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Oil, Equities, and the Zero Lower Bound

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Abstract

From late 2008 to 2017, oil and equity returns were more positively correlated than in other periods. In addition, we show that both oil and equity returns became more responsive to macroeconomic news. We provide empirical evidence and theoretical justification that these changes resulted from nominal interest rates being constrained by the zero lower bound (ZLB). Although the ZLB alters the economic environment in theory, supportive empirical evidence has been lacking. Our paper provides clear evidence of the ZLB altering the economic environment, with implications for the effectiveness of fiscal and monetary policy.

JEL Classifications: F31, F41, E30, E01, C81

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1 Introduction

We document that the behavior of oil and equity returns changed dramatically from late 2008 to 2017. During this period, oil and equity returns became highly correlated. At other times, they are typically uncorrelated. Also in contrast to historical experience, from 2008 to 2017, oil and equity returns became responsive to macroeconomic news surprises such as unanticipated changes in nonfarm payrolls. We provide both empirical evidence and theoretical justification that these changes resulted from nominal interest rates being constrained by the zero lower bound (ZLB). Although a large theoretical literature has argued that the ZLB alters the economic environment, empirical support for this proposition has been lacking, especially for measures of economic activity. As such, our paper's major contribution is to provide strong evidence of the ZLB altering the economic environment, which has implications for the effectiveness of both fiscal and monetary policy and for the desirability of a fast exit from ZLB episodes.

As can be seen in Panel (a) of Figure 1, the correlation between oil and equity returns increased sharply in 2008. Between 1983 and 2008, the correlation fluctuated around zero, only turning sharply negative in response to events such as the Gulf War in 1990. The correlation rose drastically in late 2008, reaching as high as 0.65 in 2010 and then averaging around 0.50 through late 2013. Thereafter, the correlation moved lower. We provide evidence that this correlation is broad based with equity returns for a disparate group of sectors all showing an increased correlation with oil.

Given that this observed increase in correlation coincides with the onset of the ZLB period in the U.S. economy, one might wonder whether the ZLB causes this increased correlation. We provide both theoretical and empirical evidence in favor of this causal relationship. We present a formal analysis with a New Keynesian model that is augmented to include oil.[1] Using our New Keynesian model, we show that oil and equity returns become more correlated at the ZLB. The mechanism for this increased correlation arises from the monetary authority being constrained at the ZLB. When the ZLB binds, the nominal interest rate does not respond to changes in inflation. By contrast, away from the ZLB, changes in inflation lead to more than a one-for-one change in the nominal rate. Consequently, a

[1] Our model is similar in structure to Bodenstein et al. (2013), although they do not consider equity prices.

shock that causes oil prices and inflation to rise will increase real interest rates when the ZLB does not bind, but decrease real interest rates when it does bind. The differing dynamics of real interest rates induced by the ZLB change the dynamics of equity prices and, as a result, the contemporaneous comovement of oil and equity returns.

To complement our one-country model, we consider an international extension with two countries. Motivated by Japan's experience at the ZLB since the mid-1990s and Mexico's experience away from the ZLB since 2008, we construct the model so that there is a large country (which we think of as the United States) and a small country. We argue that the predictions of the two-country version of the model are roughly consistent with data from the United States, Japan, and Mexico.

Building on our model's theoretical implications, we provide further empirical evidence of the role of the ZLB by studying how oil and equity returns respond to identified shocks. In particular, we report how much oil and equity returns change on the day of a surprise in U.S. macroeconomic announcements. We identify our shocks as the difference between actual economic announcements (such as nonfarm payrolls) and the average forecast from a week earlier. We show that, in contrast to historical experience, oil and equity returns became and remained responsive to macroeconomic news surprises, such as unanticipated changes in nonfarm payrolls, for several years. These results build on and extend the existing literature. For example, consistent with our finding that the response changes with the onset of the ZLB, Kilian and Vega (2011) report that oil prices do not have statistically significant responses to macroeconomic news surprises over the period from 1983 to 2008. Likewise, using data from 1957 to 2000, Boyd et al. (2005) claim that equities responded positively to bad news in expansions and negatively to bad news in recessions. Our results differ in that the increased responsiveness of equity returns post 2008 has outlasted the recession and instead seems to be related to the low level of nominal interest rates.

Although we provide both a consistent theoretical model and supportive empirical evidence that the ZLB caused this changing relationship between oil and equity returns, alternative explanations are conceivable. For example, the increased financialization of commodities or greater uncertainty related to the financial crisis could potentially explain this increased correlation.[2] As such, we statistically test the relative merits of explanations based on measures of the ZLB (either a Taylor-rule-implied interest rate or the shadow rate of Wu and Xia (2016)) and explanations based on other variables, including open interest in oil futures contracts, the VIX, and the uncertainty indexes of Jurado et al. (2015) and Baker et al. (2015). Overall, we find that the variation in sensitivity to macroeconomic news surprises is best explained by measures of monetary policy being constrained by the ZLB.

Finally, we provide a structural vector autoregression (VAR) analysis of the same correlation, albeit at a monthly frequency. In our structural VAR work, the reason for the change in correlation was not because aggregate demand or supply shocks became more important. Instead, consistent with our ZLB-driven hypothesis, shocks that have an immediate impact on oil returns but not on aggregate demand or oil supply went from causing a negative correlation to a positive correlation.

To summarize, we present multiple pieces of empirical evidence that are consistent with the ZLB changing the correlation between oil and equity returns. These results should help focus the debate over which economic models are most appropriate for studying recent conditions.

1.1 Relationship to literature

The increased correlation between oil and equity returns has been discussed by Lombardi and Ravazzolo (2016) and Serletis and Xu (2016). Lombardi and Ravazzolo (2016) are concerned with the implications of time-varying correlation for portfolio allocation. Serletis and Xu (2016) include a time dummy variable in their analysis starting in late 2008, which they associate with the ZLB. Relative to these earlier studies, our paper provides three contributions. First, we offer a theoretical explanation for the empirical change correlation in a DSGE model. Second, we empirically test predictions of the model beyond the increased correlation of oil and equity prices at the ZLB. Third, we test alternative hypotheses why the correlation between oil and equity returns may have increased (e.g., increased

[2] The literature on financialization of commodities is large and not wholly in agreement. For example, Tang and Xiong (2012) argue that financialization plays an important role in price movements. In contrast, Irwin and Sanders (2012), Fattouh et al. (2013), and Hamilton and Wu (2015) find a more limited role for financialization.

financialization or an increased prevelance of demand shocks) against the ZLB driving the change in correlation.

Our empirical evidence is supportive of a large literature of models in which economic outcomes are different under the ZLB. A non-exhaustive list of representative papers includes the following. Eggertsson (2011), Christiano et al. (2011), Woodford (2011), and Erceg and Linde (2014) show that, in their models, fiscal multipliers were much larger under the ZLB. Likewise, Eggertsson et al. (2014) present a theoretical model, in which structural reform, which is normally expansionary, is contractionary when monetary policy is constrained. In the model of Caballero et al. (2015), the role of capital flows changes under the ZLB.

Relative to the theoretical literature on the ZLB, the empirical literature testing for ZLB effects is less extensive. Dupor and Li (2015) present some empirical evidence, including whether professional forecasters revised their inflation expectations commensurate with their output forecast revisions in response to government stimulus measures. Plante et al. (2016) study the relationship between uncertainty and GDP growth at the ZLB and find that they have been more negatively correlated during the ZLB period, as predicted by the New Keynesian model. Wieland (Forthcoming) explores whether reductions in oil supply are contractionary at the ZLB and fails to find strong evidence. Garin et al. (Forthcoming) study how the economy responds to TFP shocks. They, too, argue that their empirical results are at odds with the New Keynesian model at the ZLB. Although these papers would seem to cast doubt on the ZLB mechanism, they are limited by the small number of observations under which the ZLB is binding because they use monthly or quarterly data. Our paper complements these previous studies by using higher frequency data based on daily price changes. In addition to providing more observations, high frequency data offers the additional benefits that the timing assumptions are more plausible and that the shocks are more likely to be unanticipated than shocks that are identified at the monthly or quarterly frequency, as discussed in Ramey (2016) and Nakamura and Steinsson (2018).

Swanson and Williams (2014) use high-frequency data to study the ZLB. They show that longer-term interest rates become less responsive to macroeconomic news surprises after 2008, which they attribute to the ZLB. Relative to that paper, a contribution of our work is showing that the ZLB affects not only interest rates by making them less responsive to surprises, but also other asset prices, including oil and equities, by making them more responsive. One important methodological contribution of our paper relative to Swanson and Williams is that, beyond reporting results for time-varying responsiveness as was done by Swanson and Williams, we estimate and test directly the hypothesis that the responsiveness varies with monetary policy conditions, as measured by an interest rate implied by a modified Taylor rule. Furthermore, we also test alternative hypotheses that attribute the change in responsiveness either to the financialization of oil markets or to increased uncertainty in the crisis era, and show that the evidence in favor of the ZLB is stronger.

2 The increased correlation between oil and equities

The correlation between daily oil and equity returns increased markedly in late 2008 (Panel (a) of Figure 1). Our measure for the price of oil is the closing value, in dollars per barrel, of the front-month futures contract for West Texas Intermediate (WTI) crude oil for delivery in Cushing, Oklahoma obtained from NYMEX.[3] For equities, we use the Fama–French value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks.[4] Table 1 presents summary statistics for these measures over our sample period, which covers April 6, 1983, through December 31, 2017.

To calculate returns, we drop days with missing values for any of our primary variables of interest: WTI futures prices, metals prices, interest rates, and the level of the equity price index implied by the Fama–French equity returns (which include dividends). Then, we calculate "daily" returns as the 100 times the log difference of these consecutive closing prices, thereby ensuring that the daily returns

[3]The series reports the official daily closing prices from the New York Mercantile Exchange, posted daily at ftp://ftp.cmegroup.com/pub/settle/nymex_future.csv. In contrast, Kilian and Vega (2011) use the daily spot price for WTI crude oil for delivery (freight on board) in Cushing, Oklahoma, as reported by the U.S. Energy Information Administration (EIA). Analyses using the EIA series, or the physical spot price for Brent Forties Oseberg crude oil, obtained from Bloomberg, generate similar results—Bloomberg Finance LP, Bloomberg Terminals (Open, Anywhere, and Disaster Recovery Licenses). Of these, we prefer the WTI nearby futures price, as its more precise timing allows us to better relate it to the macroeconomic announcements. In supplementary analysis, we also use the WTI far futures price, which we define as the price of the furthest available December contract.

[4]Fama–French data downloaded from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

are calculated over the same period for each variable. Panel (a) of Figure 1 depicts the correlation of these daily returns for oil and equities using a rolling sample of one year.[5]

Next, we use a Chow test for the simple regression of oil returns on equity returns to determine whether there is a break in the oil–equity relationship and find a statistically significant break date of September 22, 2008. Table 2 reports the estimated equity beta for three sample periods: the full sample, pre-break, and post-break. As shown in Table 2, the coefficient is slightly negative for the pre-break sample, but is large, positive, and significant for the post-break sample. The coefficient of 0.79 in the post-break sample implies that during this period, a daily return of 1 percent on the equity index is associated with an oil return of about 0.79 percent. We find similar results when using alternative measures of oil prices, including the physical spot prices for WTI and Brent crude oil. Consistent with the lower variation in far futures prices as compared to nearby futures (reported in Table 1), we find that the results when using the WTI far futures series are qualitatively similar but quantitatively smaller.

To demonstrate that the break in the relationship extends beyond the oil market, we also use the metals spot index constructed by the Commodities Research Bureau. Applying the Chow test to the regression of metals on equity returns also implies a statistically significant break date of September 30, 2008. As with oil, Table 2 shows that the slope coefficient on equity returns is essentially zero for the pre-break sample, but is much larger and statistically significant for the post-break sample. Using the standard Andrews supremum-Wald critical value based upon 15 percent trimming of the sample as in Stock and Watson (2003), all of these break dates were found to be statistically significant at the 1-percent level.

Finally, to ensure that the increased correlation between oil and equity returns is not being driven by fluctuations in the energy component of the equity market, we separately regress oil on each of the 12 Fama–French industry portfolios, determined by SIC codes, as well as on returns for the S&P 500 Ex-Energy index obtained from Bloomberg (Ticker: SPXXEGP).[6] The results of the related Chow tests are presented in Panel B of Table 2. In the pre-break sample, returns in all of the non-energy

[5]In Appendix B, we show that this sustained increase is also visible when using window sizes ranging from one month to three years, and when using returns calculated at the daily, weekly, monthly, and quarterly frequencies.

[6]Panel A of Appendix Table B.1 presents summary statistics for the industry portfolios.

related sectors are negatively associated with oil prices. Only the energy sector shows a positive, statistically significant relationship before the break in 2008. In contrast, post-break, all of the sectors display a positive and statistically significant relationship similar to that of the energy sector. These results confirm that our finding of an increased correlation between oil and equity returns is not being driven exclusively by equity prices for energy producers. Instead, the increased correlation between oil and equity returns is broad based.

2.1 The desired policy rate and the ZLB

Given that the relationship between oil and equity returns seems to be dependent on economic conditions, we now formally estimate how this relationship varies with the proximity of the stance of desired monetary policy to the ZLB. Because the federal funds rate may be censored or constrained near the ZLB, diverging from the desired stance of monetary policy, we construct an alternative measure of the stance of monetary policy. First, we define the *notional rate* as the prediction for the federal funds rate using the modified Taylor rule as in Bernanke (2015). As seen in Panel (b) of Figure 1, the notional rate closely tracks the observed federal funds rate until the ZLB era. Next, we construct our *desired policy rate* as being equal to the observed federal funds rate when the notional rate is above zero, and being equal to the notional rate when the notional rate is below zero (and the federal funds rate may be censored). The notional rate is defined as

$$\tilde{NR}_t = \pi_t + y_t + 0.5(\pi_t - 2) + 2, \tag{1}$$

where π_t is inflation and y_t is the output gap. To measure inflation, π_t, we use the deflator for core personal consumption expenditures, which excludes food and energy prices. For the output gap, y_t, we use published estimates prepared by Federal Reserve staff for FOMC meetings through 2009 and then use estimates produced and published by the Congressional Budget Office through 2017.[7] We

[7]Published estiates prepared by Federal Reserve staff for FOMC meetings through 2009 can be obtained from Bernanke (2015) at `https://www.brookings.edu/wp-content/uploads/2015/04/Taylor-Rule-Data.xlsx` or from the Philadelphai Fed's Real-Time Data Research Center at `https://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/gap-and-financial-data-set`. After 2009, potential output is measured using the CBO's estimate of potential output, available on FRED and `https://alfred.stlouisfed.org/series/downloaddata?seid=GDPPOT`, and output is measured using the Philadelphia Fed's real-time GDP series, available at `https:`

use real-time data when available. Panel (b) of Figure 1 depicts the desired monetary policy rate, $\tilde{R}_t$, showing how it is a combination of the observed federal funds rate and the notional rate.

We estimate the equity beta for oil as a function of the desired policy rate using the model

$$Oil_t = \alpha(\tilde{R}_t) + \beta(\tilde{R}_t)Equity_t + \varepsilon_t. \tag{2}$$

The estimates of α and β solve the kernel regression problem

$$\left\{\hat{\alpha}(\tilde{R}_k), \hat{\beta}(\tilde{R}_k)\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{\tilde{R}_t - \tilde{R}_k}{h}\right)(Oil_t - \alpha - \beta Equity_t)^2. \tag{3}$$

For each $\tilde{R}_k$, $\hat{\alpha}(\tilde{R}_k)$ and $\hat{\beta}(\tilde{R}_k)$ minimize the weighted regression of oil returns on equity returns, using all available observations. These weights are based on the difference $\tilde{R}_t - \tilde{R}_k$, normalized by a constant, h, evaluated using ϕ, the standard normal density function. For this regression, we set h equal to one, and results are robust to other values. For each $\tilde{R}_k$, the weights on the observations decline as the distance between $\tilde{R}_t$ and $\tilde{R}_k$ increases. The intuition for this setup is that each estimated $\hat{\beta}(\tilde{R}_k)$ places more weight on the observed $(Oil_t, Equity_t)$ when $\tilde{R}_t$ is close to $\tilde{R}_k$.

Figure 2 plots our estimate of $\beta(\tilde{R}_k)$ and provides further evidence that oil and equities have stronger co-movement (i.e., $\beta(\tilde{R}_k)$ is larger) when interest rates are low, and in particular, when the notional rate is negative. That is, when the Taylor rule would imply nominal interest rates that are lower than the ZLB, we find that the correlation between oil and equity returns is high.

Having established an empirical linkage between oil and equity returns that seems to depend on the ZLB, the next section provides a theoretical background for our work and motivates additional empirical exercises.

3 A DSGE model with oil

To study the theoretical effect of the ZLB on the relationship between oil and equity returns, we use a medium-scale, New Keynesian model augmented with oil, similar to the model in Bodenstein et al. (2013). As in the previous section, we find that the ZLB dramatically changes the behavior of oil and

//www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/routput.

equity returns. Most of our analysis is conducted in a one-country model. In Section 3.7, we extend the analysis to a multicountry setting.

3.1 Households

The representative household has an expected utility function given by

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j} - h\bar{C}_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{L_{t+j}^{1+\varphi}}{1+\varphi} + \eta_{t+j} V\left(\frac{B_{t+j}}{P_{C,t+j}}\right)\right]. \tag{4}$$

Here, $0 < \beta < 1$, C_t denotes consumption, $\bar{C}_t$ is average aggregate consumption, $0 \leq h < 1$ controls consumption habit, L_t denotes hours worked, $P_{C,t}$ is the price of consumption, and $B_t/P_{C,t}$ are real bond holdings. We include real bond holdings in the utility function as in Fisher (2015) to capture changes in the spread between risky and risk-free assets. We couple the bonds in the utility function with the preference shifter, η_t, to allow the spread to change over time. In our model, η_t plays an analogous role to the spread shock in Smets and Wouters (2007).[8] Consumption is an aggregate of non-oil goods, $Y_{C,t}$, and oil, $O_{C,t}$, where

$$C_t = \left(\omega_C^{1-\rho_C} Y_{C,t}^{\rho_C} + (1-\omega_C)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}}. \tag{5}$$

As in Bodenstein et al. (2013), $\mu_{C,t}$ is a process that affects preferences for oil consumption.

The household faces a per-period budget constraint given by

$$B_t + P_{C,t}C_t + P_{Y,t}I_t = (1+R_{t-1})^{\frac{1}{4}} B_{t-1} + R_{K,t}K_t + W_tL_t + T_t, \tag{6}$$

where $P_{Y,t}$ is the price of non-oil output, I_t is investment, K_t are capital holdings, $R_{K,t}$ is the nominal rental rate of capital, W_t is the nominal wage rate, R_t is the annualized net nominal interest rate, and T_t are lump-sum profits, taxes, and transfers. Capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right). \tag{7}$$

[8] We normalize real bonds to be in zero net supply and assume that V is increasing, concave, and has some positive and negative support.

The parameter ϕ_K controls adjustment costs to changes in investment, as in Christiano et al. (2005). To connect our model to the data, it is useful to note that the price of capital is given by

$$P_{K,t} = \beta \frac{\Lambda_{t+1} P_{C,t}}{\Lambda_t P_{C,t+1}} \left[P_{K,t+1} \left(1-\delta\right) + R_{K,t+1} \right]. \tag{8}$$

The variable Λ_t is the marginal utility of consumption in period t. The ex-post nominal returns on capital and oil (in logs) are given by

$$Equity_t = \log\left(\frac{P_{K,t}(1-\delta) + R_{K,t}}{P_{K,t-1}}\right) \text{ and } Equity_t = \log\left(P_{O,t}/P_{O,t-1}\right), \tag{9}$$

where $P_{O,t}$ is the nominal price of a unit of oil.

3.2 Firms

Perfectly competitive firms produce final output, Y_t, using intermediate inputs, $X_t\left(i\right)$. The production technology and demand curves (derived from perfect competition) are given by

$$Y_t = \left(\int_0^1 X_t(i)^{\frac{\nu-1}{\nu}} di\right)^{\frac{\nu}{\nu-1}} \text{ and } X_t(i) = \left(\frac{P_{X,t}(i)}{P_{Y,t}}\right)^{-\nu} Y_t, \tag{10}$$

where $\nu > 1$ and $P_{X,t}(i)$ is the price of $X_t(i)$. A unit measure of monopolists produce with

$$X_t(i) = \left(\omega_X^{1-\rho_X} \left(V_t(i)\right)^{\rho_X} + \left(1-\omega_X\right)^{1-\rho_X} \left(\frac{O_{X,t}(i)}{\mu_{X,t}}\right)^{\rho_X}\right)^{\frac{1}{\rho_X}}. \tag{11}$$

The variable $O_{X,t}(i)$ is oil used in production and, as in Bodenstein et al. (2013), $\mu_{X,t}$ is an exogenous and stochastic process that shifts the usefulness of oil in production. We assume $V_t(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha}$, where A_t is the economy-wide level of technology. Monopolists take demand curves as given and maximize expected discounted profits

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \left[\frac{(1+\tau_X) P_{X,t+j}\left(i\right)}{P_{C,t+j}} - MC_{t+j}\right] \left(\frac{P_{X,t+j}\left(i\right)}{P_{Y,t+j}}\right)^{-\nu} Y_{t+j}. \tag{12}$$

Here, MC_t is real marginal cost and τ_X is a subsidy to offset steady-state distortions due to monopoly power. Firms are subject to Calvo pricing frictions, as in Christiano et al. (2005). In each period, firm i has probability $1-\xi$ that it can update its price optimally. Otherwise, the firm updates its price by the inflation rate for non-oil output in the previous period.

3.3 Oil market

In each period oil supply, O_t, is exogenously determined. Households purchase oil using non-oil output. Our assumption is akin to assuming that oil must be purchased from abroad using non-oil output.[9] Market clearing in the oil market and the resource constraint imply

$$O_t = \int_0^1 O_{X,t}(i)di + O_{C,t} \text{ and } Y_t = Y_{C,t} + G_t + I_t + \frac{P_{O,t}}{P_{Y,t}} O_t, \tag{13}$$

where G_t are government purchases.

3.4 Government policy

The fiscal authority purchases G_t. Lump sum taxes are set to satisfy the government budget constraint, period-by-period, with $B_t = 0$. The monetary authority sets $R_t = \max\left\{0, \tilde{R}_t\right\}$, where

$$\left(1+\tilde{R}_t\right)^{\frac{1}{4}} = \left(\left[1+\tilde{R}_{t-1}\right]^{\frac{1}{4}}\right]^{\gamma} \left([1+R]^{\frac{1}{4}} \left(\frac{\pi_{Y,t}}{\pi^*}\right)^{\theta_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\theta_Y}\right)^{1-\gamma} \tag{14}$$

The variable $\tilde{R}_t$ is the notional interest rate that the monetary authority would set if it were not constrained by the ZLB.[10] In the Taylor-type rule for $\tilde{R}_t$, R is the steady-state annualized net nominal interest rate, γ controls the amount of interest rate smoothing in the Taylor-type rule, $\pi_{Y,t} \equiv P_{Y,t}/P_{Y,t-1}$ denotes the inflation rate of prices for non-oil output, π^* is the monetary authority's target rate of inflation, and $\theta_\pi > 1$ to satisfy the Taylor principle.

We specified the monetary policy rule in terms of $\pi_{Y,t}$ so that movements in oil prices would not feed through as quickly to movements in the notional interest rate.[11] The Taylor rule also includes the deviation of output from its potential level, Y_t^*, defined as the level of output that would prevail with no nominal rigidities. We include an interest rate smoothing term to make the rule similar to those considered in the DSGE literature. Removing the interest rate smoothing term would make the effects of the ZLB even more dramatic in two ways. First, the nominal interest rate would respond even more to changes in inflation and the output gap during normal times as compared to at the ZLB.

[9] Our main results are little changed if we assume that the oil supply is owned by the household.

[10] In the model, there is no distinction between the notional rate and the desired policy rate. In Section 2.1, the two differ because we use the observed federal funds rate when the Taylor-rule implied notional rate is above zero.

[11] Our main results are similar if the monetary authority responds to consumption-goods price inflation.

Second, with interest rate smoothing, negative values of $\tilde{R}_t$ imply low future interest rates. Without interest rate smoothing, negative levels of $\tilde{R}_t$ would have no effect on future interest rates.

In our simulations, we will also consider an alternative version of our model in which there is no ZLB constraint so that, for all t, $R_t = \tilde{R}_t$. We analyze the effects of the ZLB by comparing the results from our benchmark model with this alternative version.

3.5 Calibration and solution strategy

For the parameters that are specific to the oil market, we draw on the DSGE literature that has incorporated oil supply. Following Bodenstein et al. (2013), we set $\rho_C = \rho_X = -1.5$ so that the elasticity of substitution for oil is 0.4. We set $\omega_C = 0.03$ and $\omega_X = 0.027$. In steady state, these parameters imply that oil used in production is about 1.8 times oil used for final consumption and that the overall oil share of the economy is a little over 4 percent, consistent with evidence in Bodenstein et al. (2013).

