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Country-Level and Cross-Sectional Data Heterogeneity**

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An Output Gap Measure for the Euro Area: Exploiting Country-Level and Cross-Sectional Data Heterogeneity

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Abstract

This paper proposes a methodology to estimate the euro-area output gap by taking advantage of two types of data heterogeneity. On the one hand, the method uses information on real GDP, inflation, and the unemployment rate for each member state; on the other hand, it jointly considers this information for all the euro-area countries to extract an area-wide output gap measure. The setup is an unobserved components model that theorizes a common cycle across euro-area economies in addition to country-specific cyclical components. I estimate the model with Bayesian methods using data for the 19 countries of the euro area from 2000:Q1 through 2017:Q2 and perform model comparisons across different specifications of the output trend. The estimation of the model preferred by the data indicates that, because of negative shocks to trend output during the global financial crisis, output remained slightly above potential in that period, but an output gap of about negative $3\frac{1}{2}$ percent emerged during the European debt crisis. At the end of the sample period, output is estimated to be about 1 percent above potential.

Keywords: Unobserved components model, euro area, Okun’s law, Phillips curve, output gap

JEL Classification Numbers: C13, C32, C52

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1 Introduction

There is an ongoing debate among central bankers about the true degree of economic slack in their respective economies and about the sensitivity of inflation to the estimated levels of economic slack. In the case of the euro area, the European Central Bank (ECB) President, Mario Draghi, states the following (see [Draghi, 2017](#)):

We see growth above trend and well distributed across the euro area, but inflation dynamics remain more muted than one would expect on the basis of output gap estimates and historical patterns. An accurate diagnosis of this apparent contradiction is crucial to delivering the appropriate policy response.

In particular, President Draghi emphasizes the uncertainty surrounding the size of the output gap.

In this paper, I take advantage of the heterogeneity in the country-level information on real GDP, inflation, and the unemployment rate, as well as the heterogeneity of this information across euro-area member states to estimate the area-wide output gap, potential output, and natural rate of unemployment, along with the uncertainty around these estimates. Specifically, I set up an unobserved components (UC) model with a common component in the output gap across countries in addition to idiosyncratic country-level counterparts. The same setup takes place for potential output and the natural rate of unemployment. In the model, output, inflation, and the unemployment rate at the country level inform the estimation of each country's output gap, potential output, and natural rate of unemployment. At the same time, these country-level macroeconomic variables also inform the estimation of the common components by jointly considering the information of all member states. I aggregate the common and country-level idiosyncratic components in a suitable way to obtain estimates for the euro area. Moreover, by estimating the features of each member economy, the model allows one to determine the effect of area-wide policies on each country's unemployment and inflation rates. Hence, this paper aims to offer information that can be relevant to the monetary policy decision process in the euro area.

Given the single currency among euro-area member states and the uniform monetary policy implementation by the ECB, one could expect that at least some portion of the variation in the country-level degrees of slack can be attributed to the variation in the area-wide output gap. Of course, there may be features particular to each country that make the evolution of every economy different for what could be considered a pattern of synchronization of business cycles. In that regard, a modeling framework that takes into account the possibility of diverse degrees of strength in the relationship between the country-specific output gaps and their area-wide counterpart can be beneficial to understand how monetary policy, for example, could propagate at the national level, which in turn can provide information about the effects of these policies on the area-wide indicators. I intend to use this feedback mechanism between the area-wide business cycle and those of the member states to offer an alternative to the estimation of the output gap produced from approaches that use aggregate data at the euro-area level.

Even though there have been multiple attempts and approaches in the literature to estimate the output gap of the euro area as a whole, none of them has considered combining the two frameworks I suggest—namely a multivariate perspective at the country level along

with the cross-sectional information across countries. By imposing a common factor structure across macroeconomic variables and countries to estimate the area-wide output gap, a UC model can potentially gain efficiency and reduce the uncertainty around the estimated degree of slack.

One of the studies that contemplates the multivariate framework for the euro area is done by [Lenza and Jarociński \(2016\)](#), who investigate (through Bayesian methods) the performance of several UC models to estimate the output gap and accurately forecast inflation in the euro area. The models vary according to the information set used to estimate the output gap and the trend specification for the variables involved. The results show that a model that assumes local linear trends and includes inflation expectations as an observable variable is the most favored by the data, according to its log marginal likelihood. However, a model in which the trends are modeled as random walks with drift and which includes inflation expectations offers the best forecast performance of inflation.

The cross-sectional framework to obtain the euro-area output gap has at its core the assumption of different degrees of business cycle synchronization among countries and between countries and the area as a whole. [Lee \(2012\)](#) estimates a time-varying parameters dynamic factor model for the 11 countries that originally joined the Economic and Monetary Union (EMU), plus Greece. A variance decomposition shows that the area-wide factor explains about 70 percent of the growth rate of real GDP per capita between the mid-1990s and the late 2000s, up from about 50 percent in the early 1980s. This evidence suggests greater business cycle synchronization across European countries in the run-up to the third stage of the EMU.

The modeling approach I propose in this paper combines the two frameworks previously described. That is, it incorporates information on several macroeconomic variables, as had been suggested by [Rünstler \(2002\)](#), [Azevedo, Koopman and Rua \(2006\)](#), [Basistha and Startz \(2008\)](#), and [Fleischman and Roberts \(2011\)](#), among others, to improve the precision of output gap estimates and, at the same time, it includes cross-sectional data in a common factor structure, as originally put forward by [Kose, Otrok and Whiteman \(2003\)](#) and [Del Negro and Otrok \(2008\)](#), among others. The setup is similar to what [Gonzalez-Astudillo \(2017\)](#) proposes to estimate the output gap of the United States using state-level data on real GDP and the unemployment rate. The main difference in the present paper is that I include the inflation rate to explicitly address how inflation can inform the estimation of the output gap and how it responds to the degree of slack in the economy.

Using data from the first quarter of 2000 to the second quarter of 2017 for the 19 countries of the euro area, I estimate the proposed model using Bayesian methods. It turns out that an output trend specified as a random walk with drift outperforms in sample a local linear trend or an integrated random walk specification. As mentioned before, the structure of the UC model allows one to estimate the common components across countries, such as the euro-area output gap, potential output, and the natural rate of unemployment. Yet it also provides estimates of these indicators at the country level, although the paper does not analyze the economic fundamentals driving the evolution of these country-specific indicators.

The model estimates that the euro-area level of output was about $5\frac{1}{2}$ percent above trend right before the financial crisis and then declined to a level just above potential at the end of 2009. In the coming years, output increases to almost 3 percent above its potential level in 2011 and then declines again during the European debt crisis to about $3\frac{1}{2}$ percent

below in late 2013. At the end of the sample period, in the second quarter of 2017, the model estimates that the euro-area output is about 1 percent above trend. In the same vein, the model estimates that the area’s potential output was significantly affected during the financial crisis, with a decline of about 1¼ percent in its level between the first quarter of 2008 and the first quarter of 2009. This decline is more than enough to prevent the appearance of an output gap during this period. All in all, the estimate of the growth rate of potential output is about 1¼ percent per year, on average.

The estimates also indicate that the euro-area natural rate of unemployment started around 11 percent in 2000 and remained there until late 2006 when it starts to increase to reach a peak of about 12.5 percent in mid-2009; it declines thereafter to about 7.7 percent in the second quarter of 2017.

Regarding the estimation of the sensitivity of inflation to economic slack, the cross-sectional average of the slopes of the Phillips curves implies that inflation increases by 0.14 percentage point for every percentage increase in output above potential. However, there is heterogeneity across countries in this sensitivity. For example, the model estimates that inflation in Estonia and Latvia would react much more strongly with respect to their respective output gaps than in Germany or Slovakia. I emphasize that this heterogeneity needs to be considered when measuring the impact of economic policies designed to affect the euro area as whole, such as those usually put in place to curb inflation pressures.

The results also show that the model with a random walk specification in the output trends of the countries performs better at forecasting out of sample the three macroeconomic variables compared with other UC models and a benchmark vector autoregressive (VAR) model. Additionally, estimating the euro-area output gap using a UC model with country-level data decreases the uncertainty around the degree of economic slack compared with a similar model that uses aggregate data at the euro area level for the same macroeconomic variables.

I structure the rest of the paper as follows: Section 2 provides a more in-depth revision of the literature regarding the estimation of the output gap as well as the synchronization of business cycles across the euro area. Section 3 describes the model proposed. In Section 4, I lay out the Bayesian estimation strategy. Section 5 makes reference to the data used, while the results appear in Section 6. The next two sections, 7 and 8, investigate the forecast performance of the model and the efficiency gains in estimating the output gap compared with a model that uses aggregate data at the euro area level. Finally, Section 9 concludes.

2 Connections with the Literature

This paper relates to two main strands of the literature. Clearly, the paper connects to the existent research on the estimation of the output gap for the euro area using UC models. The second topic relates to the literature on synchronization of business cycles within Europe and the euro area using factor models and other statistical techniques. Next, I make an overview of both points separately.

2.1 Estimation of the Output Gap in the Euro Area

One of the first attempts to estimate the euro-area output gap is [Uhide and Ross \(2001\)](#), who used four different perspectives, as follows: statistical properties, economic properties, survey data, and “thick” estimates. Under the economic properties approach, there are UC models based on the Phillips curve. The authors estimate four variants of such a model for the euro area including real GDP, the unemployment rate, and consumer price inflation. They find that the best UC model to forecast inflation includes hysteresis effects in which the current NAIRU reacts to the lagged unemployment rate gap.

[Rünstler \(2002\)](#) examines the reliability of real-time estimates of the output gap for the euro area. He finds that a multivariate UC model that adds capacity utilization and factor inputs greatly reduces the filter uncertainty compared to a baseline bivariate model of inflation and output, the latter being uninformative to a great extent because of large standard errors and significant biases. Among the models, one based on the output-capital ratio and total factor productivity performs best, and its conditional forecast performance with respect to inflation is also satisfactory.

Along with VAR-based Beveridge-Nelson decompositions, [Camba-Mendez and Rodriguez-Palenzuela \(2003\)](#) specify three variants of a trivariate UC model that includes real GDP, the unemployment rate, and consumer price inflation for the euro area, exploiting Okun’s law and the Phillips curve. One variant assumes that the trend components of each of the three observable variables is a random walk with drift. Another variant specifies local linear trends in each of the three variables, whereas the last variant assumes smooth trend components. Results show that UC models perform less well than VAR models in forecasting output, the unemployment rate, and inflation.

[Fabiani and Mestre \(2004\)](#) also exploit Okun’s law and the Phillips curve to estimate the NAIRU for the euro area using UC models with local linear trends in the unemployment rate and output, but not on inflation. Four different specifications of such models are estimated in which the timing convention of the unemployment rate and output cycles, the causality channel, and the variable defining inflation in the Phillips curve are altered from the baseline model. The authors find that the area-wide NAIRU estimates are robust to changes in the underlying models, as long as the models belong to the UC class.

To check if inflation improves the reliability of real-time estimates of the output gap for the EU-11 countries, [Planas and Rossi \(2004\)](#) introduce a Phillips curve relationship in a bivariate UC model for decomposing output into trend and cycle. The results show that the bivariate model that includes inflation does not significantly improve the uncertainty around the real-time estimate of the output gap compared to a univariate model.

[Proietti, Musso and Westermann \(2007\)](#), under four variants of trend output specification—random walk with drift, local linear, damped slope, and integrated random walk—estimate a bivariate UC model with inflation and output to obtain the output gap for the euro area. Additionally, using a production function approach, they estimate a multivariate UC model that includes inflation, output, the labor force participation rate, a proxy of the unemployment rate, capacity utilization, and the consumer price index to obtain another estimate of the output gap also under the four variants of trends mentioned before. The forecasting exercises results indicate that the UC model based on the production function approach outperforms the bivariate model, in general. Furthermore, the models with higher forecasting

accuracy include a trend specification with a damped slope.

The first attempt to estimate the European Monetary Union output gap with Bayesian methods was done by [Planas, Rossi and Fiorentini \(2008\)](#), who use a UC model with real GDP and the inflation rate as observable variables. The results show that the modal period of the cycle is about nine years. In addition, the results indicate a small but positive degree of correlation between the innovations to the cycle and to inflation. The authors also illustrate the effect of a perturbation to the output gap on inflation both in the short and in the long run.

[Marcellino and Musso \(2011\)](#) perform an evaluation of the reliability of real-time output gap estimates for the euro area using a real-time data set. The technique used to estimate the output gap is through UC models in the multivariate context of [Proietti, Musso and Westermann \(2007\)](#). The findings indicate that real-time estimates of the output gap tend to be characterized by a high degree of uncertainty both on the magnitude and sign dimensions. The uncertainty is mostly due to parameter instability and model uncertainty, rather than data revisions.

[Gurin, Maurin and Mohr \(2015\)](#) estimate univariate and bivariate UC models that include output and inflation to obtain a measure of the euro-area output gap. The models vary in terms of the output trend specifications, which include the usual random walk with drift, a Markov-switching trend intercept, and Markov-switching in the intercept of the trend and in the variance of the trend shock. The inflation specification includes, besides the output gap, the nominal exchange rate, the price of oil, and a time-varying intercept that follows a random walk. The authors then combine the output gap estimates obtained from each of the different models to get a global estimate of the output gap. The forecasting exercise for inflation shows that the output gap measures help forecasting inflation over most of the sample, but fail dramatically since the last recession.

Under a different approach and with a different goal than to measure the output gap but rather to obtain insights about the sensitivity of inflation to the degree of slack in the economy, [Blanchard, Cerutti and Summers \(2015\)](#) estimate a time-varying coefficients Phillips curve that includes inflation expectations for 20 countries, including several belonging to the euro area. Results show that the median slope of the Phillips curve across countries has dropped substantially and has remained relatively constant since the early 1990s at around one-fifth of the value reached in the late 1970s, while there is evidence of a rise since the 1970s in the anchoring to long-term inflation expectations.

2.2 Common Business Cycles and Synchronization in Europe

The literature on common components of business cycles in Europe relates to the present paper in that both assume the existence of underlying factors to economic activity and let the data determine the strength of the connection between the economic activity of each country and those factors. For example, [Forni et al. \(2000\)](#) estimate a generalized dynamic factor model for 10 countries of the euro area, taking into account seven macroeconomic variables for each country. After obtaining the common component of GDP for each separate country, a weighted average of the common components, using the GDP levels as weights, is constructed and labeled as the coincident indicator for the euro area. The authors then obtain the proportion of the variance of each variable that is explained by the common

component, which reaches 85 percent for the euro-area aggregate of GDP.

In a somewhat different analysis, [Artis, Krolzig and Toro \(2004\)](#) estimate a Markov-switching VAR to identify a common cycle in Europe. Results show that there is evidence of a common unobserved component governing European business cycle dynamics, suggesting the existence of a common business cycle. The authors also explore the contribution of the European business cycle to the individual country cycles and undertake an impulse response analysis to investigate the response of each individual country to European expansions and recessions.

Another way to analyze the business cycles of a group of countries as a whole is by determining the degree of synchronization among the member states. [Aguilar-Conraria and Soares \(2011\)](#) use wavelet analysis to study the business cycle synchronization of the EU-12 countries. The wavelet analysis allows one to compare the wavelet spectra of two countries to check if the contribution of cycles at each frequency to the total variance is similar between both countries, if this contribution happens at the same time or not, and, finally, if the ups and downs of each cycle occur simultaneously. Results show that business cycles dissimilarities are highly correlated with geographical physical distances. In particular, France and Germany appear to be the most synchronized countries with the rest of Europe, while Portugal, Greece, Ireland, and Finland do not show statistically relevant degrees of synchronization with the other European countries. Some countries show a French accent, like Spain, while others have a German accent, like Austria.

However, [Camacho, Perez-Quiros and Saiz \(2006\)](#) warn that assuming that a European cycle exists and that it coincides either with the cycle of a leading European economy, the cycle of a weighted average of several European economies, or the cycle of a common factor, should be done cautiously. The authors use cluster analysis to find out if there is a “European business cycle” that links the European economies. The results show that the degree of business cycle synchronization within the group of old European Union members is higher than across the recently acceded countries, but synchronization across old members has not significantly increased since the establishment of the common currency. In particular, the results do not favor evidence of some distinct euro-economy attractor.

3 The Model

To estimate the euro-area output gap, I adopt the setup in [Basistha and Nelson \(2007\)](#) for each country of the monetary union and propose the following UC model that incorporates country-level information of output, the unemployment rate, and inflation:

$$y_{it} = \tau_{it}^y + c_{it}, \quad (1)$$

$$u_{it} = \tau_{it}^u + \theta_{1i}c_{it} + \theta_{2i}c_{i,t-1}, \quad (2)$$

$$\pi_{it} = \beta_{0i} + \beta_{1i}\pi_t^e + (1 - \beta_{1i})\pi_{i,t-1} + \kappa_i c_{it} + \eta_{it}^\pi, \quad (3)$$

where y_{it} , u_{it} , and π_{it} are (the log of) output, the unemployment rate, and the inflation rate, respectively, for country $i = 1, 2, \dots, n$ with n being the number of countries and $t = 1, 2, \dots, T$, with T being the sample size. In equation (1), output is the sum of a trend, τ_{it}^y , and a cyclical component, c_{it} . Similarly, the unemployment rate for each country in

equation (2) is the sum of a trend, τ_{it}^u and a cycle which is a linear combination of the cyclical component of output in a way resembling Okun's law with coefficients θ_{1i} and θ_{2i} .

