Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

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2019-058

Please cite this paper as:
Antinolfi, Gaetano, Francesca Carapella and Francesco Carli (2019). "Transparency and collateral: the design of CCPs' loss allocation rules," Finance and Economics Discussion Series 2019-058. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2019.058.

Transparency and collateral: the design of CCPs' loss allocation rules*

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July 23, 2019

Abstract

This paper adopts a mechanism design approach to study optimal clearing arrangements for bilateral financial contracts in which an assessment of counterparty risk is crucial for efficiency. The economy is populated by two types of agents: a borrower and lender. The borrower is subject to limited commitment and holds private information about the severity of such lack of commitment. The

*We are very grateful to Ned Prescott for his thoughtful discussion and to Guillaume Rocheteau for his valuable input. We also thank Garth Baughman, Florian Heider, Marie Hoerova, Cyril Monnet, Borghan Narajabad, and William Roberds for their comments and suggestions, and participants in the Spring 2013 Midwest Macroeconomics Meetings; 2013 European Summer Meetings of the Econometric Society; 2013 Society for Advancement of Economic Theory Conference; 2014 Chicago Fed Money workshop; System Committee Meeting on Financial Structure and Regulation at the Dallas Fed; First African Search and Matching Workshop; and seminar participants at the Federal Reserve Board, the Federal Reserve Bank of Atlanta, Bank of Portugal, Universities of Auckland, Bern, Birmingham, Católica-Lisbon, Porto, Tilburg, and UC Irvine. All errors are our own. The views expressed in this paper are solely the responsibility of the authors, and should not be interpreted as reflecting the views of the Board of Governors or the Federal Reserve System.

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lender can acquire information at a cost about the commitment of the borrower, which affects the assessment of counterparty risk. When truthful revelation by the borrower is not incentive compatible, the mechanism designer optimally trades off the value of information about the lack of commitment of the borrower with the cost of incentivizing the lender to acquire such information. Central clearing of these financial contracts through a central counterparty (CCP) allows lenders to mutualize their counterparty risks, but this insurance may weaken incentives to acquire and reveal information about such risks. If information acquisition is incentive compatible, then lenders choose central clearing. If it is not, they may prefer bilateral clearing to prevent strategic default by borrowers and to economize on costly collateral. Central clearing is analyzed under different institutional features observed in financial markets, which place different restrictions on the contract space in the mechanism design problem. The interaction between the costly information acquisition and the limited commitment friction differs significantly in each clearing arrangement and in each set of restrictions. This results in novel lessons about the desirability of central versus bilateral clearing depending on traders' characteristics and the institutional features defining the operation of the CCP.

Keywords: Limited commitment, central counterparties, collateral

JEL classification: G10, G14, G20, G23

1 Introduction

An important aspect of modern financial contracting is that financial institutions trade a variety of products bilaterally, such as over-the-counter (OTC) derivatives, repurchase agreements, and reserves held at the central bank.[1] Information about the exposure of a counterparty to various risks is necessary to select appropriate contractual terms, such as prices and collateral, in order to control the risk that a counterparty will not fulfill its future obligations. This information, however, is often confined within a bilateral relationship because of the high degree of specialization in understanding and pricing risks specific to a certain financial product, and because of the interaction between the counterparties across other financial markets.

The financial crisis of 2008 has highlighted the systemic importance of such information.[2] Both academic researchers and policymakers argued that, during the crisis, asymmetric information and lack of transparency in over-the-counter markets contributed to uncertainty over the risks that certain institutions posed, causing runs and exacerbating financial distress.[3] Consequently, particular attention has been devoted to the role of clearing institutions and to their potential in improving transparency in financial markets.[4] Mandatory clearing via a central counterparty (CCP), defined below, has been at the center of financial reforms both in the US and in Europe. However, the consequences of these reforms on the incentives of financial market participants to acquire information about each other are not well understood.

In this paper, we address the question of potential tradeoffs between bilateral and central clearing with respect to market transparency. We adopt a mechanism design approach and develop a model of financial contracting where information about a counterparty is *soft* in the sense that it can be verified only by agents within the bilateral transaction. This assumption captures the idea that soft information is often

[1]See Krishnamurthy et al. (2014), `http://www.newyorkfed.org/banking/tpr_infr_reform_data.html` (2014), `http://www.newyorkfed.org/markets/gsds/search.html` (2014); and for the Federal Funds market Afonso and Lagos (2012a), Afonso and Lagos (2012b), Bech and Atalay (2010).

[2]Among many, see Caballero and Simsek (2009), Zawadowski (2011), and Zawadowski (2013).

[3]See Acharya and Bisin (2014), Pirrong (2009), and Powell (November, 21st 2013), Duffie et al. (2010), Jackson and Miller (2013).

[4]See Acharya and Bisin (2014) on transparency, but also Biais et al. (2016), and Koeppl (2013) among others.

related to significant synergies across different projects and trades which are observable only to the agents involved in those activities. Thus, soft information cannot be easily and publicly verified by a third party, or it is difficult to summarize and aggregate.[5]

In our economy, trading is bilateral and subject to two frictions: limited pledgeability of a counterparty's future income, and private information about the degree of pledgeability of income, which we call an agent's pledgeability type. Costly monitoring reveals the extent to which a counterparty's income is pledgeable. This information, however, is not available to a third party, such as a clearing institution, which has to induce truthful reporting about the monitoring activity and its outcome by choosing contractual terms appropriately. When monitoring doesn't take place, the pledgeability type of a counterparty cannot be part of contractual terms. In this case, information is not available to financial market participants, and in particular to clearing institutions; such lack of information results in inefficient collateral by the CCP and, possibly, in strategic defaults by some of its members.

Clearing is the process of transmitting, reconciling, and confirming payment orders or instructions to transfer securities prior to settlement. Clearing is bilateral when it takes place via traders' respective clearing banks: under this arrangement each trader bears the risk that her bilateral counterparty may default. Traders manage this risk by requiring collateral to be posted. Central clearing is done by a third party, namely a central counterparty, that transforms the nature of the risk exposure of the two parties in a trade. A CCP is an entity that interposes itself between two counterparties, becoming the buyer to every seller and the seller to every buyer for the specified set of contracts.[6] The substitution of the CCP as the sole counterparty for each of the two original traders in a bilateral transaction is called novation. Novation, however, doesn't

[5]See Stein (2002), Petersen (2004), Hauswald and Marquez (2006), Mian (2003).

[6]See Capital requirements for bank exposures to central counterparties (2012), and BIS glossary of terms used in payments and settlement systems (2003).

eliminate counterparty risk: the CCP needs to use proper risk management tools and loss allocation mechanisms to guarantee that it has enough resources to perform the obligations stemming from novation, despite possible default by some of its members.[7] Through novation, the CCP observes *all* contracts traded by institutions for which it performs clearing services in a *specified* financial market. Both *all* and *specified* are important components of this definition: the first one implies that, within a specific market, the CCP has information about the network of trades across its members, which may not be available to the bilateral counterparties. The second implies that the CCP may lack information about its members, if that information is learned outside the specified set of contracts which the CCP clears. Previous research on CCPs, for example Acharya and Bisin (2014), has focused on the first component, recognizing the potential welfare benefits of CCP clearing. Instead, we focus on the second component and characterize the conditions under which CCP clearing might reduce welfare relative to bilateral clearing.

In our economy, clearing arrangements and risk management tools adopted by the CCP affect equilibrium outcomes, including incentives to acquire information about counterparties. Our model is novel in this respect: it shows that crucial information acquired in a bilateral relationship may be lost when clearing services are transferred to a central counterparty which operates under commonly used loss allocation methods. We model the institutional features of loss allocation methods adopted by modern CCPs as restrictions on the contract space of the CCP.

The tradeoff between bilateral and central clearing arises from *i*) two dimensions of risk against which traders value insurance, namely a counterparty's uncertain income and pledgeability type, *ii*) private information about a counterparty's pledgeability type, which introduces an adverse selection problem, *iii*) and the risk management

[7] See, among others, `https://www.chicagofed.org/publications/chicago-fed-letter/2017/389` (2017)

tools and loss allocation methods adopted by the CCP.

The severity of the adverse selection problem interacts with the value of insurance in different ways in each clearing arrangement. With bilateral clearing, counterparty risk is managed through collateral requirements, which are costly in terms of foregone investment opportunities. Costly monitoring provides the information necessary to tailor collateral requirements to the counterparty's pledgeability type.

With CCP clearing, uncertainty about a counterparty's income is managed through specific loss allocation methods, which define the financial resources that are used to absorb the losses caused by a default. We consider three institutional arrangements currently adopted by CCPs to absorb losses in excess of defaulting members' margins. In the first case, the CCP allocates losses pro-rata among surviving members, as in Acharya and Bisin (2014) and Koeppl and Monnet (2010). Loss mutualization enables the CCP to diversify counterparty risk and save on collateral requirements. The second institutional arrangement consists in partial tear-up of unmatched contracts. Under this loss allocation method, which is adopted under extreme default scenarios, the CCP can terminate contracts by original counterparties, thus de-facto undoing novation. The third institutional arrangement is a pro-rata loss allocation method with due diligence, which is routine operation of modern CCPs, and is equivalent to the first arrangement but with disciplinary actions. According to this method, the CCP must honor the contracts it clears, unless it detects lack of due diligence by its members, in which case it adopts disciplinary actions.[8]

The loss allocation method adopted by the CCP implies a degree of insurance against counterparty risk which interacts in an important way with the supply of information about pledgeability types. When the CCP can induce each member to monitor a counterparty and truthfully reveal her type, it can implement separating

[8] See Ice Clear Credit Clearing Rules (2018), ICE Clear Disclosure Framework (2018), and National Securities Clearing Corporation - Rules and Procedures (2018).

contracts that make central clearing Pareto superior to bilateral clearing. We call an allocation that satisfies these conditions incentive-feasible.[9]

An incentive-feasible allocation always exists if the CCP adopts, in equilibrium, a partial tear-up loss allocation method. When the CCP doesn't or cannot rely on a partial tear-up loss allocation method, incentive-feasible allocations may not exist and there is a trade-off between bilateral and central clearing. CCP clearing naturally maintains the ability to provide insurance by pooling risk over idiosyncratic shocks to income. Without the information generated by monitoring, however, the CCP cannot tailor contracts to counterparty types in a trade, resulting in either excessive or insufficient collateral. With bilateral clearing, insurance requires to post collateral. This is costly, but it is exactly this cost that preserves incentives to monitor. Intuitively, monitoring produces information useful in tailoring collateral requirements to the type of counterparty and, when collateral is costly, this information is valuable. If monitoring is not too costly, traders prefer bilateral clearing. The insurance provided by the CCP may not be sufficient to compensate for the loss of information about a counterparty's type. Note that this result is not related to the common idea that CCPs may generate moral hazard and increase risk by providing insurance. In our economy the amount of risk is fixed. Rather, it is due to the lack of incentives to acquire and transmit information about counterparties, which may result from the activity of the CCP.

As we discuss in the next section, our results and the economic mechanism at the core of our analysis are consistent with empirical findings for certain financial markets, with concerns of practitioners and regulators, and with findings from recent work based on network analysis. Because different clearing arrangements provide different incentives, the optimal clearing arrangement depends on the structure of financial assets

[9]Because monitoring and truth-telling are incentive feasible, then the CCP tailors collateral requirements to counterparty types, and is able to implement transfers that make every participant weakly better off.

traded and the information set of market participants and clearing institutions. In this respect, our model shows what characteristics of assets and trades are optimally associated with bilateral and central clearing arrangements. Specifically, for a region of the parameter space, our model implies that financial institutions with high opportunity cost of collateral, such as dealers and hedge funds, should prefer to clear their trades bilaterally, whereas institutions with a low opportunity cost of collateral, such as money market funds (MMMF), are more likely to rely on CCPs.[10]

The paper is organized as follows: the remainder of this section provides a literature review, Section 2 describes the model, Sections 3 optimal contracts without information acquisition, Section 4 optimal contracts with information acquisition, and Section 5 comparative statics. Section 6 concludes.

1.1 Related Literature

Our paper relates to the literature that studies how changes in financial market infrastructure influence the exposure of market participants to default as well as market liquidity risk.

Part of this literature has focused on the benefits of CCP clearing. Carapella and Mills (2011) focus on netting and highlight a liquidity enhancing role for CCPs, which reduce trading costs and facilitate socially desirable transactions that would not occur with bilateral clearing. Koeppl and Monnet (2010) focus on novation and counterparty risk insurance: in their framework CCP clearing is the efficient arrangement for centralized trading platforms, and it improves on bilateral clearing for OTC trades by providing a better allocation of default risk. Acharya and Bisin (2014) focus

[10]That the opportunity cost of collateral is larger for dealers than MMMF is reflected in higher returns produced by the former. That MMMF have taken up central clearing wherever possible is reflected, for example, in the increase in the rate of repos cleared at Fixed Income Clearing Corporation (FICC) between 2015 and 2017, which have tended to replace reverse repos with the Fed (see BIS Quarterly Review (2017)).

on information dissemination and stress the welfare enhancing effect of central clearing on transparency: CCP clearing can correct for an externality introduced by the non-observability of trading positions, when the exposure to third parties can cause a counterparty to default. Monnet and Nellen (2012) focus on two-sided limited commitment and show that a CCP can improve on a segregation technology (defined as a vault for collateral assets) through novation and mutualization.

We differ from these papers as in our model the provision of clearing services by a CCP is endogenously limited by the loss allocation method adopted. Duffie and Zhu (2011) also show that introducing a CCP that clears a class of derivatives may lead to an increase in average exposure to counterparty default. However, their mechanism is very different from ours, as their focus is on netting. The authors show that when a CCP is dedicated to clear only one class of derivatives, the benefits of bilateral netting between pairs of counterparties across different assets may be larger than the benefits of multilateral netting among many clearing participants but within a single class of assets. In our model, we focus on novation and mutualization of losses as the key features of central clearing.

Two models of central clearing related to ours are Biais et al. (2016) and Koeppl (2013). In both papers, there is asymmetric information between buyers and sellers of financial assets in the form of moral hazard, and collateral plays both the role of insuring counterparties and aligning incentives. Moral hazard generates the potential for excessive risk taking, which is controlled by margin requirements. In Biais et al. (2016), the risk pooling activity associated with central clearing allows the central counterparty to set margins more efficiently than in bilateral trade, and central clearing always dominates. In Koeppl (2013), margin requirements play a similar role as in Biais et al. (2016) in controlling moral hazard, but also, as in our case, generate a tradeoff between bilateral and central clearing. In Koeppl (2013), a CCP has the objective of minimizing the risk associated with trades. Moral hazard in risk-taking is potentially amplified by collusion between buyers and sellers, and the resulting margin requirements can have a negative impact on market liquidity (as measured by the probability of finding a counterparty). Thus, the tradeoff between bilateral and central clearing lies in their relative impact on market liquidity. In our environment the CCP provides insurance via loss mutualisation as well, but, via novation, it interacts with adverse selection and costly monitoring. This interaction affects traders' incentives to acquire socially valuable information about their trading partners, and transmit it to the CCP. This mechanism is similar to what Pirrong (2009) suggests: information asymmetries between the CCP and its clearing members may result in an increase in counterparty risk at the CCP, especially for complex products traded by large and opaque financial institutions. Our paper is also related to the literature on payment systems, in particular to Koeppl et al. (2012), who study the efficiency of a clearing and settlement system in an environment with information asymmetry between the clearing institution and traders. In our model, trading is subject to an information asymmetry as well: traders can costly acquire soft information about their counterparty while the clearing institution cannot. However, the focus of our paper is the endogenous effect of this information asymmetry on the credit risk faced by the clearing institution. In this respect our paper complements the one by Koeppl et al. (2012) by characterizing how central clearing can affect transparency and risk management in financial markets.

An extensive analysis of central counterparties is provided by Pirrong (2009), which describes aspects of central clearing that are reminiscent of our formal model, noting that central clearing always involves a mechanism to redistribute losses in case of default. This redistribution may be affected by asymmetric information problems, which are likely to be relatively more severe when central clearing involves members whose balance sheets are opaque as a result of trading positions outside the products that are centrally cleared. These balance sheet risks and the asymmetric information problems associated with them affect more severely centrally cleared markets than bilaterally cleared markets. A conclusion of Pirrong's discussion is that "...a CCP prices default risk as if all members are homogeneous, when in fact they are not necessarily so. Although this imposed homogeneity can contribute to liquidity, it misprices balance sheet risks and tends to encourage trading by less creditworthy firms. Thus, a variety of considerations suggests that the cost of evaluating and pricing balance sheet risks are lower in bilateral OTC markets than centrally cleared ones, especially when intermediaries are complex firms engaged in information-intensive intermediation." (Pirrong (2009), page 50.)

Our results formalize concerns expressed by practitioners and analysts about regulatory reforms of clearing arrangements. Gregory (2014), Section 1.5, discusses possible dangers of introducing mandatory central counterparty clearing: "A third potential problem [of CCP clearing] is related to loss mutualization that CCPs use whereby any losses in excess of a member's own financial resources are generally mutualized across all the surviving members. The impact of such mechanism is to homogenize the underlying credit risk such that all CCP members are more or less equal. . . . Many firms trading derivatives (e.g. large banks and hedge funds) specialize precisely in understanding risks and pricing, and hence are likely to have better information than CCPs especially for more complex derivatives." Indeed, "One of the last futures exchanges to adopt a CCP was the London Metal Exchange in 1986 (again with regulatory pressure being a key factor)." (Gregory (2014), Section 2.1.5.)

Our results and the economic mechanism at the core of our analysis are consistent with empirical findings on central clearing for credit default swaps. Although they cannot measure monitoring and transparency directly, Loon and Zhong (2014) find that trading volume increase when credit default swaps are cleared centrally. This is an equilibrium outcome of our model, despite transparency may decrease with central clearing.

The results and assumptions of our model are also consistent with the empirical evidence in Bignon and Vuillemey (2016). First, we assume that the CCP cannot directly monitor ultimate investors. Bignon and Vuillemey (2016) find evidence of this information asymmetry in the failure of the *Caisse de Liquidation des Affaires et Marchandises* (CLAM, a CCP clearing sugar futures) in Paris in 1974, as "retail investors were unsophisticated and non-diversified, did not have enough liquid financial resources" and that CLAM could not "directly monitor ultimate investors".[11] Second, we show the existence of equilibria where lenders do not have incentives to acquire information about their counterparties and/or pass it on to the CCP. In equilibrium, then, the CCP is unable to charge member-specific margins. Bignon and Vuillemey (2016) show that CLAM kept margins *at a constant level across members*, which *was not sufficient to ensure stable clearing* and ended with the failure of a large CCP member and eventually of the CCP itself.

Finally, our results about the existence of a trade-off between bilateral and central clearing based on the value of insurance provided by CCPs, resonate with those in Garratt and Zimmerman (2015). They study netting in a network model of trades and find that CCPs "can improve netting efficiency only if agents have some degree of risk aversion that allows them to trade off the reduced variance against the higher expected netted exposures." They further hypothesize that "This may explain why, in the absence of regulation, traders in a derivatives network may not develop a CCP themselves," which is also consistent with our results about bilateral clearing being preferred in some financial markets.

[11] Bignon and Vuillemey (2016) go even further, theorizing *risk-shifting* behavior on the part of the CCP once it realized it was close to bankruptcy.

2 The Model

Time is discrete and consists of two periods, $t = 1, 2$. The economy is populated by two types of agents: a unit measure of *lenders* and a unit measure of *borrowers*. Lenders and borrowers have different preferences, and have access to different technologies.

There are two goods: a consumption good and a capital good. In the first period, lenders receive an endowment of one unit of capital, while borrowers receive an endowment of ω units of consumption good. The consumption good can be stored from $t = 1$ to $t = 2$ by both lenders and borrowers. The capital good can be invested at time $t = 1$ and transformed into time $t = 2$ consumption. Only borrowers have access to this technology. The technology is indivisible, takes one unit of capital good at $t = 1$, and returns $\widetilde{\theta}$ units of consumption good at $t = 2$; $\widetilde{\theta}$ is a random variable with support $\{0, \theta\}$, whose realization is unknown at the time of investment. We define $p = Prob(\widetilde{\theta} = \theta)$ to be the probability of success of investment and assume that the law of large numbers holds so that the aggregate return on investment is $p\theta$.

Borrowers have preferences biased towards consumption in the first period relative to lenders. Specifically, borrowers' preferences are defined over $t = 1$ consumption c_1 and time $t = 2$ consumption c_2, and are represented by the utility function

$$U(c_1, c_2) = \alpha c_1 + c_2 \qquad \alpha > 1$$

Borrowers have limited commitment to repay: a borrower can repudiate a contract and, after default, consume a fraction $1 - \lambda^i$ of the output realization, where i denotes a borrower's type. There are two types of borrowers, distinguished by the extent to which they can pledge their income. A measure q of borrowers can pledge a fraction λ^H of their income, and we will refer to them as high-pledgeabilitiy borrowers, while a measure $1 - q$ can pledge a fraction $\lambda^L < \lambda^H$, and we will refer to them as lowpledgeabilitiy borrowers. The type λ^i is private information of the borrower, but can be learned by a lender by exerting monitoring effort.

The preferences of a lender are defined over second period consumption x_2, and time-1 monitoring effort e, according to the utility function

$$V(x_2, e) = u(x_2) - \gamma \cdot e$$

where u is strictly increasing and strictly concave, and $e \in \{0, 1\}$. We further assume that $\lim_{x \to 0} u'(x) = +\infty$.

The mismatch between endowments and preferences over consumption goods generates incentives to trade. Lenders have capital but they need borrowers to use their technology to transform it into consumption goods. Nevertheless, trade is subject to two frictions. First, there is limited commitment; second, each lender is randomly matched and can only contract with one borrower. Trade is bilateral.

When a lender and a borrower are matched with each other, they enter into a relationship described by a contract. The lender provides the contract to the borrower as a take-it-or- leave-it (TIOLI) offer, which also specifies a clearing and settlement arrangement.[12] In the second period, settlement takes place either bilaterally or trough a CCP, according to the lenders' choice.

Feasible contracts differ depending on the clearing arrangement initially chosen. In the next sections, we define and characterize optimal contracts with bilateral and central clearing. Our analysis includes the equilibrium characterization of economies with central clearing under different assumptions about the contract space, each corresponding to different possibilities for the CCPs' operation, rules, and procedures. Specifically, we analyze *1)* a benchmark case, where potential CCP losses are allocated

[12]When the commitment constraint is binding, the assumption of a TIOLI is without loss of generality because of transferable utility.

pro-rata among its members; *2)* a *partial tear-up* case, which is adopted by CCPs under extreme default scenarios; and *3)* a case with *pro-rata allocation of losses and due diligence*, describing the routine operation of a CCP with disciplinary actions.[13]

We model the benchmark case by assuming that, after novation, the financial obligation between the original counterparties is eliminated and the CCP allocates losses pro-rata among its members. Novation is, in fact, a counterparty substitution: from the bilateral counterparty to the CCP, which "stands in between buyers and sellers and guarantees the performance of trades ...[and]... is legally obliged to perform on the contracts it clears."[14] The assumption that the financial obligation between original counterparties is eliminated is meant to model the legal obligation of the CCP to perform on cleared contracts. The pro-rata loss allocation method implies that any losses which the CCP experiences are mutualized across its members. Hence, no information about a member's default is used to allocate losses ex-post. In the context of our model, this is achieved by allowing the CCP to pool idiosyncratic risk.[15]

We model the partial tear-up case by assuming that the CCP can offer its members fully state contingent contracts. In extreme default scenarios "loss allocation methods [that] go beyond the idea of simply using default funds on a pro-rata basis" can be implemented. One such method gives the CCP an "option [...] to "tear-up" unmatched contracts with surviving clearing members. [...] The aim of the tear-up is to return the CCP to a matched book by terminating the other side of a defaulter's trades (or at least those that cannot be auctioned). All other contracts (possibly the majority of the total contracts cleared) could remain untouched."[16] This loss allocation method, in the

[13]See Gregory (2014) section 10.3.

[14]See Gregory (2014), section 8.3, and **?**: "CCPs are best seen as commitment mechanisms that assure the performance of financial contract obligations. How they perform that function sets them apart from other infrastructures, intermediaries and financial institutions."

[15]A possible interpretation of this first case is that the contracts submitted for central clearing are liquid, and one side of the contract can sell his position to a third party, implying that the initial link between the lender and the borrower is destroyed.

[16]See Gregory (2014), pg 187-192.

context of our model, requires the CCP to terminate its contract with a lender upon default (exogenous or strategic) of her original borrower. To gain perspective into the preference of CCPs (and regulators) to avoid partial tear-ups, it is instructive to note that such loss allocation method was not adopted in the two stressful scenarios that involved CCPs in the past few decades. In September 2008, the default of Lehman did not trigger any partial tear up but was resolved worldwide with auctions and transfers of clients' accounts resulting in closing out all of Lehman's positions without affecting CCPs ability to perform on their obligations.[17] Ten years later, in September 2018, despite the default of a major member wiping out "2/3 of the default fund" of Nasdaq Clearing, the CCP did not adopt a partial tear-up. Rather, Nasdaq Clearing i) contributed its Junion Capital fund, and ii) recapitalized the Default Fund using additional contributions from its clearing members – in other words, a pro-rata loss allocation method was adopted.[18]

We model the case of pro-rata with due diligence by assuming that the CCP must honor the contracts it clears, unless it detects lack of due diligence by its members. These assumptions are equivalent to a CCP operating with a *pro-rata* loss allocation method with the ability to impose sanctions for violations of its rules or for *prohibited conduct*.[19] The *pro-rata* loss allocation method implies that any losses which the CCP experiences are mutualized across its members. In the context of our model, this is achieved by allowing the CCP to pool idiosyncratic risk, similarly to Koeppl and Monnet (2010) and Biais et al. (2016). The ability to impose sanctions for violations

[17]See Central Counterparty Default Management and the Collapse of Lehman Brothers", CCP12, The Global Association of Central Counterparties (2009). Also, see Faruqui et al. (2018), and the speech by Sir Jon Cunliffe, Deputy Governor for Financial Stability of the Bank of England, at the FIA International Derivatives Expo 2018, London (5 June 2018).

