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**Levent Altinoglu**

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# The Origins of Aggregate Fluctuations in a Credit Network Economy

Levent Altinoglu\*

*Federal Reserve Board of Governors*

## **Abstract**

I show that inter-firm lending plays an important role in business cycle fluctuations. I first build a tractable network model of the economy in which trade in intermediate goods is financed by supplier credit. In the model, a financial shock to one firm affects its ability to make payments to its suppliers. The credit linkages between firms propagate financial shocks, amplifying their aggregate effects by about 30 percent. To calibrate the model, I construct a proxy of inter-industry credit flows from firm- and industry-level data. I then estimate aggregate and idiosyncratic shocks to industries in the US and find that financial shocks are a prominent driver of cyclical fluctuations, accounting for two-thirds of the drop in industrial production during the Great Recession. Furthermore, idiosyncratic financial shocks to a few key industries can explain a considerable portion of these effects. In contrast, productivity shocks had a negligible impact during the recession.

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\*Email: levent.altinoglu@gmail.com. Phone: +1 617-817-6669. Address: Constitution Ave & 20th St NW, Washington, DC 20551. Website: <https://sites.google.com/site/leventaltinoglu/>  
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## 1. INTRODUCTION

The recent financial crisis and ensuing recession have underscored the importance of external finance for the real economy. Generally, firms borrow extensively from their suppliers in the form of trade credit, or delayed payment terms provided by suppliers to their customers. Indeed, trade credit is the single most important source of short-term external financing for firms in the US, yet it has been largely absent from the business cycle literature. In this paper, I show that trade credit plays an important role in business cycle fluctuations.

To this end, I introduce trade credit into a network model of the economy and show that the credit interlinkages between firms can generate large fluctuations from small financial disturbances. I then use the framework to empirically shed light on the sources of observed fluctuations in the US. Accounting for the effects of the interlinkages between firms turns out to be crucial for identifying the sources of aggregate fluctuations in the US. In particular, I find financial shocks to be a key driver of cyclical fluctuations, particularly during the Great Recession. In contrast, productivity shocks play only a minor role.

The credit linkages I consider take the form of trade credit relationships between non-financial firms, in which a firm purchases intermediate goods on account and pays its supplier at a later date. Trade credit accounts for more than half of firms' short-term liabilities and more than one-third of their total liabilities in most OECD countries. In the US, trade credit was three times as large as bank loans and fifteen times as large as commercial paper outstanding on the aggregate balance sheet of non-financial corporations in 2012.<sup>1</sup> These facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the financial distress of their customers.<sup>2</sup> A number of studies - including Jacobson

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<sup>1</sup> See the Federal Reserve Board Flow of Funds.

<sup>2</sup> For example, the government bailout of the US automotive industry in 2008 was precipitated

and von Schedvin (2015), Boissay and Gropp (2012), Raddatz (2010), and Kalemli-Ozcan et al. (2014) - have found that firm- and industry-level trade credit linkages are an important channel through which financial shocks are transmitted from firms to their suppliers. Yet the macroeconomic implications of trade credit have been largely overlooked in the literature.

I consider an economy similar to that of Bigio and La'O (2016), in which firms are organized in a production network and trade intermediate goods with one another. Limited enforcement problems require firms to make cash-in-advance payments to their suppliers before production takes place. As a result, firms face cash-in-advance constraints on their production. However, I assume that firms can delay part of these payments by borrowing from their suppliers. To obtain this credit, a firm can credibly pledge some fraction of its future cash flow to repay its suppliers. Whereas in Bigio and La'O (2016), the tightness of financial constraints is fixed exogenously, trade credit in this framework implies that the tightness of constraints fluctuates endogenously with the cash-flow of downstream firms. As a result, credit linkages generate rich network effects by which financial shocks propagate through the economy.

When one firm is hit with an adverse shock to its cash on hand, there are two channels by which other firms in the economy are affected. First is the standard input-output channel: the shocked firm cuts back on production, reducing the supply of its good to its customers.<sup>3</sup> Second is a new credit linkage channel which tightens the financial constraints of upstream firms. That is, the shocked firm reduces the up-front payments it makes to its suppliers. Being more cash-constrained, these suppliers may be forced to cut back on their own production, and reduce the up-front payments to their own suppliers, etc. In this way, credit interlinkages propagate the firm-level financial shock across the economy. This additional upstream propagation turns out to be a powerful mechanism by which the financial conditions

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by an acute shortage of liquidity, which came about largely due to extended delays in payment for goods already delivered.

<sup>3</sup>This channel has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2016).

in the economy are tightened endogenously.<sup>4</sup>

Next, I evaluate the quantitative relevance of the mechanism. In order to overcome the paucity of data on trade credit, I first construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis (BEA). I thus produce a map of the credit network of the US economy at the three-digit NAICS level of detail, with which I calibrate the model.

Counterfactual exercises reveal the amplification mechanism to be quantitatively significant, amplifying financial shocks by 30-40 percent. Furthermore, the aggregate impact of an idiosyncratic (industry-level) financial shock depends jointly on the underlying structures of the credit and input-output networks of the economy. Based on this analysis, certain industries emerge as systemically important to the US economy, such as auto manufacturing and petroleum and coal manufacturing. Moreover, the systemic importance of an industry is closely related to the intensity of trade credit use by its largest trading partners. Thus, credit interlinkages play a significant role in exacerbating the effects of financial shocks and amplifying their aggregate effects.

In the empirical part of the paper, I use this theoretical framework to investigate which shocks drive cyclical fluctuations once we account for the network effects created by credit interlinkages. Accounting for these effects turns out to be crucial for identifying the sources of business cycle fluctuations in the US. My framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks. To address these issues, I use two methodological approaches.

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data. To do this, I first construct

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<sup>4</sup> This upstream propagation of financial shocks is consistent with industry- and firm-level evidence on trade credit, including in Jacobson and von Schedvin (2015) and Raddatz (2010), who find that the trade credit relationships between firms transmit financial distress from firms to their suppliers.

quarterly measures of bank lending based on data from Call Reports collected by the FFIEC. I then augment an identified VAR of macro and monetary variables with this measure of bank lending, and with the excess bond premium of Gilchrist and Zakrajsek (2012), which reflects the risk-bearing capacity of the financial sector. I construct financial shocks as changes in bank lending which arise from orthogonalized innovations to the excess bond premium.<sup>5</sup> For productivity shocks, I use the quarterly, utilization-adjusted changes in total factor productivity (TFP) estimated by Fernald (2012).

Feeding these estimated shocks into the model, I find that, before 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production. However, during the Great Recession, productivity shocks had virtually no adverse effects on industrial production - in fact, they actually *mitigated* the downturn. On the other hand, two-thirds of the peak-to-trough drop in aggregate industrial production during the recession can be accounted for by financial shocks, with the remainder unaccounted for by either shock. By propagating financial shocks across firms and exacerbating the financial conditions in the economy, trade credit linkages thus amplified the drop in aggregate industrial production during the recession.

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic, industry-specific shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster, Sarte, and Watson (2011), using data on the output and employment growth of US industrial production industries. I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use standard factor methods to decompose each of these shocks into an aggregate component and an idiosyncratic component.

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<sup>5</sup> I construct the measure of bank lending in such a way that changes in the demand for bank lending are largely netted out. Therefore, changes in my measure of bank lending mostly reflect supply-side changes.

Through variance decomposition I show that, while the idiosyncratic component of productivity shocks can account for a fraction of aggregate volatility before 2007, it played virtually no role during the Great Recession. Rather, nearly three-quarters of the drop in industrial production during the recession can be accounted for by aggregate financial shocks. In addition, the remainder can be accounted for by idiosyncratic financial shocks to a few systemically important industrial production industries - namely the oil and coal, chemical, and auto manufacturing industries. Furthermore, the credit and input-output linkages between industries played a significant role in propagating these industry-level shocks across the economy.

The broad picture which emerges from these two empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession.<sup>6</sup> Thus, when we account for the amplification mechanism of trade credit and input-output interlinkages, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

### *Related Literature*

This paper contributes to several strands of the literature. A growing literature examines the importance of network effects in macroeconomics, including Acemoglu et al. (2012), Shea (2002), Dupor (1999), Horvath (2000), Acemoglu et al. (2015), Baqaee (2016), and Carvalho and Gabaix (2013). These abstract away from financial frictions. The seminal work of Acemoglu et al. (2012) show that the network structure of an economy can generate aggregate fluctuations from idiosyncratic, firm-level shocks, using a frictionless input-output model of the economy.

The notable work of Bigio and La'O (2016) explores the interaction between financial frictions and the input-output structure of an economy by introducing financial constraints to the Acemoglu et al. (2012) economy. However, they do not explicitly model any credit relationships between firms. As a result, the financial

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<sup>6</sup> While shocks to aggregate TFP have long been relied upon as a principal source of cyclical fluctuations, the lack of direct evidence for such shocks has raised questions about their empirical viability.

constraints that firms face are fixed exogenously, and do not become tighter in response to shocks. Luo (2016) embeds an input-output structure in the framework of Gertler and Karadi (2011), with a role for trade credit. However, trade credit linkages do not propagate shocks across the economy *per se*.<sup>7</sup> Kiyotaki and Moore (1997) study theoretically how a shock to a firm in a credit chain can cause a cascade of defaults in a partial equilibrium framework. Gabaix (2011), Foerster et al. (2011), and Stella (2014) evaluate the contribution of idiosyncratic shocks to aggregate fluctuations, the latter two using a structural factor approach. Jermann and Quadrini (2012) evaluate the importance of financial shocks by explicitly modeling the tradeoff between debt and equity financing. Ramirez (2017) uses an input-output model to explain certain empirical features of asset prices.

The rest of the paper is organized as follows. In section I, I introduce the stylized model and derive the analytical results. In sections II-IV, I generalize the production network structure, discuss the construction of my proxy for credit flows and calibration, and summarize the quantitative results. In section V, I perform the empirical analyses.

## 2. STYLIZED MODEL: VERTICAL PRODUCTION STRUCTURE

In this section, I build intuition with a simple model. The stylized nature of the production structure of the economy permits closed-form expressions for equilibrium variables. I will later generalize both the production structure and preferences.

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are  $M$  goods, each produced by a continuum of competitive firms with constant returns-to-scale in production. We can therefore consider each good as being produced by a representative, price-taking firm. Each good can be consumed

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<sup>7</sup> In that paper, credit linkages only affect the interest rate that the bank charges firms. As such, all network effects are due to input-output linkages, as in Bigio and La'O (2016).