For the parameters of our model not related to oil, we use parameter values commonly found in the DSGE literature. We set the parameter governing consumption habit, h, to 0.7, in line with Boldrin et al. (2001). We set $\delta = 0.025$, as in Christiano et al. (2005), who draw on Christiano and Eichenbaum (1992). The parameter α is set to 0.33 so that the steady-state labor share of payments to labor and capital is roughly 0.67. We set $\phi_K = 3$, in line with Bodenstein et al. (2013). The value $1 - \xi$ governs how often firms can update their prices optimally. We set $\xi = 0.75$. This value is slightly higher than the value implied by evidence in Nakamura and Steinsson (2008) but slightly lower than the value implied by estimates in Gust et al. (2017). As in Christiano et al. (2005), we set $\sigma = 1$, $\varphi = 1$, and we normalize steady-state labor supply to be 1. We set $\beta = 0.9975$ to imply a steady-state risk-free real interest rate of 1 percent. The parameter ν governs substitution between different monopolists' output. We set $\nu = 7$, which is within the range of values considered in Altig et al. (2011), and implies steady-state markups of 15 percent.

For government policy, we calibrate steady-state government purchases to be 20 percent of steady-state output. We set the monetary authority's inflation objective to 2 percent annual inflation. We set $\theta_\pi = 1.5$ to satisfy the Taylor principle, $\theta_Y = 0.25$, and $\gamma = 0.75$.

Our model has six exogenous processes, G_t, A_t, $\mu_{X,t}$, $\mu_{C,t}$, η_t, and O_t. We assume that each of these processes is an AR(1) in log deviations from steady state, except for η_t which is specified as an AR(1) in levels because it has a zero steady state. As in Bodenstein et al. (2013), we set $\mu_{C,t} = \mu_{X,t}$ and refer to this shock as an oil demand shock. We calibrate $\mu_{X,t}$ to have persistence 0.95 and shock volatility to 0.01. This calibration yields similar unconditional autocorrelation properties for oil demand as those in Bodenstein et al. (2011). Similar to Bodenstein et al. (2011), we specify A_t to have persistence 0.89 and shock volatility 0.015 and O_t to have persistence 0.99 and shock volatility 0.018. We calibrate G_t to have persistence 0.85 and shock volatility 0.01. We refer to η_t as a spread shock, and specify it to be an AR(1) process in levels with persistence 0.95 and shock volatility 0.0005. We normalize $V'(0)$ to be equal to Λ in steady state so that a shock to η_t plays the same role as the spread shock in Smets and Wouters (2007). Gust et al. (2017) consider a similar shock to η_t, and use persistence 0.85. However, they report that the data seem to prefer a more persistent process for η_t, so we use 0.95. We set the shock volatility for η_t so that the ZLB binds roughly 10 percent of the time.

We solve the model using the methodology of Guerrieri and Iacoviello (2015). Their solution strategy involves a first-order perturbation to the model, which is applied piecewise so as to accommodate the ZLB. Guerrieri and Iacoviello (2015) show that their solution methodology performs well, even when compared to fully non-linear numerical solutions. The main advantage of using the methodology of Guerrieri and Iacoviello (2015) is that it is able to accommodate the number of state variables implied by medium-scale DSGE models.

3.6 The effects of the ZLB on oil and equity returns

We simulate one million periods from our model to generate data that we can use to analyze how the correlation between oil and equity returns change as the notional interest rate changes. We report local correlations, constructed using the local mean, variance, and covariance of oil and equity returns, which are computed using methodology analogous to Equation 3. We simulate two different versions of our model. In one version, we include the ZLB constraint. In the other, there is no ZLB constraint, and R_t equals $\tilde{R}_t$ for all t.

Figure 3 shows the model-implied value of the local correlation between oil and equity returns as a function of $\tilde{R}_t$. This correlation can be related to the rolling correlations between oil and equity returns, as shown in Panel (a) of Figure 1. When the interest rate is above the ZLB, the correlation is negative. As reported by the black line, for the model that includes the ZLB constraint, when the interest rate is at the ZLB (and the notional interest rate is less than the ZLB) the correlation is positive. Figure 3 also shows the correlation between oil and equity returns for the model without the ZLB constraint. In this version, as when the interest rate is above zero, the correlation between oil and equity returns is negative for any value of $\tilde{R}_t$.

To understand why the correlation between oil and equity returns changes at the ZLB, we run a kernel regression similar to Equation 3, using either oil or equity returns as the dependent variable and the structural shocks in our model as explanatory variables. We scale our structural shocks by their standard deviation.

Figure 4 shows the coefficient $\hat{\beta}(\tilde{R})$ estimated using data from our model when equity returns are used as the dependent variable. Each panel of the figure shows the coefficient estimates for the two different data sets, generated by our two model versions (with and without the ZLB constraint).

Panel (a) shows that, with the ZLB constraint, positive technology shocks cause equity returns to rise away from the ZLB, but fall when the notional rate is negative. When interest rates are positive, the rental rate of capital rises with the improvement in its marginal product. At the ZLB, the positive technology shock causes inflation to fall because marginal cost falls. The nominal interest rate cannot fall in response to the decline in inflation, causing real interest rates to rise. The rise in real interest rates offsets the direct effects of the technology shock and causes equity returns to fall. Panel (b) shows that equity returns always fall in response to positive spread shocks. Equity returns fall because households have an increased desire to hold bonds rather than capital. Away from the ZLB, real interest rates fall in response to a positive spread shock because output and inflation fall. At the ZLB, the nominal interest rate cannot fall in the same way, causing real interest rates to rise and equity returns to respond more strongly.

Panel (c) shows that government spending shocks have little effect on equity returns away from the ZLB. The increase in demand due to increased government consumption is offset by an increase in real interest rates. At the ZLB, real interest rates do not rise in the same way, and equity returns rise. As shown in Panel (d), we find that the response of equity returns to oil demand shocks changes sign at the ZLB. Away from the ZLB, oil demand shocks increase the price of oil used for production, causing marginal cost, inflation, and real interest rates to rise. The rise in real interest rates leads to the low equity return. At the ZLB, the real interest rate falls in response to the rise in inflation, causing equity prices, and thus equity returns, to rise.

Panel (e) of Figure 4 shows the response of equity prices to a positive oil supply shock. Away from the ZLB, oil supply shocks cause equity prices (and also returns) to rise, because an increase in oil supply reduces marginal cost, lowering inflation and the real interest rate. At the ZLB, equity prices increase less because the nominal interest rate does not respond. Unlike for technology shocks, the sign of the response of equity returns to oil supply shocks does not change at the ZLB because, in our model, oil supply shocks are assumed to have very persistent effects that are expected to outlast a particular ZLB episode. After the ZLB no longer binds, the persistent increase in oil supply causes inflation and real interest rates to be low, increasing consumption. These longer-run effects offset the effects of the low short-term real interest rates. If the oil supply process is modeled instead with a much lower persistence of 0.10 instead of 0.99, then the response of equity prices is negative at the ZLB (shown as the dashed-dotted and dotted lines).[12]

Figure 5 shows the coefficient $\hat{\beta}(\tilde{R})$ estimated using data from our model when oil returns are used as the dependent variable. Each panel of the figure shows the coefficient estimates for the two different data sets, generated by our two model versions (with and without the ZLB constraint). Panel (a) shows that positive technology shocks always cause oil prices to fall. The reason is that less oil is needed in production, so oil demand declines. At the ZLB, the oil price decline is larger

[12]Oil supply could be modeled as in Leduc et al. (2016) as having very persistent and transitory components. For parsimony and consistency with Bodenstein et al. (2013), we elected to show separately the two cases of high and low persistence. Our calculated local correlations are similar if we use persistence 0.10 for the oil supply process.

because the decline in inflation that comes with a positive technology shock is not accompanied by a decline in the nominal interest rate, which causes demand to be relatively low. Panel (b) shows that positive spread shocks also cause oil prices to fall. The reason is that households would prefer to save in bonds than purchase oil consumption. As in the case of positive technology shock, at the ZLB the nominal interest rate does not fall, magnifying the effects of the shock. Panel (c) shows that a positive government spending shock causes oil prices to rise because output demand rises and oil is used in production. At the ZLB, the nominal interest rate does not respond to the increase in output and inflation, and the effects are larger than away from the ZLB. Panels (d) and (e) show the effects of positive oil demand and supply shocks. Oil demand shocks cause oil prices to rise and oil supply shocks cause prices to fall. There is little effect from the ZLB because, in our model, monetary policy responds to a measure of inflation that does not include oil consumption.

The change in correlation between oil and equity prices can now be understood by considering the effects of shocks on oil and equity returns jointly. The ZLB does not change the sign of the effect of each structural shock on oil prices, and in some cases magnifies the effects. The ZLB changes the sign of the response of equity returns to technology shocks and oil demand shocks. In both cases, oil and equity returns move in the same direction in response to a shock at the ZLB, whereas away from the ZLB they move in opposite directions. The effects of spread shocks and government spending shocks on equity prices are magnified at the ZLB and oil and equity returns move in the same at the ZLB. The effects of oil supply shocks are muted at the ZLB, while away from the ZLB, oil supply shocks move oil and equity returns in opposite directions. Overall, the ZLB causes greater positive co-movement between oil and equity returns, which increases the local correlation for low values of $\tilde{R}_t$.

Consistent with our empirical findings, our DSGE model shows that if monetary policy is constrained by the ZLB, then the correlation between oil and equity returns rises. Moreover, at the ZLB, the sign of the response of equity returns to certain structural shocks changes, and the effects of some shocks are magnified. In Section 4, we show that these model predictions hold in the data. But first, we present some international evidence.

3.7 International considerations

Having shown that the ZLB is theoretically consistent with the observed increase in the correlation between oil and equity returns in the United States, we now turn to international considerations. We are motivated by Japan's experience at the lower bound since the 1990s as well as Mexico's experience away from the lower bound since 2008 (see Figure 6).

Our model is, in most respects, the two-country analogue of our one-country model. We assume that non-oil output is an aggregate of home and foreign goods. Additionally, we assume that consumers prefer the good from the home country relatively more than the good from the foreign country (home bias).[13] We incorporate nominal rigidities using Calvo-style sticky prices, as in our one-country model, and assume that firms set prices in the currency in which the good is sold (so-called "local-currency pricing"). We assume that households experience preference shocks for bonds, as in our one-country model. So as to accommodate this setup, we assume either that only risk-free nominal bonds are traded in international asset markets or that there is financial autarky (we present results for both cases). In our model, the world is composed of a large country (with size 0.9), which we think of as the United States, and a small country (with size 0.1). Similar to our one-country model, we assume oil supply is exogenous and owned by neither country so that changes in oil prices do not transfer wealth across countries. A detailed description of the model is given in Appendix D.

To isolate the effects of the ZLB in either the small or the large country, we only ever impose the ZLB in one country. In the other country, we assume that the nominal interest rate is unconstrained. Panel (a) of Figure 7 shows that in the large country, the ZLB changes the correlation between oil and equity returns in similar ways to our one-country model, regardless of our assumption about international asset markets. Intuitively, because the large country comprises most of the world, adding a small country to the model has little effect. As shown in Panel (b) of Figure 7, in the large country, the correlation between oil and equity returns is unchanged if the ZLB is binding in the small country. Intuitively, the small country has little effect on the large country in general, so the ZLB binding in the small country also has little effect. Thus, our model predicts little change in the correlation between

[13] The elasticity of substitution between home and foreign goods is calibrated to be 1.5.

oil and equity returns in the large country when the small country is at the ZLB, which is consistent with our finding that the U.S. correlation changed only after 2008.

Figure 8 displays the correlation between oil and small country equity returns when the small country is constrained by the ZLB. In Panel (a), we see that the correlation between oil and small country equity returns changes in the small country's currency, regardless of our assumptions about international financial markets, but to a somewhat smaller extent under financial autarky. In Panel (b), we see that in the large country's currency, the correlation does not change if there are traded nominal bonds. The reason is that the ZLB is not binding in the large country, so nominal returns in the large country respond to shocks in similar ways, regardless of the ZLB in the small country. Under financial autarky, the correlation changes because the household in the large country is unable to purchase bonds in the small country to arbitrage nominal returns.

Figure 9 shows rolling correlations between oil and equity returns in Japan.[14] The correlations are computed in yen in Panel (a) and dollars in Panel (b). We have daily data on exchange rates, oil prices, and equity prices, and these data do not neatly match up because of the time difference between the United States and Japan. We compute the rolling correlations for monthly returns under the assumption that the time zone differences will have little effect at a monthly horizon.[15] Consistent with our model, the correlation in yen rose during the period in which the Japan was at the ZLB. There is a somewhat smaller increased correlation in dollars during that period, which is consistent with our model with no financial integration. In both currencies, there is a decline in the correlation between oil and equity returns in Japan in the late 2000s. Consistent with our model, this is around the time that the Bank of Japan temporarily raised the discount rate (Figure 6).

Figure 10 displays the correlation between oil and small country equity returns when the large country is constrained by the ZLB. In the small country, when the large country is at the ZLB, the correlation changes in the large country's currency, but not in the small country's currency. In the small country, because the ZLB is not binding, the nominal interest rate responds to shocks in similar ways, regardless of the desired policy rate in the big country. As a result, the real rate channel that

[14]Equity returns for Japan come from Bloomberg—Bloomberg Finance LP, Bloomberg Terminals (Open, Anywhere, and Disaster Recovery Licenses).

[15]Figure B.2 in Appendix B shows that the correlations look similar for weekly returns and various window lengths.

causes the change in correlation in the one-country model is not present, and the correlation in the small country's currency is unchanged when the big country is at the ZLB. However, the exchange rate is heavily influenced by shocks in the big country, and its properties change when the large country is at the ZLB. This change translates into an increase in the correlation between the small country's equity returns and oil returns measured in the big country's currency.

Figure 11 shows rolling correlations between oil and equity returns in Mexico.[16] The correlations are computed in dollars in Panel (a) and in pesos in Panel (b). As for Japan, we show here the rolling correlations for monthly returns.[17] Consistent with our model, the correlation in dollars rose during the period in which the United States was at the ZLB. Although our model predicts little change in the correlation when measured in the small country's currency, the observed correlation in pesos did increase during the first part of the ZLB period, though not by as much as correlation in dollars. One possible explanation for the increase in the oil and equity return correlation measured in pesos is that the discount rate in Mexico was held constant from 2009 to 2013 (Figure 6). Additionally, in our model the large country being at the ZLB does not constrain the small country's monetary policy. In reality this assumption may not hold.

4 Estimating the response to macroeconomic news

Having presented theoretical results and also international evidence, we now turn our attention to further empirical evidence for the United States. Our empirical evidence relates to the theoretical prediction of our DSGE model that at the ZLB, the sign of the response of equity returns to structural shocks changes and the effects of some shocks are magnified.

[16]Equity returns for Mexico come from Haver Analytics, Haver Analytics, http://www.haver.com/our_data.html.

[17]Figure B.3 in Appendix B shows the daily and weekly returns.

4.1 Macroeconomic news surprises

To test the New Keynesian model developed in the previous section, we need to identify shocks. One challenge in the existing literature is that using quarterly data limits the number of observations. Another challenge is the ongoing debate about the plausibility of identifying assumptions. We avoid these issues by looking at the response at the daily frequency to macroeconomic news, which is defined as the difference between the announced value of a macroeconomic statistic and its previously expected value from a survey. It is important to note that news about macroeconomic announcements is not what macroeconomists would call a *news shock*. A Beaudry/Portier-style news shock, as in Barsky et al. (2014), is information about the *future* state of the world. In contrast, our macroeconomic news announcements provide information about the *current* state of the world.

We measure macroeconomic news using the same approach that has been well established in the empirical literature such as Beechey and Wright (2009) and Kilian and Vega (2011). We use survey results from Action Economics as the expected U.S. macroeconomic fundamentals.[18] Macroeconomics news is defined as the difference between the announced realization of the macroeconomic aggregates and the survey expectations. We focus on the variables that Swanson and Williams (2014) use in their analysis of interest rate movements during the ZLB period: capacity utilization, consumer confidence, core CPI, GDP (advance), initial claims, ISM manufacturing, leading indicators, new home sales, nonfarm payrolls, core PPI, retail sales excluding autos, and the unemployment rate. Following Swanson and Williams (2014), our regression sample begins in January 1990, when all but two of the surprises are available. Our sample ends in December 2017, five years later than that of Swanson and Williams (2014).

Since the units of measurement differ across the news indicators, we follow the common practice in this literature and normalize the surprise component of each news announcement by its full sample standard deviation. This normalization allows the responses to be comparable across all announce-

[18] Action Economics, LLC, Action Economics Weekly Survey, `http://www.actioneconomics.com/index.php`.

ments. Therefore, for each indicator j at time t, the surprise component s_{jt} is

$$s_{jt} = \frac{(A_{jt} - E_{jt})}{\sigma_j}, \tag{15}$$

where A_{jt} denotes the announced value of indicator j and E_{jt} refers to the market's expectation of indicator j prior to the announcement. To calculate σ_j, which is the standard deviation of the surprise component $(A_{jt} - E_{jt})$, we use the entire sample period available for each surprise. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that all positive surprises represent stronger-than-expected growth. Summary statistics for the surprise component of each announcement, $(A_{jt} - E_{jt})$, can be found in Panel B of Appendix Table B.1.

As discussed in Beechey and Wright (2009), the response of asset prices to news events occurs very rapidly, often completely adjusting within 15 minutes of the announcement. However, as was also noted in Beechey and Wright, although intra-daily regressions provide more efficient estimates of the reactions to news announcements, the daily estimators also are consistent. It would seem reasonable to expect a similar result for oil prices. In addition, by using daily data, our results are most comparable to those reported in Kilian and Vega (2011).[19] Using high-frequency data, Rosa (2014) reports statistically significant results for the responses of oil prices to macroeconomic news over the 1999 to 2011 sample. However, he does not consider the role of time variation, which we emphasize here, and which may explain the difference between the results reported in Rosa and those in Kilian and Vega.

4.2 Sensitivity during the ZLB period

We now test whether the sensitivity to macroeconomic news surprises changes during the ZLB period, as would be predicted by our model. Oil and equity returns are calculated as in Section 2 as 100 times the log difference in daily prices, with an adjustment for dividend payments. For interest rates, consistent with Swanson and Williams (2014), our dependent variable is the daily change in basis

[19]Studies using higher frequency prices include Halova (2012), which looks at how oil and natural gas respond to news about oil and natural gas inventories.

points for the market yield on U.S. Treasury securities at a constant maturity of 2 years. We also include market yields at 1-year and 10-year constant maturity for comparison.

Our estimation procedure is similar to those found in earlier papers, such as Kilian and Vega (2011). We estimate the effect of news surprises using the model

$$Y_t = \alpha + \boldsymbol{\beta} \boldsymbol{s_t} + \varepsilon_t. \tag{16}$$

In this model, $\boldsymbol{s_t} = \{s_{1t}, ..., s_{12t}\}$ and $\boldsymbol{\beta} = \{\beta_1, ..., \beta_{12}\}$. Each s_{jt} refers to the standardized macroeconomic news surprise for announcement j on day t. Each β_j measures the response of the variable Y to a one-standard-deviation surprise for the corresponding announcement s_j. This regression is estimated separately for each asset that we are interested in, so $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$. By estimating the response on an announcement day, we attempt to isolate the immediate reaction of asset prices to the news announcement as much as possible. As discussed earlier, this strategy has already been applied successfully to numerous financial assets in the literature, including in Andersen et al. (2003) and Kilian and Vega (2011). The regression model is estimated using data for only those days on which at least one news announcement was made.[20]

To get a baseline estimate for responsiveness to surprises, we first report the estimates over the pre-ZLB era, which covers January 1990 through March 2009. In the pre-ZLB columns of Table 3, the generally small coefficient estimates and lack of statistical significance indicate that both oil and equity returns are not responsive to macroeconomic news. In contrast, the larger coefficients and t-statistics for interest rates indicate their responsiveness to surprises over this period, which is consistent with the results in Swanson and Williams (2014).

These pre-ZLB era estimates can be compared with estimated betas for the ZLB period, which were estimated by restricting the sample to the period when the ZLB is binding (i.e., when the notional rate implied by the Taylor rule is negative, April 2009 to December 2014 and July 2015 to December 2015). As reported in the ZLB era columns of Table 3 and in contrast to the lack of response during

[20] The regression sample includes all days with at least one announcement and with available data for our dependent variables of interest. For each day in our regression sample, we set $s_{jt} = 0$ for those variables without an announcement on that day. In order to prevent these 0's from biasing the coefficients, the s_{jt} are demeaned using the mean of the s_{jt} in the regression sample. We also considered results with all non-announcement days included. Making the change did not alter our results.

the pre-ZLB era, oil and equities respond strongly to surprises during the ZLB period. During the ZLB era and in comparison to the pre-ZLB era, interest rates respond less to news. This decline in the interest rate response is consistent with the results in Swanson and Williams (2014). Figure 12 summarizes these results.

Our next set of regressions estimates the average response to all the news announcements made during a particular era. Estimating the average response provides two benefits. First, the average response can summarize the information contained in 12 individual responses. Because the news announcements have already been standardized, the average response is a sensible statistic. Second, the average response can be estimated for the post-ZLB era of January 2016 to December 2017. With only two years of data in the post-ZLB era, the individual β_j would be estimated using relatively few observations. In contrast, pooling across the observations provides a more reliable estimate. For these reasons, we estimate an average response using a pooled model

$$Y_t = \alpha + \beta S_t + \varepsilon_t. \tag{17}$$

In the pooled model, we pool the news surprises to generate $S_t = \sum_{j=1}^{12} s_{jt}$, and then we estimate β, the average response to a one-standard-deviation surprise. By pooling the data, we increase the number of observations used to estimate β.

Table 4 reports results for the pre-ZLB era, the ZLB era, and the post-ZLB era. For both oil and equities, the average responsiveness to news surprises is low before the ZLB, jumps up during the ZLB period, and then declines thereafter. As such, these results are supportive of the conjecture that the ZLB played an important role in determining the responsiveness of oil and equity returns.

Likewise, as in Swanson and Williams (2014), short-term interest rates are less sensitive to macroeconomic news during the ZLB era than during the pre-ZLB era. In the post-ZLB era, oil and equity returns once again become less responsive to news, and short-term interest rates become slightly more responsive in the post-ZLB period. Although the post-ZLB responses for oil and equities are similar to the pre-ZLB responses, the estimated response of interest rates in the post-ZLB period remains somewhat attenuated (see Figure 12). This attenuation may reflect the mechanisms discussed in Swanson and Williams (2014) regarding low interest rate environments.

4.3 Kernel regression using the desired policy rate

Having shown that the change in the response of macroeconomic news seems to be coincident with the ZLB period in the United States, we now test this hypothesis more directly. We use a kernel regression setup similar to the one in Equation 3 to estimate coefficients on pooled surprises that vary with other underlying, or controlling variables, Z_t,

$$Y_t = \alpha(Z_t) + \beta(Z_t)S_t + \varepsilon_t. \tag{18}$$

The estimates of α and β solve the kernel regression problem

$$\left\{\hat{\alpha}(Z_k), \hat{\beta}(Z_k)\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{Z_t - Z_k}{h}\right)(Y_t - \alpha - \beta S_t)^2 . \tag{19}$$

In particular, we can estimate how the responsiveness to surprises changes based on our estimate of the desired monetary policy rate from Section 2.1. When using the desired policy rate as the kernel variable, $Z_k = \tilde{R}_k$, the coefficients $\hat{\beta}(\tilde{R}_k)$ are estimated by placing more weight on the observed responses to surprises on days when $\tilde{R}_t$ is close to $\tilde{R}_k$. Figure 13 plots the result of the pooled surprises estimation. The results provide direct evidence of the higher sensitivity of oil and equities to macroeconomic news announcements during periods with lower desired rates, and the higher sensitivity of interest rates to macroeconomic news announcements during periods with higher desired rates.

Next, to test for statistical significance of these results, we construct a test statistic $F(Z)$ that compares the sum of squared residuals for the kernel regression model to the sum of squared residuals for a restricted model, in which α and β do not vary across time or with any other controlling variables

$$F(Z) = \frac{SSR - SSR(Z)}{SSR(Z)}, \tag{20}$$

where

$$SSR(Z) = \sum_t \left(Y_t - \hat{\alpha}(Z_t) - \hat{\beta}(Z_t)S_t\right)^2 \text{ and } SSR = \sum_t \left(Y_t - \hat{\alpha} - \hat{\beta}S_t\right)^2 . \tag{21}$$

To determine the associated p-value, we compare this test statistic $F(Z)$ to a distribution of F^{sim} generated using a wild bootstrap procedure. To generate the simulated distribution, we run 1000 simulations. For each simulation i, we use the restricted model estimates for $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\varepsilon}_t$ to generate: $Y_{it}^{sim} = \hat{\alpha} - \hat{\beta}S_t + \nu_{it} * \hat{\varepsilon}_t$. Note that the Y_{it}^{sim} preserves any existing serial correlation in the explanatory variables by leaving the S_t variable fixed and preserves heteroscedasticity by scaling the residuals $\hat{\varepsilon}_t$ by $\nu_{it} = 1 - 2B_{it}$, where B_{it} is a Bernoulli random variable, $B_{it} \sim B(1, 0.5)$.[21] Using these Y_{it}^{sim}, we estimate both the restricted and unrestricted models, and generate the resulting distribution of test statistics. We use this distribution to determine how frequently one would observe in this simulated distribution the empirical test statistic computed using the actual data.

Using this test statistic and simulated distribution, we test the null hypothesis that the restricted model, in which the coefficients α and β do not vary, is equivalent to the unrestricted model, in which the coefficients are allowed to vary with the desired policy rate, $\tilde{R}_k$. We find statistically significant improvement in model fit for all three of our dependent variables. As reported in the first row of Table 5, the p-values for oil, equities, and interest rates are 0.07, 0.02, and less than 0.001, respectively.

