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Mixed Signals: Equilibrium Investment Distortions with Adverse Selection*

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Abstract

We study how adverse selection distorts equilibrium investment allocations in a Walrasian credit market with two-sided heterogeneity. Representative investor and partial equilibrium economies are special cases where investment allocations are distorted above perfect information allocations. By contrast, the general setting features a pecuniary externality that leads to trade and investment allocations below perfect information levels. The degree of heterogeneity between informed agents' type governs the direction of the distortion. Moreover, contracts that complete markets dampen the impact of pecuniary externalities and change equilibrium distortions. Implications for empirical design in credit market studies and financial stability are discussed.

JEL Classification: D52, D53, D82, E44, G32
Keywords: asymmetric information, pecuniary externality, investment, signalling, cost of capital, credit default swaps

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1 Introduction

Anonymous credit market intermediation through pools of debt securities is a rapidly growing feature in modern economies. The canonical example is the market for mortgages. Banks and other mortgage lenders initiate and screen applicants, but the loans are ultimately standardized, bundled, and sold to many diverse investors as mortgage-backed securities (MBS). The agency MBS market grew from $370b in 1996 to $1.4t in 2017.[1] In addition, corporate loans are packaged into collateralized loan obligations (CLOs) and sold to a diverse set of investors–non-bank investors such as hedge funds, mutual funds, pension funds. Notably, issuance of risky "leveraged loans" packaged into CLOs grew from $9 billion in the late 1990s to approximately $500 billion in 2018 with a total outstanding dollar amount over $1 trillion.[2]

The pooling of risk and anonymous exchange changes the way asymmetric information affects equilibrium allocations. In particular, debt pools diversify idiosyncratic risk but introduce aggregate risk that affects the average repayment rate across loans in the pool. Anonymous exchange lends itself to price taking behavior through Walrasian markets rather than strategic interaction and extensive form games. This paper draws on these features and presents a model of two-sided heterogeneity into a general equilibrium model of competitive pooling with information asymmetries. Our main purpose is to study how asymmetric information affects equilibrium allocations in this new credit environment.

The model features two types of heterogeneous agents. The first type of agents are called firms. We focus on the most parsimonious case with two firm types. Firms possess private information about their technological ability to transform inputs into output, which we interpret as cash flows. Both types of firms issue generic non-contingent promises to repay $1 tomorrow in exchange for inputs today *i.e* debt contracts. Default is an aggregate outcome when the state-specific cash flows are insufficient to cover promises in some state. The major difference between firms is that high types deliver more in default states than low types, generating firm-type specific default premia.

The second type of agents in the model are risk-neutral investors with heterogeneous beliefs about the expected value of firm output.[3] Thus, there is two-sided heterogeneity. Trade occurs between firms and investors through competitive credit markets. The credit market is modeled ala Dubey and Geanakoplos (2002) where debt is pooled and each debt pool is a market defined by a price-quantity pair. The source of adverse selection is that low cash-flow types would like to sell the same

[1]Source: SIFMA https://www.sifma.org/resources/research/us-mortgage-related-issuance-and-outstanding/.

[2]SP Global, Leveraged Loan Commentary and Data (LCD).

[3]We stress the important aspect of this framework is that investors have heterogeneous marginal utilities for consumption across states. The fact that we choose to think about differences in beliefs is not material.

promise to deliver \$1 tomorrow in exchange for the same quantity of inputs today that high cash flow firms receive, but deliver less in default states.

The first result is that the representative agent or partial equilibrium model is a special case in which the high-type firms always issue more debt and over invest in equilibrium compared to a perfect information benchmark. Too much trade is a consequence of the Spence-Mirlees single-crossing condition in the model where high types are more willing to trade than low-types. Hence, adverse selection generates a distortion above the perfect information level of trade.

The second, and main result of the paper, shows that two-sided heterogeneity in Walrasian markets can over turn the excessive trade result, despite monotonicity in the single-crossing condition. The reason is that incentive compatible allocations are impacted by a pecuniary externality in general equilibrium. More specifically, risky debt prices rise as high types trade less in debt markets. Hence, there is a *positive externality* in the credit market for low types that allows their budget set to endogenously expand. The allocation for high types is determined by a binding incentive compatibility constraint that low types prefer to trade in the debt market for which they are better off truthfully revealing their type. Therefore, the positive externality reduces the incentive for low types to mimic high types. By contrast, the pecuniary externality is absent in the representative investor or partial equilibrium setting and through standard single-crossing arguments, it is always least costly for high types to separate by over investing.

The main comparative static result shows that the direction of the trade distortion crucially depends on the relative heterogeneity across firms. A widening in the productivity distribution between firm types moves the economy from a distortion on the low to the upper end resulting in excessive trade compared to perfect information levels. The intuition is that the difference in the efficient allocations in terms of prices and investment scales increases as productivity differences widen. As a result, the incentive compatible allocation for high types becomes more distorted at the low end than the high end. In other words, the adverse selection cost of reducing the level of trade rises. By contrast, raising more debt and increasing the level of trade generates a negative pecuniary externality that lowers all risky debt prices. As the the fundamental difference between the two firm types at their efficient contracts increases, distortions at the top are both incentive compatible and least costly for high types because they are met with a relatively small distortion in the equilibrium allocation. In sum, adverse selection costs are non-monotonic in the level of trade, and depend on the relative difference between firm types.

An important implication of equilibria with too much trade is that investor losses, conditional on default, are higher on high type assets compared to perfect information. The reason that losses on high type debt rise is that the incentive constraint forces high types to trade too much at the market interest rate. They would prefer to trade less, pay lower credit spreads, and reduce investment and production. Hence, the standard marginal product and cost of capital equivalence is broken. In the aggregate default state, each individual debt claimant recovers less than she would without adverse selection. Thus too much trade ex ante leads to larger losses ex post.

The model also suggests that adverse selection can amplify credit extension and investment resulting in an "investment boom." Dell'Ariccia and Marquez (2012) argue that loose lending standards may result in more risky firms receiving credit in equilibrium, which in our model, manifests through a widening in the distribution of firm fundamentals. Under asymmetric information, high types expand investment because the marginal low type receiving credit in equilibrium has more incentive to mimic at low investment levels, amplifying the "lending boom."

Our fourth result shows that with two-sided heterogeneity, financial innovation through contracts that improve risk sharing and complete markets for one set of agents (investors) can generate too much trade for the other set of agents (firms). The characterization of equilibrium with two-sided heterogeneity will generally resemble the one with one-sided heterogeneity. The contracts we have in mind are credit default swaps (CDS). In economies with adverse selection, CDS cause equilibrium risky debt prices to become inelastic with respect to the level of trade in the credit market. However, allocations with the distortions at the low end only occur in the presence of the positive pecuniary externality. Therefore, when the pecuniary externality is muted, the cost of distorting trade below perfect information to meet incentive compatibility rises.

The intuition is the following: CDS post their endowment to sell insurance, which allows them to take leverage and hold more risk than buying bonds.[4] The high type's debt issuance decision impacts the value of debt claims in default states, and, consequently, equilibrium CDS prices. High CDS prices that result from high debt levels allow CDS sellers to sell more contracts per unit of endowment. Thus the CDS market clears without CDS sellers needing to post much additional collateral despite large bond supply changes. The debt-supply change needed to impact the *marginal CDS trader's* price must be much larger than change needed to impact the *marginal bond buyer's* price. Hence, the impact of the pecuniary externality in the credit market is greatly minimized, meaning that risky debt prices are relatively unresponsive to large fluctuations in the level of trade between firms and investors.

The final result shows that, in the presence of information asymmetries, CDS can generate a rise in *aggregate investment and trade* despite higher borrowing costs. In particular, because prices are relatively inelastic in the CDS economy, the level of

[4]Limiting the amount of CDS to the size of the underlying debt market could be done by imposing the restriction that only investors who own debt can purchase CDS. This restriction is referred to as covered-CDS (Che and Sethi (2015) and Darst and Refayet (2018)). The unrestricted case we present allows for covered positions to be taken and for naked positions that facilitate bets between agents who do not own the underlying debt.

trade required for high types to meet incentive compatibility more than compensates for the reduction in low type trade due to higher borrowing costs. This result is due to the interaction of the CDS and adverse selection, and is generally consistent with the concurrent increase in CDS market activity and investment boom in the build up to the financial crisis. General equilibrium models with CDS and perfect information cannot generate this relationship without imposing a counter-factual restriction that the amount of CDS must be tied to the size of the underlying debt market (Che and Sethi (2015) and Darst and Refayet (2018)). Relatedly, Fostel and Geanakoplos (2016) show that CDS cause investment to fall below Arrow-Debreu and economies with leverage or autarky.

Our results have important implications for testing how asymmetric information affects corporate credit markets. The dominant conceptual framework used to test for asymmetric information is the positive correlation test.[5] Crawford, Pavanini, and Schivardi (2017) recently applied the methodology to corporate loans in Italy. The null hypothesis is that a positive correlation between either *ex ante* loan demand or loan amount and *ex post* default rates is consistent with asymmetric information. Rejecting the null is consistent with, but of course does not prove, symmetric information. The logic of the test derives from the pooling equilibrium in Stiglitz and Weiss where higher interest rates only attract high-risk firms who are more likely to default. There are two problems with applying this reasoning *generally* across corporate credit markets. The first, less problematic, reason is that one cannot use Stiglitz and Weiss (1981) to reject a null hypothesis of asymmetric information with a positive correlation between the *intensive* margin of loan use and default because loan amounts are exogenously fixed.[6] The second, and more problematic, reason is that our model shows that the correlation between default and loan use can be zero under both perfect and asymmetric information assumptions. Thus, rejecting the null of positive correlation on its own cannot identify asymmetric information. The problem arises when private information concerns the first moment (expected repayment) and default is an aggregate state outcome rather than private information about the second moment where default is idiosyncratic. Our model suggests the correlation between the extensive margin of loan demand and default is more apt than the correlation between the intensive loan size and default.[7]

The rest of the paper is organized as follows. The remainder of this section is related to the literature. Section 2 presents the baseline economy and the different agents' maximization problems. We present a condensed version of the baseline

[5]The positive correlation test was pioneered by Chiappori and Salanié (2000) in insurance markets and Adams, Einav, and Levin (2009) in auto loan markets.

[6]The separating equilibria in Bester (1985) are also derived with exogenously given investment and collateral amounts.

[7]To the credit of Crawford et. al, they study credit lines which are debt contracts with the insurance-like feature of being state-contingent. Therefore, the positive correlation test can more heavily rest on the predictions of the insurance literature to mitigate the critique our model presents.

economy under perfect information against which to compare the asymmetric information economy that follows. We discuss how to deal with a perfectly competitive credit market subject to adverse selection through the concept of debt pools and competitive pooling. We then introduce the notion of CDS and solve several numerical examples to highlight our main message and mechanism. Section 3 presents the analytical results for both the baseline and the CDS economy. Section 4 offers some interpretations and discussion of our model and the empirical relevance of our results.

Related Literature

A long literature on the effect of information asymmetries on credit markets stems from Jaffee and Russell (1976) and Stiglitz and Weiss (1981) (SW). Hellwig and Gale (1985) show in partial equilibrium that credit is not rationed but investment is too low after relaxing very specific assumptions: the nature of the information asymmetry, endogenous investment scale, and all-or-nothing returns in bad states. De Maza and Webb (1987) show that investment is higher than first best in SW when projects have the same scale but different first moments. Arnold and Riley (2009) show that credit rationing in SW only exists if there are in fact two market loan rates. Credit is rationed only at the lowest rate. The framework in these papers only consider pooling equilibria with a representative lender.

The partial equilibrium papers of Bester (1985), Besanko and Thakor (1987), and Milde and Riley (1987) show that equilibria are separating when lenders have a rich set of contract instruments. In Besanko and Thakor (1987), low risk borrowers always *over borrow* but pay *higher borrowing costs* than high risk borrowers.[8] Milde and Riley (1987) show that loan size can be either a monotonically increasing or decreasing function of firm type depending on shape of the productivity function.[9] We show that the general equilibrium effects with heterogeneous lenders generate pecuniary externalities. Consequently, the equilibrium allocations in these models do not generalize to our setting. In this sense, our finding that fully-separating equilibria may involve high types engaging in too little trade in general equilibrium is similar to Guerrieri and Shimer (2014), but that is the unique outcome in their dynamic modeling framework.

Kurlat (2013) and Bigio (2015) study how shocks from the financial sector affect the real sector when asset markets are subject to information asymmetries as in Akerlof (1970). Caramp (2018) studies the incentive to produce different quality assets ex ante in the presence of ex post adverse selection. Our paper also relates to the

[8]Relatedly, Morellec and Shuerhoff (2013) show that firms with high cash flows will prematurely exercise growth options (reducing the value of the project akin to overinvestment) in a dynamic corporate finance model.

[9]Martin (2009) is also related but his focus is on how investment changes as entrepreneurial net worth changes, on which our model is silent.

literature on credit booms and busts. Lorenzoni (2008) shows that pecuniary externalities lead to *ex ante* excessive investment compared to the constrained efficient allocation. He and Kondor (2016) study over and under investment inefficiencies similar to ours where boom-bust cycles stem from private liquidity management. A series of positive (negative) shocks induces firms to turn liquid resources (illiquid capital) into illiquid capital (liquid resources), which is interpreted as an investment boom (bust). The private allocations are inefficient because firms do not internalize their decisions' to build and liquidate capital affect on equilibrium prices, resulting in a pecuniary externality. Dell'Ariccia and Marquez (2006) show that changes in the composition of borrowers can lead to pooling equilibria in which all borrowers receive credit during a credit boom. Our mechanism holds in separating equilibria when relative cost of under compared to over investment allocations rises as low types become more risky.

