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Efficient Mismatch*

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Abstract

This paper presents a model in which mismatch employment arises in a constrained efficient equilibrium. In the decentralized economy, however, mismatch gives rise to a congestion externality whereby heterogeneous job seekers fail to internalize how their individual actions affect the labor market outcomes of competitors in a common unemployment pool. We provide an analytic characterization of this distortion, assess the distributional nature of the associated welfare effects, and relate it to the relative productivity of low- and high-skilled workers competing for similar jobs.

Keywords: Labor market frictions, crowding in/out, skill-mismatch,competitive search equilibrium

JEL Classification: E24, J31, J64

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1 Introduction

Empirical evidence suggests that labor market mismatch may help to explain recent U.S. unemployment dynamics. Şahin, Song, Topa, and Violante (2014), for example, estimate that an increase in sectoral mismatch between vacant jobs and unemployed workers contributed as much as one-third of the rise in the U.S. unemployment rate after the 2007-'08 global financial crisis. Beyond labor market dynamics, mismatch may also have important distributional implications as idle workers shift job seeking behavior across different sectors, industries, and occupations within the labor market.

This paper develops a tractable theoretical framework to understand the welfare implications of skill-mismatch employment. Our starting point is to build a model in which mismatch is a feature of the constrained efficient equilibrium. The notion of mismatch considered in this paper is one in which an unemployed worker with a college degree chooses to accept a lower paying job that does not necessarily require post-secondary education to perform (i.e., an engineer can chose to work as a waiter). The alternative is to search instead for a high-tech job that requires post-secondary education and, hence, cannot be performed by a low-skilled worker (a college degree is required to get a job as an engineer).

A key contribution of the paper is to show that efficient mismatch does not generally emerge in the decentralized equilibrium. In the private economy, mismatch generates a distortion owing to competition amongst heterogeneous job seekers searching for low-skilled employment in a common unemployment pool. While our model of mismatch is stylized, a virtue is that it is tractable. This tractability allows for a complete analytic characterization of all the distortions operating in the model as well as a clear understanding of how they are shaped by the underlying structure of the labor market.

Our framework builds on the body of literature that embeds Diamond-Mortensen-Pissarides style labor search and matching frictions (Diamond, 1982; Mortensen and Pissarides, 1994; and Pissarides, 2000) into a general equilibrium setting. We deviate from the standard setup by introducing two-sided heterogeneity and segmented labor markets.[1] Low-skilled

[1]Examples of papers that focus on heterogeneity in job matching include, among others, Pissarides (1994), Mortensen and Pissarides (1999), Albrecht and Vroman (2002), Gautier (2002), Shi (2002), and Pries (2008). More recently this literature has expanded to include on-the-job search; some examples include Barlevy (2002), Krause and Lubick (2006), Dolado et al. (2009), Menzio and Shi (2010), and Lise and Marc Robin

individuals are assumed to only be qualified to work in the low-tech sector while high-skilled individuals can work in either the low- or high-tech sector. Hence, in our setup low- and high-skilled job seekers are forced to compete in a common unemployment pool for job opportunities with a low-tech firm. Skill-mismatch employment (henceforth, mismatch) is a situation in which a high-skilled worker endogenously chooses to enter into an employment relationship with the low-tech firm.

Employment relationships are formed in one of two segmented labor markets, both of which are characterized by search and matching frictions. The high-tech job market is a standard search market with high-tech firms posting costly vacancies in order to match with high-skilled job seekers. In contrast, the low-tech job market assumes that low- and high-skilled job seekers compete in order to match with a given vacancy posted by the low-tech firm. Once a vacancy is matched with an anonymous searching worker, there is random assignment to worker type. That is, a vacant low-tech job is filled with either a low-skilled worker or a high-skilled worker seeking mismatch employment with some probability. This probability is an endogenous variable which depends on the labor force participation decisions of low- and high-skilled households, respectively.

Our setup assumes only one type of low-tech job and does not consider the case in which differentiated vacancies can be posted for low-skilled and mismatched workers. These assumptions seem justified for two reasons. First, allowing for differentiated vacancies for low-tech jobs implies a level of labor market segmentation which seems unrealistic. Second, in our view, competition amongst different types of workers for a given job is at the heart of the welfare relevance of mismatch employment. This competition is captured in our model by the endogenous probability that a vacant low-tech job is filled with either a low-skilled or mismatched worker. Under what circumstances does this competition result in an efficient allocation of resources? Under what circumstances does it generate externalities? Who gains and who losses from these externalities? Our framework allows us to answer these questions by modeling job market competition explicitly through a common matching function for low-tech jobs. In this sense, search and matching frictions are essential to our analysis.

(2017). Shimer (2007) presents a model of a mismatch with heterogeneous labor markets. Finally, some recent contributions that study mismatch from an empirical standpoint include Şahin et al. (2014) and Barnichon and Figura (2015).

We show that in the constrained efficient equilibrium, a planner that takes as given the search frictions internalizes the competition for low-tech jobs by choosing the socially optimal skill-composition of the unemployment pool. The resulting efficient level of mismatch equates the productivity differential between low-skilled and mismatched workers employed in low-tech jobs to the differential in the marginal rate of substitution between consumption and leisure across the two types of households. The planner can achieve this outcome because labor allocations are chosen from the perspective of a socially optimal value of leisure.

Heterogeneous households do not necessarily share this value of leisure; hence, the level of mismatch that obtains in the decentralized economy is generally inefficient. This result is established in a series of propositions which, taken together, provide a complete analytic characterization of the distortions in the model. Intuitively, the mismatch distortion arises because high-skilled workers searching for mismatch jobs in the low-tech job market do not internalize the effect their search activity has on the labor market outcomes of low-skilled workers. By the same token, low-skilled workers do not internalize the effect their search activity has on high-skilled job seekers looking for mismatch employment. As long as individual households make participation decisions based on their own heterogeneous private valuation of leisure, the socially optimal skill-composition of the unemployment pool for low-tech jobs will not obtain.

Ultimately, the welfare implications across households are determined by relative productivity. When high-skilled workers are more productive in mismatch employment relative to their low-skilled colleagues, their participation in the market for low-tech jobs is inefficiently low. There is too little search activity and mismatch job creation is too low relative to the constrained efficient equilibrium. In contrast, low-skilled participation is inefficiently high. The resulting welfare effects are distributional in that high-skilled households experience welfare gains that come entirely at the expense of low-skilled households. The opposite intuition holds when low-skilled workers are relatively more productive.

Efficient mismatch only emerges in the private economy when low-skilled and mismatched workers are equally productive. In this knife-edge case, both types independently share a common view of the private value of leisure which happens to coincide with the social optimum. The lack of any true heterogeneity across households means that private objective functions coincide and there is no scope for competition to distort the labor market.

Our main results are derived under a labor market structure characterized by wage bargaining. It should be clear, however, that this distortion generalizes to *any environment* in which wages are determined by a time-invariant surplus sharing rule, regardless of the wage setting protocol that delivers that rule. Moreover, we show that while it is theoretically possible to decentralize efficient mismatch using a market-based method of wage determination, doing so requires a strong assumption regarding the ability of the firm to post wages. In order to obtain efficient mismatch in a competitive search equilibrium à la Moen (1997), the low-tech firm needs to be able to post *differentiated wages* in a way that allows competing job seekers to trade off their *individual* labor compensation against *type-specific* job finding probabilities. Effectively, this implies the firm has an ability to segment the low-tech labor market in a way that eliminates job competition. In this sense, it is not surprising that wage posting removes the mismatch distortion.

We close the paper by doing some simple quantitative exercises using a calibrated version of the model to illustrate the distributional effects of the mismatch distortion. The calibrated model is also used to help gauge the size of the mismatch distortion relative to more well known distortions in the search and matching literature (i.e., those created through inefficient bargaining and/or the presence of unemployment benefits).

Our paper is related to a broader literature that focuses on different notions of mismatch unemployment as well as heterogeneity in job matching. Within this strand, our work focuses more narrowly on matching efficiency. Similar papers include Albrecht, Navarro, and Vroman (2010) and Gravel (2011), both of which show that inefficiencies arise in models with endogenous participation and heterogeneity in the productivity of matches. Gavrel (2012) shows inefficiencies in a model with two-sided heterogeneity where firms can rank their applicants. In each of these papers, the inefficiency operates through the production function as the firm does not internalize how its vacancy posting decision affects average match quality. In our paper, average match quality is also distorted, but the source of the inefficiency comes from job competition amongst heterogeneous job seekers. Gautier, Teulings, and van Vuuren (2010) develop a model of mismatch with two-sided heterogeneity where, absent on-the-job search, efficiency obtains with a constant returns matching technology, provided congestion externalities are absent. They show introducing on-the-job search generates a distortion that can be removed by wage posting with commitment. Menzio and Shi (2011) develop a model with heterogeneous quality in job matches and on-the-job search where, upon meeting, a worker and a firm observe a potentially imperfect signal regarding the productivity of the potential match. In their framework, the unique decentralized equilibrium is efficient. Our paper differs from both of these in that we show mismatch itself generates a distortion which is unrelated to on-the-job search.

The remainder of the paper is organized as follows. The next section presents the model. Section 3 describes decentralized equilibrium with wages determined via bargaining. The main results are presented in Section 4 in a series of propositions. Section 5 presents results in the competitive search equilibrium. Section 6 uses a parameterized version of the model to illustrate some quantitative points. Finally, Section 7 concludes.

2 The Model

The model introduces two-sided heterogeneity for both workers and firms into the general equilibrium labor search framework of Arseneau and Chugh (2012). This framework offers a convenient benchmark for understanding the efficient equilibrium and the distortions associated with the decentralized search equilibrium. Note also that our model does not allow for mismatched workers to transit to high-tech employment through on-the-job search. We have shown in other work that on-the-job search only serves to amplify the mismatch distortion, but does so at the expense of notably complicating the model.

2.1 Production

Production is divided into two sectors: a final goods sector and an intermediate goods sector. The intermediate goods sector consists of two types of firms called high-tech and low-tech firms, respectively. Regardless of firm type, labor is the only input into production. In order to hire a unit of labor, intermediate goods producing firms must engage in costly search and matching in order to form a long-lasting employment relationship.

Firm type is differentiated by the skill set required to do the job. High-tech intermediate goods, denoted $y^H(n_t^H)$ where the production function $y^H(.)$ is increasing and concave, can only be produced by high-skilled workers, n_t^H. In contrast, low-tech intermediate goods, denoted $y_t^L(n_t^L, n_t^M)$ where $y^L(.)$ is increasing and weakly concave in both its arguments, can be produced using either low- or high-skilled workers. In our notation, n_t^L denotes a low-skilled worker employed by the low-tech firm and n_t^M denotes a high-skilled worker engaged in mismatch employment in the low-tech production sector.

The final goods producer simply aggregates the intermediate goods into a single final product, which is sold to, and ultimately consumed by, households, so $Y_t = F(y_t^L, y_t^H)$. It is helpful to differentiate between three distinct cases depending on the relative productivity of low-skilled versus mismatched workers in the production of low-tech goods. Letting y_i^L denote the derivative of with respect to the i^{th} input in the production function for the low-tech intermediate good, the cases we focus on are:

(*Case i.*) $y_1^L(n_t^L, n_t^M) < y_2^L(n_t^L, n_t^M)$;

(*Case ii.*) $y_1^L(n_t^L, n_t^M) > y_2^L(n_t^L, n_t^M)$;

(*Case iii.*) $y_1^L(n_t^L, n_t^M) = y_2^L(n_t^L, n_t^M)$.

As will become clear later in the paper, assumptions regarding the precise nature of mismatch employment turn out to play a critical role in shaping our main results.

2.2 The Labor Market

The labor market is segmented into two distinct markets for low- and high-tech employment, respectively, both of which are subject to search and matching frictions.

The search market for high-tech jobs is standard. Existing high-tech employment relationships exogenously terminate with probability ρ^H. Replenishing the stock of high-skilled jobs requires costly effort on the part of workers and firms. High-skilled households spend time searching, s_t^H, for employment opportunities while high-tech firms must pay a fixed cost, γ^H, to post vacancies in the high-tech market, v_t^H, in order to attract workers. There is free entry in high-tech vacancy postings. New high-tech jobs are formed according to a matching technology, $m(s_t^H, v_t^H)$, which is constant returns to scale and increasing and concave in both v_t^H and s_t^H.

The stock of high-tech jobs evolves according to

$$n_t^H = (1-\rho^H)n_{t-1}^H + m(s_t^H, v_t^H) \tag{1}$$

The market for low-tech jobs is non-standard due to the introduction of mismatch employment. Low-tech jobs expire at exogenous rate, ρ^L, and low-tech firms must pay a fixed cost, γ^L, in order to post a vacancy, v_t^L, to attract workers into employment in the low-tech industry. As above, there is free entry in low-tech vacancy postings. However, a key difference is that, in contrast with high-tech jobs, low-tech production can be done using labor supplied by both types of households. Let $e_t = s_t^L + s_t^M$ denote total search effort in the market for low-tech jobs, where s^L denotes search by low-skilled households for low-tech jobs and s^M denotes search by high-skilled households for mismatched jobs.

New matches are formed according to a matching technology, $m(e_t, v_t^L)$, which is constant returns to scale and increasing and concave in both e_t and v_t^L. However, because the pool of searching workers in the low-tech job market contains both low- and high-skilled job seekers, we assume random assignment regarding whether a given low-tech vacancy matches with a low- or a high-skilled worker.[2] Let $\eta_t = s_t^L/e_t$ denote the share of low-skilled job seekers in the market for low-tech jobs and $1-\eta_t$ denote the share of high-skilled workers searching for mismatched jobs.