4.4 The shadow rate

The previous section provides strong evidence that oil and equities are more sensitive to macroeconomic news surprises when the desired policy rate is negative. We now turn to testing alternative hypotheses for these findings. In this subsection, we study the Wu and Xia (2016) shadow rate rather than our desired policy rate.

The Wu–Xia shadow rate is a market-implied driver of the short-term rate that is allowed to be negative during the ZLB period. The shadow rate is estimated using a dynamic term structure model and thus incorporates information from observed longer-term rates during the ZLB era with the historical relationship between short- and longer-term rates. As seen in Panel (a) of Figure 14, in contrast to our Taylor-rule-implied notional rate, the Wu–Xia measure is positive in 2009 and most negative in 2014. The factors affecting this rate include the monetary policy rate, the expected time at the ZLB, and various risk premia. In particular, unconventional monetary policy (UMP) can lower the shadow rate whereas it would not have the same direct effect on the Taylor-rule-implied notional rate.

[21] This choice of scaling variable ν_{it} is based on Davidson and Flachaire (2008). If we were to use a scaling variable with a standard normal distribution instead, the fourth moment of the simulated residuals would be artificially exaggerated.

One potential benefit of using the shadow rate in our kernel regressions—as opposed to our notional rate implied by the Taylor rule—is that it might better capture the effect of UMP through observed longer-term rates. In theory, the use of the shadow rate in the kernel regression should help us examine the extent to which UMP is a substitute for interest rate policy. If UMP is a fully effective substitute, it would be equivalent to the model in Section 3 in which the ZLB is not binding, and we should not see heightened sensitivity to shocks in the ZLB period.[22] However, if UMP is only somewhat effective, we would see some additional sensitivity to shocks.

We test the model with the shadow rate against the alternative in which the sensitivity to news surprises does not vary and find statistically significant improvement in model fit for equity and interest rate responsiveness to surprises, though not for oil. As reported in the second row of Panel (a) in Table 5, the p-values for oil, equities, and interest rates are 0.01, 0.02, and less than 0.001, respectively.

Finally, we estimate a model that allows the coefficients on the surprises to vary with both the desired policy rate and the shadow rate.[23] Using this model, we test the null hypotheses that a model including the two rates is equivalent to a model including just the desired policy rate or just the shadow rate. We find that in general, once the model coefficients are allowed to vary with one of the two rates, the inclusion of the second rate does not result in a statistically significant improvement in model fit (Panel (b), Table 5). As such, the two rates appear to be substitutes for each other in our regressions.

4.5 Uncertainty and financialization

Although we have presented a theoretical justification for why the ZLB could induce changes in the responsiveness to macroeconomic shocks, one might speculate about alternative explanations. In particular, the common folk wisdom that all correlations go to one in a crisis suggests that increased uncertainty could be an alternative driver of the elevated oil–equity correlation and the increased responsiveness to macroeconomic news surprises during the ZLB period. To test this conjecture, we use three different measures of uncertainty. First, we use the 90-day moving average of the daily

[22] Wu and Xia (2016) provide a discussion of how UMP undoes the constraints implied by the ZLB.

[23] When using two controlling variables, the coefficients in the model $Y_t = \alpha(Z_{1t}, Z_{2t}) + \beta(Z_{1t}, Z_{2t})S_t + \varepsilon_t$ are estimated by solving $\left\{\hat{\alpha}(Z_{1k}, Z_{2k}), \hat{\beta}(Z_{1k}, Z_{2k})\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{Z_{1t}-Z_{1k}}{h_1}\right) \phi\left(\frac{Z_{2t}-Z_{2k}}{h_2}\right) (Y_t - \alpha - \beta S_t)^2$.

series for economic policy uncertainty from Baker et al. (2015). Second, we use the 90-day horizon measure of financial uncertainty from Jurado et al. (2015). Finally, we use the 90-day moving averages of the VIX, which is a measure of options-implied stock market volatility. According to this measure, market uncertainty began rising in 2007, spiked sharply in 2008 at the height of the financial crisis, and remained elevated for a few years after that. All three of these measures are depicted in Panel (b) of Figure 14, and summary statistics are reported in Panel (d) of Table 1.

A second alternative hypothesis is that with increased financialization of oil markets, the greater overlap between oil market and other financial market participants resulted in greater sensitivity of oil to general market conditions. According to this theory, the oil market would react much more strongly to events that earlier would have moved only equity markets. We proxy for financialization using the 90-day rolling average of the open interest across all futures contracts for WTI crude oil on NYMEX, as depicted in Panel (c) of Figure 14.

To test these alternative hypotheses, we re-estimate the kernel regression of our three dependent variables on macroeconomic news surprises in Equation 18 using each of our alternative controlling variables in turn. We test each of these models against the alternative in which the sensitivity to news surprises does not vary. We also estimate the models using pairs of controlling variables. We test the null hypothesis that a model including the desired policy rate along with one of the alternative controlling variables is equivalent to a model including just the desired policy rate or just the alternative controlling variable.

Table 6 summarizes the hypotheses being tested and the p-values that result from the wild bootstrap procedure for each test. In Panel (a), we generally find that the inclusion of the alternative variables tends to not improve the fit of the models for oil or equities in a statistically significant way against the restricted alternative in which the coefficients are non-varying. The exception is for the inclusion of open interest, which improves model fit for equities and interest rates—but, counter to the financialization hypothesis, not for oil. In Panel (b), we find that when allowing the coefficients to vary with the desired policy rate, the addition of a second controlling variable sometimes provides a statistically significant improvement in model fit for the interest rate, but again not for oil or for equities. Lastly, we find in Panel (c) of Table 5 that even after including the alternative kernel variables, the addition of the desired policy rate to the kernel often results in a statistically significant improvement in model fit. In conclusion, we find that the variation in sensitivity to macroeconomic news surprises for oil, equities, and interest rates is better explained by our measure of constrained monetary policy than by the alternative measures of uncertainty and oil market financialization.

5 Additional evidence: A structural VAR

Although we have shown that oil and equity returns have become more responsive to macroeconomic data announcements, we have not yet documented the role of structural shocks, such as oil supply shocks or aggregate demand shocks, in changing the correlation. As discussed in Kilian and Park (2009), the source of the shock can determine whether oil price increases are associated with increases or decreases in equity prices. For example, an increase in the size or frequency of aggregate demand shocks could also have contributed to the increase in the oil-equity correlation during the ZLB period.

In this section, we consider the role of structural shocks by estimating a monthly structural VAR. We then use the VAR's implied structural moving average representation to decompose the correlation into its contributions from the various structural shocks. This exercise is analogous to a variance decomposition but is done for a correlation between two variables rather than for the variance of a single variable.

Our estimated structural VAR is similar to that of Kilian and Park (2009), featuring four monthly variables: the log difference of global oil production ΔO, the detrended level of global real economic activity A, the log difference in the real oil price ΔP, and real U.S. equity returns ΔE.[24] Given our interest in the correlation between oil and equity returns, we estimate the VAR using the first difference of real oil prices and 12 lags, whereas Kilian and Park (2009) estimate their VAR using

[24]Global oil production data is obtained from the U.S. Energy Information Administration's Monthly Energy Review (Table 11.1b). The detrended global real economic activity index, as in Kilian and Park (2009) and Kilian (2009), was downloaded on March 27, 2018 from `http://www-personal.umich.edu/~lkilian/reaupdate.txt`. The nominal oil price is the refiner acquisition cost of imported crude oil, obtained from the U.S. Energy Information Administration's Petroleum Marketing Monthly (Table 1). Nominal U.S. equity returns reflect the market return on the Fama–French value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks. Real oil prices and real equity returns are constructed using the U.S. CPI for all urban consumers, obtained from the Bureau of Labor Statistics.

the level of real oil prices and 24 lags.[25] We identify shocks to the structural VAR using a standard recursiveness assumption, with the following identifying assumptions.

Identifying Assumptions in the Structural VAR

Shock	Immediate Response	Delayed Response
Oil Supply Shock	ΔE, ΔP, A, ΔO	
Aggregate Demand Shock	ΔE, ΔP, A	ΔO
Oil Residual Shock	ΔE, ΔP	A, ΔO
Equity Residual Shock	ΔE	ΔP, A, ΔO

We remain agnostic about the final two shocks, labeling them as generic residual shocks to the relevant variables. The estimated VAR implies the following moving average representation for the h-step ahead forecast errors,

$$y_{t+h} - y_{t+h|t} = \sum_{i=0}^{h-1} \Theta_i w_{t+h-i}, \tag{22}$$

where Θ_i is a 4-by-4 matrix of moving average coefficients implied by the estimated VAR and structural factorization of the variance-covariance matrix of the reduced form residuals.

Rather than discussing impulse responses to the structural shocks, we instead focus on our paper's central question of what causes changes in the correlation between oil and equity returns. To do so we decompose the correlation into its contribution from the various structural shocks. Although a covariance decomposition is seldom featured in the literature, to calculate a covariance decomposition is just a straightforward application of textbook calculations for a variance decomposition (Hamilton (1994)).

The covariance calculation requires two kinds of matrices. The first is $MSPE(h)$ the value of the h-step ahead forecast variance-covariance matrix conditional on all shocks

$$MSPE(h) = E\left(\left(y_{t+h} - y_{t+h|t}\right)\left(y_{t+h} - y_{t+h|t}\right)'\right) = \sum_{i=0}^{h-1} \Theta_i I \Theta_i', \tag{23}$$

[25] As shown in the appendix, estimating the VAR using the level of real oil prices and/or 24 lags does not substantively change our analysis about the correlation between oil and equity returns. Our total sample period is 1974 to 2017, whereas Kilian and Park (2009) use a sample period of 1974 to 2006.

where I is the identity matrix. The second is $MSPE_j(h)$, the h-step ahead forecast variance-covariance matrix conditional on only the j-shock

$$MSPE_j(h) = \sum_{i=0}^{h-1} \Theta_i E_j \Theta_i^{'}, \tag{24}$$

where all elements of E_j are equal to zero except the j-th, j-th element, which equals one.

The correlation between oil and equity returns can be defined using terms from these two matrices. In particular, define $\sigma_p(h)$ as the square root of the 3,3 element of $MSPE(h)$, $\sigma_e(h)$ as the square root of the 4,4 element of $MSPE(h)$, and $\sigma_{pe}(h)$ as the 3,4 element of $MSPE(h)$. Furthermore, define $\sigma_{pe,j}(h)$, the covariance conditional just on shock j, as the 3,4 element of $MSPE_j(h)$. Having defined these terms, we can then write the correlation between oil and equity returns $\rho_{pe}(h)$ as the following equation

$$\rho_{pe}(h) = \frac{\sigma_{pe}(h)}{\sigma_p(h)\,\sigma_e(h)} = \sum_{j=1}^{4} \frac{\sigma_{pe,j}(h)}{\sigma_p(h)\,\sigma_e(h)}. \tag{25}$$

For any shock, a larger $\sigma_{pe,j}(h)$ indicates a larger contribution of the j-th shock to the overall correlation.

We estimate the VAR separately for an early sample of January 1974 to March 2009 and for a late sample of April 2009 to December 2017. Table 7 reports our results for $h = 1000$, which is large enough that $\rho_{pe}(h)$ approximates well the correlation between oil and equity returns. Consistent with our results for daily data, the overall correlation $\rho_{pe}(h)$ for monthly data increases from negative 0.10 in the early sample to positive 0.33 in the late sample. As reported in Table 7, the correlation change was not driven by oil supply or aggregate demand shocks. Instead, during the ZLB period, shocks that have an immediate impact on oil prices but not oil demand and supply went from causing a negative correlation to a positive correlation. This change in the correlation is almost entirely due to the oil residual shock, which contributes negative 0.11 to the oil–equity correlation in the early sample and positive 0.24 in the late sample. The negative effect of these shocks in the pre-ZLB sample is consistent with the empirical results reported in Kilian and Park (2009), and the positive effect in the late sample is consistent with how the ZLB alters the response in our DSGE model discussed in Section 3. These monthly oil residual shocks are, by themselves, not very cleanly identified, highlighting the benefits of our work in Section 4, which uses higher frequency data. In high frequency data the timing assumptions are more plausible and the shocks are more likely to be unanticipated than shocks that are identified at the monthly or quarterly frequency (Ramey, 2016).

6 Conclusion

Starting in late 2008, the correlation between oil and equity returns, which previously had been either small or negative, increased dramatically and remained elevated thereafter. Our main argument is that this correlation change is evidence that the ZLB alters the dynamic behavior of the economy. Our argument is supported by the following observations. First and most obviously, the ZLB becomes binding and the correlation increases at the same time. Second, this jump in correlation is consistent with a standard New Keynesian model in which the ZLB is binding and is not consistent with the same model that ignores the ZLB. A multicountry version of the New Keynesian model is also consistent with international experience at the ZLB. Third, consistent with theory, we show that oil and equity returns became more responsive to macroeconomic news announcements when the ZLB binds. Fourth, we consider alternative hypotheses that could alter the responsiveness of oil and equity returns related to financialization and uncertainty. Our empirical evidence shows that these alternative explanations do not improve the fit relative to conditioning on the ZLB. Finally, we use a structural VAR on monthly data and again find evidence supportive of our ZLB-driven hypothesis.

As such, our results complement and extend the findings of Swanson and Williams (2014) for interest rates by showing that activity measures such as oil and equity prices are also affected by the ZLB. Our findings are more supportive than some previous empirical work that the ZLB alters the economic environment. We would argue that our results should be preferred given our use of daily data with clearly identified shocks.

A large literature now exists that shows that the ZLB theoretically changes the dynamic behavior of the economy. However, empirical macroeconomic evidence that the ZLB actually causes these changes has been scarce. The scarcity of evidence for the United States likely reflects that interest rates were well above zero until 2008, and consequently the number of monthly or quarterly observations of macroeconomic variables available to study is small. Our study links the predictions of the New Keynesian model to our results from daily data, and as such provides further empirical evidence in favor of our model's predictions.

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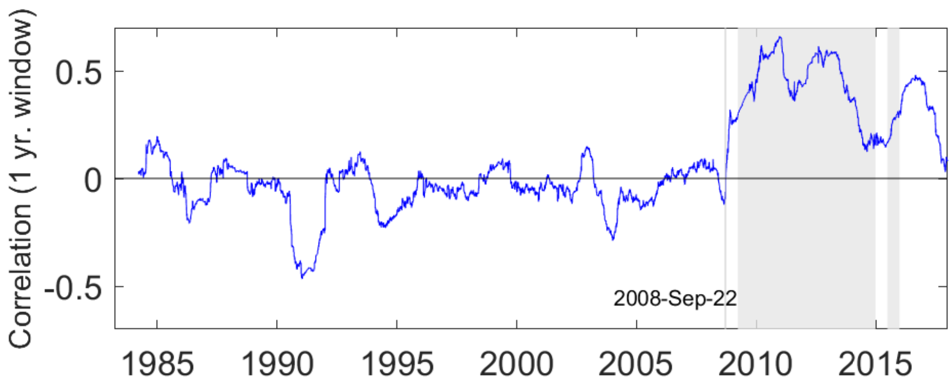
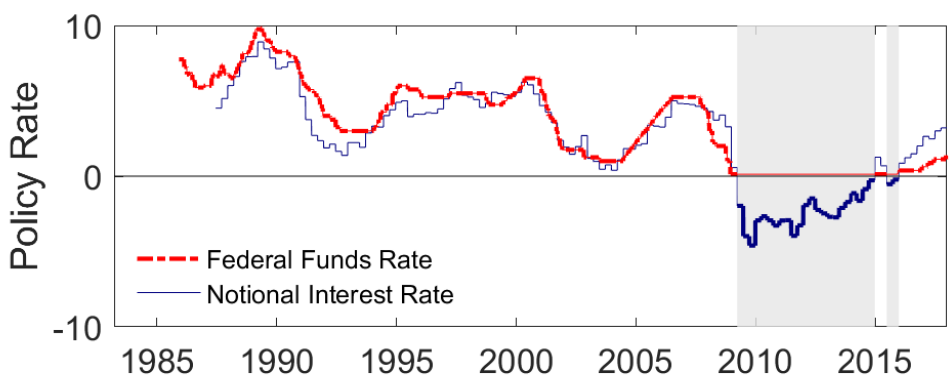
A Figures and tables

Figure 1: Oil-Equity Correlation and Policy Rates

(a) Oil-Equity Rolling Correlation

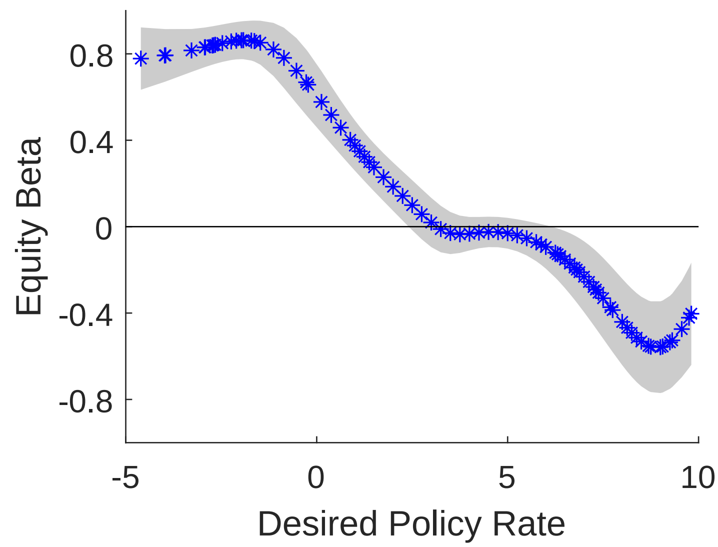


(b) Policy Rates



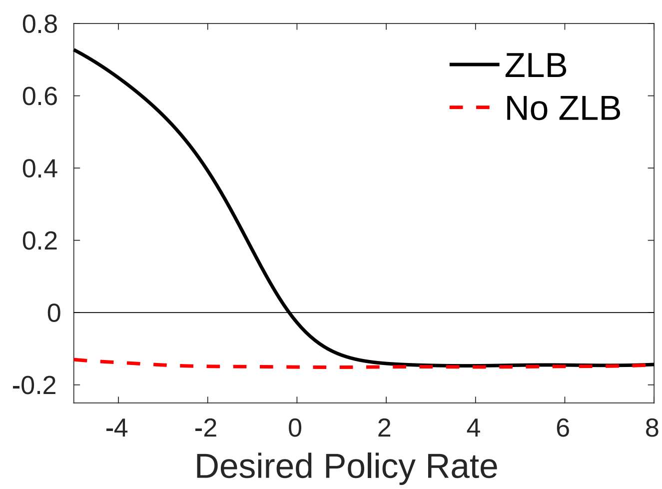
Panel (a): Rolling correlation between daily oil and equity returns. The date axis marks the end of the one-year rolling window over which the correlation is calculated. Panel (b): The federal funds rate and the notional interest rate implied by the modified Taylor rule in Equation 1 ((Bernanke, 2015)). The notional interest rate is intended to capture the target federal funds rate as implied by the current state of the economy, without censoring due to the zero lower bound. The thicker lines represent the series for the *desired policy rate*, $\tilde{R}_k$, defined as the notional rate when it is negative, and the observed federal funds rate otherwise.

Figure 2: Equity Beta for Oil



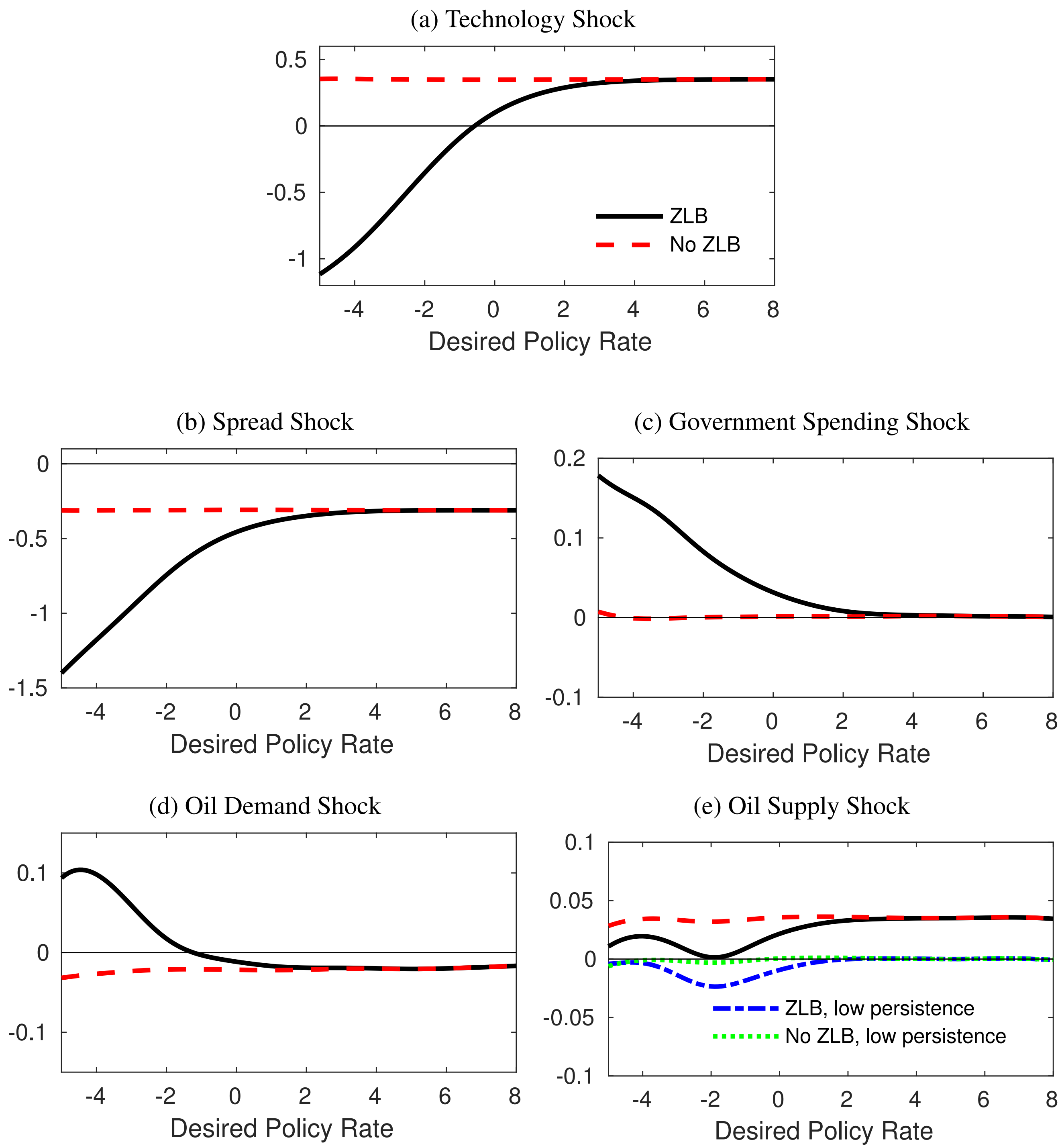
Note: The dots show the estimates of $\beta(\tilde{R}_k)$ using the kernel regression in Equation 2, $Oil_t = \alpha(\tilde{R}_t)+\beta(\tilde{R}_t)Equity_t+\varepsilon_t$. The shaded region represents a 90 percent confidence interval for the estimated $\beta(\tilde{R}_k)$, based on the wild bootstrap as described in Section 4.3.

Figure 3: Local Correlation for Oil and Equity Returns



Note: The solid black lines show the local regression slope for the model that includes the ZLB constraint. The dashed red lines show the local regression slope for the model that does not include the ZLB constraint. We compute the local correlations using one million simulated periods from our model.

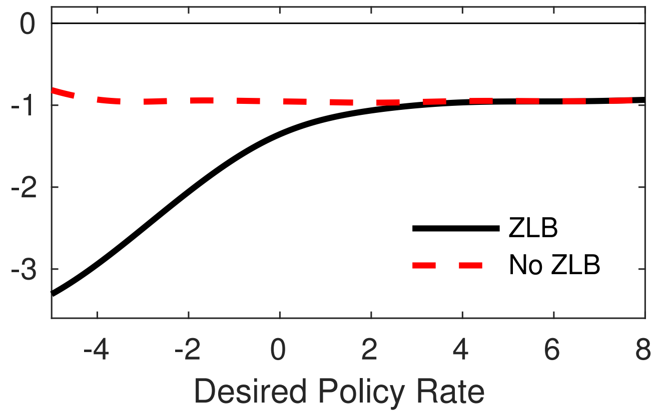
Figure 4: Equity Return Kernel Regression



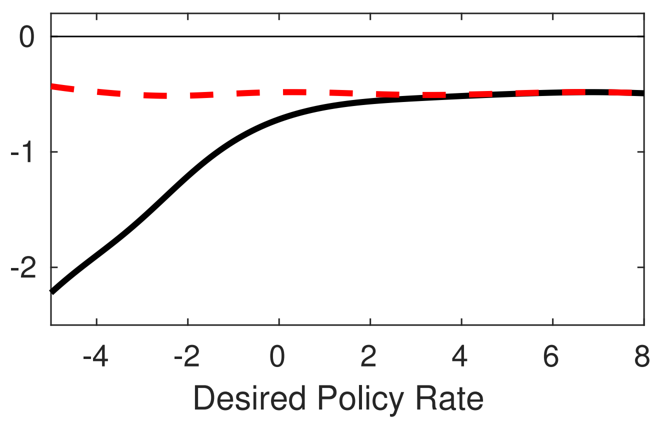
Note: The solid black lines show the local regression slope for the model that includes the ZLB constraint. The dashed red lines show the local regression slope for the model that does not include the ZLB constraint. We compute the local regression slopes using one million simulated periods from our model. In Panel (e), the dashed-dotted blue line is analogous to the black solid line, but for the model in which the persistence of oil supply is reduced from 0.99 to 0.10. The dotted green line is similarly analogous to the dashed red line.

