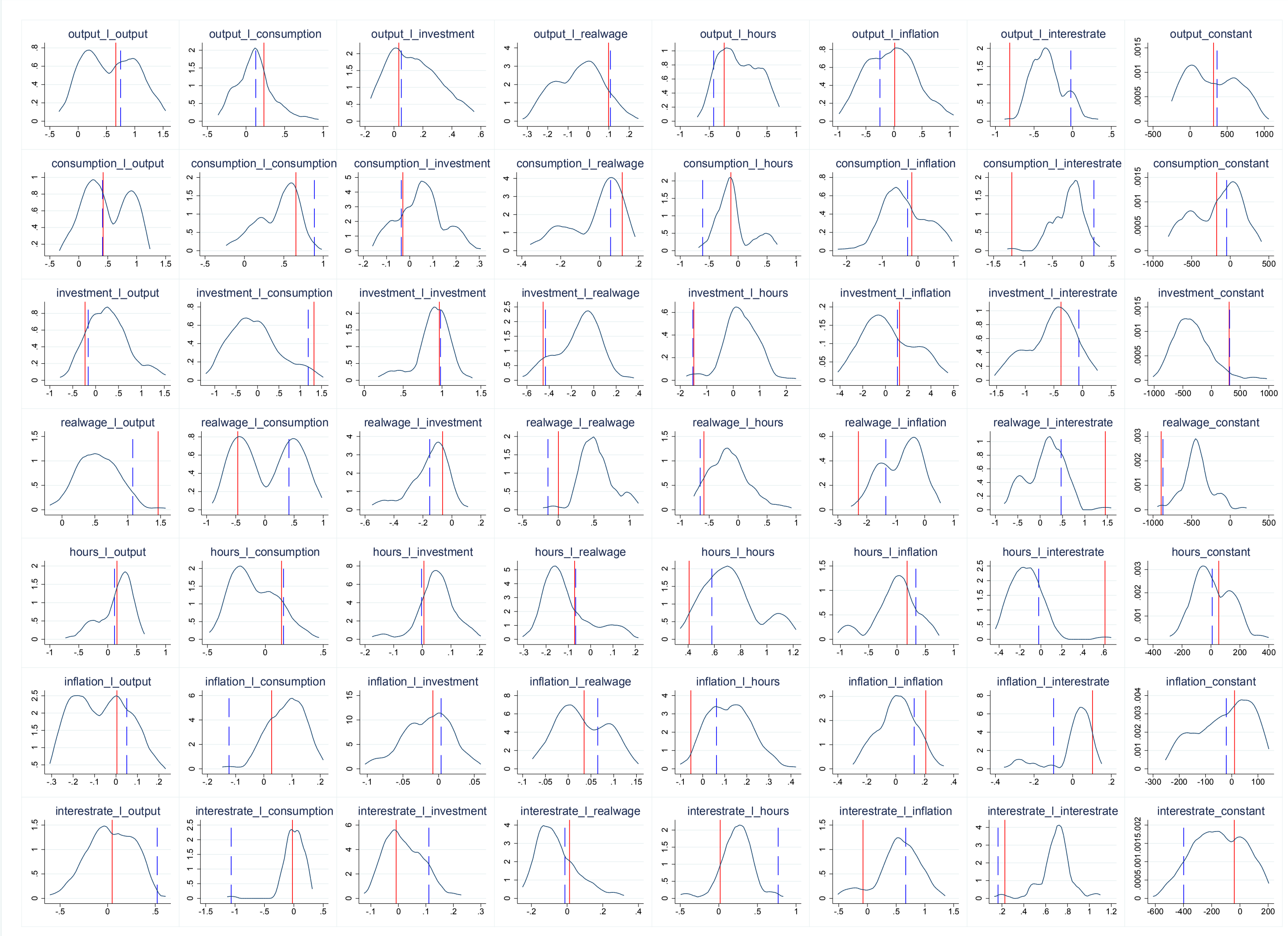
We test explicitly for parameter instability in our sample through the following thought experiment: If one were to randomly test if a period of 31 quarters was structurally different during the Great Moderation, what are the chances that one would get a distribution of parameters as extreme as the those seen during the effective lower bound?[30] To answer this question, we estimate sequential VARs over 31 quarters from 1984:Q1 through 2001:Q1. The last start date for the 31 VAR sample, 2001:Q1, is chosen such that the sample ends before the ELB.

Figure 10 plots the kernel density distributions of the estimated parameters. The parameters during the ELB period are very similar to pre-ELB parameters, and lie plausibly within the pre-ELB VAR parameter distributions. The parameters that are visibly different are the estimates of lagged interest rate coefficients.

Figure 11 presents the rolling estimates of the lagged interest rate parameters. The VAR coefficients using data from the ELB are very extreme in comparison to the pre-ELB sample. This is intuitive: if unconventional policy had significant effects, a federal funds

[30] We choose 31 quarters, as this is the length of the observed sample during the ELB.

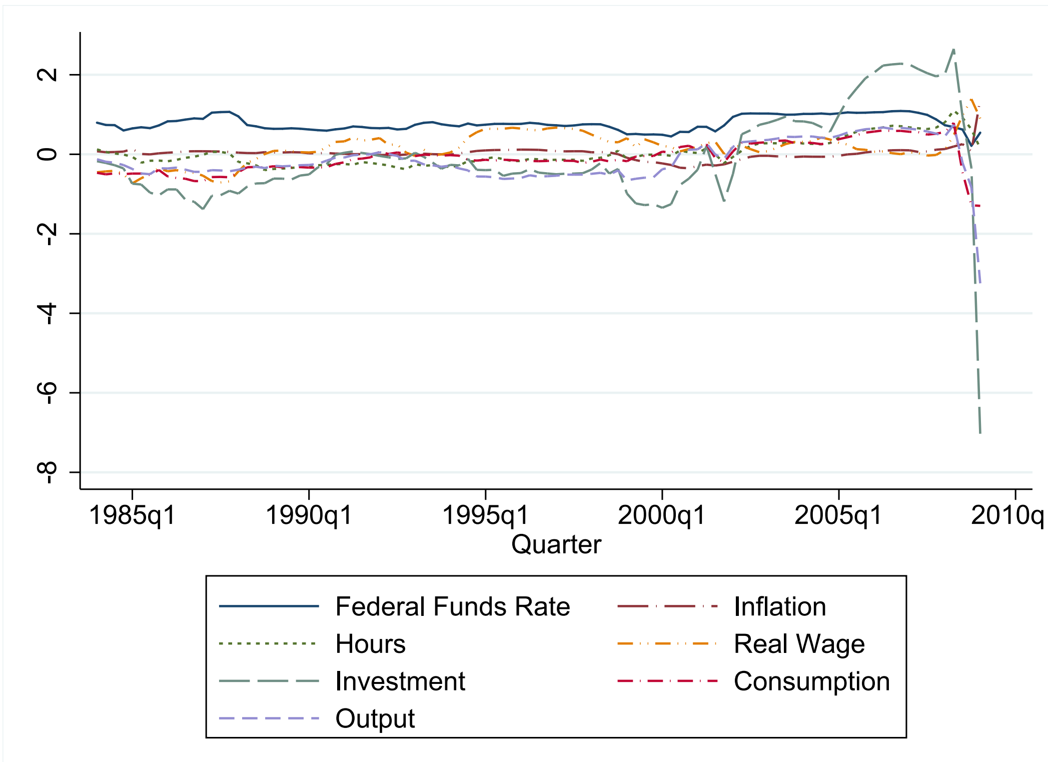
Figure 10: Kernel Density Estimates of VAR Parameters, from 31 Quarter Rolling Estimations, 1984:Q1-2008:Q3, Compared to ELB Period Parameters



