Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Short Takes on Monetary Policy Strategy: An Introduction to Some Basic Concepts

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2018-089

Please cite this paper as:
Clouse, James A. (2018). "Short Takes on Monetary Policy Strategy: An Introduction to Some Basic Concepts," Finance and Economics Discussion Series 2018-089. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2018.089.

Short Takes on Monetary Policy Strategy:

An Introduction to Some Basic Concepts

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October, 2018

[1] The views expressed in this note are those of the author and do not in any way represent an official view of the Board of Governors or its staff. Ellen Meade, Matt Luecke, David Skidmore and Joe Pavel provided very helpful feedback on earlier drafts.

Takeoff

Over recent decades, central banks have made enormous strides in enhancing transparency around many elements of the formulation and conduct of monetary policy. Still, even for an audience of seasoned policy experts, providing clarity about aspects of monetary policy strategy is a daunting task and all the more so when the audience extends to the public at large. This collection of short notes attempts to take a small step in fostering more inclusive discussion of monetary policy strategy by presenting some standard results in a way that may be useful as an introduction to basic concepts for students and nonspecialists. The discussion below relies heavily on diagrams to illustrate the main points while keeping equations to a minimum. Some mathematical details are provided in the appendix.

The paper is organized as a collection of "short takes" on a series of related issues. Take 1 presents a bare bones "baseline" economic model. Take 2 introduces a standard central bank "loss function" and discusses optimal monetary policy in the context of the baseline model. Take 3 considers how optimal policy in the baseline model is affected under different assumptions about the structure of the economy. Take 4 examines how alternative specifications of the central bank's loss function affect optimal policy. Take 5 introduces uncertainty in the baseline model and reviews two key results—the certainty equivalence principle and the policy attenuation principle. Take 6 examines how alternative specifications of the loss function affect optimal policy under uncertainty. Take 7 looks at optimal policy with an objective function that places a great deal of weight on tail risks. Take 8 extends the baseline economic model to include two periods and discusses the zero lower bound problem. Take 9 uses the extended model to discuss risk management incentives for monetary policy in connection with the zero lower bound. Take 10 discusses some special topics in the context of the baseline model and the final section concludes with a few "takeaways."

Take 1: A Baseline Economic Model

"The Federal Open Market Committee decided today to raise its target for the federal funds rate by 25 basis points to 4-1/2 percent. Although recent economic data have been uneven, the expansion in economic activity appears solid. Core inflation has stayed relatively low in recent months and longer-term inflation expectations remain contained. Nevertheless, possible increases in resource utilization as well as elevated energy prices have the potential to add to inflation pressures. The Committee judges that some further policy firming may be needed to keep the risks to the attainment of both sustainable economic growth and price stability roughly in balance."

FOMC Statement, January 2006

The FOMC statement above embeds a number of recurring themes in monetary policy analysis—a focus on economic activity and inflation, concerns that tightening resource utilization together with supply shocks (in the form of higher energy prices) could lead to inflation pressures, and careful monitoring of inflation expectations. In the case above, these observations and concerns led the FOMC to judge that policy should be tightened to foster progress toward the Federal Reserve's longer-run goals of maximum employment and price stability. A number of these basic ideas can be illustrated in the context of a very simple baseline model of the economy. The discussion below sketches the key features of this framework.

1.1 Overview

In the baseline model, inflation and output are determined by two basic equations—a Phillips curve and a so-called investment-savings (IS) equation. The Phillips curve describes the behavior of inflation in response to the level of the "output gap." The output gap is a measure of the cyclical position of the economy and is defined as the difference between the actual level of output and the level of potential output. Potential output, in turn, is the level of output that can be produced by the economy when labor and capital are fully employed. When the economy is very strong, the output gap can be large and positive. When the economy is very weak, the output gap is large and negative. In much of what follows, it is convenient to focus on a related measure of the output gap defined in terms of the unemployment rate. The unemployment rate gap is defined as the difference between the current level of the unemployment rate and the so-called "natural rate" of unemployment defined as the level of the unemployment rate when the economy is at "full employment." The unemployment rate gap is large and positive when the economy and labor market are very weak, and it is large and negative when the economy and labor market are very strong.

The IS curve describes the behavior of aggregate demand in the economy. As with the formulation of the Phillips curve, it is convenient to discuss the IS curve in terms of the unemployment rate gap. The IS curve in this model thus relates the level of the unemployment rate gap to the central bank's choice of the policy rate relative to the "neutral" policy rate. An increase in the policy rate gap—the spread between the policy rate and the neutral policy rate—depresses aggregate demand and pushes the unemployment rate gap up. Conversely, all else equal, a reduction in the policy rate gap provides some impetus to aggregate demand, lowers the unemployment rate, and pushes the unemployment rate gap lower.

1.2 Phillips Curve

A simple version of the Phillips curve expresses the deviation of inflation from the central bank's target value as a function of three basic factors. As shown in equation (1), the first of these factors is the unemployment rate gap. An unemployment rate gap, $\hat{u}$, that is positive—unemployment is above the natural rate—puts downward pressure on inflation and vice versa.

$$\hat{\pi} = \pi - \pi^* = -\tau\hat{u} + (\pi^e - \pi^*) + \varepsilon \qquad (1)$$

The "slope" of the Phillips curve, τ, measures the sensitivity of inflation to the unemployment rate gap. The second key factor affecting inflation is the level of inflation expectations relative to the central bank's target value. If the public is convinced that the Federal Reserve will take actions necessary to achieve its inflation target, this term should generally be equal to zero.

The final factor is the "error term" which captures various demand and supply shocks that can affect inflation on a temporary basis. For example, adverse oil price shocks might temporarily boost the level of inflation. Conversely, new technologies that allow firms to produce more with the same level of inputs can put temporary downward pressure on inflation.

In more advanced models, the Phillips curve can incorporate forward-looking elements. For example, inflation today could depend on expectations of the future levels of aggregate demand. In addition, inflation expectations can be modelled in various ways, either as a function of other observed variables or as a so-called "rational expectation" of future values of inflation. In the baseline model, we simply treat inflation expectations as fixed or exogenous. Even with this simple setup, inflation expectations play an important role in the determination of actual inflation. When the central bank is fully credible and the public anticipates that inflation will equal the central bank's target in the longer-run, inflation expectations and the central bank's inflation target should be equal and the actual level of inflation is affected only by the level of unemployment rate gap.

In figure 1.1, the Phillips curve is shown by the downward sloping green line. Here the unemployment rate gap is shown along the x-axis and the inflation deviation from target is measured along the y-axis. When there are no shocks to the Phillips curve and when expected inflation aligns with the central bank's target rate, the Phillips curve passes through the origin. When there are "cost push" shocks to inflation captured by the error term, ε, the Phillips curve shifts up. Conversely, if there are shocks that depress inflation, the Phillips curve shifts down. Similarly, if inflation expectations move above or below the central bank's target rate, the Phillips curve shifts up or down.

1.3 IS Curve

The second key relationship in the model is a simplified IS curve that relates the level of the unemployment rate gap to the interest rate gap. The interest rate gap term is the spread between the actual real rate of interest given by $i - \pi^e$ and the so-called neutral real rate of interest given by r^*. Intuitively, when the central bank raises the level of the real federal funds rate above the long-run neutral rate, this tends to depress aggregate demand and restrain employment and vice versa.

$$\hat{u} = un - un^* = \alpha(i - \pi^e - r^*) + \eta = \alpha(i - (\pi^e - \pi^*) - (r^* + \pi^*)) + \eta \qquad (2)$$

In this equation, un and un^* are the actual observed unemployment rate and the "natural" rate of unemployment, respectively. Under this specification, the level of the unemployment rate gap, $\hat{u}$, is unaffected by the actual level of inflation. As a result, the IS curve is shown as a vertical line in figure 1.2. When there are adverse shocks to the economy, $\eta > 0$, the IS curve shifts to the right in this diagram. And conversely when there are favorable shocks to the economy, the IS curve shifts to the left. The "slope" of the IS curve with respect to the policy rate gap, α, measures the sensitivity of the unemployment rate to deviations of the real rate from the neutral real rate, r^*.

1.4 Determination of Inflation and Unemployment and the Stance of Policy

The Phillips curve and IS curve equations together determine the unemployment rate and inflation rate in the model. Figure 1.3 displays both curves with the unemployment rate deviation shown on the horizontal axis and the inflation gap shown on the vertical axis. For a given setting of the policy rate, inflation and output are determined at the intersection of these two curves at a point shown by the black dot.

As shown by the dashed green line, if there is a cost push shock to the Phillips curve, inflation rises along the existing IS curve to a point shown by the red dot. If there is an adverse shock to the IS curve, inflation falls and the unemployment rate rises moving down along the Phillips curve to the point shown by the blue dot.

The other factor that affects the position of the IS curve is the level of the expected real interest rate relative to the neutral real interest rate, r^*. As shown in figure 1.4, a policy tightening boosts the level of the real interest rate relative to the neutral rate and pushes the IS curve to the right. Inflation falls and unemployment rises in this case as the economy moves from the initial equilibrium shown by the black dot to the new equilibrium shown by the red dot. A policy easing has the reverse effect; the unemployment rate gap falls and the inflation gap rises along the Phillips curve as the economy moves to a new equilibrium shown by the green dot.

Policymakers can thus utilize changes in the policy rate gap to offset or mitigate the effects of other types of shocks. In the case of an adverse shock to the IS curve, the central bank can completely offset the shock by a suitable reduction in the level of the policy rate. In the case of an adverse shock to the Phillips curve, the policymaker can tighten policy to mitigate the effect of the inflation shock on the level of inflation.

Of course, the real world is far more complicated than this simple model. But even through the narrow lens of this baseline model, the ability of the central bank to conduct monetary policy effectively depends critically on the confidence of the public in the central bank's determination to achieve its inflation target over time. For example, as noted above and as shown in figure 1.5, in response to an adverse shock to the IS curve, the central bank can ease policy in a way that cushions the effect of the shock on the unemployment rate. In the diagram, the adverse economic shock would initially push the unemployment rate up and the inflation rate down as the economy moved along the Phillips curve from the black dot to the red dot. In response, the central bank could ease policy and return the IS curve to its initial position, moving the economy back up to the black dot with the economy at full employment and inflation at target. However, if the public believes that this policy easing signals that the central bank is less committed to its inflation target, inflation expectations could rise, and the Phillips curve could shift up as shown by the green dotted line. Moreover, the rise in inflation expectations would lower the real policy rate and shift the IS curve further to the left. In that case, as shown in figure 1.6, the initial effect of policy easing could be successful in returning the economy to full employment. However, as inflation expectations adjusted, the level of inflation would move higher and the unemployment rate would fall below the natural rate as indicated by the yellow dot. A key implication is that the ability of the central bank to achieve its employment and price stability objectives depends critically on the public's confidence that the central bank will take the actions necessary to maintain stable inflation in the longer run. When the public is fully confident in the central bank's commitment to stable inflation, the central bank will have greater freedom to take actions necessary to stabilize the real economy.

Figure 1.1: Phillips Curve

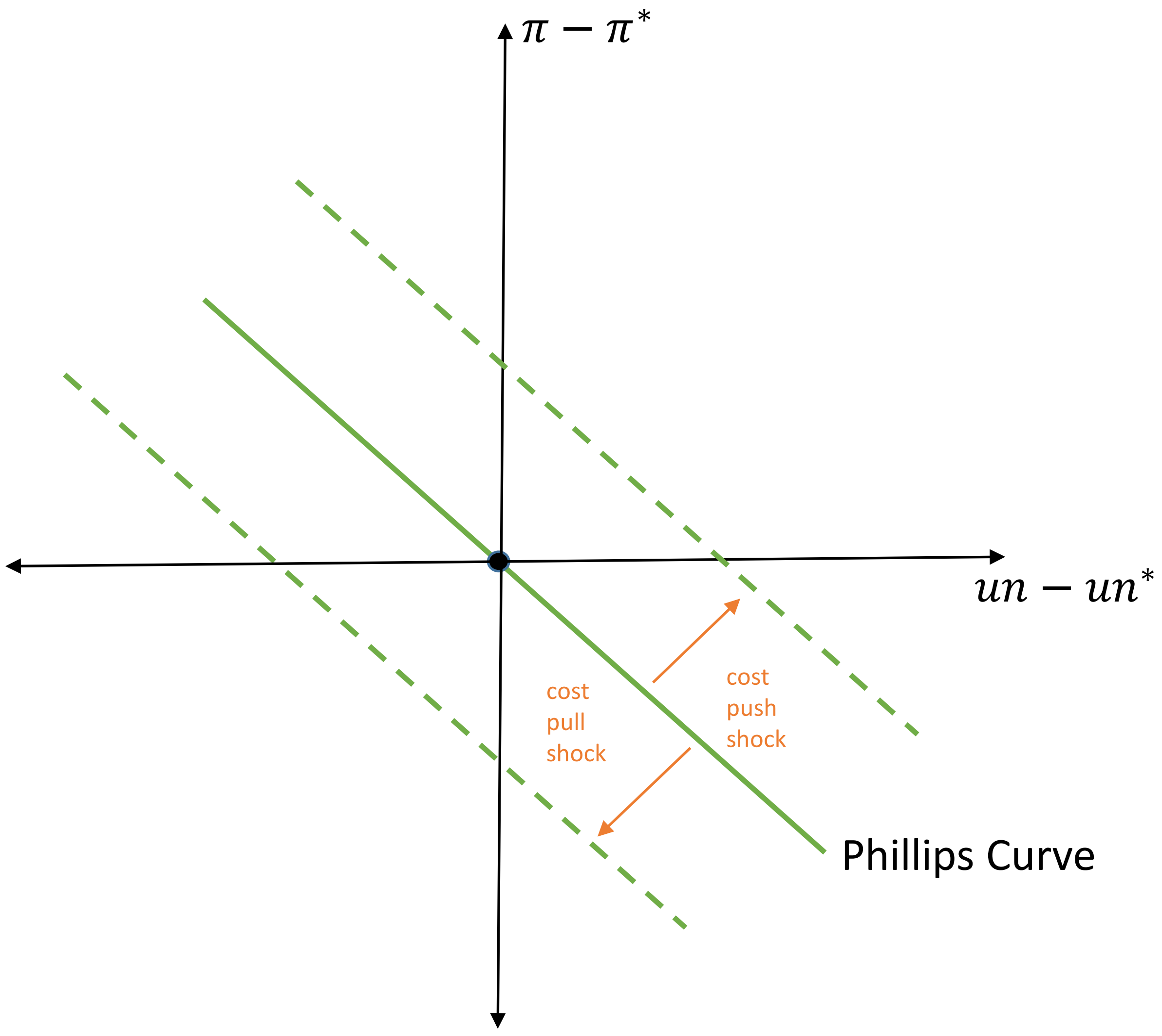


Figure 1.2: IS Curve

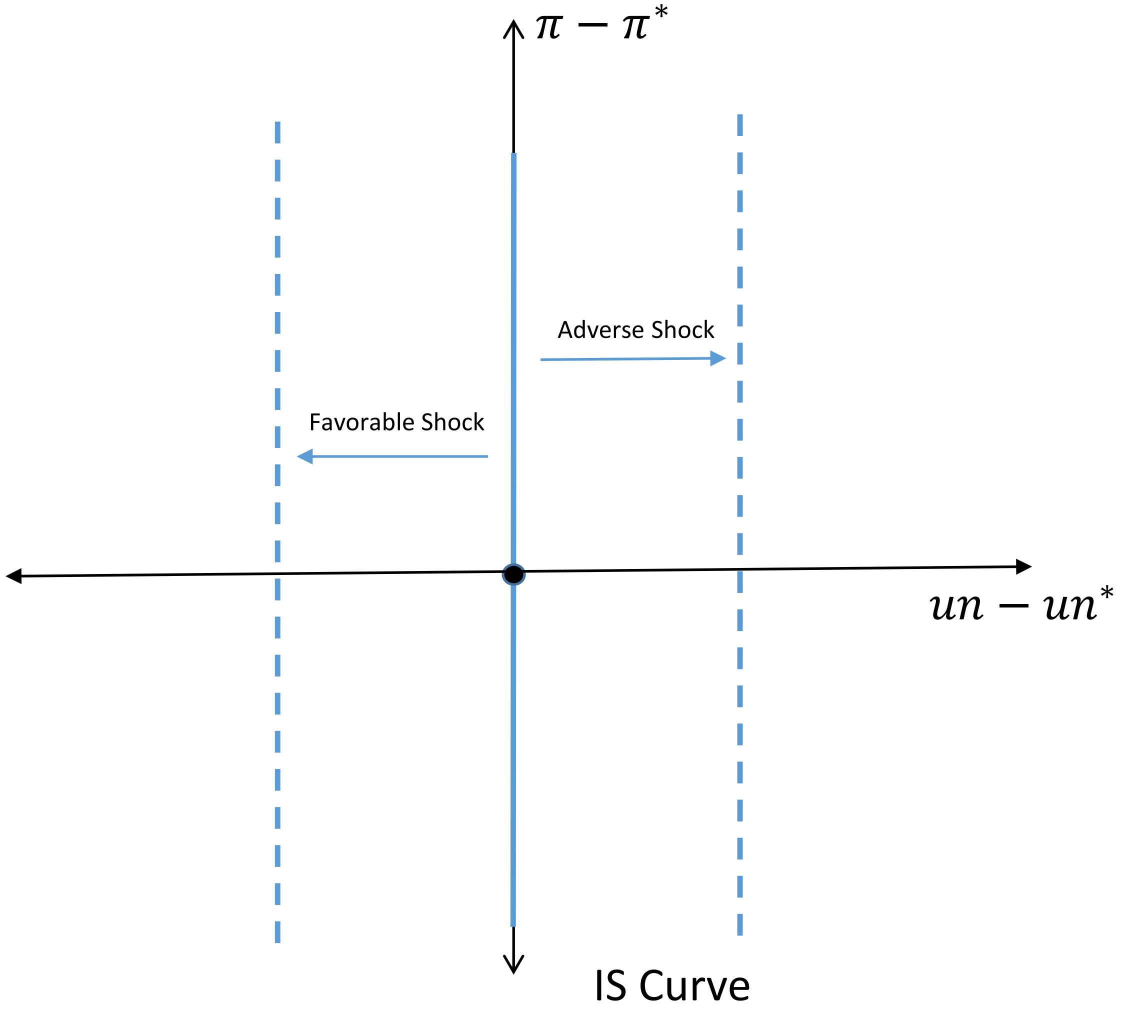


Figure 1.3: Determination of Inflation and Unemployment

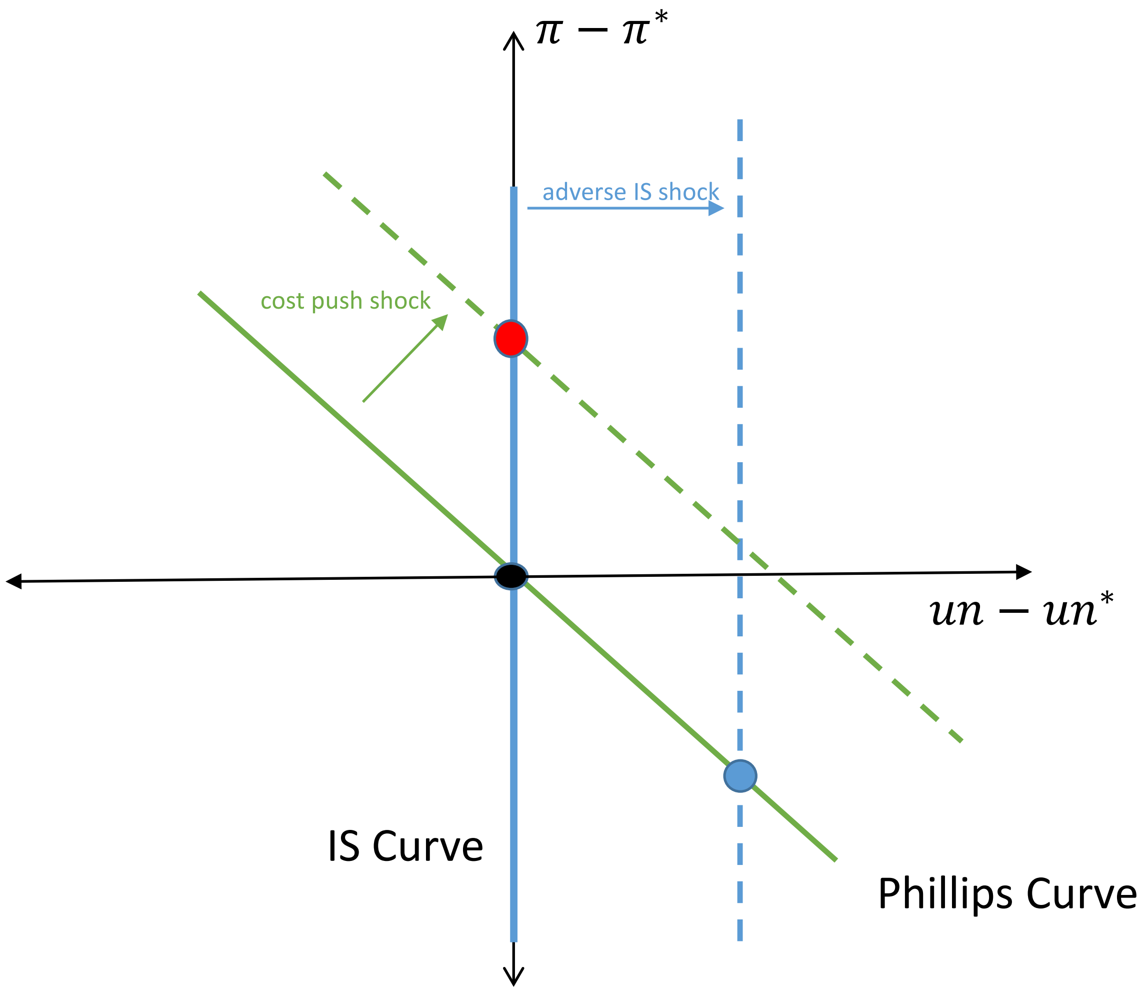


Figure 1.4: Effect of Policy Easing and Tightening

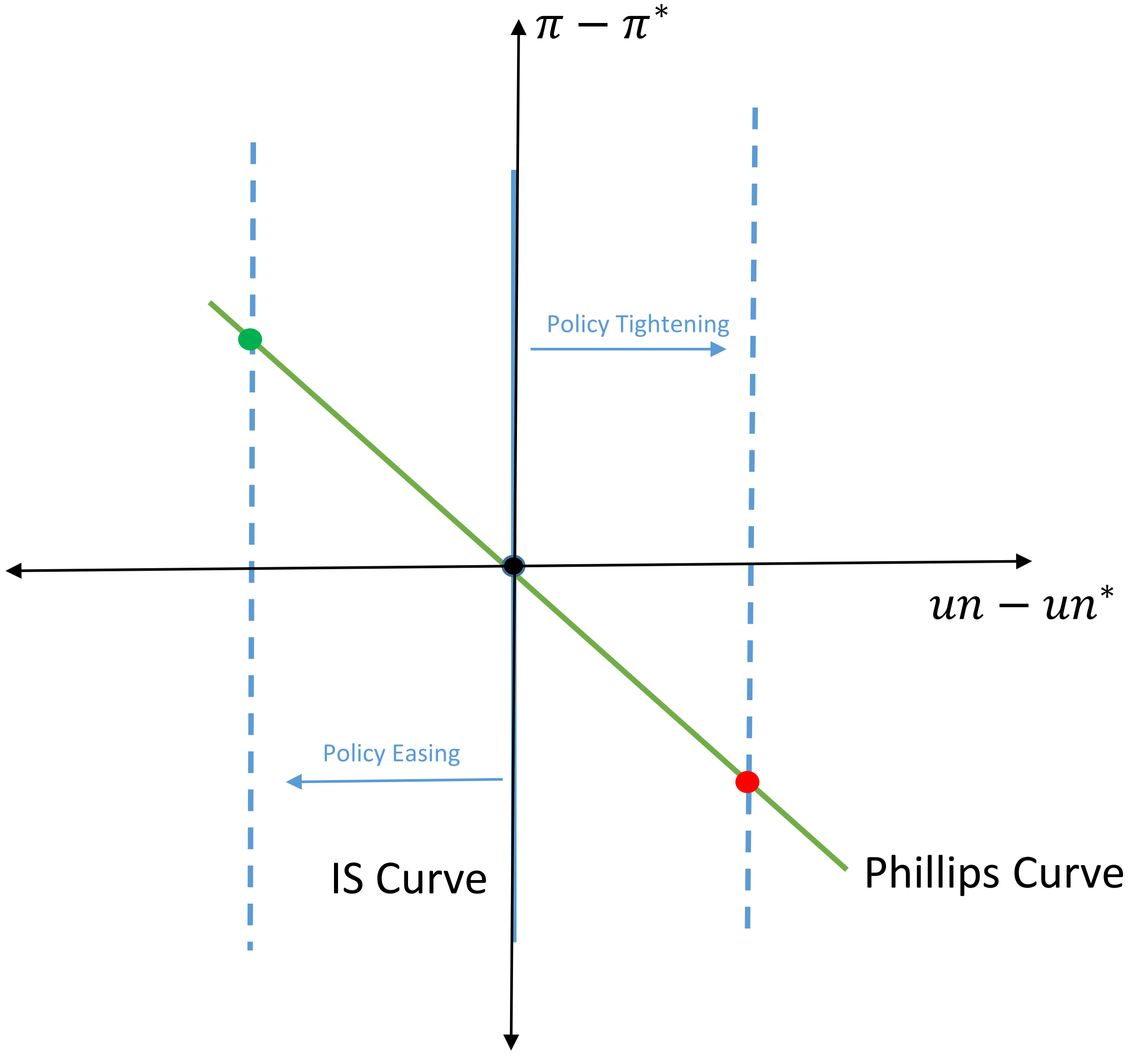


Figure 1.5: Countering an Adverse Economic Shock

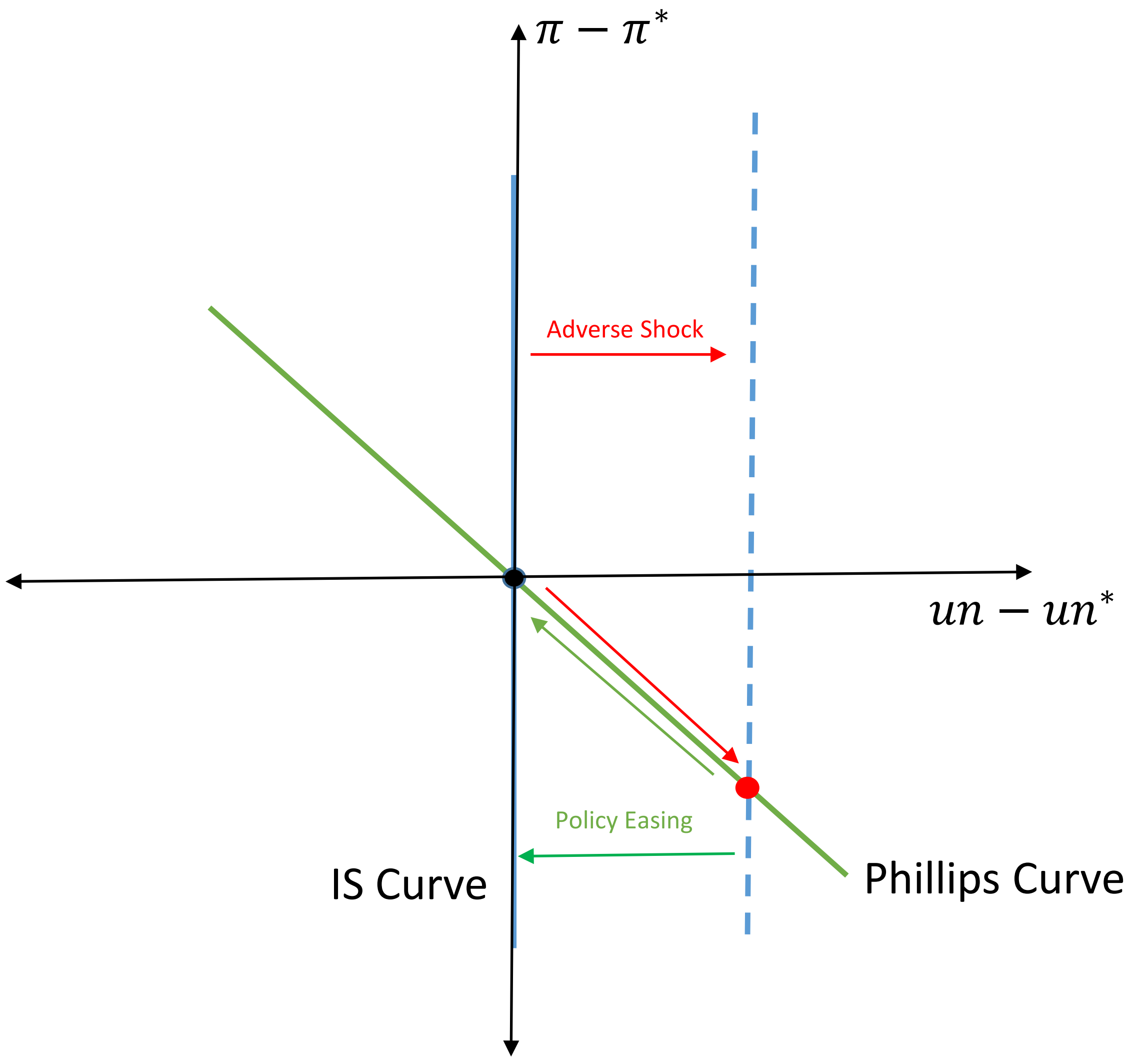
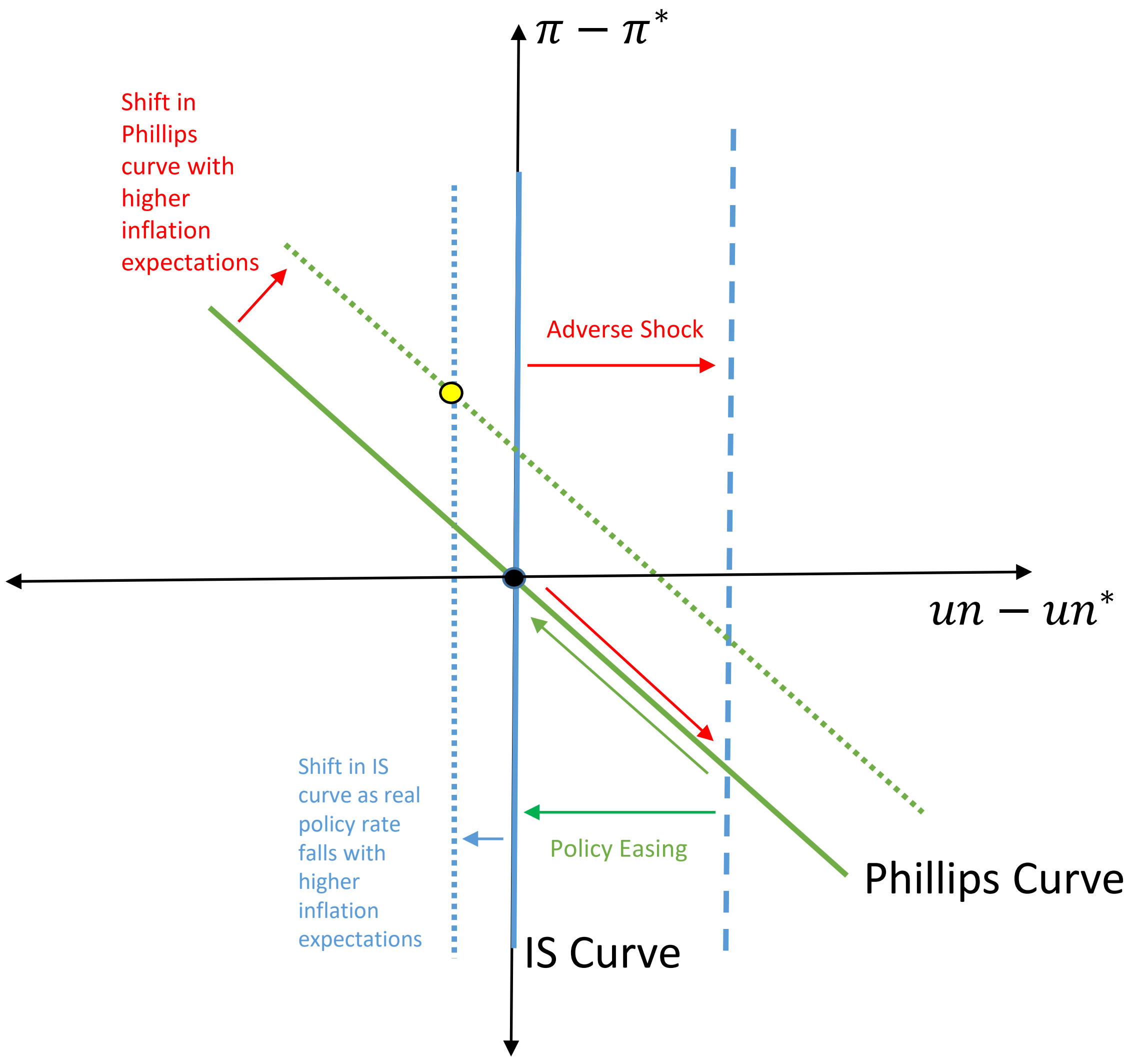


Figure 1.6: Inflation Expectations and Policy



Take 2: Optimal Policy in the Baseline Model

"In setting monetary policy, the Committee seeks to mitigate deviations of inflation from its longer-run goal and deviations of employment from the Committee's assessments of its maximum level. These objectives are generally complementary. However, under circumstances in which the Committee judges that the objectives are not complementary, it follows a balanced approach in promoting them, taking into account the magnitude of the deviations and the potentially different time horizons over which employment and inflation are projected to return to levels judged consistent with its mandate."

