Financial Heterogeneity and Monetary Union

Technical Appendix – For Online publication Only

Simon Gilchrist[*] Raphael Schoenle[†] Jae Sim[‡] Egon Zakrajšek[§]

June 17, 2018

[*]Department of Economics New York University and NBER. Email: sg40@nyu.edu
[†]Department of Economics Brandeis University. Email: schoenle@brandeis.edu
[‡]Research and Statistics, Federal Reserve Board. Email: jae.w.sim@frb.gov
[§]Monetary Affairs, Federal Reserve Board. Email: egon.zakrajsek@frb.gov

This technical appendix presents supplementary material for the paper "Financial Heterogeneity and Monetary Union." The appendix consists of two main parts. Section 1 presents the complete model and shows how the model can be modified to allow for complete risk sharing—that is, the de facto formation of a fiscal union—between the two countries. Section 2 contains some additional results that are referenced in the main text.

1 Model Appendix

1.1 Product Demands and Accounting Identities

Product Demands

In the symmetric equilibrium, all households choose the same level of consumption. Hence, we omit the household superscript j going forward. The Lagrangian associated with the cost minimization problem faced by the household in the home country is given by

$$\mathcal{L}_c = \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - \lambda_{c,t} \left\{ \left[\sum_{k=h,f} \Xi_k \left(\int_{N_k} \left(c_{i,k,t} / s^{\theta}_{i,k,t-1} \right)^{1-1/\eta} di \right)^{\frac{1-1/\varepsilon}{1-1/\eta}} \right]^{\frac{1}{1-1/\varepsilon}} - x_t \right\}.$$

The first-order condition for $c_{i,h,t}$ is given by

$$P_{i,h,t} = \Xi_h \lambda_{c,t} \frac{\left(c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{c_{i,h,t}} \left[\int_{N_h} \left(c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta} di \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon}, \tag{1-1}$$

while the first-order condition for $c_{j,h,t}$ is given by

$$P_{j,h,t} = \Xi_h \lambda_{c,t} \frac{\left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta}}{c_{j,h,t}} \left[\int_{N_h} \left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta} dj \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon}. \tag{1-2}$$

Taking the ratio of equations (1-1) and (1-2) yields

$$\frac{P_{i,h,t}}{P_{j,h,t}} = \frac{c_{j,h,t}}{c_{i,h,t}} \frac{\left(c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{\left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta}},$$

or equivalently,

$$\left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{-1/\eta} = \frac{P_{j,h,t} s^{\theta}_{j,h,t-1}}{P_{i,h,t}} \frac{\left(c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{c_{i,h,t}}.$$

Raising the above expression to the power of $1 - 1/\eta$, integrating the resulting expression with respect to j, and finally raising the resulting expression to the power of $1/(1 - 1/\eta)$ yields

$$\begin{aligned} \left[\int_{N_h} \left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta} dj \right]^{1/(1-1/\eta)} = & \left[\int_{N_h} \left(P_{j,h,t} s^{\theta}_{j,h,t-1} \right)^{1-\eta} dj \right]^{1/(1-1/\eta)} \\ & \times c_{i,h,t} \left(s^{\theta}_{i,h,t-1} \right)^{\eta-1} P^{\eta}_{i,h,t}. \end{aligned} \tag{1-3}$$

Define the following aggregates:

$$x_{h,t} = \left[\int_{N_h} \left(c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta} dj \right]^{1/(1-1/\eta)} ; \tag{1-4}$$

$$\tilde{P}_{h,t} = \left[\int_{N_h} \left(P_{j,h,t} s^{\theta}_{j,h,t-1} \right)^{1-\eta} dj \right]^{1/(1-\eta)} . \tag{1-5}$$

We can then rewrite equation (1-3) in terms of the aggregates (1-4) and (1-5) as

$$c_{i,h,t} = \left(\frac{P_{i,h,t}}{\tilde{P}_{h,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} = \left(\frac{P_{i,h,t}}{P_{h,t}} \right)^{-\eta} \left(\frac{\tilde{P}_{h,t}}{P_{h,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t}. \tag{1-6}$$

Following the same steps, we can derive demand for the foreign product by the home country's household:

$$c_{i,f,t} = \left(\frac{P_{i,f,t}}{\tilde{P}_{f,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t} = \left(\frac{P_{i,f,t}}{P_{f,t}} \right)^{-\eta} \left(\frac{\tilde{P}_{f,t}}{P_{f,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t}, \tag{1-7}$$

where

$$x_{f,t} = \left[\int_{N_f} \left(c_{j,f,t} / s^{\theta}_{j,f,t-1} \right)^{1-1/\eta} dj \right]^{1/(1-1/\eta)} ; \tag{1-8}$$

$$\tilde{P}_{f,t} = \left[\int_{N_f} \left(P_{j,f,t} s^{\theta}_{j,f,t-1} \right)^{1-\eta} dj \right]^{1/(1-\eta)} . \tag{1-9}$$

Using equations (1-4) and (1-8), the consumption/habit aggregator x_t can then be written as

$$x_t = \left[\sum_{k=h,f} \Xi_k x^{1-1/\varepsilon}_{k,t} \right]^{1/(1-1/\varepsilon)} . \tag{1-10}$$

We can then think of another cost minimization problem, namely, minimizing the cost of obtaining the bundle x_t by choosing $x_{k,t}$, when the unit price of $x_{k,t}$ is given by $\tilde{P}_{k,t}$. The Lagrangian associated with this problem is given by

$$\mathcal{L}_x = \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - \tilde{P}_t \left[\left(\sum_{k=h,f} \Xi_k x^{1-1/\varepsilon}_{k,t} \right)^{1/(1-1/\varepsilon)} - x_t \right],$$

where $\tilde{P}_t$ is the Lagrange multiplier. The first-order conditions for this program are given by

$$x_{h,t} = \Xi^{\varepsilon}_h \left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t; \tag{1-11}$$

$$x_{f,t} = \Xi^{\varepsilon}_f \left(\frac{\tilde{P}_{f,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t. \tag{1-12}$$

Substituting these conditions into equation (1-10) yields:

$$1 = \left[\sum_{k=h,f} \Xi_k \left[\Xi^{\varepsilon}_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} \right]^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} .$$

Solving the above expression for $\tilde{P}_t$ gives the expression for the welfare-based aggregate price index:

$$\tilde{P}_t = \left[\sum_{k=h,f} \Xi_k \tilde{P}_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}, \tag{1-13}$$

shown in equation (6) of the main text.

Accounting Identities

The following accounting identities are used in the main text:

$$\begin{aligned} \int_{N_k} P_{i,k,t} c_{i,k,t} di &= \int_{N_k} P_{i,k,t} \left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t} di \\ &= \tilde{P}_{k,t}^{\eta} x_{k,t} \int_{N_k} \left(P_{i,k,t} s_{i,k,t-1}^{\theta} \right)^{1-\eta} di \\ &= \tilde{P}_{k,t} x_{k,t}, \quad k = h, f; \end{aligned} \tag{1-14}$$

and

$$\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \sum_{k=h,f} \tilde{P}_{k,t} \Xi_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t = \tilde{P}_t^{\varepsilon} x_t \sum_{k=h,f} \Xi_k \tilde{P}_{k,t}^{1-\varepsilon} = \tilde{P}_t x_t. \tag{1-15}$$

For $k = h, f$, we define the following relative prices in the home country: $p_{i,k,t} = P_{i,k,t}/P_{k,t}$, $\tilde{p}_{k,t} = \tilde{P}_{k,t}/P_{k,t}$, and $p_{k,t} = P_{k,t}/P_t$; similarly, for the foreign country we define $p^*_{i,k,t} = P^*_{i,k,t}/P^*_{k,t}$, $\tilde{p}^*_{k,t} = \tilde{P}^*_{k,t}/P^*_{k,t}$, and $p^*_{k,t} = P^*_{k,t}/P^*_t$, for $k = h, f$. In a symmetric equilibrium, these definitions then imply

$$\begin{aligned} \tilde{p}_t = \frac{\tilde{P}_t}{P_t} &= \left[\sum_{k=h,f} \Xi_k \tilde{p}_{k,t}^{1-\varepsilon} p_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f} \Xi_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)} \right]^{1/(1-\varepsilon)}. \end{aligned} \tag{1-16}$$

1.2 Optimal Pricing

In this section, we derive the optimal pricing rules for both home and foreign country firms in the symmetric equilibrium. Focusing first on the home country firms, the full set of first-order conditions implied by the optimization of the Lagrangian (8) in the main text is given by:
With respect to $d_{i,t}$:

$$\xi_{i,t} = \begin{cases} 1 & \text{if } d_{i,t} \geq 0; \\ 1/(1-\varphi) & \text{if } d_{i,t} < 0. \end{cases} \tag{2-1}$$

With respect to $h_{i,t}$:

$$\xi_{i,t} w_t = \alpha \kappa_{i,t} \left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha-1}, \tag{2-2}$$

where the conditional demand for labor is given by

$$h_{i,t} = \frac{a_{i,t}}{A_t} \left(\phi + c_{i,h,t} + c^*_{i,h,t} \right)^{\frac{1}{\alpha}}. \tag{2-3}$$

With respect to $c_{i,h,t}$ and $c^*_{i,h,t}$:

$$\mathbb{E}^a_t[\nu_{i,h,t}] = \mathbb{E}^a_t[\xi_{i,t}]p_{i,h,t}p_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda_{i,h,t}; \tag{2-4}$$

$$\mathbb{E}^a_t[\nu^*_{i,h,t}] = \mathbb{E}^a_t[\xi_{i,t}]q_t p^*_{i,h,t}p^*_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda^*_{i,h,t}. \tag{2-5}$$

With respect to $s_{i,h,t}$ and $s^*_{i,h,t}$:

$$\lambda_{i,h,t} = \rho\mathbb{E}_t[m_{t,t+1}\lambda_{i,h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu_{i,h,t+1}\frac{c_{i,h,t+1}}{s_{i,h,t}}\right]\right]; \tag{2-6}$$

$$\lambda^*_{i,h,t} = \rho\mathbb{E}_t[m_{t,t+1}\lambda^*_{i,h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu^*_{i,h,t+1}\frac{c^*_{i,h,t+1}}{s^*_{i,h,t}}\right]\right]. \tag{2-7}$$