The x-axes denote parameter estimates from sequential VARs estimated over 31 quarters from 1984:Q1-2008:Q3. The y-axes denote density. The red lines indicate parameters from a VAR estimated over 2008:Q4-2016:Q2, while the blue dashed lines indicate the same VAR estimated with the baseline shadow rate in place of the federal funds rate.

rate of zero does not adequately describe the actual stance of monetary policy. A VAR estimated with just the federal funds rate would then find very biased effects of monetary policy during the ELB period, as it would be using a policy instrument with a much tighter stance than the stance which was realized due to LSAPs and forward guidance.

Figure 11: Parameter Stability of Lagged Interest Rate Coefficient: Parameter Estimates from 31 Quarter Rolling VARs

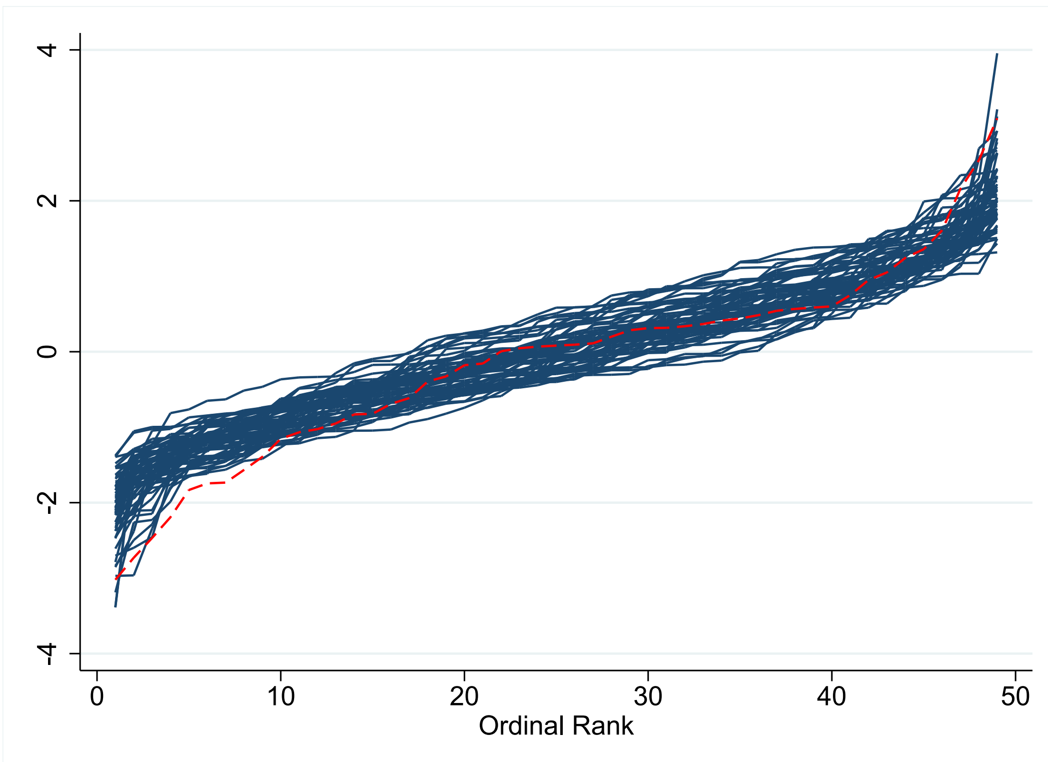


This figure presents the lagged interest rate coefficient on each variable over rolling 31 quarter VAR samples from 1984:Q1-2016:Q2. Each point indicates the parameter over a VAR beginning at that date.

We thus present blue dashed lines indicating VAR parameters estimated over the ELB data using the estimated shadow rate in the kernel density estimates of Figure 10. Validating our hypothesis regarding the bias of the federal funds rate as a policy stance, the lagged interest rate parameters using the estimated baseline shadow rate are much more consistent with the pre-ELB distribution. It is important to remember that this is not a result of the state-space model maximizing these parameters. The Kalman filter maximizes the forecasts of the measurement series.

In order to get a last sense of how the distribution of parameters during the ELB period compares to the in-sample 31 quarter estimated VARs, we present Figure 12. Each line connects the standardized parameters (ordered from smallest to largest) of each VAR estimated over 31 quarters beginning 1984:Q1 to 2001:Q1.[31] We exclude lagged interest rate predictors, as these were shown to be biased.

Figure 12: Distribution of Standardized Parameters: Rolling 31 Quarter VAR Samples



The x-axes denotes the ranking of each standardized parameter from the most negative to the most positive, excluding lagged interest rate parameters. Each blue line is a separate VAR estimated over a 31 quarter period between 1984:Q1 and 2008:Q3. The dashed red line is the distribution of standardized coefficients from a VAR estimated from 2008:Q4 to 2016:Q2.

For the most part, the ELB parameters fall in the middle of the distribution. Closer to the tails, some parameters seem to be more extreme at the ELB. We present parameters

[31]The parameters are standardized with respect to equivalent parameters estimated in each VAR. For example, every estimate of the constant for output is standardized with respect to each sequential VAR estimate of that parameter.

ranked greater than 1.95 standard deviations from the mean in the ELB sample in Table 2.[32] As shown, the most extreme parameters are those governing the investment and real wage predictions.

If these parameters changed significantly following the crisis, the forecasts of these time series, and hence the measurement equations, would result in less accurate estimates of the federal funds rate as an unobserved variable when the pre-ELB VAR was used for the ELB period. In order to gauge the influence of these series on the shadow rate estimation, we present Figure 13. This chart plots the estimated shadow rates when we exclude these series one at a time from the measurement equations. This method prevents any potentially structurally changed parameters from influencing the estimation process. We also present a specification omitting inflation, as our baseline VAR does result in a statistically insignificant price puzzle.

The estimated shadow rate is fairly similar to the baseline shadow rate regardless of which series is excluded. Since the shadow rate is measured from each of the six time series in our underlying VAR, the few parameters that show evidence of structural change are not influential enough to change the estimated shadow rate. The one exception is that while excluding investment, the shadow rate is slightly more negative before 2011. However, the estimate is still well within the confidence bounds of the baseline shadow rate.