From FOMC Statement of Longer Run Goals and Monetary Policy Strategy

In these few words, the Federal Open Market Committee (FOMC) articulates both an overarching policy strategy aimed at achieving the long-run goals of maximum employment and stable prices, and also describes its short-run tactics in conducting policy to minimize deviations of inflation and employment from their respective goals. Those few words, however, embed many layers of meaning. Indeed, the strategy and tactics of monetary policy are the focus of a vast and complicated economic literature. As noted above, in the baseline model, the central bank can adjust the position of the IS curve by changing the value of the policy rate. In doing so, it can choose to conduct policy so that inflation and output are determined at any point along the Phillips curve. But which point should the central bank choose? And how would it do so under a "balanced approach"?

A standard approach in determining the point along the Phillips curve that the central bank should choose involves specifying a "loss function" for the central bank. Similar to the language in the FOMC's statement of longer run goals and monetary policy strategy, often the loss function is stated in terms of squared deviations of inflation and the unemployment rate from their respective longer-run goals (these deviations are denoted by $\hat{\pi}$ and $\hat{u}$, respectively, in equation (3)). Intuitively, the central incurs a "loss" that is an increasing function of the magnitude of the deviation of inflation and the unemployment rate from their respective target or longer-run levels.

$$L = (\tfrac{1}{2})(\hat{u}^2 + \hat{\pi}^2) \qquad (3)$$

A plot of this loss function is shown in figure 2.1. The lowest loss is equal to zero at the point at which the unemployment rate and inflation gaps are both equal to zero. At other points, the losses are higher and the loss function specified in (3) has the shape of a cone with a rounded tip. The "level curves" for this loss function (also sometimes called indifference curves) are shown by the cross section circles in that diagram at particular levels along the z-axis. As shown in figure 2.2, it is convenient to "project" these level curves into the plane below with the unemployment rate gap on the x-axis and the inflation gap on the y-axis. Each level curve maps out all combinations of inflation and output deviations that result in the same loss to the central bank. So, for example, inflation and unemployment deviation pairs of (1 ,1), (-1, -1), (1,-1), and (-1,-1) would all lie on the same indifference curve corresponding to a loss of 1 (by equation 3). Policymakers should choose a point along the Phillips curve with an indifference curve that is closest to the origin—that is, the lowest possible loss. As shown in figure 2.3, that would occur at points like those shown by the black dots where the indifference curve is just tangent to the Phillips curve. To achieve that point, the central bank would adjust the level of the policy rate to push the IS curve so that it intersects the Phillips curve at that point of tangency. The set of all possible tangency points (corresponding to potential shocks to the Phillips curve) defines the set of all possible optimal economic outcomes. As shown by the solid orange line in figure 2.4, the locus of all such optimal economic outcomes in the baseline model is a straight line passing through the origin.

Each point along the optimal economic outcomes line corresponds to a particular setting of the central bank's policy rate. The optimal policy rate depends on all of the "exogenous" variables in the model including the central bank's inflation target, the level of the neutral real interest rate, the deviation of inflation expectations from the central bank's target rate, and the shocks to the IS curve and the Phillips curve. In the baseline model, the central bank is assumed to observe the shocks to the IS curve and Phillips curve prior to making its choice about the policy rate. The relationship between the policy rate and these exogenous variables is sometimes referred to as the policy "reaction function." The formula for the optimal policy reaction function in the baseline model is:

$$i - i^* = \left(1 + \frac{\tau}{\alpha(1+\tau^2)}\right)(\pi^e - \pi^*) + \frac{\tau}{\alpha(1+\tau^2)}\varepsilon - \frac{1}{\alpha}\eta \qquad (4)$$

As one might expect, the policy reaction function calls for the central bank to tighten policy in response to positive shocks to the Phillips curve, ε, and to ease policy in response to positive shocks to the IS curve, η. (Recall that the IS curve in equation (2) is based on the unemployment rate gap rather than the output gap, so a positive shock to the IS curve is one that depresses aggregate demand and boosts the unemployment rate). The policy rate also responds positively to increases in inflation expectations and with a coefficient greater than 1. The magnitude of the coefficient on expected inflation is an example of the "Taylor principle" which suggests that the policy rate must respond by more than one for one in reaction to increases in inflation. As shown in equation (2), in the baseline model, the policy rate must increase by as much as an increase in expected inflation, π^e, just to leave the unemployment rate unchanged. The central bank must push the policy rate up by more than this though to also counter the upward pressure on inflation stemming from the increase in inflation expectations.

It's also worth noting that the coefficients in the optimal policy reaction function in equation (4) are functions of the parameters of the IS curve and the Phillips curve. In general, lower values for the slope of the IS curve, α, imply that the central bank must move the policy rate by more in response to various shocks. That's because the lower values of this coefficient imply that any given setting of the policy rate has a less pronounced effect on the economy. So the central bank must generally move the policy rate by more in response to various shocks in order to achieve the desired outcome.

It is often helpful to have an easy graphical way of thinking about the policy reaction function. One way to do so is to note from the IS curve that the desired amount of policy stimulus or restraint, hereafter referred to as the policy stance (PS), is directly related to the optimal choice of the unemployment rate. From the IS curve, the desired PS is given by:

$$\text{PS} = \alpha(i - \pi^e - r^*) = \hat{u} - \eta \qquad (5)$$

Using this relationship, figure 2.5 plots the usual optimal economic outcomes line (in orange) along with a similar PS line (heavy purple dotted line) that shows the optimal desired level of economic stimulus or restraint associated with any given optimal outcome. This line is simply a horizontal translation of the optimal economic outcomes line by the amount of the shock to the IS curve. In this case, the shock to the IS curve is assumed to be positive, so the optimal policy stance line is a translation to the left. To read the policy stance associated with any point on the optimal economic outcomes line, one can find the point on the PS line that corresponds to the inflation rate determined by the intersection of the Phillips curve and the optimal economic outcomes line. In the diagram, the optimal levels of the unemployment rate gap and inflation rate gap are determined at the black dot. The corresponding policy stance associated with this outcome is shown by the heavy purple triangle. In this case, even though an inflation shock has pushed the Phillips curve outward, the central bank eases policy a little because there has also been a simultaneous adverse shock to the IS curve. The light purple dotted line shows the optimal policy stance line that would correspond to a case in which the IS curve experiences a very favorable shock that would otherwise push the unemployment rate much lower. In this case, as shown by the light purple triangle, the central bank would need to tighten policy very aggressively to achieve the optimal economic outcomes denoted by the black dot.

2.1 Key Characteristics of Optimal Policy in the Baseline Model

For reference, some basic properties of optimal policy in the baseline model are listed below.

Property 1: Optimal Unemployment and Inflation Combinations fall in Quadrants 1 and 3. Referring to the optimal economic outcomes line in figure 2.4, the optimal levels of inflation and unemployment occur only in quadrants 1 and 3. That is, there are no shocks to the Phillips curve or IS curve that would lead the central bank to choose to be in quadrants 2 and 4. In quadrants 2 and 4, there is no policy tradeoff. If inflation is above target and the unemployment rate is below the natural rate as in quadrant 2, the central bank should clearly tighten policy according to this model. And conversely, if inflation is below target and the unemployment rate is well above the natural rate, as in quadrant 4, the central bank should clearly ease policy.

Property 2: Optimal Choices for the Unemployment Rate and Inflation are Unaffected by Shocks to the IS Curve.

A second key result in this model is that aggregate demand shocks do not affect the optimal outcomes for inflation and unemployment. Indeed, borrowing the language from the FOMC's statement of longer-run goals, the central bank's objectives are "complementary" in the case of a shock to the IS curve because the required policy response reduces the magnitude of both the deviation of the unemployment rate and the deviation of the inflation rate from their respective longer run goals. The central bank is extraordinarily powerful in the baseline model and can fully offset shocks to the IS curve through the choice of the policy rate. Moreover, there are no costs or limits in changing the policy rate as assumed, for example, in some other common models of optimal policy. As a result, the central bank can change the policy rate by as much as necessary in the current period to achieve the optimal economic outcome. The central bank is more limited in its ability to offset shocks to the Phillips curve because it can only do so, according to the model, indirectly through influencing the unemployment rate gap. As a result, in the baseline model, the optimal economic outcomes line is largely determined by shocks to the Phillips curve and how the central bank views the tradeoff between inflation and unemployment. Of course, even though the optimal levels of the unemployment rate and inflation are unaffected by shocks to the IS curve, the policy rate itself is very much influenced by these shocks. For example, if the economy experiences a very adverse shock to the IS curve, the central bank would need to ease the stance of monetary policy by a large amount in order to achieve the "optimal" economic outcome.

Property 3: Optimal Choices of Inflation and the Unemployment Rate are Positively Correlated.

Another basic property of the model is that the optimal economic outcomes line—the collection of optimal combinations of inflation and unemployment—implies that inflation deviations and unemployment rate deviations are *positively* correlated even though the Phillips curve posits a negative relationship between the unemployment rate gap and inflation. This highlights a key practical difficulty in identifying a Phillips curve. Although the structural Phillips curve slopes downward (higher unemployment leads to lower inflation), optimal policy can alter the observed correlation of inflation and the unemployment rate gaps in the data, making it very difficult to identify the underlying Phillips curve.

Property 4: Optimal Choices for Inflation and Unemployment Inherit the Distributional Properties of Inflation Shocks. If Inflation Shocks are Symmetrically Distributed, then Inflation and Unemployment will be Symmetrically Distributed around Their Respective Long Run Goals.

The central banks' loss function is symmetric in the sense that it attaches the same costs at the margin to deviations of inflation above and below target that are of equal magnitude. This symmetry is evident in the shape of the indifference curves shown in figure 2.2. In the baseline model, these curves are simply concentric circles. As a result, under optimal policy, the solutions for the optimal unemployment rate gap and inflation gap all fall along the straight line of optimal economic outcomes shown in figure 2.4. The mean values of inflation and the unemployment rate over time are thus are equal to their respective target levels assuming that the mean value of inflation shocks is equal to zero and the level of inflation expectations is equal to the central bank's target rate. Assuming that inflation shocks are symmetrically distributed around zero, both the optimal inflation rate and optimal unemployment rate would be symmetrically distributed around their respective targets.

Property 5: Optimal Choices for Inflation and Unemployment Rates are Uncorrelated Over Time.

Yet another basic property of the optimal solutions is that there is no "serial correlation" in the optimal outcomes. That is, in each period, the unemployment rate and inflation gaps are simple functions of the observed shock to the Phillips curve in that period. The result stems partly from the very simple form of the economic model but also stems importantly from the assumption that the central bank can freely adjust the level of interest rates to achieve its objectives in the current period. In the baseline model, there is also no serial correlation in the setting of the policy rate. That result depends entirely on the very simple structure of the economy assumed in the model. In particular, absent any lags or other elements in the model that play out over time, the central bank can achieve its desired outcome in each period and every period is a "new" period, unaffected by developments in prior periods.

Property 6: Optimal Policy Results in the Price Level Rising on Average Over Time at the Central Bank's Target Rate and a Variance in the Future Price Level That Increases with the Forecast Horizon.

A final basic property of optimal policy is that the price level evolves over time as a so-called random walk with drift. In each period, the central bank lets bygones be bygones with any past inflation deviations from target. Thus, the price level increases over time at a trend rate given by the central bank's target inflation rate—that's the drift component. In addition, past shocks to inflation tend to cumulate over time—that's the random walk component. As a result, the expected average rate of inflation over any period of time is equal to the central bank's target rate. However, the conditional variance of the price level at any future date increases steadily as the forecast horizon increases.

Figure 2.1: Loss Function and Level Curves

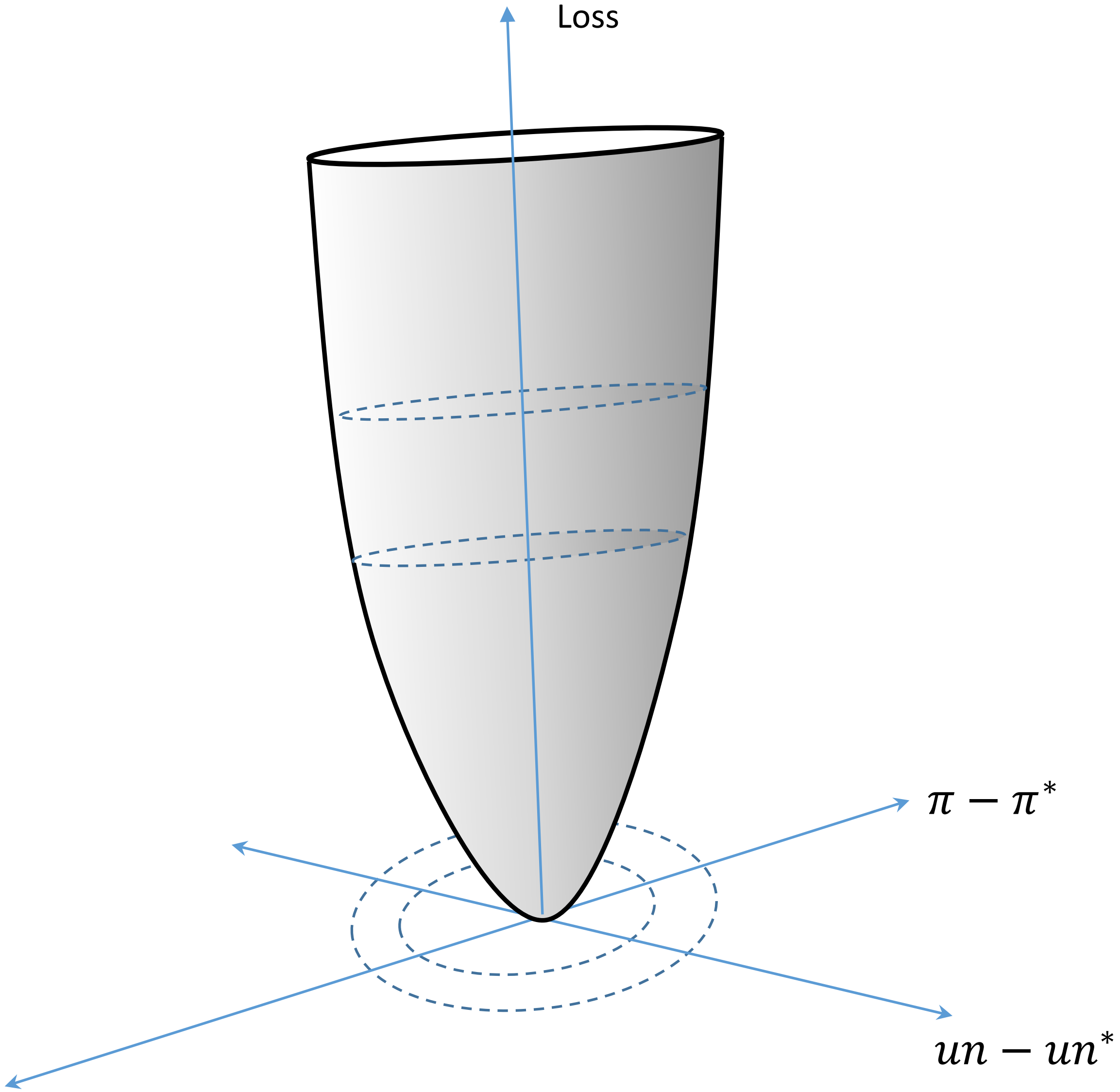


Figure 2.2: Level Curves

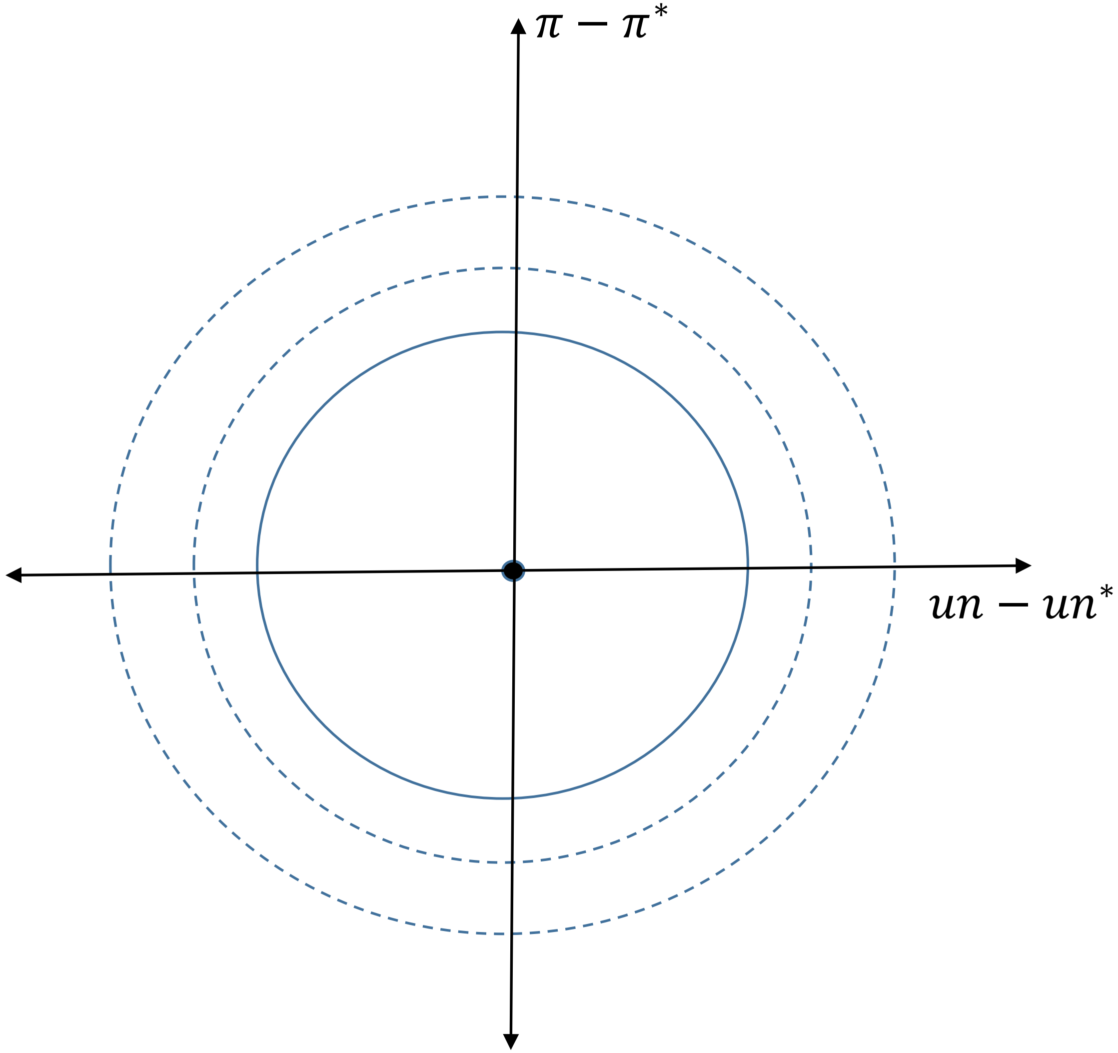


Figure 2.3: Optimal Policy

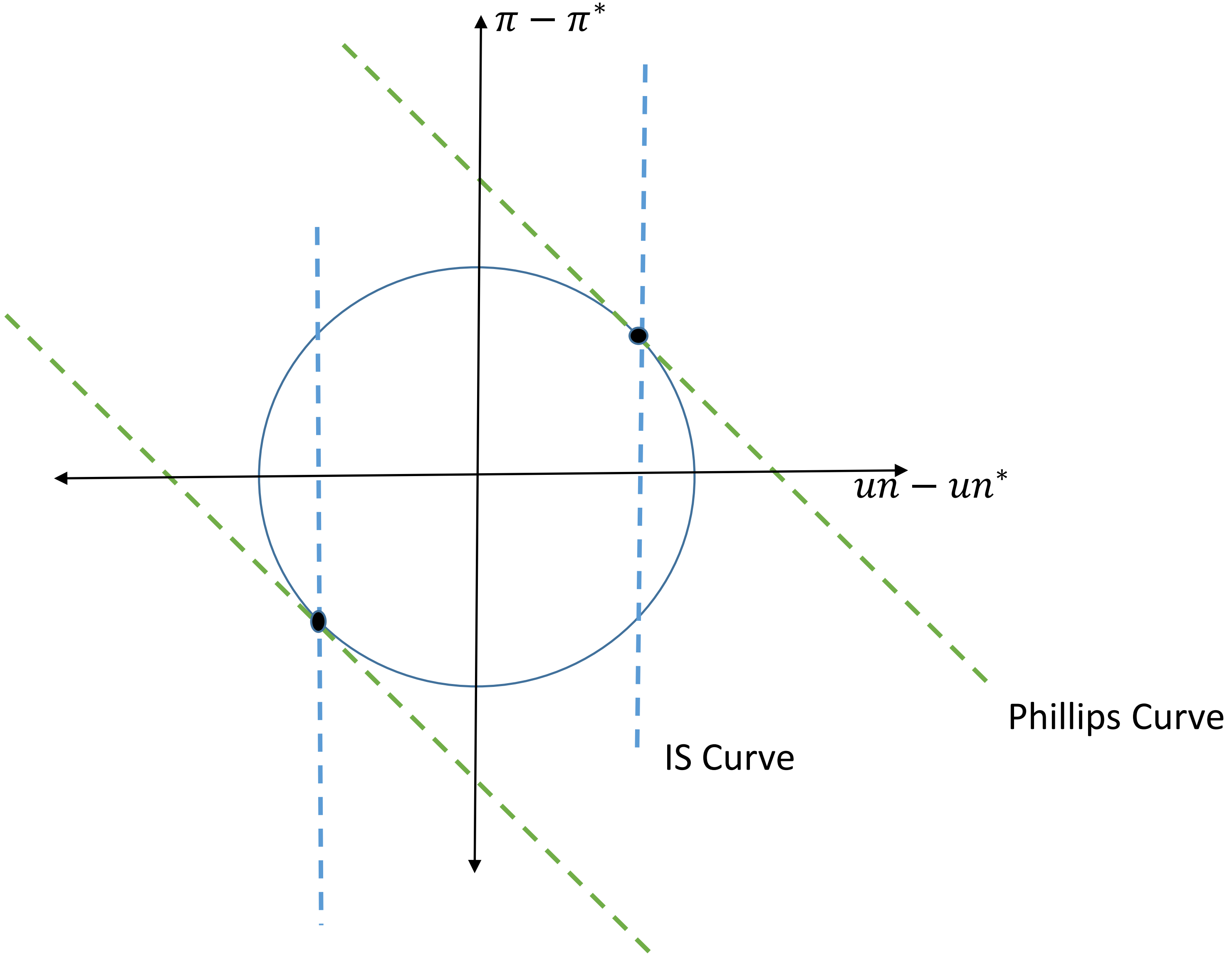


Figure 2.4: Optimal Economic Outcomes Line

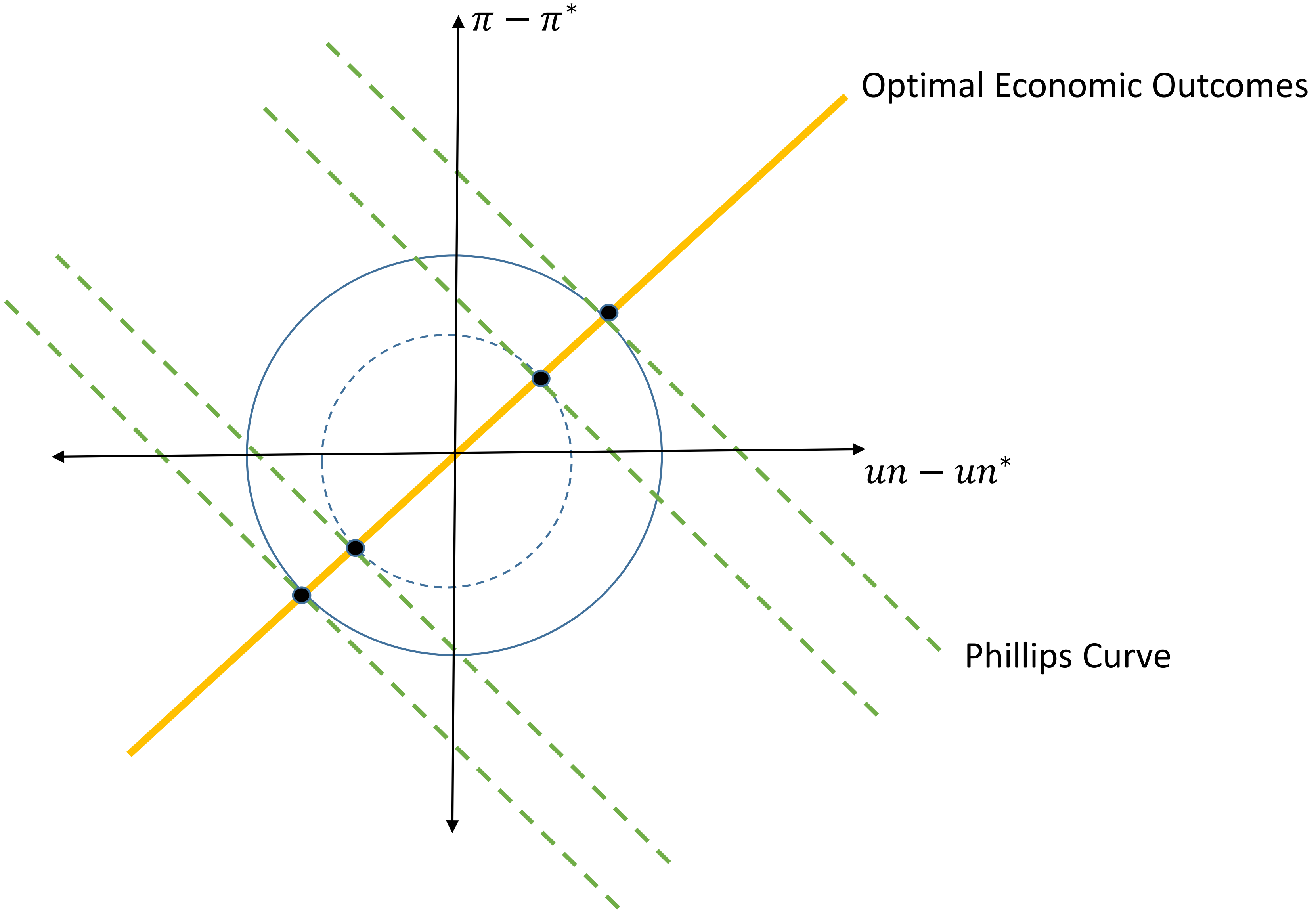
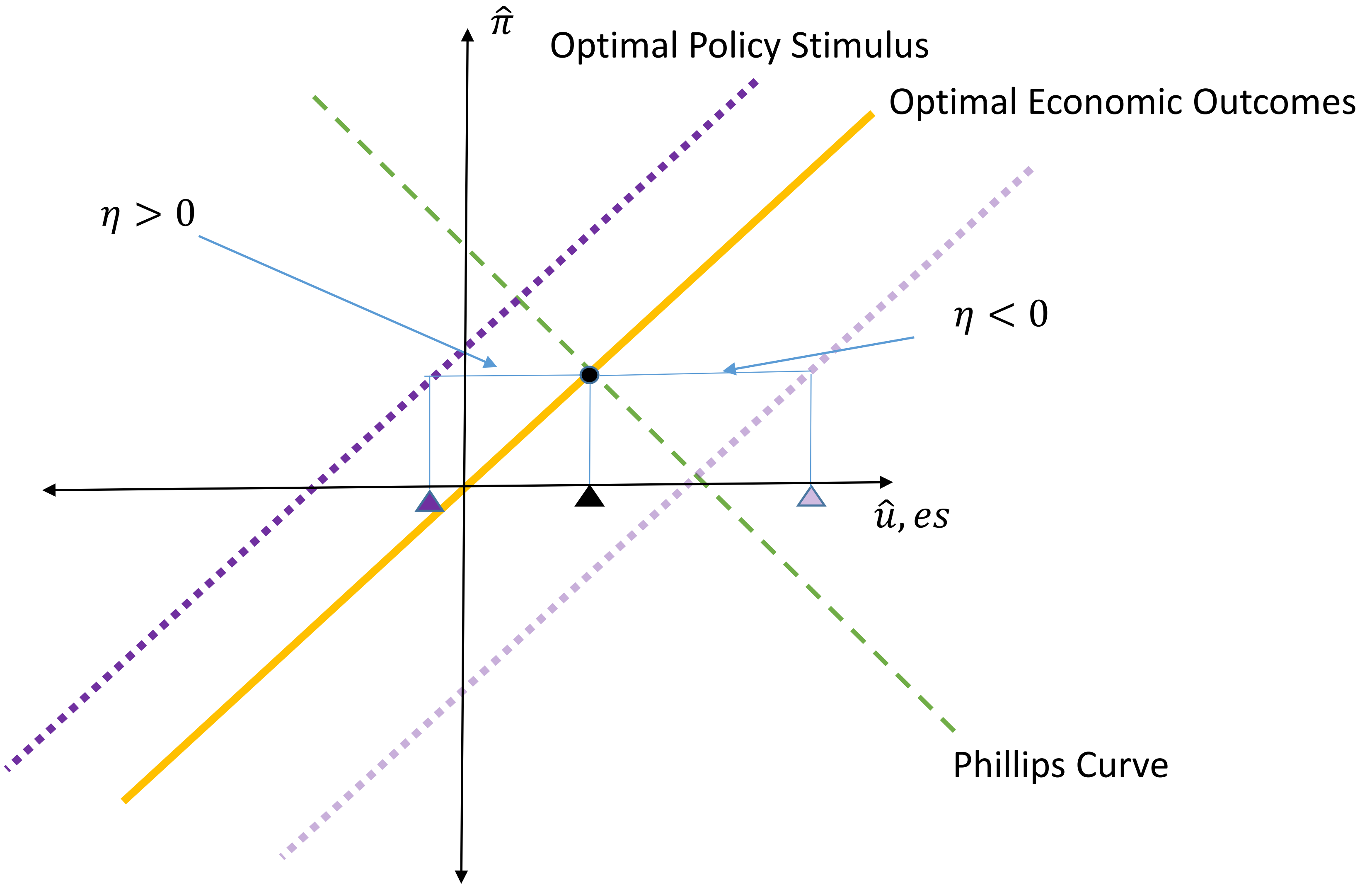