With respect to $p_{i,h,t}$ and $p^*_{i,h,t}$:

$$\begin{aligned}\eta\frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{p_{i,h,t}}c_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}]&\left[p_{h,t}c_{i,h,t} - \gamma_p\frac{\pi_{h,t}}{p_{i,h,t-1}}\left(\pi_{h,t}\frac{p_{i,h,t}}{p_{i,h,t-1}} - 1\right)c_t\right] \\ &+ \gamma_p\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}[\xi_{i,t+1}]\pi_{h,t+1}\frac{p_{i,h,t+1}}{p^2_{i,h,t}}\left(\pi_{h,t+1}\frac{p_{i,h,t+1}}{p_{i,h,t}} - 1\right)c_{t+1}\right];\end{aligned} \tag{2-8}$$

$$\begin{aligned}\eta\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{p^*_{i,h,t}}c^*_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}]&\left[q_t p^*_{h,t}c^*_{i,h,t} - \gamma_p\frac{q_t\pi^*_{h,t}}{p^*_{i,h,t-1}}\left(\pi^*_{h,t}\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}} - 1\right)c^*_t\right] \\ &+ \gamma_p\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}[\xi_{i,t+1}]q_{t+1}\pi^*_{h,t+1}\frac{p^*_{i,h,t+1}}{p^{*2}_{i,h,t}}\left(\pi^*_{h,t+1}\frac{p^*_{i,h,t+1}}{p^*_{i,h,t}} - 1\right)c^*_{t+1}\right]\end{aligned} \tag{2-9}$$

In the absence of nominal price rigidities, the first-order conditions (2-8) and (2-9) reduce to

$$p_{i,h,t}p_{h,t} = \eta\frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]}; \tag{2-10}$$

and

$$q_t p^*_{i,h,t}p^*_{h,t} = \eta\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]}. \tag{2-11}$$

Dividing the first-order conditions (2-4) and (2-5) by the expected shadow value of internal funds yields

$$\frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} = p_{i,h,t}p_{h,t} - \frac{\mathbb{E}^a_t[\kappa_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} + (1-\rho)\frac{\lambda_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}; \tag{2-12}$$

and

$$\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} = q_t p^*_{i,h,t}p^*_{h,t} - \frac{\mathbb{E}^a_t[\kappa_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} + (1-\rho)\frac{\lambda^*_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}. \tag{2-13}$$

Similarly, dividing the first-order-conditions (2-6) and (2-7) by the expected shadow value of internal funds we obtain

$$\begin{aligned}\frac{\lambda_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} = \rho\mathbb{E}_t&\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{\lambda_{i,h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &+ \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{\mathbb{E}^a_{t+1}[\nu_{i,h,t+1}]}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\frac{c_{i,h,t+1}}{s_{i,h,t}}\right];\end{aligned} \tag{2-14}$$

and

$$\begin{aligned}\frac{\lambda^*_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \rho\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{\lambda^*_{i,h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{\mathbb{E}^a_{t+1}[\nu^*_{i,h,t+1}]}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\frac{c^*_{i,h,t+1}}{s^*_{i,h,t}}\right].\end{aligned} \tag{2-15}$$

Updating equations (2-12) and (2-13) one period and substituting the resulting expressions into the right-hand sides of equations (2-14) and (2-15), we obtain

$$\begin{aligned}\frac{\lambda_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(\rho+\theta(1-\eta)(1-\rho)\frac{c_{i,h,t+1}}{s_{i,h,t}}\right)\frac{\lambda_{i,h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c_{i,h,t+1}}{s_{i,h,t}}\mathbb{E}^a_{t+1}\left[\left(p_{i,h,t+1}p_{h,t+1}-\frac{\mathbb{E}^a_{t+1}[\kappa_{i,t+1}]}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right)\right]\right];\end{aligned} \tag{2-16}$$

and

$$\begin{aligned}\frac{\lambda^*_{i,h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(\rho+\theta(1-\eta)(1-\rho)\frac{c^*_{i,h,t+1}}{s^*_{i,h,t}}\right)\frac{\lambda^*_{i,h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c^*_{i,h,t+1}}{s^*_{i,h,t}}\mathbb{E}^a_{t+1}\left[\left(q_{t+1}p^*_{i,h,t+1}p^*_{h,t+1}-\frac{\mathbb{E}^a_{t+1}[\kappa_{i,t+1}]}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right)\right]\right].\end{aligned} \tag{2-17}$$

We then impose the symmetric equilibrium conditions, $c_{i,h,t+1} = c_{h,t+1}$, $s_{i,h,t} = s_{h,t}$, $\lambda_{i,h,t} = \lambda_{h,t}$, $p_{i,h,t+1} = 1$, $c^*_{i,h,t+1} = c^*_{h,t+1}$, $s^*_{i,h,t} = s^*_{h,t}$, $\lambda^*_{i,h,t} = \lambda^*_{h,t}$, and $p^*_{i,h,t+1} = 1$, to obtain

$$\begin{aligned}\frac{\lambda_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(\rho+\theta(1-\eta)(1-\rho)\frac{s_{h,t+1}/s_{h,t}-\rho}{1-\rho}\right)\frac{\lambda_{h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{s_{h,t+1}/s_{h,t}-\rho}{1-\rho}\mathbb{E}^a_{t+1}\left[\left(p_{h,t+1}-\frac{1}{\tilde{\mu}_{t+1}}\right)\right]\right];\end{aligned} \tag{2-18}$$

and

$$\begin{aligned}\frac{\lambda^*_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(\rho+\theta(1-\eta)(1-\rho)\frac{s^*_{h,t+1}/s^*_{h,t}-\rho}{1-\rho}\right)\frac{\lambda^*_{h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{s^*_{h,t+1}/s^*_{h,t}-\rho}{1-\rho}\mathbb{E}^a_{t+1}\left[\left(q_{t+1}p^*_{h,t+1}-\frac{1}{\tilde{\mu}_{t+1}}\right)\right]\right],\end{aligned} \tag{2-19}$$

where we used the fact that $c_{h,t+1}/s_{h,t} = (s_{h,t+1}/s_{h,t}-\rho)/(1-\rho) \equiv g_{h,t+1}$, $c^*_{h,t+1}/s^*_{h,t} = (s^*_{h,t+1}/s^*_{h,t}-\rho)/(1-\rho) \equiv g^*_{h,t+1}$, and $\mathbb{E}^a_{t+1}[\kappa_{i,t+1}]/\mathbb{E}^a_{t+1}[\xi_{i,t+1}] = \tilde{\mu}^{-1}_{t+1}$. We can define the growth-adjusted, compounded discount factors, $\beta_{h,t,s}$ and $\beta^*_{h,t,s}$, as

$$\beta_{h,t,s} = \begin{cases} m_{s-1,s}g_{h,s} & \text{if } s = t+1; \\ m_{s-1,s}g_{h,s} \times \prod_{j=1}^{s-(t+1)}(\rho+\chi g_{h,t+j})m_{t+j-1,t+j} & \text{if } s > t+1; \end{cases} \tag{2-20}$$

$$\beta^*_{h,t,s} = \begin{cases} m_{s-1,s}g^*_{h,s} & \text{if } s = t+1; \\ m_{s-1,s}g^*_{h,s} \times \prod_{j=1}^{s-(t+1)}(\rho+\chi g^*_{h,t+j})m_{t+j-1,t+j} & \text{if } s > t+1, \end{cases} \tag{2-21}$$

where $\chi = \theta(1-\eta)(1-\rho)$.

Rational expectations solutions to equations (2-18) and (2-19) can then be found by iterating the two equations forward as

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right]; \tag{2-22}$$

and

$$\frac{\lambda_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}^*\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(q_s p_{h,s}^*-\frac{1}{\tilde{\mu}_s}\right)\right]. \tag{2-23}$$

After imposing the symmetric equilibrium conditions, we substitute equations (2-12) and (2-13) into equations (2-10) and (2-11), which yields

$$p_{h,t} = \eta p_{h,t} - \eta\frac{1}{\tilde{\mu}_t} + (1-\rho)\eta\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}; \tag{2-24}$$

and

$$q_t p_{h,t}^* = \eta q_t p_{h,t}^* - \eta\frac{1}{\tilde{\mu}_t} + (1-\rho)\eta\frac{\lambda_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]}. \tag{2-25}$$

Finally, substituting equations (2-22) and (2-23) into equations (2-24) and (2-25) and solving the resulting expressions for $p_{h,t}$ and $q_t p_{h,t}^*$ yields the firm's optimal pricing strategies in the domestic and foreign markets, given by equations (14) and (15) in the main text.

In the main text, we presented the profit-maximization problem from the vantage point of the firm in the home country. For the sake of completeness, we now state the corresponding problem faced by foreign firms and list the associated first-order conditions. Like a home country firm, a foreign firm chooses the sequence $\{d_{i,t}^*, h_{i,t}^*, c_{i,f,t}^*, c_{i,f,t}, s_{i,f,t}^*, s_{i,f,t}, p_{i,f,t}^*, p_{i,f,t}\}_{t=0}^{\infty}$ to optimize the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^* = \mathbb{E}_0\sum_{t=0}^{\infty} m_{0,t}^*\Bigg\{ & d_{i,t}^* + \kappa_{i,t}^*\left[\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha} - \phi^* - (c_{i,f,t}^* + c_{i,f,t})\right] \\
& + \xi_{i,t}^*\Bigg[p_{i,f,t}^* p_{f,t}^* c_{i,f,t}^* + q_t^{-1} p_{i,f,t} p_{f,t} c_{i,f,t} - w_t^* h_{i,t}^* - d_{i,t}^* + \varphi^*\min\{0, d_{i,t}^*\} \\
& - \frac{\gamma_p}{2}\left(\frac{p_{i,f,t}^*}{p_{i,f,t-1}^*}\pi_{f,t}^* - 1\right)^2 c_t^* - \frac{\gamma_p}{2}q_t^{-1}\left(\frac{p_{i,f,t}}{p_{i,f,t-1}}\pi_{f,t} - 1\right)^2 c_t\Bigg] \\
& + \nu_{i,f,t}^*\left[(p_{i,f,t}^*)^{-\eta}(\tilde{p}_{f,t}^*)^{\eta} s_{i,f,t-1}^{*\theta(1-\eta)} x_{f,t}^* - c_{i,f,t}^*\right] + \nu_{i,f,t}\left[p_{i,f,t}^{-\eta}\tilde{p}_{f,t}^{\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t} - c_{i,f,t}\right] \\
& + \lambda_{i,f,t}^*\left[\rho s_{i,f,t-1}^* + (1-\rho)c_{i,f,t}^* - s_{i,f,t}^*\right] + \lambda_{i,f,t}\left[\rho s_{i,f,t-1} + (1-\rho)c_{i,f,t} - s_{i,f,t}\right]\Bigg\},
\end{aligned} \tag{2-26}$$

where $\tilde{p}_{f,t}^* = \tilde{P}_{f,t}^*/P_{f,t}^*$ and $\tilde{p}_{f,t} = \tilde{P}_{f,t}/P_{f,t}$.