Figure 5: Oil Return Kernel Regression

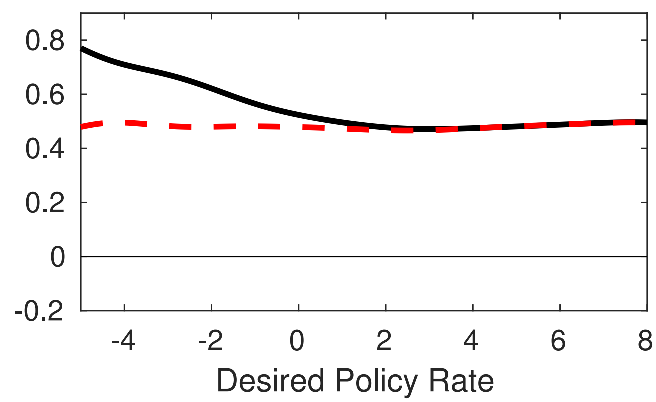
(a) Technology Shock



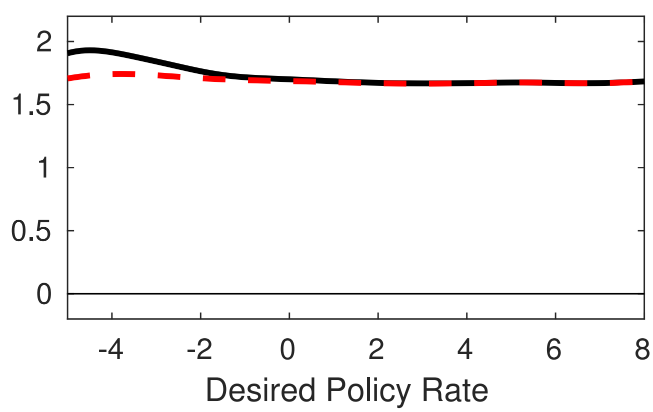
(b) Spread Shock



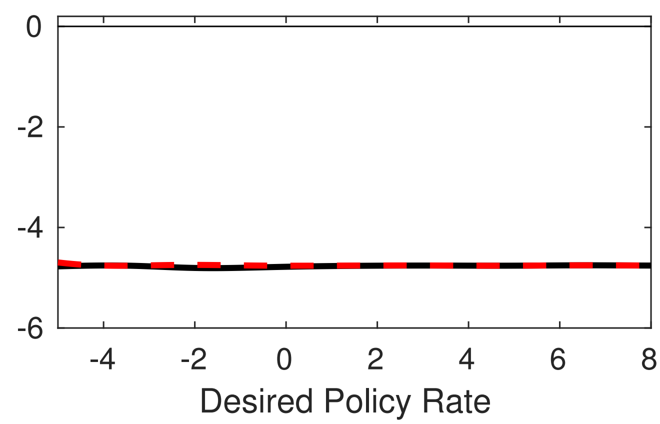
(c) Government Spending Shock



(d) Oil Demand Shock

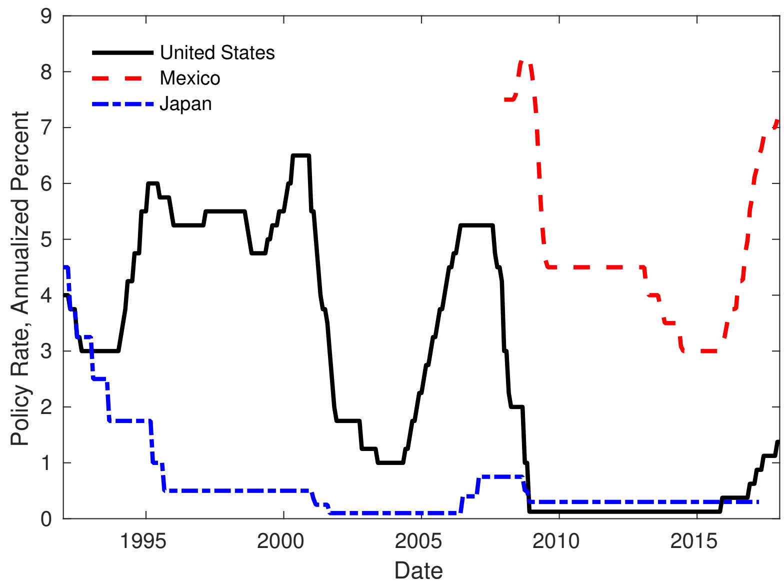


(e) Oil Supply Shock



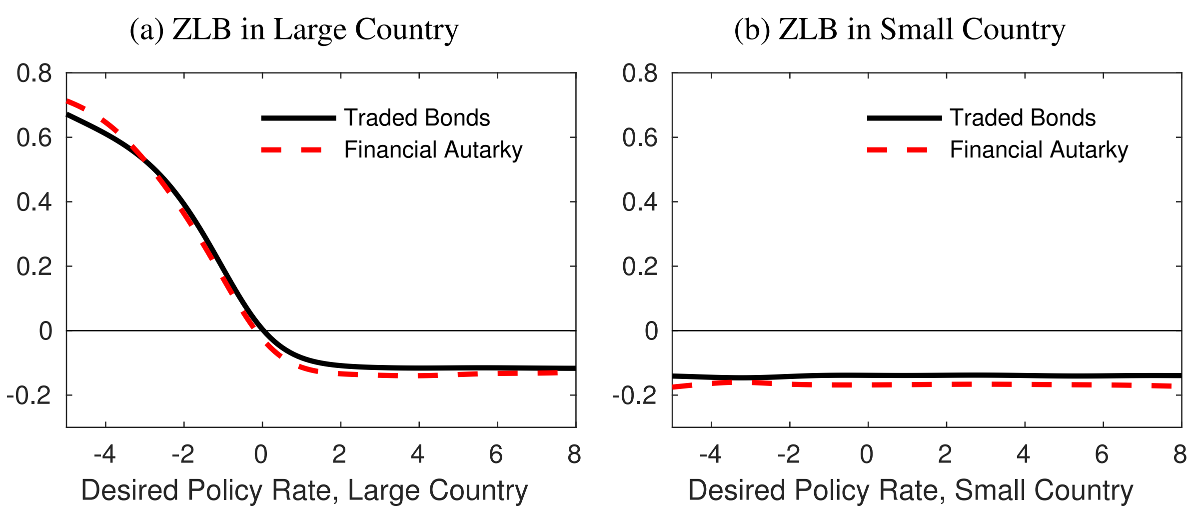
Note: The solid black lines show the local regression slope for the model that includes the ZLB constraint. The dashed red lines show the local regression slope for the model that does not include the ZLB constraint. We compute the local regression slopes using one million simulated periods from our model.

Figure 6: Policy Rates in the United States, Japan, and Mexico



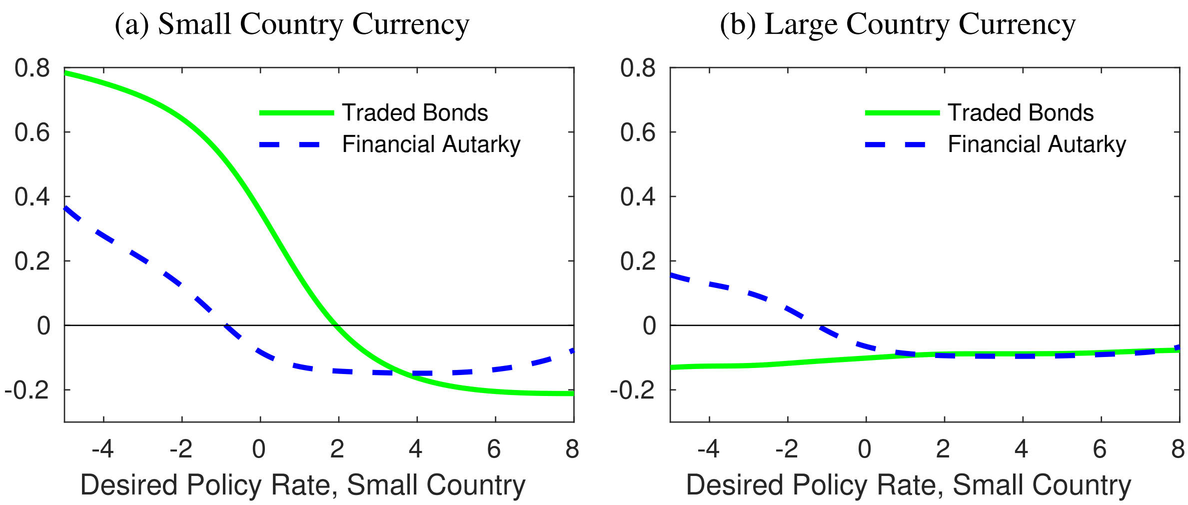
Note: The solid black line shows the midpoint of the target range for the federal funds rate. The blue dash-dotted line shows the discount rate in Japan. The red dashed line shows the discount rate in Mexico. Data for the United States and Japan come from FRED (`https://fred.stlouisfed.org`). The series for the United States is constructed from the FRED series named named DFEDTAR, DFEDTARL, and DFEDTARU. The series for Japan is the FRED series named INTDSRJPM193N. The series from Mexico comes from Banco de México (`http://www.banxico.org.mx/estadisticas/statistics.html`).

Figure 7: Oil-Equity Correlation in Large Country, International Model



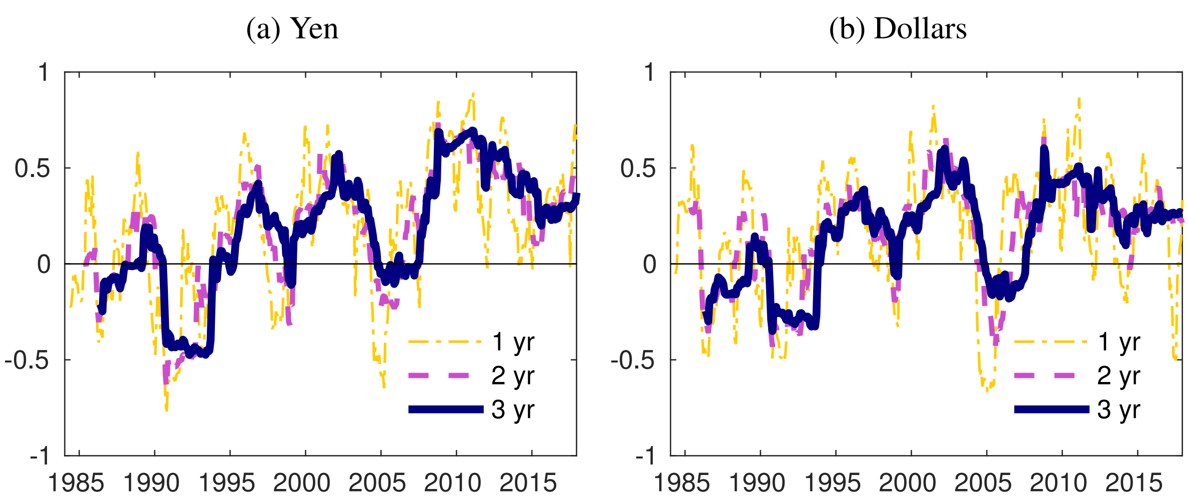
Note: The figures show the local correlation between oil returns and large country equity returns, with both series expressed in the large country currency. We assume the world size is 1, with the size of the large country equal to 0.9 and the size of the small country equal to 0.1. In Panel (a), the ZLB is imposed for the large country and the local correlation is calculated over the range of large country desired policy rates, while in Panel (b), the ZLB is imposed for the small country and the local correlation is calculated over the range of small country desired policy rates.

Figure 8: Oil-Equity Correlation in Small Country, with Small Country ZLB



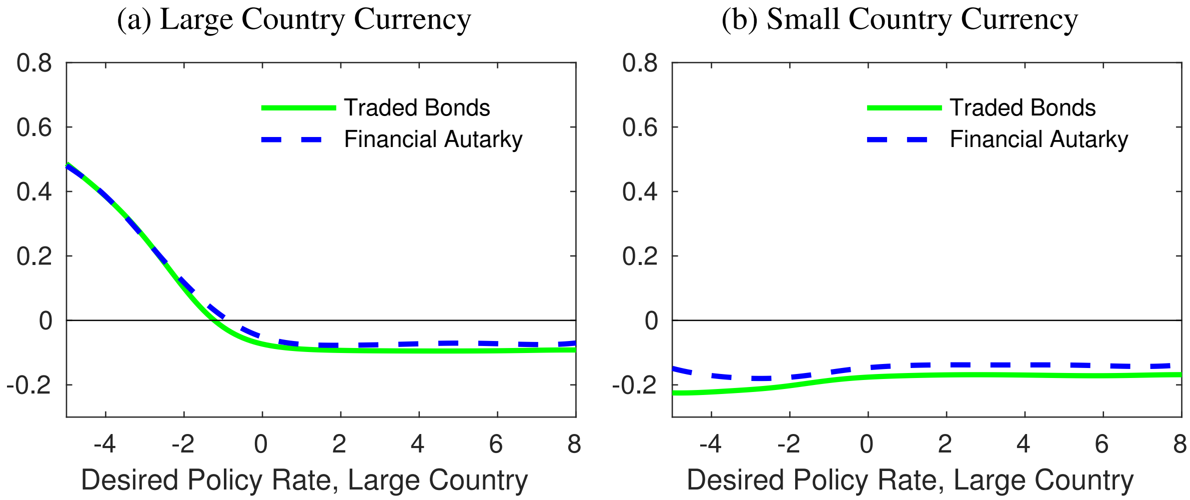
Note: The figures show the local correlation between oil returns and small country equity returns when the small country is constrained by the ZLB. We assume the world size is 1, with the size of the large country equal to 0.9 and the size of the small country equal to 0.1. In Panel (a), the returns are expressed in the small country's currency, while in Panel (b), the returns are expressed in the large country's currency.

Figure 9: Oil-Equity Rolling Correlation, Japan



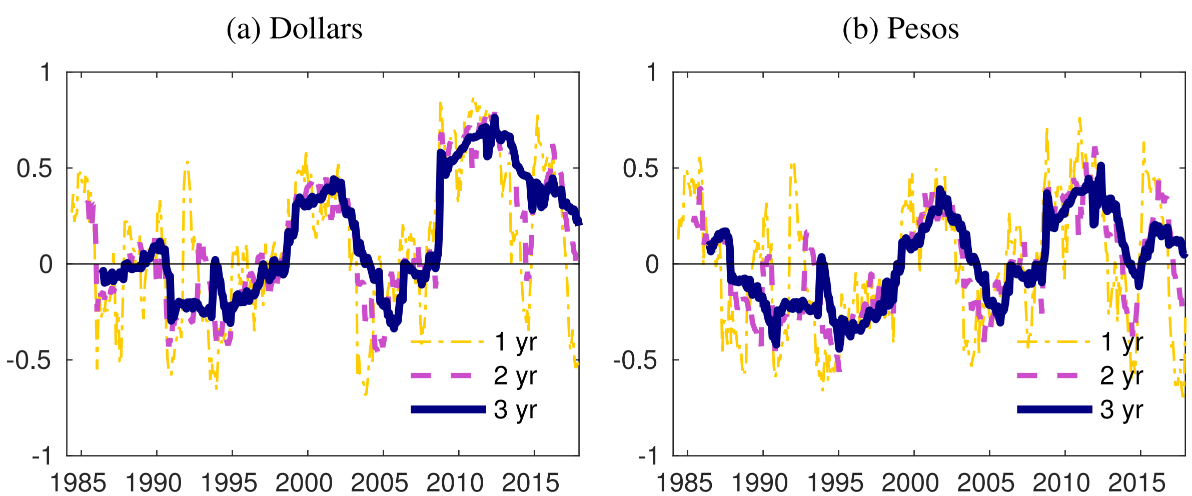
Note: Monthly returns computed as log differences in the TOPIX equity index on last trading day of each period. Oil returns computed as log differences in oil price on last trading day of each period. Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates from the H.10 release from the Federal Reserve.

Figure 10: Oil-Equity Correlation in Small Country, with Large Country ZLB



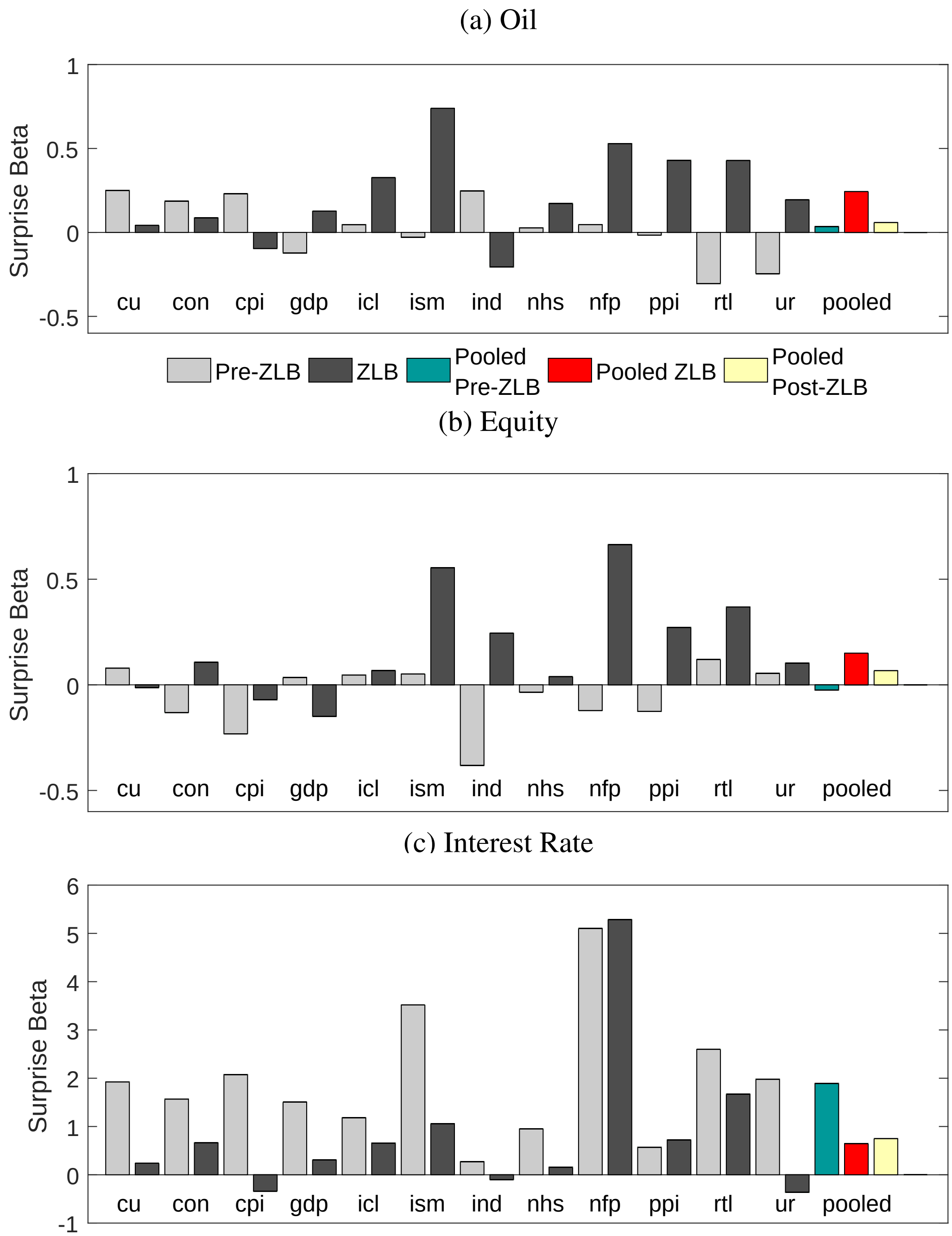
Note: The figures show the local correlation between oil returns and small country equity returns when the large country is constrained by the ZLB. We assume the world size is 1, with the size of the large country equal to 0.9 and the size of the small country equal to 0.1. In Panel (a), the returns are expressed in the large country's currency, while in Panel (b), the returns are expressed in the small country's currency.

Figure 11: Oil-Equity Rolling Correlation, Mexico



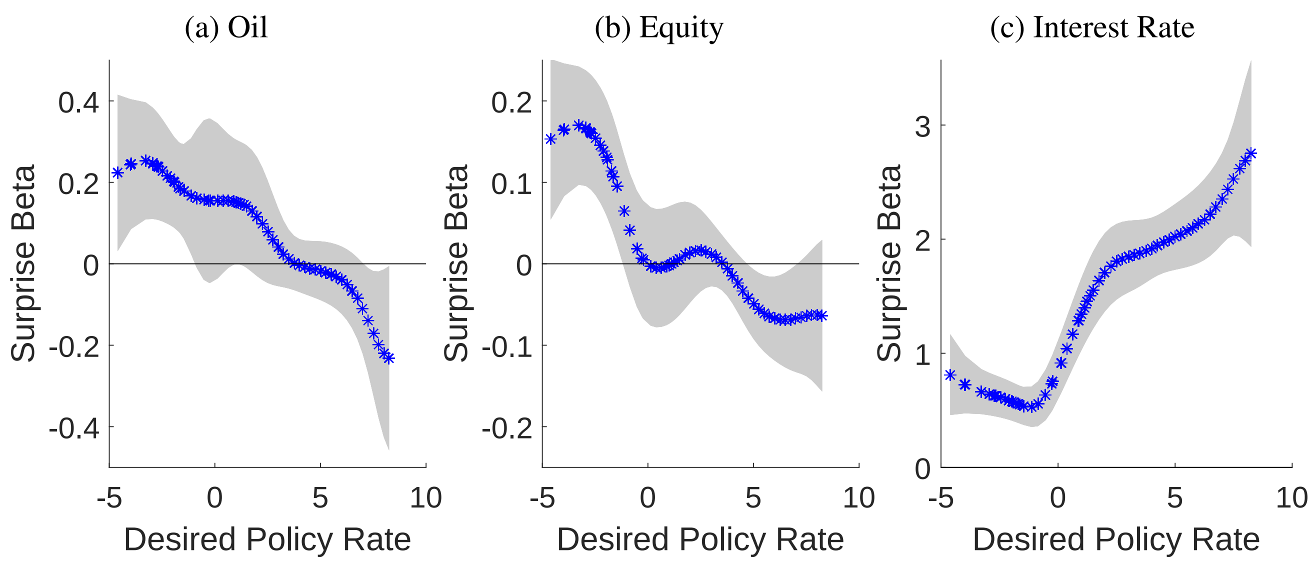
Note: Monthly returns computed as log differences in the BOLSA equity index on last trading day of each period. Oil returns computed as log differences in oil price on last trading day of each period. Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates obtained from the Bank of Mexico.

Figure 12: Responsiveness to Surprises in the ZLB Period



Note: The bars represent the estimated coefficients from the pre-ZLB, and ZLB, and post-ZLB eras for the regression $Y_t = \alpha + \boldsymbol{\beta} \boldsymbol{s_t} + \varepsilon_t$, where $\boldsymbol{s_t}$ is the vector of standardized and demeaned news on day t. The final three bars in each panel represent the β's from the regression using pooled surprises, $Y_t = \alpha + \beta S_t + \varepsilon_t$, where $S_t = \sum_{j=1}^{12} s_{jt}$. The pre-ZLB era regressions cover January 1990 to March 2009; the ZLB era regressions cover April 2009 to December 2014 and July 2015 to December 2015; and the post-ZLB era regressions cover January 2016 to December 2017. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that positive surprises represent a stronger-than-expected economy. See Section 4.2 for more detail and Table 3 for the associated announcement codes and regression statistics.

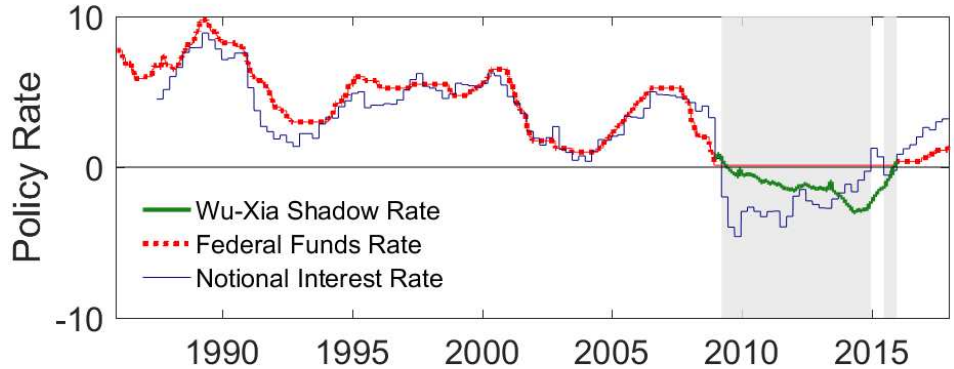
Figure 13: Surprise Betas and the Desired Policy Rate



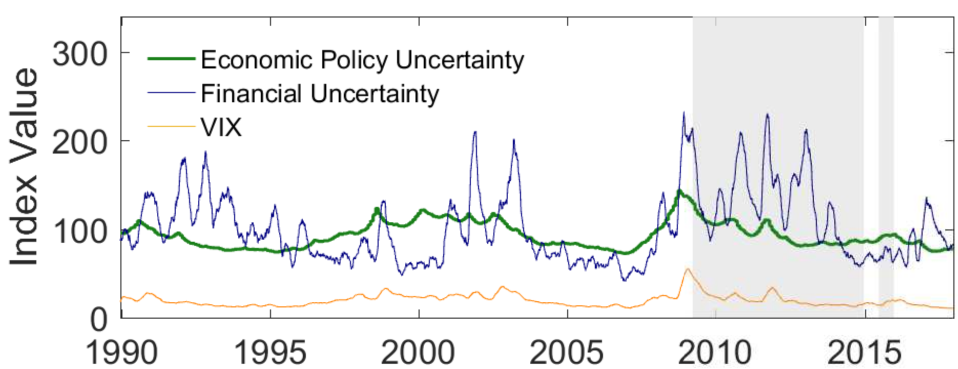
Note: For each of our dependent variables, $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$, we estimate the regression $Y_t = \alpha(\tilde{R}_t) + \beta(\tilde{R}_t)S_t + \varepsilon_t$, using the desired policy rate as the controlling variable in the kernel regression. The shaded region represents a 90 percent confidence interval for the estimated $\beta(\tilde{R}_t)$ based on the wild bootstrap.

