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Optimal Inflation Target with Expectations-Driven Liquidity Traps*

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Abstract

In expectations-driven liquidity traps, a higher inflation target is associated with lower inflation and consumption. As a result, introducing the possibility of expectations-driven liquidity traps to an otherwise standard model lowers the optimal inflation target. Using a calibrated New Keynesian model with an effective lower bound (ELB) constraint on nominal interest rates, we find that even a very small probability of falling into an expectations-driven liquidity trap lowers the optimal inflation target nontrivially. Our analysis provides a reason to be cautious about the argument that central banks should raise their inflation targets in light of a higher likelihood of hitting the ELB.

JEL: E32, E52, E61, E62, E63

Keywords: Liquidity Traps, Optimal Inflation Target, Sunspot Shock, Zero Lower Bound.

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1 Introduction

The recent experiences with the effective lower bound (ELB) constraint in advanced economies have put the question of how central banks should manage the problems associated with the ELB constraint on nominal interest rates at the forefront of the monetary policy debate. One popular policy proposal to manage the consequences of the ELB constraint is to increase the inflation target. With a higher inflation target, the nominal interest rate would be higher on average. As a result, the probability that the policy rate is constrained by the ELB would be lower. Also, a higher inflation target mitigates the declines in output and inflation when the policy rate is constrained at the ELB by pushing down the expected real rate facing households and firms. With the long-run neutral real rates expected to be lower now and in the future compared with the past, some economists have suggested that central banks may want to increase the inflation target from the current 2 percent in advanced economies (Ball (2013); Blanchard, Dell'Ariccia, and Mauro (2010)).

In this paper, we examine an argument for being cautious about raising the inflation target. The argument we examine is as follows. The policy rate may become constrained by the ELB not only because of fundamental shocks, but also because of self-fulfilling expectations (Benhabib, Schmitt-Grohe, and Uribe (2001) and Bullard (2010)). In economies in which only fundamental shocks can push the policy rate to the ELB, the possibility of being constrained by the ELB increases the optimal inflation target for the reasons stated earlier. However, as first observed by Mertens and Ravn (2014), a higher inflation target is associated with a lower output gap at the ELB if the policy rate is at the ELB because of self-fulfilling expectations, instead of fundamental shocks. One implication of this observation is that the optimal inflation target would decline if the possibility of falling into expectations-driven liquidity traps (LTs) is introduced to the standard model with the ELB. Using a standard New Keynesian model, we investigate the extent to which the possibility of expectations-driven LTs lowers the optimal inflation target.

Our main finding is that even a very small probability of expectations-driven LTs nontrivially lowers the optimal inflation target. Under various calibrations of the model, a 0.1 percent (quarterly) probability of falling into expectations-driven LTs typically lowers the optimal inflation target by more than 1 percentage point. With a 0.5 percent probability of expectations-driven LTs, the optimal inflation target is typically slightly negative.

The reason for the very high sensitivity of the optimal inflation target to the probability of expectations-driven LTs is somewhat technical. In the standard New Keynesian model, the persistence of the expectations-driven LT and the persistence of the fundamental-driven LT have to be sufficiently high and low, respectively, for the equilibrium to exist (Nakata and Schmidt (2019)). As a result, even a small probability of falling into an expectations-driven LT implies an unconditional probability of being in an expectations-driven LT that is higher than the unconditional probability of being in a fundamental-driven LT, making expectations-driven LTs the primary concern for the central bank. When expectations-driven LTs are the primary concern, the optimal inflation target is negative. Thus, even a small probability of falling into an expectations-driven LT makes the optimal inflation target negative.

Our analysis is motivated by the observation that Japan's prolonged ELB experience may well be characterized as being primarily driven by self-fulfilling expectations rather than fundamental shocks (see, among others, Aruoba, Cuba-Borda, and Schorfheide (2018); Bullard (2010)).[1] In Japan, both the output gap and inflation have been positive in the past few years. However, before that, inflation rates and the output gap had been negative for almost two decades. The combination of a slightly negative output gap and mild deflation with the nominal interest rate at the ELB constraint is consistent with the expectations-driven LT in the standard New Keynesian model. As we have learned over the past decade, what happens in Japan—though it may initially seem a theoretical curiosity for other countries—may happen in other countries years later. Thus, our analysis will be relevant for thinking about the optimal inflation target not only in Japan, but also in other countries.[2]

Myriad factors that influence the optimal inflation target of an economy are absent in our model.[3] In this paper, our goal is not to come up with a sensible policy recommendation about whether to change the inflation target of 2 percent currently adopted by many central banks. Rather, our goal is to highlight a factor that has been neglected in the literature and examine its quantitative relevance.

Our paper is related to a set of papers that analyze the implications of alternative inflation targets in the interest rate feedback rule for the dynamics and welfare of economies with the ELB constraint. Earlier research on this topic includes Reifschneider and Williams (2000) and Coenen, Orphanides, and Wieland (2004) who use the FRB/US model—a large-scale macroeconometric models of the U.S. economy—to analyze how the volatilities of output and inflation are affected by the level of the inflation target.[4] Our paper is most closely related to recent papers that compute the optimal inflation target in DSGE models, such as Coibion, Gorodnichenko, and Wieland (2012); Blanco (2018); Carreras, Coibion, Gorodnichenko, and Wieland (2016); and Andrade, Galí, Le Bihan, and Matheron (2018).[5] None of these papers allows for expectations-driven LTs. Our focus is the implications of expectations-driven LTs for the optimal inflation target.[6]

[1] Bullard (2010) makes this observation casually based on the historical constellation of the inflation rate and the policy rate in Japan. Aruoba, Cuba-Borda, and Schorfheide (2018) provide formal econometric evidence that Japan has been in expectations-driven LTs for most of the past two decades using a nonlinear DSGE models that allows for a sunspot shock.

[2] Not all economists agree that an expectations-driven LT is a good characterization of the Japanese ELB experience. See, for example, Nishizaki, Sekine, and Ueno (2014); Eichenbaum (2017).

[3] See, for example, Kiley, Mauskopf, and Wilcox (2007); Kryvtsov and Mendes (2015).

[4] See Williams (2009), Tulip (2014), and Kiley and Roberts (2017) for more recent examples of analyses based on FRB/US model.

[5] Some economists compute the average inflation rate that prevails when the interest rate policy is conducted optimally and refer to that rate of inflation as the optimal rate of inflation. See, for example, Billi (2011).

[6] There is also one methodological difference between our paper and these papers. We work with a fully

Our paper builds on Mertens and Ravn (2014), who analyze the effects of exogenous fiscal shocks in a one-time temporary expectations-driven LT. While not their focus, they note an interesting feature of the expectations-driven LT—an increase in the inflation target lowers the output gap in an expectations-driven LT. Our work examines implications of this observation for the optimal inflation target. To do so, we go beyond their framework in which an expectations-driven LT is a one-time event and allow the expectations-driven LT to be recurring, as in Aruoba, Cuba-Borda, and Schorfheide (2018).

Beyond Mertens and Ravn (2014), our paper is related to various papers analyzing expectations-driven LTs. Some papers are focused on analyzing what policies may (or may not) eliminate this LT (Alstadheim and Henderson (2006); Armenter (2017); Benhabib, Schmitt-Grohe, and Uribe (2002); Schmidt (2016); Schmitt-Grohe and Uribe (2013); Sugo and Ueda (2008); Nakata and Schmidt (2019); and Tamanyu (2019)). Others are focused on the dynamics in and out of this LT (Aruoba, Cuba-Borda, and Schorfheide (2018); Bilbiie (2018); Cuba-Borda and Singh (2018); Hirose (2007); Hirose (Forthcoming); Schmitt-Grohe and Uribe (2017)). The novelty of our paper is that we examine the implication of expectations-driven LTs for the optimal inflation target.

The rest of the paper is organized as follows. Section 2 describes the stylized model and its calibration. Section 3 describes the results from the stylize model. Section 4 concludes.

2 Model

We use a nonlinear New Keynesian model with Rotemberg pricing. Because the model is standard, we relegate the details of the model to Appendix A. The equilibrium conditions of the model are given by

$$C_t^{-\chi_c} = \beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1}, \tag{1}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c}, \tag{2}$$

$$\begin{aligned}\frac{Y_t}{C_t^{\chi_c}}\left[\varphi\left(\frac{\Pi_t}{(\Pi^{targ})^{\alpha}}-1\right)\frac{\Pi_t}{(\Pi^{targ})^{\alpha}}-(1-\theta)-(1-\tau)\theta w_t\right]&\\ = \beta\delta_t\mathrm{E_t}\frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\varphi\left(\frac{\Pi_{t+1}}{(\Pi^{targ})^{\alpha}}-1\right)\frac{\Pi_{t+1}}{(\Pi^{targ})^{\alpha}},&\end{aligned} \tag{3}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\frac{\Pi_t}{(\Pi^{targ})^{\alpha}}-1\right]^2 Y_t, \tag{4}$$

$$Y_t = N_t, \tag{5}$$

$$R_t = \max\left[R_{ELB},\quad \frac{\Pi^{targ}}{\beta\delta_t}\left(\frac{\Pi_t}{\Pi^{targ}}\right)^{\phi_\pi}\right]. \tag{6}$$

nonlinear macroeconomic model, while these authors work with semi-loglinear models in which all the equilibrium conditions are log-linearized except for the ELB constraint. The only exception is Blanco (2018).