Figure 1: Vertical Production Chain



by the household or used in the production of other goods.

The representative household supplies labor competitively to firms and consumes a final consumption good. It has preferences over consumption  $C$  and labor  $N$  given by  $U(C, N)$ , and a standard budget constraint, where  $w$  denotes the competitive wage earned from working, and  $\pi_i$  the profit earned by firm  $i$ .

$$U(C, N) = \log C - N \quad C = wN + \sum_{i=1}^M \pi_i \quad (1)$$

There are  $M$  price-taking firms who each produce a different good, for now arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with  $i = M$  denoting the producer of the final good.

The production technology of firm  $i$  is Cobb-Douglas over labor and intermediate goods, where  $x_i$  denotes firm  $i$ 's output,  $n_i$  its labor use, and  $x_{i-1}$  its use of good  $i-1$ ,  $z_i$  denotes firm  $i$ 's total factor productivity,  $\eta_i$  the share of labor in its production (and  $\eta_1 = 1$ ), and  $\omega_{i,i-1}$  the share of good  $i-1$  in firm  $i$ 's total intermediate good use (equal to 1 for now). Let  $p_s$  denote the price of good  $s$ .

$$x_i = z_i n_i^{\eta_i} x_{i-1}^{\omega_{i,i-1}(1-\eta_i)} \quad (2)$$

Limited enforcement problems between firms create a need for *ex ante* liquidity to finance working capital. The household cannot force any debt repayment. Therefore, firm  $i$  must pay the full value of wage bill,  $wn_i$ , up front to the household before production takes place. In addition, each firm  $i$  must pay for some portion of its intermediate goods purchases  $p_{i-1}x_{i-1}$  up front to its supplier. Thus, firms are required to have some funds at the beginning of the period before any revenue is realized.

Firm  $i$  can delay payment to its supplier by borrowing some amount  $\tau_{i-1}$  from

its supplier, representing the trade credit loan given from  $i - 1$  to  $i$ . In addition, I assume each firm has some other exogenous source of funds  $b_i$ , which I interpret as a cash loan from an outside bank, for ease of exposition. The net payment that firm  $i - 1$  receives from its customer at the beginning of the period is therefore  $p_{i-1}x_{i-1} - \tau_{i-1}$ . Firm  $i$ 's cash-in-advance constraint takes the form

$$\underbrace{wn_i}_{\text{wage bill}} + \underbrace{p_{i-1}x_{i-1} - \tau_{i-1}}_{\text{CIA payment to supplier}} \leq \underbrace{b_i}_{\text{bank loan}} + \underbrace{p_ix_i - \tau_i}_{\text{CIA from customer}} . \quad (3)$$

Thus, the cash that firm  $i$  is required to have in order to employ  $n_i$  units of labor and purchase  $x_{i-1}$  units of intermediate good  $i - 1$ , is bounded by the amount of cash that firm  $i$  can collect at the beginning of the period. Note that trade credit appears on both sides of the constraint.

Firms face borrowing constraints on the size of loans they can obtain from their suppliers and the bank. Firm  $i$  can obtain the loan  $b_i$  from the bank at the beginning of the period by pledging a fraction  $B_i$  of its total end-of-the-period revenue  $p_ix_i$ , and a fraction  $\alpha$  of its accounts receivable  $\tau_{i+1}$ , where  $\alpha \in (0, 1]$ .<sup>8</sup>

$$b_i \leq B_i p_i x_i + \alpha \tau_i \quad (4)$$

Firms are also constrained in their ability to obtain trade credit from their suppliers. In particular, firm  $i$  can credibly pledge a fraction  $\theta_i$  of its end-of-the-period revenue to repay its supplier.

$$\tau_{i-1} \leq \theta_i p_i x_i \quad (5)$$

Underlying this constraint is a contracting problem outlined in the seminal work on trade credit contracts in Burkart and Ellingsen (2004), in which a moral hazard problem between supplier and producer is managed by pledging the producer's

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<sup>8</sup> I will later show that  $\alpha$  parameterizes the degree of substitutability between cash and bank credit. Nevertheless, the collateralizing accounts receivables for borrowing, sometimes referred to as factoring, is prevalent. See Mian and Smith Jr. (1992) and Omiccioli (2005).

receivables as collateral for the trade credit loan.<sup>9</sup>

How do firms choose how much to lend to their customers and borrow from their suppliers? Recall that representative firm  $i$  is actually comprised of a continuum of competitive firms with CRS production. Perfect competition amongst these suppliers forces them to offer their customers the maximum amount of trade credit permitted by the constraint. This result holds even when these suppliers are cash-constrained in equilibrium.<sup>10</sup> (I leave the proof of this to an online appendix.) While this pins down the supply of trade credit, I study firms' demand for trade credit below.

We can re-write firm  $i$ 's cash-in-advance constraint as

$$wn_i + p_{i-1}x_{i-1} \leq \underbrace{\chi_i p_i x_i}_{\text{liquid funds}} \quad (6)$$

where

$$\chi_i \equiv \underbrace{\frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i}}_{\text{debt/revenue ratio}} + \underbrace{1 - \frac{\tau_i}{p_i x_i}}_{\text{cash/revenue ratio}}. \quad (7)$$

Therefore, a firm's expenditure on inputs is bounded by the amount of funds it has at the beginning of the period. The variable  $\chi_i$  describes the tightness of firm  $i$ 's cash-in-advance constraint, and will play a key role in the mechanism of the model. The tightness of a firm's cash-in-advance constraint is comprised of the firm's debt-to-revenue ratio and its cash-to-revenue. These describe how much of the firm's revenue is financed by debt, and how much of its revenue is collected as a cash-in-

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<sup>9</sup> Constraints of this form have found empirical support in studies using micro-data on trade credit, such as Petersen and Rajan (1997). That paper also find that the asset holdings of a firm are not a significant predictor of purchases made on credit as a ratio over assets, suggesting that physical capital holdings are not used as collateral for trade credit, as they might be for financing longer-term investment projects.

<sup>10</sup>These results are supported by micro-level evidence on trade credit: competition amongst suppliers is often sufficiently high that they are forced to offer their customers extended payment terms, even when they are cash-constrained. See, for instance, Barrot (2015).

advance payment, respectively. Notice that  $\chi_i$  is decreasing in  $\frac{\tau_i}{p_i x_i}$ , the amount of  $i$ 's output sold on credit: the more credit that  $i$  gives its customer, the less cash it collects at the beginning of the period.

Firm  $i$  chooses its input purchases  $n_i$  and  $x_{i-1}$ , and how much trade credit to borrow  $\tau_{i-1}$ , to maximize its profits subject to its cash-in-advance constraint. (Recall that because of perfect competition, the firm takes its trade credit lending  $\tau_i$  as given.)

$$\begin{aligned} \max_{n_i, x_{i-1}, \tau_{i-1}} \quad & p_i x_i - w n_i - p_{i-1} x_{i-1} \\ \text{s.t.} \quad & w n_i + p_i x_{i-1} \leq \chi_i(\tau_{i-1}) p_i x_i \end{aligned} \quad (8)$$

$$\tau_{i-1} \leq \theta_i p_i x_i \quad (9)$$

Denote by  $\tau_{i-1}^*$  firm  $i$ 's choice of how much trade credit to borrow from its supplier. I show in online appendix O.A1 that if firm  $i$ 's cash-in-advance constraint (8) is binding in equilibrium, then it borrows the maximum amount of trade credit offered by its supplier, pinning down  $\tau_{i-1}^* = \theta_i p_i x_i$ . For much of this paper, I consider this more interesting case in which firms are constrained in equilibrium.<sup>11</sup>

If firms are constrained in equilibrium, we can re-write the tightness  $\chi_i$  of a firm's constraint using firms' binding borrowing constraints to replace  $\tau_i$  and  $b_i$ .

$$\chi_i = \underbrace{\frac{B_i + \theta_i}{p_i x_i}}_{\text{debt/revenue ratio}} + \underbrace{1 - (1 - \alpha)\theta_{i+1} \frac{p_{i+1} x_{i+1}}{p_i x_i}}_{\text{cash/revenue ratio}} \quad (10)$$

Crucially, equation (4) shows that  $\chi_i$  is an *equilibrium object* - it is an endogenous variable which depends on the firm's forward credit linkage  $\theta_{i+1}$  and the revenue of its customer.<sup>12</sup> Hence, changes in the price of its customer's good affect the tightness of firm  $i$ 's cash-in-advance constraint.<sup>13</sup> Here, the endogeneity of  $\chi_i$  will

<sup>11</sup>Nevertheless that (9) binds in equilibrium is not crucial for the qualitative results, and may in fact understate the quantitative results.

<sup>12</sup> Notice that the firm's debt-to-revenue ratio is fixed, because firms collateralize their end-of-period revenue for borrowing.

<sup>13</sup>This a key difference with Bigio and La'O (2016), in which the tightness of each firm's cash-in-advance is an exogenous parameter because there is no inter-firm lending.

be a critical determinant of how the economy responds to shocks.

Firm  $i$ 's optimality conditions equate the ratio of expenditure on each type of input with the ratio of their share of production. I show in online Appendix O.A1 that firm  $i$ 's cash-in-advance constraint (3) binds in equilibrium if and only if  $\chi_i < 1$ . Combining the first order conditions with the cash-in-advance constraint yields the optimality conditions below.<sup>14</sup>

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}, \quad p_{i-1} = \phi_i \omega_{i,i-1} (1 - \eta_i) \frac{p_i x_i}{x_{i-1}} \quad (11)$$

Here,  $\phi_i \equiv \min \{1, \chi_i\}$  describes firm  $i$ 's shadow value of funds.<sup>15</sup>  $\phi_i$  is strictly less than one if and only if firm  $i$ 's cash-in-advance is binding in equilibrium. Equation (5) says that, if binding, the cash-in-advance constraint inserts a wedge  $\phi_i < 1$  between the marginal cost and marginal benefit of each input, representing the distortion in the firm's input use created by the constraint. A tighter cash-in-advance (lower  $\chi_i$ ) corresponds to a greater distortion, and lower output. Through  $\chi_i$ ,  $\phi_i$  *endogenously* depends on shadow value funds of downstream firms  $\phi_{i+1}$ , reflecting that firms' constraints are interdependent due to trade credit.

Note that there are two types of interlinkages between firms: input-output linkages, represented by input shares  $\omega_{i,i-1}$  in production; and credit linkages, represented by the borrowing limits  $\theta_i$  between firms. Each of these interlinkages will play a different role in generating network effects from shocks.