Under these assumptions, new low-tech jobs form according to $\eta_t m(e_t, v_t^L)$, so the stock of low-tech labor evolves according to

$$n_t^L = (1-\rho^L)n_{t-1}^L + \eta_t m(e_t, v_t^L) \tag{2}$$

and the stock of mismatch labor evolves according to

$$n_t^M = (1-\rho^L)n_{t-1}^M + (1-\eta_t)m(e_t, v_t^L). \tag{3}$$

Assuming that both low- and high-skilled individuals obtain low-tech jobs through a

[2] Random assignment reflects an assumption that the market for low-skilled jobs is not further segmented and that low-tech firms cannot rank applicants by skill level in the job matching process.

common matching technology captures the idea that mismatch creates spillovers across heterogeneous workers through competition in a common unemployment pool for these types of jobs.

Finally, in terms of notation, let $f_t^H = m(s_t^H, v_t^H)/s_t^H$; $f_t^L = m(e_t, v_t^L)/e_t$; $q_t^H = m(s_t^H, v_t^H)/v_t^H$; $q_t^L = m(e_t, v_t^L)/v_t^L$. Also, define $\theta_t^H \equiv v_t^H/s_t^H$ as market tightness in the high-tech sector and $\theta_t^L \equiv v_t^L/e_t$ as market tightness in the low-tech sector.

2.3 The Social Welfare Problem

The economy is inhabited by a unit mass of individuals, a fraction κ of which are low-skilled while the remaining $1-\kappa$ are high-skilled. Individuals are aggregated into two separate households, differentiated by type. For the sake of convenience, we assume there is aggregate risk sharing across individuals within a given household type.[3]

Regardless of type, households allocate their time between labor market activity (i.e., working or actively searching for employment) and leisure. In terms of notation, let the mass of low-skill individuals participating in the labor force be given by $lfp_t^L = n_t^L + s_t^L - \eta_t m(e, v_t^L)$.[4] Similarly, the mass of high-skill individuals participating in the labor force is given by $lfp_t^H + lfp_t^M$, where: $lfp_t^H = n_t^H + s_t^H - m(s_t^H, v_t^H)$ denotes participation in the market for high-tech jobs and $lfp_t^M = n_t^M + s_t^M - (1-\eta_t)m(e, v_t^L)$ denotes participation in the low-tech job market for mismatched jobs.

Household preferences are defined over consumption of the final good, denoted c_t^L and c_t^H for low- and high-skilled households, respectively, and the disutility of labor market participation. Any given household derives utility from consumption per the function u, with $\frac{\partial u}{\partial c} = u_c > 0$ and $\frac{\partial^2 u}{\partial c^2} < 0$. In addition, any given household derives disutility from labor force participation per the function h, with $\frac{\partial h}{\partial lfp} = h' > 0$ and $\frac{\partial^2 h}{\partial lfp^2} < 0$.

The planner chooses allocations subject to the search frictions—so that the concept of efficiency is one of constrained efficiency, or the "second best"—but *internalizes the random*

[3] The risk sharing assumption is common in search-theoretic general equilibrium models of the labor market following Merz (1995) and Andolfatto(1996).

[4] Note that the timing of the model is such that successful search within the period, given by $\eta_t m(e, v_t^L)$ for low-skilled individuals searching in the low-tech job market, is counted as part of the employment stock, so it must be netted out to avoid double counting.

assignment of low-tech jobs between low-skilled and mismatched workers. It does this by directly choosing the skill composition of the pool of unemployed workers seeking employment in the low-tech industry.

The social welfare problem involves choosing a sequence of allocations, $\{c_t^L,\ c_t^H,\ n_t^L,\ n_t^M,\ n_t^H,\ s_t^L,\ s_t^M,\ s_t^H,\ v_t^H,\ v_t^L,\ \eta_t^L\}$, to maximize an equally-weighted sum of the utility of low- and high-skilled households whose discounted lifetime expected value is denoted by U. Specifically, the planner's problem is

$$\max\ \mathcal{U} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{u(c_t^L) - h^L\left(n_t^L + s_t^L - \eta_t^L m(e^L, v_t^L)\right) \tag{4}$$

$$+u(c_t^H) - h^H\left(n_t^H + s_t^H - m(s^H, v_t^H)\right) - h^M\left(n_t^M + s_t^M - \left(1-\eta_t^L\right) m(e^L, v_t^L)\right)\}$$

where: $\mathbb{E}_t$ is the expectation operator; $\beta \in (0,1)$ is the subjective discount factor.

The planner faces a resource constraint

$$Y_t = c_t^L + c_t^H + \gamma^L v_t^L + \gamma^H v_t^H \tag{5}$$

as well as the three different laws of motion for the respective stocks of employment given by equations (1) through (3), and the definitions $e_t = s_t^L + s_t^M$ and $\eta_t = s_t^L / e_t$.

2.4 Social Efficiency

The constrained efficient equilibrium is characterized by a set of six efficiency conditions combined with the economy-wide resource constraint, the three laws of motion for the respective labor stocks, and the definition of the share of low-skilled searchers in the market for low-tech employment. All details regarding the derivation of these conditions are relegated to a supplementary online appendix for expositional purposes.

The planner chooses allocations to equate the the marginal utility of consumption across both households, so consumption risk sharing extends across as well as within households.

$$u_{c,t}^L = u_{c,t}^H = u_{c,t} \tag{6}$$

Efficiency in the high-tech job market is described by two separate efficiency conditions, one static and one dynamic. The static efficiency condition is given by

$$\frac{h_t^{H\prime}}{u_{c,t}} = \frac{m_{s,t}^H}{m_{v,t}^H}\gamma^H \tag{7}$$

This equation describes efficient job search in the market for high-tech employment. Intuitively, at the optimum, the marginal rate of substitution (MRS) between consumption and leisure for high-skilled households must equal the marginal rate of transformation of a unit of leisure into a unit of consumption, holding output constant. As discussed in greater detail in Arseneau and Chugh (2012), the term on the right hand side captures the idea that leisure can be transformed into consumption statically within the period by going through the matching function and effectively freeing up $1/m_{v,t}^H$ units of vacancies which can be transformed into consumption at the rate γ^H.

In addition, because an employment relationship is a long-lived investment, the following dynamic efficiency condition must also be satisfied

$$\frac{1}{m_{s,t}}\frac{h_t^{H\prime}}{u_{c,t}} + \sum_{s=1}^{\infty}(1-\rho^H)^s\mathbb{E}_t\left\{\beta^s\frac{u_{c,t+s}}{u_{c,t}}\frac{h_{t+s}^{H\prime}}{u_{c,t+s}}\right\} = \sum_{s=0}^{\infty}(1-\rho^H)^s\mathbb{E}_t\left\{\beta^s\frac{u_{c,t+s}}{u_{c,t}}Y_{3,t+s}\right\} \tag{8}$$

This equation describes efficient job formation in the high-tech sector. The left hand side is the discounted sum of marginal rates of substitution between consumption and labor effort over the anticipated life of the job (adjusted for the incremental search required to create the job in the initial period). The right hand side is the discounted sum of the marginal production of labor (MPL) over the anticipated life of the job.

Due to the existence of mismatch, the conditions that describe efficiency in the market for low-tech jobs are somewhat more complicated and are described by three equations. The first addresses static efficiency in the market for low-tech employment.

$$\eta_t\frac{h_t^{L\prime}}{u_{c,t}} + (1-\eta_t)\frac{h_t^{M\prime}}{u_{c,t}} = \frac{m_{s,t}^L}{m_{v,t}^L}\gamma^L \tag{9}$$

In the market for low-tech jobs, both low- and high-skilled workers are forced to compete in a common unemployment pool with random assignment of matches. The social planner internalizes this by equating a probability weighted average of the marginal rates of substitution for low- and high-skilled job seekers, respectively, to the marginal rate of transformation of a generic unit of leisure into consumption. In other words, the equation highlights the fact that the social planner has in mind a specific notion of the socially optimal value of leisure when it internalizes the composition of the low-skilled labor pool.

There are two separate dynamic efficiency conditions that govern the evolution of the stock of low-tech and mismatched jobs in the economy, respectively, given by

$$\begin{aligned} &\frac{1}{m_{s,t}^{L}}\left(\eta_t \frac{h_t^{L\prime}}{u_{c,t}} + (1-\eta_t)\frac{h_t^{M\prime}}{u_{c,t}}\right) - \frac{1-\eta_t}{f_t^{L}}\left(\frac{h_t^{M\prime}}{u_{c,t}} - \frac{h_t^{L\prime}}{u_{c,t}}\right) \\ &+ \sum_{s=1}^{\infty}(1-\rho^{L})^{s}\mathbb{E}_t\left\{\beta^{s}\frac{u_{c,t+s}}{u_{c,t}}\frac{h_{t+s}^{L\prime}}{u_{c,t+s}}\right\} = \sum_{s=0}^{\infty}(1-\rho^{L})^{s}\mathbb{E}_t\left\{\beta^{s}\frac{u_{c,t+s}}{u_{c,t}}Y_{1,t+s}\right\} \end{aligned} \tag{10}$$

and

$$\begin{aligned} &\frac{1}{m_{s,t}^{L}}\left(\eta_t \frac{h_t^{L\prime}}{u_{c,t}} + (1-\eta_t)\frac{h_t^{M\prime}}{u_{c,t}}\right) + \frac{\eta_t}{f_t^{L}}\left(\frac{h_t^{M\prime}}{u_{c,t}} - \frac{h_t^{L\prime}}{u_{c,t}}\right) \\ &+ \sum_{s=1}^{\infty}(1-\rho^{L})^{s}\mathbb{E}_t\left\{\beta^{s}\frac{u_{c,t+s}}{u_{c,t}}\frac{h_{t+s}^{M\prime}}{u_{c,t+s}}\right\} = \sum_{s=0}^{\infty}(1-\rho^{L})^{s}\mathbb{E}_t\left\{\beta^{s}\frac{u_{c,t+s}}{u_{c,t}}Y_{2,t+s}\right\} \end{aligned} \tag{11}$$

The general intuition for both expressions, which describe socially optimal low-tech and mismatch job creation, respectively, is generally similar to that for equation (8) above. Indeed, the second line of each equation equates the discounted sum of the MRS between consumption and leisure to the discounted sum of the MPL over the life of the job. But, equations (10) and (11) differ from equation (8) in two key respects. First, as with the static efficiency condition, the planner has in mind a particular notion of the socially optimal value of leisure given the nature of matching in the low-tech market. This is captured by the first term in the top line of each equation, familiar from equation (9) above. In addition, the second term on the top line of each equation captures the relative cost of shifting job search activity for low-tech jobs between the low-skilled and high-skilled household.

The interpretation is more clear if we take the difference between equations (10) and (11) to get the following expression:

$$\frac{1}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}} - \frac{h_t^{L\prime}}{u_{c,t}}\right) \tag{12}$$

$$+\sum_{s=1}^{\infty}(1-\rho^L)^s \mathbb{E}_t\left\{\beta^s \frac{u_{c,t+s}}{u_{c,t}} \frac{h_{t+s}^{M\prime} - h_{t+s}^{L\prime}}{u_{c,t+s}}\right\} = \sum_{s=0}^{\infty}(1-\rho^L)^s \mathbb{E}_t\left\{\beta^s \frac{u_{c,t+s}}{u_{c,t}}(Y_{2,t+s} - Y_{1,t+s})\right\}$$

While the planner forms jobs according to a socially optimal value of leisure, he or she also understands that the resulting composition of the low-tech unemployment pool creates spillovers across households due to the random nature of matching. This expression describes how the planner internalizes these spillovers. It says that when the composition of low-tech labor force is at the social optimum, the marginal cost of shifting the burden of labor effort between low-skilled and mismatched workers must be exactly offset by the resulting shift in productivity gains.

The relevance of *Cases i* through *iii* outlined in Section 2.1 above become clear. For the sake of intuition, consider steady state. When $Y_2 - Y_1 > 0$, the socially optimal composition of the labor force must be such that the MRS between consumption and leisure for mismatched workers exceeds that of low-skilled workers, so that $\frac{h^{M\prime} - h^{L\prime}}{u_c} > 0$. The planner wants to take advantage of the higher productivity of high-skilled individuals working in mismatched jobs by requiring them to work more (driving up $\frac{h^{M\prime}}{u_c}$ relative to $\frac{h^{L\prime}}{u_c}$ as high-skilled households take less leisure). The opposite intuition holds when $Y_1 - Y_2 > 0$ in that the planner wants to shift the burden of production onto the more productive low-skilled worker, so that $\frac{h^{L\prime} - h^{M\prime}}{u_c} > 0$. Finally, when $Y_1 - Y_2 = 0$, the planner would like to allocate labor activity such that the marginal rates of substitution between consumption and leisure are equated across low- and high-skilled households, $\frac{h^{L\prime} - h^{M\prime}}{u_c} = 0$.

3 The Decentralized Search Economy

We describe the decentralized search equilibrium under wage bargaining.

3.1 Households

Individuals in the economy are separated into low- and high-skilled households.