Figure 2.5: Optimal Economic Outcomes and Optimal Policy Stimulus



Take 3: Variations in the Baseline Economic Model

Over recent years, many authors have discussed the "flattening" of the Phillips curve—that is, the apparent decline in the sensitivity of inflation to the level of the unemployment rate gap. In addition, especially during the financial crisis and its aftermath, there were questions about whether the traditional transmission mechanism for monetary policy operating through interest rates had been damped. Below we focus on the way that changes in the Phillips curve and IS curve along these lines affect optimal policy in the baseline model.

3.1 The Curse of a Flat Phillips Curve

As shown in figure 3.1, optimal policy with a flat Phillips curve generally features a steeper optimal economic outcomes line than in the case when the Phillips curve is steeper. With a flatter Phillips curve, any given increase in the level of the unemployment rate gap generates a smaller decline in inflation. As a result, the central bank must engineer a larger increase (or decrease) in the unemployment rate in order to combat positive (or negative) inflation shocks when the Phillips curve is flat than in the case with a steeper Phillips curve.

Since the cost of combatting inflation is higher with a flat Phillips curve, optimal policy allows a larger portion of inflation shocks to show through to the level of inflation. In response to a cost push inflation shock—one that pushes the Phillips curve up and to the right—the optimal stance of policy is easier than would otherwise be the case. In response to a cost pull shock—a downward shift in the Phillips curve—policy is tighter than would otherwise be the case.

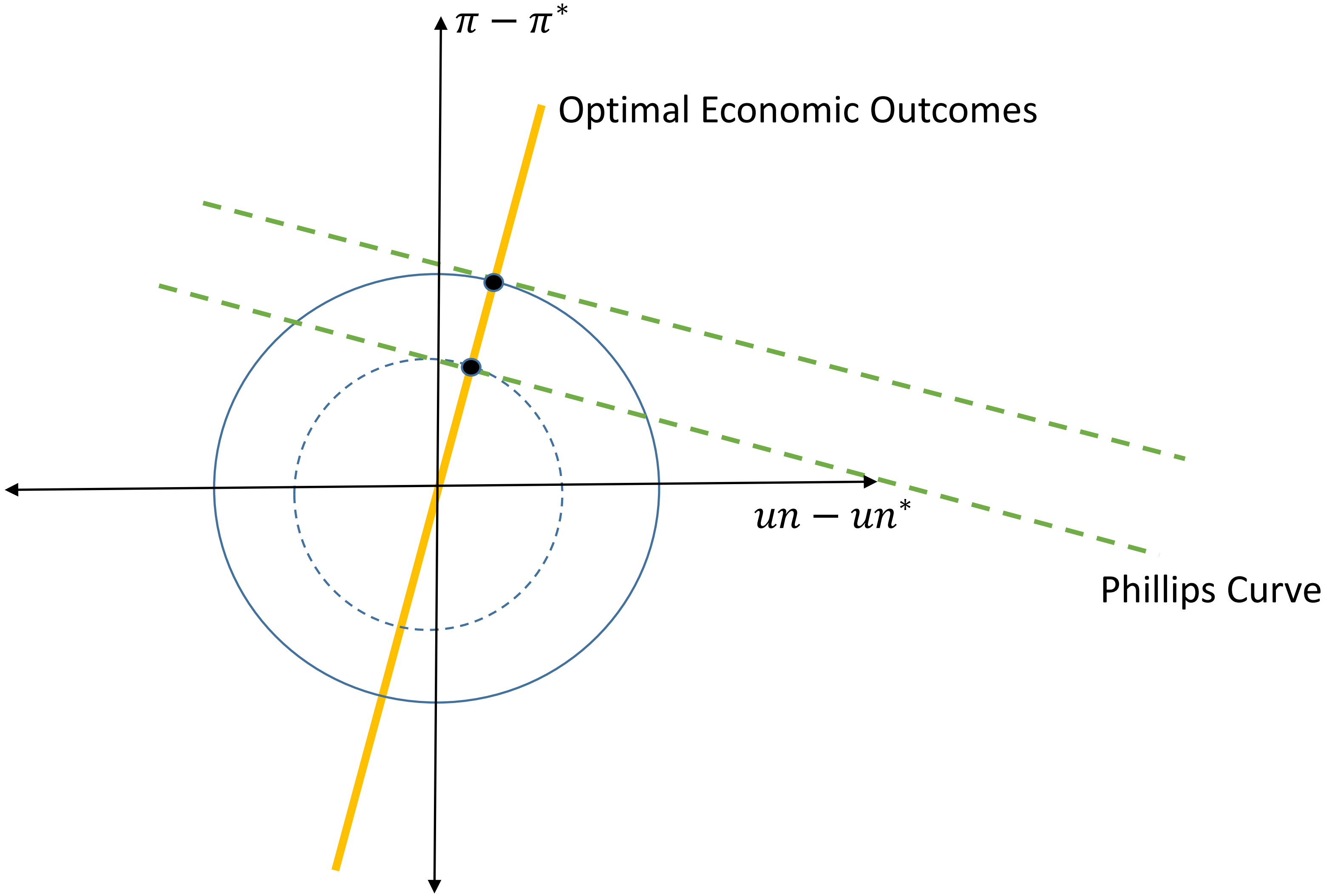
A flatter Phillips curve is generally a curse for policymakers in the baseline model. Because inflation shocks are more costly to combat with a flatter Phillips curve, the expected value of the central bank's loss function in the baseline model increases unambiguously as the slope of the Phillips curve declines.

3.2 Who's Afraid of a Flat IS Curve?

In contrast to changes in the slope of the Phillips curve, changes in α—the parameter describing responsiveness of the unemployment rate to the policy rate or the "slope" of the IS curve—have *no* implications for optimal choices of the inflation and unemployment rate gaps in the baseline model as long the slope remains positive. This result stems from the fact that the central bank adjusts the level of the policy rate in order to achieve the optimal level of the unemployment rate as shown in figure 2.3. As long as the slope is positive, and there are no constraints on the choice of the policy rate, the central bank can simply set the policy rate at the level necessary to achieve the desired position of the IS curve and the optimal combination of inflation and the unemployment rate.

Of course, the value of the parameter α does affect the setting of the policy rate necessary to achieve the optimal inflation and unemployment rates. If α is quite high, the economy is very sensitive to interest rates and the central bank would need to make only small changes in the interest rate gap in order to adjust the level of the unemployment rate. In this world, the variability of the policy rate over time would be relatively low. Conversely, if α is quite low, the economy is insensitive to interest rates and the central bank would need to make relatively large changes in the interest rate gap in order to achieve the desired level of the unemployment rate. By implication, a "flattening" in the IS curve in the baseline model results in an increase in the variability of the policy rate over time even though the optimal choices for the unemployment rate and inflation rate are unaffected by the slope of the IS curve.

Figure 3.1: Flat Phillips Curve and Optimal Policy Choices



Take 4: Variations on the Central Bank Loss Function

The baseline model assumes a very simple specification for the policymaker's loss function. Here we consider the implications of some alternative specifications for the loss function. These alternative specifications are one way to capture the idea that some policymakers are "hawks" while others are "doves." As discussed below, however, these labels can obscure some important issues in cases when inflation and the unemployment rate are above or below their respective targets.

4.1 Symmetric Hawks and Doves

Frequently, policymakers that seem to focus more on inflation deviations from target are described as inflation "hawks." Other policymakers that are especially concerned about the cost of high unemployment are described as "doves." One might view both of these types of policymakers as captured by the loss function in equation (6). Hawks in this setting might be viewed as putting relatively more weight on inflation deviations from target, so they would have an objective function with a weighting parameter γ greater than 1. Doves would put relatively more weight on the unemployment rate gap, so they would have a loss function in which γ is less than 1.

$$L = (\tfrac{1}{2})(\hat{u}^2 + \gamma\hat{\pi}^2) \qquad (6)$$

Figure 4.1 illustrates the shape of the indifference curves for this concept of balanced hawks and doves. When the weighting parameter, γ, is equal to 1 the objective function is that in the baseline model and the indifference curves, shown in figure 4.1, are circles. An inflation hawk, γ greater than 1, would have indifference curves that look like the red dotted oval while the indifference curves for a symmetric dove would look like the green dotted oval in that diagram.

As an aside, it's useful to compare the loss function specification here with the specification for a pure inflation targeting central bank. A pure inflation targeting central bank would only care about the deviations of inflation from target. The level curves for that loss function would be horizontal lines on either side of the horizontal axis as shown in figure 4.2. All points along the x-axis—that is, with an inflation gap of zero—are points where the central bank's loss is zero. For the two Phillips curves shown, the optimal points would then be determined at the point where the Phillips curve intersects the horizontal axis, shown by the blue dots. And the optimal policy response then would be the setting of the policy rate that pushes the IS curve to a point that passes through the blue dot. Figure 4.3 shows how the optimal solutions for the loss function specified in (4) move closer and closer to the solution for the pure inflation targeting bank as the weight on inflation in equation (6) increases.

The optimal economic outcomes line for hawks, shown in figure 4.4, is relatively flat, reflecting the fact that such policymakers are more willing to adjust the stance of policy and the level of the unemployment rate gap in order to keep inflation close to target. Conversely, as shown in figure 4.5, the optimal economic outcomes line for doves is relatively steep in this diagram. That steep slope reflects the fact that doves are more willing to tolerate deviations of inflation from target, if necessary, in order to keep the unemployment rate close to the long-run natural rate.

The solutions for the optimal levels of inflation and unemployment rate deviations are as shown in the appendix. The form of these expressions is very similar to those for the baseline model except that they now incorporate the weighting parameter, γ. Not surprisingly, in response to a cost push inflation shock, high values of γ lead hawkish policymakers to choose a point along the Phillips curve that involves a relatively low inflation rate and relatively high unemployment rate.

It is important to note that this concept of "symmetric" hawks and doves does not imply that hawks always run a "tight" monetary policy while doves always run an "easy" policy. In response to a cost push shock, hawks will indeed tighten policy by more than doves in order to keep inflation closer to target. However, in response to a "cost pull" shock that drives inflation below target, the "hawks" would be more willing to run an easy policy and let the unemployment rate move below the natural rate in order to push inflation back up toward target. The "doves" in this case would run a tighter policy because they would not want to see the unemployment rate fall well below the long-run natural level.

4.12 Properties of Optimal Policy for Symmetric Hawks and Doves

Optimal policy with these types of symmetric objective functions inherits many of the basic properties of optimal policy described above in the baseline model. The expected value of the unemployment rate and inflation are both equal to their long-run targets. And the distributions of both inflation and the unemployment rate around their targets are symmetric assuming that inflation shocks are symmetrically distributed. With a value of γ greater than 1, the variability of inflation falls relative to the variation of the unemployment rate. As shown in appendix equation (4.5), the feedback rule for the policy rate for hawks puts a higher weight on inflation shocks than in the baseline model. The feedback rule for the policy rate for doves puts a lower weight on inflation shocks than in the baseline model.

4.2 Asymmetric Hawks and Doves

The objective functions described above are all "symmetric" in the sense that the loss associated with deviations of equal magnitude for inflation or for unemployment receive the same weight. As shown in figure 4.1, the indifference curves for these objective functions are symmetric about the origin. In contrast, a number of objective functions are "asymmetric" in the treatment of inflation and unemployment deviations. Such objective functions have indifference curves that look "lop sided." Optimal policy with these types of objective functions can have some interesting properties.

One type of asymmetric objective function attaches a higher weight to deviations of inflation above or below the target or to unemployment rate deviations above or below the target. For example, a policymaker that attached more weight to deviations of inflation above target than for those below target could have a loss function with indifference curves like those shown in figure 4.6. Optimal policy in this case depends on whether the inflation shock is positive or negative. When the inflation shock is positive, optimal policy would move to points like those shown by the red dots. With a negative inflation shock of the same magnitude, the optimal policy outcome would be at a point like those shown by the yellow dots. The optimal economic outcomes line would look like that shown by the orange lines in figure 4.6. Thus, asymmetric preferences lead to an optimal economic outcomes line with a "kink" at the point where inflation is equal to the central bank's target rate. The flatter trajectory of the optimal economic outcomes line with positive inflation shocks reflects the fact that the policymaker attaches high costs to above-target inflation and so more aggressively adjusts the stance of policy in order to keep inflation closer to target.

Similarly, one can specify an asymmetric loss function associated with unemployment rate deviations. The deviation of the unemployment rate below the natural rate, un^*, then might have lower weight than an equivalent magnitude deviation with the unemployment rate above un^*. With this type of objective function, the optimal economic outcomes line could look as shown in figure 4.7. Just as in the case of asymmetric inflation hawks, the optimal economic outcomes line for asymmetric unemployment rate doves exhibits a kink at the point where the unemployment rate equals the long-run natural rate. When inflation shocks are positive, the optimal economic outcomes line is relatively steep in this case because the policymaker sees high levels of unemployment as especially costly.

4.21 Properties of Optimal Policy with Asymmetric Objective Functions

Even with an asymmetric objective function, many of the key properties noted above in the baseline model remain intact. However, there are some notable differences on a few points. An implication of a "kinked" optimal economic outcomes line is that property 4 from the baseline model—the symmetry result—is violated. Even when the shocks to the IS curve and Phillips curve have zero means, the expected values of inflation and the unemployment rate over time are not equal to their long-run goals. For an asymmetric inflation hawk, the expected value of the inflation rate will be below zero because the policymaker aggressively adjusts policy to damp the effect of positive inflation shocks. As a result, there is an internal inconsistency in the model. Inflation expectations that were initially anchored at the central bank's target in the model would be inconsistent with observed outcomes on average over long periods. Over time, the public would presumably learn that inflation is systematically deviating from the central bank's announced "target." In this case, inflation expectations might adjust lower over time.

Moreover, the distributions of inflation and unemployment are no longer symmetric even if the underlying shocks have symmetric distributions. In particular, the distributions for inflation and unemployment have a discontinuity at zero. Moreover, the distribution on either side of that point have different shapes corresponding to the differential policy response on either side of zero.

As in the baseline model, optimal policy implies a "rule" for setting the policy rate that responds to shocks to the IS curve and the Phillips curve. For both asymmetric hawks and doves, the response to shocks to the IS curve is as in the baseline model. However, the response to inflation shocks depends on whether those shocks are positive or negative. For an asymmetric inflation hawk, a positive inflation shock leads to an aggressive policy tightening, a sizable increase in the unemployment rate and a modest increase in inflation. A negative shock to the Phillips curve, by contrast, results in a moderate policy response with the unemployment rate and inflation both moving moderately lower. For an asymmetric unemployment dove, a positive shock to the Phillips curve leads to a modest policy tightening and an associated modest increase in the unemployment rate along with a relatively large increase in inflation. With a negative shock to the Phillips curve, an asymmetric unemployment rate dove would be willing to aggressively ease policy, pushing the unemployment rate well below the natural rate in an effort to keep inflation closer to target.