5.3.2 Forecast Evaluation

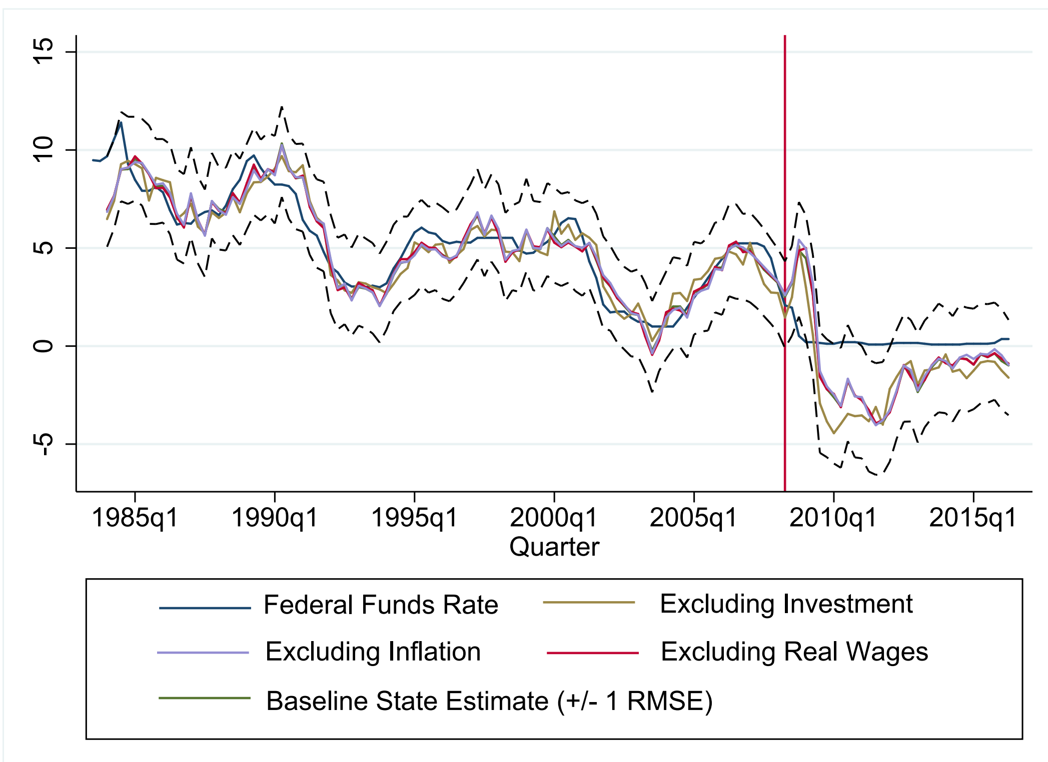
One additional way to test for parameter instability is to evaluate the forecast performance of a model during the time in which it is thought that the parameters have changed. If the model is not a good characterization of the data during that time, the out-of-sample forecast errors will be larger than the in-sample forecast residuals. Even if some parame-

[32] Note that the parameters from the 31 quarter VAR estimates have heavier tails than would be expected from a normal distribution.

Table 2: ELB Period Parameters Greater than 1.95 Standard Deviations from Respective Means

Parameter	Z-Score
Investment, Lagged Hours	-3.10
Investment, Lagged Consumption	2.43
Investment Constant	2.44
Real Wage, Constant	-2.58
Real Wage, Lagged Real Wage	-2.37
Real Wage, Lagged Inflation	-2.09
Real Wage, Lagged Output	2.87

Figure 13: Implied Monetary Stance, Excluding Series from Measurement Equations



Shadow rate estimates when excluding selected series from the measurement equations one at a time. The series excluded each have at least one parameter at a tail of the distribution in Figure 12 greater than 1.65 standard deviations from the mean.

ters change, it could be that these parameters are not influential for model dynamics and thus that the forecast accuracy of the model could be relatively unaffected.

As shown in Table 3, the in-sample, one-step-ahead forecast errors (Column a) are largely comparable to the one-step-ahead forecast residuals from after the crisis (Columns c and d).[33] The mean absolute forecast error is within one standard deviation of forecast residuals for every variable in the VAR.[34] This means that following the crisis, the VAR predictions still forecast relatively well, indicating that the model is likely to provide accurate estimates of the effective federal funds rate before and after the crisis. In addition, it is notable that the mean errors of many of the series are relatively small in an economic sense. For example, the forecasts of output and consumption are on average less than 0.5 percentage points from the actual values. Note that we use the estimated shadow rate in lieu of the federal funds rate following 2008:Q3, to avoid biasing our forecast errors by assuming that unconventional policy had no effect.

We also present a placebo test, in Column b, where we compare out-of-sample forecast errors from before the crisis with out-of-sample forecast errors during the effective bound. This exercise is based on the following logic: Using out-of-sample testing, it is clear that our methodology captures the federal funds rate well during periods when it is observed. In particular, Figure 8 shows that the federal funds rate is adequately captured out-of-sample following 2002:Q1 using VARs estimated from 1984:Q1-2002:Q1. Therefore, if out-of-sample forecast errors during that time are no larger than forecast errors during the effective bound, our VAR parameters are also adequately capturing dynamics when the monetary policy stance is unobserved. Column b presents forecast errors from 2002:Q2-2008:Q3 using VAR predictions from 1984:Q1-2002:Q1 and using the estimated out-of-

[33]We present Column d, which omits the 2008:Q4 residual, for reference because that quarter is characterized by unprecedented shocks due to the financial crisis that are difficult to capture with a statistical model. A regime-shifting VAR would be best for this quarter, but no comparable post-war data to estimate such a model on exists.

[34]We use mean absolute forecast errors because larger shocks during the crisis, as found by Stock and Watson (2012), would necessarily imply greater mean squared errors during the ELB sample regardless of model stability.

Table 3: VAR Mean Absolute Forecast Accuracy

	(a) Forecast Residuals Pre-2008:Q4	(b) Forecast Errors 2002:Q2-2008:Q3	(c) Forecast Errors Post-2008:Q3	(d) Forecast Errors Post-2008:Q4
Output	0.383	0.323	0.485	0.419
	(0.297)	(0.240)	(0.437)	(0.238)
Consumption	0.315	0.264	0.498	0.413
	(0.251)	(0.224)	(0.376)	(0.329)
Investment	0.973	1.11	1.285	1.13
	(0.758)	(0.871)	(1.35)	(1.033)
Real Wage	0.498	0.858	0.808	0.806
	(0.427)	(0.440)	(0.824)	(0.838)
Hours	0.331	0.448	0.466	0.432
	(0.260)	(0.260)	(0.404)	(0.362)
Inflation	0.132	0.488	0.185	0.174
	(0.114)	(0.254)	(0.167)	(0.128)
Observations	99	26	31	30

Mean one-step-ahead forecast errors/residuals, standard deviations in parentheses. Column (a) denotes in-sample forecast residuals from a VAR estimated from 1984:Q1-2008:Q3. Column (b) denotes out-of-sample forecast errors from a VAR estimated from 1984:Q1-2002:Q1. Column (c) denotes out-of-sample forecast errors from a VAR estimated from 1984:Q1-2008:Q3. For comparison, Column (d) repeats Column (c), but does not include the error from 2008:Q4.

sample shadow rate from the 1984:Q1-2002:Q1 VAR predictions. Comparing Column b with Columns c and d shows that the effective bound forecast errors compare well to the out-of-sample forecast errors during a time when the methodology has been demonstrated to adequately recover estimates of the federal funds rate. While the output, consumption and investment equations have slightly larger forecast errors, the real wage, hours, and inflation equations have smaller forecast errors. In conclusion, a thorough evaluation of the VAR shows that parameter stability is not an issue in the implementation of our shadow rate.

