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Carlos Ramírez

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CARLOS RAMÍREZ*

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Regulating Financial Networks Under Uncertainty

ABSTRACT

I study the problem of regulating a network of interdependent financial institutions that is prone to contagion when there is uncertainty regarding its precise structure. I show that such uncertainty reduces the scope for welfare-improving interventions. While improving network transparency potentially reduces this uncertainty, it does not always lead to welfare improvements. Under certain conditions, regulation that reduces the risk-taking incentives of a small set of institutions can improve welfare. The size and composition of such a set crucially depend on the interplay between (i) the (expected) susceptibility of the network to contagion, (ii) the cost of improving network transparency, (iii) the cost of regulating institutions, and (iv) investors' preferences.

Keywords: Financial networks, contagion, policy design under uncertainty.
JEL classification: C6, E61, G01.

The financial crisis that began in 2007 underscored the relevance of interdependencies among financial institutions—e.g., banks, money market funds, investment banks, and insurance companies—in the functioning of modern economies. While, in normal times, connections among institutions—in the form of contractual obligations or common exposures—can be beneficial, as they help institutions manage liquidity or diversity risk, they can also create channels through which shocks propagate in times of economic stress. These channels might cause problems at one institution to spread to others, potentially leading to cascades of distress with economy-wide implications.

In light of the potential harmful side effects of these connections, policymakers across the globe implemented responses that directly or indirectly take into account the interconnected nature of modern financial systems so as to preserve the benefits of connections while managing their unintended negative consequences. When designing these responses, however, policymakers are confronted with an inconvenient truth: it is hard to determine the precise structure of the network of exposures among financial institutions because of the opacity, complexity, and multifaceted nature of their connections. Importantly, this problem becomes particularly acute in times of economic stress, as spirals of fire sales may become relevant. A natural question then arises: How can policymakers regulate a network of interdependent financial institutions when those policymakers are fundamentally uncertain about its precise structure? Despite its importance, this question has been overlooked by most of the literature. This paper partially fills this gap by developing a model to study the behavior of such policymakers.

I show that uncertainty about the precise structure of the network can reduce the scope for welfare-improving interventions, as such uncertainty gives rise to substantial difficulties in determining the likelihood of systemic events. However, this lack of certainty does not necessarily justify a non-interventionist policy, considering the profound negative consequences of cascades of distress. While improving network transparency could help policymakers overcome forecasting limitations, it does not necessarily lead to welfare improving interventions as improving network transparency is also costly. For example, improving network transparency tends to be suboptimal in networks that exhibit highly symmetric structures, as detailed knowledge of the network might not be needed if institutions behave in a similar fashion from the perspective of shock propagation. Under certain conditions, regulation that reduces the risk-taking incentives of a small set of institutions can increase welfare. The size and optimal composition of that set is determined by the interplay between (i) the (expected) susceptibility of the network to contagion, (ii) the cost of improving network transparency, (iii) the cost of regulating institutions, and (iv) investors' preferences.

The model is motivated by a large economy in which financial institutions (banks, for short) are interconnected through an exogenous network of opaque exposures, on either the asset side or the liability side, that cannot be mitigated through contractual protections. In times of economic stress, some of these exposures (henceforth referred to as contagious exposures) function as propagation mechanisms, as banks become more vulnerable to distress affecting related banks (henceforth referred to as neighbors). Cascades of distress may occur as a result of contagion, as the distress affecting one bank could cause distress to that bank's neighbors, which, in turn, may cause distress to the neighbors' neighbors, and so on.

To capture policymakers' inability to ascertain the precise structure of the network in times of economic stress, I assume that the set of contagious exposures is randomly determined and unknown when designing interventions. Because banks fail to internalize the consequences of their actions on the spread of distress, introducing regulation can potentially lead to a Pareto improvement. A (social) planner seeks to maximize expected total output by imposing preemptive liquidity restrictions on a set of banks. While liquidity restrictions decrease banks' likelihood of distress, they are not costless. Liquidity restrictions adversely increase banks' cost of lending, as they limit banks' ability to allocate funds toward more productive projects, thereby introducing resource misallocation. While the planner is uncertain which exposures may propagate distress, she can improve transparency regarding the network of contagious exposures. By improving network transparency, the planner can strategically target banks to limit the spread of distress while at the same time avoiding dead-weight losses associated with regulating an excessively large number of banks. Importantly, improving network transparency is socially costly, as it might decrease banks' confidentiality, compromising their market position and potentially decreasing market efficiency.

I first analyze the behavior of the planner when the degree distribution of the network of contagious exposures—which captures the distribution of contagious exposures across banks—is known. I show that the optimal policy—which is jointly determined by a choice of network transparency and a selection of restricted banks—is shaped by the interplay between aggregate characteristics of this degree distribution, the cost of improving network transparency, and the costs of restricting banks. If the network exhibits a highly asymmetric structure, then a handful of banks play an important role in the propagation of distress. Learning the identity of those banks becomes critical to adequately avoid contagion, as regulating them effectively deters the emergence of cascades of distress. As a result, improving network transparency tends to be optimal. However, if the network exhibits a highly symmetric structure, improving network transparency tends not to be optimal. In this case, more information regarding the precise structure of the network is not necessarily informative, as every bank is likely to play a similar role in the propagation of distress when conditions deteriorate. Finally, higher costs of restricting banks lead to a smaller fraction of banks that can be restricted.

Next, I analyze the behavior of the policymaker when she is unsure about the degree distribution of the network of contagious exposures. The optimal policy intervention is then affected by investors' attitudes toward ambiguity and their beliefs regarding the susceptibility of the network to contagion. Under certain conditions, small changes in beliefs generate significant changes in the optimal fraction of restricted banks. When investors are sufficiently ambiguity averse, they are worried that the number of restricted banks might not be sufficiently large to prevent large cascades of distress when aggregate conditions deteriorate. As a result, more banks might need to be restricted as network uncertainty increases. Importantly, the (social) value of improving network transparency is intimately linked to the interplay between network uncertainty and the symmetry of the network. In symmetric structures—that is, when banks behave in a similar fashion—the higher the model uncertainty, the less informative the information regarding the precise structure of the network. However, in asymmetric structures—that is, when only a few banks play a key role in the propagation of distress—improving network transparency may be more valuable as network uncertainty increases. The reason is that more transparency allows the planner to limit the negative consequences of network uncertainty on the robustness of policy interventions.

The first set of results informs the ongoing debate regarding the optimal design of macroprudential regulations. While post-crisis reforms with a macroprudential dimension have focused principally on large financial institutions, my results underscore that the architecture of the financial system (and not just the size of institutions) matters for policy design. In addition, these results provide a rationale for regulation that seeks to improve network transparency and, in particular, improve information disclosure, as institution-level information may be critical to effectively limit the effect of cascades of distress. More broadly, these results highlight the importance of developing privacy-preserving methods for sharing financial exposures of institutions that could play an important role in the propagation of distress when a crisis manifests. The second set of results highlights that an appropriate macroprudential regulatory framework must be mindful of the uncertainty regarding the pattern of relationships among institutions.

Related literature. This paper contributes to several strands of the literature. First, this paper adds to a body of work that explores how network features of the financial system affect the likelihood of contagion. An incomplete list includes Rochet and Tirole (1996), Allen and Gale (2000), Freixas et al. (2000), Eisenberg and Noe (2001), Lagunoff and Schreft (2001), Dasgupta (2004), Leitner (2005), Nier et al. (2007), Allen and Babus (2009), Haldane and May (2011), Allen et al. (2012), Amini et al. (2013), Cont et al. (2013), Georg (2013), Zawadowski (2013), Cabrales et al. (2014), Elliott et al. (2014), Glasserman and Young (2015, 2016), Acemoglu et al. (2015), and Castiglionesi et al. (2017). Unlike these papers, my paper explicitly focuses on the planner's problem in the presence of spillovers and uncertainty regarding the pattern of connections among institutions. Second, my paper adds to recent research that explores how policy interventions affect the mechanism through which shocks propagate (see, for example, Beale et al. (2011), Gai et al. (2011), Battiston et al. (2012), Goyal and Vigier (2014), Adrian et al. (2015), Erol and Ordoñez (2017), Aldasoro et al. (2017), and Galeotti et al. (2018)). While my paper also focuses on how contagion varies with different policy interventions, it provides a tractable framework in which optimal policies can be determined under uncertainty regarding the economy's connectivity structure.

The rest of the paper is organized as follows. Section I introduces the baseline model. Section II explores how regulation affects the likelihood of contagion and, in doing so, the distribution of total output. Section III describes the optimal intervention. Section IV extends the baseline model to environments wherein investors are uncertain about aggregate characteristics of the economy's connectivity structure. Section V concludes. All derivations appear in the Appendix.

I. Baseline Model

Though stylized, the baseline model conveys the main intuition for how uncertainty about the precise structure of the network reduces the scope for welfare-improving interventions.

A. The economy

Environment. The economy consists of n banks, a continuum of entrepreneurs, and numerous investors. Banks' payoffs are linked via exogenous inter-bank exposures.[1] To keep things simple, while banks may differ in their number of exposures, they are ex ante identical in other respects, such as size and leverage.

[1] It is useful to think of the network of inter-bank exposures as a reduced form that captures the overall effect of multiple financial linkages among banks within a multilayer network. Treating the multilayer network as a single network simplifies the analysis by allowing me to circumvent the complications that arise from having to model how dependencies among banks are aggregated in equilibrium.

There are three periods, indexed by $t = \{0, 1, 2\}$. At $t = 0$, a planner imposes liquidity restrictions on a set of banks to maximize expected total output. At $t = 1$, investors endow each bank with one unit of resources. Immediately after, each bank faces a continuum of entrepreneurs with measure 1. Subject to their regulatory constraints, banks invest their funds in a (risky) portfolio consisting of entrepreneurs' projects (henceforth referred to as illiquid assets) and liquid assets. Liquid assets are (exogenously determined) investment opportunities that can be easily converted into cash. At $t = 2$, payoffs are realized and consumption occurs. Just before payoffs are realized, aggregate conditions deteriorate, and banks become vulnerable to (unanticipated) adverse liquidity shocks affecting their neighbors. Cascades of liquidity shocks might occur if the liquidity shock affecting one bank causes a liquidity shock to some of its neighbors, which may cause a liquidity shock to the neighbors' neighbors, and so on.[2]

Propagation of liquidity shocks. The propagation of liquidity shocks is determined by a simple stochastic process. First, one randomly chosen bank faces an adverse liquidity shock. Second, if that bank is affected, this shock might spread to others via randomly selected contagious exposures. Bank i faces an adverse liquidity shock if two things happen: (1) there is a sequence of contagious exposures between i and the first bank that faces the liquidity shock, and (2) every bank in that sequence is affected by the liquidity shock.[3]

[2] In practice, cascades of liquidity shocks may capture liquidity-driven crises (as in Diamond and Rajan (2011) and Caballero and Simsek (2013)) in which the liquidity shocks affecting a small set of banks induces adverse liquidity shocks for some of their neighbors. When market participants face high uncertainty, those neighbors may face a run due to solvency concerns, which, in turn, potentially causes solvency concerns about some of the neighbors' neighbors, possibly generating cascades of runs. Consequently, cascades of liquidity shocks could also capture crises of confidence. Another example of cascades relates to situations in which the liquidity shocks affecting some banks lead to write-downs in the balance sheets of some of their neighbors. If resulting losses exceed the capital of such neighbors, those neighbors will face liquidity shocks, which, in turn, may cause other banks to face liquidity shocks as well.

[3] The shock propagation mechanism is similar to the one used in Ramírez (2017). The main results continue to hold if a small set of banks is initially affected by liquidity shocks. Conditional on banks i, j, and k being connected via contagious exposures, the existence of a contagious exposure between i and j is independent of the existence of a contagious exposure between j and k. A richer model would include local dependencies among such events so that the effect that a single distressed neighbor has on a bank depends critically on whether other neighbors face liquidity shocks. For a model that introduces such dependencies, see Watts (2002). If one introduces such dependencies, the basic trade-off behind the main results should continue to appear.

For concreteness, I assume that the resulting distribution of contagious exposures across banks—which basically determines the susceptibility of the economy to contagion—can be characterized by an arbitrary distribution $\{p_k^\alpha\}_{k=0}^{n-1}$, with shape parameter α, where p_k^α denotes the likelihood that a randomly chosen bank has k contagious exposures at $t = 2$. While the number of contagious exposures per bank is unknown at $t = 0$, the planner knows the functional form of $\{p_k^\alpha\}_{k=0}^{n-1}$. Section IV discusses extending the baseline model to environments wherein this functional form is unknown, even by the planner.

Planner's information. Before restricting banks, the planner observes noisy signals about the future number of contagious exposures per bank. While signals are not informative by themselves, the planner can improve their precision—and, in doing so, improve network transparency—by collecting bank-level information at a cost $\kappa > 0$. Parameter κ can be broadly interpreted as the (social) cost of designing and implementing policies to improve information disclosure and transparency regarding the network of contagious exposures.[4] Importantly, while more transparency is important when targeting banks, it also decreases banks' confidentiality. As information might be valuable to banks, revealing it could compromise their market position, reducing their incentives to lend and potentially decreasing market efficiency.[5] Hence, acquiring information is socially costly. For simplicity, all those costs are captured by parameter κ.

Figure 1 depicts the timeline of events.

[4]Additionally, these policies may allow regulators to uncover banks that play an important role in the transmission of shocks when a crisis materializes. Two important examples of such policies are the Comprehensive Liquidity Assessment and Review (CLAR) and the Dodd-Frank Act supervisory stress test, run annually by the Federal Reserve. In these programs, regulators evaluate the liquidity risk profile of bank holding companies (BHCs) through a range of metrics and project whether BHCs would be vulnerable during times of weak economic conditions. Other examples include programs implemented by the SEC such as forms N-MFP and PF. Form N-MFP requires registered money market funds to report their portfolio holdings and other information on a monthly basis, while form PF requires private funds to report assets under management.

[5]Thinking in banks as secret keepers is also consistent with this idea; see Dang et al. (2017) for more details.

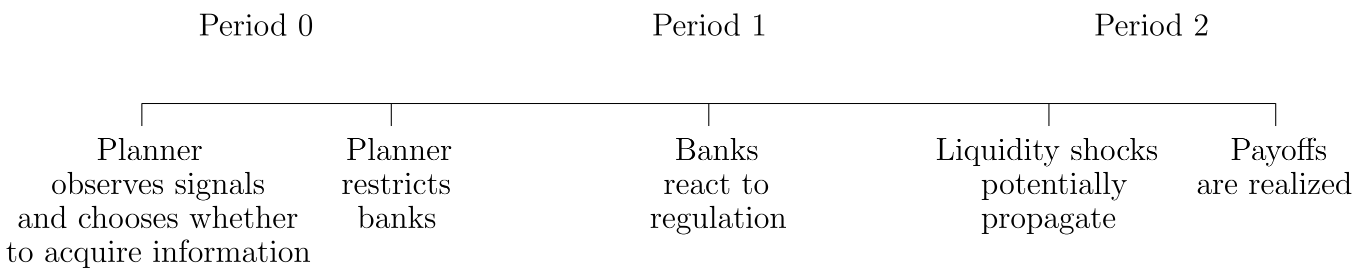

Figure 1. Model timeline

B. Banks' and the Planner's problems

Banks' problem. Banks choose the composition of their portfolio to maximize expected profits subject to their regulatory constraints. Let R_L and R_I denote the (random) payoff of liquid and illiquid assets to every bank, respectively. For each unit invested at $t = 1$, the (random) payoff of bank i at $t = 2$, π_i, equals

$$\pi_i(\omega_i) \quad = \quad \omega_i \times R_L + (1 - \omega_i) \times R_I - \beta_{\omega_i} \times \varepsilon_i, \tag{1}$$

where ω_i denotes the fraction of bank i's portfolio invested in liquid assets. Random variable ε_i equals 1 if bank i faces an adverse liquidity shock at $t = 2$; otherwise, $\varepsilon_i = 0$. The term β_{ω_i} captures the effect of liquidity shocks on bank i's payoff. For simplicity, ω_i takes two values, ω_L or ω_H, with $0 \leq \omega_L < \omega_H \leq 1$. Moreover, illiquid assets yield a higher expected payoff; hence, $\mathbb{E}[R_L] < \mathbb{E}[R_I]$.

Importantly, the liquidity of bank i's portfolio, ω_i, alters the effect of ε_i on bank i's payoff. In particular,

$$\beta_{\omega_i} \quad = \quad \begin{cases} 0, \text{ if } \omega_i = \omega_H \\ \omega_L \times R_L + (1 - \omega_L) \times R_I, \text{ otherwise.} \end{cases} \tag{2}$$

Equation (2) has two implications. First, banks with more liquid portfolios are not affected by liquidity shocks when aggregate conditions deteriorate, while banks with more illiquid portfolios default when facing a liquidity shock as they yield a payoff of zero. Consequently, when choosing ω_i, bank i faces the following trade-off: the more liquid its portfolio, the higher its resilience to liquidity shocks, but potentially the lower its future payoff. Second, banks with more liquid portfolios do not propagate liquidity shocks, as they are unaffected by such shocks. As a result, the planner's selection of restricted banks alters the spread of liquidity shocks at $t = 2$.

Within the model, restrictions take a simple form: the planner forces restricted banks to hold more liquid portfolios. By doing so, she aims to increase the likelihood that restricted banks absorb, rather than amplify, adverse liquidity shocks when a crisis manifests, mitigating the severity of cascades of liquidity shocks. The optimal portfolio allocation of bank i, ω_i^*, then solves

$$\begin{aligned} \max_{\omega_i \in \{\omega_L, \omega_H\}} \quad & \mathbb{E}_i \left[\pi_i(\omega_i) \right] \\ \text{s.t.} \quad & \omega_H \times e_i \leq \omega_i \text{ (regulatory constraint)}, \end{aligned} \tag{3}$$

where e_i equals 1 if bank i is restricted at $t = 0$ and 0 otherwise; operator $\mathbb{E}_i$ emphasizes that bank i chooses ω_i^* based on its available information and subjective beliefs. Problem (3) underscores that regulated banks optimize within the confines of regulatory constraints. Importantly, from bank i's perspective, investing in illiquid assets is more lucrative than storing funds in cash, as banks underestimate the likelihood of being affected by cascades of liquidity shocks (in a sense properly specified in Appendix A.A).[6] Consequently, bank i chooses $\omega_i^* = \omega_L$, unless the planner imposes restrictions on i.

The planner's problem. For ease of exposition, suppose the planner has decided whether to acquire bank-level information so as to improve network transparency. Let $\mathcal{I}$

[6]This feature of the model is consistent with the "underestimated risks" factor highlighted by the IGM Forum (2017) as one of the most prominent factors contributing to the 2007–2009 financial crisis as well as banks' lack of appreciation of downside risks highlighted by Gennaioli and Shleifer (2018).

denote her information set, and let $\mathcal{R}_{\mathcal{I}}$ denote a set of banks restricted based on $\mathcal{I}$, whose cardinality is denoted by $|\mathcal{R}_{\mathcal{I}}|$, with $0 \leq |\mathcal{R}_{\mathcal{I}}| \leq n$. The planner chooses $\mathcal{R}_{\mathcal{I}}$ to solve

$$\max_{\mathcal{R}_{\mathcal{I}}} \quad \mathbb{E}\left[\frac{1}{n}\text{TO}\middle|\mathcal{R}_{\mathcal{I}}\right] - \kappa \times \mathbb{1}_{\kappa} \tag{4}$$

where $\text{TO} = \sum_{i=1}^{n}[\pi_i + (1-\omega_i)y]$ denotes total output and y denotes the (random) output of projects financed at $t = 1$, with $\mathbb{E}[y] = \mu > 0$; $\left(\frac{1}{n}\right)$ is a normalization term. Variable $\mathbb{1}_{\kappa}$ is an indicator function that equals 1 if the planner acquires bank-level information and 0 otherwise. Notably, problem (4) underscores that the selection of $\mathcal{R}_I$ requires a holistic view of the health of the economy rather than of the health of individual banks, being mindful of the role the network structure plays in the propagation of liquidity shocks.

While restricting banks potentially curbs the spread of liquidity shocks—as restricted banks neither face nor propagate shocks—it also increases the losses associated with regulation. These losses arise because (a) restricted banks invest a higher fraction of their portfolio in assets that generate lower expected payoffs, and (b) fewer entrepreneurs are financed at $t = 1$, thereby decreasing their contribution to expected total output. Consequently, increasing network transparency might be important, as it could allow the planner to dampen contagion more efficiently. Although improving network transparency is socially costly, not improving transparency is also costly, as it results in welfare losses associated with regulating an excessively large number of banks.