[18]See https://business.nasdaq.com/updates-on-the-Nasdaq-Clearing-Member-Default/index.html. See Elliott (2013) for a discussion of the disadvantages of a pro-rata loss allocation method with respect to the incentives of the CCP members to participate in a default management process.

[19]See Gregory (2014) sections 10.1.1-3 on the *pro-rata* loss allocation method. Also see Ice Clear Credit Clearing Rules (2018) and ICE Clear Disclosure Framework (2018): articles $701-2$, rule 609.

of the CCP's rules and procedures implies that the net payment from the CCP to a member might differ from the payment specified in the contract submitted for central clearing. In the context of our model, the CCP detects lack of due diligence when it is able to identify whether a lender did not monitor her counterparty or did not report her counterparty's type truthfully. When this happens we assume that the CCP can use any information about the original counterparty to punish such misbehavior by the lender.

Labeling agents as *lenders* and *borrowers* and modeling the contract between them as a loan is meant to capture the counterparty (credit) risk of a financial relationship. In this respect, it should not be thought of as a restriction on the set of contracts analyzed in our model relative to the set of contracts which are bilaterally and centrally cleared in reality. A loan in our model is the analog of any financial obligation with a component of counterparty risk, which we formalize as limited commitment to honor such obligation. Whether the obligation is a repayment for a loan obtained in the past – as in a repurchase agreement or a bond – or the transfer of an asset – as in an option which is exercised by its holder – the limited commitment to keep promises previously made is intrinsically the same. Limited commitment is the pivotal friction in the model, and it introduces interesting interactions between the clearing arrangement, the terms of the contract traded, and the information acquired about the counterparty.

3 Optimal contracts without information acquisition

In this section, we characterize optimal contracts without information acquisition. The goal of this section is twofold: *1)* to introduce notation and the basic mechanics of our model; *2)* to set a benchmark for contracts to which we will refer in subsequent sections when information acquisition will not occur in equilibrium. The definition of a contract differs depending on the clearing arrangement chosen. With bilateral clearing, settlement involves only the original counterparties. Instead, when clearing is central, borrowers and lenders submit to the CCP the contract upon which they agree. The CCP then *novates* such contract. With novation, the original contract is suppressed and replaced by two contracts: one between the lender and the CCP, and one between the borrower and the CCP. The CCP takes the terms of the original contract as given, but can require borrowers to post collateral (i.e. *margin*), and lenders to contribute to a loss mutualization scheme (i.e. *default or guarantee fund*).

3.1 Bilateral Clearing without information acquisition

When clearing is bilateral, lenders commit to a mechanism that specifies a menu of contracts. Without loss of generality, we assume that lenders commit to direct revelation mechanisms, that is, a contract is executed after the borrower truthfully announces his type. Thus, a strategy for a borrower is a pair $(m^i, \sigma^i) \in \{\lambda^L, \lambda^H\} \times \{0, 1\}$, where m^i is his reporting strategy and σ^i his default decision when the idiosyncratic state is $s = h$; $\sigma^i = 1$ means that the borrower defaults in equilibrium.[20]

A mechanism with bilateral clearing is a menu of contracts $(\Sigma^i, c_1^i, c_{2,s}^i, x_{2,s}^{i,\Delta})_{i=\{L,H\}}$, where Σ^i is the lender's default recommendation (contingent on the idiosyncratic state $s = h$) to a borrower that reports his type to be λ^i. We use the notation $\Sigma^i = 1$ to mean that a lender recommends her counterparty to default in equilibrium. Also, Δ represents the public history of the borrower's default/repayment decision, where $\Delta = 1$ if the borrower defaults in equilibrium, and $\Delta = 0$ if the borrower repays. We say that a contract is incentive-compatible if a borrower's best strategy (m^i, σ^i) is to report

[20] To be more specific, a strategy for a borrower is a triple $(m^i, \sigma_l^i, \sigma_h^i)$, where σ_s^i is type-i borrower's default strategy, when the idiosyncratic state is s. However, $\sigma_l^i = 0$ is a (weakly) dominant strategy, so we can ignore it from the definition of the borrower's feasible strategies and assume that a borrower always repays when $s = l$.

truthfully his type, $m^i = \lambda^i$, and then follow the default/repayment recommendation, $\sigma^i = \Sigma^i$.

After reporting his type and accepting the ensuing contract, a borrower receives one unit of capital and transfers $\omega - c_1^i$ units of consumption good to the lender. As an example of a financial contract between the lender and the borrower, consider a repurchase agreement (repo): then we can think of the unit of capital transferred by the lender to the borrower at $t = 1$ as the *starting leg* of the repo, and of the payment x_2^i by the borrower to the lender at $t = 2$ as the *closing leg* of the repo.[21] We can think of the transfer of $\omega - c_1^i$ by the borrower to the lender at $t = 1$ as collateral, as it denotes the amount of consumption good stored by the lender to be consumed at $t = 2$. In this respect $\omega - c_1^i$ is akin to margins in financial transactions (or a house in a mortgage) as it preserves the value of the lender's investment by insuring the lender against the borrower's default.[22] The borrower then chooses to invest the unit of capital, while the lender chooses to store the consumption good $\omega - c_1^i$. In the second period, after the shock realization is known, the lender is entitled to consumption $x_{2,s}^i$, whereas the borrower is entitled to consumption $c_{2,s}^i$ and chooses whether to default ($\sigma^i = 1$) or to repay ($\sigma^i = 0$). When the realization of the borrower's idiosyncratic state at $t = 2$ is low ($s = l$) the lender can use the additional resources from collateral for her own consumption. When the realization of the borrower's idiosyncratic state at $t = 2$ is high ($s = h$) and the borrower repays ($\sigma^i = 0$) the lender returns the collateral to the borrower, who consumes it together with the return from his production technology net of the payment to the lender for the closing leg of the repo.[23] If the borrower

[21] See Garbade (2006).

[22] Notice that we are assuming one sided limited commitment, only on the side of the borrower. Therefore lenders always return the collateral to borrowers if $\tilde{\theta} = \theta$. Storage is verifiable.

[23] This example reflects the micro-foundations of a repo as derived in Antinolfi et al. (2015), where borrowers have access to an investment technology that needs lenders' capital (and borrowers' know how) to be operated, and where the possibility of borrowers' default justifies lenders to require the transfer of collateral upfront from borrowers as a means to self insure.

defaults ($\sigma^i = 1$) the lender keeps the collateral and the borrower's pledgeable income $(\omega - c_1^i + \lambda^i\theta)$, whereas borrower's consumption is equal to $(1-\lambda^i)\theta$.

The optimal mechanism solves the following problem:

$$(P_0^b) \quad V^{bil,e=0} = \max \sum_{i=L,H} q_i \left[p\left\{ \Sigma^i u(x_{2h}^{i1}) + (1-\Sigma^i) u(x_{2h}^{i0}) \right\} + (1-p) u(x_{2l}^i) \right] \tag{1}$$

$$s.t. \quad \alpha c_1^i + p\left[\Sigma^i (1-\lambda^i)\theta + (1-\Sigma^i) c_{2h}^i\right] + (1-p)\, c_{2l}^i \geq \alpha\omega \tag{2}$$

$$\omega \geq c_1^i \geq 0 \tag{3}$$

$$x_{2h}^{i0} + c_{2h}^i \leq \omega - c_1^i + \theta \tag{4}$$

$$x_{2h}^{i1} \leq \omega - c_1^i + \lambda^i\theta \tag{5}$$

$$x_{2l}^i + c_{2l}^i \leq \omega - c_1^i \tag{6}$$

$$(\lambda^i, \Sigma^i) \in \underset{(\hat{m},\hat{\sigma})}{\operatorname{argmax}} \left\{ \alpha c_1^{\hat{m}} + p\left[\hat{\sigma}(1-\lambda^i)\theta + (1-\hat{\sigma}) c_{2h}^{\hat{m}}\right] + (1-p)\, c_{2l}^{\hat{m}} \right\} \tag{7}$$

Constraint (2) is borrower i's participation constraint, for $i \in \{L, H\}$: the borrower can always refuse to trade, and consume the endowment ω in the first period. Constraint (3) is time $t = 1$ feasibility, (4) and (5) are time $t = 2$ feasibility in states $(s, \Delta) = (h, 0)$ and $(s, \Delta) = (h, 1)$ respectively; (6) is time $t = 2$ feasibility condition in state l. Finally, constraint (7) is the incentive-compatibility constraint for a borrower of type λ^i: the strategy pair (λ^i, Σ^i) is incentive compatible if there is no other strategy pair $(\hat{m}, \hat{\sigma})$ that yields a higher payoff. Notice that a borrower can deviate by reporting a different type $\hat{m} \neq \lambda^i$, by choosing a different default strategy $\hat{\sigma} \neq \Sigma^i$, or both.

In Proposition 3 of Section 3.2 we prove that bilateral clearing is never optimal when borrowers' type is private information and lenders have no technology to learn such type. Thus, we do not characterize the soloution to problem (P_0^b), because the goal of the paper is to compare bilateral versus central clearing, and the characterization of mechanisms with bilateral clearing and no monitoring is irrelevant to the question we want to address.

3.2 Central Clearing without information acquisition

A key aspect of central clearing is novation, that is the legal act of erasing the original obligations between a borrower and a lender, and the CCP becoming the sole counterparty to each of them. We model novation by assuming that the CCP commits to a mechanism at the beginning of $t = 1$, and that lenders and borrowers negotiate over the contracts in such mechanism. Each contract specifies transfers between borrowers and the CCP and transfers between lenders and the CCP, while no transfer between the borrower and the lender takes place. Transfers are a function of *i)* public information and *ii)* the restrictions on the contracting space consistent with institutional arrangements adopted by CCPs in reality.

Formally, a strategy for a type-i borrower is a pair $(m^i, \sigma^i) \in \{\lambda^L, \lambda^H\} \times \{0, 1\}$ that specifies a first period announcement strategy $m^i \in \{\lambda^i, \lambda^H\}$ and a second period default decision $\sigma^i \in \{0, 1\}$. Similarly to the bilateral clearing case, $\sigma^i = 1$ means that the borrower defaults in equilibrium. Let then $\Delta \in \{0, 1\}$ be the borrower's observed default/repayment decision and $s \in \{l, h\}$ denote his time $t = 2$ idiosyncratic state. Define $\mathcal{H}_2$ to be the set of all possible second-period histories for a borrower. Thus, an element of $\mathcal{H}_2$ is a pair (s, Δ), and $\mathcal{H}_2 = \{l, h\} \times \{0, 1\}$. A mechanism with central clearing consists of contracts between the CCP and the lender, $\{X_2^i(h_2)\}_{i=L,H}$, and between the CCP and the borrower, $\{\Sigma^i,\ C_1^i,\ C_{2,s}^i\}_{i=L,H}$, which are executed if the borrower reports his type to be λ^i. Let h_2 denote the public history of a borrower after novation, and Σ^i the default decision that the CCP recommends to a borrower who reports his type to be λ^i. A mechanism is incentive compatible if it is the borrower's best response to report truthfully his type, and then follow the default recommendation

Σ^i.

In Lemma 2 we prove that optimal contracts between the CCP and lenders are independent of public history h_2. Thus, in this section we abstract from discussing the differences in the specific institutional arrangements adopted by CCPs with respect to their loss allocation mechanism. We defer to Section 4.2 the discussion of these institutional arrangements and the analysis of the ensuing restrictions on the contracting space with central clearing.

To simplify notation, we then write $X_2^i\big((s,\Delta)\big) = X_{2,s}^{i,\Delta}$. Referring to the example in Section 3.1, if the financial contract is a repurchase agreement, the contract between the lender and the CCP involves a starting leg where the lender transfers his endowment of capital to the CCP at $t = 1$, and a closing leg where the CCP pays $X_{2,s}^{i,\Delta}$ to the lender, where (s, Δ) denotes new contractible information about the borrower, observed by the CCP in $t = 2$. The contract between the borrower and the CCP involves a starting leg where the CCP transfers one unit of capital (received from the lender) to the borrower, and the borrower transfers $\omega - C_1^i$ units of good to the CCP as a margin requirement. The closing leg of this contract involves the transfer of $\theta - C_{2,s}^i$ units of consumption good from the borrower to the CCP, of which $\theta - C_{2,s}^i - X_{2,s}^{i,\Delta}$ are default fund contributions from the borrower.[24]

Let $\hat{m}$ and $\hat{\sigma}$ be defined as in Section 3.1 for problem (P_0^b). The optimal mechanism with central clearing and no monitoring solves

$$(P_0) \quad V^{CCP,e=0} = \max \sum_i q_i \Big\{ p \Big[\Sigma^i u(X_{2,h}^{i,1}) + (1-\Sigma^i) u(X_{2,h}^{i,0}) \Big] + (1-p) u(X_{2,l}^{i,0}) \Big\} \tag{8}$$

$$s.t. \;\; \alpha C_1^i + p[\Sigma^i (1-\lambda^i)\theta + (1-\Sigma^i) C_{2h}^i] + (1-p)\, C_{2l}^i \geq \alpha\omega \tag{9}$$

$$0 \leq C_1^i \leq \omega \tag{10}$$

[24] Clearly, if the realization of the borrower's output shock is low ($s = l$) then this borrower makes no payment to the CCP at $t = 2$. The CCP can use resources from margin requirements and default fund contributions of borrowers able to pay, in order to settle payments to lenders.

$$\sum_i q_i \Big\{ p\Big[\Sigma^i\Big\{X^{i,1}_{2,h} + (1-\lambda^i)\theta\Big\} + (1-\Sigma^i)\Big\{X^{i,0}_{2,h} + C^i_{2h}\Big\}\Big]$$

$$+ (1-p)(C^i_{2l} + X^{i,0}_{2,l})\Big\} \leq p\theta + \sum_i q_i\{\omega - C^i_1\} \tag{11}$$

$$(\lambda^i, \Sigma^i) \in \underset{(\hat{m},\hat{\sigma})}{\operatorname{argmax}} \Big\{\alpha c^{\hat{m}}_1 + p\Big[\hat{\sigma}(1-\lambda^i)\theta + (1-\hat{\sigma})C^{\hat{m}}_{2h}\Big] + (1-p)\, C^{\hat{m}}_{2l}\Big\} \tag{12}$$

Constraint (9) is borrower's i participation constraint; (10) and (11) are respectively time $t = 1$ and $t = 2$ feasibility constraints. Note that the feasibility constraint in $t = 2$ is defined for the aggregate resources of the CCP in the second period, since the CCP becomes the buyer to every seller and the seller to every buyer. Also, note that CCP's resources are constant, as there is no aggregate uncertainty. Constraint (12) is the incentive compatibility constraint of a borrower who must report his type truthfully, $m^i = \lambda^i$, and then follow the default recommendation, $\sigma^i = \Sigma^i$.

Lemma 1 *Any contract preventing borrowers' default,* $\Sigma^H = \Sigma^L = 0$*, must satisfy* $\min\{C^H_{2,h},\, C^L_{2,h}\} \geq (1-\lambda^L)\theta$ *and* $\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} = \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l}$.

Lemma 1 is a direct consequence of the incentive-compatibility constraint (12), and the observation that a borrower may deviate by reporting a different type $\hat{m} \neq \lambda^i$, by choosing a different default strategy $\hat{\sigma} \neq \Sigma^i$, or both. In other words, the limited commitment friction interacts with private information in a way that forces the CCP to treat both types of borrowers as if their constraints were the same as that of the worst type. This result has important consequences for the contract which the CCP can offer, as it prevents any separation of borrowers where information about a high pledgeability type can be exploited, if at the same time the contract prevents borrowers' default.

Lemma 2 *A solution to Problem* (P_0) *is such that* $X^{i,\Delta}_{2,s} = X^{-i,\Delta'}_{2,s'}$ *for all* $s, s' \in \{l, h\}$ *and* $\Delta, \Delta' \in \{0, 1\}$.

Lemma 2 proves that the CCP optimally ignores information about a borrower in the contract with the lender who was the original counterparty to that borrower: concavity of the utility function $u(\cdot)$ implies that a solution must satisfy $X_{2,s}^{i,\Delta} = X_{2,s'}^{i,\Delta'}$, for all $s, s' \in \{l, h\}$ and $\Delta, \Delta' \in \{0, 1\}$. Thus, any restriction to the space of contracts between the CCP and lenders is irrelevant.

Comparing problem (P_0) to problem (P_0^b) we can prove the following result:

Proposition 3 *Without information acquisition, central clearing is the optimal clearing arrangement: the solution to (P_0) dominates the solution to (P_0^b).*

When lenders cannot learn the pledgeability type of their counterparty, they are no better than the CCP at evaluating the risk that a borrower will strategically default. Thus, the CCP can always replicate borrowers' optimal contracts of Section 3.1, and, in addition, insure lenders against the idiosyncratic risk associated with the original counterparty. Despite borrowers' expected repayments to lenders are the same as with bilateral clearing, counterparty risk vanishes with the CCP due to the provision of insurance. Hence, central clearing is always preferred to bilateral clearing.

To characterize the solution to problem (P_0), using Lemma 2 we simplify the notation and write $X_{2,s}^{i,\Delta} = X_2$ in (8) and in (11). Also, it is easy to see that the resource constraint (11) is binding:

$$X_2 = p\theta - \sum_i q_i \Big[\Sigma^i p(1-\lambda^i)\theta + (1-\Sigma^i)pC_{2h}^i + (1-p)C_{2l}^i\Big] + \sum_i q_i\{\omega - C_1^i\}. \quad (13)$$

Then the CCP chooses a collateral policy and a default recommendation to maximize time $t = 2$ revenues, subject to borrowers' participation and incentive-compatibility constraints. Specifically, because $\alpha > 1$, the expected revenues of the CCP are larger when borrowers consumes all of their endowment ω in $t = 1$, and consume nothing in $t = 2$. However, such a contract would induce all borrowers to default in equilibrium.

In this environment, the CCP may provide the borrower incentives to repay at $t = 2$, by storing some of his endowment from time $t = 1$ to time $t = 2$.

In particular, let $\omega(\lambda)$ be the smallest amount of collateral that a borrower with pledgeability λ needs to post in order to overcome his limited commitment problem:

$$\omega(\lambda) = \frac{(1-\lambda)p\theta}{\alpha} \tag{14}$$

Assumption 4 *Assume that borrowers' endowment ω is large enough: $\omega > \omega(\lambda^L) \equiv \frac{(1-\lambda^L)p\theta}{\alpha}$.*

To ease exposition we maintain Assumption 4 throughout the paper. However, we can relax this assumption and show that our main results holds true.[25] Assumption 4 guarantees that a borrower's participation constraint binds. In general, the interaction between the participation and the incentive constraints in problem (P_0) can give rise to solutions where the former is slack, for example if collateral is relatively scarce in the economy and the commitment problem is relatively severe (i.e. ω is relatively small and λ is relatively small). This would imply that the consumption allocation of the borrower at $t = 2$ exceeds the value of his outside option from simply consuming his endowment, which delivers utility $\alpha\omega$. In this case the borrower would earn extra rents with respect to what is necessary to satisfy his participation constraint, as $C_{2h} > \alpha\omega$. Assumption 4 guarantees that this does not happen. We can then proceed to characterize the solution to problem (P_0).

Proposition 5 *Let Assumption 4 hold. With no information acquisition, the optimal contract with CCP clearing satisfies $C_1^H = C_1^L$, and $C_{2,s}^H = C_{2,s}^L$. In particular, $C_{2,l}^H =$*

[25] The proof is available in Antinolfi et al. (2018) and upon request.

$C_{2,l}^{L} = 0$, *and*

$$C_1 = \begin{cases} \omega - \omega(\lambda^L) & \textit{if } q \leq 1/\alpha \\ \omega - \omega(\lambda^H) & \textit{if } q > \frac{1}{\alpha} \end{cases} \qquad C_{2,h} = \begin{cases} (1-\lambda^L)\theta & \textit{if } q \leq 1/\alpha \\ (1-\lambda^H)\theta & \textit{if } q > \frac{1}{\alpha} \end{cases}$$

where $\omega(\lambda)$ is defined in (14). If $q < 1/\alpha$ no borrower defaults in equilibrium ($\Sigma^L = 0$, $\Sigma^H = 0$); if $q > 1/\alpha$ low-pledgeability borrowers default in equilibrium ($\Sigma^L = 1$, $\Sigma^H = 0$).

Proposition 5 shows that optimal contracts with central clearing do not separate high-pledgeability from low-pledgeability borrowers, with the only exception of the default recommendation in a region of the parameters' space. Specifically, without information acquisition, the CCP must choose between two classes of contracts: one in which no borrower defaults in $t = 2$, and one in which λ^H borrowers repay in $t = 2$ whereas λ^L borrowers default in equilibrium. In the first scenario, the CCP offers a pooling contract that treats all borrowers as if they were the worst possible type. As a result, λ^H borrowers end up posting excessive collateral with respect to what their type would require. In the second scenario, contracts which let λ^L borrowers post too little collateral and default in equilibrium are pooling over λ^H types.[26]

Both excessive and insufficient collateral requirements play an important role in the decision of the CCP. Specifically, on the one hand, higher collateral requirements increase the amount of resources available at $t = 2$ if they prevent λ^L borrowers from defaulting in equilibrium. On the other hand, higher collateral requirements reduce the amount of resources available at the CCP in $t = 2$ through a different channel: to leave the participation constraint unaffected, if borrowers post an additional unit of collateral at $t = 1$ they must be compensated by $\alpha > 1$ units of consumption at $t = 2$.

[26] Formally, this is not a pooling contract, as the recommended default decision, which is part of the optimal contracts, differs for the two type of borrowers. However, these contracts are observationally equivalent to the CCP offering the same (pooling) contract that induces different default strategies in equilibrium.

The resolution of this trade-off depends on the cost of collateral, α, and on the measure of λ^L borrowers, $1-q$. In particular, when the population of λ^L types is relatively large, i.e. $q \leq \frac{1}{\alpha}$, the CCP benefits from preventing the default of λ^L borrowers. Thus, all borrowers post enough collateral to satisfy the limited commitment problem of λ^L types, namely $\omega(\lambda^L)$. If instead the population of λ^L types is relatively small, i.e. $q > \frac{1}{\alpha}$, it is too costly for the CCP to prevent the default of λ^L borrowers. Thus, the CCP maximizes its resources by requiring all borrowers to post collateral to satisfy the limited commitment problem of λ^H types, namely $\omega(\lambda^H)$.

Substituting the results from Proposition 5 into equation (8) we can finally write the value of central clearing with no information acquisition as:

$$V^{CCP,e=0} = \begin{cases} u\Big(\omega(\lambda^L) + p\theta\lambda^L\Big) & \text{if } q \leq \frac{1}{\alpha} \\ u\Big(\omega(\lambda^H) + p\theta[q\lambda^H + (1-q)\lambda^L]\Big) & \text{if } q \geq \frac{1}{\alpha} \end{cases} \tag{15}$$

This equation will be a useful benchmark in the following section.

4 Optimal contracts with information acquisition

Next, we introduce the possibility for lenders to engage in costly monitoring, which reveals to her the type λ^i of her counterparty. By assumption, this remains private information of the lender and the borrower. As a result, when designing a contract with monitoring, the CCP needs to take into account the incentives that lenders have to monitor their counterparty and report truthfully the information they learn.

4.1 Bilateral clearing with information acquisition

With bilateral clearing, when the lender monitors her counterparty and learns his type λ^i, $i \in \{L, H\}$, she will offer a contract that prevents the borrower from defaulting strategically in equilibrium. Therefore, a contract with bilateral clearing and information acquisition is a list $(x^i_{2,s},\, c^i_1,\, c^i_{2,s})$, where $x^i_{t,s}$ and $c^i_{t,s}$ are respectively the lender's and the borrower's consumption in time t and state s, when the borrower's type is i. The contract is indexed by the borrower's type i, and second period consumption is indexed by the idiosyncratic state $s \in \{l, h\}$.

Let V_i denote the value to a lender of a match with a borrower of type λ^i, once the lender has paid the cost γ and knows the borrower's type. Then lenders choose contracts $(x^i_{2,s},\, c^i_1,\, c^i_{2,s})_{i\in\{L,H\}}$ to solve

$$(P^i) \quad V_i = \max_{\left(x^i_{2,h}, x^i_{2,l}, c^i_1, c^i_{2,h}, c^i_{2,l}\right)\in\Re^5_+} \quad pu(x^i_{2,h}) + (1-p)u(x^i_{2,l}) - \gamma \tag{16}$$

$$s.t. \quad \alpha c^i_1 + pc^i_{2,h} + (1-p)c^i_{2,l} \geq \alpha\omega \tag{17}$$

$$\omega \geq c^i_1 \geq 0 \tag{18}$$

$$c^i_{2,h} + x^i_{2,h} \leq \omega - c^i_1 + \theta \tag{19}$$

$$c^i_{2,l} + x^i_{2,l} \leq w - c^i_1 \tag{20}$$

$$c^i_{2,h} \geq (1-\lambda^i)\theta \tag{21}$$

Constraint (17) is the borrower's participation constraint, (18) is time $t = 1$ feasibility of the consumption plan, and likewise (19) and (20) are time $t = 2$ feasibility in states h and l respectively. Constraint (21) is the borrower's limited commitment constraint: the borrower can default and consume $1 - \lambda^i$ units of consumption (in the low state $\widetilde{\theta} = 0$, and limited commitment to repay is not relevant).