## 2.1. Equilibrium

I close the model by imposing labor and goods market clearing conditions  $N = \sum_{i=1}^M n_i$  and  $C = Y \equiv x_M$ .

*Definition:* An equilibrium is a set of prices  $\{p_{i \in I}, w\}$ , and quantities  $x_i, n_i, \tau_{i \in I}$  that (i) maximize the representative household's utility, subject to its budget constraint; (ii) maximize each firm's profits subject to its cash-in-advance, bank bor-

<sup>14</sup>Since  $\tau_{i-1}$  is important only insofar as it affects the tightnesses of firms' constraints, it shows up in firm  $i$ 's first order conditions only through  $\phi_i$ .

<sup>15</sup>More precisely, the shadow value of funds of firm  $i$  is given by  $\frac{1}{\phi_i} - 1$ .

rowing, and supplier borrowing constraints; and (iii) clear goods markets and the labor market.

Equilibrium aggregate output in the economy is determined by each firm's production function and financial constraint. To see this, let  $\bar{Y}$  denote the aggregate output that would prevail in a frictionless input-output economy (à la Acemoglu et al. (2012)), given by  $\bar{Y} \equiv \prod_{i=1}^M \tilde{\eta}_i \tilde{\eta}_i z_i \tilde{\omega}_i$ .<sup>16</sup> Define aggregate liquidity in the economy as  $\bar{\Phi} \equiv \prod_{i=1}^M \phi_i^{\sum_{j=1}^i \tilde{\eta}_j}$ , an aggregation of all firm's shadow value of funds. And since the production structure of the economy is simply a supply chain, the share of firms  $i-1$ 's good in firm  $i$ 's production is  $\omega_{i,i-1} = 1$  for all  $i$ . Then an analytical expression for equilibrium aggregate output, derived in online appendix O.A1, shows output to be log-linear in  $\bar{Y}$  and the aggregate liquidity in the economy.

$$Y = \bar{Y} \bar{\Phi} \quad (12)$$

Intuitively, (12) says that equilibrium aggregate output is constrained by aggregate liquidity - the funds available to all firms to finance working capital at the beginning of the period. Note that if all firms are unconstrained, then  $\bar{\Phi} = 1$  and  $Y = \bar{Y}$ . If one firm  $i$  is constrained, aggregate output depends on how its constraint affects the supply of intermediate good  $i$  for all downstream firms, given by  $\sum_{j=1}^i \tilde{\eta}_j$ .<sup>17</sup> To summarize, firms' financial constraints distort production in a way which depends on the underlying structures of the credit and input-output networks of the economy.

## 2.2. Aggregate Impact of Firm-Level Shocks

I now examine how the economy responds to firm-level financial shocks and productivity shocks. I model a *financial shock* to firm  $i$  by a change in  $B_i$ , the fraction of firm  $i$ 's revenue that the bank will accept as collateral for the bank loan. This is a reduced-form way to capture a reduction in the supply of bank credit to

<sup>16</sup>Here,  $\tilde{\omega}_i \equiv \prod_{j=i+1}^M \omega_{j,j-1}$  denotes firm  $i$ 's share in total intermediate good use, and  $\tilde{\eta}_i \equiv \eta_i \tilde{\omega}_i$  denotes firm  $i$ 's share of labor in aggregate output.

<sup>17</sup>Note that the credit network of the economy - i.e. the set  $\{\theta_i\}_{\forall i \in I}$  - shows up implicitly in (12) through each  $\phi_i$ .

firm  $i$ , and represents an exogenous tightening in firm  $i$ 's financial constraint.<sup>18</sup>

If firm  $i$  is unconstrained in equilibrium, a marginal financial shock  $dB_i$  has no effect on its production - the firm has deep pockets and can absorb the shock. However, if the firm is constrained, then it is forced to reduce production as it can no longer finance as many inputs with up front payments. In addition to this direct effect, there are two types of network effects by which the shock affects other firms in the economy: input-output channel and the credit linkage channel.

*Network Effects: Standard Input-Output Channel:* Through the first channel, which I call the *standard input-output channel*, the shock propagates through input-output interlinkages, increasing firms' input costs. This is the standard channel analyzed in the input-output literature, including Acemoglu et al. (2012) and Bigio and La'O (2016). The reduction in firm  $i$ 's output increases the price  $p_i$  of good  $i$ . This acts as a supply shock to the customer downstream (firm  $i + 1$ ), who is now faced with a higher unit cost of its intermediate good. In response, firm  $i + 1$  cuts back on production, which causes the  $p_{i+1}$  to increase, etc. Thus, as a result of the shock to firm  $i$ , all firms downstream experience a supply shock to their intermediate goods, and cut back on production. This amplifies the shock because as firms reduce production, they cut back on employment which, in turn, reduces the wage and household consumption.<sup>19</sup> In addition, the shock travels upstream as suppliers adjust their output to respond to the fall in demand for their intermediate goods.

*Network Effects: Credit Linkage Channel:* There is also a new, additional channel of transmission - which I call the *credit linkage channel* - which describes how

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<sup>18</sup>In the general network model in the following section, each firm sells some portion of its output directly to the household. In this setting, one could alternatively interpret the fall in  $B_i$  as a failed payment by final consumer. In either case, these are idiosyncratic shocks to the firm's liquid funds such that  $\frac{dX_i}{dB_i} > 0$ , and are not well-represented by a change in its productivity or technology.

<sup>19</sup> This channel is ultimately driven by the input specificity in each firm's production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good  $i$  in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

the financial constraints of upstream firms are tightened endogenously in response to the shock.

Recall that when firm  $i$  cuts back on production, the price  $p_i$  of its good rises. This increases the collateral value of its future cash flow, allowing it to delay payment for a larger fraction of its purchase from supplier  $i - 1$ .<sup>20</sup> As a result, supplier  $i - 1$ 's cash/revenue ratio falls, meaning the fraction of its revenue collected as up front payment falls. This tightens its cash-in-advance constraint - i.e.  $\chi_{i-1}$  falls.<sup>21</sup>

$$\chi_{i-1} \downarrow \equiv \underbrace{B_{i-1} + \theta_{i-1}}_{\text{debt/revenue ratio}} + \underbrace{1 - \frac{\tau_{i-1}}{p_{i-1}x_{i-1}}}_{\text{cash/revenue ratio } \downarrow} \quad (13)$$

Thus, with less cash on-hand, the supplier  $i - 1$  is now faced with a tighter financial constraint itself. The supplier may therefore be forced to reduce production further, and thereby pass the shock to its own suppliers and customers. (This continues up the chain of firms). In this manner, the initial effect of the shock is amplified as upstream firms experience tighter financial conditions.

But why doesn't firm  $i - 1$  reduce the trade credit loans it makes in order to increase its cash holdings and relax its own constraint? Recall that representative firm  $i - 1$  consists of a continuum of firms, and that perfect competition forces them to offer the maximum trade credit, even when they are themselves constrained.<sup>22</sup> Note also that  $\alpha$  mitigates the transmission, allowing firm  $i - 1$  to partially offset the lost up-front cash payments with a larger bank loan. Thus,  $\alpha$  parameterizes the substitutability between cash and bank credit.

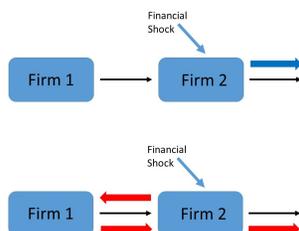
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<sup>20</sup>This is true even though the volume of trade credit  $\tau_{i-1}$  may actually fall in response to the shock.

<sup>21</sup> More precisely, there are three effects of the shock  $d B_i$  on  $\chi_{i-1}$ . Recall from (10) that firm  $i - 1$ 's cash/revenue ratio depends inversely on  $\frac{p_i x_i}{p_{i-1} x_{i-1}}$ . First, the shock increases  $p_i$ , as discussed above. Second, the fall in firm  $i$ 's output increases the ratio  $\frac{x_i}{x_{i-1}}$  due to the decreasing returns to  $x_{i-1}$ . And third, the fall in  $i$ 's demand reduces the price  $p_{i-1}$  of good  $i - 1$ . All of these effects unambiguously reduce  $\chi_{i-1}$ .

<sup>22</sup>This mechanism is in line with strong empirical evidence that firms in financial distress reduce the up-front payments they make to their suppliers, thereby transmitting the financial distress to their suppliers. See Jacobson and von Schedvin (2015), Raddatz (2010), and Boissay and Gropp (2012).

Figure 2: Feedback Effect



*Feedback Effect Created by Transmission Channels:* Importantly, the two transmission channels produce a feedback effect which amplifies the shock, as illustrated in Figure 2. Suppose that firm 2 is hit with an adverse financial shock, causing its cash-in-advance constraint to become tighter, and forcing it to cut back on production. The standard input-output channel, represented by the blue arrow, transmits the shock downstream in the form of a higher intermediate good price.

In addition, the credit linkage channel tightens the constraints of upstream firms, as firm 2 reduces the cash-in-advance payments it makes to its supplier. With a tighter financial constraint the supplier is forced to reduce production, which feeds back to firm 2 again in the form of higher price for the intermediate good. Thus, firm 2 is hit not only with a tighter financial constraint, but also endogenously higher input costs, (which it passes on to its customer, and so on). In this manner, the two channels interact to create a feedback loop represented by the red arrows, which exacerbates the initial shock.<sup>23</sup>

### 2.3. Impact of Firm-Level Shock on Aggregate Output

In light of these mechanisms, I now derive analytical expressions for how a firm-level financial shock affects aggregate output, and show that the credit network effects amplify the shock in a manner which depends on the structure of the credit linkages.

From (12), I decompose the change in aggregate output due to a financial shock to firm  $i$  into components reflecting the standard input-output channel and the credit

<sup>23</sup>A firm-level financial shock in my model therefore is isomorphic to an *aggregate* financial shock to all firms in a model with fixed constraints, e.g. Bigio and La'O (2016).

linkage channel.

$$\frac{d \log Y}{dB_i} = \sum_{j=1}^M \bar{v}_j \frac{d \log \phi_j}{dB_i} \quad (14)$$

Here, the terms  $\frac{d \log \phi_j}{dB_i}$  capture the credit linkage channel, and reflect how the financial shock to firm  $i$  affects the shadow value of funds of every other firm  $j$  in the network. The terms  $\bar{v}_j$  capture the standard input-output channel, and map these changes in each  $\phi_j$  into aggregate output. ( $\bar{v}_j \equiv \sum_{k=1}^j \tilde{\eta}_k$  depends on the share of labor in aggregate output of each firm.) This decomposition will allow me to quantify the aggregate effects of each channel later on.