Using the previous analysis, I now fully formulate the planner's problem. Let $\mathcal{I}_1$ denote the planner's information set after acquiring bank-level information; otherwise, her information set is denoted by $\mathcal{I}_0$. The planner chooses $\mathcal{I} \in \{\mathcal{I}_0, \mathcal{I}_1\}$ and $\mathcal{R}_{\mathcal{I}}$ to solve

$$\max_{\mathcal{I}\,\in\,\{\mathcal{I}_0,\mathcal{I}_1\}} \left\{ \max_{\mathcal{R}_{\mathcal{I}_0}} \mathbb{E}\left[\frac{1}{n}\text{TO}\middle|\mathcal{R}_{\mathcal{I}_0}\right],\ \max_{\mathcal{R}_{\mathcal{I}_1}} \left(\mathbb{E}\left[\frac{1}{n}\text{TO}\middle|\mathcal{R}_{\mathcal{I}_1}\right] - \kappa\right)\right\}. \tag{5}$$

C. Discussion of Assumptions and Equilibrium Outcomes

Why are contagious exposures randomly determined? The random selection of contagious exposures serves as a metaphor for the planner having difficulty assessing how cross-exposures will react when aggregate conditions deteriorate, as even regulators rarely have a comprehensive view of all relationships among banks. This difficulty makes the planner fundamentally uncertain which banks are more prone to propagate liquidity shocks in times of economic stress. Of course, this random selection provides a crude approximation of how liquidity shocks propagate when a crisis manifests. Yet, it allows me to provide a tractable analysis of cascades of liquidity shocks within the model.

What happens if contagious exposures are known? If contagious exposures are known, banks' actions would be strategic substitutes: an increase in the liquidity of bank i's portfolio reduces the incentives of its neighbors to increase the liquidity of their portfolios. Because bank i is resilient to liquidity shocks, its neighbors are less vulnerable to shocks that propagate through i, and, thus, i's neighbors have fewer incentives to increase the liquidity of their portfolios. It follows from Galeotti et al. (2018) that optimal interventions would target banks that do not necessarily share exposures so as to move neighbors' incentives in opposite directions. However, within my model, contagious exposures are unknown, and, thus, banks are ex ante identical—which, to an extent, facilitates the analysis, as banks' strategic considerations do not play a critical role.

Mapping liquidity restrictions to other policy tools. In a broad sense, liquidity restrictions within the model conceptually capture a diverse set of regulatory tools implemented after the 2007–2009 financial crisis. In particular, ω_H is assumed to be sufficiently large so that banks with more liquid portfolios are not vulnerable to liquidity shocks when aggregate conditions deteriorate. As a result, restrictions implicitly provide banks with better incentives for prudent risk-taking, generating greater buffers to support their operations when a crisis manifests.[7]

[7] In reality, liquidity restrictions take different forms. For example, with the aim of promoting the

While different forms of regulation, such as the risk-weighted capital ratio (RWC) and the liquidity coverage ratio (LCR) seek to address different distortions, arguments in favor of them share a common ground with the benefits of liquidity restrictions in the model. For example, the RWC aims to increase banks' skin in the game, effectively reducing their incentives to engage in excessive risk-taking, therefore decreasing banks' insolvency risk. Likewise, the LCR seeks to curb banks' incentives to engage in risky funding activities, thereby decreasing the likelihood of runs due to solvency concerns.

Inefficiency of the market equilibrium. Without regulation, every bank holds a fraction ω_L of its portfolio in liquid assets, as banks underestimate the likelihood of being affected by cascades of liquidity shocks at $t = 2$. To ensure that regulation potentially leads to a Pareto improvement, I assume there exists at least one bank that if restricted would cause expected total output to increase. Intuitively, an increase in the resilience of this bank—and the resilience of its (direct and indirect) neighbors—more than compensates the dead-weight losses associated with regulating such a bank. Consequently, the market equilibrium is not efficient and regulatory interventions are potentially welfare-improving. Appendix A.B provides detailed derivations.

II. Welfare Effects of Regulation

This section studies how regulation alters expected total output. Because restricted banks are forced to change their risk-taking behavior, regulation reshapes the way that liquidity shocks propagate at $t = 2$. In doing so, regulation modifies the distribution of total output when a crisis manifests.

Choosing how many banks to regulate. Within the model, restricted banks become resilient to liquidity shocks and their contagious exposures do not contribute to the spread

short-term resilience of banks, the liquidity coverage ratio requires banks to have enough liquidity to cover a 30-calendar-day liquidity stress period. Another example is the net stable funding ratio, which seeks to address significant maturity mismatches between assets and liabilities, providing banks with better buffers to absorb losses when affected by adverse liquidity shocks. See Tarullo (2019) for a broad description of the post-crisis approach to prudential regulation.

of shocks. Consequently, imposing restrictions on a set of banks can be represented by the removal of such banks (and their exposures) from any realized network of contagious exposures.

For ease of exposition, suppose the planner has decided whether to acquire bank-level information and restricts all banks belonging to an arbitrary set $\mathcal{R}$ of size $n \times x$, with $0 \leq x \leq 1$. Then,

$$\begin{aligned}\left(\frac{1}{n}\right)\text{TO} &= \left(\frac{1}{n}\right)\left(\sum_{i\in\mathcal{R}}\pi_i + (1-\omega_H)y\right) + \left(\frac{1}{n}\right)\left(\sum_{i\notin\mathcal{R}}\pi_i + (1-\omega_L)y\right) \\ &= x(R_I - \omega_H \Delta R) + \left(\frac{1}{n}\right)\left(\sum_{i\notin\mathcal{R}}\pi_i\right) + y(1-\omega_L - x\Delta\omega),\end{aligned}$$

where $\Delta\omega \equiv \omega_H - \omega_L$ and $\Delta R \equiv R_I - R_L$. To determine the distribution of $\left(\frac{1}{n}\right)\left(\sum_{i\notin\mathcal{R}}\pi_i\right)$, it is illustrative to analyze how shocks propagate when aggregate conditions deteriorate. Because the bank that initially faces a liquidity shock is selected uniformly at random, the probability that such a bank was restricted at $t=0$ is x. In this case, contagion is prevented from its onset, as restricted banks do not propagate liquidity shocks. However, if such a bank was not restricted, then at least one bank faces a liquidity shock and that shock might spread. Therefore,

$$\left(\frac{1}{n}\right)\sum_{i\notin\mathcal{R}}\pi_i = \begin{cases}(1-x)[R_I - \omega_L\Delta R], \text{ with probability } x \\ \left(1 - x - \frac{m}{n}\right)[R_I - \omega_L \Delta R], \text{ with probability } (1-x)\phi_m^x \text{ with } m = 1, \cdots, n(1-x),\end{cases}$$

where ϕ_m^x denotes the probability that m banks are affected by a liquidity shock at $t=2$ once a fraction x of banks have been restricted. After some algebra, it can be shown that (see Appendix A.C)

$$\left(\frac{1}{n}\right)\mathbb{E}[\text{TO}|x] = \eta - \underbrace{\nu(1-x)\frac{\langle\phi^x\rangle}{n}}_{\text{costs of contagion}} - \underbrace{x\Delta\omega(\mathbb{E}[\Delta R] + \mu),}_{\text{dead-weight losses}} \tag{6}$$

where $\nu \equiv \mathbb{E}[R_I] - \omega_L \mathbb{E}[\Delta R]$ and $\eta \equiv \nu + \mu(1-\omega_L)$. The term $\langle \phi^x \rangle \equiv \left(\sum_{m=1}^{n(1-x)} m \phi_m^x \right)$ denotes the expected number of banks affected by liquidity shocks when aggregate conditions deteriorate once a fraction x of banks have been restricted.

It directly follows from (6) that increasing x not only alters the costs arising from the spread of liquidity shocks—captured by $\nu(1-x)\frac{\langle \phi^x \rangle}{n}$—but also increases the losses arising from liquidity restrictions—captured by $x\Delta\omega(\mathbb{E}[\Delta R] + \mu)$. Intuitively, once a fraction x of banks are restricted, (a) banks' expected payoffs decrease by $x\Delta\omega\mathbb{E}[\Delta R]$, as restricted banks invest a higher fraction of their portfolio in assets with lower expected returns, and (b) the expected return from entrepreneurs drops from $(1-\omega_L)\mu$—when all banks invest a higher fraction of their portfolios in illiquid assets—to $(1-\omega_L)\mu - x\mu\Delta\omega$, when a fraction x of banks invests a higher proportion of their portfolios in liquid assets. As a result, the dead-weight losses associated with restricting a fraction x of banks is $x\Delta\omega(\mathbb{E}[\Delta R] + \mu)$.

Although increasing x directly increases the aforementioned losses, an increase in x does not necessarily decrease the cost arising from the spread of liquidity shocks. While increasing x decreases $(1-x)$, increasing x might increase or decrease $\frac{\langle \phi^x \rangle}{n}$. It is then pivotal to determine probabilities $\{\phi_m^x\}_{m=1}^{n(1-x)}$. While computing these probabilities is challenging—as liquidity shocks may propagate in intricate ways—these probabilities can be analytically determined within my model for economies with arbitrary sizes and connectivity structures; see Proposition 1 in Appendix A.C for more details.

Notably, the dependence of $\frac{\langle \phi^x \rangle}{n}$ on x hinges on the interplay between (a) the susceptibility of the economy to contagion—which is encoded in the distribution $\{p_k^\alpha\}_k$—and (b) how restricted banks are selected, as the composition of banks in $\mathcal{R}$ profoundly affects probabilities $\{\phi_m^x\}_{m=1}^{n(1-x)}$. Figures 2 and 3 illustrate this result, assuming $\{p_k^\alpha\}_{k=1}^n$ follows a Poisson distribution.[8] Figure 2(a) depicts $\langle \phi^x \rangle$ as a function of x if the planner were to restrict banks at random. When $\alpha = 1$, $\langle \phi^x \rangle$ is a weakly decreasing function of x, as increasing x tilts the distribution $\{\phi_m^x\}_{m=1}^{n(1-x)}$ (see figure 3(a)), making cascades of liquidity shocks relatively

[8]For more details about this result, see Proposition 2 in Appendix A.C.

less likely, thereby decreasing $\langle\phi^x\rangle$. However, when $\alpha > 1$, $\langle\phi^x\rangle$ may increase or decrease with x. For small values of x, increasing x tends to isolate banks with only few contagious exposures, making cascades relatively more likely (see figure 3(b)), which, in turn, increases $\langle\phi^x\rangle$. But, when x is relatively large, increasing x isolates a sufficiently large number of banks to tilt the distribution $\{\phi^x_m\}_{m=1}^{n(1-x)}$, curbing the likelihood of large cascades, thereby decreasing $\langle\phi^x\rangle$ (see figure 3(c)). Figure 2(b) highlights the importance of how restricted banks are selected by depicting $\langle\phi^x\rangle$ as a function of x if the planner were to restrict all banks with contagious exposures above a certain threshold—assuming she could rank banks based on their number of contagious exposures. As figure 2(b) shows, $\langle\phi^x\rangle$ continues to vary with x, but not necessarily in a continuous fashion.

Improving network transparency. It follows from (6) that a planner's decision to collect bank-level information so as to improve network transparency depends on how much that information helps reduce $\frac{\langle\phi^x\rangle}{n}$. If only a few banks play a key role in the propagation of shocks, targeting those banks substantially reduces $\frac{\langle\phi^x\rangle}{n}$, and, hence, collecting information about their identities may be worth the cost. Consequently, improving network transparency has an intrinsic social value to the extent that it allows the planner to dampen cascades of liquidity shocks more effectively. As the next section shows, this value is determined by aggregate characteristics of the distribution $\{p^\alpha_k\}_k$ and dictates the optimal choice of information.

III. Optimal Intervention

This section describes the optimal intervention—which is jointly determined by a choice of network transparency and a selection of restricted banks—and explores how that policy varies with the primitives of the model. For ease of exposition, I first explore the optimal selection of restricted banks, given a choice of network transparency. I then study the optimal choice of information.

A. *Selecting the optimal set of restricted banks*

Given a choice of network transparency, an optimal choice of x exists under somewhat general conditions; see Proposition 3 in Appendix A.D. To help illustrate the planner's trade-off when choosing x, I rewrite the first order condition of her optimization problem as

$$\underbrace{\nu\left(\frac{\langle\phi^{x^*}\rangle}{n}-(1-x^*)\frac{\partial}{\partial x}\left(\frac{\langle\phi^{x}\rangle}{n}\right)\bigg|_{x=x^*}\right)}_{\text{marginal benefit}} = \underbrace{\Delta\omega(\mathbb{E}[\Delta R]+\mu).}_{\text{marginal cost}} \tag{7}$$

Equation (7) shows that the optimal fraction of restricted banks, x^*, is deliberately selected so that the benefits of restricting the last bank are equal to the dead-weight losses associated with restricting such a bank. The marginal benefit arises because restricting the last bank not only increases the resilience of that bank but also increase the resilience of its (direct and indirect) neighbors, thereby decreasing the expected number of banks affected by the spread of liquidity shocks. The marginal cost arises because (a) the last restricted bank is forced to hold a higher fraction of its portfolio in assets with lower expected returns, and (b) a lower fraction of productive projects are financed at $t = 1$. To sum up, x^* is selected so as to limit the spread of liquidity shocks while at the same time avoiding excessive dead-weight losses from liquidity restrictions.[9]

B. *Optimal choice of network transparency*

Before choosing x^*, the planner decides whether to collect bank-level information to improve transparency regarding the structure of the network of contagious exposures. If she decides not to collect information, then banks are ex-ante identical from her point of view. By collecting information, however, she can strategically choose x^* to ensure that the

[9]Proposition 4 in Appendix A.D shows that, under certain conditions, the solution of the planner's problem is interior. More importantly, the optimal fraction of restricted banks gets arbitrarily close to x^*—the solution of equation (7)—as the economy grows large. For details about the general case, see Proposition 5 in Appendix A.D.

smallest number of banks possible is affected by the spread of liquidity shocks. Naturally, her decision to collect information will depend on how much that information helps her to mitigate contagion more effectively.

Let x_1 denote the optimal fraction of restricted banks chosen after acquiring certain bank-level information, and let x_0 denote the optimal fraction of restricted banks chosen when no information is acquired. The (social) value of improving network transparency, SVI, is then

$$\begin{aligned} \text{SVI} \equiv \left(\frac{1}{n}\right)(\mathbb{E}[\text{TO}|x_1] - \mathbb{E}[\text{TO}|x_0]) &= (x_0 - x_1)\Delta\omega(\mathbb{E}[\Delta R] + \mu) \\ &+ \nu\left((1-x_0)\frac{\langle\phi^{x_0}\rangle}{n} - (1-x_1)\frac{\langle\phi^{x_1}\rangle}{n}\right). \end{aligned} \tag{8}$$

Equation (8) highlights the two components of the value of information. Notably, these two components relate to the fact that improving network transparency potentially allows the planner to limit the spread of liquidity shocks more effectively. The first term, $(x_0 - x_1)\Delta\omega(\mathbb{E}[\Delta R] + \mu)$, captures the idea that more transparency might allow the planner to restrict fewer banks, decreasing the dead-weight losses associated with regulation. The second term, $\nu\left((1-x_0)\frac{\langle\phi^{x_0}\rangle}{n} - (1-x_1)\frac{\langle\phi^{x_1}\rangle}{n}\right)$, captures the idea that more transparency might allow the planner to decrease the spread of liquidity shocks more effectively, as she strategically chooses x_1 to ensure that the smallest number of banks possible are affected by contagion.

Importantly, different connectivity structures yield different values of network transparency, as SVI varies with $\{p_k^\alpha\}_k$, because x_0, x_1, $\langle\phi^{x_0}\rangle$, and $\langle\phi^{x_1}\rangle$ are implicit functions of $\{p_k^\alpha\}_k$. For example, when $\{p_k^\alpha\}_k$ is regular—that is, all banks exhibit the same number of contagious exposures at $t = 2$—banks behave in a similar fashion in times of economic stress. Thus, improving network transparency is not optimal, as more transparency does not allow the planner to uncover differences across banks in their role spreading liquidity shocks. However, when the number of contagious exposures varies across banks, improving transparency is potentially valuable.

Optimal rule. Within the model, the optimal choice of network transparency follows a simple rule. If SVI $\geq \kappa$, then the social benefit of more transparency outweighs its cost, and, thus, it is optimal to improve network transparency. Namely, improving network transparency is optimal if and only if

$$(x_0 - x_1)\Delta\omega(\mathbb{E}[\Delta R] + \mu) + \nu\left((1 - x_0)\frac{\langle \phi^{x_0} \rangle}{n} - (1 - x_1)\frac{\langle \phi^{x_1} \rangle}{n}\right) \quad \geq \quad \kappa.$$

Intuitively, different pieces of information yield different levels of network transparency, allowing the planner to target banks differently. As a result, the planner's decision to collect granular information is closely linked to (a) how much transparency the planner is able to obtain from such information, and (b) how useful that transparency is to limit the negative effects of cascades of liquidity shocks. Importantly, how useful transparency is fundamentally depends on the pattern of contagious exposures across banks.

C. Optimal interventions in large economies

While I can numerically solve for the optimal intervention, this section derives the optimal intervention in closed form under certain conditions in order to illustrate the importance of $\{p_k^\alpha\}_k$ for policy design. Within this section, I focus on an economy of infinite size. This is convenient for two reasons: (1) it considerably facilitates computations, and (2) it provides results that are potentially useful, as modern economies are comprised of a large number of institutions. Additionally, I assume that bank-level information allows the planner to rank banks based on their future number of contagious exposures. While, in reality, the planner might collect different pieces of information and, thus, decide between intermediate levels of network transparency, this assumption simplifies computations even further. Appendix A.E provides detailed derivations.

Under the aforementioned assumptions, the optimal fraction of restricted banks, x^*, equals

$$x^* = \begin{cases} x_r, \text{ if } \Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\} \\ x_t, \text{ if } \min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\} < \Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \frac{\nu-\kappa}{x_t} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{9}$$

where x_t and x_r denote the smallest fraction of banks that must be restricted to prevent large cascades of liquidity shocks if the planner targets banks with or without bank-level information, respectively; $\Delta x \equiv x_r - x_t$. Large cascades of liquidity shocks are defined as events in which a finite fraction of banks in an economy of infinite size face a liquidity shock as a result of any one bank initially facing a liquidity shock.[10]

Intuitively, when $\Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\}$, dead-weight losses associated with less efficient targeting—which results from not acquiring bank-level information—are sufficiently small. It is then optimal not to improve network transparency, and, thus, $x^* = x_r$. When $\min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\} < \Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \frac{\nu-\kappa}{x_t}$, less efficient targeting becomes sufficiently costly, as it involves restricting an excessively large fraction of banks. It is then optimal to improve network transparency so as to strategically select banks, and, thus, $x^* = x_t$. Finally, when $\Delta\omega(\mathbb{E}[\Delta R] + \mu) > \frac{\nu-\kappa}{x_t}$, dead-weight losses associated with regulation are too large, and, thus, a non-interventionist policy is optimal; that is, $x^* = 0$. Notably, preventing large cascades of liquidity shocks is not optimal in this case, regardless of how susceptible to contagion the economy might be.

The social value of improving network transparency, SVI, equals

$$\text{SVI} = \begin{cases} \Delta x \Delta\omega(\mathbb{E}[\Delta R] + \mu), \text{ if } \Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \frac{\nu}{x_r} \\ \nu - x_t \Delta\omega(\mathbb{E}[\Delta R] + \mu), \text{ if } \frac{\nu}{x_r} < \Delta\omega(\mathbb{E}[\Delta R] + \mu) \leq \frac{\nu}{x_t} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{10}$$

[10]In practice, these cascades capture situations in which a non-negligible fraction of banks in a large but finite economy face liquidity shocks as a result of a small set of banks initially facing liquidity shocks.