C_t, N_t, Y_t, w_t, Π_t, and R_t are consumption, labor supply, output, real wage, inflation, and the policy rate, respectively. Equation 1 is the consumption Euler equation, equation 2 is the intratemporal optimality condition of the household, and equation 3 is the optimality condition of the intermediate good producing firms relating today's inflation to real marginal cost today and expected inflation tomorrow (forward-looking Phillips curve). Equation 4 is the aggregate resource constraint capturing the resource cost of price adjustment, equation 5 is the aggregate production function, and equation 6 is the interest-rate feedback rule where Π^{targ} is the central bank's inflation target.

Note that the intercept of the interest-rate feedback rule is time-varying and depends on δ_t. Under this policy rule, the effect the discount rate shock has on the economy through the consumption Euler equation is fully offset by a corresponding movement in the policy rate, as under optimal policy, unless the ELB constraint binds. This specification of the policy rule is often used in the ELB literature (see, for example, Boneva, Braun, and Waki (2016); Eggertsson (2011)).

We allow for a form of indexation in the specification of the price adjustment cost. Specifically, the price adjustment cost is a quadratic function of $\Pi_t/(\Pi^{targ})^{\alpha}$. If the indexation parameter, α, is 1, there is no steady state cost of a non-zero inflation target. The smaller the indexation parameter is, the larger the steady state cost of a non-zero inflation target becomes. Thus, holding all other parameter values fixed, an increase in α increases the optimal inflation target. The key ingredient of our model—a higher inflation target lowers consumption and inflation in the deflationary regime—does not depend on the value of α, as shown in Appendix D. However, as we will discuss shortly in Section 2.1, one of our calibration principles is to make the optimal inflation target 2 percent in the model without expectations-driven LTs; Thus, it is necessary to allow for some degree of price indexation to achieve that goal.[7]

δ_t is an exogenous shock to the household's discount rate and follows a two-state Markov process. It takes two values, $\delta_N = 1$ and $\delta_C > 1$, where N and C stand for **normal** and **crisis** states, respectively. The persistence of each state is given by

$$\text{Prob}(\delta_{t+1} = \delta_N | \delta_t = \delta_N) = p_N, \tag{7}$$

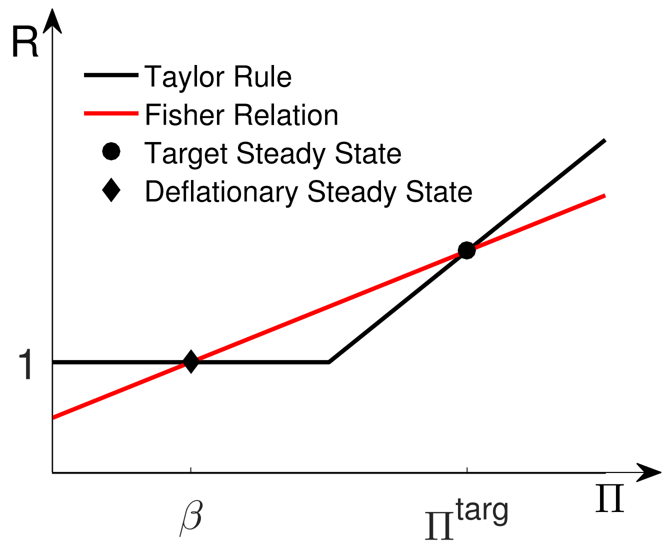
$$\text{Prob}(\delta_{t+1} = \delta_C | \delta_t = \delta_C) = p_C. \tag{8}$$

As pointed out by Benhabib, Schmitt-Grohe, and Uribe (2002) and as illustrated in Figure 1, the coexistence of the Euler equation—which implies a Fisher relation—and the truncated Taylor rule means that there are two steady states: one in which the policy rate is above zero and inflation is at the target (the target steady state), and one in which the

[7] It is common to not allow for any degree of price indexation in the literature computing the optimal inflation target (see, for example, Coibion, Gorodnichenko, and Wieland (2012); Andrade, Galí, Le Bihan, and Matheron (2018)). In our fully nonlinear model, we find that the optimal inflation target is only slightly positive under a wide range of parameter configurations unless we allow for some degree of price indexation.

policy rate is zero and the gross rate of inflation is β (the deflationary steady state). With an exogenous crisis shock, we have one equilibrium associated with each steady state: one that fluctuates around the target steady state, and one that fluctuates around the deflationary steady state.

Figure 1: Target and Deflationary Steady States



As in Mertens and Ravn (2014) and Aruoba, Cuba-Borda, and Schorfheide (2018), we introduce a two-state Markov sunspot shock, s_t, that allows the economy to transition between the target regime and the deflationary regime—the regime of an expectations-driven LT. s_t takes two values, T and D. When $s_t = T$, the economy is in the target regime. When $s_t = D$, the economy is in the deflationary regime. The persistence of each regime is given by

$$\text{Prob}(s_t = T | s_t = T) = p_T, \tag{9}$$

$$\text{Prob}(s_t = D | s_t = D) = p_D. \tag{10}$$

As discussed in Appendix B and analytically shown in Nakata and Schmidt (2019), there are restrictions on these transition probabilities for the sunspot equilibrium to exist. In particular, the persistence parameters for both target and deflationary regimes must be sufficiently high for the sunspot equilibrium to exist. Throughout the paper, we restrict our attention to the set of parameter values consistent with the existence of the sunspot equilibrium.

The value function associated with an equilibrium is given by the the expected discounted sum of future utility flows to the household. We can recursively write the value function as follows:

$$V_t = u(C_t, N_t) + \beta\delta_t \mathrm{E_t} V_{t+1}, \tag{11}$$

where the per-period utility flow is given by

$$u(C_t, N_t) := \Big[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n}\Big]. \tag{12}$$

The welfare of the economy is measured by the unconditional expected value.

A recursive sunspot equilibrium of this stylized economy is given by a set of value and policy functions for $\{V(\delta, s),\ C(\delta, s),\ N(\delta, s),\ Y(\delta, s),\ w(\delta, s),\ \Pi(\delta, s),\ R(\delta, s)\}$ such that (i) the equilibrium conditions described above are satisfied, (ii) the policy rate in the normal state of the target regime is above the ELB, and (iii) the policy rate in the normal state of the deflationary regime is at the ELB. That is, we require that

$$R(\delta = \delta_N, s = T) = \frac{\Pi^{targ}}{\beta\delta_t}\left(\frac{\Pi(\delta = \delta_N, s = T)}{\Pi^{targ}}\right)^{\phi_\pi} > 1, \tag{13}$$

$$R(\delta = \delta_N, s = D) = 1, \tag{14}$$

$$\frac{\Pi^{targ}}{\beta\delta_t}\left(\frac{\Pi(\delta = \delta_N, s = D)}{\Pi^{targ}}\right)^{\phi_\pi} < 1. \tag{15}$$

Following Mertens and Ravn (2014) and Aruoba, Cuba-Borda, and Schorfheide (2018), we focus on a recursive equilibrium in which the allocations today depend only on the realization of the crisis shock and the sunspot shock. In the deflationary regime, the Taylor principle is violated. As a result, there are infinitely many equilibria in the deflationary regime in which allocations today depend on past allocations and sunspot shocks—that are unrelated to the sunspot shock in this paper that dictates whether the economy is in the target or deflationary regime. Our equilibrium definition rules out these equilibria.[8]

2.1 Calibration

We set χ_C, χ_N, and θ to 1, 1, and 11, respectively. These are in line with the standard values in the literature. A production subsidy, τ, is set to $1/\theta$ so as to eliminate the distortion associated with monopolistic competition in the product market. With this value of τ, the level of consumption and labor supply are efficient if the inflation target is 0 percent and if there are no shocks. For the policy rule, the inflation response coefficient, ϕ_π, is set to 2 and the ELB, R_{ELB}, is set to 1. While we solve our model under a number of different values of the inflation target parameter to determine the optimal inflation target, we will closely examine the dynamics of the model under 0 percent and 2 percent inflation targets to understand the key forces of the model.

[8] See Hirose (2007) and Hirose (Forthcoming) for in-depth analyses on these equilibria.

Conditional on the aforementioned parameters and the persistence parameters that will be discussed shortly, the price adjustment cost parameter (φ), the degree of indexation (α), and the size of the crisis shock (δ_C) are chosen so that (i) consumption falls about 7 percent and inflation declines by about 2 percentage points in the crisis state of the target regime, and (ii) the optimal inflation target is 2 percent in the absence of the sunspot shock. The severity of a crisis is in line with those considered in Boneva, Braun, and Waki (2016) and Hills and Nakata (2018).

Table 1: Baseline Parameter Values for the Stylized Model

Parameter	Description	Parameter Value
β	Discount rate	$\frac{1}{1+0.0025}$
χ_c	Inverse intertemporal elasticity of substitution for C_t	1
χ_n	Inverse labor supply elasticity	1
θ	Elasticity of substitution among intermediate goods	11
τ	Production subsidy	$1/\theta$
φ	Price adjustment cost	1038
α	Degree of indexation	0.893
$400(\Pi^{targ} - 1)$	Inflation target in the Taylor rule	[0, 2]
ϕ_π	Coefficient on inflation in the Taylor rule	2
R_{ELB}	Effective lower bound	1
δ_C	Size of the crisis shock	1.0165
p_N	Persistence of the normal state	0.995
p_C	Persistence of the crisis state	0.75
p_T	Persistence of the target regime	0.995
p_D	Persistence of the deflationary regime	0.975

The persistence of the normal state and the persistence of the crisis state are set to 0.995 and 0.75, respectively. The normal state persistence of 0.995 implies that the crisis shock hits the economy, on average, once in 50 years. The crisis state persistence of 0.75 implies that the crisis shocks lasts for one year on average. We will also consider other values of p_C and p_N for sensitivity analyses.