Figure 14: Alternative Theories

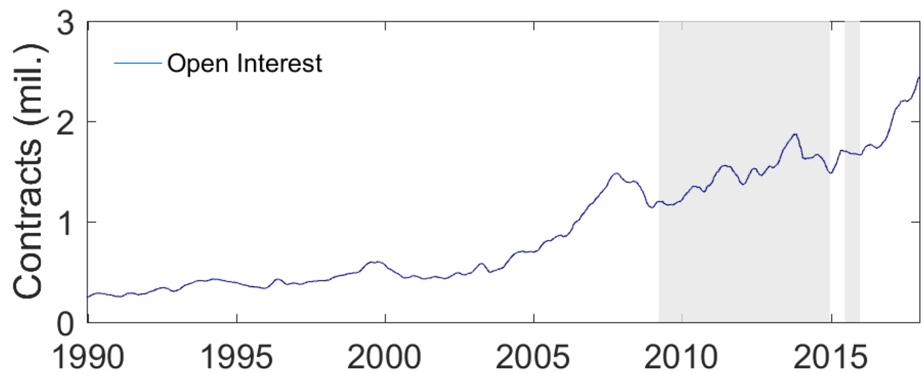
(a) Policy Rates



(b) Uncertainty



(c) Financialization



Panel (a): Policy rate measures include the Wu and Xia (2016) shadow rate, the notional interest rate implied by the modified Taylor rule in Equation 1 ((Bernanke, 2015)), and the federal funds rate. Panel (b): Uncertainty measures include the 90-day moving average of the daily series for economic policy uncertainty from Baker et al. (2015), the 90-day horizon measure of financial uncertainty from Jurado et al. (2015), and the 90-day moving average of the VIX. The economic policy uncertainty measure is multiplied by 100 in this panel for ease of comparison to the other two series. Panel (c): The financialization measure is the 90-day moving average of open interest across all maturities of WTI futures contracts, expressed in millions of contracts.

Table 1: Summary Statistics

Variable	Obs.	Start Date	Mean	St. Dev.	Min.	Max.
Panel A: Primary Variables of Interest						
Oil returns (WTI nearby futures)	8584	1983-Apr-06	0.01	2.40	-40.05	22.80
Equity returns	8584	1983-Apr-06	0.04	1.09	-19.13	9.89
Δ Interest rate (2 year)	8584	1983-Apr-06	-0.09	6.15	-84.00	38.00
Panel B: Alternative Measures						
WTI physical spot returns	7910	1986-Jan-03	0.01	2.56	-40.64	21.70
Brent physical spot returns	8388	1983-May-17	0.01	2.32	-40.71	27.82
WTI far futures returns	8584	1983-Apr-06	0.01	1.37	-10.35	10.80
Metals returns	8584	1983-Apr-06	0.02	0.86	-10.29	9.40
Δ Interest rate (1 year)	8584	1983-Apr-06	-0.09	5.45	-83.00	52.00
Δ Interest rate (10 year)	8584	1983-Apr-06	-0.09	6.39	-75.00	39.00
S&P 500 excl. energy	4946	1998-Jan-02	0.02	1.22	-9.11	10.10
Panel C: International Equity Returns						
TOPIX (in dollars)	7920	1983-Apr-06	0.02	1.40	-17.26	10.53
TOPIX (in yen)	7926	1983-Apr-06	0.01	1.29	-15.81	12.86
BOLSA (in dollars)	8100	1983-Apr-06	0.07	2.10	-31.23	23.29
BOLSA (in pesos)	8106	1983-Apr-06	0.13	1.75	-20.24	23.58
Panel D: Controlling Variables						
Desired policy rate ($\tilde{R}$)	8187	1985-Dec-19	3.10	3.45	-4.60	9.81
Wu–Xia shadow rate (SR)	8250	1985-Dec-19	3.24	3.18	-2.99	9.81
Economic policy uncert. (EPU)	9914	1980-Jan-01	0.94	0.14	0.72	1.43
Financial uncertainty (FNU)	12051	1985-Jan-01	101.50	38.50	40.95	232.72
VIX	7304	1990-Jan-02	19.45	6.97	10.37	55.03
Open interest (OI)	8338	1986-Jan-15	0.81	0.59	0.07	2.44

Notes: The price of oil is the closing value, in dollars per barrel, of the front-month futures contract for West Texas Intermediate (WTI) crude oil for delivery in Cushing, Oklahoma obtained from NYMEX. Equity returns are obtained from the Fama-French value-weighted daily return on all NYSE, AMEX, and NASDAQ stocks (which include dividends), and are converted to price levels.

The WTI physical spot price is for WTI crude oil for delivery (freight on board) in Cushing, Oklahoma, as reported by the U.S. Energy Information Administration. The Brent physical spot price is for Brent Forties Oseberg crude oil, obtained from Bloomberg. The WTI far futures price is the price of the furthest available December contract for WTI crude oil, and is obtained from NYMEX. The metals price is the Commodities Research Bureau Metals Index obtained from Bloomberg. The S&P 500 Ex-Energy and TOPIX are also obtained from Bloomberg, while the BOLSA is obtained from Haver Analytics. The yen exchange rate is obtained from the Federal Reserve H.10. release, and the peso exchange rate is obtained from the Bank of Mexico.

The desired policy rate is defined as the notional rate when it is negative, and the observed federal funds rate otherwise. See Equation 1 for details on construction of the notional interest rate. For the remaining controlling variables, we use the shadow rate constructed in Wu and Xia (2016), the 90-day moving average of the daily series for economic policy uncertainty from Baker et al. (2015), the 90-day horizon measure of financial uncertainty from Jurado et al. (2015), the 90-day moving averages of the VIX obtained from Bloomberg, and the 90-day moving average of open interest measured in millions of contracts obtained from the CFTC.

Table 2: Chow Test Results

	Obs.	Break Date	Full Sample β	Full Sample t-stat	Pre-Break β	Pre-Break t-stat	Post-Break β	Post-Break t-stat
Panel A: Regressions on the Full Index of Equity Returns								
WTI nearby futures returns	8584	2008-Sep-22	0.19	(8.15)	-0.16	(-5.33)	0.79	(21.34)
WTI physical spot returns	7910	2008-Sep-22	0.21	(8.19)	-0.15	(-4.72)	0.80	(21.53)
Brent physical spot returns	8388	2008-Sep-22	0.25	(10.87)	-0.11	(-3.68)	0.85	(26.49)
WTI far futures returns	8584	2008-Sep-22	0.10	(7.55)	-0.06	(-3.21)	0.37	(20.62)
Metals returns	8584	2008-Sep-30	0.10	(11.91)	0.02	(2.14)	0.24	(15.14)
Panel B: Regressions of Oil on Various Equity Sectors and Indexes								
Consumer nondurables	8584	2008-Sep-22	0.02	(0.90)	-0.29	(-9.31)	0.85	(16.71)
Consumer durables	8584	2008-Sep-22	0.11	(6.38)	-0.17	(-7.50)	0.51	(19.29)
Manufacturing	8584	2008-Sep-22	0.22	(10.24)	-0.14	(-4.97)	0.70	(22.77)
Energy	8584	2008-Nov-21	0.62	(37.38)	0.47	(22.99)	0.97	(37.88)
Chemicals	8584	2008-Sep-22	0.13	(5.33)	-0.21	(-7.49)	0.84	(21.21)
Business equipment	8584	2008-Sep-22	0.08	(4.57)	-0.07	(-3.91)	0.66	(18.26)
Telecommunications	8584	2008-Sep-22	0.10	(4.53)	-0.17	(-6.66)	0.68	(18.51)
Utilities	8584	2008-Sep-22	0.28	(10.36)	-0.03	(-0.97)	0.75	(18.04)
Shops	8584	2008-Sep-22	-0.03	(-1.33)	-0.27	(-10.54)	0.67	(15.14)
Health care	8584	2008-Sep-22	0.00	(0.22)	-0.20	(-7.97)	0.61	(13.79)
Finance	8584	2008-Sep-22	0.10	(5.43)	-0.21	(-8.60)	0.42	(16.67)
Other	8584	2008-Sep-22	0.15	(6.85)	-0.18	(-6.50)	0.68	(20.04)
S&P 500 excl. energy	4946	2008-Sep-22	0.26	(9.27)	-0.18	(-4.58)	0.74	(18.99)
TOPIX Equity Index	7920	1991-Aug-21	0.02	(1.02)	-0.19	(-4.23)	0.08	(3.99)
BOLSA Equity Index	5655	2008-Aug-29	0.22	(13.47)	0.03	(1.62)	0.64	(22.39)

Notes: Panel A reports equity betas from the regression $Y_t = \alpha + \beta Equity_t + \varepsilon_t$. Panel B reports equity sector betas from the regression $Oil_t = \alpha + \beta EquitySector_t + \varepsilon_t$. Full sample observations, β, and t-statistics are reported. The break date, pre-break, and post-break results are estimated after applying the standard Chow test to determine the break date which minimizes the sum of squared errors for regressions run on the pre- and post-break samples. All of these break dates were found to be statistically significant at the 1% level ($F_{crit} = 7.8$) when using the standard Andrews supremum-Wald critical value based upon 15% trimming of the sample as in Stock and Watson (2003).

Table 3: Oil, Equity, and Interest Rate Surprise Beta Estimates

	Oil				Equity				Interest rate			
	pre-ZLB era		ZLB era		pre-ZLB era		ZLB era		pre-ZLB era		ZLB era	
	β	t-stat	β	t-stat	β	t-stat	β	t-stat	β	t-stat	β	t-stat
Capacity utilization (cu)	0.25	(1.64)	0.04	(0.19)	0.08	(1.03)	-0.01	(-0.11)	1.92	(4.49)	0.24	(0.70)
Consumer confidence (con)	0.19	(1.14)	0.09	(0.41)	-0.13	(-1.59)	0.11	(0.90)	1.57	(3.43)	0.66	(2.01)
Core CPI (cpi)	0.23	(1.53)	-0.10	(-0.34)	-0.23	(-3.06)	-0.07	(-0.44)	2.08	(4.94)	-0.34	(-0.76)
GDP advance (gdp)	-0.12	(-0.49)	0.13	(0.27)	0.04	(0.28)	-0.15	(-0.55)	1.51	(2.13)	0.31	(0.41)
Initial claims (clm)	0.05	(0.62)	0.33	(2.73)	0.05	(1.24)	0.07	(0.99)	1.18	(5.64)	0.66	(3.49)
ISM manufacturing (ism)	-0.03	(-0.19)	0.74	(3.11)	0.05	(0.67)	0.55	(4.08)	3.52	(8.31)	1.06	(2.83)
Leading indicators (ind)	0.25	(0.96)	-0.21	(-0.81)	-0.38	(-2.94)	0.24	(1.67)	0.27	(0.38)	-0.10	(-0.25)
New home sales (nhs)	0.03	(0.20)	0.17	(0.51)	-0.03	(-0.50)	0.04	(0.20)	0.95	(2.45)	0.16	(0.30)
Nonfarm payrolls (nfp)	0.05	(0.31)	0.53	(1.51)	-0.12	(-1.61)	0.66	(3.31)	5.10	(12.14)	5.28	(9.57)
Core PPI (ppi)	-0.02	(-0.11)	0.43	(1.51)	-0.13	(-1.74)	0.27	(1.68)	0.57	(1.42)	0.72	(1.61)
Retail sales ex. autos (rtl)	-0.31	(-1.80)	0.43	(1.15)	0.12	(1.40)	0.37	(1.74)	2.60	(5.47)	1.67	(2.85)
Unemployment rate (ur)	-0.25	(-1.38)	0.19	(0.77)	0.05	(0.61)	0.10	(0.71)	1.98	(3.99)	-0.36	(-0.91)
R-squared		0.01		0.03		0.01		0.05		0.15		0.15

Notes: The table reports the individual elements β_j of the vector $\boldsymbol{\beta}$, from the regression $Y_t = \alpha + \boldsymbol{\beta} \boldsymbol{s_t} + \varepsilon_t$, where $\boldsymbol{s_t}$ is the vector of standardized and demeaned news on day t. The parameters β_j measure the responses of each dependent variable to a one standard deviation news surprise. 2330 observations are included in the pre-ZLB era regressions (January 1990 to March 2009), and 638 observations are included in the ZLB era regressions (April 2009 to December 2014 and July 2015 to December 2015). The ZLB era is defined as the period during which the Taylor rule notional rate is negative. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that positive surprises represent a stronger-than-expected economy. See Section 4.2 for more detail.

Table 4: Estimated Average Response to Macroeconomic News

Dependent Variable Y_t		Pre-ZLB	ZLB	Post-ZLB
Oil	β	0.04	0.24	0.06
	t-stat	0.87	3.59	0.32
Equity	β	-0.03	0.15	0.07
	t-stat	-1.23	3.85	1.24
Interest Rate (2 years)	β	1.89	0.65	0.75
	t-stat	16.46	5.75	2.96
Interest Rate (1 year)	β	1.47	0.21	0.37
	t-stat	15.52	4.03	2.28
Interest Rate (10 years)	β	1.48	1.20	0.74
	t-stat	13.19	5.78	2.42
Observations		2330	763	230

Notes: The table reports the value of β from the regression using pooled surprises, $Y_t = \alpha + \beta S_t + \varepsilon_t$, where S_t is the average of standardized and demeaned news on day t. β measures the response of each dependent variable to a one standard deviation news surprise. The ZLB era is defined as the period during which the Taylor-rule implied notional rate is negative. See Section 4.2, and Figure 12, which plots these values for Oil, Equity, and Interest Rate.

Table 5: Do the ZLB measures improve model fit?

		Dependent Variable (Y_t)		
Null Hypothesis		Oil	Equity	Int. Rate
a.	$H_0 : \beta = \beta(\tilde{R}_k)$	0.07	0.02	0.00
	$H_0 : \beta = \beta(SR_k)$	0.01	0.02	0.00
b.	$H_0 : \beta(SR_k) = \beta(SR_k, \tilde{R}_k)$	0.90	0.47	0.23
	$H_0 : \beta(\tilde{R}_k) = \beta(\tilde{R}_k, SR_k)$	0.57	0.77	0.60
c.	$H_0 : \beta(FNU_k) = \beta(FNU_k, \tilde{R}_k)$	0.22	0.13	0.00
	$H_0 : \beta(EPU_k) = \beta(EPU_k, \tilde{R}_k)$	0.16	0.07	0.00
	$H_0 : \beta(VIX_k) = \beta(VIX_k, \tilde{R}_k)$	0.11	0.18	0.00
	$H_0 : \beta(OI_k) = \beta(OI_k, \tilde{R}_k)$	0.09	0.04	0.00

Notes: The table reports for each dependent variable $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$ the p-values for the test of each null hypothesis listed. Rejection of the null hypothesis provides evidence that the variable being tested is able to improve the fit of the kernel regression $Y_t = \alpha(.) + \beta(.)S_t + \varepsilon_t$, relative to the alternative listed. Panel (a) tests models in which the sensitivity to news surprises does not vary against models in which the sensitivity varies with a single policy rate. Panel (b) tests the null hypothesis that a model including the two rates is equivalent to a model including just one of the two rates. Panel (c) tests the null hypothesis that a model including the desired policy rate and one of the alternative variables is equivalent to a model including just the alternative controlling variable. See Table B.1 for controlling variable summary statistics, and Sections 4.3 through 4.5 for more detail.

Table 6: Do the alternative controlling variables improve model fit?

Null Hypothesis		Dependent Variable (Y_t) Oil	Equity	Int. Rate
a.	$H_0 : \beta = \beta(FNU_k)$	0.11	0.43	0.18
	$H_0 : \beta = \beta(EPU_k)$	0.41	0.36	0.00
	$H_0 : \beta = \beta(VIX_k)$	0.89	0.87	0.00
	$H_0 : \beta = \beta(OI_k)$	0.13	0.01	0.00
b.	$H_0 : \beta(\tilde{R}_k) = \beta(\tilde{R}_k, FNU_k)$	0.29	0.45	0.04
	$H_0 : \beta(\tilde{R}_k) = \beta(\tilde{R}_k, EPU_k)$	0.41	0.31	0.04
	$H_0 : \beta(\tilde{R}_k) = \beta(\tilde{R}_k, VIX_k)$	0.81	0.83	0.00
	$H_0 : \beta(\tilde{R}_k) = \beta(\tilde{R}_k, OI_k)$	0.54	0.40	0.24

Notes: The table reports for each dependent variable $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$ the p-values associated with the test of each null hypothesis listed. Rejection of the null hypothesis provides evidence that the controlling variable being tested is able to improve the fit of the kernel regression $Y_t = \alpha(.) + \beta(.)S_t + \varepsilon_t$, relative to the alternative listed. Panel (a) tests models in which the sensitivity to news surprises does not vary against models in which the sensitivity varies with one of the alternative controlling variables. Panel (b) tests whether a model including just the desired policy rate is equivalent to a model including one of the alternative controlling variables along with the desired policy rate. See Table B.1 for controlling variable definitions and summary statistics, and Sections 4.3 through 4.5 for more detail.

Table 7: Structural VAR Decomposition of the Correlation between Oil and Equity Returns

		Contribution of			
	Correlation	Oil Supply	Agg. Demand	Oil Resid.	Equity Resid.
	$\rho_{pe}(h)$	$\frac{\sigma_{pe,1}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,2}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,3}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,4}(h)}{\sigma_p(h)\sigma_e(h)}$
Jan. 1974 – Mar. 2009	-0.100	0.000	0.020	-0.114	-0.006
Apr. 2009 – Dec. 2017	0.327	0.043	0.013	0.240	0.031

Notes: The table reports the structural decomposition of the correlation between monthly oil and equity returns based on the vector autoregression (VAR) described in Section 5. The decomposition is based on Equation 25. The VAR is estimated independently for each reported sample, and the value of $h = 1000$.

For Online Publication

Oil, Equities, and the Zero Lower Bound

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August 16, 2018

Abstract

The following appendixes are supplementary material for our paper "Oil, Equities, and the Zero Lower Bound." Appendix B contains additional tables and figures from our empirical work. Appendix C contains details about our benchmark New Keynesian model with oil. Appendix D contains details about our two-country New Keynesian model with oil.

*The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. We thank Martin Bodenstein, Craig Burnside, François Gourio, Luca Guerrieri, Lee Smith, Johannes Wieland, and Jing Cynthia Wu, for helpful comments and discussion. We thank Anastacia Dialynas for her contributions to the initial empirical investigations. Comments and suggestions can be directed to robert.j.vigfusson@frb.gov.

B Additional figures and tables

Figure B.1: Oil and Equity Correlation - Robustness

The rolling window correlations between oil and equity returns are presented here. The four panels illustrate the correlations for returns calculated over daily, weekly, monthly, and quarterly frequencies. The lines in each panel show the rolling windows of various lengths (1 month up to 3 years).

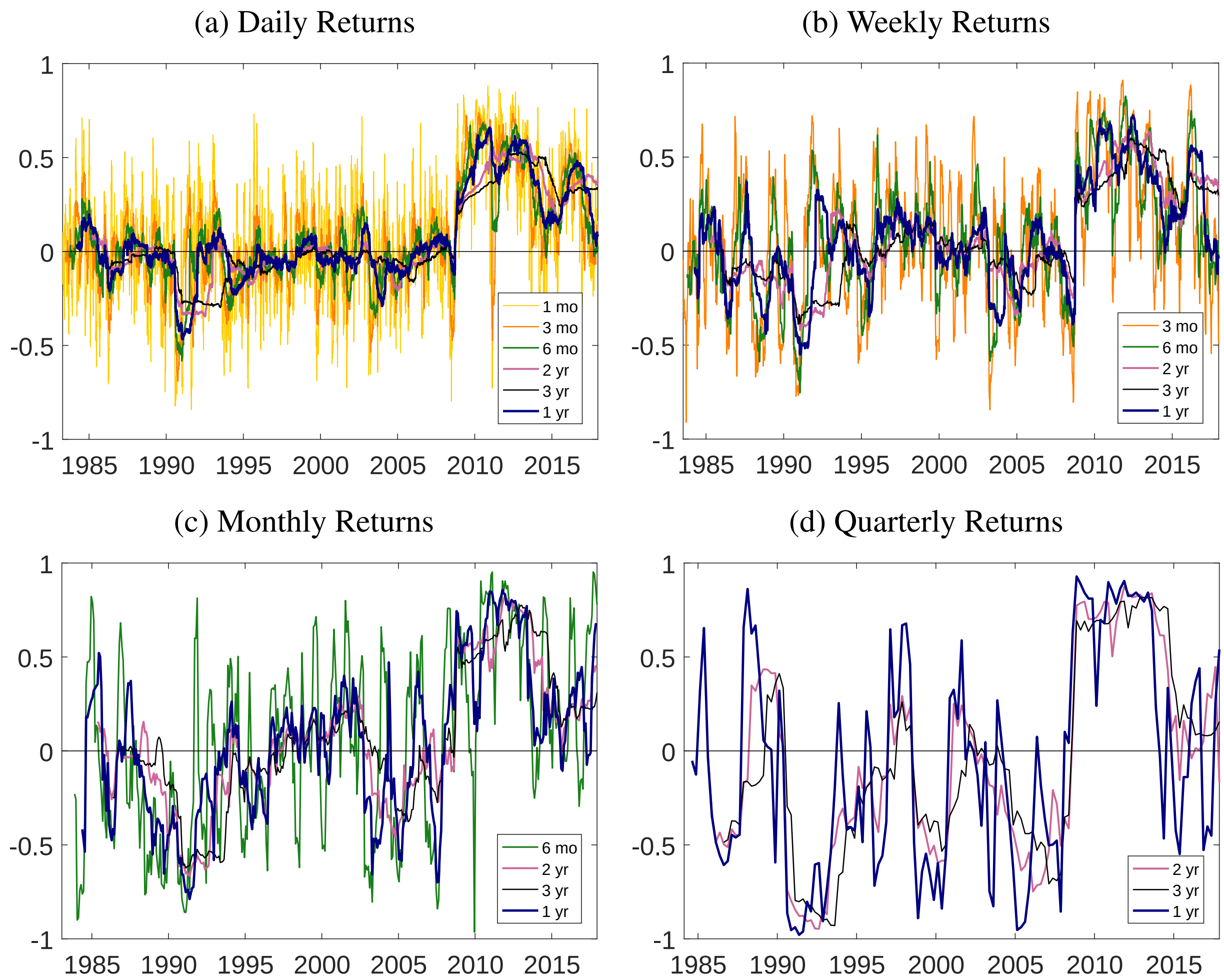
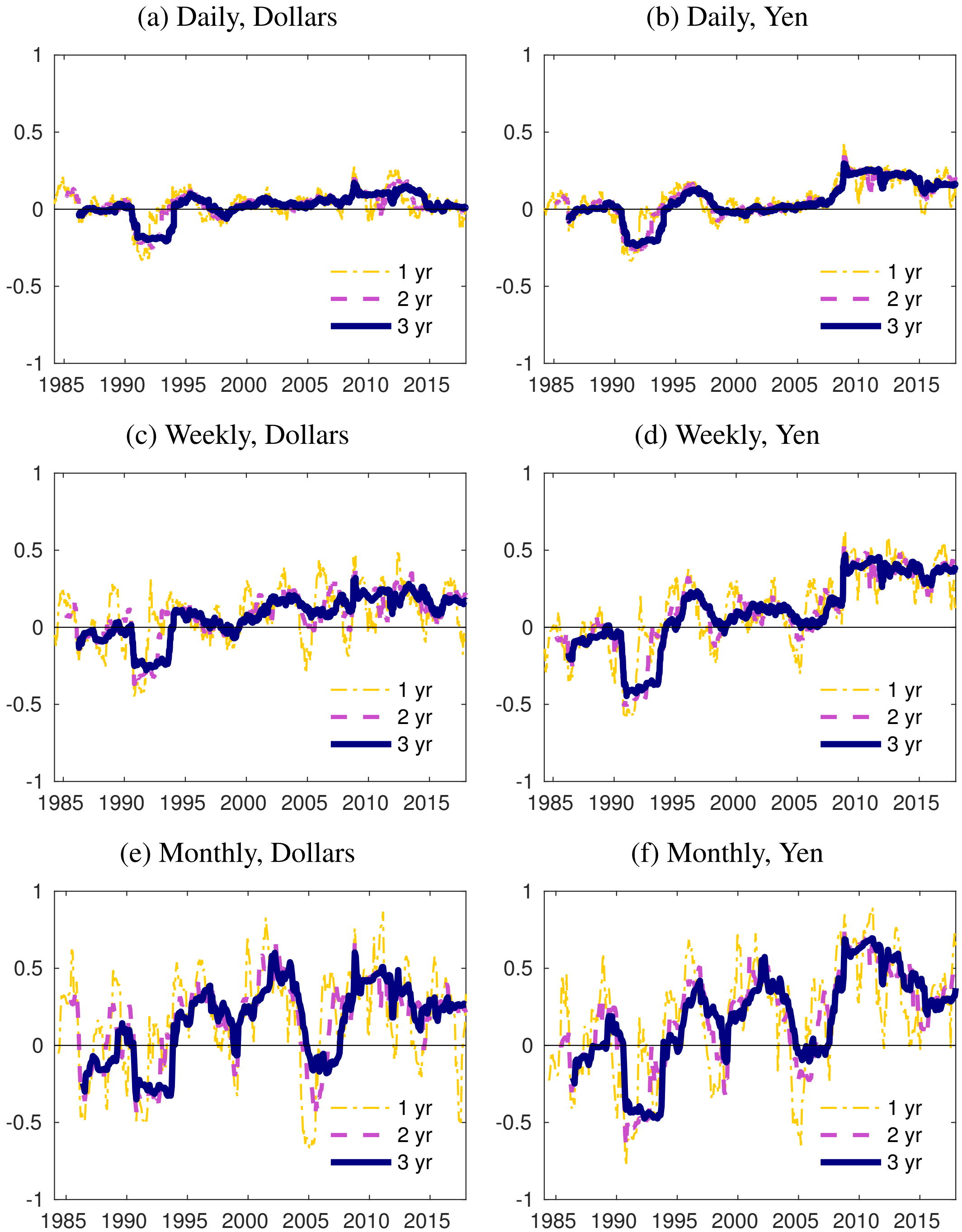
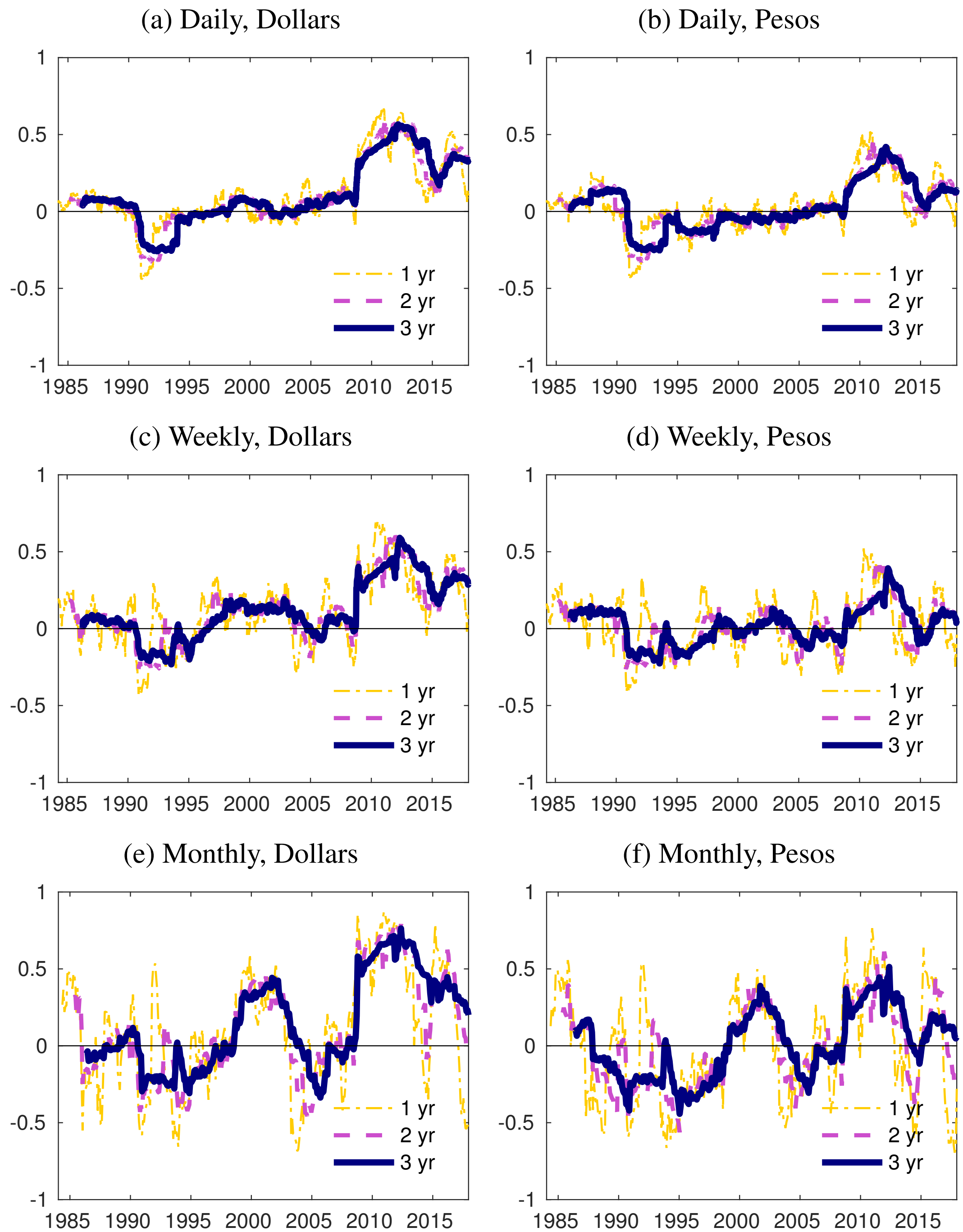