That is, if the marginal cost of regulation, $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$, is sufficiently small, i.e., $\Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu}{x_r}$, the value of improving network transparency is proportional to Δx, as more transparency allows the planner to target banks more effectively. Importantly, the value of improving transparency increases with $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$, as strategically targeting banks directly reduces the losses associated with excessive regulation. For intermediate values of $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$, i.e., $\frac{\nu}{x_r} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu}{x_t}$, it becomes optimal to improve network transparency as long as the cost of doing so is sufficiently small. The value of improving transparency then decreases with $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$ because, as $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$ increases, any intervention becomes more costly to begin with. For sufficiently large values of $\Delta\omega(\mathbb{E}[\Delta R]+\mu)$, a non-interventionist policy is optimal, and, thus, network transparency adds no efficiency gains from a policy perspective; therefore, SVI $= 0$.

Importantly, expressions (9) and (10) show that the optimal intervention crucially depends on the interplay between

- the marginal cost of regulation, $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$,
- x_t and x_r, and
- the (social) cost of improving network transparency, κ.

As a result, the interaction between these variables determines the optimal policy within the model. Importantly, this interaction is shaped by the nature of the network of contagious exposures, $\{p_k^\alpha\}_k$, as x_t and x_r are functions of $\{p_k^\alpha\}_k$.

To better illustrate how the optimal intervention varies with changes in the network structure, I now provide comparative statics under two distinct families of networks: Poisson and Power-law. In symmetric structures, such as Poisson networks, banks behave in a similar fashion from a shock propagation perspective. Consequently, the benefit of improving network transparency is capped by the network structure, as more transparency does not necessarily allow the planner to target banks more effectively. However, asymmetric structures, such as Power-law networks, exhibit a fundamentally different behavior, as a small fraction of banks drives the spread of liquidity shocks. As a result, the scope for welfare-improving interventions is intimately linked to the symmetry of the network of contagious exposures.

Poisson networks. Suppose p_k^α follows a Poisson distribution with parameter $\alpha > 1$—that is, $p_k^\alpha = \frac{\alpha^k e^{-\alpha}}{k!}$. Then,

$$x_r = 1 - \frac{1}{\alpha} \quad \text{and} \quad x_t = x_r - \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!},$$

where K_α solves $\frac{1}{\alpha} = \sum_{j=0}^{(K_\alpha-2)} \frac{e^{-\alpha}\alpha^j}{j!}$. Thus, $\Delta x = \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}$.

Notably, $\Delta x \to 0$ as $\alpha \to \infty$. While more transparency allows the planner to deliberately target banks, strategic targeting becomes less effective as α grows large. When $\{p_k^\alpha\}_k$ follows a Poisson distribution, banks behave in a similar fashion from the perspective of shock propagation. As α grows large, contagious exposures become more frequent. Consequently, there is less room for policy improvement, as more banks can transmit shocks widely as they exhibit an excessively high number of contagious exposures.

Figure 4(a) depicts x_r and x_t as a function of α. As α grows, the number of contagious exposures increases on average. Thereby, with or without bank-level information, a larger fraction of banks must be restricted to prevent large cascades.

Substituting x_r and x_t into (9) yields

$$x^* = \begin{cases} \left(1-\frac{1}{\alpha}\right), \text{ if } \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \min\left\{\frac{\nu\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} \\ \left(1-\frac{1}{\alpha}\right) - \left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right), \text{ if } \min\left\{\frac{\nu\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu-\kappa}{\left(1-\frac{1}{\alpha}\right)-\left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)} \\ 0, \quad \text{otherwise.} \end{cases} \tag{11}$$

Expression (11) highlights the critical role that the interaction between the network structure, parameterized by α, and the marginal cost of regulation, $\Delta\omega\,(\mathbb{E}[\Delta R]+\mu)$, plays in the determination of x^*. Figure 5(a) depicts x^* as a function of α for different values of $\Delta\omega\,(\mathbb{E}[\Delta R]+\mu)$, assuming $\kappa = 1/10$. When α is sufficiently small, contagious exposures are less frequent. Because banks behave in a similar fashion at $t = 2$, no single bank plays a determinant role in the spread of liquidity shocks. As a result, the planner has little incentive to improve network transparency if $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$ is sufficiently small, and, thus, $x^*=x_r$. However, as α increases, the economy becomes more prone to contagion, as more banks exhibit a higher number of contagious exposures, thereby increasing the planner's incentives to identify the set of most contagious banks. Consequently, $x^*=x_t$, unless $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$ is sufficiently small. When $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$ is sufficiently large, the dead-weight losses associated with excessive regulation are considerable. Thus, for larger values of α, $x^*=0$ as the costs associated with liquidity restrictions overcompensate the benefits associated with the prevention of large cascades.

Substituting x_r and x_t into (10) yields

$$\mathrm{SVI} = \begin{cases} \left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } \Delta\omega(\mathbb{E}[\Delta R]+\mu)\leq\frac{\nu\alpha}{\alpha-1} \\ \nu-\left(\left(1-\frac{1}{\alpha}\right)-\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } \frac{\nu\alpha}{\alpha-1}<\Delta\omega(\mathbb{E}[\Delta R]+\mu)\leq\frac{\nu}{\left(1-\frac{1}{\alpha}\right)-\left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{12}$$

Expression (12) shows how SVI depends on the interplay between the network structure and the marginal cost of regulation. Consistent with the above results, SVI $\to 0$ as $\alpha\to\infty$, as strategic targeting becomes less effective as α grows large.

To better illustrate how the network structure reshapes SVI, figure 5(b) depicts the value of improving network transparency as a function of α. Notably, when $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$ is sufficiently small, SVI increases with α in certain regions while it decreases in others. Intuitively, in regions where SVI is an increasing function of α, more transparency allows the planner to mitigate the spread of liquidity shocks more effectively. Hence, as the susceptibility of the economy to contagion increases, so does the value of network transparency. Why does SVI sharply decrease with α in certain regions? The reason lays in the behavior of $x_t(\alpha)$. Because more transparency allows the planner to rank banks based on contagious exposures, the optimal selection strategy takes a simple form within the model: restrict any bank with contagious exposures above a certain threshold. Because restricting the most contagious banks results in the removal of a large fraction of contagious exposures, the fraction of restricted banks might not always increase with α. This is because the number of banks with contagious exposures above a certain threshold does not monotonically increase with α. As a result, Δx might decrease with α in certain regions, and, in doing so, SVI might decrease with α. Finally, when $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$ is sufficiently large, improving network transparency tends to be optimal. However, strategic targeting becomes less effective as contagious exposures become more frequent. Hence, SVI weakly decreases with α for sufficiently high values of $\Delta\omega\left(\mathbb{E}[\Delta R]+\mu\right)$.

Power-law networks. Suppose p_k^α follows a Power-law distribution—that is, $p_k^\alpha \propto k^{-\alpha}$, with $\alpha > 1$. To keep things simple, assume further that the minimum number of contagious exposures per bank equals one. Then,

$$
\begin{aligned}
x_r &= \begin{cases} 1-\left(\left(\frac{2-\alpha}{3-\alpha}\right)-1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 < \alpha \leq 3. \end{cases} \\
x_t &= K_\alpha^{(1-\alpha)}
\end{aligned}
$$

where K_α satisfies $K_\alpha^{2-\alpha} - 2 = \left(\frac{2-\alpha}{3-\alpha}\right)\left(K_\alpha^{3-\alpha}-1\right)$. Thus,

$$
\Delta x = \begin{cases} 1-\left(\left(\frac{2-\alpha}{3-\alpha}\right)-1\right)^{-1} - K_\alpha^{(1-\alpha)} & \text{if } \alpha > 3 \\ 1 - K_\alpha^{(1-\alpha)} & \text{if } 1 < \alpha \leq 3. \end{cases}
$$

Figure 4(b) depicts x_r and x_t as a function of α. Notably, the difference between x_r and x_t in the Power-law case is visibly larger than in the Poisson case. Importantly, if $\alpha \leq 3$, preventing large cascades of liquidity shocks without improving network transparency requires the planner to restrict every bank, which might prove to be too difficult if the planner faces (exogenous political) restrictions on the maximum number of banks she can restrict. When $\alpha \leq 3$, there is sufficiently high variation in the number of contagious exposures across banks.

This ensures that liquidity shocks affecting one bank almost surely affect a non-negligible fraction of them through contagion. Because only a small fraction of banks exhibit an excessively large number of contagious exposures when $\alpha \leq 3$, the planner is likely to miss such banks if selecting banks without granular information, as she acts as if she were to restrict banks uniformly at random. Consequently, network transparency proves to be more helpful in the Power-law than in the Poisson case.

Figure 4(b) also highlights that x_t is a non-monotonic function of α. When $\alpha \leq 2$, an extremely small fraction of banks plays a key role in the propagation of liquidity shocks; hence, restricting these banks dampens the onset of contagion. Next, for larger values of α, the fraction of banks driving the propagation of shocks increases with α, thereby increasing the fraction of banks that must be restricted to avoid large cascades. Finally, for even larger values of α, the fraction of restricted banks decreases with α. This is because the size of the largest set of banks that are (potentially) affected by the spread of liquidity shocks decreases with α, even before restrictions are implemented. The same argument explains the behavior of $x_r(\alpha)$ when $\alpha \geq 3$.

As before, substituting x_r and x_t into expressions (9) and (10) yields the optimal fraction of restricted banks, x^*, and the social value of improving network transparency, SVI, as a function of α. For conciseness, I omit those expressions and illustrate the importance of the network structure via figures. Figure 6(a) depicts x^* as a function of α for different values of $\Delta\omega\,(\mathbb{E}[\Delta R] + \mu)$. Consistent with the previous analysis, when $\alpha \leq 3$, preventing large cascades without granular information requires restricting every bank. If $\Delta\omega(\mathbb{E}[\Delta R] + \mu)$ is not negligible, the planner might prefer to improve network transparency to minimize the dead-weight losses associated with excessive regulation; hence, $x^* = x_t$. For larger values of α, the size of the largest set of banks that are (potentially) affected by contagion decreases with α, and so does the incentive to improve network transparency. Consequently, when the marginal costs of regulation are sufficiently small, $x^* = x_r$. Otherwise, $x^* = x_t$. However, when the marginal cost of regulation is sufficiently large, a non-interventionist policy is optimal, and, thus, $x^* = 0$. Importantly, the region of the parameter space when a non-interventionist policy is optimal is much smaller in the Power-law case than in the Poisson case.

Figure 6(b) illustrates how the network structure reshapes SVI in Power-law networks. Because improving network transparency is optimal in a large region of the parameter space, the behavior of SVI(α) is mostly driven by how Δx changes with α. When $\alpha \leq 3$, large cascades are effectively prevented by improving transparency, as, without granular information, large cascades can only be prevented by restricting every bank. Thus, the value of improving network transparency is proportional to $(1 - x_t)$ in this case. Now, for larger values of α, there is no need to restrict every bank to prevent large cascades. However, restricting banks without information is less efficient, as a considerably larger fraction of banks must be restricted. Finally, when α is sufficiently large, the value of improving network transparency decreases with α. This is because the size of the largest set of banks that are potentially affected by the spread of liquidity shocks decreases with α, even before restrictions are implemented, and, as a consequence, there is less room for policy improvements as α increases.

The network structure matters. The marked differences between the connectivity structures of Poisson and Power-law networks underscore three important results. First, different network structures exhibit different susceptibility to contagion, imposing distinct challenges to planners when trying to mitigate cascades of liquidity shocks. Second, the ability of the planner to prevent cascades of liquidity shocks in certain networks heavily depends on how restricted banks are selected, and, thus, the knowledge available to the planner may be critical. Third, the scope for welfare-improving interventions is intimately linked to the symmetry of the network of contagious exposures.

IV. Extended Model

This section extends the baseline model to environments wherein the planner is uncertain about the distribution $\{p_k^\alpha\}_k$. This type of uncertainty fundamentally differs from the uncertainty captured by the baseline model. In the baseline model, the planner does not know the exact number of contagious exposures per bank, but she knows $\{p_k^\alpha\}_k$. Here, however, the planner is unable to pin down such a distribution. Consequently, she faces model uncertainty, as she is unsure about the distribution of contagious exposures, which ultimately determines how the economy behaves when aggregate conditions deteriorate.[11]

To better appreciate the implications of model uncertainty, suppose parameter α is random and unknown, while the functional form of $\{p_k^\alpha\}_k$ continues to be known. Because α affects how liquidity shocks spread, it is now difficult to analyze the distribution of total output. For a given value of α, if a fraction x of banks were restricted, the previous analysis shows that

$$\left(\frac{1}{n}\right)\mathbb{E}_\alpha[\text{TO}|x] \;=\; \eta - \nu(1-x)\left(\sum_{m=1}^{n(1-x)}\frac{m}{n}\phi_m^x(\alpha)\right) - x\Delta\omega(\mathbb{E}[\Delta R]+\mu)$$

where probabilities ϕ_m^x are written as $\phi_m^x(\alpha)$ to emphasize their dependence on the distribution $\{p_k^\alpha\}_k$. Importantly, probabilities $\phi_m^x(\alpha)$ are now random variables, as α is random, which, in turn, makes expected total output a random variable. As a result, the previous framework—in which x is selected to maximize expected total output—is incapable of dealing with this type of uncertainty.

To capture model uncertainty in a tractable way consistent with the previous analysis, I extend the baseline model along two dimensions. First, I consider a representative investor who owns all assets in the economy and has preferences that can be characterized by the smooth ambiguity model of Klibanoff et al. (2005). In a broad sense, these preferences capture circumstances in which investors are uncertain about the "true model" that determines the

[11]See Routledge and Zin (2009) and Easley and OHara (2010) for models that connect liquidity and model uncertainty. See Ruffino (2014) for a discussion of some implications of model uncertainty for regulation.

behavior of the economy. Given the uncertainty about the model, investors may exhibit aversion to (or preference for) that uncertainty. For example, if investors are averse to such uncertainty, they worry about making non-optimal decisions ex ante because they do not know the "true model." Importantly, with these preferences, investors' tastes for risk and model uncertainty (henceforth referred to as ambiguity) can be separated in a simple form that makes it tractable to nest the baseline model into the new framework.

Second, parameter α is unknown, but, to keep things simple, the exact distribution of contagious exposures across banks is assumed to belong to a known family of distributions—say, $\left\{\{p_k^{\alpha}\}_{k=0}^{n-1}\right\}_{\alpha\in\mathcal{A}}$—where $\mathcal{A}$ denotes the set of plausible values for α. While investors do not know the exact value of α, the observation of noisy signals allows them to generate subjective beliefs over $\mathcal{A}$. These beliefs are captured by a distribution f, which denotes a Borel probability measure on $\mathcal{A}$ with barycenter $\bar{\alpha} \equiv \int_{\alpha\in\mathcal{A}} \alpha df$. For consistency, the barycenter $\bar{\alpha}$ is assumed to satisfy

$$\sum_{k=0}^{s} p_k^{\bar{\alpha}} = \frac{1}{n}\sum_{i=1}^{n} 1_{s_i\leq s}, \quad -\infty < s < \infty, \tag{13}$$

where $1_{s_i\leq s}$ equals 1 if $s_i \leq s$, and 0 otherwise. Variable s_i represents a noisy signal about the future number of contagious exposures of bank i. Equation (13) implies that investors are able to infer $\bar{\alpha}$ from observing signals $\{s_i\}_{i=1}^{n}$.

A. Optimal Intervention

Optimal selection of restricted banks. Taking these two modifications into account, I now reformulate the planner's problem. Given a choice of information, the planner chooses a set of banks to regulate, $\mathcal{R}$, to solve

$$\begin{aligned} \max_{\mathcal{R}} \quad & \mathbb{E}_{\bar{\alpha}}\left(\frac{1}{n}\text{TO}_{\alpha}\middle|\mathcal{R}\right) - \left(\frac{\theta}{2}\right)\times\mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_{\alpha}\left(\text{TO}_{\alpha}\middle|\mathcal{R}\right)\right) - \kappa\times\mathbb{1}_{\kappa} \\ \text{s.t.} \quad & 0 \leq |\mathcal{R}| \leq n, \end{aligned} \tag{14}$$

where operator $\mathbb{E}_{\bar{\alpha}}(\cdot)$ denotes the expectation when $\alpha = \bar{\alpha}$. Operator $\mathbb{V}_f(\cdot)$ denotes the variance of expected total output, computed using distribution f. Parameter θ is a non-negative coefficient capturing the representative investor's attitude toward ambiguity. Notably, the extended model is equivalent to the baseline model when $\mathcal{A}$ is singleton or $\theta = 0$. When $\mathcal{A}$ is singleton, there is no model uncertainty, as the exact value of α is known. When $\theta = 0$, investors are ambiguity neutral. Thus, they do not mind not knowing α and behave as if $\alpha = \bar{\alpha}$.

To better illustrate the planner's trade-off in this new environment, I now rewrite the first order condition of her optimization problem (see Appendix A.F for more details)

$$\underbrace{\nu\left(\frac{\langle\phi_{\bar{\alpha}}^{x^a}\rangle}{n} - (1-x^a)\frac{\partial}{\partial x}\left(\frac{\langle\phi_{\bar{\alpha}}^{x}\rangle}{n}\right)\bigg|_{x=x^a}\right)}_{\text{marginal benefit}} = \Delta\omega(\mathbb{E}[\Delta R]+\mu) + \underbrace{\left(\frac{\theta}{2}\right)\nu^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi_{\alpha}^{x}\rangle}{n} - \frac{\langle\phi_{\bar{\alpha}}^{x}\rangle}{n}\right)^2 df(\alpha)\right)\bigg|_{x=x^a}}_{\text{marginal cost}}.$$

where $\langle\phi_{\bar{\alpha}}^{x^a}\rangle \equiv \sum_{m=1}^{n(1-x^a)}\phi_m^x(\bar{\alpha})$, and $\langle\phi_{\bar{\alpha}}^{x}\rangle \equiv \sum_{m=1}^{n(1-x)}\phi_m^x(\bar{\alpha})$. Proposition 7 in Appendix A.F shows that, under certain conditions, the optimal fraction of restricted banks gets arbitrarily close to x^a—which denotes the solution of the above equation—as the economy grows large.[12]

To appreciate the importance of model uncertainty, it is illustrative to emphasize the similarities between the above equation and equation (7). While the marginal benefit in both equations arises from the fact that the planner seeks to limit the spread of liquidity shocks, the marginal cost now has an extra component—the second term in the RHS of the above equation. Importantly, this component is unrelated to the dead-weight losses arising from liquidity restrictions but captures that the representative investor dislikes making non-optimal decisions ex-ante. This cost arises solely from the fact that (a) investors do not know α, and (b) they exhibit aversion to ambiguity, as $\theta > 0$. Consequently, the optimal fraction of

[12]The above equation characterizes the optimal fraction of restricted banks when the solution of the planner's problem is interior; for details about the general case, see Proposition 8 in Appendix A.F.

restricted banks, x^a, now also hinges on investors' subjective beliefs, captured by distribution f, and their attitudes toward ambiguity, captured by θ.

Social Value of Improving Transparency. Let x_1 denote the optimal fraction of restricted banks chosen after acquiring certain bank-level information, and let x_0 denote the optimal fraction of restricted banks chosen without acquiring information. The social value of improving transparency, SVI, is then

$$
\begin{aligned}
\text{SVI} &\equiv \left(\frac{1}{n}\right)\left(\mathbb{E}_{\bar{\alpha}}[\text{TO}|x_1] - \mathbb{E}_{\bar{\alpha}}[\text{TO}|x_0]\right) - \left(\frac{\theta}{2}\right)\left(\mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_{\alpha}\left(\text{TO}|x_1\right)\right) - \mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_{\alpha}\left(\text{TO}|x_0\right)\right)\right) \\
&= (x_0 - x_1)\Delta\omega(\mathbb{E}[\Delta R] + \mu) + \nu\left((1-x_0)\frac{\langle \phi_{\bar{\alpha}}^{x_0}\rangle}{n} - (1-x_1)\frac{\langle \phi_{\bar{\alpha}}^{x_1}\rangle}{n}\right) \\
&+ \left(\frac{\theta}{2}\right)\nu^2\left((1-x_0)^2\left[\int_{\alpha\in\mathcal{A}}\left(\frac{\langle \phi_{\alpha}^{x_0}\rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^{x_0}\rangle}{n}\right)^2 df(\alpha)\right] - (1-x_1)^2\left[\int_{\alpha\in\mathcal{A}}\left(\frac{\langle \phi_{\alpha}^{x_1}\rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^{x_1}\rangle}{n}\right)^2 df(\alpha)\right]\right).
\end{aligned}
$$

Thus, the social value of network transparency now has three components. The first two terms in the RHS of the above equation capture ideas similar to the two components described in the RHS of equation (8). The first term arises from the fact that, on average, more transparency might help decrease dead-weight losses generated from excessive regulation, as the planner might be able to restrict fewer banks. The second term captures the fact that, on average, more transparency might allow the planner to limit the spread of liquidity shocks more effectively. However, the third term is new and captures that α is unknown and investors are ambiguity averse. Intuitively, it represents the extent to which more transparency allows the planner to hedge the risk of making non-optimal decisions ex-ante as a result of not knowing α. Importantly, the perception of such risk is intimately linked to the underlying family of distributions $\{\{p_k^{\alpha}\}_k\}_{\alpha\in\mathcal{A}}$ as well as investors' subjective beliefs over $\mathcal{A}$, f.

