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Voluntary Reserve Targets

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Abstract

This paper updates the standard workhorse model of banks' reserve management to include frictions inherent to money markets. We apply the model to study monetary policy implementation through an operating regime involving voluntary reserve targets (VRT). When reserves are abundant, as is the case following the unconventional policies adopted during the recent financial crisis, operating regimes based on reserve requirements may lead to a collapse in interbank trade. We show that, no matter the relative abundance of reserves, VRT encourage market activity and support the central bank's control over interest rates. In addition to this characterization, we consider (i) the impact of routine and non-routine liquidity injections by the central bank on market outcomes and (ii) a comparison with the implementation framework currently adopted by the Federal Reserve. Overall, we show that a VRT framework may provide several advantages over other frameworks.

Keywords: Monetary Policy, Reserve Targets, Money Markets

JEL classification: E42, E43, E44, E51, E52, E58

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1 Introduction

This paper considers a framework for monetary policy implementation termed Voluntary Reserve Targets (VRT). A version of this framework was first operated by the Bank of England (BoE) from 2006 to 2009, and the Federal Reserve (Fed) continues to consider VRT for its long-run framework for implementing monetary policy.[1] The specifics of implementation can have various implications for money markets, and old frameworks pose a number of difficulties. We show that VRT could overcome a number of these difficulties, improving over other frameworks.

Ten years on from the heart of the crisis, focus has shifted from the extraordinary measures taken by central banks around the world in response to the crisis to the design of steady-state operating procedures and frameworks for monetary policy implementation.[2] Monetary policy concerns a central bank's strategy for affecting the economy, such as setting the short term interest rate. The tactics employed to effect that strategy, however, comprise monetary policy implementation – the systems, methods, and operations by which a central bank intervenes in the economy.[3] Whatever framework for monetary policy implementation central banks ultimately adopt – and different jurisdictions will employ different frameworks based on their specific context – good frameworks allow for the effective and efficient exercise of policy while providing flexibility to accommodate changing financial conditions. Here, we argue that a VRT framework possesses these and other characteristics which make it worthy of consideration, alongside existing alternatives, for any central bank.

A VRT framework differs from traditional frameworks in that it does not rely on required reserves, but asks banks to commit to voluntary targets for their central bank deposits. A target-dependent non-linear interest rate schedule incentivizes banks to choose positive targets. This paper explores the operation of a VRT framework in a model of banks' reserve management with over-the-counter (OTC) trade, and is the first to provide a characterization of equilibrium target choices in this framework. Specifically, this paper shows that target choices depend directly on banks' expectations regarding the supply of central bank balances. Because targets track the supply of reserves, a VRT program incentivizes money market trade between banks. These features distinguish a VRT framework from other frameworks and make it particularly appealing in settings where interbank trade is desirable.

Indeed, in April 2008, a meeting of the FOMC considered potential operating frameworks

[1] See BoE (2006), FOMC (2016).

[2] This shift is reflected, for example, in the 2016 Jackson Hole conference, as well as the ongoing discussions concerning the draw downs of quantitative easing in the U.S. and Europe.

[3] For a simple introduction to monetary policy implementation with special reference to the US, see Ennis and Keister (2008). In the appendix, we provide further discussion of some experiences of different central banks around the world with special relevance for this paper.

in light of a change in law granting the Fed the authority to pay interest on reserves. In part due to the success of the BoE, all but 4 of the 17 Fed governors and presidents at the time expressed support for a focus on voluntary reserve targets in considerations for a future framework, with only one expressing a definite preference for another policy (FOMC (2008a)).[4] Despite this interest, there existed no formal modelling of the strategy that banks might follow in setting their targets, a potentially concerning uncertainty.[5] This paper resolves that uncertainty, providing a detailed characterization of banks' choice of targets, and their implications for money markets.

2 Summary of Results and Literature

In a VRT framework, banks commit in advance to a target, and then the central bank pays high interest on balances up to the target, low interest on balances exceeding the target, and charges a fee on shortages relative to the target.

With banks earning a low rate when overshooting and paying a penalty when undershooting their targets, banks are incentivized to set targets which reflect their expected end of day holdings. This simple intuition is preserved even when complicated by the risk introduced by customers' withdrawals and deposits throughout the day, as well as the risk of mismatch and the potential for hold-up in an over-the-counter (OTC) money market. For example, by raising targets, banks absorb any expected abundance of reserves. Hence, we say that VRT generates endogenous scarcity in the interbank market and drives trade in the money market. Endogenous scarcity and the behavior of targets drive a number of novel and important results.

First, as long as banks have a fairly good understanding of the factors impacting their reserve holdings, voluntary targets remove the need for the central bank to constantly adjust the supply of reserves in order to meet their interest rate objective. This reduces the administrative and operational costs of so-called fine tuning operations. Indeed, one way to think of a VRT framework is as a standard required reserves framework where, instead of adjusting supply to meet requirements, the requirements automatically adjust to supply.[6] Despite this

[4]In November 2016, the FOMC again discussed possibilities for its framework. The minutes reference VRT (FOMC (2016)), indicating its active consideration, but detailed transcripts remain classified.

[5]After Fed Staff presented a variety of options to the FOMC, including two variants of VRT, Governor Mishkin asked how banks would decide their targets. To this, staff responded "The determination of voluntary targets... is a really big issue..." In discussing the BoE, Fed staff reported that "They [the BoE] were extremely worried. They had no idea what they were going to get [in terms of targets]." See FOMC (2008a), 159.

[6]In the canonical model for policy implementation in the US, the central bank would require minimum reserves and open market operations to affect the interbank rate. By selling assets, the central bank could drain reserve balances from the economy, thus resulting in a scarcity of liquidity relative to the reserve

adjustment, and as expected in an OTC market, the model predicts dispersion in rates. But rates between banks are always bounded between the VRT framework's rates on excesses and shortages of reserves. Moreover, in a symmetric environment, the rate exactly averages to hit the middle of the corridor defined by the interest paid on reserves and the penalty charged for shortages.

Second, the level of targets does not yield useful information about banks' demand for reserves, as suggested by some authors (e.g. Clews et al. (2010)). During the night, reserve balances sit on the central bank's books and can not be used for any purpose. Uncertainty over payment flows coupled with overdraft penalties induce some precautionary holdings of reserves when the payment system is open, which allows transactions to be cleared and settled. Absent interest earnings or reserve requirements, banks would do their best to minimize their holdings when the payment system closes. But the supply of reserves is determined by the central bank, and the banking system must hold whatever quantity is supplied. In this sense – that there is no use for overnight reserves, per se – there is no natural notion of "fundamental" demand for reserves that might be revealed by targets. With VRT, banks simply target whatever they expect the central bank to supply, with an adjustment for risk. The desire of banks to target supply can create tenuous effects if, in turn, the central bank attempts to adjust supply to match targets – a policy pursued by the BoE. We formalize this situation in the context of our model, explaining the fact that aggregate targets decreased fairly steadily from the start of the BoE's VRT framework up until the beginnings of the crisis in 2007.

Third, VRT allow central banks the flexibility to design their balance sheet independently of their targets for interbank rates, and still encourage an active interbank market no matter the quantity of reserves held by an individual bank. An issue that materialized during the recent crisis concerned the ability of central banks to separately control the quantity of liquidity in the market and the interest rate. Central banks may have preferred, at least initially, to provide liquidity to financial institutions without significantly reducing interest rates to approach the zero lower bound. In a standard framework based on reserve requirements, increasing the supply of reserves would tend to decrease rates. A floor system, where the central bank pays interest on deposits while oversupplying the market with funds, pushes the market rate down to the rate offered by the central bank on deposits. Hence, under a floor system, a central bank can control market rates by varying its deposit rate. This works independently of the supply of reserves, so long as this supply exceeds the quantity required to exhaust demand for reserves due to requirements and precautionary motives, see Keister et al. (2008) and Goodfriend (2002). Because targets respond to the supply of

requirement, and in a higher price for such liquidity in the interbank market.

funds, VRT deliver the same separation of rates and quantities as a floor but at both high and low levels of aggregate or individual reserves. Moreover, a floor system can substantially reduce trade in the interbank market, while VRT always incentivize trade. Such trade, however, is not obviously desirable, and variants of floor systems where reserve supply just barely satiates demand could resolve other issues (Potter (2016)). If, as suggested by Bindseil (2016), the separation of rates from quantities and active trade are both useful characteristics of a monetary policy implementation framework, then VRT deserve consideration.

Fourth, VRT give a central bank flexibility in choosing its counterparties. With VRT, voluntary targets replace standard liability-based reserve requirements. This means that non-depository institutions (non-DIs) can meaningfully participate. For example, when adopting VRT, the BoE included building societies, an important class of money market participant. In the U.S., statute prevents the Fed from paying interest directly to non-DIs, such as the government sponsored enterprises (GSEs). This made rate control difficult in the recent crisis because of the important role played by the GSEs in US money markets (Bech and Klee (2011)) and lead to the establishment of an alternative deposit-like facility available to non-DIs termed the Overnight Reverse Repurchase (ON RRP) operation which succeeded in providing a firm floor under interbank rates (Armenter and Lester (2017)). By endogenously generating scarcity in the interbank market, however, VRT would indirectly affect non-DIs by creating competition for their deposits, lifting the rate even when the central bank is barred from paying interest to non-DIs. Indeed, depending on the relative quantity of funds held by non-DIs and a number of other parameters, our analysis shows that VRT may be less costly to the Fed, in terms of interest payments, than the current floor and ON RRP system and as effective at achieving a given federal funds rate.

To formalize some of these ideas, we consider the behavior of banks in response to interventions by the central bank. An anticipated monetary injection, one to which banks can adjust targets, has no effect on rates. The size of the central bank balance sheet and, in general, the provision of liquidity to the banking system are independent of the interbank rate. In contrast, unanticipated injections decrease rates. Our findings justify the BoE's abandonment of VRT during their rapid and uncertain balance sheet expansions in response to the crisis. But, now that most central banks around the world have largely stabilized policies, the results outlined above indicate that neither the BoE nor any other central bank would need wait until having normalized its balance sheet to implement VRT. In 2006 the BoE adopted VRT despite uncertainty over banks' target setting strategy because target levels were not its first order concern (BoE (2006), BoE (2015), Clews (2005a)).[7] Given

[7]The BoE has only de minimis reserve requirements, mainly for the purposes of funding its operations. This prevented it from implementing strategies to stabilize interest rates that depend on a significant re-

targets, however, the BoE could implement both averaging and tolerance bands, which reduced rate variability (Osborne (2016)). These strategies, period averaging and tolerance bands, are well understood, so we abstract from them in our analysis, focusing instead on the implications of target choices.[8]

Our model is firmly in the style of Poole (1968), with the inclusion of over-the-counter trade subject to a search and matching friction in the form of random matching with bargaining. A burgeoning literature explores different specifications of OTC trade in money markets, and finds that each specification can explain a number of otherwise puzzling patterns in these markets. See, e.g. Afonso and Lagos (2015), Armenter and Lester (2017), Bech and Klee (2011), Bech and Monnet (2016), and Ennis and Weinberg (2013). We focus on deriving the determinants of targets, their consequences for money markets, and the implications for central bank costs. Hence, we employ the simplest form of OTC model – single-shot random matching with bargaining. Like the papers just cited, we abstract from long-run macroeconomic effects in order to allow for rich interactions in the money market. A different literature, however, highlights the potential for the choice of monetary policy implementation framework to affect macroeconomic variables in unexpected ways. These include Berentsen and Monnet (2008a), Berentsen et al. (2014), Keister et al. (2015), Williamson (2015).

The next section details our model, followed by solution and characterization. These are followed by a section considering central bank interventions, another comparing VRT to the current US framework, and the conclusion. An appendix contains the less economically relevant derivations as well as a section providing more background on implementation in general.

3 Model

This section develops a model of the market for central bank reserves in the spirit of Poole Poole (1968). It is a one-period model where agents borrow and lend in response to shocks to their reserves holdings in order to maximize profit. The key difference from standard Poole-type models is in the money market which, here, involves OTC trades with random matching and bargaining.

quirement, like period averaging and tolerance bands.

[8] On reserve averaging, see Clouse and Dow (2002) and Whitesell (2006a). On tolerance bands, see Whitesell (2006a), Whitesell (2006b), and Armenter (2016). Whitesell (2006b) discusses a framework similar in spirit to VRT, but with substantial differences. Moreover, our primary contribution concerns the implications of endogenous targets on rates and other variables, while he derives choices given an exogenous rate distribution.

3.1 Agents and Preferences

A measure one of banks and non-banks populate the economy with a proportion q_N of non-banks and $1 - q_N$ of banks. In the U.S. these represent DIs and the GSEs. There is a single good – central bank reserves. Banks and non-banks have linear utility over their final reserve holdings. Banks are distinguished from non-banks in that (a) banks face uncertain reserve balances derived from demand deposits and other business lines, while non-banks have a deterministic endowment of reserves, and (b) the central bank is barred from offering reserve remuneration to non-banks, but can remunerate banks.[9] The central bank is a non-strategic technology that simply runs an exogenous deposit facility, described below.

Banks' uncertainty over their reserve balances derives from two shocks. One shock is early and can potentially be undone through borrowing and lending. The other is a late shock that comes after the market for reserves has closed, so too late to be undone. The early shock results in a bank holding some quantity of reserves D, with the population distribution denoted $F(D)$ and its support $\mathcal{D}$. The late shock, denoted ε, is additive to the first, occurring after the market for loans closes. To get smoothness, assume it follows a continuous distribution $G(\varepsilon)$ with support $\mathcal{E}$. Also assume pairwise independence across shocks and banks, and that a law of large numbers holds. We interpret both D and ε as payment shocks deriving from withdrawals and deposits that every bank must honor and settle. From a technical perspective, the first shock is used to generate heterogeneity among banks, which is necessary to obtain interbank trade in a match between two banks, and the second shock induces curvature in the payoff function of a bank, whose preferences are linear over end of period reserve holdings. Non-banks do not face this uncertainty and simply have access to a quantity C of reserves.