5.3.3 Full-Sample Estimation

As a final check, a bound on our baseline results is presented in Figure 14 where we estimate the VAR over the whole sample (1984:Q1-2016:Q2), including the effective bound period.[35] The resulting state estimate is now a result of parameters partly estimated during the effective bound, which assuages concerns about structural change. On the other hand, this also introduces a very large bias in the results: The parameters are fit to minimize deviations from predictions where the stance of monetary policy is set to a federal funds rate of essentially zero during the effective bound period. The VAR parameters are thus heavily biased against finding an effect of unconventional policy, and also cause severe bias in the measurement of monetary shocks from before the effective bound. Nonetheless, the full-sample VAR parameters still result in a state estimate that lies within the baseline estimate confidence interval, and still result in monetary accommodation in excess of an effective federal funds rate of zero, though less accommodation than in the baseline estimation. The full-sample state estimate also depicts similar dynamics.

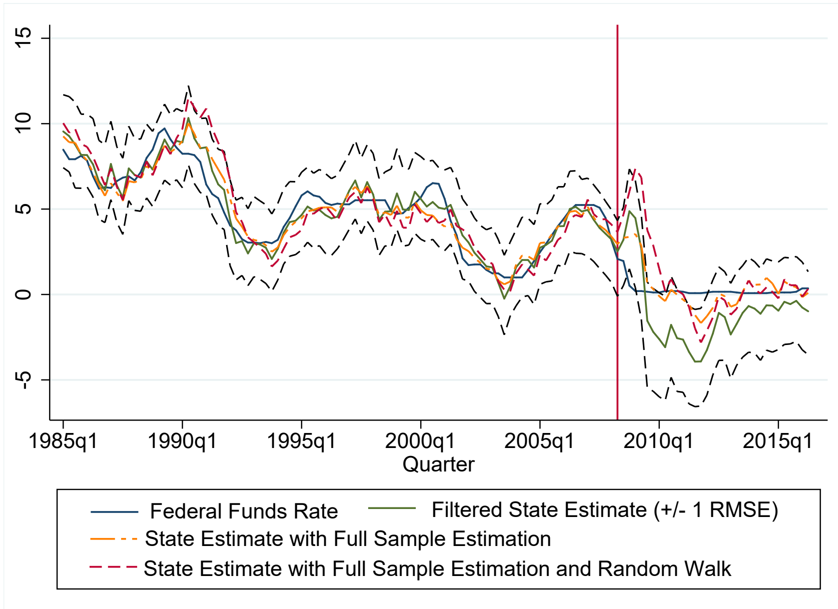
Figure 14 also presents a second variation of the full-sample VAR state estimate using a random walk reformulation of the state equation, as in Section 5. By imposing a random walk rather than using estimated parameters, we do not bias the state estimates with a reaction function that predicts a federal funds rate of zero at the ELB. The state equation is especially problematic at the ELB as the persistence of the federal funds rate will be biased by a period of an unchanging interest rate that is unlikely without the ELB constraint.

The random walk reformulation results in a state estimate that is more accommodative than the state estimate with estimated parameters in the state equation. The shadow rate troughs just 1 percent less negative than the baseline estimate. Overall, the full-sample estimation procedures provide important evidence that a federal funds rate of zero did not capture the effects of policy during the ELB, meaning that unconventional policy had significant effects. Severely biased estimation procedures still result in shadow rate

[35]It is not possible to estimate a shadow rate over the ELB period only, as there are too few observations for the Kalman filter to converge. Furthermore, the bias of using well-measured parameters from before the ELB could be preferable to the increase in variance from such a small sample (see Clark and McCracken (2009)).

troughing below a -1 percent policy stance.

Figure 14: Implied Monetary Stance, Full-Sample Results



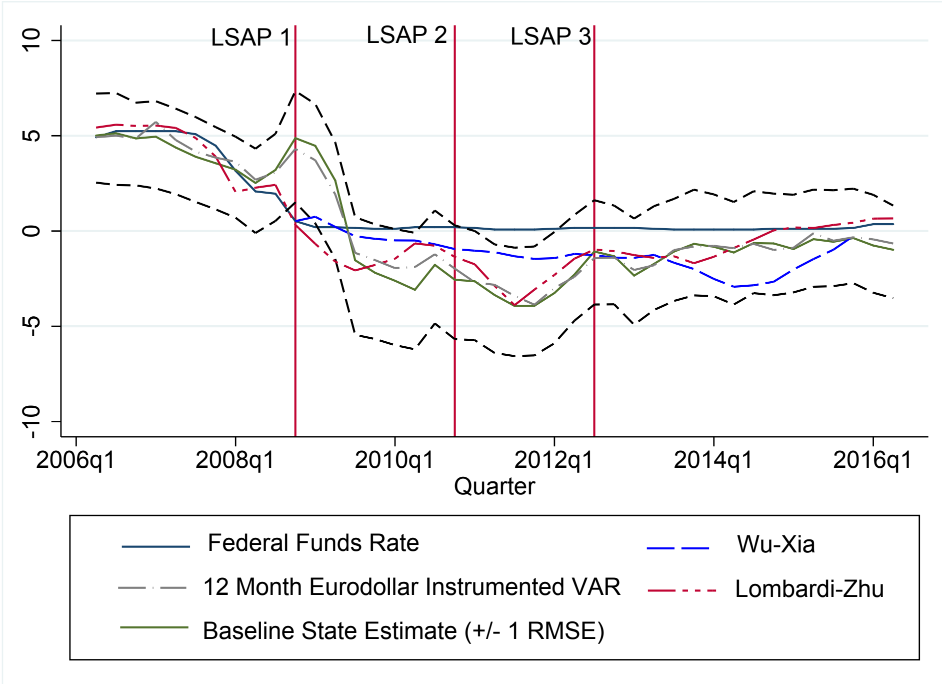
State estimates with random walk estimated with the federal funds rate equation replaced by $FFR_t = FFR_{t-1} + \epsilon_t^{FFR}$. Full-sample estimation: 1984:Q1-2016:Q2.