Optimal choice of network transparency. As before, the optimal choice of network transparency follows a simple rule. If SVI $\geq \kappa$, the social benefit of more network transparency outweighs its cost, and, thus, it is optimal to improve network transparency. Notably, the extent of uncertainty regarding the network now plays a key role in the decision of whether to improve network transparency.

B. Importance of network uncertainty

To appreciate the importance of network uncertainty for policy design, I now explore (via numerical solutions) how the optimal intervention varies with (a) investors' subjective beliefs, $f(\alpha)$, and (b) investors' aversion to ambiguity, θ. To facilitate the comparison between the baseline and extended models, I focus on Poisson and Power-law networks. For concreteness, I assume hereafter that $\nu = 1$, $\Delta\omega(\mathbb{E}[\Delta R] + \mu) = 2$, $\kappa = 1/10$, and $n = 50$, while $f(\alpha)$ follows a truncated normal distribution with mean $\bar{\alpha} = 3$, variance $\sigma^2 > 0$, and $\mathcal{A} = [2, 4]$. In what follows, I capture variation in investors' subjective beliefs through variation in σ^2.

Poisson networks. Figure 7 highlights the implications for policy design of investors' subjective beliefs and their aversion to ambiguity. Figure 7(a) depicts the optimal fraction of restricted banks, x^*, as a function of σ^2. Unless θ is sufficiently large, variation in investors' beliefs does not generate variation in x^*. When investors do not experience sufficiently large disutility from making non-optimal decisions ex ante, it is optimal for the planner to choose her policy as if investors were to behave as if $\alpha = \bar{\alpha}$. Because $\bar{\alpha} = 3$ and $\Delta\omega(\mathbb{E}[\Delta R] + \mu) = 2$, a non-interventionist policy is optimal regardless of the extent of network uncertainty (which is consistent with figure 5(a)).

However, when investors exhibit sufficiently high aversion to ambiguity, the optimal policy is heavily affected by changes in investors' beliefs. As figure 7(a) shows, for even small values of σ^2, it becomes optimal to restrict a much larger fraction of banks. As σ^2 increases, the extent of uncertainty regarding the network structure increases. When facing high network uncertainty, investors are worried that the fraction of restricted banks may not be sufficiently large to prevent large cascades when a crisis manifests. As a result, it is optimal to weakly increase the fraction of restricted banks as network uncertainty increases. In other words, the lack of certainty about the network structure is not a justification for inaction, but rather the opposite, considering the considerable negative consequences of large cascades of distress. Importantly, figure 7(b) shows that in Poisson networks the value of information decreases as σ^2 increases. The reason is simple. The higher σ^2, the higher network uncertainty and, thus, the less informative network transparency is in an economy where banks behave in a similar fashion in times of economic stress. Consequently, improving network transparency is not optimal when θ and σ^2 are sufficiently high.

Power-law networks. Figure 8 shows that differences in the nature of the network of contagious exposures continue to have important implications for policy making under network uncertainty. Consistent with the previous results, figure 8(a) shows that x^* is heavily affected by changes in network uncertainty when investors exhibit sufficiently high aversion to ambiguity. Notably, figure 8(b) shows that the value of network transparency weakly increases with σ^2 in Power-law networks. Intuitively, because $\Delta\omega(\mathbb{E}[\Delta R] + \mu)$ is not negligible, it is costly to make mistakes by restricting an excessively large fraction of banks. Consequently, the higher the network uncertainty, the higher the planner's incentives to identify banks that drive the propagation of liquidity shocks so as to make her intervention more robust. Because banks dramatically differ in their role spreading liquidity shocks in Power-law networks, the value of network transparency (weakly) increases as σ^2 increases. Finally, for relatively large values of σ^2, the extent of network uncertainty is too large. As a result, SVI does not vary with σ^2, as the informativeness of network transparency is capped by network uncertainty.

V. Conclusion

My primary goal has been to show that it is possible to define an optimal solution of the problem of regulating a network of interdependent financial institutions under uncertainty regarding its precise structure. By incorporating results from the literature on random graphs, my model makes it possible to compute the optimal policy in economies with arbitrary sizes and network structures. In particular, when the network degree distribution is known, the baseline model provides a complete description of the optimal policy. When such a distribution is unknown, an extension of the model describes the optimal policy by drawing insights from the literature on decision-making under ambiguity. As the size of the economy grows large, policies that prevent large cascades of distress can be analytically determined.

While the proposed framework does not capture the economic incentives underlying the formation of interdependencies or the reasons some institutions may be more prone to propagating shocks than others, it provides a simple, yet general, approximation of the problem faced by policymakers nowadays, where the lack of detailed information and the high complexity of interactions among institutions besets the regulation and supervision of financial networks. In doing so, the proposed framework provides a benchmark to which other models can be compared to.

My emphasis on the relevance of network uncertainty should not be understood as downplaying the important role that leverage, size, and short-term funding play in the design of optimal policies. As the network structure interacts with these variables, policy interventions should be mindful of such an interaction so as to take into consideration how financial (and non financial) institutions react to regulation and how such reactions contribute to financial stability.

Finally, network uncertainty is not only a problem for regulators as it also gives rise to uncertainty for market participants, especially in times of economic stress. In doing so, network uncertainty itself can be a source of cascades of distress. Future research in this area is certainly called for.

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For Online Publication: Appendix for "Regulating Financial Networks Under Uncertainty"

Appendix A Mathematical Derivations

This section contains detailed derivations of results and propositions mentioned in the body of the paper.

A Banks' beliefs and their behavior

The payoff of bank i is given by

$$\pi_i = \omega_i R_L + (1-\omega_i) R_I - \beta_{\omega_i} \varepsilon_i,$$

where

$$\beta_{\omega_i} \quad = \quad \begin{cases} (\omega_L R_L + (1-\omega_L) R_I) \text{ with probability } p_i \text{ if } \ \omega_i = \omega_L, \\ 0, \qquad\qquad\qquad\qquad\qquad\quad \text{otherwise,} \end{cases}$$

with $p_i = \mathbb{P}[i \text{ faces a liquidity shock} | \omega_i = \omega_L]$. Given how liquidity shocks propagate among banks,

$$p_i(\mathcal{C}_i) = \frac{1}{n} + \left(1 - \frac{1}{n}\right) \frac{\mathbb{E}[|\mathcal{C}_i|]}{n}$$

where $\mathcal{C}_i$ denotes the set of banks (directly or indirectly) connected to bank i —via a sequence of contagious exposures at $t = 2$—whose portfolio contains a fraction ω_L in liquid assets. From bank i's perspective,

$$\mathbb{E}_i[\pi_i | \omega_i = \omega_H] - \mathbb{E}_i[\pi_i | \omega_i = \omega_L] \quad = \quad \nu \left[\frac{1}{n} + \left(1 - \frac{1}{n}\right) \frac{\mathbb{E}_i[|\mathcal{C}_i|]}{n} \right] - \Delta\omega \mathbb{E}[\Delta R].$$

Consequently, if bank i tends to underestimate the likelihood of being affected by cascades of liquidity shocks as n grows large—i.e., $\mathbb{E}_i[|\mathcal{C}_i|] = o(n)$—then

$$\lim_{n\to\infty} \left(\mathbb{E}_i[\pi_i | \omega_i = \omega_H] - \mathbb{E}_i[\pi_i | \omega_i = \omega_L] \right) \quad < \quad 0.$$

Namely, investing in illiquid assets is more lucrative than storing funds in cash.

B Inefficiency of the market equilibrium

Without regulation, every bank holds a fraction ω_L of its portfolio in liquid assets. As a result, the expected total output generated in the market equilibrium, $\mathbb{E}[TO_E]$, equals

$$\begin{aligned}\mathbb{E}[\mathrm{TO_E}] &= \mathbb{E}\left[\sum_{i=1}^{n}\pi_i + (1-\omega_L)y\right] \\ &= \underbrace{n\left(\omega_L\mathbb{E}[R_L] + (1-\omega_L)\mathbb{E}[R_I]\right) - \left(\sum_{i=1}^{n}\mathbb{E}_0[\beta_{\omega_i}\varepsilon_i]\right)}_{\text{banks' profits}} + \underbrace{n(1-\omega_L)\mu}_{\text{projects' payoffs}}\end{aligned}$$

where $\mathbb{E}_0[\beta_{\omega_i}\varepsilon_i] = \mathbb{E}[\beta_{\omega_i}\varepsilon_i|\omega_i = \omega_{-i} = \omega_L]$. The first term in the above expression corresponds to the sum of banks' expected profits, while the second term corresponds to the expected payoffs of projects financed at $t = 1$.

To appreciate the potential benefits of regulation, suppose only one bank, say bank i, is forced to hold a fraction ω_H of its portfolio in liquid assets. Let $\mathbb{E}[TO_i]$ denote expected total output in this case. Then, $\mathbb{E}[TO_i]$ equals

$$\begin{aligned}\mathbb{E}[\mathrm{TO}_i] = &\overbrace{\omega_H\mathbb{E}[R_L] + (1-\omega_H)\mathbb{E}[R_I] - \mathbb{E}'[\beta_{\omega_i}\varepsilon_i] + (1-\omega_H)\mu}^{\text{output derived from bank } i\text{'s response to regulation}} + \\ &\underbrace{(n-1)\left(\omega_L\mathbb{E}[R_L] + (1-\omega_L)\mathbb{E}[R_I]\right) - \left(\sum_{j\neq i}^{n}\mathbb{E}''[\beta_{\omega_j}\varepsilon_j]\right) + (n-1)(1-\omega_L)\mu,}_{\text{output derived from other banks' actions}}\end{aligned}$$

with $\mathbb{E}'[\beta_{\omega_i}\varepsilon_i] = \mathbb{E}[\beta_{\omega_i}\varepsilon_i|\omega_i = \omega_H \text{ and } \omega_{-i} = \omega_L]$ and $\mathbb{E}''[\beta_{\omega_j}\varepsilon_j] = \mathbb{E}[\beta_{\omega_j}\varepsilon_j|\omega_i = \omega_H \text{ and } \omega_{-i} = \omega_L]$. Thus, the difference $(\mathbb{E}[\mathrm{TO}_i] - \mathbb{E}[\mathrm{TO}_E])$ describes the welfare effects of imposing liquidity requirements on bank i. This difference can be written as

$$\underbrace{\left(\mathbb{E}_0[\beta_{\omega_i}\varepsilon_i] - \mathbb{E}'[\beta_{\omega_i}\varepsilon_i]\right) + \sum_{j\neq i}^{n}\left(\mathbb{E}_0[\beta_{\omega_j}\varepsilon_j] - \mathbb{E}''[\beta_{\omega_j}\varepsilon_j]\right)}_{\text{benefit}} - \underbrace{\Delta\omega\left[\mathbb{E}[\Delta R] + \mu\right]}_{\text{cost}}.$$

Thus, the benefit of increasing the liquidity of bank i's portfolio is composed of two terms. The first term captures the increase in bank i's resilience to liquidity shocks. The second term captures the increase in the resilience of bank i's neighbors (and the neighbors of those neighbors, and so on), as bank i no longer propagates shocks when conditions deteriorate. Importantly, bank i fails to internalize this term when choosing ω_i. The cost of increasing bank i's liquidity is also composed of two terms. The first term captures the decrease in the expected payoff of bank i, as illiquid assets yield a higher expected payoff. The second term—which is also not considered by bank i—captures the decrease in the amount of external financing available to productive projects, as a fraction $\Delta\omega$ of projects are no longer financed as a result of i being force to hold a more liquid portfolio.

To ensure that regulation can potentially lead to a Pareto improvement, I assume that there exists at least one bank, say bank l, so that the following inequality is satisfied

$$(\mathbb{E}_0[\beta_{\omega_l}\varepsilon_l] - \mathbb{E}'[\beta_{\omega_l}\varepsilon_l]) + \sum_{j\neq l}^{n}(\mathbb{E}_0[\beta_{\omega_j}\varepsilon_j] - \mathbb{E}''[\beta_{\omega_j}\varepsilon_j]) \quad > \quad \Delta\omega\,[\mathbb{E}[\Delta R] + \mu]\,.$$

Namely, the increase in resilience of bank l, its neighbors, and the neighbors of those neighbors, $(\mathbb{E}_0[\beta_{\omega_l}\varepsilon_l] - \mathbb{E}'[\beta_{\omega_l}\varepsilon_l]) + \sum_{j\neq l}^{n}(\mathbb{E}_0[\beta_{\omega_j}\varepsilon_j] - \mathbb{E}''[\beta_{\omega_j}\varepsilon_j])$, more than compensates the dead-weight losses associated with regulating bank l, $\Delta\omega\,[\mathbb{E}[\Delta R] + \mu]$. Consequently, the market equilibrium is not efficient, and regulatory interventions are potentially welfare-improving.

C *Welfare Effects of Regulation*

Suppose the planner restricts all banks in $\mathcal{R}$, with $|\mathcal{R}| = nx$. Then

$$\left(\frac{1}{n}\right)\mathbb{E}[\mathrm{TO}|x] \quad = \quad x(\mathbb{E}[R_I] - \omega_H\mathbb{E}[\Delta R]) + \left(\frac{1}{n}\right)\sum_{i\notin\mathcal{R}}\mathbb{E}[\pi_i] + \mu(1-\omega_L - x\Delta\omega)$$

It is worth noting that

$$\left(\frac{1}{n}\right)\sum_{i\notin\mathcal{R}}\pi_i \quad = \quad \begin{cases}(1-x)[R_I - \omega_L\Delta R], & \text{with probability } x\\ \left(1-x-\frac{m}{n}\right)[R_I - \omega_L\Delta R], & \text{with probability } (1-x)\phi^x_m \text{ with } m = 1,\cdots,n(1-x).\end{cases}$$

where ϕ^x_m denotes the probability that m nonrestricted banks are affected by liquidity shocks, once $n\times x$ banks have been restricted. Consequently,

$$\begin{aligned}\left(\frac{1}{n}\right)\mathbb{E}[\mathrm{TO}|x] \quad &= \quad x(\mathbb{E}[R_I] - \omega_H\mathbb{E}[\Delta R])\\ &+ \quad (\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R])\left(x(1-x) + (1-x)^2\underbrace{\left(\sum_{m=1}^{n(1-x)}\phi^x_m\right)}_{=1} - \frac{(1-x)}{n}\underbrace{\sum_{m=1}^{n(1-x)}m\phi^x_m}_{=\langle\phi^x\rangle}\right)\\ &+ \quad \mu(1-\omega_L - x\Delta\omega)\\ &= \quad x(\mathbb{E}[R_I] - \omega_H\mathbb{E}[\Delta R]) + (1-x)(\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R])\left(1 - \frac{\langle\phi^x\rangle}{n}\right) + \mu(1-\omega_L - x\Delta\omega)\\ &= \quad (\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R]) + \mu(1-\omega_L) - (1-x)\left(\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R]\right)\frac{\langle\phi^x\rangle}{n} - x\Delta\omega(\mathbb{E}[\Delta R] + \mu)\\ &= \quad \eta - \nu(1-x)\frac{\langle\phi^x\rangle}{n} - x\Delta\omega(\mathbb{E}[\Delta R] + \mu)\end{aligned}$$

where $\langle\phi^x\rangle$ denotes the expected number of nonrestricted banks affected by liquidity shocks once $n\times x$ banks have been restricted.

To pin down the optimal set of restricted banks, it is pivotal to determine probabilities $\{\phi^x_m\}_{m=1}^{n(1-x)}$. While computing these probabilities is challenging, the following two propositions tell us that, within the model, these probabilities can be analytically determined.

PROPOSITION 1 (Probabilities $\phi^{x_{\mathcal{I}}}_m$): *Suppose all banks within an arbitrary set $\mathcal{R}_{\mathcal{I}}$ are restricted, with $\frac{|\mathcal{R}_{\mathcal{I}}|}{n} = x_{\mathcal{I}}$. Let $\theta^{x_{\mathcal{I}}}_k = \theta^{x_{\mathcal{I}}}_k(p^{\alpha}_k)$ denote the probability that a nonrestricted bank shares k contagious exposures with other nonrestricted banks, with $k = \{0, \cdots, n - |\mathcal{R}_{\mathcal{I}}| - 1\}$. Then*

$$\phi^{x_{\mathcal{I}}}_m = \begin{cases} \frac{\langle \theta^{x_{\mathcal{I}}} \rangle}{(m-1)!} \left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta^{x_{\mathcal{I}}}_k\}_k\right)^m \right] \right) \Big|_{z=0}, & \text{with } m = \{2, \cdots, n - |\mathcal{R}_{\mathcal{I}}|\} \\ \theta^{x_{\mathcal{I}}}_0 & , \text{with } m = 1, \end{cases}$$

where $\left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta^{x_{\mathcal{I}}}_k\}_k\right)^m \right] \right) \Big|_{z=0}$ denotes the $(m-2)$ derivative of $g\left(z, \{\theta^{x_{\mathcal{I}}}_k\}_k\right)^m$ evaluated at $z = 0$, with

$$g(z, \{\theta^{x_{\mathcal{I}}}_k\}_k) = \sum_{k=0}^{n-|\mathcal{R}_{\mathcal{I}}|-2} \left(\frac{(k+1)\theta^{x_{\mathcal{I}}}_{k+1}}{\langle \theta^x \rangle} \right) z^k \quad \text{and} \quad \langle \theta^{x_{\mathcal{I}}} \rangle = \sum_{k=0}^{n-|\mathcal{R}_{\mathcal{I}}|-1} k \theta^{x_{\mathcal{I}}}_k.$$

Proof. It is worth noting that $\{\theta^r_k\}_{k=0}^{n-r-1}$ represents the degree distribution of a randomly generated network among $(n-r)$ banks. Consequently, ϕ^r_m is equivalent to the probability that a randomly chosen nonrestricted bank belongs to a connected subgraph of size m. Therefore, one can directly apply results in Newman (2007) and show that

$$\phi^r_m = \begin{cases} \frac{\langle \theta^r \rangle}{(m-1)!} \left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta^r_k\}_k\right)^m \right] \right) \Big|_{z=0}, & \text{with } m = \{2, \cdots, n - r\} \\ \theta^r_0 & , \text{with } m = 1. \end{cases}$$

where $\langle \theta^r \rangle$ denotes the average number of contagious exposures among nonrestricted banks and $\left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta^r_k\}_k\right)^m \right] \right) \Big|_{z=0}$ denotes the $(m-2)$ derivative of $g\left(z, \{\theta^r_k\}_k\right)^m$ evaluated at $z = 0$, where $g(z, \{\theta^r_k\}_k)$ represents the excess degree distribution function of $\{\theta^r_k\}_k$, defined as

$$g(z, \{\theta^r_k\}_k) \equiv \sum_{k=0}^{n-r-2} \left(\frac{(k+1)\theta^r_{k+1}}{\langle \theta^r \rangle} \right) z^k.$$

□

REMARK 1 (Numerical solutions): *When solving the model numerically, the $(m-2)$ derivative of $g\left(z, \{\theta^r_k\}_k\right)^m$ evaluated at $z = 0$, can be approximated by*

$$\frac{1}{\epsilon^{m-2}} \left[\sum_{j=0}^{m-2} (-1)^{m-2-j} \binom{m-2}{j} g\left(j\epsilon, \{\theta^r_k\}_k\right)^m \right]$$

with $\epsilon > 0$ sufficiently small.

Given probabilities $\{\theta_k^{x_\mathcal{I}}\}_k$, Proposition 1 determines the likelihood of cascades of liquidity shocks for any given size m. In doing so, it allows me to keep track of the entire size distribution of cascades of liquidity shocks at $t = 2$. More importantly, it allows me to characterize the distribution of total output.