Figure 4.1: Loss Function Variations: Symmetric Hawks and Doves

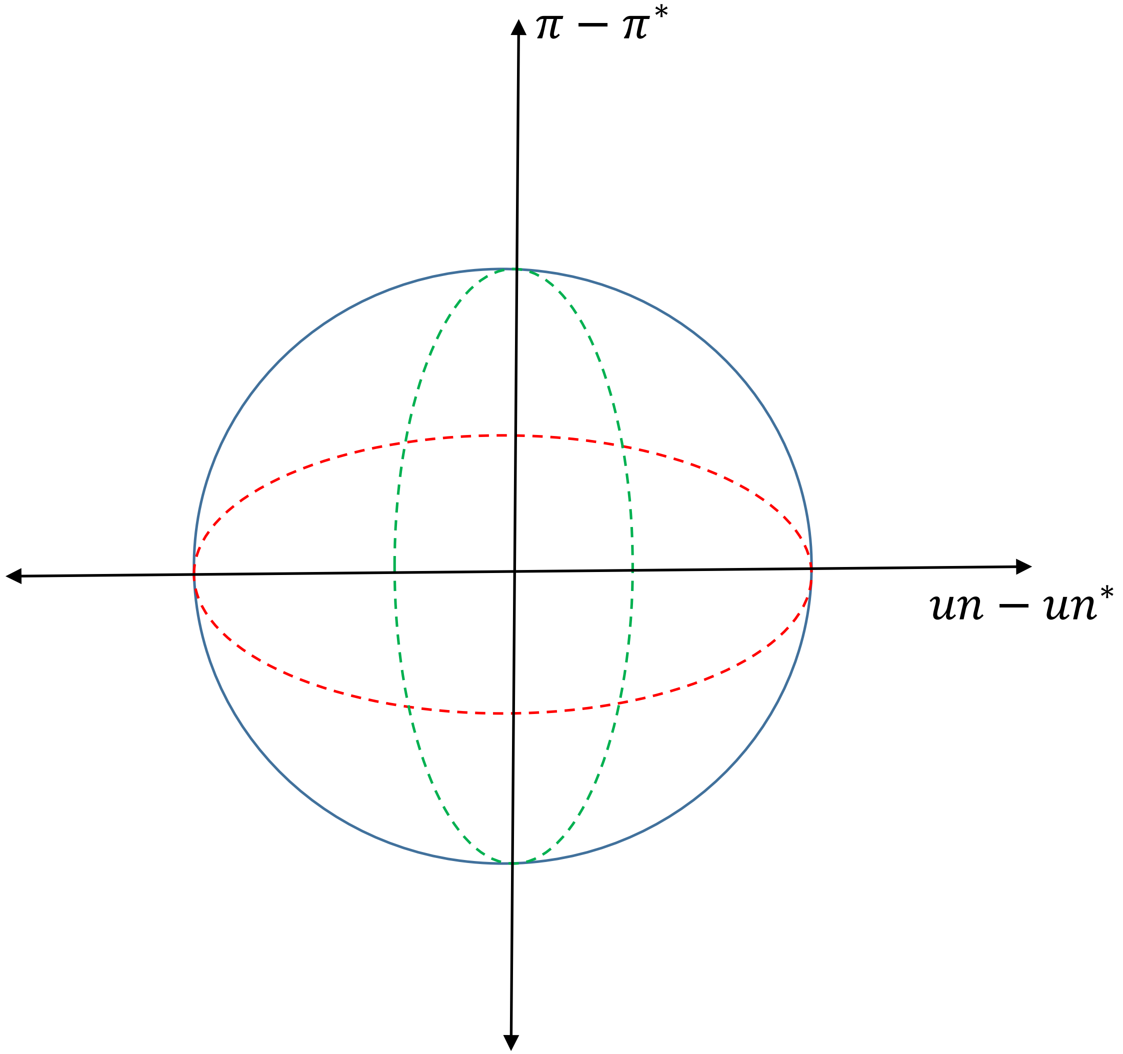


Figure 4.2: Level Curves: Pure Inflation Target

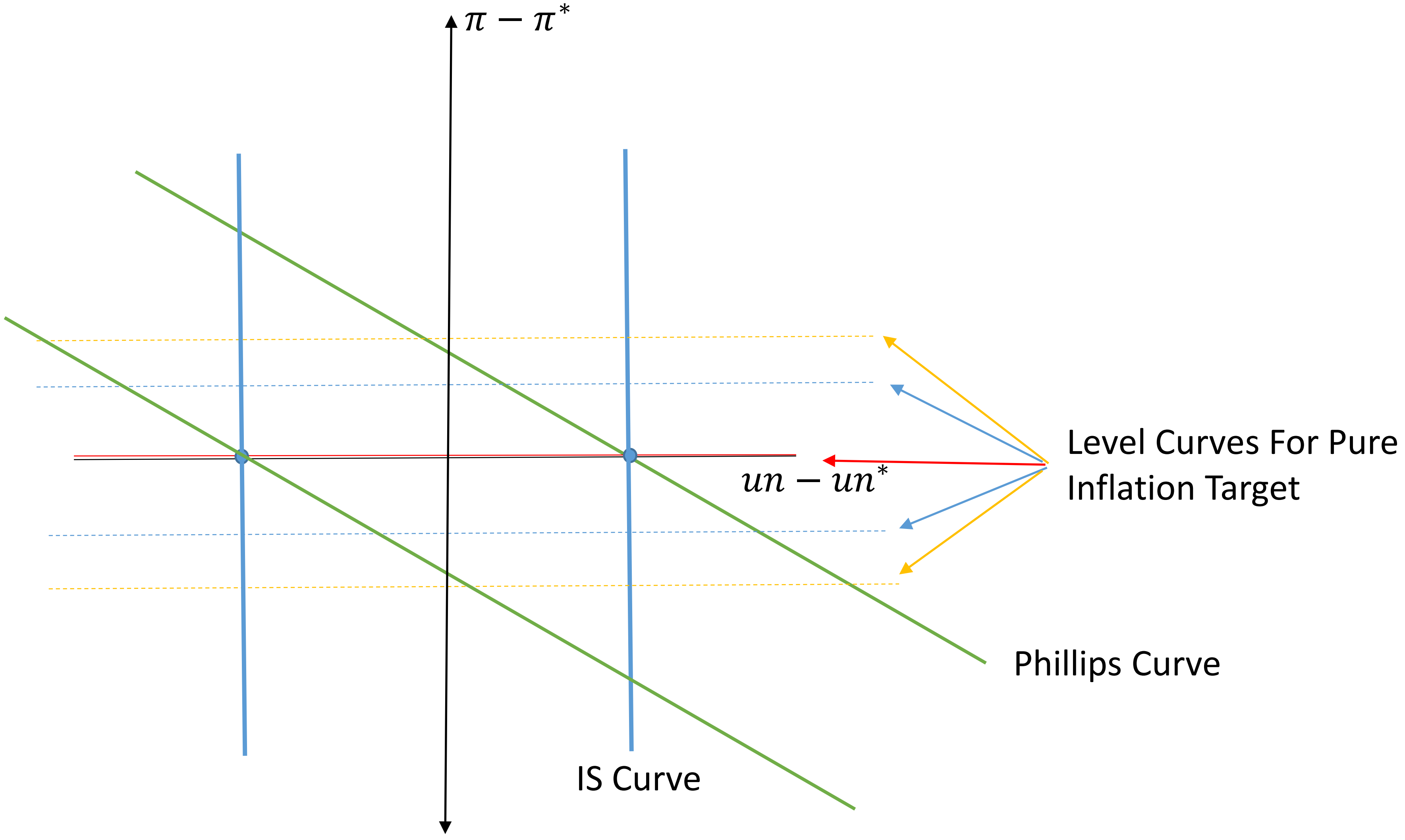


Figure 4.3: Symmetric Hawks and Doves

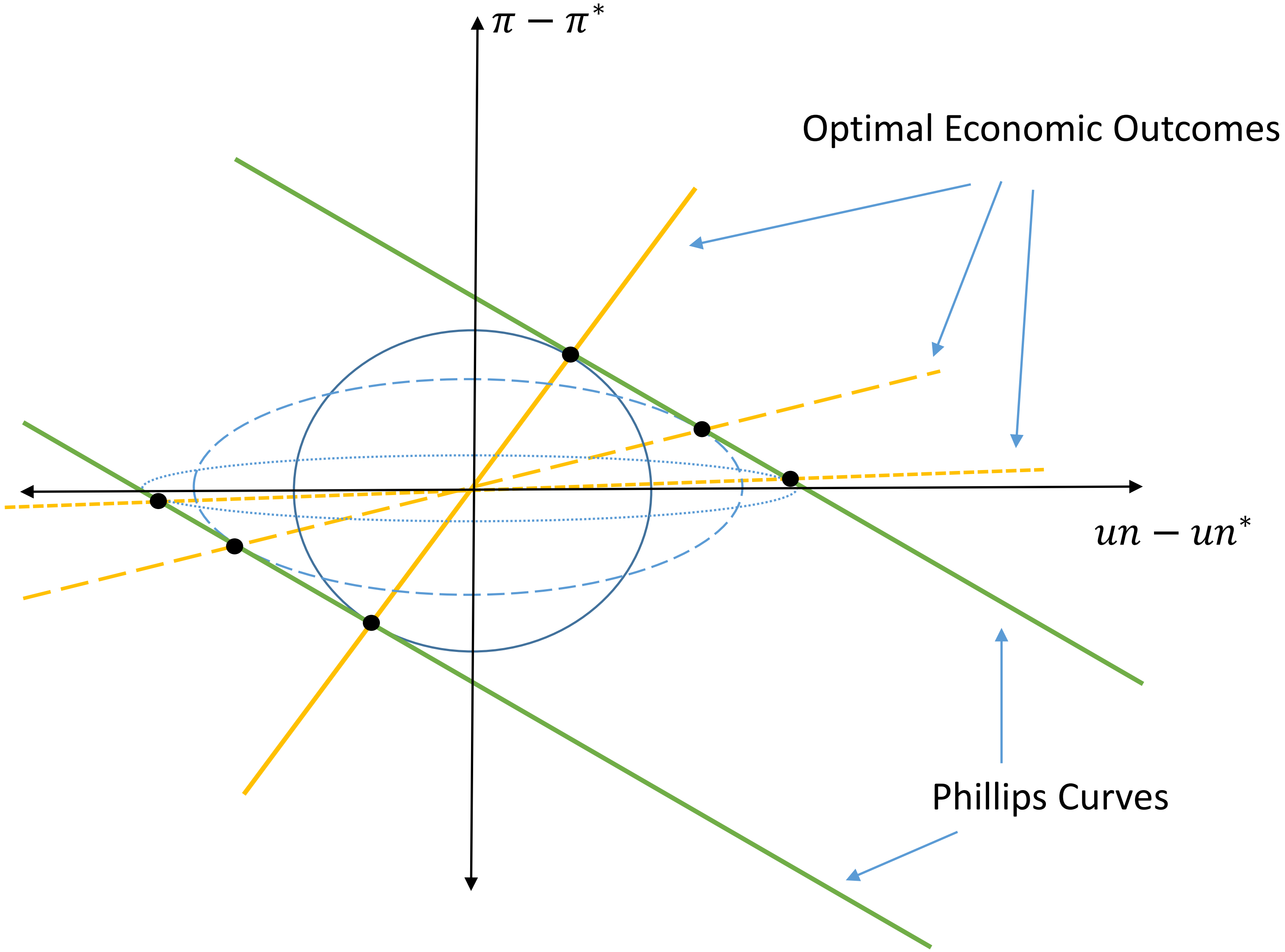


Figure 4.4: Optimal Economic Outcomes Line: Symmetric Hawks

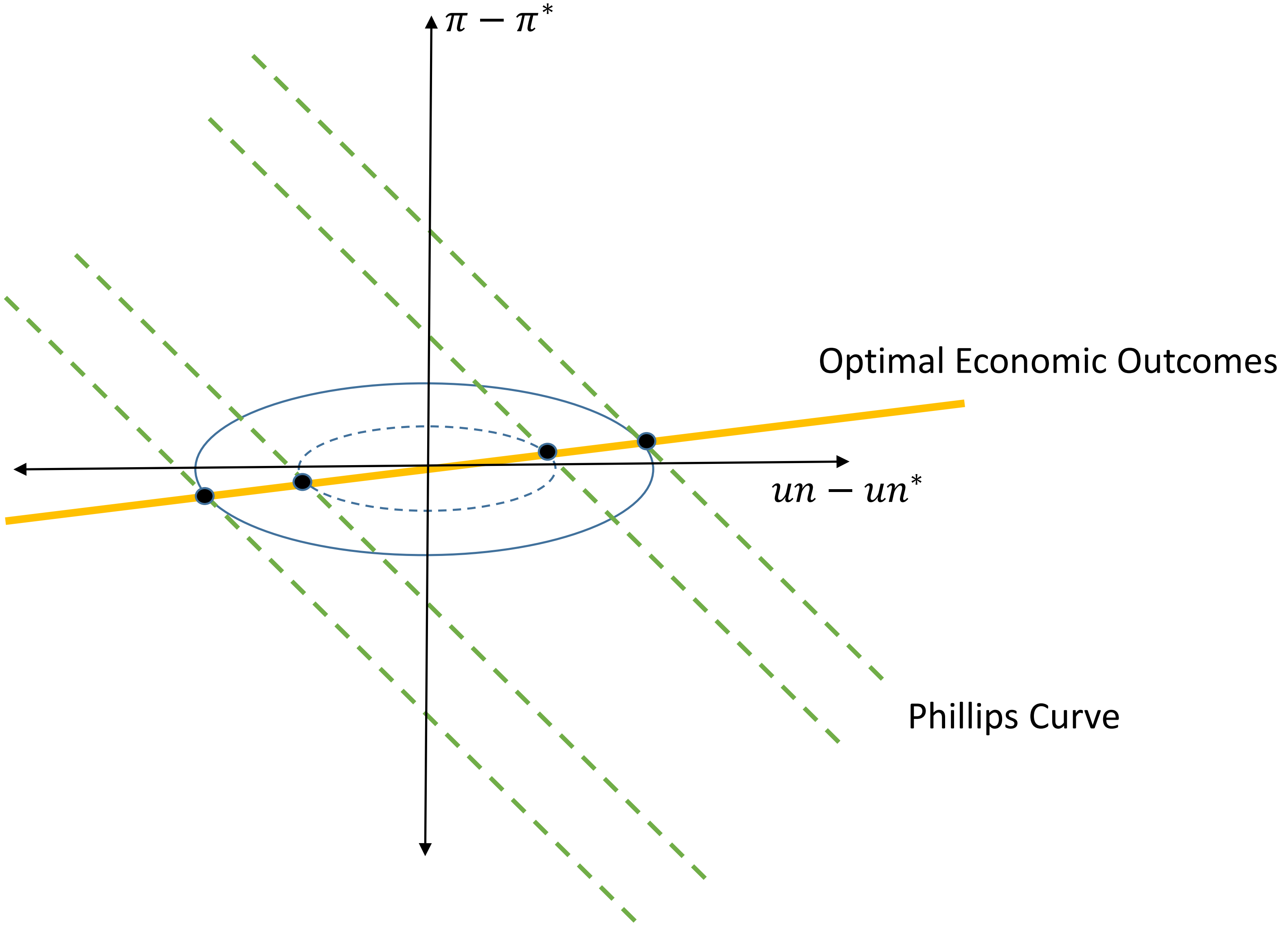


Figure 4.5: Optimal Economic Outcomes Line: Symmetric Doves

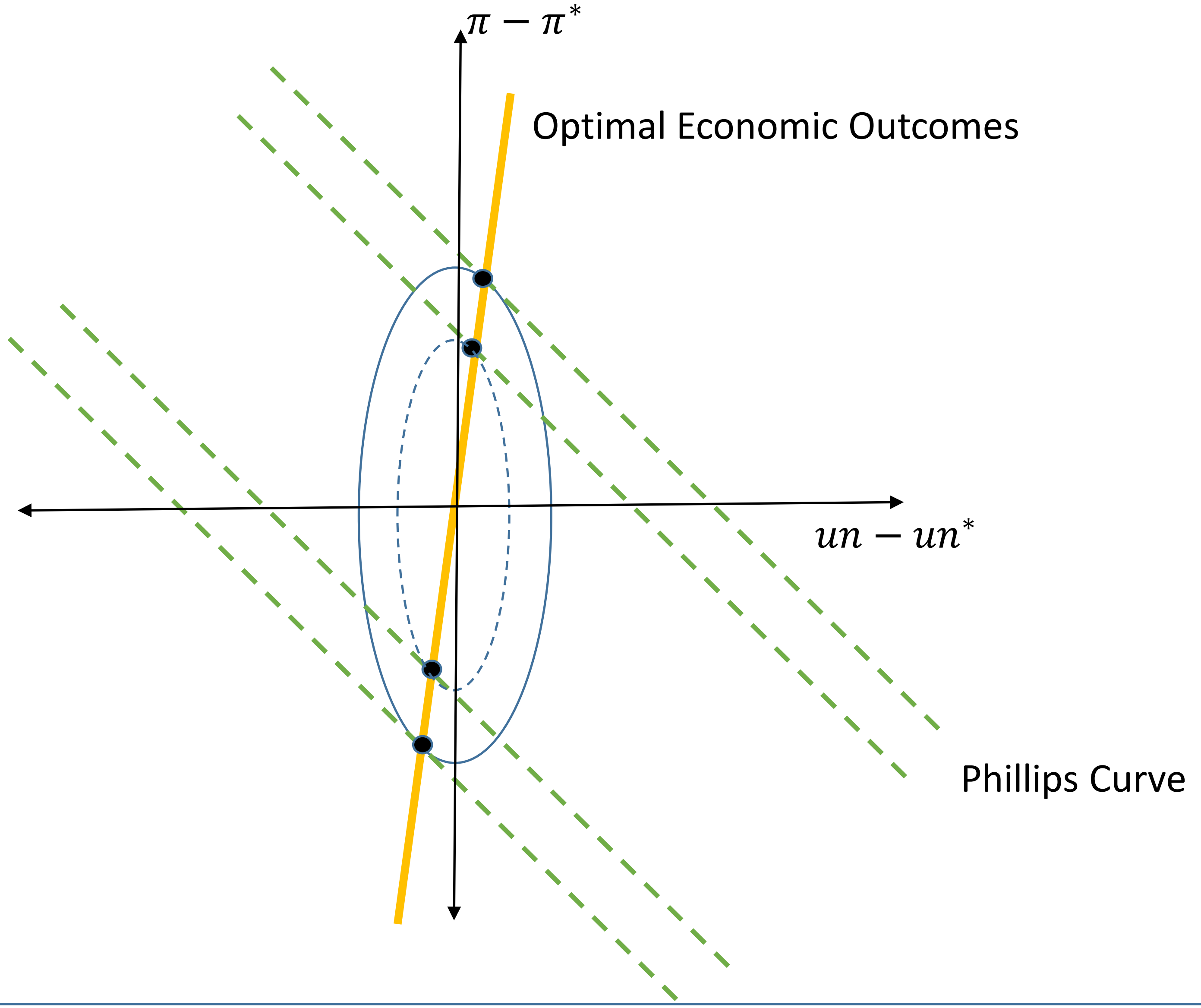


Figure 4.6: Optimal Economic Outcomes Line: Asymmetric Hawks

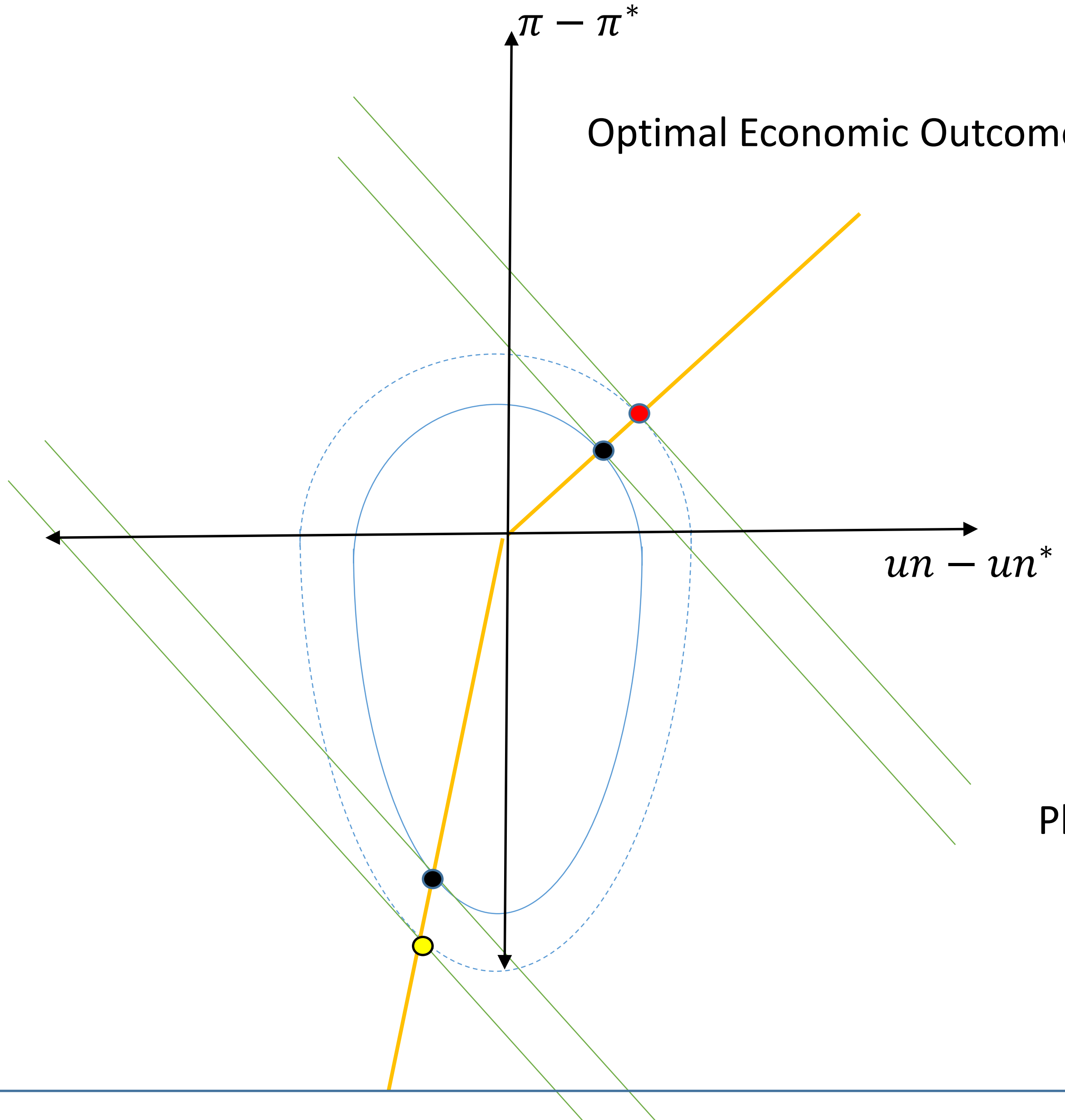
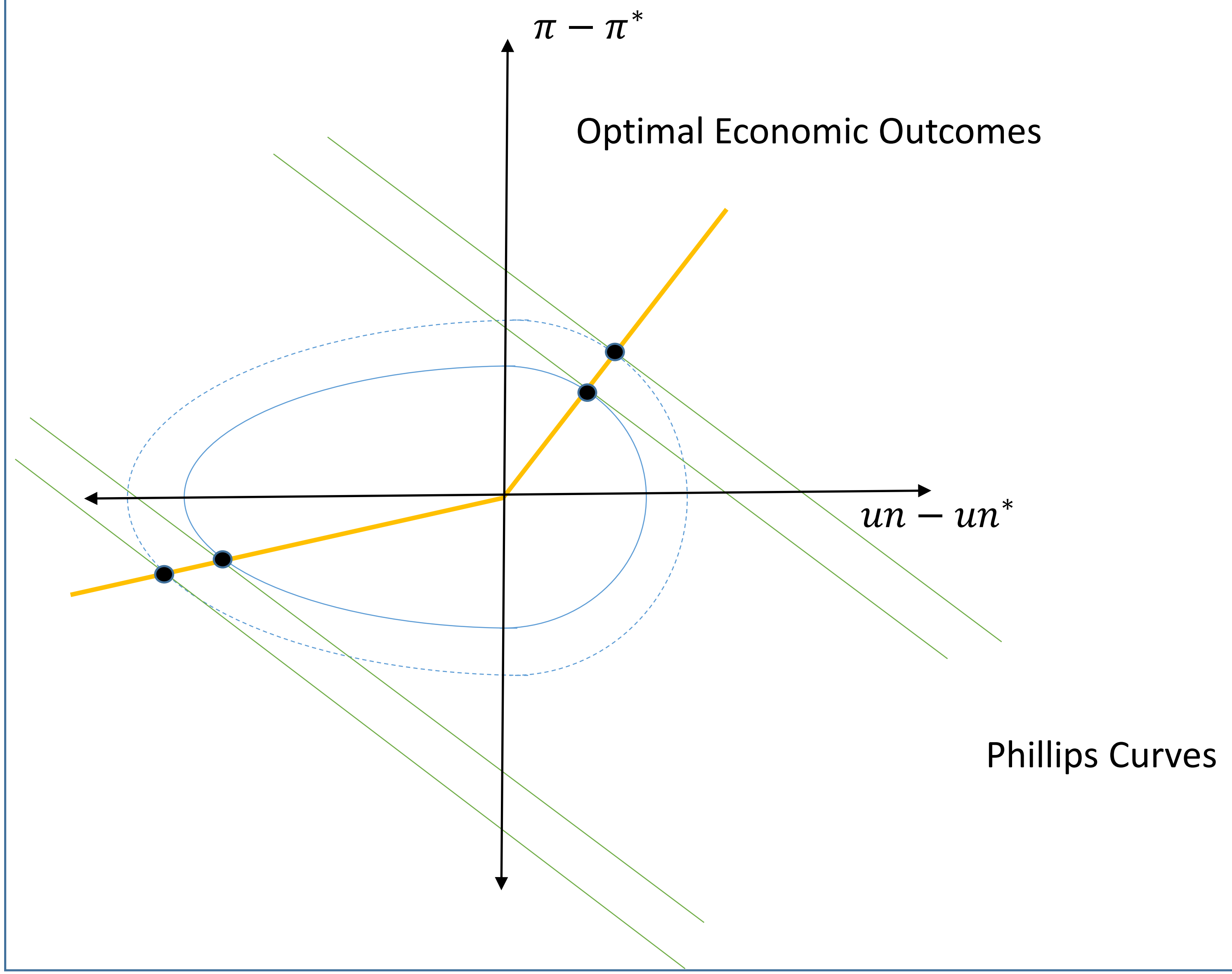


Figure 4.7: Optimal Economic Outcomes Line: Asymmetric Doves



Take 5: Uncertainty in the Baseline Model

"Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape. As a consequence, the conduct of monetary policy in the United States at its core involves crucial elements of risk management, a process that requires an understanding of the many sources of risk and uncertainty that policymakers face and the quantifying of those risks when possible."

Chairman Greenspan, 2004

Under the baseline model and all of the variations considered above, the policymaker enjoys perfect information about the state of the economy and, in particular, is assumed to observe the shocks to the IS curve and the Phillips curve prior to making a choice regarding the setting of the policy rate. In keeping with the spirit of the quote from Chairman Greenspan above, a useful extension of the baseline model is to consider how optimal policy should respond in the absence of complete information about the shocks affecting the economy.

5.1 Additive Shocks and Certainty Equivalence

A simple variation on the baseline model is to assume that both the Phillips curve and IS curve are subject to a second round of shocks, ε' and η', after the policymaker commits to the choice of the policy rate in each period.

Given the uncertainty in this case, it is natural assume that the central bank seeks to minimize the expected value of the loss function. As shown in equation (7), the expected value of the loss function can be calculated for any distribution for the second round shocks, $\eta'$and ε'. The uncertainty about the shocks in the model increases the value of the central bank's loss function. So the central bank is clearly worse off when the second round shocks affecting the economy are unknown and uncertain. However, the extra terms in the loss function are simply constants and do not affect the shape of the indifference curves. This implies the remarkable result that optimal policy in the baseline model with uncertainty about shocks to the IS curve and Phillips curve is identical to optimal policy in the baseline model with complete certainty.

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2\} = \left(\frac{1}{2}\right)((\hat{u}^2 + \hat{\pi}^2) + (1+\tau^2)\sigma_{\eta'}^2 + \sigma_{\varepsilon'}^2\,)) \qquad (7)$$

This so-called "certainty equivalence" result is a key property present in models in which the central bank minimizes a quadratic objective function in connection with an underlying economic model with a linear structure. A corollary of the basic certainty equivalence result in the baseline model is that skews in the underlying distribution of the second round shocks to the Phillips curve and IS curve also are of little consequence for the conduct of policy. Again, the only factors that affect the optimal choice for the policy rate and the optimal choices for the ex-ante values of inflation and unemployment are the expected values of the second round shocks. All higher moments of the distributions of the second round shocks including variances, skews, kurtosis (fat tails) and so on do not affect optimal policy.

The certainty equivalence result seems to call into question the quote from Chairman Greenspan above citing the challenges of conducting monetary policy under uncertainty. According to the certainty equivalence principle, policymakers just need to compute the expected value of various shocks, incorporate those expected values as appropriate in any behavioral relationships in the economy, and then conduct policy exactly as if the central bank had perfect information about the shocks affecting the economy.

This result is a consequence of the quadratic form of the objective function and the linear specification of the key behavioral relationships of the economy. As we'll see in later sections, Chairman Greenspan had excellent reasons to underscore the difficulties of conducting monetary policy under uncertainty. Indeed, relaxing the key assumptions of the baseline model can produce very different results.

5.2 Parameter Uncertainty and the Attenuation Principle

The discussion of certainty equivalence focused on the shock terms to the Phillips curve and the IS curve or on other terms that cannot be influenced by the setting of the policy rate. Another common type of uncertainty focuses on the parameters of the underlying economic model, including those that may affect the economy in a "multiplicative" way. For example, policymakers could be uncertain about the "slope" parameters α and τ for the IS curve and the Phillips curve, respectively.

Uncertainty about these parameters does affect the central bank's optimal policy choice because the setting of the policy rate affects the level of uncertainty about inflation and unemployment. In these cases, a key result is the so-called "attenuation" principle noted originally in the seminal paper by Brainard (1967). In a nutshell, the attenuation principle suggests that in the presence of this type of parameter uncertainty, policymakers should adjust their policy choice relative to the case with perfect certainty in a way that reduces the uncertainty about the goal variables. In many cases, this can imply that the central bank should respond less aggressively to economic shocks. In other cases, the central bank should respond more aggressively to economic shocks. Examples of these situations are described in more detail below.

5.21 Uncertainty Regarding the Slope of IS Curve

Often, policymakers may be uncertain about just how sensitive the economy is to the setting of the policy rate. For example, suppose the value of the IS curve parameter α is not known with certainty. As noted in the appendix, with some simple algebra, the form of the objective function when there is uncertainty about the slope of the IS curve takes the form shown in equation (8). As in the case with uncertainty about "additive" factors discussed above, there is a term that captures the uncertainty regarding the post-shock values of the unemployment rate and inflation. But now that term is not simply a constant. It depends on the squared value of the interest rate gap.

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2) = \left(\frac{1}{2}\right)((\hat{u}^2 + \hat{\pi}^2) + \bar{\alpha}^2(1+\tau^2)\sigma_\alpha^2(i - i^*)^2) \qquad (8)$$

Intuitively, this type of uncertainty provides an incentive for the policymaker to keep the policy rate gap, $i - i^*$, close to zero because that lowers the variance of the unemployment rate gap. And because the variation of the inflation gap stems partly from the variation of the unemployment gap, keeping i close to i^*also lowers the uncertainty associated with the inflation gap. The attenuation principle in this case suggests that the central bank should respond less aggressively to inflation shocks than in the case with perfect certainty. That result stems from the fact that uncertainty about the unemployment rate in the model becomes larger as the level of the interest gap increases. Under complete information, the policymaker might want to raise the policy rate significantly above i^* in order to address an inflation shock. However, in the presence of uncertainty about the slope of the IS curve, the potential for a policy-induced increase in uncertainty about the inflation rate and unemployment rate stemming from a widening in the policy rate gap causes the central bank to take a more cautious approach.

Similarly, the policymaker should respond less aggressively to observed shocks to the rate of unemployment than in the case with perfect certainty. Again, this stems from the fact that there is now a cost to adjusting the level of the policy rate relative to i^*in the form of increased uncertainty about the ex-post levels of the unemployment rate and inflation rate gaps.

The appendix shows that this form of the objective function can also be viewed as increasing the weight on the unemployment rate gap and shifting the indifference curve to the right or left depending on the observed value of the shock to the IS curve. Figure 5.1 displays the optimal economic outcomes line in the case with an observed adverse shock to the IS curve. The level curves in this case shift to the right and place relatively high weight on unemployment rate gaps. The optimal economic outcomes line in this case passes through quadrant 4—something that never happens in the baseline model. The intuition is that in the case of an adverse first-round shock to the IS curve, the policymaker will not choose to completely offset that shock as in the baseline model because doing so would increase the magnitude of the interest gap and thus boost the level of uncertainty about the post-shock values of the unemployment rate and inflation.

5.22 Uncertainty Regarding the Slope of Phillips Curve

Another interesting case occurs when there is uncertainty about the slope of the Phillips curve. In this case, the expected value of the objective function is as shown in equation 9. The term that captures the effect of uncertainty about the slope of the Phillips curve depends on the value of the unemployment rate gap.

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2)\} = \left(\frac{1}{2}\right) ((\hat{u}^2 + \hat{\pi}^2) + \bar{\tau}^2 \hat{u}^2 \sigma_\tau^2 \ ) \qquad (9)$$

So the attenuation principle suggests that the central bank should shade its policy choices relative to the case with perfect certainty in a way that results in a lower level of the unemployment rate gap.

As in the case with certainty about the slope of the Phillips curve, the central bank should completely offset any shocks to the IS curve. Doing so reduces the magnitude of the expected value the unemployment and inflation rates from their respective targets and also reduces the ex-post uncertainty about the inflation rate stemming from the uncertainty about the slope of the Phillips curve.

The attenuation principle in this case implies that the central bank should respond less aggressively to observed shocks to the Phillips curve than in the case with perfect certainty. In response to an inflation shock, the policymaker might otherwise wish to push the level of the unemployment rate relatively high in order to damp inflation pressures. But doing so when there is a lot of uncertainty about the slope of the Phillips curve would result in an increase in the ex-post variance of inflation and that would be costly according to equation (9). As result, the policymaker adjusts the policy rate to keep the unemployment rate gap relatively small and allows more of the observed shock to the Phillips curve to show through to the rate of inflation.

As a result, the attenuation principle in this case implies the central bank should be quite aggressive in offsetting adverse (or favorable) shocks to the unemployment rate. Indeed, the form of the objective function in equation (7) is just a variation on the "balanced dove" specification discussed above in Take 4.

A graphical representation is shown in figure 5.2. In the diagram, the uncertainty about the slope of the Phillips curve results in higher weight being placed on unemployment rate gaps. The optimal economic outcomes line is thus steeper than in the baseline model but still passes through the origin.

5.23 The Faults in Our Stars: Uncertainty Regarding the Level of r^* and un^*

"Experience has revealed two realities about the relation between inflation and unemployment, and these bear directly on the two questions I started with. First, the stars are sometimes far from where we perceive them to be. In particular, we now know that the level of the unemployment rate relative to our real-time estimate of u will sometimes be a misleading indicator of the state of the economy or of future inflation."*

Chairman Powell, 2018

Central bankers spend a great deal of time and effort in attempting to reach judgments regarding the levels of the key "star" parameters in the basic model—the neutral real interest rate, r^*, and the natural rate of unemployment, un^*. These two parameters are central to the conduct of monetary policy in the baseline model but in the real world they are unobserved variables and must be estimated. Moreover, the range of uncertainty around such estimates is typically very large.

Similar to the analysis above, one could view the uncertainty surrounding these parameters as a "shock" that is only observed after the central bank has committed to its choice of the policy rate. In that case, uncertainty about r^* is just another example of "additive" uncertainty discussed above. In the baseline model, policymakers faced with this uncertainty would form their best estimate of the expected value of r^* and then conduct policy as if this value was known with certainty. Greater uncertainty about r^* would increase the expected value of the central bank's loss function but would have no effect on optimal outcomes for the unemployment rate and inflation gaps or on the optimal policy rate.

The case with uncertainty about the natural rate of unemployment, un^*, is even simpler. In the baseline model, a shock to un^* passes through one for one to a corresponding change in the actual unemployment rate. As a result, the unemployment rate gap is completely unaffected by this type of shock; the expected value of the central bank's loss function is unaffected as are all other aspects of the optimal policy outcomes.

Another type of uncertainty that policymakers can face in reaching judgments about the star variables is the inability to completely identify the values of these parameters in the current period. For example, in the baseline model, the policymaker observes the value of the shock to the IS curve and the Phillips curve in the current period. However, in practice, it may be quite difficult to exactly determine the extent to which observed levels of the unemployment rate and inflation rate reflect the shocks to the IS curve, the Phillips curve, or shocks to the level of the natural rate un^*, or all of them. As discussed in the appendix, under some assumptions, it's possible to make estimates of what those shocks might be based on the observed values of the unemployment rate and inflation rate and some sense of the uncertainty about the natural rate of unemployment and the IS and

Phillips curve shocks. In that sort of "signal extraction" exercise, the *relative* variances of the IS and Phillips curve shocks and the shocks to the natural rate of unemployment determine the estimated values of these shocks in the current period. Intuitively, if the variance of IS curve shocks is very high, chances are that an observed high level of the unemployment rate is primarily attributable to a large contemporaneous shock to the IS curve. Conversely, if uncertainty about the natural rate of unemployment is relatively high, the policymaker might conclude that a relatively large factor affecting the unemployment rate in the current period could be a higher than usual level of the natural rate.

The signal extraction process described above does depend on the *relative* variances of the IS curve and Phillips curve shocks and the shocks to the natural rate of unemployment, but it is purely a statistical procedure. In the baseline model, policymakers faced with this type of identification risk would, as always, form their best estimate of the expected value of the IS curve shock, the Phillips curve shock and the natural rate of unemployment and then conduct policy as if these values were known with certainty. The policy reaction function in this case is:

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \mathrm{E}\{\varepsilon\})}{1+\tau^2} - \frac{\mathrm{E}\{\eta\}}{\alpha} = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \mathrm{E}\{\varepsilon\})}{1+\tau^2} - \frac{E\{\eta\}}{\alpha} \qquad (10)$$

This expression is identical to the policy reaction function for the baseline model shown in equation (4) except that the actual values of the shocks to the Phillips curve and IS curves in equation (4) are replaced with their expected values *conditional* on the observed values of inflation and unemployment based on the statistical signal extraction procedure.

The upshot of this analysis is that, in the baseline model, uncertainty about the level of the "star" parameters, $r^*$and un^*, does not have major implications for optimal monetary policy. However, as discussed in more detail below, some variations on the assumptions in the standard model can provide a strong rationale for focusing on the uncertainty associated with r^* and un^*. As noted above in the quote from Chairman Powell, in the presence of identification risk, the central bank may be quite concerned that its estimates of these parameters may be inaccurate, especially when the cost of a policy mistake is quite high. For example, if a central bank mistakenly estimates that the natural rate of unemployment is quite high, it might provide less policy accommodation than desired in response to an adverse shock to the IS curve. And there may be situations in which the cost of that type of policy mistake is especially high. Conversely, there may be cases in which a central bank mistakenly assumes that the natural rate of unemployment is quite low and therefore provides a great deal of policy accommodation to push the unemployment rate down to this low estimate of the natural rate. Particularly if inflation expectations become unanchored, maintaining a very accommodative policy based on this misperception of the natural rate could also be very costly.