As a baseline, we set the persistence of the deflationary regime to 0.975, which implies an expected duration of the deflationary regime of 10 years. With this persistence, the decline of output in the normal state of the deflationary regime is about 1 percent.[9] We will also consider some alternative values for the deflationary regime persistence for a sensitivity analysis. We use the target regime persistence of 0.995—which implies an expected duration of the target regime of 50 years—as our baseline, but we compute the optimal inflation target for a wide range of values to understand how the probability of moving from the target regime to the deflationary regime affects the optimal inflation target.

In the model, there are four states (two states for the sunspot shock and two states for the natural rate shock); thus, solving for the policy functions amounts to solving a system of

[9]This level of the output gap during the expectations-driven LT is broadly in line with the average output gap during the prolonged ELB episode in Japan.

nonlinear equations. We used Matlab's built-in nonlinear equation solver, `fsolve`, to solve our model.

3 Results

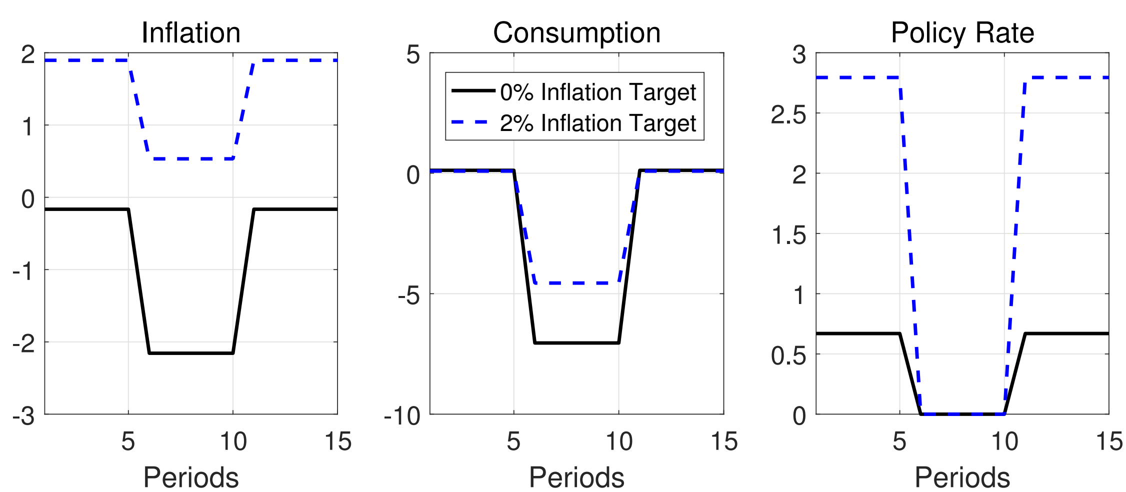
We first discuss how the inflation target affects allocations and welfare in a version of the model with a crisis shock only. Next, we discuss how the inflation target affects allocations and welfare in a version of the model with a sunspot shock only. Finally, we discuss the optimal inflation target in the model with both a crisis shock and a sunspot shock.

3.1 Optimal inflation target in the model with a crisis shock only

Figure 2 shows the dynamics of the economy in the model with a crisis shock only under 0 percent and 2 percent inflation targets. The economy is in the normal state from period 1 to period 5. The crisis shock hits the economy at period 6, and stays there until period 10. The economy is back in the normal state thereafter.

When the inflation target is zero—the case shown by the solid black lines—inflation and the output gap are close to the target and the policy rate is positive in the normal state.[10] When the crisis shock hits the economy, the central bank lowers the policy rate, trying to offset the adverse effects of the shock, but the ELB constraint prevents the central bank from fully neutralizing the effects of the shock: Inflation and consumption decline and the policy rate is at the ELB.

Figure 2: Allocations under a crisis shock



[10]Inflation and the output gap are not exactly zero because of the anticipation effects of being constrained by the ELB in the future. See Hills, Nakata, and Schmidt (2016) and Nakata and Schmidt (Forthcoming) for detailed analyses on the anticipation effects associated with the ELB constraint.

Figure 3: AD and AS Curves in the Crisis State and in the Deflationary Regime

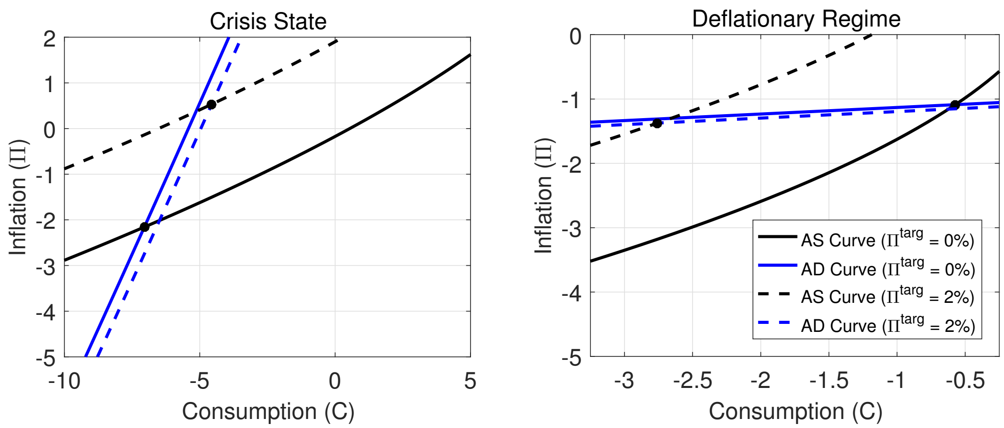
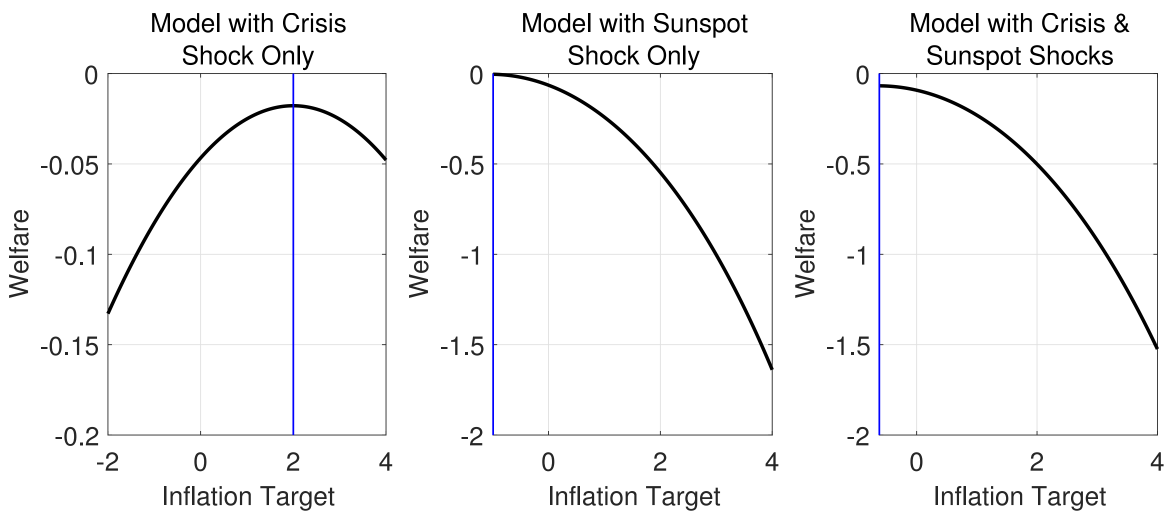


Figure 4: Welfare and the Inflation Target



Note: For each model, welfare is measured by the perpetual consumption transfer—expressed as a percentage of consumption at the deterministic steady state of the target regime—we need to give to the household in the version of the economy without the ELB so that it is as well-off as the household in the economy with the ELB.

The dashed blue lines in Figure 2 show the dynamics of the economy when the inflation target is 2 percent. In the normal state, a higher inflation target implies higher inflation and a higher policy rate because the Taylor rule is operative. Higher inflation increases the price adjustment cost and thus lowers consumption, though these effects are very small in our baseline calibration.[11] Higher normal state inflation leads to a lower expected real interest rate in the crisis state in which the policy rate is constrained at the ELB, mitigating

[11] In models with a Calvo-pricing, a higher inflation lowers output by increasing higher price dispersion. The output cost of higher inflation in Rotemberg and Calvo pricing models can be seen as capturing various costs associated with high inflation in reduced-form ways.

the declines in inflation and consumption in the crisis state: In the crisis state, inflation and consumption are higher under the 2 percent inflation target than under the 0 percent inflation target.

These favorable effects of a higher inflation target on crisis state inflation and consumption can be fully understood by examining how an increase in the inflation target affects crisis state AD and AS curves—the set of consumption-inflation pairs consistent with the consumption Euler equation and the Phillips curve in the crisis state, respectively. A higher inflation target means that inflation is higher in the normal state, because the Taylor rule operates in the normal state. When there is a positive probability of returning to the normal state—holding the crisis state inflation rate fixed—higher normal state inflation leads to higher inflation expectations and thus a lower expected real rate in the crisis state in which the ELB binds. The consumption Euler equation requires that crisis state consumption increases when the expected real rate declines. Thus, the AD curve shifts to the right. At the same time, the Phillips curve requires that crisis state inflation increases with normal state inflation—holding crisis state consumption fixed—because firms are forward-looking in their pricing decision. Thus, the AS curve shifts up. Taken together, these shifts in the AD and AS curves mean that, in equilibrium, an increase in the inflation target leads to higher inflation and consumption in the crisis state.