Figure B.2: Oil-Equity Rolling Correlation, Japan



Note: Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates from the H.10 release from the Federal Reserve. Period returns computed as log changes in equity index on last trading day of each period. Oil returns computed as log changes in oil price on last trading day of each period.

Figure B.3: Oil-Equity Rolling Correlation, Mexico



Note: Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates from the Bank of Mexico. Period returns computed as log changes in equity index on last trading day of each period. Oil returns computed as log changes in oil price on last trading day of each period.

Table B.1: Summary Statistics: Equity Sector Returns and Macroeconomic News Surprises

Variable	Obs.	Start Date	Mean	St. Dev.	Min.	Max.
Panel A: Equity Sector Returns						
Consumer nondurables	8584	1983-Apr-06	0.05	0.95	-18.67	8.83
Consumer durables	8584	1983-Apr-06	0.03	1.46	-20.27	9.12
Manufacturing	8584	1983-Apr-06	0.05	1.21	-22.61	9.55
Energy	8584	1983-Apr-06	0.04	1.46	-21.60	17.24
Chemicals	8584	1983-Apr-06	0.05	1.10	-21.33	9.40
Business equipment	8584	1983-Apr-06	0.04	1.54	-22.43	14.95
Telecommunications	8584	1983-Apr-06	0.04	1.23	-18.26	13.21
Utilities	8584	1983-Apr-06	0.04	0.97	-13.77	12.67
Shops	8584	1983-Apr-06	0.05	1.15	-18.32	10.43
Healthcare	8584	1983-Apr-06	0.05	1.15	-19.71	10.29
Finance	8584	1983-Apr-06	0.04	1.42	-16.08	15.62
Other	8584	1983-Apr-06	0.03	1.18	-18.13	9.43
Panel B: Macroeconomic News Surprises						
Capacity utilization (cu)	357	1988-Apr-18	-0.01	0.35	-1.57	1.40
Consumer confidence (con)	316	1991-Jul-30	0.25	5.12	-14.00	13.30
Core CPI (cpi)	341	1989-Aug-18	-0.01	0.11	-0.34	0.40
GDP advance (gdp)	123	1987-Apr-23	0.08	0.74	-1.68	1.80
Initial claims (clm)	1303	1991-Jul-18	0.05	18.08	-85.00	94.00
ISM manufacturing (ism)	333	1990-Feb-01	0.03	1.97	-6.30	7.40
Leading indicators (ind)	455	1980-Feb-29	0.02	0.31	-1.80	2.00
New home sales (nhs)	353	1988-Mar-29	5.43	56.77	-166.00	249.00
Nonfarm payrolls (nfp)	395	1985-Feb-01	-8.29	100.29	-328.00	408.50
Core PPI (ppi)	337	1989-Aug-11	-0.02	0.24	-1.20	1.07
Retail sales ex. autos (rtl)	454	1980-Feb-13	-0.03	0.66	-2.40	5.13
Unemployment rate (ur)	453	1980-Feb-07	0.04	0.16	-0.60	0.60

Notes: In Panel (a), the 12 industry-specific equity returns series are obtained from the Fama-French data library, and are converted to levels. To calculate returns, we drop days with missing values for oil, metals, interest rates, or equities, and then calculate “daily” returns as the 100 times the log difference of these consecutive closing prices. For Panel (b) only, news surprises are defined as the difference between the announced realization of the macroeconomic aggregates and the survey expectations. Prior to use in regression analysis, each surprise is divided by the full sample standard deviation reported above. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements throughout the paper, so that positive surprises represent a stronger-than-expected economy.

Table B.2: Structural VAR Decomposition of the Correlation between Oil and Equity Returns

			Contribution of			
		Corr.	Oil Supply	Agg. Demand	Oil Resid.	Equity Resid.
	Lags	$\rho_{pe}(h)$	$\frac{\sigma_{pe,1}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,2}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,3}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,4}(h)}{\sigma_p(h)\sigma_e(h)}$
Oil Price in Differences						
Jan. 1974 – Mar. 2009	**12**	**-0.100**	**0.000**	**0.020**	**-0.114**	**-0.006**
Jan. 1974 – Mar. 2009	24	-0.125	0.001	0.011	-0.129	-0.008
Jan. 1974 – Dec. 2006	12	-0.172	-0.003	0.005	-0.165	-0.010
Jan. 1974 – Dec. 2006	24	-0.176	0.001	0.008	-0.178	-0.008
Apr. 2009 – Dec. 2017	**12**	**0.327**	**0.043**	**0.013**	**0.240**	**0.031**
Oil Price in Levels						
Jan. 1974 – Mar. 2009	12	-0.096	0.004	0.014	-0.111	-0.003
Jan. 1974 – Mar. 2009	24	-0.138	-0.000	0.001	-0.132	-0.007
Jan. 1974 – Dec. 2006	12	-0.172	-0.000	0.002	-0.166	-0.008
Jan. 1974 – Dec. 2006	24	-0.183	-0.001	0.003	-0.179	-0.006
Apr. 2009 – Dec. 2017	12	0.326	0.046	0.013	0.242	0.025

Notes: The table reports the structural decomposition of the correlation between monthly oil and equity returns based on the monthly VAR described in Section 5. The decomposition is based on Equation 25. The VAR is estimated independently for each reported sample. When the VAR is estimated using the log-level of the oil price (instead of the log difference), we calculate the correlation and decompositions for oil and equity returns using the implied moving average representation for oil returns. The value of $h = 1000$. Bolded rows denote our benchmark results, as reported in the main text.

C Benchmark New Keynesian model

In this appendix, we describe our benchmark New Keynesian model. We use a medium-scale New Keynesian model and add endogenous oil demand along with exogenous oil supply along the lines of Bodenstein et al. (2013).

C.1 Household

The representative household maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C_{t+j} - h\bar{C}_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \log\left(\eta_t\right) V\left(\frac{B_t}{P_{C,t}}\right)\right) \tag{C.1}$$

where C_t is consumption, $\bar{C}_t$ is average aggregate consumption, L_t is hours worked, B_t is nominal bond holdings, and $P_{C,t}$ is the price of the consumption good. The stochastic variable η_t is a preference shifter than captures increased desire to hold safe nominal assets. The budget constraint is

$$B_t + P_{C,t}C_t + P_{Y,t}I_t = \left(1 + R_{t-1}\right)^{1/4} B_{t-1} + R_{K,t}K_t + W_tL_t + T_t \tag{C.2}$$

where $P_{Y,t}$ is the price of non-oil output, R_t is the net annual nominal interest rate, W_t is the wage rate, $R_{K,t}$ is the rental rate on capital, K_t, I_t is investment, and T_t are lump-sum profits and taxes. The capital accumulation equation is

$$K_{t+1} = I_t \left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + \left(1 - \delta\right) K_t. \tag{C.3}$$

The definition of consumption is

$$C_t = \left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}}. \tag{C.4}$$

The household creates the consumption good to minimize the cost of consumption. That is, the household solves

$$\min_{Y_{C,t},O_{C,t}} P_{Y,t}Y_{C,t} + P_{O,t}O_{C,t} \tag{C.5}$$

subject to the constraint that

$$\left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}} \geq C_t. \tag{C.6}$$

Here $Y_{C,t}$ is non-oil output used for consumption and $O_{C,t}$ is oil that is consumed by the household. Then the first-order conditions are

$$Y_{C,t} = \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} \omega_C C_t, \tag{C.7}$$

$$O_{C,t} = \left(\frac{P_{O,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} C_t\left(1-\omega_C\right)\mu_{C,t}^{\frac{\rho_C}{\rho_C-1}}. \tag{C.8}$$

The ideal price index for final consumption is given by

$$P_{C,t} = \left(\omega_C\left(P_{Y,t}\right)^{\frac{\rho_C}{\rho_C-1}} + \left(1-\omega_C\right)\left(P_{O,t}\mu_{C,t}\right)^{\frac{\rho_C}{\rho_C-1}}\right)^{\frac{\rho_C-1}{\rho_C}}. \tag{C.9}$$

The first-order conditions of the household are

$$\left(C_t - h\bar{C}_{t-1}\right)^{-\sigma} = \Lambda_t, \tag{C.10}$$

$$\Lambda_t W_t / P_{C,t} = \chi L_t^{\phi}, \tag{C.11}$$

$$\Lambda_t = \log\left(\eta_t\right) V'\left(\frac{B_t}{P_{C,t}}\right) + \beta\left(1+R_t\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}}, \tag{C.12}$$

$$\frac{P_{Y,t}}{P_{C,t}}\Lambda_t = Q_t\left[\left(1-\frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right)-\frac{I_t}{I_{t-1}}\phi_K\left(\frac{I_t}{I_{t-1}}-1\right)\right] + \beta E_t Q_{t+1}\phi_K\left(\frac{I_{t+1}}{I_t}-1\right)\frac{I_{t+1}^2}{I_t^2}, \tag{C.13}$$

$$Q_t = \beta E_t\left[Q_{t+1}\left(1-\delta\right)+\Lambda_{t+1}\frac{R_{K,t+1}}{P_{C,t+1}}\right]. \tag{C.14}$$

C.2 Goods aggregators

Perfectly competitive firms aggregate intermediate inputs into non-oil output, Y_t. Non-oil output is a composite of goods purchased from monopolists. We denote the quantity purchased form monopolist i by $X_t\left(i\right)$. The intermediate inputs are aggregated according to

$$Y_t = \left(\int_0^1 X_t\left(i\right)^{\frac{\nu-1}{\nu}}\,di\right)^{\frac{\nu}{\nu-1}}. \tag{C.15}$$

Demand curves are then of the form

$$X_t\left(i\right) = \left(\frac{P_{X,t}\left(i\right)}{P_{Y,t}}\right)^{-\nu} Y_t. \tag{C.16}$$

Here, $P_{X,t}\left(i\right)$ is the price of $X_t\left(i\right)$. Perfect competition implies the ideal price index for Y_t is given by

$$P_{Y,t} = \left(\int_0^1 P_{X,t}\left(i\right)^{1-\nu}\,di\right)^{\frac{1}{1-\nu}}. \tag{C.17}$$

C.3 Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists are only able to optimize their price with probability ξ in each period. If monopolist i can update its price, it chooses $\tilde{P}_{X,t}\left(i\right)$ to maximize

$$E_t\sum_{j=0}^{\infty}\Lambda_{t+j}\left(\frac{\tilde{P}_{X,t}\left(i\right)}{P_{C,t+j}}\tilde{X}_{t,j}\left(1+\tau_X\right)-MC_{t+j}\right)\left(\frac{\tilde{P}_{X,t}\left(i\right)}{P_{Y,t+j}}\tilde{X}_{t,j}\right)^{-\nu} Y_{t+j} \tag{C.18}$$

where

$$\tilde{X}_{t,j} = \begin{cases} 1 & j = 1 \\ \pi_{Y,t} \times \pi_{Y,t+1} \times \cdots \times \pi_{Y,t+j-1} & else \end{cases}. \tag{C.19}$$

Here, $\tilde{X}$ captures indexation to past price changes. The FOC with respect to $\tilde{P}_{X,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{\tilde{P}_{X,t}}{P_{C,t}} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{t,j} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \tilde{X}_{t,j} \right)^{-\nu} Y_{t+j} \right.$$
$$\left. - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \tilde{X}_{t,j} \right)^{-\nu} Y_{Y,t+j} \right] = 0. \tag{C.20}$$

Here we set $\tilde{P}_{X,t}(i) = \tilde{P}_{X,t}$ for all firms that can update their price because they all face the same problem. Then we have

$$F_{1,t} \tilde{p}_{X,t} = F_{2,t} \tag{C.21}$$

where $\tilde{p}_{X,t} \equiv \tilde{P}_{X,t}/P_{C,t}$ and $F_{1,t}$ and $F_{2,t}$ are defined as

$$F_{1,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{t,j}^{1-\nu} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \right)^{-\nu} Y_{t+j} \tag{C.22}$$

$$F_{2,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \tilde{X}_{t,j}^{-\nu} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \right)^{-\nu} Y_{t+j}. \tag{C.23}$$

The variables $F_{1,t}$ and $F_{2,t}$ can be expressed as

$$F_{1,t} = \Lambda_t Y_t + \beta\xi E_t \pi_{Y,t}^{1-\nu} \pi_{C,t+1}^{-1} \pi_{Y,t+1}^{\nu} F_{1,t+1}, \tag{C.24}$$

$$F_{2,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y_t + \beta\xi E_t \pi_{Y,t}^{-\nu} \pi_{Y,t+1}^{\nu} F_{2,t+1}, \tag{C.25}$$

where

$$\pi_{Y,t} \equiv P_{Y,t}/P_{Y,t-1} \tag{C.26}$$

and

$$\pi_{C,t} \equiv P_{C,t}/P_{C,t-1}. \tag{C.27}$$

The ideal price index for retail goods evolves according to

$$P_{Y,t} = \left((1-\xi)\,\tilde{P}_{X,t}^{1-\nu} + \xi \pi_{Y,t-1}^{1-\nu} P_{Y,t-1}^{1-\nu} \right)^{\frac{1}{1-\nu}} \tag{C.28}$$

so that

$$p_{Y,t} = \left((1-\xi)\,\tilde{p}_{X,t}^{1-\nu} + \xi \pi_{Y,t-1}^{1-\nu} \frac{p_{Y,t-1}^{1-\nu}}{\pi_{C,t}^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \tag{C.29}$$

C.4 Marginal cost

In this subsection we drop the i index from firm-specific quantities. The firm solves the following cost minimization problem

$$\min_{K_t, L_t} W_t L_t + R_{K,t} K_t + P_{O,t} O_t \tag{C.30}$$

subject to the constraint that

$$\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(\frac{O_{X,t}}{\mu_{X,t}} \right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \geq X_t. \tag{C.31}$$

Here

$$V_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{C.32}$$

and A_t is a stochastic process that represents aggregate technology. The first-order conditions are

$$\frac{R_{Kt}}{P_{Ct}} = MC_t\,(X_t)^{1-\rho_V} (\omega_V)^{1-\rho_V} V_t^{\rho_V - 1} \alpha A_t \left(\frac{L_t}{K_t} \right)^{1-\alpha}, \tag{C.33}$$

$$\frac{W_t}{P_{C,t}} = MC_t\,(X_t)^{1-\rho_V} (\omega_V)^{1-\rho_V} V_t^{\rho_V - 1} (1-\alpha)\, A_t \left(\frac{L_t}{K_t} \right)^{-\alpha}, \tag{C.34}$$

$$\frac{P_{O,t}}{P_{C,t}} = MC_t\,(X_t)^{1-\rho_V} (1-\omega_V)^{1-\rho_V} \left(\frac{O_{X,t}}{\mu_{X,t}} \right)^{\rho_V - 1} \frac{1}{\mu_{X,t}}. \tag{C.35}$$

C.5 Oil Market

There is an exogenous supply of oil, O_t. Oil-market clearing implies

$$O_{C,t} + O_{X,t} = O_t. \tag{C.36}$$

C.6 Goods market clearing

We assume that oil is paid for using non-oil output. So, goods market clearing implies

$$Y_{C,t} + G_t + I_t + \left(O_{C,t} + O_{X,t}\right)\frac{P_{O,t}}{P_{Y,t}} = Y_t. \tag{C.37}$$

C.7 Aggregation

Aggregating across firms yields

$$\int_0^1 \left(\frac{P_{X,t}(i)}{P_{Y,t}}\right)^{-\nu} Y_t di = \int_0^1 \left((\omega_V)^{1-\rho_V} V_t(i)^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} di \tag{C.38}$$

$$= \int_0^1 \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{V_t(i)\,\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} V_t(i)\, di \tag{C.39}$$

$$= \int_0^1 \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{V_t(i)\,\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} A_t\left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i)\, di. \tag{C.40}$$

From cost minimization, the ratios $\frac{O_{X,t}(i)}{V_t(i)\mu_{X,t}}$ and $\frac{K_t(i)}{L_t(i)}$ are common across firms. Then

$$\int_0^1 \left(\frac{P_{X,t}(i)}{P_{Y,t}}\right)^{-\nu} Y_t di = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{C.41}$$

so that

$$d_t^{-1} Y_t = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{C.42}$$

where the last equation follows without the (i)'s because all firms choose the same capital-to-labor ratio from the constant-returns-to-scale production technology. Here the dispersion term, d_t, represents the resource costs of price dispersion and can be written recursively as

$$d_t = (1-\xi)\left(\tilde{p}_{X,t}\right)^{-\nu} + \xi \pi_{Y,t-1}^{-\nu} \pi_{Y,t}^{\nu} d_{t-1}. \tag{C.43}$$

C.8 Government

The monetary authority follows a truncated Taylor rule. The desired policy rate, $\tilde{R}_t$ evolves according to

$$\left[1+\tilde{R}_t\right]^{1/4} = \left(\left[1+\tilde{R}_{t-1}\right]^{1/4}\right)^{\gamma} \left(\left([1+R]^{1/4}\right)\left(\frac{\pi_{Y,t}}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y_t^N}\right)^{\theta_Y}\right)^{1-\gamma} \text{ where } \theta_\pi > 1. \tag{C.44}$$

Here, R is the steady-state annualized net nominal interest rate, π is the target rate of inflation. The natural rate of output, Y_t^N is defined as the level of output that would prevail under flexible prices, given the entire history of shocks. The fiscal authority balances its budget with lump sum taxes so that $B_t = 0$. Government purchases, G_t, follows an $AR(1)$. To incorporate the zero lower bound,

$$R_t = \max\left\{0, \tilde{R}_t\right\}. \tag{C.45}$$

C.9 Equilibrium

A rational expectations equilibrium is a sequence of prices and quantities that have the property that the household and firm optimality conditions are satisfied, the goods market, labor market, and oil markets clear, and the nominal interest rate and government purchases evolve as specified. To solve for a rational expectations equilibrium, we solve for the following 24 endogenous objects: C_t, Λ_t, L_t, $w_t \equiv \frac{W_t}{P_{C,t}}$, Y_t, R_t, MC_t, $\pi_{C,t}$, K_t, I_t, Q_t, $r_{K,t} \equiv \frac{R_{K,t}}{P_{C,t}}$, $p_{Y,t} \equiv \frac{P_{Y,t}}{P_{C,t}}$, $\tilde{p}_{X,t}$, $F_{1,t}$, $F_{2,t}$, d_t, $\pi_{Y,t}$, V_t, $O_{X,t}$, $Y_{C,t}$, $O_{C,t}$, $p_{O,t} \equiv \frac{P_{O,t}}{P_{C,t}}$, $\tilde{R}_t$. To determine these variables, we require that the following 24 equations hold: (C.3), (C.7), (C.8), (C.9), (C.10), (C.11), (C.12), (C.13), (C.14), (C.37), (C.21), (C.24), (C.25), (C.26), (C.29), (C.32), (C.33), (C.34), (C.35), (C.36), (C.42), (C.43), (C.44), (C.45). The budget constraint of the household clears by Walras' law. We linearize the model around non-stochastic steady state. We incorporate the zero lower bound using the methodology of Guerrieri and Iacoviello (2015). We utilize the OccBin solver from Guerrieri and Iacoviello (2015), which interacts with Dynare (see Adjemian et al. (2011)).