The first-order conditions implied by this program are given by

$$d_{i,t}^*: \quad \xi_{i,t}^* = \begin{cases} 1 & \text{if } d_{i,t}^* \geq 0; \\ 1/(1-\varphi^*) & \text{if } d_{i,t}^* < 0. \end{cases} \tag{2-27}$$

$$h_{i,t}^*: \quad \xi_{i,t}^* w_t^* = \alpha\kappa_{i,t}^*\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha-1}, \quad \text{where } h_{i,t}^* = \frac{a_{i,t}^*}{A_t^*}(\phi^* + c_{i,f,t}^* + c_{i,f,t})^{\frac{1}{\alpha}}. \tag{2-28}$$

$$c^*_{i,f,t}: \quad \nu^*_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}]p^*_{i,f,t}p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{i,f,t}. \tag{2-29}$$

$$c_{i,f,t}: \quad \nu_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}]q_t^{-1}p_{i,f,t}p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{i,f,t}. \tag{2-30}$$

$$s^*_{i,f,t}: \quad \lambda^*_{i,f,t} = \rho\mathbb{E}_t[m^*_{t,t+1}\lambda^*_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu^*_{i,f,t+1}\frac{c^*_{i,f,t+1}}{s^*_{i,f,t}}\right]\right]. \tag{2-31}$$

$$s_{i,f,t}: \quad \lambda_{i,f,t} = \rho\mathbb{E}_t[m^*_{t,t+1}\lambda_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu_{i,f,t+1}\frac{c_{i,f,t+1}}{s_{i,f,t}}\right]\right]. \tag{2-32}$$

$$\begin{aligned} p^*_{i,f,t}: \quad 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}]\left[p^*_{f,t}c^*_{i,f,t} - \gamma_p\frac{\pi^*_{f,t}}{p^*_{i,f,t-1}}\left(\pi^*_{f,t}\frac{p^*_{i,f,t}}{p^*_{i,f,t-1}} - 1\right)c^*_t\right] - \eta\frac{\nu^*_{i,f,t}}{p^*_{i,f,t}}c^*_{i,f,t} \\ & + \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]\pi^*_{f,t+1}\frac{p^*_{i,f,t+1}}{(p^*_{i,f,t})^2}\left(\pi^*_{f,t+1}\frac{p^*_{i,f,t+1}}{p^*_{i,f,t}} - 1\right)c^*_{t+1}\right]. \end{aligned} \tag{2-33}$$

$$\begin{aligned} p_{i,f,t}: \quad 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}]\left[q_t^{-1}p_{f,t}c_{i,f,t} - \gamma_p\frac{q_t^{-1}\pi_{f,t}}{p_{i,f,t-1}}\left(\pi_{f,t}\frac{p_{i,f,t}}{p_{i,f,t-1}} - 1\right)c_t\right] - \eta\frac{\nu_{i,f,t}}{p_{i,f,t}}c_{i,f,t} \\ & + \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]q^{-1}_{t+1}\pi_{f,t+1}\frac{p_{i,f,t+1}}{p^2_{i,f,t}}\left(\pi_{f,t+1}\frac{p_{i,f,t+1}}{p_{i,f,t}} - 1\right)c_{t+1}\right]. \end{aligned} \tag{2-34}$$

1.3 Nonstochastic Steady State

In this section, we describe how relative prices are determined in various markets and how they are related to each other in the symmetric equilibrium. Recall that the assumptions of risk neutrality, i.i.d. idiosyncratic cost shocks, and our within-period sequence of decisions together imply that all home country firms set identical prices in the domestic and foreign markets: $P_{i,h,t} = P_{h,t}$ and $P^*_{i,h,t} = P^*_{h,t}$, for all i. By the symmetry of our setup, $P_{i,f,t} = P_{f,t}$ and $P^*_{i,f,t} = P^*_{f,t}$, for all i as well. However, because both sets of firms "price to market," $P_{i,h,t} \neq Q_tP^*_{i,h,t}$ and $P_{i,f,t} \neq Q_t^{-1}P^*_{i,f,t}$, in general. The symmetric equilibrium implies that $p_{i,h,t} = p^*_{i,h,t} = p_{i,f,t} = p^*_{i,f,t} = 1$ or equivalently, $(P_{i,h,t}/P_{h,t}) = (P^*_{i,h,t}/P^*_{h,t}) = (P_{i,f,t}/P_{f,t}) = (P^*_{i,f,t}/P^*_{f,t}) = 1$.

In any symmetric equilibrium, the type-specific ratios of the habit-adjusted price index ($\tilde{P}_{k,t}$) to the corresponding CPI ($P_{k,t}$) must satisfy the following conditions:

$$\begin{aligned} \tilde{p}_{h,t} &= \tilde{P}_{h,t}/P_{h,t} = s^{\theta}_{h,t-1}; \\ \tilde{p}^*_{h,t} &= \tilde{P}^*_{h,t}/P^*_{h,t} = s^{*\theta}_{h,t-1}; \\ \tilde{p}_{f,t} &= \tilde{P}_{f,t}/P_{f,t} = s^{\theta}_{f,t-1}; \\ \tilde{p}^*_{f,t} &= \tilde{P}^*_{f,t}/P^*_{f,t} = s^{*\theta}_{f,t-1}. \end{aligned}$$

These relative prices can then be used to derive the equilibrium demands for the habit-adjusted consumption baskets. For example, in the home country, the symmetric equilibrium condition $x^j_{h,t} = x_{h,t}$, for all j, implies

$$\begin{aligned} x_{h,t} &= \Xi_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \Xi_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{P_{h,t}}\frac{P_{h,t}}{P_t}\frac{P_t}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \Xi_h^{\varepsilon} p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t, \end{aligned}$$

where

$$\begin{aligned} \tilde{p}_t &= \left[\sum_{k=h,f} \Xi_k \left(\frac{\tilde{P}_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f} \Xi_k \left(\frac{\tilde{P}_{k,t}}{P_{k,t}}\frac{P_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f} \Xi_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \end{aligned}$$

Similarly, it can be shown that

$$\begin{aligned} x_{f,t} &= \Xi_f^{\varepsilon} p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t; \\ x^*_{h,t} &= \Xi_h^{*\varepsilon} (p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t; \\ x^*_{f,t} &= \Xi_f^{*\varepsilon} (p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t; \end{aligned}$$

and

$$\tilde{p}^*_t = \left[\sum_{k=h,f} \Xi_k^* s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.$$

As is typical in such models, only relative prices $p_{h,t}$, $p^*_{h,t}$, $p_{f,t}$, $p^*_{f,t}$, $\tilde{p}_{h,t}$, $\tilde{p}^*_{h,t}$, $\tilde{p}_{f,t}$, $\tilde{p}^*_{f,t}$, $\tilde{p}_t$, and $\tilde{p}^*_t$—along with the real exchange rate q_t—are determined in equilibrium.

Relative Prices and Quantities in the Symmetric Equilibrium

In the steady state, the four Phillips curves implied by our model are given by

$$p_h = \eta \frac{\nu_h}{\mathbb{E}_t^a[\xi_i]}; \tag{3-1}$$

$$q p^*_h = \eta \frac{\nu^*_h}{\mathbb{E}_t^a[\xi_i]}; \tag{3-2}$$

$$p^*_f = \eta \frac{\nu^*_f}{\mathbb{E}_t^a[\xi^*_i]}; \tag{3-3}$$

$$p_f q^{-1} = \eta \frac{\nu_f}{\mathbb{E}_t^a[\xi^*_i]}. \tag{3-4}$$

Equations (3-1)–(3-4) correspond to the steady-state Phillips curves for the price of home-country goods in the home country, the price of home-country goods in the foreign country, the price of foreign-country goods in the home country, and the price of foreign-country goods in the foreign country, respectively.[1]

The symmetric equilibrium conditions and the law of motion for habit stocks imply that $c_h = s_h$, $c_h^* = s_h^*$, $c_f = s_f$, and $c_f^* = s_f^*$. Using these conditions, together with the first-order conditions for habit stocks, we obtain

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_h}{\mathbb{E}^a[\xi_i]}; \tag{3-5}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]}; \tag{3-6}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]}; \tag{3-7}$$

$$\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]}. \tag{3-8}$$

Combining equations (3-1)–(3-4) and equations (3-5)–(3-8) yields

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = p_h\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{3-9}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^*\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{3-10}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^*\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{3-11}$$

$$\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q}\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}, \tag{3-12}$$

which imply that $qp_h^*/p_h = \lambda_h^*/\lambda_h$ and $qp_f^*/p_f = \lambda_f^*/\lambda_f$. By combining the first-order conditions (2-2), (2-4), and (2-5) of home country firms with the corresponding first-order conditions of foreign country firms (see equations (2-28), (2-29), and (2-30)), we obtain

$$\frac{\nu_h}{\mathbb{E}^a[\xi_i]} = p_h - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h}{\mathbb{E}^a[\xi_i]}; \tag{3-13}$$

$$\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^* - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]}; \tag{3-14}$$

$$\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^* - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]}; \tag{3-15}$$

$$\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q} - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]}. \tag{3-16}$$