Inflation is central to the notion of potential GDP and, hence, to that of the output gap. The New Keynesian Phillips curve with forward-looking expectations implies that the output gap drives the dynamics of inflation relative to expected inflation. Equation (3) is a 'hybrid' Phillips curve that allows one to estimate the output gap using UC models. The specified Phillips curve is both forward- and backward-looking as a way to incorporate possible expectation mismeasurements, non-rational adaptive expectation formation, or price rigidities not fully captured by a pure forward-looking Phillips curve, which can affect the estimation of the output gap. In said equation, π_{it} is the inflation rate of country i , $\pi_{i,t-1}$ is its lagged value, π_t^e denotes the one-year ahead (survey) euro-area inflation expectations, and η_{it}^π is a composite unobserved variable that plays a role in both expected inflation and supply shocks. Long-run neutrality is ensured by restricting $\beta_{1i} \in [0, 1]$ for all i . The parameter κ_i represents the slope of the Phillips curve for country i , which theory suggests is positive. [Lenza and Jarociński \(2016\)](#) also consider inflation expectations in a UC model that includes a Phillips curve-style equation to estimate the output gap. Their results show that the data favor models in which the inflation trend is informed by inflation expectations data.¹

In equation (1), following [Gonzalez-Astudillo \(2017\)](#), I assume that each country's output trend is a function of a trend that is common to all the countries in the monetary union and of an idiosyncratic trend, as follows:

$$\tau_{it}^y = \delta_i^y \tau_t^y + \xi_{it}^y, \quad (4)$$

where τ_t^y and ξ_{it}^y are the common and idiosyncratic trends, respectively. The coefficient δ_i^y determines how intensely a country's i output trend is affected by the common output trend, which is a random walk with drift, as shown below:

$$\tau_t^y = \mu + \tau_{t-1}^y + \eta_t^y. \quad (5)$$

Then, I take an approach similar to [Lenza and Jarociński \(2016\)](#) to specify the idiosyncratic output trend as one of the following three alternatives:²

- Random walk (RW):

$$\xi_{it}^y = \mu_i + \xi_{i,t-1}^y + \eta_{it}^y. \quad (6)$$

- Local linear trend (LLT):

$$\begin{aligned} \xi_{it}^y &= \mu_{i,t-1} + \xi_{i,t-1}^y + \eta_{it}^y, \\ \mu_{it} &= \mu_{i,t-1} + \nu_{it}. \end{aligned} \quad (7)$$

¹This setup takes a stand on the way to model the trend inflation. [Clark and Doh \(2014\)](#) assess the inflation forecasting ability of different inflation trend specifications and find that the accuracy of the local linear trend setup of [Stock and Watson \(2007\)](#) or the survey-based specification are about equal.

²In [Lenza and Jarociński \(2016\)](#), the three alternatives are proposed for the trend of the euro area aggregate output instead of for that of each country.

- Integrated random walk (IRW):

$$\begin{aligned}\xi_{it}^y &= \mu_{i,t-1} + \xi_{i,t-1}^y, \\ \mu_{it} &= \mu_{i,t-1} + \nu_{it}.\end{aligned}\tag{8}$$

The reasoning behind having these three alternatives is that it may be too restrictive to assume that the conditional average growth rate of trend output has remained constant over time, as it happens with the RW specification. Both the LLT and the IRW specifications allow the conditional average growth rate of trend output to change over time. These variations can be potentially useful to incorporate the possibility of periods with sustained low growth rates of output like those experienced during episodes of secular stagnation (see [Summers, 2015](#)).³

In a similar manner as for the trend of output, the trend of the unemployment rate is modeled as a function of a common trend with a loading coefficient that determines the strength of the feedback to the country-specific trend, and an idiosyncratic unemployment rate trend that follows a random walk without drift, as shown below:

$$\tau_{it}^u = \delta_i^u \tau_t^u + \xi_{it}^u, \tag{9}$$

$$\xi_{it}^u = \xi_{i,t-1}^u + \eta_{it}^u, \tag{10}$$

where the common unemployment rate trend follows a random walk as well, as specified below:

$$\tau_t^u = \tau_{t-1}^u + \eta_t^u. \tag{11}$$

In this specification, neither the common nor the idiosyncratic unemployment trends include a drift term. These assumptions are based on the reasoning that the unemployment rate would not be a trending variable, hence I set the drifts to zero.⁴ This type of specification is used by [Clark \(1989\)](#), [Sinclair \(2009\)](#), [Basistha and Nelson \(2007\)](#), and [Gonzalez-Astudillo and Roberts \(2016\)](#), among others, for the U.S. economy.

Each country's cycle is a linear combination of a euro-area common cycle and an idiosyncratic cyclical component, as follows:

$$c_{it} = \alpha_i c_t + v_{it}, \tag{12}$$

where the loading coefficient α_i determines if a country is procyclical, acyclical, or counter-cyclical with respect to the common cycle, and the strength of the cyclical. The idiosyn-

³There are two other possibilities for incorporating a time-varying intercept in the output trends. The first is to adopt the three variants on the common output trend in equation (5) and leave the idiosyncratic trend as a random walk specification as in (6). I prefer to have time-varying intercepts in the idiosyncratic trends because of the more sensible model identification restrictions to be described below with respect to the intercepts of the output trends. The second possibility is to allow time-varying intercepts in both the common and the idiosyncratic output trends. However, the model-fit comparisons would have become much more cumbersome in that case.

⁴[Lenza and Jarociński \(2016\)](#) find that the posterior mean of the intercept of their unemployment trend is undistinguishable from zero.

cratic component follows an AR(1) process, as indicated below:

$$v_{it} = \rho_i v_{i,t-1} + \zeta_{it}, \quad (13)$$

with $|\rho_i| < 1$, whereas the common cyclical component is specified as a stationary AR(2) process, as follows:

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t. \quad (14)$$

Finally, the error terms $\varepsilon_t, \eta_t^y, \eta_t^u, \nu_t, \eta_{it}^y, \eta_{it}^u, \eta_{it}^\pi, \zeta_{it}, \nu_{it}$ are assumed to be white noise, uncorrelated with each other and normally distributed.⁵ Appendix A presents the state-space models in matrix form.

4 Estimation Strategy

To obtain the euro-area output gap, I estimate with Bayesian methods the three UC models in equations (1)-(14), which are distinguished by the three idiosyncratic output trend specifications (RW, LLT, and IRW).

I need to impose some identification conditions to estimate the coefficients of the model and the output gap. First, I set to one the variances of the error terms of every common component, that is, $\sigma_\varepsilon^2 = \sigma_{\eta^y}^2 = \sigma_{\eta^u}^2 = 1$. These restrictions are necessary to identify the scale of the loading coefficients, α_i , δ_i^y and δ_i^u , respectively. I set $\mu = 0$ to allow the unrestricted estimation of the country-specific drifts, μ_i , in the case of the random walk specification, or their initial values, μ_{i0} , in the cases of the local linear and the integrated random walk trends. Generally speaking, to avoid having country-level GDP and unemployment trends that diverge in expectation with respect to their common counterparts, I restrict the loading coefficients, δ_i^y and δ_i^u , to be positive. The common cycle, c_t , is identified only up to its sign, therefore I restrict one of the loading coefficients $\alpha_i > 0$ for $i = \text{Germany}$, without loss of generality. Lastly, to be consistent with the theoretical aspects of the Phillips curve, I restrict $\beta_{1i} \in [0, 1]$ and $\kappa_i > 0$ for all i . Basistha and Nelson (2007) adopt the first restriction to guarantee long-run neutrality while Matheson and Stavrev (2013) and Blanchard, Cerutti and Summers (2015) restrict the slope of the Phillips curve to be positive in a time-varying coefficient setup; I incorporate said constraint in the constant coefficient setup proposed.

The model has 268 coefficients to be estimated when the identification restrictions are imposed. I use the Gibbs sampler to obtain sequential draws from the posterior distribution between latent states, using the Durbin and Koopman (2002) simulation smoother, and coefficients. Appendix C describes in detail the sampling procedure.

⁵Morley, Nelson and Zivot (2003), Basistha and Nelson (2007), and Sinclair (2009), among others, allow for correlation between the trend and cycle perturbations. In the current context, that type of specification would increase significantly the number of estimated coefficients, increasing the parameter uncertainty of the model. Lenza and Jarociński (2016) argue that, by allowing more flexible specifications in the output trend, as done here, it is possible to account for the consequences of the trend-cycle correlation assumption.

5 Data

I employ data at a quarterly frequency from the first quarter of 2000 to the second quarter of 2017 for the 19 countries of the euro area.⁶ Real GDP is expressed in chained 2010 million euro. The unemployment rate is the quarterly average of the percentage of unemployed persons aged 15 to 74. The inflation rate corresponds to the annualized quarterly percent change in the overall harmonized index of consumer prices (HICP). Eurostat provides all the data, which are seasonally adjusted either directly by the statistical agency or manually using conventional seasonal adjustment techniques. Inflation expectations correspond to the mean point estimate of the year-over-year percentage change of the HICP one year ahead obtained from the survey of professional forecasters. The ECB provides this information. Appendix B describes the data and their transformations in more detail.

6 Model Estimation Results

This section lays out the estimation results of the UC models of equations (1)-(8). It briefly describes the choice of prior distributions for the parameters of the models. Then, the section discusses which of the three specifications for the idiosyncratic trend (RW, LLT, or IRW) is preferred by the data using marginal data density measures. Once the preferred model is chosen according to this criterion, the section describes the features of the euro-area output gap, potential output, and natural rate of unemployment, as well as the features of the country-specific counterparts without discussing the economic reasons behind their features because they are beyond the scope of the present study.

6.1 Prior Distributions

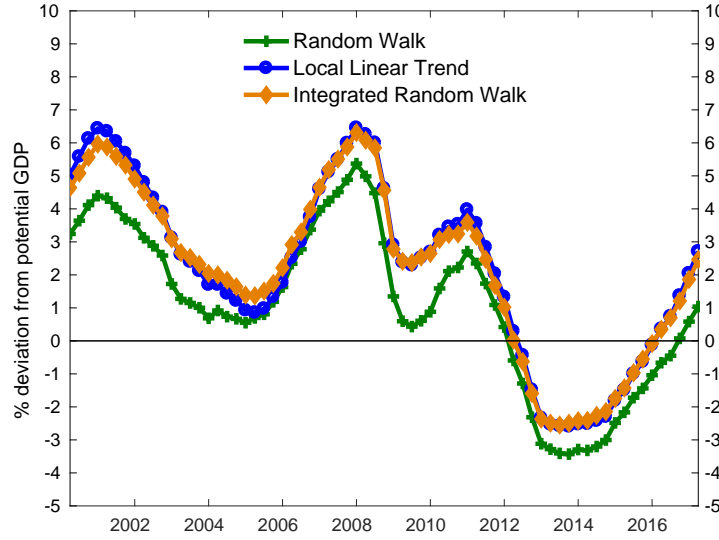
The prior distributions used are consistent with the application of the Gibbs sampler for linear regression models with independent conditional mean and variance components. That is, I assume that the coefficients of the conditional means of both the observation and transition equations of each state-space model have a normal prior distribution, whereas the variance coefficients of the shocks have inverse gamma prior distributions. Appendix D describes the choice of hyperparameters.

6.2 Estimate of the Euro Area Output Gap

The Gibbs sampler produced 300,000 draws. After burning in the first 100,000 and thinning every 100th draw, I kept 2,000 draws from the posterior distribution. Appendix E shows the posterior means and standard deviations of the parameters, as well as the convergence diagnostics of the sampler.

⁶I consider a balanced panel of information to estimate the model. Implicitly, I assume that the countries that joined the euro area after its original inception, namely Cyprus, Greece, Estonia, Latvia, Lithuania, Malta, Slovakia, and Slovenia were sharing at least part of the business cycle properties with the rest of euro-area members before joining.

Figure 1: Smoothed Estimates of the Euro-Area Output Gaps Using Country-level Data for Three Specifications of the Output Trend



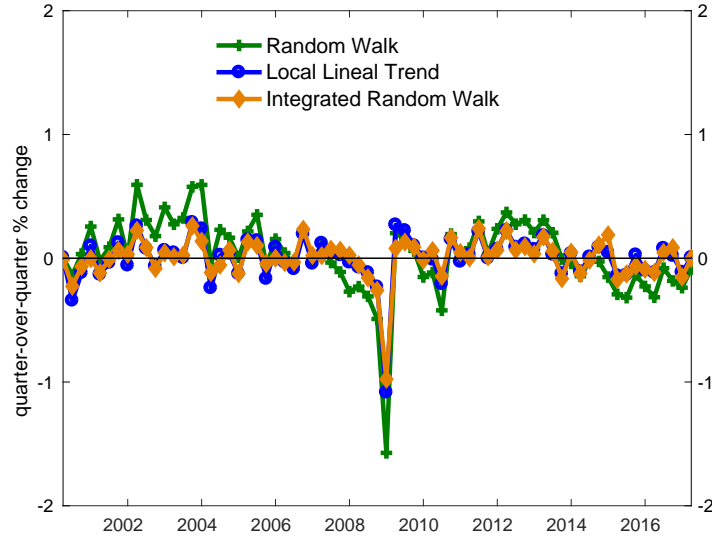
Note: Time series are the averages of the posterior draws of the euro area output gap using the [Durbin and Koopman \(2002\)](#) simulator smoother.

To obtain the euro-area output gap, I average the smoothed estimate of the country-specific output gaps, c_{it} in equation (12), using the nominal GDP weights in each period. The smoothed estimates of the euro-area output gaps for each of the three output trend specifications appear in Figure 1.

As expected, the estimated output gaps from the LLT and IRW trend specifications look very similar, whereas the output gap estimated from the RW specification shows significant differences with respect to the other two. For example, the LLT- and IRW-implied output gaps indicate that the euro-area output started the 2000s about 5 percent above potential, while the RW-implied output gap indicates that the economy was only about 3 percent above trend. Another difference occurs around the period of the global financial crisis when the LLT and IRW trend models estimate output stayed around 2.5 percent above potential, whereas the RW trend model estimates that real and potential GDP were almost at the same level. The other difference occurs at the end of the sample, in the second quarter of 2017, when the first two trend assumptions imply that output is almost 3 percent above potential while the RW specification yields an estimate of only 1 percent. In general, the RW specification estimates deviations of output from trend that are less positive and more negative than those of the LLT and IRW specifications.

Regardless of the trend specification, all the models estimate that output remained above potential around the onset of the global financial crisis and that a negative output gap started to appear only around the events of the European debt crisis. Through the lens of the models, this situation means that the estimates of potential output were severely affected during the financial crisis, causing output not to go below its potential counterpart. Figure 2 shows the evolution of the smoothed common output trend shock, η_t^y , for each of the three trend

Figure 2: Smoothed Estimates of the Aggregate Output Trend Perturbations



Note: Time series are averages of the posterior draws of the common output trend shocks, η_t^y , using the [Durbin and Koopman \(2002\)](#) simulator smoother weighted by the posterior means of the output trend loadings, δ_t^y , and nominal GDP of each country.

specifications.

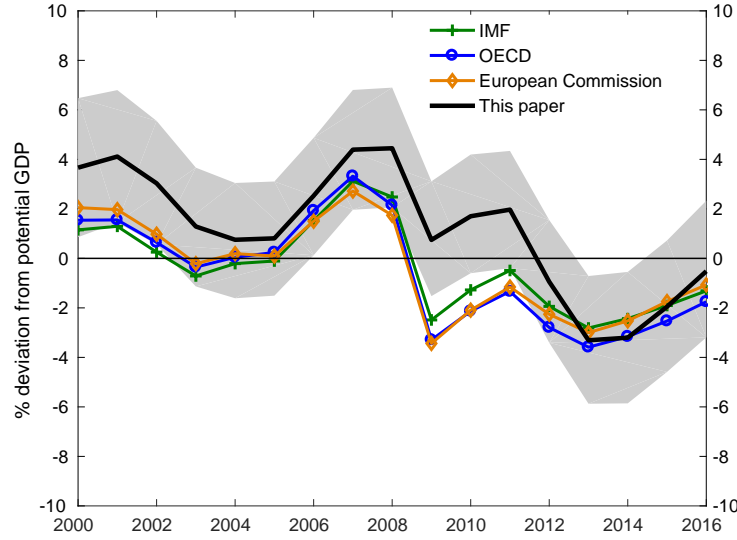
As the figure illustrates, under any of the three trend output assumptions, the model estimates that all the aggregate output trends suffered significantly negative shocks (the largest was about negative 1.5 percent) between the second quarter of 2008 and the first quarter of 2009. The effect of these shocks on the estimate of potential implies that, even though real GDP declined markedly during the global financial crisis, a euro-area output gap did not emerge.⁷

To determine which of the three output trend specifications is more favored by the data, I perform a marginal data density analysis. I use the method outlined in [Chib \(1995\)](#) to obtain the marginal likelihood of the data, which encompasses real GDP, the unemployment rate, and the HICP inflation rate for each of the 19 countries of the euro area. The results are as follows:

- Random walk output trends: -5,615.2
- Local linear output trends: -6,016.0
- Integrated random walk output trends: -6,048.1

⁷The results about the shocks to the output trend are consistent with the differences among the output gaps from the three idiosyncratic trend specifications. In particular, the common output trend under the RW specification experiences upward level perturbations between 2001 and 2006 that are larger than under the IRW and LLT alternatives. As a consequence, the common output trend in the RW case is higher than the other two cases, implying a lower estimated euro-wide output gap throughout.

Figure 3: Smoothed Estimate of the Euro-Area Output Gap Using Country-Level Data and Other Estimates



Source: World Economic Outlook Database of the International Monetary Fund (IMF), Economic Outlook of the Organisation for Economic Co-operation and Development (OECD), Annual Macro-economic Database of the European Commission (European Commission), and author's calculations (This paper). Shaded areas denote the 90 percent confidence sets. The frequency is annual and the quarterly estimated output gap is averaged over the four quarters of each year. The last data point is 2016.

According to the results, the data favor a specification in which the idiosyncratic output trend follows a random walk.⁸ Based on these results, the forthcoming discussion focuses on the results for real GDP, the unemployment rate, and inflation obtained from the specification in which the idiosyncratic trends follow a random walk specification.

To further put in perspective the results of the model proposed in this paper, Figure 3 shows the smoothed estimate of the output gap for the euro area under the preferred output trend specification (RW), along with the estimates by the International Monetary Fund, the Organisation for Economic Co-operation and Development, and the European Commission. Although the 90 percent confidence set of the estimated deviation of output from potential includes those of the three institutions over most of the sample, there is a stark difference around the onset of the global financial crisis in 2009 through 2010. The three institutions estimate a large output gap (around negative 3 to negative 4 percent) during this period, whereas the estimate in this paper indicates that output was still above potential, even though the confidence set includes zero. The difference is likely that the aforementioned institutions have not estimated as negative a shock to potential output as the one I obtain using the model proposed. Other than that, the estimates of the euro-area output gap at the end of 2016 for the model and the institutions are relatively close. This paper estimates the output gap to be negative $\frac{1}{2}$ percent, whereas the European Commission, the IMF, and the

⁸In contrast, the marginal likelihood measures in [Lenza and Jarociński \(2016\)](#) indicate that, for a UC model with aggregate data and several macroeconomic variables, a specification with local linear trends in the observables is the most favored by the data.

OECD estimate output gaps of about negative 1 percent, negative $1\frac{1}{4}$ percent, and negative $1\frac{3}{4}$ percent, respectively.