All told, there is a simple trade-off in adjusting the inflation target in this model with a crisis shock only. On the one hand, a higher inflation target is associated with inefficiently low consumption in the normal state. On the other hand, a higher inflation target is associated with better stabilization outcomes in the crisis state.[12] Reflecting this trade-off, welfare is a concave function of the inflation target, as shown in the left panel of Figure 4. As discussed earlier, the degree of indexation is chosen so that welfare attains its maximum at 2 percent, indicated by the vertical blue line.

3.2 Optimal inflation target in the model with a sunspot shock only

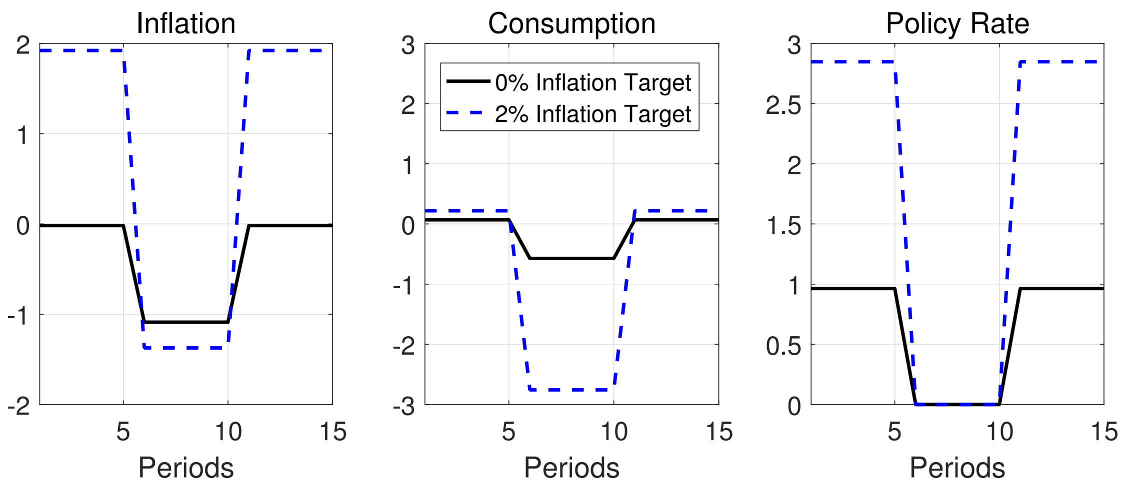
Figure 5 shows the dynamics of the model with a sunspot shock only. The economy is in the target regime from period 1 to period 5. The sunspot shock hits the economy at period 6 and stays there until period 10 – that is, the economy is in the deflationary regime from period 6 to 10. The economy is back in the target regime thereafter.

When the inflation target is zero—shown by the solid black lines—the policy rate is positive, inflation is essentially zero, and consumption is slightly above the efficient level in the target regime. When the economy moves to the deflationary regime, the policy rate hits the ELB, inflation falls by 1 percentage point, and consumption declines by 1/2 percentage point.

When the inflation target is 2 percent—shown by the dashed blue lines—the policy rate

[12] In models with a continuous shock, there is an additional benefit of raising the inflation target, which is that a higher inflation target lowers the probability of being constrained by the ELB.

Figure 5: Allocations under a sunspot shock



is close to 3 percent, inflation is slightly below 2 percent, and consumption is slightly above the efficient level in the target regime. When the economy moves to the deflationary regime, the policy rate hits the ELB and inflation and consumption fall. Inflation is about negative 1.5 percent and consumption is about 3 percent below the efficient level under the 2 percent inflation target. Inflation and consumption are nontrivially lower in the deflationary regime under the 2 percent inflation target than under the 0 percent inflation target.

These adverse effects of a higher inflation target on the deflationary regime inflation and consumption can be understood through AS and AD curves in the deflationary regime. As in the model with a crisis shock only, a higher inflation target means higher inflation in the target regime. When there is a positive probability of returning to the target regime—holding the deflationary regime inflation rate fixed—higher target regime inflation means higher inflation expectations, a lower expected real rate, and higher consumption in the deflationary regime.[13] Thus, the AD curve shifts to the right. Similar to what we saw in the model with a crisis shock only, the Phillips curve requires that higher target regime inflation leads to higher deflationary regime inflation, holding the deflationary regime consumption fixed, causing the AS curve to shift up. These shifts in the AD and AS curves mean that, in equilibrium, a higher inflation target leads to lower inflation and consumption in the deflationary regime.

Although the effects of a higher inflation target on the AD and AS curves in the deflationary regime are the same as those in the crisis state we examined earlier, they have the opposite equilibrium implications in the deflationary regime. In the crisis state, the AD curve is steeper than the AS curve. However, in the deflationary regime, the AD curve is flatter than the AS curve. Thus, a shift of the AD or AS curve in the same direction leads to the opposite equilibrium effects in inflation and consumption.

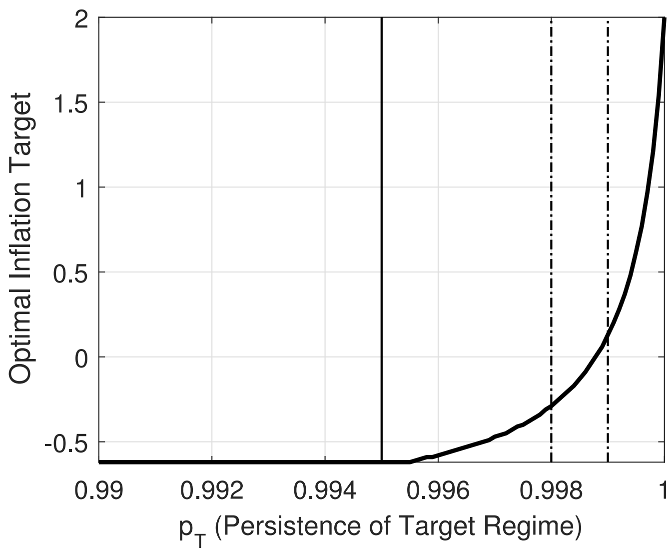
[13] If the deflationary regime is an absorbing regime, a change in the inflation target does not affect inflation and output in the deflationary regime, as pointed out by Cuba-Borda and Singh (2018).

All told, in the model with a sunspot shock only, a higher inflation target worsens the allocations in both the target and deflationary regimes. Thus, welfare is higher when the inflation target is lower, as shown in the middle panel of Figure 4.[14]

3.3 Model with both a crisis shock and a sunspot shock

Because a higher inflation target is associated with lower welfare in the model with a sunspot shock, if we introduce the sunspot shock to the model with a crisis shock only, the optimal inflation target declines. The solid line in the right panel of Figure 4 shows how the welfare of the economy depends on the inflation target in the model with both a crisis shock and a sunspot shock. The welfare is maximized at negative 0.6 percent, the lowest value of the inflation target consistent with the existence of the sunspot equilibrium and 2.6 percentage points lower than that in the economy with a crisis shock only and essentially the same as that in the economy with a sunspot shock only.

Figure 6: Optimal Inflation Target with Alternative Probabilities of Moving to the Deflationary Regime



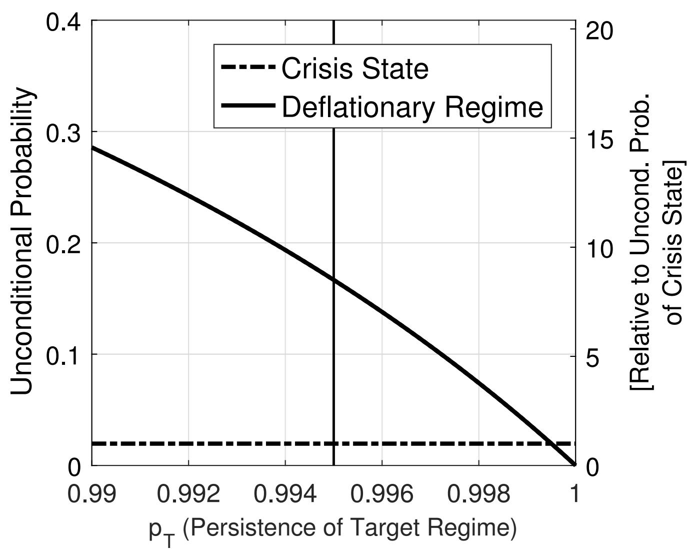
The extent to which the sunspot shock lowers the inflation target depends on how likely it is for the economy to be in the deflationary regime. In Figure 6, we show how the optimal inflation target varies with the probability of moving from the target regime to the deflationary regime under our baseline calibration. According to the figure, the optimal inflation declines as the persistence of the target regime declines—in other words, the probability of moving from the target regime to the deflationary regime increases—for any given persistence

[14]Sunspot equilibrium does not exist when the inflation target is sufficiently low, as discussed in Appendix B. The lowest value of the inflation target in this panel is the lowest value of the inflation target consistent with the existence of the sunspot equilibrium.

of the deflationary regime. With the target regime persistence of 0.998 and 0.999, the optimal inflation target is 0.1 percent and negative 0.3 percent, respectively.

The effect of the sunspot shock on the optimal inflation target dominates that of the crisis shock because the unconditional probability of being in the deflationary regime is much higher than the unconditional probability of being in the crisis state, unless the transition probability of moving from the target regime to the deflationary regime (p_T) is very small. To see this point, Figure 7 shows the unconditional probabilities of being in the deflationary regime for a range of p_T (solid black line) with other transition probability parameters fixed at their baseline values. Under our baseline value, $p_T = 0.995$, the unconditional probability of the deflationary regime is 16.6 percent, about 8 times as large as the unconditional probability of the crisis state, which is 2 percent as shown by the dashed black line. Only when p_T is very close to 1 (higher than 0.9995, to be specific) is the unconditional probability of the crisis state higher than the unconditional probability of the deflationary regime.