It is easy to see that at a solution both second-period feasibility constraints (19) and (20) should bind. Solving for $x^i_{2,h}$ and $x^i_{2,l}$ and replacing their values in the objective function (16), we can solve for $(c^i_1,\, c^i_{2,h},\, c^i_{2,l})$.

Similarly to Section 3.2, because $\alpha > 1$, a lender's expected consumption is larger when the borrower consumes his whole endowment ω in $t = 1$, and nothing in $t = 2$. However, such a contract violates the limited commitment constraint (21), and leaves the lender with no consumption in the second period when the output realization is low, as implied by constraint (20). Therefore, the lender will always store some of the borrower's endowment from time $t = 1$ to time $t = 2$. Collateral with bilateral clearing plays two roles. First, it provides insurance to the lender against the risk of the low-consumption state at $t = 2$ when $s = l$. Second, it provides the borrower incentives to repay at $t = 2$. It does so indirectly, by storing consumption goods up to $t = 2$. The larger this amount, the easier it is for the borrower to satisfy the limited commitment constraint (21). To characterize the solution to (P^i) let λ^* be the unique value satisfying

$$\frac{\alpha - p}{1 - p} = \frac{u'\left(\frac{(1-\lambda^*)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^*)\theta\right)}. \tag{22}$$

Intuitively, λ^* is the smallest value of λ such that the limited commitment constraint is slack. For any $\lambda \leq \lambda^*$, the limited commitment constraint (21) is binding because the quality of the counterparty is relatively low, which is equivalent to a high borrower's temptation to default.

In the rest of the paper, we make the following assumption:

Assumption 6 *Assume that the commitment problem of λ^L borrowers is severe enough:*

$$\frac{\alpha - p}{1 - p} > \frac{u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^L)\theta\right)}.$$

Assumption 6 guarantees that, with bilateral clearing, information about the quality of counterparties has positive value: in the Appendix we show that when Assumption 6 is violated, information about the quality of a counterparty has no value and no trade-off exists between bilateral and central clearing. Under this additional assumption we can characterize the solution to problem (P^i).[27]

Lemma 7 *Let $\omega(\lambda)$ be defined in (14) and λ^* in (22) and let Assumptions 4 and 6 hold. Then, optimal contracts with bilateral clearing and monitoring satisfy* $c^i_{2,l} = 0$, $x^i_{2,h} = \theta - c^i_{2,h} + \omega - c^i_1$, $x^i_{2,l} = \omega - c^i_1$, $c^L_{2,h} = (1-\lambda^L)\theta$, $c^L_1 = \omega - \omega(\lambda^L)$, *and*

(1) $c^H_1 = \omega - \omega(\lambda^*)$, $c^H_{2,h} = (1-\lambda^*)\theta$, *if* $\frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{\alpha-p}{1-p}$.

(2) $c^H_1 = \omega - \omega(\lambda^H)$, $c^H_{2,h} = (1-\lambda^H)\theta$, *if* $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$.

To understand the intuition behind Lemma 7, recall that the limited commitment constraint of a λ^L borrower binds by Assumption 6. Thus, collateral always provides λ^L borrowers with incentives to repay at $t = 2$. Differently, the limited commitment constraint (21) of a λ^H borrower may or may not bind. If $\lambda^H > \lambda^*$, which corresponds to Case (1) in Lemma 7, λ^H borrowers' temptation to default is low and the limited commitment constraint is slack. In this case, the more important role of collateral is insurance against the low realization of $\tilde{\theta}$. If instead $\lambda^H < \lambda^*$, which corresponds to Case (2) in Lemma 7, the limited commitment constraint of λ^H borrowers binds and collateral provides them with incentives to repay at $t = 2$.

Lemma 7 also shows that collateral is the only tool that lenders can use to manage counterparty risk when clearing is bilateral. Because collateral is costly, lenders optimally choose to bear part of counterparty risk, namely lenders' consumption is higher in the state of nature where her counterparty can repay.

Corollary 8 *A solution to problem (P^i) is such that insurance is incomplete,* $x^i_{2,h} > x^i_{2,l}$.

[27] We characterize the entire set of solutions to the bilateral problem with informatino acquisition in Appendix 6.1.

Although insurance is incomplete, lenders can customize collateral requirements to the specific risk of their counterparty, after acquiring information about his pledgeability type. Such information may be valuable and, considering central clearing without information acquisition as a benchmark, as characterized in Section 3.2, we can prove the following:

Proposition 9 *Let Assumption 4 and Assumption 6 hold. Then, if*

$$\frac{1}{\alpha}\left[pu\Big(\min\{\lambda^H\theta+\omega(\lambda^H),\lambda^*\theta+\omega(\lambda^*)\}\Big)+(1-p)u(\max\{\omega(\lambda^H),\omega(\lambda^*)\})\right]$$
$$+\frac{\alpha-1}{\alpha}\left[pu\Big(\lambda^L\theta+\omega(\lambda^L)\Big)+(1-p)u\Big(\omega(\lambda^L)\Big)\right]>u\Big(\omega(\lambda^L)+\lambda^Lp\theta\Big), \tag{23}$$

there exists an interval $(\underline{q},\overline{q})$ *and a function* $\overline{\gamma}(q):(\underline{q},\overline{q})\to\Re_+$ *such that bilateral clearing with information acquisition is preferred to central clearing without information acquisition, if and only if* $q\in(\underline{q},\overline{q})$ *and* $\gamma<\overline{\gamma}(q)$.

Proposition 9 can be understood in terms of the value of information under bilateral clearing. In particular, we can interpret bilateral clearing with monitoring as a lottery $\mathcal{L}^{bil}=(pq,p(1-q),(1-p)q,(1-p)(1-q)$ over outcomes $(x^H_{2,h},\, x^H_{2,l},\, x^L_{2,h},\, x^L_{2,l})$, whereas central clearing results in a degenerate lottery over X_2^{CCP}. The threshold $\overline{\gamma}(q)$ can be rewritten as

$$\overline{\gamma}=u(E_{\mathcal{L}^{bil}}-RP_{\mathcal{L}^{bil}})-u(X_2^{CCP}) \tag{24}$$

where $E_{\mathcal{L}^{bil}}$ and $RP_{\mathcal{L}^{bil}}$ are, respectively, the expected value and the risk-premium of the lottery $\mathcal{L}^{bil}$.[28] When Assumption 4 and Assumption 6 are satisfied, $X_2^{CCP}<E_{\mathcal{L}^{bil}}$ and a trade-off between bilateral and central clearing may exist, provided that lenders are not too risk-averse and the population of borrowers is sufficiently heterogeneous.

[28]The risk premium of a lottery is a measure of how many resources, in expectation, an agent is willing to give up to get rid of uncertainty: $RP_{\mathcal{L}}=E_{\mathcal{L}}-CE_{\mathcal{L}}$.

In fact, central clearing has the advantage of providing insurance by pooling risk over idiosyncratic uncertainty. However, the CCP requires all traders to post the same amount of collateral. Thus, central clearing has the limitation of requiring a fraction of the borrowers' population to post either excessive or insufficient collateral necessary to provide incentives to repay. On the one hand, the value of insurance provided by the CCP is smaller the less risk-averse lenders are. On the other hand, the benefits from collateral customization are larger if the population of borrowers contains both high-pledgeability and low-pledgeability types, hence the necessary and sufficient condition is $q \in (\underline{q}, \overline{q})$.

Risk-aversion may change the desirability of bilateral vs. central clearing through its effect on $\overline{\gamma}$. In particular, from equation (24), risk-aversion has an effect on $\overline{\gamma}$ through its effect on i) the risk-premium $RP_{\mathcal{L}^{bil}}$, ii) expected consumption under optimal contracts with bilateral clearing, and iii) the utility function $u(\cdot)$. In the next lemma we provide sufficient conditions which guarantee that an increase in risk-aversion results in central clearing becoming relatively more desirable.

Lemma 10 *Let $u(x)$ and $v(x)$ be v.N-M utility functions. Consider two economies: economy A populated by lenders with utility $u(\cdot)$ and economy B populated by lenders with utility $v(\cdot)$. Suppose there exists a concave function $\rho : \Re_+ \to \Re_+$ such that $v(x) = \rho(u(x))$ and $|\rho(a)-\rho(b)| \leq \zeta|a-b|$, for $\zeta \leq 1$. Also, suppose that $\frac{\alpha-p}{1-p} > \frac{v'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)}{v'\left(\theta-\frac{\alpha-p}{\alpha}(1-\lambda^L)\theta\right)}$ (the equivalent of Assumption 6 for economy B). Then, if lenders in economy B prefer bilateral clearing with information acquisition to central clearing with no information acquisition, then also lenders in economy A prefer bilateral clearing with information acquisition to central clearing with no information acquisition.*

Lemma 10 can be understood through the different effects of an increase in risk-aversion. First, more risk-averse lenders are willing to give up more resources to avoid uncertainty over consumption, i.e. $RP_{\mathcal{L}^{bil}}$ increases with risk-aversion. Second, optimal contracts with bilateral clearing may change with risk-aversion, and so does $E_{\mathcal{L}^{bil}}$. If the Arrow-Pratt coefficient of absolute risk-aversion increases uniformly with risk-aversion, the demand of collateral for insurance becomes larger and expected consumption under bilateral clearing decreases, i.e. $E_{\mathcal{L}^{bil}}$ decreases.[29] Finally, risk-aversion affects the desirability of bilateral vs. central clearing through the form of the utility function: that is the difference $u(E_{\mathcal{L}^{bil}} - RP_{\mathcal{L}^{bil}}) - u(X_2^{CCP})$ is not in general scale invariant, but depends on the function $u(\cdot)$. In Lemma 10 we show that when we consider transformation in risk-aversion that result in a concave contraction of the original utility function, central clearing becomes relatively more desirable with risk-aversion.[30]

4.2 Central clearing with information acquisition

The optimality of bilateral clearing in Proposition 9 hinges on lenders' ability to acquire information about the quality of their counterparty and to customize collateral accordingly. A natural question is whether a central counterparty could aggregate such information. Specifically, can lenders credibly acquire information about counterparty quality and transfer it to the CCP, which afterwards could customize collateral requirements to borrowers' pledgeability types. The answer to this question depends on the consequences of novation and the risk management adopted by the CCP.

Similarly to Section 3.2, we model novation by assuming that the CCP commits to a mechanism at the beginning of $t = 1$, and that borrowers and lenders negotiate over

[29] Assuming that the Arrow-Pratt coefficient increases uniformly is equivalent to assume a concave transformation of the original utility function: $v(x) = \rho(u(x))$ for $\rho(\cdot)$ concave.

[30] As an example of such an increase in risk-aversion, consider the function $u : \Re_+ \to \Re_+$ defined as $u(x) = 3x^{\frac{1}{3}}$, and the transformation $\rho : \Re_+ \to \Re_+$, $\rho(y) = 2\,(1+y)^{\frac{1}{2}} - 2$. Then, the more risk-averse lender has utility $v : \Re_+ \to \Re_+$, $v(x) = g\big(u(x)\big)$. It is easy to check that $u(0) = v(0) = 0$. Also, $u'(0) = v'(0) = \infty$. Moreover, $g'(y) > 0 > g''(y)$, and $g'(0) = 1$. Then, $g(\cdot)$ is a concave contraction that defines a new utility function, $v(\cdot)$, that satisfies all the assumptions that we made in the paper about the utility function $u(\cdot)$.

these contracts. A contract specifies transfers between the CCP and lenders and between the CCP and borrowers. Transfers are a function of *i)* public information and *ii)* the restrictions on the contracting space consistent with the institutional arrangements adopted by CCPs in reality.

A strategy for a lender is a pair $(e, m) \in \{0, 1\} \times \{\lambda^H, \lambda^L\}$ of effort e and message reported to the CCP m; a strategy for a borrower is a default decision $\sigma_s(\lambda, m)$: $\{\lambda^L, \lambda^H\}^2 \times \{l, h\} \to \{0, 1\}$. For example, $\sigma_h(\lambda^L, \lambda^H)$ is the default decision of a λ^L-borrower at the node corresponding to lender's first-period message $m = \lambda^H$, when the idiosyncratic state is $s = h$. Note that strategic default is relevant only when the idiosyncratic state is $s = h$, and easily $\sigma_s(\lambda, m) = 0$ when realization of the idiosyncratic state $s = l$.

Using the same notation as in Section 3.2, let $h_2 = (s, \Delta)$ be the public history of a borrower after novation, where $\Delta \in \{0, 1\}$ is his observed repayment/default decision and $s \in \{l, h\}$ his idiosyncratic state at $t = 2$. Let $\mathcal{H}_2$ denote the set of all such histories, and define $\hat{\mathcal{H}}_2 \subseteq \mathcal{H}_2$ to be the set of all second-period histories of a borrower upon which the contract between the CCP and the original lender can be contingent on. Finally, let $\hat{h}_2 \in \hat{\mathcal{H}}_2$ be an element of this set. A mechanism with central clearing and monitoring consists of contracts between the CCP and lenders, $\{X_2^m(\hat{h}_2)\}$, and contracts between borrowers and the CCP, $\{C_1^m, C_{2,s}^m\}$.

We consider three specifications for $\hat{\mathcal{H}}_2$: *1)* a benchamrk case where $\hat{\mathcal{H}}_2 = \{\emptyset\}$; *2)* $\hat{\mathcal{H}}_2 = \{(s, \Delta) : s \in \{l, h\}, \Delta \in \{0, 1\}\}$; and *3)* $\hat{\mathcal{H}}_2 = \{\Delta : \Delta \in \{0, 1\}\}$. The three cases correspond to different assumptions about the CCP rules and procedures. Specifically, we focus on the rules defining the loss allocation methods, member's due diligence, and prohibited conduct. These rules impose a constraint on the contract the CCP itself can offer.

Consistently with our description in Section 2, case *1)* describes a scenario where the CCP allocates losses pro-rata among its members. Thus, the contract between a lender and the CCP must be independent of the second-period history of the original borrower. Therefore, in this case we write $\hat{\mathcal{H}}_2 = \emptyset$. Case *2)* corresponds to a scenario where the CCP adopts a partial tear-up loss allocation method, by which the CCP can select and terminate certain contracts upon default of a member. In practice, and as described in Section 2, CCPs adopts this loss allocation method under extreme default scenarios, and typically terminate contracts by original counterparty. In the context of our model this is equivalent to imposing no restrictions on second period repayments by the CCP to lenders, thus $\hat{\mathcal{H}}_2 = \{(s, \Delta) \in \{l, h\} \times \{0, 1\}\}$. Case *3)* describes a scenario where the CCP adopts a pro-rata loss allocation method with due-diligence, which are key features of standard loss mutualization schemes.[31] Pro-rata allocation of losses implies that even if the original counterparty of a lender defaults, the CCP cannot refuse to perform on her obligation and pay the lender. In our model, this requires that payments to a lender are independent of her borrower's default. Due diligence, however, implies that the CCP can assess fees and impose sanctions for the violation of its Rules and Procedures.[32] In our model, because strategic default by a borrower signals that his lender did not monitor him or did not report his type truthfully, the CCP is able to identify a violation of its rules and punish the lender. Thus, a pro rata loss allocation method with due diligence requires $\hat{\mathcal{H}}_2 = \{\Delta \in \{0, 1\}\}$ and allows the CCP to choose the punishment for a lender who did not report truthfully her counterparty type. We assume that the CCP chooses the harshest feasible punishment as this sets a lower bound on the measure of economies for which our results hold.[33] Due to non-negativity of consumption, the CCP allocates zero consumption to a lender

[31]See National Securities Clearing Corporation - Rules and Procedures (2018), and UBS Response to the Committee on Payment and Settlement Systems (2012)

[32]See National Securities Clearing Corporation - Rules and Procedures (2018), and Ice Clear Credit Clearing Rules (2018).

[33]A more lenient punishment would results in a larger set of economies for which our results hold.

who misbehaved.[34]

Let $w^i(\hat{h}_2) = u\big(X_2^i(\hat{h}_2)\big)$ be the lender's payoff when she reports $m = \lambda^i$ in $t = 1$ and, in addition to this message, the contract between the CCP and the lender is contingent on history $\hat{h}_2$. Define next $\pi(\hat{h}_2|\sigma(\lambda^i, m))$ to be the probability distribution over histories $\hat{h}_2$ conditional on the default strategy of a borrower of type λ^i, when the lender's message in $t = 1$ is m. Let $\pi(\emptyset|\sigma(\lambda^i, m)) = 1$.[35] The CCP contracts which induce monitoring and result in borrowers' separation solve the following maximization problem:

$$(P_1) \qquad \max \quad \sum_{i\in\{L,H\}} q_i \left[\sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^i)\Big) w^i(\hat{h}_2) \right] - \gamma \tag{25}$$

$$s.t. \quad \alpha C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i \geq \alpha\omega \tag{26}$$

$$C_{2,h}^i \geq (1-\lambda^i)\theta \tag{27}$$

$$\omega - \sum_{i\in\{L,H\}} q_i\, C_1^i \geq 0 \tag{28}$$

$$\sum_{i\in\{L,H\}} q_i \left[\sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^i)\Big) \;\; u^{-1}(w^i(\hat{h}_2)) \right] + \sum_{i\in\{L,H\}} \Big\{ q_i[C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i] \Big\} \leq \omega + p\theta \tag{29}$$

$$\sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^i)\Big) w^i(\hat{h}_2) \geq \sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^{-i})\Big) w^{-i}(\hat{h}_2) \tag{30}$$

$$-\gamma + \sum_{i\in\{L,H\}} q_i \left[\sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^i)\Big) w^i(\hat{h}_2) \right] \geq \max_{j\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{\hat{h}_2\in\hat{\mathcal{H}}_2} \pi\Big(\hat{h}_2|\sigma(\lambda^i,\lambda^j)\Big) w^j(\hat{h}_2) \right] \right\} \tag{31}$$

$$\sigma_s(\lambda^i,\lambda^j) = 0 \;\text{ iff }\; C_{2,h}^j \geq (1-\lambda^i)\theta \tag{32}$$

[34] In principle, the CCP could adopt a pro-rata loss allocation method without due diligence. However notice that this would be de-facto equivalent to the case where original counterparties are not ex-post identifiable which we analyze in the benchmark case.

[35] Note that in the definition of $\sigma(\lambda, \lambda^i)$, the lender may know λ if she monitored her counterparty, but it can be as well private information of the borrower if monitoring did not occur.

where we use the notation $q^H = q$ and $q^L = 1-q$. One can easily see that $\sigma_l(\lambda^i, \lambda^j) = 0$ for $i, j \in \{L, H\}$, because borrowers have no incentive to default in state $s = l$. Constraint (26) is borrower i's participation constraint and (27) his limited commitment constraint. Equations (28) and (29) are time $t = 1$ and $t = 2$ feasibility constraints, and (30) and (31) are, respectively, the ex-post and ex-ante incentive-compatibility constraint for lenders. Specifically, when (30) is satisfied lenders prefer, after monitoring, to report truthfully their counterparty's type. When constraint (31) is satisfied a lender prefers, ex-ante, to monitor her counterparty (and then report her type truthfully) rather than not to monitor and report that her counterparty is either a high-pledgeability or a low-pledgeability type. Constraint (32) defines optimal borrowers' default decision.

4.2.1 Contracts with pro-rata loss allocation method: $\hat{\mathcal{H}}_2 = \emptyset$

This section characterizes equilibrium contracts with central clearing when the CCP allocates losses pro-rata among its members, resulting in $\hat{\mathcal{H}}_2 = \emptyset$. In this case, the contract between the CCP and a lender is independent of any information available at $t = 2$ *after* novation, such as information about the lender's original counterparty. With $\hat{\mathcal{H}}_2 = \emptyset$ we can write $w^i(\hat{h}_2) = w^i$. The incentive-compatibility constraint (31) thus becomes:

$$-\gamma + qw^H + (1-q)w^L \geq \max\{w^H, w^L\} \tag{33}$$

The left-hand side of (33) is the payoff for a lender after monitoring and truthtelling. The right-hand side of (33) is the payoff of a lender who does not monitor her counterparty and reports the type associated with the larger repayment. One can easily see that there exists no incentive-compatible contract: for any non-negative (w^H, w^L) constraint (33) is violated. We summarize this result in the next proposition.

Lemma 11 *For any $\gamma > 0$, if $\hat{\mathcal{H}}_2 = \emptyset$ there exists no solution to problem (P_1).*

A consequence of Lemma 11 is that lenders cannot credibly transfer information about their counterparty to the CCP. Thus, the CCP must adopt a homogeneous collateral policy that treats all borrowers either as a low-pledgeability or as a high-pledgeability type. Thus, bilateral clearing is optimal for the same conditions as in Proposition 9.

Corollary 12 *For any $\gamma > 0$, if $\hat{\mathcal{H}}_2 = \emptyset$ bilateral clearing is optimal when the assumptions in Proposition 9 are satisfied.*

4.2.2 Contracts with partial tear-up: $\hat{\mathcal{H}}_2 = \mathcal{H}_2 = \{(s, \Delta) \in \{l, h\} \times \{0, 1\}\}$

This section characterizes equilibrium contracts with central clearing that can be contingent not only the first period message by the lender, but also on second period histories: $\hat{\mathcal{H}}_2 = \mathcal{H}_2$. As discussed in Section 2 we refer to these contracts as associated with *a partial tear-up allocation of losses.*[36] In this case, contracts between a lender and the CCP can depend on the message reported by lender to the CCP at $t = 1$, the realization of the borrower's returns, and the borrowers' default decision. Hence, we write $w^i(\hat{h}_2) = w_s^{i,\Delta}$.

Because the objective of the CCP is to customize collateral requirements to borrowers' types, it is optimal to use the information available to lenders after monitoring, and punish lenders who misreport the type of their counterparty. This is feasible for the CCP when a borrower strategically defaults: if at $t = 1$ a lender reports that her counterparty is a high-pledgeability type, and at $t = 2$ the borrower defaults when $s = h$, this default signals the misbehavior of the lender. Indeed, from constraint (32), strategic default of a borrower ($\Delta = 1$) can only be the consequence of inappropriate collateral requirement. Also, by constraint (27), collateral requirements can only be

[36] See Gregory (2014) section 10.3.

insufficient because the lender did not monitor or report truthfully her counterparty type. Hence, it is feasible for the CCP to identify a lender who misreported her counterparty type, and punish her. Constraint (31) implies that the optimal punishment is the hardest feasible one: in fact choosing $w_h^{i,1}$ as low as it is feasible relaxes the incentive constraint (31). By non negativity of consumption, the lowest feasible punishment is $w_h^{i,1} = 0$.

Using this result, we can prove the following proposition.

Proposition 13 *Let Assumption 4 and Assumption 6 hold, and let $\hat{\mathcal{H}}_2 = \mathcal{H}_2$ in problem (P_1). If $\gamma < \overline{\gamma}$, for $\overline{\gamma}$ defined in Proposition 9, there exist contracts $(w_s^{i,0}, C_1^i, C_{2,s}^i)$ feasible and incentive-compatible in problem (P_1), such that*

$$\sum_{i \in \{L,H\}} \left\{ q_i \Big[\sum_{s=l,h} p_s w_s^{i,0} \Big] \right\} \geq \sum_{i \in \{L,H\}} \left\{ q_i \Big[\sum_{s=l,h} p_s u(x_{2,s}^{i^*}) \Big] \right\}$$

where $x_{2,s}^{i^*}$ is lenders' consumption in the optimal contract with bilateral clearing and monitoring of Lemma 7.

Proposition 13 proves that the CCP can replicate any contracts with bilateral clearing when it can make payments contingent on the message by the lender at $t = 1$ and on the borrower's history at $t = 2$. As a result, when information acquisition is valuable under bilateral clearing, such information can be acquired and credibly transmitted to the CCP.

Intuitively, when $\hat{\mathcal{H}}_2 = \mathcal{H}_2$, the set of contingencies spans the set of states of nature and any bilateral contract becomes feasible with central clearing, including the optimal bilateral contract with information acquisition. Importantly, Proposition 13 shows that such contracts are also incentive compatible for the parameter space where information acquisition is valuable with bilateral clearing. That is, when $\gamma < \overline{\gamma}$, bilateral contracts with information acquisition satisfy incentive-compatibility constraints

(30) and (31). This happens for two reasons: first, constraint (30) is always satisfied by constraint (31), and can then be ignored. In other words, lenders' best deviation is always ex-ante, before monitoring, rather than lying after acquiring information about the counterparty type. Hence, the CCP needs to consider only two kinds of deviations, which are considered in the right-hand side of constraint (31): 1) not monitoring and reporting a high-pledgeability counterparty; 2) not monitoring and reporting a low-pledgeability counterparty. Second, these two deviations induce lotteries over time $t = 2$ consumption for lenders that are inferior to the one induced by optimal contracts in Proposition 5, which are pooling contracts with central clearing. As a result, bilateral contracts with information acquisition satisfy the incentive-compatibility constraint (31), because in economies with $\gamma < \bar{\gamma}$ they are preferred to contracts with central clearing and no information acquisition, as shown in Proposition 9. In conclusion, Proposition 13 shows that central clearing is always preferred to bilateral clearing when CCP contracts are state contingent.