6 Implications

As the validity of the estimation has been established, the estimates can be used to answer questions about policy effectiveness. Figure 15 presents the baseline results from Figure 1, shown from 2006 forward, with vertical lines indicating the implementations of the three LSAP programs. We observe that the estimated shadow rate decreases immediately after each LSAP program was implemented, providing evidence for the effectiveness of those programs. For reference, we also present the very similar shadow rate resulting from the VAR identified with 12-month-ahead-Eurodollar surprises.

In our estimation, LSAPs 1 and 2 seem to have had large effects. In particular, LSAP 1 seems to have immediately decreased the effective interest rate environment, which is plausible given that it helped to unfreeze credit markets. While our baseline shadow rate

Figure 15: Shadow Rates during the ELB Period



The red lines denote the starts of LSAPs 1, 2, and 3.

decreases the quarter before the announcement of LSAP 2, that program was widely anticipated by market participants.[36] This indicates that the LSAP programs could have important signaling effects. In our baseline estimation, LSAP 3 seems to have had the smallest effect, as the shadow rate decreases by less than 1 percent after its implementation. However, the confidence bands around this estimate are wide. Furthermore, as exhibited by Figure 3, there is much greater variance in point estimates across VAR specifications following LSAP 3.

For comparison, Figure 15 also presents the estimated shadow rates from Lombardi and Zhu (2014) and Wu and Xia (2016). When compared with that of Wu and Xia (2016), both our shadow rate and that of Lombardi and Zhu (2014) indicate quite different effects of non-traditional policy. Wu and Xia's use of no-arbitrage restrictions and estimation

[36]See Ben S. Bernanke (2010), "What the Fed did and why: supporting the recovering and sustaining price stability," *Washington Post*, November 2010.

based purely on yields imply very different shadow rate dynamics.[37] In contrast, our results are determined by the levels of real activity in the six VAR measurement equations. Our methodology reflects the impact of unconventional policy on real activity, while Wu and Xia (2016) show its effects on financial instruments. In principle, these two shadow rate methodologies could result in very different values. For example, if the transmission from financial conditions to real activity was impaired, the size of the Federal Reserve's balance sheet and the size of unconventional monetary policy's real effects would not have to be correlated. While the Wu and Xia shadow rate is correlated with the size of the Federal Reserve's balance sheet, our shadow rate depicts immediate and visible effects of the LSAP programs, pointing to strong flow or signaling rather than to stock effects. One caveat to these comparisons, however, is that our standard error bands do encompass the Wu and Xia shadow rate.

Our shadow rate exhibits strong co-movement with that of Lombardi and Zhu (2014). This is interesting given the very different methodologies and underlying data of our respective shadow rates. The Lombardi and Zhu shadow rate is based on a dynamic factor model derived from yields, monetary aggregates, and Federal Reserve balance sheet items. Their shadow rate is built to provide a policy instrument summary statistic at the ELB. In contrast, our shadow rate measures the causal effect of monetary policy on the real economy. The fact that the two shadow rates are characterized by such similar dynamics is comforting as it indicates that changes in the policy instruments of the Federal Reserve, as measured by Lombardi and Zhu, are mirrored by changes in policy effects on real activity, as measured by our shadow rate. Monetary loosenings, as measured by Lombardi and Zhu through reductions in yields, increases in the size of the Fed's balance sheet, and increases in monetary aggregates, are highly correlated with changes in our shadow rate

[37] It is useful to note that two-factor dynamic term structure models models are robust to many specification choices and result in more accommodative shadow rates (Krippner, 2015) that are closer to that of our baseline estimate than three-factor shadow rate dynamic term structure models such as Wu and Xia's.

estimated from macroeconomic time series.

7 Conclusion

This paper provides new estimates of the real effects of unconventional monetary policy by creating a shadow rate inferred directly from real economic activity. We show that monetary policy was significantly more accommodative than a federal funds rate of zero would imply, indicating real beneficial effects from LSAPs and forward guidance.

We are confident that our results accurately capture the dynamics and broad magnitudes of the real effects of policy. Our methodology correctly forecasts the federal funds rate out-of-sample before the ELB. After the ELB, we show that our VAR accurately forecasts variables other than the federal funds rate. Our estimates using high-frequency identification with external instruments show similar results. We also obtain similar estimates when incorporating credit spreads and fiscal policy, and when using a missing data procedure with ELB data for estimation. Finally, explicit evaluation of the ELB VAR parameters shows that they are drawn from a similar distribution as parameters from the Great Moderation. The one exception we find is that the parameters governing the effects of the interest rate on other variables appear severely biased when not incorporating the shadow rate as the policy stance.

References

Aastveit, K. A., Carriero, A., Clark, T. E., and Marcellino, M. (2016). Have standard vars remained stable since the crisis? *Journal of Applied Econometrics*.

Caldara, D. and Herbst, E. (2018). Monetary policy, real activity, and credit spreads: Evidence from bayesian proxy svars. *American Economic Journal: Macroeconomics*.

Chan, J. C. and Strachan, R. (2014). The zero lower bound: Implications for modeling the interest rate. *MPRA Paper*, 39360.

Christensen, J. H. and Rudebusch, G. D. (2016). Modeling yields at the zero lower bound: Are shadow rates the solution? In *Dynamic Factor Models*, pages 75–125. Emerald Group Publishing Limited.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of macroeconomics*, 1:65–148.

Clark, T. E. and McCracken, M. W. (2009). Improving forecast accuracy by combining recursive and rolling forecasts. *International Economic Review*, 50(2):363–395.

Coenen, G. and Straub, R. (2005). Does government spending crowd in private consumption? theory and empirical evidence for the euro area. *International Finance*, 8(3):435–470.

Coenen, G., Straub, R., and Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the euro area. *Journal of Economic Dynamics and Control*, 37(2):367–386.

De Jong, P. (1988). The likelihood for a state space model. *Biometrika*, 75(1):165–169.

Engen, E. M., Laubach, T., and Reifschneider, D. (2015). The macroeconomic effects of the federal reserve's unconventional monetary policies. *Finance and Economics Discussion Series 2015-00*.

Ferrara, L., Marcellino, M., and Mogliani, M. (2015). Macroeconomic forecasting during the great recession: The return of non-linearity? *International Journal of Forecasting*, 31(3):664–679.

Gagnon, J. (2016). Quantitative easing: An underappreciated success. *PIIE Policy Brief*, 16.

Gagnon, J., Raskin, M., Remache, J., and Sack, B. (2011). The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases. *International Journal of Central Banking*, 7(1):3–43.

Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.

Gilchrist, S., López-Salido, D., and Zakrajšek, E. (2015). Monetary policy and real borrowing costs at the zero lower bound. *American Economic Journal: Macroeconomics*, 7(1):77–109.

Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. *The American Economic Review*, 102(4):1692–1720.