The following proposition determines probabilities $\{\theta_k^{x_\mathcal{I}}\}_k$ as a function of the planner's information set and the network of contagious exposures, $\{p_k^\alpha\}_k$. Importantly, Proposition 2 highlights the importance of the selection of restricted banks on the distribution of probabilities $\{\theta_k^{x_\mathcal{I}}\}_k$, and, thus, on the susceptibility of the economy to contagion, after restrictions have been implemented.

PROPOSITION 2 (Probabilities $\{\theta_k^{x_\mathcal{I}}(p_k^\alpha)\}_k$): *Let $\langle k\rangle = \sum_{k=0}^{n-1} kp_k^\alpha$ denote the expected number of contagious exposures per bank when a crisis manifests and no bank has been restricted. Suppose the planner restricts a fraction $x_\mathcal{I}$ of banks, with $\mathcal{I} = \{\mathcal{I}_0, \mathcal{I}_1\}$. Here $\mathcal{I}_0$ denotes the planner's information set when no information is acquired while $\mathcal{I}_1$ denotes the planner's information set when bank-level information is acquired. Suppose that information allows the planner to rank banks based on the future number of contagious exposures.*

- *If $\mathcal{I} = \mathcal{I}_0$, then*

$$\theta_k^{x_{\mathcal{I}_0}} = \begin{cases} \sum_{j=k}^{n-1} p_j^\alpha \binom{j}{k} (1 - x_{\mathcal{I}_0})^k \, x_{\mathcal{I}_0}^{j-k} & \text{if } k = \{0, \cdots, n(1 - x_{\mathcal{I}_0}) - 1\} \\ 0 & \text{otherwise.} \end{cases}$$

- *If $\mathcal{I} = \mathcal{I}_1$, then*

$$\theta_k^{x_{\mathcal{I}_1}} = \begin{cases} \sum_{j=k}^{k_x} p_j^\alpha \binom{j}{k} (1 - \kappa_x)^k \kappa_x^{j-k} & \text{if } k = \{0, \cdots, k_x\} \\ 0 & \text{otherwise,} \end{cases}$$

where k_x and κ_x satisfy

$$x_{\mathcal{I}_1} = 1 - \sum_{k=0}^{k_x} p_k^\alpha \quad , \quad \kappa_x = 1 - \frac{1}{\langle k\rangle}\left(\sum_{k=0}^{k_x} kp_k^\alpha\right).$$

Proof. There are two cases.

(a) $\mathcal{I} = \mathcal{I}_0$. Here, banks are ex ante identical from the point of view of the planner, as she is unable to identify whether some banks will exhibit more contagious exposures than others. Hence, when solving her problem, the planner effectively acts as if she restricts banks uniformly at random.
It is then illustrative to explore the distribution of contagious exposures among non-restricted banks after imposing restrictions on a fraction $\frac{r}{n}$. Consider a bank with k_0 contagious exposures. After imposing restrictions, that bank may have k contagious exposures, with $k \leq k_0$, as some of its neighbors may be restricted. Additionally, the probability that a subset of k neighbors is not restricted is $\left(1 - \frac{r}{n}\right)^k$, whereas the probability that the remaining neighbors are restricted is $\left(\frac{r}{n}\right)^{k_0 - k}$. Because there are

$\binom{k_0}{k}$ different subsets of k neighbors, the distribution of contagious exposures among nonrestricted banks is

$$\theta_k^r \;=\; \begin{cases} \sum_{j=k}^{n-1} p_j^\alpha \binom{j}{k} \left(1-\frac{r}{n}\right)^k \left(\frac{r}{n}\right)^{j-k} & \text{if } k=\{0,\cdots,n-r-1\} \\ 0 & \text{otherwise.} \end{cases} \tag{A1}$$

In other words, θ_k^r captures the probability that a nonrestricted bank shares k contagious exposures with other nonrestricted banks, once r banks are restricted.

(b) $\mathcal{I} = \mathcal{I}_1$. Here, the planner is able to identify which banks will exhibit the highest number of contagious exposures. Then, she can use that information and restrict such banks first to prevent contagion more efficiently than restricting at random, as such intervention results in the removal of a larger fraction of contagious exposures.
Suppose the planner imposes restrictions on all banks with more than k_x contagious exposures, with $k_x \geq s^*$. Restricting those banks is equivalent to restricting a fraction x of banks with the highest number of contagious exposures. The relationship between x and k_x is given by

$$x \;=\; \sum_{k_x<k} p_k^\alpha \quad\Longrightarrow\quad x \;=\; 1-\sum_{k=0}^{k_x} p_k^\alpha. \tag{A2}$$

Implementing the above policy results in an approximate random removal of contagious exposures from nonrestricted banks, as contagious exposures of restricted banks no longer propagate liquidity shocks. The probability κ_x that a contagious exposure leads to a restricted bank equals

$$\kappa_x \;=\; \sum_{k_x<k} \frac{kp_k^\alpha}{\sum_k kp_k^\alpha} \;=\; \frac{1}{\langle k\rangle}\left(\sum_{k_x<k} kp_k^\alpha\right) \quad\Longrightarrow\quad \kappa_x \;=\; 1-\frac{1}{\langle k\rangle}\left(\sum_{k=0}^{k_x} kp_k^\alpha\right). \tag{A3}$$

It is important to note that the network of contagious exposures that remains after implementing the above policy is equivalent to a network in which the maximum number of contagious exposures per bank is k_x and a fraction κ_x of banks is restricted uniformly at random. It follows from the previous analysis that the probability that a nonrestricted bank has k contagious exposures once a fraction x of banks has been restricted, φ_k^x, is given by

$$\varphi_k^x \;=\; \begin{cases} \sum_{j=k}^{k_x} p_j^\alpha \binom{j}{k} (1-\kappa_x)^k \kappa_x^{j-k} & \text{if } k=\{0,\cdots,k_x\} \\ 0 & \text{otherwise.} \end{cases} \tag{A4}$$

□

Proposition 2 illustrates the importance of bank-level information for the selection of restricted banks. When $\mathcal{I} = \mathcal{I}_0$, the planner cannot identify whether some banks are more prone to propagate liquidity shocks than others, and, thus, she effectively acts as if she restricts banks uniformly at random. However, when $\mathcal{I} = \mathcal{I}_1$, the planner is able to identify the most contagious banks. She then restricts such banks first to prevent contagion more efficiently.

The computation of probabilities $\{\phi_m^{x_\mathcal{I}}\}_{m=1}^{n(1-x_\mathcal{I})}$ in propositions 1 and 2 requires calculating sums and derivatives, which can always be done numerically for any finite number of banks. However, for certain network structures, it is possible to derive a closed-form expression for these probabilities, as example 1 shows.

EXAMPLE 1 (Poisson Distribution): *Suppose* $\{p_k^\alpha\}_{k=0}^{n-1}$ *follows a Poisson distribution, that is,* $p_k^\alpha = e^{-\alpha}\frac{\alpha^k}{k!}$*, and a fraction* $x_\mathcal{I}$ *of banks is restricted.*

- *If* $\mathcal{I} = \mathcal{I}_0$*, then*

$$\phi_m^{x_{\mathcal{I}_0}} = \frac{e^{-\left(1-x_{\mathcal{I}_0}\right)\alpha m}\left(\left(1-x_{\mathcal{I}_0}\right)\alpha m\right)^{m-1}}{m!}, \quad m = \{1, \cdots, n(1-x_{\mathcal{I}_0})\}.$$

- *If* $\mathcal{I} = \mathcal{I}_1$*, then* $\kappa_x = 1 - \frac{1}{\alpha}\left(e^{-\alpha}\sum_{k=0}^{k_x}\frac{k\alpha^k}{k!}\right)$*,* $x_{\mathcal{I}_1} = e^{-\alpha}\sum_{k=k_x}^{n-1}\frac{\alpha^k}{k!}$*, and*

$$\phi_m^{x_{\mathcal{I}_1}} = \frac{e^{-(1-\kappa_x)\alpha m}\left((1-\kappa_x)\alpha m\right)^{m-1}}{m!}, \quad m = \{1, \cdots, \lceil n\left(1-x_{\mathcal{I}_1}\right)\rceil\}.$$

Proof. There are two cases.

(a) $\mathcal{I} = \mathcal{I}_0$. When $\{p_k^\alpha\}_{k=0}^{n-1}$ follows a Poisson distribution with parameter α, $\{\theta_k^r\}_{k=0}^{n-r-1}$ approximately follows a Poisson distribution of parameter $\left(1-\frac{r}{n}\right)\alpha$. As a result,

$$g(z, \{\theta_k^r\}_{k=0}^{n-r-1}) = e^{\left(1-\frac{r}{n}\right)\alpha(z-1)},$$

and, thus,

$$\phi_m^r = \frac{e^{-\left(1-\frac{r}{n}\right)\alpha m}\left(\left(1-\frac{r}{n}\right)\alpha m\right)^{m-1}}{m!}, \quad m = \{1, \cdots, n-r\}.$$

(b) $\mathcal{I} = \mathcal{I}_1$. Here, $p_k^\alpha = e^{-\alpha}\frac{\alpha^k}{k!}$, with $k = \{0, n-1\}$. The result follows directly from (a) after substituting p_k^α into

$$\kappa_x = \sum_{k_x<k}\frac{kp_k^\alpha}{\sum_k kp_k^\alpha} = \frac{1}{\langle k\rangle}\left(\sum_{k_x<k}kp_k^\alpha\right) \implies \kappa_x = 1 - \frac{1}{\langle k\rangle}\left(\sum_{k=0}^{k_x}kp_k^\alpha\right).$$

□

D Optimal Interventions

PROPOSITION 3 (Existence of the maximum): *Suppose the planner only considers interventions in which* $\frac{\langle\phi^x\rangle}{n}$ *is a lower semi-continuous function of* x*—that is,*

$$\langle\phi^{x_0}\rangle \leq \liminf_{x\to x_0}\langle\phi^x\rangle, \quad \textit{with} \quad \frac{1}{n} \leq x_0 \leq 1.$$

Then, there exists a point $\frac{1}{n} \leq x^* \leq 1$ *that solves*

$$\max_{x} \quad \left(\frac{1}{n}\right) \mathbb{E}\left[TO \middle| x\right] - \kappa \times \mathbb{1}_\kappa$$
$$s.t. \quad \frac{1}{n} \leq x \leq 1,$$

Proof. Note that $\left(\frac{1}{n}\right) \mathbb{E}[\text{TO}|x]$ is bounded for all $0 \leq x \leq 1$, as $0 \leq \frac{\langle \phi^x \rangle}{n} \leq (1-x)$. If $g(x) \equiv \frac{\langle \phi^x \rangle}{n}$ is a lower semi-continuous function of x, then the image points near x under $g(\cdot)$ never fall below $g(x)$ too much. Importantly, these images points can still be vastly greater than $g(x)$.

Because $g(x)$ is lower semi-continuous, $\left(\frac{1}{n}\right) \mathbb{E}[\text{TO}|x]$ is an upper semi-continuous function of x. Baire's generalization of the Weierstrass' theorem ensures that $\left(\frac{1}{n}\right) \mathbb{E}[\text{TO}|x]$ always attains its maximum over a compact set; see (Ok, 2007, Chapter 4). □

Proposition 3 ensures there exists a maximum. The following proposition ensures the solution is interior.

PROPOSITION 4 (Interior Solution): *Let* $z = (1-\alpha)x_0 + \alpha x_1 \in \left(\frac{1}{n}, 1\right)$, *with* $\frac{1}{n} \leq x_0 < x_1 \leq 1$ *and* $\alpha \in (0,1)$. *If*

$$\frac{(1-\alpha)(1-x_0)\langle \phi^{x_0} \rangle + \alpha(1-x_1)\langle \phi^{x_1} \rangle}{(1-\alpha)(1-x_0) + \alpha(1-x_1)} \quad > \quad \langle \phi^z \rangle$$

for all $z \in \left(\frac{1}{n}, 1\right)$, *then the optimal intervention,* x^*, *gets arbitrarily close to the solution of the following equation*

$$\nu \left(\frac{\langle \phi^{x^*} \rangle}{n} - (1-x^*) \frac{\partial}{\partial x} \frac{\langle \phi^x \rangle}{n} \bigg|_{x=x^*} \right) \quad = \quad \Delta\omega(\mathbb{E}[\Delta R] + \mu)$$

as n *gets large.*

Proof. It is worth noting that

$$\begin{aligned}
\left(\frac{1}{n}\right) \mathbb{E}[\text{TO}|z] &= \eta - (1-z)\left(\frac{\nu}{n}\right)\langle \phi^z \rangle - z\Delta\omega(\mathbb{E}[\Delta R] + \mu) \\
&= (1-\alpha)\left(\eta - (1-x_0)\left(\frac{\nu}{n}\right)\langle \phi^z \rangle - x_0\Delta\omega(\mathbb{E}[\Delta R] + \mu)\right) \\
&+ \alpha\left(\eta - (1-x_1)\left(\frac{\nu}{n}\right)\langle \phi^z \rangle - x_1\Delta\omega(\mathbb{E}[\Delta R] + \mu)\right) \\
&= (1-\alpha)\left(\eta - (1-x_0)\left(\frac{\nu}{n}\right)\langle \phi^{x_0} \rangle - x_0\Delta\omega(\mathbb{E}[\Delta R] + \mu)\right) \\
&+ \alpha\left(\eta - (1-x_1)\left(\frac{\nu}{n}\right)\langle \phi^{x_1} \rangle - x_1\Delta\omega(\mathbb{E}[\Delta R] + \mu)\right) \\
&+ \left(\frac{\nu}{n}\right)((1-\alpha)(1-x_0)\langle \phi^{x_0} \rangle + \alpha(1-x_1)\langle \phi^{x_1} \rangle - [(1-\alpha)x_0 + \alpha(1-x_1)]\langle \phi^z \rangle) \\
&> \left(\frac{1}{n}\right)((1-\alpha)\mathbb{E}[\text{TO}|x_0] + \alpha\mathbb{E}[\text{TO}|x_1])
\end{aligned}$$

where the last inequality holds if and only if

$$\frac{(1-\alpha)(1-x_0)\langle\phi^{x_0}\rangle+\alpha(1-x_1)\langle\phi^{x_1}\rangle}{(1-\alpha)(1-x_0)+\alpha(1-x_1)} \quad > \quad \langle\phi^{z}\rangle.$$

As a result, $\left(\frac{1}{n}\right)\mathbb{E}[\mathrm{TO}]$ is a strictly concave function of x, and thus, x^* approximately satisfies

$$\frac{\partial}{\partial x}\left(\left(\frac{1}{n}\right)\mathbb{E}[\mathrm{TO}]\right)\Bigg|_{x=x^*} \quad = \quad 0 \tag{A5}$$

as n grows large.[13] Equation (A5) can be rewritten as

$$\nu\left(\frac{\langle\phi^{x^*}\rangle}{n}-(1-x^*)\frac{\partial}{\partial x}\left(\frac{\langle\phi^{x}\rangle}{n}\right)\Bigg|_{x=x^*}\right) \quad = \quad \Delta\omega(\mathbb{E}[\Delta R]+\mu)$$

□

The next proposition characterizes the optimal fraction of restricted banks if the solution of the planner's problem is not interior.

PROPOSITION 5 (Selection of Restricted Banks): *Suppose the planner has decided whether to acquire bank-level information. Define*

$$\begin{aligned}
\Delta_{1/n} &\equiv \eta-\nu\left(1-\frac{1}{n}\right)\frac{\langle\phi^{1/n}\rangle}{n}-\frac{\Delta\omega}{n}(\mathbb{E}[\Delta R]+\mu),\\
\Delta_1 &\equiv \eta-\Delta\omega(\mathbb{E}[\Delta R]+\mu),\\
\Delta_{\mathcal{I}} &\equiv \max_{x\,\in\,(1/n,1)}\left\{\eta-\nu\,(1-x)\,\frac{\langle\phi^{x}\rangle}{n}-x\Delta\omega(\mathbb{E}[\Delta R]+\mu)\right\},
\end{aligned}$$

The optimal fraction of restricted banks, $x^*_{\mathcal{I}}$, is given by

$$x^*_{\mathcal{I}} \quad = \quad \begin{cases} 1/n \text{ if } \ \Delta_{1/n}>\max\left\{\Delta_1,\Delta_{\mathcal{I}}\right\}\\ 1 \text{ if } \ \Delta_1>\max\left\{\Delta_{1/n},\Delta_{\mathcal{I}}\right\}\\ x^*_{\mathcal{I}} \text{ if } \ \Delta_{\mathcal{I}}>\max\left\{\Delta_1,\Delta_{1/n}\right\},\end{cases}$$

where $x^*_{\mathcal{I}}\in(1/n,1)$ solves

$$\nu\left(\frac{\langle\phi^{x^*}\rangle}{n}-(1-x^*)\frac{\partial}{\partial x}\left(\frac{\langle\phi^{x}\rangle}{n}\right)\Bigg|_{x=x^*}\right) \quad = \quad \Delta\omega(\mathbb{E}[\Delta R]+\mu)$$

Proof. If $\Delta_{1/n}>\max\{\Delta_1,\Delta_{\mathcal{I}}\}$, then $x^*_{\mathcal{I}}=1/n$ as expected total output is maximized with almost no regulation. If $\Delta_{\mathcal{I}}>\max\left\{\Delta_{1/n},\Delta_1\right\}$, then the planner's problem has an interior solution—that is, $x^*_{\mathcal{I}}\in(1/n,1)$. In this case, $x^*_{\mathcal{I}}$ satisfies the first order condition of the

[13] The fraction of restricted banks must be a rational number because n is a natural number. However, the solution of equation (A5) could be an irrational number. Nonetheless, the optimal intervention x^* gets arbitrarily close to the solution of equation (A5) as n grows large.

planner's problem,

$$\nu\left(\frac{\langle\phi^{x^*}\rangle}{n}-(1-x^*)\frac{\partial}{\partial x}\left(\frac{\langle\phi^{x}\rangle}{n}\right)\Bigg|_{x=x^*}\right) \quad = \quad \Delta\omega(\mathbb{E}[\Delta R]+\mu)$$

Finally, if $\Delta_1 > \max\left\{\Delta_{1/n}, \Delta_{\mathcal{I}}\right\}$, then $x_{\mathcal{I}}^* = 1$, as expected total output is maximized by restricting as many banks as possible. □

E Optimal Interventions in Large Economies

When designing optimal interventions, it is pivotal to understand under which conditions cascades of liquidity shocks are non-negligible. As I focus on a system of infinite size, any cascade of finite size becomes negligible as the economy grows large. The following section provides a detailed definition and analysis of cascades of liquidity shocks in large economies.

E.1 Large Cascades of Liquidity Shocks

The Rise of Large Cascades of Liquidity Shocks. To fix notation, let $\mathcal{G}_n$ denote a network of contagious exposures among n banks and $\{\mathcal{G}_n\}_{n\in\mathbb{N}}$ denote a sequence of such networks, indexed by the number of banks n. Let $\mathcal{S}(\mathcal{G}_n)$ denote the largest subset of connected banks in $\mathcal{G}_n$, and let $|\mathcal{S}(\mathcal{G}_n)|$ denote the cardinality of such a set. To determine the condition under which large cascades of liquidity shocks arise, one can use the following idea, similar to the one proposed by Molloy and Reed (1995) and Cohen et al. (2000). Let n_0 denote a large natural number. Suppose there are two banks belonging to each element in the subsequence $\{\mathcal{S}(\mathcal{G}_n)\}_{n\geq n_0}$—say, i and j, which are directly connected. If bank i (or j) is also directly connected to another bank—and loops of contagious exposures can be ignored—then the size of the largest sequence of connected banks is proportional to the size of the system—i.e., $\lim_{n\to\infty}\frac{\mathbb{E}|\mathcal{S}(\mathcal{G}_n)|}{n} > 0$—and, thus, large cascades of liquidity shocks occur; otherwise, the largest sequence of connected banks is fragmented, and, thus, $\lim_{n\to\infty}\frac{\mathbb{E}|\mathcal{S}(\mathcal{G}_n)|}{n} = 0$.[14] Therefore, the condition that determines the emergence of large cascades of liquidity shocks is given by

$$\lim_{n\to\infty}\mathbb{E}_n\left[k_i|i\leftrightarrow j\right] \quad = \quad \lim_{n\to\infty}\sum_{k_i}k_i\mathbb{P}_n\left[k_i|i\leftrightarrow j\right] \;\leq\; 2, \tag{A6}$$

where $\mathbb{P}_n\left[k_i|i\leftrightarrow j\right]$ denotes the probability that bank i has k_i contagious exposures, given that i and j are connected via one contagious exposure. It follows from Bayes' rule that

$$\mathbb{P}_n\left[k_i|i\leftrightarrow j\right] \quad = \quad \frac{\mathbb{P}_n\left[i\leftrightarrow j|k_i\right]\mathbb{P}_n\left[k_i\right]}{\mathbb{P}_n\left[i\leftrightarrow j\right]}.$$

Because contagious exposures are randomly determined,

$$\mathbb{P}_n\left[i\leftrightarrow j\right] = \frac{\mathbb{E}_n[k]}{n-1} \quad \text{and} \quad \mathbb{P}_n\left[i\leftrightarrow j|k_i\right] = \frac{k_i}{n-1}.$$

[14] As n grows large, loops of contagious exposures can be ignored for $\frac{\mathbb{E}_n(k^2)}{\mathbb{E}_n(k)} < 2$. For more details, see Cohen et al. (2000).