C.10 Steady state

To determine steady state, we assume that target inflation is π. So, $\pi_C = \pi_Y = \pi$. The intertemporal Euler equation determines $(1+\tilde{R})^{1/4} = (1+R)^{1/4} = \pi\beta^{-1}$. We normalized $L = 1$. Firm optimality and symmetry of the equilibrium imply $\tilde{p}_X = 1$. Because of our indexation assumption, there is no price dispersion in steady state, so $d = 1$. We will normalize the price of oil to be $p_O = 1$ (we have to find O instead). As a result, $p_Y = 1$, meaning $Q = \Lambda$. Marginal cost is given by

$$MC = \frac{\nu - 1}{\nu}(1+\tau_X) = 1 \tag{C.46}$$

From pricing optimality

$$F_1 = F_2 = (1-\beta\xi)^{-1}\Lambda Y \tag{C.47}$$

The rental rate of capital is

$$r_K = \frac{1-\beta(1-\delta)}{\beta} \tag{C.48}$$

From our normalization of p_O

$$O_C = (1-\omega_C)\,C \tag{C.49}$$

and

$$Y_C = \omega_C C \tag{C.50}$$

The marginal utility of consumption gives

$$([1-h]\,C)^{-\sigma} = \Lambda \tag{C.51}$$

Note that

$$I = \delta K \tag{C.52}$$

and

$$Y = I + Y_C + G + O_C + O_X \tag{C.53}$$

From the definition of V we have

$$V = K^{\alpha} \tag{C.54}$$

This means

$$\delta K + C + G + O_X = \left((\omega_V)^{1-\rho_V}(K^{\alpha})^{\rho_V} + (1-\omega_V)^{1-\rho_V}(O_X)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{C.55}$$

We know that cost minimization implies

$$r_K = MC\left((\omega_V)^{1-\rho_V}V^{\rho_V} + (1-\omega_V)^{1-\rho_V}(O_X)^{\rho_V}\right)^{\frac{1-\rho_V}{\rho_V}}(\omega)^{1-\rho_V}V^{\rho_V-1}\alpha\left(\frac{1}{K}\right)^{1-\alpha} \tag{C.56}$$

$$w = MC\left((\omega_V)^{1-\rho_V}V^{\rho_V} + (1-\omega_V)^{1-\rho_V}(O_X)^{\rho_V}\right)^{\frac{1-\rho_V}{\rho_V}}\left(\frac{\omega_V}{V}\right)^{1-\rho_V}(1-\alpha)\left(\frac{1}{K}\right)^{-\alpha} \tag{C.57}$$

$$1 = MC\left((\omega_V)^{1-\rho_V}V^{\rho_V} + (1-\omega_Y)^{1-\rho_V}(O_X)^{\rho_V}\right)^{\frac{1-\rho_V}{\rho_V}}(1-\omega_V)^{1-\rho_V}(O_X)^{\rho_V-1} \tag{C.58}$$

Meaning

$$r_K = \left(\frac{\omega_V}{1-\omega_V}\right)^{1-\rho_V}\left(\frac{K^{\alpha}}{O_X}\right)^{\rho_V-1}\alpha\left(\frac{1}{K}\right)^{1-\alpha} \tag{C.59}$$

and

$$O_X = r_K^{-\frac{1}{\rho_V-1}}\left(\frac{\omega_V}{1-\omega_V}\right)^{-1}(K^{\alpha})\,\alpha^{\frac{1}{\rho_V-1}}\left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V-1}} \tag{C.60}$$

So,

$$\begin{aligned} r_K = &\left((\omega_V)^{1-\rho_V}(K^{\alpha})^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(r_K^{-\frac{1}{\rho_V-1}}\left(\frac{1-\omega_V}{\omega_V}\right)^{-1}(K^{\alpha})\,\alpha^{\frac{1}{\rho_V-1}}\left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V-1}}\right)^{\rho_V}\right)^{\frac{1-\rho_V}{\rho_V}} \\ &\times MC\,(1-\omega_V)^{1-\rho_V}(K^{\alpha})^{\rho_V-1}\,\alpha\left(\frac{1}{K}\right)^{1-\alpha} \end{aligned} \tag{C.61}$$

with r_K known and MC known, we can solve for K. With K we get O_X, V, and then w. With the intratemporal Euler equation, we get χ. We have Y from production technology, C from market clearing, O_C from $O_C = (1 - \omega_C)\, C$. With both O_X and O_C we have O. The rest follows easily.

D Two-country model

Here we extend our one-country model to a two-country environment. We assume that there are two countries, home and foreign. The home country is size n and the foreign country is size $1-n$. We are only going to allow non-state-contingent home and foreign nominal bonds to be traded internationally. Our model features Calvo-style sticky prices and so-called "local-currency pricing." Again we add endogenous oil demand along with exogenous oil supply along the lines of Bodenstein et al. (2013).

D.1 Household

The representative household in the home country maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C_{t+j} - h\bar{C}_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \log\left(\eta_t\right) V\left(\frac{B_{H,t}}{P_{C,t}}\right) + \log\left(\eta_t^*\right) V\left(\frac{B_{F,t} NER_t}{P_{C,t}}\right) \right). \tag{D.1}$$

Here C_t is per-capita consumption, $\bar{C}_t$ is average aggregate per-capita consumption, L_t is per-capita hours worked, $B_{H,t}$ is per-capita nominal home bond holdings, $B_{F,t}$ is per-capita nominal foreign bond holdings, $P_{C,t}$ is the price of the home consumption good in the home currency unit, and NER_t is the nominal exchange rate quoted as the price of the foreign currency unit. The stochastic variables η_t and η_t^* are preference shifters than capture the desire to hold safe nominal assets in the home and foreign currency. The budget constraint is

$$B_{H,t} + B_{F,t} + P_{C,t}C_t + P_{Y,t}I_t + \frac{\phi_b}{2}\left(\frac{B_{F,t}NER_t}{P_{C,t}}\right)^2 P_{Y,t} = \left(1+R_{t-1}\right)^{1/4} B_{H,t-1} + \left(1+R_{t-1}^*\right)^{1/4} B_{F,t-1} NER_t + R_{K,t}K_t + W_tL_t + T_t \tag{D.2}$$

where $P_{Y,t}$ is the price of non-oil output in the home country, R_t is the annualized net nominal interest rate on the home bond, R_t^* is the annualized net nominal interest rate on the home bond, W_t is the wage rate, $R_{K,t}$ is the rental rate on capital, K_t is per-capita capital holdings, I_t is per-capita investment, and T_t are per-capita lump-sum profits and taxes. The term $\frac{\phi_b}{2}\left(\frac{B_{F,t}NER_t}{P_{C,t}}\right)^2 P_{Y,t}$ is a carrying cost of holding the home-country bond. From a practical perspective, ϕ_b is set to a small number and this term ensures stationarity in the model. See Schmitt-Grohé and Uribe (2003). The capital accumulation equation is

$$K_{t+1} = I_t\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + (1-\delta)\,K_t. \tag{D.3}$$

The definition of consumption is

$$C_t = \left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{O_C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}}. \tag{D.4}$$

The household creates the consumption good to minimize costs

$$\min_{Y_{C,t},O_{C,t}} P_{Y,t}Y_{C,t} + P_{O,t}O_{C,t} \tag{D.5}$$

subject to the constraint that

$$\left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{O_C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}} \geq C_t \tag{D.6}$$

where $Y_{C,t}$ is non-oil output used for consumption and $O_{C,t}$ is oil that is consumed by the household. Then the first-order conditions are

$$Y_{C,t} = \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C - 1}}\omega_C C_t \tag{D.7}$$

$$O_{C,t} = \left(\frac{P_{O,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C - 1}} C_t\left(1-\omega_C\right)\mu_{O_C,t}^{\frac{\rho_C}{\rho_C - 1}} \tag{D.8}$$

The ideal price index for final consumption is given by

$$P_{C,t} = \left(\omega_C\left(P_{Y,t}\right)^{\frac{\rho_C}{\rho_C-1}} + \left(1-\omega_C\right)\left(P_{O,t}\mu_{O_C,t}\right)^{\frac{\rho_C}{\rho_C-1}}\right)^{\frac{\rho_C-1}{\rho_C}}. \tag{D.9}$$

The household-wide first-order conditions are

$$\left(C_t - h\bar{C}_{t-1}\right)^{-\sigma} = \Lambda_t \tag{D.10}$$

$$\Lambda_t W_t / P_{C,t} = \chi L_t^{\phi} \tag{D.11}$$

$$\Lambda_t = \log\left(\eta_t\right) V'\left(\frac{B_{H,t}}{P_{C,t}}\right) + \beta\left(1+R_t\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}} \tag{D.12}$$

$$\begin{aligned}\Lambda_t + \phi_B \frac{B_{F,t}}{P_{C,t}} NER_t \frac{P_{Y,t}}{P_{C,t}} =& \log\left(\eta_t^*\right) V'\left(\frac{B_{F,t}}{P_{C,t}} NER_t\right) \\ &+ \beta\left(1+R_t^*\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}} \frac{NER_{t+1}}{NER_t}\end{aligned} \tag{D.13}$$

$$\begin{aligned}\frac{P_{Y,t}}{P_{C,t}} \Lambda_t =& Q_t \left[\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \frac{I_t}{I_{t-1}} \phi_K \left(\frac{I_t}{I_{t-1}} - 1\right)\right] \\ &+ \beta E_t Q_{t+1} \phi_K \left(\frac{I_{t+1}}{I_t} - 1\right) \frac{I_{t+1}^2}{I_t^2}\end{aligned} \tag{D.14}$$

$$Q_t = \beta E_t \left[Q_{t+1}\left(1-\delta\right) + \Lambda_{t+1} \frac{R_{K,t+1}}{P_{C,t+1}}\right]. \tag{D.15}$$

The representative foreign household maximizes

$$\begin{aligned}E_t \sum_{j=0}^{\infty} \beta^j \Bigg(& \frac{\left(C_{t+j}^* - h\bar{C}_{t+j-1}^*\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi}\left(L_{t+j}^*\right)^{1+\phi} \\ &+ \log\left(\eta_t\right) V\left(\frac{B_{H,t}^*}{P_{C,t}^* NER_t}\right) + \log\left(\eta_t^*\right) V\left(\frac{B_{F,t}^*}{P_{C,t}^*}\right)\Bigg)\end{aligned} \tag{D.16}$$

where C_t^* is per-capita consumption, $\bar{C}_t^*$ is average aggregate per-capita consumption, L_t^* is per-capita hours worked, $B_{H,t}^*$ is per-capita home nominal bond holdings, $B_{F,t}^*$ is per-capita foreign nominal bonds, and $P_{C,t}^*$ is the price of the foreign consumption good in the foreign currency unit. Note that η_t and η_t^* are the same preference shifters as for the home household. In this way, we capture global demand for the desire to hold safe nominal assets in one currency or another. The budget constraint is

$$B^*_{F,t} + P^*_{C,t}C^*_t + P^*_{Y,t}I^*_t + B^*_{H,t}NER^{-1}_t + \frac{\phi_b}{2}\left(\frac{B^*_{H,t}}{P^*_{Ct}NER_t}\right)^2 P^*_{Y,t} =$$
$$\left(1 + R^*_{t-1}\right)^{1/4} B^*_{F,t-1} + \left(1 + R_{t-1}\right)^{1/4} B^*_{H,t-1}NER^{-1}_t + R^*_{K,t}K^*_t + W^*_tL^*_t + T^*_t \tag{D.17}$$

The term $\frac{\phi_b}{2}\left(\frac{B^*_{H,t}}{P^*_t NER_t}\right)^2 P^*_{C,t}$ is a carrying cost of holding the home-country bond. The capital accumulation equation is

$$K^*_{t+1} = I^*_t\left(1 - \frac{\phi_K}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2\right) + (1-\delta)\,K^*_t. \tag{D.18}$$

Then the first-order conditions for the composite consumption good are

$$Y^*_{C,t} = \left(\frac{P^*_{Y,t}}{P^*_{C,t}}\right)^{\frac{1}{\rho_C - 1}} \omega_C C^*_t, \tag{D.19}$$

$$O^*_{C,t} = \left(\frac{P^*_{O,t}}{P^*_{C,t}}\right)^{\frac{1}{\rho_C - 1}} C^*_t\left(1-\omega_C\right)\left(\mu^*_{O_C,t}\right)^{\frac{\rho_C}{\rho_C - 1}}. \tag{D.20}$$

The ideal price index for final consumption is given by

$$P^*_{C,t} = \left(\omega_C\left(P^*_{Y,t}\right)^{\frac{\rho_C}{\rho_C - 1}} + (1-\omega_C)\left(P^*_{O,t}\mu^*_{O_C,t}\right)^{\frac{\rho_C}{\rho_C - 1}}\right)^{\frac{\rho_C - 1}{\rho_C}}. \tag{D.21}$$

The household-wide first-order conditions are

$$\left(C^*_t - h\bar{C}^*_{t-1}\right)^{-\sigma} = \Lambda^*_t \tag{D.22}$$

$$\Lambda^*_t W^*_t / P^*_{C,t} = \chi\left(L^*_t\right)^{\phi} \tag{D.23}$$

$$\Lambda^*_t + \phi_b \frac{B^*_{H,t}}{NER_t P^*_{C,t}}\frac{P^*_{Y,t}}{P^*_{C,t}} = \log\left(\eta_t\right) V'\left(\frac{B^*_{H,t}}{P^*_{C,t}}NER^{-1}_t\right) + \beta\left(1+R_t\right)^{1/4} E_t \frac{\Lambda^*_{t+1}}{\pi^*_{C,t+1}}\frac{NER_t}{NER_{t+1}} \tag{D.24}$$

$$\Lambda_t^* = \log(\eta_t^*) V'\left(\frac{B_{F,t}^*}{P_{C,t}^*}\right) + \beta (1+R_t^*)^{1/4} E_t \frac{\Lambda_{t+1}^*}{\pi_{C,t+1}^*} \tag{D.25}$$

$$\begin{aligned}\frac{P_{Y,t}^*}{P_{C,t}^*}\Lambda_t^* =& Q_t^*\left[\left(1-\frac{\phi_K}{2}\left(\frac{I_t^*}{I_{t-1}^*}-1\right)^2\right)-\frac{I_t^*}{I_{t-1}^*}\phi_K\left(\frac{I_t^*}{I_{t-1}^*}-1\right)\right] \\ &+\beta E_t Q_{t+1}\phi_K\left(\frac{I_{t+1}^*}{I_t^*}-1\right)\left(\frac{I_{t+1}^*}{I_t^*}\right)^2\end{aligned} \tag{D.26}$$

$$Q_t^* = \beta E_t\left[Q_{t+1}^*(1-\delta)+\Lambda_{t+1}^*\frac{R_{K,t+1}^*}{P_{C,t+1}^*}\right]. \tag{D.27}$$

Note that we define the real exchange rate, RER_t, so that

$$RER_t = \frac{NER_t P_{C,t}^*}{P_{C,t}}. \tag{D.28}$$

D.2 Goods aggregators

In each country, perfectly competitive firms aggregate country-specific intermediate inputs into $Y_{H,t}$, $Y_{F,t}$, $Y_{H,t}^*$, and $Y_{F,t}^*$. The values $Y_{H,t}$ and $Y_{F,t}$ are composites of goods purchased from monopolists by perfectly competitive firms who produce using

$$Y_{H,t} = \left(\frac{1}{n}\right)^{\frac{1}{\nu}}\left(\int_0^n X_{H,t}(i)^{\frac{\nu-1}{\nu}}\, di\right)^{\frac{\nu}{\nu-1}} \tag{D.29}$$

$$Y_{F,t} = \left(\frac{1}{1-n}\right)^{\frac{1}{\nu}}\left(\int_0^{1-n} X_{F,t}(i)^{\frac{\nu-1}{\nu}}\, di\right)^{\frac{\nu}{\nu-1}} \tag{D.30}$$

Demand curves are then of the form

$$X_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t} \tag{D.31}$$

and

$$X_{F,t}(i) = \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\nu} Y_{F,t}. \tag{D.32}$$

The zero profit condition, along with these demand curves, implies the ideal price index is give by

$$P_{H,t} = \left(\frac{1}{n} \int_0^n P_{H,t}(i)^{1-\nu} \, di \right)^{\frac{1}{1-\nu}}. \tag{D.33}$$

Similarly,

$$P_{F,t} = \left(\frac{1}{1-n} \int_0^{1-n} P_{F,t}(i)^{1-\nu} \, di \right)^{\frac{1}{1-\nu}}. \tag{D.34}$$

The foreign country is symmetric. Demand curves are of the form

$$X^*_{H,t}(i) = \frac{1}{n} \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\nu} Y^*_{H,t} \tag{D.35}$$

and

$$X^*_{F,t}(i) = \frac{1}{1-n} \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}} \right)^{-\nu} Y^*_{F,t}. \tag{D.36}$$

The zero profit conditions, along with these demand curves, imply ideal price indexes

$$P^*_{H,t} = \left(\frac{1}{n} \int_0^n P^*_{H,t}(i)^{1-\nu} \, di \right)^{\frac{1}{1-\nu}} \tag{D.37}$$

and

$$P^*_{F,t} = \left(\frac{1}{1-n} \int_0^{1-n} P^*_{F,t}(i)^{1-\nu} \, di \right)^{\frac{1}{1-\nu}}. \tag{D.38}$$

D.3 Retailers

Non-oil output, Y_t, is created by combining goods from countries H and F ($Y_{H,t}$ and $Y_{F,t}$) using

$$Y_t = \left(\omega^{1-\rho} (Y_{H,t})^{\rho} + (1-\omega)^{1-\rho} (Y_{F,t})^{\rho} \right)^{\frac{1}{\rho}} \tag{D.39}$$

where $\omega \equiv 1 - (1-n)\,\Omega$. The value $0 < \Omega \leq 1$ captures home bias if it is less than one (see Faia and Monacelli (2008)). Profits are given by

$$P_{Y,t} \left(\omega^{1-\rho} (Y_{H,t})^{\rho} + (1-\omega)^{1-\rho} (Y_{F,t})^{\rho} \right)^{\frac{1}{\rho}} - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \tag{D.40}$$

where $P_{H,t}$ is the nominal price of $Y_{H,t}$, $P_{F,t}$ is the nominal price of $Y_{F,t}$. Demand curves are then

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_{Y,t}}\right)^{\frac{1}{\rho-1}} \omega Y_t \tag{D.41}$$

and

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_{Y,t}}\right)^{\frac{1}{\rho-1}} (1-\omega)\, Y_t. \tag{D.42}$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields the ideal price index

$$P_{Y,t} = \left(\omega P_{H,t}^{\frac{\rho}{\rho-1}} + (1-\omega)\,(P_{F,t})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{D.43}$$

Non-oil output in the foreign country, Y_t^*, are created by combining goods for countries H and F ($Y_{H,t}^*$ and $Y_{F,t}^*$) using

$$Y_t^* = \left((\omega^*)^{1-\rho}\left(Y_{F,t}^*\right)^{\rho} + (1-\omega^*)^{1-\rho}\left(Y_{H,t}^*\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{D.44}$$

where $\omega^* \equiv 1 - n\Omega^*$. The value $0 < \Omega^* \leq 1$ captures home bias if it is less that one. Profits are given by

$$P_{Y,t}^*\left((\omega^*)^{1-\rho}\left(Y_{F,t}^*\right)^{\rho} + (1-\omega^*)^{1-\rho}\left(Y_{H,t}^*\right)^{\rho}\right)^{\frac{1}{\rho}} - P_{F,t}^* Y_{F,t}^* - P_{H,t}^* Y_{H,t}^* \tag{D.45}$$

where $P_{H,t}^*$ is the nominal price of $Y_{H,t}^*$, $P_{F,t}^*$ is the nominal price of $Y_{F,t}^*$. Demand curves are given by

$$Y_{H,t}^* = \left(\frac{P_{H,t}^*}{P_{Y,t}^*}\right)^{\frac{1}{\rho-1}} (1-\omega^*)\, Y_t^* \tag{D.46}$$

and

$$Y_{F,t}^* = \left(\frac{P_{F,t}^*}{P_{Y,t}^*}\right)^{\frac{1}{\rho-1}} \omega^* Y_t^*. \tag{D.47}$$

The ideal price index for Y_t^* is given by

$$P_{Y,t}^* = \left(\omega^*\left(P_{F,t}^*\right)^{\frac{\rho}{\rho-1}} + (1-\omega^*)\left(P_{H,t}^*\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \tag{D.48}$$

We define,

$$\pi_{Y,t} \equiv P_{Y,t}/P_{Y,t-1} \tag{D.49}$$

and

$$\pi^*_{Y,t} \equiv P^*_{Y,t}/P^*_{Y,t-1}. \tag{D.50}$$

D.4 Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists set their price in the currency where their goods are sold (so-called "local-currency pricing"). Monopolists are only able to optimally update their price with probability ξ in each period. If monopolist i in the country H can optimally update its price, it chooses $\tilde{P}_{H,t}(i)$ and $\tilde{P}^*_{H,t}(i)$ to maximize

$$\begin{aligned} E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \Bigg\{ & \left(\frac{\tilde{P}_{H,t}(i)}{P_{C,t+j}} \tilde{X}_{H,t,j} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j} \\ & + \left(\frac{NER_{t+j} \tilde{P}^*_{H,t}(i)}{P_{C,t+j}} \tilde{X}^*_{H,t,j} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}^*_{H,t}(i)}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j} \Bigg\} \end{aligned} \tag{D.51}$$

where

$$\tilde{X}_{H,t,j} = \begin{cases} 1 & j=1 \\ \pi_{H,t} \times \pi_{H,t+1} \times \cdots \times \pi_{H,t+j-1} & else \end{cases}, \tag{D.52}$$

and

$$\tilde{X}^*_{H,t,j} = \begin{cases} 1 & j=1 \\ \pi^*_{H,t} \times \pi^*_{H,t+1} \times \cdots \times \pi^*_{H,t+j-1} & else \end{cases}. \tag{D.53}$$

Here,

$$\pi_{H,t} \equiv P_{H,t}/P_{H,t-1} \tag{D.54}$$

and

$$\pi^*_{H,t} \equiv P^*_{H,t}/P^*_{H,t-1}. \tag{D.55}$$

The variables $\tilde{X}_{H,t,j}$ and $\tilde{X}^*_{H,t,j}$ capture indexation to past price changes. The first-order condition with respect to $\tilde{P}_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{\tilde{P}_{H,t}}{P_{C,t}} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{H,t,j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \right] \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j} = 0 \quad \text{(D.56)}$$

Then we have

$$F_{H,t} \tilde{p}_{H,t} = K_{H,t} \quad \text{(D.57)}$$

where $\tilde{p}_{H,t} \equiv \tilde{P}_{H,t}/P_{C,t}$ and $F_{H,t}$ and $K_{H,t}$ are given by

$$F_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{H,t,j} \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j} \quad \text{(D.58)}$$

and

$$K_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j}. \quad \text{(D.59)}$$

These can be written as

$$F_{H,t} = \Lambda_t Y_{H,t} + \beta\xi E_t \pi_{H,t}^{1-\nu} \pi_{C,t+1}^{-1} \pi_{H,t+1}^{\nu} F_{H,t+1} \quad \text{(D.60)}$$

and

$$K_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y_{H,t} + \beta\xi E_t \pi_{H,t}^{-\nu} \pi_{H,t+1}^{\nu} K_{H,t+1} \quad \text{(D.61)}$$

The ideal price index for home goods in the home market is given by

$$P_{H,t} = \left((1-\xi) \tilde{P}_{H,t}^{1-\nu} + \xi \pi_{H,t-1}^{1-\nu} P_{H,t-1}^{1-\nu} \right)^{\frac{1}{1-\nu}}. \quad \text{(D.62)}$$

Then

$$p_{H,t} = \left((1-\xi) \tilde{p}_{H,t}^{1-\nu} + \xi \pi_{H,t-1}^{1-\nu} \frac{p_{H,t-1}^{1-\nu}}{\pi_{C,t}^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \quad \text{(D.63)}$$

The first-order condition with respect to $\tilde{P}^*_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{NER_{t+j}}{NER_t} \frac{NER_t P^*_{C,t}}{P_{C,t}} \tilde{p}^*_{H,t} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}^*_{H,t,j} - \frac{MC_{t+j}}{1+\tau_X} \frac{\nu}{\nu-1} \right] \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j} = 0 \tag{D.64}$$

where $\tilde{p}^*_{H,t} \equiv \tilde{P}^*_{H,t}/P^*_{C,t}$. Then we have

$$F^*_{H,t} RER_t \tilde{p}^*_{H,t} = K^*_{H,t} \tag{D.65}$$

where

$$F^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{NER_{t+j}}{NER_t} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}^*_{H,t,j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j} \tag{D.66}$$

$$K^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j}. \tag{D.67}$$

These variables can be written as

$$F^*_{H,t} = \Lambda_t Y^*_{H,t} + \beta\xi E_t \frac{NER_{t+1}}{NER_t} \left(\pi^*_{H,t}\right)^{1-\nu} \pi^{-1}_{C,t+1} \left(\pi^*_{H,t+1}\right)^{\nu} F^*_{H,t+1} \tag{D.68}$$

$$K^*_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y^*_{H,t} + \beta\xi E_t \left(\pi^*_{H,t}\right)^{-\nu} \left(\pi^*_{H,t+1}\right)^{\nu} K^*_{H,t+1} \tag{D.69}$$

The ideal price index for home goods in the foreign market is given by

$$P^*_{H,t} = \left((1-\xi) \left(\tilde{P}^*_{H,t}\right)^{1-\nu} + \xi \left(\pi^*_{H,t-1} P^*_{H,t-1}\right)^{1-\nu} \right)^{\frac{1}{1-\nu}} \tag{D.70}$$

so that

$$p^*_{H,t} = \left((1-\xi) \left(\tilde{p}^*_{H,t}\right)^{1-\nu} + \xi \left(\pi^*_{H,t-1}\right)^{1-\nu} \frac{\left(p^*_{H,t-1}\right)^{1-\nu}}{\left(\pi^*_{C,t}\right)^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \tag{D.71}$$