Gürkaynak, R. S., Sack, B., and Swansonc, E. T. (2005). Do actions speak louder than words? the response of asset prices to monetary policy actions and statements. *International Journal of Central Banking*.

Gust, C., Herbst, E., López-Salido, D., and Smith, M. E. (2017). The empirical implications of the interest-rate lower bound. *American Economic Review*, 107(7):1971–2006.

Hirano, K. and Wright, J. H. (2017). Forecasting with model uncertainty: Representations and risk reduction. *Econometrica*, 85(2):617–643.

Holston, K., Laubach, T., and Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*.

Iwata, S. and Wu, S. (2006). Estimating monetary policy effects when interest rates are close to zero. *Journal of Monetary Economics*, 53(7):1395–1408.

Jackson Young, L. (2014). Monetary policy, macro factors, and the term structure at the zero lower bound.

Johannsen, B. K. and Mertens, E. (2016). A time series model of interest rates with the effective lower bound.

Krippner, L. (2013). Measuring the stance of monetary policy in zero lower bound environments. *Economics Letters*, 118(1):135 – 138.

Krippner, L. (2015). A comment on wu and xia (2015), and the case for two-factor shadow short rates.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2011). The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy. *Brookings Papers on Economic Activity*, 43(2 (Fall)):215–287.

Kroft, K., Lange, F., Notowidigdo, M. J., and Katz, L. F. (2016). Long-term unemployment and the great recession: the role of composition, duration dependence, and nonparticipation. *Journal of Labor Economics*, 34(S1):S7–S54.

Kuttner, K. N. (2001). Monetary policy surprises and interest rates: Evidence from the fed funds futures market. *Journal of monetary economics*, 47(3):523–544.

Laubach, T. and Williams, J. C. (2016). Measuring the natural rate of interest redux. *Business Economics*, 51(2):57–67.

Leduc, S. and Wilson, D. (2017). Has the wage phillips curve gone dormant? *FRBSF Economic Letter*, 2017:30.

Lombardi, M. J. and Zhu, F. (2014). A shadow policy rate to calibrate us monetary policy at the zero lower bound.

Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the united states. *The American Economic Review*, 103(4):1212–1247.

Miranda-Agrippino, S. (2016). Unsurprising shocks: information, premia, and the monetary transmission.

Miranda-Agrippino, S. and Ricco, G. (2017). The transmission of monetary policy shocks.

Mouabbi, S. and Sahuc, J.-G. (2016). Evaluating the macroeconomic effects of the ecb's unconventional monetary policies.

Nakajima, J. (2011). Monetary policy transmission under zero interest rates: An extended time-varying parameter vector autoregression approach. *The BE Journal of Macroeconomics*, 11(1).

Nakamura, E. and Steinsson, J. (2018). High frequency identification of monetary non-neutrality.

Ng, S. and Wright, J. H. (2013). Facts and challenges from the recession for forecasting and macroeconomic modeling. *Journal of Economic Literature*, 51(4):1120–1154.

Rossi, B. and Zubairy, S. (2011). What is the importance of monetary and fiscal shocks in explaining us macroeconomic fluctuations? *Journal of Money, Credit and Banking*, 43(6):1247–1270.

Skaperdas, A. (2016). How effective is monetary policy at the zero lower bound? identification through industry heterogeneity. *SSRN working paper*.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *The American Economic Review*, 97(3):586–606.

Stock, J. H. and Watson, M. W. (2012). Disentangling the channels of the 2007-2009 recession. *Brookings Papers on Economic Activity: Spring 2012*.

Stock, J. H., Wright, J. H., and Yogo, M. (2002). A survey of weak instruments and weak identification in generalized method of moments. *Journal of Business & Economic Statistics*, 20(4):518–529.

Swanson, E. T. (2017). Measuring the effects of federal reserve forward guidance and asset purchases on financial markets. Technical report, National Bureau of Economic Research.

Wu, J. C. and Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, 48(2-3):253–291.

Appendix

SVAR Identification through External Instruments

We estimate the SVARs using external instruments building on Gertler and Karadi (2015).[38] For purposes of exposition, assign the federal funds rate equation to be the first row of

$$\mathbf{A}_0\mathbf{Y_t} = \mathbf{A}_1\mathbf{Y_{t-1}} + \epsilon_{\mathbf{t}} \tag{11}$$

Let $\boldsymbol{\phi}$ be causal estimates resulting from the two-stage least squares regressions of the reduced form residuals on exogenous innovations in the policy shocks

$$\mathbf{u_t^q} = \boldsymbol{\phi}\hat{u}_t{}^p + \boldsymbol{\xi_t} \tag{12}$$

Identification of the structural form (11) uses estimates of $\boldsymbol{\phi}$ and the restrictions imposed by $\boldsymbol{\Sigma} = E[\boldsymbol{u_t u_t}']$, the variance covariance matrix of the reduced form model. Partition this matrix such that

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_{11} & \boldsymbol{\Sigma_{12}} \\ \boldsymbol{\Sigma_{21}} & \boldsymbol{\Sigma_{22}} \end{bmatrix}$$

Partition the matrix $\mathbf{B}$ identically, where $\mathbf{B} = \mathbf{A}_0^{-1}$, and the reduced form errors are mapped from the structural shocks such that $\mathbf{u_t} = \mathbf{B}\boldsymbol{\epsilon_t}$,

$$\mathbf{B} = \begin{bmatrix} B_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$$

[38] Gertler and Karadi (2015) do not solve for a full structural VAR, as they only characterize impulse responses of a monetary shock. In our case, we require the full structural form of the VAR to create our state-space measurement equations.

Note that estimates of $\boldsymbol{\phi}$ are the estimates of $\mathbf{B}_{21}$ identified up to scale.[39] Identification of scale follows from the following expressions[40]

$$B_{11}^2 = \Sigma_{11} - \mathbf{B}_{12}\mathbf{B}_{12}' \tag{13}$$

where

$$\mathbf{B}_{12}\mathbf{B}_{12}' = (\boldsymbol{\Sigma_{21}} - \Sigma_{11} \cdot \boldsymbol{\phi})' \cdot \Gamma^{-1} \cdot (\boldsymbol{\Sigma_{21}} - \Sigma_{11} \cdot \boldsymbol{\phi}) \tag{14}$$

and

$$\Gamma = \boldsymbol{\Sigma_{22}} + \Sigma_{11}(\boldsymbol{\phi} \cdot \boldsymbol{\phi}') - \boldsymbol{\Sigma_{21}}\boldsymbol{\phi}' - \boldsymbol{\phi}\boldsymbol{\Sigma_{21}}' \tag{15}$$

The first row of $\mathbf{A}_0$, denoted a, can be derived using results for the inverse of a partitioned matrix as

$$a' = (B_{11} - \mathbf{B}_{12}'\mathbf{B}_{22}^{-1} \cdot \mathbf{B}_{21})^{-1} \cdot \begin{pmatrix} 1 \\ -\mathbf{B}_{22}'^{-1}\mathbf{B}_{12} \end{pmatrix} \tag{16}$$

Where the bottom vector $-\mathbf{B}_{22}'^{-1}\mathbf{B}_{12}$ is obtained as the transpose of

$$\mathbf{B}_{12}'\mathbf{B}_{22}^{-1} = (\boldsymbol{\Sigma_{21}} - B_{11}\mathbf{B}_{21})'(\Sigma_{22} - \mathbf{B_{21}}\mathbf{B_{21}}')^{-1} \tag{17}$$

We impose a recursive ordering on the rest of $\mathbf{A}_0$ and estimate the SVAR using maximum likelihood. In practice, we rescale the federal funds rate row of $\mathbf{A}_0$ such that $A_0[1,1] = 1$ and reestimate the variance of structural shocks to the federal funds rate in SVAR likelihood maximization. We can do this since the model would otherwise be overidentified.