Thus, equation (A6) is equivalent to

$$\lim_{n\to\infty} \frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]} \quad \leq \quad 2, \tag{A7}$$

It is important to note that the derivation of equation (A7) does not rely on the functional form of $\mathbb{P}_n[k]$ and applies to any distribution of links in which banks are randomly connected to each other. Equation (A7) establishes that if, in the limit, there is enough variation in the number of contagious exposures among banks, liquidity shocks affecting one bank almost surely affects a non-negligible fraction of them. High variation in the number of contagious exposures makes the economy more prone to contagion, as banks with a large number of contagious exposures can effectively reach a large fraction of banks.

Preventing large cascades of liquidity shocks. Because restricting a bank not only precludes that bank from facing liquidity shocks, but also precludes that bank from propagating liquidity shocks, restricting a sufficiently large fraction of banks can potentially prevent the emergence of large cascades of liquidity shocks. When x exceeds a certain threshold, x^*, large cascades of liquidity shocks can be prevented, as the network of contagious exposures disintegrates into smaller and disconnected parts, keeping liquidity shocks locally confined. Importantly, the value of x^* critically depends on how restricted bank are selected, as the ratio in (A7) varies across policies.

First, suppose a fraction x of bank are restricted uniformly at random. After imposing restrictions, a bank with k_0 contagious exposures may only have k contagious exposures, with $k \leq k_0$, as some of its neighbors may be restricted. In addition, the probability that a subset of k neighbors is not restricted is $(1-x)^k$, whereas the probability that the remaining neighbors are restricted is x^{k_0-k}. Because there are $\binom{k_0}{k}$ different subsets of k neighbors, the distribution of contagious exposures among nonrestricted banks is

$$\mathbb{P}'_n\left(k\right) \quad = \quad \sum_{k\geq k_0} p^{\alpha}_{k_0} \binom{k_0}{k} (1-x)^k x^{k_0-k},$$

and, thus,

$$\mathbb{E}'_n[k] = \langle k\rangle(1-x) \quad \text{and} \quad \mathbb{E}'_n[k^2] = \langle k^2\rangle(1-x)^2 + \langle k\rangle x(1-x), \tag{A8}$$

where expectations with superscript prime denote expectations after implementing restrictions. After banks are restricted, large cascades of liquidity shocks arise if and only if

$$\lim_{n\to\infty} \frac{\mathbb{E}'_n[k^2]}{\mathbb{E}'_n[k]} \quad \leq \quad 2. \tag{A9}$$

It then directly follows from substituting equation (A8) into equation (A9) that x^* must satisfy

$$x^* = 1 - \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}.$$

Now, suppose banks with the highest number of contagious exposures are restricted. The following computations closely follow the ideas in Cohen et al. (2001). Restricting banks with more than $K(x^*)$ contagious exposures is approximately equivalent to restricting a fraction x^* of banks, where x^* satisfies

$$x^* = 1 - \sum_{k=0}^{K(x^*)} p_k^{\alpha}.$$

Take a bank with k contagious exposures. The fraction of contagious exposures attached to all banks with k contagious exposures equals $\frac{kp_k^{\alpha}}{\langle k \rangle}$. As a consequence, the fraction of contagious exposures attached to restricted banks is

$$s(x^*) \;=\; \frac{1}{\langle k \rangle}\left(\sum_{k=K(x^*)+1}^{n-1} kp_k^{\alpha}\right) \;=\; 1 - \frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K(x^*)} kp_k^{\alpha}\right)$$

Because imposing restrictions on a set of banks can be represented by the removal of such banks and their exposures, the optimal policy x^* must satisfy

$$s(x^*) \;=\; x^* \;=\; 1 - \frac{\langle k(x^*) \rangle}{\langle k(x^*)^2 \rangle - \langle k(x^*) \rangle}.$$

Therefore,

$$\begin{aligned}
1 - s(x^*) &= \frac{\langle k(x^*) \rangle}{\langle k(x^*)^2 \rangle - \langle k(x^*) \rangle} \\
\frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K(x^*)} kp_k^{\alpha}\right) &= \frac{\sum_{k=0}^{K(x^*)} kp_k^{\alpha}}{\sum_{k=0}^{K(x^*)} k^2 p_k^{\alpha} - \sum_{k=0}^{K(x^*)} kp_k^{\alpha}} \\
\frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K(x^*)} kp_k^{\alpha}\right) &= \frac{\sum_{k=0}^{K(x^*)} kp_k^{\alpha}}{\sum_{k=0}^{K(x^*)} k(k-1) p_k^{\alpha}} \\
\langle k \rangle &= \sum_{k=0}^{K(x^*)} k(k-1)p_k^{\alpha}. \qquad \text{(A10)}
\end{aligned}$$

which determines the condition under which large cascades of liquidity shocks emerge.

Preventing large cascades in Poisson networks. If the planner cannot identify whether some banks will exhibit more contagious exposures than others, then

$$x^* \;=\; 1 - \left(\frac{1}{\alpha}\right).$$

However, if the planner is able to identify which banks will exhibit the highest number of contagious exposures, then

$$x^* \;=\; 1 - e^{-\alpha}\left(\sum_{k=0}^{K(x^*)} \frac{\alpha^k}{k!}\right), \text{ where } K(x^*) \text{ satisfies } \alpha \;=\; e^{-\alpha}\left(\sum_{k=2}^{K(x^*)} \frac{\alpha^k}{(k-2)!}\right).$$

Derivation. The derivation of the above equations uses the following arguments. For a Poisson distribution with parameter α, the first two moments are given by $\langle k \rangle = \alpha$ and $\langle k^2 \rangle = \alpha^2 + \alpha$. Thus, when restricting at random, the optimal policy is given by $x^* = 1 - \frac{1}{\alpha}$. Provided that $p_k^\alpha = e^{-\alpha}\frac{\alpha^k}{k!}$, a direct application of condition (A10) yields

$$x^* \;=\; 1 - e^{-\alpha}\left(\sum_{k=0}^{K(x^*)} \frac{\alpha^k}{k!}\right), \text{ where } K(x^*) \text{ satisfies } \alpha \;=\; e^{-\alpha}\left(\sum_{k=2}^{K(x^*)} \frac{\alpha^k}{(k-2)!}\right).$$

Preventing large cascades in Power-law networks. If the planner cannot identify whether some banks will exhibit more contagious exposures than others, then

$$x^* \;=\; \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) k_0 - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$

However, if the planner is able to identify which banks will exhibit the highest number of contagious exposures, then

$$x^* \;=\; 1 - \sum_{k=0}^{K(x^*)} k^{-\alpha}.$$

where $K(x^*)$ satisfies

$$\left(\frac{K(x^*)}{k_0}\right)^{2-\alpha} - 2 \;=\; \left(\frac{2-\alpha}{3-\alpha}\right) k_0 \left(\left(\frac{K(x^*)}{k_0}\right)^{3-\alpha} - 1\right).$$

Derivation. The derivation of the above equations uses the following arguments. A continuous Power-law distribution with parameter α, minimal value k_0, and maximum value K, satisfies

$$\begin{aligned}
\langle k \rangle = k_0^{\alpha-1} K^{2-\alpha}\left(\frac{\alpha-1}{\alpha-2}\right) &\quad \text{and} \quad \langle k^2 \rangle \;=\; k_0^{\alpha-1} K^{3-\alpha}\left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 1 < \alpha < 2 \\
\langle k \rangle = k_0\left(\frac{\alpha-1}{\alpha-2}\right) &\quad \text{and} \quad \langle k^2 \rangle \;=\; k_0^{\alpha-1} K^{3-\alpha}\left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 2 < \alpha < 3 \\
\langle k \rangle = k_0\left(\frac{\alpha-1}{\alpha-2}\right) &\quad \text{and} \quad \langle k^2 \rangle \;=\; k_0^{2}\left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 3 < \alpha
\end{aligned}$$

As a consequence, when K grows large,

$$\langle k \rangle = k_0 \left(\frac{\alpha - 1}{\alpha - 2}\right) \text{ if } \alpha > 2 \quad \text{and} \quad \langle k^2 \rangle = k_0^2 \left(\frac{\alpha - 1}{\alpha - 2}\right) \text{ if } \alpha > 3$$

and they diverge in all other cases.

Now, consider the case when the planner cannot differentiate among banks before implementing her policy. Using the above equations and the condition that determines the emergence of large cascades of liquidity shocks, it is easy to show that

$$x^* = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) k_0 - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$

as n grows large.

When the planner can identify banks with the highest number of contagious exposures, the following equation

$$\sum_{k=k_0}^{k_x} k(k-1)p_k = \langle k \rangle$$

determines the emergence of large cascades of liquidity shocks. Because the network follows a Power-law distribution with parameter α, the above equation is equivalent to

$$(\alpha - 1)k_0^{\alpha-1} \left(\frac{k_x^{3-\alpha} - k_0^{3-\alpha}}{3 - \alpha} - \frac{k_x^{2-\alpha} - k_0^{2-\alpha}}{2 - \alpha}\right) = k_0 \left(\frac{\alpha - 1}{\alpha - 2}\right)$$

which is equivalent to

$$\left(\frac{k_0}{3 - \alpha}\right)\left(\left(\frac{k_x}{k_0}\right)^{3-\alpha} - 1\right) - \left(\frac{1}{2 - \alpha}\right)\left(\left(\frac{k_x}{k_0}\right)^{2-\alpha} - 2\right) = 0,$$

and, thus, k_x can be derived from α.

E.2 Interventions in Large Economies

Suppose the planner restricts all banks in $\mathcal{R}$, with $|\mathcal{R}| = nx$. Then

$$\left(\frac{1}{n}\right) \mathbb{E}[\text{TO}|x] = x(\mathbb{E}[R_I] - \omega_H \mathbb{E}[\Delta R]) + \left(\frac{1}{n}\right) \sum_{i \notin \mathcal{R}} \mathbb{E}[\pi_i] + \mu(1 - \omega_L - x\Delta\omega)$$

For a given information set, let x^* denote the smallest fraction of banks that must be restricted to prevent the emergence of large cascades of liquidity shocks. As the economy grows large, it is worth noting that

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{i\notin\mathcal{R}}\pi_i = \begin{cases}(1-x)[R_I-\omega_L\Delta R], \text{ with probability } x \text{ if } x<x^* \\ 0, \text{ with probability } (1-x) \text{ if } x<x^* \\ (1-x)[R_I-\omega_L\Delta R], \text{ with probability } 1 \text{ if } x\geq x^*.\end{cases}$$

Let $\mathcal{R}^c$ denote the complement set of $\mathcal{R}$. The above expression follows from the fact that if $x \geq x^*$, then the size of the largest connected component in $\mathcal{R}^c$ is almost surely of order $\rho^2\log(n)$, where ρ is the highest degree within $\mathcal{R}^c$. However, if $x < x^*$ the size of the largest connected component is of order n—and the size of the second largest connected component is of order $\log(n)$; for more details, see Molloy and Reed (1998). As a result,

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\mathbb{E}[\text{TO}|x] = \begin{cases}x(\mathbb{E}[R_I]-\omega_H\mathbb{E}[\Delta R])+(1-x)(\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R])+\mu(1-\omega_L-x\Delta\omega), \text{ if } x\geq x^* \\ x(\mathbb{E}[R_I]-\omega_H\mathbb{E}[\Delta R])+x(1-x)(\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R])+\mu(1-\omega_L-x\Delta\omega), \text{ if } x<x^*.\end{cases}$$

Define $\Delta x = (x^* - x)$. To determine the optimal policy, it is worth noting that $H(x) \equiv \lim_{n\to\infty}\left(\frac{1}{n}\right)(\mathbb{E}[\text{TO}|x^*]-\mathbb{E}[\text{TO}|x])$ equals

$$H(x) = \begin{cases}-\Delta x\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } x\geq x^* \\ (1-x)^2(\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R])-\Delta\omega(\mathbb{E}[\Delta R]+\mu)\Delta x, \text{ if } x<x^*.\end{cases}$$

If $\left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) \geq x^*$, then $H(x) \geq 0$, $\forall$ x in $[0, x^*]$. Thus, x^* generates higher expected total output than any other fraction $0 \leq x \leq 1$, as $H(x)$ is strictly positive when $x > x^*$.

However, if $\left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) < x^*$, then $H(0) < 0$. Consequently, $H(x) < 0$, when $0 \leq x < x^*$, as $H(x)$ is an increasing function of x when $x < x^*$ and $H(x^*) = 0$. Therefore, $x = 0$ maximizes expected total output. Then, as n grows large, the optimal policy converges to the following intervention:

$$x^{\text{optimal}} = \begin{cases}x^*, \text{ if } \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right)\geq x^* \\ 0, \qquad \text{otherwise.}\end{cases}$$

Importantly, the value of x^* does not depend on the values of $\mathbb{E}[R_I]$, ω_L, $\mathbb{E}[\Delta R]$, μ, or $\Delta\omega$. However, x^* does depend on the distribution $\{p_k^\alpha\}_{k=1}$ and how banks are targeted. In particular, if bank-level information is not acquired, then $x^* = 1 - \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}$. If bank-level information is acquired, then $x^* = 1 - \sum_{k=k_{\min}}^{K(x^*)} p_k^\alpha$—where $K(x^*)$ is the solution of the following equation $\langle k\rangle = \sum_{k=k_{\min}}^{K(x^*)} k(k-1)p_k^\alpha$ and $k_{\min}$ is the smallest number of links that a bank might have.

Let x_t and x_r denote the smallest fraction of banks that must be restricted to prevent large cascades of liquidity shocks when targeting with and without bank-level information, respectively. Because $x_t \leq x_r$, then $\Delta x \geq 0$.

EXAMPLE 2 (Poisson Networks): *If the network exhibits a Poisson degree distribution of* *parameter α, then*

$$x_r = 1 - \frac{1}{\alpha}$$
$$x_t = x_r - \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}$$

where K_α solves $\frac{1}{\alpha} = \sum_{j=0}^{(K_\alpha-2)} \frac{e^{-\alpha}\alpha^j}{j!}$. Thus, $\Delta x = \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}$.

Proof. If bank-level information is not acquired, then $x_r = 1 - \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}$. The result follows directly from the fact a Poisson network with parameter α yields $\langle k\rangle = \alpha$ and $\langle k^2\rangle = \alpha^2 + \alpha$. If bank-level information is acquired, then $x_t = 1 - \sum_{k=k_{\min}}^{K^*} p_k^\alpha = \sum_{k=K^*+1}^{\infty} p_k^\alpha$—where K^* is the solution of the equation $\langle k\rangle = \sum_{k=k_{\min}}^{K^*} k(k-1)p_k^\alpha$, with $p_k^\alpha = \frac{e^{-\alpha}\alpha^k}{k!}$. It is worth noting that the fraction of exposures (links) attached to restricted banks, p, equals $p = \frac{1}{\langle k\rangle}\sum_{k=K^*+1}^{\infty} kp_k^\alpha$. Importantly, if $p = x_r$, then large cascades of liquidity shocks are prevented. Because $p = \frac{1}{\langle k\rangle}\sum_{k=K^*+1}^{\infty} kp_k^\alpha = \sum_{k=K^*}^{\infty}\frac{e^{-\alpha}\alpha^k}{k!} = (\sum_{k=K^*+1}^{\infty}\frac{e^{-\alpha}\alpha^k}{k!}) + \frac{e^{-\alpha}\alpha^{K^*}}{K^*!} = x_t + \frac{e^{-\alpha}\alpha^{K^*}}{K^*!}$, then $x_t = x_r - \frac{e^{-\alpha}\alpha^{K^*}}{K^*!}$. □

EXAMPLE 3 (Power-law Networks): *If the network exhibits a Power-law degree distribution of parameter α and $k_{min} = 1$, then*

$$x_r = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$
$$x_t = K_\alpha^{(1-\alpha)}$$

where K_α satisfies

$$K_\alpha^{2-\alpha} - 2 = \left(\frac{2-\alpha}{3-\alpha}\right)\left(K_\alpha^{3-\alpha} - 1\right).$$

As a result,

$$\Delta x = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} - K_\alpha^{(1-\alpha)} & \text{if } \alpha > 3 \\ 1 - K_\alpha^{(1-\alpha)} & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$

To determine the (endogenous) value of bank-level information and optimal intervention, it is illustrative to analyze how $\left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right)$ compares to x_t and x_r. First, suppose $x_r \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right)$. Then, $\lim_{n\to\infty}\left(\frac{1}{n}\right)(\mathbb{E}[\text{TO}|x_t] - \mathbb{E}[\text{TO}|x_r]) = \Delta x\Delta\omega(\mathbb{E}[\Delta R]+\mu)$, which represents the value of bank-level information. Consequently, if $\Delta x\Delta\omega(\mathbb{E}[\Delta R]+\mu) \geq \kappa$, then acquiring bank-level information is optimal and $x^{\text{optimal}} = x_t$. Otherwise, it is optimal not to acquire information and $x^{\text{optimal}} = x_r$. Second, suppose $x_t \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) < x_r$. Then $\lim_{n\to\infty}\left(\frac{1}{n}\right)(\mathbb{E}[\text{TO}|x_t] - \mathbb{E}[\text{TO}|x=0]) = (\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R]) - x_t\Delta\omega(\mathbb{E}[\Delta R]+\mu)$ represents the value of bank-level information. Thus, if $(\mathbb{E}[R_I] - \omega_L\mathbb{E}[\Delta R]) - x_t\Delta\omega(\mathbb{E}[\Delta R]+\mu) \geq \kappa$, then acquiring bank-level information is optimal and $x^{\text{optimal}} = x_t$. Otherwise, acquiring information is suboptimal and $x^{\text{optimal}} = 0$. Finally, suppose $\left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) < x_t$. Then, $x^{\text{optimal}} = 0$ no matter what, and, thus, the value of information is 0. As a result, acquiring bank-level information is sub-optimal.