Figure 7: Unconditional Probability of Being in a Crisis State or Deflationary Regime

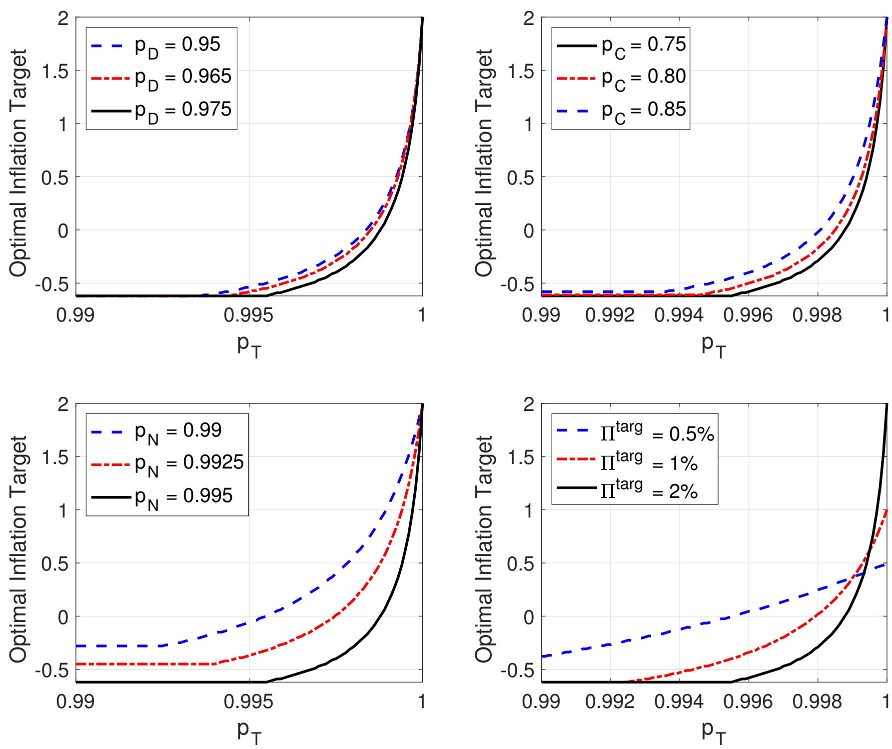


The unconditional probability of the deflationary regime being much higher than the unconditional probability of the crisis state is a necessary by-product of the equilibrium existence conditions on the transition probabilities of the crisis shock and the sunspot shock. For an equilibrium to exist in the model with a crisis shock, the persistence of the crisis state has to be sufficiently low (Nakata and Schmidt (Forthcoming); Appendix B); for an equilibrium to exist in the model with a sunspot shock, the persistence of the deflationary regime has to be sufficiently high (Nakata and Schmidt (2019); Appendix B). Thus, unless the persistence of the target regime is very high—that is, it is very unlikely for an economy to fall into expectation-driven LTs—or the persistence of the normal state is very low—that is, it is very likely for an economy to be hit by a crisis shock—the unconditional probability of the deflationary regime is higher than the unconditional probability of the crisis state.

3.4 Sensitivity analysis

We have just seen that the extent to which the possibility of falling into an expectations-driven LT lowers the optimal inflation target depends on the likelihood of being in an expectations-driven LT relative to the likelihood of being in a fundamental-driven LT. Thus, the optimal inflation target is higher under alternative calibrations in which the unconditional probability of being in an expectations-driven LT is lower than that in our baseline calibration calibration or those in which the unconditional probability of being in a fundamental-driven LT is higher than that in our baseline.

Figure 8: Optimal Inflation Target with Alternative Probabilities of Moving to the Deflationary Regime



In the top-right, top-left, and bottom-left panels of Figure 8, we show two alternative calibrations of the model with lower deflationary regime persistence (p_D), lower crisis shock persistence (p_C), and lower crisis shock frequency—equivalently, higher normal state persistence (p_N)—respectively. In each panel, these alternative calibrations are displayed by the dashed and dash-dotted lines. In the top-right and bottom-left panels, when we vary the crisis shock persistence and frequency, the crisis shock size, price adjustment cost (φ), and the degree of indexation (α) are adjusted to be consistent with the calibration principle outlined earlier.

The top-left panel shows that the lower deflationary regime persistence, the higher the optimal inflation target. The top-right and bottom-left panels show that the higher the crisis persistence or the crisis frequency—both of which implies a higher unconditional probability of being in the crisis state—the higher the optimal inflation target.

Finally, in the bottom-right panel of Figure 8, we show two alternative calibrations—shown by the dashed and dash-dotted lines—in which the crisis shock size, φ, and α are chosen so that the optimal inflation target is 0.5 percent and 1 percent in the model with the crisis shock only, respectively. This exercise is motivated by the possibility that the ELB is the only reason why the central bank aims for a positive inflation target and that the ELB accounts for only a part of the stated inflation target by central banks.[15] In these alternative calibrations, because the optimal inflation target is lower to begin with, the possibility of falling into the expectations-driven LT reduces the optimal inflation target by less than it does in the baseline calibration.

4 Conclusion

In this paper, we examine how the possibility of falling into expectations-driven LTs affects the optimal inflation target. Using a calibrated New Keynesian model, we find that even a very small probability of expectations-driven LTs nontrivially lowers the optimal inflation target under a wide range of parameter values. Our paper highlights a factor that has been neglected in the literature and the policy debate regarding the optimal inflation target. Because myriad factors influence the judgment on whether central banks should increase their inflation target in light of the ELB consideration, caution is of course warranted in drawing any policy implications from our exercise.

One limitation of our analysis—shared by other papers on this topic—is that we are silent about why an economy may fall into the expectations-driven LT or why it may escape from it. In particular, we assume that the transition probabilities governing the sunspot shock are exogenous to the conduct of monetary policy. According to a commonly told narrative of the Japanese policy by seasoned observers (see, for example, Hayakawa (2016)), one rationale for the aggressive monetary policy easing in Japan that started in 2013—including an official adaptation of the 2 percent inflation target—is that the aggressive easing may help push the Japanese economy out of the expectations-driven LT. While it is plausible that a change in some aspects of the monetary policy can affect the likelihood of falling into an expectations-driven LT, we followed the literature and excluded such a possibility in this paper. We leave the investigation of such a possibility to future research.

[15] One commonly cited reason for having a positive inflation target, besides the ELB, is an upward bias in the measured rate of inflation. See, for example, Kiley, Mauskopf, and Wilcox (2007).

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Technical Appendix for Online Publication

This technical appendix is organized as follows:

- Section A describes the details of the stylized model.
- Section B describes the conditions under which the equilibrium exists for the model with a demand shock and/or sunspot shock.
- Section C presents some analytical results on the effect of a higher inflation target in a semi-loglinear New Keynesian model.
- Section D discusses the effect of a higher inflation target in the nonlinear model with alternative values of α.

A Details of the Stylized Model

This section describes a stylized DSGE model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and government policies.

A.1 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_t,N_t,B_t} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s \Big] \Big[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \Big], \tag{16}$$

subject to the budget constraint

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t, \tag{17}$$

or equivalently

$$C_t + \frac{B_t}{R_t P_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + \Phi_t, \tag{18}$$

where C_t is consumption, N_t is the labor supply, P_t is the price of the consumption good, W_t (w_t) is the nominal (real) wage, Φ_t is the profit share (dividends) of the household from the intermediate goods producers, B_t is a one-period risk free bond that pays one unit of money at period t+1, and R_t^{-1} is the price of the bond.

The discount rate at time t is given by $\beta\delta_t$ where δ_t is the discount factor shock altering the weight of future utility at time $t+1$ relative to the period utility at time t. This increase in δ_t is a preference imposed by the household to increase the relative valuation of future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

A.2 Firms

There is a final good producer and a continuum of intermediate goods producers indexed by $i \in [0,1]$. The final good producer purchases the intermediate goods $Y_{i,t}$ at the intermediate price $P_{i,t}$ and aggregates them using CES technology to produce and sell the final good Y_t to the household and government at price P_t. Its problem is then summarized as

$$\max_{Y_{i,t}, i\in[0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di, \tag{19}$$

subject to the CES production function

$$Y_t = \Big[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \Big]^{\frac{\theta}{\theta-1}}. \tag{20}$$

Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{i,t} = N_{i,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[16] by setting the price of its own good. We can assume that each firm receives a production subsidy τ so that the economy is fully efficient in the steady state.[17] In our baseline, we set $\tau = 1/\theta$. Price changes are subject to quadratic adjustment costs.

$$\max_{P_{i,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s \Big] \lambda_t \Big[P_{i,t} Y_{i,t} - (1-\tau) W_t N_{i,t} - P_t \frac{\varphi}{2} [\frac{P_{i,t}}{P_{i,t-1}(\Pi^{targ})^{\alpha}} - 1]^2 Y_t \Big], \tag{21}$$

such that

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t.^{18} \tag{22}$$

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s \Big] \lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

A.3 Monetary Policy

It is assumed that the central bank determines the short term nominal interest rate according to a Taylor rule

$$R_t = \max \left[1, \quad \frac{\Pi^{targ}}{\beta \delta_t} \left(\frac{\Pi_t}{\Pi^{targ}} \right)^{\phi_\pi} \right], \tag{23}$$

[16]Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level P_t which we take care of further along in the document.