The resulting matrices $\hat{\mathbf{A}}_0$ and $\hat{\mathbf{A}}_1$ are then used to create a state-space system, where the state equation for the federal funds rate is reformulated as a random walk in order to measure the federal funds rate from the measurement equations. Estimates with the endogenously estimated federal funds rate equation are shown in this appendix.

[39] The estimated coefficients of $\boldsymbol{\phi}$ are the effects of a one unit shock to the policy rate. For an estimate of $\mathbf{B}_{21}$, $\hat{\boldsymbol{\phi}}$ needs to multiplied by $\hat{B_{11}}$.

[40] Helpful notes on the derivation are available from Michele Piffer at https://sites.google.com/site/michelepiffereconomics/other

Additional Figures

Figure 1: Interbank Spreads

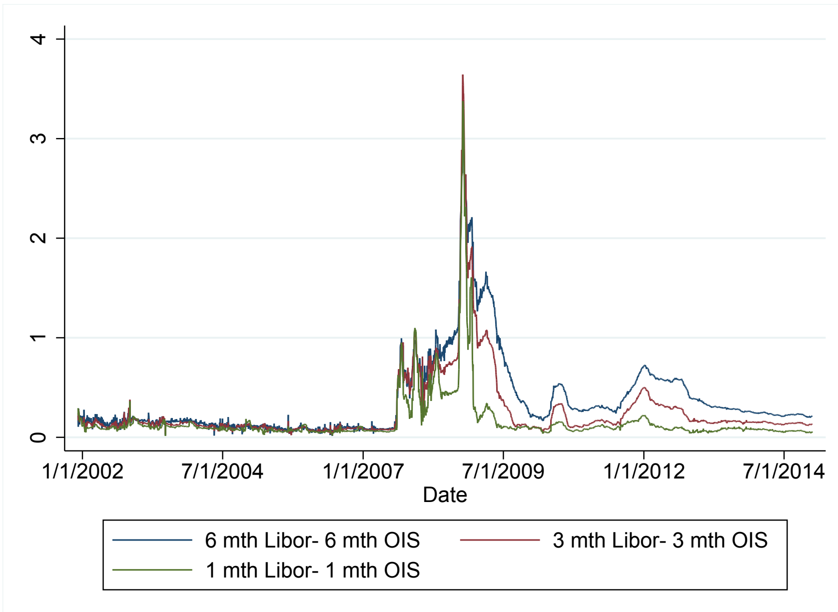
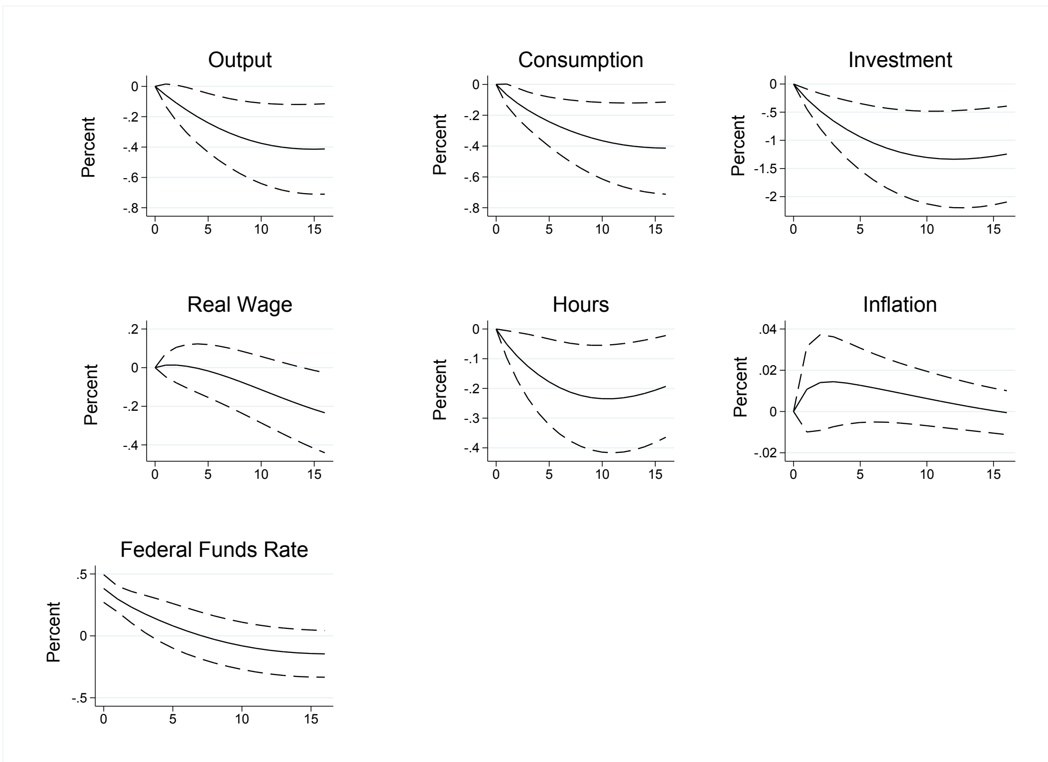
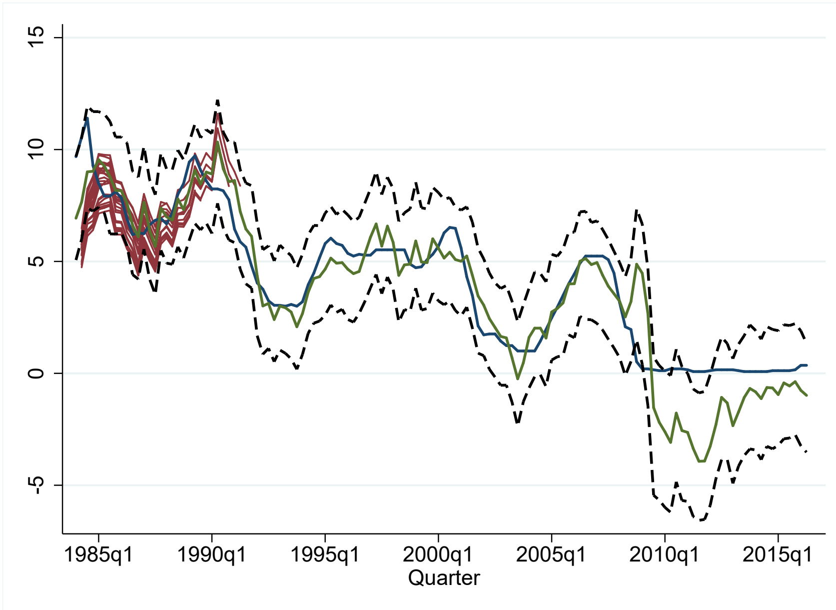