6.3 Estimates of Country-level Output Gaps and Potential Outputs

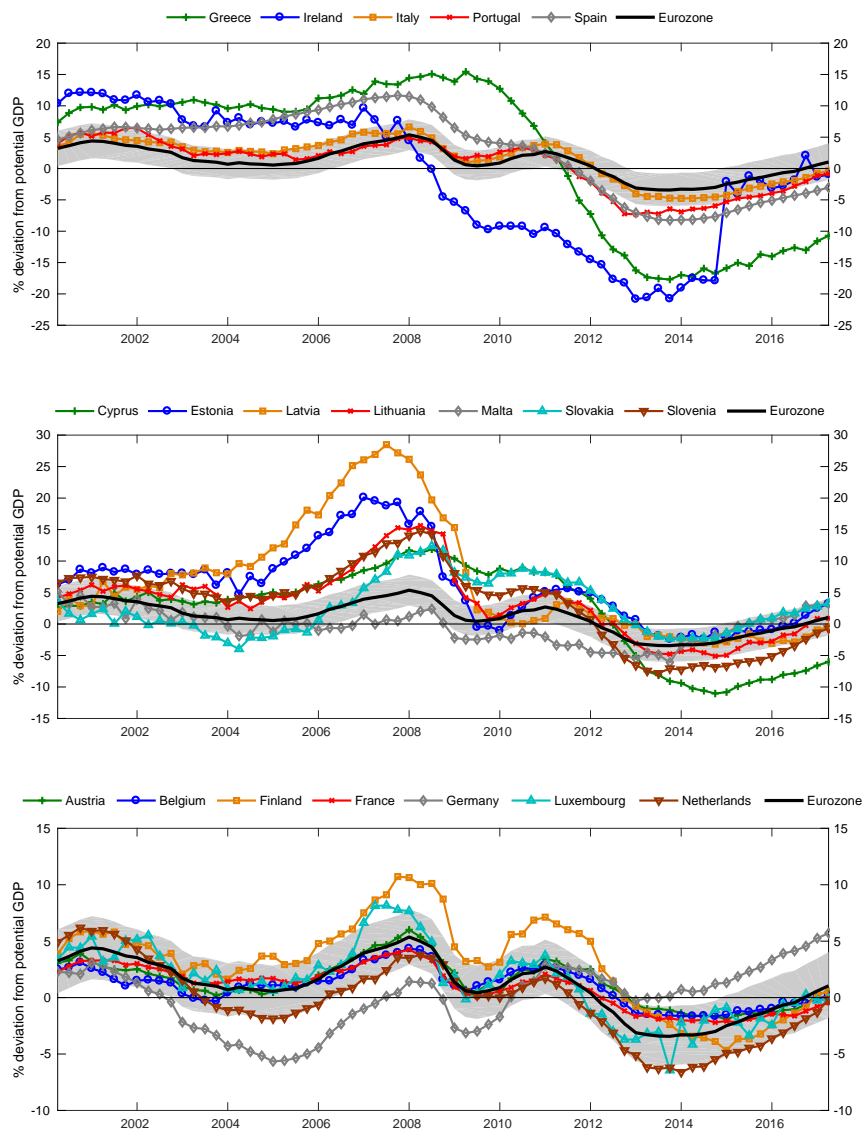
As a byproduct of the model proposed, one can estimate the country-level output gaps and potential outputs. In addition, the model allows one to estimate how connected the state-level cycles and trends are with their common counterparts. Figure 4 shows the estimated output gaps for three groups of countries. The first group is composed of the countries considered to be weaker economically following the global financial crisis: Greece, Ireland, Italy, Portugal, and Spain. In the second group, I include the countries that joined the euro area later than its original inception in 1999 and which are not in the first group: Cyprus, Estonia, Latvia, Lithuania, Malta, Slovakia, and Slovenia. Finally, the third group is formed by the countries that are original members of the euro area and which are not in the first group: Austria, Belgium, Finland, France, Germany, Luxembourg, and Netherlands.

The results show that the three groups of countries can have very different features in their output gaps. For instance, some of the countries in the first group experienced output above potential significantly beyond than what is estimated for the euro area (labeled “Eurozone” in each panel) since 2000 and prior to the global financial crisis: Greece’s output was about 10 percent above potential from 2000 to 2010, Ireland’s output hovers around 5 percent above trend from 2003 to 2007, whereas Spain has a degree of over utilization between 5 percent and 10 percent from 2000 to 2009. This situation reverses around late 2011 for most countries, the exception being Ireland which started to experience large negative output gaps as early as 2008. In recent years, all the countries in this group have estimated output gaps that are mostly larger than the euro-area estimate, with Greece going through an estimated output gap of about negative 10 percent in the second quarter of 2017, and Ireland being affected by positive idiosyncratic perturbations since late 2014 that bring its output gap close to the euro-wide estimate at the end of the sample.

The second group of countries seems to have converged to the euro-area estimate of the output gap more rapidly than the first group. There are countries such as Estonia and Latvia with output significantly above potential estimates in the run-up to the financial crisis between late 2004 and late 2007. The same is true for Cyprus, Lithuania, Slovakia, and Slovenia, but to a lesser extent. Starting in 2013, countries such as Cyprus and Slovenia begin to experience output gaps larger than the euro-area as a whole. Slovenia has virtually closed its output gap by the second quarter of 2017, but Cyprus still maintains an elevated degree of economic slack of the order of about 6 percent.

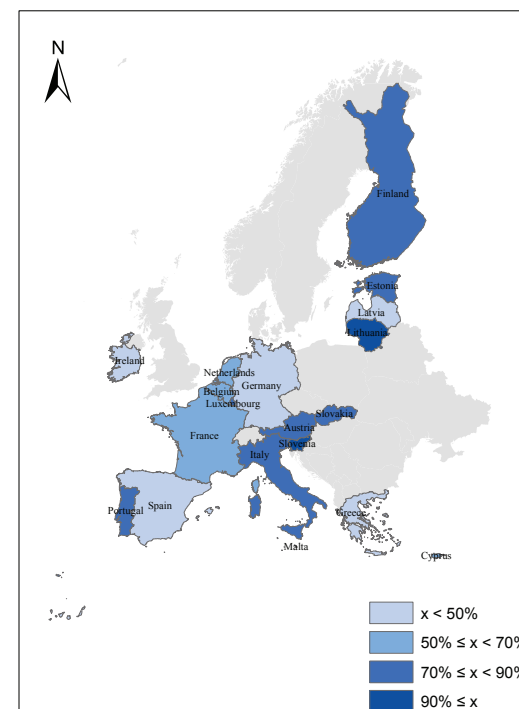
The most homogeneous group appears to be the third one. Apart from Germany and Finland, the cyclical positions of the other countries in this group are mostly inside the confidence set of the estimate of the euro area. Germany presents a relatively large degree of slack that reaches 5 percent between 2004 and 2006. The output gap then materially closes by mid-2010 and starts to deviate from the euro-wide estimate starting in 2014 to put output about 6 percent above potential in the second quarter of 2017. Regarding Finland, its estimated cyclical position is outside the confidence set of that for the euro-area between

Figure 4: Smoothed Estimates of the Country-Level Output Gaps



Note: Time series are the averages of the posterior draws of the euro area output gap using the [Durbin and Koopman \(2002\)](#) simulator smoother. Shaded areas denote the 90 percent confidence sets.

Figure 5: Variance Decomposition of Country-Level Output Gaps



Note: "x" is the proportion of the variance of the country's output gap that is explained by the variance of the common cyclical component. Coefficients are evaluated at the posterior mean.

2005 and 2008 when output reaches about 10 percent above trend. After that, this economy experiences less tight resource utilization, but still has output more than 5 percent above potential in 2011. By the end of the sample period, Finland’s output gap is estimated to have closed.

Another way to characterize the economies of the member states is to break down their output cyclical and trend components in their respective common and idiosyncratic counterparts. Recall that for each country, its cycle is a linear combination of common cyclical and idiosyncratic components. The same is true for the output trend. Figure 5 depicts the contribution of the variation in the common cyclical component to the variance of each country’s output gap. This contribution is a measure of how close a member state’s business cycle has been to that of the euro area in the analyzed period.

In line with the results shown in Figure 4, the map shows that, on the one hand, the countries whose cyclical positions are least influenced by the common euro-area cycle are Greece, Ireland, Spain, Latvia, and Germany. These countries experience idiosyncratic perturbations that make their economies more unique and less connected with the cyclical position of the euro area as a whole. On the other hand, the member states that more strongly co-move with the common cycle are Estonia, Lithuania, Slovenia, and Finland.

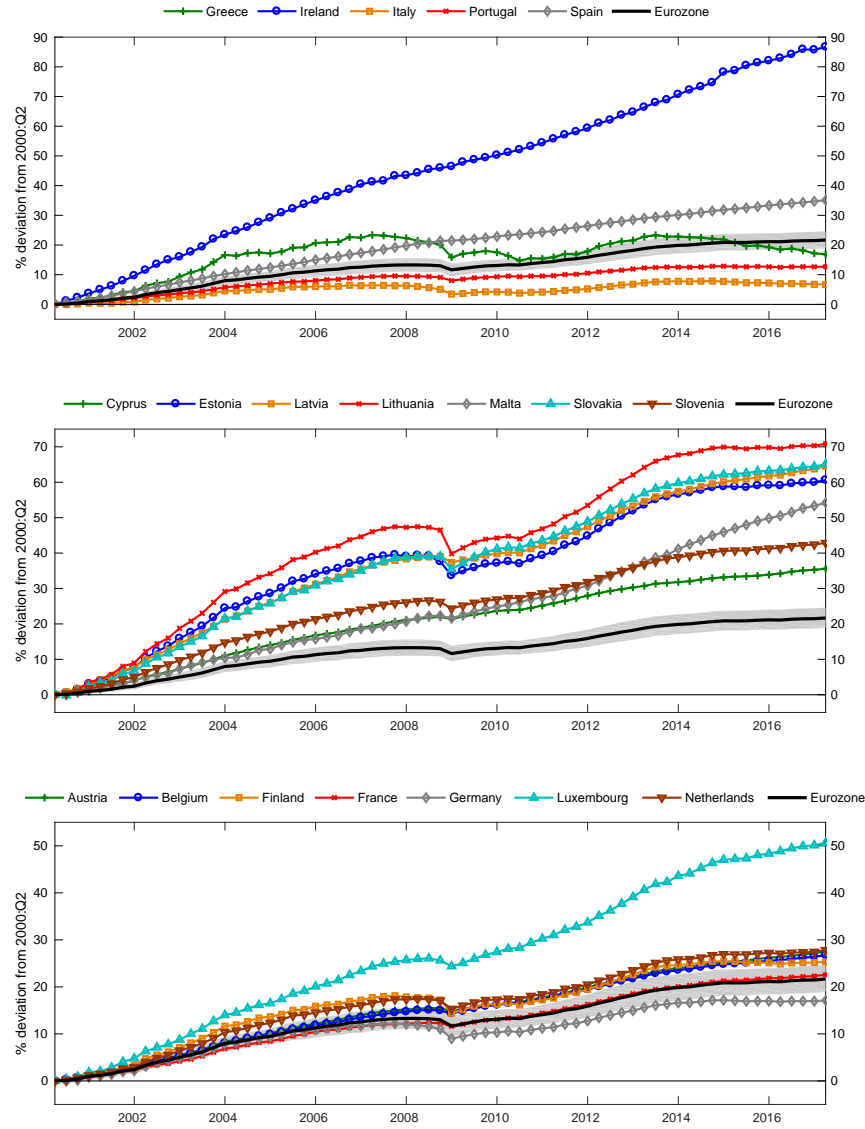
Few countries are estimated to have experienced a negative output gap during the years of the global financial crisis. Among them, there are Ireland, Malta, and Germany, indicating that the model estimates negative shocks to the output trend for most countries during the crisis. Figure 6 illustrates the evolution of the estimated output trends for the euro area and for individual countries divided in the three groups described previously.

The results indicate that the euro-area potential output (labeled “Eurozone” in each panel) increased about twenty percent between early 2000 and early 2017, slightly below 1.2 percent per year, on average. As evidenced before, negative shocks to the common output trend between the second quarter of 2008 and the first quarter of 2009 make potential output to decline during that period by about 1.7 percent. In fact, apart from Ireland and Spain, all countries are estimated to have experienced a decline in their potential outputs during the global financial crisis.

The first panel shows that the member states with the lowest potential output growth rates are Italy and Portugal, reaching approximately 7 percent and 13 percent, respectively, since 2000. The fastest growing potential output is estimated to be in Ireland, where it almost doubles since 2000. As can be inferred from the middle panel, the countries that joined the euro area in the most recent years have experienced higher potential output growth rates, on average, than the other member states, with Lithuania having an estimated increase of about 70 percent compared with its level in 2000. In the third group of countries, Luxembourg has reached an increase of 50 percent in its potential output with respect to the level in 2000. All the other countries are clustered with potential output growth rates between 20 percent and 30 percent, except Germany, which is estimated to have experienced a growth rate of only 17 percent (roughly 1 percent per year, on average).

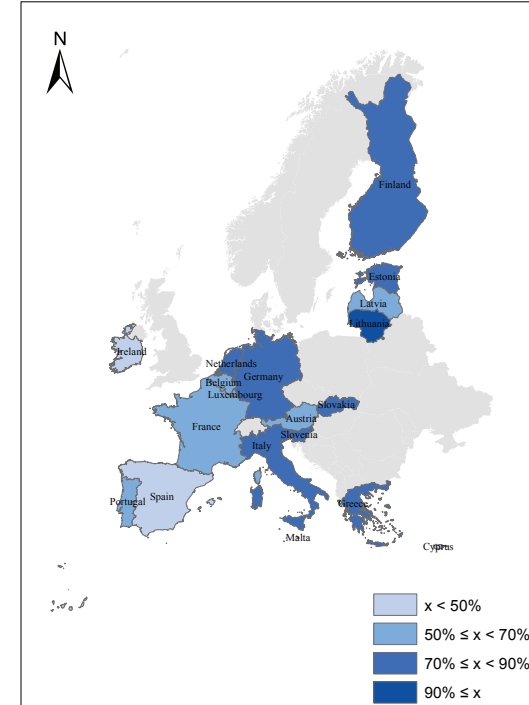
Figure 7 shows the proportion of the variation in potential output at the country level that has been related to the variation of the common trend for each member state during the period of analysis. The variations in the potential outputs of Ireland, Spain, Malta, and Cyprus are the least affected by the variations of the trend common to the euro area, meaning that these countries have idiosyncratic output trend perturbations that make them

Figure 6: Smoothed Estimates of the Country-Level Potential Outputs



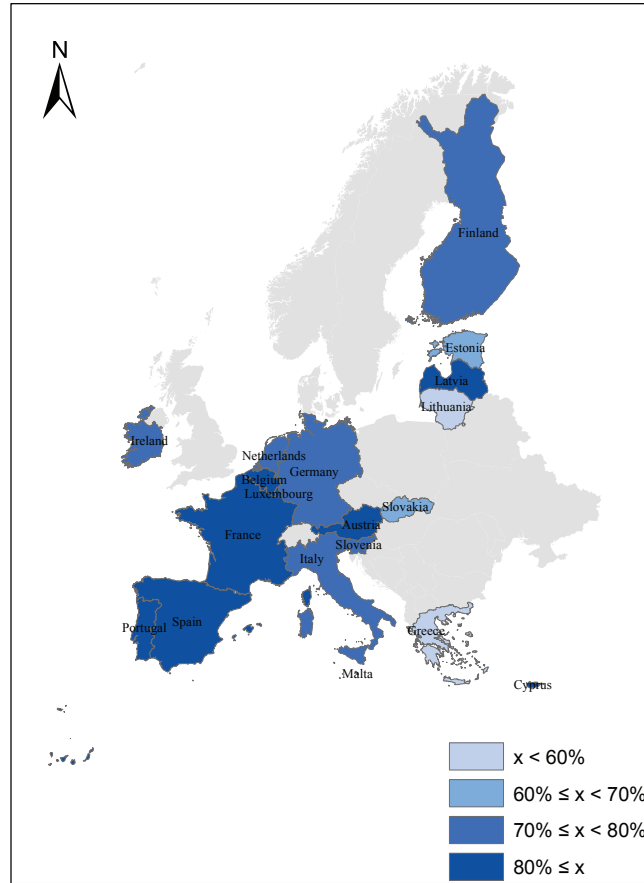
Note: Time series are the averages of the posterior draws of the euro area output gap using the [Durbin and Koopman \(2002\)](#) simulator smoother. Shaded areas denote the 90 percent confidence sets.

Figure 7: Variance Decomposition of Country-Level Potential Outputs



Note: "x" is the proportion of the variance of the country's potential output that is explained by the variance of the common trend component. Coefficients are evaluated at the posterior mean.

Figure 8: Variance Decomposition of Country-Level Real GDP Growth Rates



Note: “x” is the proportion of the variance of the country’s real GDP growth rate that is explained by the variance of its cyclical component. Coefficients are evaluated at the posterior mean.

distinct from a euro-wide aggregate. In contrast, Estonia, Lithuania, Slovenia, and Finland show variations in their trend outputs that are highly affected by variations in the common component of trend output.

6.3.1 A Further Look at the Characteristics of the Country-Level Output Gaps

Finally, the model also allows a more detailed exploration to decompose the variations of real GDP growth for each country into variations that are due to either the country-specific cycle or trend. There may be countries that are “more cyclical” than others in that they can experience relatively more perturbations in the growth of real GDP that can be thought of transitory rather than permanent, which have a more lasting effect on real GDP growth. Figure 8 illustrates the results.

The countries that can be referred to as more cyclical in the euro area are Portugal, Spain, Cyprus, and Austria, whereas the countries that are relatively more influenced in their real GDP growth rates by perturbations to the trend are Greece, Estonia, Lithuania, and Slovakia.

6.3.2 A Further Look at the Characteristics of the Euro-Area Output Gap

With the information at the country level about cycles and trends, and considering the covariance between potential output and the output gap within and across countries, one can estimate the proportion of the euro-area real GDP growth that is due to changes in either its cyclical or trend components (see [Gonzalez-Astudillo, 2017](#), for details on the derivation). Using the posterior means of the parameters, the model implies that about 69 percent of the variations in real GDP growth are due to the cycle and the remainder to potential output. This percentage is somewhat above estimates for the United States that put the contribution of the cycle at about 60 to 65 percent (see [Gonzalez-Astudillo and Roberts, 2016](#)).