Figure 5.1: Optimal Economic Outcomes Line: Uncertainty about Slope of IS Curve

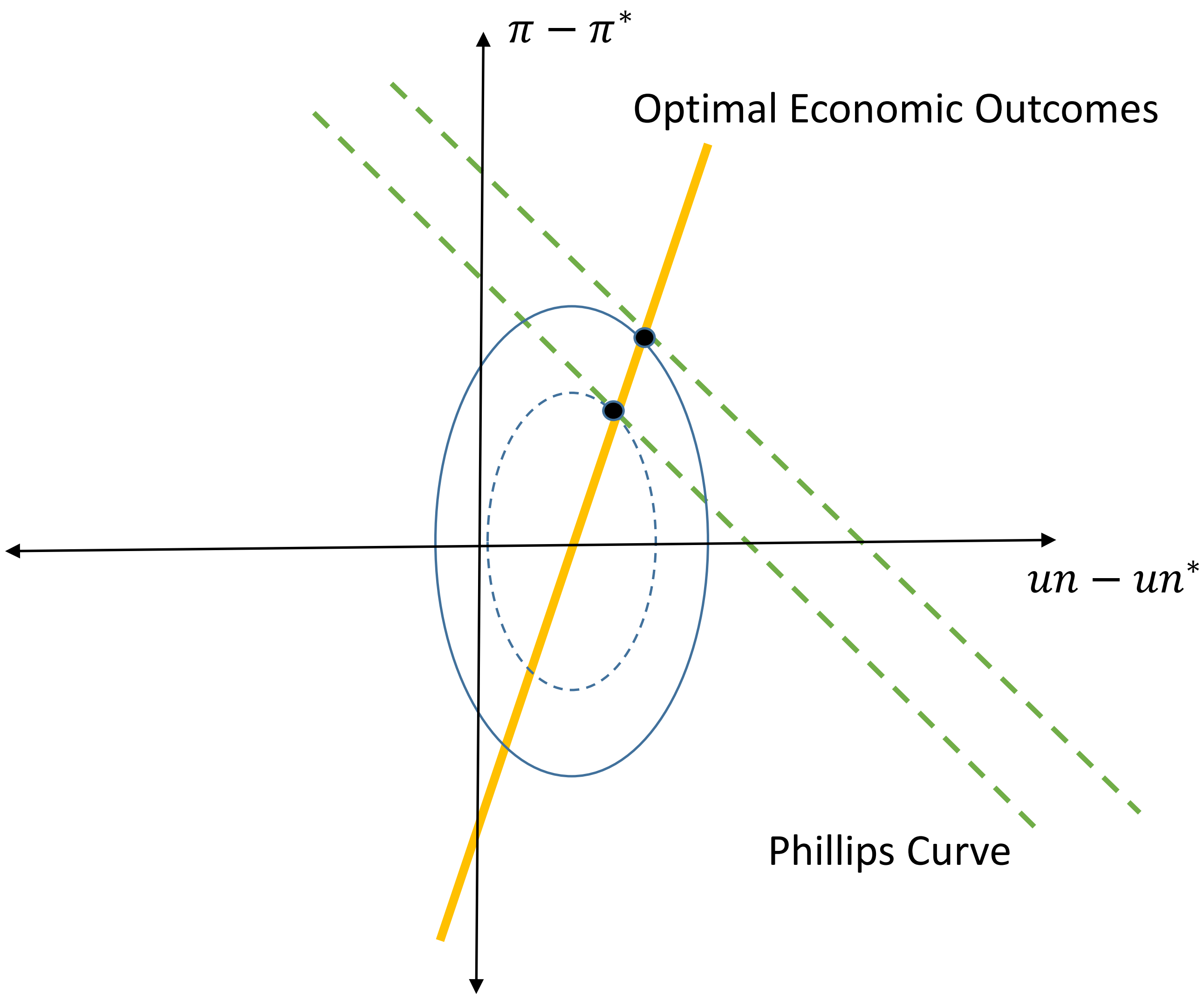
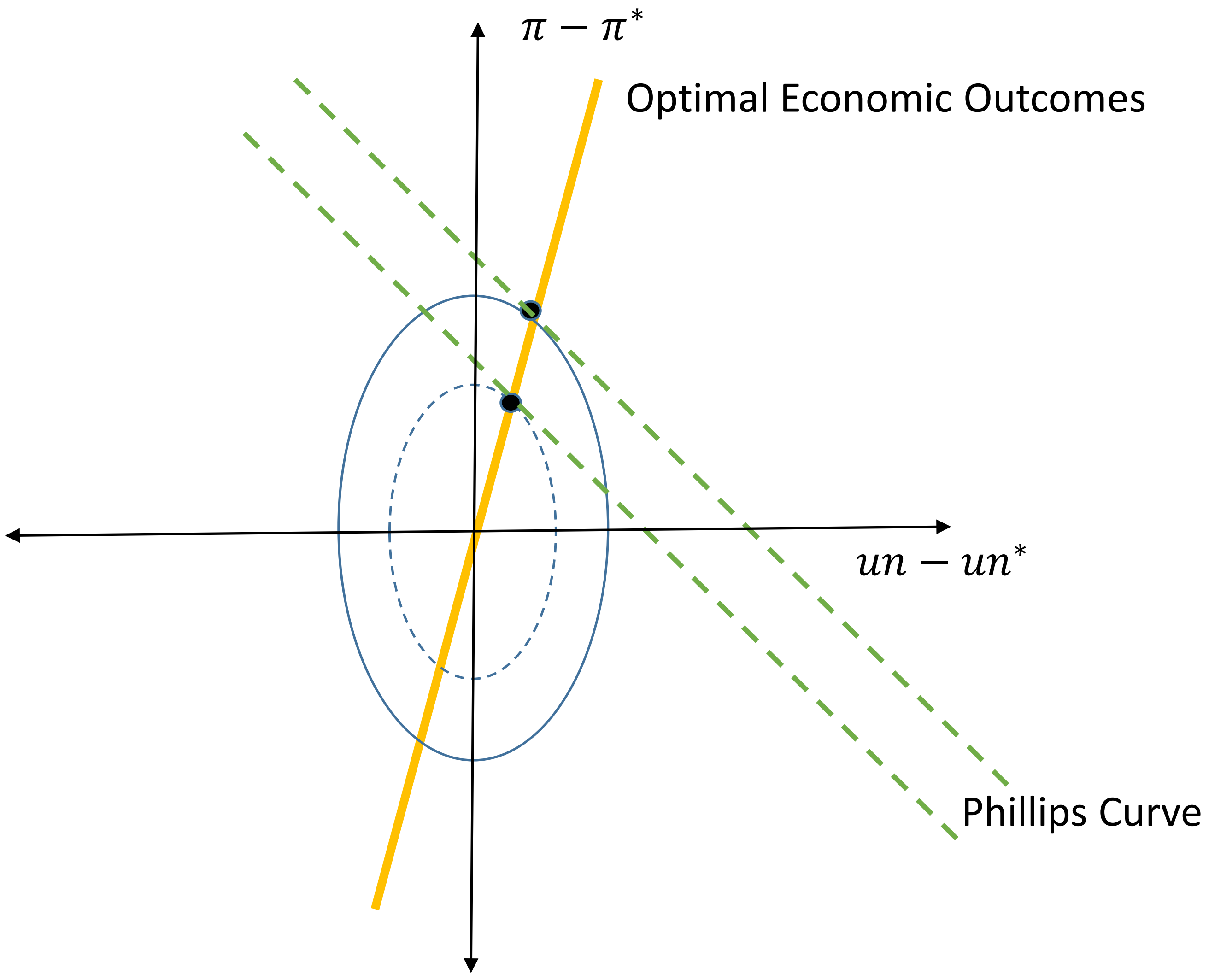


Figure 5.2 Optimal Economic Outcomes Line: Uncertainty about Slope of Phillips Curve



Take 6: Uncertainty and Asymmetric Objective Functions

"Uncertainty--about the state of the economy, the economy's structure, and the inferences that the public will draw from policy actions or economic developments--is a pervasive feature of monetary policy making. [….] Notably, we now appreciate that policy decisions under uncertainty must take into account a range of possible scenarios about the state or structure of the economy, and those policy decisions may look quite different from those that would be optimal under certainty. For example, policy actions may be attenuated or augmented relative to the "no-uncertainty benchmark," depending on one's judgments about the possible outcomes and the costs associated with those outcomes. "

Chairman Benanke, 2007

As Chairman Bernanke noted above, optimal policy responses must take account of outcomes that may be especially costly. The piece-wise "asymmetric" objective functions described above are one way to capture the idea of outcomes that may be viewed as especially costly. In this case, even in the simple baseline economic model, policymakers will assign higher weights to some ex-post outcomes than to others. As a result, they will have an incentive to adjust the stance of policy ex-ante to reduce the risk of an especially costly ex-post outcome.

6.1 Asymmetric Hawks and Uncertainty

Consider the case of the objective function for the "asymmetric hawk" described above in which the policymaker attaches greater weight to inflation realizations above target than to those below target. In this case, the expected value of the objective function is as shown in appendix equation 6.1. The cost associated with uncertainty about inflation is split into two parts—one that captures the expected cost of positive shocks that may push inflation above target and a second that captures the expected cost of negative shocks that push inflation below target. As shown in figure 6.1, the optimal economic outcomes line for this objective function with uncertainty about shocks to the IS curve and Phillips curves (shown by the smooth yellow line) is a probability weighted average of the kinked optimal economic outcomes lines from the case with perfect certainty (as described above in section 4.2). (See the appendix for details).

The optimal choice of the ex-ante inflation rate for a policymaker with these preferences is even lower than in the case with perfect certainty because the policymaker wants to guard against an outcome in which the second round shock to inflation turns out to be large and negative. As described above, in the case with no uncertainty, a policymaker with asymmetric hawk preferences will be quite aggressive in adjusting the stance of policy to offset an observed positive shock to inflation. It turns out that an asymmetric hawk will be even more aggressive in taking steps to combat inflation in the case with uncertainty about inflation shocks.

If there are second round shocks to the IS curve, as discussed in Take 5, uncertainty about those second round shocks will also matter to an asymmetric inflation hawk. An asymmetric inflation hawk would be especially concerned about an unexpected shock to the IS curve that could push the unemployment rate much lower than expected. In that case, the realized value of inflation would turn out to be higher than expected and that outcome would be especially costly for an asymmetric inflation hawk. As a result, an increase in uncertainty about shocks to the IS curve will also make the asymmetric inflation hawk run a tighter ex-ante policy in order to guard against especially adverse inflation outcomes.

Similar analysis shows that, unlike the baseline model, an asymmetric hawk will tend to run a tighter stance of policy in the presence of uncertainty about r^*. In particular, an asymmetric hawk will want to shade the stance of policy toward higher levels of the policy rate to guard against outcomes in which r^* turns out to be much higher than expected. In that case, the policymaker would have provided more stimulus than desired and the policy consequences of that mistake are more costly than the alternative case in which r^* is lower than expected and less policy stimulus than intended was provided.

6.2 Asymmetric Doves and Uncertainty

A similar type of analysis applies in the case of an asymmetric dove facing uncertainty about a second round shock to the unemployment rate. The optimal economic outcomes line for a policymaker with these preferences is shown by the smooth yellow line in figure 6.2. The optimal economic outcomes line is again a probability weighted average of the "kinked" dotted optimal economic outcomes line that arises with asymmetric preferences of this form in the case with perfect certainty.

Policymakers with these preferences are especially concerned about outcomes with the unemployment rate above the natural rate of unemployment. In the case with perfect certainty, such policymakers allow a large portion of a positive shock to inflation to show through so as to limit the effect of such a shock on the level of the unemployment rate. Conversely, in the presence of a negative shock to inflation, the policymaker is willing to allow the unemployment rate to fall to levels substantially below the natural rate of unemployment.

In the presence of uncertainty about an ex-post shock to the unemployment rate, an asymmetric dove would shade the ex-ante choice of policy to provide more stimulus. That policy helps to guard against outcomes in which the ex-post shock pushes the unemployment rate well above the natural rate of unemployment.

An asymmetric dove will also want to respond to uncertainty about the levels of r^*. In particular, they would be especially concerned about outcomes in which the level of r^* is lower than expected. In this case, the stance of policy chosen ex-ante could be too tight and the potential for an ex-post level of the unemployment rate above the natural rate of unemployment is higher. As a result, an asymmetric dove facing uncertainty about r^* will want to adopt a policy stance that is even more accommodative than the one they would choose in the case of perfect certainty.

Figure 6.1: Optimal Economic Outcomes Line: Asymmetric Hawks and Uncertainty

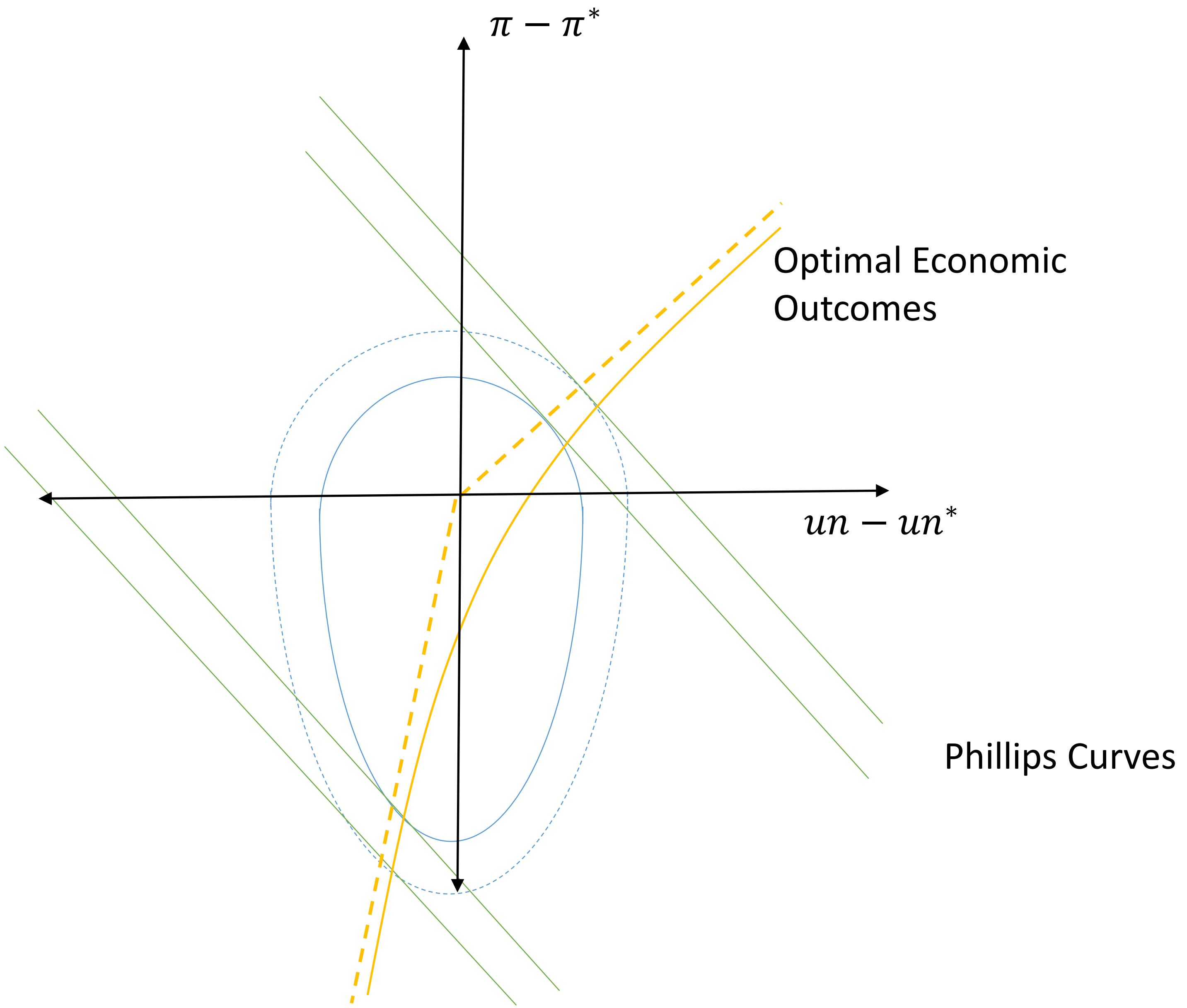
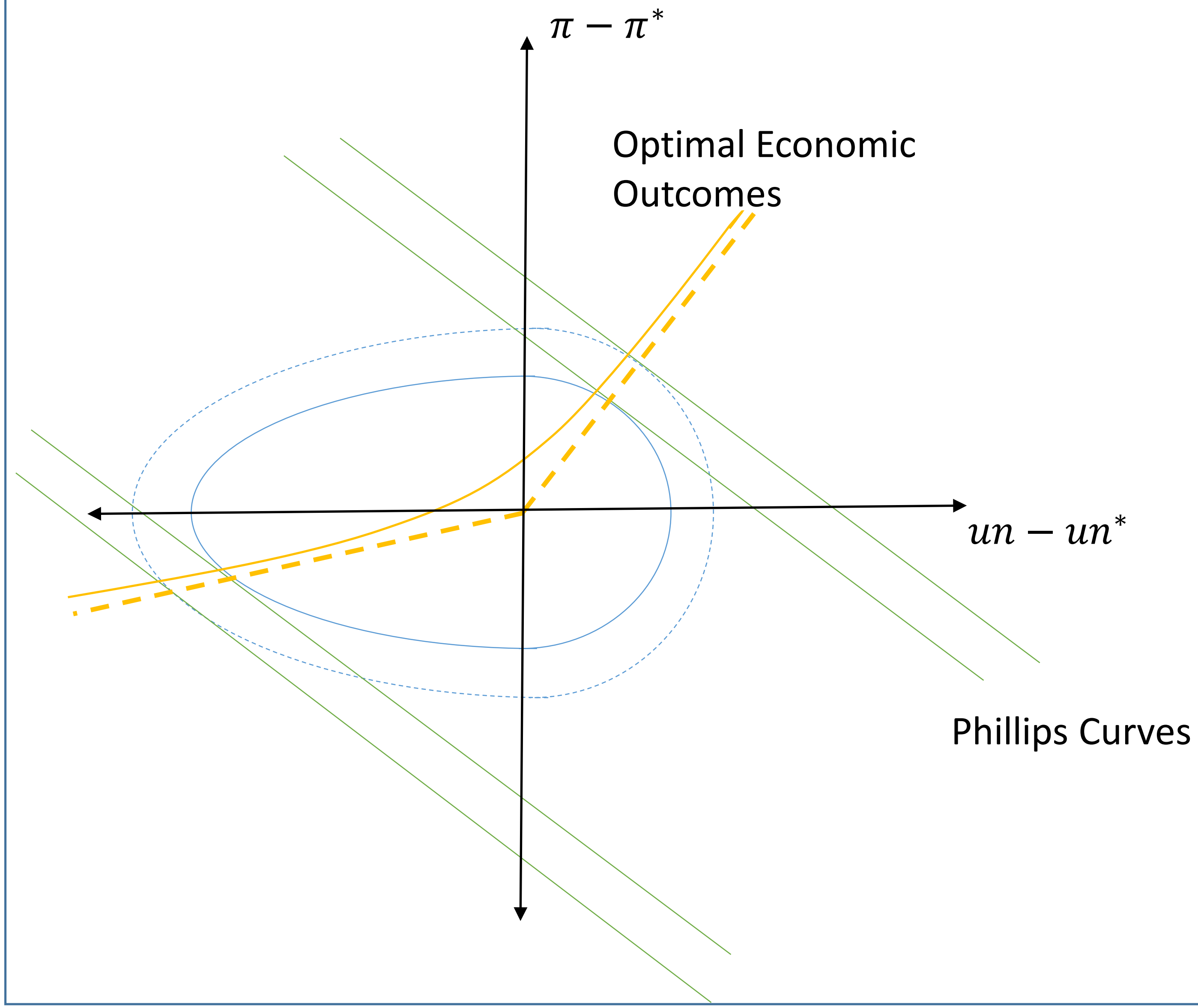


Figure 6.2: Optimal Economic Outcomes Line:
Asymmetric Doves and Uncertainty



Take 7: Appealing to a Higher Power—Tail Risk Avoidance

"At times, policy practitioners operating under a risk-management paradigm may be led to undertake actions intended to provide some insurance against the emergence of especially adverse outcomes. For example, following the Russian debt default in the fall of 1998, the Federal Open Market Committee (FOMC) eased policy despite our perception that the economy was expanding at a satisfactory pace and that, even without a policy initiative, was likely to continue to do so. We eased policy because we were concerned about the low-probability risk that the default might severely disrupt domestic and international financial markets, with outsized adverse feedback to the performance of the U.S. economy."

Chairman Greenspan, 2004

The quote from Chairman Greenspan describing the policy response to the global distress in the fall of 1998 seems far removed from the world of certainty equivalence. As we've seen, certainty equivalence breaks down in the case of objective functions that are asymmetric. However, departures from certainty equivalence can arise even in the case of symmetric objective functions.

A case of some interest is one in which the policymaker is especially concerned about "tail risks" that would lead to large deviations of the unemployment rate or the inflation rate from their respective targets. For example, suppose the objective function is based on the unemployment rate and inflation deviations raised to the fourth power rather than just the squared values as in the baseline model. In this case, policymakers very much wish to avoid large deviations in either the unemployment rate or inflation from their respective targets.

In the case of certainty, the indifference curves for this objective function are symmetric as in figure 7.1 and look a bit like a "rounded off" square. Moreover, the optimal economic outcomes line is again a straight line through the origin just as in the simple baseline model.

However, in the presence of uncertainty about the shocks to unemployment and inflation, optimal policy for preferences of this form must take account of higher moments of the distribution of possible outcomes and the possible interactions between the ex-ante level of the unemployment rate and inflation and the ex-post levels of these variables. In contrast to the case with a quadratic objective function, the variance and skew of potential shocks can affect the shape of the indifference curves and thus the shape of the optimal economic outcomes line.

It's straightforward to show that the optimal economic outcomes line with the quartic objective function coupled with uncertainty about shocks to inflation and unemployment assumes the form shown in appendix equation 7.7. In this case, the variance and skew terms for the shocks enter directly into the expression for the optimal economic outcomes line. The intuition is that policymakers are especially concerned about large deviations with this objective function. As a result, they adjust the ex-ante values of the unemployment rate and inflation to guard against especially bad outcomes. If, for example, the variance of shocks to inflation is very large, policymakers will try to keep the ex-ante value of inflation as close to zero as possible. The same is true in the case of uncertainty about output shocks. Similarly, if inflation shocks are skewed to the upside, then policymakers will guard against especially large ex-post values of inflation by tightening policy and keeping the ex-ante value of inflation a little below zero.

These effects are shown graphically in figure 7.2. The top panels in figure 7.2 display the shape of the optimal economic outcomes line when the variance of shocks to the IS curve is substantially larger than the variance of shocks to the Phillips curve. In this case, the optimal economic outcomes line has a sort of "s" shape with a relatively steep section as the unemployment rate gap approaches zero. As noted above, with a high variance of shocks to the IS curve, the policymaker is very intent on keeping the ex-ante level of the unemployment rate gap close to zero. So it takes a sizable inflation gap to induce the policymaker to move away from that position. That policy implies that the policymaker will respond very little to observed modest inflation shocks, preferring instead to allow those shocks to show through almost entirely to higher inflation.

The green dashed lines in these panels show three distinct Phillips curves. And, as always, the optimal point for policy would be determined at the intersection of the optimal economic outcomes line and the Phillips curve. Note that once the Phillips curve is quite far away from zero, the "nonlinear" effects stemming from uncertainty about shocks become far less pronounced. That occurs because once the Phillips curve is sufficiently far away from zero, the costs that matter most are the "certain" costs associated with the ex-ante deviations of inflation and unemployment from their respective goals.

When the shocks to the unemployment rate are skewed to the upside, the optimal economic outcomes line shifts to the left. That occurs because the policymaker is willing to incur a small cost in expectation in operating with a negative ex-ante unemployment rate gap in order to help cushion the blow in the event of an especially large, positive ex-post shock to the unemployment rate. With the optimal economic outcomes line assuming that position, the optimal point for the economy for some Phillips curves would be in quadrant 2 as shown by the yellow dot. As noted above, in the baseline model with a quadratic objective function, the policymaker would never choose a point in quadrant 2. But with quartic preferences that put a lot of weight on tails risks and with skewed shocks to the economy, that result no longer holds. Conversely, as shown in the top right panel, when the shocks to the IS curve are skewed to the downside (that is, large favorable shocks to the economy are possible), then then optimal economic outcomes line shifts to the right. The optimal economic outcomes line with the middle Phillips curve now leaves the economy in quadrant 4. Again, this is a position that the policymaker would never choose in the baseline model. In this case, the policymaker is inclined to tighten policy and keep the unemployment rate a little on the high side ex-ante to provide a cushion in case the ex-post shock to the IS curve is very favorable.

The bottom row of the figure illustrates the case in which the variance of inflation shocks is substantially larger than the variance of unemployment shocks. In this case, the optimal economic outcomes line again has an "s" shape, but now one with a flat portion around the horizontal axis. That occurs because the possibility of a large and costly ex-post inflation shock provides strong incentives for the policymaker to keep the ex-ante inflation gap very close to zero. The policymaker in this case is willing to tolerate a relatively large unemployment rate gap in order to keep the inflation gap close to zero. That strategy, in turn, implies that the policymaker will tighten very aggressively in response to observed positive inflation shocks and will ease very aggressively in response to observed negative inflation shocks.

The remaining bottom panels show the cases when inflation shocks are skewed to the upside and downside. In these cases, the optimal economic outcomes line shifts down and up, respectively. As before, that result arises because in the presence of skewed shocks, the policymaker is willing to incur some costs in setting the ex-ante level of the inflation gap above or below zero in order to cushion the blow in the event of an especially large shock in the long tail of the distribution for inflation shocks. As shown by the yellow dots, the skews in the distribution can lead the policymaker then to choose points in quadrants 2 and 4 as a form of "insurance" against these large ex-post shocks.

As discussed above, the optimal policy stance line is a horizontal translation of the optimal economic outcomes line with the translation amount reflecting the magnitude of the observed shock to the IS curve in the period. The nonlinear shape of the optimal economic outcomes line in these panels thus implies a nonlinear response in the stance of policy to a shock to the Phillips curve. In particular, in the top row of panels, the policymaker would adjust the stance of policy quite aggressively in response to first round shocks to the Phillips curve in the "flat" regions of the curves. Again, that would reflect a strong desire to keep the ex-ante inflation gap as low as possible. Conversely, in the bottom rows, the policymaker would not respond much at all to first round observed shocks to the Phillips curve in the "steep" region of the economic outcomes line. That again reflects the strong desire to keep the magnitude of the unemployment rate gap as small as possible in this scenario.

Figure 7.1: Optimal Economic Outcomes Line: Tail Risks

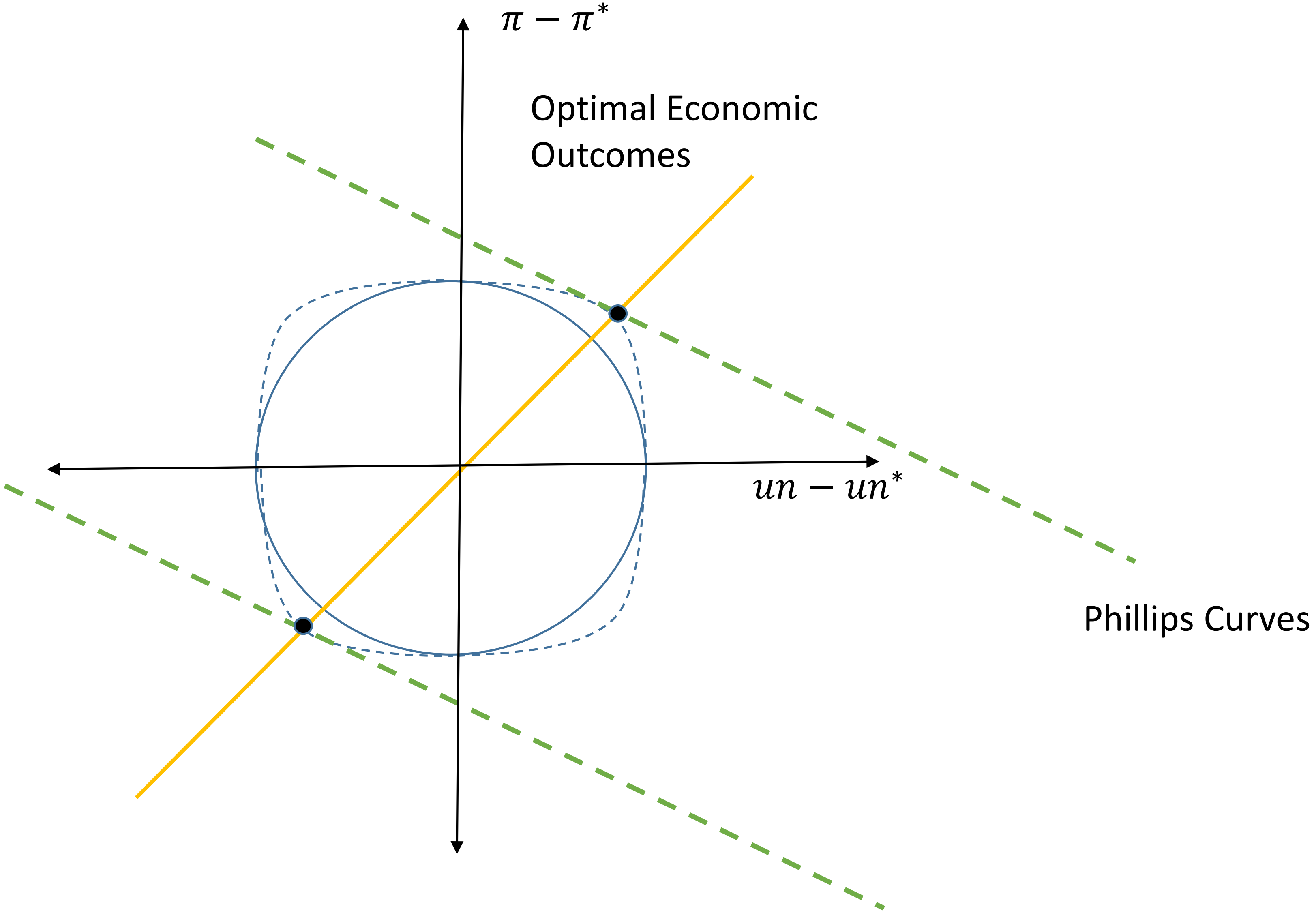
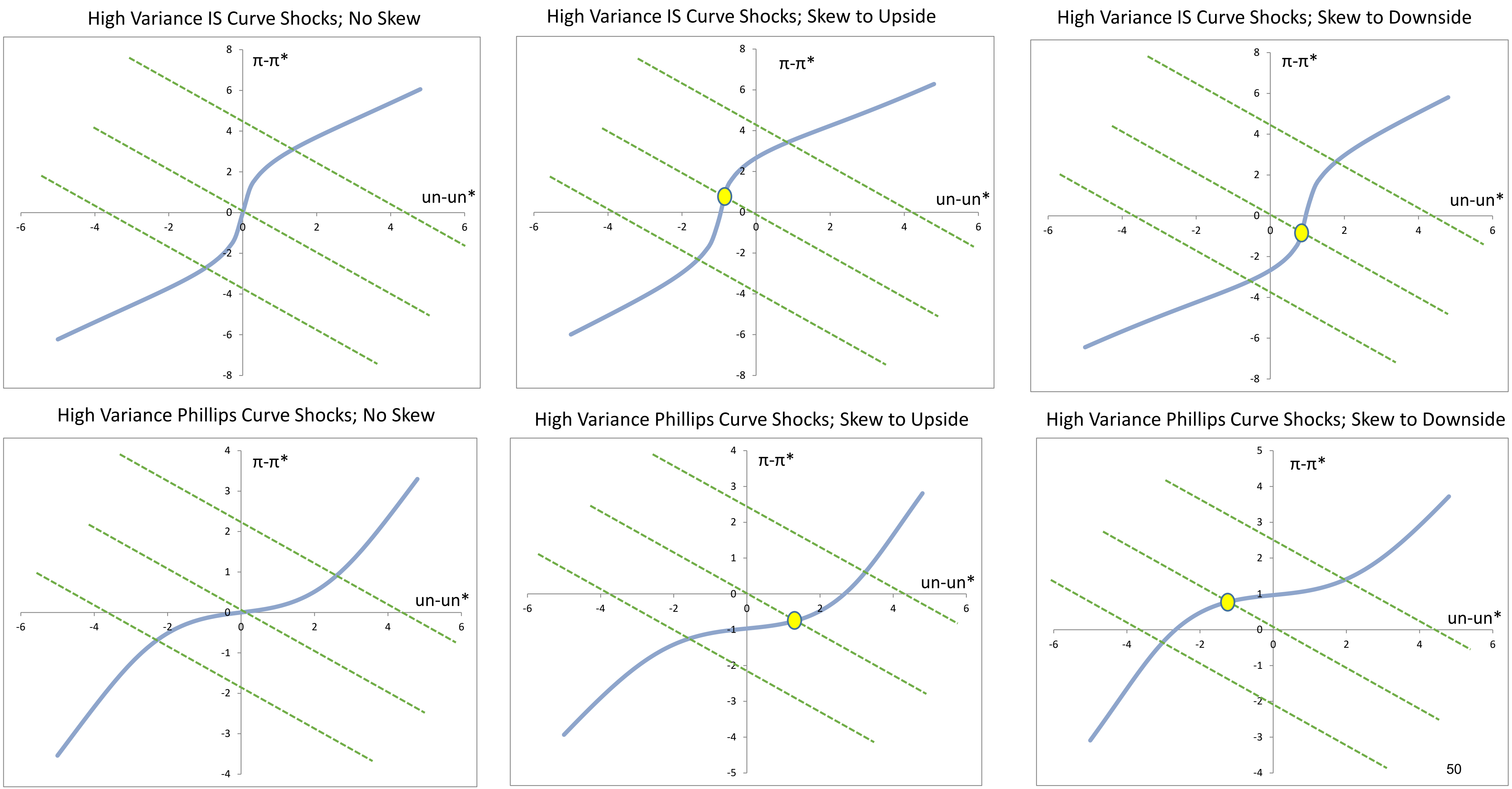


Figure 7.2: Variance and Skew Effects



Take 8: Extended Baseline Model and The Zero Lower Bound

There are considerable downside risks to the near-term outlook as well. […] Turning to inflation, […] the considerable slack in labor and product markets will put downward pressure on the underlying rate of inflation over the next few years. […] Given the sizable downside risk to the forecast for growth, the risks to the inflation forecast are likewise weighted to the downside. In conclusion, I think the present situation obviously calls for an easing of policy, as I assumed in my forecast. Given the seriousness of the situation, I believe that we should put as much stimulus into the system as we can as soon as we can.