3.1.1 Low-Skilled Households

The low-skilled household chooses sequences of consumption, c_t^L, real non state-contingent bond, B_t^L, and search activity to achieve a desired low-tech employment stock in order to maximize discounted lifetime utility $\mathcal{U}^L = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (u(c_t^L) - h\left(lfp_t^L\right))$. Low-skilled households face the following budget constraint:

$$c_t^L + B_t^L = w_t^L n_t^L + \chi^L(1 - f_t^L)s_t^L + R_t B_{t-1}^L + \kappa(\Pi_t^L + \Pi_t^H) - T_t^L$$

where: w_t^L is the wage received by a low-skilled individual employed in a low-tech job; χ^L is an exogenous unemployment benefit; the real non state-contingent bond pays an interest rate of R_t; Π_t^L and Π_t^H denote the profits of intermediate low- and high-tech goods producing firms paid to the household in the form of a dividend, and T_t^L is a lump sum tax levied by a government to finance the unemployment benefit. We assume that households pay the lump sum tax and receive a dividend from firm ownership in proportion to their share of the total population.

In addition to the budget constraint, the household also faces a constraint on the perceived law of motion for the stock of employment given by:

$$n_t^L = (1 - \rho^L)n_{t-1}^L + f_t^L s_t^L$$

The first order conditions for c_t^L and B_t^L can be manipulated into a standard bond Euler equation:

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} R_{t+1} \right\}, \tag{13}$$

which defines the stochastic discount factor for pricing the one-period, risk-free government bond, $\Xi_{t+1|t} \equiv \beta u_{c,t+1}^L / u_{c,t}^L$.

We can also use the first order conditions on s_t^L and n_t^L to obtain the optimal labor-force participation condition for low-skilled individuals:

$$\frac{h_t^{L\prime}}{u_{c,t}^L} = (1-f_t^L)\chi^L + f_t^L \left[w_t^L + (1-\rho^L)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1-f_{t+1}^L}{f_{t+1}^L} \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) \right\} \right] \tag{14}$$

which says that the low-skilled household will search for low-tech employment up until the point at which the probability-weighted cost of doing so, the disutility of search effort net of the outside option, χ^L, is exactly offset by the probability weighted expected benefit of getting a low-tech job. The expected benefit of low-tech employment is the wage plus the continuation value of the long-lived employment relationship.

3.1.2 High-Skilled Households

High-skilled households choose sequences of consumption, c_t^H, non state-contingent bond holdings, B_t^H, and search activity in both the market for low- and high-tech jobs, given by s_t^M and s_t^H, in order to achieve a desired stock of mismatch and high-tech employment, given by n_t^M and n_t^H, respectively. These quantities are chosen in order to maximize discounted lifetime utility, $\mathcal{U}^H = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (u(c_t^H) - h\left(lfp_t^H, lfp_t^M\right))$.

The high-skilled household faces the following budget constraint:

$$\begin{aligned} c_t^H + B_t^H = w_t^H n_t^H + w_t^M n_t^M + \chi^H \left[(1-f_t^L)s_t^M + (1-f_t^H)s_t^H \right] \\ +R_t B_{t-1}^H + (1-\kappa)\left(\Pi_t^L + \Pi_t^H\right) - T_t^H, \end{aligned}$$

and perceived laws of motion for the stocks of mismatch and high-tech employment:

$$n_t^M = (1-\rho^L)n_{t-1}^M + f_t^L s_t^M,$$

and

$$n_t^H = (1-\rho^H)n_{t-1}^H + f_t^H s_t^H,$$

where: w_t^H and w_t^M are the wages received by high-skilled individuals employed in high-tech and mismatch jobs, respectively; χ^H is an exogenous unemployment benefit paid to high-skill workers who searched for jobs but did not find one, and T_t^H is a lump sum tax.

The first-order conditions over c_t^H and B_t^H can be combined to yield a standard consumption Euler equation:

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^H}{u_{c,t}^H} R_{t+1} \right\}. \tag{15}$$

Noting that trade of the non-contingent real bond equates the marginal utility of consumption across the low- and high-skilled household, so that $\beta u^L_{c,t+1}/u^L_{c,t} = \beta u^H_{c,t+1}/u^H_{c,t} = \Xi_{t+1|t}$, we can use the first order condition for n^H_t to write the optimal participation condition in the market for high tech employment as:

$$\frac{h^{H\prime}_t}{u^H_{c,t}} = (1 - f^H_t)\chi^H + f^H_t \left[w^H_t + (1 - \rho^H)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1 - f^H_{t+1}}{f^H_{t+1}} \left(\frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H \right) \right\} \right], \tag{16}$$

Finally, the condition governing optimal participation for high-skilled individuals in the market for low-tech jobs can be written as:

$$\frac{h^{M\prime}_t}{u^H_{c,t}} = \left(1 - f^L_t\right) \chi^H + f^L_t \left[w^M_t + (1 - \rho^L)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1 - f^L_{t+1}}{f^L_{t+1}} \left(\frac{h^{M\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H \right) \right\} \right] \tag{17}$$

Intuitively, the interpretation of equations (16) and (17) are very similar to that of equation (14) above for the low-skilled household.

The tradeoff faced by the high-skilled household when deciding how to allocate search activity across the two segmented labor markets can be highlighted by comparing equations (16) and (17) in the steady state. For simplicity, assume no unemployment benefits, $\chi^H = 0$, and that employment relationships form in the presence of search frictions, so $f^L < 1$ and $f^H < 1$, but they only last for a single period, so $\rho^L = \rho^H = 1$. In this case, the high-skilled household allocates search activity so that the marginal rate of substitution between participation in the high-tech and mismatch labor markets is equal to the probability adjusted wage ratio across the two markets:

$$\frac{h^{H\prime}}{h^{M\prime}} = \frac{f^H}{f^L} \frac{w^H}{w^M}.$$

Holding $h^{H\prime}/h^{M\prime}$ constant, a larger wage premium for working in high-tech employment (higher w^H/w^M) must be compensated by improved job finding prospects in the market for mismatch employment (lower f^H/f^L). In this sense, our model captures the idea that high-skilled individuals are willing to accept a lower quality job to move out of unemployment more quickly, but doing so comes at the cost of accepting a lower wage.

3.2 Production

Production is divided into a final goods and an intermediate goods sector.

3.2.1 Final Goods Production

The representative final goods producer purchases both low- and high-tech intermediate inputs and aggregates them into a final good using the technology, $Y(y_t^L, y_t^H)$. This final good is then sold to households in a perfectly competitive market. The final goods producer chooses intermediate inputs to solve the following problem:

$$\max \ \mathbb{E}_t \sum_{t=0}^{\infty} \Xi_{t+1|t} \left[Y(y_t^L, y_t^H) - p_t^L y_t^L - p_t^H y_t^H \right],$$

where: p_t^L and p_t^H are the prices of the low- and high-tech intermediate inputs, respectively, relative to the final good. The demand for each intermediate input equates the marginal product to the price, so that $Y_{L,t} = p_t^L$ and $Y_{H,t} = p_t^H$.

3.2.2 Intermediate Goods Production

Two types of firms engage in the production of intermediate goods. Each operates separately in either the low- or the high-tech sector.

Low-tech Firms. For a given low-tech vacancy, the low-tech firm can match with and hire either a low- or a high-skilled worker. Accordingly, the low-tech firm chooses the desired stock of low-skill employees, n_t^L, the desired stock of high-skill employees, n_t^M, and vacancies, v_t^L, to solve:

$$\max \ \mathbb{E}_t \sum_{t} \Xi_{t+1|t} \left[p_t^L y^L(n_t^L, n_t^M) - w_t^L n_t^L - w_t^M n_t^M - \gamma^L v_t^L \right],$$

subject to the perceived laws of motion for low-skill and mismatch employment stocks, respectively:

$$n_t^L = (1-\rho^L) n_{t-1}^L + \eta_t^L q_t^L v_t^L,$$

and

$$n_t^M = (1-\rho^L) n_{t-1}^M + \left(1-\eta_t^L\right) q_t^L v_t^L,$$

where: q_t^L is the probability that a given vacancy posted in the market for low-tech jobs is successful in finding a worker, regardless of whether the worker is low- or high-skill.

Furthermore, the fraction of low-skill workers in the total pool of individuals searching for low-skill jobs is given by $\eta_t^L \equiv s_t^L/(s_t^L + s_t^M)$. With this notation, the per period probability that a low-tech vacancy turns into an employment match with a low-skill worker is $\eta_t q_t^L$ and the probability that a low-tech vacancy turns into a mismatch employment relationship with a high-skill worker is $(1 - \eta_t)q_t^L$.

The optimal vacancy posting condition is given by:

$$\frac{\gamma^L}{q_t^L} = \eta_t^L \mathbf{J_t^L} + \left(1 - \eta_t^L\right) \mathbf{J_t^M}, \tag{18}$$

where: $\mathbf{J_t^L}$ and $\mathbf{J_t^M}$ are defined by the Lagrangian multipliers on the perceived laws of motion for low-tech and mismatch employment, respectively. Free entry implies that low-tech firm posts vacancies up until the point at which the cost, γ^L, is exactly offset by the expected gain from making a match. The expected gain is the probability that a match is made in the low-tech market, q_t^L, times a probability weighted average of the value of a match with a low-tech worker, $\eta_t^L \mathbf{J_t^L}$, and a (mismatched) high-tech worker, $(1 - \eta_t^L)\mathbf{J_t^M}$. Note that free entry implies that it is always optimal for the low-tech firm to fill an open position with any type of worker it encounters as long as the surplus associated with the employment relationship is positive.

The job creation conditions for low-skill and mismatch employment are given by:

$$\mathbf{J_t^L} = p_t^L y_{1,t}^L - w_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \left\{\Xi_{t+1|t} \mathbf{J_{t+1}^L}\right\}, \tag{19}$$

and

$$\mathbf{J_t^M} = p_t^L y_{2,t}^L - w_t^M + \left(1 - \rho^L\right) \mathbb{E}_t \left\{\Xi_{t+1|t} \mathbf{J_{t+1}^M}\right\}. \tag{20}$$

Both equate the value of a (low-skilled or mismatched, respectively) employee working in the low-tech job to the present discounted value of the stream of marginal revenues that the job produces, net of the wage, over the expected duration of the employment relationship.

High-tech Firms. High-tech firms can only employ high-skilled workers because they are the only ones qualified to do the work. The high-tech firm chooses the stock of high-skill employees, n_t^H, and vacancies, v_t^H, to solves the following profit maximization problem:

$$\max \ \mathbb{E}_t \sum_t \Xi_{t+1|t} \left[p_t^H y^H(n_t^H) - w_t^H n_t^H - \gamma^H v_t^H\right],$$

subject to the perceived law of motion for high-tech employment:

$$n_t^H = (1 - \rho^H) n_{t-1}^H + q_t^H v_t^H,$$

where: q_t^H is the probability that a given vacancy posted in the market for high-tech jobs is successful in finding a worker.

The optimal vacancy posting condition is given by:

$$\gamma^H / q_t^H = \mathbf{J_t^H}, \tag{21}$$

where: $\mathbf{J_t^H}$ is the Lagrangian multiplier on the law of motion for high-tech employment.

The job creation condition for high-skill employment is given by:

$$\mathbf{J_t^H} = p_t^H y_{1,t}^H - w_t^H + \left(1 - \rho^H\right) \mathbb{E}_t \left\{\Xi_{t+1|t} \mathbf{J_{t+1}^H}\right\}. \tag{22}$$

3.3 The Labor Market

In order to close the model, we need to address matching and wage determination in each of the two segmented labor markets.

3.3.1 Matching

Labor market matches are formed according to the same constant returns matching technologies described in Section 2 for low-tech, mismatched, and high-tech jobs, respectively.

3.3.2 Wage Determination

Wages are determined through Nash bargaining. Let $\psi^i \in (0, 1)$ for $i \in (L, H)$ denote the exogenous bargaining power of workers. Nash bargaining we well known in the literature, so for the sake of brevity we present only the wage solution, leaving the details to the Appendix.

The wage for a low-skilled worker employed in a low-tech job is given by:

$$w_t^L = \psi^L p_t^L y_{1,t}^L + \left(1 - \psi^L\right) \chi^L + \psi^L (1 - \rho^L) \mathbb{E}_t \left\{\Xi_{t+1|t} f_{t+1}^L \mathbf{J_{t+1}^L}\right\}. \tag{23}$$

The wage paid by low-tech firms to low-skilled workers is a weighted average of the present discounted value of the stream of marginal revenue that accrues to the low-tech firm from hiring the additional employee and the outside option that accrues to the worker, given by the unemployment benefit.

The mismatch wage is given by the expression:

$$w_t^M = \psi^H p_t^L y_{2,t}^L + \left(1 - \psi^H\right) \chi^H + \psi^H (1 - \rho^L) \mathbb{E}_t \left\{ \Xi_{t+1|t} f_{t+1}^L \mathbf{J}_{\mathbf{t+1}}^{\mathbf{M}} \right\}. \tag{24}$$

Finally, the wage for a high-skilled worker employed in a high-tech job is given by:

$$w_t^H = \psi^H p_t^H y_{1,t}^H + \left(1 - \psi^H\right) \chi^H + \psi^H (1 - \rho^H) \mathbb{E}_t \left[\Xi_{t+1|t} \left(f_{t+1}^H \mathbf{J}_{\mathbf{t+1}}^{\mathbf{H}} \right) \right]. \tag{25}$$

Note that because free entry into vacancy postings drives $\mathbf{J}_{\mathbf{t+1}}^{\mathbf{H}} = \gamma^H / q_{t+1}^H$, the continuation value in equation (25) can also be expressed as $\mathbb{E}_t\{\Xi_{t+1|t}(f_{t+1}^H / q_{t+1}^H)\gamma^H]$.