In the last step, we substitute equations (3-1)–(3-4) and equations (3-9)–(3-12) into equa-

[1] Recall that in our notation, subscripts h and f indicate the origin of the good, while the superscript "*," or its absence, indicate the destination of the good, with "*" indicating the foreign country. For example, p_f^* is the (relative) price of goods produced by foreign firms and sold in the foreign country, whereas p_f is the price of goods produced by foreign firms, but sold in the home country in the local currency—hence the multiplication by $1/q$, which converts it to the currency of the foreign country.

tions (3-13)–(3-16) and solve for p_k and p_k^*, $k = h, f$, which yields:

$$
\begin{aligned}
p_h &= \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]} \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]} \frac{w}{\alpha A}\big(\phi + c_h + c_h^*\big)^{\frac{1-\alpha}{\alpha}}; && \text{(3-17)}\\
p_h^* &= \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]} q^{-1} \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]} \frac{w}{\alpha A}\big(\phi + c_h + c_h^*\big)^{\frac{1-\alpha}{\alpha}}; && \text{(3-18)}\\
p_f^* &= \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]} \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]} \frac{w^*}{\alpha A^*}\big(\phi^* + c_f^* + c_f\big)^{\frac{1-\alpha}{\alpha}}; && \text{(3-19)}\\
p_f &= \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]} q \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]} \frac{w^*}{\alpha A^*}\big(\phi^* + c_f^* + c_f\big)^{\frac{1-\alpha}{\alpha}}. && \text{(3-20)}
\end{aligned}
$$

Note that the law of one price holds in the nonstochastic steady state: $p_h = qp_h^*$ and $p_f^* = p_f/q$, which imply that $\lambda_h^*/\lambda_h = 1$ and $\lambda_f^*/\lambda_f = 1$. This follows from the assumed symmetry of the two markets, in terms of the elasticity of substitution, the strength of the deep-habit mechanism, and so on. However, the law of one price is generally violated in stochastic simulation because in those circumstances, the two countries experience different sequences of asymmetric shocks, which affect the intensity of customer-market relationships, leading to different long-run demand elasticities in the two countries, as well as to different internal liquidity positions. In general, firms will optimally exploit any differences in customer-market relationships and financial positions between the two countries by price discriminating across the border.

In the steady state, the external financing triggers are given by

$$
\begin{aligned}
a^E &= \frac{A}{w\big(\phi + c_h + c_h^*\big)^{\frac{1}{\alpha}}}\big(p_h c_h + q p_h^* c_h^*\big); && \text{(3-21)}\\
a^{*E} &= \frac{A^*}{w^*\big(\phi^* + c_f^* + c_f\big)^{\frac{1}{\alpha}}}\big(p_f^* c_f^* + q^{-1} p_f c_f\big), && \text{(3-22)}
\end{aligned}
$$

which can be used to compute $\mathbb{E}^a[\xi_i]$, $\mathbb{E}^a[\xi_i a_i]$, $\mathbb{E}^a[\xi_i^*]$, and $\mathbb{E}^a[\xi_i^* a_i^*]$:

$$
\begin{aligned}
\mathbb{E}^a[\xi_i] &= 1 + \frac{\varphi}{1-\varphi}\big[1 - \Phi(z^E)\big]; && \text{(3-23)}\\
\mathbb{E}^a[\xi_i a_i] &= 1 + \frac{\varphi}{1-\varphi}\big[1 - \Phi(z^E - \sigma)\big]; && \text{(3-24)}\\
\mathbb{E}^a[\xi_i^*] &= 1 + \frac{\varphi^*}{1-\varphi^*}\big[1 - \Phi(z^{*E})\big]; && \text{(3-25)}\\
\mathbb{E}^a[\xi_i^* a_i^*] &= 1 + \frac{\varphi^*}{1-\varphi^*}\big[1 - \Phi(z^{*E} - \sigma)\big], && \text{(3-26)}
\end{aligned}
$$

where

$$
\begin{aligned}
z^E &= \sigma^{-1}\big(\ln a^E + 0.5\sigma^2\big); && \text{(3-27)}\\
z^{*E} &= \sigma^{-1}\big(\ln a^{*E} + 0.5\sigma^2\big). && \text{(3-28)}
\end{aligned}
$$

The following ratios should be satisfied in the steady state:

$$
\begin{aligned}
\frac{c_{i,h}}{c_{i,f}} &= \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta} s_{i,h}^{\theta(1-\eta)} x_h}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta} s_{i,f}^{\theta(1-\eta)} x_f} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta} s_{i,h}^{\theta(1-\eta)} \Xi_h^{\varepsilon}\tilde{p}_h^{-\varepsilon} p_h^{-\varepsilon}\tilde{p}^{\varepsilon} x}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta} s_{i,f}^{\theta(1-\eta)} \Xi_f^{\varepsilon}\tilde{p}_f^{-\varepsilon} p_f^{-\varepsilon}\tilde{p}^{\varepsilon} x};\\
\frac{c_{i,h}^*}{c_{i,f}^*} &= \frac{p_{i,h}^{-\eta}\tilde{p}_h^{*\eta} s_{i,h}^{*\theta(1-\eta)} x_h^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta} s_{i,f}^{*\theta(1-\eta)} x_f^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta} s_{i,h}^{*\theta(1-\eta)} \Xi_h^{*\varepsilon}\tilde{p}_h^{*-\varepsilon} p_h^{*-\varepsilon}\tilde{p}^{*\varepsilon} x^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta} s_{i,f}^{*\theta(1-\eta)} \Xi_f^{*\varepsilon}\tilde{p}_f^{*-\varepsilon} p_f^{*-\varepsilon}\tilde{p}^{*\varepsilon} x^*}.
\end{aligned}
$$

Imposing the symmetric equilibrium conditions and using the fact that $\tilde{p}_k = s_k^\theta$, for $k = h, f$, then yields

$$\frac{c_h}{c_f} = \left(\frac{\Xi_h}{\Xi_f}\right)^{\varepsilon}\left(\frac{p_h}{p_f}\right)^{-\varepsilon}\left(\frac{s_h^\theta}{s_f^\theta}\right)^{1-\varepsilon}; \quad (3\text{-}29)$$

$$\frac{c_h^*}{c_f^*} = \left(\frac{\Xi_h^*}{\Xi_f^*}\right)^{\varepsilon}\left(\frac{p_h^*}{p_f^*}\right)^{-\varepsilon}\left(\frac{s_h^{*\theta}}{s_f^{*\theta}}\right)^{1-\varepsilon}. \quad (3\text{-}30)$$

Because $c_{i,k} = c_k = s_k = s_{i,k}$ and $c_{i,k}^* = c_k^* = s_k^* = s_{i,k}^*$, equation (6) from the main text and its foreign counterpart imply

$$x = \left[\sum_{k=h,f} \Xi_k (c_k^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}; \quad (3\text{-}31)$$

$$x^* = \left[\sum_{k=h,f} \Xi_k^* (c_k^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \quad (3\text{-}32)$$

Aggregate (conditional) labor demands in the home and foreign countries are given by

$$h = \left[\frac{\phi + c_h + c_h^*}{A^\alpha \exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}; \quad (3\text{-}33)$$

$$h^* = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha} \exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}, \quad (3\text{-}34)$$

while the first-order conditions for hours worked from the households' problems can be expressed as

$$h = U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right], \quad (3\text{-}35)$$

$$h^* = U_h^{*-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right]. \quad (3\text{-}36)$$

Combining these expressions yields the conditions that clear the domestic and foreign labor markets and determine equilibrium wages:

$$U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right] = \left[\frac{\phi + c_h + c_h^*}{A^\alpha \exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}; \quad (3\text{-}37)$$

$$U_h^{*-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right] = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha} \exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}. \quad (3\text{-}38)$$

Finally, equilibrium consistency requires that

$$1 = \left[\sum_{k=h,f} \Xi_k p_k^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}; \quad (3\text{-}39)$$

$$1 = \left[\sum_{k=h,f} \Xi_k^* p_k^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \quad (3\text{-}40)$$

The Real Exchange Rate

We assume that the equilibrium interest rates are determined by the households' rate of time preferences: $r = r^* = \delta^{-1} - 1$. In the case of incomplete risk sharing, this assumption determines the equilibrium holdings of international bonds $B_h = B_f = 0$. Via the bond market clearing conditions $B_h + B_h^* = 0$ and $B_f + B_f^* = 0$, this implies that $B_h^* = B_f^* = 0$. In that case, the real exchange rate is determined such that $b_h = b_f = 0$, which, together with equation (27) in the main text, implies

$$0 = wh - qw^*h^* + \tilde{d} - q\tilde{d}^* - (\tilde{p}x - q\tilde{p}^*x^*),$$

or equivalently,

$$q = \frac{wh + \tilde{d} - \tilde{p}x}{w^*h^* + \tilde{d}^* - \tilde{p}^*x^*}. \tag{3-41}$$

To model a complete risk-sharing arrangement in a monetary union, we allow for state-contingent bonds that are traded internationally, along with government bonds that are in zero net supply. With complete risk sharing, we no longer need to rely on imperfections in the government bond market to induce a long-run stationary equilibrium—accordingly, we set $\tau = 0$ in that case. The presence of a complete set of state-contingent bonds implies the following risk-sharing condition:

$$q_t = \varrho_0 \frac{U_{x,t}^*/\tilde{p}_t^*}{U_{x,t}/\tilde{p}_t}, \quad \text{where } \varrho_0 = q_0 \frac{U_{x,0}/\tilde{p}_0}{U_{x,0}^*/\tilde{p}_0^*}. \tag{3-42}$$

In this case, equation (3-42) replaces the bond-holding condition (27) in the main text, which was derived under incomplete markets. Therefore, in the case of complete risk sharing between the two countries, the real exchange rate should at any point in time satisfy

$$q = \varrho_0 \frac{U_x^*}{U_x} \left[\frac{\sum_{k=h,f} \Xi_k^* (p_k^* s_k^{*\theta})^{(1-\varepsilon)}}{\sum_{k=h,f} \Xi_k (p_k s_k^{\theta})^{(1-\varepsilon)}} \right]^{-1/(1-\varepsilon)}. \tag{3-43}$$

1.4 The Log-Linear Phillips Curves

As shown by equations (16) and (17) in the main text, the log-linearized CPI inflation dynamics in the home and foreign country are given by

$$\begin{aligned} \hat{\pi}_t &= \Xi_h p_h (\hat{p}_{h,t-1} + \hat{\pi}_{h,t}) + \Xi_f p_f (\hat{p}_{f,t-1} + \hat{\pi}_{f,t}); \\ \hat{\pi}_t^* &= \Xi_h^* p_h^* (\hat{p}_{h,t-1}^* + \hat{\pi}_{h,t}^*) + \Xi_f^* p_f^* (\hat{p}_{f,t-1}^* + \hat{\pi}_{f,t}^*), \end{aligned}$$

where the dynamics of $\hat{\pi}_{h,t}$, $\hat{\pi}_{h,t}^*$, $\hat{\pi}_{f,t}^*$, and $\hat{\pi}_{f,t}$ are determined by the four log-linearized Phillips curves.