Vice Chair Yellen, October 2008 FOMC Meeting

As the quote from Chair Yellen above attests, concerns about downside risks to the outlook and the implications for policy were at the forefront of policy discussions at the peak of crisis and over the long, slow period of recovery. And yet, as discussed above, with a quadratic objective function for the central bank and a linear model for the economy, the role for uncertainty in the conduct of optimal monetary policy is quite limited. While some types of "multiplicative" uncertainty do matter for monetary policy, uncertainty about many factors such as the equilibrium real rate, the natural rate of unemployment rate, and the magnitude of shocks to the economy and inflation does not affect optimal policy choices in the baseline model. Moreover, skews in the distribution of shocks to the economy also have no important implications for policy in the baseline model.

The discussion below shows how optimal policy choices change when the key assumptions underlying the baseline model are significantly changed to encompass an important nonlinearity in the economy—the zero lower bound (ZLB) on short-term nominal interest rates. The zero lower bound constraint affects many of the standard results from the baseline model. In particular, the zero lower bound constraint provides an important reason for central bankers to care a great deal about uncertainty even when the loss function for the central bank is the simple quadratic form assumed in the baseline model.

In the baseline model, the central bank is very powerful and can perfectly offset all observed shocks to aggregate demand. As a result, the central bank does not respond now to uncertainty related to future values of aggregate demand; there may be a lot of uncertainty about future shocks to the economy but the central bank can be completely confident that it will be able to address those shocks through appropriate setting of the policy rate. In contrast, with the zero lower bound constraint, the ability to easily offset shocks can be constrained. In that case, policymakers may have an incentive to take actions in the current period that will help avoid becoming constrained in the future. In effect, they will be willing to take out some "insurance" against possible future shocks by providing more accommodation in the current period. This insurance has an associated cost in the form of larger unemployment rate and inflation rate gaps in the current period than would be chosen in the absence of a ZLB constraint. The policymaker then must weigh the gain from reducing the likelihood of a future episode in which the ZLB is binding with the cost incurred today by intentionally running a policy that is different than the one that would minimize today's objective function.

8.1 Baseline Two-Period Model

To illustrate some of the issues associated with the zero lower bound, it's useful to extend the baseline model to include two periods with a simple form of persistence in the unemployment rate across the two periods. This extension makes it possible to examine how the central bank should adjust the stance of monetary policy to guard against the possibility of becoming constrained by the zero lower bound in a future period.

8.11 Structure of the Two Period Model

As shown in equation (11) below, the two period version of the baseline model considered here specifies an IS curve in period 2 in which the unemployment rate gap depends on the policy rate as before but also on the level of the unemployment rate from period 1. That type of relationship captures the effects of lags in the economy. As in the baseline model, we assume that the policymaker observes all the shocks to the IS curve and the Phillips curve in each period.

$$\hat{u}_2 = un_2 - un^* = \hat{u}_1 + \alpha(i_2 - \pi^e - r^*) + \eta_2 \qquad (11)$$

To solve the model, it is instructive to consider the policy choices in period 2 in the case when the policymaker does not face the ZLB constraint. In this case, the policymaker sets the policy rate in period 2 to fully offset the effect of the unemployment rate gap from the prior period as well as the shock to the unemployment rate in period. The result is that optimal economic outcomes in period 2 are identical to those in the baseline model and, in particular, do not depend on any outcomes from the first period.

The expected value of the loss function in period 1 is thus essentially the same as in the baseline model. As a result, the shape of the indifference curves for the loss function in the first period is the same as in the baseline model and optimal economic outcomes are identical to the baseline model. The bottom line is that when the policymaker is unconstrained in the choice of the policy rate, optimal economic outcomes in the two period model are identical to optimal economic outcomes in the baseline one period model. There is a difference, however, in the rule for the optimal setting of the policy rate that is necessary to achieve the optimal economic outcomes. In particular, as shown in the appendix, the optimal setting of the policy rate in period 2 depends on outcomes from the first period. That dependency, in turn, arises because of the lag structure embedded in the modified IS curve in equation (11).

8.2 The Zero Lower Bound in the Two Period Model

A key aspect of the baseline two period model is that the central bank is able to adjust the level of short term interest rates in a way that positions the IS curve to achieve the optimal point. However, the situation is more complicated if the policymaker cannot push the nominal federal funds rate below zero. When that constraint is binding, the level of output and inflation will be determined at the intersection of the Phillips curve and the IS curve with the policy rate pinned at zero. This point will be at a position that could be much inferior to the unconstrained optimal policy choice. For example, as shown in figure (8.1), suppose the economy is subject to both an adverse shock to the IS curve and a shock that pushes the Phillips curve lower. Absent any adjustment in the policy rate, the IS curve could shift well to the right as shown by the dashed blue line. In the baseline model, that presents no real difficulties because the central bank can simply lower the policy rate as much as desired to achieve the optimal outcome denoted by the black dot. However, with a zero lower bound constraint, the central bank may only be able to move the IS curve to the left as far as the red dashed line. In this case, the unemployment rate and inflation rate are determined at the intersection of the constrained IS curve and the Phillips curve at a point like the red dot. That outcome is very poor from the central bank's perspective. Indeed, that point lies on an indifference curve that is much farther out than the one corresponding to the unconstrained optimum at the black dot.

8.21 Characterizing the Zero Lower Bound

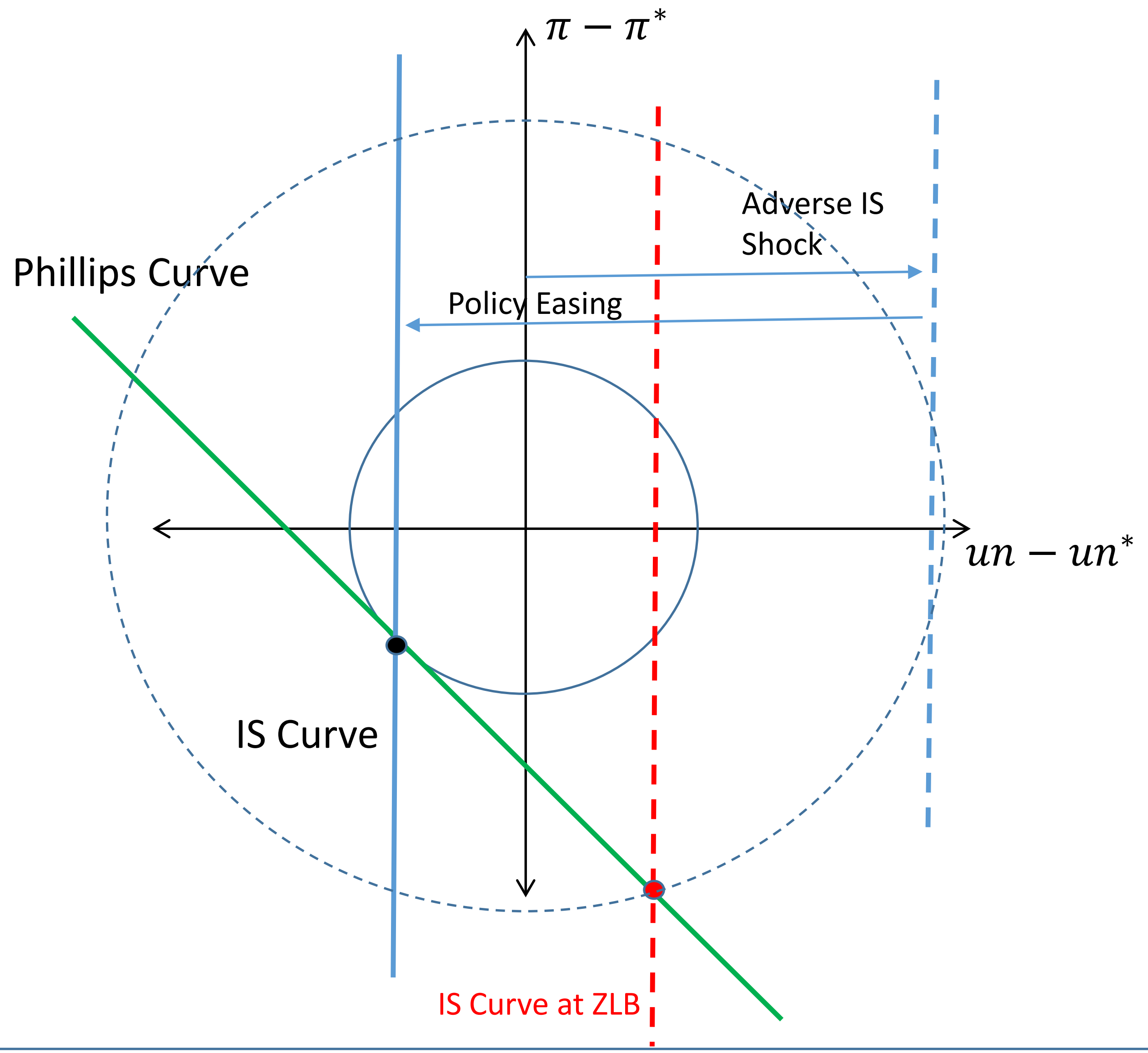
What kinds of situations give rise to a binding zero lower bound constraint? As discussed above, the central bank encounters the ZLB when the optimal unconstrained choice of the policy rate in period 2 is less than or equal to zero. As shown in the appendix, in the two period model, the probability that the optimal unconstrained policy rate is less than or equal to 0 in period 2 is given by:

$$P(ZLB) = \mathrm{N}((\hat{u}_1 - \alpha i^*)/\sigma_z) \qquad (12)$$

For convenience, the shocks to the IS curve and Phillips curve are assumed to be normally distributed, and the function N() is just the cumulative distribution function for the normal distribution. The term in the numerator represents the gap between the inherited unemployment rate gap from the first period and the maximum amount of stimulus that the central bank can deliver in period 2 by reducing the policy rate all the way to zero. The maximum stimulus, in turn, is a function of the level of the neutral policy rate, $i^* = r^* + \pi^*$. The higher the values of these parameters, the more stimulus the central bank can provide in response to adverse shocks to aggregate demand and the lower the probability that the zero lower bound will become a constraint. Similarly, the slope of the IS curve, α, translates the maximum amount of rate stimulus into an economic measure comparable to the unemployment rate. So the more sensitive the economy is to interest rates, the higher the value of α, and the lower the probability that the zero lower bound constraint will become binding. Conversely, an elevated level of the unemployment rate gap from the first period, $\hat{u}_1$, carried into the second period makes it more likely that the zero lower bound constraint will become binding. The denominator, σ_z, is the standard deviation of the shock term that matters in determining whether the zero lower bound becomes binding. As discussed in the appendix, the variance of this shock term is a function of the variances of both the shocks to the Phillips curve and the IS curve. Assuming that the numerator is negative, higher levels of the variance of shocks to the unemployment rate and inflation make it more likely that the zero lower bound will become binding.

This simple expression helps to explain why central bankers around the world have been so concerned about the apparent global trend toward lower levels of the neutral real rate, r^*. All else equal, a low value for r^* increases the probability of encountering the zero lower bound. The expression also explains why some have suggested that central banks should raise the level of the inflation target. All else equal, a higher inflation target would increase the maximum amount of economic stimulus that could be delivered in the event of an adverse shock to aggregate demand and thus reduces the probability of a binding ZLB constraint. The expression also makes clear why economists are so interested in developing new tools such as forward guidance or asset purchases that may be effective in providing stimulus but that would not be subject to the zero lower bound constraint.

Figure 8.1: The ZLB Constraint



Take 9: Risk Management Near the Zero Lower Bound

The risk of a binding ZLB changes the nature of optimal policy. In period 1, optimal policy depends partly on the level of inflation and unemployment gaps in the current period. But now the level of the unemployment rate gap in the current period has implications for the starting point in the second period and the possibility of being constrained by the ZLB. In particular, there are incentives to strengthen the economy in the first period to reduce the likelihood that the ZLB constraint will become binding in period 2. Like insurance, the extent to which such actions are taken depends on probability of the future adverse outcome and the cost of the insurance. The cost of insurance in this case is the need to move away from a policy setting that would otherwise minimize the loss function for just the current period.

The appendix works through the algebra of how the zero lower bound constraint affects the shape of the indifference curves in period 1. As shown in figure (9.1), the effect of the ZLB constraint in period 2 alters the shape and position of the indifference curves in period 1. Each indifference curve is "flattened" as the unemployment rate gap in period 1 moves above zero. That's because a positive unemployment rate gap in period 1 imposes the usual cost in period 1 and also carries with it a higher potential for a cost in period 2 associated with a binding zero lower bound constraint.

The indifference curve also shifts to the left. The net result in that the optimal economic outcomes line now has a non-linear shape with a relatively "steep" slope for positive values of the unemployment gap in period 1 and a somewhat flatter slope for negative values of the unemployment rate gap. So in response to a positive inflation shock in period 1, the policymaker would tend to allow more of that shock to show through to the inflation rate than in the baseline model. Moving to counter that inflation shock in the baseline model would involve tightening policy and tolerating a period with a positive unemployment rate gap. With the threat of the zero lower bound in period 2, however, the policymaker would want to keep the unemployment rate gap smaller in the current period because carrying a sizable positive unemployment rate gap into the second period would create a higher probability of being constrained by the zero lower bound.

The optimal economic outcomes line now does not pass through the origin and "crosses" into quadrant 2. In the baseline model, optimal policy never occurred at points in quadrant 2 because with the unemployment rate below target and inflation above target, it always made sense for the policymaker to tighten policy. But with the possibility of a binding zero lower bound in period 2, the policymaker may be willing to tolerate a period with above target inflation and below target unemployment in order to reduce the probability and expected cost of encountering the zero lower bound in the next period.

9.1 Certainty Equivalence Revisited

Uncertainty that mattered little in the baseline model can be quite important in the presence of ZLB risks. In particular, certainty equivalence does not apply even for a simple quadratic objective function. For example, as discussed above, the variance of output shocks has an important bearing on the risk of being trapped at the ZLB. The larger the variance of output shocks, the more the policymaker should want to enter the second period with an unemployment rate below the NAIRU and inflation above π^*. The variance of shocks to inflation similarly affects the possibility of encountering the zero lower bound.

Uncertainty about the level of r^* now presents asymmetric policy risks. If the level of r^* is lower than the central bank believes, the chances of being trapped at the zero lower bound are higher. As a result, unlike the baseline model, uncertainty about the level of r^* matters a lot to policymakers when the stance of monetary policy could be constrained by the zero lower bound. In terms of the indifference curves, greater uncertainty about r^* increases the potential risks of a zero lower bound episode and "flattens" the portion of the curves with positive unemployment rate gaps. So the higher the uncertainty about r^*, the larger the "risk management" incentives are associated with the zero lower bound.

The risk of a binding ZLB constraint also implies that higher moments of the distribution of shocks to the unemployment rate and inflation such as skews and fat tails will affect optimal policy choices. For example, if the distribution of shocks to the unemployment rate is skewed to higher values or simply has "fat" tails, that will imply larger risks that that the ZLB could be binding. As a result, the policymaker will have greater incentives to implement a risk-management approach to policy in the current period.

9.2 The Curse of the Flat Phillips Curve Revisited

In the baseline model, a flat Phillips curve was unambiguously a bad thing for central bankers. A flat Phillips curve makes it more costly for the central bank to combat shocks to inflation because it must engineer larger changes in the unemployment rate to offset any shock to the Phillips curve. In the baseline model, the expected value of the central bank's loss function thus was larger with a flat Phillips curve.

That basic result is more complicated in the presence of a zero lower bound constraint. In the case when the central bank is constrained by the zero lower bound, a flat Phillips curve can help to keep inflation anchored near the central bank's inflation target. Thus while a flat Phillips curve is unambiguously a curse when the central bank can freely adjust the stance of policy to offset any shock to the real economy, it can be a blessing of sorts when adjustments to the stance of policy are constrained by the zero lower bound. This situation is illustrated in figure (9.2). Here, when the policymaker is unconstrained by the ZLB, the combination of a downward shift in the Phillips curve and a large adverse shock to the IS curve would lead to policy adjustments that put the economy at the point shown by the black dot. If the Phillips curve is relatively flat, the policymaker could again ease policy aggressively to arrive at the optimal economic outcomes shown by the grey dot. In the case with the flat Phillips curve, the policymaker is worse off because the grey dot lies on an indifference curve that is "farther out" than for the black dot.

However, if the policymaker cannot offset the adverse shock to the IS curve because of the ZLB, the economy could end up at a point like the red dot with very large unemployment rate and inflation rate gaps. In this case, the outcome with the flat Phillips curve is a better than in the case with a steeper curve because the inability to completely adjust the policy rate as desired has a smaller impact on the rate of inflation.

9.3 Fear the Flat IS Curve

In the baseline model, policymakers had nothing to fear in the case of a relatively flat IS curve. They could easily achieve any desired level of the unemployment rate by simply implementing larger adjustments in the stance of monetary policy. But that conclusion also needs to be tempered greatly when there is a risk of a binding ZLB constraint. A flat IS curve means that smaller shocks to the unemployment rate can drive the central bank to the zero lower bound. That effect is evident in the expression for the probability of encountering the zero lower bound in equation (11). There, the parameter α enters with a negative sign because the higher the value of α, the more economic stimulus the central bank can impart by cutting the federal funds rate all the way to zero. When α is quite small, this capability to provide economic stimulus is much reduced, thus greatly increasing the odds of encountering the zero lower bound.

Summary

The zero lower bound presents very serious challenges for central banks. The ZLB constrains the amount of economic stimulus that the central bank can provide in response to adverse economic shocks. In extreme cases, the inability to provide adequate stimulus can lead to economic conditions that prolong the episode of being constrained by the ZLB. For example, a central bank that is constrained by the ZLB may not be able to counter deflationary pressures. As inflation falls, the real debt burdens of households and businesses increase and their spending may decline further. Many authors have pointed to such debt-deflation dynamics during periods like the Great Depression and, more recently, in the long period of declining prices in Japan.

As discussed above, the risk that the ZLB constraint will become binding is importantly influenced by the long-run value of the neutral real rate as well as the central bank's inflation target and the interest sensitivity of aggregate demand. Given the decline in real interest rates observed over recent decades, many have suggested that there may be less scope than in the past for central banks to respond to adverse economic shocks before encountering the ZLB. Moreover, the possibility that the central bank could be constrained by the ZLB can affect optimal policy even when the economy is performing reasonably well.

Figure 9.1 Optimal Economic Outcomes Line: Zero Lower Bound

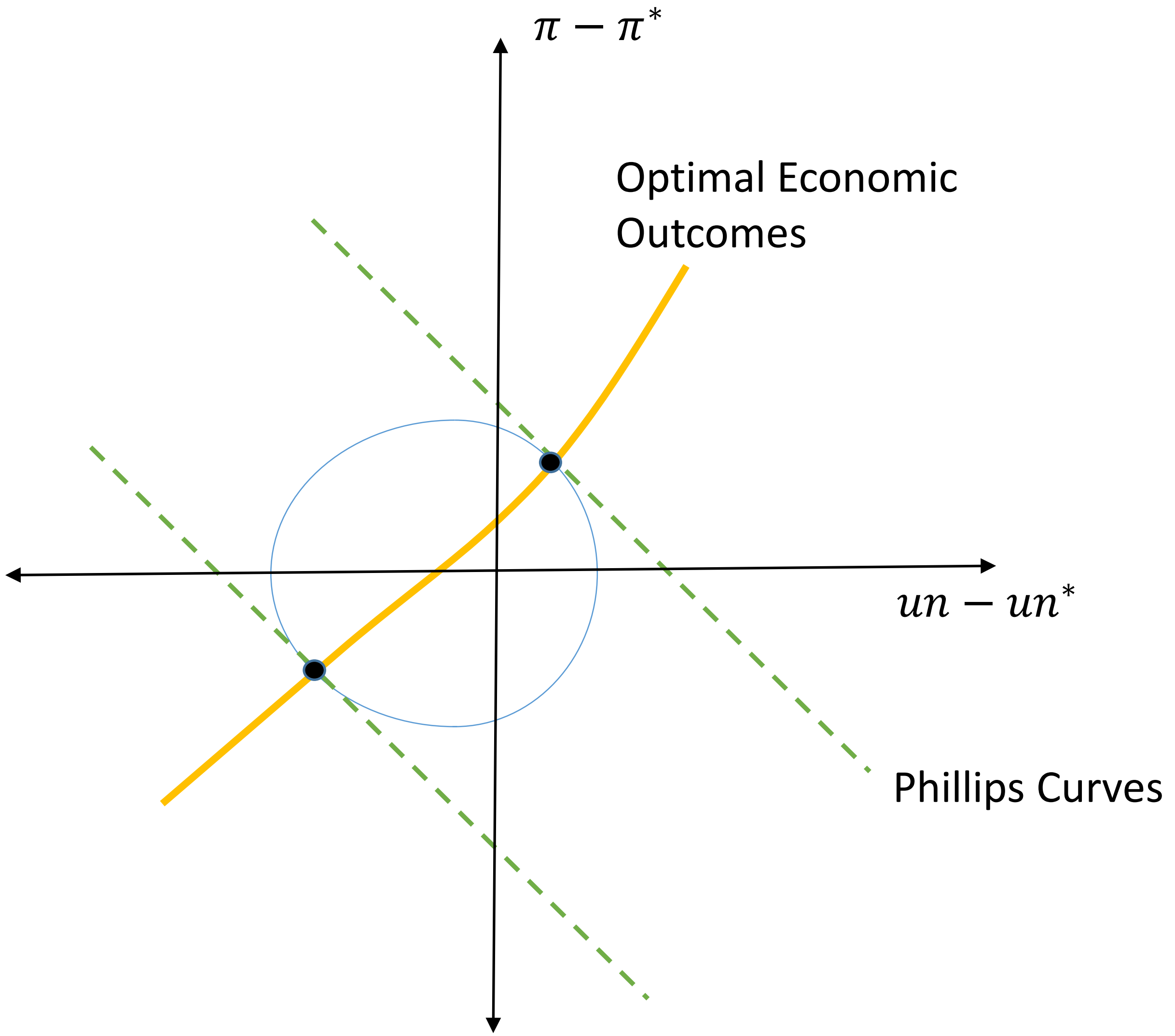
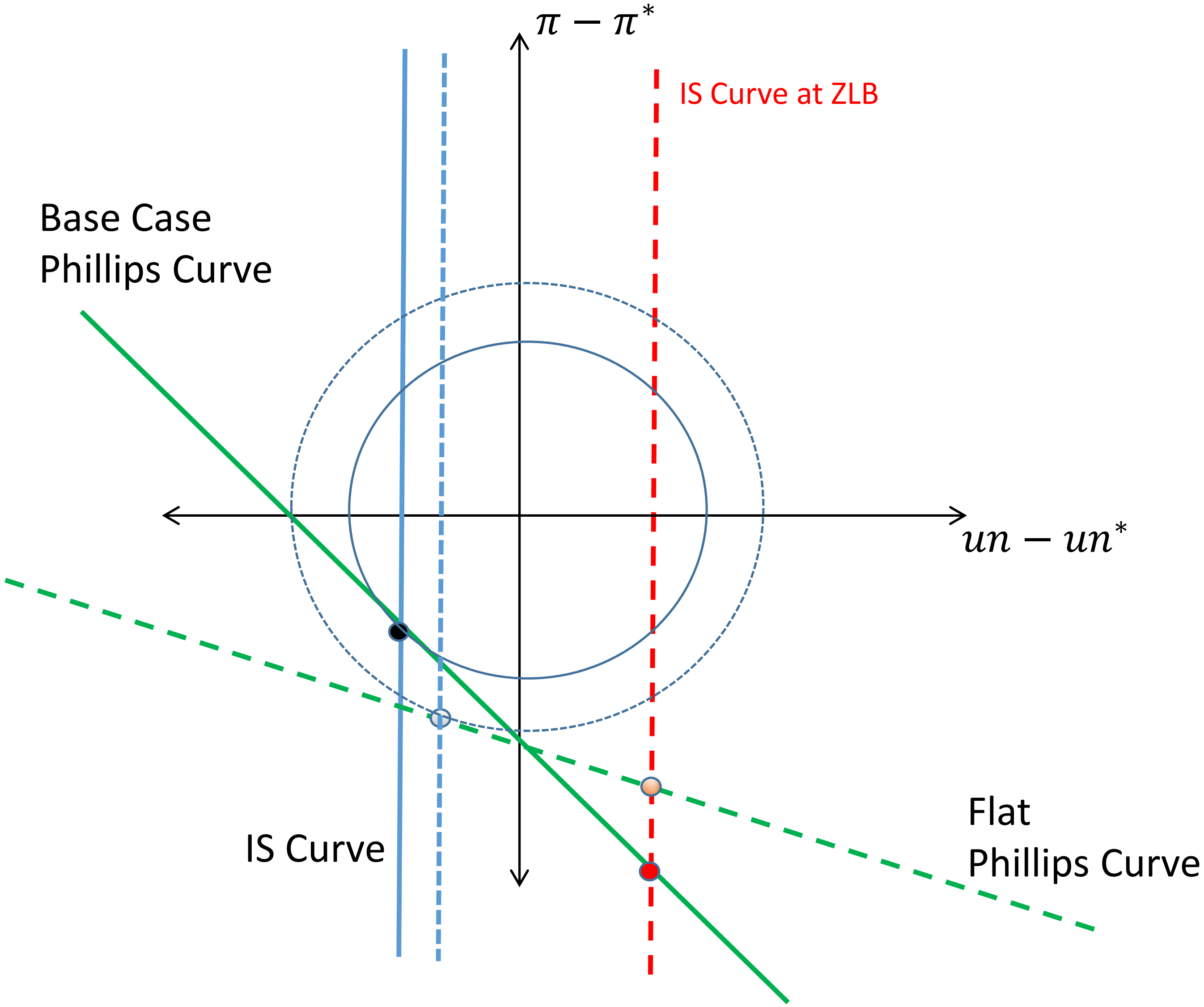


Figure 9.2: The Partial Blessing of a Flat Phillips Curve



Take 10: Special Topics

The basic framework discussed in previous sections is helpful in thinking about many other types of topics. The discussion below touches on ways that "endogenous" risks such as financial stability and risks to inflation expectations can be examined in the context of the baseline model. In addition, simple variations on the baseline framework help to illustrate the potential connections between concepts of "fairness" and the conduct of monetary policy.