3.2 Remuneration Schedule and Timing

At the beginning of the day, the Central Bank announces a remuneration schedule and solicits announcements of reserve targets from banks. The remuneration schedule pays at the end of the day as a function of reserve holdings and targets. The schedule involves three rates including interest paid on reserves up to the target, i_T; interest paid on reserves held over the target, i_E; and a fee charged on shortages of reserves with respect to the target, ϕ. Specifically, the post-remuneration balances of a bank holding pre-remuneration deposits D with a target T is given by

[9] To be precise, in the U.S. non-banks are not legally allowed to hold accounts at the Federal Reserve, but they hold cash accounts at banks, which have accounts at the Federal Reserve. Thus, the cash owned by non-banks is ultimately in the form of central bank reserves for our purposes.

$$R(D,T) = \begin{cases} i_T T + i_E(D-T) & \text{if } D \geq T \\ i_T D - \phi(T-D) & \text{if } D < T. \end{cases}$$

Note that this can also be written in terms of targets and reserves in excess of targets as $R(D,T) = \tilde{R}(D-T) + i_T T$ with

$$\tilde{R}(x) = \begin{cases} i_E x & \text{if } x \geq 0 \\ (i_T + \phi)x & \text{if } x < 0. \end{cases}$$

Rewriting the remuneration schedule in this way makes clear that, given a target, the effective marginal interest rate is $i_T + \phi$ when short of the target, and i_E when in excess. This formulation also highlights the separability between targets and excess reserves under this remuneration schedule which, in turn, drives some key results below. While this schedule is not without loss of generality, it offers some tractability and follows the BoE's system and the Fed's considerations (Bank of England (2015), FOMC (2008b)).

The timing of events within the period follows several stages:

1. Each bank announces its target, T.
2. Each bank's early shock, D, realizes.
3. The money market opens, where F denotes the loan size and t the gross repayment.
4. Each bank's late shock, ε, realizes.
5. The central bank remunerates reserves.
6. The money market settles, i.e. a borrowing bank repays t to its lender.

4 Solving the Model

We solve the model backwards from the settlement stage to characterize value functions, trades and transfers, and finally target choices.

4.1 Late Shock

After trade, but before settlement, banks get a late shock to their reserve holdings ε, so, if they carried $\hat{D}$ balances after the trading round, they will hold $\tilde{D} = \hat{D} + \varepsilon$ when entering the settlement stage, where $\hat{D}$ comprises reserves holdings after the first liquidity shock, and any lending or borrowing. Thus $R(\hat{D}+\varepsilon, T)$ denotes the end of period reserves holdings, after remuneration. In advance of the late shock and ignoring any private loans, the expected value of a bank holding $\hat{D}$ balances is simply the quantity of reserves it holds after remuneration, and can be written as

$$\begin{aligned} V(\hat{D}, T) &= \mathbb{E}_{\varepsilon}[R(\hat{D}+\varepsilon, T)] \\ &= (i_T - i_E)T + i_E\hat{D} + (i_T - i_E + \phi)(\hat{D} - T)G(T - \hat{D}) \\ &\quad + (i_T + \phi)\int_{-\infty}^{T-\hat{D}} \varepsilon dG(\varepsilon) + i_E \int_{T-\hat{D}}^{\infty} \varepsilon dG(\varepsilon). \end{aligned}$$

It is easy to show that $V(\hat{D}, T)$ is strictly concave in $\hat{D}$ if the support of ε is unbounded.[10] This implies that the equilibrium contract traded in the money market is unique, as will be shown later.

Below it will be valuable to define the surplus generated by a given change in deposits F. As shown in the appendix, §8.2.1, the surplus from having additional reserves F for a bank with initial reserves D and target T is:

$$V(D+F, T) - V(D, T) = i_E \int_{T-D-F}^{T-D} (1 - G(\varepsilon))d\varepsilon + (i_T + \phi)\int_{T-D-F}^{T-D} G(\varepsilon)d\varepsilon.$$

The above equation shows that the value of borrowing F for a bank with initial reserves D and target T is the sum of two terms. First, the interest on excess reserves weighted by the average probability the bank holds reserves in excess of its target at the end of the period. Second, the effective interest when it is short of its target weighted by the average probability of being short.

Letting $\tilde{i} = i_T + \phi - i_E$ denote the width of the interest rate corridor, the value of a loan F can also be written as

$$V(D+F, T) - V(D, T) = i_E F + \tilde{i}\int_{T-D-F}^{T-D} G(\varepsilon)d\varepsilon, \tag{1}$$

which will be useful for further calculations.

[10] And, generally, for any $\hat{D} \in [T + \underline{\varepsilon}, T + \overline{\varepsilon}]$ where $\underline{\varepsilon}$ and $\overline{\varepsilon}$ are the upper and lower supports of $\mathcal{E}$.

4.2 Money Market

In the money market, banks and non-banks are matched in pairs to bargain, borrow, and lend. The simplest matching process is assumed. All agents are paired with another at random according to the population distribution. With a unit mass of agents, then, the mass of the pairs is 1/2. Of these, $q_n^2/2$ are non-bank pairs, $q_n(1-q_n)$ are bank to non-bank pairs, and $(1-q_n)^2/2$ are bank pairs. Notice that the matching technology allows for matches between agents of the same type. Matches between two non-banks feature no trade as both share equal linear values. Gains from trade for the other two kinds of matches derive from different sources. Banks differ from non-banks in access to the central bank, so the former can profitably borrow from the latter. Moreover, banks differ from each other as a result of the early shock, $\tilde{D}$, so would like to rebalance.

Assume both agents' reserve holdings are observable and there is full commitment to repayment of principal and interest at the end of the period (thus obviating limited information bargaining or borrowing limits). Under these assumption, it is natural for Nash bargaining to set the terms of trade – loan size and interest rate – with no trade as the threat point. Let (t_{ij}, F_{ij}) denote the gross end of day transfer of principal and interest, and loan size, respectively, in a meeting between agents i and j for $i, j \in \mathcal{D} \cup \{N\}$.[11] As non-banks have no incentive to trade with one another, we simply set $(t_{NN}, F_{NN}) = (0, 0)$.[12] This leaves one to find the bargaining solution for contracts traded in matches between a non-bank ($i = N$) and a bank with initial shock D ($j = D$), denoted shortly ND, and in matches between two banks, one with initial shock D ($i = D$) and another with initial shock D' ($j = D'$), denoted shortly DD'. Let β denote the bargaining power of the borrowing agent – so the bank with initial shock D in ND meetings and the bank with initial shock $D' < D$ in DD' meetings.[13]

4.2.1 Bank to Non-Bank Meetings

First, consider meetings between banks and non-banks. As Nash bargaining is efficient, and non-banks make no interest on reserves, loans from non-banks to banks are always of

[11] In principle, banks may differ in both deposits and also their target choice, so a bank should be identified by a vector (D, T). All equilibria, however, feature symmetric targets, see Proposition 2. So, to ease notation, we assume it from the outset in order to ease exposition and let $i \in \mathcal{D}$ denote a bank entering the market with reserves $D \in \mathcal{D}$. Since non-banks are identical they are simply indexed by $i = N$.

[12] No such assumption is necessary for bank to bank meetings with equal deposits as banks will end up having concave values as a result of the concave remuneration schedule. Non-banks, however, are purely linear, so their trade volumes are indeterminate but would be zero with any positive transaction cost.

[13] More precisely define the borrowing agent is the one with the higher marginal value of funds. Due to the concavity of $V(\hat{D}, T)$ in $\hat{D}$ and the assumption that the distribution of the second shock is the same for every bank, the borrowing agent is always the one with the larger shortages of reserves relative to his target, $T - \hat{D}$. Because in equilibrium all banks choose the same T, the borrowing agent is simply the one with fewer reserves at the time of trading, that is the one with initial shock $D' < D$ in DD' meetings.

maximal size: $F_{ND} = C$. All that is left to solve for is the transfer:

$$t_{ND} \in \text{argmax} \left\{ (V(D + C, T) - t_{ND} - V(D, T))^{\beta} (t_{ND} - C)^{1-\beta} \right\}.$$

The solution, derived in the appendix §8.3, is

$$t_{ND} = [i_E(1 - \beta) + \beta] C + (1 - \beta)\tilde{i} \int_{T-D-C}^{T-D} G(\varepsilon) d\varepsilon \tag{2}$$

4.2.2 Bank to Bank Meetings

In meetings between two banks, the contract chosen will depend on the targets of the separate banks. While we will solve for a symmetric target choice below, one must take care to account for off-equilibrium choices of target. Let D_L, D_B denote the reserve holdings of the would-be lender and borrower banks respectively. For the sake of notation, we will write T for the choice of the bank with reserves holdings D_L and T' for the choice of a bank with reserves holdings D_B in this meeting. The Nash problem is:

$$\max_{(t_{LB}, F_{LB})} (V(D_B + F_{LB}, T') - t_{LB} - V(D_B, T'))^{\beta} (V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T))^{1-\beta},$$

where t_{LB}, F_{LB} denote the transfer of principal and interest, and the loan amount, respectively, in a contract between banks with reserve holdings D_L, D_B. The solution, derived in the appendix, is

$$t_{LB} = (1 - \beta)[V(D_B + F_{LB}, T') - V(D_B, T')] - \beta[V(D_L - F_{LB}, T) - V(D_L, T)] \tag{3}$$

and

$$F_{LB} = \frac{D_L - T - (D_B - T')}{2}. \tag{4}$$

This is the familiar result that, with transferable utility, Nash bargaining awards each party a proportion of the joint surplus equal to their bargaining power. Moreover, reserve shortages (relative to targets) are equalized by inter-bank trade. This derives from two facts. First, that the remuneration function is separable in the target T and the shortage with respect to the target $D - T$. Second, that the remuneration function is concave, so the expected surplus from trade is maximized when banks smooth their reserve shortages relative to their targets in anticipation of the late shock ε.

4.2.3 Special Case: Required Reserves

The model described so far can also be used to characterize the role of reserve requirements in the traditional framework employed by the Federal Reserve before the crisis. Allow T to vary across banks and consider T_i as the exogenously given reserve requirement for every bank i. Let the remuneration of reserves still follow the schedule $R(D, T_i)$, where $i_T, i_E \geq 1$ with equality meaning no interest on reserves, as held before 2008. The penalty rate ϕ can be interpreted as the rate charged on discount window borrowing.

Proposition 1. *A symmetric increase in targets strictly raises rates so long as shortages are interior to the support of ε. The same is true for a symmetric decrease in D.*

Proof. See appendix §8.6.1. □

The pre-crisis framework for monetary policy implementation in the United States was based on the combination of exogenous reserve requirements, set as a fraction of the total deposits at a bank, and open market operations. *Ceteris paribus*, and as long as $\phi > 0$, exogenously raising reserve requirements causes banks to be more eager to borrow in order to minimize the expected penalty for falling short of the requirement. This is equivalent to an increase in the surplus from trade which then results in higher interest rates on interbank loans.

In the pre-crisis framework in the United States, however, if banks hold sufficient liquidity to meet reserve requirements without needing to borrow, it may be challenging for the central bank to maintain control over interbank and money market rates. In these situations, central banks have historically seemed prone to turn to floor systems for policy implementation, where all reserves are remunerated at the same rate, implying that interbank loans, if any, must be repaid at a higher rate than the rate of remuneration of reserves.[14] When a significant fraction of market participants is ineligible to hold reserves at the central bank, however, money market rates may fall below the interest on reserves. Thus the central bank may fail to set a floor on rates. A tool which has been recently adopted in the United States to address this issue is the Overnight Reverse Repurchase facility, which we discuss in detail in section 6. Because the list of counterparties eligible to access the ON RRP facility includes non-bank financial institutions, then the ON RRP is effectively a deposit facility at the central bank for non-banks.[15]

[14]Examples of central banks that adopted similar changes in their implementation frameworks are the BoE and the Federal Reserve after the recent financial crisis, when both central banks increased the size of their balance sheet in an effort to provide liquidity to the financial system.

[15]In the ON RRP facility the Federal Reserve sells a security to a counterparty with an agreement to buy the security back at a pre-specified date and price, with the interest rate computed from the difference between the original purchase price and the (higher) repurchase price.

4.3 Choice of Target

This section solves for an equilibrium in target choices. Let $\tilde{H}(T)$ denote the distribution of target choices across banks and $x = T - D$ a bank's reserves shortages relative to its target. Given a distribution of banks' early shock $F(D)$, let $H(x)$ denote the implied distribution of reserves shortages relative to targets across banks.

The objective function of a bank when choosing its target is

$$U(T|H) = \mathbb{E}_D\left[V(D,T) + q_N\beta S_N(D,T) + (1-q_N)\,\mathbb{E}_x\left[\hat{\beta}(x, T-D)S(T-D,x)\right]\right], \quad (5)$$

where we write

$$\hat{\beta}(x,y) = \begin{cases}(1-\beta) & \text{if } x > y\\ \beta & \text{if } x \leq y\end{cases}$$

because we assume the borrower has bargaining power β.

The first term in (5) is the value of a bank with target T and reserves D before trade. The second is the share of surplus retained by a bank in a meeting with a non-bank. The third is the share of surplus retained by a bank in a meeting with another bank, which depends on whether one is a lender or borrower. This, in turn, depends on the excess reserves of the two banks in a match. The following proposition contains our existence and uniqueness result.

Proposition 2. *If G is strictly increasing and $i_T > i_E$, there exists an essentially unique equilibrium. In it, almost all banks choose a symmetric target.*

Proof. See appendix §8.6.2. □

Intuitively, banks' first order condition reveals that targets are set to balance expected marginal values from holding reserves above the target against expected marginal values from falling short of the target, appropriately weighted.

$$\begin{aligned}\frac{i_T - i_E}{\tilde{i}} = \mathbb{E}_D\Bigg[&G(T-D) - q_N\beta[G(T-D) - G(T-D-C)]\\ &-(1-q_N)\int \hat{\beta}(D,D')\left(G(T-D) - G\left(T - \frac{D+D'}{2}\right)\right)dF(D')\Bigg]. \quad (6)\end{aligned}$$

One can rewrite the right hand side of (6) as

$$P^{short} = G(T-D)\left[1 - q_N\beta - (1-q_N)\int \hat{\beta}(D,D')\,dF(D')\right] \\ + q_N\beta G(T-D-C) + (1-q_N)\int \hat{\beta}(D,D')G\left(T - \frac{D+D'}{2}\right)dF(D'). \quad (7)$$

This is exactly the bargaining power-weighted probability of missing the target, thus having a reserve shortage; the first term reflects no trade, the second trade with non-banks, and the third trade with banks. This leads to the central contribution of this paper – an interpretable condition for the choice of target.

$$(i_T - i_E)(1 - P^{short}) = \phi P^{short}. \quad (8)$$

Banks set targets so as to equalize the interest rate earned at the margin between targeted and excess reserves multiplied by the (bargaining power weighted) probability of holding excess reserves, with the penalty rate charged at the margin on reserves shortages multiplied by the probability of having a shortage.