As a consequence, the optimal intervention is given by

$$x^{\text{optimal}} = \begin{cases} x_r, \text{ if } x_r \leq \min\left\{\left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right), x_t + \frac{\kappa}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right\} \\ x_t, \text{ if } \frac{\kappa}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)} + x_t \leq x_r \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) \text{ or} \\ \qquad x_t \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]-\kappa}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) \text{ and } \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) < x_r \\ 0, \qquad \text{otherwise,} \end{cases} \tag{A11}$$

which can be rewritten as

$$x^{\text{optimal}} = \begin{cases} x_r, \text{ if } \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\} \\ x_t, \text{ if } \min\left\{\frac{\nu}{x_r}, \frac{\kappa}{\Delta x}\right\} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu-\kappa}{x_t} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{A12}$$

The endogenous value of information is

$$\text{SVI} = \begin{cases} \Delta x\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } x_r \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) \\ (\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]) - x_t\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } x_t \leq \left(\frac{\mathbb{E}[R_I]-\omega_L\mathbb{E}[\Delta R]}{\Delta\omega(\mathbb{E}[\Delta R]+\mu)}\right) \leq x_r \\ 0, \qquad \text{otherwise,} \end{cases} \tag{A13}$$

which is equivalent to

$$\text{SVI} = \begin{cases} \Delta x\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu}{x_r} \\ \nu - x_t\Delta\omega(\mathbb{E}[\Delta R]+\mu), \text{ if } \frac{\nu}{x_r} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu}{x_t} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{A14}$$

It directly follows from the above analysis

EXAMPLE 4 (Optimal Intervention and Value of Information with Poisson Networks): *If the network exhibits a Poisson degree distribution of parameter α, then*

$$x^{optimal} = \begin{cases} \left(1-\frac{1}{\alpha}\right), \text{ if } \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \min\left\{\frac{\nu\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} \\ \left(1-\frac{1}{\alpha}\right) - \left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right), \text{ if } \min\left\{\frac{\nu\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu-\kappa}{\left(1-\frac{1}{\alpha}\right)-\left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)} \\ 0, \qquad \text{otherwise.} \end{cases}$$

where K_α *solves* $\frac{1}{\alpha} = \sum_{j=0}^{(K_\alpha - 2)} \frac{e^{-\alpha}\alpha^j}{j!}$. *The endogenous value of information is*

$$SVI = \begin{cases} \left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)\Delta\omega(\mathbb{E}[\Delta R]+\mu), & \text{if } \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu\alpha}{\alpha-1} \\ \nu - \left(\left(1-\frac{1}{\alpha}\right) - \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)\Delta\omega(\mathbb{E}[\Delta R]+\mu), & \text{if } \frac{\nu\alpha}{\alpha-1} < \Delta\omega(\mathbb{E}[\Delta R]+\mu) \leq \frac{\nu}{\left(1-\frac{1}{\alpha}\right)-\left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)} \\ 0, & \text{otherwise.} \end{cases}$$

F Optimal Interventions under Network Uncertainty

PROPOSITION 6 (Existence of the maximum): *Given* $\mathcal{A}$ *and* f, *suppose the planner only considers interventions in which* $\frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}$ *and* $\int_{\alpha\in\mathcal{A}} \left(\frac{\langle \phi_{\alpha}^x \rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}\right)^2 df(\alpha)$ *are lower semi-continuous functions of* x*—that is,*

$$\langle \phi_{\bar{\alpha}}^{x'} \rangle \leq \liminf_{x\to x'} \langle \phi_{\bar{\alpha}}^x \rangle \text{ and } \int_{\alpha\in\mathcal{A}} \left(\frac{\langle \phi_{\alpha}^{x'} \rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^{x'} \rangle}{n}\right)^2 df(\alpha) \leq \liminf_{x\to x'} \int_{\alpha\in\mathcal{A}} \left(\frac{\langle \phi_{\alpha}^x \rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}\right)^2 df(\alpha).$$

with $\frac{1}{n} \leq x' \leq 1$. *Then, there exists a point* $\frac{1}{n} \leq x^* \leq 1$ *that solves*

$$\begin{aligned} \max_x \quad & \mathbb{E}_{\bar{\alpha}}\left(\frac{1}{n}TO_\alpha \middle| x\right) - \left(\frac{\theta}{2}\right) \times \mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_\alpha\left(TO_\alpha | x\right)\right) - \kappa \times \mathbb{1}_\kappa \\ s.t. \quad & \frac{1}{n} \leq |x| \leq 1, \end{aligned}$$

Proof. If $\frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}$ and $\int_{\alpha\in\mathcal{A}} \left(\frac{\langle \phi_{\alpha}^x \rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}\right)^2 df(\alpha)$ are lower semi-continuous functions of x, then the objective function of the aforementioned optimization problem is an upper semi-continuous function of x. Baire's generalization of the Weierstrass' theorem ensures that such an objective function always attains its maximum over a compact set; see (Ok, 2007, Chapter 4). □

PROPOSITION 7 (Interior Solution): *Given* $\mathcal{A}$ *and* f, *suppose*

$$\mathbb{E}_{\bar{\alpha}}\left(\frac{1}{n}TO_\alpha \middle| x\right) - \left(\frac{\theta}{2}\right) \times \mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_\alpha\left(TO_\alpha | x\right)\right)$$

is a strictly concave function of x *within the interval* $\left[\frac{1}{n}, 1\right]$. *Then, the optimal intervention,* x^*, *gets arbitrarily close to the solution of the equation*

$$\begin{aligned} \nu\left(\frac{\langle \phi_{\bar{\alpha}}^{x^a} \rangle}{n} - (1-x^a)\frac{\partial}{\partial x}\left(\frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}\right)\Bigg|_{x=x^a}\right) &= \Delta\omega(\mathbb{E}[\Delta R]+\mu) \\ &+ \left(\frac{\theta}{2}\right)\nu^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle \phi_{\alpha}^x \rangle}{n} - \frac{\langle \phi_{\bar{\alpha}}^x \rangle}{n}\right)^2 df(\alpha)\right)\Bigg|_{x=x^a}. \end{aligned}$$

as n *get large.*

Proof. If

$$\mathbb{E}_{\bar{\alpha}}\left(\frac{1}{n}\text{TO}_{\alpha}\middle|x\right) - \left(\frac{\theta}{2}\right) \times \mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_{\alpha}\left(\text{TO}_{\alpha}\middle|x\right)\right)$$

is a strictly concave function of x within the interval $\left[\frac{1}{n}, 1\right]$, then the solution of the optimization problem is interior. Notably,

$$\begin{aligned}
\mathbb{E}_{\bar{\alpha}}\left(\frac{1}{n}\text{TO}_{\alpha}\middle|x\right) &= \eta - (1-x)\nu\frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n} - x\Delta\omega(\mathbb{E}(\Delta R)+\mu) \\
\mathbb{V}_f\left(\frac{1}{n}\mathbb{E}_{\alpha}\left(\text{TO}_{\alpha}\middle|x\right)\right) &= (1-x)^2\nu^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi^x_{\alpha}\rangle}{n} - \frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n}\right)^2 df(\alpha).
\end{aligned}$$

As a result, the first order condition of the planner's problem can be rewritten as

$$\begin{aligned}
\nu\left(\frac{\langle\phi^{x^a}_{\bar{\alpha}}\rangle}{n} - (1-x^a)\frac{\partial}{\partial x}\left(\frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n}\right)\Bigg|_{x=x^a}\right) &= \Delta\omega(\mathbb{E}[\Delta R]+\mu) \\
&+ \left(\frac{\theta}{2}\right)\nu^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi^x_{\alpha}\rangle}{n} - \frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n}\right)^2 df(\alpha)\right)\Bigg|_{x=x^a}.
\end{aligned}$$

Because the fraction of restricted banks must be a rational number, x^* gets arbitrarily close to the solution of the above equation as n gets large. □

PROPOSITION 8 (Selection of Restricted Banks): *Given $\mathcal{A}$ and f, suppose the planner has decided whether to acquire bank-level information. Define*

$$\begin{aligned}
\Delta_{1/n} &\equiv \eta - \nu\left(1-\frac{1}{n}\right)\frac{\langle\phi^{1/n}_{\bar{\alpha}}\rangle}{n} - \frac{\Delta\omega}{n}(\mathbb{E}[\Delta R]+\mu) - \frac{\theta}{2}\nu^2\left(1-\frac{1}{n}\right)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi^{1/n}_{\alpha}\rangle}{n} - \frac{\langle\phi^{1/n}_{\bar{\alpha}}\rangle}{n}\right)^2 df(\alpha), \\
\Delta_1 &\equiv \eta - \Delta\omega(\mathbb{E}[\Delta R]+\mu), \\
\Delta_{\mathcal{I}} &\equiv \max_{x\,\in\,(1/n,1)}\left\{\eta - \nu(1-x)\frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n} - x\Delta\omega(\mathbb{E}[\Delta R]+\mu) - \frac{\theta}{2}\nu^2(1-x)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi^x_{\alpha}\rangle}{n} - \frac{\langle\phi^x_{\bar{\alpha}}\rangle}{n}\right)^2 df(\alpha)\right\},
\end{aligned}$$

The optimal fraction of restricted banks, $x^*_{\mathcal{I}}$, is given by

$$x^*_{\mathcal{I}} = \begin{cases} 1/n \text{ if } \Delta_{1/n} > \max\{\Delta_1, \Delta_{\mathcal{I}}\} \\ 1 \text{ if } \Delta_1 > \max\{\Delta_{1/n}, \Delta_{\mathcal{I}}\} \\ x^*_{\mathcal{I}} \text{ if } \Delta_{\mathcal{I}} > \max\{\Delta_1, \Delta_{1/n}\}, \end{cases}$$

where $x^*_{\mathcal{I}} \in (1/n, 1)$ gets arbitrarily close to the solution of the following equation, x^a,

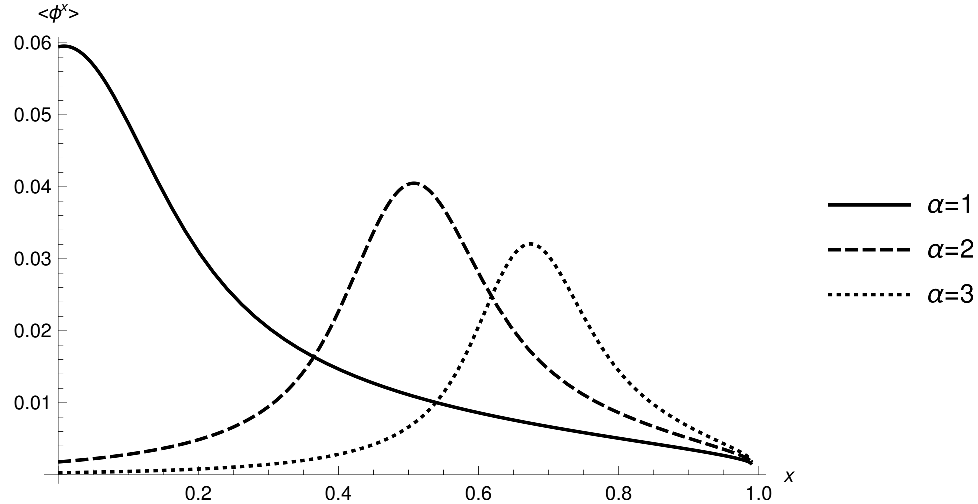
$$\begin{aligned} \nu\left(\frac{\langle\phi^{x^a}_{\bar{\alpha}}\rangle}{n} - (1-x^a)\frac{\partial}{\partial x}\left(\frac{\langle\phi^{x}_{\bar{\alpha}}\rangle}{n}\right)\Bigg|_{x=x^a}\right) &= \Delta\omega(\mathbb{E}[\Delta R]+\mu) \\ &+ \left(\frac{\theta}{2}\right)\nu^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}\left(\frac{\langle\phi^{x}_{\alpha}\rangle}{n} - \frac{\langle\phi^{x}_{\bar{\alpha}}\rangle}{n}\right)^2 df(\alpha)\right)\Bigg|_{x=x^a}. \end{aligned}$$

as n grows large.

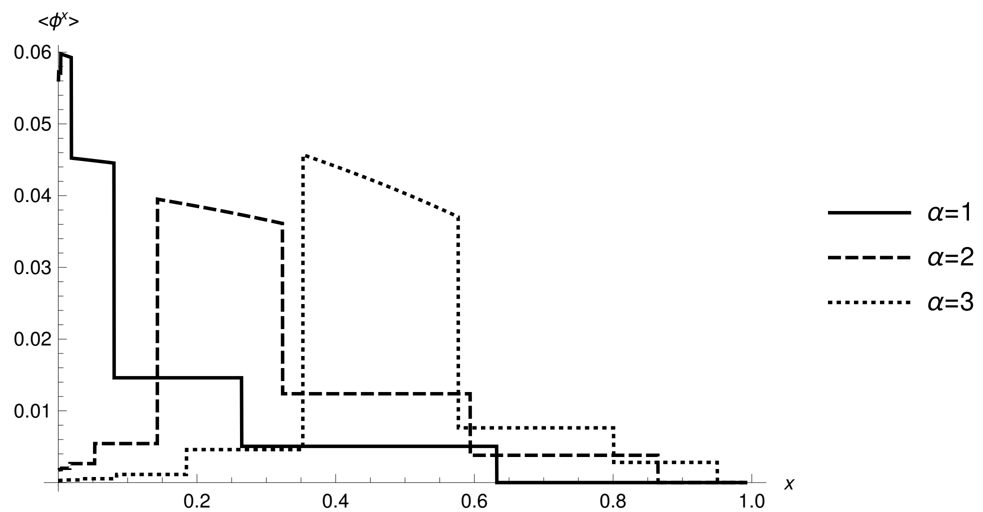
Proof. If $\Delta_{1/n} > \max\{\Delta_1, \Delta_{\mathcal{I}}\}$, then $x^*_{\mathcal{I}} = 1/n$, as expected total output is maximized with almost no regulation. If $\Delta_{\mathcal{I}} > \max\left\{\Delta_{1/n}, \Delta_1\right\}$, then the planner's problem has an interior solution—that is, $x^*_{\mathcal{I}} \in (1/n, 1)$. In this case, as n grows large, $x^*_{\mathcal{I}}$ gets arbitrarily close to the solution of the first order condition of the planner's problem. Finally, if $\Delta_1 > \max\left\{\Delta_{1/n}, \Delta_{\mathcal{I}}\right\}$, then $x^*_{\mathcal{I}} = 1$, as expected total output is maximized by restricting as many banks as possible. □

Appendix B Figures

This section contains figures mentioned in the body of the paper.

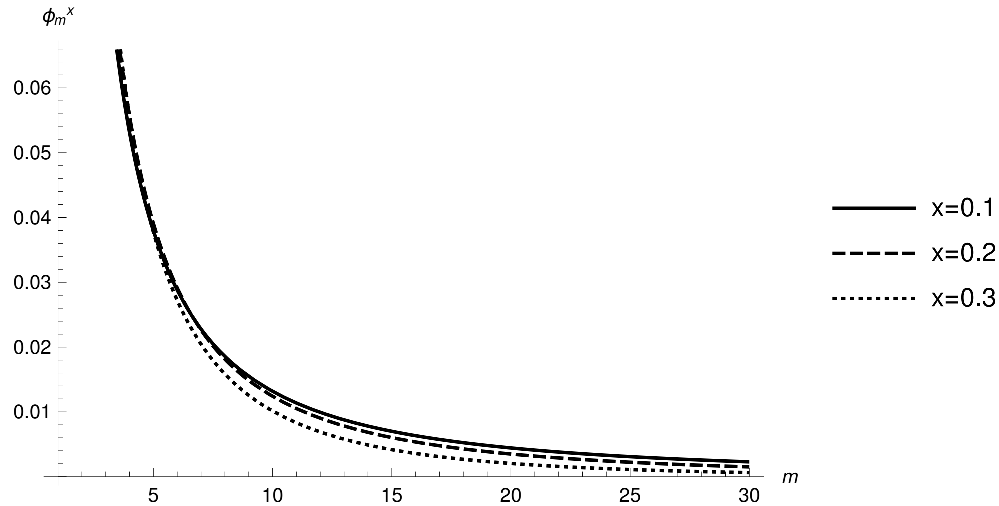


(a) $\langle \phi^x \rangle$ as a function of x. Random targeting.

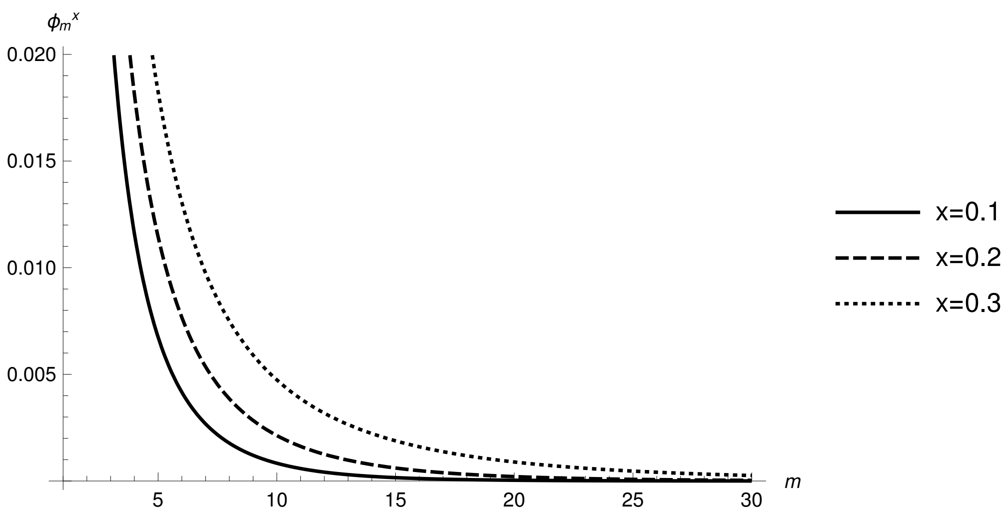


(b) $\langle \phi^x \rangle$ as a function of x. Targeting based on banks' future number of contagious exposures.

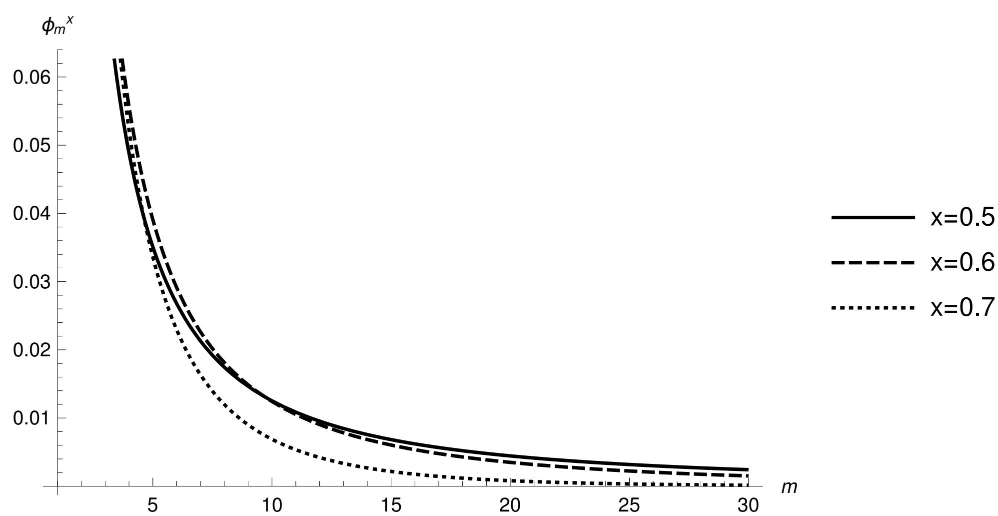
Figure 2. $\{p_k^\alpha\}_k$ follows a Poisson distribution.



(a) $\{\phi_m^x\}_m$ if $\alpha = 1$, $x \in \{0.1, 0.2, 0.3\}$, and $n = 100$.

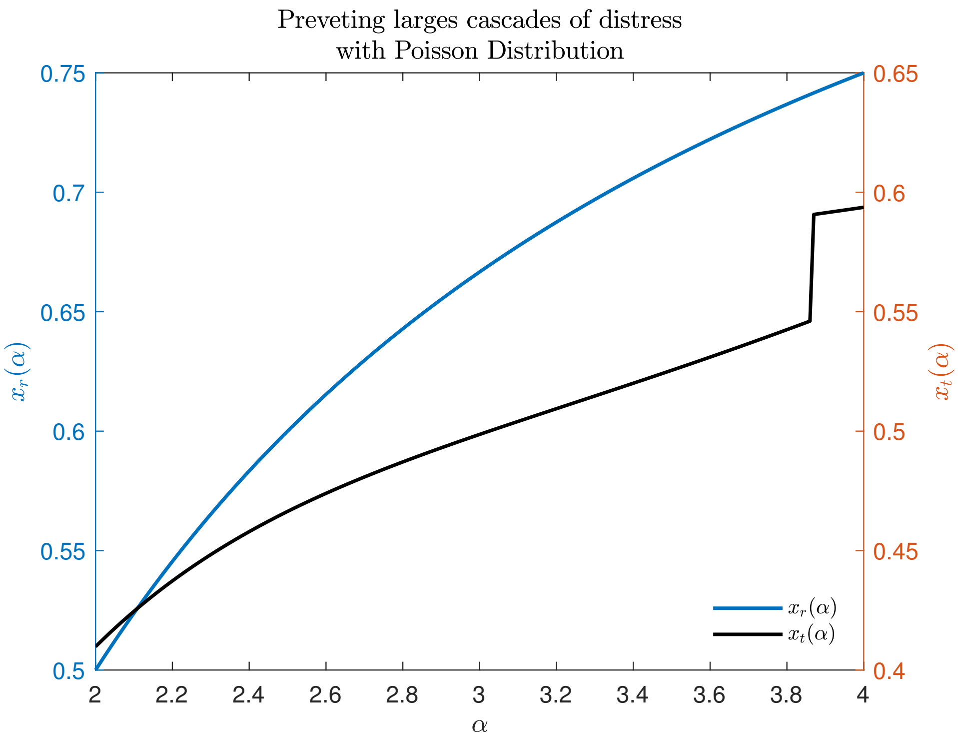


(b) $\{\phi_m^x\}_m$ if $\alpha = 2$, $x \in \{0.1, 0.2, 0.3\}$, and $n = 100$.