Home country: The Phillips curve governing the behavior of $\hat{\pi}_{h,t}$, the domestic inflation in the home country, is given by

$$\begin{aligned} \gamma_p \pi_{h,t}(\pi_{h,t} - \pi) &= p_{h,t}\frac{c_{h,t}}{c_t} - \eta \frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} \frac{c_{h,t}}{c_t} \\ &\quad + \gamma_p \mathbb{E}_t \left[m_{t,t+1} \frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]} \pi_{h,t+1}(\pi_{h,t+1} - \pi) \frac{c_{t+1}}{c_t} \right]; \end{aligned} \tag{4-1}$$

while the Phillips curve governing the behavior of domestic producers' inflation in the foreign country is given by

$$\begin{aligned}\gamma_p q_t \pi^*_{h,t}(\pi^*_{h,t}-\pi^*) &= q_t p^*_{h,t}\frac{c^*_{h,t}}{c^*_t} - \eta\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c^*_{h,t}}{c^*_t} \\ &+ \gamma_p\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}q_{t+1}\pi^*_{h,t+1}(\pi^*_{h,t+1}-\pi^*)\frac{c^*_{t+1}}{c^*_t}\right].\end{aligned} \tag{4-2}$$

In the steady state, equations (4-1) and (4-2) reduce to

$$0 = \frac{p_h c_h}{c} - \eta\frac{\mathbb{E}^a[\nu_h]}{\mathbb{E}^a[\xi]}\frac{c_h}{c}; \tag{4-3}$$

and

$$0 = \frac{q p^*_h c^*_h}{c^*} - \eta\frac{\mathbb{E}^a[\nu^*_h]}{\mathbb{E}^a[\xi]}\frac{c^*_h}{c^*}, \tag{4-4}$$

respectively. Note that the ratios $p_h c_h/c$ and $q p^*_h c^*_h/c^*$ correspond to the market shares of home country firms in the domestic and foreign markets, respectively. Imposing these conditions, one can derive the log-linearized Phillips curves as

$$\hat{\pi}_{h,t} = \frac{1}{\gamma_p}\frac{p_h c_h}{c}\left[\hat{p}_{h,t} - (\hat{\nu}_{h,t} - \hat{\xi}_t)\right] + \delta\mathbb{E}_t[\hat{\pi}_{h,t+1}]; \tag{4-5}$$

and

$$\hat{\pi}^*_{h,t} = \frac{1}{\gamma_p} q p^*_h \frac{c^*_h}{c^*}\left[\hat{q}_t + \hat{p}^*_{h,t} - (\hat{\nu}^*_{h,t} - \hat{\xi}_t)\right] + \delta\mathbb{E}_t[\hat{\pi}^*_{h,t+1}]. \tag{4-6}$$

The dynamics of the terms $\hat{\nu}_{h,t} - \hat{\xi}_t$ and $\hat{\nu}^*_{h,t} - \hat{\xi}_t$ are governed by

$$\frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}\frac{\mathbb{E}^a_s[\xi_{i,s}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right]; \tag{4-7}$$

$$\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} = q_t p^*_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta^*_{h,t,s}\frac{\mathbb{E}^a_s[\xi_{i,s}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(q_s p^*_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right], \tag{4-8}$$

where recall that $\tilde{\mu}_t = \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]}\frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}$ is the financially adjusted (gross) markup, $\chi = \theta(1-\eta)(1-\rho)$, and the growth-adjusted, compounded discount factors, $\beta_{h,t,s}$ and $\beta^*_{h,t,s}$, are given by equations (2-20) and (2-21), respectively.

In the steady state, $g_{h,s+1} = g^*_{h,s+1} = 1$, which implies that $\beta_{h,t,s} = \beta^*_{h,t,s} = [\delta(\rho+\chi)]^{s-t}$. Hence, given that $\mathbb{E}^a[\nu_h]/\mathbb{E}^a[\xi] = p_h/\eta$ and $\mathbb{E}^a[\nu^*_h]/\mathbb{E}^a[\xi] = q p^*_h/\eta$, the log-linear dynamics of $\hat{\nu}_{h,t} - \hat{\xi}_t$ and $\hat{\nu}^*_{h,t} - \hat{\xi}_t$ are given by

$$\begin{aligned}\hat{\nu}_{h,t} - \hat{\xi}_t &= \eta\left(\hat{p}_{h,t} - \frac{\hat{\mu}_t}{p_h\tilde{\mu}}\right) + \eta\chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left(\hat{p}_{h,s} - \frac{\hat{\mu}_s}{p_h\tilde{\mu}}\right)\right] \\ &+ \eta\chi\left(1 - \frac{1}{p_h\tilde{\mu}}\right)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left[\hat{\beta}_{h,t,s} - (\hat{\xi}_t - \hat{\xi}_s)\right]\right]\end{aligned} \tag{4-9}$$

and

$$\begin{aligned}\hat{\nu}^*_{h,t} - \hat{\xi}_t = \eta\left((\hat{q}_t + \hat{p}_{h,t}) - \frac{\hat{\mu}_t}{qp^*_h\tilde{\mu}}\right) \\ + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\big[\delta(\rho+\chi)\big]^{s-t}\left((\hat{q}_s + \hat{p}^*_{h,s}) - \frac{\hat{\mu}_s}{qp^*_h\tilde{\mu}}\right)\right] \\ + \eta\chi\left(1 - \frac{1}{qp^*_h\tilde{\mu}}\right)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\big[\delta(\rho+\chi)\big]^{s-t}\big[\hat{\beta}^*_{h,t,s} - (\hat{\xi}_t - \hat{\xi}_s)\big]\right],\end{aligned} \tag{4-10}$$

where the steady-state values of p_h, p^*_h, and $\tilde{\mu}$ satisfy

$$p_h = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta) - \theta\delta(1-\rho)\big]}\frac{1}{\tilde{\mu}};$$

and

$$p^*_h = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta) - \theta\delta(1-\rho)\big]}\frac{1}{q\tilde{\mu}}.$$

Foreign country: The two Phillips curves pertaining to foreign firms are given by

$$\begin{aligned}0 = \frac{p^*_{f,t}c^*_{f,t}}{c^*_t} - \gamma_p\pi^*_{f,t}(\pi^*_{f,t} - \pi^*) - \eta\frac{\mathbb{E}^a_t[\nu^*_{i,f,t}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c^*_{f,t}}{c^*_t} \\ + \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}\pi^*_{f,t+1}(\pi^*_{f,t+1} - \pi^*)\frac{c^*_{t+1}}{c^*_t}\right];\end{aligned} \tag{4-11}$$

and

$$\begin{aligned}0 = \frac{q_t^{-1}p_{f,t}c_{i,f,t}}{c_t} - \gamma_p q_t^{-1}\pi_{f,t}(\pi_{f,t} - \pi) - \eta\frac{\mathbb{E}^a_t[\nu_{i,f,t}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c_{f,t}}{c_t} \\ + \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}q^{-1}_{t+1}\pi_{f,t+1}(\pi_{f,t+1} - \pi)\frac{c_{t+1}}{c_t}\right].\end{aligned} \tag{4-12}$$

In the steady state, these become

$$0 = \frac{p^*_f c^*_f}{c^*} - \eta\frac{\nu^*_f}{\xi^*}\frac{c^*_f}{c^*}; \tag{4-13}$$

and

$$0 = \frac{q^{-1}p_f c_f}{c} - \eta\frac{\nu^*_f}{\xi^*}\frac{c_f}{c}. \tag{4-14}$$

Imposing these conditions, one can derive the log-linear versions of the foreign Phillips curves as

$$\hat{\pi}^*_{f,t} = \frac{1}{\gamma_p}\frac{p^*_f c^*_f}{c^*}\big[\hat{p}^*_{f,t} - (\hat{\nu}^*_{f,t} - \hat{\xi}^*_t)\big] + \delta\mathbb{E}_t[\hat{\pi}^*_{f,t+1}]; \tag{4-15}$$

and

$$\hat{\pi}_{f,t} = \frac{1}{\gamma_p}\frac{q^{-1}p_f c_f}{c}\big[\hat{p}_{f,t} - \hat{q}_t - (\hat{\nu}_{f,t} - \hat{\xi}^*_t)\big] + \delta\mathbb{E}_t[\hat{\pi}_{f,t+1}]. \tag{4-16}$$