Contrast this to the original Poole (1968) model, where a bank demands reserves to equate the expected opportunity cost of falling short of requirements with the expected opportunity cost of holding excess reserves. In that model, a central bank must shift reserve supply in order to adjust the target rate given a demand relationship fixed by requirements. In our model, no matter the expected supply, banks adjust their targets to satisfy the same trade-off defined by the first order condition (6). This is the precise sense in which, instead of adjusting supply as needed in the traditional framework, demand adjusts to meet supply with VRT.

The result that banks balance probability weighted margins leads to the natural result that, if the two margins are equal, then targets are chosen to equalize these probabilities. In particular, when the bargaining power of the borrower and lender are also equal, and the model is otherwise symmetric, then banks target the expected supply of reserves.

Lemma 1. *Banks set $T = \mathbb{E}[D+\varepsilon]$ if the following hold: $\beta = 1/2$, the corridor is symmetric, $i_T - i_E = \phi$, both F and G are symmetric about their mean, and there are no non-banks ($q_N = 0$).*

Proof. See appendix §8.6.4. □

Notice that $i_T > i_E$ is a necessary condition for an equilibrium to feature a choice of target interior to the set of feasible reserves holdings, as is $\phi > 0$. If there were no benefit for setting positive targets, or no cost of setting achievable ones, targets would be set to corners and would not matter. The following corollary formalizes both this result and the notion of endogenous scarcity mentioned in the introduction.

Corollary 1. *Let $\mathcal{DE}$ denote the set of all possible end-of-period reserve holdings for a bank after both shocks, $\mathcal{DE} = \{d + e : d \in \mathcal{D}, e \in \mathcal{E}\}$. In any equilibrium where $T \in int(\mathcal{DE})$, then $i_T > i_E$.*

Proof. See appendix §8.6.3. □

The results obtained in this section fully characterize the functioning of a VRT framework and its effect on money market rates.

5 Central Bank Liquidity Injections

The goal of this section is twofold: first, it formalizes the experience of the BoE, which ran a version of a VRT framework between 2006 and 2009. Second, it highlights an important feature of the VRT framework for policy making: the separation between two distinct objectives for central banks, namely interest rate control and liquidity provision.

In order to formalize these ideas we study the effects of anticipated and unanticipated monetary interventions by the central bank on the equilibrium outcome. A monetary intervention by the central bank is defined as a transfer of reserves to all banks, so we also refer to this intervention as a liquidity injection.[16] As shown below, the key difference between the equilibrium outcomes induced by the two types of interventions is the choice of targets by banks.

In the first exercise, where the details of the monetary intervention are publicly announced before banks choose their targets, banks respond by raising (reducing) targets to maximize expected profits, if transfers are positive (negative). Intuitively, banks' profit maximization depends on their expected reserve shortages. Banks have an incentive to set high targets due to the high rate of remuneration they earn on reserves up to the target, but they also want to minimize their shortage of reserves relative to the target in light of the penalty charged when falling below target. Every bank resolves this trade-off by choosing the target to balance its costs and benefits, as characterized by equation (8). A transfer of reserves alters this balance, decreasing the chance of falling below the *pre-intervention* target. As a consequence, anticipating larger reserve holdings *post-intervention*, a higher target is more profitable. If transfers are also symmetric across banks then all banks shift their targets exactly by the

[16] Although our framework is sufficiently flexible to allow for any distribution of transfers to banks, we focus on symmetric transfers in order to derive clear results.

amount of the transfer, thus leaving expected reserve shortages unchanged. Then, since trade depends on expected shortages of reserves relative to the target, a symmetric monetary intervention has no effect on the pattern of trade in money markets.

In the second exercise the monetary intervention is unanticipated in the sense that it is announced after banks choose their targets. If transfers are positive, then, when they enter the money market, banks expect to hold *too many* reserves relative to the target they had already chosen. This implies that their surplus from trade decreases, causing rates in the money market to fall.

Combining the results from both exercises, it follows that an injection of liquidity in the banking system announced before banks' choice of target is innocuous to the money market – rates and volumes do not respond to the monetary intervention because targets do. In other words, in order to inject liquidity without affecting money markets, it is important for the central bank to credibly announce the details of the intervention before banks choose their targets, or, alternatively, to allow banks to adjust their targets more frequently when more frequent interventions are foreseen.

First, these results show that the VRT framework can be designed to separate liquidity provision from its impact on interest rates in money market. This allows for rate control without decreasing trade in money markets. Second, these results support the BoE's decision to abandon its VRT in 2009 due to the frequent and sizable monetary interventions that the BoE expected to adopt.[17] Alternatively, the BoE could have allowed more frequent target adjustment, if it was aiming to provide liquidity to the banking system without affecting money market rates.

5.1 Anticipated Liquidity Injections

A liquidity injection by the central bank is anticipated when it takes place before banks' choice of reserve targets. The key requirement is that the liquidity injection is credible and perfectly anticipated when banks choose their targets, rather than requiring the actual injection to occur before banks' choice of targets.

For simplicity, let the liquidity injection be symmetric across all banks, so that $D^{\Delta} = D + \Delta$ for all $D \in \mathcal{D}$, and let Δ denote the units of cash that the central bank prints and transfers to each bank's reserve accounts. Let F^{Δ}_{Nj}, i^{Δ}_{Nj} and F^{Δ}_{LB}, i^{Δ}_{LB} denote the loan amounts and interest rates in the contract between a non bank and a bank with liquidity position D_j, and between two banks with liquidity positions D_L and D_B. Similarly, let T^{Δ} denote the banks' choice of target. Then the liquidity injection has no effect on the equilibrium in the

[17] See Bank of England (2015).

money market, the only effect being an increase in targets equal to the liquidity injection. The following proposition formalizes this result.

Proposition 3. *Let G be strictly increasing. If the central bank transfers reserves to banks in the amount $\Delta_i = \Delta$ for all banks $i \in [0, (1 - q_N)]$, and if the transfer is anticipated when banks choose their reserve targets, then $T^{\Delta} = T + \Delta$, $F^{\Delta}_{LB} = F_{LB}$, $F^{\Delta}_{Nj} = F_{Nj} = C$, $i^{\Delta}_{LB} = i_{LB}$, $i^{\Delta}_{ND} = i_{ND}$.*

Proof. See appendix §8.7. □

Intuitively, when banks learn the details of the policy interventions, they learn both that they will possess more reserves, and that all other banks will as well. This is equivalent to a shift in the support of the distribution of their early shock, and in the support of the distribution of potential borrowers and lenders in the money market, for a given distribution of targets chosen by other banks. Since a bank effectively makes profits by earning a high return on reserves up to the target (i.e. i_T), expecting an increase in other banks' targets after the transfer is announced implies that the unique best response by an individual bank is to raise its target. Thus, believing that the distribution of other banks' reserve shortages does not change, an individual bank expects money market conditions (i.e. the distribution of potential borrowers and lenders) not to change. Thus, because raising its target increases its profits without worsening its position in money markets, a bank finds it optimal to maintain the *pre-intervention* expected reserve shortages.

Although we prove this result for symmetric transfers of reserves across banks, it is robust to asymmetric transfers, as long as the distribution of potential trading partners (i.e. the distribution of reserve shortages) implied by banks' response to the intervention does not change. For example, this holds if only one bank is affected by the injection or if different banks are affected differently, but the differences are perfectly anticipated by each.

5.2 Unanticipated Liquidity Injections

Consider now a liquidity injections that is not anticipated when banks choose their targets. As in the previous section, let F^{Δ}_{Nj}, i^{Δ}_{Nj} and F^{Δ}_{LB}, i^{Δ}_{LB} denote, respectively, the loan amounts and interest rates in the contract between a non-bank and a bank with liquidity position D_j, and between two banks with liquidity positions D_L and D_B. In this case, banks' targets are unaffected by the liquidity injection by assumption, so $T^{\Delta} = T$. However, banks enter the money market with smaller shortages relative to their targets. In other words banks are now more likely to hold reserves above the target relative to what they would hold if they could adjust their targets. This decreases the surplus from trade in money markets and so the interest rate, which allocates to the lender its share of the surplus. The following proposition formalizes this discussion.

Proposition 4. *Let G be strictly increasing. If a symmetric liquidity injection of size Δ is carried out, and it is unanticipated when banks choose their reserve targets, then $T^{\Delta} = T$, $F_{LB}^{\Delta} = F_{LB}$, $F_{Nj}^{\Delta} = F_{Nj} = C$, $i_{LB}^{\Delta} \leq i_{LB}$, $i_{ND}^{\Delta} \leq i_{ND}$.*

Proof. See appendix §8.7. □

5.3 The Role of Supply of Reserves by the Central Bank

In this section we characterize in further detail the interaction between the strategy followed by the central bank in supplying reserves and the equilibrium choice of targets by banks, in a VRT framework. We propose a stylized model of a central bank supplying reserves to match banks' targets, and show that this behavior would lead to the decline in reserve balances during the period before the financial crisis.

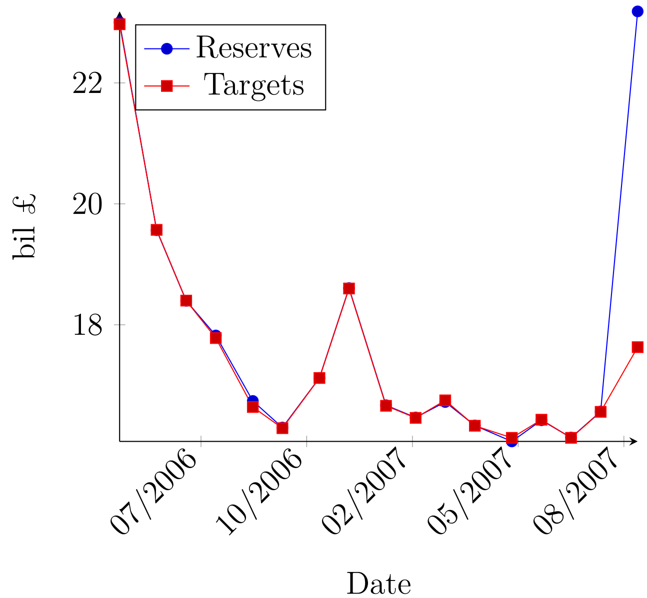
From establishing VRT in April 2006 until June 2007, the BoE followed a policy of conducting periodic interventions to ensure that its supply of reserves would equal aggregate targets. Over this period, as shown in Figure 1, reserves drifted down by more than a quarter, from over 24 to under 18 bil £. Starting in August 2007, in response to the beginnings of the crisis, the BoE began increasing reserves significantly, and eventually abandoned VRT altogether, so we exclude this later period.

The official announcements describing the adoption of a VRT at the BoE make no reference to target setting behavior (See, e.g. Bank of England (2006)), but the articles circulated to discuss the rationale for voluntary targets refer to the notion that banks would set targets to reflect their need for central bank balances (Tucker (2004), Clews (2005b)).[18]

However, as equation (8) reveals, this is not the case. Instead, in a VRT banks set targets equal to expected supply of reserves subject to a wedge. This wedge, in turn, reflects the asymmetry of the corridor, as measured by $(i_T - i_E)/\phi$ and the asymmetry of the two shock distributions, summarized by P^{short}. Hence, the only information revealed by targets which could be of use to a central bank is the latter. But, as made clear in Proposition 3, the size of the wedge between targets and supply is invariant to an anticipated injection or removal of funds. This dynamic stands awkwardly with the policy of a central bank that injects reserves sufficient to equate aggregate balances with targets at various points during its monetary policy cycle.

[18] A later note makes clear the view that targets "reveal banks' demand for reserves" (Clews et al. (2010) pp. 299).

Figure 1: Sterling Reserves and Targets



Source: Bank of England, Statistical Interactive Database, `https://www.bankofengland.co.uk`

Suppose there are no non-banks and consider the following central bank policy. Given the average targets of banks, the central bank will conduct a uniform monetary injection to equalize total supply of reserves, $\mathbb{E}[D+\epsilon]$, with aggregate targets, $\overline{T}$. That is, in the notation of Proposition 3, the central bank commits to an anticipated monetary injection of $\Delta = \Gamma(\mathbb{T})$, with $\mathbb{T} \in \mathbb{R}_+^{(1-q_N)}$ denoting the vector of individual banks' targets and $\Gamma : \mathbb{R}_+^{(1-q_N)} \to \mathbb{R}$ defined as $\Gamma(\mathbb{T}) = \overline{T} - \mathbb{E}[D+\epsilon]$. Given Proposition 3, an anticipated, uniform injection of reserves results in an equal increase in targets. That is, if a bank would have set a target of T_0 without the injection, it should set its target to $T_i = T_0 + \Delta$. Given this new target, the cycle could continue, defining another injection attempting to equalize the new targets with supply, etc. The fixed point would solve a game where banks anticipate that their targets will always be met by symmetric injection.[19] In equilibrium, we must have $T_i = T_{-i}$, hence, a finite solution to this game only exists if $T_0 = \mathbb{E}[D+\epsilon]$. Indeed, in such a case, any set of targets could be supported if banks could coordinate. If banks are willing to set targets equal to supply, but supply moves to exactly follow their targets, any level would suffice. Alternately, if $T_0 < \mathbb{E}[D+\epsilon]$ this dynamic would result in zero targets, while $T_0 > \mathbb{E}[D+\epsilon]$ would result in infinite ones.

These three cases depend, of course, on the trade-off described in equation (8). According to Lemma 1, a sufficient condition for the knife-edge case $T_0 = \mathbb{E}[D+\epsilon]$ is full symmetry of

[19]This assumes each institution is too small to manipulate the market.

the model. In any other case, attempting to adjust supply of reserves to aggregate targets leads either to infinite or zero targets, thus impairing the smooth functioning of a VRT. The following proposition formalizes this discussion, showing that, unless policy rates are chosen very carefully as a function of primitives in the economy, equilibrium targets are either zero or infinity.

Using the same notation as in the previous section, let Δ denote the central bank injection of reserve to every bank, and let D^{Δ} denote the liquidity position of bank with early liquidity shock D after the monetary injection Δ.