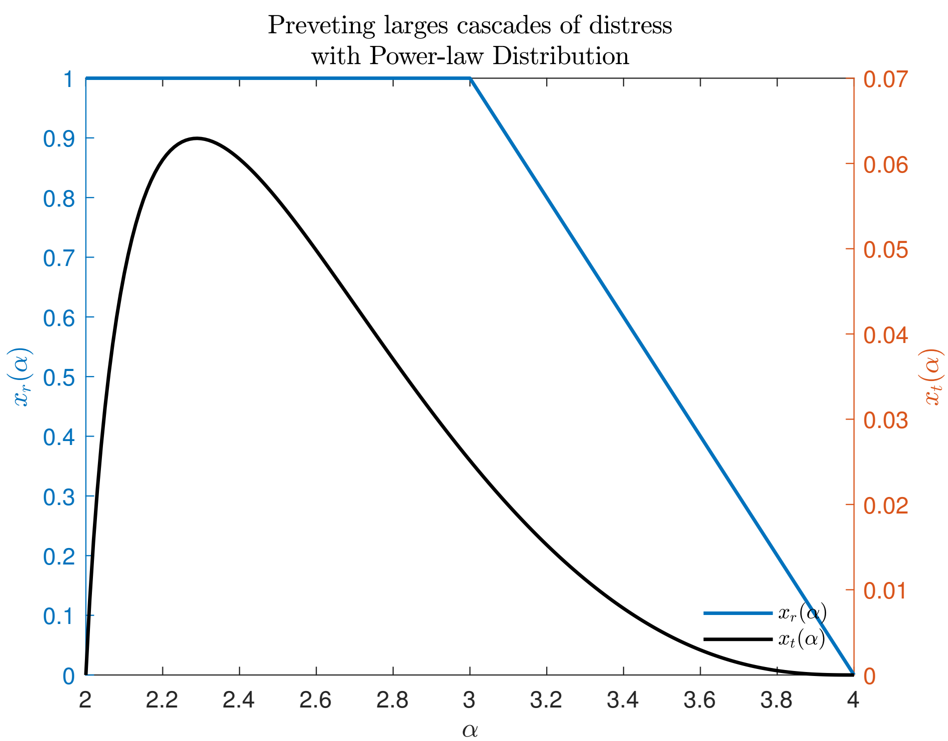


(c) $\{\phi_m^x\}_m$ if $\alpha = 2$, $x \in \{0.5, 0.6, 0.7\}$, and $n = 100$.

Figure 3. $\{p_k^\alpha\}_k$ follows a Poisson distribution and restricted banks are selected at random

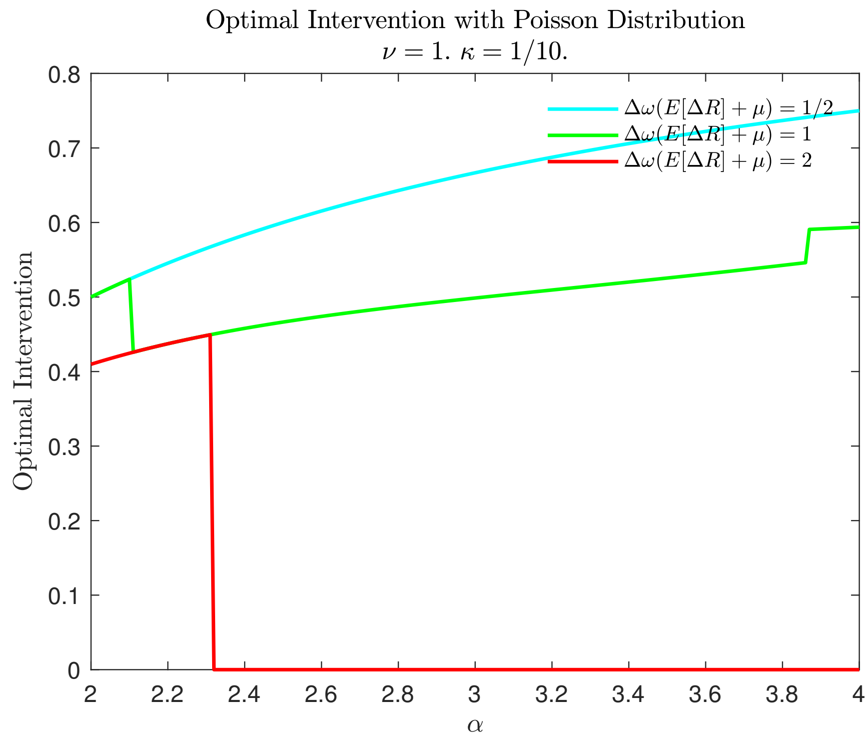


(a) Poisson

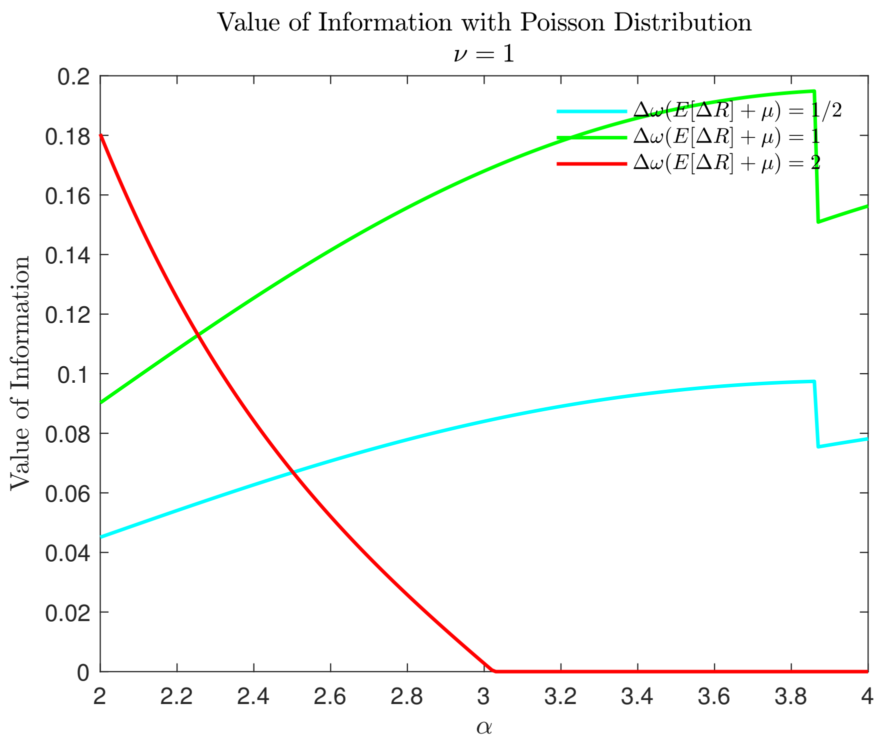


(b) Power-law

Figure 4. Preventing large cascades of distress

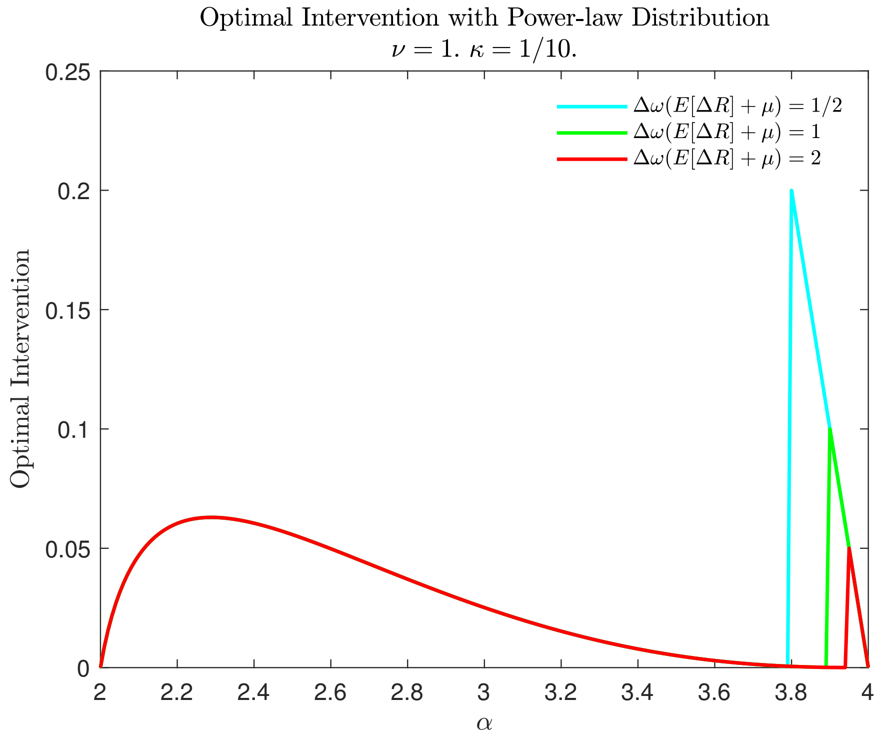


(a) Optimal fraction of restricted banks

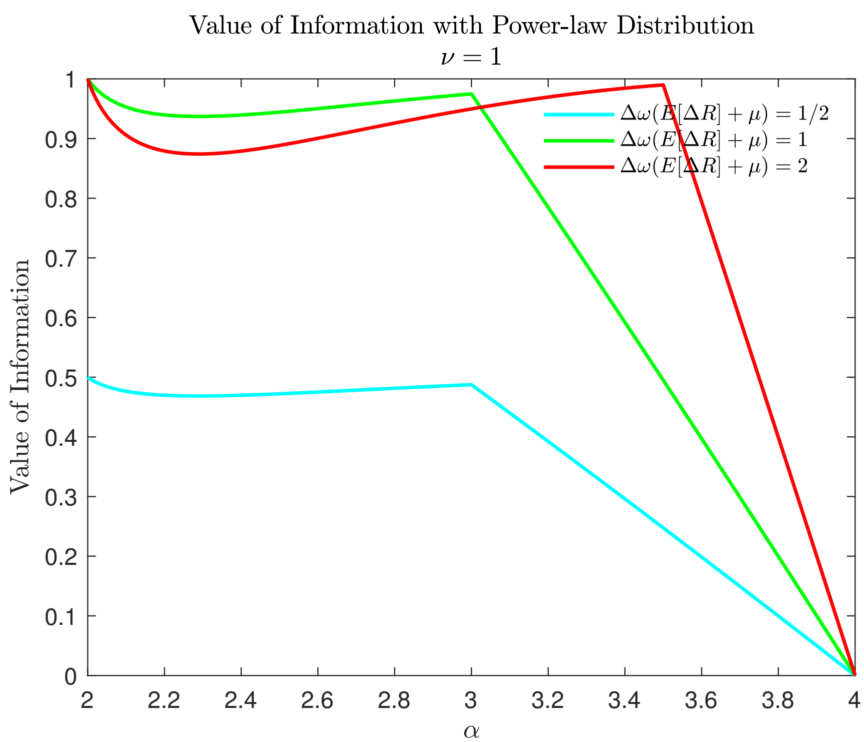


(b) Social Value of Information

Figure 5. Optimal intervention as a function of α in Poisson networks.

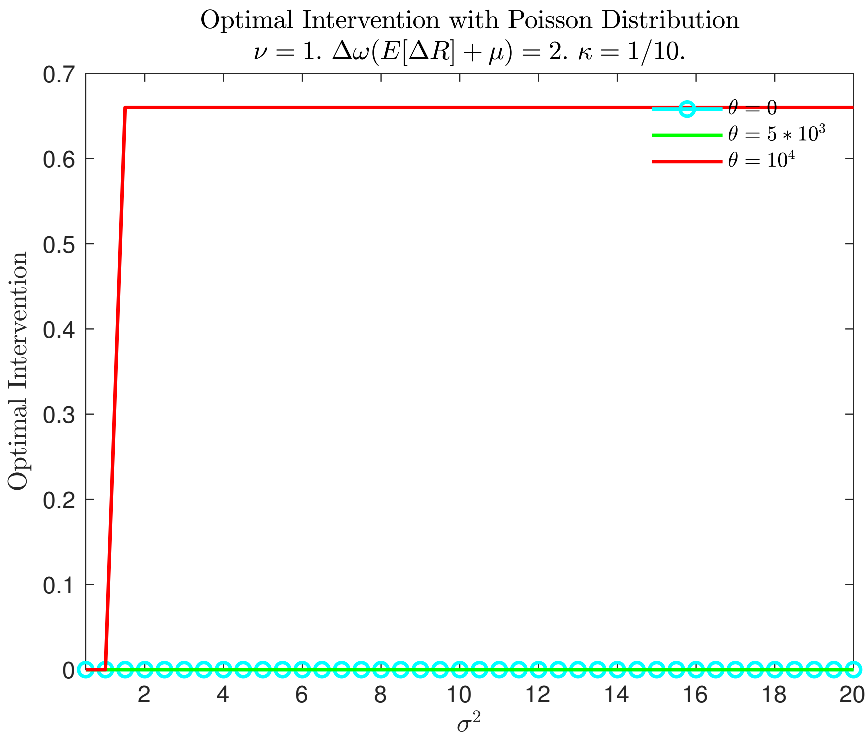


(a) Optimal fraction of restricted banks

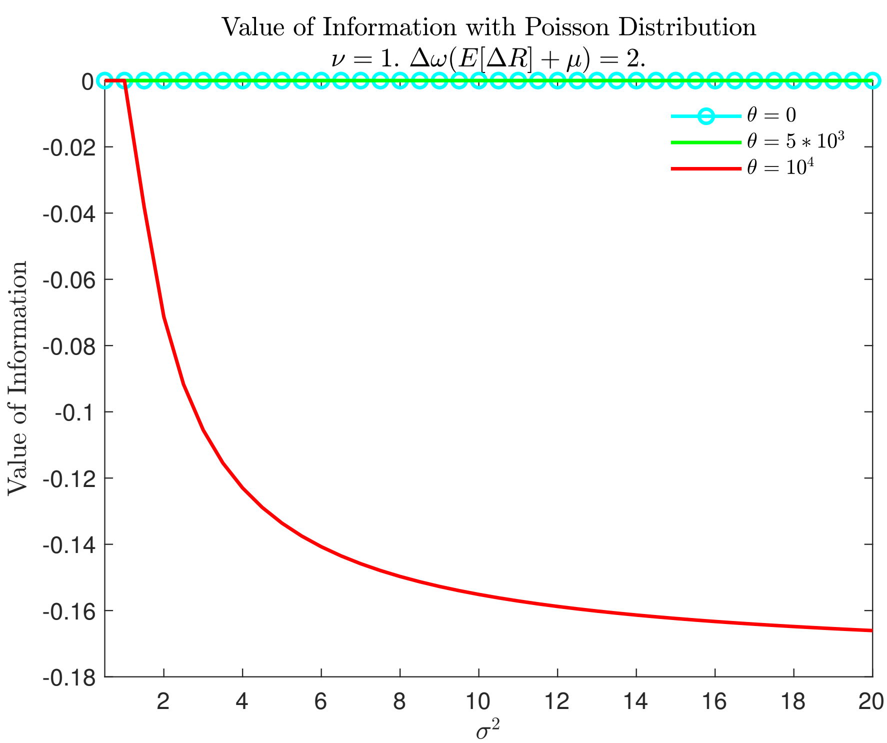


(b) Social Value of Information

Figure 6. Optimal intervention as a function of α in Power-law networks.

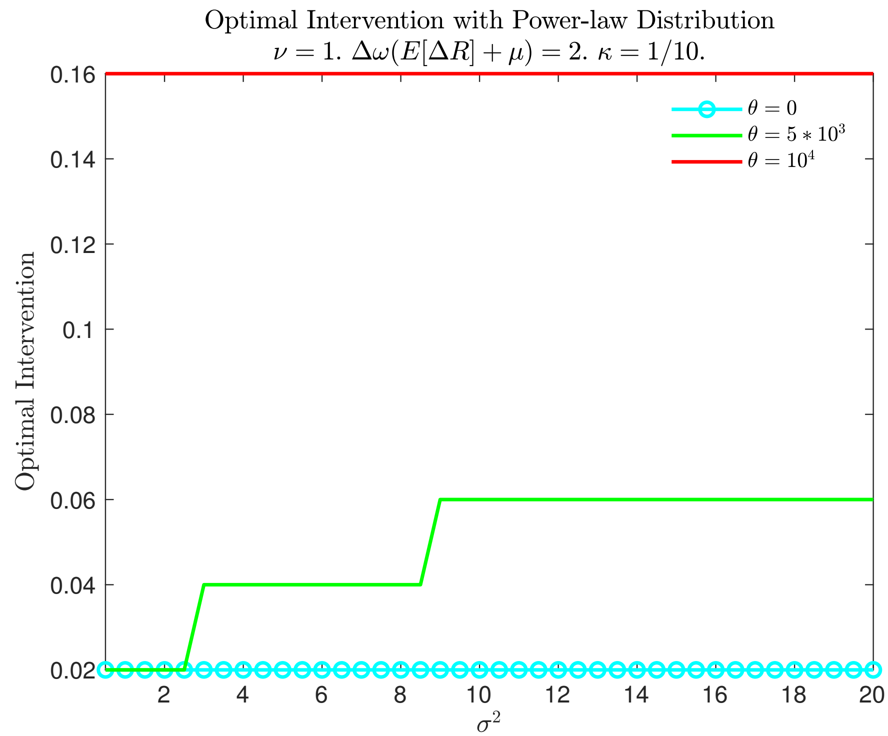


(a) Optimal fraction of restricted banks

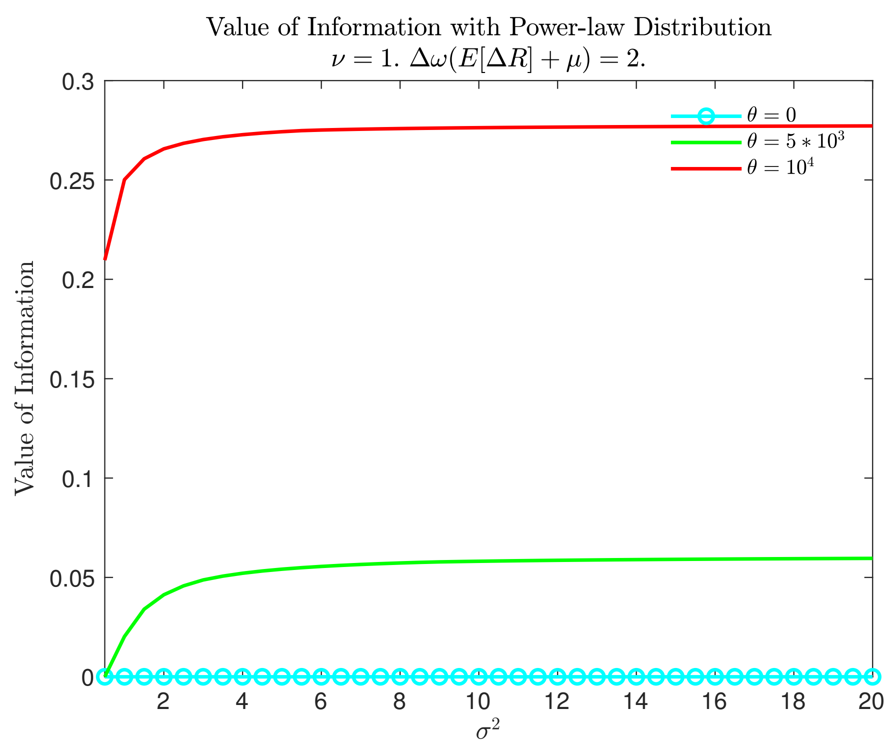


(b) Social Value of Information

Figure 7. Optimal intervention as a function of σ^2 in Poisson networks with model uncertainty.



(a) Optimal fraction of restricted banks



(b) Social Value of Information

Figure 8. Optimal intervention as a function of σ^2 in Power-law networks with model uncertainty.