In an economy without the credit linkage channel, such Bigio and La'O (2016), each  $\phi_j$  is fixed so that  $\frac{d \log \phi_j}{dB_i} = 0$  for all  $j \neq i$ . In words, financial constraints would not respond endogenously to a shock. Therefore, (14) would reduce to  $\frac{d \log Y}{dB_i} = \bar{v}_i$ .

However, credit network effects *amplify* the effects of the firm-level financial shock on aggregate output. This is because  $\frac{d \log \phi_j}{dB_i} \geq 0$  and therefore  $\frac{d \log Y}{dB_i} = \sum_{j=1}^M \bar{v}_j \frac{d \log \phi_j}{dB_i} > \bar{v}_i$  (proved in online appendix O.A2). In addition, the credit network effects  $\frac{d \log \phi_j}{dB_i}$  are weakly increasing in  $\theta_{jk}$  for all firms  $i, j$ , and  $k$ . Thus, the aggregate impact of the financial shock depends on the location of firm  $i$  within the networks, and the strength of input-output and credit linkages between firms.

#### 2.4. Impact of Firm-Level Productivity Shock on Aggregate Output

Now consider a productivity shock to firm  $i$ , represented by a fall in  $i$ 's total factor productivity (TFP)  $z_i$ . It turns out that, due to Cobb-Douglas production, each firm's cash/revenue ratio, and therefore the tightness of their constraint  $\phi_j$ , is independent of the productivity of firms  $z_i$ .<sup>24</sup> As a result, while the standard input-output channel amplifies the productivity shock just as in Acemoglu et al. (2012), the credit linkage channel does not.

*Summary of Theoretical Results:* To summarize, the credit linkages between firms create a multiplier effect which amplifies the aggregate effects of firm-level

<sup>24</sup>Acemoglu, Akcigit, and Kerr (2015) argue that Cobb-Douglas is a good approximation for production at the industry level.

shocks. The aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

### 3. GENERAL MODEL

To capture more features of the economy, I now allow for an arbitrary network structure so that each firm may trade with and borrow from or lend to any other firm in the economy.

I assume that each of the  $M$  goods can be consumed by the representative household or used in the production of other goods. The household's total consumption  $C$  is Cobb-Douglas over the  $M$  goods, and it has GHH preferences.<sup>25</sup>

$$U(C, N) = \frac{1}{1-\gamma} \left( C - \frac{1}{1+\varepsilon} N^{1+\varepsilon} \right)^{1-\gamma}, \quad C \equiv \prod_{i=1}^M c_i^{\beta_i} \quad (15)$$

Here,  $\varepsilon$  and  $\gamma$  respectively denote the Frisch and income elasticity of labor supply. The household maximizes its utility subject to its budget constraint (1). This yields optimality conditions which equate the ratio of expenditure on each good with the ratio of their marginal utilities, and the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j}, \quad N^{1+\varepsilon} = C \quad (16)$$

Each firm can trade with all other firms. Firm  $i$ 's production function is again Cobb-Douglas over labor and intermediate goods.

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left( \prod_{j=1}^m x_{ij}^{\omega_{ij}} \right)^{1-\eta_i} \quad (17)$$

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<sup>25</sup> Quantitatively similar results hold for preferences which are additively separable in aggregate consumption  $C$  and labor  $N$ .

Here,  $x_i$  denotes firm  $i$ 's output and  $x_{ij}$  denotes firm  $i$ 's use of good  $j$ . Since  $\omega_{ij}$  denotes the share of  $j$  in  $i$ 's total intermediate good use, I assume  $\sum_{j=1}^M \omega_{ij} = 1$  so that each firm has constant returns to scale. The input-output structure of the economy can be summarized by the matrix  $\Omega$  of intermediate good shares  $\omega_{ij}$ .<sup>26</sup>

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & & \\ \vdots & & \ddots & \\ \omega_{M1} & & & \omega_{MM} \end{bmatrix}$$

Note that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence,  $\Omega$  describes how firms would trade with each other in the absence of frictions.

Each firm's cash-in-advance constraint takes the same form as in the stylized model, with the exception that each firm has  $M$  suppliers and  $M$  customers instead of just one of each.  $\tau_{is}$  denotes the trade credit loan that firm  $i$  receives from each of its suppliers  $s$ .

$$wn_i + \underbrace{\sum_{s=1}^M (p_s x_{is} - \tau_{is})}_{\text{net CIA payment to suppliers}} \leq b_i + \underbrace{p_i x_i - \sum_{c=1}^M \tau_{ci}}_{\text{net CIA received from customers}} \quad (18)$$

Firm  $i$  faces borrowing constraints with each of its suppliers, to which it can pledge fractions  $\theta_{is}$  of its future cash flow to repay the loans. Each firm can also borrow  $b_i$  from the bank by pledging  $B_i$  of its revenue and  $\alpha$  of its accounts receivable  $\sum_{c=1}^M \tau_{ci}$ .

$$\tau_{is} \leq \theta_{is} p_i x_i \quad b_i \leq B_i p_i x_i + \alpha \sum_{c=1}^M \tau_{ci} \quad (19)$$

As before, competition amongst suppliers in industry  $s$  forces them to offer the maximum trade credit permitted by the limited enforcement problem, so that the trade

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<sup>26</sup>This is simply a generalization of the input-output structure in the stylized model. In that case, the  $\Omega$  would be given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.

credit borrowing constraint always binds when industries are cash-constrained in equilibrium. The structure of the credit network between firms can be summarized by the matrix of  $\theta_{ij}$ 's.

$$\Theta \equiv \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & & \\ \vdots & & \ddots & \\ \theta_{M1} & & & \theta_{MM} \end{bmatrix}$$

Plugging the binding borrowing constraints into (18) yields a constraint on  $i$ 's total input purchases, where  $\chi_i$  describes the tightness of  $i$ 's cash-in-advance constraint.

$$wn_i + \sum_{s=1}^M p_s x_{is} \leq \chi_i p_i x_i \quad (20)$$

Just as in the stylized version,  $\chi_i$  is an equilibrium object, where firm  $i$ 's cash/revenue ratio depends on the prices  $p_c$  of its customer's goods and its forward credit linkages  $\theta_{ci}$ .

$$\chi_i = \underbrace{B_i + \sum_{s=1}^M \theta_{is}}_{\text{debt/revenue ratio}} + 1 - \underbrace{(1 - \alpha) \sum_{c=1}^M \theta_{ci} \frac{p_c x_c}{p_i x_i}}_{\text{cash/revenue ratio}} \quad (21)$$

Firms choose labor and intermediate goods to maximize profits subject to their cash-in-advance constraint. Again, firm  $i$ 's constraint inserts a wedge  $\phi_i$  between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \quad x_{ij} = \phi_i (1 - \eta_i) \omega_{ij} \frac{p_i}{p_j} x_i \quad (22)$$

where the wedge  $\phi_i = \min\{1, \chi_i\}$  is determined by the firm's shadow value of funds. Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^M n_i \quad x_i = c_i + \sum_{c=1}^M x_{ci} \quad (23)$$

The equilibrium conditions of this generalized model take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model. When taking this model to industry-level data, the calibration of the model will allow industries to differ in how financially constrained they are.

### *Relationship Between Firm Influence and Size*

A well-known critique of frictionless input-output models such as Acemoglu et al. (2012) is that the size of a firm, as measured by its share  $s_i$  of aggregate sales, is sufficient to determine the aggregate impact of a shock to sector  $i$ , and one does not need to know anything about the underlying input-output structure of the economy. Bigio and La'O (2016), however, show that this result breaks down when the economy has financial frictions. My model shows that when credit linkages between firms propagate shocks across the economy, the aggregate impact of an idiosyncratic shock depends also on the underlying structure of the credit network of the economy, summarized by the matrix  $\Theta$ .

## 4. QUANTITATIVE ANALYSIS

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

### *4.1. Mapping the US Credit Network*

Calibration of the trade credit parameters  $\theta_{ij}$  requires data on credit flows between industry pairs; but data on credit flows at any level of detail is scarce. To overcome

Figure 3: Constructing Proxy for Trade Credit Flows



this paucity of data, I construct a proxy for trade credit flows  $\tau_{ij}$  between industry pairs using industry-level input-output data and firm-level balance sheet data. I use input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the period 1997-2013. The BEA publishes annual input-output data at the three-digit NAICS level, at which there are 58 industries, excluding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to  $p_j x_{ij}$  in my model for every industry pair  $\{i, j\}$ . Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm’s total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample.

My strategy for constructing the proxy is illustrated in Figure 3. From the payables and receivables data, I observe how much, on average, firms in each industry have borrowed from all of their suppliers collectively, and lent to all of their customers collectively.<sup>27</sup> However, I do not observe how an industry’s stock of trade credit and debt breaks down across each of its suppliers and customers. Therefore, I combine the input-output data with the payables and receivables data to approximate the fraction of sales from firms in industry  $j$  to firms in industry  $i$  made on credit, on average, yielding a proxy for trade credit flows  $\tau_{ij}$  between each industry pair.

Many studies have found that large firms on average use trade credit less intensively than their smaller counterparts, presumably because they have greater access to other forms of financing.<sup>28</sup> Since the publicly-traded firms in the Compustat database tend to be large, my use of this data likely biases downward the extent of trade credit linkages between firms, and therefore potentially underestimates their quantitative importance in amplifying business cycles.

<sup>27</sup>The vast majority of accounts receivables and payables of US corporations consists of trade credit.

<sup>28</sup>See, for instance, Petersen and Rajan (1997).

## 4.2. Calibration

With the proxy for trade credit flows at hand, I calibrate the general model to match US data. I calibrate technology parameters  $\eta_i$  and  $\omega_{ij}$  to match the BEA input-output tables of the median year in my sample, 2005. The firm optimality conditions and CRS technology imply

$$\phi_i = \frac{wn_i + \sum_{j=1}^M p_j x_{ij}}{p_i x_i}. \quad (24)$$

The right-hand side of (12) is directly observable from the BEA's Direct Requirements table.

Looking through the lens of the model, the observed input-output tables reflect both technology parameters and distortions created by the financial constraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm  $i$ 's optimality conditions for each input and my calibrated  $\phi_i$ 's.

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \quad \omega_{ij} = \frac{p_j x_{ij}}{(1 - \eta_i) \phi_i p_i x_i} \quad (25)$$

Again the ratios  $\frac{wn_i}{p_i x_i}$  and  $\frac{p_j x_{ij}}{p_i x_i}$  are directly observable from the Direct Requirements tables for every industry  $i$  and  $j$ .