Proposition 5. *Fix $\hat{T}_i \in (0,\infty), \forall i \in [0,(1-q_N)]$. Let $\overline{T} = \int_0^{1-q_N} \hat{T}_i di$ and $\Delta = \overline{T}$. Assume there exists $\hat{D} \in \mathcal{D}$ such that $-\hat{D} \in \mathcal{E}$, then the solution to (6) is $T^* = \hat{T} = \hat{T}_i, \forall i$ if and only if $K\tilde{i} = i_T - i_E$, where*

$$K = \mathbb{E}_D \Bigg[G(-D) - q_N \beta [G(-D) - G(-D-C)] \\ -(1-q_N) \int \hat{\beta}(D,D') \left(G(-D) - G\left(-\frac{D+D'}{2}\right)\right) dF(D') \Bigg]. \quad (9)$$

Moreover, if $K\tilde{i} < i_T - i_E$ then $T^* = \infty$, and if $K\tilde{i} > i_T - i_E$ then $T^* = 0$.
If $\nexists \hat{D}$ such that $-\hat{D} \in \mathcal{E}$ then a solution to (6) is $T^* = \hat{T}$ if and only if $i_T = i_E$. If $i_T > i_E$ then $T^* = \infty$ and if $i_T < i_E$ then $T^* = 0$.

Proof. See appendix §8.7. □

Intuitively, K is defined in (9) as the probability of falling short of the target if the target is set to zero, or, equivalently, if the central bank is expected to make a reserve transfer and the target is set to equal such transfer. Then, Proposition 5 shows that, under appropriate assumptions on the support of the shocks, the choice of targets is indeed equal to the transfer of reserves by the central bank if and only if policy rates are set to satisfy the first order condition (6). The sufficient conditions for this result concern the probability that a bank falls below its target, even when the central bank supplies reserves exactly equal to the target. If falling below the target is a zero probability event when the central bank supplies reserves exactly equal to the target, then targets equal infinity when the net (of the penalty rate) remuneration rate of reserves is positive, and equal zero otherwise. In other words, if banks are sure to never fall below the target, given the policy of supplying reserves followed by the central bank, then their choice of target is determined by the relative marginal remuneration rate of reserves below and above the target. This result highlights an additional channel that makes the choice of policy rates critical for the well functioning of a policy implementation mechanism. Here policy rates need to be chosen carefully not only for the purpose of achieving a target rate in the interbank market, but also to avoid the collapse of the implementation mechanism itself. In fact, if equilibrium targets were zero then no trade would take place in the money market, and no transmission of policy rates to financial markets would materialize. If, on the contrary, equilibrium targets converged to infinity, so would the amount of reserves in the economy. Proposition 5 shows that, in order to appropriately choose policy rates, knowledge of banks' liquidity positions and distribution of shocks is particularly relevant when the central bank links its supply of reserves to the aggregate level of targets.

Finally, two operational details rationalize the declining, but seemingly convergent, targets experienced by the BoE. The BoE operated a symmetric corridor – the penalty for shortages equaled the difference between target and excess rates. Shortages, however, were funded by secured loans, while excess reserves were simply deposited. Hence, in addition to the explicit penalty rate, a bank facing a shortage relative to its target would have to post costly collateral. This explains the decreasing targets: the BoE's corridor was asymmetric towards the top. Collateral costs, however, are not fixed. Banks carry some collateral for portfolio purposes, and having to post it imposes little cost. As reserve supply decreased, so did the expected magnitude of shortages. Hence, the extra penalty imposed by collateral costs also shrinks. Hence, the corridor converges to be symmetric. This would lead to a convergent evolution of targets.

5.4 Discussion

The effects of monetary interventions, formalized in Propositions 3 and 4 do not depend crucially on the simplicity of the portfolio problem of banks in our model. In a richer version of the model where banks can also invest in productive opportunities besides depositing their balances at the central bank, the increase in the reserve target (and in aggregate reserve holdings) following a monetary injection would not be one-to-one but it would still take place.

The stark result in Proposition 3, showing that banks raise targets one-to-one in response to the monetary intervention, do not crucially depend on the assumption of uniform distribution of the money injections across banks. As long as the distribution of reserve shortages is unchanged after the liquidity injections, so are the results in Proposition 3. More generally, asymmetric liquidity injections, although anticipated, would affect the distribution of initial liquidity shocks banks face and banks' expected profits. Reserve targets would adjust asymmetrically, as the intervention affects the distribution of potential borrowers and lenders in the money market. As a consequence, loan values and rates would change since banks' expected reserve shortages changes. Nonetheless, the results formalized in Propositions 3 and 4 are qualitatively robust, as the change in money market rates induced by an anticipated monetary injection is smaller than the one induced by an unanticipated liquidity injection.

Under a VRT the central bank is largely free to separate the provision of liquidity to banks from its effect on interest rates in money markets. This feature of the VRT framework disappears, to some extent, in the presence of extraordinary policy measures, such as sudden and sizable interventions, which affect equilibrium interest rates. These results are consistent with the the BoE's decision to abandon the Sterling monetary framework in the wake of the recent financial crisis, when the BoE recognized that it would have lost control over market rates *had banks been required to continue to set and meet* reserve targets. [20]

6 Comparing VRT to the U.S. System

The previous section illustrated the effects of changing the stock of reserves under VRT. During the crisis, many central banks injected large quantities of liquidity and moved to a floor system for implementing monetary policy. The Fed currently operates a variant of a floor system where DIs receive interest on reserves and non-DIs receive a lower rate through the so-called ON RRP facility (see Frost et al. (2015)). This latter facility allows the Fed to put a firm floor under market rates despite a large balance sheet and a statutory prohibition against paying interest on reserves to non-banks. In this section, we model such a system and compare it to VRT. We find that, when implementing the same effective market rate, VRT can be less costly to the central bank in terms of interest rates than the floor with ON RRP if non-banks are large.

In the following subsections we *i)* characterize equilibrium loans and rates in the economy with a floor system and ON RRP, *ii)* characterize effective market rates under VRT, and *iii)* derive sufficient conditions which guarantee that the cost to a central bank of implementing a given effective rate with a VRT framework is lower than doing so with a floor system and ON RRP, the current U.S. framework.

6.1 A Floor with ON RRP

For consistency, we express the floor with ON RRP in terms of the model presented in section3 but with three alterations. First, under the floor with ON RRP, we fix targets exogenously so that they never bind, $T = \inf \mathcal{D} \bigcup \mathcal{E}$. Second, the rates on excess and targeted balances are set equal to one another, so banks face a linear rate of remuneration termed

[20] See BoE (2015).

simply interest on reserves, i_E^O. Finally, we suppose that instead of zero, non-banks have access to the ON RRP rate, i_O which is assumed weakly less than i_E^O.

These assumptions result in an inactive interbank market because every bank earns the rate i_E^O on their reserve balances and there are no gains from trade.[21] The following lemma characterizes the equilibrium allocation and interest rate, and it provides necessary and sufficient conditions on the ON RRP rate to implement the same effective interest rate in money markets as in the economy with a VRT framework. For notation, write r_O and r for these effective market rates under a floor with ON RRP and VRT, respectively.

Lemma 2. *An equilibrium with trade exists if and only if*

$$\frac{(2-\beta)\,r-1}{(1-\beta)} \quad > \quad i_E^O > 1+\frac{r-1}{(1-\beta)}. \tag{10}$$

In equilibrium $i_{ND} = 1+(1-\beta)(i_E^O - i^O)$, $F^N = C$, $i_F = i_E$, *and* $F = 0$ *without loss of generality. Further,* $r_O = r$ *if and only if*

$$i_O \quad = \quad i_E^O + \frac{1-r}{(1-\beta)}. \tag{11}$$

Proof. See appendix 6. □

Intuitively, banks are indifferent between trading and not trading because they always earn the same rate i_E^O on all their reserve balances. Also, trade between banks and non-banks takes place as long as the ON RRP rate is lower than the remuneration rate of reserve, as required by equation (11). Only banks matched with non-banks actively trade with non-banks lending their entire endowment to banks in order to earn part of the higher rate offered to banks, i_E^O. Unmatched non-banks access the ON RRP facility and earn i^O.

Equation (11) derives the necessary ON RRP rate to generate an effective money market rate equal to that with the VRT (i.e. $r_O = r$). Equation (11) also implies that the ON RRP rate and the effective money market rate r are negatively correlated. This follows from the effect of the ON RRP rate on the surplus from trade in money markets: the higher the ON RRP rate, the higher the outside option of lenders, which reduces the surplus from trade for a given interest on reserves and is therefore associated with a lower interest rate in the money market. In other words, the equilibrium money market rate increases only if the interest on reserves increases. The ON RRP rate guarantees that no trade can take place at a lower rate in equilibrium (i.e. it sets a floor on money market rates), but it does not

[21] Although there are equilibria with interbank trade, these equilibria are such that any loan takes place at the rate i_E^O and are payoff equivalent to the no trade equilibrium.

guarantee that trade does take place in equilibrium. In the extreme case in which the ON RRP rate equals the interest on reserves (i.e. $i_E^O = i^O$) every agent is indifferent between trading and not; the surplus from trade is zero. Linear preferences imply, therefore, that there exist a continuum of equilibria characterized by different trade sizes, but all at the same rate $r = i_E^O = i^O$. All of these equilibria are payoff equivalent to the equilibrium with no trade in money market. Indeed, the surplus from trade is strictly positive if and only if $i_E^O > i^O$. As a consequence, an increase in the ON RRP rate is unable to induce an increase in the effective money market rate unless the interest on reserves increases as well.

6.2 Effective Rate under VRT

The equilibrium in the economy with VRT is characterized above in section 4. The only remaining step is to characterize the effective money market rate which is used in (11) to define the ON RRP rate.

In a match between two banks with initial liquidity shocks D_L and D_B, respectively, the first order conditions for the bargaining problem yield:

$$t_{LB} = i_E F_{LB} + (1-\beta)\,\tilde{i}\int_{T-D_B-F_{LB}}^{T-D_B} G(\varepsilon)\,d\varepsilon + \beta\tilde{i}\int_{T-D_L}^{T-D_L+F_{LB}} G(\varepsilon)\,d\varepsilon.$$

Recall that $F_{LB} = \frac{D_L - D_B}{2}$ in a symmetric equilibrium where all banks choose the same reserve target, so

$$i_{LB} = i_E + (1-\beta)\,\frac{2\tilde{i}}{D_L - D_B}\int_{T-D_B-F_{LB}}^{T-D_B} G(\varepsilon)\,d\varepsilon + \beta\frac{2\tilde{i}}{D_L - D_B}\int_{T-D_L}^{T-D_L+F_{LB}} G(\varepsilon)\,d\varepsilon.$$

In a match between a non-bank and a bank with early liquidity shock D_B, the first order conditions yield

$$t_{ND_B} = (\beta + (1-\beta)\,i_E)\,C + (1-\beta)\,\tilde{i}\int_{T-D_B-C}^{T-D_B} G(\varepsilon)\,d\varepsilon.$$

The money market rate, mirroring the effective federal funds rate, is then calculated as a volume weighted average of the rates on all loans between a bank and a non-bank, and between two banks.

To characterize the weights, notice that a bank with early liquidity shock D_B is matched with a non-bank with probability q_N. In this match, the bank borrows C units of cash at the rate $\frac{t_{ND_B}}{C}$. Overall there is a measure $q_N(1-q_N)$ of these contracts. A bank with reserve balances D_L is randomly matched with another bank with probability $(1-q_N)$. In this case it lends F_{LB} at rate $\frac{t_{LB}}{F_{LB}}$ if the other bank has liquidity position $D_B < D_L$, while it borrows

F_{LB} at rate $\frac{t_{LB}}{F_{LB}}$ if the other bank has liquidity position $D_B > D_L$. Overall there is a measure $\frac{(1-q_N)^2}{2}$ of these contracts in the economy.

Let $\mathbb{F}$ denote the aggregate volume of loans in the money market. Then $\mathbb{F}$ is defined as follows:

$$\mathbb{F} = (1-q_N)\int_{\underline{D}}^{\overline{D}}\left\{q_N C + \frac{(1-q_N)}{2}\left[\int_{\underline{D}}^{D} F_{DD'}dF(D') + \int_{D}^{\overline{D}} F_{D'D}dF(D')\right]\right\}dF(D),$$

and the effective money market rate is defined as

$$\begin{aligned} r = {} & (1-q_N)\int_{\underline{D}}^{\overline{D}} q_N \frac{t_{ND}}{\mathbb{F}} dF(D) + \\ & (1-q_N)\int_{\underline{D}}^{\overline{D}}\left\{\frac{(1-q_N)}{2}\left[\int_{\underline{D}}^{D}\frac{t_{DD'}}{\mathbb{F}}dF(D')\right]\right\}dF(D) + \\ & (1-q_N)\int_{\underline{D}}^{\overline{D}}\left\{\frac{(1-q_N)}{2}\left[\int_{D}^{\overline{D}}\frac{t_{D'D}}{\mathbb{F}}dF(D')\right]\right\}dF(D). \end{aligned}$$

The first term accounts for the bank to non-bank meeting, where a bank borrows C units of cash and repays $t_{ND} = Ci_{ND}$. The second term accounts for a bank to bank meeting where a bank with reserve holdings D after its early shock lends $F_{DD'}$ to a bank with reserve holdings $D' < D$ and is receives $t_{DD'}$ as repayment at the end of the period. The third term accounts for a bank to bank meeting where a bank with reserve holdings D borrows $F_{D'D}$ from a bank with reserve holdings $D' > D$ and repays $i_{D'D} = \frac{t_{D'D}}{F_{D'D}}$ at the end of the period.

Letting $\omega_{NB} = (1-q_N)\,q_N$ denote the mass of bank to non-bank matches and $\omega_{LB} = \frac{(1-q_N)^2}{2}$ denote the bank to bank matches, the effective money market rate can then be rearranged as

$$r = \frac{\omega_{NB}}{\mathbb{F}}\int_{\underline{D}}^{\overline{D}} t_{ND}dF(D) + \frac{\omega_{LB}}{\mathbb{F}}\mathbb{E}_{DD'}(t_{DD'}). \tag{12}$$

Combining (12) with (11) allows a comparison of the central bank costs of implementing the same effective money market rate with the two frameworks.

6.3 Central Bank Costs

Central banks face – and impose on the financial sector – substantial costs related to monetary policy implementation. Interest payments represent a transfer from the government to the financial sector, but there are real costs involved as well. For example, the central bank's staffing and transactions costs associated with open market operations, including market surveillance and analysis, can be large. Reporting information to the central bank can also prove costly for private institutions, especially when a central bank requires frequent reporting of detailed deposit or other statistics from every bank branch, as is done in the U.S. Reducing these costs might be an important motive in the choice of monetary policy implementation frameworks.