Kurlat and Scheuer (2018) are the first to introduce two-sided heterogeneity and uncertainty into a standard signaling environment. Sellers differ in quality and buyers have different technological abilities to screen sellers. Thus, sellers care with whom they interact. In their model, some high types do not incur signaling costs because they are matched to agents possessing the best screening technologies. Buyer and seller decisions are strategic compliments, which lead to multiplicity. There are equilibria in which some markets feature trade with multiple-types and full separation in others. The model produces a novel theory of price dispersion for similar types. Though we have two-sided heterogeneity, we only consider one-sided uncertainty. Guerrieri and Shimer (2018) and Williams (2017) study price formation in markets with multi-dimensional private information.

Our paper is the first to study the relationship between CDS markets and information asymmetries between borrowers and lenders. Duffee and Zhou (2001) argue that CDS allow banks to overcome the "lemons" problem in the loan sale market. Parlour and Winton (2013) show how CDS alter monitoring incentives with moral hazard between borrowers and lenders and asymmetric information between lenders seeking to transfer credit risk. Thompson (2010) shows that information asymmetries between counterparties in OTC insurance markets affects the quality of collateral used to back insurance contracts. Perfect information models that study the relationship between CDS and investment find that investment is low when investors are free to make speculative bets on borrower default (Fostel and Geanakoplos (2016) and Darst and Refayet (2018)). We show that the interaction between perfect risk sharing with CDS and asymmetric information generates inefficiently high investment levels.

Lastly, our paper relates to models of competitive pooling stemming from Dubey and Geanakoplos (2002) and Dubey, Geanakoplos, and Shubik (2005). Fostel and Geanakoplos (2008) study the effects of information asymmetries on issuance rationing. Nevov (2016) studies how asymmetric information affects asset prices and credit supply in pooling equilibria by assumption. Bengui and Phan (2018) study leveraged asset pricing bubbles. Related modeling approaches have been explored in Bisin and Gottardi (1999) and (2006) to study equilibrium efficiency, Guerrieri and Shimer (2018) to study price dispersion and efficiency with multidimensional private information, and by Azevedo and Gottlieb (2017) to study the effects of mandates on insurance markets.

2 Baseline Economy

2.1 Model

2.1.1 Time and Uncertainty

Consider a competitive credit market with two periods $t = \{0, 1\}$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature $s = \{U, D\}$ at time 1. The economy at time 1 is characterized by the realization of a binary productivity shock, common to all firms, denoted $A_s \in \{A_U, A_D\}$, $1 = A_U > A_D$. The economy has one durable consumption good.

2.1.2 Agents

The economy is populated by two sets of heterogeneous agents that can be thought of as firms and investors. Firms have no endowment, but possess a technology to turn consumption goods at time 0 into an uncertain amount of consumption goods at time 1 that depends on the realization of A_s. Investors hold all initial endowment that they can risklessly store and asset markets are incomplete for the time being. We describe the agents in more detail below.

Firms

There are two firm types, $i = \{g, b\}$, and a large number of each type.[10] All firms of the same type are identical, and we consider a representative firm of each type. Firms are risk-neutral expected profit maximizers that take the price of the durable consumption good as given.

The production technologies are strictly increasing concave functions in the input variable, $f^{i'}(I^i) > 0$, $f^{i''}(I^i) < 0$, of the following form: $f^i(I^i) = A_s (I^i)^{\alpha^i}$. It will become clear that the domain of the input variable is bounded between 0 and 1, $I^i \in (0, 1)$, and in general, $I^i << 1$. The key difference between firm types is that the expected value of their output differs, which determines the amount investors are willing to pay to hold claims on their output. The parameters α^i, $i = \{g, b\}$ are idiosyncratic production parameters. Because $I^i < 1$, the types are parameterized

[10] Two types is the common assumption in the literature. See Bester (1985), Milde and Riley (1987), Besanko and Thakor (1987), and Nevov (2016)).

by $0 < \alpha^g < \alpha^b < 1$. Lastly, each firm knows that $s = U$ with probability γ and $s = D$ with probability $(1-\gamma)$. The stochastic structure and productions functions meet assumptions A1-C1 in Milde and Riley (1998) where it is shown that under asymmetric information, the level of trade in firm inputs is an increasing function of firm type.

Investors

There is a unit mass of uniformly distributed, risk-neutral investors, $h \in H \sim U(0,1)$, who do not discount the future. Investors have linear utility for the single consumption good x_s at time 1. Each investor $h \in H = (0,1)$ is endowed with one unit of the consumption good, e^h, and assigns probability h to $s = U$ and $(1-h)$ to $s = D$. A higher h denotes more "optimism". The von-Neumann-Morgenstern expected utility function for investor h is given by:

$$U^h(x_U, x_D) = hx_U + (1-h)x_D. \tag{1}$$

A representative agent model with $h = \gamma$ is included as a special case that we will analyze in section 2.3.

2.1.3 Frictions and Debt Contracts

There are two main frictions in the economy. First, there is a payment enforcement friction where investors cannot coerce firms to pay their debts. All promises will therefore require collateral. Firms issue promises backed by the future value of their output as in Fostel and Geanakoplos (2016) and Vishwanathan and Rampini (2010), and can be interpreted as a cash-flow based borrowing constraint. These models implicitly assume there are no collateral cash flow problems. This assumption does not hold in debt financing models when effort affects the value of output as with moral hazard (Holstrom and Tirole (1997)) or when creditors can only claim a portion of an asset's value–the land but not the fruit (Kiyotaki and Moore (1997)). In our model, the expected value of the fruit determines borrowing and investment limits. Lian and Ma (2018) show that the most prevalent type of borrowing constraint in the cross-section of corporate indentures is cash-flow based. Lenders have the right to seize collateral up to the value of the promise, but no more. Promises are interpreted as debt instruments such as bonds or loans. Cao and Lagunoff (2018) justify focusing on collateralized debt contracts by showing that the optimal contract is non state-contingent when there is enough heterogeneity in the private information dimension. Each promise has a face value of 1 upon maturity.

Second, we assume an information asymmetry between firms and investors. Firms are privately informed about their production technology (α^g or α^b). As we will see, debt contracts with endogenous price-quantity pairs allow firms to choose their level of trade to signal their type. The information asymmetry coupled with firm differences generates an adverse selection problem where type b firms may want to trade contracts in the same markets as type g firms.

Firms raise capital by trading a quantity of bonds, q^i, with investors for capital at a price, p^i, both of which are endogenously determined for each firm type. A debt contract is characterized by the ordered pair $c^i \equiv (q^i, p^i)$.

2.1.4 Perfect Information benchmark

We first characterize debt contracts and the efficient level of trade in the perfect information benchmark. We assume that investors can distinguish between the two firm types. In equilibrium, debt contracts for each firm result in efficient levels of trade and investment levels. More precisely, debt contracts equate the marginal cost and marginal product of capital. We refer the interested reader to Darst and Refayet (2018) for a thorough description and analysis of equilibrium in this model. For brevity, we extract the key points as they relate to debt contract and the equilibrium level of trade for each firm type.

A key object in this class of economies are the debt delivery functions because they affect the borrowing constraint. The delivery function is defined as follows: $d_s^i\left(q^i\right) \equiv \min\left[1, \frac{A_s\left(I^i\right)^{\alpha^i}}{q^i}\right]$. Each promise delivers the minimum of its face value or the pro rata asset value of the firm in default. Debt prices reflect the fact that investors correctly anticipate the respective value of firm production based on knowing to whom the different αs belong.

Each investor, h, decides at time 0 what portion of his cash endowment he wishes to store for time 1 consumption and how much to trade in the credit market. Given market prices for bonds, p^i for $i \in \{g, b\}$, investors choose cash holdings, $\left\{x_0^h\right\}$, and bond quantities, $\left\{\left(q^i\right)^h\right\}$, that maximize expected utility given by (1) subject to the following budget set:

$$\begin{aligned} B^h\left(p^i\right) \quad = \quad & \left\{\left(x_0^h, \left(q^i\right)^h, x_s^h\right) \in R_+ \times R_+ \times R_+ : \right. \\ & x_0^h + \sum_i \left(p^i\left(q^i\right)\right)\left(q^i\right)^h = e^h, \\ & \left. x_s^h = x_0^h + \sum_i d_s^i\left(q^i\right)^h\right\},\ s = \{U, D\}, \ i \in \{g, b\}. \end{aligned}$$

At time 0, initial endowments can be used to buy bonds or stored for consumption. At time 1, investors consume from two potential sources in either state of nature: consumption based on risk-less cash holdings and consumption from total bond holdings.

Perfect competition in the credit market assures that the marginal investor breaks even in expectation. However, since there are two different debt contracts being issued, there will be two different break even conditions. The first condition is that a marginal investor must be indifferent to purchasing either of the two bonds. The second condition is that a marginal investor must be indifferent between buying one of the bonds and holding cash. Specifically,

$$\frac{h^1 \times d_U^i + \left(1-h^1\right) \times d_D^i}{p^i} = \frac{h^1 \times d_U^j + \left(1-h^1\right) \times d_D^j}{p^j} \tag{2}$$

$$\frac{h^2 \times d_U^j + \left(1-h^2\right) \times d_D^j}{p^j} = 1 \tag{3}$$

where the superscripts denote different marginal buyers. With heterogeneous investors, there will be two marginal buyers, $h^1 > h^2$. All investors up to and including the more optimistic marginal buyer, $h \geq h^1$, trade with type b firms. Investors less optimistic than marginal buyer h^1 upto and including marginal buyer h^2 trade with by type g firms. All other investors do not trade and remain in cash.

Turning to the firms' problem, we focus the analysis on economies for which debt is risky in equilibrium, which is at the heart of the pecuniary externality. Moreover, besides being empirically relevant, focusing on risky debt equilibria has two additional advantages: 1) one can think about derivative contracts on which risky debt is based (see Section 3), and 2) due to non-linearity, it allows us to sharpen our analytical results in the propositions that follow. We make the following parameter assumptions to ensure that candidate allocations will not be risk-free.[11]

Assumption 1 *We restrict the parameter set,* $\Gamma\left(\alpha^i, A_D, \gamma\right) \in R^3_{[0,1]\times[0,1]\times[0,1]}$, *such that,*

- $0 < A_D < \alpha^g < \alpha^b < 1$
- $\gamma > \bar{\gamma}$

where $\bar{\gamma} \equiv inf\left\{\gamma \in [0,1] \,|\, d_D^i(\cdot) < 1\right\}$.

Each firm maximizes expected profits by choosing its investment scale taking as given the market price of debt.

$$\begin{aligned} \max_{I^i} E_0[\Pi] \equiv \pi^i &= \left\{\gamma\left[A_U\left(I^i\right)^{\alpha^i} - q^i d_U^i\right] + (1-\gamma)\left[A_D\left(I^i\right)^{\alpha^i} - q^i d_D^i\right]\right\} \quad (4) \\ &\text{s.t.} \\ I^i &= p^i q^i, \end{aligned}$$

The first order conditions for a maximum, which determine the optimal level of trade and investment scale, equate the marginal product and cost of capital. Under Assumption 1, the first order conditions are simply:

[11] These are derived in Darst and Refayet (2018) Proposition 1.

$$\alpha^i \left(I^i\right)^{\alpha^i - 1} = \frac{1}{p^i},\ i \in \{g, b\}. \tag{5}$$

Under perfect information, debt contracts and investment are efficient. To close the model, debt prices are determined in equilibrium by market clearing. The supply of debt issued must equal the amount of funds investors allocate to the debt market,

$$\frac{1 - h^1}{p^b} = q^b \tag{6}$$

$$\frac{h^1 - h^2}{p^g} = q^g \tag{7}$$

The system of equations (2), (3), (5) for $i \in \{g, b\}$, (6), and (7) determine the six unknowns $\{p^i, q^i, h^1, h^2\}$ for $i \in \{g, b\}$. The definition of equilibrium in this economy is the following: Given market prices for bonds, $\{p^i\}$, firms choose the level of trade in the credit market, $\{q^i\}$, to maximize (4), investors choose consumption allocations, $\{x_s\}$, to maximize (1) subject to their budget sets, and the bond market clears. The following proposition characterizes some important properties of the perfect information equilibrium.

Proposition 1 *Perfect Information Equilibrium*

The equilibrium debt contracts in the credit economy with perfect information have the following properties:

1. $d_D\left(q^i\right) = \frac{A_D}{\alpha^i}$*; and*
2. $MPK^i = MC^i$ *given by (5)*
3. $p^{g^*} > p^{b^*},\ q^{g^*} > q^{b^*},\quad I^{g^*} > I^{b^*}$.

Condition 1 is derived by plugging equation (5) and $I^i = p^i q^i$ into $d_s^i\left(q^i\right)$. It shows that the two debt contracts have to be priced differently because $d_D\left(q^b\right) = \frac{A_D}{\alpha^b} \neq d_D\left(q^g\right) = \frac{A_D}{\alpha^g}$. The delivery functions conditional on bad states contain two components. The first is the aggregate component, A_D. The second is the idiosyncratic production component that governs production or cash flow, α^i. In equilibrium, good types pay lower credit spreads, $r^i = 1 - p^i$, and generally trade more at higher prices, $p^g > p^b$, consistent with observable differences between investment grade and high yield debt for example.