6.4 Estimates of the Euro Area and Country-Level Natural Rates of Unemployment

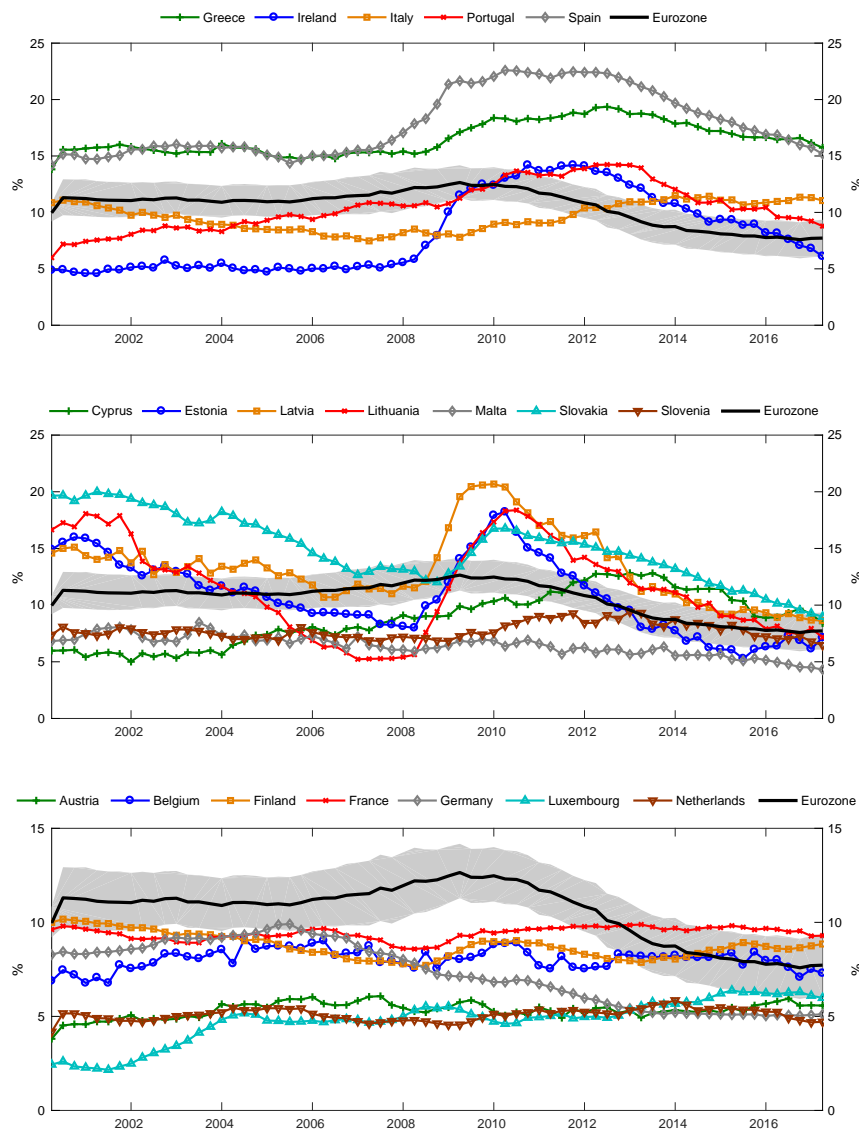
The model presented also allows the estimation of the natural unemployment rate for each country. I use the labor market size of each country to construct an estimate of the euro-area natural rate of unemployment. Figure 9 depicts the estimated natural unemployment rates for the three groups of countries described before and the respective euro-area estimate (labeled “Eurozone” in each panel).

The euro-area natural unemployment rate is estimated to have started around 11 percent in 2000 and remained there until late 2006 when it starts to increase to reach a peak of about 12.5 percent in mid-2009; it declines thereafter to about 7.7 percent in the second quarter of 2017 with a 90 percent confidence set between 6.1 percent and 9.4 percent.⁹

The results show again very distinctive patterns for the three groups of countries. The group in the first panel has estimated natural unemployment rates higher than the area-wide equivalent, whereas the second group is closer to the euro-area estimate, and the third group has, in general, lower estimated rates than the euro area-wide measure. In the first group, Greece and Spain stand out with estimates of the natural unemployment rate around 15 percent, reaching almost 20 percent in the case of Greece and surpassing that level in the case of Spain in the years following the global financial crisis. Italy shows a relatively stable natural unemployment rate that is around 11 percent at the end of the sample. Portugal and Ireland show more volatility around the years of the global financial crisis, but not as much as Greece and Spain, and have gone back roughly to pre-crisis levels.

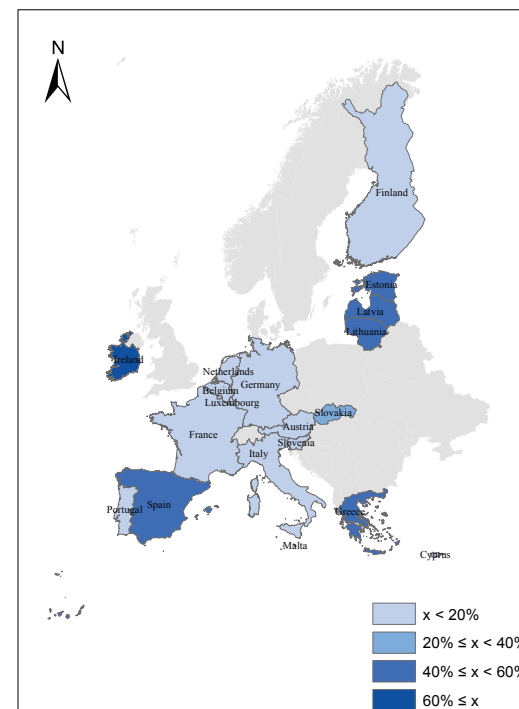
⁹The OECD offers a measure of the NAIRU or equilibrium unemployment rate (not shown in the figure) that is much less variable than the estimate offered by the proposed model in this paper. The OECD estimates the NAIRU of the sixteen euro-area countries that belong to the organization to have started around 8.8 percent in 2000, increase to almost 9.3 percent in 2005 and then decline slightly until 2008, when it begins to increase to reach almost 9.2 percent in 2010. The NAIRU estimated by the OECD declines thereafter to end at about 8.7 percent in 2017, its lowest level since 2000, which is qualitatively in line with the results in this paper.

Figure 9: Estimates of the Euro-area Natural Unemployment Rate



Note: Time series are the averages of the posterior draws of the euro-area unemployment trend using the [Durbin and Koopman \(2002\)](#) simulator smoother. Shaded areas denote the 90 percent confidence sets.

Figure 10: Variance Decomposition of Country-Level Natural Unemployment Rates



Note: "x" is the proportion of the variance of the country's natural unemployment rate that is explained by the variance of the common trend component. Coefficients are evaluated at the posterior mean.

In the second group, Estonia, Latvia, Lithuania, and Slovakia show downward trends in their estimated natural unemployment rates from very high levels until 2008 when they start to increase significantly until late 2009 or early 2010, at which moment they trend downward again to end in a range between 7 percent and 9 percent. Cyprus, Malta, and Slovenia show less volatile natural unemployment rate estimates and seem to have returned to pre-crisis levels or even lower, except for Cyprus whose natural rate remains elevated at the end of the sample at a level near 8.5 percent, about 2.5 percentage points higher than in 2000.

The countries in the third group show very stable estimates of the natural rate of unemployment, except for Germany, whose estimate mostly trends downward from an original level above 8 percent in 2000 to about 5 percent in 2017. This latter level is around which the estimates of Austria, Luxembourg, and Netherlands have hovered for most of the sample period. In contrast, Belgium, Finland, and France show natural unemployment rate estimates that fluctuate around a relatively higher level of about 9 percent.

One can further analyze how closely related the structural features of the country-level labor markets have been to the structure of the common labor market, in particular with respect to the natural rate of unemployment during the period analyzed. Figure 10 shows the proportions of the variation in the natural rates of unemployment of each country that are due to variations in the common component of the unemployment rate trend. A higher proportion means that the trend component of the country-level unemployment rate is more affected by changes in the common component.

The results indicate that only about a fourth of the member states have their natural rates of unemployment significantly connected with the area-wide natural rate. The countries in this group are: Ireland, Spain, Estonia, Latvia, and Lithuania. The rest of the countries have a very low proportion of the variation in their natural unemployment rates explained by variations in the common unemployment rate trend, with about half of the member states having a proportion below 10 percent. These findings would imply that the structural component of the country-level unemployment rates cannot be easily affected by policies at the euro area level designed to affect the common structural unemployment rate.¹⁰

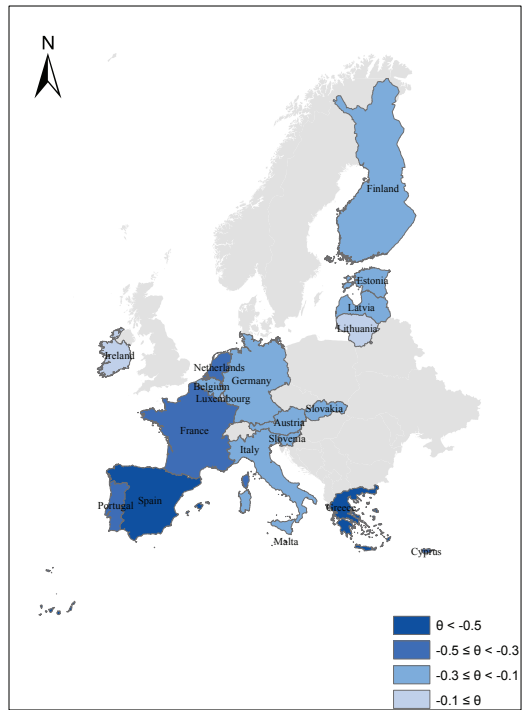
6.4.1 Characterizing the Cyclical Properties of Country-Level Labor Markets

Based on the estimates of the Okun's law coefficients, θ_{1i} and θ_{2i} , the model allows one to characterize how cyclical the labor markets of the member states are. Figure 11 shows the countries whose unemployment rates react more strongly to changes in their cyclical positions of output for given unemployment rate trends. I plot the sum of the Okun's law coefficients whose interpretation is the decline in percentage points in the unemployment rate for an increase of one percentage point in the level of output above potential.

The results show that the countries with most cyclical labor markets are Spain and Greece with Okun's law coefficients larger (in absolute value) than 0.5, followed by Cyprus and Portugal with coefficients above 0.4 (in absolute value). In contrast, the least cyclical labor markets would be in Malta, Ireland, Luxembourg, and Lithuania, all of them with coefficients below 0.1 (in absolute value).

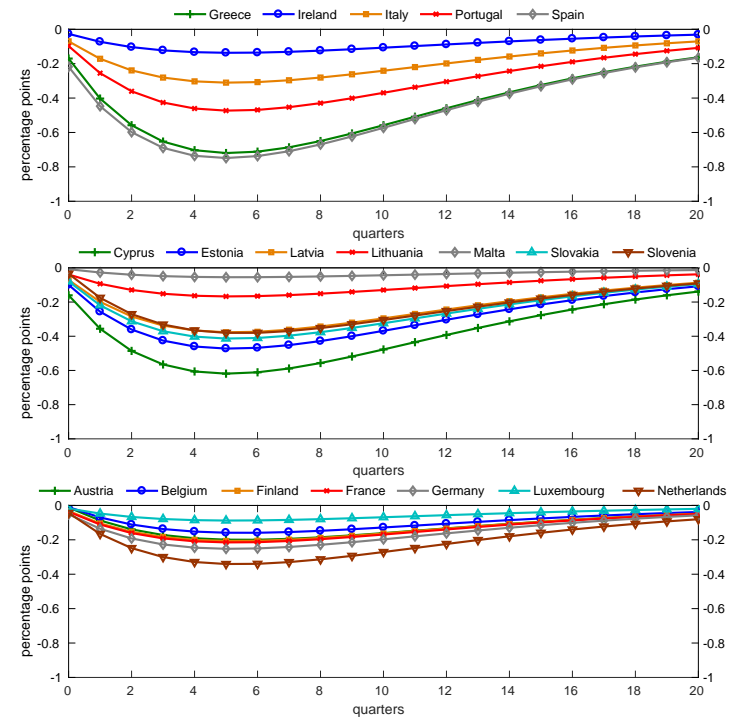
¹⁰Labor market or social insurance policies can vary significantly across the country members. These differences could explain why structural unemployment remains so country-specific even as potential output seems to be affected by euro-area wide policies.

Figure 11: Okun's Law Coefficient Estimates



Note: θ in the figure is the sum $\theta_1 + \theta_2$ for each country evaluated at the posterior mean.

Figure 12: Impulse-Response Analysis for the Cyclical Component of the Unemployment Rate



Note: Response to a 1/2 percentage point increase in the euro-area output gap. Coefficients are evaluated at the posterior mean.

One can also use the model to simulate the response of the country-level unemployment rates after a shock to the euro-wide output gap, keeping constant the idiosyncratic trends and cycles as well as the common trend. This simulation is a way to understand how economic policies (monetary or fiscal) designed to affect the business cycle of the euro area would propagate geographically. The shock is an initial widening of the euro-area output gap by $\frac{1}{2}$ percentage point. The responses of the country-level unemployment rates appear in Figure 12.

In line with the estimated Okun's law coefficients, the impulse-response analysis shows that the cyclical component of the unemployment rates of Greece and Spain would respond more significantly to an area-wide increase of output above trend. In the same vein, those countries with smaller coefficients mentioned before have more muted unemployment rate responses from the aggregate shock. However, there can be countries with relatively high estimated Okun's law coefficients, but with only moderate unemployment rate responses to a common cycle shock. That is the case of France.¹¹ In this case, the unemployment rate cycle would respond more to the idiosyncratic component of the cycle of output than to its common counterpart.

6.5 Estimates of Country-Level Phillips Curves

A concept central to that of potential output and, hence, to that of the output gap is the Phillips curve. Understanding the dynamics of inflation with respect to the degree of slack in the economy is central to the design and implementation of economic policies in general, and of monetary policy in particular. This section explores the cyclicity of inflation in the euro-area member states.

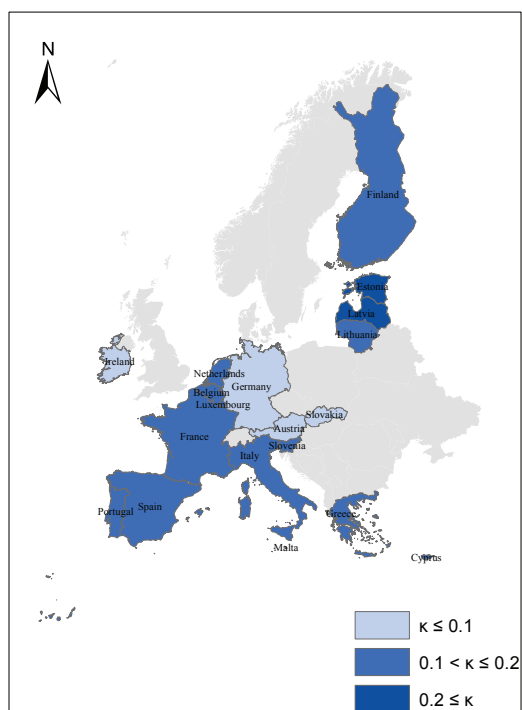
Figure 13 classifies the countries according to the posterior mean estimates of the slope of the Phillips curve, κ_i , for a given level of inflation expectations and lagged inflation. The higher the coefficient, the stronger the response of the country's inflation to its cyclical position. In general, the posterior mean estimates of the slope of the Phillips curve for the countries are centered around a value close to 0.14. However, there are countries with slopes larger than twice this value and others with coefficients about half the cross-sectional average. The countries with the largest Phillips curve slope are estimated to be Estonia and Latvia, whereas those with the smallest response of inflation to the output gap would be Germany and Slovakia.

As with the country-level unemployment rates, one can simulate the propagation of the effects of common cyclical component changes on the inflation rates of the euro-area country members. Figure 14 depicts the impulse-response analysis of an initial deviation of output $\frac{1}{2}$ percentage point above trend, keeping inflation expectations fixed.¹² The first

¹¹Recall that under the model specification, a perturbation to the common cycle affects each country's cycle depending on the loading coefficients, α_i . The response of each country's unemployment rate to the country-specific cycle on impact, in turn, depends on the Okun's law coefficients, θ_{1i} and θ_{2i} .

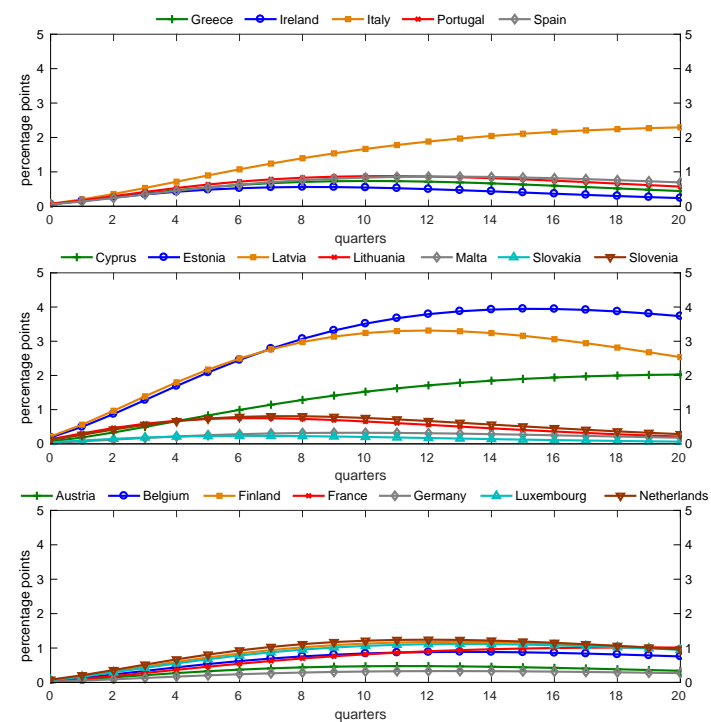
¹²Recall that under the model specification, a perturbation to the common cycle affects each country's cycle depending on the loading coefficients, α_i . The response of each country's inflation rate to the country-specific cycle on impact, in turn, depends on the Phillips curve slope coefficients, κ_i . The dynamic response of inflation depends on the persistence coefficient, $1 - \beta_{1i}$. The higher this coefficient, the lower the dependence of inflation on inflation expectations.

Figure 13: Phillips Curve Slope Coefficient Estimates



Note: Coefficients are evaluated at the posterior mean.

Figure 14: Impulse-Response Analysis for the Cyclical Component of the Inflation Rate



Note: Response to a 1/2 percentage point increase in the euro-area output gap. Coefficients are evaluated at the posterior mean.

panel shows that, except for Italy because of a highly persistent inflation, those countries that experienced weaker economies following the global financial crisis have very uniform responses of inflation, which are relatively muted. The group of countries in the middle panel has more varied responses, with the inflation rates of Latvia and Estonia reacting more strongly and persistently to the common shock, whereas Slovakia’s inflation shows one of the least significant responses. The countries in the last group show some variability in the responses of inflation, with Germany at the bottom because of its very low estimated Phillips curve slope coefficient and Netherlands at the top, albeit with a response still relatively small.