Whether interest payments represent a cost, however, is more subtle. On the one hand, interest payments reduce seigniorage, so they increase distortionary taxation given fixed spending. On the other hand, interest payments may reduce other distortions, such as the regulatory burden faced by the traditional banking sector in a framework based on required reserves. A more fundamental argument, however, indicates that central bank interest may pose either no or negative cost. When a central bank creates reserves through the purchase (and so redemption and destruction of) other government debt, it swaps one interest bearing government liability for another (Keister et al. (2015)). Indeed, if the liquidity premium on reserves were sufficiently high, this substitution of reserves for longer dated debt might prove profitable. But where this mechanism is misunderstood or underappreciated, or it is deemed unpalatable for the monetary authority to manipulate the maturity structure of government liabilities, there might exist political or other forces which implicitly limit a central bank's interest payments (Berentsen et al. (2014)).

With this constraint in mind, the following sections characterize and compare the burden of interest payment under the floor system with ON RRP and VRT.

6.3.1 Cost of the Floor System with ON RRP

In the economy with a floor system T is exogenously set so that $T \leq \inf \mathcal{D} \bigcup \mathcal{E}$. The gross rate paid on banks' excess reserves is denoted i_E^O, while the gross rate paid by the ON RRP facility is denoted i_O and defined in (11). The interest cost to the central bank of running this system includes both interest on reserves paid to banks and the ON RRP rate paid to non-banks.

Let K^O denote the cost to the central bank of the floor system with ON RRP, and let K_B^O and K_N^O denote the components of such cost stemming from paying interest on reserves to banks and ON RRP rate to non-banks, respectively, net of principal repayments. Then $K^O = (1 - q_N) K_B^O + q_N K_N^O$ with

$$
\begin{aligned}
K_B^O &= \left(i_E^O - 1\right) \int_{\underline{D}}^{\overline{D}} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (D + \varepsilon)\, dG(\varepsilon)\, dF(D) \\
&= \left(i_E^O - 1\right) \int_{\underline{D}}^{\overline{D}} \left[D + \left(\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon dG(\varepsilon) \right) \right] dF(D) \\
&= \left(i_E^O - 1\right) \left\{ \int_{\underline{D}}^{\overline{D}} D dF(D) + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon dG(\varepsilon) \right\}.
\end{aligned}
$$

Here, $\int_{\underline{D}}^{\overline{D}}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}(D+\varepsilon)\,dG(\varepsilon)\,dF(D)$ is the the average quantity of reserves endowed to each bank and (i_E-1) is the (net of principle) interest rate paid on banks' reserves. As for interest payments associated with non-banks' endowment,

$$K_N^O \;=\; Ci_E^O(1-q_N)+Ci_Oq_N-C,$$

where $Ci_E^O(1-q_N)$ is the cost stemming from the fraction of non-banks that matched with a bank, $(1-q_N)$, and Ci_Oq_N is the cost stemming from the fraction of non-banks, q_N, that did not match with a bank. Thus,

$$K^O=(1-q_N)\,i_E^O\left(\int_{\underline{D}}^{\overline{D}}DdF(D)+\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\varepsilon dG(\varepsilon)\right)+q_NC\left(i_E^O(1-q_N)+i_Oq_N\right)-P^O \tag{13}$$

where $P^O=q_NC+(1-q_N)\left(\int_{\underline{D}}^{\overline{D}}DdF(D)+\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\varepsilon dG(\varepsilon)\right)$ denotes the principal repayment.

6.3.2 Cost of the VRT system

Using similar notation as in the case of a floor system and ON RRP, let K^V denote the total cost to the central bank of implementing the effective money market rate r with a VRT framework. Similarly, let K_B^V and K_N^V denote, respectively, the components of such cost stemming from trades between banks and trades between banks and non-banks. Thus, the total cost of the VRT framework net of principal repayment, is

$$K^V=(1-q_N)^2K_B^V+(1-q_N)\,q_NK_N^V.$$

Consider first the component stemming from trades between banks and non-banks, K_N^V, and let $P_N^V=\int_{\underline{D}}^{\overline{D}}DdF(D)+\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\varepsilon dG(\varepsilon)+C$ denote the principal repayment. Then

$$\begin{aligned}K_N^V \;=\; &\int_{\underline{D}}^{\overline{D}}\left(i_T\int_{\underline{\varepsilon}}^{T-D-C}(D+C+\varepsilon)\,dG(\varepsilon)\right)dF(D)\\ &+\int_{\underline{D}}^{\overline{D}}\left[\int_{T-D-C}^{\overline{\varepsilon}}(i_TT+i_E(D+C+\varepsilon-T))\,dG(\varepsilon)\right]dF(D)\\ &-\int_{\underline{D}}^{\overline{D}}\left(\phi\int_{\underline{\varepsilon}}^{T-D-C}(T-D-C-\varepsilon)\,dG(\varepsilon)\right)dF(D)-P_N^V.\end{aligned}$$

The first two terms are interest payments on targeted and excess balances, including the loans $F_{ND}=C$ from non-banks to banks, while the third term denotes the revenue which the central bank earns on reserve shortages with respect to announced targets.

Consider now the component of the central bank cost stemming from bank to bank trades, K_B^V. For the $\frac{(1-q_N)^2}{2}$ trades between two banks, there are two components of cost to the central bank, one from each bank, as the reallocation of funds through trade matters for the remuneration of reserves (which motivates trade in the first place). In bank to non-bank matches, in contrast, one need only consider the bank's balances because non-banks do not earn interest. As in the previous subsection, $P_B^V = \int_{\underline{D}}^{\overline{D}} DdF(D) + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon dG(\varepsilon)$ gives the expected endowment of banks . Given this, the cost to the central bank, net of principal repayment, can be written as

$$\begin{aligned} K_B^V &= \mathbb{E}_{DD'}\left\{\int_{\underline{\varepsilon}}^{T-\hat{D}} (i_T+\phi)\left(\hat{D}+\varepsilon\right) - \phi T dG(\varepsilon)\right\} + \\ &\quad \mathbb{E}_{DD'}\left\{\int_{T-\hat{D}}^{\overline{\varepsilon}} (i_T - i_E) T + i_E \left(\hat{D}+\varepsilon\right) dG(\varepsilon)\right\} - P_B^V, \end{aligned} \tag{14}$$

where we write $\hat{D} = D_L - \frac{D_L - D_B}{2} = D_B + \frac{D_L - D_B}{2}$, with $D_L = \max\{D, D'\}$ and $D_B = \min\{D, D'\}$ denoting the holdings of the lender and borrower banks, respectively.

Under the assumption that F and G are uniform we can characterize the cost to the central bank of either framework, and derive sufficient conditions for the VRT to be cheaper.

Proposition 6. *Assume* $(i_T+\phi) \geq \left(\underline{i_T}+\underline{\phi}\right)$ *and* $C \geq \underline{C}$*, with* $\left(\underline{i_T}+\underline{\phi}\right)$ *and* $\underline{C}$ *defined in (31)and (33) respectively. Then* $K^V < K^O$*.*

Proof. See appendix 6. ☐

Intuitively, the floor system and ON RRP is more expensive for the central bank than the VRT if 1) the amount lent by non-banks to banks, C, is sufficiently large; and 2) the effective remuneration rate under the VRT, which includes the penalty rate for falling short of the target, is sufficiently large.

The former condition guarantees that the reserve balances that the central bank remunerates in the floor system with ON RRP are sufficiently large. Part of these are remunerated at the interest of reserve, when non-banks are matched with banks, and part are remunerated at the ON RRP rate, for unmatched non-banks. The latter condition can be interpreted as a lower bound on the penalty rate ϕ for shortages with respect to the target. In an equilibrium with VRT it is necessary that $i_T > i_E$ in order to induce banks to choose positive targets. The higher the remuneration rate i_T the higher the targets. A higher penalty rate ϕ partially compensates this effect of i_T on targets. Therefore, given reserve shortages, the total revenue to the central bank increases in the penalty rate ϕ because, by reducing the target, it reduces the balances on which the central bank must pay interest, and because it raises the marginal revenue to the central bank per unit of reserve shortage.

7 Conclusion

This paper analyzes a novel framework for monetary policy implementation, termed Voluntary Reserve Targets (VRT), that overcomes several difficulties inherent in other frameworks. A version of a VRT framework was pioneered by the BoE between 2006 and 2009, and abandoned upon the onset of the recent financial crisis. We show that the voluntary nature of setting banks' targets for reserve holdings allows the central bank to separate the objectives of interest rate control and liquidity provision to the banking system, while effectively transmitting changes in policy rates to money market rates. We also study the effects on money markets of anticipated and unanticipated liquidity injections to the banking system. Our results are consistent with the BoE decision to abandon their version of the VRT framework during the recent financial crisis, when sizeable and potentially unpredictable monetary interventions were foreseen. Our analysis further reveals that VRT can be less costly to the central bank than the implementation framework currently adopted in the US, based on a floor and ON RRP system, and equally effective at achieving a given money market rate.

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8 Appendix

8.1 Value of a bank before late shock

$$
\begin{aligned}
V(\hat{D},T) &= \mathbb{E}_\epsilon[R(\hat{D}+\epsilon,T)] \\
&= i_T T + \int_{-\infty}^{T-\hat{D}} (i_T+\phi)(\hat{D}-T+\epsilon) + \int_{T-\hat{D}}^{\infty} i_E(\hat{D}-T+\epsilon)dG(\epsilon) \\
&= i_T T + (\hat{D}-T)\left[(i_T+\phi)G(T-\hat{D}) + i_E(1-G(T-\hat{D}))\right] \\
&\quad + (i_T+\phi)\int_{-\infty}^{T-\hat{D}} \epsilon dG(\epsilon) + i_E \int_{T-\hat{D}}^{\infty} \epsilon dG(\epsilon) \\
&= (i_T-i_E)T + i_E\hat{D} + (i_T-i_E+\phi)(\hat{D}-T)G(T-\hat{D}) \\
&\quad + (i_T+\phi)\int_{-\infty}^{T-\hat{D}} \epsilon dG(\epsilon) + i_E \int_{T-\hat{D}}^{\infty} \epsilon dG(\epsilon).
\end{aligned}
$$

8.2 Derivatives of Value before late shock

Some derivatives may be of interest. First, with respect to policy parameters:

$$
\begin{aligned}
\frac{\partial V}{\partial i_E} &= (\hat{D}-T)(1-G(T-\hat{D}) + \int_{T-\hat{D}}^{\infty} \epsilon dG(\epsilon) \\
\frac{\partial V}{\partial i_T} &= T + (\hat{D}-T)G(T-\hat{D}) + \int_{-\infty}^{T-\hat{D}} \epsilon dG(\epsilon) \\
\frac{\partial V}{\partial \phi} &= (\hat{D}-T)G(T-\hat{D}) + \int_{-\infty}^{T-\hat{D}} \epsilon dG(\epsilon).
\end{aligned}
$$

Next with respect to endogenous variables:

$$
\begin{aligned}
\frac{\partial V}{\partial T} &= (\hat{D}-T)(i_T+\phi-i_E)g(T-\hat{D}) + (i_T+\phi-i_E)(T-\hat{D})g(T-\hat{D}) \\
&\quad + i_T - \left[(i_T+\phi)G(T-\hat{D}) + i_E(1-G(T-\hat{D}))\right] \\
&= (i_T-i_E) - (i_T+\phi-i_E)G(T-\hat{D})
\end{aligned}
$$

and, similarly,

$$
\frac{\partial V}{\partial \hat{D}} = \left[(i_T+\phi)G(T-\hat{D}) + i_E(1-G(T-\hat{D}))\right] = i_E + (i_T+\phi-i_E)G(T-\hat{D}).
$$

8.2.1 Manipulating Differences of Value Functions

Below, the important factors will be differences between value functions before and after a loan.

Write $\tilde{i} = i_T + \phi - i_E$. Write $\Delta V(F) \equiv V(D+F,T) - V(D,T)$. Looking above, we get

$$\begin{aligned}
\Delta V(F) &= i_E F + \tilde{i}(D+F-T)G(T-D-F) - \tilde{i}(D-T)G(T-D) \\
&\quad + (i_T+\phi)\int_{-\infty}^{T-D-F} \epsilon\, dG(\epsilon) + i_E \int_{T-D-F}^{\infty} \epsilon\, dG(\epsilon) \\
&\quad - (i_T+\phi)\int_{-\infty}^{T-D} \epsilon\, dG(\epsilon) - i_E \int_{T-D}^{\infty} \epsilon\, dG(\epsilon) \\
&= i_E F + \tilde{i} F G(T-D-F) + \tilde{i}(D-T)[G(T-D-F) - G(T-D)] \\
&\quad - \tilde{i}\int_{T-D-F}^{T-D} \epsilon\, dG(\epsilon) \\
&= i_E F + \tilde{i} F G(T-D-F) + \tilde{i}(D-T)[G(T-D-F) - G(T-D)] \\
&\quad - \tilde{i}\left[(T-D)G(T-D) - (T-D-F)G(T-D-F) - \int_{T-D-F}^{T-D} G(\epsilon)\, d\epsilon\right] \\
&= i_E F + \tilde{i}\int_{T-D-F}^{T-D} G(\epsilon)\, d\epsilon
\end{aligned}$$

8.3 Bank to Non-Bank Transfer

The first order condition with respect to t_{ND} gives

$$0 = -\frac{\beta}{V(D+C,T) - t_{ND} - V(D,T)} + \frac{1-\beta}{t_{ND} - C}$$

$$\implies \beta(t_{ND} - C) = (1-\beta)(V(D+C,T) - t_{ND} - V(D,T))$$

$$\implies t_{ND} = C + (1-\beta)(V(D+C,T) - V(D,T) - C)$$

and, finally, substituting for $V(D+C,T) - V(D,T)$ from above gives

$$t_{ND} = [i_E(1-\beta) + \beta]C + (1-\beta)\tilde{i}\int_{T-D-F}^{T-D} G(\epsilon)\, d\epsilon$$

8.4 Bank to Bank Transfer

The first order condition with respect to the transfer t_{LB} is

$$0 = \frac{-\beta}{V(D_B + F_{LB}, T') - t_{LB} - V(D_B, T')} + \frac{1-\beta}{V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T)}$$

$$\implies \beta[V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T)] = (1-\beta)[V(D_B + F_{LB}, T') - t_{LB} - V(D_B, T')]$$

$$\implies t_{LB} = (1-\beta)[V(D_B + F_{LB}, T') - V(D_B, T')] - \beta[V(D_L - F_{LB}, T) - V(D_L, T)]$$