I calibrate the parameters  $\theta_{ij}$ , representing the credit linkages between industries  $j$  and  $i$ , to match my proxy of inter-industry trade credit flows  $\hat{\tau}_{ij}$  using industry  $i$ 's binding borrowing constraint.

$$\theta_{ij} = \frac{\hat{\tau}_{ij}}{p_i x_i} \quad (26)$$

Industry  $i$ 's total revenue  $p_i x_i$  is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005).

To calibrate  $B_i$ , the parameters reflecting the agency problem between firm  $i$  and the bank, recall the definition of  $\phi_i$  given by (11), which depends on the technology parameters (calibrated as described above) and the tightness  $\chi_i$  of each industry's cash-in-advance, where

$$\chi_i = B_i + \sum_{s=1}^M \theta_{is} + 1 - (1 - \alpha) \sum_{c=1}^M \theta_{ci} \frac{p_c x_c}{p_i x_i}. \quad (27)$$

The total revenue of each industry  $p_i x_i$  is observable from the Uses by Commodity tables, and  $\phi_i$  and  $\theta_{is}$  for all  $s$  were calibrated as described above. I therefore use (13) and (11) to back out  $B_i$  for each industry. Thus, the calibration of  $B_i$  ensures that  $\phi_i < 1$ , so that all industries are constrained to some degree in equilibrium.<sup>29</sup>

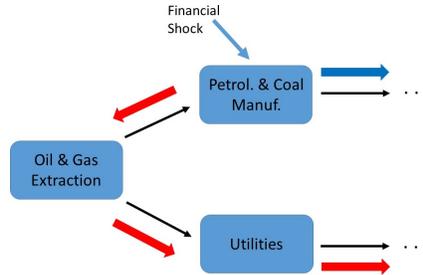
I follow the standard literature and set  $\varepsilon = 1$  and  $\gamma = 2$ , which represent the Frisch and income elasticity, respectively. I set  $\alpha = 0.2$  in my baseline calibration, but check the sensitivity of the quantitative results to varying  $\alpha$ .<sup>30</sup>

## 5. A QUANTITATIVE EXPLORATION OF THE MODEL

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and financial shocks.

In this more general setting, the presence of higher-order linkages means there are now additional spillover effects. To illustrate, consider Figure 4. The petroleum and coal manufacturing industry and the utilities industry are linked by a common supplier, the oil and gas extraction industry. Suppose that firms in petroleum and coal manufacturing experience tighter financial constraints, forcing some to reduce production, and raising the price of petroleum. This corresponds to the standard input-output channel represented by the blue arrow.<sup>31</sup> In the absence of the credit linkage channel of transmission, firms in the utilities industry will remain largely

Figure 4: Transmission Mechanism in the General Model



unaffected by the shock.

However, the shock causes petroleum and coal manufacturers to reduce the up front payments they make to their oil and gas suppliers. With tighter financial constraints, these suppliers reduce production, raising the price of oil and gas. As a result utilities firms pass these higher input costs downstream in the form of higher energy prices. These additional credit network effects further amplify the effects of the shock.

How large are these credit network effects likely to be? To answer this, I hit the US economy with an aggregate financial shock, and industry-level financial shocks, and measure the response in aggregate output to a log-linear approximation.

### 5.1. Response to an Aggregate Financial Shock

Suppose that the economy is hit with a one percent aggregate financial shock: each industry  $i$ 's cash-in-advance constraint is tightened by one percent.<sup>32</sup> Under my conservative, baseline calibration, I find that US GDP falls by 2.92 percent - a

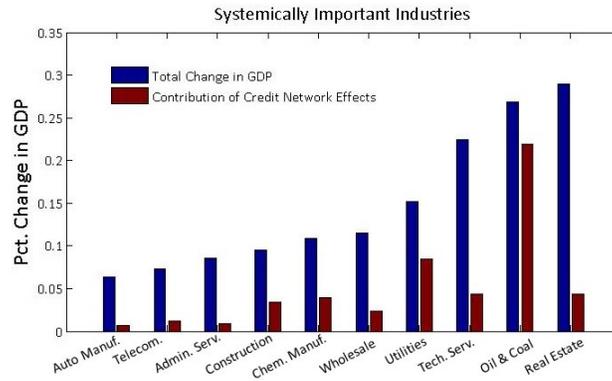
<sup>29</sup>Recall that for a credit supply shock to have any effect on industry  $i$ , a necessary condition is that the industry be constrained in equilibrium.

<sup>30</sup> Recall that  $\alpha$  is the fraction of receivables that industries can collateralize to borrow from the bank. Omiccioli (2005) finds that the median Italian firm in a sample collateralizes 20 percent of its accounts receivable for bank borrowing.

<sup>31</sup>In addition, the suppliers in the oil and gas industry will face lower demand from their customers, and reduce production accordingly.

<sup>32</sup> More specifically  $dB_i = 0.01$  for all industries  $i$ . This can be interpreted as a one percent fall in the aggregate supply of credit.

Figure 5:



Notes: This chart shows the ten most systemically important US industries based on the counterfactual drop in GDP in response to a 1 percent industry-level financial shock. The contribution of credit network effects is computed numerically. This exercise is done excluding financial industries.

large drop. Shutting off the credit linkage channel, I find that GDP falls by only 2.28 percent in response to the same aggregate shock. Thus, the credit network effects amplify the fall in GDP by about 30 percent. This is a conservative estimate of the quantitative relevance of the mechanism, given that the calibration uses data on only large, publicly-traded firms who use trade credit less intensively than other. Table 3 in the appendix reports the sensitivity of these results to the specification of  $\alpha = 0.2$ , the parameter controlling the substitutability of cash and bank credit.

## 5.2. Response to Industry-Level Financial Shocks

Next, I ask which industries are likely to be systemically important to the US economy, in light of these network effects. I measure the systemic importance of industry  $i$  by the how much GDP falls in response to a 1 percent financial shock to industry  $i$ . This industry-specific shock should be interpreted as an exogenous tightening of the financial constraints of at least some firms in the industry.

Figure 5 shows a bar graph of the ten most systemically important industries in the US, based on this exercise. For each industry  $i$ , the blue bars show the elasticity of GDP with respect to  $B_i$ , or the percentage change in GDP in response to a 1

percent financial shock to industry  $i$ .

The model implies that an industry-level financial shock can have a strong impact on US GDP. For example, although the technical services industry accounts for only 0.069 percent of US GDP, a one percent financial shock this industry causes a fall in GDP of 0.19 percent - a multiplier of 2.75. The red bars indicate the magnitude of the credit network effects of the shock.<sup>33</sup> These credit network effects contribute substantially to this amplification, accounting for between one-fifth to half of the fall in GDP in response to an industry-level shock, depending on the industry.

### 5.3. Mapping the Model to the Data

In order to map the model to the data, I extend the static model to be a repeated cross-section. Let  $X_t$ ,  $N_t$ ,  $B_t$ , and  $z_t$  denote the  $M$ -by-1 vectors of output growth, employment growth, financial shocks, and productivity for each industry respectively, in quarter  $t$ . The log-linearized model yields closed-form expressions for how the output and employment of each industry respond to financial and productivity shocks.

$$X_t = G_X B_t + H_X z_t \quad N_t = G_N B_t + H_N z_t \quad (28)$$

The  $M$ -by- $M$  matrices  $G_X$  and  $H_X$  ( $G_N$  and  $H_N$ ) map industry-level financial and productivity shocks, respectively, into output growth (employment growth), and capture the effects of input-output and credit interlinkages in propagating shocks across industries. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct the observed, quarterly cyclical fluctuations in the output  $\hat{X}_t$  and employment  $\hat{N}_t$  of US industrial production industries using data from the Federal

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<sup>33</sup> This is computed by subtracting the drop in GDP that occurs with credit linkage channel shut off, from the total drop in GDP. I shut off the credit linkage channel by imposing that financial constraints do not respond endogenously to financial shocks, i.e.  $\frac{d \log \phi_j}{d B_i} = 0$  for all  $j \neq i$ .

Reserve Board's Industrial Production Indexes, which includes data on the output growth of these industries, and the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, from which I observe the number of workers employed by each of these industries. At the three-digit NAICS level there are 23 such industries.<sup>34</sup> For each dataset, I take 1997 Q1 through 2013 Q4 as my sample period, and seasonally-adjust and de-trend each series. In the empirical analysis to follow, I use this data and the expressions (28) to decompose observed cyclical fluctuations into various components.

## 6. EMPIRICAL ANALYSES

In the empirical part of the paper, I use my theoretical framework to investigate which shocks drive observed cyclical fluctuations in the US, once we account for the network effects created by credit and input-output linkages between industries. The framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks.

To this end, I use two methodological approaches to identifying shocks. In the first, I identify shocks without imposing the structure of my model on the data. This permits a cleaner identification of financial and productivity shocks, and estimates a residual component of fluctuations which are not explained by either of these shocks. In the second approach, I identify shocks using a structural estimation of the model. While this attributes all fluctuations to financial and productivity shocks only, it allows for a decomposition between aggregate versus industry-level shocks.

### *6.1. First Method: Estimating Shocks without the Model*

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data - the identifying assumptions are

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<sup>34</sup>Hours worked is not directly available at this level of industry detail and this frequency.

completely independent of the model. An added advantage of this method is that it permits the estimation of a residual component of observed fluctuations - a component which is not explained by either shock. However, the shocks estimated using this method are assumed to be common to all industries.

*6.1.1. Estimating financial shocks* To identify credit supply shocks to the US economy, I estimate an identified VAR using a similar approach as Gilchrist and Zakrajsek (2011). To do this requires first constructing a measure of bank-intermediated business lending.

I construct a measure of aggregate business lending by US financial intermediaries using quarterly Call Report data collected by the FFIEC. To capture lending to the business sector, I use commercial and industrial loans outstanding and unused loan commitments - a cyclically-sensitive component of bank lending.<sup>35</sup> I thus construct a measure called the *business lending capacity* of the financial sector, as the sum of unused commitments and commercial and industrial loans outstanding in each quarter.<sup>36</sup>

To empirically identify credit supply shocks, I augment a standard VAR of macroeconomic and financial variables with the measure of business lending capacity, and the excess bond premium of Gilchrist and Zakrajsek (2012) - a component of corporate credit spreads designed to capture changes in the risk-bearing capacity of financial intermediaries.<sup>37</sup> The endogenous variables included in the VAR, ordered recursively, are: (i) the log-difference of real business fixed investment; (ii) the log-difference of real GDP; (iii) inflation as measured by the log-difference of

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<sup>35</sup>Gilchrist and Zakrajsek (2011) show that the contraction in unused loan commitments was concomitant with onset of the financial crisis in 2007, while business loans outstanding contracted only with a lag of about four quarters.