Corollary 14 *With partial tear-up allocation of losses, central clearing is always the optimal clearing arrangement.*

Notice that the results in Proposition 13, and its implications in Corollary 14, are non trivial. One might be induced to believe that the CCP is solving the same problem of as that of an unconstrained social planner in the case of a partial tear-up loss allocation method. This is, however, incorrect as the CCP is not endowed with the monitoring technology with which lenders are endowed. Rather, the CCP has to induce lenders to exert effort and monitor their borrowers. Using the replicability results in Proposition 13, the CCP can offer lenders fully state contingent contracts. This, in turn, allows the CCP to reduce the provision of ex ante insurance to lenders typical of a pro-rata loss allocation method, and to favor the provision of incentives to acquire information about borrowers. For our result it is sufficient that these incentives are just enough to implement the bilateral contract.

4.2.3 Contracts with pro-rata loss allocation and due diligence: $\hat{\mathcal{H}}_2 = \{\Delta \in \{0,1\}\}$

This section characterizes equilibrium contracts where the CCP adopts a pro-rata loss allocation method and due diligence. In practice, CCPs impose sanctions for violation of their rules, and assess fines if a member shows "prohibited conduct" or conduct which is inconsistent with "just and equitable principles of trade." Violations of the rules of a CCP include failure to provide information regarding the businesses and operations of the member and its risk management practices, or reporting its financial or operational condition. Additionally, members may need to submit information to the CCP "as the Corporation from time to time may reasonably require." In this respect, lenders' costly information acquisition in our model captures the idea that members might learn their counteparties' financial and operation condition by, for example, monitoring their counterparties.

Formally, we allow contracts between the CCP and lenders to depend on observed lack of due diligence, which can be revealed by the strategic default of a lender's counterparty. Contracts, however, are not contingent on the realization of a borrower's idiosyncratic return, as the CCP is legally obliged to honor the contracts it clears. To do so it may have to absorb losses, in which case it uses the resources contributed pro-rata to the default fund by surviving members. Hence, $\hat{\mathcal{H}}_2 \subset \mathcal{H}_2$, as $\hat{\mathcal{H}}_2 = \{\Delta \in \{0,1\}\}$, and we write $w^i(\hat{h}_2) = w^{i,\Delta}$.

We solve problem (P_1) in two steps. In the first step the CCP determines the contracts offered to borrowers, $\{C_1^i, C_{2,h}^i, C_{2,l}^i\}_{i=L,H}$, that maximizes time $t = 2$ resources. It is optimal to solve for $\{C_1^i, C_{2,h}^i, C_{2,l}^i\}_{i=L,H}$ that maximize time $t = 2$ resources because it relaxes constraints (29), (30), and (31), while satisfying (26), (27), and (28). Then, in the second step, the CCP determines the contracts it offers to lenders, $\{w^{i,\Delta}\}_{i=L,H}$, given resources available.

In the next lemma we characterize optimal contracts offered to borrowers.

Lemma 15 *Let $\omega(\lambda)$ be defined in (14). Optimal contracts offered to borrowers, $\{C_1^i$, $C_{2,h}^i$, $C_{2,l}^i\}_{i=L,H}$, satisfy $C_{2h}^i = (1-\lambda^i)\theta$, $C_{2l}^i = 0$, $C_1^i = \omega - \omega(\lambda^i)$.*

To gain intuition for Lemma 15, note that, when contracts are cleared centrally, there is no need for collateral for insurance purposes, because the CCP can fully insure lenders by pooling borrowers idiosyncratic risk. Hence, the CCP's objective is to minimize collateral requirements, and the limited commitment constraint of both types of borrowers is binding. Recall that Assumption 4 guarantees that the participation constraint is always binding. Then the consumption allocation is determined as a residual from the borrowers' participation constraint in $t = 1$.

Next, we can characterize optimal contracts between lenders and the CCP: define the function

$$\phi(\gamma) = qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right) + (1-q)u^{-1}\left(\gamma\left[\frac{1-p(1-q)}{pq(1-q)}\right]\right)$$

which maps any value of $\gamma \geq 0$ to the minimum aggregate resources (i.e. $t = 2$ consumption goods) consistent with the existence of a solution to the CCP problem. Further, define the threshold $\hat{\gamma}$ as the unique solution to

$$\phi(\hat{\gamma}) = \sum_{i\in\{L,H\}} q_i\left[\lambda^i p\theta + \omega(\lambda^i)\right] \tag{34}$$

for $\omega(\lambda)$ defined in (14). Thus, $\hat{\gamma}$ denotes the largest value of γ such that a solution to problem (P_1) with pro-rata allocation of losses and due diligence exists. For economies where such a solution exists we can prove the following result.

Proposition 16 *With pro-rata allocation of losses and due diligence, a solution to problem* (P_1) *exists and is unique if and only if* $\gamma \leq \hat{\gamma}$. *If* $\gamma \leq \hat{\gamma}$, $w^{H,0^*} = w^{L,0^*} + \frac{\gamma}{q}$, *where* $w^{L,0^*}$ *solves*

$$qu^{-1}\left(w^{L,0^*} + \frac{\gamma}{q}\right) + (1-q)u^{-1}(w^{L,0^*}) = \sum_{i \in \{L,H\}} q_i \left[\lambda^i p\theta + \omega(\lambda^i)\right]$$

Finally, if $\gamma \leq \hat{\gamma}$, *then*

$$\max\left\{u(X_2^*), \sum_{i \in \{L,H\}} \left\{q_i w^{i,0^*}\right\} - \gamma\right\} \geq \sum_{i \in \{L,H\}} \left\{q_i \Big[\sum_{s=l,h} p_s u(x_{2,s}^{i^*})\Big]\right\} - \gamma$$

where $x_{2,s}^{i^*}$ *is lenders' consumption in the optimal contract with bilateral clearing and monitoring of Lemma 7, and* X_2^* *is lenders consumption in (13) for the optimal contract with CCP clearing and no monitoring in Proposition 5.*

The first part of Proposition 16 shows that a solution to the CCP problem (P_1) with pro-rata allocation of losses and due diligence, if it exists, is such that lenders' incentive-compatibility constraint (31) binds for $\hat{i} = L$. The second part of Proposition 16 proves that information is more valuable with central clearing, because the CCP can provide full insurance against the idiosyncratic return risk and partial insurance against the counterparty-type risk. More precisely, in the proof of Proposition 16 we show that lenders would prefer the contract with bilateral clearing and monitoring over the contract with central clearing and pooling over λ^L only if, given the monitoring cost γ, the value of facing a λ^H counterparty is significantly higher than the value of facing a λ^L counterparty. However, if this is the case and $\gamma \leq \hat{\gamma}$, a CCP can replicate such bilateral contracts and obtain enough resources at $t = 2$ to induce lenders to monitor their counterparties and report truthfully their type. Further, the CCP can transfer some resources from lenders facing a λ^H counterparty to lenders facing a λ^L counterparty, without violating lenders' incentive compatibility constraints. As a result, central clearing improves on bilateral clearing by providing insurance against the risk of facing a counterparty type λ^L.

Following a similar argument, we prove the following result for economies that do not satisfy the conditions in Proposition 16.

Proposition 17 *Let Assumption 4 and Assumption 6 hold. Also, let $\overline{\gamma}$ be defined in Proposition 9 and $\hat{\gamma}$ in (34). With pro-rata allocation of losses and due diligence, bilateral clearing (with monitoring) is the optimal clearing arrangement if and only if $\gamma \in (\hat{\gamma}, \overline{\gamma})$.*

The conditions in Proposition 17 are necessary and sufficient for optimality of bilateral clearing with pro-rata allocation of losses and due diligence. Central clearing has the advantage of providing insurance by pooling risk over idiosyncratic risks and, as a result, has the potential to economize on the use of collateral necessary to insure against idiosyncratic risk in bilateral clearing. However, without the information generated by monitoring, the CCP must offer contracts that require all traders to post the same amount of collateral, which is associated either with a low-pledgeability or a high-pledgeability counterparty. Thus, central clearing has the limitation of requiring a fraction of the borrowers' population to post either excessive or insufficient collateral necessary to provide incentives to repay. When $\gamma \in (\hat{\gamma}, \overline{\gamma})$, the benefits of central clearing do not compensate for its costs: the insurance against uncertain returns does not compensate lenders for the distortion in the use of collateral due to the lack of information about the counterparty quality. Thus lenders choose to clear contracts bilaterally and acquire information about their borrowers.[37]

[37]Notice that the assumption that monitoring activity is costly for lenders is necessary for the

5 Implications for Collateral and Default

The goal of this section is to illustrate the implications of our model for the collateral policies chosen with each clearing arrangement and for the associated default rates in equilibrium. We combine results from the previous sections and analyze the role of some primitives of the model for equilibrium outcomes. We focus on economies where there exists a trade-off between bilateral and central clearing, since this is the main focus of our paper. Therefore, we restrict our analysis to economies where the CCP operates with a pro-rata loss allocation method with due diligence (these are the economies of Section 4.2.3), and where $\gamma > \hat{\gamma}$, with $\hat{\gamma}$ defined in (34). In these economies information acquisition is not incentive-compatible with central clearing, and, as a consequence, bilateral clearing might be preferred. Table 1 summarizes results about default rates and collateral requirements.

		CCP			Bilateral			
Parameteres		collateral	default		collateral		default	
		λ^L λ^H	λ^L	λ^H	λ^L	λ^H	λ^L	λ^H
$q \leq \frac{1}{\alpha}$	$\lambda^H < \lambda^*$	$\omega(\lambda^L)$	$\sigma^L = 0$	$\sigma^H = 0$	$\omega(\lambda^L)$	$\omega(\lambda^H)$	$\sigma^L = 0$	$\sigma^H = 0$
	$\lambda^H > \lambda^*$	$\omega(\lambda^L)$	$\sigma^L = 0$	$\sigma^H = 0$	$\omega(\lambda^L)$	$\omega(\lambda^*)$	$\sigma^L = 0$	$\sigma^H = 0$
$q \geq \frac{1}{\alpha}$	$\lambda^H > \lambda^*$	$\omega(\lambda^H)$	$\sigma^L = 1$	$\sigma^H = 0$	$\omega(\lambda^L)$	$\omega(\lambda^*)$	$\sigma^L = 0$	$\sigma^H = 0$
	$\lambda^H < \lambda^*$	$\omega(\lambda^H)$	$\sigma^L = 1$	$\sigma^H = 0$	$\omega(\lambda^L)$	$\omega(\lambda^H)$	$\sigma^L = 0$	$\sigma^H = 0$

Table 1: Collateral policy and strategic default strategies under bilateral clearing with monitoring and central clearing with no information acquisition, assuming that $\gamma > \hat{\gamma}$, where $\hat{\gamma}$ is defined in (34) and λ^* in (22).

In the economies we consider, collateral requirements with bilateral clearing are

existence of a trade-off between bilateral and central clearing. More specifically, in a simpler set-up where lenders always have the information about the quality of their counterparty, which is equivalent to $\gamma = 0$, information transmission is always incentive-compatible ($0 = \gamma < \hat{\gamma}$) and central clearing is always the optimal arrangement.

tailored to borrowers' type. However, because information acquisition is not incentive compatible with central clearing ($\gamma > \hat{\gamma}$), the CCP must then choose a homogenoues collateral policy to maximize resources at $t = 2$, trading-off collateral costs with the cost of default. Equilibrium default measures the opportunity cost of collateral: lowering collateral requirements and tailoring them to λ^H borrowers comes at the cost of equilibrium default of λ^L types. In economies where the measure of λ^H borrowers is small ($q \leq \frac{1}{\alpha}$), average collateral with central clearing is larger, as λ^H borrowers post more collateral than their type would require. As a consequence, default does not occur in equilibrium, both with central and bilateral clearing. Differently, in economies where the measure of λ^H borrowers is large, the CCP saves resources by requiring less collateral and allowing default by λ^L borrowers. Thus, average collateral is lower than with bilateral clearing, but equilibrium default is larger. Lemma 18 summarizes these results.

Lemma 18 *Suppose the CCP adopts a pro-rata allocation of losses method with due diligence. Let Assumption 4 and Assumption 6 hold, and assume that $\gamma > \hat{\gamma}$, for $\hat{\gamma}$ defined in (34). If $q \leq \frac{1}{\alpha}$, average collateral requirements are lower with bilateral clearing, whereas average defaults are the same under the two clearing arrangements. If instead $q \geq \frac{1}{\alpha}$, average collateral requirements are larger and average defaults are smaller with bilateral clearing than they are with central clearing.*

Using the results from Lemma 18, we then investigate the effects of an increase in the cost of collateral, α. The next lemma shows that when the measure of λ^H borrowers is small, and under some additional assumptions, an increase in the cost of collateral results in a larger set of economies where bilateral clearing is preferred to central clearing.

Lemma 19 *Suppose that* $q \leq \frac{1}{\alpha}$*, Assumption 4 holds, and* $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta-\frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$*. Also, assume that lenders are not prudent, i.e.* $u'''(x) \leq 0$*. Then* $\frac{d\overline{\gamma}}{d\alpha} > 0$ *and* $\frac{d\hat{\gamma}}{d\alpha} < 0$*.*

Lemma 19 provides sufficient conditions for an increase in the cost of collateral to result in a larger set of economies where bilateral clearing is preferred. When $q < \frac{1}{\alpha}$, the CCP adopts the homogeneous collateral policy that treats all borrowers as low-pledgeability types. Therefore, high-pledgeability borrowers are required to post excessive collateral with respect to what their type would require. Bilateral clearing, on the other hand, features information acquisition, which allows lenders to tailor collateral requirements to the type of their counterparty. Thus, when $q < \frac{1}{\alpha}$, bilateral clearing has the advantage of economizing over the average collateral requirement. As a result, *ceteris paribus*, an increase in the cost of collateral strengthens the relative advantage of bilateral clearing. Formally, an increase in α works as a negative income effect that lowers lenders' expected consumption at $t = 2$, because more resources must be allocated to satisfy borrowers' participation constraint. Due to the distortion induced by the homogeneous collateral policy of the CCP, on average, this mechanism is stronger with central clearing. However, the fact that average collateral requirements are larger with central clearing is not enough to reach the conclusion that bilateral clearing is preferred for a larger set of economies when α increases. The reason is that, with bilateral clearing, lenders' consumption depends also on the return of their counterparty's technology. Thus, with bilateral clearing, the effect of an increase in the cost of collateral must be weighted by the marginal utility of consumption at different levels of consumption. The assumption that lenders are not prudent in Lemma 19, i.e. $u''' \leq 0$, is sufficient to guarantee that this second-order effect on lenders' payoffs works in the same direction as the first-order effect on average collateral requirement. Thus, under the assumption that lenders are not prudent we can conclude that bilateral clearing is preferred to central clearing for a larger set of economies.

Lemma 20 shows that the opposite result holds when we consider economies that do not fall under the assumptions of Lemma 19.

Lemma 20 *Suppose that $q > \frac{1}{\alpha}$, Assumption 4 holds, $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta-\frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$, and $\gamma > \hat{\gamma}$ for $\hat{\gamma}$ defined in (34). Also, assume that lenders prudent enough, i.e. $u'''(x) > 0$ and*

$$pu'\left(\frac{(1-\lambda^H)p\theta}{\alpha} + \lambda^H\theta\right) + (1-p)u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right) > u'\left(\frac{(1-\lambda^L)p\theta}{\alpha} + \lambda^L p\theta\right)$$

Then, and increase in the cost of collateral α makes central clearing more desirable, i.e. $\frac{d\overline{\gamma}}{d\alpha} < 0$.

Lemma 20 considers economies where the optimal contract with central clearing adopts the homogeneous collateral policy that treats all borrowers as λ^H types. In this scenario, under some additional assumptions, an increase in the cost of collateral results in a larger set of economies where central clearing is preferred. The reasoning mirrors the arguments in Lemma 19. With central clearing, low-pledgeability borrowers are required to post too little collateral with respect to what their type would require, and default. In an economy where central clearing is preferred, the benefit from economizing on collateral compensates lenders for the cost of λ^L borrowers defaulting in equilibrium. In this scenario, an increase in the cost of collateral must strengthen, *ceteris paribus*, this effect. As in the proof of Lemma 19, this is not enough to reach the conclusion that central clearing is preferred for a larger set of economies, because an increase in α has a second-order effect on the payoff under bilateral clearing due to the interaction between collateral and uncertain returns. The assumption that lenders are prudent, i.e. $u''' \geq 0$, is sufficient conclude that this second order effect works in the same direction as the direct first order effect on economizing on collateral by letting λ^L borrowers default in equilibrium.

Large values of α can be associated with financial institutions such as hedge funds or broker-dealers, whose opportunity cost of collateral is higher than, say, that of money market funds.[38] In this respect, our results are broadly consistent with evidence of dealers and hedge funds clearing a substantial share of their trades bilaterally, whereas money market funds are more likely to rely on financial market infrastructure (e.g. General Collateral Finance Repo Service (GCF Repo) and triparty settlement).[39] Analogously, our results are consistent with central clearing arising endogenously in markets where participants are homogenous in terms of their business type (in the model, q close to 1 or 0), when we interpret the pledgeability parameter λ as the riskiness in a counterparty's set of activities.[40]

6 Conclusions

This paper characterizes optimal clearing arrangements for financial transactions in a model where insurance is valuable because of uncertain returns to investment and heterogeneous quality of trading counterparties. Using a mechanism design approach, we consider the institutional arrangements of modern CCPs and model them as constraints on the contract space of the CCP. The contribution of the analysis is the identification of a trade-off between clearing bilaterally and channeling clearing services through a CCP. This trade-off arises when incentives to monitor bilateral trades are incompatible

[38] At least under normal circumstances, disregarding events as money market funds *breaking the buck*.

[39] As an example, for evidence related to the US repo market see the Office of Financial Research Brief Paper no. 17-04, *Benefits and Risks of Central Clearing in the Repo Market*. See also footnote 10 in the introduction for more details on MMMF and central clearing.

[40] As an example, recall that the first central counterparties originated next to grain and coffee exchanges, where *farmers* and *bakers* traded futures. Among many, for references see Kroszner (2006), and Gregory (2014).

with the risk pooling activity of the CCP, and under loss allocation methods commonly adopted by CCPs for their routine operations. Thus, even though the motivation for central clearing might arise from reasons outside the model, such as systemic risk consequences of opaque bilateral positions, the consequence of mandatory CCP clearing is a potential loss of information across markets due to decreased incentives to monitor trading partners. This result should not of course lead to the conclusion that CCP's are not useful in sharing risk in markets. It rather highlights the limits inherent in different loss allocation mechanisms and the importance of the risk of the underlying assets and of the degree of heterogeneity of market participants in determining whether CCP's can perform their risk sharing function effectively.

Appendix

Proof of Lemma 1

Consider problem (8) - (12) where the recommended default decision $\Sigma^H = 0$ and $\Sigma^L = 0$. These two conditions require in (12) that $C^H_{2,h} \geq (1-\lambda^H)\theta$ and $C^L_{2,h} \geq (1-\lambda^L)\theta$ respectively. Constraint (12) for λ^H-borrowers can be rewritten as

$$C^H_{2,h} \geq (1-\lambda^H)\theta \tag{35}$$

$$\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} \geq \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l} \tag{36}$$

whereas constraint (12) for λ^L-borrowers becomes

$$C^L_{2,h} \geq (1-\lambda^L)\theta \tag{37}$$

$$\alpha C^L_1 + p(1-\lambda^L)\theta + (1-p)C^L_{2,l} \geq \alpha C^H_1 + p\max\{(1-\lambda^L)\theta, C^H_{2,h}\} + (1-p)C^H_{2,l} \tag{38}$$

The optimal contract should satisfy $C^H_{2,h} \geq (1-\lambda^L)\theta$. Then (35) can be ignored. Furthermore both (38) and (36) bind.

Combine (38) with (36):

$$\begin{aligned}\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} &\geq \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l}\\ &\geq \alpha C^H_1 + p\max\{(1-\lambda^L)\theta, C^H_{2,h}\} + (1-p)C^H_{2,l}\\ &\geq \alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l}\end{aligned}$$

Then all weak inequalities have to hold with equality, $C^H_{2,h} \geq (1-\lambda^L)\theta$, and both (38) and (36) bind.

Proof of Lemma 2

The conclusion follows directly from concavity of the utility function $u(\cdot)$ and linearity of constraint (11) in $X_{2,s}^{i,\Delta}$.

Proof of Proposition 3

Proof. Let $(\Sigma^{i*},\, c_1^{i*},\, c_{2,s}^{i*},\, x_{2,s}^{i,\Delta*})_{i=\{L,H\}}$ be the solution to problem (1)-(7):

$$V^{bil,e=0} = \sum_{i=L,H} q_i \left[p\Big\{ \Sigma^{i*} u(x_{2h}^{i1*}) + (1-\Sigma^i) u(x_{2h}^{i0*}) \Big\} + (1-p) u(x_{2l}^{i*}) \right]$$

Define then the following contracts with CCP clearing for problem (8)-(12): $\{\hat{X}_2^{i,\Delta}\}_{i=L,H}$ and $\{\hat{\Sigma}^i,\, \hat{C}_1^i,\, \hat{C}_{2s}^i\}_{i=L,H}$ where $\hat{\Sigma}_i = \Sigma^{i*}$, as well as $\hat{C}_1^i = c_1^{i*}$, $\hat{C}_{2s}^i = c_{2s}^{i*}$, and

$$\begin{aligned}
\hat{X}_2^{i,\Delta} &= u^{-1}\left(\sum_{i=L,H} q_i \left[p\Big\{ \Sigma^{i*} u(x_{2h}^{i1*}) + (1-\Sigma^{i*}) u(x_{2h}^{i0*}) \Big\} + (1-p) u(x_{2l}^{i*}) \right] \right) \\
&< \sum_{i=L,H} q_i \left[p\Big\{ \Sigma^{i*} x_{2h}^{i1*} + (1-\Sigma^{i*}) x_{2h}^{i0*} \Big\} + (1-p) x_{2l}^{i*} \right] \qquad (39) \\
&= p\theta + \omega - \sum_{i=L,H} \left\{ \hat{C}_1^i + p\Big[\hat{\Sigma}^i (1-\lambda^i)\theta + (1-\hat{\Sigma}^i)\hat{C}_{2h}^i + (1-p)\hat{C}_{2l}^i \Big] \right\}
\end{aligned}$$

where the inequality follows from concavity of $u(\cdot)$ (convexity of $u^{-1}(\cdot)$) and the equality in the last line follows from constraints (4), (5), and (6). By construction, constraints (9) and (10) are satisfied by (2) and (3). Constraint (11) is satisfied by (39) and, also by construction, constraint (12) is satisfied by (7).

Then, the contracts $\{\hat{X}_2^{i,\Delta}\}_{i=L,H}$ and $\{\hat{\Sigma}^i,\, \hat{C}_1^i,\, \hat{C}_{2s}^i\}_{i=L,H}$ are feasible for problem (8)-(12). By optimality of $\{\hat{X}_2^{i,\Delta}\}_{i=L,H}$ and $\{\hat{\Sigma}^i,\, \hat{C}_1^i,\, \hat{C}_{2s}^i\}_{i=L,H}$ it needs to be

$$V^{CCP,e=0} \geq u(X_2^{i,\Delta}) = V^{bil,e=0}$$

meaning that when borrowers pledgeability type is private information, central clearing is the optimal clearing arrangement: the solution to (8)-(12) dominates the solution to (1)-(7).

The last expression shows that (8)-(12) ■

Proof of Proposition 5

Proof. We prove the proposition in four steps: in the first step, we show that we can ignore contracts that recommend $\Sigma^H = 1$. In the second step we characterize optimal contracts where no borrower defaults in equilibrium, $\Sigma^H = \Sigma^L = 0$. In the third step we characterize optimal contracts where λ^L borrowers default in equilibrium, $\Sigma^H = 0$ and $\Sigma^L = 1$. In the fourth and last step we compare the two classes of contracts and we determine the optimal contracts.

Step 1. Without loss of generality, in problem (8)-(12) we can ignore all contracts that recommend the strategy $\Sigma^H = 1$. Therefore, λ^H borrowers never default in equilibrium.

Suppose by contradiction that the optimal contracts X_2 and $\{\Sigma^i,\ C_1^i,\ C_{2s}^i\}_{i=L,H}$ recommend $\Sigma^H = 1$.[41] Then, by (12) it must be that λ^H-borrowers prefer the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^H, 1)$ to the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 0)$:

$$\alpha C_1^H + p(1-\lambda^H)\theta + (1-p)C_{2,l}^H \geq \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L \tag{40}$$

We need to consider two cases separately: i) the case when the recommended default strategy to a λ^L borrower is $\Sigma^L = 0$ and ii) when the recommended strategy is $\Sigma^L = 1$.

[41] We are using the property that $X_2^{i,\Delta}$ is the same for all i and all Δ.

Suppose first that optimal contracts recommend $\Sigma^L = 0$: from (12) λ^L-borrowers need to prefer the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 0)$ over the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 1)$:

$$\alpha C_1^L + C_{2,h}^L + (1-p)C_{2,l}^L \geq \alpha C_1^H + p(1-\lambda^L)\theta + (1-p)C_{2,l}^H$$

Combining this expression with (40) we obtain a contradiction. Therefore, it is not possible for the contracts to recommend $\Sigma^L = 0$. Suppose then optimal contracts recommend $\Sigma^L = 1$. Define a new contract $\{(\tilde{C}_1^i, \tilde{C}_{2,s}^i), \tilde{X}_2\}$ as $\tilde{X}_2 = X_2$, $\tilde{C}_{2,h}^H = (1-\lambda^H)\theta$, $\tilde{C}_{2,s}^i = C_{2,s}^i$ if either $i \neq H$ and $s \neq h$, $\tilde{C}_1^i = C_1$ for $i = L, H$. Let such a contract recommend $\tilde{\Sigma}^H = 0$, $\tilde{\Sigma}^L = 1$. It is easy to check that all constraints in problem (8) - (12) are satisfied, and as $\tilde{X}_2^i = X_2^i$, the new contract is payoff equivalent to the original (optimal) one, which concludes the proof of Step 1.