[17]$(\theta - 1) = (1-\tau)\theta$ which implies zero profits in the zero inflation steady state. In a welfare analysis, this would extract any inflation bias from the second-order approximated objective welfare function. τ therefore represents the size of a steady state distortion (see Chapter 5 Appendix, Galí (2008)).

[18]This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \Big[\int_0^1 P_{i,t}^{1-\theta} di \Big]^{\frac{1}{1-\theta}}$.

where $\Pi_t = \frac{P_t}{P_{t-1}}$.

A.4 Market clearing conditions

The market clearing conditions for the final good, labor, and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi}{2}\Big[\frac{P_{i,t}}{P_{i,t-1}(\Pi^{targ})^{\alpha}} - 1\Big]^2 Y_t di, \tag{24}$$

$$N_t = \int_0^1 N_{i,t} di, \tag{25}$$

and

$$B_t = 0. \tag{26}$$

A.5 Recursive equilibrium

Given P_0 and a two-state Markov shock process establishing δ_t, an equilibrium consists of allocations $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}, G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{i,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that (i) given the determined prices and policies, allocations solve the problem of the household, (ii) $P_{i,t}$ solves the problem of firm i, (iii) R_t follows a specified rule, and (iv) all markets clear.

Combining all of the results from (i)-(iv), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$C_t^{-\chi_c} = \beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1}, \tag{27}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c}, \tag{28}$$

$$\begin{aligned} \frac{Y_t}{C_t^{\chi_c}} \left[\varphi\left(\frac{\Pi_t}{(\Pi^{targ})^{\alpha}} - 1\right)\frac{\Pi_t}{(\Pi^{targ})^{\alpha}} - (1-\theta) - \theta(1-\tau) w_t\right] \\ = \beta\delta_t \mathrm{E_t} \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi\left(\frac{\Pi_{t+1}}{(\Pi^{targ})^{\alpha}} - 1\right)\frac{\Pi_{t+1}}{(\Pi^{targ})^{\alpha}}, \end{aligned} \tag{29}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\frac{\Pi_t}{(\Pi^{targ})^{\alpha}} - 1\right]^2 Y_t, \tag{30}$$

$$Y_t = N_t, \tag{31}$$

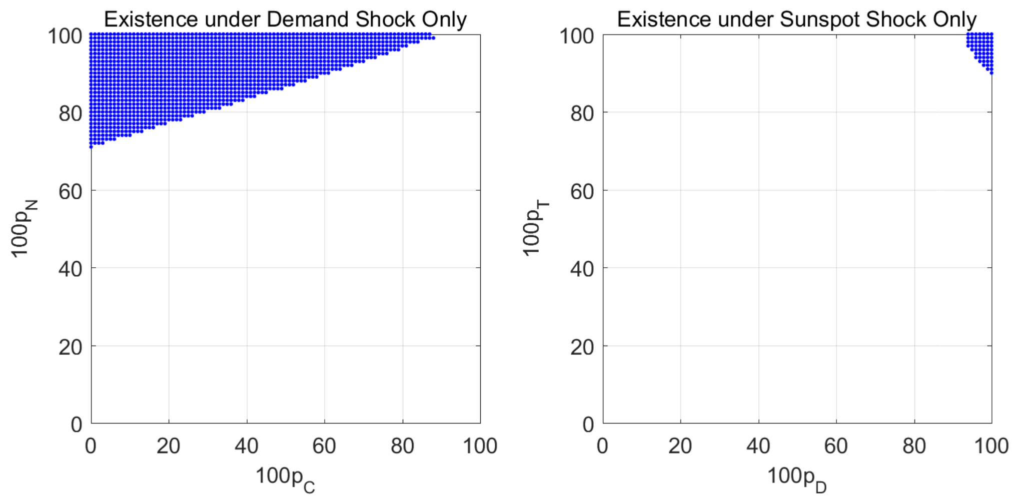
$$R_t = \max\left[1, \quad \frac{\Pi^{targ}}{\beta\delta_t}\left(\frac{\Pi_t}{\Pi^{targ}}\right)^{\phi_\pi}\right]. \tag{32}$$

Equation 27 is the consumption Euler equation, Equation 28 is the intratemporal optimality condition of the household, Equation 29 is the optimal condition of the intermediate good producing firms (forward-looking Phillips Curve) relating today's inflation to real marginal cost today and expected inflation tomorrow, Equation 30 is the aggregate resource constraint capturing the resource cost of price adjustment, and Equation 31 is the aggregate production function. Equation 32 is the interest-rate feedback rule.

B On the existence and multiplicity of a sunspot equilibrium

The left panel of Figure 9 shows pairs of p_N and p_C that are consistent with the equilibrium existence in the model with a demand shock only, whereas the right panel shows pairs of p_T and p_D that are consistent with the equilibrium existence in the model with a sunspot shock only.

Figure 9: Transition probabilities and the equilibrium existence



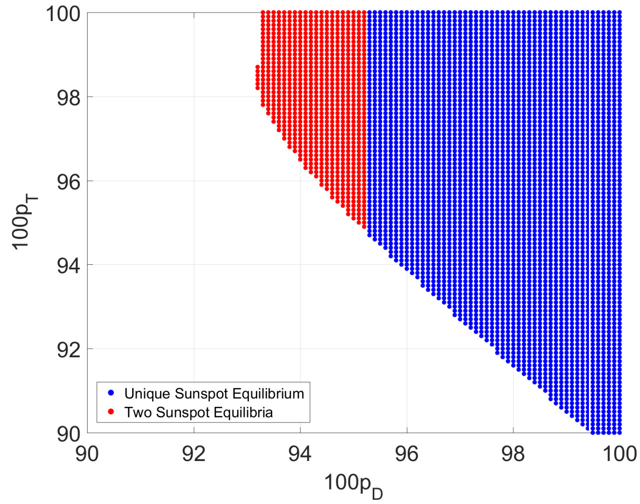
Consistent with the analytical result based on a semi-loglinear New Keynesian model in Nakata and Schmidt (Forthcoming), in the model with a demand shock only, the equilibrium exists if and only if p_N is sufficiently high and p_C is sufficiently low. When p_N is low, there is a high probability of moving to the crisis state in the next period when the economy is currently in the normal state. The anticipation effect of moving to the crisis state in the next period causes the policy rate to decline. When p_N is sufficiently low, the policy rate in the normal state becomes negative, violating the conditions for the equilibrium existence. When p_C is high, there is a high probability of staying in the crisis state in the next period when the economy is currently in the crisis state. When p_C is too high, inflation and output consistent with the private sector equilibrium conditions are positive, which implies a positive policy rate and thus inconsistent with the equilibrium existence.[19]

In the model with a sunspot shock only, the equilibrium exists if and only if p_T and p_D are both sufficiently high (see Nakata and Schmidt (2019) for a proof of this statement in a semi-loglinear model). The reason for why p_T needs to be high for the equilibrium to exist in this model is the same as the reason for why p_N needs to be high for the equilibrium to exist in the model with a demand shock only. Why does p_D have to be sufficiently high for the equilibrium to exist? When p_D is low, there is a low probability of staying in the deflationary state in the next period when the economy is currently in the target state. When p_D is too low, inflation and output consistent with the private sector equilibrium conditions are positive, which implies a positive policy rate and thus inconsistent with the equilibrium existence.[20]

[19]A version of this equilibrium existence result with $p_N = 1$ is well known in the literature. See, for example, Eggertsson (2011) and Boneva, Braun, and Waki (2016).

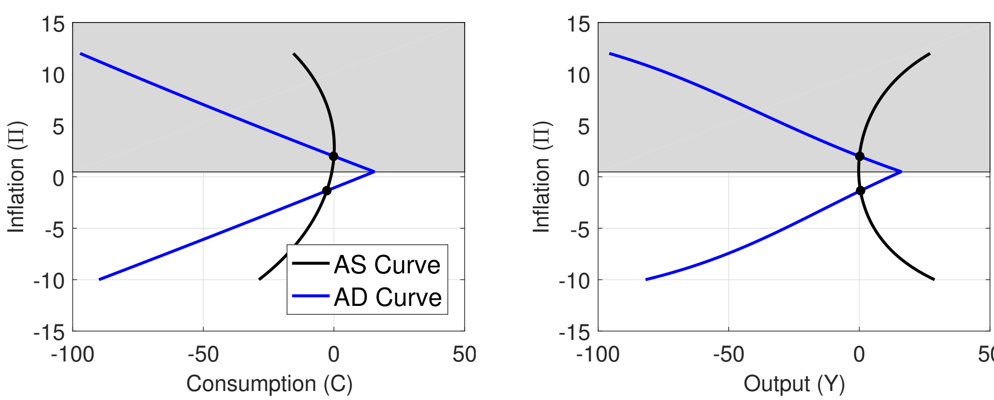
[20]A version of this equilibrium existence result with $p_T = 1$ is well known in the literature. See, for example,

Figure 10: Transition probabilities and the sunspot equilibria multiplicity



Consistent with Boneva, Braun, and Waki (2016) and as shown by the red dots in Figure 10, there are two pairs of inflation and the output gap in the deflationary regime satisfying the equilibrium conditions when p_D is sufficiently small, but large enough so that an equilibrium exists. To illustrate the multiplicity of the sunspot equilibrium, Figure 11 and 12 show the AD and AS curves in the deflationary regime when $p_D = 0.975$ and $p_D = 0.95$, respectively. Focusing on the part of the AD curve consistent with the binding ELB constraint, there is only one intersection of the AS and AD curves when $p_D = 0.975$, as shown by Figure 11. When $p_D = 0.95$, there are two intersection of the AS and AD curves, as shown by Figure 12. One intersection features moderate declines in inflation and consumption, as well as a moderate increase in output (output increases due to price-adjustment costs associated with the mild decline in inflation). The other intersection features very large declines in inflation and consumption, as well as a very large increase in output.