<sup>36</sup>Changes in business lending capacity mostly reflect supply-side changes. To see why, consider the following example. Suppose that a business draws down an existing line of credit it has with its bank. This is recorded as a fall in unused commitments, but reflects an increase in demand for credit rather than a contraction in the supply of credit. However, the loan is now recorded as an on-balance sheet commercial or industrial loan. Therefore, the fall in unused commitments is exactly offset by the increase in commercial and industrial loans outstanding, leaving bank lending capacity unchanged. So this measure of business lending capacity is largely unresponsive to firms drawing down their lines of credit.

<sup>37</sup>I thank Simon Gilchrist for kindly sharing the excess bond premium data.

the GDP price deflator; (iv) the quarterly average of the excess bond premium; (v) the log difference business lending capacity (vi) the quarterly (value-weighted) excess stock market return from CRSP; (vii) the ten-year (nominal) Treasury yield; and (viii) the effective (nominal) federal funds rate. The identifying assumption implied by this ordering is that stock prices, the risk-free rate, and bank lending can react contemporaneously to shocks to the excess bond premium, while real economic activity and inflation respond with a lag. I estimate the VAR using two lags of each endogenous variable.

To map the orthogonalized innovations in the excess bond premium into the financial shocks  $\tilde{B}_t$  of my model, I make use of the impulse response function of business lending capacity, and construct financial shocks as changes in the supply of bank lending which arise due to orthogonalized innovations in the risk-bearing capacity of the financial sector. Figure 6 plots the time series of this shock.

To allow for credit supply shocks to affect industries differentially depending on their dependence on external finance, I also load the financial shocks onto each industry based on a measure of the the industry’s external finance dependence, constructed according to Rajan and Zingales (1998).<sup>38</sup> However, the results reported hereafter are for financial shocks  $\hat{B}_t$  which load equally onto all industries.

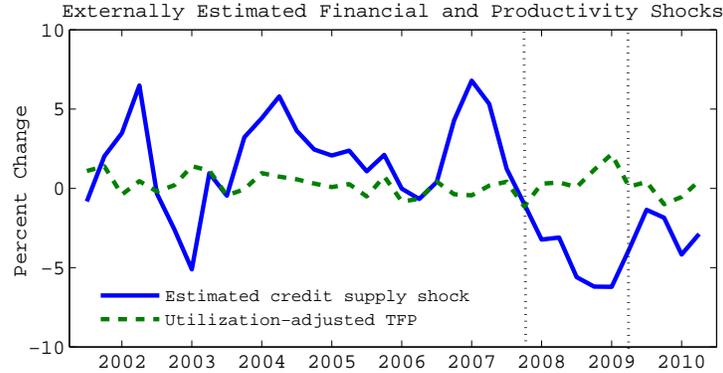
*6.1.2. Estimating productivity shocks* The Federal Reserve Bank of San Francisco produces a quarterly series on TFP for the US business sector, adjusted for variations in factor utilization, according to Fernald (2012). As such, this series is readily mapped into my model as an aggregate productivity shock  $\tilde{z}_t$ . Figure 6 plots time series for this productivity shock. Let  $\hat{z}_t \equiv \tilde{z}_t \vec{1}$  denote the  $M$ -by-1 vector of these shocks.

*6.1.3. Decomposing Observed Fluctuations in Industrial Production* With the estimated shocks at hand, I use log-linearized expression (28) to decompose observed

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<sup>38</sup>In this manner, I obtain a time-varying, industry-specific financial shock  $\tilde{B}_{it}$  which can be fed into the model. Although they varies across industries in any given quarter, these shocks to each industry are perfectly correlated across time, and so should not be interpreted as idiosyncratic shocks.

Figure 6:



Notes: This figure shows the series of quarter-to-quarter growth in utilization-adjusted TFP measure of Fernald (2012) and the credit supply shocks, estimated as changes in the business lending capacity of the financial sector which are due to orthogonalized innovations to the excess bond premium. Financial shocks were estimated using an identified VAR.. TFP data was obtained from the San Francisco Fed database.

cyclical fluctuations in industrial production into components coming from the financial shocks, productivity shocks, and a residual.

$$\hat{X}_t = G_X \hat{B}_t + H_X \hat{z}_t + \varepsilon_t \quad (29)$$

The residual  $\varepsilon_t$  is the component of these fluctuations which is unexplained by either of these shocks. I then feed these shocks into the model and perform a variance decomposition of aggregate industrial production.

The variance decomposition of output before 2007 is given in Table 1. In the period 2001 - 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production. The remaining half is unaccounted for by either type of shock.

However, the story is different for the Great Recession. Figure 7 plots the time series of aggregate industrial production during the Great Recession, as well as a simulation for each of its components.<sup>39</sup> These counterfactual series are constructed by feeding each of the estimated components through the model one at a time, and thus represents how aggregate industrial production would have evolved in the

<sup>39</sup>The time series for observed aggregate IP is constructed from the cyclical component of IP growth. It is constructed as an aggregate index of the observed industry-level growth rates.

Table 1: Variance Decomposition of IP: 2001Q4:2007Q3

	Share of Aggregate Volatility
Productivity Shocks	0.205
Financial Shocks	0.279
Residual	0.516

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the pre-recessionary period 2001 Q4 - 2007 Q3. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Financial shocks were estimated using an identified VAR, and capture quarterly credit supply shocks to the productive sector. Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database. The residual is the component of aggregate industrial production which is unexplained after these shocks are fed through the log-linearized model.

absence of other shocks, beginning in 2007 Q3.

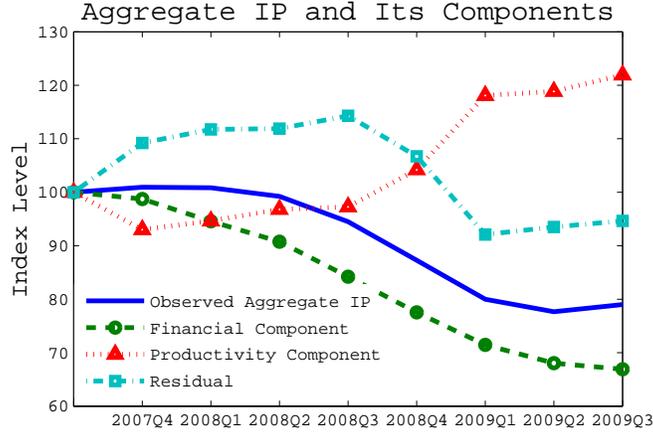
During the recession, productivity shocks had virtually no adverse effects on industrial production - in fact, they actually *mitigated* the downturn. Rather, financial shocks are the main culprit, accounting for two-thirds of the peak-to-trough drop in aggregate industrial production during the recession. The remaining one-third is not accounted for by either shock. Furthermore, the credit network of these industries played a quantitatively significant role during this period, amplifying the effects of the financial shocks by about 15% (i.e. adding 3.98 percentage points to the peak-to-trough drop in the financial component of aggregate industrial production).

## 6.2. Second Method: Structural Factor Analysis

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster et al. (2011)<sup>40</sup>, using data on the output and employment growth of US IP industries. The procedure involves two steps. I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use dynamic factor methods to decompose each of these shocks into an

<sup>40</sup>Foerster et al. (2011) allow only for productivity shocks in driving observed fluctuations.

Figure 7:



Notes: This figure shows the time series of aggregate industrial production and its components. Observed aggregate industrial production is an index constructed from the de-trended, seasonally-adjusted industry-level quarter-to-quarter growth rates in the output of the 23 industrial production industries at the three-digit NAICS level, obtained from FRB IP Indexes. Each of the other series depict counterfactual indexes constructed from the respective components of the observed series, beginning in 2007 Q3, and represent how aggregate IP would have evolved in the absence of other shocks. Financial shocks were estimated using an identified VAR. Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database.

aggregate component and an idiosyncratic, industry-specific component.<sup>41</sup>

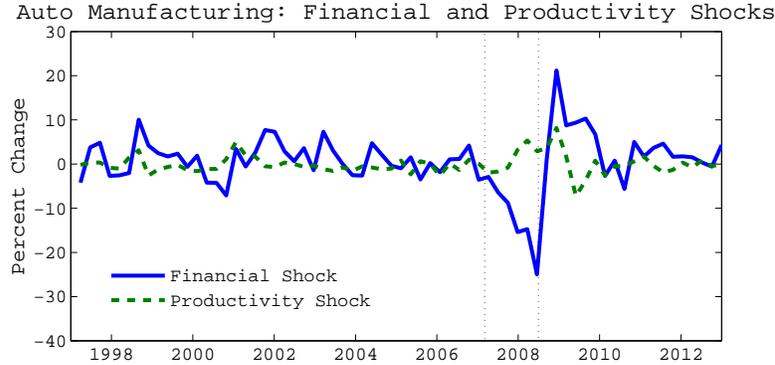
### 6.2.1. Step 1: Structural Estimation of Shocks

I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. To do this, recall that from equations (28) I have an exactly identified system of equations. Given the observations  $\hat{X}_t$  and  $\hat{N}_t$ , I then invert the system to back-out industry-level each quarter over my sample period 1997 Q1 to 2013 Q4. Denote by  $\check{B}_t$  and  $\check{z}_t$  the  $M$ -by-1 vectors of financial and productivity shocks estimated with this procedure in quarter  $t$ . And let  $Q \equiv H_X - G_X G_N^{-1} H_N$ .

$$\check{B}_t = G_N^{-1} (\hat{N}_t - H_N \check{z}_t) \quad \check{z}_t = Q^{-1} \hat{X}_t - Q^{-1} G_X G_N^{-1} \hat{N}_t \quad (30)$$

<sup>41</sup>As in Foerster et al. (2011), these latter shocks are specific to each industry, but idiosyncratic in the sense that they are uncorrelated across industries.

Figure 8:



Notes: This figure shows the quarterly time series of the productivity and financial shocks to the auto manufacturing industry over the sample period. Financial shocks are captured by percent changes in parameters  $B_t$  in the model, and thus represent exogenous tightening in the cash-in-advance constraint of an industry. Productivity shocks are changes in TFP. These shocks were estimated using the log-linearized model, and quarterly data on the employment and output growth of IP industries, obtain from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively.