Below, it will be valuable to substitute this quantity into the surplus of each party:

$$\begin{aligned} S_L(T, T') &\equiv V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T) \\ &= V(D_L - F_{LB}, T) - V(D_L, T) + (1-\beta)[V(D_B + F_{LB}, T') - V(D_B, T')] \\ &\quad - \beta[V(D_L - F_{LB}, T) - V(D_L, T)] \\ &= (1-\beta)[V(D_L - F_{LB}, T) - V(D_L, T) + V(D_B + F_{LB}, T') - V(D_B, T')]. \end{aligned}$$

Similarly,

$$S_B(T, T') = \beta[V(D_L - F_{LB}, T) - V(D_L, T) + V(D_B + F_{LB}, T') - V(D_B, T')].$$

8.5 Bank to Bank Loan

The first order condition to the inter-bank bargaining problem with respect to the loan size is

$$0 = \beta \frac{V_1(D_B + F_{LB}, T')}{V(D_B + F_{LB}, T') - t_{LB} - V(D_B, T')} - (1-\beta) \frac{V_1(D_L - F_{LB}, T)}{V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T)}$$

$$\implies \frac{V_1(D_B + F_{LB}, T')}{V_1(D_L - F_{LB}, T)} = \frac{1-\beta}{\beta} \frac{V(D_B + F_{LB}, T') - t_{LB} - V(D_B, T')}{V(D_L - F_{LB}, T) + t_{LB} - V(D_L, T)} = \frac{1-\beta}{\beta} \frac{S_B}{S_L}$$

$$\implies \frac{V_1(D_B + F_{LB}, T')}{V_1(D_L - F_{LB}, T)} = 1$$

Using the expression for V_1 derived above, we see that

$$i_E + (i_T + \phi - i_E) G(T' - D_B - F_{LB}) = i_E + (i_T + \phi - i_E) G(T - D_L + F_{LB}).$$

And so, by monotonicity of G, we must have

$$F_{LB} = \frac{D_L - T - (D_B - T')}{2}$$

8.6 Proofs of Section 4

8.6.1 Proof of Proposition 1

Proof. Write x_0^B and x_0^L for the shortages $(T-D)$ of borrower and lender respectively. Now suppose targets are raised by some amount ΔT yielding new shortages $x_1^B = x_0^B + \Delta T$ and similarly for x_1^L. First, note that for this symmetric increase loan sizes are unaffected

$$F_0 = \frac{x_0^B - x_0^L}{2} = \frac{x_0^B + \Delta T - (x_0^L + \Delta T)}{2} = \frac{x_1^B - x_1^L}{2} = F_1.$$

Turn now to transfers. Writing $y = (x^L + x^B)/2$ for average shortage, from above we have

$$t = i_E F + (1-\beta)\int_y^{x^B} G(\epsilon)\,d\epsilon + \beta\int_{x^L}^{y} G(\epsilon)\,d\epsilon.$$

Thus, since $y_1 = y_0 + \Delta T$, we can write

$$\begin{aligned}
t_1 - t_0 &= \\
&= (1-\beta)\left[\int_{y_1}^{x_1^B} G(\epsilon)\,d\epsilon - \int_{y_0}^{x_0^B} G(\epsilon)\,d\epsilon\right] + \beta\left[\int_{x_1^L}^{y_1} G(\epsilon)\,d\epsilon - \int_{x_0^L}^{y_0} G(\epsilon)\,d\epsilon\right] \\
&= (1-\beta)\left[\int_{y_0+\Delta T}^{x_0^B+\Delta T} G(\epsilon)\,d\epsilon - \int_{y_0}^{x_0^B} G(\epsilon)\,d\epsilon\right] + \beta\left[\int_{x_0^L+\Delta T}^{y_0+\Delta T} G(\epsilon)\,d\epsilon - \int_{x_0^L}^{y_0} G(\epsilon)\,d\epsilon\right] \\
&= (1-\beta)\left[\int_{x_0^B}^{x_0^B+\Delta T} G(\epsilon)\,d\epsilon - \int_{y_0}^{y_0+\Delta T} G(\epsilon)\,d\epsilon\right] + \left[\int_{y_0}^{y_0+\Delta T} G(\epsilon)\,d\epsilon - \int_{x_0^L}^{x_0^L+\Delta T} G(\epsilon)\,d\epsilon\right] \\
&= (1-\beta)\int_0^{\Delta T}\left[G(\epsilon + x_0^B) - G(\epsilon + y_0)\right]d\epsilon + \beta\int_0^{\Delta T}\left[G(\epsilon + y_0) - G(\epsilon + x_0^L)\right]d\epsilon \\
&\geq 0.
\end{aligned}$$

The last inequality holds weakly because G is increasing and $x_0^B > y_0 > x_0^L$. It will hold strictly if the support of G intersects the interval $[x_0^L, x_0^B + \Delta T]$ with positive measure.

Finally, that changes in D give the same result is Proposition 4. □

8.6.2 Proof of Proposition 2

Proof. Taking a derivative of the above with respect to T yields

$$\begin{aligned}
U'(T|H) = i_T - i_E - \tilde{i}\,\mathbb{E}_D\Bigg[&G(T-D) - q_N\beta[G(T-D) - G(T-D-C)] \\
&-(1-q_N)\int \hat{\beta}(x, T-D)\left(G(T-D) - G\left(\frac{T-D+x}{2}\right)\right)dH(x)\Bigg] \quad (15)
\end{aligned}$$

Some inspection reveals that $U'(T|H)$ is strictly decreasing in T if G is. Thus, U is strictly concave and so, for any H, there is a unique best response. Hence, in equilibrium, H is degenerate and all agents choose a symmetric target. $\square$

8.6.3 Proof of Corollary 1

Proof. Suppose by contradiction that $i_T = i_E$ and $T \in \mathcal{DE}$. By definition of equilibrium T satisfies $(i_T - i_E)(1 - P^{short}) = \phi P^{short}$, which, evaluated at $i_T = i_E$ implies $\phi P^{short} = 0$, which, in turn, is satisfied if and only if $P^{short} = 0$. With P^{short} defined in (7), however, notice that the last two terms are always non negative. Thus $P^{short} = 0$ only if the first term in (7) is negative, which can be rearranged as

$$\begin{aligned}
G(T-D)\left[1 - q_N\beta - (1-q_N)\int \hat{\beta}(D,D')\,dF(D')\right] &= \\
G(T-D)\left[1 - q_N\beta - (1-q_N)\left(\beta\int_D^{\overline{D}} dF(D') + (1-\beta)\int_{\underline{D}}^{D} dF(D')\right)\right] &= \\
G(T-D)\left[1 - q_N\beta - (1-q_N)\left(\beta\,[F(D')]_D^{\overline{D}} + (1-\beta)\,[F(D')]_{\underline{D}}^{D}\right)\right] &= \\
G(T-D)\left[1 - q_N\beta - (1-q_N)\left(\beta\left[F\left(\overline{D}\right) - F(D)\right] + (1-\beta)\,[F(D) - F(\underline{D})]\right)\right] &= \\
G(T-D)\left[1 - q_N\beta - (1-q_N)\left(\beta\,[1 - F(D)] + (1-\beta)\,F(D)\right)\right] &= \\
G(T-D)\left[1 - \beta + (1-q_N)\,F(D)\,(2\beta - 1)\right] &
\end{aligned}$$

Define $f_D(\beta) = [1 - \beta + (1-q_N)F(D)(2\beta-1)]$ and notice that $f_D(\beta) > 0$ for all β, D. In fact

$$\begin{aligned}
f_D(0) &= 1 - (1-q_N)F(D) > 0 \\
f_D(1) &= (1-q_N)F(D) > 0 \\
\frac{\partial f_D(\beta)}{\partial \beta} &= -1 + 2(1-q_N)F(D)
\end{aligned}$$

Thus $\frac{\partial f_D(\beta)}{\partial \beta} > 0$ if and only if $F(D) > \frac{1}{2(1-q_N)}$. Let $\mathcal{D}^+$ denote the set of values of the first liquidity shock such that $F(D) > \frac{1}{2(1-q_N)}$, that is to say $\mathcal{D}^+ = \{D \in \mathcal{D} : F(D) > \frac{1}{2(1-q_N)}\}$. Then $f_D(\beta) > 0$ for all $\beta \in (0,1)$ and $D \in \mathcal{D}^+$. Analogously, let $\mathcal{D}^-$ denote the set of values of the first liquidity shock such that $F(D) < \frac{1}{2(1-q_N)}$, that is to say $\mathcal{D}^- = \{D \in \mathcal{D} : F(D) < \frac{1}{2(1-q_N)}\}$. Then $\frac{\partial f_D(\beta)}{\partial \beta} < 0$ for all $D \in \mathcal{D}^-$. Because $f_D(1) > 0$ and f_D is continuous and monotonically decreasing in β for all $D \in \mathcal{D}^-$ then $f_D(\beta) > f_D(1)$ all $D \in \mathcal{D}^-$. Thus $f_D(\beta) > 0$ for all $D \in \mathcal{D}$.

$\square$

8.6.4 Proof of Lemma 1

Proof. Under the above conditions, the T equation reduces to

$$1 = \int G(T - D)dF(D) + \int\int G\left(T - \frac{D + D'}{2}\right) dF(D')dF(D)$$

Considering the first integral and substituting $T = \mathbb{E}[D] + \mathbb{E}[\epsilon]$ and using the fact that $G(\mathbb{E}[\epsilon] - x) = 1 - G(\mathbb{E}[\epsilon] + x)$ for all x with symmetric G,

$$\begin{aligned}
\int G(T - D)dF(D) &= \int_{\mathbb{E}[D]}^{\infty} G(\mathbb{E}[\epsilon] + (\mathbb{E}[D] - D)) + G(\mathbb{E}[\epsilon] - (\mathbb{E}[D] - D))dF(D) \\
&= \int_{\mathbb{E}[D]}^{\infty} G(\mathbb{E}[\epsilon] + (\mathbb{E}[D] - D)) + (1 - G(\mathbb{E}[\epsilon] + (\mathbb{E}[D] - D)))dF(D) \\
&= \int_{\mathbb{E}[D]}^{\infty} dF(D) = \frac{1}{2}
\end{aligned}$$

If F is symmetric, the mean of two independent draws from F is also symmetric so precisely the same calculation shows that

$$\int\int G\left(T - \frac{D + D'}{2}\right) dF(D')dF(D) = \frac{1}{2}.$$

□

8.7 Proofs of section 5

8.7.1 Proof of proposition 3

Proof. Let T^* denote the solution to (6) when the support of the distribution of banks initial liquidity position is $[\underline{D}, \overline{D}]$. Then $T^{\Delta} = T^* + \Delta$ solves (6) when the support of the distribution of banks initial liquidity position is $[\underline{D} + \Delta, \overline{D} + \Delta]$. Optimality conditions to the bargaining problems imply that $F_N^{\Delta} = C$ and $F_{LB}^{\Delta} = \frac{D_L + \Delta - T^{\Delta} - (D_B + \Delta - T'^{\Delta})}{2}$. By lemma 2 $T^{\Delta} = T'^{\Delta}$ in any equilibrium, implying $F_{LB}^{\Delta} = F_{LB}$. Moreover, the solution to the bargaining problems is such that

$$i_{LB}^{\Delta} = \frac{t_{LB}}{F_{LB}^{\Delta}} = \frac{(1 - \beta)\Delta V(F_{LB}^{\Delta}) - \beta \Delta V(-F_{LB}^{\Delta})}{F_{LB}^{\Delta}} \quad (16)$$

$$i_{ND}^{\Delta} = \frac{t_{ND}}{C} = \frac{C + (1 - \beta)(\Delta V(C) - C)}{C} \quad (17)$$

where

$$\Delta V(F) = i_E F + \tilde{i}\int_{T-D-F}^{T-D} G(\varepsilon)d\varepsilon \tag{18}$$

$$\Delta V(-F) = -i_E F - \tilde{i}\int_{T-D}^{T-D+F} G(\varepsilon)d\varepsilon \tag{19}$$

Therefore, with F_{LB} and F_N unchanged by the liquidity injection, only the shortage of reserves with respect to the target, $T-D$ determines equilibrium interest rates. Since targets adjust exactly by the amount of the liquidity injection, then $T^{\Delta} - D^{\Delta} = T - D$. With unchanged reserves shortage relative to the target equilibrium interest rates are unchanged. □

8.7.2 Proof of Proposition 4

Proof. By definition of unanticipated liquidity injection, banks' reserve targets do not change, thus $T^{\Delta} = T$. When bargaining in the federal funds market, a bank which would have a reserve shortage of $T-D$ relative to its target in absence of the liquidity injection, has now a reserve shortage of $T-D-\Delta$ relative to its target. In the federal funds market the solution to the bargaining problem between two banks satisfies:

$$F_{LB}^{\Delta} = \frac{D_L^{\Delta} - T - (D_B^{\Delta} - T)}{2} = F_{LB} \tag{20}$$

and

$$i_{LB}^{\Delta} = i_E + \frac{\tilde{i}\Big[\int_{T-D_B^{\Delta}-F_{LB}^{\Delta}}^{T-D_B^{\Delta}}(1-\beta)G(\varepsilon)d\varepsilon + \int_{T-D_L^{\Delta}}^{T-D_L^{\Delta}+F_{LB}^{\Delta}}\beta G(\varepsilon)d\varepsilon\Big]}{F_{LB}^{\Delta}} \tag{21}$$

Because loan amounts are unchanged, and $T - D_j^{\Delta} < T - D_j$ for $j = L, B$ then:

$$\int_{T-D_B^{\Delta}-F_{LB}^{\Delta}}^{T-D_B^{\Delta}} G(\varepsilon)d\varepsilon < \int_{T-D_B-F_{LB}}^{T-D_B} G(\varepsilon)d\varepsilon \tag{22}$$

$$\int_{T-D_L^{\Delta}}^{T-D_L^{\Delta}+F_{LB}^{\Delta}} G(\varepsilon)d\varepsilon < \int_{T-D_L}^{T-D_L+F_{LB}} G(\varepsilon)d\varepsilon \tag{23}$$

The solution to the bargaining problem of a bank matched with a non bank satisfies:

$$F_N^{\Delta} = C = F_N \tag{24}$$

since non banks' outside option (i.e. storage) has a lower rate of return than the interest on a loan, and

$$i_{ND}^{\Delta} \;=\; \beta + (1-\beta)i_E + \frac{\tilde{i}\int_{T-D_j^{\Delta}-F_{Nj}^{\Delta}}^{T-D_j^{\Delta}} G(\varepsilon)d\varepsilon}{F_N^{\Delta}} \tag{25}$$

Because loan amounts are unchanged and $T - D_j^{\Delta} < T - D_j$ for all j then:

$$\int_{T-D_j^{\Delta}-F_{Nj}^{\Delta}}^{T-D_j^{\Delta}} G(\varepsilon)d\varepsilon \;<\; \int_{T-D_j-F_{Nj}}^{T-D_j} G(\varepsilon)d\varepsilon \tag{26}$$

□

8.7.3 Proof of proposition 5

Proof. Suppose first there exists $\hat{D} \in [\underline{D}, \overline{D}]$ such that $-\hat{D} \in \mathcal{E}$. The result follows from substituting $\Delta = \hat{T}$ in equation (6) and requiring $T^* = \hat{T}$. If $\tilde{i}K < i_T - i_E$ then $\frac{\partial U}{\partial T} > 0$, yielding $T^* = \infty$. If $\tilde{i}K > i_T - i_E$ then $\frac{\partial U}{\partial T} < 0$, yielding $T^* = 0$. Suppose now that $\nexists \hat{D}$ such that $-\hat{D} \in \mathcal{E}$. Then the right hand side of (6) equals zero. Thus a solution to (6) is $T^* = \hat{T}$ if and only if the left hand side of (6) equals zero, which requires $i_T = i_E$. If $i_T > i_E$ then $\frac{\partial U}{\partial T} > 0$, yielding $T^* = \infty$. If $i_T < i_E$ then $\frac{\partial U}{\partial T} < 0$, yielding $T^* = 0$. □

8.8 Proofs of section 6

8.8.1 Proof of lemma 2

Proof. In the economy with a floor system and ON RRP the first order conditions to the bargaining problem of a bank and a non bank yield:

$$i_{ND} \;=\; 1 + (1-\beta)\left(i_E^O - i^O\right) \tag{27}$$

where we have substituted out $i_{ND} = \frac{t_{ND}}{C} = 1 + \frac{(1-\beta)\left[\Delta V(C) - i^O C\right]}{C}$ with $\Delta V(C) = i_E^O C$.