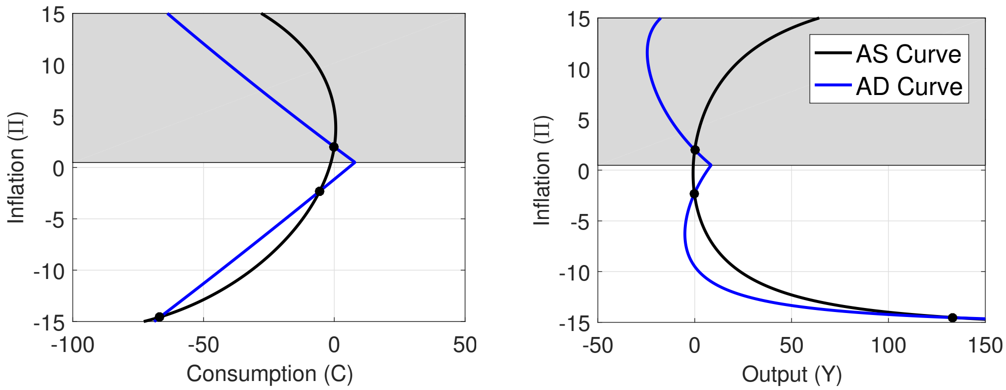
Figure 11: AD and AS Curves in the Deflationary Regime
—High p_D—



Note: Grey shades indicate the region in which the ELB is not binding.

Mertens and Ravn (2014) and Boneva, Braun, and Waki (2016).

Figure 12: AD and AS Curves in the Deflationary Regime
—Low p_D—



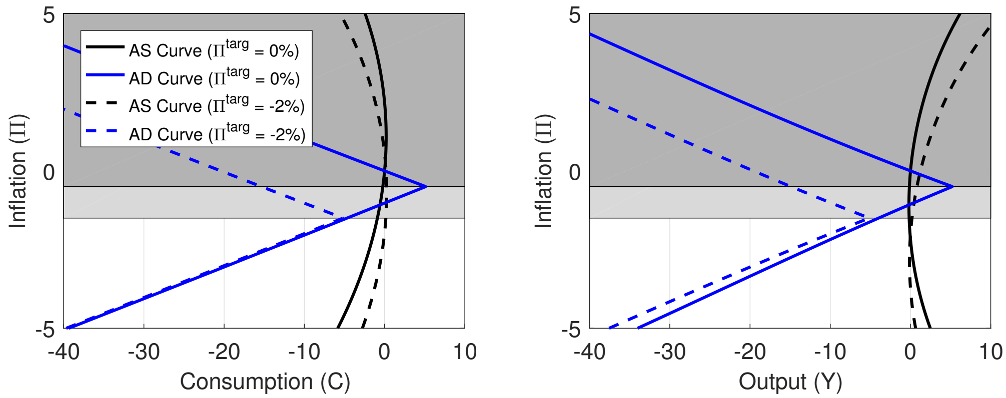
Note: Grey shades indicate the region in which the ELB is not binding.

When there are more than one sunspot equilibrium, we focus on the equilibrium with moderate declines in inflation and consumption in the deflationary regime for two reasons. First, the allocation in this equilibrium changes continuously with a change in p_D even at the threshold value of p_D above which there is one sunspot equilibrium and below which there are two sunspot equilibrium. In contrast, the other sunspot equilibrium with very large declines in inflation and consumption "suddenly" shows up when p_D declines below the threshold. Second, we would like our deflationary regime to look like the Japanese economy over the past two decades, which features mild deflation.

B.1 Equilibrium existence and the inflation target

As noted in Section 3, the sunspot equilibrium ceases to exist when the inflation target is sufficiently low. To see this result, Figure 13 shows the AS and AD curves when the inflation target is 0 percent and -2 percent. According to the figure, the AD curve shifts to the left and the AS curve shifts to the right when the inflation target declines. With the inflation target of -2 percent, there is no intersection for the region of inflation rate consistent with the binding ELB constraint.

Figure 13: AD and AS Curves in the Deflationary Regime
—Low Π^{targ}—



Note: Grey shades indicate the region in which the ELB is not binding.

C Some analytical results from a semi-loglinear model

In this section, we will investigate the effect of raising the inflation target in a semi-loglinear model that allows for analytical results.[21]

C.1 Model

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \sigma\left[i_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\} - (r^* + \pi^*) + \delta_t\right], \tag{33}$$
$$\hat{\pi}_t = \kappa y_t + \beta \mathbb{E}_t\{\hat{\pi}_{t+1}\}, \tag{34}$$
$$\hat{\pi}_t = \pi_t - \pi^*, \tag{35}$$
$$i_t = \max\left[0, (r^* + \pi^*) + \phi_\pi \hat{\pi}_t\right]. \tag{36}$$

Here, equation (33) is the log-linearized consumption Euler Equation, equation (34) is the forward looking Phillips curve, and equation (36) is the interest rate feedback rule with a zero lower bound (ZLB) constraint. Notice that $\hat{\pi}_t$ is the inflation gap—deviation of the inflation rate (π_t) from the inflation target (π^*). y_t is the deviation of output from its deterministic steady state level. Note that, in deriving these semi-loglinear equilibrium conditions, we assume that the the price "indexation" parameter in the original fully nonlinear model is unity so that the level of the inflation target does not affect the deterministic steady state level of output.

As in the main body of the paper, we will first discuss the effect of an increase in the inflation target parameter in the version of the model with a crisis shock only, and then move on to the analysis in the version of the model with a sunspot shock only.

[21] See Bilbiie (2018) for a related analysis. He analytically shows that a *temporary* increase in inflation in the target regime lowers inflation and output in a deflationary regime in a similar setup. We analytical shows that a permanent increase in inflation in the target regime lowers inflation and output in a deflationary regime.

C.2 Model with a crisis shock only

As in the main text, we assume that the demand shock, δ_t, follows a two-state Markov shock process. $Prob(N|N) := p_N$ is the probability of staying in the normal state in the next period when the economy is in the normal state today. $Prob(C|C) := p_C$ is the probability of staying in the crisis state in the next period when the economy is in the crisis state today.

The equilibrium conditions of the economy with a crisis shock only is given by

$$y_N = p_N y_N + (1-p_N)y_C + \sigma\left[p_N\pi_N + (1-p_N)\pi_C - \pi^*\right] - \sigma\left[i_N - (r^* + \pi^*) + \delta_N\right], \tag{37}$$

$$\pi_N - \pi^* = \kappa y_N + \beta\left(p_N\pi_N + (1-p_N)\pi_C - \pi^*\right), \tag{38}$$

$$i_N = r^* + \pi^* + \phi_\pi(\pi_N - \pi^*), \tag{39}$$

$$y_C = p_C y_C + (1-p_C)y_N + \sigma\left[p_C\pi_C + (1-p_C)\pi_N - \pi^*\right] - \sigma\left[i_C - (r^* + \pi^*) + \delta_C\right], \tag{40}$$

$$\pi_C - \pi^* = \kappa y_C + \beta\left(p_C\pi_C + (1-p_C)\pi_N - \pi^*\right), \tag{41}$$

$$i_C = \max\left[0, r^* + \pi^* + \phi_\pi\pi_C - \pi^*\right]. \tag{42}$$

To analyze how an increase in the inflation target affects the crisis shock state in a transparent way, we assume that the normal state is an absorbing state (i.e., $p_N = 1$). Then,

$$y_N = 0, \tag{43}$$

$$\pi_N = \pi^*, \tag{44}$$

$$i_N = r^* + \pi^*, \tag{45}$$

$$y_C = \frac{\sigma(1-\beta p_C)}{(1-\beta p_C)(1-p_C) - \kappa\sigma p_C}\pi^* + \frac{\sigma(r^* - \delta_C)(1-\beta p_C)}{(1-\beta p_C)(1-p_C) - \kappa\sigma p_C}, \tag{46}$$

$$\pi_C = \left[\frac{\kappa\sigma}{(1-\beta p_C)(1-p_C) - \kappa\sigma p_C} + 1\right]\pi^* + \frac{\kappa\sigma(r^* - \delta_C)}{(1-\beta p_C)(1-p_C) - \kappa\sigma p_C}, \tag{47}$$

$$i_C = 0. \tag{48}$$

As shown by Nakata and Schmidt (Forthcoming) and Nakata and Schmidt (2019), for the equilibrium to exist, $(1-\beta p_C)(1-p_C) - \kappa\sigma p_C > 0$. Thus, the coefficients in front of π^* in equations (46 and 47) are both positive, meaning that y_C and π_C increase with the inflation target.