Figure 2: Baseline VAR Impulse Responses to a Federal Funds Rate Shock



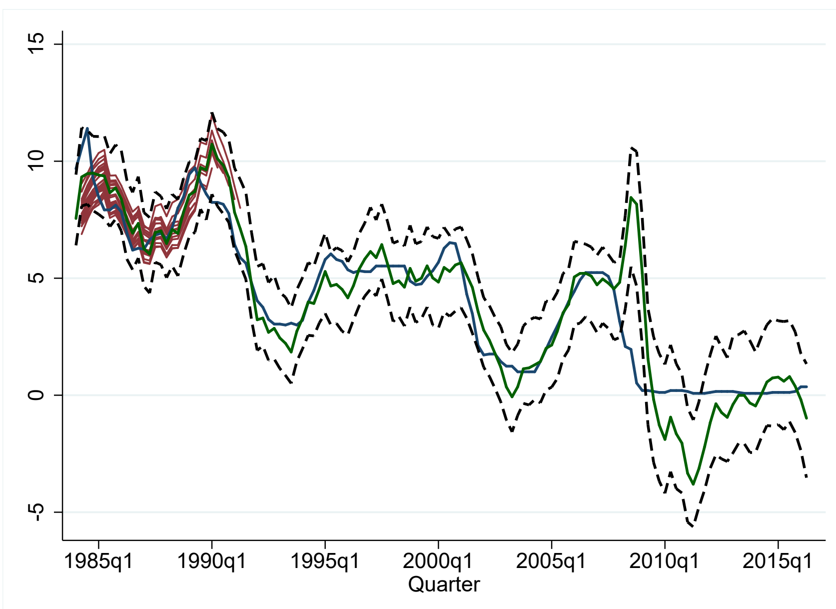
Bootstrapped one standard error bands using Cholesky identification, 1984:Q1-2008:Q3 sample used for shadow rate estimation.

Figure 3: Implied Monetary Stance, Filtered Out-of-Sample Forecasts Iterated Backwards with Baseline VAR



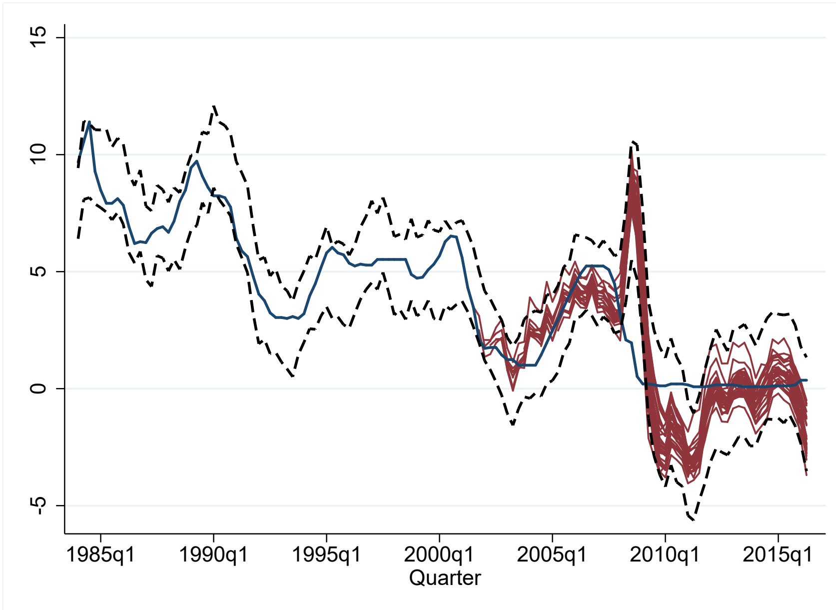
Blue line: federal funds rate. Red lines: out-of-sample state estimates. Dashed black lines: 1984Q1-2008:Q3 baseline state estimate one standard error band.

Figure 4: Implied Monetary Stance, Smoothed Out-of-Sample Forecasts Iterated Backwards with Baseline VAR



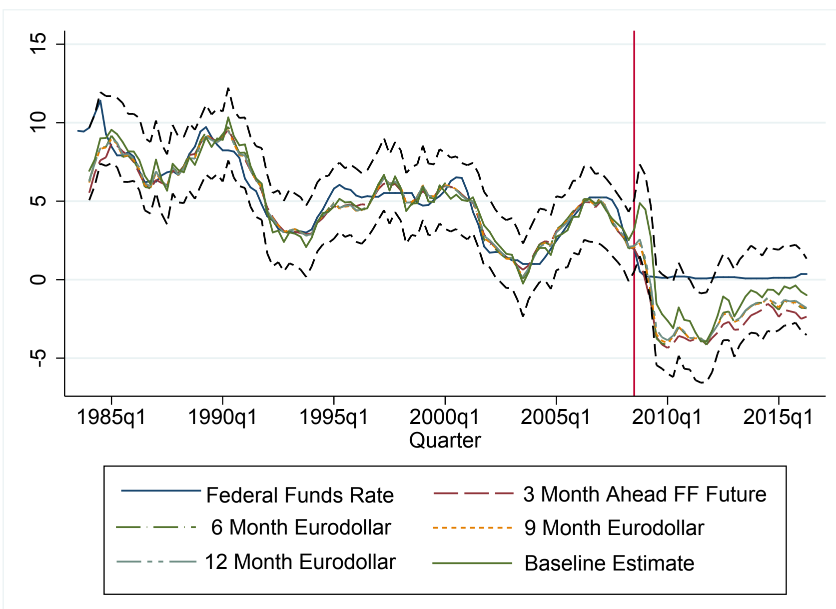
Blue line: federal funds rate. Red lines: out-of-sample state estimates. Dashed black lines: 1984Q1-2008:Q3 baseline state estimate one standard error band.

Figure 5: Implied Monetary Stance, Smoothed Out-of-Sample Forecasts Iterated Forward with Baseline VAR



Blue line: federal funds rate. Red lines: out-of-sample state estimates. Dashed black lines: 1984Q1-2008:Q3 state estimate one standard error band.

Figure 6: Implied Monetary Stance, VAR Identified with External Instruments and Endogenously Estimated State Equation



Each estimate is using a VAR identified with the noted instrument. Dashed black lines: 1984Q1-2008:Q3 baseline state estimate one standard error band.