3.4 Search Equilibrium

The equilibrium of the system is a sequence of allocations and prices $\{c_t^L,\ c_t^H,\ R_t,\ n_t^L,\ n_t^M,\ n_t^H,\ s_t^L,\ s_t^M,\ s_t^H,\ v_t^L,\ v_t^H,\ \mathbf{J}_{\mathbf{t}}^{\mathbf{L}},\ \mathbf{J}_{\mathbf{t}}^{\mathbf{M}},\ \mathbf{J}_{\mathbf{t}}^{\mathbf{H}},\ w_t^L,\ w_t^M,\ w_t^H,\ p_t^L,\ p_t^H, B_t^H, B_t^L, T_t^L, T_t^H\}$ that solves the optimality conditions for: low-skilled households, summarized by equations (13) through (14); high-skilled households, summarized be equations (15) through (17); low-tech intermediate goods producers, summarized by equations (18) through (20); high-tech intermediate goods producers, summarized by equations (21) and (22).

We also have the demand for the low- and high-tech intermediate input, given by $Y_{L,t} = p_t^L / Z_t$ and $Y_{H,t} = p_t^H / Z_t$, respectively; the laws of motion for respective employment stocks, equations (1) through (3); the wages are pinned down by equations (23) through (25).

The bond market clearing condition is given by $B_t^L + B_t^H = 0$. There is a government budget constraint that must be satisfied in order to finance the unemployment benefit, $\kappa T_t^L + (1-\kappa) T_t^H = \chi^L (1 - f_t^L) s_t^L + \chi^H \left[(1 - f_t^L) s_t^M + (1 - f_t^H) s_t^H\right]$. The household budget constraint pins down the level of bond holdings.

Finally, the economy-wide resource constraint is given by:

$$Y_t = c_t^L + c_t^H + \gamma^L v_t^L + \gamma^H v_t^H \tag{26}$$

4 Characterizing the Distortions

The distortions in the decentralized search equilibrium are characterized in a series of propositions. These propositions represent the main contribution of the paper. Taken as a whole, they establish the mismatch distortion and show that it is unique from more standard sources of inefficiency that commonly arise in search and matching models, such as from the presence of unemployment benefits and/or inefficient wage bargaining.

Throughout the remainder of the paper, we assume the matching functions in both the low- and high-tech labor markets are Cobb-Douglas and the parameter governing the elasticity of matches with respect to search unemployment is given by ξ^L and ξ^H, respectively.

The first proposition establishes the conditions that ensure static and dynamic efficiency in the high-tech labor market.

Proposition 1 *In a search equilibrium with bargaining, the necessary and sufficient conditions for static and dynamic efficiency in the high-tech labor market are given by:*

(i.) No unemployment benefits, $\chi^H = 0$;

(ii.) The Hosios (1990) condition, $\xi^H = \psi^H$.

Proof. See Appendix A.1. ∎

The labor market for high-tech employment is a standard search and matching market and, as such, the conditions that deliver efficiency under bargaining are well documented in the literature. Accordingly, we keep our discussion brief.

The next proposition addresses static efficiency in the market for low-tech jobs.

Proposition 2 *In a search equilibrium with bargaining, the necessary and sufficient conditions for static efficiency in the low-tech labor market are given by:*

(i.) No unemployment benefits, $\chi^L = \chi^H = 0$;

(ii.) The Hosios (1990) condition, $\xi^L = \psi^L$;

(iii.) Symmetry across labor markets, so that $\psi^L = \psi^H$.

Proof. See Appendix A.2. ∎

As with Proposition 1, conditions (*i.*) and (*ii.*) are standard in the literature. In contrast, the third condition regarding symmetry is new and deserves further discussion. Intuitively, condition (*ii.*) delivers the efficient wage for low-skilled workers endowed with bargaining power, ψ^L, while condition (*iii.*) delivers the efficient wage for mismatched workers endowed with bargaining power, ψ^H. Given that the elasticity of the matching function with respect to search unemployment, ξ^L, is common for both types of jobs (i.e., they are formed through a common matching function), it must be that efficiency requires $\psi^L = \psi^H = \xi^L$. In this sense, the symmetry condition is nothing more than an additional constraint that is required to ensure bargaining efficiency across both types of jobs. When taken in conjunction with Proposition 1, it highlights the fact that asymmetries in labor market institutions (i.e., $\psi^L \neq \psi^H$ and/or $\xi^L \neq \xi^H$) can lead to distortions that spill over across segmented labor markets. This is true even in the case where $\psi^L = \xi^L$ and $\psi^H = \xi^H$ holds for each market individually.

The next proposition establishes the unique instance in which the decentralized economy is able to achieve the constrained efficient outcome.

Proposition 3 *The search equilibrium with bargaining achieves the constrained efficient equilibrium if and only if* $y^L_{n^L}(n^L_t, n^M_t) = y^L_{n^M}(n^L_t, n^M_t)$ *and Propositions 1 and 2 both hold.*

Proof. See Appendix A.3. ■

The following corollary is a direct result of Proposition 3 and establishes a link between productivity differentials and dynamic efficiency in low-tech and mismatch employment. This is the mismatch distortion, which is a key result of the paper.

Corollary 1 *In a search equilibrium with bargaining, productivity differential in low-tech production (i.e.,* $y^L_1(n^L_t, n^M_t) \neq y^L_2(n^L_t, n^M_t)$*) violate dynamic efficiency for both low-tech and mismatch employment. In this case, the private equilibrium is not constrained efficient, regardless of whether or not Proposition 1 and 2 both hold.*

The mismatch distortion is a form of congestion externality that arises because low- and high-skilled workers compete to match with a fixed number of low-tech vacancies. This competition is distortionary as long as there is heterogeneity in the private valuation of leisure. This heterogeneity only occurs when there is a productivity gap between the two labor inputs. In this case, high-skilled workers searching for mismatched jobs do not internalize the fact that increasing search effort can crowd in or crowd out the job finding prospects of low-skilled workers who are looking to match with the same set of vacancies. Similarly, low-skilled workers do not internalize the fact that their search behavior affects the job finding prospects of high-skilled workers looking for mismatch employment. Neither agent participates in the labor market in a way that is consistent with the planner's view of the social valuation of leisure.

When there is no productivity gap low- and high-skilled workers value leisure symmetrically in a way that aligns with the social value of leisure. Accordingly, the intuition behind Proposition 3 is straightforward: the mismatch distortion is eliminated due to the lack of any meaningful heterogeneity in the objective functions of the two types of households.

5 Competitive Search Equilibrium

The mismatch distortion is derived assuming wages are determined via bargaining. This raises the question of whether it is robust to other wage determination protocols.

In this section, we show that while it is *technically* feasible to decentralize the efficient set of wages, doing so requires an extreme—an we think unreasonable—assumption about the ability of the firm to effectively segment the low-skilled labor market.

We consider wage posting in a competitive search equilibrium (CSE) as described in Moen (1997). The firm understands there is a tradeoff between the wage that it posts in a given que and the number of workers that will join that que in order to try to match with that particular posting. While posting a lower wage may increase the value of a potential match to the firm, doing so makes that same match harder to fill because the lower wage is less attractive to searching workers.

In order to implement wage posting, the high-tech firm chooses w_t^H and θ_t^H to maximize the value of a high-tech vacancy, given by equation (21), subject to optimal participation condition for high-skilled households, equation (16). The resulting surplus sharing rule is

$$\frac{\mathbf{W_t^H} - \mathbf{U_t^H}}{\mathbf{J_t^H}} = \frac{\xi^H}{1 - \xi^H} \tag{27}$$

where: $\mathbf{W_t^H} - \mathbf{U_t^H} = \frac{h_t^{H\prime} - \chi^H u_{c,t}^H}{f_t^H u_{c,t}^H}$ is the value of a high-tech job to a high-skilled worker as defined by equation (16). This equation implicitly determines the high-tech wage.

The low-tech firm chooses differentiated wages, w_t^L and w_t^M, as well as the effective que lengths for each type of worker, by choosing θ_t^L and η_t, in order to maximize the expected value of a low-tech vacancy, given by equation (18), subject to the participation constraints for low-tech and mismatch search activity, given by equations (14) and (17), respectively.

The solution to this problem gives rise to two conditions which, together, implicitly pin down the wages for low-skilled and mismatch jobs. The first condition states that the firm posts wages in a way that equates the value of low-skilled and mismatch workers.

$$\mathbf{J_t^L} = \mathbf{J_t^M} \tag{28}$$

The second condition gives rise to the following surplus sharing rule:

$$\frac{\eta_t(\mathbf{W_t^L} - \mathbf{U_t^L}) + (1-\eta_t)(\mathbf{W_t^M} - \mathbf{U_t^M})}{\eta_t \mathbf{J_t^L} + (1-\eta_t)\mathbf{J_t^M}} = \frac{\xi^L}{1-\xi^L} \tag{29}$$

where: $\mathbf{W_t^L} - \mathbf{U_t^L} = \frac{h_t^{L\prime} - \chi^L u_{c,t}^L}{f_t^L u_{c,t}^L}$ is the value of a low-tech job to a low-skilled worker as defined by equation (14) and $\mathbf{W_t^M} - \mathbf{U_t^H} = \frac{h_t^{M\prime} - \chi^H u_{c,t}^H}{f_t^L u_{c,t}^M}$ is the value of a low-tech job to a high-skilled worker as defined by equation (17).

In order to solve for the CSE, we replace equations (23) through (25), which determine wages in the search equilibrium with bargaining, with equations (27) through (29), which determine the posted wages.

The results are summarized by the following proposition.

Proposition 4 *As long as $\chi^L = \chi^H = 0$, the competitive search equilibrium with wage posting gives rise to a constrained efficient equilibrium. This is true regardless of the relative productivity of low-skilled and mismatch workers.*

Proof. See Appendix A.4. ■

The proposition establishes that, in absence of an unemployment benefit, it is *technically feasible* decentralize the constrained efficient equilibrium. Intuitively, the low-tech firm's objective is to post differentiated wages that maximize the expected value of a match. In solving this problem, the firm internalizes the distortion stemming from job competition and posts wages in a way that elicits optimal search on the part of heterogeneous job seekers.

One reaction might be that Proposition 4 implies the mismatch distortion is nothing more than an artifact of an arbitrary assumption regarding the structure of the frictional labor market (i.e., it reflects wage bargaining). In other words, as long as wages are posted in a competitive search equilibrium, the novelty of the result disappears.

This reaction is warranted provided the assumptions underlying the firm's ability to post wages in the CSE are reasonable. We argue they are not. In particular, implementing the CSE requires the firm to post wages in a way that allows competing job seekers to trade off their *individual* labor compensation against *type-specific* job finding probabilities.[5] Effectively, this amounts to giving the firm the ability to segment the low-tech labor market. It is not at all surprising that this resolves the mismatch distortion. Ultimately, the inefficiency due to mismatch depends on job competition and segmentation eliminates this competition.

While Proposition 4 establishes that it is *technically feasible* to decentralize the efficient equilibrium, doing so requires what we view as an unrealistic assumption with regard to the power the firm has in posting wages. Viewed this way, it is more difficult to dismiss the distortion as imply a function of the bargaining assumption. Indeed, the mismatch distortion extends to any equilibrium in which wages are determined by a time invariant surplus sharing rule, be it the solution of a bargaining problem or otherwise.

6 Quantitative Results

A calibrated version of the model is used to illustrate how the mismatch distortion affects the labor market outcomes and welfare of low- and high-skilled households.

[5]Specifically, it does this by choosing not only w_t^L and w_t^M, but also the type-specific labor market tightness through its simultaneous choice of θ_t^L and η_t.

6.1 The Benchmark Economy

The benchmark economy is socially efficient. There are no unemployment benefits, a symmetric Hosios condition is imposed across both labor markets, and low-skilled and mismatched workers are equally productive, so there is no mismatch distortion.

6.1.1 Parameterization

Our calibration, summarized in Table 1, is at the weekly frequency. We use data on educational attainment to calibrate worker heterogeneity and data on employment by occupation to calibrate firm heterogeneity. Where applicable, we also use aggregate labor market data.

The empirical counterparts to our low- and high-tech sectors correspond to routine and non-routine occupations, respectively, as per BLS occupational classifications. With this dichotomy, we use the BLS occupational outlook handbook to obtain educational attainment requirements for entry-level positions by occupation. Only 14 percent of routine jobs require at least some post-secondary education, while the same is true for roughly 82 percent of non-routine jobs. Accordingly, we interpret low-skill workers as those with at most a high school degree and high-skill workers as those with at least some post-secondary education. Moreover, data from the BLS shows that about one-half the U.S. population has at most a high school degree, so we set $\kappa = 0.5$.

With regard to preferences, because we assume that the time period is equal to one week we set the discount factor $\beta = 0.999$, which is consistent with an annual interest rate of 5 percent. We assume a standard functional form for the sub-utility over consumption for both low- and high-skilled individuals:

$$u(c_t^i) = \frac{1}{1-\sigma}\left(c_t^i\right)^{1-\sigma} \text{ for } i \in (H, L).$$

and set $\sigma = 1$ for $i \in (H,L)$.