7 Evaluating the Forecasting Performance of the Model

Even though the marginal data densities favor the model in which the country output trends are modeled as random walks, an additional consideration to take into account to choose the best specification and to diagnose the features of a chosen model against other alternatives is to evaluate its forecasting capabilities. This section performs a pseudo out-of-sample forecasting exercise in which I use each of the specifications proposed in Section 3 to predict the three variables of interest (real GDP growth, unemployment rate, and inflation rate) at the euro area level. The forecast horizon is one and four quarters ahead.

The forecasting exercise is also useful for determining whether there is any gain in using disaggregated data at the country level for real GDP growth, the unemployment rate, and the inflation rate to obtain the euro-area output gap compared to using euro-area aggregate data for the three variables of interest. A priori, there is no reason to believe that using disaggregate data would offer an advantage in terms of forecasting capabilities, so it is worth to explore how the model proposed in this paper performs. To that end, I set up a UC model for real GDP growth, the unemployment rate, and the inflation rate at the aggregate euro area level. The structure is basically the same as for each country in the model of equations (1)-(8) and resembles the model proposed by Basistha and Nelson (2007): output is the sum of a trend and a cycle, the unemployment rate is the sum of a trend and a cycle that is a function of the cycle of output following an Okun’s law, and the inflation rate is a ‘hybrid’ Phillips curve. However, I setup the aggregate output trend under the three specifications used before: random walk (RW), local linear trend (LLT), and integrated random walk (IRW). The description of the models and the estimation results appear in Appendix G.

In the exercise, I assume that the data, both at the country as well as the aggregate euro area level, are available through the first quarter of 2009, so that the out-of-sample forecasting exercise starts in the second quarter of 2009. At each subsequent quarter, I re-estimate the model with the new data and obtain 10,000 draws from the posterior distribution and the Durbin and Koopman (2002) simulator smoother to construct the forecasts. To forecast the three euro-area aggregate variables (real GDP growth, unemployment rate, and inflation rate), I first forecast the country-level variables using the UC models proposed (with the three output trend specifications—RW, LLT, and IRW—) and then aggregate them to obtain the euro-area-level equivalent (these models are labeled “Country-level”).¹³ These forecasts are then compared with those of (i) three UC models corresponding to the three output

¹³To aggregate the country-level forecasts to the euro-area aggregates of real GDP growth, the unemployment rate, and inflation, I use nominal GDP, the labor force, and HICP weights, respectively.

trend specifications based on the three aggregate variables (labeled “Aggregate”), and (ii) a VAR(4) model for the three aggregate variables. Table 1 shows the mean squared forecast errors for all the models and their standard deviations across posterior draws.

In general, according to the results, the UC models either with aggregate or country-level data forecast any of the three variables better than the VAR model, one or four quarters ahead. Between the UC models that use aggregate or country-level data, there is no significant difference in terms of forecasting inflation at any horizon. However, the models with country-level data, generally speaking, have more forecast accuracy in terms of real GDP growth and the unemployment rate than the models with aggregate data. Additionally, almost everywhere, the UC models with output trends specified as random walks have the lowest mean squared forecast errors. In particular, among the UC models with country-level data, the RW specification in the idiosyncratic output trends forecasts any of the three variable at least as well as the other models at all horizons. More specifically, at the one-quarter-ahead horizon, the model with RW idiosyncratic output trends is significantly better at forecasting real GDP growth and the unemployment rate than the other two trend alternatives, given the small standard deviations and the relatively large differences in the mean squared forecast errors. That does not happen with inflation, in which case the differences are rather small. The significantly better performance of the RW specification at forecasting all the variables four quarters ahead is even more noticeable. These results, along with those about the marginal data density, favor the country-level UC model with RW idiosyncratic output trends as the best modeling choice.

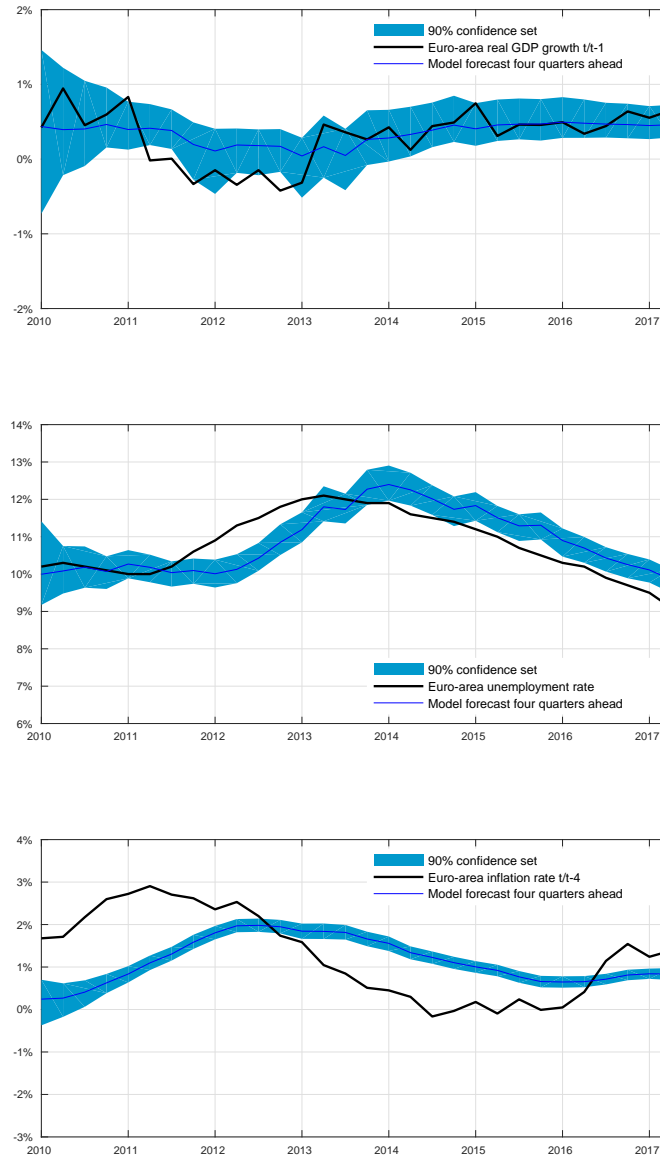
As a way to visualize how well the RW specification forecasts the variables of interest, Figure 15 illustrates its four-quarter-ahead forecasts. Each of the panels shows the observed macroeconomic variable since the first quarter of 2010 in the black line, the median of model forecasts in the thin blue line, and the 90 percent confidence set in the light blue shaded area. The first panel shows that the confidence sets of the model forecast include the realized real GDP growth rate almost everywhere, except for the four quarters of 2011 and the second and fourth quarters of 2012. The unemployment rate forecast appears in the second panel, which shows the model has a relatively hard time forecasting the increase in the unemployment rate around the events of the European debt crisis between mid-2011 and early 2013. After that, the forecast misses to the upside the downward trajectory of the observed series. Finally, the third panel evidences that the inflation forecast is much less volatile than the actual series. The reason is that the predictable component of inflation puts more weight on expected inflation (a much less volatile variable) four quarters ahead than on lagged realized inflation for each country. In fact, the model forecast of inflation is qualitatively very similar to the observed core HICP inflation rate (not shown) in the period of analysis. That means that the unpredictable component, through the lens of the model, may be given by perturbations to the more volatile categories of energy and unprocessed food price inflation. The performances of the other two trend specifications (LLT and IRW) appear in Appendix F.

Table 1: Mean Squared Forecast Errors for Out-Of-Sample Forecasts

Real GDP growth rate		
Model	1 quarter ahead	4 quarters ahead
VAR(4)	0.74 (—)	1.12 (—)
Aggregate (RW)	0.51 (0.07)	0.62 (0.09)
Aggregate (LLT)	0.68 (0.08)	0.87 (0.12)
Aggregate (IRW)	0.67 (0.08)	0.88 (0.12)
Country-level (RW)	0.43 (0.06)	0.36 (0.05)
Country-level (LLT)	0.57 (0.08)	0.65 (0.10)
Country-level (IRW)	0.62 (0.10)	0.69 (0.10)
Unemployment rate		
Model	1 quarter ahead	4 quarters ahead
VAR(4)	0.23 (—)	1.08 (—)
Aggregate (RW)	0.32 (0.02)	1.02 (0.09)
Aggregate (LLT)	0.29 (0.02)	0.90 (0.08)
Aggregate (IRW)	0.28 (0.02)	0.89 (0.08)
Country-level (RW)	0.17 (0.01)	0.65 (0.04)
Country-level (LLT)	0.22 (0.01)	0.77 (0.05)
Country-level (IRW)	0.22 (0.01)	0.76 (0.06)
Inflation rate		
Model	1 quarter ahead	4 quarters ahead
VAR(4)	2.15 (—)	2.19 (—)
Aggregate (RW)	1.54 (0.06)	1.39 (0.09)
Aggregate (LLT)	1.54 (0.06)	1.38 (0.07)
Aggregate (IRW)	1.54 (0.06)	1.39 (0.07)
Country-level (RW)	1.58 (0.03)	1.34 (0.04)
Country-level (LLT)	1.57 (0.03)	1.44 (0.07)
Country-level (IRW)	1.56 (0.03)	1.47 (0.08)

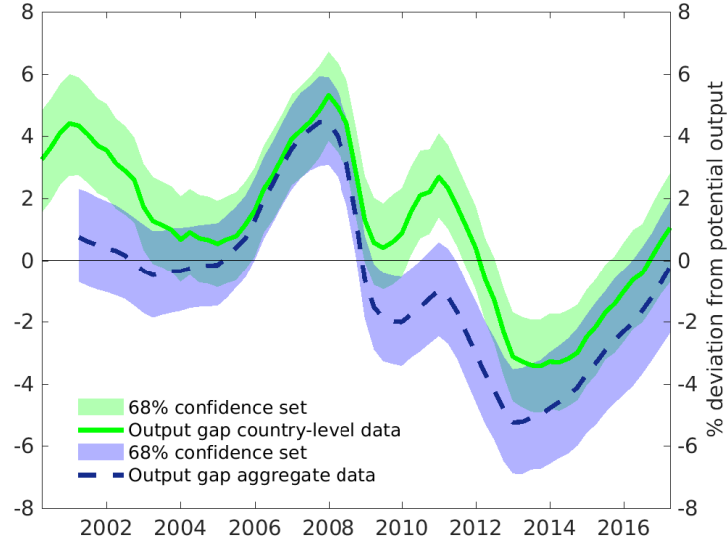
Note: Aggregate denotes the model estimated with euro-area-level data on real GDP, the unemployment rate, and the inflation rate. Country-level denotes the model estimated with country-level data. RW denotes the output trend follows a random walk process. LLT denotes the output trend is specified as a local linear trend. IRW denotes the output trend follows and integrated random walk. Numbers in parenthesis are the standard deviation of the mean square forecast error across posterior draws. The VAR(4) is estimated with classical methods.

Figure 15: Model Forecasts under the RW Specification and Observed Variables



Note: The model forecast four quarters ahead is the median of the model forecast across draws. *Source:* Eurostat and author's calculations.

Figure 16: Output Gap Estimates Comparison: Country-Level vs. Aggregate Data



Note: Time series are the averages of the posterior draws of the euro-area output gap using the [Durbin and Koopman \(2002\)](#) simulator smoother. The aggregate estimate is obtained with data from 2001:Q2 because of data availability.

8 Does Introducing Country-Level Data Reduce the Uncertainty around the Estimate of the Output Gap?

According to the reasoning provided by [Stock and Watson \(2016\)](#), the uncertainty around the estimate of the euro-area output gap would have two opposing forces at work when one uses country-level as opposed to aggregate data. First, a model with heterogeneous units provides more precise estimates of a given common component among units and, hence, makes the signal extraction uncertainty smaller compared with a model that uses a single unit in which the data are aggregated. Second, a model with several heterogeneous units has many more parameters to estimate than an equivalent model with aggregate data, which makes the parameter uncertainty larger in the former. It is not clear which of the two forces dominates in a general framework. Figure 16 compares the estimates of the output gap using the model with country-level data of equations (1)-(8) with that of the aggregate data model used for the forecast comparisons of the previous section along with the corresponding confidence sets. Both estimates correspond to the models with RW output trends.

Based on the results, the use of country-level data somewhat reduces the uncertainty around the estimate of the euro-area output gap compared with the estimate obtained using aggregate data. The confidence sets in the model proposed in this paper are 5 percent narrower, on average, than those from the UC model with aggregate data. Hence, one can conclude that, in this case, the signal extraction uncertainty channel dominates the parameter uncertainty one, and it is beneficial to use disaggregated data.

9 Conclusion

This paper proposes a method to estimate the output gap of the euro area using data on real GDP, the unemployment rate, and the inflation rate of the member states through the Bayesian estimation of an unobserved components model. A key feature of the model is the assumption of common cycle and trend components in real GDP and the unemployment rate across countries. This feature allows one to obtain measures of the output gap, potential output, and the natural unemployment rate of the euro area. Additionally, the framework proposed also permits the characterization of the degree of slack, potential output, and natural rate of unemployment for each member state. This characterization is important for understanding how policies that affect the area as a whole, for example with the goal to curb inflation pressures, propagate across countries.

Among other results, the model's estimation of the data-preferred specification indicates that output was about 5 percent above potential prior to the financial crisis at the end of 2007, then declined to almost its potential level at the end of 2009. After this period, output surpassed its potential counterpart by about 3 percent in 2011 and then declined to nearly 4 percent below trend at the end of 2013, during the European debt crisis. In the second quarter of 2017, it is estimated that output stands at about 1 percent above potential. A negative output gap did not emerge during the global financial crisis because the model estimates potential output to have declined about $1\frac{3}{4}$ percent in that period, which is more than enough to prevent real GDP to fall below trend. In addition, the estimate of the natural unemployment rate is 7.7 percent in the second quarter of 2017, after having reached about 12.5 percent in mid-2009.

Appendix

A State-Space Model in Matrix Form

The model in Equations (1)-(14) can be written in matrix form as follows:

$$\mathbf{z}_{it} = \mathbf{C}(\boldsymbol{\Theta}_{\text{mi}}) + \mathbf{H}(\boldsymbol{\Theta}_{\text{mi}})\mathbf{x}_{it} + \mathbf{w}_{it}, \quad \mathbf{w}_{it}|\mathfrak{F}_{t-1} \sim \text{iid } \mathbf{N}(0, \mathbf{R}(\boldsymbol{\Theta}_{\text{mi}})) \quad (15)$$

$$\mathbf{x}_{it} = \mathbf{F}(\boldsymbol{\Theta}_{\text{si}})\mathbf{x}_{i,t-1} + \mathbf{G}\mathbf{v}_{it}, \quad \mathbf{v}_{it}|\mathfrak{F}_{t-1} \sim \text{iid } \mathbf{N}(0, \mathbf{Q}(\boldsymbol{\Theta}_{\text{si}})), \quad (16)$$

where \mathfrak{F}_{t-1} is the sigma-field containing information until period $t - 1$, with

$$\mathbf{z}_{it} = \begin{bmatrix} \Delta y_{it} \\ \Delta u_{it} \\ \Delta \pi_{it} - \beta_{1i}(\pi_t^e - \pi_{i,t-1}) \end{bmatrix}, \quad \mathbf{w}_{it} = \begin{bmatrix} \eta_{it}^y \\ \eta_{it}^u \\ \eta_{it}^\pi \end{bmatrix}, \quad \mathbf{v}_{it} = \begin{bmatrix} \varepsilon_t \\ \eta_t^y \\ \eta_t^u \\ \nu_{it} \\ \zeta_{it} \end{bmatrix}.$$

For the RW specification, we have the following matrices:

$$\mathbf{x}_{it} = \begin{bmatrix} c_t \\ c_{t-1} \\ c_{t-2} \\ \eta_t^y \\ \eta_t^u \\ v_{it} \\ v_{i,t-1} \\ v_{i,t-2} \end{bmatrix}, \quad \mathbf{C}(\boldsymbol{\Theta}_{\text{mi}}) = \begin{bmatrix} \mu_i + \delta_i^y \mu \\ 0 \\ \beta_{0i} \end{bmatrix},$$

$$\mathbf{H}(\boldsymbol{\Theta}_{\text{mi}}) = \begin{bmatrix} \alpha_i & -\alpha_i & 0 & \delta_i^y & 0 & 1 & -1 & 0 \\ \alpha_i \theta_{1i} & \alpha_i(\theta_{2i} - \theta_{1i}) & -\alpha_i \theta_{2i} & 0 & \delta_i^u & \theta_{1i} & \theta_{2i} - \theta_{1i} & -\theta_{2i} \\ \alpha_i \kappa_i & 0 & 0 & 0 & 0 & \kappa_i & 0 & 0 \end{bmatrix},$$

$$\mathbf{F}(\boldsymbol{\Theta}_{\text{si}}) = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For the LLT and RW specifications, the matrices are the following:

$$\mathbf{x}_{it} = \begin{bmatrix} c_t \\ c_{t-1} \\ c_{t-2} \\ \eta_t^y \\ \eta_t^u \\ \mu_{it} \\ \mu_{i,t-1} \\ v_{it} \\ v_{i,t-1} \\ v_{i,t-2} \end{bmatrix}, \quad \mathbf{C}(\boldsymbol{\Theta}_{\text{mi}}) = \begin{bmatrix} \delta_i^y \mu \\ 0 \\ \beta_{0i} \end{bmatrix},$$

$$\mathbf{H}(\boldsymbol{\Theta}_{\text{mi}}) = \begin{bmatrix} \alpha_i & -\alpha_i & 0 & \delta_i^y & 0 & 0 & 1 & 1 & -1 & 0 \\ \alpha_i \theta_{1i} & \alpha_i (\theta_{2i} - \theta_{1i}) & -\alpha_i \theta_{2i} & 0 & \delta_i^u & 0 & 0 & \theta_{1i} & \theta_{2i} - \theta_{1i} & -\theta_{2i} \\ \alpha_i \kappa_i & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_i & 0 & 0 \end{bmatrix},$$

$$\mathbf{F}(\boldsymbol{\Theta}_{\text{si}}) = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For all specifications, the variance matrices are as follows:

$$\mathbf{R}(\boldsymbol{\Theta}_{\text{vi}}) = \begin{bmatrix} \sigma_{\eta_i^y}^2 & 0 & 0 \\ 0 & \sigma_{\eta_i^u}^2 & 0 \\ 0 & 0 & \sigma_{\eta_i^\pi}^2 \end{bmatrix}, \quad \mathbf{Q}(\boldsymbol{\Theta}_{\text{vi}}) = \begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta^y}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\eta^u}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\nu_i}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\zeta_i}^2 \end{bmatrix},$$

where for the IRW trend setup, $\sigma_{\eta_i^y}^2 = 0$. Finally,

$$\boldsymbol{\Theta}_{\text{mi}} = \{\mu, \mu_i, \alpha_i, \delta_i^y, \sigma_{\eta_i^y}^2, \theta_{1i}, \theta_{2i}, \delta_i^u, \sigma_{\eta_i^u}^2, \beta_{0i}, \beta_{1i}, \kappa_i, \sigma_{\eta_i^\pi}^2\},$$

$$\boldsymbol{\Theta}_{\text{si}} = \{\phi_1, \phi_2, \rho_i, \sigma_\varepsilon^2, \sigma_{\eta^y}^2, \sigma_{\eta^u}^2, \sigma_{\zeta_i}^2, \sigma_{\nu_i}^2\},$$

for $i = 1, \dots, n$.