There are pecuniary externalities in this economy with risky debt due to investor heterogeneity. The only parameter that differentiates firms is α^i. Suppose low types' ability to generate cash flow and produce relative to high types falls so that the difference between α^is increases. Investors rationally expect lower delivery from low types. The market price of low type debt before the change in α^b is too high to clear the market and demand must fall. In equilibrium, some investors who preferred low-type debt before the change in α^b strictly prefer high-type after the change. The equilibrating forces lead to higher prices, more trade, investment, and profits for high type firms. The same logic holds in the reverse case to arrive at a negative pecuniary externality. As we will see, the pecuniary externalities have very different effects on debt contracts and the level of trade and investment in the asymmetric information economy.

Remark 1 *A note on the correlation between loan amount and default under Perfect Information*

Current research uses the positive correlation test to infer evidence of asymmetric information in different markets.[12] The logic is that a positive correlation between loan amount and default is consistent with asymmetric information, and rejecting the null suggests symmetric information. Proposition 1 shows that the different firm types borrow different amounts. Moreover, the economy considers risky debt for both firms that default in the same state. Therefore, there is no correlation between loan amount and default under perfect information, which is consistent with the positive correlation test. However, we show in the next section that, when defaults are clustered and are better viewed as an aggregate outcome, different firm types continue to borrow different amounts in the presence of asymmetric information. There is no correlation between loan amount and default under *both symmetric and asymmetric information*. In sum, the positive correlation test on the intensive margin cannot statistically reject asymmetric information.

2.2 Asymmetric Information

We now describe the model with asymmetric information and proceed to characterize equilibrium debt contracts and investment properties. We assume that firm types are now private information. Lenders only know the distribution of types and cannot distinguish between them. Consequently, lenders know there are high and low type firms.

Debt Pools: The strategy we adopt to analyze asymmetric information in a perfectly competitive credit market is based on the notion of competitive debt pools developed by Dubey and Geanakoplos (2002) and Dubey, Geanakoplos, and Shubik (2005).[13] The idea is that firms issue claims into pools that investors purchase. Banks or underwriters serve as basic intermediaries that verify the eligibility and credit limits

[12] Specific applications of the positive correlation test are used by Crawford et al. (2017) in corporate lending markets, Adams et al. (2009) in subprime auto markets, and Chiaporri and Selanié (2000) in insurance markets.

[13] Many recent papers have adopted similar strategies to analyze asymmetric information in a perfectly competitive setting (see Azevedo and Gottlieb (2017), Bengui and Phan (2017), Fostel and Geanakoplos (2008), Nenov (2017)).

set by each pool. They also function as servicers to collect and deliver payments to investors.[14]

Initially, there are an infinite number of potential pools into which firms can sell promises. Each pool is characterized by a competitive price that all agents take as given, a delivery function, and a limit on the amount of debt that can be raised. For each pool, there is an exogenous quantity limit, q, that no individual firm can exceed, and an endogenously determined price, p.

The delivery functions of the different pools are determined on a pro rata basis as with perfect information. The only difference now is that investors do not *ex ante* know whether the high or low firm type sells claims into a given pool. Pools contain promises with face value totaling q^i deliver according to

$$d_s^i \equiv \min \left[1, \frac{A_s \left(I^i\right)^{\alpha^i}}{q^i} \right]. \tag{8}$$

If low types sell claims into pools with high types, the proportional delivery rate will be less than the proportion of good types in the economy and there is adverse selection. We make two further simplifying assumptions to ensure that the quantity limits for each pool reflect one and only one average delivery.

Assumption 2 *Investors are rational and form correct expectations about each debt pools' delivery rates.*

Assumption 3 *Individual debt pools are exclusive. Firms can sell debt claims into only one pool.*

There will be no cross-subsidization of types between pools because of exclusivity. The exclusivity assumption opens the door for signaling to play an important role in our analysis.

The collection of pools characterized with associated quantity limits and market prices comprise a menu of price-quantity contracts:

$$\overrightarrow{p} = \{ (p(q), q) ; \; q \in (0,1), \; p \in \mathbb{R}_+ \} .$$

[14]We are abstracting away from intermediary functions of banks to focus on the anonymous interactions between firms and investors. An interesting question we leave for future research is when do we see credit pools materialize versus arm's length and relationship issuance in the bond and private loan market.

2.2.1 Agents' Problems

Firm Maximization Problem

The source of adverse selection is that low types would like to raise debt at the low credit spreads meant for high types while repaying less in expectation. To prevent adverse selection, incentive compatibility requires that all contracts be designed in a way that firms truthfully reveal their type. Formally, the incentive compatibility constraints are:

$$\left(I^b\right)^{\alpha^b} - q^b \geq \left(I^g\right)^{\alpha^b} - q^g \qquad ICC\,1 \tag{9}$$

$$\left(I^g\right)^{\alpha^g} - q^g \geq \left(I^b\right)^{\alpha^g} - q^b \qquad ICC\,2 \tag{10}$$

ICC 1 (ICC 2) states that firm b (g) can do no better by acting as firm g (b). Furthermore, feasibility and limited liability constraints imply that rational firms only participate in production by issuing debt if they earn non-negative expected profits: $\Pi^i \geq 0$. Firms solve the following problem:

$$\max_{q^i} \Pi^i = \left\{\gamma\left[A_U\left(I^i\right)^{\alpha^i} - q^i d_U^i\right] + (1-\gamma)\left[A_D\left(I^i\right)^{\alpha^i} - q^i d_D^i\right]\right\} \tag{11}$$

$$\text{s.t.} \begin{cases} \left(I^i\right)^{\alpha^i} - q^i \geq \left(I^i\right)^{\alpha^j} - q^i \\ I^i = p^i q^i \end{cases}$$

for $i = \{g, b\}$, $j \neq i$ and $d_s^i\left(q^i\right)$ corresponds to the state-specific delivery rule in (8). Lastly, given the price-quantity schedule, define the set of feasible contracts as those that satisfy the incentive compatibility constraints as
$\mathcal{C}\left(\overrightarrow{p}\right) \equiv \left\{\left(q^g, q^b\right) \in [0,1] \times [0,1] : \pi^b\left(q^b; q^g\left(\Gamma\right)\right) \geq \pi^b\left(q^g; q^g\left(\Gamma\right)\right)\right\}$.

Investor Maximization Problem

The investors' problem is the same as before except that now the price-quantity schedule is taken as given. Formally, given the price-quantity schedule, $\overrightarrow{p}$, investors choose cash holdings, $\left\{x_0^h\right\}$, and bond quantities, $\left\{\left(q^i\right)^h\right\}$, that maximize expected utility given by (1), subject to the budget set:

$$\begin{aligned} B^h\left(\overrightarrow{p}\right) = & \left\{\left(x_0^h, \left(q^i\right)^h, x_s^h\right) \in R_+^3 : \right. \\ & x_0^h + \sum_i \overrightarrow{p}\left(q^i\right)^h = e^h, \\ & \left. x_s^h = x_0^h + \sum_i d_s^i\left(q^i\right)^h\right\},\ s = \{U, D\},\ i = \{g, b\}. \end{aligned}$$

2.2.2 Equilibrium concept

Dubey and Geanakoplos (2002) (DG) show that a unique refined separating equilibrium always exists. The techniques used in DG are valid for existence of a separating equilibrium in this model. The key issue in defining equilibrium in perfectly competitive models of adverse selection is how to deal with the prices of all contracts, not just those contracts traded in equilibrium. Specifically, we must confront the problem that (8) is undefined for all markets with no trade, $q^i = 0$. We follow DG and assume that there are extremely productive agents who contribute a small and safe promise into each market. The promises are safe in the sense that the delivery on each promise strictly exceeds face value.[15] This will ensure that for inactive pools, agents' beliefs will not become unduly pessimistic. Without this refinement, agents' arbitrary beliefs about contracts that are not traded in equilibrium could result in many equilibria with non-traded contracts due to the expectation that only low types sell promises in those markets. A similar assumption to ours is common in signaling environments (see Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014), Kurlat and Scheuer (2018), Nevov (2016), Azevedo and Gottlieb (2017)).[16]

Definition 1: *A separating equilibrium is a collection of debt prices, firm investment decisions, investor consumption plans and debt holdings,* $\mathcal{E} = \left[p^i, I^i, (x_0, x_s, q^i)^{h \in H} \in R^5_+\right]$ *such that the following are satisfied:*

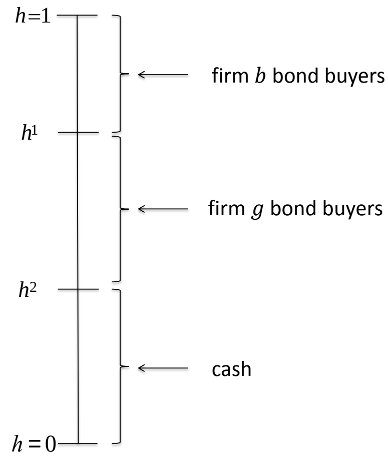
1. $\int_0^1 x_0^h dh + \sum\limits_i \int_0^1 p^i \left(q^i\right)^h dh = \int_0^1 e^h dh$
2. $\sum_i \int_0^1 \left(q^i\right)^h d_s^i dh + \sum_i \pi_s^i = \sum_i A_s \left(I^i\right)^{\alpha^i}, \; s = \{U, D\}$
3. $I^i = \int_0^1 p^i \left(q^i\right)^h dh$
4. $\pi^i \left(I^i\right) \geq \pi^i \left(\hat{I}^i\right), \text{ for } \forall I,$
5. $\left(x_0^h, \left(q^i\right)^h, x_s^h\right) \in B^h \left(\overrightarrow{p}\left(q^i\right)\right) \Longrightarrow U^h(x) \leq U^h\left(x^h\right), \; \forall h$
6. $\left(p^g, p^b\right) = \left(\overrightarrow{p}\left(q^g\right), \overrightarrow{p}\left(q^b\right)\right)$
7. $q^g \neq q^b$.[17]

[15] One may think of these agents as the government or publicly traded investment grade firms who typically do not issue leveraged loans because they issue commercial paper or publicly traded debt.

[16] Alternatively, Bisin and Gottardi (2006) show that Walrasian equilibrium always exists in adverse selection models when agents are only allowed to trade incentive-compatible contracts with linear prices over the restricted set of incentive-compatible trades. Our refinement effectively restricts beliefs rather than the set of contracts.

[17] The definition that all prices $\overrightarrow{p}$ for the markets q including ones not actively traded have to be determined. Let firm $i \neq j$ be the constrained firm that must signal its type. When firm i chooses to under-invest, it will be that $q^i < q^j$. In this case, $\forall q < q^i$ the price $\overrightarrow{p}(q)$ is determined such that firm i is indifferent between issuing q and q^i. For $q \geq q^i$ the price $\overrightarrow{p}(q)$ is determined such that

Figure 1: Baseline Economy



Conditions (1) and (2) are the respective goods market clearing conditions at time 0 and 1. At time 0 all initial endowment is either stored for consumption or used to purchase debt. Total firm output is consumed either by firm managers as profits or investors as debt deliveries. The debt market clearing is given by (3). Condition (4) implies that firms choose investment to maximize profits and (5) states that portfolio choices are optimal in the budget set. Condition (6) states that equilibrium debt market prices are in the price-quantity menu. Lastly, condition (7) states that the two firms must always issue different debt quantities in a separating equilibrium.

Equilibrium is characterized by two *marginal buyers*, $h^1 > h^2$. Every agent $h > h^1$ purchases debt issued by type b firms, every agent $h^2 < h < h^1$ purchases debt issued by type g firms, and every agent $h < h^2$ remains in cash. This regime is shown in Figure 1.

2.2.3 Numerical Examples

We first highlight our main results and intuition through numerical examples. We consider different values of the productivity parameters to compare perfect and asymmetric information economies. In Section 3 we analyze in detail how equilibrium is characterized and provide formal proofs where possible.

There are four parameters in the model: productivity parameters, α^i, the likelihood of a good state at time 1, γ, and the aggregate shock at time 1, A_D. The

firm j is indifferent between issuing q and q^j . Alternatively, when firm i chooses to over-invest, it will be the case that $q^i > q^j$. In this case, $\forall q > q^i$ the price $\overrightarrow{p}(q)$ is determined such that firm i is indifferent between issuing q and q^i. For $q \leq q^i$ the price $\overrightarrow{p}(q)$ is determined such that firm j is indifferent between issuing q and q^j. As shown by Dubey and Geanakoplos (2002) this separating equilibrium is robust to refinements.