Given our symmetric setup, the log-linear dynamics of the terms $\hat{\nu}^*_{f,t} - \hat{\xi}^*_t$ and $\hat{\nu}^*_{f,t} - \hat{\xi}^*_t$ are governed by

$$\begin{aligned}\hat{\nu}^*_{f,t} - \hat{\xi}^*_t = \eta\left(\hat{p}^*_{f,t} - \frac{\hat{\mu}^*_t}{p^*_f\tilde{\mu}^*}\right) + \eta\chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left(\hat{p}^*_{f,s} - \frac{\hat{\mu}_s}{p^*_f\tilde{\mu}^*}\right)\right] \\ + \eta\chi\left(1 - \frac{1}{p^*_f\tilde{\mu}^*}\right)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left[\hat{\beta}^*_{f,t,s} - (\hat{\xi}^*_t - \hat{\xi}^*_s)\right]\right]\end{aligned} \tag{4-17}$$

and

$$\begin{aligned}\hat{\nu}_{f,t} - \hat{\xi}^*_t = \eta\left((\hat{p}_{f,t} - \hat{q}_t) - \frac{\hat{\mu}^*_t}{q^{-1}p_f\tilde{\mu}^*}\right) \\ + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left((\hat{p}_{f,s} - \hat{q}_s) - \frac{\hat{\mu}^*_s}{q^{-1}p_f\tilde{\mu}^*}\right)\right] \\ + \eta\chi\left(1 - \frac{1}{q^{-1}p_f\tilde{\mu}^*}\right)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left[\delta(\rho+\chi)\right]^{s-t}\left[\hat{\beta}_{f,t,s} - (\hat{\xi}^*_t - \hat{\xi}^*_s)\right]\right],\end{aligned} \tag{4-18}$$

where p^*_f, p_f, and $\tilde{\mu}^*$ satisfy the following steady-state relationships:

$$\begin{aligned}p^*_f &= \frac{\eta(1-\rho\delta)}{(\eta-1)\left[(1-\rho\delta) - \theta\delta(1-\rho)\right]}\frac{1}{\tilde{\mu}^*} \\ p_f &= \frac{\eta(1-\rho\delta)}{(\eta-1)\left[(1-\rho\delta) - \theta\delta(1-\rho)\right]}\frac{q}{\tilde{\mu}^*}.\end{aligned}$$

1.5 Equilibrium System of Equations

In this section, we provide the set of equations characterizing the symmetric equilibrium in the case of flexible exchange rates and when the two countries have a complete risk-sharing arrangement. All equilibrium equations are expressed in their symmetric equilibrium forms:

$$\begin{aligned}0 = -\frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t} - \pi_w)\pi_{w,t} \\ - \delta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}(\pi_{w,t+1} - \pi_w)\pi_{w,t+1}\frac{\pi_{w,t+1}}{\pi_{t+1}}\frac{h_{t+1}}{h_t}\right];\end{aligned} \tag{5-1}$$

$$\begin{aligned}0 = -\frac{h_t^{*1/\zeta}/U^*_{x,t}}{w^*_t/\tilde{p}^*_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi^*_{w,t} - \pi^*_w)\pi^*_{w,t} \\ - \delta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}(\pi^*_{w,t+1} - \pi^*_w)\pi^*_{w,t+1}\frac{\pi^*_{w,t+1}}{\pi^*_{t+1}}\frac{h^*_{t+1}}{h^*_t}\right].\end{aligned} \tag{5-2}$$

$$0 = -\frac{c_{h,t}}{c_{f,t}} + \left(\frac{\Xi_h}{\Xi_f}\right)^{\varepsilon}\left(\frac{p_{h,t}}{p_{f,t}}\right)^{-\varepsilon}\left(\frac{s^{\theta}_{h,t-1}}{s^{\theta}_{f,t-1}}\right)^{1-\varepsilon}; \tag{5-3}$$

$$0 = -\frac{c^*_{h,t}}{c^*_{f,t}} + \left(\frac{\Xi^*_h}{\Xi^*_f}\right)^{\varepsilon}\left(\frac{p^*_{h,t}}{p^*_{f,t}}\right)^{-\varepsilon}\left(\frac{s^{*\theta}_{h,t-1}}{s^{*\theta}_{f,t-1}}\right)^{1-\varepsilon}. \tag{5-4}$$

$$
\begin{aligned}
0 &= -\tilde{p}_{h,t} + s^{\theta}_{h,t-1}; && (5\text{-}5)\\
0 &= -\tilde{p}_{f,t} + s^{\theta}_{f,t-1}; && (5\text{-}6)\\
0 &= -\tilde{p}^*_{h,t} + s^{*\theta}_{h,t-1} && (5\text{-}7)\\
0 &= -\tilde{p}^*_{f,t} + s^{*\theta}_{f,t-1}. && (5\text{-}8)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -x_{h,t} + \Xi^{\varepsilon}_h p^{-\varepsilon}_{h,t}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t; && (5\text{-}9)\\
0 &= -x_{f,t} + \Xi^{\varepsilon}_f p^{-\varepsilon}_{f,t}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t; && (5\text{-}10)\\
0 &= -x^*_{h,t} + \Xi^{*\varepsilon}_h (p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t; && (5\text{-}11)\\
0 &= -x^*_{f,t} + \Xi^{*\varepsilon}_f (p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t. && (5\text{-}12)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\tilde{p}_t + \left[\sum_{k=h,f} \Xi_k s^{\theta(1-\varepsilon)}_{k,t-1} p^{1-\varepsilon}_{k,t}\right]^{\frac{1}{1-\varepsilon}}; && (5\text{-}13)\\
0 &= -\tilde{p}^*_t + \left[\sum_{k=h,f} \Xi^*_k s^{*\theta(1-\varepsilon)}_{k,t-1} p^{*(1-\varepsilon)}_{k,t}\right]^{\frac{1}{1-\varepsilon}}. && (5\text{-}14)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\pi_{h,t} + \frac{p_{h,t}}{p_{h,t-1}}\pi_t; && (5\text{-}15)\\
0 &= -\pi^*_{h,t} + \frac{p^*_{h,t}}{p^*_{h,t-1}}\pi^*_t; && (5\text{-}16)\\
0 &= -\pi_{f,t} + \frac{p_{f,t}}{p_{f,t-1}}\pi_t; && (5\text{-}17)\\
0 &= -\pi^*_{f,t} + \frac{p^*_{f,t}}{p^*_{f,t-1}}\pi^*_t. && (5\text{-}18)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -h^S_t + h^D_t; && (5\text{-}19)\\
0 &= -h^{*S}_t + h^{*D}_t. && (5\text{-}20)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\mathbb{E}^a_t[\kappa_{i,t}] + \mathbb{E}^a_t[\xi_{i,t} a_{i,t}]\frac{w_t}{\alpha A_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1-\alpha}{\alpha}}; && (5\text{-}21)\\
0 &= -\mathbb{E}^a_t[\kappa^*_{i,t}] + \mathbb{E}^a_t[\xi^*_{i,t} a^*_{i,t}]\frac{w^*_t}{\alpha A^*_t}\left(\phi^* + c_{f,t} + c^*_{f,t}\right)^{\frac{1-\alpha}{\alpha}}. && (5\text{-}22)
\end{aligned}
$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}]p_{h,t} - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t}; \tag{5-23}$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}]q_t p_{h,t}^* - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t}^*; \tag{5-24}$$

$$0 = -\nu_{f,t}^* + \mathbb{E}_t^a[\xi_{i,t}^*]p_{f,t}^* - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t}^*; \tag{5-25}$$

$$0 = -\nu_{f,t} + \mathbb{E}_t^a[\xi_{i,t}^*]q_t^{-1} p_{f,t} - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t}. \tag{5-26}$$

$$0 = -\lambda_{h,t} + \rho\mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu_{h,t+1}\frac{c_{h,t+1}}{s_{h,t}}\right]\right]; \tag{5-27}$$

$$0 = -\lambda_{h,t}^* + \rho\mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}^*] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu_{h,t+1}^*\frac{c_{h,t+1}^*}{s_{h,t}^*}\right]\right]; \tag{5-28}$$

$$0 = -\lambda_{f,t}^* + \rho\mathbb{E}_t[m_{t,t+1}^*\lambda_{f,t+1}^*] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}^*\mathbb{E}_{t+1}^a\left[\nu_{f,t+1}^*\frac{c_{f,t+1}^*}{s_{f,t}^*}\right]\right]; \tag{5-29}$$

$$0 = -\lambda_{f,t} + \rho\mathbb{E}_t[m_{t,t+1}^*\lambda_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}^*\mathbb{E}_{t+1}^a\left[\nu_{f,t+1}\frac{c_{f,t+1}}{s_{f,t}}\right]\right]. \tag{5-30}$$

$$\begin{aligned} 0 = &-p_{h,t}\frac{c_{h,t}}{c_t} + \gamma_p\pi_{h,t}(\pi_{h,t}-\pi) + \eta\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}}{c_t} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\pi)\frac{c_{t+1}}{c_t}\right]; \end{aligned} \tag{5-31}$$

$$\begin{aligned} 0 = &-q_t p_{h,t}^*\frac{c_{h,t}^*}{c_t^*} + \gamma_p q_t\pi_{h,t}^*(\pi_{h,t}^*-\pi^*) + \eta\frac{\nu_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}^*}{c_t^*} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}q_{t+1}\pi_{h,t+1}^*(\pi_{h,t+1}^*-\pi)\frac{c_{t+1}^*}{c_t^*}\right]; \end{aligned} \tag{5-32}$$

$$\begin{aligned} 0 = &-p_{f,t}^*\frac{c_{f,t}^*}{c_t^*} + \gamma_p\pi_{f,t}^*(\pi_{f,t}^*-\pi^*) + \eta\frac{\nu_{f,t}^*}{\mathbb{E}_t^a[\xi_{i,t}^*]}\frac{c_{f,t}^*}{c_t^*} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}^*\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*]}\pi_{f,t+1}^*(\pi_{f,t+1}^*-\pi^*)\frac{c_{t+1}^*}{c_t^*}\right]; \end{aligned} \tag{5-33}$$

$$\begin{aligned} 0 = &-q_t^{-1}p_{f,t}\frac{c_{f,t}}{c_t} + \gamma_p q_t^{-1}\pi_{f,t}(\pi_{f,t}-\pi) + \eta\frac{\nu_{f,t}}{\mathbb{E}_t^a[\xi_{i,t}^*]}\frac{c_{f,t}}{c_t} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}^*\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*]}q_{t+1}^{-1}\pi_{f,t+1}(\pi_{f,t+1}-\pi)\frac{c_{t+1}}{c_t}\right]. \end{aligned} \tag{5-34}$$