Thus, I construct industry-level shocks as the observed fluctuations, filtered for the the network effects created by interlinkages. The model is able to separately identify these shocks because each type of shock has quantitatively differential effects on an industry's output and employment.<sup>42</sup>

Figure 8 shows the time series of the estimated financial and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

Between 2007 and 2009, the output and employment of industrial production industries took a sharp drop for a number of quarters. As illustrated in the figure, this contraction shows up in the model as an acute tightening in the financial constraints of these firms, reaching up to a 25 percent decline in a single quarter.<sup>43</sup>

### 6.2.2. Step 2: Dynamic Factor Analysis

Next, I use factor methods to decompose the financial and productivity shocks,  $\check{B}_t$  and  $\check{z}_t$ , into aggregate and idiosyncratic components.

<sup>42</sup>Namely, productivity shocks affect an industry's output relative to its employment through Cobb-Douglas production functions. On the other hand, financial shocks do not affect production functions, but tightens the cash-in-advance constraints.

<sup>43</sup>These features broadly hold across most industries in industrial production.

$$\check{B}_t = \Lambda_B F_t^B + u_t, \quad \check{z}_t = \Lambda_z F_t^z + v_t \quad (31)$$

Here,  $F_t^B$  and  $F_t^z$  are scalars denoting the common factors affecting the output and employment growth of each industry at quarter  $t$ , and are assumed to follow an AR(1) process; the residual components,  $u_t$  and  $v_t$ , are the idiosyncratic shocks. Hence, I estimate two dynamic factor models; one for the financial shocks  $\check{B}_t$  and one for the productivity shocks  $\check{z}_t$ .<sup>44</sup>

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks to the excess bond premium. The large aggregate financial shocks estimated by the structural factor analysis is broadly reflective of the severe credit crunch that occurred during this period.

### 6.2.3. Decomposing Observed Fluctuations in Industrial Production

To perform a variance decomposition of observed industrial production from 1997 Q1 to 2013 Q4, I follow the procedure described in Appendix A3. For the full sample period, aggregate volatility is about 0.19%.<sup>45</sup> The results are summarized in Table 2.

Before the Great Recession, aggregate volatility was driven primarily by aggregate financial shocks and idiosyncratic productivity shocks; aggregate financial shocks account for nearly a half of aggregate volatility. Nevertheless, idiosyncratic productivity shocks account for a quarter of aggregate volatility.

Furthermore, the credit network of industrial production industries amplified these shocks, accounting for nearly one-fifth of observed aggregate volatility.

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<sup>44</sup> I use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more information in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

<sup>45</sup>This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.

Table 2: Pre-Recession Composition of Agg. Vol.: 1997Q1:2006Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	0.365
Agg. Component	0.133
Idios. Component	0.232
Financial Shocks	0.635
Agg. Component	0.45
Idios. Component	0.185

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the period 1997 Q1 - 2006 Q4. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Shocks to industrial production industries were estimated using the structural factor analysis of these industries' quarterly output and employment growth, obtained from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively. The aggregate and idiosyncratic components were estimated by dynamic factor analysis of the industry-level financial shocks, where the common components are assumed to follow an AR(1) process.

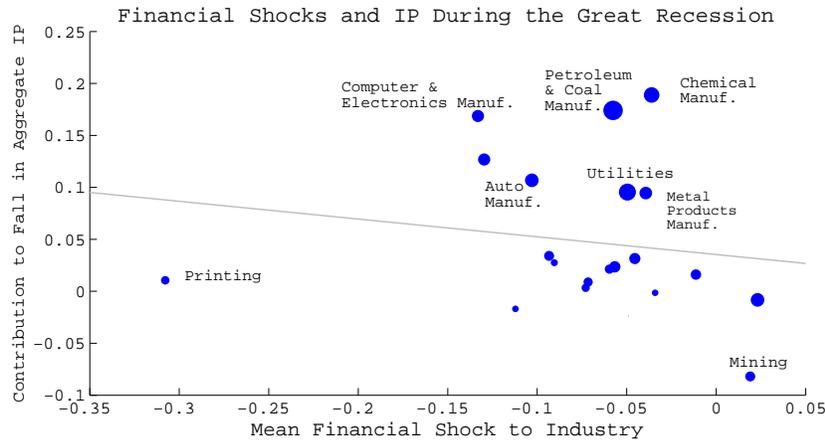
Aggregate financial shocks were the primary driver of the Great Recession. I perform an accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production over 2007Q4: 2009Q2 can be explained by each type of shock. I find that changes in productivity did not contribute to the decline in aggregate industrial production during the recession. In contrast, 73 percent of the drop in aggregate industrial production is due to an aggregate financial shock, and a sizable fraction of the the remainder can be accounted for by idiosyncratic financial shocks to the three most systemically important industries

Figure 9 depicts the relationship between industry-level financial shocks and an industry's contribution to aggregate output, for industrial production industries during the Great Recession.

Large financial shocks to a few systemically important industries can explain the bulk of the decline in aggregate industrial production during the Great Recession. In fact, idiosyncratic shocks to the oil and coal products manufacturing, chemical products manufacturing, and auto manufacturing industries account for about 9 percent of the decline (or one-third of the decline unaccounted for by aggregate shocks), despite comprising only about 25 percent of aggregate industrial production. This suggests that idiosyncratic financial shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

In contrast, both the aggregate and idiosyncratic components of productivity shocks were slightly positive during this period on average. As such, changes in

Figure 9:



Notes: This figure shows a scatter plot of industrial production industries by the mean quarterly financial shock to each industry during the recessionary period 2007 Q4 - 2009 Q2, and by each industry's contribution to the peak-to-trough fall in aggregate industrial production observed over this period. Each industry's spot is weighted by a measure of the industry's systemic importance to the US economy, computed using numerical simulations. The figure includes a least-squares line. Shocks were estimated using a structural factor approach and quarterly data on the employment and output growth of industrial production industries, obtained from the BLS Quarterly Census of Employment and Wages and the FRB Industrial Production Indexes, respectively. The shocks consist of both the aggregate and idiosyncratic components. An industry's contribution to the peak-to-trough drop in aggregate industrial production is computed by simulating the path of an index of each industry-level component of aggregate industrial production (i.e. how aggregate industrial production would evolve with shocks to exactly one industry) and computing the peak-to-trough change between 2007 Q4 and 2009 Q2.

productivity did not contribute to the decline in aggregate industrial production during the recession.

### 6.3. Take-Aways from the Two Empirical Analyses

The broad picture which emerges from these empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession. While much of the previous literature has relied on shocks to aggregate TFP drive the business cycle, the dearth of direct evidence for such shocks has raised concerns about their empirical viability. I have argued that the credit and input-output interlinkages of firms can create a powerful mechanism by which a shock to one firm's financial constraint propagates across the economy. The confluence of my empirical results suggest that once we account for these interlinkages, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

## 7. CONCLUSION

In this paper, I showed that inter-firm lending plays an important role in business cycle fluctuations. First, I introduced supplier credit into a network model of the economy and show that trade credit interlinkages can create a powerful amplification mechanism. To evaluate the model quantitatively, I constructed a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data.

Finally, I used the model to investigate which shocks drive the US business cycle when we account for the linkages between industries. To do so, I identified shocks both structurally and without the use of my model. Feeding these shocks through the model showed financial shocks to be a key driver of aggregate fluctuations, particularly during the Great Recession, and productivity shocks to play only a minor role. Thus, accounting for the role that credit and input-output interlinkages play helps to capture the empirical importance of financial shocks in US business cycle fluctuations.

## Appendix

### A1. Demand for Trade Credit

Firm  $i$ 's problem is to choose its input purchases and trade credit borrowing to maximize its profits, subject to its cash-in-advance constraint. Recall that competition amongst suppliers forces each firm to offer the maximum trade credit allowed by the borrowing constraint. Therefore, firm  $i$  takes  $\tau_i$  as given.

$$\begin{aligned} \max_{n_i, x_{i-1}, \tau_{i-1}} \quad & p_i x_i - w n_i - p_{i-1} x_{i-1} \\ \text{s.t.} \quad & w n_i + p_i x_{i-1} \leq \chi_i(\tau_{i-1}) p_i x_i \end{aligned}$$

$$\tau_{i-1} \leq \theta_i p_i x_i$$

Notice that in general, there is a tradeoff to taking more trade credit (i.e. to increasing  $\tau_{i-1}$ ). A higher  $\tau_{i-1}$  relaxes firm  $i$ 's cash-in-advance constraint, allowing it to purchase more inputs *ceteris paribus*. But a higher  $\tau_{i-1}$  may also tighten its

supplier's cash in advance constraint, causing the price of its intermediate good  $p_{i-1}$  to increase. Let  $\tau_{i-1}^*$  denote the optimal amount of trade credit borrowing. We can solve for optimal  $\tau_{i-1}^*$  separately from  $n_i$  and  $x_{i-1}$ . In particular, there are three relevant cases.

*Case 1)* If both  $i$  and  $i-1$  are unconstrained in equilibrium, then there is no tradeoff to firm  $i$  taking marginally more  $\tau_{i-1}$ . So there is a continuum of  $\tau_{i-1}$  between which firm  $i$  is indifferent: the set of all  $\tau_{i-1}$  such that both firm  $i$  and firm  $i-1$  are unconstrained in equilibrium, i.e.  $\chi_i, \chi_{i-1} < 1$ . Without loss of generality, we can take  $\tau_{i-1}^* = \min \tau_{i-1} \mid \chi_i, \chi_{i-1} < 1$ .

*Case 2)* If  $i$  is unconstrained in equilibrium, but  $i-1$  is constrained in equilibrium, then the tradeoff mentioned above applies. The optimal  $\tau_{i-1}$  will be the minimum such that  $i$ 's cash-in-advance constraint is not binding. Any  $\tau_{i-1} > \tau_{i-1}^*$  will further constrain supplier  $i-1$ , and therefore  $i$  will face a higher input price  $p_{i-1}$ . And any  $\tau_{i-1} < \tau_{i-1}^*$  will mean that firm  $i$  will be constrained in equilibrium and will have to reduce production.

*Case 3)* If firm  $i$  is constrained in equilibrium,  $\tau_{i-1}^*$  is the maximum allowable by the trade credit borrowing constraint:  $\tau_{i-1}^* = \theta_i p_i x_i$ . To see this, first recall that firm  $i$  actually consists of a continuum of identical firms with CRS production. Being constrained, each individual firm has an incentive to take the maximum amount of trade credit. They do not internalize the fact that, when all firms do this, they may increase the price  $p_{i-1}$  of inputs that they face.<sup>46</sup> Thus, in any equilibrium in which firm  $i$  is constrained (i.e. its cash-in-advance constraint is binding), the trade credit borrowing constraints bind and  $\tau_{i-1} = \theta_i p_i x_i$ .