Table 1: Representative Investor Equilibrium: $\alpha^g = 0.5$, $\alpha^b = 0.7$, $\gamma = 0.50$, and $A_D = 0.30$

	Asymmetric Info.		*Perfect Info.*	
	$i = g$	$i = b$	$i = g$	$i = b$
Price: p^i	.7546	.7143	.8000	.7143
Quantity: q^i	.2619	.1389	.2000	.1389
Investment: I^i	.1977	.0992	.1600	.0992
Marginal Buyers:γ	.5000	.5000	.5000	.5000
Distortion.Wedge: λ^i	.8486	1	1	1
Profit: π^i	.0913	.0298	.1000	.0298

parameters (γ, A_D) are not integral for the main results outside of the restrictions we impose through Assumption 1 to focus the analysis on risky debt issuance. For now, let $A_D = 0.30$, and $\gamma = 0.50$. The key parameters for our analysis are the αs. Begin by setting $\alpha^g = 0.5$ and $\alpha^b = 0.7$. [18]

We begin with the special case, representative investor, version of the model. In particular, fix investor beliefs at $Pr(s = U) = \gamma$. This special case maps to the literature's traditional assumption that a representative investor prices all debt in equilibrium. The equilibrium values are in Table 1. Notice two things: 1) high types trade and invest more under asymmetric than perfect information, and investment is distorted from its efficient level. The investment distortion captures the effect of asymmetric information. Formally, define $\lambda^i \equiv \frac{MPK^i}{MC^i} \neq 1$ as a measure of the investment distortion. $\lambda^i > 1$ ($\lambda^i < 1$) implies too little trade or under (over) investment at market prices because the efficient investment level under perfect information is higher (lower) than the asymmetric information equilibrium outcome. 2) low types receive the exact same equilibrium contracts regardless of the information asymmetry. Hence, there is no pecuniary externality in the special case.

Example 2 is the general case with heterogeneous investors using the same parameters are example 1. Table 2 shows equilibrium values for both the perfect and asymmetric information economies. The key results are the following: 1) high types trade and invest *less* than the perfect information benchmark, $I^{*g}_{AI} < I^{*g}_{PI}$, *i.e.* there is an under investment distortion; 2) the low type investment is efficient, $MPK^b = MC^b$; and 3) the financial conditions for low types improve under asymmetric information, $p^{*b}_{AI} > p^{*b}_{PI}$, and $\Pi^b_{AI} > \Pi^b_{PI}$, a positive pecuniary externality. The positive externality arises because high types' trade less with investors than with perfect information. This pushes up contract prices for all firms types because market clearing is achieved by an investor who is more willing to hold risky debt.

[18]The reason is that A_D and γ have proportional second order effects on prices. The productivity parameters have first order effects on both prices and quantities.

Table 2: Baseline Economy: $\alpha^g = 0.5$, $\alpha^b = 0.7$, $\gamma = 0.50$, and $A_D = 0.30$

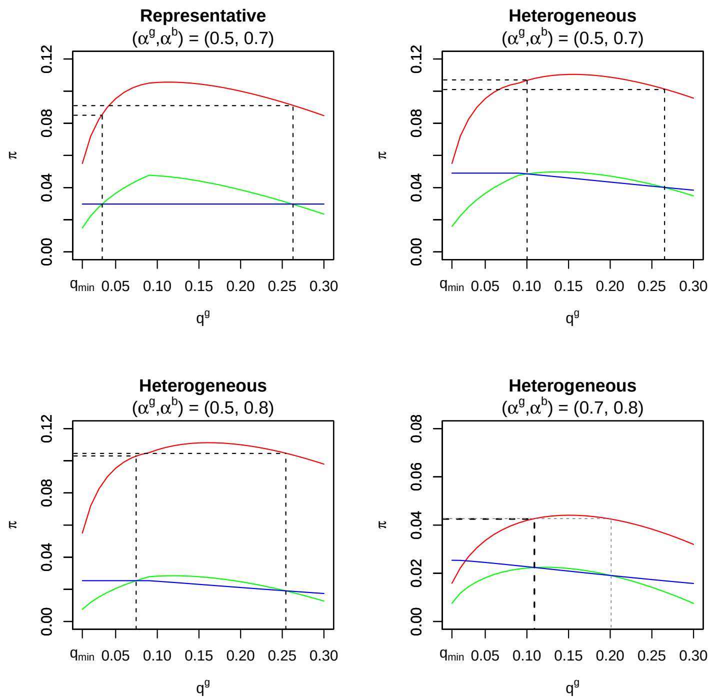
	Asymmetric Info.		*Perfect Info.*	
	$i = g$	$i = b$	$i = g$	$i = b$
Price: p^i	.9821	.8807	.8597	.8335
Quantity: q^i	.1001	.2264	.2149	.1991
Investment: I^i	.0983	.1994	.1848	.1660
Marginal Buyers: h	.7023	.8006	.6493	.8340
Distortion.Wedge: λ^i	1.5662	1	1	1
Profit: Π^i	.1067	.0485	.1075	.0427

Figure 2 graphically depicts equilibrium debt contracts. The first graph on the top left is the representative investor equilibrium. High type profits are the red line, low type truthful separating profits are the blue line, and low type mis-representing profits are in green. The horizontal axis are different debt markets in q^g-space. The low type separating and mis-representing profit functions intersect at two different markets $q^g = \left\{\underline{q}^g, \overline{q}^g\right\}$. These markets represent the infimum and supremum quantity limits in the set of incentive compatible contracts defined by $\mathcal{C}\left(\overrightarrow{p}\right)$–the roots of (9). All markets defined by quantities $\underline{q}^g < q < \overline{q}^g$ are not incentive compatible because low types prefer to pool with high types in those markets. The vertical lines show the high type profit levels for the two incentive compatible markets. The incentive efficient equilibrium as defined by Bisin and Gottardi (2006) is determined by $\pi^g\left(\overline{q}^g\right) > \pi^g\left(\underline{q}^g\right)$.

The top right figure is the heterogeneous agent equilibrium in example 2 using the same parameter values as example 1. The image shows that incentive efficient allocation changes to high types engaging in less trade and investment : $\pi^g\left(\underline{q}^g\right) > \pi^g\left(\overline{q}^g\right)$. The reason is that, due to the pecuniary externality, low type contracts when truthfully revealing are a function of high type actions shown by the downward sloping blue line.

Example 3 shows the effect of increasing productivity dispersion across types. Low types are less productive than in example 2, ceteris paribus, $\alpha^b = 0.8 > 0.7$. Table 3 shows the new equilibrium values. First, the difference between the firm types' efficient investment levels widens. Second, the incentive efficient allocation again moves high types to engage in too much trade and investment–$\lambda^g < 1$. The marginal product is less than the marginal cost at market prices. Finally, the incentive efficient allocation for low types involves less trade at lower prices when high types over invest because of a negative pecuniary externality. The incentive efficient allocation is shown in the bottom left panel of figure 2 where $\pi^g\left(\underline{q}^g\right) < \pi^g\left(\overline{q}^g\right)$. Compared to example 2, both of the incentive compatible contracts have lower quantities. This suggests that the investment distortion due to adverse selection is more punitive for under than over investment allocations as firm differences grow.

Figure 2: Investment Distortions



The final example, example 4, shows that the investment distortion is due to the *relative heterogeneity* between types. The parameters are the same as example 3 except α^g is higher to make the firms more similar, $\alpha^g = 0.7 > 0.5$. The results in table 4 show that the investment distortion reverts to under investment. Lastly, note that the measure of the investment distortion, λ^g, declines in example 4 relative to example 2. The interpretation is that adverse selection distorts investment less when cash flows and productivities are similar. Graphically, the bottom right panel of figure 2 shows the equilibrium allocation from example 4 where high types restrict rather than expand their level of trade and investment.

2.3 Representative Investor and Partial Equilibrium

Before moving on to the main comparative static results in the general model, we analyze the central assumption driving the results from examples 1 and 2.[19] Specifically, we show that equilibrium is *always* characterized by high types engaging in

[19]There are numerous assumptions in the literature that vary from model to model making it difficult to pin-point exactly which are most important for different results. The most common points of departure between models are what form the information asymmetry takes and how the production process is modelled. Specifically, the information asymmetry may be about expected returns (first moment) or risk (second moment). This difference has implications for whether credit

Table 3: Increased Heterogeneity: $\alpha^g = 0.5$, $\alpha^b = 0.8$, $\gamma = 0.50$, and $A_D = 0.30$

	Asymmetric Info.		*Perfect Info.*	
	$i = g$	$i = b$	$i = g$	$i = b$
Price: p^i	.8457	.8262	.8693	.8408
Quantity: q^i	.2540	.1527	.2173	.1638
Investment: I^i	.2148	.1261	.1889	.1377
Marginal Buyers: h	.6591	.8739	.6733	.8623
Distortion.Wedge: λ^i	.9123	1	1	1
Profit: π^i	.1047	.0191	.1087	.0205

Table 4: Relative Heterogeneity: $\alpha^g = 0.7$, $\alpha^b = 0.8$, $\gamma = 0.50$, and $A_D = 0.30$

	Asymmetric Info.		*Perfect Info.*	
	$i = g$	$i = b$	$i = g$	$i = b$
Price: p^i	.8833	.8599	.8330	.8269
Quantity: q^i	.0966	.1792	.1988	.1532
Investment: I^i	.1093	.1541	.1656	.1267
Marginal Buyers: h	.7494	.8459	.7077	.8733
Distortion.Wedge: λ^i	1.2031	1	1	1
Profit: π^i	.0427	.0224	.0426	.0191

too much trade and investment when facing a representative investor or equivalent partial equilibrium economy. The representative investor assumption renders bad-type debt prices as purely a function of parameters, which can be defined as a partial equilibrium absent market clearing, completely shutting down the pecuniary externality.

Consider the asymmetric information economy described in Section 2, but assume a representative investor who also knows that $Pr\,(s = U) = \gamma$ as do the firms. A common set of beliefs and the zero profit conditions imply that debt is priced purely as a function of parameters. To see this, both firm types' debt is priced by equating expected returns to holding riskless cash: $\gamma + (1 - \gamma)\, d_D\,(q^i) = p^i$. Investor expected returns to all assets must therefore be equal to 1:

$$\frac{\gamma + (1 - \gamma)\, d_D\,(q^i)}{p^i} \equiv \chi^i \left(q^i\right) = 1. \tag{12}$$

In equilibrium, low types receive a debt contract that equates the marginal prod-

is rationed or not ala Stiglitz and Weiss. Moreover, production is typically either decreasing returns to scale or fixed, and in both cases, production returns all or nothing (Hellwig and Gale being the notable exception to the latter). Whether or not equilibrium is characterized by pooling versus separating depends on whether debt contracts permit screening devices in addition to interest rates that clear the market (see Bester (1985) and Milde and Riley (1987)).

uct with the marginal cost, so by Proposition 1, $d_D\left(q^b\right) = \frac{A_D}{\alpha^b}$. Moreover, low type profits also depend only on parameters. As such, one can write the ICC constraint that high types must satisfy, (9), as

$$\kappa = \gamma\left(\left(I^g\right)^{\alpha^b} - q^g\right) \tag{13}$$

where $\kappa \equiv \Pi^b_{PI}$ is the low type profit level under perfect information. What this shows is that the two roots that solve (13) are associated with the same profit level for good firm types, Π^g_{AI}, which the flat blue line with slope equal to 0 depicted in figure 2. We first formalize the notion of partial equilibrium in a similar spirit to Guerrieri and Shimer (2014). The representative investor economy is equivalent to a "partial equilibrium" problem where the financial conditions low types face are taken as given.

Definition 2: *A separating partial equilibrium with expected return to risky debt, $\chi^i\left(q^i\right) = 1$, is a collection of debt prices, firm investment decisions, investor consumption plans and debt holdings, $\mathcal{E} = \left[p^i, I^i, \left(x_0, x_s, q^i\right) \in R^5_+\right]$, such that the following are satisfied*:

(i) conditions (2)-(7) hold from the separating equilibrium, and

(ii) investor beliefs are exogenously fixed at $h = \gamma$.

A partial equilibrium can be found through the following procedure: 1) choose the parameter set $\Gamma(\cdot)$; 2) solve for low-type debt prices and investment levels jointly from (5) and (8); 3) the associated solution will determine low-type profits, κ, in (13); 4) for the derived κ, high-type endogenous variables, $\left(I^g, p^g, q^g\right)$ are determined by simultaneously solving (8), (12), and (13).

Denote these candidate solutions to (13), relative to the perfect information equilibrium, by $c^g\left(\underline{q}^g\right) < c^g\left(q^{g^*}_{PI}\right) < c^g\left(\overline{q}^g\right)$. Equilibrium will be determined by profit maximization of the good firm type via

$$c^g\left(\overrightarrow{p}\left(q^g\right), q^g\right) \equiv \underset{q^g \in \left\{\underline{q}^g, \overline{q}^g\right\}}{\arg\max}\, \pi^g\left(c^g\left(\cdot\right)\right).$$

Proposition 2 *A separating partial equilibrium exists. High types engage more trade and investment in equilibrium relative to the perfect information benchmark:* $c^g\left(\cdot, \overline{q}^g\right) > c^g\left(\cdot, q^{q^*}_{PI}\right)$.

Existence follows from standard arguments from the discussion above. The complete proof is in the appendix.

The over-investment result is a consequence of the single crossing property in signalling games in which the marginal gain for each unit of trade is increasing the agent's type. In our model, the high type is more productive on the margin and trades with investors at a lower cost for each unit of input. Therefore, high types are more willing to trade a unit claim on its future cash flows in exchange for capital at any given market price. The single crossing property can be seen in figure 2 by the growing difference between high and low type mimicking profits (the difference between the red and green curves) as q^g rises. Formally, $\frac{\partial^2 \Pi(q;\alpha)}{\partial\alpha\partial q}<0$ where the sign is flipped because the high type is the low α. Thus, if low type profits are fixed and independent of high type decisions while the difference between mimicking profits grows in q^g, it is always the case that $\pi^g\left(\overline{q}^g;\cdot\right) > \pi^g\left(\underline{q}^g;\cdot\right)$.