According to Step 1, we have to consider only two classes of contracts: contracts in which no borrower defaults in $t = 2$, that is $\Sigma^H = \Sigma^L = 0$, and contract in which only λ^H borrowers repay in $t = 2$, whereas λ^L borrowers default in equilibrium, that is $\Sigma^H = 0$ and $\Sigma^L = 1$.

Step 2. A solution to problem (8)-(12) with $\Sigma^H = 0$ and $\Sigma^L = 0$ is such that

$$C_{2,l}^H = C_{2,l}^L = 0, \qquad C_1^H = C_1^L = \omega - \frac{(1-\lambda^L)p\theta}{\alpha},$$
$$C_{2h}^H = C_{2,h}^L = (1-\lambda^L)\theta \qquad X_2 = \omega(\lambda^L) + p\theta\lambda^L$$

Step 2.1: From Lemma 1 we know that, given $\Sigma^H = 0$ and $\Sigma^L = 0$, constraint (12) requires $\min\{C_{2,h}^H, C_{2,h}^L\} \geq (1-\lambda^L)\theta$ and $\alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L$.

Step 2.2: W.l.o.g we can ignore the participation constraint (9) of the λ^H borrower. The result follows immediately from the previous step.

Step 2.3: We have that $C^L_{2,l} = 0$.
Suppose not: $C^L_{2,l} > 0$. Then it must be $C^L_1 = \omega$. If not we could reduce $C^L_{2,l}$ by ϵ, increase C^L_1 by $\frac{(1-p)\epsilon}{\alpha}$, and increase X_2 by $(1-q)(1-p)\epsilon[1-\frac{1}{\alpha}] > 0$. The new contract would be feasible and expected utility would increase. Then $C^L_1 = \omega$, and therefore as $C^L_{2,l} > 0$ and $C^L_{2,h} \geq (1-\lambda^L)\theta$, the participation constraint (9) of λ^L borrowers can be ignored as well. Moreover, it must be $C^H_{2,h} = (1-\lambda^L)\theta$, otherwise we could reduce $C^L_{2,l}$ by ϵ and $C^H_{2,h}$ by $\frac{(1-p)\epsilon}{p}$ and increase X_2 by $p\epsilon$. The new contract would still satisfy all constraints and the expected utility would increase. Similarly it should be $C^H_{2,l} = 0$. If not we could reduce $C^L_{2,l}$ and $C^H_{2,l}$ by ϵ, and increase X_2 by $(1-p)\epsilon$. Finally, it should be $C^H_1 = 0$, otherwise we could reduce $C^L_{2,l}$ by ϵ, reduce C^H_1 by $\frac{(1-p)\epsilon}{\alpha}$ and increase X_2 by $(1-p)\epsilon[\frac{1}{\alpha} + (1-q)]$. Combing $C^H_1 = C^H_{2,l} = C^L_{2,l} = 0$, $C^H_{1,h} = (1-\lambda^L)\theta$, $C^L_1 = \omega$, we obtain that the binding (12) becomes

$$\alpha\omega + pC^H_{2,h} + (1-p)C^L_{2,l} = (1-\lambda^L)\theta$$

which can never be satisfied for $C^L_{2,l} > 0$ and $C^L_{2,h} \geq (1-\lambda^L)\theta$, which is a contradiction.

Step 2.4: We have that $C^H_{2,l} = 0$.
Suppose not: suppose $C^H_{2,l} > 0$. Then it should be $C^H_1 = \omega$, otherwise we could educe $C^H_{2,l}$ by ϵ, increase C^H_1 by $\frac{(1-p)\epsilon}{\alpha}$, and increase X_2 by $q(1-p)\epsilon[1-\frac{1}{\alpha}] > 0$. Moreover the participation constraint (9) of λ^L borrowers should bind: if not following the same arguments of the previous step it should be $C^L_1 = 0$ and $C^L_{2,h} = (1-\lambda^L)\theta$. But the

participation constraint (9) of λ^L borrowers and the binding (12) would give

$$(1-\lambda^L)p\theta = \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L = \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H$$
$$= \alpha\omega + pC_{2,h}^H + (1-p)C_{2,l}^L > (1-\lambda^L)p\theta$$

which is a contradiction. Then it should be

$$\alpha C_1^L + pC_{2,h}^L = \alpha\omega$$

This implies that $C_1^L < \omega$, as $C_{2,h}^L > 0$. Then (9) of λ^L borrowers and (12) give

$$\alpha\omega = \alpha C_1^L + pC_{2,h}^L = \alpha\omega + pC_{2,h}^H > \alpha\omega$$

which is a contradiction.

Step 2.5: $C_{2,h}^H = C_{2,h}^L = (1-\lambda^L)\theta$.

Suppose $C_{2,h}^i > (1-\lambda^L)\theta$. Reduce $C_{2,h}^i$ by ϵ, increase C_1^i by $\frac{p\epsilon}{\alpha}$ and X_2 by $q_i p\epsilon[1-\frac{1}{\alpha}]$, and the expected utility would increase.

Step 2.6: $C_1^H = C_1^L$.

Follows from (12) holding with equality.

Step 2.7: We have that $C_1^i = \omega - \frac{(1-\lambda^L)p\theta}{\alpha}$.

Suppose by contradiction that $C_1^i > \omega - \frac{(1-\lambda^L)p\theta}{\alpha} > 0$, where the last inequality (> 0) comes from Assumption 4. Then, we could modify the allocation by defining allocation defined by $\hat{C}_1 = C_1 - \varepsilon$ for $\varepsilon > 0$ arbitrarily small, and $\hat{X}_2 = X_2 + \alpha\varepsilon$. This allocation is still in the constraint set of problem problem (8)-(12), and contradicts optimality of

$C_1^i > \omega - \frac{(1-\lambda^L)p\theta}{\alpha}$.

Step 3. A solution to problem (8)-(12) with $\Sigma^H = 0$ and $\Sigma^L = 1$ is such that $C_{2,l}^H = C_{2,l}^L = 0$, $C_1^H = C_1^L$, where

$$C_{2,h}^i = \begin{cases} (1-\lambda^H)\theta & \text{if } q \geq \frac{1}{\alpha} \\ (1-\lambda^L)\theta & \text{if } q < \frac{1}{\alpha} \end{cases} \qquad C_1^i = \begin{cases} \omega - \omega(\lambda^H) & \text{if } q \geq \frac{1}{\alpha} \\ \omega - \omega(\lambda^L) & \text{if } q < \frac{1}{\alpha} \end{cases}$$

Consider problem (8) - (12). In (12), the recommended default decision $\Sigma^H = 0$ and $\Sigma^L = 1$ require $C_{2,h}^H \geq (1-\lambda^H)\theta$ and $C_{2,h}^L < (1-\lambda^L)\theta$ respectively. Constraint (12) for λ^H-borrowers can be rewritten as

$$C_{2,h}^H \geq (1-\lambda^H)\theta \tag{41}$$

$$\alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H \geq \alpha C_1^L + p\max\{(1-\lambda^H)\theta, C_{2,h}^L\} + (1-p)C_{2,l}^L \tag{42}$$

whereas constraint (12) for λ^L-borrowers becomes

$$C_{2,h}^L \leq (1-\lambda^L)\theta \tag{43}$$

$$\alpha C_1^L + p(1-\lambda^L)\theta + (1-p)C_{2,l}^L \geq \alpha C_1^H + p\max\{(1-\lambda^L)\theta, C_{2,h}^H\} + (1-p)C_{2,l}^H \tag{44}$$

Step 3.1: W.l.o.g. we can choose $C_{2,h}^L = (1-\lambda^H)\theta$, and ignore constraint (43).
This choice satisfies (43) and relaxes (44) as much as possible. Since $\Sigma^L = 1$ is the recommended (i.e. incentive compatible) deafult choice, $C_{2,h}^L$ does not appear in any other constraint. This means that we can assume $C_{2,h}^L = (1-\lambda^H)\theta$.

Step 3.2: We can ignore the participation constraint of λ^L-borrowers.

From (44) and the participation constraint of λ^H-borrowers,

$$\alpha C_1^L + p(1-\lambda^L)\theta + (1-p)C_{2,l}^L \geq \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H \geq \alpha\omega$$

<u>Step 3.3</u>: The optimal contract requires $C_{2,h}^H \leq (1-\lambda^L)\theta$.

Suppose by contradiction the optimal contracts $\{(C_1^i, C_{2,h}^i, C_{2,l}^i),\ X_2\}$ satisfies $C_{2,h}^H > (1-\lambda^L)\theta > (1-\lambda^H)\theta$. Then we can ignore (41). Moreover it needs to be that $C_1^H = \omega$: if not, the CCP could reduce $C_{2,h}^H$ by ϵ, increase C_1^H by $\frac{p}{\alpha}\epsilon$, and increase X_2 by $\frac{\alpha-1}{\alpha}qp\epsilon > 0$. All constraints are still satisfied but the expected utility of lenders increases. But then, since $C_1^H = \omega$, we can also ignore the participation constraint of λ^H-borrowers. From constraint (42), we can ignore (44). Therefore the only constraints left are (42), the resource constraint (10) for $i = L$, and the second-period resource constraint (11). Note that (42) should bind or the CCP could reduce $C_{2,h}^H$ and increase X_2 accordingly, without violating any constraint:

$$\alpha\omega + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + p(1-\lambda^H)\theta + (1-p)C_{2,l}^L \tag{45}$$

From this expression and (10) it needs to be $C_{2,l}^L > C_{2,l}^H \geq 0$. Then it has to be $C_{2,l}^H = 0$, otherwise we could decrease both $C_{2,l}^L$ and$C_{2,l}^H$ by ϵ, and increase X_2 by $(1-p)\epsilon$ it needs to be $C_{2,l}^L > C_{2,l}^H \geq 0$. Then it has to be $C_{2,l}^H = 0$, otherwise we could decrease both $C_{2,l}^L$ and$C_{2,l}^H$ by ϵ, and increase X_2 by $(1-p)\epsilon$. Replacing $C_{2,l}^H = 0$ we obtain that

$$(1-p)C_{2,l}^L = \alpha(\omega - C_1^L) + p[C_{2,h} - (1-\lambda^H)\theta] > 0$$

But then it has to be that $C_1^L = \omega$: if $C_1^L < \omega$, the CCP can decrease $C_{2,l}^L$ by ϵ and increase C_1^L by $\frac{p}{\alpha}\epsilon$, and increase X_2 by $\frac{\alpha-1}{\alpha}(1-q)p\epsilon > 0$. All constraints are still satisfied but the expected utility of lenders increases. Moreover it needs to be that

$C^L_{2,l} = 0$. If not, the CCP could reduce C^H_1 by ϵ, $C^L_{2,l}$ by $\frac{p}{1-p}\epsilon$, and increase X_2 by $p\epsilon$. All constraints are still satisfied but the expected utility of lenders strictly increase. But then, equation (45) becomes

$$(1-\lambda^L)p\theta < pC^H_{2,h} = p(1-\lambda^H)\theta$$

which is not possible. This proves that it must be that $C^H_{2,h} \leq (1-\lambda^L)\theta$. Replacing this value in (44), the latter becomes

$$\alpha C^L_1 + (1-p)C^L_{2,l} \geq \alpha C^H_1 + (1-p)C^H_{2,l}$$

<u>Step 3.4</u>: At the optimal solution, equation (44) holds with equality: $\alpha C^L_1 + (1-p)C^L_{2,l} = \alpha C^H_1 + (1-p)C^H_{2,l}$.
Suppose not: suppose that (44) is slack. The only active constraints are then the resource constraint in $t = 1$, (10) the resource constraint in $t = 2$, (11), and the incentive compatibility constraints (41) and (42). But then it should easily be that it $C^L_1 = C^L_{2,l} = 0$. As a result, (44) can only hold if $C^H_1 = C^H_{2,l} = 0$, and equation (44) holds with equality.

<u>Step 3.5</u>: Constraint (42) can be ignored.
Use the fact that (44) binds and (43), we obtain

$$\begin{aligned}\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} &= \alpha C^L_1 + pC^H_{2,h} + (1-p)C^L_{2,l}\\ &\geq \alpha C^L_1 + p(1-\lambda^H)\theta + (1-p)C^L_{2,l}\end{aligned}$$

<u>Step 3.6</u>: It is optimal to choose $C^H_{2,l} = C^L_{2,l} = 0$.
Suppose not: suppose w.l.o.g. that $C^H_{2,l} \geq C^L_{2,l} \geq 0$ with one inequality holding has a

strict inequality. If $C^H_{2,l} = C^L_{2,l} > 0$, then we could decrease both by ϵ, increase X_2 by $(1-p)\epsilon$, satisfying all the relevant constraints and increasing the expected utility. If instead $C^H_{2,l} > C^L_{2,l} = 0$, it has to be $0 \leq C^H_1 < C^L_1$. But then we could reduce $C^H_{2,l}$ by ϵ, reduce C^L_1 by $\frac{(1-p)\epsilon}{\alpha}$, and increase X_2 by $(1-p)\epsilon[q + \frac{1-q}{\alpha}]$. All constraints would be satisfied, and the expected utility would increase.

Step 3.7: It is optimal to choose $C^H_1 = C^L_1$.

It follows immediately by the binding (44) once we replace $C^H_{2,l} = C^L_{2,l} = 0$.

$$\alpha C^H_1 = \alpha C^L_1$$

Step 3.8: The feasibility constraint (10) can be ignored.

By Assumption 4, $\omega > \omega(\lambda^L) > \omega(\lambda^H)$. Suppose first by contradiction that $C_1 = \omega$. Then, the participation constraint (9) can be ignored for both type of borrowers. Then, we could decrease slightly C_1 and increase slightly X_2, to satisfy (11). Since $C^H_1 = C^L_1$, constraint (12) is unaffected by such modification. Suppose next that $C_1 = 0$. Then, it means that $C^H_{2,h} > \frac{(1-\lambda^L)p\theta}{\alpha} > \frac{(1-\lambda^H)p\theta}{\alpha}$ for the participation constraint (9) to hold for $i = H$. But then, we can decrease $C^H_{2,h}$ and $C^L_{2,h}$ slightly by the same amount and increase X_2 to leave (11) unchanged and to induce the same borrowers' default choice. This increases lenders' consumption, and contradicts that $C_1 = 0$ can be optimal.

Step 3.9: The participation constraint of a λ^H type binds: $\alpha C_1 + pC^H_{2h} = \alpha\omega$.

Suppose that the participation constraint is slack. Then easily $C_{2h} = (1-\lambda^H)\theta$ and $C_1 = 0$. This can be a solution if $\omega < \frac{(1-\lambda^H)p\theta}{\alpha}$.

Step 3.10: Rewrite the residual problem"

$$
\begin{aligned}
\max \quad & u\left(p\theta + \frac{pC_{2h}}{\alpha} - qpC_{2h} - (1-q)p(1-\lambda^L)\theta\right) \\
s.t. \quad & \omega - \frac{pC_{2h}}{\alpha} \geq 0 \\
& (1-\lambda^L)\theta \geq C_{2h} \geq (1-\lambda^H)\theta
\end{aligned}
$$

If $q \geq \frac{1}{\alpha}$, the objective is decrasing in C_{2h}, so the solution is $C_{2h} = (1-\lambda^H)\theta$. This can be a solution only if $\omega \geq \frac{(1-\lambda^H)p\theta}{\alpha}$. If $q < \frac{1}{\alpha}$, then the solution is increasing in C_{2h}, so the solution is $C_{2h} = \min\left\{(1-\lambda^L)\theta, \frac{\alpha\omega}{p}\right\} = (1-\lambda^L)\theta$, where the last inequality follows by Assumption 4.

Step 4. The optimal contract induces no borrower to default in equilibrium, $\Sigma^H = \Sigma^L = 0$, if $q < \frac{1}{\alpha}$, and induces λ^L borrowers to default in equilibrium, $\Sigma^H = 0$, $\Sigma^L = 1$, if $q \geq \frac{1}{\alpha}$.

This follows just by comparing the payoffs of the two contracts.

■

Proof of Lemma 7

Proof. Consider problem (16)-(21). By Assumption 4, $\omega > \omega(\lambda^L) > \omega(\lambda^H)$. Replace $x^i_{2,h}$ and $x^i_{2,l}$ from the binding constraints (19) and (20), and rewrite the problem

$$
\begin{aligned}
(P^i) \quad V_i = \max_{\left(c^i_1, c^i_{2,h}, c^i_{2,l}\right) \in \Re^3_+} \quad & pu\Big(\theta - c^i_{2,h} + \omega - c^i_1\Big) + (1-p)u\Big(\omega - c^i_1 - c^i_{2,l}\Big) - \gamma \\
s.t. \quad & \alpha c^i_1 + pc^i_{2,h} + (1-p)c^i_{2,l} \geq \alpha\omega \qquad (17) \\
& \omega \geq c^i_1 \geq 0 \qquad (18)
\end{aligned}
$$

$$c^i_{2,h} \geq (1-\lambda^i)\theta \tag{21}$$

Step 1. We have that $c^i_1 < \omega$.

Suppose by contradiction that $c_1 = \omega$. Then, because of (21),

$$\alpha c^i_1 + pc^i_{2,h} + (1-p)c^i_{2,l} \geq \alpha\omega + p(1-\lambda^i)\theta > \alpha\omega$$

and the participation constraint (17) is slack. But then, we can slightly decrease c^i_1 without violating any constraint and increasing the objective function, which proves that $c_1 = \omega$ is not possible.

Step 2. We have that $c^i_{2,l} = 0$.

Next, we show that second period borrowers' consumption in the low state equals zero, i.e. $c^i_{2,l} = 0$. To prove this, first notice that it must be that $x^i_{2,h} \geq x^i_{2,l}$. If not, i.e. if $x^i_{2,h} < x^i_{2,l}$, combining equations (19) and (20) (with equality) we obtain

$$c^i_{2,h} = c^i_{2,l} + \theta + (x^i_{2,l} - x^i_{2,h}) > c^i_{2,l} + \theta > (1-\lambda^i)\theta$$

Then, the lender could reduce $c^i_{2,h}$ by ϵ, increase $x^i_{2,h}$ by the same amount, increase $c^i_{2,l}$ by $\frac{p}{1-p}\epsilon$, and reduce $x^i_{2,l}$ by the same amount. All constraints would be satisfied, and by concavity of $u(\cdot)$ the lender would increase her expected utility. Now that we established that $x^i_{2,h} \geq x^i_{2,l}$, suppose by contradiction that $c^i_{2,l} > 0$. Then it should be that $x^i_{2,h} = x^i_{2,l}$. If not, i.e. if $x^i_{2,h} > x^i_{2,l}$, the lender could increase $c^i_{2,h}$ by ϵ, reduce $x_{2,h}$ by the same amount, reduce $c^i_{2,l}$ by $\frac{p}{1-p}\epsilon$, and increase $x_{2,l}$ by the same amount. All constraints would be satisfied, and by concavity of $u(\cdot)$ the lender would increase her expected utility. Since $x^i_{2,h} = x^i_{2,l}$, combining (19) and (20) (with equality) we obtain

$$c^i_{2,h} = c^i_{2,l} + \theta > (1 - \lambda^i)\theta$$

But then the lender could reduce $c^i_{2,h}$ and $c^i_{2,l}$ by ϵ, increase c_1 by $\frac{\epsilon}{\alpha}$, and increase both $x_{2,h}$ and $x_{2,l}$ by the same amount $\frac{\alpha-1}{\alpha}\epsilon$. All constraints would be satisfied and the lender expected revenues would increase. Therefore it can not be that $c^i_{2,l} > 0$, and we conclude that it should be that $c^i_{2,l} = 0$.

Step 3. The participation constraint (17) binds.

Suppose by contradiction that the participation constraint (17) is slack. Then, the limited commitment constraint (21) should bind, i.e. $c^i_{2,h} = (1 - \lambda^i)\theta$. Indeed, both constraints (17) and (21) can not be slack, because then we could decrease $c^i_{2,h}$ without violating any constraint. Then, since $\omega > \omega(\lambda^L)$, it must be that $c^i_1 > 0$. Indeed, if we had $c^i_1 = 0$, then $\alpha c^i_1 + pc^i_{2,h} = p(1 - \lambda^i)\theta < \alpha\omega$ and the participation constraint (17) would be violated. But then, if $c^i_1 > 0$ and (17) is slack, we could just decrease c^i_1 slightly, and increase lenders' consumption, which proves that the participation constraint (17) should bind.

Step 4. A solution to problem (16)-(21) is such that $c^i_{2,h} = (1-\lambda^i)\theta$, $c^i_1 = \omega - \frac{(1-\lambda^i)p\theta}{\alpha}$ if $\lambda \leq \lambda^*$, whereas $c^i_{2,h} = (1-\lambda^*)\theta$, $c^i_1 = \omega - \frac{(1-\lambda^*)p\theta}{\alpha}$ if $\lambda \leq \lambda^*$, for λ^* defined in equation (22).

Ignore (18) and replace $c^i_{2,h} = \alpha\frac{\omega - c^i_1}{p}$ in the objective function and in (21), and rewrite the residual problem as

$$(P^i) \quad V_i = \max_{(c_1^i)\in\Re_+} \quad pu\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right) + (1-p)u\Big(\ \omega - c_1^i\Big) - \gamma$$

$$c_1^i \le \omega - \frac{(1-\lambda^i)p\theta}{\alpha} \tag{21}$$

Ignore constraint (21): the first order condition for optimality is

$$(\alpha - p)u'\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right) \le (1-p)u'(\omega - c_1^i) \tag{46}$$

with equality if $c_1^i > 0$. Notice that the left-hand side is decreasing in c_1^i and the right-hand side is increasing in c_1^i.

Suppose first that $c_1^i = 0$: equation (46) requires

$$(\alpha - p)u'\left(\theta - (\omega)\frac{\alpha - p}{p}\right) \le (1-p)u'(\omega)$$

Since $\omega > \frac{(1-\lambda^L)p\theta}{\alpha}$ by Assumption 4,

$$(\alpha-p)u'\left(\theta - \frac{(1-\lambda^L)\theta}{\alpha}[\alpha - p]\right) < (\alpha-p)u'\left(\theta - (\omega)\frac{\alpha - p}{p}\right) \le (1-p)u'(\omega) < (1-p)u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)$$

which violates Assumption 6. Then, there exists a unique $c_1^{i*} > 0$ that solves (46). Given this c_1^*, the solution to problem (16)-(21) depends on λ^i: either $c_1^i = c_1^*$ and (21) holds for c_1^*, $\alpha\frac{\omega - c_1^*}{p\theta} \ge (1-\lambda^i)$, or (21) binds, $\alpha\frac{\omega - c_1^i}{p\theta} = (1-\lambda^i)$. Notice that (21) is decreasing in c_1^i; thus, there exists unique λ^* such that the solution is c_1^* if and only if $\lambda \ge \lambda^*$, whereas the solution is pinned down by the binding constraint (21) if $\lambda < \lambda^*$. Specifically, λ^* solves (46) for $\omega - c_1^i = \frac{(1-\lambda^*)p\theta}{\alpha}$:

$$(\alpha - p)u'\left(\theta - \frac{(1-\lambda^*)p\theta}{\alpha}[\alpha - p]\right) = (1-p)u'\left(\frac{(1-\lambda^*)p\theta}{\alpha}\right)$$

It is easy to see that such λ^* is defined in equation (22). Define then the function

$$F(\lambda) = \frac{u'\left(\frac{(1-\lambda)p\theta}{\alpha}\right)}{u'\left(\theta + (1-\lambda)\frac{p\theta}{\alpha}(1-\frac{\alpha}{p})\right)}$$

Notice that $F(0) = 1 < \frac{\alpha-p}{1-p}$ and $F'(\lambda) > 0$. Thus, there exists a unique $\lambda^* \in (0,1)$ such that $F(\lambda^*) = \frac{\alpha-p}{1-p}$. Then, we can conclude that $c_1^i = c_1^*$ if $\lambda^i \geq \lambda^*$, and $c_1^i = \omega - \frac{(1-\lambda^i)p\theta}{\alpha}$ if $\lambda^i < \lambda^*$.

Step 5. We have $c_{2,h}^L = (1-\lambda^L)\theta$ and $c_1^L = \omega - \frac{(1-\lambda^L)p\theta}{\alpha}$.

It follows from Step 4 and Assumption 6.

Step 6. We have that

(1) $c_1^H = \omega - \omega(\lambda^*)$, $c_{2,h}^H = (1-\lambda^*)\theta$, if $\frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{\alpha-p}{1-p}$.

(2) $c_1^H = \omega - \omega(\lambda^H)$, $c_{2,h}^H = (1-\lambda^H)\theta$, if $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$.