B Data Details

The information on real GDP, the unemployment rate, the HICP inflation rate at the country- and euro-level come from Eurostat. The Survey of Professional Forecasters (SPF) HICP inflation rate expectations come from the ECB. The data are as follows:

- Real GDP: Both at the country and at the euro area level, I use GDP in million 2010 euro seasonally and calendar adjusted from 2000:Q1 to 2017:Q2. Slovakia does not have this information available, so I perform an X-12 seasonal adjustment to the available data for this country in million 2010 euro not seasonally adjusted. The growth rate of real GDP is obtained as the quarterly percent change.
- Unemployment rate: It is the percentage of active population (15 - 74 years old) that is unemployed. Both at the country and at the euro-level, the data are the quarterly average of the monthly indicator and seasonally adjusted but not calendar adjusted from 2000:Q1 to 2017:Q2.
- HICP: It is the harmonized index of consumer prices with base year 2015 at the monthly frequency which I seasonally adjust using an X-12 procedure and then pick the last value of every quarter to calculate the annualized quarterly rate of inflation. At the country level, the monthly data runs from 1999:M12 to 2017:M06. At the euro area level, the monthly data runs from 2000:M12 to 2017:M06.
- SPF HICP inflation rate expectations: It is the mean point estimate of the year-over-year percentage change one year ahead in the HICP. The data cover 2000:Q1 to 2017:Q2.
- GDP, labor market, and price inflation weights: To obtain the euro-area real GDP growth rate, I weight the real GDP growth rates across countries using nominal GDP weights at the country level, where GDP is million current euro not seasonally adjusted, which I seasonally adjust using an X-12 procedure. To obtain the euro-area unemployment rate from the country-level rates, I aggregate the unemployment rates from the countries using the shares of active population over the total of the euro area as weights. These data are in an annual frequency, which I assume remains constant every quarter of a given year at the annual figure. Finally, to obtain the aggregate HICP inflation rate, I construct the euro-area inflation rate by aggregating the country-level rates weighted by the HICP weights for each country. These data are also in annual frequency and the same treatment as the active population is given within each quarter.

C Details on the Gibbs Sampler

Let \mathbf{z}_{it} , \mathbf{x}_{it} , Θ_{mi} , and Θ_{si} for $i = 1, 2, \dots, n$, be defined as in Appendix A. Let $\mathbf{Z}_T = \{\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2, \dots, \tilde{\mathbf{z}}_T\}$ denote the observed data and let $\mathbf{X}_T = \{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_T\}$. Here, $\tilde{\mathbf{z}}_t = \{\mathbf{z}_{1t}, \mathbf{z}_{2t}, \dots, \mathbf{z}_{nt}\}$ and $\tilde{\mathbf{x}}_t = \{\mathbf{x}_{1t}, \mathbf{x}_{2t}, \dots, \mathbf{x}_{nt}\}$. Denote $\Theta_m = \bigcup_{i=1}^n \Theta_{mi}$ and $\Theta_s = \bigcup_{i=1}^n \Theta_{si}$.

Partition $\Theta_{\text{si}} = \Theta_{\text{s}}^1 \cup \Theta_{\text{si}}^2 \cup \Theta_{\text{si}}^3$, with

$$\begin{aligned}\Theta_{\text{s}}^1 &= \{\phi_1, \phi_2\}, \\ \Theta_{\text{si}}^2 &= \{\rho_i, \sigma_{\zeta_i}^2, \sigma_{\nu_i}^2\}, \\ \Theta_{\text{si}}^3 &= \{\sigma_e^2, \sigma_{\eta^y}^2, \sigma_{\eta^u}^2\}.\end{aligned}$$

Notice that the identification conditions imply that Θ_{si}^3 is fixed.

Also, partition $\Theta_{\text{mi}} = \Theta_{\text{mi}}^1 \cup \Theta_{\text{mi}}^2 \cup \Theta_{\text{mi}}^3$, with

$$\begin{aligned}\Theta_{\text{mi}}^1 &= \{\mu_i, \alpha_i, \delta_i^y, \sigma_{\eta_i^y}^2\}, \\ \Theta_{\text{mi}}^2 &= \{\theta_{1i}, \theta_{2i}, \delta_i^u, \sigma_{\eta_i^u}^2\}, \\ \Theta_{\text{mi}}^3 &= \{\beta_{0i}, \beta_{1i}, \kappa_i, \sigma_{\eta_i^\pi}^2\},\end{aligned}$$

where μ has been excluded because it is fixed under the identification conditions. For the RW output trend specification there is no sampling of $\sigma_{\nu_i}^2$, whereas for the LLT specification there is no sampling of the parameters μ_i , and for the IRW specification there is no sampling of either μ_i or $\sigma_{\eta_i^y}^2$.

The Gibbs sampler operates as follows:

1. Start with initial values for the model's parameters, $\Theta = \Theta_{\text{m}} \cup \Theta_{\text{s}}$.
2. Draw \mathbf{X}_T from $p(\mathbf{X}_T | \mathbf{Z}_T, \Theta_{\text{m}}, \Theta_{\text{s}})$ using the [Durbin and Koopman \(2002\)](#) simulation smoother.
3. Draw Θ_{s}^1 from $p(\Theta_{\text{s}}^1 | \mathbf{Z}_T, \mathbf{X}_T, \Theta_{\text{m}}, \Theta_{\text{si}}^2, \Theta_{\text{si}}^3)$ using the conditional distributions implied by the independent normal-inverse-gamma prior.
4. For $i = 1, 2, \dots, n$, sample as follows:
 - (a) Draw Θ_{si}^2 from $p(\Theta_{\text{si}}^2 | \mathbf{Z}_T, \mathbf{X}_T, \Theta_{\text{m}}, \Theta_{\text{s}}^1, \Theta_{\text{si}}^3)$ using the conditional distributions implied by the independent normal-inverse-gamma prior.
 - (b) Draw Θ_{mi}^1 from $p(\Theta_{\text{mi}}^1 | \mathbf{Z}_T, \mathbf{X}_T, \Theta_{\text{mi}}^2, \Theta_{\text{mi}}^3, \Theta_{\text{m}(\cdot/i)}, \Theta_{\text{s}})$ using the conditional distributions implied by the independent normal-inverse-gamma prior. Repeat similarly for Θ_{mi}^2 and Θ_{mi}^3 and sample from
 - $p(\Theta_{\text{mi}}^2 | \mathbf{Z}_T, \mathbf{X}_T, \Theta_{\text{mi}}^1, \Theta_{\text{mi}}^3, \Theta_{\text{m}(\cdot/i)}, \Theta_{\text{s}})$ and
 - $p(\Theta_{\text{mi}}^3 | \mathbf{Z}_T, \mathbf{X}_T, \Theta_{\text{mi}}^1, \Theta_{\text{mi}}^2, \Theta_{\text{m}(\cdot/i)}, \Theta_{\text{s}})$,
respectively.
5. Return to step 2.

D Choice of Prior Distributions

The choice of prior distributions of the parameters of the UC models appears in Table 2. For convenience, I have included the equation to which each of the parameters belongs.

Table 2: Prior Distributions of the Parameters - Country Level Data

Parameter	Distribution	Mean	Sd.	Equation
ϕ_1	Truncated Normal	1.7	0.2	$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t$
ϕ_2	Truncated Normal	-0.75	0.2	$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t$
μ_i	Normal	0.5	0.5	$\xi_{it}^y = \mu_i + \xi_{i,t-1}^y + \eta_{it}^y$
α_i	Normal	1	1	$c_{it} = \alpha_i c_t + v_{it}$
δ_i^y	Truncated Normal	1	1	$\tau_{it}^y = \delta_i^y \tau_t^y + \xi_{it}^y$
δ_i^u	Truncated Normal	1	1	$\tau_{it}^u = \delta_i^u \tau_t^u + \xi_{it}^u$
θ_{1i}	Normal	-0.2	0.2	$u_{it} = \tau_{it}^u + \theta_{1i} c_{it} + \theta_{2i} c_{i,t-1}$
θ_{2i}	Normal	-0.2	0.2	$u_{it} = \tau_{it}^u + \theta_{1i} c_{it} + \theta_{2i} c_{i,t-1}$
β_{0i}	Normal	0	0.5	$\pi_{it} = \beta_0 + \beta_1 \pi_t^e + (1 - \beta_1) \pi_{i,t-1} + \kappa c_{it} + \eta_{it}^\pi$
β_{1i}	Normal	0.5	0.5	$\pi_{it} = \beta_0 + \beta_1 \pi_t^e + (1 - \beta_1) \pi_{i,t-1} + \kappa c_{it} + \eta_{it}^\pi$
κ_i	Normal	0.1	0.1	$\pi_{it} = \beta_0 + \beta_1 \pi_t^e + (1 - \beta_1) \pi_{i,t-1} + \kappa c_{it} + \eta_{it}^\pi$
ρ_i	Truncated Normal	0	1	$v_{it} = \rho_i v_{i,t-1} + \zeta_{it}$
$\sigma_{\eta_i^y}^2$	Inverse Gamma	0.25	Inf	$\xi_{it}^y = \mu_i + \xi_{i,t-1}^y + \eta_{it}^y, \quad \text{var}(\eta_{it}^y) = \sigma_{\eta_i^y}^2$
$\sigma_{\eta_i^u}^2$	Inverse Gamma	0.25	Inf	$\xi_{it}^u = \xi_{i,t-1}^u + \eta_{it}^u, \quad \text{var}(\eta_{it}^u) = \sigma_{\eta_i^u}^2$
$\sigma_{\eta_i^\pi}^2$	Inverse Gamma	1	Inf	$\pi_{it} = \beta_0 + \beta_1 \pi_t^e + (1 - \beta_1) \pi_{i,t-1} + \kappa c_{it} + \eta_{it}^\pi, \quad \text{var}(\eta_{it}^\pi) = \sigma_{\eta_i^\pi}^2$
$\sigma_{\zeta_i}^2$	Inverse Gamma	10.145	Inf	$v_{it} = \rho_i v_{i,t-1} + \zeta_{it}, \quad \text{var}(\zeta_{it}) = \sigma_{\zeta_i}^2$
$\sigma_{\nu_i}^2$	Inverse Gamma	1	Inf	$\mu_{it} = \mu_{i,t-1} + \nu_{it}, \quad \text{var}(\nu_{it}) = \sigma_{\nu_i}^2$

For several parameters, I choose prior means roughly consistent with the posterior mean and variance results in [Lenza and Jarociński \(2016\)](#) for the comparable parameters, although I chose larger standard deviations. That is the way in which I specify the prior distributions of the mean growth rate of trend GDP, μ_i , in the RW case, the parameters of the common cycle, ϕ_1 and ϕ_2 , the Okun's law coefficients, θ_{1i} and θ_{2i} , and the slope of the Phillips curve, κ_i .¹⁴ The latter parameter choice is also informed by the results in [Blanchard, Cerutti and Summers \(2015\)](#). Regarding the remaining parameters of the Phillips curve, β_{0i} and β_{1i} , given the lack of previous estimates for this parameters, I choose a somewhat agnostic distribution in which the mean of the intercept is zero and the persistence coefficient is centered at 0.5. Under the same criteria, I set the persistence coefficient of the idiosyncratic component of the country's cycle, ρ_i , to be centered at zero. I choose a truncated distribution in some of the cases given the stationary or the identification restrictions.

The prior distributions of the variance parameters is inverse gamma in all cases. I set the means as follows: For the variances of the shocks of the idiosyncratic trends, $\sigma_{\eta_i^y}^2$ and $\sigma_{\eta_i^u}^2$, I specify that 80 percent of the variation of the output or unemployment trend of a particular country is explained by the common respective trend and the rest by the idiosyncratic component. I apply the same criterion to the proportion of the variation in the country's cycle that is due to the common cycle; that is how the mean of the prior distribution of $\sigma_{\zeta_i}^2$ is picked. Additionally, given the absence of previous estimates in the literature, I assume $\sigma_{\eta_i^\pi}^2$ and $\sigma_{\nu_i}^2$ to have a prior mean of one. In all cases, the prior distributions of the variances do not have a well defined variance.

¹⁴[Gurin, Maurin and Mohr \(2015\)](#) find smaller estimated coefficients for ϕ_1 and ϕ_2 in a Markov-switching specification of the output trend than the choice made in this paper.

Finally, I set the prior distribution of the loading coefficients α_i , δ_i^y , and δ_i^u to be normally distributed and centered at one with unity standard deviation as a way to incorporate the lack of prior knowledge about these parameters.

E Parameter Results

In this appendix, I lay out the full results from the Bayesian estimation showing the estimates of the posterior mean and standard deviation of the parameters of the model, as well as the first and fiftieth order autocorrelation coefficient, the relative numerical efficiency (RNE) using a 4 percent taper, and the p-value of the Geweke (1991) convergence diagnostics using a 4 percent taper as well in which the null hypothesis considers equality of the means of the first 20 percent of draws with that of the last 50 percent. Following Lenza and Jarociński (2016), I include the value of the output gap at the end of the sample among the parameters for which convergence diagnostics are reported. Given the large number of parameters, the results appear in Tables 3-9. In the tables, the parameters μ_i , α_i , δ_i^y , δ_i^u , θ_{1i} , θ_{2i} , β_{0i} , β_{1i} , κ_i , ρ_i , $\sigma_{\zeta_i}^2$, $\sigma_{\eta_i^y}^2$, $\sigma_{\eta_i^u}^2$, $\sigma_{\eta_i^\pi}^2$ are numbered from $i = 1, \dots, 19$ according to the number of countries in the euro area in the following order: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain. The estimated value of the output gap at the end of the sample period is denoted c_T . ϕ_1 and ϕ_2 are the parameters of the AR(2) specification of the common component of the cycle.

Recall that the draws from the posterior distribution used to produce the results of the model are based on 300,000 draws after burning in the first 100,000 and thinning every 100th draw, which left us with 2,000 draws from the posterior distribution. The results of the diagnostics tests show that these 2,000 draws do not evidence significant autocorrelation of first order and almost no autocorrelation of order fifty. Apart from 2 out of 268 coefficients, the p-values of the test of equality of means between the first 20 percent of the draws and the last 50 percent are all above 1 percent, which indicate that the null hypothesis is not rejected for any of the parameters and the sampler has converged.

An analysis of the results indicates that the weighted average mean growth rate of trend output is about 1.2 percent per year.¹⁵

F Four-Quarters-Ahead Forecasts for the Local Linear Trend and Integrated Random Walk Trend Models

Figures 17 and 18 show the four-quarter ahead forecasts of the macroeconomic variables for the LLT and IRW specifications, respectively. Each of the panels in the figures shows the observed macroeconomic variable since the first quarter of 2010 in the black line, the median of model forecasts in the thin blue line, and the 90 percent confidence set in the light blue shaded area. The first panel refers to real GDP growth, the second panel depicts the unemployment rate, and the third panel, the inflation rate.

¹⁵This estimate is the result of taking the posterior mean estimates of μ_i for all i and weight them by the average relative importance of each country on the euro-area nominal GDP over the sample period.

Table 3: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
ϕ_1	1.686	0.100	0.032	0.003	0.956	0.460
ϕ_2	-0.713	0.100	0.034	0.009	0.957	0.441
c_T	1.039	1.808	0.005	0.011	1.460	0.900
μ_1	0.387	0.058	0.104	0.048	0.924	0.287
μ_2	0.375	0.052	0.081	0.006	0.933	0.540
μ_3	0.503	0.084	-0.000	-0.017	1.310	0.561
μ_4	0.819	0.156	0.160	0.026	0.635	0.630
μ_5	0.329	0.094	0.136	0.033	0.813	0.129
μ_6	0.314	0.048	0.069	0.001	0.837	0.022
μ_7	0.220	0.086	0.100	0.017	0.814	0.666
μ_8	0.187	0.151	0.123	0.067	1.135	0.172
μ_9	1.224	0.226	0.116	0.036	0.956	0.900
μ_{10}	0.071	0.064	0.068	0.050	1.090	0.202
μ_{11}	0.909	0.123	0.094	0.013	0.688	0.001
μ_{12}	0.944	0.176	0.155	0.026	0.903	0.152
μ_{13}	0.709	0.094	0.024	0.025	1.067	0.551
μ_{14}	0.780	0.122	0.000	-0.015	1.120	0.135
μ_{15}	0.375	0.070	0.141	0.030	0.633	0.029
μ_{16}	0.169	0.067	0.047	0.047	1.100	0.084
μ_{17}	0.889	0.163	0.123	0.022	0.793	0.274
μ_{18}	0.590	0.087	0.104	0.032	0.935	0.270
μ_{19}	0.508	0.060	0.022	-0.016	1.288	0.517
α_1	0.343	0.090	0.204	-0.010	0.633	0.274
α_2	0.283	0.084	0.140	0.036	0.682	0.971
α_3	0.528	0.134	0.133	0.013	0.729	0.403
α_4	0.868	0.227	0.271	0.001	0.558	0.166
α_5	0.631	0.140	0.302	0.040	0.464	0.789
α_6	0.269	0.077	0.183	0.052	0.885	0.620
α_7	0.423	0.119	0.232	0.002	0.671	0.332
α_8	0.541	0.199	0.188	0.047	0.598	0.084
α_9	0.717	0.318	0.198	0.047	0.496	0.855
α_{10}	0.446	0.097	0.244	0.029	0.623	0.750
α_{11}	0.748	0.235	0.200	0.022	0.673	0.463
α_{12}	0.864	0.210	0.304	-0.006	0.488	0.974
α_{13}	0.457	0.146	0.174	0.045	0.727	0.895
α_{14}	0.303	0.159	0.101	-0.021	1.206	0.453
α_{15}	0.443	0.103	0.266	0.031	0.481	0.893
α_{16}	0.440	0.114	0.208	0.043	0.728	0.551
α_{17}	0.632	0.178	0.248	0.018	0.649	0.118
α_{18}	0.794	0.148	0.366	0.073	0.499	0.629
α_{19}	0.442	0.097	0.190	0.033	0.979	0.719

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error. c_T denotes the output gap in 2017:Q2.