It is also true that the separating equilibrium is unique. In particular Milde and Riley (1987) and Riley (1985) show that the pareto-efficient separating set of no loss contracts is unique among the set of all no-loss contracts as long as the marginal cost of signaling is non-decreasing in α_i. Intuitively, the high type always chooses to separate as long as doing so is less costly on the margin.[20]

3 General Equilibrium Analysis and Analytical Results

In this section we fully characterize the general version of the baseline model with asymmetric information. Subsection 3.1 is the baseline economy and highlights how the presence of pecuniary externalities interacts with information asymmetries to generate different equilibrium levels of trade and investment. Section 3.2 introduces financial contracts that allow investors to more efficiently trade risk called credit default swaps. It is shown that investor trade in the CDS market mitigates the pecuniary externality in the credit market leading to allocations characterized by too much trade and investment relative to both perfect and asymmetric information economies without CDS.

3.1 Baseline Economy

The difference between general and partial equilibrium in the model is that in general equilibrium markets clear (condition 1) and marginal investors are endogenously determined, $(h_{1,2}(\cdot) \neq \gamma)$. Market clearing requires that the supply of investor capital funding the respective firms equal each firms' investment demand:

$$1 - h_1 = p^b q^b \tag{14}$$
$$h_1 - h_2 = p^g q^g \tag{15}$$

[20]Additional uniqueness arguments are found in Dubey and Geanakoplos (2002) and Bisin and Gottardi (2006).

The respective marginal buyers, h_1 and h_2, determine equilibrium debt pricing according to their indifference equations. Specifically, the more optimistic marginal buyer, h_1, must be indifferent between buying debt issued by either firm type. The less optimistic marginal buyer will be indifferent between purchasing high-type debt and cash.

$$\frac{h_1 + (1 - h_1)\, d_D\left(q^b\right)}{p^b} = \frac{h_1 + (1 - h_1)\, d_D\left(q^g\right)}{p^g} \tag{16}$$

$$\frac{h_2 + (1 - h_2)\, d_D\left(q^g\right)}{p^g} = 1. \tag{17}$$

Equations (14)-(16) close the model from Section 2. Equation (17) is the technically the same.

Proposition 3 *A separating general equilibrium defined by* $\mathcal{E} = \left\{ \left(\overrightarrow{p}\left(q^i\right), q^i, I^i\right)_{i=\{g,b\}} \left(x_0, x_s\right)_{h \in H} \right\}$ *exists and is unique. It is characterized by the solution to (5) for* $i = b$*, (9), (14), (15), (16), (17) and marginal buyers* $h_1 > h_2$*.*

Proof: See appendix A

Equations (14)-(17) show that market clearing determines which marginal buyers pin down risky debt prices. Multiple risky-debt types and investors with different marginal utilities are the sources of the pecuniary externalities in the model. The marginal buyer h_2 and market clearing determines prices for high types. However, the equilibrium price for high-types affects the willing for marginal buyer h_1 to trade with low-types and the debt contracts it obtains in equilibrium. The information asymmetry in the credit market requires that high type choices be incentive compatible with low type choices. Thus, high types' level of trade affects the allocations available to low types, which in turn affects the incentive constraints. If either firm-type's debt were risk free, then all investors would price all debt equal to 1. One firm-type's level of trade in the credit market would not affect relative prices and the risk-free return. Hence, equilibrium default is crucial. Moreover, when debt prices are determined by a representative marginal utility, each firm type's equilibrium debt prices are only function of that firm's level of trade in the credit market. Both assets' expected returns remain equal to the representative investor's outside option to remain in cash.

What are the equilibrium characteristics of the allocations in the model? To answer this question, we first use the following Lemma.

Lemma 1 ***Under investment*** *High types trade and invest less in aggregate than low types in any under investment equilibrium allocation;*
$\hat{q}^g < q^{*b}, q^g \in \mathcal{C}\left\{q : \pi\left(c^g\left(\overrightarrow{p}\left(\underline{q}\right), \underline{q}\right)\right) > \pi\left(c^g\left(\overrightarrow{p}\left(\overline{q}\right), \overline{q}\right)\right)\right\}.$

Proof: See appendix A

Lemma 1 shows that in any incentive efficient allocation in which high types trade and invest less than the perfect information benchmark, they also trade and invest less than low types. At first glance it is surprising that high types trade in lower quantity debt markets than low types given that Proposition 2 shows that the model's single-crossing property implies the opposite. After all, single-crossing is governed by technological constraints between agents' (firms') types and their marginal utilities (profits) for trade (investment). In particular, single-crossing is given by $-\frac{\partial^2 \Pi}{\partial \alpha \partial I} < 0$, $\forall I \in [0, e^{-1}]$, which implies that trade and investment is *increasing* in firm type.[21] Importantly, the maximum equilibrium investment level the high types choose is determined by plugging the risk-free rate, $p^g = 1$, into the first order condition, yielding $I^g = (\alpha^g)^{\frac{1}{1-\alpha^g}}$. Thus, the set of admissible trade and corresponding investment levels is capped by $\lim_{\alpha \to 1} (\alpha^g)^{\frac{1}{1-\alpha^g}} = e^{-1}$, and the level of trade should be increasing in agents' type for all admissible quantities. In sum, the Spence-Mirlees single crossing condition is not sufficient to characterize equilibrium allocations when pecuniary externalies are present.

In Walrasian equilibria, the decision high types make affects the contract space available to low types through its resource constraint $I^g = p^b(p^g)\, q^b$, which then feeds back into the high types incentive constraint. In particular, from the ICC constraint, (9): $\Pi^b\left(q^b; q^g, \Gamma\right) \geq \gamma\left((I^g)^{\alpha^b} - q^g\right)$, the left hand side is the low types' reveal profit level as a function of its own contractual terms, high type decision (q^g), and parameters. The right hand side is the low types' mimicking profit function from choosing to issue claims with high types. The left hand side is decreasing in high type quantities when the pecuniary externality is present because investors require more (less) compensation for default losses due to high (low) debt levels. The right hand side is a strictly concave function of high type quantities. A distortion on the lower end–under investing–raises the left and lowers the right hand side of (9) while over investing lowers both sides. Thus, distortions on the lower end relax the constraint from both sides while a distortion on the upper end only relaxes the constraint from the single-crossing property that high types are more willing to trade than low types (the right hand side falls more than the left as q^g increases.)

The equilibrium allocations and the direction of the distortion when pecuniary externalities are present depend on the efficient trade and investment levels across firm types, not just the willingness for high types to raise debt and invest implied by the single crossing condition. The efficient investment levels from the perfect information allocations depend on the $\left(\alpha^g, \alpha^b\right)$-pair, which are the upper and lower bounds of the productivity distribution. Take the case of similar α^is. The level of investment that maximizes low type profits is similar to the investment level on

[21]The sign of the single-crossing condition is the opposite from Proposition 4 in Mide and Riley (1988), $\frac{\partial}{\partial \theta}\left(-\frac{\frac{\partial A}{\partial L}}{\frac{\partial A}{\partial R}}\right) > 0$, because types are increasing in θ in their model.

the right hand side of (9), $I^g \approx I^{*b}$. Therefore, a relatively small distortion on the low end raises the low types truthful profit function and pushes $I^b > I^g$ satisfying incentive compatibility for high types. By Lemma 1, only when high types operate at a smaller scale can under investing be incentive compatible; otherwise, low types can expand investment up to the under investment quantity high types choose and receive financing at lower costs. Examples 2 and 4 confirm the characterization of Lemma 1 with similar αs.

Now consider a parameterization with more heterogeneity between types, $\alpha_g \ll \alpha_b$. The difference between both efficient investment levels and debt prices is large, $p^{*^g} \gg p^{*^b}$ and $I^{*^g} \gg I^{*^b}$. From Lemma 1, a distortion on the lower end needs to be significantly larger than when firms are similar. This implies that the cost of under investing is increases in heterogeneity as the level of trade needed to satisfy incentive constraints is much lower, $\underline{q}^g < q^{*b} \ll q^{*g}$. In addition, there is a strong price incentive for low types to pool with high types, which expands the range of contracts with distortions on the lower end, $q^g \in \left[\underline{q}^g, q^{*g}\right]$, that are not incentive compatible. By contrast, marginally over investing *always lowers* equilibrium debt prices, reducing the incentive to pool through both the efficient scale and price effects through standard single crossing arguments. In sum, for high types, the relative cost of a distortion on the high end falls as agent heterogeneity rises. Example 3 confirms that allocations involve high types engaging in too much trade when heterogeneity increases. We can prove the following:

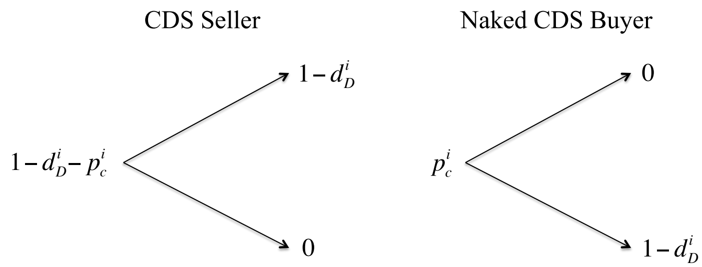
Proposition 4 *Let $\Delta_\alpha \equiv \alpha^b - \alpha^g$ measure the heterogeneity between the upper and lower bound of firm types in an economy's productivity distribution, where $\Delta_\alpha \in [0, \overline{\Delta}_\alpha]$ with $\overline{\Delta}_\alpha \equiv \max\left(\alpha^b - \alpha^g\right)$ such that (9) binds, $\forall \left(\alpha^g, \alpha^b\right)$. Let an under investment equilibrium allocation $\mathcal{E}^U$ be given by Proposition 3 such that $q^g \in \mathcal{C}\left\{q : \pi\left(c^g\left(\overrightarrow{p}\left(\underline{q}\right), \underline{q}\right)\right) > \pi\left(c^g\left(\overrightarrow{p}\left(\overline{q}\right), \overline{q}\right)\right)\right\}$ and an over investment equilibrium allocation $\mathcal{E}^O$ be given by Proposition 3 such that $q^g \in \mathcal{C}\left\{q : \pi\left(c^g\left(\overrightarrow{p}\left(\underline{q}\right), \underline{q}\right)\right) < \pi\left(c^g\left(\overrightarrow{p}\left(\overline{q}\right), \overline{q}\right)\right)\right\}$. The following properties hold:*

1. *Choose any Δ_α for which the equilibrium allocation is given by $\mathcal{E}^U$. Equilibrium will remain $\mathcal{E}^U$ as $\Delta_\alpha \to 0$;*
2. *Choose any Δ_α for which the equilibrium allocation is given by $\mathcal{E}^O$. Equilibrium will remain $\mathcal{E}^O$ as $\Delta_\alpha \to \overline{\Delta}_\alpha$*

Proof: See appendix A

Proposition 4 says that allocations remaine characterized by distortions on the low (high) end as agents become more (less) similar. We cannot provide a formal characterization of precisely when distortion flips because of the non-linearity in the model. In general, it will be a function of both the difference and level of the α-pair. However, our examples and simulations confirm that the economy switches from under to over investment as heterogeneity or cash-flows differences increase and vice versa.

Figure 3: CDS Payouts



3.2 CDS Economy

In this section, we show that financial innovation and market completeness interacts with the adverse selection problem leading to allocations that generally can be characterized by high types engaging in too much trade and investment. In particular, we show that credit default swaps (CDS) facilitate efficient risk sharing, which mitigates the pecuniary externality in the credit market and can move the economy from too little to too much trade and investment despite no change in firm fundamentals or investor beliefs.

A CDS is a derivative contract that compensates the buyer for losses given default of an underlying debt security. In particular, CDS contracts compensate buyers the difference between a debt contract's face and default values, $1 - d_D^i$. A CDS will pay 0 at $s = U$ when firms honor debt contracts. Let $\left(q_c^i\right)^h$ be the number of CDS contracts sold by each CDS seller, and let p_c^i be the price of the CDS contract. Figure 3 shows the CDS contract payout for CDS sellers and buyers.

3.2.1 Investor Maximization Problem

The only change in this economy relative to the baseline is the set of assets available for investors to trade. Therefore, the firms' maximization problem remains unchanged. CDS contracts typically require collateral in the form of cash. We assume the CDS seller must post enough cash collateral to cover all CDS payments in the worst case scenario to rule out any counter-party risk. The investors' own cash collateral, *i.e.* skin in the game, required to sell CDS is the loss given default on debt less the CDS premium. The total number of CDS contracts an individual investor can sell is:

$$\left(q_c^i\right)^h = \frac{1}{(1 - d_D^i) - p_c^i}. \tag{18}$$

The economics of CDS are the following: CDS allow investors to fully transfer consumption into the states they find most likely by effectively purchasing Arrow securities (Fostel and Geanakoplos (2012)). In particular, CDS sellers use their endowment to collateralize CDS contracts that pay only when $s = U$. Conversely, CDS purchasers use their endowment to buy an asset that delivers only when $s = D$. The return to selling CDS is the expected repayment in the upstate divided by the skin in the game required to cover losses in the down state: $\frac{h\left(1-d_D^i\right)}{\left(1-p_c^i-d_D^i\right)}$. Similarly, the return on purchasing a CDS is the payout conditional on default, divided by the price: $\frac{(1-h)\left(1-d_D^i\right)}{p_c^i}$. Lastly, note that holding the debt and corresponding CDS is equivalent to owning a risk-free asset that delivers 1 in both states.