Given its choice of  $\tau_{i-1}^*$ , firm  $i$  then chooses its inputs to solve the problem outlined in the text.

$$\begin{aligned} & \max_{n_i, x_{i-1}} p_i x_i - w n_i - p_{i-1} x_{i-1} \\ & s.t. \quad w n_i + p_i x_{i-1} \leq \chi_i (\tau_{i-1}^*) p_i x_i. \end{aligned}$$

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<sup>46</sup>Therefore, even if there is a  $\tilde{\tau}_{i-1} < \theta_i p_i x_i$  such that industry-wide profits will be higher (taking into account tradeoff of lower input price  $p_{i-1}$ ), these firms are unable to coordinate on that  $\tilde{\tau}_{i-1}$ .

Table 3:

$\alpha$	$P(\hat{\alpha} \leq \alpha)$	% Change in GDP	Credit Network Amplification
0	0.18	4.04%	77.2%
0.1	0.32	3.26%	43.0%
<b>0.2</b>	<b>0.5</b>	<b>2.92%</b>	<b>28.1%</b>
0.4	0.66	2.59%	13.6%
0.5	0.75	2.50%	9.6%
1	0.97	2.28%	0%

Notes: This table reports the results of the sensitivity analysis. Recall that  $\alpha$  is the fraction of accounts receivable that banks can collateralize to borrow from the bank, and controls the substitutability of cash and bank credit for firms in the model. The first column indicates the value of  $\alpha$  used. The second column yields the fraction of Italian firms which collateralizes less than  $\alpha$  of their receivables to borrow from banks, as estimated by Omiccioli (2005). The third column lists the total percentage change in GDP in response to a 1 percent financial shock to all US industries. The fourth column lists by how much the credit network effects amplify the drop in GDP in response to the shock. The bold row indicates the baseline calibration.

## A2. Sensitivity Analysis

In the quantitative analysis, I computed the change in GDP to a counterfactual one percent aggregate financial shock. Table 3 reports these results for different values of  $\alpha$ .

While the multiplier effect of the credit network indeed falls as  $\alpha$  approaches 1, credit network effects are quantitatively significant for reasonable values of  $\alpha$ .

## A3. Structural Factor Analysis: Aggregate Volatility

Assume the shocks  $B_t$  and  $z_t$  in (28) are composed of an aggregate and idiosyncratic components.

$$B_t = \Lambda_B F_t^B + u_t \quad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B \quad (32)$$

$$z_t = \Lambda_z F_t^z + v_t \quad F_t^z = \gamma_z F_{t-1}^z + \iota_t^z \quad (33)$$

Then letting  $\Sigma_{XX}$  denote the variance-covariance matrix of  $X_t$ , and  $\bar{s}$  a vector of industry shares of aggregate output, aggregate volatility (of output) is approximately

$$\sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s} = \bar{s}' G_X \Sigma_{BB} G_X' \bar{s} + \bar{s}' H_X \Sigma_{zz} H_X' \bar{s}. \quad (34)$$

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### O.A1. Solution for Stylized Model

I solve in closed form for aggregate output in the stylized (vertical) economy. I proceed recursively, beginning with the final firm in the chain, firm  $M$ .

#### **Firm $M$**

Recall that firm  $M$  collects none of its sales from the household up front (does not give the household any trade credit,  $\tau_M = 0$ ). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from  $M - 1$ , to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

$$\begin{aligned} & \max_{n_M, x_{M-1}, b_M, \tau_{M-1}} p_M x_M - w n_M - p_{M-1} x_{M-1} \\ \text{s.t. } & w n_M + p_{M-1} x_{M-1} \leq b_M + \tau_{M-1} + p_M x_M - \tau_M \\ & b_M \leq B_M p_M x_M + \alpha \tau_M \\ & \tau_{M-1} p_{M-1} x_{M-1} \leq \theta_M p_M x_M \end{aligned}$$

We can combine the constraints to re-write the problem.

$$\begin{aligned} & \max_{n_M, x_{M-1}, b_M, \tau_{M-1}} p_M x_M - w n_M - p_{M-1} x_{M-1} \\ \text{s.t. } & w n_M + p_{M-1} x_{M-1} \leq \chi_M p_M x_M \end{aligned}$$

where  $\chi_M = \frac{\tau_{M-1}^*}{p_M x_M} + B_M$ . Here,  $\tau_{M-1}^*$  denotes firm  $M$ 's choice of trade credit borrowing, based on the arguments given in Appendix A1. (Notice that when firm  $M$  is constrained,  $\chi_M = \theta_{M, M-1} + B_M$ ).

If firm  $M$  is unconstrained in equilibrium, then the optimality conditions equate the marginal cost of each type of input with the marginal revenue.

$$w = \eta_M \frac{p_M x_M}{n_M} \quad p_{M-1} = (1 - \eta_M) \frac{p_M x_M}{x_{M-1}} \quad (35)$$

Firm  $M$ 's expenditure in inputs is then

$$w n_M + p_{M-1} x_{M-1} = (\eta_M + (1 - \eta_M)) p_M x_M. \quad (36)$$

Therefore, firm M is then unconstrained in equilibrium if and only if its expenditure at its unconstrained optimum is less than its liquidity at this optimum.

$$p_M x_M < \chi_M p_M x_M \quad i.e. \quad \chi_M > 1 \quad (37)$$

If firm M is constrained in equilibrium, then its binding cash-in-advance pins down its level of output. The only choice left to make is how much labor to hire  $n_M$  versus how much intermediate goods  $x_{M-1}$  to purchase, given its level of output  $x_M$ . Because  $\chi_M$  is independent of M's choice of  $n_M$  and  $x_{M-1}$ , the problem of maximizing profits subject to the binding cash-in-advance is equivalent to minimizing its expenditure  $n_M + x_{M-1}$  subject to producing  $x_M$ . Thus, it solves the following cost-minimization problem.

$$\begin{aligned} & \min_{n_M, x_{M-1}} w n_M + p_{M-1} x_{M-1} \\ & s.t. \quad x_M = z_M n_M^{\eta_M} x_{M-1}^{(1-\eta_M)} \end{aligned}$$

Then firm M's optimality condition equates the ratio of expenditure on each input with the ratio of each input's share in production.

$$\frac{w n_M}{p_{M-1} x_{M-1}} = \frac{\eta_M}{(1-\eta_M)} \quad (38)$$

Using this, we can rewrite M's binding cash-in-advance as

$$w = \eta_M \chi_M \frac{p_M x_M}{n_M}.$$

Together, the constrained and unconstrained cases imply that we can write the optimality condition as

$$w = \phi_M \eta_M \frac{p_M x_M}{n_M}. \quad (39)$$

$\phi_M \equiv \min \{1, \chi_M\}$  represents the distortion in firm M's optimal labor usage due to its cash-in-advance. Financial frictions introduce wedge between firm's marginal benefit and cost of production. The wedge between these two objects is increasing in the tightness  $\chi_M$  of M's constraint, and decreasing in the returns-to-scale of firm M's production function. Given firm M's solution, we can continue recursively and solve firm M-1's problem.

## Equilibrium

Each other firm's problem is symmetric. Continuing recursively, I obtain the closed-form solution for each firm. To summarize, I have, for each firm  $i$   $w = \phi_i \eta_i \frac{p_i x_i}{n_i}$ . Market clearing conditions are given by  $C = Y \equiv x_M$  and  $N = \sum_{i=1}^M n_i$

Given the firm optimality conditions, we can write each  $n_i$  as a function of aggregate output  $x_M$

$$w n_i = p_M x_M \left( \prod_{j=i}^M \phi_j \right) \left( \prod_{j=i}^{M-1} \omega_{j+1,j} (1 - \eta_j) \right) \eta_i \quad (40)$$

The household's preferences and optimality conditions imply

$$w = \frac{V'(N)}{U'(x_M)} = x_M \quad (41)$$

Let good M be the numeraire. Combining (40) with (41) yields a closed-form expression for each firm's labor use.

$$n_i = \eta_i \prod_{j=i+}^M \omega_{j,j-1} (1 - \eta_j) \phi_j \quad (42)$$

By recursively plugging in the production functions into one another, we can obtain aggregate output as a function of the labor use of each firm, where  $\delta_{M-i} \equiv \prod_{j=0}^{i-1} (1 - \eta_{M-j})$ .

$$Y = \left( \prod_{i=0}^{M-1} z_{M-i}^{\delta_{M-i}} \right) \left( \prod_{i=0}^{M-1} n_{M-i}^{\eta_{M-i} \delta_{M-i}} \right) \quad (43)$$

Then combining (42) and (43) yields a closed-form expression for aggregate output (12).

### O.A2. Proof of Amplification

From the definitions of  $\chi_i$  and  $\phi_i$ , we have

$$\phi_i = \min \left\{ 1, \frac{1}{r_i} \left( B_i + \theta_{i,i-1} - \theta_{i+1,i} \frac{1}{\phi_{i+1} \omega_{i+1,i} (1 - \eta_{i+1})} \right) \right\}.$$

Here,  $r_i = 1$  denotes firm  $i$ 's returns-to-scale. It follows that

$$\frac{d\phi_{i-1}}{dB_i} = \begin{cases} \frac{1}{r_i} \frac{\alpha\theta_{i,i-1}}{\phi_i\omega_{i,i-1}(1-\eta_i)} > 0 & \text{if } \phi_{i-1} < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d\phi_j}{dB_i} = 0 \forall j > i \text{ and } \frac{d\phi_j}{dB_i} = \frac{1}{r_i} > 0 \text{ for } j = i.$$

Putting these cases together, we can write  $\frac{d\log\phi_j}{dB_i}$  for any  $j$ .

$$\frac{d\log\phi_j}{dB_i} = \begin{cases} \frac{1}{r_i} > 0 & \text{if } j = i \\ \frac{1}{\phi_j} \frac{1}{r_j} \frac{\theta_{kj}}{\phi_k\omega_{kj}(1-\eta_k)} \frac{d\phi_k}{dB_i} \geq 0 \forall k & \text{if } j < i \\ 0 & \text{otherwise} \end{cases}$$

It follows that  $\frac{d\log\phi_j}{dB_i} \geq 0$  and  $\frac{d}{d\theta_{ij}} \left( \frac{d\log\phi_j}{dB_i} \right) \geq 0$ .

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