The sub-utilities over labor activity for low- and high-skilled individuals, respectively are:

$$h(lfp_t^L) = \frac{\phi^L}{1+1/\varepsilon}\left(n_t^L + (1-f_t^L)s_t^L\right)^{1+1/\varepsilon},$$

and

$$h\left(lfp_t^H\right) + h\left(lfp_t^M\right) = \begin{bmatrix} \frac{\phi^H}{1+1/\varepsilon}\left(n_t^H + (1-f_t^H)s_t^H\right)^{1+1/\varepsilon} \\ +\frac{\phi^M}{1+1/\varepsilon}\left(n_t^M + (1-f_t^L)s_t^M\right)^{1+1/\varepsilon} \end{bmatrix},$$

where: $\phi^i > 0$, for $i \in (H,L)$, and $\varepsilon > 1$ are parameters.

In parameterizing preferences over labor market activity, quadratic labor disutility (so that $\varepsilon = 1$) implies that aggregate labor force participation rate is highly inelastic with respect to output per worker, which is in line with the data.[6] The average labor force participation rate in the U.S. is 0.631. Moreover, BLS data show that the average participation rate of individuals with at least some post-secondary education is 1.33 times higher than the participation rate of individuals with at most a high school education. We calibrate the scaling parameters, ϕ^L and ϕ^H, to target these participation rates. The scaling parameter for the disutility of mismatch employment for high-skilled individuals, ϕ^M, is calibrated to target a steady-state ratio of total employment in high- to low-tech jobs of $n^H/(n^L + n^M) = 1.11$, which is the average ratio of employment in non-routine to routine occupations in the U.S.

Table 1: Baseline parameterization

Preference parameters	
Discount factor, β	0.999
Utility curvature, σ	1
Elasticity of participation, ε	1
Scaling for disutility of low-skill participation, ϕ^L	9.592
Scaling for disutility of mismatch participation, ϕ^M	79.732
Scaling for disutility of high-skill participation, ϕ^H	11.091
Production parameters	
Aggregate technology, Z	0.278
Input-specific technologies, $z^H = z^M = z^L$	1
High-skill share in final goods, ϱ	0.582
Final goods input substitutability, ω	0.400
Labor market parameters	
Fraction of low-skill population, κ	0.500
Vacancy flow costs, $\gamma^H = \gamma^L$	0.200
Low-tech job destruction probability, ρ^L	0.012
High-tech job destruction probability, ρ^H	0.007
Low-tech matching efficiency, A^L	0.185
High-tech matching efficiency, A^H	0.158
Matching function elasticity, $\xi^L = \xi^H$	0.500
Worker bargaining power, $\psi^H = \psi^L$	0.500
Unemployment benefits, $\chi^L = \chi^H$	0

For production, we assume that output of final goods is a CES aggregate of the low- and high-tech intermediate good, so that:

$$Y_t = Z_t \left(\varrho \left(y_t^H\right)^{\omega} + (1-\varrho)\left(y_t^L\right)^{\omega_F}\right)^{1/\omega},$$

where: Z_t is aggregate productivity; $\varrho \in (0{,}1)$ is the share of the high-tech intermediate input

[6]Our assessment of this elasticity comes from using quarterly data on real GDP from the Bureau of Economic Analysis and data on aggregate employment and the aggregate labor force participation rate from the BLS.

in final goods production; and ω governs the degree of substitutability between the high- and low-tech goods in final goods production. In turn, production of the high-tech good is given by $y_t^H = z_t^H n_t^H$, where: z^H is an input-specific technology parameter. Similarly, production of the low tech good is linear in low-skill and mismatch employment relationships; in other words, this good can be produced with only low-skill labor, only mismatch labor, or both:

$$y_t^L = z_t^L n_t^L + z_t^M n_t^M,$$

where: z_t^L and z_t^M are input-specific technology parameters. The steady state values of z^H, z^L, and z^M are normalized to 1. In contrast, the value of Z is chosen to normalize steady state aggregate output so that at quarterly frequency $Y = 1$.

The remainder of the production parameters are either chosen based on the existing literature or calibrated to match empirically observed wage differentials. For final output we follow Krusell, Ohanian, Rios-Rull, and Violante (2000) and set ω equal to 0.4. Setting $z^L = z^M$ equates the mismatch and low-skill wage and eliminates the mismatch distortion ($z^H = 1$ is a normalization). For the share parameter in the final goods aggregator ϱ we draw on occupational wage data from the BLS. The employment-weighted median wages of individuals employed in nonroutine occupations is 1.35 times that of employment-weighted median wages of individuals employed in routine occupations. Accordingly, we choose ϱ so that $w^H/W^L = 1.35$, where $W^L = (w_t^L n_t^L + w_t^M n_t^M)/(n_t^L + n_t^M)$.

Turning to the labor market, we assume that both the low- and high-tech job markets are characterized by a standard Cobb-Douglas matching function:

$$m_t^i = A^i \left(e_t^i\right)^{\xi^i} \left(v_t^i\right)^{1-\xi^i}, \text{ for } i \in \{L,H\},$$

where: A^i is matching efficiency; and ξ^i is the elasticity of the matching function with respect to total search search activity in a market, which we denote by e^i. We set $\xi^i = 0.5$ for $i \in \{L, H\}$, which is broadly in line with Petrongolo and Pissarides (2001).

The matching efficiency parameters, A^L and A^H, are jointly calibrated to hit empirical targets that we obtain from both aggregate and sector-specific data on job finding probabilities. Starting with the aggregate data and following the methodology in Elsby, Michaels, and Solon (2009) and Shimer (2012), monthly data on unemployment since 1951 reveal that the probability that an average unemployed individual matches with a job within a week is

0.132. Thus, one calibrating target for the two matching efficiency parameters is the steady-state value $\frac{m^H + m^L}{s^L + s^M + s^H} = 0.132$. Moving to the sector-specific data, we find that since 2000 the average job-finding probability of individuals last employed in routine occupations is 0.99 times that of individuals last employed in nonroutine occupations. Assuming that an individual's last occupation is roughly indicative of their skill level, our second calibrating target for the matching efficiency parameters is the steady-state value: $\frac{m^L/(s^L + s^M)}{m^H/s^H} = 0.99$.

The exogenous job destruction probabilities ρ^L and ρ^H are calibrated using BLS data on aggregate and occupation-specific unemployment rates. These data show that the average U.S. unemployment rate since 1951 is 0.058, so one of the job destruction rates is pinned down by targeting the steady-state ratio $(u^L + u^H)/(lfp^L + lfp^H) = 0.058$. In addition, these data also show that the average unemployment rate of individuals last employed in nonroutine occupations is about 1.62 times as high as that of individuals last employed in nonroutine occupations. So, we pin down the second job destruction rate by targeting the steady-state ratio $\frac{u^L}{lfp^L}/\frac{u^H}{lfp^H} = 1.62$.

We assume symmetry in the vacancy posting costs, $\gamma^H = \gamma^L$, and calibrate these costs to target the ratio of aggregate vacancies to aggregate unemployment: $\frac{v^L + v^H}{(1 - f^L)s^L + (1 + f^H)s^H} = 0.68$. The target for this ratio results from using data on aggregate job openings from the BLS Job Openings and Labor Turnover Survey since 2000 (when first available) combined with the Conference Board's Help-Wanted Index from 1951 through 2000 together with time series for aggregate U.S. unemployment.

Unemployment benefits are set to zero, $\chi^L = \chi^H = 0$, to ensure that the private equilibrium in the benchmark economy is efficient. Similarly, we assume symmetry in bargaining power, so that $\psi^H = \psi^L = 0.5$. This parameterization has the virtue that, absent any other distortion in the model, $\psi^H = \psi^L = \xi^H = \xi^L$ delivers both an efficient split of match surplus (see Hosios (1990)) as well as cross-market efficiency.

6.1.2 Allocations in the Efficient Equilibrium

Table 2 presents allocations in the baseline economy in which the private equilibrium coincides with the socially efficient equilibrium (that is, *Case iii.* in Section 2.1).

Table 2: Benchmark Economy (Quarterly Frequency)

	Efficient Private Equilibrium	
	Low-skilled Household	*High-skilled Household*
	Aggregate Variables	
1. c^L, c^H	0.358	0.358
2. LFP rate (L,H)	0.542	0.719
	Labor Market Variables	
3. n^L, n^H	0.251	0.313
n^M	–	0.030
4. s^L, s^H	0.023	0.016
s^M	–	0.003
5. v^L, v^H	0.013	0.012
6. θ^L, θ^H	0.509	0.710
7. f^L, f^H	0.748	0.782
8. q^L, q^H	0.989	0.937
9. Unemp. rate (L,H)	0.075	0.046
10. η	0.893	–

Consumption is equal across the two households. Accordingly, all of the heterogeneity is forced into the labor market with the participation rate for low-skilled households at 54.2% as opposed to 71.9% for high-skilled households. Vacancy postings are broadly similar across the two segmented labor markets, but there is a notable difference in labor market tightness reflecting relatively more job seeking activity in the market for low-tech jobs. The last line of the table shows that low-skilled workers constitute only 89.3% of the unemployment pool for low-tech jobs, with high-skilled workers seeking mismatch jobs making up the rest. High-skilled participation in the low-tech labor market pushes down the effective job finding rate for low-skilled workers. This (efficient) crowding out in the unemployment pool results in a notable difference in the unemployment rate across the two types of workers, which stands at 7.5% for the low-skilled household and 4.6% for the high-skilled household.

6.2 The Mismatch Distortion

We turn now to a quantitative illustration of the mismatch distortion, for which we focus on steady states. The exercises that follow focus on how private allocations change as the productivity of mismatched workers is altered relative to low-skilled workers, spanning the three cases highlighted in Section 2.1.

Table 3 shows selected allocations in the presence of the mismatch distortion under two different assumptions regarding relative productivity. Panel A presents the case where mismatch workers are 10% more productive than low-skilled counterparts in low-tech employment (that is, $z^M/z^L = 1.1$, corresponding to *Case i.* in Section 2.1). The first two columns of Panel A present the efficient allocations while the second two show the allocations in the private economy. When mismatched workers have comparative advantage in the production of the low-tech good, the high-skilled household experiences a welfare gain on the order of 0.9% of steady state consumption. (Welfare differentials are measured in terms of consumption equivalence: see table footnote for details.) This welfare gain comes almost entirely at the expense of the low-skilled household, which suffers a cost of roughly equal magnitude implying the distributional nature of the mismatch distortion washes out in aggregate.

The remaining rows in Panel A show the loss in consumption is negligible and shared evenly across households. In contrast, the mismatch distortion operates largely through differences in labor market activity. Participation for the (more productive) high-skilled household is inefficiently low in the private equilibrium and for the low-skilled household it is inefficiently high. Accordingly, the skill-composition of the low-tech unemployment pool shifts in such a way that there are too many low-skilled workers. A social planner would like to increase the participation of high-skilled households in mismatch job activity.

The distributional nature of the welfare effects stems from the fact that, on the one hand, both households share the welfare costs associated with lower consumption in the private equilibrium. But, at the same time, the high-skilled worker is able to exploit of his/her comparative advantage in productivity by enjoying leisure and shifting an inefficiently high burden of production onto the low-skilled worker.

Panel B presents the case where low-skilled workers are 10% more productive than mismatched workers ($z^M/z^L = 0.9$, corresponding to *Case ii.* in Section 2.1). The results are somewhat similar quantitatively but, in this case, low-skilled households gain 0.8% percent of steady state consumption at the expense of the high-skilled households.

Figures 1 through 5 shed additional light on the distributional nature of the welfare effects. In all of the figures, we report differences in steady state allocations between the efficient and the private equilibrium (either in levels or in percent, as noted) for $z^M/z^L \in [0.2, 1.4]$. This range for relative productivity includes the baseline economy (Case *iii.* presented in Table 2), which is denoted by the solid dots in each panel, as well as *Cases i* and *ii* (as presented in Panels A and B of Table 3), denoted by the hollow dots to the right and the left, respectively, of the solid dot in each panel.

Table 3: Allocations under the mismatch distortion (Quarterly Frequency)

	Panel A. $y_1^L(n_t^L, n_t^M) < y_2^L(n_t^L, n_t^M)$				Panel B. $y_1^L(n_t^L, n_t^M) > y_2^L(n_t^L, n_t^M)$			
	Efficient Equilibrium		Private Equilibrium		Efficient Equilibrium		Private Equilibrium	
	Low	*High*	*Low*	*High*	*Low*	*High*	*Low*	*High*
Welfare Costs								
1. Hh. welfare	−−	−−	0.896	−0.885	−−	−−	−0.768	0.777
2. Agg welfare	−−		0.002		−−		0.002	
Aggregate and Labor Market Variables								
3. c^L, c^H	0.360	0.360	0.360	0.360	0.357	0.357	0.357	0.357
4. LFP rate (L,H)	0.476	0.761	0.484	0.754	0.486	0.750	0.480	0.757
5. Unemp. rate (L,H)	0.091	0.052	0.091	0.052	0.092	0.051	0.092	0.052
6. η^L	0.882	–	0.895	–	0.903	–	0.891	–

Notes: Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low-and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains.

Figure 1 shows how the change in z^M/z^L affects the marginal product of labor for low-skilled, mismatch, and high-skilled workers (left panel) as well as relative wages (right panel). In the baseline economy, the MPL of low-skilled and mismatched workers is equal and the corresponding wage ratio is given by $w^M/w^L = 1$. At the same time, the wage premium for high-tech employment is $w^H/w^M = 1.6$. Moving to the right of the baseline, an increase in z^M/z^L pushes up the wage premium for mismatch over low-skilled workers, so $w^M/w^L > 1$ while the high-tech premium declines getting closer to 1 as z^M/z^L approaches the upper bound of its range in the exercise. Moving to the left of the baseline, as z^M/z^L decreases low-skilled workers command a wage premium over mismatched workers, so that $w^M/w^L < 1$, and the high-tech wage premium increases sharply.