Given debt and CDS prices (p^i, p_c^i), each investor chooses cash, debt and CDS holdings $\left\{x_0^h, (q^i)^h, (q_c^i)^h\right\}$ to maximize utility given by (1) subject to the following budget set:

$$\begin{aligned} B^h\left(\overrightarrow{p}\left(q^i\right), p_c^i\right) \;=\; & \left\{\left(x_0^h, \left(q^i\right)^h, \left(q_c^i\right)^h, x_s^h\right) \in R_+^4\right\}: \\ & x_0^h + \sum_i \overrightarrow{p}\left(q^i\right)\left(q^i\right)^h + \sum_i p_c^i \left({q_c}^i\right)^h = e^h, \\ & x_s^h = x_0^h - \sum_i p_c^i \left(q_c^i\right)^h + \sum_i \left(q^i\right)^h d_s^i + \sum_i \left(q_c^i\right)^h \left(1 - d_s^i\right)\Big\} \\ & s = \{U, D\}. \end{aligned}$$

The budget sets place no quantity restriction on the amount of CDS that can be bought or sold. In other words, both covered and naked CDS are permitted. All endowment is used to construct portfolios of cash, debt, and CDS holdings from which all investor consumption derives. We make use of the following two lemmas to characterize equilibrium.

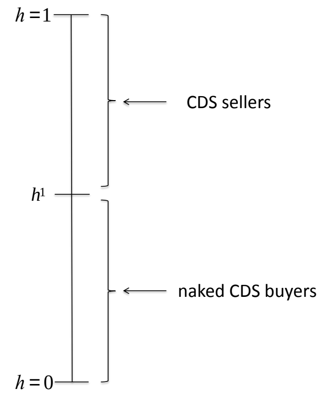
Lemma 2 *If $0 < d_D^i < 1$, then no debt for which CDS are sold will be purchased without CDS.*

Proof: See Appendix A.

Lemma 2 says that any investor optimistic enough to hold debt without a CDS will be better off selling CDS. The intuition is that optimists prefer leveraging their cash endowment via selling CDS rather than buying bonds. The bounds on debt delivery follow from the fact that if the debt delivery is zero, then CDS and debt contracts have identical payoffs and CDS are redundant. In addition, if debt delivery is 1, then debt contracts are risk free and CDS will not trade.

Lemma 3 *No investor holds a cash equivalent asset in equilibrium.*

Figure 4: CDS Economy



Proof: See Appendix A.

Lemma 3 is the compliment to lemma 2. It says that any investor pessimistic enough to remain in cash or hold debt with a CDS will be better off buying CDS without holding debt.

Putting lemmas 2 and 3 together, equilibrium is characterized by a single marginal investor indifferent to buying and selling CDS. Figure 4 shows the marginal buyer characterization of equilibrium in the CDS economy. The production economy with derivatives faces a well-known existence problem. All optimists sell CDS and all pessimists buy CDS and neither hold debt. CDS are derivatives whose value is based on an underlying debt instrument, and if that instrument does not trade, then CDS cannot exist. Hence, there is no fixed point in the economy (See Fostel and Geanakoplos (2016) and Darst and Refayet (2018)).

We introduce a refinement in the economy that does not perturb the baseline and nevertheless allows us to define and characterize equilibrium with derivatives. Following Darst and Refayet (2018), we assume there is an institutional investor, M, in the economy that will always purchase debt. In addition, the investor is subject to risk-based capital requirements that make holding a risky portfolio prohibitively expensive.[22] The baseline economy allocations are not affected by including investor M because she is not strictly a risky debt investor.

Let investor M's utility function be given by $U^M\left(x_s, c_0^i\right)\epsilon$, $\epsilon > 0$. The object x_s is the consumption plan for time 1 and c_0^i is a bundle consisting of debt with CDS contract, $c_0^i = q^i + q_c^i$.[23] Finally, $U^{M\prime}\left(c_i\right) > 0$, meaning that the investor always

[22]One may think of insurance companies that are subject to capital requirements set by National Association of Insurance Commissioners (NAIC).

[23]The micro-foundations of the investor's preferences are not explicitly modeled, though it is

prefers to invest in debt when possible. We can now define the *institutional investor's* budget set.

$$\begin{aligned} B^M\left(\overrightarrow{p}^i, p_c^i\right) &= \Big\{\left(x_0^M, q^{i,M}, q_c^{i,M}, x_s^M\right) \in R_+^4 : \\ & x_0^M + \sum_i \left(\overrightarrow{p}^i q^{i,M} + p_c^i q_c^{i,M}\right) = e^M, \\ & x_s^M = x_0^M + \sum_i c_0^{i,M}\Big\}. \end{aligned}$$

The institutional investor uses her endowment to purchase bonds and CDS and consumes from the proceeds. The investor takes the price-quantity schedule in the debt market and CDS prices as given.

3.2.2 Equilibrium

Definition 3: *A separating equilibrium in the CDS Economy is a collection of debt prices, CDS prices, firm investment decisions, and investor consumption decisions,* $\mathcal{E}_c = \left[p^i, p_c^i, I^i, (x_0, q^i, q_c^i, x_s)^{h\in(H,M)} \in R_+^7\right]$ *such that the following are satisfied*:

1. $\int_0^1 x_0^h dh + x_0^M + \sum_i p^i \left(q^i\right)^M + \sum_i p_c^i \left(q_c^i\right)^M = \int_0^1 e^h dh + e^M$
2. $\sum_i \left(q^i\right)^M + \sum_i \pi_s^i = \sum_i A_s \left(I^i\right)^{\alpha^i}, \; s = \{U, D\}$
3. $I^i = p^i \left(q^i\right)^M$
4. $\pi_i\left(I_i\right) \geq \pi_i(\hat{I}_i), \; \forall \hat{I}_i > 0$
5. $\left(x_0^h, \left(q^i\right)^h, \left(q_c^i\right)^h, x_s^h\right) \in B^h\left(\overrightarrow{p}\left(q^i\right), p_c^i\right) \Rightarrow U^h(x) \leq U^h\left(x^h\right), \; \forall h, M$
6. $p^i = \overrightarrow{p}\left(q^i\right)$
7. $q^g \neq q^b$
8. $\int_0^1 \left(q_c^i\right)^h dh + \left(q^i\right)^M = 0$

Conditions 1-7 correspond to the equilibrium conditions described for the *Baseline Economy* and includes the institutional investor. Condition 8 states that the CDS market must be in zero net supply including the institutional investor's CDS positions.

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easy to do so. For example, one could assume the investor manages a pension fund or an insurance company's assets and invests in safe assets for which an investment fee is administered given by ϵ.

The debt delivery functions, (8), first order conditions for low types, (5), and incentive compatibility for high types, (9), are unchanged. What changes in the CDS economy are the marginal buyer pricing equations and market clearing conditions. In the remainder of the section, we analyze the key pricing and market clearing equations that determine the level of trade, investment, and the direction of the investment in the CDS economy.

Selling a CDS written on either firm-type's debt is an equivalent way to purchase the Arrow-Up security. Therefore, the return to selling either CDS must be the same. The prices of the two securities are different because the collateral necessary to sell the two CDS differ based on the different debt delivery functions. In equilibrium, CDS prices are set in order to equate the expected returns. Similarly, buying a CDS on either firm-type's debt is an equivalent way to purchase the Arrow-Down security, where the price difference again equates the expected returns.

$$\frac{h_1\left(1-d_D\left(q^g\right)\right)}{1-p_c^g-d_D\left(q^g\right)}=\frac{h_1\left(1-d_D\left(q^b\right)\right)}{1-p_c^b-d_D\left(q^b\right)} \tag{19}$$

$$\frac{\left(1-h_1\right)\left(1-d_D\left(q^g\right)\right)}{p_c^g}=\frac{\left(1-h_1\right)\left(1-d_D\left(q^b\right)\right)}{p_c^b} \tag{20}$$

The marginal buyer's beliefs, h_1, cancel out in both transactions. That is because each asset pays in exactly the same state, irrespective of which underlying debt instrument serves as the CDS reference entity. Combining the two above CDS pricing equations, equilibrium requires that the relative CDS prices are equal to the relative losses given default;

$$\frac{p_c^b}{p_c^g}=\frac{1-d_D\left(q^b\right)}{1-d_D\left(q^g\right)}. \tag{21}$$

In addition, the marginal buyer in the economy is indifferent between selling and buying CDS on either firm-type's debt:

$$\frac{h_1\left(1-d_D^i\left(q^i\right)\right)}{1-p_c^i-d_D^i\left(q^i\right)}=\frac{\left(1-h_1\right)\left(1-d_D^i\left(q^i\right)\right)}{p_c^i}. \tag{22}$$

Moreover, because the institutional investor purchases CDS with debt–a risk-free asset–the no-arbitrage conditions require that CDS and debt prices must sum to 1: $p^i+p_c^i=1$. Using the no-arbitrage condition to re-arrange and simplify (22), we see that the marginal CDS investor prices all firm debt in equilibrium:

$$h_1+\left(1-h_1\right)d_D^i\left(q^i\right)=p^i. \tag{23}$$

Notice the similarity in how debt gets priced in the CDS and the representative investor or partial equilibrium economies. Equation (23) implies that the same marginal buyer's beliefs will pin down all debt prices, similar to equation (12). The difference is that expectations are exogenously specified in the partial equilibrium, but are endogenously determined in the full general equilibrium setting through market clearing *i.e.* $\mathrm{h}_{1,2}(\Gamma) \neq \gamma$. In particular, the supply of CDS sold by optimists equals their total endowment over total skin in the game needed to sell all contracts. The skin in the game is the portion of their endowment needed to cover losses for each contract, $\omega^i \left(1 - \left(p_c^i + d_D^i\right)\right)$, weighted by each contracts' share in the portfolio, $\omega^i = \frac{q_c^i}{\sum_i q_c^i}$ with $\sum_i \omega^i = 1$. The demand for CDS equals all of the contracts purchased by the institutional investor–equal to total bond supply–plus the total CDS contracts purchased by pessimists weighted by the respective portfolio shares.

$$\frac{1 - h_1}{1 - \frac{\sum_i q_c^i \left(p_c^i + d_D^i(q^i)\right)}{\sum_i q_c^i}} = \sum_i q^i + \frac{h_1}{\frac{\sum_i q_c^i p_c^i}{\sum_i q_c^i}}. \tag{24}$$

The relationships in (21)-(24) illustrate how changes in the relative supply of high-type debt affect market prices for low-type debt in the CDS economy, hence the pecuniary externalities that arrise.

Remark 2 *The general equilibrium effects of asymmetric information in debt prices tend to be subdued in the CDS economy.*

The market prices of risky debt are less responsive to changes in the supply of debt when investors more efficiently share risk by trading CDS than when they only purchase bonds. To see why, recall that selling CDS allows optimists to leverage their endowment and hold more credit risk for every dollar of collateral compared to using the dollar to purchase bonds. For example, consider a single firm economy and suppose that the recovery value on debt is 0.20 with the corresponding debt price of 0.85. The marginal buyer in this economy is $h_1 = .8125$. Each investor can purchase $\frac{1}{0.85} \approx 1.18$ debt contracts. By comparison, all CDS sellers would be able to sell $\frac{1}{(1-d_D)-p_c} = \frac{1}{1-.2-.15} \approx 1.53$ CDS contracts with the same dollar. In fact, it is easy to show that investors can always sell more CDS than they can puchase bonds as long as CDS are not redundant and no-arbitrage holds, *i.e* $d_D^i\left(q^i\right) > 0$ and $p_c^i + p^i = 1$ respectively.

Adverse selection affects debt recovery values through the investment distortion, which in turn impacts how investors price risky debt. Debt prices in equilibrium are determined by marginal buyer expectations and the recovery value. The effect of better risk sharing in the CDS economy is that changes in recovery values need to be much larger to induce an equal sized shift in marginal buyer expectations to have a commensurate impact on debt prices compared to the baseline economy. However, CDS prices adjust and mitigate the changes in recovery values that ultimately determine the skin in the game required to sell contracts. Specifically, issuing more debt with a lower recovery value raises the price that CDS sellers receive and partially offsets the collateral value needed to compensate the CDS buyers for the lower recovery value in default. Thus, for a given investment distortion due to adverse selection, both positive and negative pecuniary externalities are muted in CDS economies.

Table 5: CDS Economy: $\alpha^g = 0.5$, $\alpha^b = 0.7$, $\gamma = 0.50$, and $A_D = 0.30$

	Asymmetric Info.		*Perfect Info.*	
	$i = g$	$i = b$	$i = g$	$i = b$
Price: p^i	.7428	.6999	.8096	.7279
Quantity: q^i	.2568	.1325	.2024	.1452
Investment: I^i	.1907	.0927	.1639	.1057
Marginal Buyers: h	.4749	.4749	.4352	.4342
Info.Wedge: λ^i	.8504	1	1	1
Profit: π^i	.0900	.0284	.1012	.0311

A direct consequence of Remark 2 is that the direction of the distortion in the CDS economy is similar to the partial equilibrium economy where the pecuniary externalities are absent. However, because price effects, though muted, remain in the CDS economy, one cannot prove in general that trade is *always* distorted on the upper end. The reason is twofold: 1) The logic of Proposition 2 used to show over investment in partial equilibrium can only be applied when the pecuniary externality is absent; 2) as in almost all general equilibrium models, the behavior of prices cannot be easily stated in terms of fundamentals. Therefore, the logic of Proposition 4 cannot be used to characterize the relative signaling costs based on price behavior in the CDS economy. We provide simulation results for parameter values in the baseline economy that generate under investment distortions. The results confirm that prices generally *diverge* in CDS economies, an affect similar to increasing firm heterogeneity. When prices diverge, one can use the logic of Proposition 4 to conclude that allocations are distorted on the upper end for high types and there is too much trade and investment relative to perfect information.