Consider a match between two banks with initial liquidity shocks D_L, D_B, such that $D_L > D_B$: for any feasible loan amount $F \in [0, D_L - T]$ condition (1) implies that $V(D_B + F, T) - V(D_B, T) = i_E^O F$ and $V(D_L - F, T) - V(D_L, T) = -i_E^O F$. Thus the surplus from trade is zero: $S(T - D_L, T - D_B) = V(D_B + F, T) - V(D_B, T) + V(D_L - F, T) - V(D_L, T) = 0$. If trade took place, the lending bank would be willing to trade only if $i_F \geq i_E^O$ and the borrowing bank would be willing tro trade only if $i_F \leq i_E^O$, since both banks earn i_E^O on their reserve balances. As a consequence, any post-trade allocation of reserves across banks is payoff equivalent to the no trade allocation. Thus, without loss of generality, banks do not trade with each other in equilibrium. In a match between a bank and a non bank, however, there is positive surplus from trade as long as $i_{ND} > i_O$ for all D, which in turn requires $i_E^O > i^O$. Condition (1), combined with the fact that the outside option of a non bank is simply C, implies that the surplus from trade is $V(D_B + C, T) - V(D_B, T) - C = (i_E - 1)C > 0$. Because trades between banks and non banks are the only trades taking place the aggregate volume of loans in the money market is $\mathbb{F} = (1 - q_N)\, q_N C$. Then the volume weighted effective money market rate is

$$r_{ONRRP} \quad = \quad (1 - q_N)\, q_N \int_{\underline{D}}^{\overline{D}} \frac{t_{Nj}}{\mathbb{F}} dF\,(D) = 1 + (1 - \beta) \left(i_E^O - i^O \right)$$

Requiring that this volume weighted effective money market rate equals the effective money market rate in the VRT identifies the ON RRP rate necessary to implement the same rate, namely r. This yields (11). A necessary and sufficient condition for an equilibrium with trade to exists in this economy is that i^O in (11) is such that $i_O < i_{ND}$ for all D, which is satisfied if and only if $\frac{(2-\beta)r-1}{(1-\beta)} > i_E^O$. Similarly, a necessary and sufficient condition for the ON RRP takeup to be positive is that $i_O > 1$, which is satisfied if and only if $i_E^O > 1 + \frac{r-1}{(1-\beta)}$. Thus a necessary and sufficient condition for an equilibrium with trade to exists is $(2 - \beta)\, r - 1 > (1 - \beta)\, i_E^O > r - \beta$. If $r > 1$ in equilibrium it is always feasible to choose i_E^O satisfying (10). Since r denotes the effective money market interest rate in the VRT economy, then $r > 1$ always if $t_{ND} > C$ and $t_{LB} > F_{LB}$. From the solution to the bargaining problem t_{ND} and t_{LB} must satisfy the following conditions:

$$\begin{aligned} t_{ND} &= [i_E\,(1 - \beta) + \beta]\, C + (1 - \beta)\, \tilde{i} \int_{T-D-C}^{T-D} G\,(\varepsilon)\, d\varepsilon \\ t_{LB} &= i_E F_{LB} + \tilde{i} \left[(1 - \beta) \int_{T-D_B-F}^{T-D_B} G\,(\varepsilon)\, d\varepsilon + \beta \int_{T-D_L}^{T-D_L+F} G\,(\varepsilon)\, d\varepsilon \right] \end{aligned}$$

Because $i_E \geq 1$ and G is strictly increasing on its support, then $t_{ND} > C$ and $t_{LB} > F_{LB}$, implying that $r > 1$. □

8.8.2 Proof of proposition 6

Proof. Notice that $K_N^V + P_N^V$ can be simplified to:

$$\begin{aligned} K_N^V + P_N^V \quad &= \quad \left(C + \tfrac{(\overline{D}+\underline{D})}{2} \right) i_E + CT\,(i_T - i_E) + \\ & \frac{\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})} \left[(T - C - \underline{\varepsilon}) \left(\frac{(\overline{D}+\underline{D})}{2} + \underline{\varepsilon} \right) + \frac{\underline{\varepsilon}^2}{2} - \frac{(T-C)^2}{2} - \frac{\left(\overline{D}+\underline{D}\right)^2 - \overline{D}\underline{D}}{6} \right] \end{aligned} \qquad (28)$$

and that the cost to the central bank of a bank to bank match, net of principal repayment, is:

$$
\begin{aligned}
K_B^V = & \int_{\underline{D}}^{\overline{D}} \int_{\underline{D}}^{D} \left[\int_{\underline{\varepsilon}}^{T-D+\frac{D-D'}{2}} \left((i_T+\phi) \left(D+\varepsilon-\frac{D-D'}{2} \right) - \phi T \right) dG(\varepsilon) \right] dF(D') \, dF(D) + \\
& \int_{\underline{D}}^{\overline{D}} \int_{\underline{D}}^{D} \left[\int_{T-D+\frac{D-D'}{2}}^{\overline{\varepsilon}} \left((i_T-i_E) T + i_E \left(D+\varepsilon-\frac{D-D'}{2} \right) \right) dG(\varepsilon) \right] dF(D') \, dF(D) - \\
& \int_{\underline{D}}^{\overline{D}} \int_{D}^{\overline{D}} \left[\int_{\underline{\varepsilon}}^{T-D-\frac{D'-D}{2}} \left((i_T+\phi) \left(D+\varepsilon+\frac{D'-D}{2} \right) - \phi T \right) dG(\varepsilon) \right] dF(D') \, dF(D) + \\
& \int_{\underline{D}}^{\overline{D}} \int_{D}^{\overline{D}} \left[\int_{T-D-\frac{D'-D}{2}}^{\overline{\varepsilon}} \left((i_T-i_E) T + i_E \left(D+\varepsilon+\frac{D'-D}{2} \right) \right) dG(\varepsilon) \right] dF(D') \, dF(D) - P_B^V
\end{aligned}
$$

Rearranging (13), (28) and (14) yields:

$$
\begin{aligned}
K^O = & (1-q_N) \left(i_E^O - 1 \right) \{ \mathbb{E}D + \mathbb{E}\varepsilon \} + q_N C \left(i_E^O (1-q_N) + i_O q_N - 1 \right) \\
K^V = & (1-q_N)^2 \, \mathbb{E}_{DD'} \left\{ \int_{\underline{\varepsilon}}^{T-\hat{D}} (i_T+\phi) \left(\hat{D}+\varepsilon \right) - \phi T dG(\varepsilon) \right\} + \\
& (1-q_N)^2 \, \mathbb{E}_{DD'} \left\{ \int_{T-\hat{D}}^{\overline{\varepsilon}} (i_T-i_E) T + i_E \left(\hat{D}+\varepsilon \right) dG(\varepsilon) \right\} \\
& (1-q_N) \left(\left[q_N \left(\frac{\tilde{i} (T-C-\underline{\varepsilon})}{(\overline{\varepsilon}-\underline{\varepsilon})} + i_E \right) - 1 \right] \mathbb{E}D - \mathbb{E}\varepsilon \right) + \\
& (1-q_N) q_N C \left[i_E - 1 + T (i_T - i_E) \right] + \\
& \frac{(1-q_N) q_N \tilde{i}}{2(\overline{\varepsilon}-\underline{\varepsilon})} \left[2\underline{\varepsilon} \left(T-C-\frac{\underline{\varepsilon}}{2} \right) - (T-C)^2 - \frac{\left(\overline{D}^2 + \overline{D}\underline{D} + \underline{D}^2 \right)}{3} \right]
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
\frac{K^V - K^O}{(1-q_N)} = & \left[q_N C + (1-q_N) \right] (i_T - i_E) T + (1+q_N) \frac{\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})} \frac{\overline{D}\underline{D}}{12} + \\
& \left[(1-q_N) \left(i_E - \frac{(i_T+\phi)}{8(\overline{\varepsilon}-\underline{\varepsilon})(\overline{D}-\underline{D})} \underline{D}^2 - \frac{\tilde{i}\underline{\varepsilon}}{(\overline{\varepsilon}-\underline{\varepsilon})} \right) \right] \mathbb{E}D + \\
& [+ \frac{q_N \tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})} \left[(1+C)(T-C-\underline{\varepsilon}) + \frac{C^2}{2} \right] + q_N i_E - i_E^O] \mathbb{E}D + \\
& - \frac{\tilde{i}}{2(\overline{\varepsilon}-\underline{\varepsilon})} (T-\underline{\varepsilon})^2 \left[(1-q_N) + q_N (\mathbb{E}D) \right] + \left[(1-q_N) i_E - i_E^O \right] (\mathbb{E}\varepsilon) + \\
& - \left[\frac{(1-q_N)(i_T+\phi)}{4(\overline{\varepsilon}-\underline{\varepsilon})} + \frac{(1+q_N)\tilde{i}}{3(\overline{\varepsilon}-\underline{\varepsilon})} \right] (\mathbb{E}D)^2 + \\
& q_N C \left[i_E - 1 - i_E^O + \frac{(1-q_N i_O)}{(1-q_N)} \right]
\end{aligned}
$$

and, using $i_O > 1$ in the economy with ON RRP we have that

$$\begin{aligned}
\frac{K^V - K^O}{(1-q_N)} \quad < \quad & \left[q_N C + (1-q_N)\right](i_T - i_E)\,T + (1+q_N)\frac{\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})}\frac{\overline{D}\underline{D}}{12} + \\
& \mathbb{E}D\left[(1-q_N)\left(i_E - \frac{(i_T+\phi)}{8(\overline{\varepsilon}-\underline{\varepsilon})(\overline{D}-\underline{D})}\underline{D}^2 - \frac{\tilde{i}\underline{\varepsilon}}{(\overline{\varepsilon}-\underline{\varepsilon})}\right)\right] + \\
& \mathbb{E}D[\frac{q_N\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})}\left[(T-C-\underline{\varepsilon}) + C\,(T-\underline{\varepsilon}) - \frac{C^2}{2}\right] + q_N i_E - i_E^O] \\
& \left[(1-q_N)\,i_E - i_E^O\right]\mathbb{E}\varepsilon) - \frac{\tilde{i}}{2(\overline{\varepsilon}-\underline{\varepsilon})}(T-\underline{\varepsilon})^2\left[(1-q_N) + q_N\mathbb{E}D\right] + \\
& q_N C\left(i_E - i_E^O - \left[\frac{(1-q_N)(i_T+\phi)}{4(\overline{\varepsilon}-\underline{\varepsilon})} + \frac{(1+q_N)\,\tilde{i}}{3(\overline{\varepsilon}-\underline{\varepsilon})}\right](\mathbb{E}D)^2\right)
\end{aligned} \tag{29}$$

Therefore a sufficient condition for $K^V < K^O$ is that the right hand side of (29) is non positive. Notice that that $(1+q_N)\frac{\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})}\left(\frac{\overline{D}\underline{D}}{12} - \frac{(\mathbb{E}D)^2}{3}\right) < 0$. Thus, the right hand side of (29) is non positive if:

$$\begin{aligned}
&\left[(1-q_N)\left(1 - \frac{(i_T+\phi)}{8(\overline{\varepsilon}-\underline{\varepsilon})(\overline{D}-\underline{D})}\underline{D}^2 - \frac{\tilde{i}\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right) + \frac{q_N\tilde{i}}{\overline{\varepsilon}-\underline{\varepsilon}}\left[-C - \underline{\varepsilon} - C\underline{\varepsilon} - \frac{C^2}{2}\right] + q_N i_E - i_E^O\right]\mathbb{E}D \quad + \\
&\qquad \left\{\left[q_N C + (1-q_N)\right](i_T - 1) + \frac{q_N\tilde{i}\mathbb{E}D\,(1+C)}{\overline{\varepsilon}-\underline{\varepsilon}}\right\}T + \left[1 - q_N - i_E^O\right]\mathbb{E}\varepsilon \quad < \\
&\qquad q_N C\left(i_E^O - 1\right) + \frac{\tilde{i}}{2(\overline{\varepsilon}-\underline{\varepsilon})}(T-\underline{\varepsilon})^2\left[(1-q_N) + q_N\mathbb{E}D\right] + \frac{(1-q_N)(i_T+\phi)}{4(\overline{\varepsilon}-\underline{\varepsilon})}(\mathbb{E}D)^2
\end{aligned} \tag{30}$$

The following lemma characterizes sufficient conditions such that (30) is satisfied for $T = 0$. Using this result and the observation that both sides of (30) are increasing in T, the rest of the proof argues that if the right hand side of (30) increases faster than the left hand side then (30) is always satisfied. Otherwise, if the left hand side of (30) increases faster than the right hand side, (30) is satisfied if C is sufficiently large.