$$0 = -\mu_t + \frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c_{h,t}^*\right)^{\frac{\alpha-1}{\alpha}}; \tag{5-35}$$

$$0 = -\mu_t^* + \frac{\alpha A_t^*}{w_t^*}\left(\phi^* + c_{f,t} + c_{f,t}^*\right)^{\frac{\alpha-1}{\alpha}}. \tag{5-36}$$

$$0 = -\tilde{\mu}_t + \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\mu_t; \tag{5-37}$$

$$0 = -\tilde{\mu}_t^* + \frac{\mathbb{E}_t^a[\xi_{i,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*a_{i,t}^*]}\mu_t^*. \tag{5-38}$$

$$\begin{aligned} 0 = -a_t^E + &\frac{A_t}{w_t\big(\phi + c_{h,t} + c_{h,t}^*\big)^{\frac{1}{\alpha}}} \\ &\times \left[c_t\left[\frac{p_{h,t}c_{h,t}}{c_t} - \frac{\gamma_p}{2}(\pi_{h,t} - \pi)^2\right] + q_t c_t^*\left[\frac{p_{h,t}^*c_{h,t}^*}{c_t^*} - \frac{\gamma_p}{2}(\pi_{h,t}^* - \pi^*)^2\right]\right]; \end{aligned} \tag{5-39}$$

$$\begin{aligned} 0 = -a_t^{*E} + &\frac{A_t^*}{w_t^*\big(\phi^* + c_{f,t} + c_{f,t}^*\big)^{\frac{1}{\alpha}}} \\ &\times \left[c_t^*\left[\frac{p_{f,t}^*c_{f,t}^*}{c_t^*} - \frac{\gamma_p}{2}(\pi_{f,t}^* - \pi)^2\right] + q_t^{-1} c_t\left[\frac{p_{f,t}c_{f,t}}{c_t} - \frac{\gamma_p}{2}(\pi_{f,t} - \pi)^2\right]\right]. \end{aligned} \tag{5-40}$$

$$0 = -z_t^E + \sigma^{-1}\big(\ln a_t^E + 0.5\sigma^2\big); \tag{5-41}$$

$$0 = -z_t^{*E} + \sigma^{-1}\big(\ln a_t^{*E} + 0.5\sigma^2\big). \tag{5-42}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}\big[1 - \Phi(z_t^E)\big]; \tag{5-43}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}\big[1 - \Phi(z_t^{*E})\big]. \tag{5-44}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}a_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}\big[1 - \Phi(z_t^E - \sigma)\big]; \tag{5-45}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*a_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}\big[1 - \Phi(z_t^{*E} - \sigma)\big]. \tag{5-46}$$

$$0 = -h_t^D + \left[\frac{\phi + c_{h,t} + c_{h,t}^*}{A_t^\alpha \exp\big[0.5\alpha(1+\alpha)\sigma^2\big]}\right]^{\frac{1}{\alpha}}; \tag{5-47}$$

$$0 = -h_t^{*S} + \left[\frac{\phi^* + c_{f,t} + c_{f,t}^*}{A_t^{*\alpha} \exp\big[0.5\alpha(1+\alpha)\sigma^2\big]}\right]^{\frac{1}{\alpha}}. \tag{5-48}$$

$$0 = -U_{x,t} + (x_t - \omega_t)^{-\gamma_x}; \tag{5-49}$$

$$0 = -U_{x,t}^* + (x_t^* - \omega_t^*)^{-\gamma_x}. \tag{5-50}$$

$$0 = -y_t + \exp\big[0.5\alpha(1+\alpha)\sigma^2\big](A_t h_t)^\alpha - \phi; \quad (5\text{-}51)$$
$$0 = -y_t^* + \exp\big[0.5\alpha(1+\alpha)\sigma^2\big](A_t^* h_t^*)^\alpha - \phi^*. \quad (5\text{-}52)$$

$$0 = -1 + \mathbb{E}_t\left[\delta \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R_t}{\pi_{t+1}}\right]; \quad (5\text{-}53)$$
$$0 = -1 + \mathbb{E}_t\left[\delta \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R^*_t}{\pi^*_{t+1}}\right]. \quad (5\text{-}54)$$

$$0 = -R_t + R\left(\frac{y_t}{y}\right)^{\psi_y}\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi}; \quad (5\text{-}55)$$
$$0 = -R^*_t + R^*\left(\frac{y^*_t}{y^*}\right)^{\psi_y}\left(\frac{\pi^*_t}{\pi^*}\right)^{\psi_\pi}. \quad (5\text{-}56)$$

$$0 = -c_t + p_{h,t}c_{h,t} + p_{f,t}c_{f,t}; \quad (5\text{-}57)$$
$$0 = -c^*_t + p^*_{h,t}c^*_{h,t} + p^*_{f,t}c^*_{f,t}. \quad (5\text{-}58)$$

$$0 = -\pi_t + \left[\sum_{k=h,f}\Xi_k(p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)}; \quad (5\text{-}59)$$
$$0 = -\pi^*_t + \left[\sum_{k=h,f}\Xi^*_k(p^*_{k,t-1}\pi^*_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)}; \quad (5\text{-}60)$$
$$0 = -x_t + \left[\sum_{k=h,f}\Xi_k(c^{1-\theta}_{k,t})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}; \quad (5\text{-}61)$$
$$0 = -x^* + \left[\sum_{k=h,f}\Xi^*_k(c^{*1-\theta}_{k,t})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \quad (5\text{-}62)$$

$$0 = -s_{h,t} + \rho s_{h,t-1} + (1-\rho)c_{h,t}; \quad (5\text{-}63)$$
$$0 = -s_{f,t} + \rho s_{f,t-1} + (1-\rho)c_{f,t}; \quad (5\text{-}64)$$
$$0 = -s^*_{h,t} + \rho s^*_{h,t-1} + (1-\rho)c^*_{h,t}; \quad (5\text{-}65)$$
$$0 = -s^*_{f,t} + \rho s^*_{f,t-1} + (1-\rho)c^*_{f,t}. \quad (5\text{-}66)$$

Complete risk sharing: As noted above, with complete risk sharing, the real exchange rate is determined by the risk-sharing condition:

$$0 = -q_t + \varrho_0\frac{U^*_{x,t}/\tilde{p}^*_t}{U_{x,t}/\tilde{p}_t}, \quad \text{where } \varrho_0 = q_0\frac{U_{x,0}/\tilde{p}_0}{U^*_{x,0}/\tilde{p}^*_0}.$$

This condition should hold regardless of the exchange rate arrangement between the two countries (i.e., flexible exchange rate regime or common currency). However, only one of the two consumption Euler equations (5-53) and (5-54) can be included in the system of equations that characterize the equilibrium in a monetary union with complete risk sharing. This is because the combination of common monetary policy and the assumption of complete risk sharing introduces linear dependence into the two Euler equations. Hence, in the monetary union with complete risk sharing, equation (5-54) is deleted and only the efficiency condition (5-53) enters the equilibrium system of equations.

Incomplete risk sharing with flexible exchange rates: With incomplete risk sharing, the following conditions govern the equilibrium in the international capital markets:

$$0 = -(1+\tau b_{h,t+1}) + \delta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R_t}{\pi_{t+1}}\right]; \tag{5-67}$$

$$0 = -(1+\tau b_{f,t+1}) + \delta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R^*_t}{\pi^*_{t+1}}\frac{q_{t+1}}{q_t}\right]; \tag{5-68}$$

$$0 = -(1+\tau b^*_{h,t+1}) + \delta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R_t}{\pi_{t+1}}\frac{q_t}{q_{t+1}}\right]; \tag{5-69}$$

$$0 = -(1+\tau b^*_{f,t+1}) + \delta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R^*_t}{\pi^*_{t+1}}\right]. \tag{5-70}$$

$$0 = b_{h,t+1} + b^*_{h,t+1}; \tag{5-71}$$

$$0 = b_{f,t+1} + b^*_{f,t+1}. \tag{5-72}$$

$$\begin{aligned} 0 = {} & -(b_{h,t+1} + q_t b_{f,t+1}) + \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} \\ & + \frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t); \end{aligned} \tag{5-73}$$

where

$$0 = -\tilde{d}_t + \tilde{d}^+_t + (1-\varphi_t)\tilde{d}^-_t; \tag{5-74}$$

$$0 = -\tilde{d}^*_t + \tilde{d}^{*+}_t + (1-\varphi^*_t)\tilde{d}^{*-}_t; \tag{5-75}$$

and

$$\begin{aligned} 0 = -\tilde{d}^+_t + \Phi(z^E_t)\Bigg[& p_{h,t}c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t}\frac{\Phi(z^E_t - \sigma)}{\Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{\frac{1}{\alpha}} \\ & - \frac{\gamma_p}{2}(\pi_{h,t} - \pi)^2 c_t - \frac{\gamma_p}{2} q_t (\pi^*_{h,t} - \pi^*)^2 c^*_t \Bigg]; \end{aligned} \tag{5-76}$$

$$\begin{aligned} 0 = -\tilde{d}^-_t + \frac{1-\Phi(z^E_t)}{1-\varphi_t}\Bigg[& p_{h,t}c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t}\frac{1-\Phi(z^E_t - \sigma)}{1-\Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{\frac{1}{\alpha}} \\ & - \frac{\gamma_p}{2}(\pi_{h,t} - \pi)^2 c_t - \frac{\gamma_p}{2} q_t (\pi^*_{h,t} - \pi^*)^2 c^*_t \Bigg]; \end{aligned} \tag{5-77}$$

$$0 = -\tilde{d}_t^{*+} + \Phi(z_t^{*E}) \Bigg[q_t^{-1} p_{f,t} c_{f,t} + p_{f,t}^* c_{f,t}^* - \frac{w_t^*}{A_t^*} \frac{\Phi(z_t^{*E} - \sigma)}{\Phi(z_t^{*E})} \left(\phi^* + c_{f,t} + c_{f,t}^*\right)^{\frac{1}{\alpha}} - \frac{\gamma_p}{2} q_t^{-1} (\pi_{f,t} - \pi)^2 c_t - \frac{\gamma_p}{2} (\pi_{f,t}^* - \pi^*)^2 c_t^* \Bigg]; \tag{5-78}$$

$$0 = -\tilde{d}_t^{*-} + \frac{1 - \Phi(z_t^{*E})}{1 - \varphi_t^*} \Bigg[q_t^{-1} p_{f,t} c_{f,t} + p_{f,t}^* c_{f,t}^* - \frac{w_t^*}{A_t^*} \frac{1 - \Phi(z_t^{*E} - \sigma)}{1 - \Phi(z_t^{*E})} \left(\phi^* + c_{f,t} + c_{f,t}^*\right)^{\frac{1}{\alpha}} - \frac{\gamma_p}{2} q_t^{-1} (\pi_{f,t} - \pi)^2 c_t - \frac{\gamma_p}{2} (\pi_{f,t}^* - \pi^*)^2 c_t^* \Bigg]. \tag{5-79}$$