Components of the utility function are shown in Figure 2. Private consumption, in the left panel, is equal across households and is inefficiently low for any point other than the baseline. In contrast, the right panel shows heterogeneity in the response of labor force participation. When mismatch workers have a comparative advantage in low-tech production participation for high-skilled households is inefficiently low in the market for low-tech jobs. In contrast, participation for low-skilled agents is inefficiently high. The opposite is true when low-skilled workers have a comparative advantage.

Figure 3 shows the distributional welfare effects. Low-skilled household suffer welfare costs from mismatch when they are relatively less productive in low-tech production. When low-skilled workers are relatively more productive they gain at the expense of mismatch workers. That said, the dotted grey line in the center shows that these costs and benefits wash out at the aggregate level suggesting that the mismatch distortion is largely distributional.

The implications for labor market tightness and unemployment rates are shown in the bottom two panels of Figure 4. Market tightness is always inefficiently low in the low-tech sector, which translates into excess unemployment for low-skilled workers. For the high-skilled household, whether or not unemployment is too high or too low relative to the constrained efficient equilibrium depends on relative productivity in the low-tech market.

Finally, Figure 5 shows that any deviation in the efficient composition of the low-tech labor pool (right panel) shows up in the form of inefficiently low average match quality in the low-tech firm (left panel). Together, the two panels suggest the planner's objective is to manage the composition of the low-tech labor pool to maximize expected match quality.

6.3 Wage Differentials: Bargaining vs. CSE

The differential between the bargained wage and the posted wage under the competitive search equilibrium is shown in Figure 6 for each of the three job types as relative productivity varies over the interval $z^M/z^L \in [0.75, 1.25]$. The high-tech wage differential, shown by the solid line, is flat at zero because there is no distortion in the high-tech labor market, hence under this parameterization the Nash wage and the efficient competitive wage coincide. In the case of the low-tech labor market, the wage differential for low-skilled and mismatch jobs both cross zero at $z^M/z^L = 1$, where the mismatch distortion is eliminated. To the right of this point, where $z^M/z^L > 1$, the Nash wage is too high relative to the efficient wage for low-skilled workers and it is too low relative to the efficient wage for mismatched workers. The opposite is true for $z^M/z^L < 1$.

One way to interpret the wage differentials is that they implicitly reveal something about how optimal labor market policy would operate in the model. For example, it is feasible to introduce a set of proportional wage taxes/subsidies that would allow the bargained wages to replicate their efficient counterpart in the CSE. For cases in which $z^M/z^L > 1$, it is clear that the optimal tax policy would call for a tax on the low-skilled wage (to reduce inefficiently high participation of the low-skilled household) and a subsidy on the mismatch wage (to increase inefficiently low participation of the high-skilled household in the low-tech labor market).[7] Note that, in order to restore efficiency, an optimizing fiscal authority would need the power to levy differentiated taxes/subsidies on the two types of labor. This type of distributional fiscal policy might not be feasible to implement.

6.4 The Size of the Mismatch Distortion

As a final exercise, we measure the welfare costs associated with each of the three different sources of inefficiency in the model in order to gauge their relative size. For this section, let $\chi^H = \chi^L = \chi$ and $\psi^H = \psi^L = \psi$.

Table 4: Welfare costs associated with distortions in the model

	Parameterization		
	$\chi = 0$; $\psi = 0.5$	$\chi = 0$; $\psi = 0.55$	$\chi = 0.05$; $\psi = 0.5$
		Panel A: $z^M/z^L = 1$	
Low-skilled Hh.	0	1.7	30
High-skilled Hh.	0	0.2	2.4
		Panel B: $z^M/z^L = 0.9$	
Low-skilled Hh.	0.9	1.0	28.8
High-skilled Hh.	-0.9	0.9	3.8
		Panel C: $z^M/z^L = 1.1$	
Low-skilled Hh.	-0.8	2.6	31.3
High-skilled Hh.	0.8	-0.6	0.9

Notes: Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Positive numbers indicate welfare costs and negative numbers indicate gains.

Results are presented in Table 4. In the top panel, where $z^M/z^L = 1$, the first column shows that welfare costs are zero in the efficient equilibrium with $\chi = 0$ and $\psi = 0.5$. Moving to the second column, keeping the unemployment benefit at zero, $\chi = 0$, and introducing the bargaining distortion in isolation, $\psi = 0.55 > \xi$ results in a roughly 2% welfare cost for the low-skilled household, while the effect on the high-skilled household is more muted. Finally, the third column shows the case in which the bargaining distortion is zeroed out, so

[7] Typically, optimal fiscal policy in a general equilibrium labor search model has the result that a proportional labor tax and a vacancy subsidy are redundant tax instruments (i.e., both appear simultaneously in the same efficiency conditions). It is interesting to note that this redundancy breaks down in this model. Though its beyond the scope of the paper, it is clear that optimal labor market policy here calls for *two separate instruments* to elicit optimal search from firms as well as heterogeneous job seekers.

that $\psi = \xi$, and only the distortion from the unemployment benefit operate, $\chi = 0.05$. The welfare costs are considerably larger for both types of households. Panels B and C present the same set of results varying relative productivity to $z^M/z^L = 0.9$ and $z^M/z^L = 1.1$, respectively. The size of the mismatch distortion is similar in magnitude to the bargaining distortion.

Finally, Figure 7 shows there is an important nonlinear interaction between the three distortions. For the case in which $z^M/z^L = 1$, the welfare costs for both types of agents blow up as χ gets larger and as ξ gets farther away from its efficient parameterization.

7 Conclusion

A model is presented in which mismatch employment exists in a constrained efficient equilibrium. However, in the private economy the presence of mismatch employment creates a form of congestion externality. We provide an analytic characterization of the mismatch distortion, assess the distributional nature of the associated welfare effects, and relate it to the relative productivity of low- and high-skilled workers competing for similar jobs.

Our model emphasizes tractability. The assumptions that deliver this tractability naturally open the door for a number of possible extensions. To keep the model simple, we employ two-by-two heterogeneity in households and firms. It might be interesting to incorporate richer heterogeneity. A natural extension would be to allow firms to rank applicants and select the ones most suitable for a job. Finally, it is also interesting to allow for on-the-job search, which we examine in Arseneau and Epstein (2014), as well as the potential for lock-in to mismatch employment due to skill deterioration in the spirit of Pissarides (1994).

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A Proofs

The general approach for all proofs involves deriving a set of efficiency conditions in the decentralized equilibrium and then comparing them to their socially efficient counterparts. We sketch the derivation for both sets of efficiency conditions and relegate the algebraic details to an online appendix. The proofs then amount to simply establishing the conditions under which the private and social efficiency conditions coincide.

A.1 Proof of Proposition 1

To establish the necessary and sufficient conditions for efficiency in the high-tech labor market, we need to show the conditions under which the both the static and dynamic private efficiency conditions coincide with their socially optimal counterparts.

We begin by establishing static efficiency in high-tech job search. Combine the planner's first order conditions for v_t^H and s_t^H to get the following expression:

$$\gamma^H \frac{m_{s,t}^H}{m_{v,t}^H} = \frac{h_t^{H\prime}}{u_{c,t}^H} \tag{A.1.1}$$

which is equation (7) in the paper.

Deriving the privately optimal counterpart involves combining the high-tech firm's first order conditions for v_t^H and s_t^H along with the expression for the Nash wage, making use of the fact that the high-skilled household's optimal participation condition defines the value of a high-tech job, $\mathbf{W_t^H} - \mathbf{U_t^H}$. The resulting expression yields:

$$\gamma^H \frac{m_{s,t}^H}{m_{v,t}^H} = \frac{1-\psi^H}{\psi^H} \frac{\xi^H}{1-\xi^H} \left(\frac{h_t^{H\prime}}{u_{c,t}^H} - \chi^H \right) \tag{A.1.2}$$

Comparing (A.1.1) and (A.1.2), it is clear that static efficiency requires $\chi^H = 0$ and $\psi^H = \xi^H$ for the two to coincide, which are conditions ($i.$) and ($ii.$) in the proposition.

All we have left to do is to show that these same two conditions also deliver dynamic efficiency. Start by combining the planner's first order conditions for n_t^H and v_t^H to get:

$$\frac{\gamma^H}{m_{v,t}^H} = Y_{3,t} + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1} \left[\frac{\gamma^H}{m_{v,t+1}^H} - \frac{h_{t+1}^{H\prime}}{u_{c,t+1}^H} \right] \tag{A.1.3}$$

Note that using A.1.1 to substitute in for $\gamma^H / m_{v,t}^H$ on the left hand side and iterating the resulting expression ahead recursively gives equation (8) in the paper.

The privately optimal analog is derived by combining the firm's first order conditions for v_t^H and s_t^H into a single job creation condition and then substituting the Nash wage into the resulting expression. Finally, the last step involves substituting in equation (A.1.2) and rearranging terms to get:

$$\frac{\gamma^H}{m_{v,t}^H} = \frac{1-\psi^H}{1-\xi^H} \left(Y_{3,t} - \chi^H\right) + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1|t} \left[\frac{\gamma^H}{m_{v,t+1}^H} - \frac{1-\psi^H}{1-\xi^H} \left(\frac{h_t^{H\prime}}{u_{c,t}^H} - \chi^H \right) \right] \tag{A.1.4}$$

Comparing equation (A.1.3) and (A.1.4), it is clear that $\chi^H = 0$ and $\psi^H = \xi^H$ are necessary and sufficient conditions for efficiency along both the static and dynamic high-tech margins. ∎

A.2 Proof of Proposition 2

We establish the necessary and sufficient conditions for static efficiency in the low-tech job market following a similar approach as above. In the socially optimal equilibrium, combine the planner's first order conditions on s_t^L, s_t^M, e_t, and v_t^L to get the following expression:

$$\gamma^L \frac{m_{s,t}^L}{m_{v,t}^L} = \eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1 - \eta_t^L\right) \frac{h_t^{M\prime}}{u_{c,t}^H}, \tag{A.2.1}$$

which is equation (9) in the paper.

Deriving the private analog to equation (A.2.1) is a multi-step process. First, combine the first order condition for v_t^L with the low-skilled household's participation condition, which defines $\mathbf{W_t^L} - \mathbf{U_t^L}$. Next, substitute in the Nash wage paid to low-skilled workers and multiply the resulting expression by the weight, η_t. In the second step, we do the same thing with the high-skilled household's optimal participation condition, which defines $\mathbf{W_t^M} - \mathbf{U_t^H}$, and apply the Nash wage for mismatched workers. Multiply the resulting expression by $1 - \eta_t$ and add it to the expression obtained in the first step.

The result is the static efficiency condition for search activity in the low-tech job market in the private equilibrium:

$$\gamma^L \frac{m_{s,t}^L}{m_{v,t}^L} = \frac{\xi^L}{1 - \xi^L} \left[\eta_t^L \left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \chi^L \right) \frac{1 - \psi^L}{\psi^L} + \left(1 - \eta_t^L\right) \left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \chi^H \right) \frac{1 - \psi^H}{\psi^H} \right] \tag{A.2.2}$$

Equations (A.2.1) and (A.2.2) coincide only when $\chi^L = \chi^H = 0$, $\xi^L = \psi^L$, and $\psi^L = \psi^H$, which are conditions (*i.*), (*ii.*), and (*iii.*) in the proposition. ∎

A.3 Proof of Proposition 3

Dynamic efficiency in the low-tech job market is characterized by the social planner's job creation conditions for low-skilled and mismatched jobs, separately.

Substitute the planner's first order condition for v_t^L into the first order condition for n_t^L and then substitute in the first order conditions for s_t^M, e_t^L, and η_t. After rearranging and making use of the static efficiency condition, equation (A.2.1), the result is the following efficient condition for low-skilled job creation:

$$\frac{\gamma^L}{m_{v,t}^L} - \left(1 - \eta_t^L\right) \Gamma_t^L = Y_{1,t} + \left(1 - \rho^L\right) \mathbb{E}_t \Xi_{t+1} \left\{ \frac{\gamma^L}{m_{v,t+1}^L} - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \left(1 - \eta_{t+1}^L\right) \Gamma_{t+1}^L \right\} \tag{A.3.1}$$

The derivation of the planner's efficiency condition for mismatch job creation follows an analogous approach starting with the first order conditions for v_t^L and n_t^M and ultimately results in the following:

$$\frac{\gamma^L}{m^L_{v,t}} + \eta^L_t \Gamma^L_t = Y_{2,t} + \left(1-\rho^L\right) \mathbb{E}_t \Xi_{t+1} \left\{ \frac{\gamma^L}{m^L_{v,t+1}} - \frac{h^{M\prime}_{t+1}}{u^L_{c,t+1}} - \left(1-\eta^L_{t+1}\right) \Gamma^L_{t+1} \right\}, \tag{A.3.2}$$

In both equations (A.3.1) and (A.3.2), define $\Gamma^L_t \equiv (1/\eta^L_t f^L_t)\left(h^{M\prime}_t/u^H_{c,t} - \gamma^L m^L_{s,t}/m^L_{v,t}\right)$. Note that substituting in the static efficiency condition on the left side of either expression and iterating forward gives equations (10) and (11) in the main text.