Endogenous Risks

Most of the types of risks discussed above were associated with a set of exogenous shocks to the economy; the statistical distributions for these shocks were largely unaffected by other economic variables. The potential for some degree of interaction or "endogeneity" between the nature of shocks to the economy and economic variables adds yet another layer of complexity to the analysis of optimal policy.

Financial Stability Risks

Policymakers often worry that the stance of monetary policy may contribute to financial strains or imbalances in a way that could set the stage for a large adverse shock to the economy in the future. One simple way of interpreting that concern in the context of the models discussed in previous sections is to hypothesize that the variance of unobserved shocks may partly depend on the stance of policy or on some other endogenous variable. For example, "Take 4" above examined the effect of additive uncertainty in the baseline model. The general conclusion under the certainty equivalence principle was that this form of uncertainty had no implications for the conduct of monetary policy. However, if the variance of the additive shocks depended on, say, the magnitude of the deviation of the policy rate from its neutral level, then the optimal policy choices would be affected. For example, some observers have worried that a lengthy period with the federal funds rate close to zero would contribute to financial imbalances that would set the stage for another economic downturn.

Optimal policy in the face of this dependence of the variance of the shocks to the IS equation or the Phillips curve equation on the policy rate is analytically similar to the case of "multiplicative" parameter uncertainty noted above in Take 4. To the extent that a larger interest rate gap (in absolute magnitude) implies a larger variance in the shocks to the IS curve or the Phillips curve, the policymaker will tend to keep the magnitude of the interest rate gap smaller than would otherwise be the case—a variation on the attenuation principle noted above. It is important to note, however, that the policy conclusions to be drawn in this type of analysis depend very much on the specification of the financial stability risk. For example, if the variance of shocks to the IS curve depends not on the interest rate gap but on the unemployment rate gap, then policymakers would move aggressively to counter shocks to the IS curve.

Risks to Inflation Expectations

A major shortcoming of the baseline model developed above is that inflation expectations are treated as determined outside the model. Policymakers often worry, however, that changes in the stance of monetary policy could have outsized and persistent effects on inflation expectations. Those sorts of effects are difficult to model, but a (very) crude variation on the baseline model captures the flavor of that type of risk. For example, suppose there is some probability that inflation expectations will jump up (or down) by a fixed amount δ in the next period and that this probability depends on the current setting of the policy rate relative to the neutral rate. In the two period model developed in Take 8 above, that "jump risk" for inflation expectations would affect the expected value of the loss function in the second period. As a result, the expected value of the loss function in the first period would assume the form:

$$L = L_1 + L_2 = \left(\frac{1}{2}\right)(\hat{u}_1^2 + \hat{\pi}_1^2) + \frac{\left(\frac{1}{2}\right)\delta^2}{1+\tau^2}\text{Prob}(i_1 - i^*) + \frac{\left(\frac{1}{2}\right)\sigma_\varepsilon^2}{1+\tau^2}$$

Here, the term $\text{Prob}(i_1 - i^*)$ is the probability that inflation jumps by the amount δ in period 2 as a function of the interest rate gap in period 1. In this case, in judging the appropriate policy action today, the policymaker would need to weigh the benefits in terms of adjusting today's inflation and unemployment rate gaps versus the potential future costs of experiencing persistently higher (or lower) inflation expectations in future periods. This type of risk is similar to the endogenous financial stability risk noted above in that it can act as a form of restraint on policymaker's desired choice of the policy in the current period. However, also similar to the case of endogenous financial stability risks, the nature of the policy conclusions depend very much on the nature of the assumed risks to inflation expectations. For example, if the probability of a future jump in inflation expectations depends not on the current interest rate gap but on the current realized value of inflation, then the central bank today would tend to be quite aggressive in keeping today's inflation close to target. The objective function for that policymaker in that case might be reasonably approximated by the "balanced hawk" objective function described above. As noted in Take 4 above, a balanced hawk tends to ease aggressively in response to downward shifts in the Phillips curve.

Fairness and the Central Bank Objective Function

Central banks often conduct policy to promote price stability as well as strong performance for the economy and labor market. The labor market is heterogeneous, and the experiences of individuals in different groups can be very strongly affected by biases based on group characteristics such as ethnicity, gender, and social status. Central banks typically are focused on the performance of the overall labor market and generally do not have the tools or the public mandate to address issues of discrimination or bias in the workplace. That said, there are ways these issues can intersect with the conduct of monetary policy.

In the baseline model above, the variable $\hat{u}$ was defined as the deviation of the unemployment rate from the natural rate. One might view $\hat{u}$ instead as a more general indicator of the "state of the labor market." The labor market experiences of different groups then might be viewed as driven by this aggregate variable but in somewhat different ways. For example, suppose that the unemployment rate gap for group 1 is $\hat{g}_1 = \lambda_1\hat{u}$ while that for group 2 is $\hat{g}_2 = \lambda_2\hat{u}$. In this setup, if the overall indicator of the state of the labor market $\hat{u}$ is equal to zero, then the unemployment rate gap in each group would be equal to zero as well. So it seems that everything in this slightly different setup would be very similar to the baseline model. Indeed, that would be the case if the objective for the central bank included the squared value of the overall labor market conditions indicator $\hat{u}$ in the loss function as in the baseline model:

$L = (\frac{1}{2})(\hat{u}^2 + \hat{\pi}^2)$ (13)

If the state of the labor market variable $\hat{u}$ is unobserved, the central bank might choose to use the weighted average of the unemployment rate gaps for each group in the loss function as the measure of the aggregate unemployment rate gap, $\hat{g} = \omega\hat{g}_1 + (1-\omega)\hat{g}_2$. Here, the weight ω is the share of group 1 in the total labor force. In this case, the loss function would be given by:

$$L = (\frac{1}{2})(\hat{g}^2 + \hat{\pi}^2)$$

Simplifying, this loss function can be expressed as:

$$L = \left(\frac{1}{2}\right)((\omega\lambda_1 + (1-\omega)\lambda_2)^2\hat{u}^2 + \hat{\pi}^2) = \left(\frac{1}{2}\right)(\mathrm{a}^2\hat{u}^2 + \hat{\pi}^2) \qquad (14)$$

In this specification, the policymaker focuses only on the aggregate measure of the unemployment rate gap, $\hat{g}$. As shown in equation 14, this turns out to be the same as just focusing on the overall labor market indicator $\hat{u}$ weighted by the squared value of the term $a = \omega\lambda_1 + (1-\omega)\lambda_2$. If the value of a in equation (14) is equal to 1, the loss function would be exactly that same as in the baseline model.

One might ask whether the implicit weighting of the individual group unemployment rate gaps in equation (14) is consistent with common interpretations of "fairness." For example, taken literally, the focus on the aggregate index implies that the policymaker would be equally happy with an employment outcome in which both groups were at full employment, $\hat{g}_1 = \hat{g}_2 = 0$, and another in which one group had a high unemployment rate while another had a very low unemployment rate so that the overall index $\hat{g}$ remained at zero.

Another loss function that might be viewed as a fairer treatment of individual groups would focus directly on the unemployment rate gaps for the two individual groups with each squared unemployment rate gap weighted by the share of that group in the total labor force. In that case, the loss function would take the form:

$$L = (\frac{1}{2})(\omega\hat{g}_1^2 + (1-\omega)\hat{g}_1^2 + \hat{\pi}^2)$$

Using the definitions for the group unemployment rate gaps above, this loss function can be expressed as:

$$L = \left(\frac{1}{2}\right)\left((\omega\lambda_1^2 + (1-\omega)\lambda_2^2)\hat{u}^2 + \hat{\pi}^2\right) = (\frac{1}{2})(b^2\hat{u}^2 + \hat{\pi}^2) \qquad (15)$$

As before, this loss function again turns out to be a function of the aggregate state of the labor market indicator $\hat{u}$, but the weight on that variable, b^2, is larger than in the case when the policymaker focuses only on the aggregate unemployment index $\hat{g}$. Indeed, the weight on the state of the labor market indicator in this case is quite sensitive to the group that has a relatively high response to the overall labor market indicator. So, for example, if one group's unemployment rate goes up by twice as much as another group when the overall labor market indicator increases, that group's experience will be weighted more heavily by this version of the central bank objective function relative to the more common specifications in equations (13) and (14). In this example, the objective function in (15) is similar to that for the "balanced dove" objective function discussed above in Take 3. Other things equal, a balanced dove would take account of the relatively large "elasticity" of the unemployment rate gap for an individual group by allowing a larger portion of inflation shocks to show through to the rate of inflation. In effect, a "high elasticity" unemployment rate group makes it expensive for the central bank to combat inflation shocks. Intuitively, in the case when the central bank focuses solely on the aggregate unemployment rate as in (14), the effect of the "high elasticity" labor force group matters only to the extent that the experience of that group affects the weighted average unemployment rate gap. In the case when the central bank focuses directly on the squared unemployment rate gaps for individual groups, the experience of the high elasticity labor force group becomes relatively more important in determining the shape of the central bank's objective function and the conduct of optimal policy.

As discussed above, over time, a central bank with a "balanced dove" objective function would achieve maximum employment and stable prices over time. The volatility of inflation over time would be larger than in the baseline model and the variability of the unemployment rate (and those for each individual group) would be lower than in the baseline model.

Takeaways: What to Make of All This?

The aim of this collection of short takes on monetary policy was to convey some of the basic principles of monetary policy strategy in a standard bare bones version of a commonly used policy framework. The two key ingredients in the framework are a model of the economy and a central bank objective function.

One key theme that emerges from the various topics discussed above is that the nature of the central bank loss function assumed in any optimal policy analysis is critical in defining the appropriate policy strategy. The nature of the policy response to inflation and output shocks, changes in inflation expectations, and uncertainty and risks surrounding the economic outlook all hinge critically on the assumed objective function for the central bank. Another key theme that emerges is that in the presence of constraints on the central bank's ability to adjust its policy instruments, optimal policy usually calls for the central bank to adjust the stance of policy in a way that helps to mitigate the potential economic costs of those constraints. In the case of the zero lower bound constraint, that incentive led to the central bank to choose to provide more accommodation than would otherwise be appropriate in order to avoid a binding ZLB constraint in the future. In the case of uncertainty about the slope of the IS curve or financial stability risks, those incentives led the central bank to attenuate its policy response to shocks in an effort to reduce the variance of future shocks to the economy.

Another important theme running beneath the surface of much of the analysis in these notes is the essential role of anchoring inflation expectations at the central bank's target. The model employed here is highly stylized and, in particular, largely assumed that inflation expectations are exogenous and pegged at the central bank's inflation target. That approach completely shuts down a central feature of more sophisticated models—the connection between inflation expectations, the actual path of inflation generated by the model, and the central bank's policy strategy. Many important questions and issues arise in models that incorporate these types of linkages.

Finally, many of the examples above underscore the importance and the challenges of incorporating risk and uncertainty in the formulation of monetary policy. Indeed, central banks in practice operate with far less information and far more uncertainty than is captured in the simple framework developed in these notes. In practice, the structure of the economy is evolving over time, and policymakers may be unsure about even the basic form of key structural relationships in the economy. In addition, the structure of the economy may be very complex, incorporating many aspects of forward-looking behavior and various types of "frictions." In this environment, calculating an "optimal" stance of policy as in the notes above may be all but impossible in the real world. Against that backdrop, central banks have increasingly come to recognize the importance of evaluating approaches to policy that perform well across a variety of economic models, and in response to a wide range of shocks and scenarios.

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Appendix: Mathematical Details

The notes below provide the mathematical details underlying the pictures and analytical points noted in the main text.

Take 1: Baseline Model

In the baseline model, the IS curve and Phillips curve specifications are as shown in equations (1.1) and (1.2).

$$\hat{u} = un - un^* = \alpha(i - \pi^e - r^*) + \eta = \alpha(i - (r^* + \pi^*)) - \alpha(\pi^e - \pi^*) + \eta \quad (1.1)$$

$$\hat{\pi} = \pi - \pi^* = -\tau\hat{u} + (\pi^e - \pi^*) + \varepsilon \quad (1.2)$$

Unless noted otherwise, we generally assume that the shock terms η and ε are normally distributed with variances σ_η^2 and σ_ε^2, respectively.

The reduced form expression for the inflation rate gap is then:

$$\hat{\pi} = -\tau\hat{u} + (\pi^e - \pi^*) + \varepsilon = -\tau\alpha(i - i^*) + (1 + \alpha\tau)(\pi^e - \pi^*) - \tau\eta + \varepsilon \quad (1.3)$$

Where

$$i^* = r^* + \pi^*$$

In the reduced form expression for the inflation gap, the coefficient on the interest rate gap term is negative—tighter policy boosts the unemployment rate and pushes inflation down. The coefficient on deviations of inflation expectations above the central bank's target rate is positive and greater than 1. This model has very high sensitivity to inflation expectations through two channels. In the Phillips curve equation (1.2), an increase in inflation expectations shows through one for one to an increase in the inflation rate. In addition, all else equal, an increase in inflation expectations pushes down real interest rates in the IS curve, stimulates demand and puts downward pressure on the unemployment rate. The pickup in aggregate demand adds to the upward pressure on inflation in the model.

Take 2: Optimal Policy in the Baseline Model

The loss function shown for the central bank (equation 2.1) is a function of the squared deviations of the unemployment rate and inflation from their respective goals.

$$L = (\tfrac{1}{2})(\hat{u}^2 + \hat{\pi}^2) \quad (2.1)$$

The first order condition for minimizing this loss function generates the expression for the "optimal economic outcomes line" shown in 2.2.

$$\hat{u} = \tau\hat{\pi} \quad (2.2)$$

Combining the first order condition in (2.2) with the Phillips curve (1.2) yields expressions for the unemployment rate and inflation gaps as functions of the exogenous variables (equations 2.3 and 2.4)

$$\hat{u} = \tau(\delta + \varepsilon)/(1 + \tau^2) \quad (2.3)$$

$$\hat{\pi} = (\delta + \varepsilon)/(1 + \tau^2) \tag{2.4}$$

where $\delta = (\pi^e - \pi^*)$

Combining the solution for the optimal unemployment rate gap in (2.3) with the IS curve equation (1.1) yields the equation for the policy rule for the interest rate gap:

$$i - i^* = \left(1 + \frac{\tau}{\alpha(1+\tau^2)}\right)\delta + \frac{\tau}{\alpha(1+\tau^2)}\varepsilon - \frac{1}{\alpha}\eta \tag{2.5}$$

The expected value of the loss function can be computed using the expressions for the optimal unemployment rate and inflation gaps as:

$$E\{L\} = (\tfrac{1}{2})E\{\hat{u}^2 + \hat{\pi}^2\} = (\tfrac{1}{2})(\delta^2 + \sigma_\varepsilon^2)/(1 + \tau^2) \tag{2.6}$$

As shown in equation (2.3) and (2.4), the optimal unconstrained setting of the funds rate completely insulates the unemployment rate and inflation rate from the effects of shocks to the IS curve. Shocks to the Phillips curve affect the unemployment rate and inflation in the same direction. Similarly, any gap between inflation expectations and the central bank's target rate captured by δ pushes the unemployment rate and inflation gaps in the same direction.

The policy rule responds positively to inflation shocks and to inflation expectations deviations from target. Positive shocks to the unemployment rate in the IS curve push the optimal setting of the funds rate lower and vice versa.

The expected value of the loss function increases with the variance of shocks to the Phillips curve. Higher values for the slope of the Phillips curve, τ, reduce the expected loss for the central bank because the central bank can offset inflation shocks with relatively small adjustments in the unemployment rate gap.

Take 4: Variations on the Central Bank Loss Function

Symmetric Hawks and Doves

The baseline model assumes that policymakers assign equal weights to unemployment rate deviations and inflation deviations from their respective targets. The model with "balanced" hawks and doves involves a modification of the loss function for the central bank as:

$$L = (\tfrac{1}{2})(\hat{u}^2 + \gamma\hat{\pi}^2) \tag{4.1}$$

Values of γ greater than one might be associated with inflation "hawks" that place greater weight on minimizing inflation deviations from target. Values of γ less than one might be interpreted as representing inflation "doves" who place relatively greater weight on minimizing unemployment rate deviations from target.

The first order condition leads to an optimal economic outcomes line of the form:

$$\hat{u} = \tau\gamma\hat{\pi} \tag{4.2}$$

The optimal levels of the unemployment rate and inflation gaps are given by:

$$\hat{u} = \gamma\tau(\delta + \varepsilon)/(1 + \gamma\tau^2) \tag{4.3}$$

$$\hat{\pi} = (\delta + \varepsilon)/(1 + \gamma\tau^2) \tag{4.4}$$

As above, the policy rule for the funds rate is based on equation 3.3 and the IS curve:

$$i - i^* = \frac{\left(\frac{\gamma\tau}{\alpha}\right)(\delta + \varepsilon)}{1 + \gamma\tau^2} - \eta/\alpha \tag{4.5}$$

For inflation hawks, these solutions imply that the policymaker would be relatively aggressive in taking actions to combat inflation and would tolerate larger swings in the unemployment rate in order to accomplish that goal. In contrast, inflation doves would generally respond less aggressively to inflation shocks in order to keep the unemployment rate close target.

Asymmetric Hawks and Doves

The model variations considered in this section focus on specifications in which policymakers have different views about deviations of inflation above or below target or about deviation of the unemployment rate above or below target.

Asymmetric Inflation Hawk

For an asymmetric inflation hawk, the central bank objective function is specified as:

$$L = (\tfrac{1}{2})(\hat{u}^2 + \gamma^b\hat{\pi}^2 I(\hat{\pi} < 0) + \gamma^a\hat{\pi}^2 I(\hat{\pi} \geq 0)) \tag{4.6}$$

Here the parameters γ^b and γ^a are the costs the central bank attaches to deviations of inflation below or above target, respectively. The indicator function $I()$ takes on a value of 1 when inflation satisfies the specified criterion and zero otherwise.

For convenience, we assume here that the gap between inflation expectations and the target inflation rate is zero. In this case, when the shock to the Phillips curve is positive, the equations for the optimal economic outcomes line, the optimal levels of the unemployment and inflation gaps, and the policy rule are:

$$\hat{u} = \tau\gamma^a\hat{\pi} \tag{4.7}$$

$$\hat{u} = \gamma^a\tau\varepsilon/(1 + \gamma^a\tau^2) \tag{4.8}$$

$$\hat{\pi} = \varepsilon/(1 + \gamma^a\tau^2) \tag{4.9}$$

$$i - i^* = \frac{\left(\frac{\gamma^a\tau}{\alpha}\right)(\delta + \varepsilon)}{1 + \gamma^a\tau^2} - \eta/\alpha \tag{4.10}$$

When the shock to the Phillips curve is negative, the corresponding equations are:

$$\hat{u} = \tau\gamma^b\hat{\pi} \tag{4.11}$$

$$\hat{u} = \gamma^b\tau\varepsilon/(1 + \gamma^b\tau^2) \tag{4.12}$$

$$\hat{\pi} = \varepsilon/(1 + \gamma^b\tau^2) \tag{4.13}$$

$$i - i^* = \frac{\left(\frac{\gamma^b \tau}{\alpha}\right)(\delta + \varepsilon)}{1+\gamma^b \tau^2} - \eta/\alpha \qquad (4.14)$$

As one would expect, an asymmetric inflation hawk responds quite aggressively to positive inflation shocks but less so in the case of negative inflation shocks. As a result, the response of the policy rate to positive inflation shock is relatively large leading to a relatively damped response of inflation to the shocks and to a relatively large response of the unemployment rate to inflation shocks.

In contrast to the cases discussed so far, the expected values of the unemployment and inflation rate gap are not zero with an asymmetric objective function. In this case, over time, the expected values of these variables would be:

$$E(\hat{u}) = \frac{\left\{\frac{\gamma^a \tau}{1+\gamma^a \tau^2} - \frac{\gamma^b \tau}{1+\gamma^b \tau^2}\right\}\sigma_\varepsilon}{(2\pi)^{.5}} > 0 \qquad (4.15)$$

$$E(\hat{\pi}) = \frac{\left\{\frac{\tau}{1+\gamma^a \tau^2} - \frac{\tau}{1+\gamma^b \tau^2}\right\}\sigma_\varepsilon}{(2\pi)^{.5}} < 0 \qquad (4.16)$$

Similarly, the expected deviation of the policy rate from the natural rate is not zero:

$$E(i - i^*) = \frac{\left\{\frac{\frac{\gamma^a \tau}{\alpha}}{1+\gamma^a \tau^2} - \frac{\frac{\gamma^b \tau}{\alpha}}{1+\gamma^b \tau^2}\right\}}{(2\pi)^{.5}} > 0 \qquad (4.17)$$

Asymmetric Dove

The case for an asymmetric unemployment dove involves a similar modification to the standard central bank loss function:

$$L = \left(\tfrac{1}{2}\right)\left(\theta^b \hat{u}^2 I(\hat{u} < 0) + \theta^a \hat{u}^2 I(\hat{u} \geq 0) + \hat{\pi}^2\right) \qquad (4.18)$$

As above, we assume that the gap between inflation expectations and the central bank's inflation target is zero. In this case, when the shock to the Phillips curve is positive, the optimal economic outcomes line takes the form:

$$\theta^a \hat{u} = \tau \hat{\pi} \qquad (4.19)$$

The corresponding optimal levels of the unemployment and inflation rate gaps are:

$$\hat{u} = \tau\, \varepsilon/(\theta^a + \tau^2) \qquad (4.20)$$

$$\hat{\pi} = \theta^a \varepsilon/(\theta^a + \tau^2) \qquad (4.21)$$

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon)}{\theta^a + \tau^2} - \eta/\alpha \qquad (4.22)$$

In the case with a negative shock to the Phillips curve, the corresponding equations are:

$$\theta^b \hat{u} = \tau \hat{\pi} \qquad (4.23)$$

$$\hat{u} = \tau\varepsilon/(\theta^b + \tau^2) \tag{4.24}$$

$$\hat{\pi} = \theta^b\varepsilon/(\theta^b + \tau^2) \tag{4.25}$$

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon)}{\theta^b + \tau^2} - \eta/\alpha \tag{4.26}$$

The expected values of the variables are not zero:

$$E(\hat{u}) = \frac{\left\{\frac{\tau}{\theta^a + \tau^2} - \frac{\tau}{\theta^b + \tau^2}\right\}}{(2\pi)^{.5}} < 0 \tag{4.27}$$

$$E(\hat{\pi}) = \frac{\left\{\frac{\tau\theta^a}{\theta^a + \tau^2} - \frac{\tau\theta^b}{\theta^b + \tau^2}\right\}}{(2\pi)^{.5}} > 0 \tag{4.28}$$

$$E(i - i^*) = \frac{\left\{\frac{\frac{\tau}{\alpha}}{\theta^a + \tau^2} - \frac{\frac{\tau}{\alpha}}{\theta^b + \tau^2}\right\}}{(2\pi)^{.5}} < 0 \tag{4.29}$$

Take 5: Uncertainty in the Baseline Model

This section introduces uncertainty in the baseline model by positing a second round of shocks, η' *and* ε', that hit the IS curve and Phillips curves after the central bank has chosen the desired level of the policy rate.

Additive Shocks Certainty Equivalence

With the assumed second round shocks, the IS curve and Phillips curve equations are given by:

$$\tilde{u} = un - un^* = \alpha(i - \pi^e - r^*) + \eta + \eta' = \hat{u} + \eta' \tag{5.1}$$

$$\tilde{\pi} = \pi - \pi^* = -\tau\tilde{u} + (\pi^e - \pi^*) + \varepsilon + \varepsilon' = \hat{\pi} - \tau\eta' + \varepsilon' \tag{5.2}$$

In the presence of uncertainty, the policymaker is assumed to minimize the expected value of the loss function given by:

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2\} = \left(\frac{1}{2}\right)((\hat{u}^2 + \hat{\pi}^2) + (1 + \tau^2)\sigma_{\eta'}^2 + \sigma_{\varepsilon'}^2\,)) \tag{5.3}$$

Similar to the baseline model without uncertainty, the optimal economic outcomes line is given by:

$$\hat{u} = \tau\hat{\pi} \tag{5.4}$$

Where $\hat{u}$ *and* $\hat{\pi}$ are the values of the unemployment and inflation gaps prior to the second round shocks, η' *and* ε'. The optimal values for these "ex-ante" variables are given by:

$$\hat{u} = \tau(\delta + \varepsilon)/(1 + \tau^2) \tag{5.5}$$

$$\hat{\pi} = (\delta + \varepsilon)/(1 + \tau^2) \tag{5.6}$$

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon)}{1 + \tau^2} - \eta/\alpha \tag{5.7}$$

The upshot here is that these equations are identical to those for the baseline model in the absence of uncertainty. Note that uncertainty does affect the level of the loss function as shown in (5.3).

However, this type of uncertainty just results in a constant in the loss function and thus has no effect on the optimal solutions. This is the so-called "certainty equivalence" principle.

Parameter Uncertainty and the Attenuation Principle

The type of uncertainty discussed above focused on uncertainty about "additive" factors in the underlying economic model. Another type of uncertainty that has attracted a great deal of attention focuses on uncertainty about factors that enter the model in a nonlinear fashion. To prominent examples of this type of uncertainty are uncertainty about the slope of the IS curve and uncertainty about the slope of the Phillips curve. These two cases are discussed in more detail below.

Uncertainty about the Slope of the IS Curve

Suppose that the policymaker is uncertain about the slope of the IS curve and must set the stance of policy before that uncertainty is resolved. One way of modeling this type of uncertainty is to specify the slope coefficient as:

$$\alpha = \bar{\alpha} + \bar{\alpha} w_{\alpha} \quad (5.8)$$

With this specification, the coefficient α has a fixed mean $\bar{\alpha}$ and variance equal to $\bar{\alpha}^2 \sigma_{\alpha}^2$. To simplify some of the equations, here we simply assume that the gap between inflation expectations and the target inflation rate is equal to zero. The equation for the IS curve then becomes:

$$\tilde{u} = un - un^* = \alpha(i - \pi^e - r^*) + \eta = \bar{\alpha}(i - (r^* + \pi^*)) + \eta + \bar{\alpha}(i - i^*) w_{\alpha} \quad (5.9)$$

The Phillips curve equation is:

$$\tilde{\pi} = \pi - \pi^* = -\tau \tilde{u} + \varepsilon = \hat{\pi} - \tau \bar{\alpha}(i - i^*) w_{\alpha} \quad (5.10)$$

It is convenient to write the inflation and output gaps in terms of the variables that are known at the time the policymaker chooses a setting for the interest rate and those that are not. These two values are given in (5.11) and (5.12) below.

$$\hat{u} = \mathrm{E}\{\tilde{u}\} = \bar{\alpha}(i - (r^* + \pi^*)) + \eta \quad (5.11)$$

$$\hat{\pi} = \mathrm{E}\{\tilde{\pi}\} = -\tau \hat{u} + \varepsilon \quad (5.12)$$

The policymaker is assumed to minimize the expected value of the loss function which can be written as:

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2) = \left(\frac{1}{2}\right) ((\hat{u}^2 + \hat{\pi}^2) + \bar{\alpha}^2 (1 + \tau^2) \sigma_{\alpha}^2 (i - i^*)^2) \quad (5.13)$$

The form of this expression clarifies the way in which uncertainty about the slope coefficient affects the incentives faced by the central bank. Now the uncertainty term entering the loss function is affected by the central bank's choice of the policy rate gap $i - i^*$. Intuitively, if there is uncertainty about the slope coefficient in the IS curve, policymakers can mitigate that uncertainty by keeping the magnitude of the interest rate gap close to zero. The first order condition in this case is:

$$\bar{\alpha}(\hat{u} - \tau \hat{\pi}) + \bar{\alpha}^2 (1 + \tau^2) \sigma_{\alpha}^2 (i - i^*) = 0 \quad (5.14)$$

This can be further rewritten as:

$$(\hat{u} - \tau\hat{\pi}) + (1 + \tau^2)\sigma_\alpha^2(\hat{u} - \eta) = 0 \quad (5.15)$$

Which leads to an expression for the optimal economic outcomes line:

$$\hat{\pi} = (\frac{1}{\tau})((1 + (1 + \tau^2)\sigma_\alpha^2)\hat{u} - (1 + \tau^2)\sigma_\alpha^2\eta) \quad (5.16)$$

When the uncertainty about the slope of the IS curve falls to zero, this expression collapses to the optimal economic outcomes line for the baseline model. When the variance of the slope term is positive, the optimal economic outcomes line is steeper than in the baseline model. That stems from the fact that that adjusting the policy rate in response to a shock to a Phillips curve is more costly than in the baseline model. As a result, it is optimal to allow more of the inflation shock to show through to the level of inflation. Conversely, a smaller portion of the inflation shock shows through to the unemployment rate because the central bank is not adjusting the stance of monetary policy as much as in the baseline model.

Another notable aspect of equation 5.13 is that the optimal line no longer passes through the origin. Now the position of the optimal line is affected by the shock to the IS curve. When there is a positive shock to the IS curve ($\eta > 0$), the policymaker now does not adjust the stance of policy to completely offset the shock as in the baseline model. As a result, an adverse shock to the IS curve puts downward pressure on the optimal inflation rate and vice versa.

As before, we can combine the optimal economic outcomes line and the Phillips curve to obtain the optimal solutions for the unemployment rate and inflation gaps:

$$\hat{u} = \tau\varepsilon/((1 + \tau^2)(1 + \sigma_\alpha^2)) + \sigma_\alpha^2\eta/(1 + \sigma_\alpha^2) \quad (5.17)$$

$$\hat{\pi} = (1 + (1 + \tau^2)\sigma_\alpha^2)\varepsilon/((1 + \tau^2)(1 + \sigma_\alpha^2)) - \tau\sigma_\alpha^2\eta/(1 + \sigma_\alpha^2) \quad (5.18)$$

The rule for the policy rate implied by these equations is:

$$i - i^* = \left(\frac{1}{\bar{\alpha}}\right)\{\frac{\tau\varepsilon}{(1+\tau^2)(1+\sigma_\alpha^2)} - \eta/(1 + \sigma_\alpha^2)\} \quad (5.19)$$

As noted above, the coefficients in the policy rule on shocks to the IS curve and shocks to the Phillips curve are both smaller than in the baseline model. This is the so-called "attenuation" effect.