Lemma 3. *There exist* $\left(\underline{i_T} + \underline{\phi}\right) \in \mathbb{R}$ *such that (30) is satisfied at* $T = 0$ *if* $(i_T + \phi) \geq \left(\underline{i_T} + \underline{\phi}\right)$.

Proof. Evaluating (30) at $T = 0$ yields

$$\begin{aligned}
&\left[(1-q_N)\left(1 - \tfrac{(i_T+\phi)}{8(\overline{\varepsilon}-\underline{\varepsilon})(\overline{D}-\underline{D})}\underline{D}^2 - \tfrac{(i_T+\phi-1)\underline{\varepsilon}}{(\overline{\varepsilon}-\underline{\varepsilon})}\right) + \tfrac{q_N(i_T+\phi-1)}{(\overline{\varepsilon}-\underline{\varepsilon})}\left[-C - \underline{\varepsilon} - C\underline{\varepsilon} - \tfrac{C^2}{2}\right] + q_N i_E - i_E^O\right]\mathbb{E}D \quad < \\
&\left[i_E^O - (1-q_N)\right]\mathbb{E}\varepsilon + q_N C\left(i_E^O - 1\right) + \tfrac{(i_T+\phi-1)}{2(\overline{\varepsilon}-\underline{\varepsilon})}\underline{\varepsilon}^2\left[(1-q_N) + q_N\mathbb{E}D\right] + \tfrac{(1-q_N)(i_T+\phi)}{4(\overline{\varepsilon}-\underline{\varepsilon})}(\mathbb{E}D)^2
\end{aligned}$$

The right hand side is increasing and the left hand side is decreasing in $(i_T + \phi)$. Thus (30) is satisfied at $T=0$ if and only if $(i_T+\phi) \geq \left(\underline{i_T}+\underline{\phi}\right)$, where $\left(\underline{i_T}+\underline{\phi}\right)$ is defined to solve:

$$\begin{aligned}\left(\underline{i_T}+\underline{\phi}\right)&\left\{\tfrac{\underline{\varepsilon}^2}{2}+\tfrac{(\mathbb{E}D)^2}{4}+\tfrac{\underline{D}^2}{8(\overline{D}-\underline{D})}+\underline{\varepsilon}+q_N\left[\tfrac{\mathbb{E}D\underline{\varepsilon}^2}{2}+C+C\underline{\varepsilon}+\tfrac{C^2}{2}-\tfrac{\underline{\varepsilon}^2}{2}-\tfrac{(\mathbb{E}D)^2}{4}-\tfrac{\underline{D}^2}{8(\overline{D}-\underline{D})}\right]\right\}=\\&\left[(1-q_N)\left(1+\tfrac{\underline{\varepsilon}}{(\overline{\varepsilon}-\underline{\varepsilon})}\right)+\tfrac{q_N}{(\overline{\varepsilon}-\underline{\varepsilon})}\left[C+\underline{\varepsilon}+C\underline{\varepsilon}+\tfrac{C^2}{2}\right]+q_N i_E-i_E^O\right](\overline{\varepsilon}-\underline{\varepsilon})\mathbb{E}D\\&-\left[i_E^O-(1-q_N)\right](\overline{\varepsilon}-\underline{\varepsilon})\mathbb{E}\varepsilon-(\overline{\varepsilon}-\underline{\varepsilon})q_N C\left(i_E^O-1\right)+\tfrac{\underline{\varepsilon}^2[(1-q_N)+q_N\mathbb{E}D]}{2}\end{aligned} \tag{31}$$

□

Notice that both sides of (30) are increasing in T. Differentiating (30) with respect to T, we conclude that if

$$[q_N C+(1-q_N)](i_T-1)+\frac{q_N\tilde{i}\mathbb{E}D(1+C)}{(\overline{\varepsilon}-\underline{\varepsilon})}<\frac{\tilde{i}}{(\overline{\varepsilon}-\underline{\varepsilon})}(T-\underline{\varepsilon})[(1-q_N)+q_N\mathbb{E}D] \tag{32}$$

then $K^V-K^O<0$ for any $T>0$. Alternatively, if (32) is violated there exists $\overline{T}>0$ such that $K^V-K^O<0$ for all $T<\overline{T}$, where

$$\overline{T}=\frac{(\overline{\varepsilon}-\underline{\varepsilon})}{\tilde{i}\,[(1-q_N)+q_N\mathbb{E}D]}\left([q_N C+(1-q_N)](i_T-1)\right)+\frac{q_N\mathbb{E}D(1+C)}{[q_N(\mathbb{E}D-1)+1]}+\underline{\varepsilon}$$

Using the equilibrium characterization of T in (6), we conclude that $T<\overline{T}$ if and only if $C\geq\underline{C}$, where $\underline{C}$ is defined as:

$$\begin{aligned}\underline{C}=&\frac{\left\{\tilde{i}(1-q_N)+q_N\left[(\overline{\varepsilon}-\underline{\varepsilon})(i_T-i_E)+\frac{\tilde{i}(1-q_N)\left(\beta-\frac{1}{2}\right)\left(\overline{D}-\underline{D}\right)}{6}\right]\right\}\mathbb{E}D}{q_N\left[(\overline{\varepsilon}-\underline{\varepsilon})(i_T-1)+\tilde{i}\left(\mathbb{E}D-\beta(1-q_N+q_N\mathbb{E}D)\right)\right]}+\\&\frac{(1-q_N)\left[(\overline{\varepsilon}-\underline{\varepsilon})(1-i_E)+\frac{\tilde{i}(1-q_N)\left(\beta-\frac{1}{2}\right)\left(\overline{D}-\underline{D}\right)}{6}\right]}{q_N\left[(\overline{\varepsilon}-\underline{\varepsilon})(i_T-1)+\tilde{i}\left(\mathbb{E}D-\beta(1-q_N+q_N\mathbb{E}D)\right)\right]}\end{aligned} \tag{33}$$

□

9 Extended Background

Between April and September 2014 the Federal Open Market Committee (FOMC) discussed ways to normalize the stance of monetary policy and the Federal Reserve's securities holdings following the response to the 2008-2010 financial crisis. The committee has concluded that some aspects of the eventual normalization process will likely differ from those specified earlier.[22]

In the United States, the canonical model of monetary policy implementation involves the central bank requiring minimum reserves and conducting open market operations to affect the federal funds rate, the current target for monetary policy. By selling assets the central bank can drain reserve balances from the economy, thus resulting in scarcity of liquidity relative to the reserve requirement, and in a higher price for such liquidity in the interbank market. This traditional framework for policy implementation, however, may raise concerns regarding the amount of assets that the central bank would need to sell in order to achieve the desired target for the federal funds rate. There could be two potential issues with this traditional framework: The first is related to the availability of enough assets on the central bank balance sheet. The second is related to possible unintended consequences on the price of those assets if they were to be sold in large volumes. Changes in asset prices may affect financial institutions' balance sheets in ways to which the central bank may not necessarily be able to respond, or even predict (Frost et al. (2015)).

Reserve requirements served several roles in the traditional model of monetary policy implementation. Initially, reserves backed notes and were thought to help fortify banks and increase stability, but from the start reserves have proved ineffective at preventing runs or even effectively smoothing short-run demands for cash (Feinman (1993)). Also, some argue that required reserves provided short term liquidity and generally contribute to the smooth functioning of payment and settlement systems.[23] Moreover, required reserves allow operation of classical money multiplier effects which the central bank can exploit. However, the tight linkage between open market operations and inflation that the classical money multiplier provides may not be desirable if, for example, a central bank seeks to increase liquidity without affecting interest rates. In response to these and other criticisms, several central banks around the world have abandoned simple reserve requirements and have opted for alternative schemes.

According to the Policy Normalization Principles and Plans adopted by the FOMC

[22]See the Minutes of the Federal Open Market Committee, April (FOMC (2014a)), June (FOMC (2014c)) and July 2014 (FOMC (2014b)).

[23]Although a dollar cannot simultaneously serve as reserves and in a payment therefore required reserves may not alone improve payments (Feinman (1993))

(FOMC (2014d)), during monetary policy normalization the Federal Reserve intends to move the federal funds rate into the target range set by the FOMC primarily by adjusting the rate of interest on excess reserves (IOER). A possible concern with such a method to implement monetary policy is that the amount of liquidity banks currently have on their balance sheet is large enough to not generate any demand for reserves in the interbank market. This means that, despite changes in the IOER, the effective federal funds rate may not change due to lack of actual demand of liquidity by banks in the interbank market. Similar concerns would arise in operating frameworks based on required reserves or in a channel system.[24] In addition to these concerns, some financial institutions (among which government sponsored enterprises) hold large quantities of federal funds, but the central bank is barred by law from paying interest to these institutions. Hence, under a standard floor system, where banks have abundant reserves and earn the policy rate on their reserves, the central bank may lose control over interest rates as institutions not eligible for these interest rates seek to lend their holdings to eligible banks at rates below the policy rate. The floor becomes "leaky." The Federal Reserve's response to this issue has involved the creation of a standing facility for non-banks offering overnight reverse repurchase agreements (ON RRP). In an ON RRP the Federal Reserve borrows overnight from non-eligible institutions, effectively providing a floor on the federal funds rate. A large volume of ON RRP, however, can be very costly to the central bank and have adverse consequences such as crowding out the private repo market, as the central bank stands as a major counterparty in the market.[25]

Alternately, the framework proposed in this paper is aimed at solving these potential problems by endogenously generating demand for liquidity in the interbank market and, thus, guaranteeing that changes in IOER will affect the federal funds rate. Versions of such a program were operated by the Bank of England between 2006 and 2008 (Clews (2005a)), on a small scale by the Federal Reserve between 1980 and 2012 (Meulendyke (1998, 152)), and with centrally dictated targets from 2007 onward by the Reserve Bank of New Zealand (Nield (2008)).

The program in the U.S., known as contractual or required clearing balances, was set forth in the Monetary Control Act of 1980 which recognized that some participants' required reserves would not be sufficient to cover regular clearing and settlement. Hence, the Federal Reserve Banks offered credits on certain services to institutions who agreed to target small

[24] The European Central Bank adopts a combination of minimum reserve requirements, open market operations and standing facilities. Similarly to the U.S. experience, reserves holdings have long been in excess of the minimum requirement (See ECB (2016)). The Bank of England Sterling Monetary Framework is based on standing facilities and asset purchases, which, during 2016-17, were extended to include a Corporate Bond Purchase Scheme (CBPS) and a Term Funding Scheme (TFS) (See the BoE (2015)).

[25] See Frost et al. (2015) and Narajabad and Kotliar (2017).

agreed levels of excess reserves.[26]

Similarly, the primary focus of the scheme in New Zealand was to promote health in the payment and settlement system by incentivizing banks to hold reserves for the purposes of payments and liquidity. The program there is much larger and interest at the policy rate is paid up to the target or "tier" which is assigned by the central bank to each participating institution. No penalty is paid by banks falling below their target, and reserves in excess of the target are still remunerated but at a rate 100bp less than the policy rate. This, of course, puts a floor on the interbank rate, but not a ceiling as a pure corridor mechanism would (see, e.g. Berentsen and Monnet (2008b)). Indeed the Reserve Bank of New Zealand operates a number of additional facilities to target alternative channels including foreign exchange and liquidity priorities.

The Bank of England's program, known as the *Sterling monetary framework*, was more central to its exercise of monetary policy. A key feature of the system is that voluntary reserve targets which are frequently renegotiated produces stable demand for reserves. If market participants expect a glut of reserves over the coming monitoring period, they can choose to set higher targets and thus reverse the glut. While active open market operations can correct deviations from the policy rate as they happen, the voluntary adjustment of targets in a VRT allows for a *self-correcting* mechanism of active market operations (which was an explicit design goal in the New Zealand system). A demand for reserves that is predictable and stable results in two desirable features of monetary policy implementation: First the central bank is able to meet the interest rate target precisely. Second, it produces less volatile money market rates because of the following mechanism. With market rates above the remuneration rate for reserve balances, a bank might be expected to place more (or borrow less) in the market and to place less in its reserve account, thus holding reserve balances towards the bottom of its target for reserve requirement. But if this bank were uncertain about their liquidity position it might not aim to hit the very bottom of its target range, for fear that an unexpected outflow would force it into paying a penalty (either by having to pay a fee for failing to meet the target or by having to borrow at the central bank's lending facility, at a rate of interest above the market rate). The more uncertain it is, the less willing it would be to aim for the bottom of the range near the target.[27] This would result in a relatively flat demand for reserve around the target, allowing the central bank to control rates more precisely.

Another motivation for adopting this framework in the UK concerns a key difference

[26]For a more complete description of the program in its' midlife, see Stevens (1993). The program was made largely defunct in 2008 with the introduction of interest on excess reserves and was finally dissolved in 2012 with changes to Regulation D, (Federal Reserve Board (2015)).

[27]See Clews (2005a, 217).

between voluntary and required reserve programs: voluntary reserves can be easily defined for non-depository institutions and these alternative institutions, insofar as they play similar roles in money markets and the payment system, can be treated the same as banks with regards to the reserve targeting. In this sense, access to central bank interest can naturally be extended and, moreover, monetary policy can directly influence institutions traditionally outside of the central bank's control. This may, in turn, allow for more effective implementation of monetary policy as alternative institutions grow in importance to the global financial system.

The model developed in this paper formalizes several of these ideas: it links the market interest rate to the reserve target chosen by banks and it studies its sensitivity to policy rates and to features of the liquidity/payment shocks which banks may receive. It also studies the effectiveness of voluntary reserve targeting in the presence of different kinds of liquidity policy by the central bank. In the face of normal, anticipated, open market operations, the central bank is largely free to separately manipulate liquidity – in terms of banks' reserve balances – and interest rates. In the presence of extraordinary monetary policy, however, when interventions may be sudden and sizable, this desirable features of voluntary reserve targeting dissolve to some extent. This result is consistent with the Bank of England's decision to abandon its Sterling monetary framework in the wake of the recent financial crisis.[28] As central banks across the world contemplate normalization of monetary policy, or even simply conducting extraordinary policies with transparency and forward guidance, it becomes crucial to understand the operation of these modern methods of monetary implementations.

[28]The framework was abandoned in March 2009, as discussed, among others, in BoE (2015).