Incomplete risk sharing in a monetary union: In a monetary union with incomplete risk sharing, equations (5-67)–(5-70) are replaced with

$$0 = -(1 + \tau b_{h,t+1}) + \delta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t^U}{\pi_{t+1}} \right]; \tag{5-80}$$

$$0 = -(1 + \tau b_{h,t+1}^*) + \delta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} \frac{R_t^U}{\pi_{t+1}^*} \right]. \tag{5-81}$$

In addition, the bond market clearing condition $0 = b_{f,t+1} + b_{f,t+1}^*$ is deleted, and the following identity is added to the system:

$$\frac{\mathbb{E}_t[q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[S_{t+1}]}{S_t} \frac{\mathbb{E}_t[\pi_{t+1}^*]}{\mathbb{E}_t[\pi_{t+1}]}. \tag{5-82}$$

Note that S_t is not a model variable, as the level of nominal exchange rate cannot be determined in the steady state. However, $\pi_{t+1}^S \equiv S_{t+1}/S_t$ is a well-defined model variable.

Exogenous variables: There are six exogenous aggregate variables in the model, all of which are assumed to follow AR(1) processes.

Technology shocks:

$$0 = -\ln A_t + \rho_A \ln A_{t-1} + \epsilon_{A,t}; \tag{5-83}$$

$$0 = -\ln A_t^* + \rho_A \ln A_{t-1}^* + \epsilon_{A,t}^*. \tag{5-84}$$

Demand shocks:

$$0 = -\omega_t + \rho_\omega \omega_{t-1} + \epsilon_{\omega,t}; \tag{5-85}$$

$$0 = -\omega_t^* + \rho_\omega \omega_{t-1}^* + \epsilon_{\omega,t}^*. \tag{5-86}$$

Financial shocks:

$$0 = -\ln \varphi_t + (1 - \rho_f) \ln \varphi + \rho_f \ln \varphi_{t-1} + \epsilon_{f,t}; \tag{5-87}$$

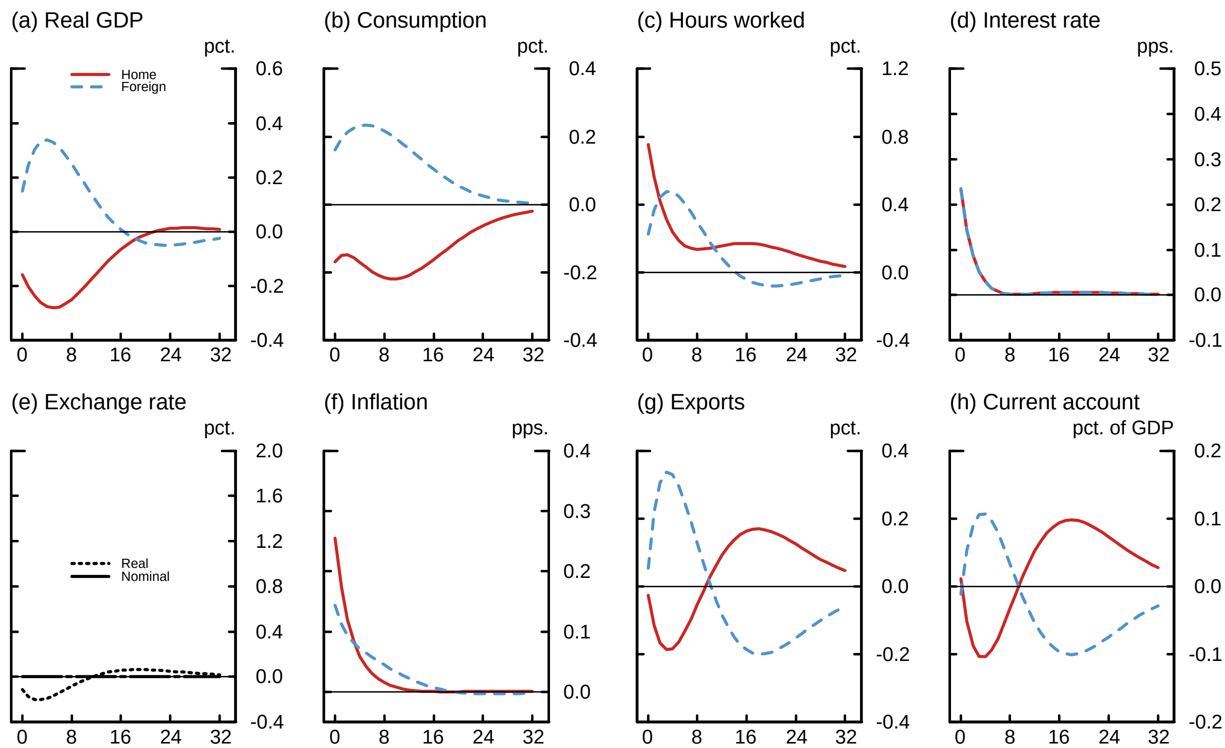
$$0 = -\ln \varphi_t^* + (1 - \rho_f) \ln \varphi^* + \rho_f \ln \varphi_{t-1}^* + \epsilon_{f,t}^*. \tag{5-88}$$

2 Additional Results

2.1 Currency Regimes and the Impact of Asymmetric Technology Shocks

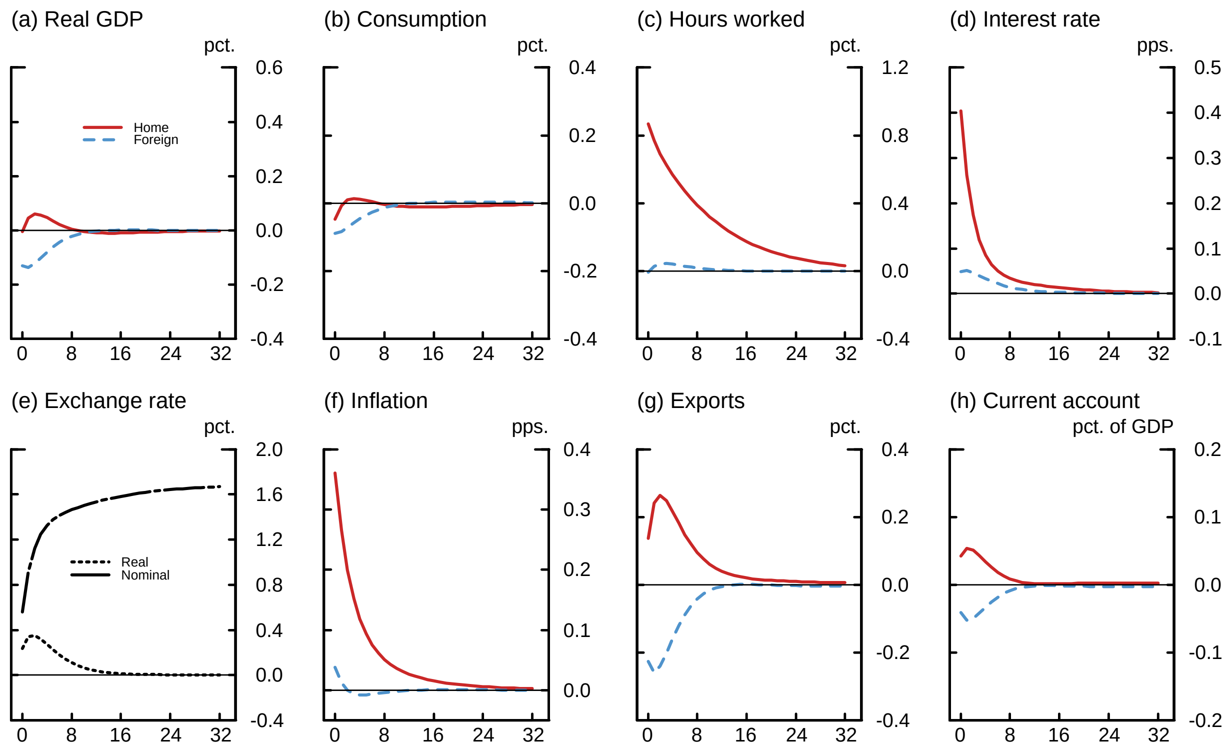
In this section, we examine the model dynamics in response to aggregate technology shocks under different currency regimes. Figure 2-1 depicts the dynamics of selected macroeconomic variables in response to an adverse technology shock in the home country when the two countries are in a monetary union. Figure 2-2, by contrast, shows the dynamics of the same variables in the case of flexible exchange rates.

FIGURE 2-1 – Asymmetric Technology Shock in Monetary Union



NOTE: The panels of the figure depict the model-implied responses of selected variables to an adverse technology shock in the home country in period 0. Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates are expressed as home currency relative to foreign currency.

Figure 2-2 – Asymmetric Technology Shock With Flexible Exchange Rates



Note: The panels of the figure depict the model-implied responses of selected variables to an adverse technology shock in the home country in period 0. Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates are expressed as home currency relative to foreign currency.