Table 5 contains the CDS economy results under both perfect and asymmetric information assumptions using the same parameter values as example 1. The results show high types engage in too much trade and investment in the asymmetric-information CDS economy, $\lambda^g < 1$, compared to the perfect-information CDS benchmark. Comparing the results of examples 1 and 5 highlights the effect of CDS on investment distortions conditional on adverse selection. The CDS economy with adverse selection is characterized by over investment but the baseline adverse selection economy is characterized by under investment, hence contracts that improve risk sharing can also change the direction of equilibrium distortions.

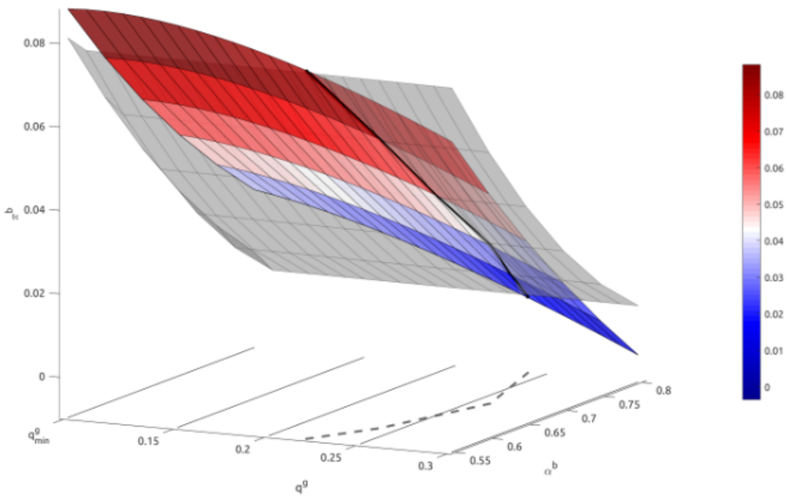
Figure 5 plots the low type mimicking profits (in color) and revealing profits (in gray) as a function of high type debt issuance in various CDS economies for which the baseline economy is characterized by under investment ($0.55 < \alpha^b \leq 0.75$) and over investment ($\alpha^b = 0.80$). The solid black line is the set of over investment equilibrium contracts and its projection onto the (q^g, α^b)-space is the dashed line. The simulations show that there are no incentive compatible under investment allocations for $q^g > q^g_{min}$ as the two surfaces do not intersect. High type debt is risk free for all quantities less than or equal to $q^g_{min}(\Gamma)$ because the firm is productive enough to repay debt in both states at time 1. Perfect risk sharing makes the positive pecuniary externality is so small that no level of risky debt satisfies the under investment condition in Lemma 1 that $I^g < I^b$. The figure also shows that the quantities associated with incentive compatible over investment contracts are non-monotonic as firm differences grow. Quantities initially rise and then fall as firm heterogeneity increases. The reason is that the adverse selection problem gets worse as firm fundamentals diverge leading to higher adverse selection costs, but is limited by the fundamental difference between firms. In particular, once the fundamental difference between firms gets large enough, low types become over-extended at high type contracts.

An additional implication of the over investment distortion is that losses given default are higher compared to the efficient benchmark, specifically for *high-type* debt contracts. In particular, figure 6 plots high-type losses given default for both the perfect and asymmetric information CDS-economies as a function of α^b.[24] Losses given default are not only higher in the over investment economies, but their movement mirrors the over investment quantities from figure 5. Losses rise (fall) as investment becomes more (less) distored relative to the perfect information benchmark. Intuitively, quantities rise as the adverse selection problem intensifies, leading to higher adverse selection costs. The result of higher adverse selection costs manifest both as a deviation from efficient investment *ex ante* and larger losses for investors *ex post*.

Our last result is that asymmetric information in a CDS economy can lead to an *increase* in aggregate investment. Table 5 shows that $\sum_i I^{*^i}_{AI} > \sum_i I^{*^i}_{PI}$. This contrasts existing theories that find unrestricted CDS economies have less investment (Darst and Refayet (2018), Fostel and Geanakopolos (2016), and Che and Sethi (2015)). The key in overturning the previous results is the presence of asymmetric information and adverse selection. The existing literature highlights that CDS can raise the cost of capital, which in turn leads to less investment. However, under asymmetric information, investment must be even lower in order for it to be incentive compatible in CDS economies because efficient risk sharing limits the impact of the positive pecuniary externality associated with under investment. High types generally over invest in equilibrium. Because the negative pecuniary externality is also muted, investment must rise in order to meet incentive compatibility. These predictions provide a novel unified theory for various empirical findings on the effect of CDS on price versus non-price terms. For example, Ashcraft and Santos (2009) find that CDS have very little impact on corporate spreads. Saretto and Tookes (2013) argue that quantities such as leverage and investment are the margins through which CDS affect debt contracts rather than prices.

[24]Losses given default are computed by 1 minus the delivery rule in (8). Low type losses are the same across information structures because they operate at their efficient level and do not incur an adverse selection cost in terms of investment distortions.

Figure 5: CDS Over Investment Simulations



4 Discussion

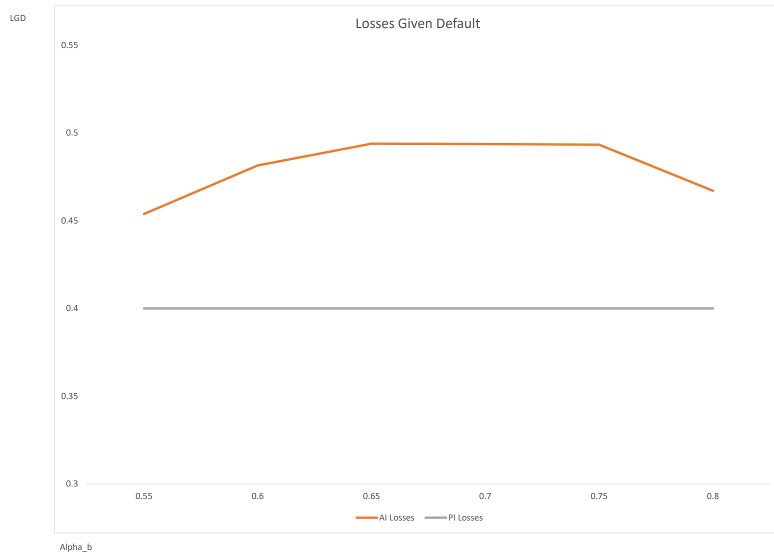
We conclude with remarks on how one may think about the various aspects of our model and applications. Debt pools can be interpreted as CLOs. CLOs are debt pools for corporate loans akin to mortgage backed securities in residential real estate, and have become an important funding source for U.S. corporations since the financial crisis.[25] The red line in figure 7 is annual CLO volume and the blue line is the total outstanding CLO amount. In addition, CLO investors are widespread and diverse. Paligorova and Santos (2018) highlight the increasing importance of non-bank intermediaries in the CLO market including pension funds, mutual funds, and hedge funds. Though reduced form, investor heterogeneity in our model captures the basic idea that different investors have diverse strategies, valuation methods, and business models.

CDS on debt pools may represent CDS indexes such as investment grade, high yield, or industry specific CDS indexes composed of various firms within an industry. Figure 8 shows the percent of all corporate CDS trades (single name and index trades) by gross notional amount that index CDSs represent. The structural break in the series is likely due to the requirement in Dodd-Frank that many derivatives move to central clearing. The dollar value of the share of transactions in CDS indexes has steadily increased post-crisis and currently stands close to 70% of all corporate CDS transactions.

The model captures the investment implications of adverse selection with financial constraints that are consistent with the data. The collateral constraint in our model is

[25] Many CLO securities until recently also contained a small fraction of corporate bonds.

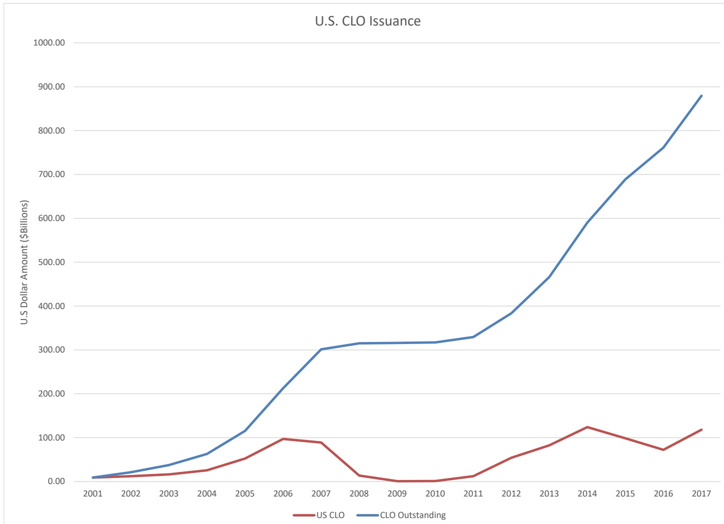
Figure 6: High Type Losses in Default



effectively a cash flow or earning based constraint. Firms that generate more output or cash flow receive better debt pricing leading to investment expansion. Hence, there is a positive relationship between cash flow, debt/leverage, and investment. Lian and Ma (2018) find exactly these relationships studying earnings-based constraints for a large cross-section of corporate borrowers. In our model lower α^i leads to more production for every unit of input and better pricing. They also find that firms issue more debt and invest more in response to positive earnings due to a loosening of the constraint. This is captured in our model by higher A_D. By contrast, positive investment shocks in the traditional Kiyotaki-Moore (1997) model generate the opposite co-movements between debt, leverage, and investment. Lastly, our framework can be used to study the implications of adverse selection in any credit market that financial innovation continues to "commoditize" through pooling and securitization.

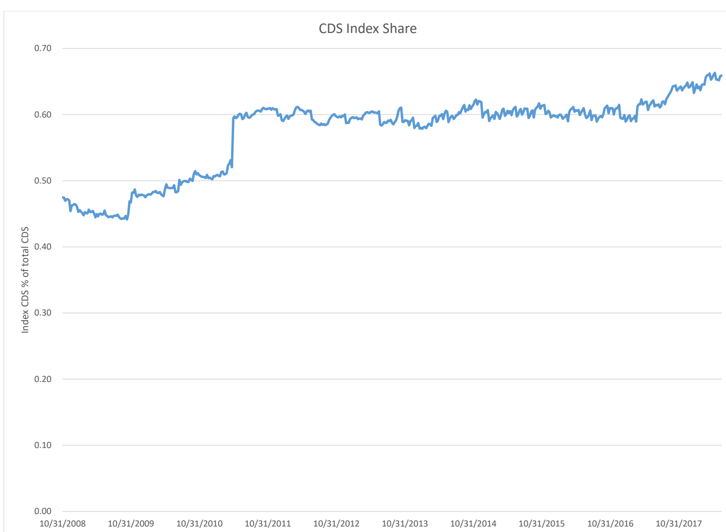
Lastly, Lemma 1 and Proposition 4 show that different firms receive different contracts in asymmetric information equilibrium. Proposition 1 also shows that firms receive different contracts in perfect information equilibria. Importantly, in both information settings, all firms default in the same aggregate states but borrow different amounts. Thus, there is no correlation between default and loan amount in either the symmetric or asymmetric information economies. The lack of correlation between default and quantity implies that using the so-called positive correlation test between loan amount and default cannot statistically reject a null hypothesis of asymmetric information as no correlation is consistent with both information structures.

Figure 7: U.S. CLO Issuance



Source: SP Global, Leveraged Loan Commentary and Data - Volume-History.

Figure 8: U.S. Corporate CDS Index Share



Source: The data are authors' calculation from International Swaps and Derivatives Association: http://www.swapsinfo.org/charts/swaps/notional-outstanding.