Uncertainty About the Slope of the Phillips Curve

Following the same approach, one can also examine the implication of uncertainty about the slope of the Phillips Curve. In this case, the slope parameter could be written as:

$$\tau = \bar{\tau} + \bar{\tau}w_\tau \quad (5.20)$$

With that specification, the IS curve would be unchanged while the Phillips Curve can be written as in equation (5.19). We assume inflation expectations equal the target rate.

$$\hat{u} = un - un^* = \alpha(i - \pi^e - r^*) + \eta = \alpha(i - (r^* + \pi^*)) + \eta \quad (5.21)$$

$$\tilde{\pi} = \pi - \pi^* = -\tau\hat{u} + \varepsilon - \bar{\tau}\hat{u}w_\tau = \hat{\pi} - \bar{\tau}\hat{u}w_\tau \quad (5.22)$$

The expected value of the central bank loss function would then be given by:

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\hat{u}^2 + \tilde{\pi}^2)\} = \left(\frac{1}{2}\right) ((\hat{u}^2 + \hat{\pi}^2) + \bar{\tau}^2 \hat{u}^2 \sigma_\tau^2 \ ) \quad (5.23)$$

The expression for the optimal economic outcomes line would be:

$$\hat{u}(1 + \bar{\tau}^2 \sigma_\tau^2) = \bar{\tau}\hat{\pi} \quad (5.24)$$

If uncertainty about the slope coefficient is zero, this expression collapses to the expression for the optimal economic outcomes line in the baseline model. The optimal policy choices for the unemployment and inflation gaps are given by:

$$\hat{u} = \bar{\tau}\, \varepsilon/(1 + \bar{\tau}^2(1 + \sigma_\tau^2)) \quad (5.25)$$

$$\hat{\pi} = (1 + \bar{\tau}^2 \sigma_\tau^2)\varepsilon/(1 + \bar{\tau}^2(1 + \sigma_\tau^2)) \quad (5.26)$$

And the policy rule in this case is given by:

$$i - i^* = (\frac{\bar{\tau}}{\alpha})(\delta + \varepsilon)/(1 + \bar{\tau}^2(1 + \sigma_\tau^2)) - \eta/\alpha \quad (5.27)$$

The policy rule in this case again tends to respond less aggressively to inflation shocks than in the baseline model because the policymaker is uncertain about how much of an effect any given change in the unemployment rate will have on inflation. This risk can be mitigated by keeping the unemployment rate gap closer to zero than in the baseline model. In contrast to the case with uncertainty about the slope of the IS curve, the policymaker continues to completely offset any shocks to the IS curve in order to keep the unemployment rate gap small.

Uncertainty about UN* and R*

The discussion above focused on the effects of uncertainty associated with exogenous shocks to the IS curve and Phillips Curve. Many of the same conclusions arise in the case with uncertainty about parameters of the underlying economic model. For example, consider the case when there is uncertainty about the neutral real rate, r^*, and the natural rate of unemployment un^* of the form:

$$un^* = \overline{un} + w_{un^*} \text{ and } r^* = \bar{r} + w_{r^*} \quad (5.28)$$

Assume again that the shocks to the neutral rate r^* and the natural rate of unemployment un^* are not known when the policymaker chooses the level of the policy rate. The IS curve and Phillips curve can then be written as:

$$\tilde{u} = un - \overline{un} - w_{un^*} = \alpha(i - \pi^e - \bar{r}) + \eta - \alpha w_{r*}$$

$$\hat{u} = E\{\tilde{u}\} = \alpha(i - \pi^e - \bar{r}) + \eta \quad (5.29)$$

$$\tilde{\pi} = \pi - \pi^* = -\tau\tilde{u} + (\pi^e - \pi^*) + \varepsilon = -\tau\hat{u} + (\pi^e - \pi^*) + \varepsilon + \tau\alpha w_{r*} = \hat{\pi} + \tau\alpha w_{r*} \quad (5.30)$$

The policymaker in this case again minimizes the expected value of the loss function given as:

$$L = \left(\frac{1}{2}\right) \mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2)\} = \left(\frac{1}{2}\right) ((\hat{u}^2 + \hat{\pi}^2) + (1 + \tau^2)\alpha^2 \sigma_{w_{r^*}}^2 \ ) \quad (5.31)$$

The effects of uncertainty again only add a constant term to the loss function. Moreover, because the shock to the natural rate passes through to the actual unemployment rate, the unemployment rate gap is unaffected by this shock.

As a result, the optimality condition is unaffected and remains:

$$\hat{u} = \tau\hat{\pi} \tag{5.32}$$

This expression together with the Phillips curve then determines the ex-ante level of the unemployment rate and inflation gaps as usual:

$$\hat{u} = \tau(\delta + \varepsilon)/(1 + \tau^2) \tag{5.33}$$

$$\hat{\pi} = (\delta + \varepsilon)/(1 + \tau^2) \tag{5.34}$$

The interest rate rule is also identical to that for the baseline model without uncertainty.

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon)}{1+\tau^2} - \eta/\alpha \tag{5.35}$$

Contemporaneous Misperception Risks

A basic problem confronting policymakers is that they may not have complete information about all shocks affecting the economy during the current period. For example, suppose that the policymaker does not observe the contemporaneous values of the shock to the natural rate of unemployment, and the shocks to the IS curve and Phillips curve. In this case, the policymaker can estimate the contemporaneous values of the natural rate of unemployment and the shocks to the IS and Phillips curve assuming that the means and variances of the unobserved variables are known. As above, assume that the natural rate in the current period is given by $un^* = \overline{un} + w_{un^*}$. And assume that the policymaker knows the value of $\overline{un}$ and the variance of the shock term w_{un^*}. Assume also that the policymaker does not observe the shock to the IS curve directly but knows that the shock term has mean zero and variance σ_η^2. Similarly, assume that the shock to the Phillips curves is not known but that it also has mean zero and variance of σ_ε^2

Then the IS curve is given by:

$$\tilde{u} = un - \overline{un} - w_{u^*} = \alpha(i - \pi^e - r^*) + \eta \tag{5.36}$$

Collecting all the known terms on one side of the equation yields:

$$V = un - \overline{un} - \alpha(i - \pi^e - r^*) = \eta + w_{u^*} \tag{5.37}$$

Similarly, the Phillips curve is given by:

$$\tilde{\pi} = \pi - \pi^* = -\tau\tilde{u} + (\pi^e - \pi^*) + \varepsilon = -\tau(un - \overline{un}) + (\pi^e - \pi^*) + \varepsilon + \tau w_{u^*} \tag{5.38}$$

Again collecting the known terms on one side of the equation yields:

$$W = (\pi - \pi^*) - \tau(un - \overline{un}) + (\pi^e - \pi^*) = \varepsilon + \tau w_{u^*} \tag{5.39}$$

By assumption everything on the left sides of 5.37 and 5.38 expression is observed. Conditional on these observed values for V and W, the expected values of w_{u^*}, η and w_{u^*} are given by:

$$E\{w_{u^*}|V,W\} = (V\sigma^2_{w_{u^*}}\sigma^2_{\varepsilon} + \tau W\sigma^2_{w_{u^*}}\sigma^2_{\eta})/(\sigma^2_{\varepsilon}\sigma^2_{\eta} + \sigma^2_{w_{u^*}}\sigma^2_{\varepsilon} + \tau^2\sigma^2_{w_{u^*}}\sigma^2_{\eta}) \quad (5.40)$$

$$E\{\varepsilon|V,W\} = W - \tau E\{w_{u^*}|V,W\} \quad (5.41)$$

$$E\{\eta|V,W\} = V - E\{w_{u^*}|V,W\} \quad (5.42)$$

These expressions can be obtained by maximizing the likelihood function for the three shocks subject to constraints of 5.37 and 5.39. Alternatively, the coefficients on V and W in equation 5.40 can be calculated as the regression coefficients of w_{u^*} on the variables $\eta + w_{u^*}$ and $\varepsilon + \tau w_{u^*}$.

Regarding equation 5.40, it's useful to note that when the variance of the shock to the natural unemployment rate falls to zero, the expected value of the shock also falls to zero. And by equations 5.41 and 5.42, the values of the shocks to the Phillips curve and IS curve may then also be determined exactly. Another aspect of equation 5.40 to note is that the value of the expected values of the shocks does not depend on the magnitude of the three variances—only the relative variances of the three shocks matter. For example, equation 5.40 can be expressed equivalently as:

$$E\{w_{u^*}|V,W\} = (V + \tau W(\frac{\sigma^2_{\eta}}{\sigma^2_{\varepsilon}}))/(1 + (\frac{\sigma^2_{\eta}}{\sigma^2_{w_{u^*}}}) + \tau^2(\frac{\sigma^2_{\eta}}{\sigma^2_{\varepsilon}})) \quad (5.43)$$

The objective function then is given by:

$$L = \left(\frac{1}{2}\right)\mathrm{E}\{(\tilde{u}^2 + \tilde{\pi}^2)|V,W\} \quad (5.44)$$

$$\hat{u} = E\{\tilde{u}|V,W\} = un - \overline{un} - \mathrm{E}\{w_{un^*}|V,W\} \quad (5.45)$$

$$\hat{\pi} = E\{\tilde{\pi}|V,W\} = -\tau\hat{u} + (\pi^e - \pi^*) + \mathrm{E}\{\varepsilon|V,W\}$$

The optimal economic outcomes line again is given by:

$$\hat{u} = \tau\hat{\pi}$$

$$i - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \mathrm{E}\{\varepsilon|V,W\})}{1+\tau^2} - \frac{\mathrm{E}\{\eta|V,W\}}{\alpha} = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \mathrm{E}\{\varepsilon|V,W\})}{1+\tau^2} - \frac{E\{\eta|V,W\}}{\alpha} \quad (5.46)$$

The end result in 5.46 is that the policy rule has a form identical to that in the baseline model. The only difference is that now the actual values of the shocks to the Phillips curve and IS curve are replaced with their estimated values conditional on V and W as shown above in equations 5.40 to 5.42.

Take 6: Uncertainty and Asymmetric Objective Functions

A basic conclusion in the discussion above is that uncertainty about "additive" factors affecting the economy does not affect optimal policy choices. This "certainty equivalence" result rests importantly on the assumed quadratic form of the central bank's objective function and the linear structure of the economic model. Below we consider how uncertainty regarding "additive" factors may affect optimal policy outcomes for cases in which policymakers' preferences depart from the standard quadratic specification. For convenience, the analysis below assumes that inflation expectations are equal to the central bank's target rate.

Asymmetric Inflation Hawk

One departure from the standard quadratic specification would be the asymmetric loss function discussed above for an inflation hawk. A policymaker with these preferences would attach greater weight to outcomes with inflation above target relative to outcomes with inflation below target. As above, we assume that the unemployment rate and inflation rate gaps are subject to a second round of shocks after the policymaker has committed to the choice of policy rate in that period.

$$\tilde{u} = \hat{u} + \eta'$$

$$\tilde{\pi} = \hat{\pi} - \tau\eta' + \varepsilon'$$

Formally, the specification of the loss function in this case would be:

$$E\{L\} = \left(\frac{1}{2}\right) E\{\tilde{u}^2 + \gamma^b \tilde{\pi}^2 I(\tilde{\pi} < 0) \ + \gamma^a \tilde{\pi}^2 I(\tilde{\pi} \geq 0)\} =$$

$$\left(\frac{1}{2}\right)\left(\hat{u}^2 + \sigma_{\eta'}^2\right) + \left(\frac{1}{2}\right)\gamma^b \int_{-\infty}^{v} (\hat{\pi}^2 + 2\sigma_z\hat{\pi}u_z + \sigma_z^2 u_z^2) + \ \left(\frac{1}{2}\right)\gamma^a \int_{v}^{\infty} (\hat{\pi}^2 + 2\sigma_z\hat{\pi}u_z + \sigma_z^2 u_z^2) =$$

$$\left(\frac{1}{2}\right)\left(\hat{u}^2 + \sigma_{\eta'}^2\right) +$$

$$\left(\frac{1}{2}\right)\gamma^b \left(\sigma_z^2 v^2 \mathrm{N}(v) + \ 2\sigma_z^2 v n(v) + \sigma_z^2\big(N(v) - v n(v)\big)\right) +$$

$$\left(\frac{1}{2}\right)\gamma^a \ \left(\sigma_z^2 v^2\big(1 - \mathrm{N}(v)\big) - \ 2\sigma_z^2 v n(v) + \sigma_z^2\big(1 - N(v) + v n(v)\big)\right) \qquad (6.1)$$

where

$$v = -\hat{\pi}/\sigma_z$$

$$z = -\tau\eta' + \varepsilon'$$

The equation for the optimal economic outcomes line is given by the expression:

$$\hat{u} = \tau\gamma^a\hat{\pi} + \tau(\gamma^b - \gamma^a)\{\big(\hat{\pi}N(v) - \sigma_z n(v)\big)\} \qquad (6.2)$$

Note that if the weights on positive and negative inflation gaps are equal, then this expression reduces to the equation for the optimal economic outcomes line in the baseline model.

Asymmetric Unemployment Dove

A similar departure from the standard quadratic specification would be the asymmetric loss function discussed above for an unemployment dove. A policymaker with these preferences would attach greater weight to outcomes with unemployment above target relative to outcomes with unemployment below target. Formally, the specification of the loss function in this case would be:

$$E\{L\} = \left(\frac{1}{2}\right) E\{\tilde{\pi}^2 + \theta^b \tilde{u}^2 I(\tilde{u} < 0) \ + \theta^a \tilde{u}^2 I(\tilde{u} \geq 0)\} =$$

$$\left(\frac{1}{2}\right)(\hat{\pi}^2 + \sigma_z^2) + \left(\frac{1}{2}\right)\theta^b \int_{-\infty}^{v} (\hat{u}^2 + 2\sigma_{\eta'}\hat{u}u_{\eta'} + \sigma_z^2 u_{\eta'}^2) +$$

$$\left(\frac{1}{2}\right)\theta^a \int_{v}^{\infty} (\hat{u}^2 + 2\sigma_{\eta'}\hat{u}u_{\eta'} + \sigma_{\eta'}^2 u_{\eta'}^2) =$$

$\left(\frac{1}{2}\right)(\hat{\pi}^2 + \sigma_z^2) +$

$\left(\frac{1}{2}\right)\theta^b\left(\sigma_{\eta'}^2 v^2 \mathrm{N}(v) + 2\sigma_{\eta'}^2 v n(v) + \sigma_{\eta'}^2\big(N(v) - v n(v)\big)\right) +$

$$\left(\tfrac{1}{2}\right)\theta^a\left(\sigma_{\eta'}^2 v^2\big(1 - \mathrm{N}(v)\big) - 2\,\sigma_{\eta'}^2 v n(v) + \sigma_{\eta'}^2\big(1 - N(v) + v n(v)\big)\right) \tag{6.3}$$

The optimal economic outcomes line in this case takes the form:

$$\tau\hat{\pi} = \theta^a\hat{u} + (\theta^b - \theta^a)\left(\hat{u}N(v) - \sigma_{\eta'}n(v)\right) \tag{6.4}$$

$$v = -\hat{u}/\sigma_{\eta'}$$

Note that if the weights on unemployment gaps above and below zero are equal, this expression collapses to the expression for the optimal economic outcomes line in the baseline model.

Take 7: Appealing to a Higher Power--Tail Risk Avoidance

The discussion above focuses on uncertainty about additive factors with asymmetric objective functions. In contrast to the case for symmetric quadratic objective functions, uncertainty about additive factors does affect the optimal policy outcomes in this case. However, uncertainty about additive factors can matter even with objective functions that are symmetric. For example, consider the case when the policymaker's objective function is based on unemployment rate gaps and inflation gaps raised to the fourth power. Such preferences might be consistent with policymakers that are especially concerned about very large inflation gaps or unemployment rate gaps.

In the absence of uncertainty, this type of objective function yields solutions that are very similar to those for the baseline model. The objective function is given by:

$$L = \left(\tfrac{1}{4}\right)(\hat{u}^4 + \hat{\pi}^4) \tag{7.1}$$

The optimal economic outcomes line is given by:

$$\hat{u}^3 - \tau\hat{\pi}^3 = 0 \tag{7.2}$$

or

$$\hat{\pi} = \tau^{-1/3}\hat{u} \tag{7.3}$$

And the optimal policy choices for the unemployment rate gaps and inflation gaps are given by:

$$\hat{u} = \tau^3\varepsilon/(1 + \tau^4) \tag{7.4}$$

$$\hat{\pi} = \tau^{8/3}\varepsilon/(1 + \tau^4) \tag{7.5}$$

As in the baseline model, the optimal economic outcomes line is a straight line through the origin and all of the basic properties of the baseline model hold. When the IS curve and Phillips curve are subjected to second round additive shocks, however, the solution becomes considerably more complicated. In this case, the expected value of the objective function is given by:

$E\{L\} = \left(\frac{1}{4}\right)\mathrm{E}\{\tilde{u}^4 + \tilde{\pi}^4\} = \left(\frac{1}{4}\right)(\hat{u}^4 + 6\hat{u}^2\sigma_u^2 + 4\hat{u}\sigma_u^3 s_u + \sigma_u^4 k_u) +$

$$\left(\frac{1}{4}\right)(\hat{\pi}^4 + 6\hat{\pi}^2\sigma_\pi^2 + 4\hat{\pi}\sigma_\pi^3 s_\pi + \sigma_\pi^4 k_\pi) \qquad (7.6)$$

Where

s_u, s_π are measures of skew for the shocks to the IS curve and Phillips curve and

k_u, k_π are measures of the kurtosis for these same shocks.

The optimal economic outcomes line in this case is given by:

$$(\hat{u}^3 + 3\hat{u}\sigma_u^2 + \sigma_u^3 s_u) = \tau(\hat{\pi}^3 + 3\hat{\pi}\sigma_\pi^2 + \sigma_\pi^3 s_\pi) \qquad (7.7)$$

This is a particular type of cubic equation—a so-called "depressed cubic"—with a solution given by:

$$\hat{\pi}^3 + 3\hat{\pi}\sigma_\pi^2 = (\hat{u}^3 + 3\hat{u}\sigma_u^2 + \sigma_u^3 s_u) - \sigma_\pi^3 s_\pi = D \qquad (7.8)$$

$$a^3 - b^3 = D \qquad (7.9)$$

$$ab = \sigma_\pi^2 \qquad (7.10)$$

$$a = ((\mathrm{D} + \frac{(D^2+4\sigma_\pi^6)^{\frac{1}{2}}}{2})^{1/3} \text{ and } b = \sigma_\pi^2/a$$

Figure 7.2 displays a range of optimal economic outcomes lines that can arise for different settings of the relative variances of the shocks and the skews to the shocks. In general, when the variance of the unemployment rate shocks is relative large,that tends to an optimal economic outcomes line that is very steep around an unemployment rate gap of zero. That is because deviations of the unemployment rate gap from zero pose greater risks of an elevated ex-post value of the unemployment rate gap this is very costly. A skew in the distribution of the shocks toward high or low unemployment rates shifts the optimal economic outcomes line to the left or the right. This reflects the incentives for the policymaker to reduce the odds of an especially large ex-post value of the unemployment rate shocks. Analogous results are shown when the variance of inflation is relative large and when there are skews in the shocks to the inflation rate.

Take 8: Extended Baseline Model and the Zero Lower Bound Problem

The discussion above focused on the role of uncertainty in the baseline model and with alternative specification of the central bank's objective function. This section focuses on a "nonlinearity" in the economy that has received a great deal of attention over recent years—the zero lower bound on nominal interest rates. In order to study this in more detail, it is helpful to expand the baseline model to encompass two periods. Moreover, we modify the IS curve in the second period so that the unemployment rate in the first period affects the initial conditions in period 2.

Structure of Two-Period Model

In the two period model, the IS curve is identical to the baseline model specification except that the unemployment rate from the first period influences the initial conditions in that period.

$$\hat{u}_2 = un_2 - un^* = \hat{u}_1 + \alpha(i_2 - \pi^e - r^*) + \eta_2 \qquad (8.1)$$

$$\hat{\pi}_2 = \pi_2 - \pi^* = -\tau\hat{u}_2 + \varepsilon_2 \qquad (8.2)$$

As before, the central bank minimizes a standard quadratic loss function. Starting in period 2, optimal policy looks a great deal like optimal policy in the baseline model.

$$L_2 = (\tfrac{1}{2})(\hat{u}_2^2 + \hat{\pi}_2^2) \tag{8.3}$$

The unemployment and inflation rate gaps have the same form as the baseline model. This result obtains because the central bank can set the funds rate in period 2 to offset any influence from the unemployment rate in period 1.

$$\hat{u}_2 = \tau(\delta + \varepsilon_2)/(1 + \tau^2) \tag{8.4}$$

$$\hat{\pi}_2 = (\delta + \varepsilon_2)/(1 + \tau^2) \tag{8.5}$$

The policy rule then is:

$$i_2 - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon_2)}{1+\tau^2} - (\hat{u}_1 + \eta_2)/\alpha \tag{8.6}$$

Plugging these expressions back into the period 2 loss function and computing expectations based on the information set in period 1 yields.

$$E\{L_2\} = E\{\left(\frac{1}{2}\right)(\hat{u}_2^2 + \hat{\pi}_2^2)\} = (\tfrac{1}{2})\sigma_\varepsilon^2/(1 + \tau^2) \tag{8.7}$$

This is just a constant, so the minimization of the loss function in period 1 looks exactly like that in the baseline model.

$$L = L_1 + L_2 = \left(\frac{1}{2}\right)(\hat{u}_1^2 + \hat{\pi}_1^2) + L_2 \tag{8.8}$$

As a result, the solutions for the optimal choices for the unemployment rate and inflation in period 1 are identical to the baseline model as is the policy rule for setting the funds rate in period 1.

$$\hat{u}_1 = \tau(\delta + \varepsilon_1)/(1 + \tau^2) \tag{8.9}$$

$$\hat{\pi}_1 = (\delta + \varepsilon_1)/(1 + \tau^2) \tag{8.10}$$

$$i_1 - i^* = \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon_1)}{1+\tau^2} - \eta_1/\alpha \tag{8.11}$$

Characterizing the ZLB

The solution is much more complicated, however, if the central bank is constrained by the zero lower bound on nominal interest rates. Returning to equation (8.6), cases in which the unconstrained optimal interest rate in period 2 is less than or equal to zero are given by:

$$i_2 = i^* + \frac{\left(\frac{\tau}{\alpha}\right)(\delta + \varepsilon_2)}{1+\tau^2} - (\hat{u}_1 + \eta_2)/\alpha \leq 0 \tag{8.12}$$

This expression, in turn, implies that the zero lower bound will be binding whenever:

$$\frac{\tau\varepsilon_2}{1+\tau^2} - \eta_2 \leq \hat{u}_1 - \alpha i^* \tag{8.13}$$

Assuming that the shocks to the IS curve and Phillips curve are normally distributed, the probability that the zero lower bound is binding is given by:

$$N((\hat{u}_1 - \alpha i^*)/\sigma_z) \tag{8.14}$$

where

$$z = \frac{\tau\varepsilon_2}{1+\tau^2} - \eta_2 \tag{8.15}$$

All else equal, higher unemployment in period increase the likelihood of encountering the zero lower bound. Similarly, lower levels of the long-run normal funds rate, i^*, or the level of the slope coefficient for the IS curve, α, also increase the likelihood of encountering the zero lower bound. In addition, a higher variance for the shocks to the IS curve and Phillips curve in period 2 also increase the ZLB risk. Intuitively, high values of the unemployment rate from the prior period would imply that the central bank would need to reduce the funds rate by a large amount to offset that effect. But that increases ZLB risk. Similarly, higher variance shocks in period 2 increase the likelihood of circumstances that could lead central bank to ease policy.

Take 9: Risk Management Near the Zero Lower Bound

In cases when the zero lower bound is binding, the unemployment rate and inflation rate are determined at the intersection of the IS curve and Phillips curve with the funds rate set at zero:

$$\hat{u}_2^c = un_2^c - un^* = \hat{u}_1 - \alpha i^* + \eta_2 \tag{9.1}$$

$$\hat{\pi}_2^c = \pi_2^c - \pi^* = -\tau\hat{u}_2^c + \varepsilon_2 \tag{9.2}$$

The expected value of the loss function in period 2 is given by:

$$E\{L_2\} = E\left\{\left(\frac{1}{2}\right)(\hat{u}_2^2 + \hat{\pi}_2^2)\right\} = \iint_{z=\varphi}^{\infty}\left(\frac{1}{2}\right)(\hat{u}_2^2 + \hat{\pi}_2^2) + \iint_{z=-\infty}^{\varphi}\left(\frac{1}{2}\right)\left(\hat{u}_2^{c^2} + \hat{\pi}_2^{c^2}\right) \tag{9.3}$$

The first term in this expression integrates across all outcomes in which the zero lower bound is not binding and the second integrates across all shock combinations for which the zero lower bound is binding. Computing this expectation is aided by orthogonalizing the underlying shocks as:

$$z = \left(\frac{1}{1+\tau^2}\right)\varepsilon - \eta$$

$$\varepsilon = \beta_\varepsilon z + x$$

$$x = \varepsilon - \beta_\varepsilon z$$

$$\eta = \beta_\eta z + \gamma_\eta x$$

where

$$\beta_\varepsilon = \frac{cov(\varepsilon,z)}{\sigma_z^2} = \left(\frac{1}{1+\tau^2}\right)\left(\frac{\sigma_\varepsilon^2}{\sigma_z^2}\right)$$

$$\beta_\eta = \frac{cov(\eta,z)}{\sigma_z^2} = -\left(\frac{\sigma_\eta^2}{\sigma_z^2}\right)$$

$$\gamma_\eta = \frac{cov(\eta, x)}{\sigma_x^2} = \beta_\varepsilon(\frac{\sigma_\eta^2}{\sigma_x^2})$$

With this orthogonalization, integrating across realizations of z/σ_z that are associated with a binding ZLB constraint involves computing the expectations for values of z/σ_z that are above and below the critical value:

$$\varphi = (\hat{u}_1 - \alpha(r^* + \pi^*))/\sigma_z \quad (9.4)$$

The integration over the orthogonal error term x is taken over the full range from minus to plus infinity.

Tedious algebra leads to an expression for the first term of equation (A.102) as:

$$\iint_{z=\varphi}^{\infty} \left(\frac{1}{2}\right)(\hat{u}_2^2 + \hat{\pi}_2^2) = (\frac{1}{1+\tau^2})((\beta_\varepsilon^2 \sigma_z^2(1 - N(\varphi) + \varphi n(\varphi)) + (1 - N(\varphi))\sigma_x^2)) \quad (9.5)$$

Similarly, the equation for the second term in the expression can be written as:

$$\iint_{z=-\infty}^{\varphi} \left(\frac{1}{2}\right)\left(\hat{u}_2^{c^2} + \hat{\pi}_2^{c^2}\right) = \mathrm{A} + \mathrm{B} \quad (9.6)$$

with

$$\mathrm{A} = (\sigma_z^2\varphi^2 + \gamma_\eta^2\sigma_x^2)N(\varphi) - 2\beta_\eta\sigma_z^2\varphi n(\varphi) + \beta_\eta^2\sigma_z^2(N(\varphi) - \varphi n(\varphi)) \quad (9.7)$$

$$\mathrm{B} = (\tau^2\sigma_z^2\varphi^2 + q_x^2\sigma_x^2)N(\varphi) - 2\tau q_z\sigma_z^2\varphi n(\varphi) + q_z^2\sigma_z^2(N(\varphi) - \varphi n(\varphi)) \quad (9.8)$$

$$q_z = (\beta_\varepsilon - \tau\beta_\eta)$$

$$q_x = (1 - \tau\gamma_\eta)$$

Combining these results, the expected value of the loss function in period 2 based on the information set in period 1 is:

$$E\{L_2\} = \left(\frac{1}{1+\tau^2}\right)\left(\beta_\varepsilon^2\sigma_z^2(1 - N(\varphi) + \varphi n(\varphi)) + (1 - N(\varphi))\sigma_x^2\right) +$$

$$(\sigma_z^2\varphi^2 + \gamma_\eta^2\sigma_x^2)N(\varphi) - 2\beta_\eta\sigma_z^2\varphi n(\varphi) + \beta_\eta^2\sigma_z^2(N(\varphi) - \varphi n(\varphi)) +$$

$$(\tau^2\sigma_z^2\varphi^2 + q_x^2\sigma_x^2)N(\varphi) - 2\tau q_z\sigma_z^2\varphi n(\varphi) + q_z^2\sigma_z^2(N(\varphi) - \varphi n(\varphi)) \quad (9.9)$$

The derivative of this expression with respect to $\hat{u}_1$ given by:

$$\Psi(\varphi) = (1 + \tau^2)\varphi N(\varphi) - \beta_\eta(1 + \tau^2)n(\varphi) + \tau\beta_\varepsilon n(\varphi)$$

The optimal policy condition then is:

$$\hat{u}_1 + \Psi\left(\frac{\hat{u}_1 - \alpha(r^* + \pi^*)}{\sigma_z}\right) = \tau\hat{\pi}_1 \quad (9.10)$$

Where

$$\Psi(\hat{u}_1) = d(E\{L_2\})/d\hat{u}_1 \quad (9.11)$$

The function $\Psi(\hat{u}_1)$ is positive and increasing. As a result, the optimal economic outcomes line crosses the x-axis with an unemployment rate gap below zero and passes through quadrant 2 as shown in figure 9.1. This reflects the incentives for the policymaker to hedge against the risk of being constrained by the zero lower bound in period 2.