Table 4: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
δ_1^y	0.288	0.109	0.105	0.022	0.816	0.757
δ_2^y	0.293	0.109	0.066	0.021	0.836	0.231
δ_3^y	0.338	0.154	0.084	0.050	0.743	0.232
δ_4^y	1.335	0.334	0.202	-0.019	0.657	0.844
δ_5^y	0.745	0.169	0.141	0.003	0.678	0.368
δ_6^y	0.285	0.088	0.066	0.030	1.463	0.939
δ_7^y	0.539	0.132	0.082	0.008	0.913	0.909
δ_8^y	1.207	0.226	0.098	0.014	1.049	0.351
δ_9^y	0.436	0.308	0.053	0.015	0.906	0.285
δ_{10}^y	0.441	0.114	0.091	-0.003	0.747	0.024
δ_{11}^y	0.773	0.328	0.198	-0.011	0.850	0.488
δ_{12}^y	1.707	0.305	0.245	0.029	0.928	0.920
δ_{13}^y	0.578	0.204	0.081	0.062	1.042	0.217
δ_{14}^y	0.261	0.178	0.008	-0.022	1.566	0.881
δ_{15}^y	0.547	0.109	0.034	-0.012	0.706	0.296
δ_{16}^y	0.333	0.123	0.008	0.032	0.758	0.599
δ_{17}^y	1.103	0.232	0.085	-0.029	0.924	0.274
δ_{18}^y	0.648	0.181	0.217	-0.005	1.067	0.917
δ_{19}^y	0.153	0.100	0.070	0.048	0.927	0.884
δ_1^u	0.040	0.029	0.029	-0.013	1.492	0.454
δ_2^u	0.112	0.063	0.027	0.035	0.920	0.175
δ_3^u	0.123	0.077	-0.007	-0.004	0.760	0.192
δ_4^u	0.575	0.216	0.207	-0.010	0.529	0.228
δ_5^u	0.065	0.038	0.236	-0.017	0.874	0.956
δ_6^u	0.063	0.030	-0.006	-0.006	0.906	0.928
δ_7^u	0.019	0.017	0.038	-0.027	1.012	0.666
δ_8^u	0.246	0.094	-0.007	-0.025	1.131	0.953
δ_9^u	0.461	0.099	0.058	0.001	0.783	0.499
δ_{10}^u	0.045	0.034	0.074	-0.002	1.271	0.962
δ_{11}^u	0.706	0.214	0.197	-0.033	0.523	0.711
δ_{12}^u	0.725	0.229	0.247	-0.008	0.454	0.627
δ_{13}^u	0.017	0.015	-0.008	0.028	1.312	0.675
δ_{14}^u	0.108	0.061	-0.020	-0.001	1.284	0.380
δ_{15}^u	0.029	0.024	0.050	0.003	1.225	0.076
δ_{16}^u	0.144	0.071	0.055	-0.001	1.008	0.977
δ_{17}^u	0.313	0.105	0.111	-0.049	1.187	0.597
δ_{18}^u	0.081	0.055	-0.006	-0.021	1.111	0.944
δ_{19}^u	0.322	0.098	0.025	0.002	1.054	0.430

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

Table 5: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\theta_{1,1}$	-0.035	0.055	-0.010	0.006	1.191	0.520
$\theta_{1,2}$	-0.034	0.109	0.029	0.018	1.043	0.827
$\theta_{1,3}$	-0.304	0.068	0.021	-0.004	1.043	0.832
$\theta_{1,4}$	-0.115	0.060	0.159	-0.001	0.716	0.812
$\theta_{1,5}$	-0.052	0.022	0.056	-0.013	0.637	0.194
$\theta_{1,6}$	-0.138	0.056	-0.006	0.007	1.054	0.643
$\theta_{1,7}$	-0.122	0.036	0.014	-0.000	1.186	0.647
$\theta_{1,8}$	-0.315	0.047	0.035	0.026	1.148	0.173
$\theta_{1,9}$	-0.038	0.035	0.227	0.004	0.486	0.728
$\theta_{1,10}$	-0.158	0.065	-0.000	0.030	1.026	0.826
$\theta_{1,11}$	-0.095	0.064	0.065	-0.016	0.532	0.930
$\theta_{1,12}$	-0.045	0.090	0.182	0.013	0.434	0.701
$\theta_{1,13}$	-0.040	0.019	-0.021	-0.009	1.016	0.260
$\theta_{1,14}$	-0.026	0.045	0.016	0.008	0.820	0.766
$\theta_{1,15}$	-0.099	0.042	0.034	0.022	0.841	0.442
$\theta_{1,16}$	-0.222	0.065	0.053	0.009	1.598	0.515
$\theta_{1,17}$	-0.126	0.055	0.036	0.024	0.970	0.130
$\theta_{1,18}$	-0.044	0.056	0.002	-0.011	1.075	0.980
$\theta_{1,19}$	-0.494	0.125	-0.011	0.003	1.896	0.717
$\theta_{2,1}$	-0.197	0.054	0.018	0.010	0.871	0.831
$\theta_{2,2}$	-0.190	0.106	0.044	0.010	0.979	0.951
$\theta_{2,3}$	-0.162	0.067	-0.002	-0.014	0.874	0.468
$\theta_{2,4}$	-0.102	0.062	0.135	-0.006	0.579	0.765
$\theta_{2,5}$	-0.078	0.021	0.035	0.048	1.051	0.980
$\theta_{2,6}$	-0.180	0.054	-0.018	0.046	1.336	0.125
$\theta_{2,7}$	-0.115	0.035	0.013	0.029	1.403	0.481
$\theta_{2,8}$	-0.214	0.043	0.021	0.036	0.722	0.961
$\theta_{2,9}$	-0.038	0.032	0.237	-0.033	0.637	0.079
$\theta_{2,10}$	-0.119	0.061	-0.022	0.004	1.371	0.211
$\theta_{2,11}$	-0.105	0.071	0.151	-0.017	0.699	0.955
$\theta_{2,12}$	-0.032	0.082	0.186	-0.028	0.903	0.976
$\theta_{2,13}$	-0.037	0.019	0.012	0.014	1.119	0.060
$\theta_{2,14}$	-0.046	0.047	0.000	-0.010	1.209	0.291
$\theta_{2,15}$	-0.208	0.043	0.045	0.003	0.862	0.233
$\theta_{2,16}$	-0.205	0.064	0.002	0.010	0.980	0.367
$\theta_{2,17}$	-0.135	0.057	0.093	0.024	1.002	0.558
$\theta_{2,18}$	-0.148	0.056	0.007	-0.029	1.139	0.149
$\theta_{2,19}$	-0.179	0.120	0.002	-0.022	1.229	0.842

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

Table 6: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\beta_{0,1}$	0.051	0.228	0.000	-0.013	1.362	0.193
$\beta_{0,2}$	0.095	0.279	-0.008	0.024	1.288	0.285
$\beta_{0,3}$	-0.160	0.386	0.015	0.008	1.066	0.409
$\beta_{0,4}$	0.353	0.464	-0.010	-0.010	1.350	0.401
$\beta_{0,5}$	-0.277	0.286	0.033	0.016	0.714	0.950
$\beta_{0,6}$	-0.230	0.240	0.008	-0.005	1.217	0.630
$\beta_{0,7}$	-0.123	0.228	-0.028	-0.025	1.593	0.716
$\beta_{0,8}$	0.147	0.397	0.017	0.014	1.241	0.178
$\beta_{0,9}$	-0.000	0.366	0.027	-0.047	1.064	0.621
$\beta_{0,10}$	-0.048	0.271	0.020	0.027	1.202	0.674
$\beta_{0,11}$	0.016	0.469	-0.001	0.006	0.976	0.399
$\beta_{0,12}$	-0.047	0.378	0.026	0.004	1.163	0.257
$\beta_{0,13}$	0.223	0.310	0.010	-0.001	0.895	0.979
$\beta_{0,14}$	0.250	0.278	0.043	-0.049	0.847	0.619
$\beta_{0,15}$	0.207	0.312	0.049	0.017	0.849	0.762
$\beta_{0,16}$	0.178	0.317	0.016	0.019	0.960	0.206
$\beta_{0,17}$	0.340	0.350	0.065	0.033	1.300	0.521
$\beta_{0,18}$	0.287	0.360	0.014	-0.005	1.038	0.763
$\beta_{0,19}$	0.103	0.345	-0.043	0.009	1.450	0.305
$\beta_{1,1}$	0.888	0.078	0.006	0.021	0.865	0.133
$\beta_{1,2}$	0.927	0.048	0.012	0.009	1.197	0.919
$\beta_{1,3}$	0.987	0.011	0.002	0.026	1.081	0.006
$\beta_{1,4}$	0.954	0.026	0.025	-0.002	1.196	0.332
$\beta_{1,5}$	0.921	0.045	0.029	-0.035	0.837	0.064
$\beta_{1,6}$	0.977	0.018	-0.006	0.005	1.346	0.965
$\beta_{1,7}$	0.918	0.064	0.016	0.039	0.740	0.867
$\beta_{1,8}$	0.852	0.077	0.051	0.017	0.663	0.051
$\beta_{1,9}$	0.763	0.110	0.032	-0.021	0.739	0.951
$\beta_{1,10}$	0.992	0.006	-0.007	-0.031	0.927	0.483
$\beta_{1,11}$	0.904	0.040	-0.002	0.028	1.481	0.243
$\beta_{1,12}$	0.574	0.124	-0.007	-0.017	1.574	0.573
$\beta_{1,13}$	0.927	0.060	-0.017	-0.014	1.442	0.915
$\beta_{1,14}$	0.832	0.085	-0.017	-0.030	1.324	0.732
$\beta_{1,15}$	0.906	0.064	0.022	-0.041	1.317	1.000
$\beta_{1,16}$	0.870	0.088	0.013	-0.015	1.046	0.957
$\beta_{1,17}$	0.556	0.109	-0.016	-0.035	1.256	0.539
$\beta_{1,18}$	0.696	0.109	-0.004	0.001	1.180	0.029
$\beta_{1,19}$	0.914	0.055	-0.013	-0.033	1.032	0.512

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

Table 7: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
κ_1	0.098	0.058	0.011	0.001	1.481	0.974
κ_2	0.180	0.074	-0.004	0.029	0.888	0.274
κ_3	0.132	0.050	0.036	0.015	0.616	0.187
κ_4	0.216	0.057	0.024	-0.015	0.885	0.419
κ_5	0.111	0.052	-0.018	-0.020	1.273	0.370
κ_6	0.146	0.064	0.006	-0.006	0.973	0.226
κ_7	0.047	0.037	0.030	-0.015	0.932	0.321
κ_8	0.113	0.024	-0.026	0.030	0.891	0.284
κ_9	0.089	0.049	0.234	0.056	0.616	0.415
κ_{10}	0.167	0.047	-0.084	0.001	1.392	0.082
κ_{11}	0.289	0.053	0.013	0.002	1.105	0.835
κ_{12}	0.156	0.072	0.035	-0.017	0.837	0.947
κ_{13}	0.140	0.065	-0.038	-0.023	1.101	0.395
κ_{14}	0.096	0.073	0.152	-0.010	1.006	0.917
κ_{15}	0.181	0.052	0.002	0.001	0.860	0.133
κ_{16}	0.154	0.055	0.016	-0.029	1.178	0.710
κ_{17}	0.069	0.052	0.035	-0.010	0.967	0.523
κ_{18}	0.140	0.053	0.005	0.002	1.142	0.551
κ_{19}	0.120	0.039	-0.031	-0.016	1.517	0.643
ρ_1	0.918	0.078	-0.009	-0.013	1.150	0.852
ρ_2	0.912	0.076	0.013	-0.009	0.966	0.235
ρ_3	0.978	0.020	0.009	0.003	1.279	0.037
ρ_4	0.914	0.108	0.050	0.020	0.987	0.865
ρ_5	0.891	0.107	-0.006	0.012	1.452	0.802
ρ_6	0.912	0.085	-0.001	0.022	1.568	0.432
ρ_7	0.985	0.014	-0.007	-0.003	1.124	0.752
ρ_8	0.984	0.013	-0.014	0.017	1.410	0.259
ρ_9	0.939	0.106	0.041	-0.013	1.048	0.522
ρ_{10}	0.917	0.075	-0.004	-0.021	1.368	0.548
ρ_{11}	0.963	0.028	0.037	-0.024	0.993	0.003
ρ_{12}	0.827	0.149	0.080	-0.005	0.949	0.829
ρ_{13}	0.619	0.190	-0.006	-0.040	1.368	0.635
ρ_{14}	0.694	0.357	0.235	-0.019	0.496	0.709
ρ_{15}	0.961	0.035	0.025	0.025	0.808	0.963
ρ_{16}	0.936	0.057	-0.019	0.059	1.138	0.945
ρ_{17}	0.943	0.085	0.012	0.022	1.143	0.364
ρ_{18}	0.897	0.105	0.023	-0.015	1.106	0.327
ρ_{19}	0.983	0.015	0.025	-0.007	0.941	0.446

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

Table 8: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\sigma_{\zeta_1}^2$	0.487	0.088	-0.037	0.018	1.597	0.199
$\sigma_{\zeta_2}^2$	0.449	0.082	0.033	0.014	1.265	0.301
$\sigma_{\zeta_3}^2$	0.904	0.182	-0.006	0.001	1.250	0.900
$\sigma_{\zeta_4}^2$	2.672	0.557	-0.014	-0.003	1.175	0.060
$\sigma_{\zeta_5}^2$	0.830	0.168	0.025	0.003	1.582	0.987
$\sigma_{\zeta_6}^2$	0.393	0.070	-0.015	0.019	0.668	0.561
$\sigma_{\zeta_7}^2$	0.644	0.124	-0.063	0.006	1.257	0.654
$\sigma_{\zeta_8}^2$	1.932	0.437	0.004	0.081	0.959	0.029
$\sigma_{\zeta_9}^2$	7.364	2.892	0.488	0.062	0.271	0.112
$\sigma_{\zeta_{10}}^2$	0.479	0.087	0.023	0.049	1.109	0.400
$\sigma_{\zeta_{11}}^2$	3.088	0.636	0.020	-0.052	1.564	0.287
$\sigma_{\zeta_{12}}^2$	1.545	0.358	0.043	-0.015	1.088	0.379
$\sigma_{\zeta_{13}}^2$	1.803	0.409	0.001	0.016	1.459	0.852
$\sigma_{\zeta_{14}}^2$	1.763	0.587	0.174	-0.016	0.508	0.259
$\sigma_{\zeta_{15}}^2$	0.498	0.092	-0.037	-0.048	0.793	0.434
$\sigma_{\zeta_{16}}^2$	0.648	0.126	-0.002	0.020	0.758	0.005
$\sigma_{\zeta_{17}}^2$	1.400	0.313	0.021	0.027	1.042	0.837
$\sigma_{\zeta_{18}}^2$	0.613	0.120	-0.014	-0.005	0.855	0.005
$\sigma_{\zeta_{19}}^2$	0.495	0.096	-0.003	0.007	1.093	0.662
$\sigma_{\eta_1^y}^2$	0.065	0.027	-0.016	-0.000	0.993	0.048
$\sigma_{\eta_2^y}^2$	0.061	0.023	0.039	0.041	1.090	0.513
$\sigma_{\eta_3^y}^2$	0.130	0.074	0.030	-0.011	0.958	0.394
$\sigma_{\eta_4^y}^2$	0.217	0.194	0.046	-0.002	0.722	0.004
$\sigma_{\eta_5^y}^2$	0.100	0.053	0.010	0.022	1.014	0.352
$\sigma_{\eta_6^y}^2$	0.056	0.021	0.021	-0.014	1.128	0.452
$\sigma_{\eta_7^y}^2$	0.077	0.033	0.021	-0.019	0.967	0.200
$\sigma_{\eta_8^y}^2$	0.401	0.261	0.051	-0.012	1.021	0.333
$\sigma_{\eta_9^y}^2$	2.044	2.831	0.622	0.077	0.264	0.284
$\sigma_{\eta_{10}^y}^2$	0.068	0.029	0.003	0.016	0.807	0.012
$\sigma_{\eta_{11}^y}^2$	0.263	0.262	0.014	0.000	1.527	0.443
$\sigma_{\eta_{12}^y}^2$	0.163	0.126	-0.003	0.001	1.326	0.921
$\sigma_{\eta_{13}^y}^2$	0.318	0.227	-0.007	-0.017	2.000	0.962
$\sigma_{\eta_{14}^y}^2$	0.600	0.464	0.274	0.024	0.670	0.565
$\sigma_{\eta_{15}^y}^2$	0.063	0.024	0.001	-0.001	1.262	0.878
$\sigma_{\eta_{16}^y}^2$	0.083	0.039	0.004	-0.022	1.122	0.921
$\sigma_{\eta_{17}^y}^2$	0.272	0.227	0.022	-0.009	1.326	0.717
$\sigma_{\eta_{18}^y}^2$	0.085	0.041	0.009	-0.018	0.807	0.323
$\sigma_{\eta_{19}^y}^2$	0.085	0.041	0.018	0.023	1.103	0.057