To understand the mechanism behind this result , it is useful to investigate the aggregate demand curve—a set of pairs of inflation and output in the crisis state consistent with the Euler equation—and the aggregate supply curve—a set of pairs of inflation and output in the crisis state consistent with the Phillips curve. They are given by

$$\pi_C = \frac{1-p_C}{\sigma p_C}y_C - \frac{1-p_C}{p_C}\pi^* - \frac{r^* - \delta_C}{p_C}, \tag{49}$$

$$\pi_C = \frac{\kappa}{1-\beta p_C}y_C + \pi^*. \tag{50}$$

According to equation (49), an increase in the inflation target shifts down the AD curve. An increase in the inflation target parameter translates into an increase in the normal state inflation. According to the Euler equation, to support the same level of output in the crisis

Table 2: Calibrations for the Semi-Loglinear Model

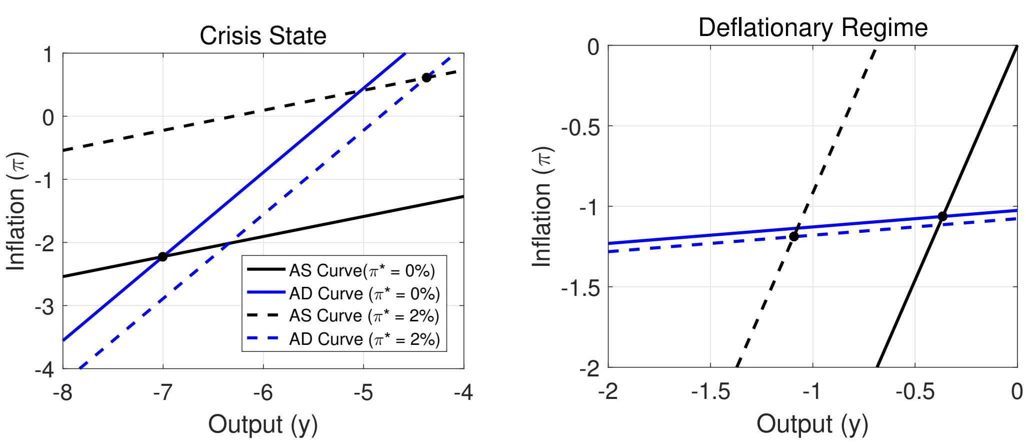
Parameter	Calibration	Description
β	1/1.0025	Discount factor
σ	1	Elasticity of Intertemporal Substitution
κ	0.02	Slope of the Phillips Curve
ϕ_π	2	Coefficient on inflation in Taylor Rule
r^*	1%	Annualized steady state nominal interest rate
δ_N	0	Demand shock in normal state
δ_C	1.6/100	Demand shock in crisis shock state
p_T	1	Persistence of Target Regime
p_D	0.975	Persistence of Deflationary Regime
p_N	1	Persistence of Normal State
p_C	0.75	Persistence of Crisis Shock State
π^*	$\{0\%, 2\%\}$	Annualized steady state nominal interest rate

state, the expected real interest rate has to remain unchanged. Thus, if the normal state inflation is higher, the crisis state inflation has to decline.

According to equation (50), an increase in the inflation target shifts up the AS curve. An increase in the inflation target parameter translates into an increase in the normal state inflation. According to the Phillips curve, the crisis state inflation positively depends on the expected inflation in the next period, which also positively depends on the normal state inflation. As a result, a higher normal state inflation leads to a higher crisis state inflation for any given level of the crisis state output.

As shown by the left panel of Figure 14 which plots the effect of an increase in π^* from 0 percent to 2 percent on the AS and AD curves, these shifts in the AS and AD curves mean a higher inflation and output in the crisis state. Table 2 shows the calibration of the semi-loglinear model used to generate the left panel of Figure 14.

Figure 14: AD and AS Curves in the Crisis State and in the Deflationary Regime
—Semi-Loglinear Model—



C.3 Model with a sunspot shock only

We now examine the effect of an increase in the inflation target on the deflationary regime output and inflation using the version of the semi-loglinear model with a sunspot shock only.

Let $Prob(T|T) = p_T$ be the probability of staying in the target regime in the next period when the economy is in the target regime today and let $Prob(D|D) = p_D$ be the probability of staying in the deflationary regime in the next period when the economy is in the deflationary regime today.
The equilibrium conditions of the model with a sunspot shock only are given by

$$\begin{aligned}
y_T =& p_T y_T + (1-p_T)y_D + \sigma\left[p_T\pi_T + (1-p_T)\pi_D - \pi^*\right] - \sigma\left[i_T - (r^* + \pi^*)\right], && (51)\\
\pi_T - \pi^* =& \kappa y_T + \beta\left(p_T\pi_T + (1-p_T)\pi_D - \pi^*\right), && (52)\\
i_T =& r^* + \pi^* + \phi_\pi(\pi_T - \pi^*), && (53)\\
y_D =& p_D y_D + (1-p_D)y_T + \sigma\left[p_D\pi_D + (1-p_D)\pi_T - \pi^*\right] - \sigma\left[i_D - (r^* + \pi^*)\right], && (54)\\
\pi_D - \pi^* =& \kappa y_D + \beta\left(p_D\pi_D + (1-p_D)\pi_T - \pi^*\right), && (55)\\
i_D =& 0. && (56)
\end{aligned}$$

To analyze how an increase in the inflation target affects the deflationary regime in a transparent way, we assume that the target regime is an absorbing state (i.e., $p_T = 1$). Then,

$$\begin{aligned}
y_T =& 0, && (57)\\
\pi_T =& \pi^*, && (58)\\
i_T =& r^* + \pi^*, && (59)\\
y_D =& \frac{\sigma(1-\beta p_D)}{(1-\beta p_D)(1-p_D) - \kappa\sigma p_C}\pi^* + \frac{\sigma r^*(1-\beta p_D)}{(1-\beta p_D)(1-p_D) - \kappa\sigma p_D}, && (60)\\
\pi_D =& \left[\frac{\kappa\sigma}{(1-\beta p_D)(1-p_D) - \kappa\sigma p_D} + 1\right]\pi^* + \frac{\kappa\sigma r^*}{(1-\beta p_D)(1-p_D) - \kappa\sigma p_D}, && (61)\\
i_D =& 0. && (62)
\end{aligned}$$

As shown by Nakata and Schmidt (2019), for the equilibrium to exist, $(1-\beta p_D)(1-p_D) - \kappa\sigma p_D < 0$. Thus, the coefficient in front of π^* in equation (60) is negative, meaning that y_D increases with the inflation target. Provided that p_D is sufficiently large, the coefficient in front of π^* in equation (61) is negative, and thus π_D increases with the inflation target.

To understand the mechanism behind this result, it is useful to investigate the aggregate demand curve and the aggregate supply curve in the deflationary regime:

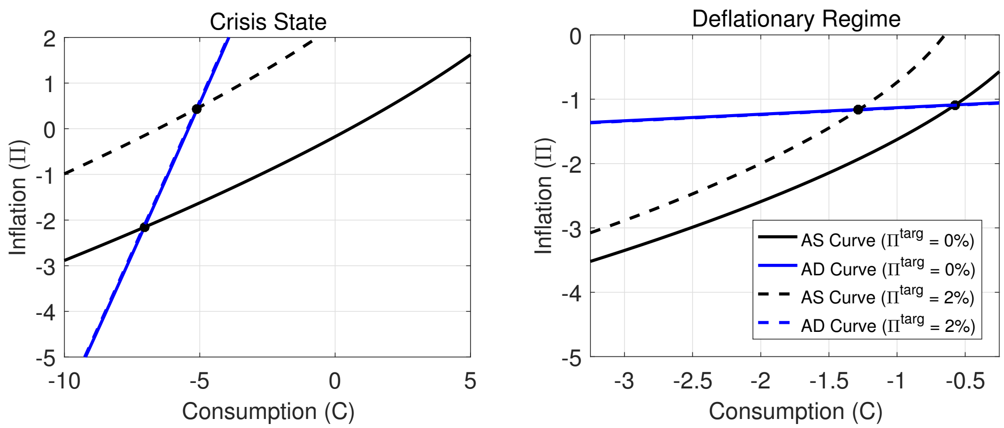
$$\begin{aligned}
\pi_D =& \frac{1-p_D}{\sigma p_D}y_D - \frac{1-p_D}{p_D}\pi^* - \frac{r^*}{p_D}, && (63)\\
\pi_D =& \frac{\kappa}{1-\beta p_D}y_D + \pi^*. && (64)
\end{aligned}$$

As in the model with a crisis shock only, an increase in the inflation target shifts down the AD curve and shifts up AS curves. Because the AS curve is steeper than the AD curve in the model with a sunspot shock only, these shifts in the AS and AD curves mean lower inflation and output in the crisis state, as shown by the right panel of Figure 14.

D Effects of a Higher Inflation Target with $\alpha = 0$ and $\alpha = 1$

While we set α to 0.893 in our baseline calibration, the key property of the model—a higher inflation target leads to lower inflation and consumption in the deflationary regime—does not hinge on the specific value of α. In this section, we examine the effect of a higher inflation target under $\alpha = 0$ (no price indexation) and $\alpha = 1$ (full price indexation). The assumption of no price indexation is common in papers analyzing the effect of a non-zero trend inflation, including those papers on the optimal inflation target (see, for example, Ascari (2004) and Ascari, Castelnuovo, and Rossi (2011)). The assumption of full price indexation is widespread in the literature estimating DSGE models and in papers using policy-oriented, medium-scale DSGE models, as the full price indexation make the dynamics of the loglinear version of those models invariant to the level of the inflation target (see, for example, Smets and Wouters (2007)).

Figure 15: AD and AS Curves: $\alpha = 0$



Figures 15 and 16 shows the effect of a higher inflation target with $\alpha = 0$ and $\alpha = 1$, respectively. When $\alpha = 0$, the effect of a higher inflation target is negligible for the AD curve. When $\alpha = 1$, the AD curves in both the crisis state and the deflationary regime shift down by a small amount, as in the case discussed in Section 3. For both $\alpha = 0$ and $\alpha = 1$, the AS curves shift up in both the crisis state and the deflationary regime, as in the case discussed in Section 3. Thus, a higher inflation target leads to lower inflation and consumption regardless of the value of α.

Figure 16: AD and AS Curves: $\alpha = 1$