It follows from Step 4. ■

Proof of Proposition 9

Proof. Let $\omega(\lambda)$ be defined in equation (14). Consider the payoff ensuing the optimal contract with central clearing in equation (15), $V^{CCP,e=0} = u(X_2)$, where

$$X_2 = \begin{cases} \omega(\lambda^L) + p\theta\lambda^L & \text{if } q \leq \frac{1}{\alpha} \\ \omega(\lambda^H) + p\theta[q\lambda^H + (1-q)\lambda^L] & \text{if } q \geq \frac{1}{\alpha} \end{cases} \tag{47}$$

and the payoff associated with the optimal contracts with bilateral clearing and screening

$$V^{bil,e=1} = \sum_{i=L,H} \Big\{ q_i \sum_{s=l,h} p_s u(x^i_{2,s}) \Big\} - \gamma$$

where

$$\begin{aligned} x^H_{2,h} &= \min\{\lambda^H \theta + \omega(\lambda^H), \lambda^* \theta + \omega(\lambda^*)\}, \qquad & x^H_{2,l} &= \max\{\omega(\lambda^H), \omega(\lambda^*)\}, \\ x^L_{2,h} &= \lambda^L \theta + \omega(\lambda^L), & x^L_{2,l} &= \omega(\lambda^L) \end{aligned} \tag{48}$$

where λ^* is defined in (22), and from Lemma 7 we know that $\lambda^H\theta+\omega(\lambda^H) = \min\{\lambda^H\theta+\omega(\lambda^H), \lambda^*\theta+\omega(\lambda^*)\}$ if and only if $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta-\frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$. Notice that $x^i_{2,s}$ as well as λ^* are independent of q. Easily, bilateral clearing with information acquisition is preferred to central clearing if and only if $V^{bil,e=1} \geq V^{CCP,e=0}$. Let then $\overline{\gamma}(q) : [0,1] \rightarrow \Re$

$$\overline{\gamma}(q) = \sum_{i=L,H} \Big\{ q_i \sum_{s=l,h} p_s u(x^i_{2,s}) \Big\} - u(X_2) \tag{49}$$

for X_2 defined in (47) and $x^i_{2,s}$ defined in (48).

Step 1. The function $\overline{\gamma}(q)$ defined in (49) satisfies $\overline{\gamma}(0) < 0$ and $\overline{\gamma}(1) < 0$.

Consider then such function: at $q = 0$ we have

$$\overline{\gamma}(0) = \Big[pu\Big(\lambda^L\theta + \omega(\lambda^L)\Big) + (1-p))u\Big(\omega(\lambda^L)\Big)\Big] - u\Big(\omega(\lambda^L) + p\theta\lambda^L\Big) < 0$$

where the inequality comes from concavity of $u(\cdot)$. Consider next the same function at

$q = 1$:

$$\begin{aligned}
\overline{\gamma}(1) &= \Big[pu\Big(\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}\Big) + (1-p))u\Big(\max\{\omega(\lambda^H), \omega(\lambda^*)\}\Big)\Big] - u\Big(\omega(\lambda^H) + p\theta\lambda^H\Big) \\
&< u\left(p\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\} + (1-p)\max\{\omega(\lambda^H), \omega(\lambda^*)\}\right) - u\Big(\omega(\lambda^H) + p\theta\lambda^H\Big) \\
&\leq u\left(p[\lambda^H\theta + \omega(\lambda^H)] + (1-p)\omega(\lambda^H)\right) - u\Big(\omega(\lambda^H) + p\theta\lambda^H\Big) = 0
\end{aligned}$$

where the first inequality comes from concavity of $u(\cdot)$, the second one from and the definition of λ^*. Thus, we also have that $\overline{\gamma}(1) < 0$ as well.

Step 2. The function $\overline{\gamma}(q)$ defined in (49) is monotone increasing for $q < \frac{1}{\alpha}$.

Consider the first derivative of the function $\overline{\gamma}(q)$ defined in (49), for $q < \frac{1}{\alpha}$:

$$\begin{aligned}
\frac{\partial\overline{\gamma}(q)}{\partial q} &= pu\Big(\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}\Big) + (1-p))u\Big(\max\{\omega(\lambda^H), \omega(\lambda^*)\Big) \\
&\quad - pu\Big(\lambda^L\theta + \omega(\lambda^L)\Big) + (1-p))u\Big(\omega(\lambda^L)\Big) > 0
\end{aligned}$$

where the inequality comes from the fact that the contract $x^L_{2,h} = \lambda^L\theta + \omega(\lambda^L)$, $x^L_{2,l} = \omega(\lambda^L)$ is feasible, but not optimal, for the problem of a lender that faces a λ^H borrower. Then, the function $\overline{\gamma}(q)$ defined in (49) is monotone increasing for $q < \frac{1}{\alpha}$.

Step 3. The function $\overline{\gamma}(q)$ defined in (49) is convex for $q > \frac{1}{\alpha}$.

Consider now the first and second derivatives of the function $\overline{\gamma}(q)$ defined in (49), for $q > \frac{1}{\alpha}$:

$$\begin{aligned}
\frac{\partial\overline{\gamma}(q)}{\partial q} &= pu\Big(\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}\Big) + (1-p))u\Big(\max\{\omega(\lambda^H), \omega(\lambda^*)\Big) \\
&\quad - pu\Big(\lambda^L\theta + \omega(\lambda^L)\Big) + (1-p))u\Big(\omega(\lambda^L)\Big) - u'(X_2)p\theta(\lambda^H - \lambda^L)
\end{aligned}$$

and

$$\frac{\partial^2 \overline{\gamma}(q)}{\partial q^2} = -u''(X_2)[p\theta(\lambda^H - \lambda^L)]^2 > 0$$

where the inequality comes from concavity of $u(\cdot)$, thus $u''(\cdot) < 0$ and $-u''(\cdot) > 0$.

Step 4. The function $\overline{\gamma}(q) > 0$ for some $q \in [0,1]$ if and only if $\overline{\gamma}(\frac{1}{\alpha}) > 0$. Moreover, if $\overline{\gamma}(\frac{1}{\alpha}) > 0$, then there exist $\underline{q}, \overline{q} \in [0,1]$, where $\underline{q} < \overline{q}$, such that $\overline{\gamma}(q) > 0$ for $\gamma \in [\underline{q}, \overline{q}]$.

The conclusion follows from Steps 1,2, and 3. Suppose first that $\overline{\gamma}(\frac{1}{\alpha}) > 0$. Since $\overline{\gamma}(q)$ is strictly increasing for $q < \frac{1}{\alpha}$ and $\overline{q}(0) < 0$, by the intermediate value theorem there should exists a unique $\underline{q} \in (0, \frac{1}{\alpha})$ such that $\overline{\gamma}(\underline{q}) = 0$. Also, since $\overline{\gamma}(1) < 0$ and $\overline{\gamma}(q)$ is convex for $\gamma < \frac{1}{\alpha}$, given that $\overline{\gamma}(\frac{1}{\alpha}) > 0$ it must be that for $q > \frac{1}{\alpha}$ the function $\overline{\gamma}(q)$ is initially decreasing, crosses the horizontal axes for a unique $\overline{q}$ where $\overline{\gamma}(\overline{q}) = 0$, and then stays negative. Then, we proved the if direction: if $\overline{\gamma}(\frac{1}{\alpha}) > 0$, there exist $\underline{q}, \overline{q} \in [0,1]$ such that $\overline{\gamma}(q) > 0$ for $\gamma \in [\underline{q}, \overline{q}]$.

Next, we prove the only if direction. Suppose that $\overline{\gamma}(q) > 0$ for some $q \in (0,1)$, but $\overline{\gamma}(\frac{1}{\alpha}) < 0$. Since the function $\overline{\gamma}(q)$ is strictly increasing for $q < \frac{1}{\alpha}$, then $\overline{\gamma}(q) < 0$ for all $q \leq \frac{1}{\alpha}$, and it must be that $\overline{\gamma}(q) > 0$ for some $q > \frac{1}{\alpha}$. But then, since $\overline{\gamma}(\frac{1}{\alpha}) < 0$, and the function $\overline{\gamma}(\frac{1}{\alpha})$ is continuous, there should exist an interval $[q', q''] \subset (\frac{1}{\alpha}, 1]$ such that $\overline{\gamma}(q) > 0$ and $\overline{\gamma}'(q) > 0$ for $q \in [q', q'']$. But then, since the function $\overline{\gamma}(q)$ is convex, it must be that $\overline{\gamma}'(q) > 0$ also for all $q > q''$, and therefore $\overline{\gamma}(1) > \overline{\gamma}(q'') > 0$, which is a contradiction.

Step 5. Replacing $q = \frac{1}{\alpha}$ in (49) we obtain the condition in (23). Also, the function $\overline{\gamma}(q) : (\underline{q}, \overline{q}) \rightarrow \Re_+$ is the$\overline{\gamma}(q)$ defined in (49), restricting the domain to the interval $(\underline{q}, \overline{q})$

■

Proof of Lemma 10

Proof. Let $(x_{2,h}^{i^u}, x_{2,l}^{i^u})$ be the optimal contract in the economy populated by lenders with utility $u(\cdot)$ and $(x_{2,h}^{i^v}, x_{2,l}^{i^v})$ be the optimal contract in the economy populated by lenders with utility $v(\cdot)$. Also, let X_2 be the optimal contract with central clearing. Note that X_2 is the same with utility $u(\cdot)$ and with utility $v(\cdot)$. Let then λ_u^* and λ_v^* be the threshold λ^* defined in (22) when the utility function is $u(\cdot)$ and $v(\cdot)$ respectively. Notice that

$$\frac{u'\left(\frac{(1-\lambda_u^*)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda_u^*)\theta\right)} = \frac{\alpha-p}{1-p} = \frac{v'\left(\frac{(1-\lambda_v^*)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda_v^*)\theta\right)}$$
$$= \frac{\rho'\left(v\left(\frac{(1-\lambda_v^*)p\theta}{\alpha}\right)\right)}{\rho'\left(v\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda_v^*)\theta\right)\right)} \frac{u'\left(\frac{(1-\lambda_v^*)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda_v^*)\theta\right)} > \frac{u'\left(\frac{(1-\lambda_v^*)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda_v^*)\theta\right)}$$

and therefore $\lambda_u^* > \lambda_v^*$. Then, since we assumed that $\frac{\alpha-p}{1-p} > \frac{v'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^L)\theta\right)}$, using the same argument as above we can easily prove that

$$\frac{\alpha-p}{1-p} > \frac{v'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^L)\theta\right)} > \frac{u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^L)\theta\right)}$$

and therefore the limited commitment constraint of λ^L borrowers binds under both lenders' utility functions. Hence,

$$x_{2,h}^{L^u} = x_{2,h}^{L^v} = \lambda^L\theta + \frac{(1-\lambda^L)p\theta}{\alpha}$$
$$x_{2,l}^{L^u} = x_{2,l}^{L^v} = \frac{(1-\lambda^L)p\theta}{\alpha}$$

Also, regarding $x_{2,s}^{H^u}$ and $x_{2,s}^{H^v}$ there are three possible cases.

<u>Case 1</u>: $\frac{\alpha-p}{1-p} > \frac{v'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$.

In this case $x_{2,h}^{H^u} = x_{2,h}^{H^v} = \lambda^H\theta + \frac{(1-\lambda^H)p\theta}{\alpha}$ and $x_{2,l}^{H^u} = x_{2,l}^{H^v} = \frac{(1-\lambda^H)p\theta}{\alpha}$. Then, since lenders in economy B prefer bilateral clearing, it must be that $\gamma < \overline{\gamma}_v$, where

$$\begin{aligned}
\overline{\gamma}_v &= q\Big[pv(x_{2,h}^{H^v}) + (1-p)v(x_{2,l}^{H^v})\Big] + (1-q)\Big[pv(x_{2,h}^{L^v}) + (1-p)v(x_{2,l}^{L^v})\Big] - v(X_2) \\
&= q\Big[p\rho\Big(u(x_{2,h}^{H^v})\Big) + (1-p)\rho\Big(u(x_{2,l}^{H^v})\Big)\Big] + (1-q)\Big[p\rho\Big(u(x_{2,h}^{L^v})\Big) + (1-p)\rho\Big(u(x_{2,l}^{L^v})\Big)\Big] - \rho\Big(u(X_2)\Big) \\
&\leq \rho\Bigg(q\Big[pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})\Big] + (1-q)\Big[pu(x_{2,h}^{L^v}) + (1-p)u(x_{2,l}^{L^v})\Big]\Bigg) - \rho\Big(u(X_2)\Big) \\
&\leq q\Big[pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})\Big] + (1-q)\Big[pu(x_{2,h}^{L^v}) + (1-p)u(x_{2,l}^{L^v})\Big] - u(X_2) \\
&= q\Big[pu(x_{2,h}^{H^u}) + (1-p)u(x_{2,l}^{H^u})\Big] + (1-q)\Big[pu(x_{2,h}^{L^u}) + (1-p)u(x_{2,l}^{L^u})\Big] - u(X_2) = \overline{\gamma}_u
\end{aligned}$$

where the first inequality follows from concavity, and the second one from the contraction property of $\rho(\cdot)$. Thus, when $\gamma < \overline{\gamma}_v$, then $\gamma < \overline{\gamma}_u$, and so agents in economy A as well prefer bilateral clearing over central clearing.

<u>Case 2</u>: $\frac{v'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$.

In this case $x_{2,h}^{H^u} = \lambda^H\theta + \frac{(1-\lambda^H)p\theta}{\alpha}$ and $x_{2,l}^{H^u} = \frac{(1-\lambda^H)p\theta}{\alpha}$, $x_{2,h}^{H^v} = \lambda_v^*\theta + \frac{(1-\lambda_v^*)p\theta}{\alpha}$, and $x_{2,l}^{H^v} = \frac{(1-\lambda_v^*)p\theta}{\alpha}$. In this case, since $\lambda_v^* \leq \lambda^H$, thus the contract $(x_{2,h}^{H^v}, x_{2,l}^{H^v})$ is feasible and incentive-compatible for the problem that lenders with utility $u(\cdot)$ face. But since such lenders prefer the contract $(x_{2,h}^{H^u}, x_{2,l}^{H^u})$, it must be that

$$pu(x_{2,h}^{H^u}) + (1-p)u(x_{2,l}^{H^u}) > pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})$$

Now, since lenders in economy B prefer bilateral clearing, it must be that $\gamma < \overline{\gamma}_v$, where

$$
\begin{aligned}
\overline{\gamma}_v &= q\Big[pv(x_{2,h}^{H^v}) + (1-p)v(x_{2,l}^{H^v})\Big] + (1-q)\Big[pv(x_{2,h}^{L^v}) + (1-p)v(x_{2,l}^{L^v})\Big] - v(X_2) \\
&= q\Big[p\rho\Big(u(x_{2,h}^{H^v})\Big) + (1-p)\rho\Big(u(x_{2,l}^{H^v})\Big)\Big] + (1-q)\Big[p\rho\Big(u(x_{2,h}^{L^v})\Big) + (1-p)\rho\Big(u(x_{2,l}^{L^v})\Big)\Big] - \rho\Big(u(X_2)\Big) \\
&\leq \rho\Bigg(q\Big[pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})\Big] + (1-q)\Big[pu(x_{2,h}^{L^v}) + (1-p)u(x_{2,l}^{L^v})\Big]\Bigg) - \rho\Big(u(X_2)\Big) \\
&\leq q\Big[pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})\Big] + (1-q)\Big[pu(x_{2,h}^{L^v}) + (1-p)u(x_{2,l}^{L^v})\Big] - u(X_2) \\
&< q\Big[pu(x_{2,h}^{H^u}) + (1-p)u(x_{2,l}^{H^u})\Big] + (1-q)\Big[pu(x_{2,h}^{L^u}) + (1-p)u(x_{2,l}^{L^u})\Big] - u(X_2) = \overline{\gamma}_u
\end{aligned}
$$

where the first inequality follows from concavity, the second one from the contraction property of $\rho(\cdot)$, and the third one from the previous argument. Thus, when $\gamma < \overline{\gamma}_v$, then $\gamma < \overline{\gamma}_u$, and so agents in economy A as well prefer bilateral clearing over central clearing.

<u>Case 3</u>: $\frac{v'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{v'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)} > \frac{\alpha-p}{1-p}$.

In this case $x_{2,h}^{H^u} = \lambda_u^*\theta + \frac{(1-\lambda_u^*)p\theta}{\alpha}$ and $x_{2,l}^{H^u} = \frac{(1-\lambda_u^*)p\theta}{\alpha}$, $x_{2,h}^{H^v} = \lambda_v^*\theta + \frac{(1-\lambda_v^*)p\theta}{\alpha}$, and $x_{2,l}^{H^v} = \frac{(1-\lambda_v^*)p\theta}{\alpha}$. We proved earlier that $\lambda_v^* < \lambda_u^*$. Then, thus the contract $(x_{2,h}^{H^v}, x_{2,l}^{H^v})$ is feasible and incentive-compatible for the problem that lenders with utility $u(\cdot)$ face. But since such lenders prefer the contract $(x_{2,h}^{H^u}, x_{2,l}^{H^u})$, it must be that

$$
pu(x_{2,h}^{H^u}) + (1-p)u(x_{2,l}^{H^u}) > pu(x_{2,h}^{H^v}) + (1-p)u(x_{2,l}^{H^v})
$$

Following the same argument as in Case 2, we can prove that when $\gamma < \overline{\gamma}_v$, then $\gamma < \overline{\gamma}_u$. So, when agents in economy B prefer bilateral with information acquisition to central clearing without information acquisition, also agents in economy A prefer bilateral with information acquisition to central clearing without information acquisition. ■

Proof of Proposition 13

Proof. Let ($x^{i*}_{2,s}$, c^{i*}_1, $c^{i*}_{2,s}$) be the optimal contract with bilateral clearing and screening in Lemma 7, for $i \in \{L, H\}$ and $s \in \{l, h\}$. Also, let $\overline{\gamma}$ be defined in Proposition 9.

Rewrite problem (P_1) for $\hat{\mathcal{H}}_2 = \{(s, \Delta) : s \in \{l, h\}, \Delta \in \{0, 1\}\}$:

$$(P_1) \quad \max \quad q\Big[\sum_{s=l,h} p_s w_s^{H,0}\Big] + (1-q)\Big[\sum_{s=l,h} p_s w_s^{L,0}\Big] - \gamma \tag{25}$$

$$s.t. \quad \alpha C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i \geq \alpha\omega \tag{26}$$

$$C_{2,h}^i \geq (1-\lambda^i)\theta \tag{27}$$

$$\omega - \sum_{i\in\{L,H\}} q_i\, C_1^i \geq 0 \tag{28}$$

$$\sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s u^{-1}(w_s^{i,0}) + C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i \right\} \leq \omega + p\theta \tag{29}$$

$$\sum_{s\in\{l,h\}} p_s w_s^{i,0} \geq \sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{-i}) w_s^{-i,1} + [1-\sigma_s(\lambda^i, \lambda^{-i})] w_s^{-i,0}\Big] \tag{30}$$

$$\begin{aligned} &-\gamma + \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s w_s^{i,0} \right\} \\ &\quad \geq \max_{\hat{i}\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{\hat{i}}) w_s^{\hat{i},1} + [1-\sigma_s(\lambda^i, \lambda^{\hat{i}})] w_s^{\hat{i},0}\Big] \right] \right\} \end{aligned} \tag{31}$$

$$\sigma_s(\lambda^i, \lambda^j) = 0 \text{ iff } \quad C_{2,h}^j \geq (1-\lambda^i)\theta \tag{32}$$

where we use the notation $p_h = p$, $p_l = 1-p$, $q^H = q$ and $q^L = 1-q$. Consider the contracts ($\hat{w}_s^{i,\Delta}$, $\hat{C}_1^i$, $\hat{C}_{2,s}^i$) which, on the equilibrium replicate the optimal contracts in Lemma 7, and that assign maximum punishment for lenders in the off-equilibrium (when the original borrower counterparty strategically defaults, $\Delta = 1$):

$$\hat{w}_s^{i,0} = u^{-1}(x_{2,s}^{i*}), \qquad \hat{w}_s^{i,1} = 0,$$
$$\hat{C}_1^i = c_1^{i*}, \qquad \hat{C}_{2,s}^i = c_{2,s}^{i*}.$$

Step 1. The contract $(\hat{w}_s^{i,\Delta}, \hat{C}_1^i, \hat{C}_{2,s}^i)$ is feasible and satisfies borrowers' individually rationality: constraints (26)-(29) are satisfied.

Constraint (26) is directly satisfied by (17). Similarly (27) is directly satisfied by (21). Also, (28) is satisfied by (18)

$$\omega - \sum_{i\in\{L,H\}} q_i \hat{C}_1^i = \omega - \sum_{i\in\{L,H\}} q_i c_1^{i*} \geq 0$$

and (29) is satisfied by (19) and (20):

$$\begin{aligned}
&\sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s \Big[u^{-1}(\hat{w}_s^{H,0}) + \hat{C}_{2,s}^i \Big] \right\} \\
&\quad = \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s=l,h} p_s \left[x_{2,s}^{i*} + c_{2,s}^{i*} \right] \right\} \leq \sum_{i\in\{L,H\}} q_i \left\{ p\Big[\omega - c_1^{i*} + \theta\Big] + (1-p)\Big[\omega - c_1^{i*}\Big] \right\} \\
&\quad = p\theta + \omega - \sum_{i\in\{L,H\}} q_i c_1^{i*} = p\theta + \omega - \sum_{i\in\{L,H\}} q_i \hat{C}_1^i
\end{aligned}$$

Step 2. Only ex-ante incentive-compatibility matters: constraint (30) is satisfied by (31).[42]

We prove this by contraposition: we prove that when (30) is not satisfied, then (31)

[42] This is true in general and not only for the replication contract.

is violated as well. Assume that (30) is violated for $\tilde{i}, -\tilde{i} \in \{L, H\}$, where $-\tilde{i} \neq \tilde{i}$:

$$pw_h^{\tilde{i},0} + (1-p)w_l^{\tilde{i},0} < \sum_{s\in\{l,h\}} p_s \Big\{\sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})w_s^{-\tilde{i},1} + [1-\sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})]w_s^{-\tilde{i},0}\Big\}$$

From the definition of the max operator, the expression above, the fact that $\sigma_s(\lambda^i, \lambda^i) = 0$, and the fact that $\gamma > 0$, we obtain

$$\begin{aligned}
&\max_{\hat{i}\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{\hat{i}})w_s^{\hat{i},1} + [1-\sigma_s(\lambda^i, \lambda^{\hat{i}})]w_s^{\hat{i},0}\Big]\right]\right\} \\
&\geq \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{-\tilde{i}})w_s^{-\tilde{i},1} + [1-\sigma_s(\lambda^i, \lambda^{-\tilde{i}})]w_s^{-\tilde{i},0}\Big]\right\} \\
&= q_{-\tilde{i}}\Big[pw_h^{-\tilde{i},0} + (1-p)w_l^{-\tilde{i},0}\Big] + q_{\tilde{i}} \sum_{s\in\{l,h\}} p_s \Big\{\sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})w_s^{-\tilde{i},1} + [1-\sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})]w_s^{-\tilde{i},0}\Big\} \\
&> q_{-\tilde{i}}\Big[pw_h^{-\tilde{i},0} + (1-p)w_l^{-\tilde{i},0}\Big] + q_{\tilde{i}}\Big[pw_h^{\tilde{i},0} + (1-p)w_l^{\tilde{i},0}\Big] \\
&= \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s w_s^{i,0} \right\} > -\gamma + \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s w_s^{i,0} \right\}
\end{aligned}$$

which proves that equation (31) is also violated.

Step 3. If $\gamma < \overline{\gamma}$, for $\overline{\gamma}$ defined in Proposition 9, the contract $(\hat{w}_s^{i,\Delta}, \hat{C}_1^i, \hat{C}_{2,s}^i)$ is incentive compatible: constraint (31) is satisfied.