Deriving the analogous private efficiency conditions requires substituting the appropriate Nash wage expression into the firm's first order condition for n^L_t and n^M_t, respectively, and combine the two using the firm's first order condition for v^L_t. The resulting expression can be simplified using the Nash bargaining solution and ultimately gives:

$$\begin{aligned} \frac{\gamma^L}{m^L_{v,t}} &= \eta^L_t \frac{1-\psi^L}{1-\xi^L}\left(Y_{1,t} - \chi^L\right) + \left(1-\eta^L_t\right) \frac{1-\psi^H}{1-\xi^L}\left(Y_{2,t}-\chi^H\right) \\ &+\eta^L_t(1-\rho^L)\mathbb{E}_t \left[\Xi_{t+1|t}\left(1 - \frac{\psi^L}{\xi^L} m^L_{s,t+1}\right) \frac{1-\psi^L}{1-\xi^L} \frac{\xi^L}{\psi^L} \frac{1}{m^L_{s,t+1}} \left(\frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}} - \chi^L\right)\right] \\ &+ \left(1-\eta^L_t\right)\left(1-\rho^L\right)\mathbb{E}_t \left[\Xi_{t+1|t}\left(1 - \frac{\psi^H}{\xi^L} m^L_{s,t+1}\right) \frac{1-\psi^H}{1-\xi^L} \frac{\xi^L}{\psi^H} \frac{1}{m^L_{s,t+1}} \left(\frac{h^{M\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H\right)\right] \end{aligned}$$

Apply the conditions in Proposition 1 and 2, so that $\chi^L = \chi^H = 0$, $\xi^L = \psi^L$, and $\psi^L = \psi^H$ allowing us to write the privately optimal low-tech job creation condition as:

$$\begin{aligned} \frac{\gamma^L}{m^L_{v,t}} &= \eta^L_t Y_{1,t} + \left(1-\eta^L_t\right) Y_{2,t} + \eta^L_t(1-\rho^L)\mathbb{E}_t \left[\Xi_{t+1|t}\left(1 - m^L_{s,t+1}\right) \frac{1}{m^L_{s,t+1}} \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}} \right] \\ &+ \left(1-\eta^L_t\right)\left(1-\rho^L\right)\mathbb{E}_t \left[\Xi_{t+1|t}\left(1 - m^L_{s,t+1}\right) \frac{1}{m^L_{s,t+1}} \frac{h^{M\prime}_{t+1}}{u^H_{c,t+1}} \right] \end{aligned} \tag{A.3.3}$$

As long as $0 < \eta_t < 1$, the only way in which equation (A.3.3) jointly coincides with both equations (A.3.1) and (A.3.2) is when $Y_{1,t} = Y_{2,t} \equiv Y_{L,t}$. ∎

A.4 Proof of Proposition 4

Establishing efficiency in the competitive search equilibrium requires showing the conditions under which the static and dynamic private efficiency conditions for both the high- and low-tech labor markets coincide with their socially optimal counterparts.

Static and Dynamic Efficiency in the High-tech Labor Market. Equation (27) implicity pins down the competitive wage. Substituting in the definitions of $\mathbf{J^H_t}$ and $\mathbf{W^H_t} - \mathbf{U^H_t}$ and solving the resulting expression for w^H_t reveals that the posted wage in the CSE is equivalent to the Nash wage under the restriction $\psi^H = \xi^H$. As such, the proof to Proposition 1 applies directly. As long as $\chi^H = 0$, both the static and dynamic private efficiency conditions coincide with their socially efficient counterparts.

Static Efficiency in the Low-tech Labor Market. We first show that equation (29) is equivalent to the static efficiency condition in the social planner's problem given by equation (9). Use the fact that with a CRTS matching function, $\frac{m_{e,t}}{m_{v,t}}\frac{q^L_t}{f^L_t} = \frac{\xi^H}{1-\xi^H}$, to substitute this in on the right side of equation (29) and rearrange to get the following

$$f_t^L \left[\eta_t(\mathbf{W_t^L} - \mathbf{U_t^L}) + (1-\eta_t)(\mathbf{W_t^M} - \mathbf{U_t^M})\right] = \frac{m_{e,t}}{m_{v,t}} q_t^L \left[\eta_t \mathbf{J_t^L} + (1-\eta_t)\mathbf{J_t^M}\right]$$

We can then use the free entry condition, given by equation (18), to write this as

$$f_t^L \left[\eta_t(\mathbf{W_t^L} - \mathbf{U_t^L}) + (1-\eta_t)(\mathbf{W_t^M} - \mathbf{U_t^M})\right] = \frac{m_{e,t}}{m_{v,t}} \gamma^L$$

Finally, using the definition of $\mathbf{W_t^L} - \mathbf{U_t^L}$ and $\mathbf{W_t^M} - \mathbf{U_t^H}$, we have

$$\eta_t^L \left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \chi^L \right) + \left(1-\eta_t^L\right) \left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \chi^H \right) = \gamma^L \frac{m_{s,t}^L}{m_{v,t}^L} \tag{A.4.1}$$

The private static efficiency condition coincides with its socially efficient counterpart as long as $\chi^L = \chi^H = 0$.

Dynamic Efficiency in the Low-tech Labor Market. Next, we need to show that as long as the static efficiency condition holds, equation (28) implies that the dynamic efficiency conditions given by equations (10) and (11) must also be satisfied.

In the competitive search equilibrium, free entry implies $\mathbf{J_t^L} = \mathbf{J_t^M} = \frac{\gamma^L}{q_t^L}$. We can use this, and the fact that $q_t^L = \frac{1}{1-\xi} m_{v,t}$ to rewrite equation (19) as

$$(1-\xi)\frac{\gamma^L}{m_{v,t}} = Y_{1,t}^L - w_t^L + (1-\rho^L)\mathbb{E}_t \left(\Xi_{t+1|t}(1-\xi)\frac{\gamma^L}{m_{v,t+1}} \right)$$

Use equation (A.4.1) to plug in for $\frac{\gamma^L}{m_{v,t}}$ on both sides of the above equation to get

$$(1-\xi)\frac{1}{m_{s,t}} \left(\eta_t^L \left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \chi^L \right) + \left(1-\eta_t^L\right) \left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \chi^H \right) \right) = Y_{1,t}^L - w_t^L$$
$$+(1-\rho^L)\mathbb{E}_t \left[\Xi_{t+1|t}(1-\xi)\frac{1}{m_{s,t+1}} \left(\eta_{t+1}^L \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) + \left(1-\eta_{t+1}^L\right) \left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H} - \chi^H \right) \right) \right]$$

Next, solve the low-skilled household's optimal participation condition, equation (14), for the low-tech wage, w_t^L, and substitute the resulting expression in above to get

$$(1-\xi)\frac{1}{m_{s,t}} \left(\eta_t^L \left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \chi^L \right) + \left(1-\eta_t^L\right) \left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \chi^H \right) \right) = Y_{1,t}^L + \chi^L$$
$$-\frac{1}{f_t^L} \left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \chi^L \right) - (1-\rho^L)\mathbb{E}_t \left[\Xi_{t+1|t} \frac{1-f_{t+1}^L}{f_{t+1}^L} \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) \right]$$
$$+(1-\rho^L)\mathbb{E}_t \left[\Xi_{t+1|t}(1-\xi)\frac{1}{m_{s,t+1}} \left(\eta_{t+1}^L \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) + \left(1-\eta_{t+1}^L\right) \left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H} - \chi^H \right) \right) \right]$$

Using the fact that $m_{s,t} = \xi f_t^L$, we can simplify and rearrange to get

$$\frac{1}{m_{s,t}}\left(\eta_t^L\left(\frac{h_t^{L\prime}}{u_{c,t}^L}-\chi^L\right)+\left(1-\eta_t^L\right)\left(\frac{h_t^{M\prime}}{u_{c,t}^H}-\chi^H\right)\right)-(1-\eta_t)\frac{1}{f_t^L}\left(\frac{h_t^{L\prime}}{u_{c,t}^L}-\chi^L-\frac{h_t^{M\prime}}{u_{c,t}^H}+\chi^H\right)=$$

$$Y_{1,t}^L+\chi^L+(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L\right)\right)$$

$$+(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}\frac{1}{m_{s,t+1}}\left(\eta_{t+1}^L\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L\right)+\left(1-\eta_{t+1}^L\right)\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}-\chi^H\right)\right)\right)$$

$$-(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}(1-\eta_{t+1})\frac{1}{f_{t+1}^L}\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L-\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}+\chi^H\right)\right)$$

As long as $\chi^L=\chi^H=0$, we can iterate this expression forward to get

$$\frac{1}{m_{s,t}^L}\left(\eta_t\frac{h_t^{L\prime}}{u_{c,t}}+(1-\eta_t)\frac{h_t^{M\prime}}{u_{c,t}}\right)-\frac{1-\eta_t}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}}-\frac{h_t^{L\prime}}{u_{c,t}}\right)$$

$$+\sum_{s=1}^{\infty}(1-\rho^L)^s\mathbb{E}_t\left\{\beta^s\frac{u_{c,t+s}}{u_{c,t}}\frac{h_{t+s}^{L\prime}}{u_{c,t+s}}\right\}=\sum_{s=0}^{\infty}(1-\rho^L)^s\mathbb{E}_t\left\{\beta^s\frac{u_{c,t+s}}{u_{c,t}}Y_{1,t+s}\right\}$$

which is equation (10), proving dynamic efficiency for low-skill jobs.

An identical approach is used to establish dynamic efficiency of mismatch jobs starting with equation (20). After plugging in the expression for the mismatch wage, w_t^M, derived from the high-skilled household's optimal participation condition, equation (17), we get

$$(1-\xi)\frac{1}{m_{s,t}}\left(\eta_t^L\left(\frac{h_t^{L\prime}}{u_{c,t}^L}-\chi^L\right)+\left(1-\eta_t^L\right)\left(\frac{h_t^{M\prime}}{u_{c,t}^H}-\chi^H\right)\right)=Y_{2,t}^L+\chi^H$$

$$-\frac{1}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}^H}-\chi^H\right)-(1-\rho^L)\mathbb{E}_t\left[\Xi_{t+1|t}\frac{1-f_{t+1}^L}{f_{t+1}^L}\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}-\chi^H\right)\right]$$

$$+(1-\rho^L)\mathbb{E}_t\left[\Xi_{t+1|t}(1-\xi)\frac{1}{m_{s,t+1}}\left(\eta_{t+1}^L\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L\right)+\left(1-\eta_{t+1}^L\right)\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}-\chi^H\right)\right)\right]$$

Using the fact that $m_{s,t}=\xi f_t^L$, simplifying and rearranging gives side, gives

$$\frac{1}{m_{s,t}}\left(\eta_t^L\left(\frac{h_t^{L\prime}}{u_{c,t}^L}-\chi^L\right)+\left(1-\eta_t^L\right)\left(\frac{h_t^{M\prime}}{u_{c,t}^H}-\chi^H\right)\right)+\frac{\eta_t}{f_t^L}\left(\frac{h_t^{L\prime}}{u_{c,t}^L}-\chi^L-\frac{h_t^{M\prime}}{u_{c,t}^H}+\chi^H\right)=$$

$$Y_{2,t}^L+\chi^H+(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}-\chi^H\right)\right)$$

$$+(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}\frac{1}{m_{s,t+1}}\left(\eta_{t+1}^L\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L\right)+\left(1-\eta_{t+1}^L\right)\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}-\chi^H\right)\right)\right)$$

$$+(1-\rho^L)\mathbb{E}_t\left(\Xi_{t+1|t}\frac{\eta_{t+1}}{f_{t+1}^L}\left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}-\chi^L-\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H}+\chi^H\right)\right)$$

As long as $\chi^L = \chi^H = 0$, we can iterate this expression forward to obtain

$$\frac{1}{m^L_{s,t}}\left(\eta_t \frac{h^{L\prime}_t}{u_{c,t}} + (1-\eta_t)\frac{h^{M\prime}_t}{u_{c,t}}\right) + \frac{\eta_t}{f^L_t}\left(\frac{h^{M\prime}_t}{u_{c,t}} - \frac{h^{L\prime}_t}{u_{c,t}}\right)$$
$$+\sum_{s=1}^{\infty}(1-\rho^L)^s \mathbb{E}_t\left\{\beta^s \frac{u_{c,t+s}}{u_{c,t}}\frac{h^{M\prime}_{t+s}}{u_{c,t+s}}\right\} = \sum_{s=0}^{\infty}(1-\rho^L)^s \mathbb{E}_t\left\{\beta^s \frac{u_{c,t+s}}{u_{c,t}} Y_{2,t+s}\right\}$$

which is equation (11), proving dynamic efficiency for mismatch jobs. ∎

Figure 1: Marginal product of labor and wage ratios, by relative productivity (z^M/z^L)