The foreign firms are symmetric. If monopolist i can update its price, it chooses $\tilde{P}^*_{F,t}(i)$ and $\tilde{P}_{F,t}(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} \Lambda^*_{t+j} \Bigg\{ \left(\frac{\tilde{P}^*_{F,t}(i)}{P^*_{t+j}} \tilde{X}^*_{F,t,j} (1+\tau_X) - MC^*_{t+j} \right) \left(\frac{\tilde{P}^*_{F,t}(i)}{P^*_{F,t+j}} \tilde{X}^*_{F,t,j} \right)^{-\nu} Y^*_{F,t+j} + \left(\frac{\tilde{P}_{F,t}(i)}{NER_{t+j} P^*_{t+j}} \tilde{X}_{F,t,j} (1+\tau_X) - MC^*_{t+j} \right) \left(\frac{\tilde{P}_{F,t}(i)}{P_{F,t+j}} \tilde{X}_{F,t,j} \right)^{-\nu} Y_{F,t+j} \Bigg\} \tag{D.72}$$

where

$$\tilde{X}_{F,t,j} = \begin{cases} 1 & j=1 \\ \pi_{F,t} \times \pi_{F,t+1} \times \cdots \times \pi_{F,t+j-1} & else \end{cases}, \tag{D.73}$$

and

$$\tilde{X}^*_{F,t,j} = \begin{cases} 1 & j=1 \\ \pi^*_{F,t} \times \pi^*_{F,t+1} \times \cdots \times \pi^*_{F,t+j-1} & else \end{cases}. \tag{D.74}$$

where

$$\pi^*_{F,t} \equiv P^*_{F,t}/P^*_{F,t-1} \tag{D.75}$$

and

$$\pi_{F,t} \equiv P_{F,t}/P_{F,t-1}. \tag{D.76}$$

The first-order condition with respect to $\tilde{P}^*_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda^*_{t+j} \left[\frac{\tilde{P}^*_{F,t}}{P^*_{C,t}} \frac{P^*_{C,t}}{P^*_{C,t+j}} \tilde{X}^*_{F,t,j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_{t+j} \right] \left(\frac{P^*_{F,t}}{P^*_{F,t+j}} \tilde{X}^*_{F,t,j} \right)^{-\nu} Y^*_{F,t+j} = 0. \tag{D.77}$$

We write this as

$$F^*_{F,t} \tilde{p}^*_{F,t} = K^*_{F,t} \tag{D.78}$$

where $\tilde{p}^*_{F,t} \equiv \tilde{P}^*_{F,t}/P^*_{C,t}$,

$$F^*_{F,t} = \Lambda^*_t Y^*_{F,t} + \beta\xi E_t \left(\pi^*_{F,t}\right)^{1-\nu} \left(\pi^*_{C,t+1}\right)^{-1} \left(\pi^*_{F,t+1}\right)^{\nu} F^*_{F,t+1} \tag{D.79}$$

and

$$K^*_{F,t} = \Lambda^*_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_t Y^*_{F,t} + \beta\xi E_t \left(\pi^*_{F,t}\right)^{-\nu} \left(\pi^*_{F,t+1}\right)^{\nu} K^*_{F,t+1}. \tag{D.80}$$

The ideal price index implies

$$p^*_{F,t} = \left((1-\xi)\left(\tilde{p}^*_{F,t}\right)^{1-\nu} + \xi \left(\pi^*_{F,t-1}\right)^{1-\nu} \frac{\left(p^*_{F,t-1}\right)^{1-\nu}}{\left(\pi^*_{C,t}\right)^{1-\nu}}\right)^{\frac{1}{1-\nu}}. \tag{D.81}$$

where $p^*_{F,t} \equiv P^*_{F,t}/P^*_{C,t}$. The first-order condition with respect to $\tilde{P}_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda^*_{t+j} \left[\frac{NER_t}{NER_{t+j}} \frac{P_{C,t}}{NER_t P^*_{C,t}} \tilde{p}_{F,t} \frac{P^*_{C,t}}{P^*_{C,t+j}} \tilde{X}_{F,t,j} - \frac{MC^*_{t+j}}{1+\tau_X} \frac{\nu}{\nu-1}\right] \left(\frac{P_{F,t}}{P_{F,t+j}} \tilde{X}_{F,t,j}\right)^{-\nu} Y_{F,t+j} = 0. \tag{D.82}$$

We can write this as

$$F_{F,t} \frac{\tilde{p}_{F,t}}{RER_t} = K_{F,t} \tag{D.83}$$

where $p_{F,t} \equiv P_{F,t}/P_{C,t}$,

$$F_{F,t} = \Lambda^*_t Y_{F,t} + \beta\xi E_t \frac{NER_t}{NER_{t+1}} \pi^{1-\nu}_{F,t} \left(\pi^*_{C,t+1}\right)^{-1} \left(\pi_{F,t+1}\right)^{\nu} F_{F,t+1} \tag{D.84}$$

and

$$K_{F,t} = \Lambda^*_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_t Y_{F,t} + \beta\xi E_t \pi^{-\nu}_{F,t} \left(\pi_{F,t+1}\right)^{\nu} K_{F,t+1}. \tag{D.85}$$

The price index implies that

$$p_{F,t} = \left((1-\xi)\left(\tilde{p}_{F,t}\right)^{1-\nu} + \xi \left(\pi_{F,t-1}\right)^{1-\nu} \frac{\left(p_{F,t-1}\right)^{1-\nu}}{\left(\pi_{C,t}\right)^{1-\nu}}\right)^{\frac{1}{1-\nu}}. \tag{D.86}$$

D.5 Marginal cost

In this subsection we drop the i index because it should be understood that all quantities are the quantity purchased by firm i. The firm solves the following cost minimization problem

$$\min_{K_t,L_t} W_t L_t + R_{Kt} K_t + P_{O,t} O_t \tag{D.87}$$

subject to the constraint that

$$\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(\frac{V_{O,t}}{\mu_{V_O,t}} \right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \geq X_t \tag{D.88}$$

where

$$V_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{D.89}$$

and A_t and A_t^* are stochastic processes. The first-order conditions for the home firms are

$$\frac{R_{Kt}}{P_{Ct}} = MC_t (X_t)^{1-\rho_V} (\omega_V)^{1-\rho_V} V_t^{\rho_V - 1} \alpha A_t \left(\frac{L_t}{K_t} \right)^{1-\alpha} \tag{D.90}$$

$$\frac{W_t}{P_{C,t}} = MC_t (X_t)^{1-\rho_V} (\omega_V)^{1-\rho_V} V_t^{\rho_V - 1} (1-\alpha) A_t \left(\frac{L_t}{K_t} \right)^{-\alpha} \tag{D.91}$$

$$\frac{P_{O,t}}{P_{C,t}} = MC_t (X_t)^{1-\rho_V} (1-\omega_V)^{1-\rho_V} \left(\frac{V_{O,t}}{\mu_{V_O,t}} \right)^{\rho_V - 1} \frac{1}{\mu_{V_O,t}}. \tag{D.92}$$

Foreign firms minimize

$$\min_{K_t^*,L_t^*} W_t^* L_t^* + R_{Kt}^* K_t^* + P_{O,t}^* O_t^* \tag{D.93}$$

subject to the constraint that

$$\left((\omega_V)^{1-\rho_V} (V_t^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(\frac{V_{O,t}^*}{\mu_{V_O,t}^*} \right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \geq X_t^* \tag{D.94}$$

where

$$V_t^* = A_t^* (K_t^*)^{\alpha} (L_t^*)^{1-\alpha}. \tag{D.95}$$

The first-order conditions for the foreign firms are

$$\frac{R_{Kt}^*}{P_{Ct}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V_t^*\right)^{\rho_V - 1} \alpha A_t^* \left(\frac{L_t^*}{K_t^*}\right)^{1-\alpha} \tag{D.96}$$

$$\frac{W_t^*}{P_{C,t}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V_t^*\right)^{\rho_V - 1} \left(1-\alpha\right) A_t^* \left(\frac{L_t^*}{K_t^*}\right)^{-\alpha} \tag{D.97}$$

$$\frac{P_{O,t}^*}{P_{C,t}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(1-\omega_V\right)^{1-\rho_V} \left(\frac{V_{O,t}^*}{\mu_{V_O,t}^*}\right)^{\rho_V - 1} \frac{1}{\mu_{V_O,t}^*}. \tag{D.98}$$

D.6 Oil Market

There is an exogenous supply of oil, O_t. Oil-market clearing implies

$$n \left(V_{O,t} + O_{C,t}\right) + \left(1-n\right) \left(V_{O,t}^* + O_{C,t}^*\right) = O_t. \tag{D.99}$$

The price is set flexibly so that the market clears and

$$P_{O,t} = NER_t P_{O,t}^*. \tag{D.100}$$

D.7 Goods market clearing

We assume that oil is paid for using non-oil output. So, goods market clearing implies

$$Y_{C,t} + G_t + I_t + \left(O_{C,t} + V_{O,t}\right) \frac{P_{O,t}}{P_{Y,t}} + \frac{\phi_b}{2} \left(\frac{B_{F,t} NER_t}{P_{C,t}}\right)^2 = Y_t \tag{D.101}$$

and

$$Y_{C,t}^* + G_t^* + I_t^* + \left(O_{C,t}^* + V_{O,t}^*\right) \frac{P_{O,t}^*}{P_{Y,t}^*} + \frac{\phi_b}{2} \left(\frac{B_{H,t}^*}{P_{C,t}^* NER_t}\right)^2 = Y_t^*. \tag{D.102}$$

The quadratic costs of bond holdings show up in the resource constraint because we assume that non-oil output is used to pay those costs.

D.8 Bond market clearing

We assume that only the home bond can be traded internationally and that both home and foreign bonds are in zero net supply. So,

$$nb_{H,t} + (1-n)\, b^*_{H,t} = 0 \tag{D.103}$$

and

$$nb_{F,t} + (1-n)\, b^*_{F,t} = 0. \tag{D.104}$$

D.9 Aggregation

Aggregating across home firms yields

$$\begin{aligned} & n\int_0^n \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t}di + (1-n)\int_0^n \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t}di \\ & = \int_0^n \left((\omega_V)^{1-\rho_V} V_t(i)^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} di & \text{(D.105)} \\ & = \int_0^n \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{V_t(i)\,\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} V_t(i)\, di & \text{(D.106)} \\ & = \int_0^n \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{V_t(i)\,\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} A_t\left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i)\, di. & \text{(D.107)} \end{aligned}$$

Due to constant-returns-to-scale, the ratios $\frac{V_{O,t}(i)}{V_t(i)\mu_{V_O,t}}$ and $\frac{K_t(i)}{L_t(i)}$ are common across firms. Then

$$\begin{aligned} & n\int_0^n \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t}di + (1-n)\int_0^n \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t}di \\ & = n\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} & \text{(D.108)} \end{aligned}$$

so that

$$d_{H,t}Y_{H,t} + d^*_{H,t}\frac{1-n}{n}Y^*_{H,t} = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{D.109}$$

where $d_{H,t}$ and $d^*_{H,t}$ are appropriately defined. Similarly,

$$d_{F,t}\frac{n}{1-n}Y_{F,t}+d^*_{F,t}Y^*_{F,t}=\left((\omega_V)^{1-\rho_V}\left(V^*_t\right)^{\rho_V}+(1-\omega_V)^{1-\rho_V}\left(\frac{V^*_{O,t}}{\mu^*_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \tag{D.110}$$

Here the dispersion terms can be written recursively as

$$d_{H,t}=(1-\xi)\,p^{\nu}_{H,t}\left(\tilde{p}_{H,t}\right)^{-\nu}+\xi\pi^{-\nu}_{H,t-1}\pi^{\nu}_{H,t}d_{H,t-1}, \tag{D.111}$$

$$d^*_{H,t}=(1-\xi)\left(p^*_{H,t}\right)^{\nu}\left(\tilde{p}^*_{H,t}\right)^{-\nu}+\xi\left(\pi^*_{H,t-1}\right)^{-\nu}\left(\pi^*_{H,t}\right)^{\nu}d^*_{H,t-1}, \tag{D.112}$$

$$d^*_{F,t}=(1-\xi)\left(p^*_{F,t}\right)^{\nu}\left(\tilde{p}^*_{F,t}\right)^{-\nu}+\xi\left(\pi^*_{F,t-1}\right)^{-\nu}\left(\pi^*_{F,t}\right)^{\nu}d^*_{F,t-1}, \tag{D.113}$$

$$d_{F,t}=(1-\xi)\left(p_{F,t}\right)^{\nu}\left(\tilde{p}_{F,t}\right)^{-\nu}+\xi\pi^{-\nu}_{F,t-1}\left(\pi_{F,t}\right)^{\nu}d_{F,t-1}. \tag{D.114}$$

D.10 Government

In each country, the monetary authority follows a truncated Taylor rule. The desired policy rates, $\tilde{R}_t$ and $\tilde{R}^*_t$ evolves according to

$$\left(1+\tilde{R}_t\right)^{1/4}=\left(\left(1+\tilde{R}_{t-1}\right)^{1/4}\right)^{\gamma}\left((1+R)^{1/4}\left(\frac{\pi_{Y,t}}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y^N_t}\right)^{\theta_Y}\right)^{1-\gamma} \tag{D.115}$$

where $\theta_\pi>1$. Here, R is the steady-state annualized net nominal interest rate, π is the target rate of inflation. In the foreign country

$$\left(1+\tilde{R}_t{}^*\right)^{1/4}=\left(\left(1+\tilde{R}_t{}^*\right)^{1/4}\right)^{\gamma^*}\left((1+R^*)^{1/4}\left(\frac{\pi^*_{Y,t}}{\pi^*}\right)^{\theta^*_\pi}\left(\frac{Y^*_t}{Y^{N*}_t}\right)^{\theta^*_Y}\right)^{1-\gamma^*} \tag{D.116}$$

where $\theta_\pi>1$. Here, R^* is the steady-state annualized net nominal interest rate, π^* is the target rate of inflation. The natural rate of output, Y^{N*}_t is defined as the level of output that would prevail under flexible prices, given the entire history of shocks. The fiscal authorities balances its budget with lump sum taxes so bonds are in zero net supply. Government purchases, G_t and G^*_t, follow independent

$AR(1)$ processes. To incorporate the zero lower bound,

$$R_t = \max\left\{0, \tilde{R}_t\right\}. \tag{D.117}$$

For the foreign country, we ignore the zero lower bound, so that

$$R_t^* = \tilde{R}_t^*. \tag{D.118}$$

We ignore the zero lower bound for the foreign country because we want to study how a binding lower bound in the home country affects the foreign country.

D.11 Equilibrium

A rational expectations equilibrium is a sequence of prices and quantities that have the property that the household and firm optimality conditions are satisfied, the goods market, labor market, and oil markets clear, and the nominal interest rate and government purchases evolve as specified. To solve for a rational expectations equilibrium, we solve for the following 35 endogenous objects: C_t, Λ_t, L_t, $w_t \equiv \frac{W_t}{P_t}$, $Y_{H,t}$, $Y_{F,t}$, R_t, MC_t, $\pi_{C,t}$, K_t, I_t, Q_t, $r_{K,t} \equiv \frac{R_{K,t}}{P_{C,t}}$, Y_t, $p_{F,t} \equiv \frac{P_{F,t}}{P_{C,t}}$, $p_{H,t} \equiv \frac{P_{H,t}}{P_{C,t}}$, $\tilde{p}_{H,t}$, $F_{H,t}$, $K_{H,t}$, $d_{H,t}$, $\pi_{H,t}$, $\tilde{p}_{F,t}$, $F_{F,t}$, $K_{F,t}$, $d_{F,t}$, $\pi_{F,t}$, $b_{H,t}$, $b_{F,t}$, $p_{Y,t} \equiv \frac{P_{Y,t}}{P_{C,t}}$, V_t, $V_{O,t}$, $Y_{C,t}$, $O_{C,t}$, $p_{O,t} \equiv \frac{P_{O,t}}{P_{C,t}}$, $\pi_{Y,t}$, the 35 star versions, as well as $\Delta NER_t \equiv \frac{NER_t}{NER_{t-1}}$ and RER_t.

We linearize the model around non-stochastic steady state. Given parameter values, we study the unique bounded rational expectations equilibrium from the linearized model. To determine these variables, we require that the linearized versions following 72 equations hold: (D.3), (D.7), (D.8), (D.9), (D.10), (D.11), (D.12), (D.13), (D.14), (D.15), (D.18), (D.19), (D.20), (D.21), (D.22), (D.23), (D.24), (D.25), (D.26), (D.27), (D.28), (D.41), (D.42), (D.43), (D.46), (D.47), (D.48), (D.101), (D.102), (D.103), (D.104), (D.57), (D.60), (D.61), (D.54), (D.63), (D.65), (D.68), (D.69), (D.55), (D.71), (D.78), (D.79), (D.80), (D.75), (D.81), (D.83), (D.84), (D.85), (D.76), (D.86), (D.89), (D.90), (D.91), (D.92), (D.95), (D.96), (D.97), (D.98), (D.99), (D.100), (D.109), (D.110), (D.111), (D.112), (D.113), (D.114), (D.115), (D.116), (D.49), (D.50), (D.17). The home household budget constraint (D.2) clears by Walras' law. Note that to solve for the natural rate of output, we find the equilibrium of a similar economy where $\xi = 0$. We linearize the model around non-stochastic steady state. We incorporate the zero lower bound using the methodology of Guerrieri and Iacoviello (2015). We utilize the OccBin solver from Guerrieri and Iacoviello (2015), which interacts with Dynare (see Adjemian et al. (2011)).

D.12 Steady State

We assume that government policy is symmetric between the home and foreign county and that the target inflation rate is π. So, $\pi_C = \pi_C^* = \pi_H = \pi_H^* = \pi_F = \pi_F^* = \pi_Y = \pi_Y^* = \pi$. The intertemporal Euler equations determine $(1+R)^{1/4} = (1+R^*)^{1/4} = \pi\beta^{-1}$. We normalized $L = L^* = 1$ (we will have to find χ instead of L). From the definition of steady state, with symmetric inflation targets $\Delta NER = 1$. We define initial conditions so that $RER = 1$. In our steady state, there are no net home bond holdings in the foreign country because of the quadratic costs of holding them. Similarly, there are no net foreign bond holdings in the home country. From firm optimality and symmetry of the equilibrium, $p_H = p_H^* = p_F = p_F^* = 1$. This also gives us that $\tilde{p}_H = \tilde{p}_H^* = \tilde{p}_F = \tilde{p}_F^* = 1$. Because of our inflation indexation assumption, there is no price dispersion in steady state, so $d_H = d_F = d_H^* = d_F^* = 1$. We will normalize the price of oil to be $p_O = p_O^* = 1$ (we have to find O instead). As a result, $p_Y = p_Y^* = 1$, meaning $Q = \Lambda$ and $Q^* = \Lambda^*$. Marginal cost is given by

$$MC = MC^* = \frac{\nu - 1}{\nu}(1 + \tau_X). \tag{D.119}$$

From pricing optimality

$$F_F = K_F = (1 - \beta\xi)^{-1}\Lambda^* Y_F \tag{D.120}$$

$$F_F^* = K_F^* = (1 - \beta\xi)^{-1}\Lambda^* Y_F^* \tag{D.121}$$

$$F_H = K_H = (1 - \beta\xi)^{-1}\Lambda Y_H \tag{D.122}$$

$$F_H^* = K_H^* = (1 - \beta\xi)^{-1}\Lambda Y_H^*. \tag{D.123}$$

The rental rate of capital is

$$r_K = r_K^* = \frac{1 - \beta(1 - \delta)}{\beta}. \tag{D.124}$$

From our normalization of p_O and p_O^*,

$$O_C = (1 - \omega_C) C \tag{D.125}$$

$$O_C^* = (1 - \omega_C) C^* \tag{D.126}$$

and

$$Y_C = \omega_C C \tag{D.127}$$

$$Y_C^* = \omega_C C^*. \tag{D.128}$$

The marginal utility of consumption implies

$$(C[1 - h])^{-\sigma} = \Lambda \tag{D.129}$$

$$(C^*[1 - h])^{-\sigma} = \Lambda^* \tag{D.130}$$

Note that

$$Y_H = (1 - (1 - n)\Omega) Y \tag{D.131}$$

$$Y_F = (1 - n)\Omega Y \tag{D.132}$$

$$Y_H^* = n\Omega^* Y^* \tag{D.133}$$

$$Y_F^* = (1 - n\Omega^*) Y^* \tag{D.134}$$

and

$$I = \delta K \tag{D.135}$$

$$I^* = \delta K^* \tag{D.136}$$

$$Y = I + Y_C + G + O_C + V_O \tag{D.137}$$

$$Y^* = I^* + Y_C^* + G^* + O_C^* + V_O^*. \tag{D.138}$$

Our aggregate variables are expressed in per-capita terms, and we are going to consider a symmetric steady state where $Y = Y^*$. From

$$d_H Y_H + d_H^* \frac{1-n}{n} Y_H^* = \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{D.139}$$

we get

$$(1 - (1-n)\,\Omega)\, Y + (1-n)\,\Omega^* Y^* = Y = Y^* \tag{D.140}$$

where

$$Y = \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}}. \tag{D.141}$$

We can see this from

$$d_F \frac{n}{1-n} Y_F + d_F^* Y_F^* = \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{D.142}$$

which yields

$$n\Omega Y + (1 - n\Omega^*)\, Y^* = \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{D.143}$$

which means the equalities above hold. This means

$$\delta K + \omega_C C + G = \left((\omega_V)^{1-\rho_V} (K^\alpha)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}}. \tag{D.144}$$

From the definition of V we have

$$V = K^\alpha \tag{D.145}$$

and

$$V^* = (K^*)^\alpha. \tag{D.146}$$

Define

$$X \equiv \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \tag{D.147}$$

From cost minimization, we know that

$$r_K = MC\,(X)^{1-\rho_V} (\omega_V)^{1-\rho_V} V^{\rho_V-1} \alpha \left(\frac{1}{K}\right)^{1-\alpha} \tag{D.148}$$

$$w = MC\,(X)^{1-\rho_V} (\omega_V)^{1-\rho_V} V^{\rho_V-1} (1-\alpha) \left(\frac{1}{K}\right)^{-\alpha} \tag{D.149}$$

$$1 = MC\,(X)^{1-\rho_V} (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V-1}. \tag{D.150}$$

Define

$$X^* \equiv \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \tag{D.151}$$

From cost minimization, we know that

$$r_K^* = MC^*\,(X^*)^{1-\rho_V} (\omega_V)^{1-\rho_V} (V^*)^{\rho_V-1}\, \alpha \left(\frac{1}{K^*}\right)^{1-\alpha} \tag{D.152}$$

$$w^* = MC^*\,(X_t^*)^{1-\rho_V} (\omega_V)^{1-\rho_V} (V^*)^{\rho_V-1} (1-\alpha) \left(\frac{1}{K^*}\right)^{-\alpha} \tag{D.153}$$

$$1 = MC^*\,(X_t^*)^{1-\rho_V} (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V-1}. \tag{D.154}$$

Then

$$r_K = \left(\frac{\omega_V}{1-\omega_V}\right)^{1-\rho_V} \left(\frac{K^\alpha}{V_O}\right)^{\rho_V-1} \alpha \left(\frac{1}{K}\right)^{1-\alpha}. \tag{D.155}$$

Then

$$V_O = r_K^{-\frac{1}{\rho_V-1}} \left(\frac{\omega_V}{1-\omega_V}\right)^{-1} (K^\alpha)\, \alpha^{\frac{1}{\rho_V-1}} \left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V-1}}. \tag{D.156}$$

So

$$r_K = \left((\omega_V)^{1-\rho_V} (K^\alpha)^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(r_K^{-\frac{1}{\rho_V - 1}} \left(\frac{\omega_V}{1-\omega_V} \right)^{-1} (K^\alpha)\, \alpha^{\frac{1}{\rho_V - 1}} \left(\frac{1}{K} \right)^{\frac{1-\alpha}{\rho_V - 1}} \right)^{\rho_V} \right)^{\frac{1-\rho_V}{\rho_V}}$$
$$\times MC\, (\omega_V)^{1-\rho_V} (K^\alpha)^{\rho_V - 1} \alpha \left(\frac{1}{K} \right)^{1-\alpha} \tag{D.157}$$

with r_K known and MC known, we can solve for K. With K we get V and then w. With the household intratemporal Euler equation, we get χ. We have Y from

$$Y = \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{D.158}$$

We know

$$Y_C + G + I + (O_C + V_O) = Y \tag{D.159}$$

and $Y_C = \omega_C C$ and $O_C = (1-\omega_C)\, C$ meaning

$$C + G + I + V_O = Y \tag{D.160}$$

With C, we get Y_C and O_C. Combined with V_O (and the star versions), we get O. The rest follows easily.

D.13 Calibration and solution strategy

For parameters that are common with our one-country model, we use the same values as in our one-country model, which are specified in Section 3.5. We set $n = 0.9$ so that the large country has size 0.9 and the small country has size 0.1. We assume that monetary policy is symmetric across the two countries and that the target level of inflation is 2 percent at an annualized rate. We set $\Omega = \Omega^* = 0.4$ to incorporate home bias. This value implies that in steady state 96 percent of non-oil expenditure in the big country is on goods from the big country. In the small country, in steady state 64 percent of non-oil expenditure is on goods from the small country. We set $\rho = 1/3$ so that the elasticity between domestic and foreign goods is 1.5.

We compute the natural rate of output as the level of output under flexible prices in both countries. As in our one-country model, we solve the mode using the methodology of Guerrieri and Iacoviello (2015). Their solution strategy involves a first-order perturbation to the model, which is applied piecewise so as to accommodate the ZLB. We only ever impose the ZLB in one country or the other. The main advantage of using the methodology of Guerrieri and Iacoviello (2015) is that it is able to accommodate the number of state variables implied by medium-scale DSGE models. In our case, the number of state variables is even larger because of the second country.