For $(x_{2,s}^{i*}, c_1^{i*}, c_{2,s}^{i*})$ the optimal contract with bilateral clearing and screening in Lemma 7, consider the following contract $(\tilde{X}_2^{i,\Delta}, \tilde{C}_1^i, \tilde{C}_{2s}^i, \tilde{\Sigma}_i)$ in (8)-(12):

$$\begin{aligned}
&\tilde{C}_{2s}^i = c_{2,h}^{H*}, \qquad \tilde{C}_1^i = c_1^{H*}, \qquad \tilde{\Sigma}^i = 1 \text{ iff } c_{2,h}^{H*} < (1-\lambda^i)\theta, \\
&\tilde{X}_2^{i,1} = \tilde{X}_2^{i,0} \equiv \tilde{X}_2 = \sum_{i\in\{L,H\}} q_i \Big\{p\Big[(1-\tilde{\Sigma}^i)x_{2,h}^{H*} + \tilde{\Sigma}^i\lambda^H\theta\Big] + (1-p)x_{2,l}^{H*}\Big\}
\end{aligned}$$

It is easy to show that such contract satisfies constraints (9)-(12). Thus, by optimality, it must be that

$$
\begin{aligned}
V^{CCP,e=0} \geq u(\tilde{X}_2) &= u\left(\sum_{i\in\{L,H\}} q_i\Big\{p\Big[(1-\tilde{\Sigma}^i)x_{2,h}^{H*}+\tilde{\Sigma}^i\lambda^H\theta\Big]+(1-p)x_{2,l}^{H*}\Big\}\right)\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{u\left(p\Big[(1-\tilde{\Sigma}^i)x_{2,h}^{H*}+\tilde{\Sigma}^i\lambda^H\theta\Big]+(1-p)x_{2,l}^{H*}\right)\Big\}\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{pu\Big((1-\tilde{\Sigma}^i)x_{2,h}^{H*}+\tilde{\Sigma}^i\lambda^H\theta\Big)+(1-p)u\left(x_{2,l}^{H*}\right)\Big\}\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{p\Big[\tilde{\Sigma}^i u(\lambda^H\theta)+(1-\tilde{\Sigma}^i)u\left(x_{2,h}^{H*}\right)\Big]+(1-p)u\left(x_{2,l}^{H*}\right)\Big\}
\end{aligned}
\tag{50}
$$

Similarly, consider the following contract ($\bar{X}_2^{i,\Delta}$, $\bar{C}_1^i$, $\bar{C}_{2s}^i$, $\bar{\Sigma}_i$) in (8)-(12):

$$
\begin{aligned}
&\bar{C}_{2s}^i = c_{2,h}^{L*}, \qquad \tilde{C}_1^i = c_1^{L*}, \qquad \bar{\Sigma}^i = 1 \text{ iff } c_{2,h}^{L*} < (1-\lambda^i)\theta,\\
&\bar{X}_2^{i,1} = \bar{X}_2^{i,0} \equiv \bar{X}_2 = \sum_{i\in\{L,H\}} q_i\Big\{p\Big[(1-\bar{\Sigma}^i)x_{2,h}^{H*}+\bar{\Sigma}^i\lambda^L\theta\Big]+(1-p)x_{2,l}^{L*}\Big\}
\end{aligned}
$$

It is also easy to show that such contract satisfies constraints (9)-(12). Thus, by optimality, it must be that

$$
\begin{aligned}
V^{CCP,e=0} \geq u(\bar{X}_2) &= u\left(\sum_{i\in\{L,H\}} q_i\Big\{p\Big[(1-\bar{\Sigma}^i)x_{2,h}^{L*}+\bar{\Sigma}^i\lambda^L\theta\Big]+(1-p)x_{2,l}^{L*}\Big\}\right)\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{u\left(p\Big[(1-\bar{\Sigma}^i)x_{2,h}^{L*}+\bar{\Sigma}^i\lambda^L\theta\Big]+(1-p)x_{2,l}^{L*}\right)\Big\}\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{pu\Big((1-\bar{\Sigma}^i)x_{2,h}^{L*}+\bar{\Sigma}^i\lambda^L\theta\Big)+(1-p)u\left(x_{2,l}^{L*}\right)\Big\}\\
&\geq \sum_{i\in\{L,H\}} q_i\Big\{p\Big[\bar{\Sigma}^i u(\lambda^L\theta)+(1-\bar{\Sigma}^i)u\left(x_{2,h}^{L*}\right)\Big]+(1-p)u\left(x_{2,l}^{L*}\right)\Big\}
\end{aligned}
\tag{51}
$$

Because $\gamma < \overline{\gamma}$, from Proposition 9 we know that

$$\sum_{i\in\{L,H\}} q_i \Big\{ \sum_{s\in\{l,h\}} \Big[p_s u(x_{2,s}^{i*}) \Big] \Big\} - \gamma \geq V^{CCP,e=0}$$

Combining these expressions together with the contract $(\hat{w}_s^{i,\Delta},\, \hat{C}_1^i,\, \hat{C}_{2,s}^i)$ we obtain

$$\begin{aligned}
&-\gamma + \sum_{i\in\{L,H\}} q_i \left\{ \sum_{s\in\{l,h\}} p_s \hat{w}_s^{i,0} \right\} = -\gamma + \sum_{i\in\{L,H\}} q_i \Big\{ \sum_{s\in\{l,h\}} \Big[p_s u(x_{2,s}^{i*}) \Big] \Big\} \\
&\geq V^{CCP,e=0} \\
&\geq \max \Bigg\{ \sum_{i\in\{L,H\}} q_i \Big\{ p \Big[\bar{\Sigma}^i u(\lambda^L \theta) + (1-\bar{\Sigma}^i) u\left(x_{2,h}^{L*}\right) \Big] + (1-p) u\left(x_{2,l}^{L*}\right) \Big\}, \\
&\qquad \sum_{i\in\{L,H\}} q_i \Big\{ p \Big[\tilde{\Sigma}^i u(\lambda^H \theta) + (1-\tilde{\Sigma}^i) u\left(x_{2,h}^{H*}\right) \Big] + (1-p) u\left(x_{2,l}^{H*}\right) \Big\} \\
&> \max_{\hat{i}\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{\hat{i}}) w_s^{\hat{i},1} + [1-\sigma_s(\lambda^i, \lambda^{\hat{i}})] w_s^{\hat{i},0} \Big] \right] \right\} \Bigg\}
\end{aligned}$$

where the last inequality follows from $w_s^{\hat{i},1} < u(\lambda^L \theta) < u(\lambda^H)\theta$, and the fact that the definition of $\bar{\Sigma}^i$ and $\tilde{\Sigma}^i$ is equivalent to the definition of $\sigma_s(\lambda^i, \lambda^j)$ in (32), and the fact that $\sigma_l(\lambda^i, \lambda^j) = 0$.

This proves that, if $\gamma < \overline{\gamma}$, the incentive-compatibility constraint (31) is satisfied.

Step 4. The contract $(\hat{w}_s^{i,\Delta},\, \hat{C}_1^i,\, \hat{C}_{2,s}^i)$ is feasible, incentive-compatible, and

$$\sum_{i\in\{L,H\}} \left\{ q_i \Big[\sum_{s=l,h} p_s \hat{w}_s^{i,0} \Big] \right\} = \sum_{i\in\{L,H\}} \left\{ q_i \Big[\sum_{s=l,h} p_s u(x_{2,s}^{i^*}) \Big] \right\}$$

Feasibility and incentive-compatibility follows from the previous steps. The last equation is true by construction, and concludes the proof of Proposition 13. ■

The optimal contract with pro-rata allocation of losses and due diligence

Rewrite the problem (P_1) for the case where $\hat{\mathcal{H}}_2 = \{\Delta \in \{0,1\}\}$:

$$\max \quad qw^{H,0} + (1-q)w^{L,0} - \gamma \tag{25}$$

$$s.t. \quad \alpha C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i \geq \alpha\omega \tag{26}$$

$$C_{2,h}^i \geq (1-\lambda^i)\theta \tag{27}$$

$$\omega - \sum_{i\in\{L,H\}} q_i\, C_1^i \geq 0 \tag{28}$$

$$\begin{aligned} &qu^{-1}(w^{H,0}) + (1-q)u^{-1}(w_s^{L,0}) \\ &\quad + \sum_{i\in\{L,H\}} \left\{ q_i[C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i] \right\} \leq \omega + p\theta \end{aligned} \tag{29}$$

$$w^{i,0} \geq \sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i, \lambda^{-i})w^{-i,1} + [1-\sigma_s(\lambda^i,\lambda^{-i})]w^{-i,0} \Big] \tag{30}$$

$$-\gamma + \sum_{i\in\{L,H\}} q_i w^{i,0} \geq \max_{\hat{i}\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{s\in\{l,h\}} p_s \Big[\sigma_s(\lambda^i,\lambda^{\hat{i}})w^{\hat{i},1} + [1-\sigma_s(\lambda^i,\lambda^{\hat{i}})]w^{\hat{i},0} \Big] \right] \right\} \tag{31}$$

$$\sigma_s(\lambda^i,\lambda^j) = 0 \;\text{ iff }\; C_{2,h}^j \geq (1-\lambda^i)\theta \tag{32}$$

where we use the notation $p_s = p$, $p_l = 1-p$, $q^H = q$, and $q^L = 1-q$. It is easy to see that maximum punishment for lack of due diligence is optimal: $w^{i,1} = 0$. Replace $w^{i,1} = 0$, and simplify notation rewriting $w^i = w^{i,0}$, that is lenders' promised utility when the first period message was $m_L = \lambda^i$ and the original borrower counterparty does not strategically default in equilibrium.

Claim 1 Constraint (30) can be ignored.

Proof. The proof is identical to Step 2 in the proof of Proposition 13. ■

Claim 2: With pro-rata allocation of losses and due diligence, optimality of central clearing with information acquisition requires that $C^H_{2,h} < (1-\lambda^L)\theta$.

Proof. Let (w^H_2, w^L_2), $(C_1,^i, C^i_{2,s})_{i=L,H,s=h,l}$ be the solution to problem (26)-(**??**), and suppose $C^H_{2h} \geq (1-\lambda^L)\theta$. Consider now the contract with central clearing, no monitoring in problem (8)-(12) defined as $\hat{X}_2 = qu^{-1}(X^H_2) + (1-q)u^{-1}(X^L_2)$, $\hat{C}_{2,s} = qC^H_{2,s} + (1-q)C^L_{2,s}$, and $\hat{C}_1 = qC^H_1 + (1-q)C^L_1$. Easily this contract induces strategies $\Sigma^L = \Sigma^H = 0$ in (12), and satisfies (9)-(11). Concavity of $u(\cdot)$ gives $u(\hat{X}^2) \geq qw^H + (1-q)w^L > qw^H + (1-q)w^L - \gamma$, so it is strictly better than the original contract with information acquisition.

■

Claim 3: The optimal contract satisfies $\sigma_h(\lambda^H, \lambda^L) = 1$ and $\sigma_h(\lambda^L, \lambda^H) = 0$.

Proof. The conclusion $\sigma_h(\lambda^H, \lambda^L) = 1$ follows from Claim 2 above. On the other hand, the conclusion $\sigma_h(\lambda^L, \lambda^H) = 0$ follows easily from (27), since $C^L_{2,h} \geq (1-\lambda^L)\theta > (1-\lambda^H)\theta$. ■

Ignoring constraint (30) and replacing $\sigma_h(\lambda^H, \lambda^L) = 1$ and $\sigma_h(\lambda^L, \lambda^H) = 0$ in (31) and (32), we can rewrite problem (25)-(32) as follows:

$$(\hat{P}^{FI}) \quad \max \quad qw^H + (1-q)w^L - \gamma \tag{25}$$

$$s.t. \quad \alpha C^i_1 + pC^i_{2,h} + (1-p)C^i_{2,l} \geq \alpha\omega \tag{26}$$

$$C^i_{2,h} \geq (1-\lambda^i)\theta \tag{27}$$

$$\omega - \sum_{i\in\{L,H\}} q_i\, C^i_1 \geq 0 \tag{28}$$

$$\begin{aligned} &qu^{-1}(w^H) + (1-q)u^{-1}(w^L_s) \\ &\qquad + \sum_{i\in\{L,H\}} \Big\{ q_i[C^i_1 + pC^i_{2,h} + (1-p)C^i_{2,l}] \Big\} \leq \omega + p\theta \end{aligned} \tag{29}$$

$$-\gamma + qw^H + (1-q)w^L \geq \max\Big\{ w^L, \Big[q + (1-q)(1-p)\Big] w^L \Big\} \tag{31}$$

We can solve problem $(\hat{P}^{FI})$ in two steps. In the first step, the CCP determines the contracts offered to borrowers, $\{C^i_1, C^i_{2s}\}_{i=H,L,s=h,l}$, to provide the maximal amount of resources in the second period. We denote such resources by Ω; they consist of the amount of consumption good stored by the CCP from $t = 1$ to $t = 2$ and of all $t = 2$ borrowers' net payments. The contracts $\{C^i_1, C^i_{2s}\}_{i=H,L,s=h,l}$ must be feasible: they should satisfy the participation and the limited commitment constraints of the borrowers. Thus, contracts $\{C^i_1, C^i_{2s}\}_{i=H,L,s=h,l}$ solve the following problem:

$$\begin{aligned} (\hat{P}b^{FI}) \qquad \Omega = \max_{\{C^i_1, C^i_{2h}, C^i_{2l}\}} \quad & \Big[\omega - qC^H_1 - (1-q)C^L_1\Big] + p\theta \\ & - q[pC^H_{2,h} + (1-p)C^H_{2,l}] - (1-q)[pC^L_{2h} + (1-p)C^L_{2l}] \\ s.t. \quad & \alpha C^i_1 + pC^i_{2h} + (1-p)C^i_{2l} \geq \alpha\omega \\ & \omega \geq C^i_1 \geq 0 \\ & C^i_{2,h} \geq (1-\lambda^i)\theta \end{aligned}$$

In the second step, the CCP determines the contracts it offers to lenders, for a given amount of resources Ω. Such contracts should persuade lenders to monitor their counterparty and report truthfully the information that they learn; thus they solve

$$
\begin{aligned}
(\hat{P}a_{\Omega}^{FI}) \qquad & \max_{\{w^H,w^L\}\in\Re^2_+} \quad qw^H + (1-q)\,w^L - \gamma \\
& s.t. \quad qu^{-1}\left(w^H\right) + (1-q)\,u^{-1}\left(w^L\right) \leq \Omega \\
& -\gamma + qw^H + (1-q)\,w^L \geq \\
& \max\left\{w^L, (q + (1-q)(1-p))\,w^H\right\} \qquad (52)
\end{aligned}
$$

Claim 4: $(C_1^i, C_{2h}^i, C_{2l}^i, w^i)_{i=L,H}$ solve the problem $(\hat{P}^{FI})$ if and only if $(C_1^i$, C_{2h}^i, $C_{2l}^i)_{i=L,H}$ solve $(\hat{P}b^{FI})$ and, letting Ω^* denote the value of the objective in $(\hat{P}b^{FI})$ at its solution, (w^H, w^L) solve $(\hat{P}a_{\Omega^*}^{FI})$.

Proof. First we show the only if direction. Suppose that $(C_1^i, C_{2h}^i, C_{2l}^i, w^i)_{i=L,H}$ is the solution to problem $(\hat{P}^{FI})$, but either $(C_1^i, C_{2h}^i, C_{2l}^i)_{i=L,H}$ does not solve $(\hat{P}b^{FI})$, or for Ω^* the solution to $(\hat{P}b^{FI})$, (w_H, w_L) solve $(\hat{P}a_{\Omega^*}^{FI})$.

If $(C_1^i, C_{2h}^i, C_{2l}^i)_{i=L,H}$ does not solve $(\hat{P}b^{FI})$, let $(C_1^{i'}, C_{2h}^{i'}, C_{2l}^{i'})_{i=L,H}$ be the solution to $(\hat{P}b^{FI})$. From problem $(\hat{P}b^{FI})$, it must be that for some i, either $C_1^{i'} < C_1^i$, or $C_{2h}^{i'} < C_{2h}$, or $C_{2l}^{i'} < C_{2l}^i$ Suppose w.l.o.g. that $C_1^{H'} < C_1^H$. Then, in problem $(\hat{P}^{FI})$ consider a new contract $(C_1^{i''}, C_{2h}^{i''}, C_{2l}^{i''}, w^{i''})_{i=L,H}$ where $C_{2h}^{i''} = C_{2h}^i$, $C_{2l}^{i''} = C_{2l}^i$, $C_1^{i''} = C_1^i - \epsilon$. If $w^L > [q+(1-q)(1-p)]w^H$, then choose $w^{H''}$ to solve $u^{-1}(w^{H''}) = u^{-1}(w^H)+\epsilon$; if instead $w^L < [q+(1-q)(1-p)]w^H$, choose $w^{L''}$ to solve $u^{-1}(w^{L''}) = u^{-1}(w^L) + \frac{q}{1-q}\epsilon$. In both cases, it is easy to show that $(C_1^{i''}, C_{2h}^{i''}, C_{2l}^{i''}, w^{i''})_{i=L,H}$ satisfies constraints (26)-(31) in problem $(\hat{P}^{FI})$, and $qw^{H''} + (1-q)w^{L''} > qw^H + (1-q)w^L$, that contradicts optimality of the original contract in problem $(\hat{P}^{FI})$. If instead $w^L = [q+(1-q)(1-p)]w^L$, then choose $w^{H''}$ and $w^{L''}$ to solve $u^{-1}(w^{H''}) = u^{-1}(w^H)+q\epsilon$, and $u^{-1}(w^{L''}) = u^{-1}(w^L)+q\epsilon$. It is easy to show that $w^{L''} > [q+(1-q)(1-p)]w^{H''}$, that $(C_1^{i''}, C_{2h}^{i''}, C_{2l}^{i''}, w^{i''})_{i=L,H}$ satisfies constraints (26)-(31) in problem $(\hat{P}^{FI})$, and $qw^{H''} + (1-q)w^{L''} > qw^H + (1-$

$q)w^L$, which contradicts again optimality of the original contract in problem $(\hat{P}^{FI})$.

If instead $(C_1^i, C_{2h}^i, C_{2l}^i, w^i)_{i=L,H}$ solve problem $(\hat{P}^{FI})$, but for Ω^* the solution to $(\hat{P}b^{FI})$, (w_H, w_L) does not solve $(\hat{P}a_{\Omega^*}^{FI})$, let $(w^{H'}, w^{L'})$ solve $(\hat{P}a_{\Omega^*}^{FI})$. It is straightforward to show that $(C_1^i, C_{2h}^i, C_{2l}^i, w^{i'})_{i=L,H}$ satisfies constraints (26)-(31) in problem $(\hat{P}^{FI})$, and $qw^{H'} + (1-q)w^{L'} > qw^H + (1-q)w^L$, which contradicts optimality of the original contract in problem $(\hat{P}^{FI})$.

Next, we show the if direction. Let $(C_1^i, C_{2h}^i, C_{2l}^i)_{i=L,H}$ solve $(\hat{P}b^{FI})$, and for Ω^* the solution to $(\hat{P}b^{FI})$, (w^H, w^L) solve $(\hat{P}a_{\Omega^*}^{FI})$. Suppose by contradiction that $(C_1^i, C_{2h}^i, C_{2l}^i, w^i)_{i=L,H}$ does not solve problem $(\hat{P}^{FI})$. Let $(C_1^{i'}, C_{2h}^{i'}, C_{2l}^{i'}, w^{i'})$ be the solution to $(\hat{P}^{FI})$. Then easily it must be that either $C_1^{i'} \neq C_1^i$, or $C_{2h}^{i'} \neq C_{2h}^i$, or $C_{2l}^{i'} \neq C_{2l}^i$: if not it must be $w^H = w^{H'}$ and $w^{L'} = w^L$ by comparing $(\hat{P}a_{\Omega^*}^{FI})$ with $(\hat{P}^{FI})$. By definition of problem $(\hat{P}b^{FI})$, then it should be that either $C_1^{i'} > C_1^i$, or $C_{2h}^{i'} > C_{2h}^i$, or $C_{2l}^{i'} > C_{2l}^i$. Suppose ,w.l.o.g. that $C_1^{H'} > C_1^H$. Then, following the same argument as in the only if part, we can prove that $(C_1^{i'}, C_{2h}^{i'}, C_{2l}^{i'}, w^{i'})$ can not be the solution to $(\hat{P}^{FI})$, which is a contradiction. ■

Claim 5: Let $\Omega \in \Re_+$. For any $(w_H, w_L) \in \Re_+^2$ such that

$$qu^{-1}\left(w^H\right) + (1-q)\,u^{-1}\left(w^L\right) \leq \Omega \tag{53}$$

$$[q + (1-q)(1-p)]\,w^H = \max\left(w^L, (q + (1-q)(1-p))\,w^H\right) \tag{54}$$

$$-\gamma + qw^H + (1-q)\,w^L \geq (q + (1-q)(1-p))\,w^H \tag{55}$$

there exist $(w^{H'}, w^{L'}) \in \Re_+^2$ such that

$$qu^{-1}\left(w^{H'}\right) + (1-q)\,u^{-1}\left(w^{L'}\right) \leq \Omega \tag{56}$$

$$w^{L'} = \max\left(w^{L'}, \left(q + \left(1-q\right)\left(1-p\right)\right) w^{H'}\right) \tag{57}$$

$$-\gamma + qw^{H'} + \left(1-q\right) w^{L'} \geq w^{L'} \tag{58}$$

and

$$qw^{H'} + (1-q)w^{L'} > qw^{H} + (1-q)w^{L} \tag{59}$$

Proof. Let $(w^H, w^L) \in \Re_+^2$ satisfy equations (53), (54), and (55). Define X as

$$qu^{-1}\left(w^H\right) + (1-q)u^{-1}\left(w^L\right) = X$$

and $(w^{H'}, w^{L'})$ as the unique solution to

$$[q + (1-q)(1-p)]w^{H'} = w^{L'}$$
$$qu^{-1}\left(w^{H'}\right) + (1-q)u^{-1}\left(w^{L'}\right) = X$$

We want to show that $(w^{H'}, w^{L'})$ satisfy equations (56), (57), (58), and (59). Notice that equation (56) and equation (57) are satisfied by construction.

Now, suppose by contradiction that equation (58) is violated. Therefore

$$w^{H'} < w^{L'} + \frac{\gamma}{q}$$
$$w^{L'} = [q + (1-q)(1-p)]w^{H'}$$

It is easy to show that the two conditions can hold only if $w^{L'} < \frac{q+(1-q)(1-p)}{pq(1-q)}$, therefore $w^{H'} = \frac{w^{L'}}{q+(1-q)(1-p)} < \frac{\gamma}{pq(1-q)}$. Since u^{-1} is increasing, by the definition of $w^{H'}$ and $w^{L'}$

we have

$$X = qu^{-1}\left(w^{H'}\right)+(1-q)u^{-1}\left(w^{L'}\right) < qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right)+(1-q)u^{-1}\left(\frac{q+(1-q)(1-p)}{pq(1-q)}\gamma\right) \tag{60}$$

It is easy to show that equations (54) and (55) can hold only if $w^H \geq \frac{\gamma}{pq(1-q)}$ and $w^L \geq \frac{q+(1-q)(1-p)}{pq(1-q)}\gamma$. Then, since u^{-1} is increasing, from the definition of X we have

$$X \geq qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right) + (1-q)u^{-1}\left(\frac{q+(1-q)(1-p)}{pq(1-q)}\gamma\right)$$

that contradicts equation (60). Therefore equation (58) can not be violated.

Finally notice that we can rewrite

$$\begin{aligned} qw^{H'} + (1-q)w^{L'} &= q\left(w^H + \int_{w_L}^{w^{L'}} \left[-u'\left(\frac{X-(1-q)u^{-1}(s)}{q}\right)\frac{1-q}{q}\frac{1}{u'(s)}\right]\mathrm{d}s\right) \\ &+ (1-q)\left(w^L + \int_{w^L}^{w^{L'}} 1\mathrm{d}s\right) \\ &= qw^H + (1-q)w^L + (1-q)\int_{w^L}^{w^{L'}}\left[1 - \frac{u'\left(\frac{X-(1-q)u^{-1}(s)}{q}\right)}{u'(s)}\right]\mathrm{d}s \\ &> qw^H + (1-q)w^L \end{aligned}$$

where the last inequality follows from concavity of u together with the fact that $\frac{X-(1-q)u^{-1}(s)}{q} > s$ for all $s \in [w^L, w^{L'}]$. Therefore equation (59) is as well satisfied. ■

Proof of Lemma 15

Proof. Since from Claim 4 $(C_1^i,\, C_{2h}^i,\, C_{2l}^i)_{i=L,H}$ need to solve $(\hat{P}b^{FI})$, the conclusion follows easily from linearity of the objective function in $(\hat{P}b^{FI})$ and the fact taht $\alpha > 1$.

Sepcifically, from Assumption 4 the participation constraint should bind: $\alpha C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i = \alpha\omega$. From linearity of the objective function and $\alpha > 1$ we obtain $C_{2,l}^i = 0$ and the fact that the limited commitment constraint binds, $C_{2,h}^i = (1-\lambda^i)\theta$. ■

Proof of Proposition 16

Proof. From Claim 5, we can simplify and rewrite problem $(\hat{P}a_\Omega^{FI})$ as follows:

$$(\hat{P}a_\Omega^{FI})' \qquad \max_{\{w^H, w^L\}\in\Re_+^2} \quad qw^H + (1-q)\,w^L - \gamma$$

$$s.t. \qquad qu^{-1}\left(w^H\right) + (1-q)\,u^{-1}\left(w^L\right) \leq \Omega \tag{61}$$

$$-\gamma + qw^H + (1-q)\,w^L \geq w^L \tag{62}$$

$$w^L - [q + (1-q)(1-p)]w^H \geq 0 \tag{63}$$

Step 1: A solution to problem $(\hat{P}a_\Omega^{FI})'$ exists (and is unique) if and only if $\Omega \geq \hat{\Omega}$, for $\hat{\Omega}$ which solves

$$\hat{\Omega} = qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right) + (1-q)u^{-1}\left(\frac{\gamma[q+(1-q)(1-p)]}{pq(1-q)}\right) \tag{64}$$

Moreover, at the solution, equations (61) and (62) hold with equality.

Proof. The smallest values of w^H and w^L that jointly satisfy (62) and (63) are $w^H = \frac{\gamma}{pq(1-q)}$ and $w^L = \frac{\gamma[q+(1-q)(1-p)]}{pq(1-q)}$. Then constraint (61) can be satisfied jointly with (62) and (63) only if $\Omega \geq \hat{\Omega}$ as defined above.

Easily, when $\Omega \geq \hat{\Omega}$ both (62) and (63) have to bind. If (62) does not bind, we can increase w^H and w^L by ϵ and all constraints are still satisfied. If (63) is not binding, we can construct a mean-preserving contraction on $u^{-1}(w^H)$ and $u^{-1}(w^L)$ so that (62) is unaffected, but by convexity of $u^{-1}(\cdot)$ the objective function strictly increases. □

Step 2: The solution to $(\hat{Pb}^{FI})$ gives

$$\Omega = \sum_{i \in \{L,H\}} q_i \Big[\lambda^i p\theta + \omega(\lambda^i) \Big] \tag{65}$$

Proof. It follows easily from Lemma 15. $\square$

Step 3: A solution to problem $(\hat{P}^{FI})$ exists and is unique if and only if $\gamma \leq \hat{\gamma}$, for $\hat{\gamma}$ defined in (34). Then, for Ω defined in (65),

$$qw^H + (1-q)w^L - \gamma = w^L,$$

for w^L solving

$$qu^{-1}\left(w^L + \frac{\gamma}{q}\right) + (1-q)u^{-1}(w^L) = \Omega.$$

Proof. The conclusion follows from Claim 4 and Claim 5 above, Lemma 15 and Step 1 and Step 2 in the proof of the current proposition. $\square$

Step 4: If $\gamma \leq \hat{\gamma}$ defined in (34), then

$$\max\left\{ u(X_2), \sum_{i \in \{L,H\}} \left\{ q_i w^{i,0^*} \right\} \right\} \geq \sum_{i \in \{L,H\}} \left\{ q_i \Big[\sum_{s=l,h} p_s u(x_{2,s}^{i^*}) \Big] \right\}$$

where $x_{2,s}^{i^*}$ is lenders' consumption in the optimal contract with bilateral clearing and monitoring of Lemma 7, and X_2 is lenders consumption in (13) for the optimal contract with CCP clearing and no monitoring in Proposition 5.