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A Appendix

Proof. *Lemma 1*:An incentive compatible allocation must have the low type just indifferent to pooling with the high type. Suppose to the contrary that the under investment equilibrium allocation is such that the high type invests more than the low type and less than its perfect information benchmark, ${q^*_{PI}}^g > \hat{q}^g > q^{*b}$. Investing $\hat{q}^g - \epsilon$ does two things: 1) it reduces the total supply of risky debt in the economy, and will raise all debt prices; 2) is causes $\hat{q} \to q^{*b}$. Both of these effects will make the low type *more likely* to mimic. Investing $\hat{q}^g + \epsilon$ does the opposite, lowering the incentive to mimic. However, $\frac{\partial \pi^g}{\partial q^g} > 0, \forall q < q^*_{PI}$. Thus the high type can raise $\hat{q}^g + \epsilon$ without violating the ICC and yield a profitable deviaion. Thus, an under investment equilibrium must be characterized by ${q^*_{PI}}^g > q^{*b} > \hat{q}^g$. ■

Proof. *Lemma 2*: Suppose to the contrary that bonds are purchased unprotected. Then it must be the case that the utility of the agent who buys the unprotected bond is given by $u^b\left(h^1\right) = \frac{h^1+(1-h^1)d^b_D}{p^b} > 1$, which can be written as $\frac{h^1\left(1-d^b_D\right)+d^b_D}{p^b} > 1$. Note that the utility of the CDS seller is given by $u^s(h_{cds}) = \frac{h_{cds}\left(1-d^b_D\right)}{p^b-d^b_D}$. Now suppose that the investor h^1 who purchases the bad bond unprotected instead writes the CDS. His utility would be given by $u^s(h^1) = \frac{h_1\left(1-d^b_D\right)}{p^b-d^b_D}$. To finish the proof it suffices to show that h^1 prefers to write CDS over buying unprotected bonds. Let $h^1\left(1-d^b_D\right) = X$, $p_b = Y$, and $d^b_D = \Lambda$. We can then rewrite the utilities in the following way: $u^b\left(h^1\right) = \frac{X+\Lambda}{Y}$ and $u^s(h^1) = \frac{X}{Y+\Lambda}$. If $u^s(h^1) > u^b\left(h^1\right)$ then,

$$
\begin{aligned}
&\implies \frac{h^1\left(1-d^b_D\right)+d^b_D}{p_b} < \frac{h^1\left(1-d^b_D\right)}{p_b - d^b_D} \\
&\implies \left(h^1\left(1-d^b_D\right)+d^b_D\right)\left(p_b - d^b_D\right) < h^1\left(1-d^b_D\right)p_b. \\
&\implies -h^1\left(1-d^b_D\right)d^b_D + \Lambda p_b - \left(d^b_D\right)^2 < 0 \\
&\implies \left(h^1\left(1-d^b_D\right) - p_b + d^b_D\right) > 0.
\end{aligned}
$$

We see that $h^1\left(1-d^b_D\right) - p^b + d^b_D > 0$, which is the same as $\frac{h^1+(1-h^1)d^b_D}{p^b} > 1$. Thus, any agent who would buy unprotected bonds would be better off selling CDS. ■

Proof. *Lemma 3*:Suppose to the contrary that h^1 holds cash. It must be the case then that the investor prefers holding cash to any other instrument int the economy. Thus we can say

$$h^1 + \left(1-h^1\right)d^b_D < p^b, \tag{25}$$

and

$$1 > \frac{(1-h^1)\left(1-d^b_D\right)}{1-p^b}. \tag{26}$$

Inserting (25) into the denominator of the *r.h.s* of (26) we do not perturb the inequality

$$1 > \frac{(1-h^1)\left(1-d^b_D\right)}{1-\left[h^1+(1-h^1)\,d^b_D\right]}.$$

Rearranging and regrouping we get

$$\left(1-h^{1}\right)\left(1-d_{D}^{b}\right)>\left(1-h^{1}\right)\left(1-d_{D}^{b}\right)\ \otimes$$

a contradiction. ■

Proof of Proposition 1.
Proof. See Proposition 1 Darst and Refayet (2018) ■

Proof of Proposition 2
Proof. Existence follows from standard convexity and fixed point arguments. Uniqueness follows from Riley (1985) and the single crossing property that the high type is more productive on the margin, and hence as a lower marginal cost of seperation than the low type. Assume by contradition that equilibrium is not unique because both candidate incentive compatible solutions yield the same profit to high types. Formally it must be that $\pi^{g}\left(\underline{q}^{g};\Gamma\right)=\pi^{g}\left(\overline{q}^{g};\Gamma\right)$. Note also that bad firm profits are fixed for both contracts and given by $\kappa\left(\Gamma\right)$. Therefore, the bad firm would also be indifferent to either seperating allocation and would have no incentive to deviate. Note that there is a unique product of $p\times q=\hat{I}$ for which $MPK^{g}=MPK^{b}$, and $\forall I^{g}<\hat{I}\Rightarrow MPK^{g}>MPK^{b}$. The difference in marginal products implies $\frac{\partial\pi^{g}}{\partial I}>\frac{\partial\pi^{b}}{\partial I},\forall I<\hat{I}$. This then implies that the difference between good and bad types' profits for the same inputs grows as well: $\pi^{g}\left(\underline{q}^{g}\right)-\pi^{b}\left(\underline{q}^{g}\right)<\pi^{g}\left(\overline{q}^{g}\right)-\pi^{b}\left(\overline{q}^{g}\right)$. But if $\kappa\left(\Gamma\right)=\pi^{b}\left(\underline{q}^{g}\right)=\pi^{b}\left(\overline{q}^{g}\right)$, then $\pi^{g}\left(\underline{q}^{g}\right)<\pi^{g}\left(\overline{q}^{g}\right)$, a contradiction. Thus, any seperating equilibrium must be unique. Moreover, the fact that the difference in profits is increasing $\forall I<\hat{I}^{g}$ but low type profits are fixed by $\kappa\left(\Gamma\right)$ implies that the unique equilibrium is characterized by over investment. ■

Proof of Proposition 3
Proof.
Existence follows directly from the existence of partial equilibrium with the added provisio that there are two marginal buyers and market clearing. Existence of the marginal buyers is given by the connectedness of the set agents $h\in H$. All investors $h>h_1$ purchase debt claims from bad type debt pools, all investors $h_1>h>h_2$ purchase debt claims from good type debt pools, and all investors $h<h_2$ remain in cash. Market clearing is as follows. Because all agens hold 1 unit of endowment and pay the market price p for claims of a particular debt pool, each investor can hold $\frac{1}{p}$ bonds. Thus total debt funding, for type b firms for example, is $1-h_1$ and the total quantity of claims type b firms issue is q.

Proving uniqueness requires different logic than the partial equlibrium proof becasue bad firm profits are no long fixed by κ. However, this actually simplifies the proof. Low type profit maximization ensures uniques even if two candidate solutions are identical to high types. Suppose there are two incentive compatible debt contracts that yield the same profts as before: $\pi^{g}\left(\underline{q}^{g};\Gamma\right)=\pi^{g}\left(\overline{q}^{g};\Gamma\right)$. At these two allocations, bad firm types are no longer indifferent because $\pi^{b}\left(\left(q^{g},q^{b}\right);\Gamma\right)$. Key step is to notice that low type profits are monotonically decreasing in high type quantities, $\frac{\partial \pi^b(q^g)}{\partial q^g} < 0$. This stems from the fact that $p^g \downarrow$ as $q^g \uparrow$ due to the dilution effect of issuing additional debt relative to the efficient contract. In equilibrium, p^b must also fall because the expected return to holding good type debt for the marginal buyer indifferent to either contract, h_1, is higher because she puts less weight on $s = D$ than the marginal buyer who prices good type debt relative to cash, h_2. Hence demand for claims in the bad type debt pools falls as well. The first order conditions for maximization require that bad types reduce investment and retain lower profits. But incentive compatibility requires that the bad type be just indifferent to pooling and seperating with high types at the two different allocations. Therefore, for any two separating quantities, $\underline{q}^g < \overline{q}^g$, bad types will always prefer to issue claims into the market corresponding to lower quantities and higher prices for which its own profts will be higher. Single crossing in the proof of Proposition 2 implies that if two allocations yield the same profits for low types, then they cannot yield the same profits for high types completing the proof. ■

Proof of Proposition 4:

Proof. The proof of the proposition proceeds by first examining the comparative statics of the perfect information contracts as productivity changes. This will help clarify how productivity differences influence the incentives for low types to pool with high type and how the high types separate. We then proceed to examine how productivity convergence or divergence affects the incentive to maintain or deviate from either over or under investment allocation.

Perfect Infomation Comparative Statics: Fix α^g and take $\alpha^b \to (\alpha^g, 1)$. As the low types approaches the productivity of the high type, the marginal products of capital converge for any investment level. In addition, from proposition 1, the delivery rate of the low type is also rising. The rising delivery rate implies the any investor, h, is willing to pay a higher price for low type debt. Therefore, for a given investment level, I^{*b}, the marginal product is rising and marginal cost is falling. The low type must expand investment to restore equilibrium, $I^{*'b} > I^{*b}$ by supplying additional debt to the market leading to lower risky debt prices. Market clearing implies that the new marginal buyer is less optimistic, $h_1' < h_1$, which means that high type debt prices must also fall, $p^{*'g} < p^{*g}$. Optimization requires lower high type investment, $I^{*'g} < I^{*g}$. Putting these arguments together, the optimal perfect information contracts are more similar in terms of investment levels. Prices must also converge since there is a representative price at $\alpha^b = \alpha^g$. The opposite comparative statics obtain as αs diverge.

Case 1:

We first establish that starting from an under investment equilibrium, the economy will remain in an under investment equilibruim as producitivies converge. Assume that for some productivity pair, $\left(\alpha^g, \alpha^b\right)$, the equilibrium allocation is characterized by under investment: $\left(\hat{c}^g\left(\overrightarrow{p}\left(\hat{\underline{q}}^g\right), \hat{\underline{q}}^g\right), c^{*b}\left(\overrightarrow{p}\left(q^{*b}\right), q^{*b}\right)\right)$ where hats on good type contracts denote constrained investment levels and stars on low types denote unconstrained levels. $\alpha^g = \alpha^b$ implies a representative firm and adverse selection is not a relevant concern. We will compare the allocation of two economies with different low type productivities: $\alpha^g < \alpha^{b'} < \alpha^b$. Note the economy denoted with primes referes to less heterogeneity.

Let $\mathcal{E}^U(\Delta_\alpha)$ be an underinvestment allocation where where $\pi^g\left(\cdot, \underline{\hat{q}}^g\right) > \pi^g\left(\cdot, \hat{\overline{q}}^g\right)$. Lemma 1 ensures that $\underline{\hat{q}}^g < q^{*b}$ and $\underline{\hat{q}}^g < q^{*b} < q^{*g}$ where q^{*g} is the uncontrained high type quantity. Define the relative under investment distortion as the difference between the constrained and unconstrained high type quantity: $q^{*g} - \underline{\hat{q}}^g$. Let $\mathcal{E}^{U'}(\Delta_{\alpha'})$ be an underinvestment allocation with less heterogeneity between types $i.e$ $\Delta_{\alpha'} < \Delta_\alpha$. The equivalent debt quantity ordering after making the low type more productive is $\underline{\hat{q}}^g < q^{*b} < q^{*'b} < q^{*'g} < q^{*g}$ where the two prime quantities reflect the fact that the low type expands while the high type contracts from the the unconstrained comparative statics. The under investment distortion is smaller after improving the low-type's productivity: $\left(q^{*g} - \underline{\hat{q}}^g\right) > \left(q^{*'g} - \underline{\hat{q}}^g\right)$. This also reflects the fact that the relative benefit to the low type of pooling falls as the debt prices converge. Therefore, under investing can only be easier to sustain in terms of the relative investment distortion required to seperate as productivities converge. By contrast, over investing is more difficult because the relative efficient investment scales are more similar as $q^{*'b} \to q^{*'g} \implies I^{*'b} \to I^{*'g}$. The high type would need to push investment even further beyond efficient scale to fully seperate via over investing. Since $\frac{\partial \pi^g}{\partial \overline{q}^g} < 0, \forall q > q^*_{PI}$, choosing a $\hat{\overline{q}}^g$ further from q^*_{PI} only lower profits further. Therefore, the cost of underinvesting falls while the cost of overinvesting rises as parameters governing producitivites and cash-flow converge until there is a single, representative firm. This completes point 1 in the propostion.

Case 2:
Let $\mathcal{E}^U(\Delta_\alpha)$ be an over investment allocation where $\pi^g\left(\cdot, \underline{\hat{q}}^g\right) < \pi^g\left(\cdot, \hat{\overline{q}}^g\right)$. Define the over investment distortion as: $\hat{\overline{q}}^g - q^{*g}$. Again, let $\mathcal{E}^{U'}(\Delta_{\alpha'})$ be an over investment allocation with more heterogeneity between types $i.e$ $\Delta_{\alpha'} > \Delta_\alpha$, and $\alpha^g < \alpha^b < \alpha^{b'}$. It is trival to show that over investment must be characterized by $\hat{\overline{q}}^g > q^{*b}$ so we do not state it as a formal proof. The comparative statics of the productivity change imply the following relationships: $\hat{\overline{q}}^g > q^{*'g} > q^{*g} > q^{*b} > q^{*'b}$. The low types invest less than before while the high types invest more, and the low type has an even lower capacity to extend investment and pool with the high type. This will make the new over investment separating allocation less punative for the high type. This implies the over investment distortion falls after the productivity divergence: $\left(\hat{\overline{q}}^g - q^{*'g}\right) < \left(\hat{\overline{q}}^g - q^{*g}\right)$. By contrast, an under investment contract raises debt prices and brings the debt issuance levels closer in line strengthening the incentive to mimic along both the price and investment level effects. As a result, the fully separating under investment contract must be further way from the efficient contract after the productivity change: $\left(q^{*g} - \underline{\hat{q}}^g\right) < \left(q^{*'g} - \underline{\hat{q}}^g\right)$. And since $\frac{\partial \pi^g}{\partial \underline{q}^g} > 0, \forall q < q^*_{PI}$ choosing a $\underline{\hat{q}}^g$ further from q^*_{PI} only lower profits further. The under investment cost

rises as the over investment cost falls until the difference between types becomes so great that low types cannot feasibly pool with high types and the ICC does not bind.
■