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

Table 9: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\sigma_{\eta_1^u}^2$	0.056	0.011	0.005	-0.027	1.109	0.800
$\sigma_{\eta_2^u}^2$	0.179	0.032	0.021	-0.029	0.738	0.460
$\sigma_{\eta_3^u}^2$	0.217	0.043	-0.012	0.026	0.980	0.816
$\sigma_{\eta_4^u}^2$	0.396	0.082	0.017	0.002	0.810	0.247
$\sigma_{\eta_5^u}^2$	0.024	0.005	0.006	-0.010	1.189	0.078
$\sigma_{\eta_6^u}^2$	0.025	0.005	-0.003	-0.026	0.718	0.962
$\sigma_{\eta_7^u}^2$	0.032	0.007	0.024	-0.000	1.256	0.747
$\sigma_{\eta_8^u}^2$	0.089	0.031	0.043	0.003	0.998	0.099
$\sigma_{\eta_9^u}^2$	0.139	0.041	0.085	0.009	0.964	0.297
$\sigma_{\eta_{10}^u}^2$	0.070	0.013	0.009	-0.001	0.881	0.020
$\sigma_{\eta_{11}^u}^2$	0.477	0.108	0.017	-0.019	0.672	0.156
$\sigma_{\eta_{12}^u}^2$	0.378	0.114	0.180	0.012	0.585	0.650
$\sigma_{\eta_{13}^u}^2$	0.039	0.007	0.009	0.014	1.255	0.919
$\sigma_{\eta_{14}^u}^2$	0.148	0.027	-0.026	0.034	1.234	0.157
$\sigma_{\eta_{15}^u}^2$	0.027	0.006	-0.003	-0.006	1.164	0.490
$\sigma_{\eta_{16}^u}^2$	0.128	0.025	-0.019	0.022	1.000	0.661
$\sigma_{\eta_{17}^u}^2$	0.167	0.042	0.038	0.000	0.717	0.748
$\sigma_{\eta_{18}^u}^2$	0.130	0.024	0.042	0.032	0.768	0.395
$\sigma_{\eta_{19}^u}^2$	0.134	0.038	0.059	-0.002	1.224	0.931
$\sigma_{\eta_1^\pi}^2$	1.981	0.342	-0.021	0.003	1.399	0.679
$\sigma_{\eta_2^\pi}^2$	3.189	0.561	-0.035	-0.010	1.261	0.861
$\sigma_{\eta_3^\pi}^2$	10.123	1.758	-0.030	-0.022	0.849	0.738
$\sigma_{\eta_4^\pi}^2$	9.089	1.607	0.026	-0.011	1.008	0.085
$\sigma_{\eta_5^\pi}^2$	2.627	0.467	-0.005	-0.000	0.763	0.225
$\sigma_{\eta_6^\pi}^2$	1.874	0.331	-0.003	-0.012	0.685	0.694
$\sigma_{\eta_7^\pi}^2$	1.895	0.327	0.010	0.011	0.932	0.214
$\sigma_{\eta_8^\pi}^2$	3.141	0.539	-0.020	-0.001	0.804	0.906
$\sigma_{\eta_9^\pi}^2$	4.167	0.782	0.014	-0.001	1.380	0.372
$\sigma_{\eta_{10}^\pi}^2$	1.453	0.258	-0.054	-0.012	1.481	0.850
$\sigma_{\eta_{11}^\pi}^2$	9.731	1.775	-0.021	-0.039	1.359	0.096
$\sigma_{\eta_{12}^\pi}^2$	7.484	1.367	0.042	0.016	1.257	0.208
$\sigma_{\eta_{13}^\pi}^2$	6.782	1.170	0.013	0.016	1.301	0.504
$\sigma_{\eta_{14}^\pi}^2$	4.239	0.735	-0.033	0.035	1.017	0.765
$\sigma_{\eta_{15}^\pi}^2$	2.099	0.385	0.020	-0.005	0.805	0.264
$\sigma_{\eta_{16}^\pi}^2$	4.166	0.746	0.013	-0.014	1.091	0.421
$\sigma_{\eta_{17}^\pi}^2$	9.164	1.599	-0.011	-0.005	0.869	0.085
$\sigma_{\eta_{18}^\pi}^2$	7.636	1.344	0.011	0.028	1.004	0.066
$\sigma_{\eta_{19}^\pi}^2$	4.822	0.818	-0.001	-0.004	1.077	0.550

Notes: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4% taper; p-value: p-value of the null hypothesis that the mean of the first 20% of draws is equal to the mean of the last 50% of draws using a 4% taper for the standard error.

The confidence sets for the real GDP growth forecast in both specifications are wider than in the RW case due to the time-varying nature of the expected growth rate of potential output and both cover almost everywhere the observed series. The unemployment rate forecasts are not very different across the three specifications, but the inflation rate forecast is. In particular, at the beginning of the forecasting exercise, the densities of the inflation rate forecasts cover negative grounds.

G Unobserved Components Models with Aggregate Data

This appendix describes the UC models with aggregate data and the results obtained from their estimation over the same sample period. The model is given by the following equations:

$$\begin{aligned} y_t &= \tau_t^y + c_t, \\ u_t &= \tau_t^u + \theta_1 c_t + \theta_2 c_{t-1}, \\ \pi_t &= \beta_0 + \beta_1 \pi_t^e + (1 - \beta_1) \pi_{t-1} + \kappa c_t + \eta_t^\pi, \\ \tau_t^u &= \tau_{t-1}^u + \eta_t^u, \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t, \end{aligned}$$

with trend output following one of the three specifications listed below:

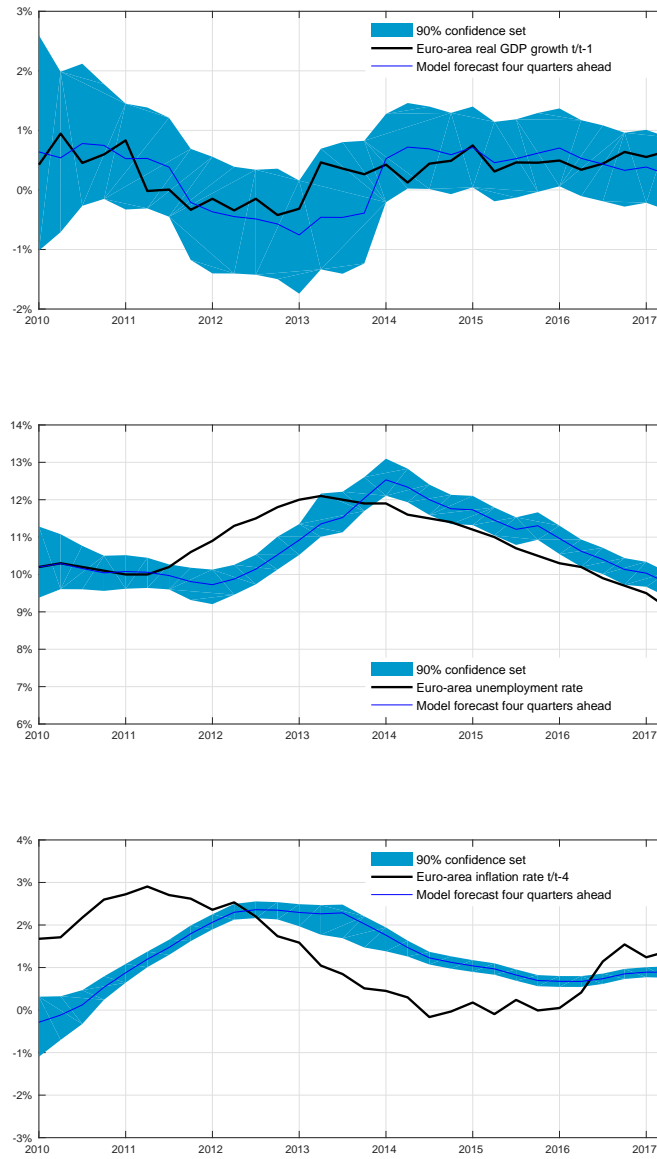
- RW: $\tau_t^y = \mu + \tau_{t-1}^y + \eta_t^y$,
- LLT: $\tau_t^y = \mu_{t-1} + \tau_{t-1}^y + \eta_t^y$, $\mu_t = \mu_{t-1} + \nu_t$,
- IRW: $\tau_t^y = \mu_{t-1} + \tau_{t-1}^y$, $\mu_t = \mu_{t-1} + \nu_t$,

and where the error terms ε_t , η_t^y , η_t^u , and ν_t are assumed to be white noise, uncorrelated with each other, and normally distributed.

The prior distributions of the coefficients are similar to those in the specification with data at the country level. However, in this case it is possible to estimate the variances of the aggregate cycle and trend shocks, σ_ε^2 , $\sigma_{\eta^y}^2$, and $\sigma_{\eta^u}^2$. The sources for the priors, as before, are the results in [Lenza and Jarociński \(2016\)](#) for real GDP and the unemployment rate and [Blanchard, Cerutti and Summers \(2015\)](#) for inflation. Prior distributions and posterior distribution results appear in Tables 10-12. Convergence diagnostics (not shown) indicate the Gibbs sampler has reached a stable posterior distribution. The estimated output gaps appears in Figure 19.

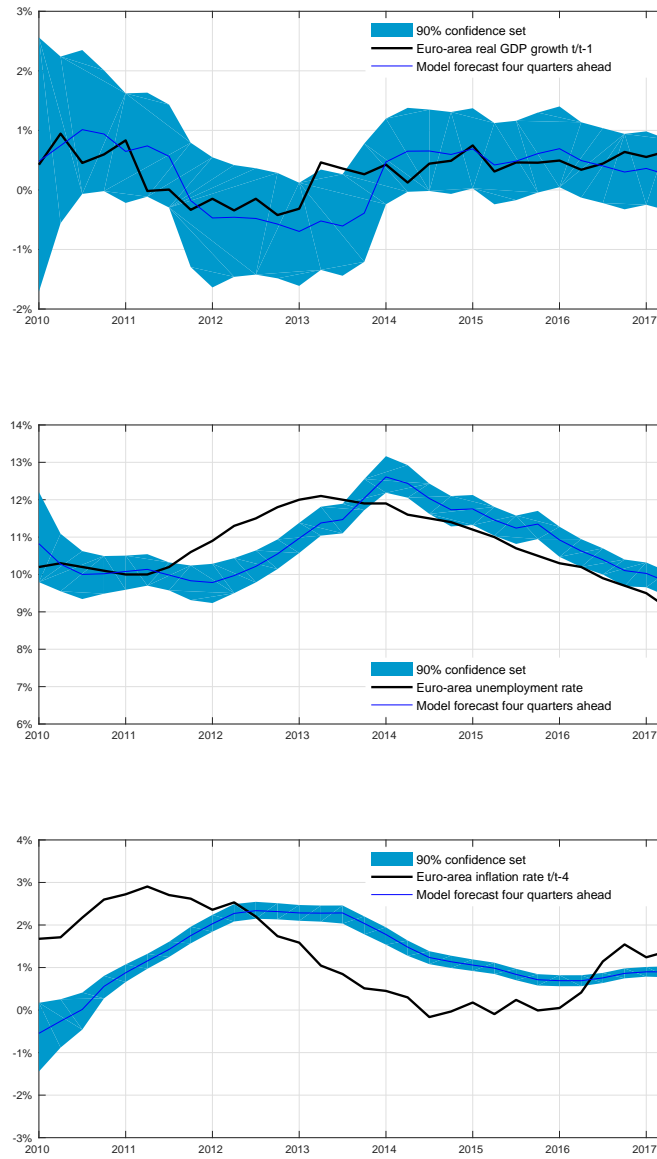
Regardless of the choice of the specification of the output trend, the three models estimate that an output gap emerged starting in 2009, when the global financial crisis hit, and reached between negative 2 percent and negative 3 percent around 2010. The double-dip recession consequence of the European debt crisis causes the output gap to become even further negative around 2013. However, the model with a RW output trend reaches only about negative 5 percent, compared with about negative 7 percent to negative 8 percent for the LLT and IRW specifications. At the end of the sample period, all three specifications yield estimated output gaps close to zero.

Figure 17: Model Forecasts and Observed Variables (LLT)



Note: The four-quarters-ahead forecast is the median of the model forecast across draws obtained from the [Durbin and Koopman \(2002\)](#) simulation smoother. *Source:* Eurostat and author's calculations.

Figure 18: Model Forecasts and Observed Variables (IRW)



Note: The four-quarters-ahead forecast is the median of the model forecast across draws obtained from the [Durbin and Koopman \(2002\)](#) simulation smoother. *Source:* Eurostat and author's calculations.

Table 10: Parameter Prior Distributions and Posterior Distribution Results
(RW)

Parameter	Prior Distribution	Prior Mean	Prior SD	Posterior Mean	[5th,95th]
ϕ_1	Truncated Normal	1.7	0.2	1.74	[1.61,1.87]
ϕ_2	Truncated Normal	-0.75	0.2	-0.78	[-0.91,-0.63]
μ	Normal	0.5	0.5	0.30	[0.10,0.50]
θ_1	Normal	-0.2	0.2	-0.22	[-0.32,-0.12]
θ_2	Normal	-0.2	0.2	-0.21	[-0.31,-0.11]
β_0	Normal	0	0.5	0.11	[-0.22,0.46]
β_1	Truncated Normal	0.5	0.5	0.23	[0.05,0.41]
κ	Truncated Normal	0.1	0.1	0.11	[0.03,0.19]
σ_ε^2	Inverse Gamma	0.1	Inf	0.11	[0.06,0.17]
$\sigma_{\eta^y}^2$	Inverse Gamma	0.1	Inf	0.10	[0.06,0.16]
$\sigma_{\eta^u}^2$	Inverse Gamma	0.1	Inf	0.01	[0.01,0.02]
$\sigma_{\eta^\pi}^2$	Inverse Gamma	0.25	Inf	1.48	[1.13,1.96]
σ_ν^2	Inverse Gamma	1	Inf	—	—

According to the forecasting results in Table 1, the three output trend specifications perform almost equally well with respect to the inflation rate. However, the RW specification forecasts real GDP growth significantly better than the other two specifications, while it does not perform significantly worse with respect to the unemployment rate. Hence, I pick the random walk specification for the output trend as the benchmark model and interpret the results according to this choice, which are shown in Table 10.

The posterior mean estimate of the drift of the output trend indicates that the average growth rate of the euro-area potential GDP would be around 1.2 percent per year, almost exactly the same value as with country-level data. Taking the estimates of the AR(2) coefficients of the common cycle at their posterior mean, one can estimate the period of the cycle, which would be about 11 years for the euro area as whole. The Okun's law coefficients are in line with those usually cited in the literature, in which the sum of the two coefficients would be around -0.5 (see [Abel, Bernanke and Croushore, 2013](#), for example), implying that for every percentage point increase in with respect to potential, the unemployment rate would be reduced a little more than 0.4 percentage point. The posterior mean estimate of the inflation expectations anchoring coefficient of the hybrid Phillips curve implies that realized inflation would be about 0.5 percentage point above expected inflation in the long run under a zero output gap scenario. The estimated slope of the hybrid Phillips curve indicates that inflation would increase about 0.1 percentage point for every percentage point increase in output with respect to potential. This estimate is well in line with the results in the literature (see [Blanchard, Cerutti and Summers, 2015](#), for example). In general, the estimated residual variance of the inflation equation is the largest, indicating that forecasting inflation for these type of models can be harder, relatively speaking, than forecasting the other two variables of interest.

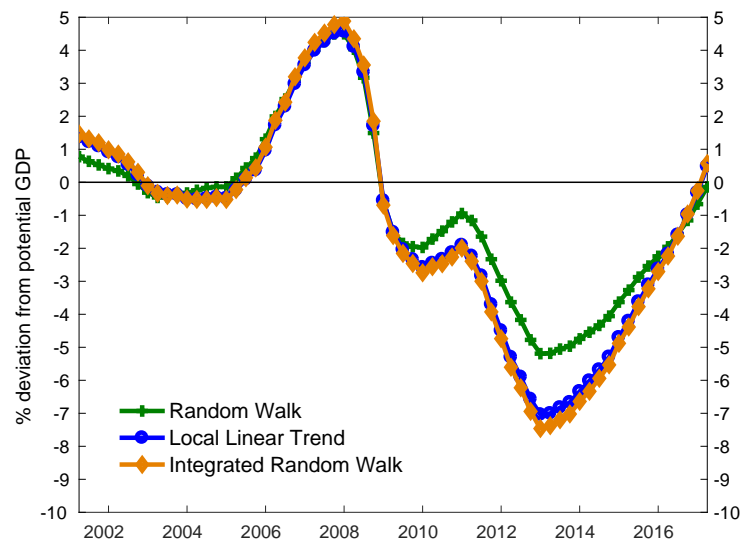
Table 11: Parameter Prior Distributions and Posterior Distribution Results
(LLT)

Parameter	Prior Distribution	Prior Mean	Prior SD	Posterior Mean	[5th,95th]
ϕ_1	Truncated Normal	1.7	0.2	1.74	[1.53,1.88]
ϕ_2	Truncated Normal	-0.75	0.2	-0.76	[-0.90,-0.57]
μ	Normal	0.5	0.5	—	—
θ_1	Normal	-0.2	0.2	-0.23	[-0.35,-0.13]
θ_2	Normal	-0.2	0.2	-0.16	[-0.28,-0.06]
β_0	Normal	0	0.5	0.13	[-0.18,0.48]
β_1	Truncated Normal	0.5	0.5	0.24	[0.05,0.44]
κ	Truncated Normal	0.1	0.1	0.09	[0.02,0.17]
σ_ε^2	Inverse Gamma	0.1	Inf	0.14	[0.06,0.27]
$\sigma_{\eta^y}^2$	Inverse Gamma	0.1	Inf	0.03	[0.01,0.06]
$\sigma_{\eta^u}^2$	Inverse Gamma	0.1	Inf	0.01	[0.01,0.02]
$\sigma_{\eta^\pi}^2$	Inverse Gamma	0.25	Inf	1.49	[1.13,1.96]
σ_ν^2	Inverse Gamma	1	Inf	0.12	[0.08,0.18]

Table 12: Parameter Prior Distributions and Posterior Distribution Results
(IRW)

Parameter	Prior Distribution	Prior Mean	Prior SD	Posterior Mean	[5th,95th]
ϕ_1	Truncated Normal	1.7	0.2	1.74	[1.57,1.87]
ϕ_2	Truncated Normal	-0.75	0.2	-0.76	[-0.90,-0.59]
μ	Normal	0.5	0.5	—	—
θ_1	Normal	-0.2	0.2	-0.21	[-0.32,-0.13]
θ_2	Normal	-0.2	0.2	-0.16	[-0.25,-0.07]
β_0	Normal	0	0.5	0.14	[-0.20,0.50]
β_1	Truncated Normal	0.5	0.5	0.24	[0.06,0.44]
κ	Truncated Normal	0.1	0.1	0.08	[0.02,0.16]
σ_ε^2	Inverse Gamma	0.1	Inf	0.16	[0.08,0.28]
$\sigma_{\eta^y}^2$	Inverse Gamma	0.1	Inf	—	—
$\sigma_{\eta^u}^2$	Inverse Gamma	0.1	Inf	0.01	[0.01,0.02]
$\sigma_{\eta^\pi}^2$	Inverse Gamma	0.25	Inf	1.49	[1.11,1.99]
σ_ν^2	Inverse Gamma	1	Inf	0.13	[0.08,0.18]

Figure 19: Smoothed Estimate of the Euro-Area Output Gap Using Aggregate Data for Three Specifications of the Output Trend



Note: Time series are the averages of the posterior draws of the euro area output gap using the [Durbin and Koopman \(2002\)](#) simulator smoother.

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