Proof. Suppose not: suppose that the optimal contract with bilateral clearing and screening dominates both the optimal contract with central clearing and screening and the optimal contract with CCP clearing and no information acquisition:

$$\max\left\{u(X_2^*), \sum_{i\in\{L,H\}}\left\{q_i w^{i,0^*}\right\}-\gamma\right\} < \sum_{i\in\{L,H\}}\left\{q_i\Big[\sum_{s=l,h} p_s u(x_{2,s}^{i^*})\Big]\right\}-\gamma \qquad (66)$$

Let (x_{2h}^{i*}, x_{2l}^{i*}, c_1^{i*}, c_{2h}^{i*}, c_{2l}^{i*}) be the optimal contracts with bilateral clearing, when the lender upon screening learns that her counterparty is of type i. Similarly, let ($w^{i,0^*}$, $C_{2h}^{i^*}$, $C_{2l}^{i^*}$, $C_1^{i^*}$) be the optimal contract with CCP clearing and screening. Define w^H and w^L as follows:

$$w^H = pu(x_{2h}^{H*}) + (1-p)u(x_{2l}^{H*})$$

$$w^L = pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*})$$

and consider, in problem ($\hat{P}^{FI}$), the contract with CCP clearing and screening (w^i, C_{2h}^i, C_{2l}^i, C_1^i), where w^H and w^L are defined above, $C_1^i = c_1^{i*}$, $C_{2h}^{i*} = c_{2h}^{i*}$, $C_{2l}^{i*} = c_{2l}^{i*}$.

Step a: The contract (w^i, C_{2h}^i, C_{2l}^i, C_1^i) satisfies $qw^H + (1-q)w^L - \gamma \geq w^L$.

Consider, in problem (8)-(12), the contract (X_2, C_1^i, C_{2h}^i, C_{2l}^i) with $C_1^i = c_1^{L*}$, $C_{2,s}^i = c_{2,s}^{L*}$, and $X_2 = u^{-1}(pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*}))$. Such contract is feasible in (8)-(12), therefore it must be

$$u(X_2^*) \geq u(X_2) = pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*}) \qquad (67)$$

Moreover, since the contract with bilateral clearing and screening dominates the contract with CCP clearing and no screening,

$$q\Big[\underbrace{pu(x_{2h}^{H*})+(1-p)u(x_{2l}^{H*})}_{w^H}\Big]+(1-q)\Big[\underbrace{pu(x_{2h}^{L*})+(1-p)u(x_{2l}^{L*})}_{w^L}\Big]-\gamma\geq u(X_2^*)$$

Combining the last expression with (66) we obtain

$$\begin{aligned}&qw^H+(1-q)w^L-\gamma\geq u(X_2^*)\geq pu(x_{2h}^{L*})+(1-p)u(x_{2l}^{L*})=w^L\\&\Rightarrow w^H\geq w^L+\frac{\gamma}{q}\end{aligned}$$

Step b: The contract $(w^i,\, C_{2h}^i,\, C_{2l}^i,\, C_1^i)$ must be that $[q+(1-q)(1-p)]w^H>w^L$.

From concavity of $u(\cdot)$ we have that

$$u^{-1}(w^H)<px_{2h}^{H*}+(1-p)x_{2l}^{H*}=\omega-c_1^{H*}+p\theta-pc_{2h}^{H*}-(1-p)c_{2l}^{H*}$$

Similarly,

$$u^{-1}(w^L)<px_{2h}^{L*}+(1-p)x_{2l}^{L*}=\omega-c_1^{L*}+p\theta-pc_{2h}^{L*}-(1-p)c_{2l}^{L*}$$

Then the contract $(w^i,\, C_{2h}^i,\, C_{2l}^i,\, C_1^i)$ easily satisfies (26)-(29). Suppose that we also had $[q+(1-q)(1-p)]w^H\leq w^L$. Then from step a, constraint (31) would also be satisfied in $(\hat{P}^{FI})$. By definition of optimality, it must be that

$$\begin{aligned}\sum_{i\in\{L,H\}}\left\{q_iw^{i,0^*}\right\}&=qw^{H,0^*}+(1-q)w^{L,0^*}\geq qw^H+(1-q)w^L\\&=q\Big[pu(x_{2h}^{H*})+(1-p)u(x_{2l}^{H*})\Big]+(1-q)\Big[pu(x_{2h}^{L*})+(1-p)u(x_{2l}^{L*})\Big]\end{aligned}$$

which contradicts equation (66).

Step c: The contract (w^i, C^i_{2h}, C^i_{2l}, C^i_1) must be such that $-\gamma + qw^H + (1-q)w^L < [q + (1-q)(1-p)]w^H$.

If not, constraint (31) would be satisfied in ($\hat{P}^{FI}$) and the CCP can always find a pair (w^H, w^L) that violates $[q + (1-q)(1-p)]w^H > w^L$, satisfies the incentive constraint $-\gamma + qw^H + (1-q)w^L \geq w^L$, and yields strictly higher utility to lenders than the optimal bilateral contract.

Step d: Condition (66) must be violated.

Consider then the solution to problem ($\hat{P}b^{FI}$): we know from Claim 4 that ($C^{i^*}_1$, $C^{i^*}_{2h}$, $C^{i^*}_{2l}$) solve problem ($\hat{P}b^{FI}$). Moreover, by definition of the maximization problem, it has to be that

$$
\begin{aligned}
\Omega^* &= p\theta + \omega - q[C^{H^*}_1 + pC^{H^*}_{2h} + (1-p)C^{H*}_{2l}] - (1-q)[C^{L^*}_1 + pC^{L^*}_{2h} + (1-p)C^{L*}_{2l}] \\
&\geq p\theta + \omega - q[c^{H*}_1 + pc^{H*}_{2h} + (1-p)c^{H*}_{2l}] - (1-q)[c^{L*}_1 + pc^{L*}_{2h} + (1-p)c^{L*}_{2l}] \\
&= q[p(\theta - c^{H*}_{2h} + \omega - c^{H*}_1) + (1-p)(\omega - c^{H*}_{2l})] + (1-q)[p(\theta - c^{L*}_{2h} + \omega - c^{L*}_1) + (1-p)(\omega - c^{L*}_{2l})] \\
&= q[px^{H*}_{2h} + (1-p)x^{H*}_{2l}] + (1-q)[px^{L*}_{2h} + (1-p)x^{L*}_{2l}] \\
&> qu^{-1}(w^H) + (1-q)u^{-1}(w^L)
\end{aligned}
$$

Define then

$$\delta = \Omega - qu^{-1}(w^H) + (1-q)u^{-1}(w^L)$$

and define $w^{H'}$ such that

$$u^{-1}(w^{H'}) = u^{-1}(w^H) + \frac{\delta}{q} \tag{68}$$

Since $u^{-1}(\cdot)$ is increasing, $w^{H'} \geq w^H$. Define now the operator

$$T(y) = u\left(\frac{qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^L) - qu^{-1}\left(\frac{y}{q+(1-p)(1-q)}\right)}{1-q}\right) - y$$

Notice that $T(y)$ is monotone decreasing in y, that for $y = \overline{y} \equiv [q + (1-p)(1-q)]u\left(\frac{qu^{-1}(w^{H'})+(1-q)u^{-1}(w^L)}{q}\right) > 0$, it is

$$T(\overline{y}) = u(0) - \overline{y} < 0$$

Furthermore, the two conditions $w^{H'} \geq w^L + \frac{\gamma}{q}$ and $w^{H'} \geq \frac{w^L}{1-p} - \frac{\gamma}{(1-q)(1-p)}$, imply that $w^{H'} \geq \frac{w^L}{q+(1-p)(1-q)}$. where the second inequality follows from $w^{H'} \geq w^H > \frac{w^L}{1-p} - \frac{\gamma}{(1-q)(1-p)}$, which results from the assumption that the incentive constraint is violated, $-\gamma + qw^H + (1-q)w^L < [q + (1-q)(1-p)]w^H$, and from the definition of $w^{H'}$ that implies $w^{H'} \geq w^H$. Then for $y = w^L$ it is true that

$$\begin{aligned} T(w_L) &= u\left(\frac{qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^L) - qu^{-1}\left(\frac{w^L}{q+(1-p)(1-q)}\right)}{1-q}\right) - w^L \\ &\geq u(u^{-1}(w^L)) - w^L = 0 \end{aligned}$$

By the intermediate value theorem, there must be a $w^{L''} \geq w^L$ such that $T(w^{L''}) = 0$. Define then $w^{L''} \in [w^L, \overline{y})$ to be the value that satisfies $T(w^{L''}) = 0$, and then define $w^{H''}$ as the solution to

$$w^{H''} = \frac{w^{L''}}{q + (1-p)(1-q)}$$

Notice that $w^{H''} \leq w^{H'}$, since $w^{L''} \geq w^L$.

Consider then the contract ($w^{H''}$, $w^{L''}$, $C_1^{i^*}$, C_{2h}^{i*}, C_{2l}^{i*}), where $w^{H''}$ and $w^{L''}$ are defined above, and $C_1^{i^*}$, $C_{2h}^{i^*}$, $C_{2l}^{i^*}$ solve problem ($\hat{P}b^{FI}$). Notice that this contract is feasible and satisfy the limited commitment constraint in problem ($\hat{P}^{FI}$): participation, limited commitment and feasibility constraints are easily satisfied by the definition of $C_1^{i^*}$, C_{2h}^{i*}, C_{2l}^{i*}. Moreover, by construction $[1+(1-q)(1-p)]w^{H''} = w^{L''}$. All is left to show is that this contract is incentive compatible. By construction, via the operator T

$$qu^{-1}(w^{H''}) + (1-q)u^{-1}(w^{L''}) = qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^{L}) = \Omega^* \geq \hat{\Omega}$$

Replacing $w^{H''}$, $w^{L''}$ and $\hat{\Omega}$ with their definitions we can rewrite

$$qu^{-1}\left(\frac{w^{L''}}{q+(1-p)(1-q)}\right)+(1-q)u^{-1}(w^{L''}) \geq qu^{-1}\left(\frac{\hat{w}^{L}}{q+(1-p)(1-q)}\right)+(1-q)u^{-1}(\hat{w}^{L})$$

Notice that this can hold if and only if $w^{L''} \geq \hat{w}^L$ and therefore $w^{H''} \geq \hat{w}^H$. Moreover, recall that $\hat{w}^H = \hat{w}^L + \frac{\gamma}{q}$. Therefore, for $w^{L''} \geq \hat{w}^L$ and $w^{H''} \geq \hat{w}^H$, the following hold:

$$\begin{aligned}
w^{H''} &= \hat{w}^H + \frac{1}{q+(1-q)(1-p)}(w^{L''} - \hat{w}^L) \\
&= \hat{w}^L + \frac{\gamma}{q} + \frac{1}{q+(1-q)(1-p)}(w^{L''} - \hat{w}^L) \\
&= \hat{w}^L + \frac{\gamma}{q} + \frac{1}{q+(1-q)(1-p)}(w^{L''} - \hat{w}^L) + w^{L''} - w^{L''} \\
&= w^{L''} + \frac{\gamma}{q} + (w^{L''} - \hat{w}^L)\left[\frac{1}{q+(1-q)(1-p)} - 1\right] \geq w^{L''} + \frac{\gamma}{q}
\end{aligned}$$

that proves that the contract $(w^{H''}, w^{L''}, C_1^{i^*}, C_{2h}^{i*}, C_{2l}^{i*})$ satisfies as well the incentive compatibility constraint. Then, by the definition of optimality, it must be

$$\begin{aligned}
&\sum_{i\in\{L,H\}}\left\{q_i w^{i,0^*}\right\} \geq qw^{H''} + (1-q)w^{L''} \\
&= qw^{H''} + (1-q)u\left(\frac{qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^{L}) - qu^{-1}(w^{H''})}{1-q}\right) \\
&= qw^{H''} + (1-q)u\left(\frac{qu^{-1}(w^{H}) + \delta + (1-q)u^{-1}(w^{L}) - qu^{-1}(w^{H''})}{1-q}\right)
\end{aligned}$$

$$
\begin{aligned}
&= qw^{H''} + (1-q)u\left(\frac{\Omega - qu^{-1}(w^{H''})}{1-q}\right) \\
&= q\left(w^{H'} - \int_{w^{H''}}^{w^{H'}} 1\mathrm{d}s\right) + (1-q)\left(w^{L} + \int_{w^{H''}}^{w^{H'}} \left[u'\left(\frac{\Omega - qu^{-1}(s)}{1-q}\right)\frac{q}{1-q}\frac{1}{u'(s)}\right]\mathrm{d}s\right) \\
&= qw^{H'} + (1-q)w^{L} + q\int_{w^{H''}}^{w^{H'}} \left[\frac{u'\left(\frac{\Omega - qu^{-1}(s)}{1-q}\right)}{u'(s)} - 1\right]\mathrm{d}s \\
&\geq qw^{H'} + (1-q)w^{L} \geq qw^{H} + (1-q)w^{L} = \sum_{i\in\{L,H\}}\left\{q_i\Big[\sum_{s=l,h} p_s u(x_{2,s}^{i*})\Big]\right\}
\end{aligned}
$$

where the first inequality in the last line follows from the fact that $\frac{\Omega - qu^{-1}(s)}{1-q} < s$ for all $s \in (w^{H''}, w^{H'}]$, and the inequality in the last line follows from the fact that $w^{H'} \geq w^{H}$, given the definition in (68). But this contradicts (66). $\square$ $\blacksquare$

Proof of Proposition 17

Proof. The conclusion follows from Proposition 9 and Proposition 16. Specifically, when $\gamma > \hat{\gamma}$, from Proposition 16, central clearing with information acquisition is not feasible. When $\gamma < \underline{\gamma}$, then from Proposition 9, bilateral clearing with information acquisition is preferred to central clearing with no information acquisition. $\blacksquare$

Proof of Lemma 19

Proof. To show that bilateral clearing is more desirable, we show that $\frac{d\hat{\gamma}}{d\alpha} < 0$ and $\frac{d\overline{\gamma}}{d\alpha} > 0$. Thus, if $\gamma \in (\hat{\gamma}, \overline{\gamma})$, then $\gamma \in (\hat{\gamma}', \overline{\gamma}')$, where $\hat{\gamma}'$ and $\overline{\gamma}'$ are the thresholds computed for $\alpha' = \alpha + \epsilon$, for ϵ small.

The conclusion $\frac{d\hat{\gamma}}{d\alpha} < 0$ comes directly from the defintion of $\hat{\gamma}$ in (34).On the other hand, since $\frac{\alpha - p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta - \frac{\alpha - p}{\alpha}(1-\lambda^H)\theta\right)}$, from Lemma 7 we have that $x_{2,h}^{H} = \frac{(1-\lambda^H)p\theta}{\alpha} + \lambda^H\theta$

and $x_{2,l}^H = \frac{(1-\lambda^H)p\theta}{\alpha}$. Thus,

$$\begin{aligned}
\frac{d\overline{\gamma}}{d\alpha} &= -q\frac{(1-\lambda^H)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^H)p\theta}{\alpha}+\lambda^H\theta\right)+(1-p)u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)\right] \\
&\quad -(1-q)\frac{(1-\lambda^L)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L\theta\right)+(1-p)u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)\right] \\
&\qquad +u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L p\theta\right)\frac{(1-\lambda^L)p\theta}{\alpha^2} \\
&\geq -q\frac{(1-\lambda^H)p\theta}{\alpha^2}u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}+\lambda^H p\theta\right)-(1-q)\frac{(1-\lambda^L)p\theta}{\alpha^2}u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L p\theta\right) \\
&\qquad +u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L p\theta\right)\frac{(1-\lambda^L)p\theta}{\alpha^2} \\
&> \frac{u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L p\theta\right)p\theta}{\alpha^2}[-q(1-\lambda^H)-(1-q)(1-\lambda^L)+(1-\lambda^L)] > 0
\end{aligned}$$

where the first inequality follows from $u'''(\cdot) \leq 0$, the second one from $u''(\cdot) < 0$, and the last one from $\lambda^H > \lambda^L$. ■

Proof of Lemma 20

Since $\frac{\alpha-p}{1-p} > \frac{u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)}{u'\left(\theta-\frac{\alpha-p}{\alpha}(1-\lambda^H)\theta\right)}$, from Lemma 7 we have that $x_{2,h}^H = \frac{(1-\lambda^H)p\theta}{\alpha} + \lambda^H\theta$ and $x_{2,l}^H = \frac{(1-\lambda^H)p\theta}{\alpha}$. Also, since $q > \frac{1}{\alpha}$, from equation (15) we have that $X_2 = \frac{(1-\lambda^H)p\theta}{\alpha} + p\theta[q\lambda^H + (1-q)\lambda^L]$. Then,

$$\begin{aligned}
\frac{d\overline{\gamma}}{d\alpha} &= -q\frac{(1-\lambda^H)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^H)p\theta}{\alpha}+\lambda^H\theta\right)+(1-p)u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)\right] \\
&\quad -(1-q)\frac{(1-\lambda^L)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L\theta\right)+(1-p)u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)\right] \\
&\qquad +u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}+p\theta[q\lambda^H+(1-q)\lambda^L]\right)\frac{(1-\lambda^H)p\theta}{\alpha^2} \\
&< -q\frac{(1-\lambda^H)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^H)p\theta}{\alpha}+\lambda^H\theta\right)+(1-p)u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right)\right] \\
&\quad -(1-q)\frac{(1-\lambda^L)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^L)p\theta}{\alpha}+\lambda^L\theta\right)+(1-p)u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&+ u'\left(\frac{(1-\lambda^L)p\theta}{\alpha} + p\theta\lambda^L\right)\frac{(1-\lambda^H)p\theta}{\alpha^2} \\
&< (1-q)\frac{(1-\lambda^H)p\theta}{\alpha^2}\left[u'\left(\frac{(1-\lambda^L)p\theta}{\alpha} + p\theta\lambda^L\right)\right] \\
&- (1-q)\frac{(1-\lambda^L)p\theta}{\alpha^2}\left[pu'\left(\frac{(1-\lambda^L)p\theta}{\alpha} + \lambda^L\theta\right) + (1-p)u'\left(\frac{(1-\lambda^L)p\theta}{\alpha}\right)\right] \\
&\leq 0
\end{aligned}
$$

where the first inequality comes from $u''(x) < 0$ and $q > \frac{1}{\alpha}$, thus $u'\left(\frac{(1-\lambda^H)p\theta}{\alpha} + p\theta[q\lambda^H + (1-q)\lambda^L]\right)$ is maximized when $q = \frac{1}{\alpha}$, the second inequality comes from the assumption

$$
pu'\left(\frac{(1-\lambda^H)p\theta}{\alpha} + \lambda^H\theta\right) + (1-p)u'\left(\frac{(1-\lambda^H)p\theta}{\alpha}\right) > u'\left(\frac{(1-\lambda^L)p\theta}{\alpha} + \lambda^L p\theta\right),
$$

and the third inequality from prudence, i.e. $u''' \geq 0$, and from $\lambda^H > \lambda^L$.

6.1 Bilateral clearing with information acquisition

Consider the problem (8)-(12) when we do not impose the Assumption 4 and Assumption 6. Assumption 4 guaranteed that $\omega > \frac{(1-\lambda^L)p\theta}{\alpha} > \frac{(1-\lambda^H)p\theta}{\alpha}$. Consider then first a generic solution for the case when $\omega < \frac{(1-\lambda^i)p\theta}{\alpha}$. The next lemma characterizes the solution to this problem.

Lemma 21 *If $\omega < \frac{(1-\lambda^i)p\theta}{\alpha}$, the participation constraint (17) is slack. In addition, the limited commitment constraint (21) is binding and $c_1^i = 0$. This is the area shaded in yellow in Figure 1.*

Proof. It is easy to see that both the participation constraint (17) and the limited commitment constraint (21) can not be slack: if this was the case, the lender could increase her revenues just by decreasing $c_{2,h}^i$.

Suppose then that $\omega < \frac{(1-\lambda^i)p\theta}{\alpha}$. Because $c^i_{2,h} \geq (1-\lambda^i)\theta$ and $c^i_1 \geq 0$, the participation constraint (17) is slack. Since both (17) and (21) can not be slack, it must be that (21) binds: $c^i_{2,h} = (1-\lambda^i)\theta$. Easily, $c^i_1 = 0$: if not, the lender could decrease c^i_1, satisfy all constraints, and increase her expected utility. ■

Next, consider the case when $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$. Suppose we do not impose Assumption 6 and let μ and η be the multipliers associated with (17) and (21) respectively. The first order conditions for optimality are

$$-pu'(\omega - c^i_1 + \theta - c^i_{2,h}) + p\mu + \eta = 0 \tag{69}$$

$$-pu'(\omega - c^i_1 + \theta - c^i_{2,h}) - (1-p)u'(\omega - c^i_1) + \alpha\mu \leq 0 \tag{70}$$

with equality if $c^i_1 > 0$. Together with the complementary slackness conditions

$$\mu\{\alpha c^i_1 + pc^i_{2,h} - \alpha\omega\} = 0 \tag{71}$$

and

$$\eta\{c^i_{2,h} - (1-\lambda^i)\theta\} = 0 \tag{72}$$

they fully characterize the solution to the problem.

Lemma 22 *If $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$, then the participation constraint (17) binds. Moreover, if we do not impose Assumption 6*

a) If $\lambda^i < \lambda^*$, then $c^i_{2,h} = (1-\lambda^i)\theta$ and $c^i_1 = \omega - \frac{(1-\lambda^i)p\theta}{\alpha}$. This is area a) in Figure 1.

b) If $\lambda^i > \lambda^*$, and $\omega < \frac{(1-\lambda^*)p\theta}{\alpha}$, then $c^i_{2,h} = \frac{\alpha\omega}{p} > (1-\lambda^i)\theta$ and $c^i_1 = 0$. This is area b) in Figure 1.

c) If $\lambda^i > \lambda^*$, and $\omega \geq \frac{(1-\lambda^*)p\theta}{\alpha}$, then $c_1^i = \omega - \frac{(1-\lambda^*)p\theta}{\alpha}$ and $c_{2,h}^i = (1-\lambda^*)\theta > (1-\lambda^i)\theta$. This is area 1) in Figure 1.

Proof. We know from Lemma 7 that when $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$, the participation constraint (17) always binds. From the same lemma we also know that if $\lambda^i < \lambda^*$, the limited liability constraint (21) binds. This proves case (a). Also from Lemma 7, we know from that if $\lambda^i > \lambda^*$, the limited commitment constraint (21) is slack. Thus, when $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$ and $\lambda^i > \lambda^*$, the only thing left to determine is whether $c_1^i > 0$ or $c_1^i = 0$. The consumption of the lender is

$$\begin{aligned} x_{2,h}^i &= \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha} \\ x_{2,l}^i &= \frac{(1-\lambda^i)p\theta}{\alpha} \end{aligned}$$

Since (21) is slack, therefore $\eta = 0$, and (17) binds, therefore $c_{2,h}^i = \frac{\alpha(\omega - c_1^i)}{p}$, condition (69) gives

$$\mu = u'\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right)$$

replaced in (70) gives

$$(\alpha - p)u'\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right) - (1-p)u'(\omega - c_1^i) \leq 0$$

with equality if $c_1^i > 0$. Then, by the definition of λ^* in (22), it is clear that $c_1^i > 0$ if and only if $\omega > \frac{(1-\lambda^*)p\theta}{\alpha}$, and $c_1^i = 0$ if $\omega < \frac{(1-\lambda^*)p\theta}{\alpha}$. This characterizes cases (b) and (c), and concludes the proof. ∎

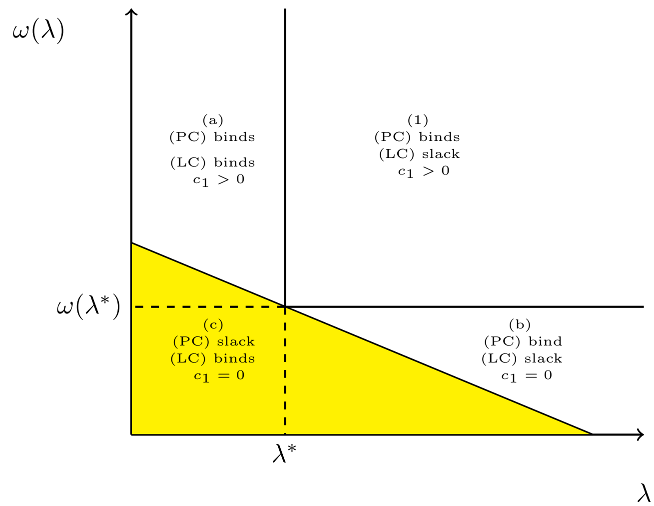

Figure 1: Solution to bilateral problem with info acquisition.

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