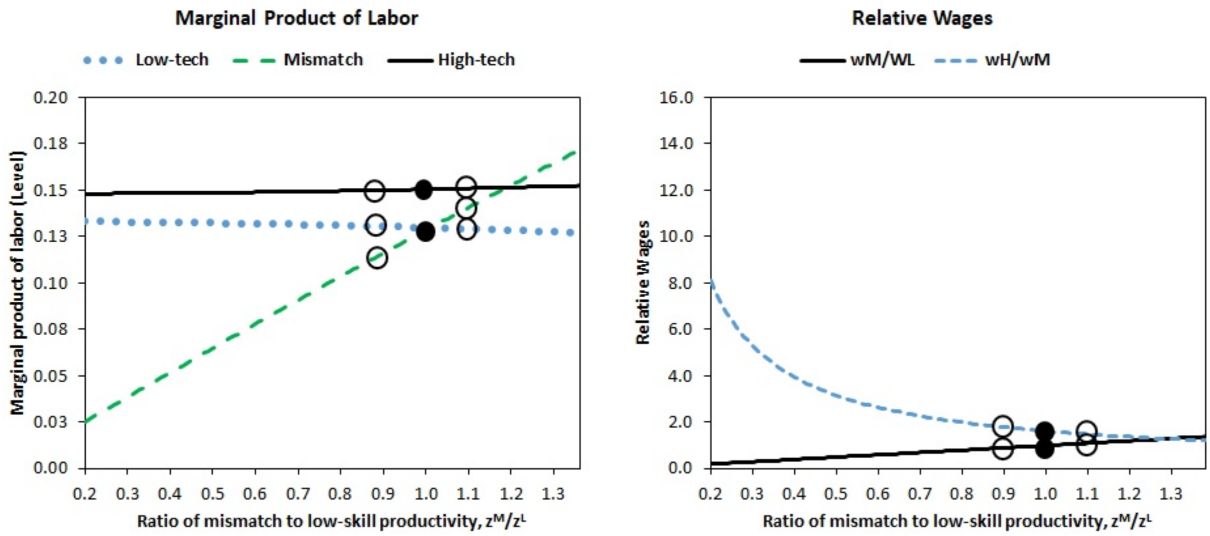
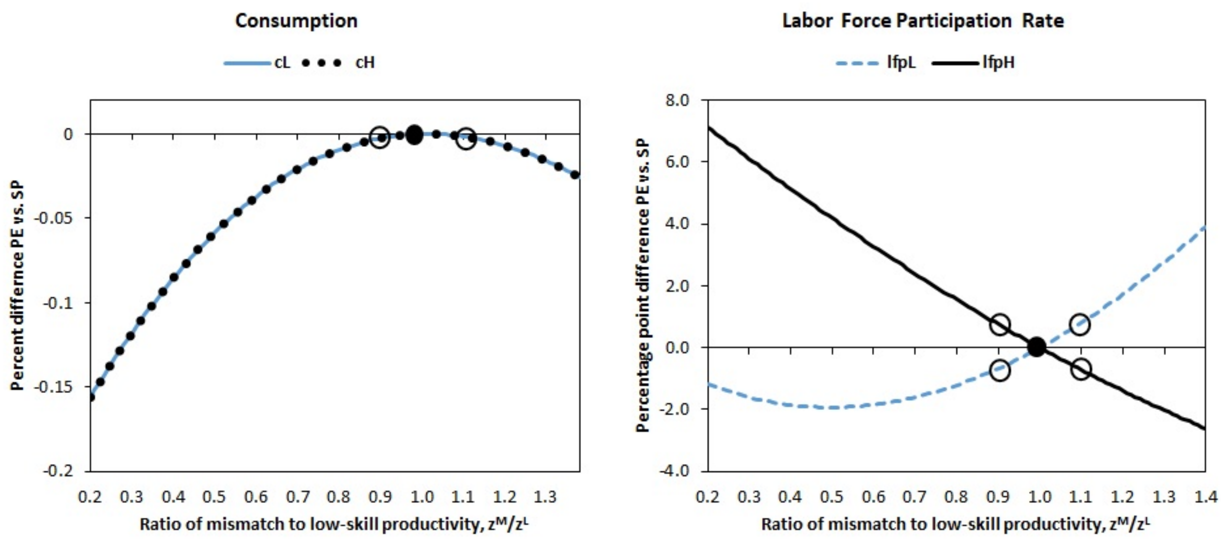


Figure 2: Consumption and labor force participation, by relative productivity (z^M/z^L)



Notes: In each panel, a solid dot denotes private outcomes in the benchmark efficient calibration reported in Table 2. The hollow dots denote outcomes in reported in Panels A and B of Table 3. In all cases, $\psi^H = \psi^L = \xi^H = \xi^L$ and $\chi^L = \chi^H = 0$.

Figure 3: Welfare costs, by relative productivity (z^M/z^L)

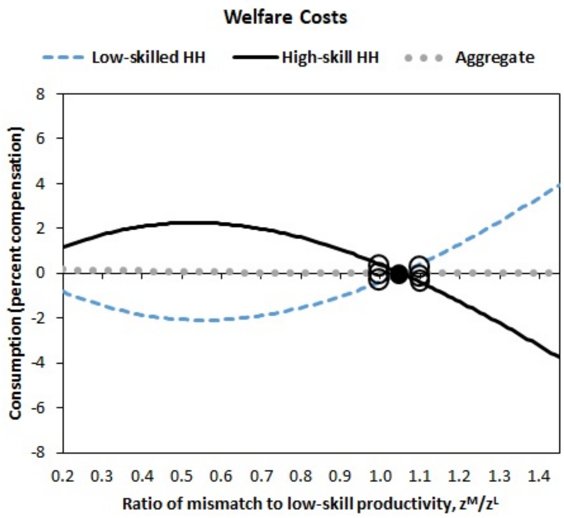
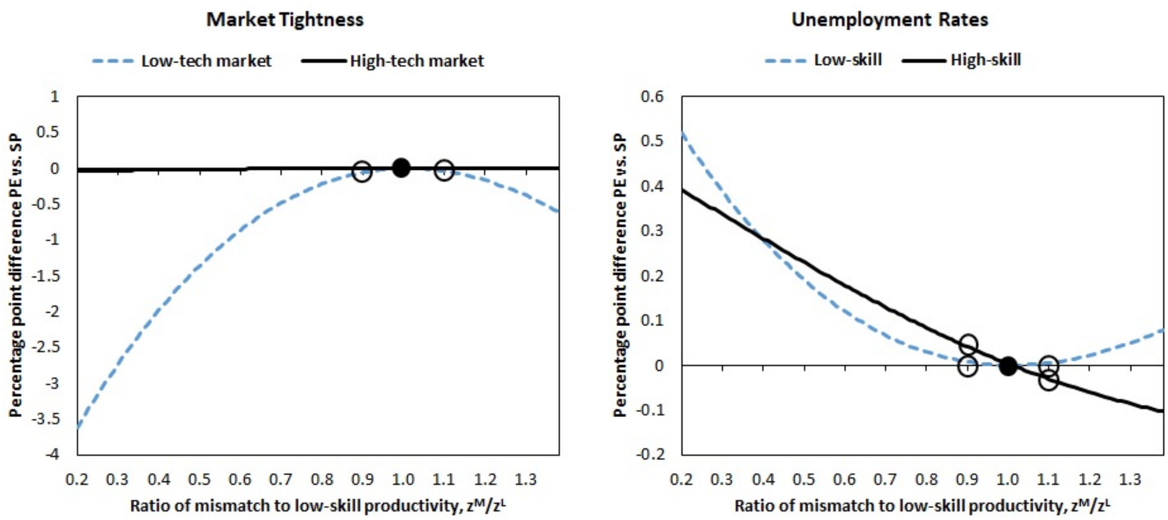
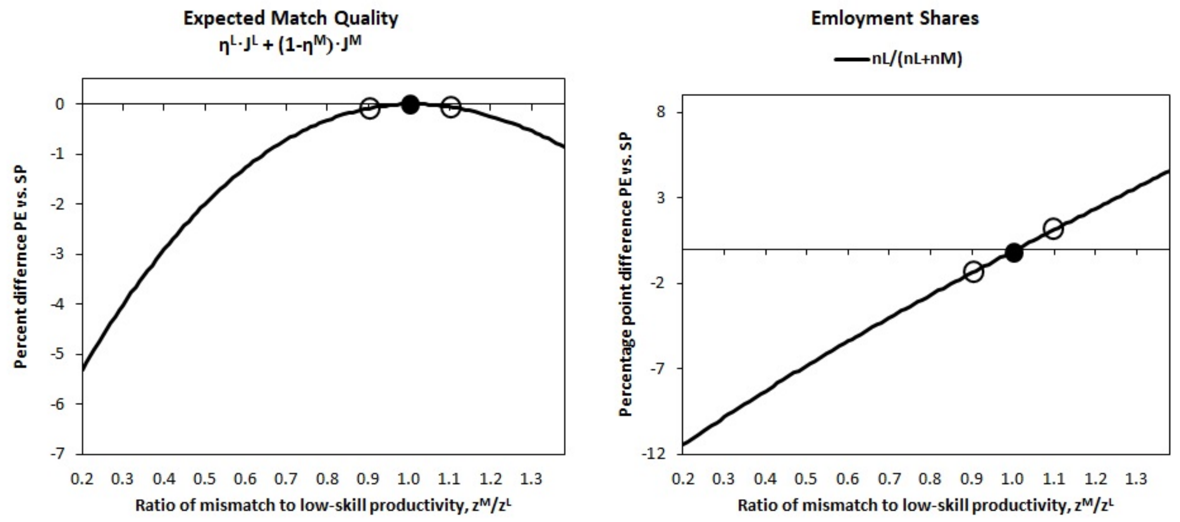


Figure 4: Labor market allocations, by relative productivity (z^M/z^L)



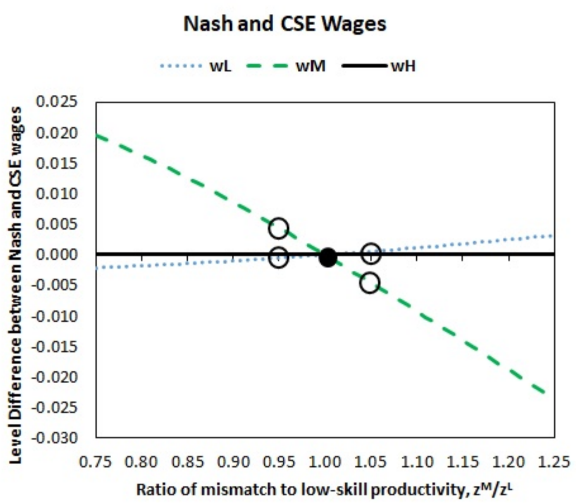
Notes: In each panel, a solid dot denotes private outcomes in the benchmark efficient calibration reported in Table 2. The hollow dots denote outcomes in reported in Panels A and B of Table 3. In all cases, $\psi^H = \psi^L = \xi^H = \xi^L$ and $\chi^L = \chi^H = 0$.

Figure 5: Match quality and composition of labor pool, by relative productivity (z^M/z^L)



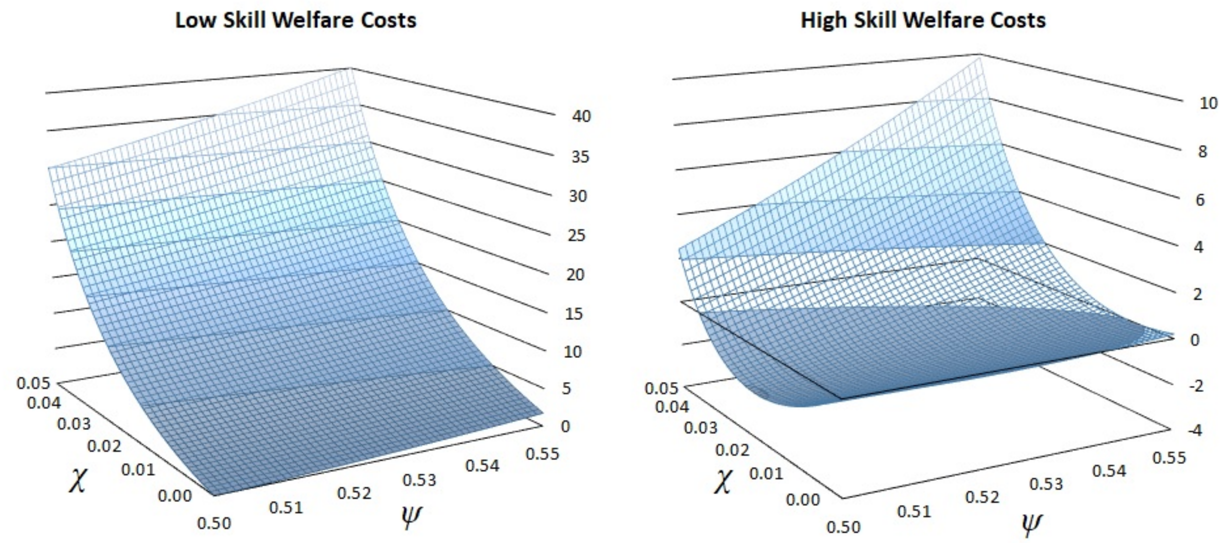
Notes: In each panel, a solid dot denotes private outcomes in the benchmark efficient calibration reported in Table 2. The hollow dots denote outcomes in reported in Panels A and B of Table 3. In all cases, $\psi^H = \psi^L = \xi^H = \xi^L$ and $\chi^L = \chi^H = 0$.

Figure 6: Match quality and composition of labor pool, by relative productivity (z^M/z^L)



Notes: In each panel, a solid dot denotes private outcomes in the benchmark efficient calibration reported in Table 2. The hollow dots denote outcomes in reported in Panels A and B of Table 3. In all cases, $\psi^H = \psi^L = \xi^H = \xi^L$ and $\chi^L = \chi^H = 0$.

Figure 7: Match quality and composition of labor pool, by relative productivity (z^M/z^L)



Notes: In each panel, a solid dot denotes private outcomes in the benchmark efficient calibration reported in Table 2. The hollow dots denote outcomes in reported in Panels A and B of Table 3. In all cases, $\psi^H = \psi^L = \xi^H = \xi^L$ and $\chi^L = \chi^H = 0$.