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Effective Lower Bound Risk*

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Abstract

Even when the policy rate is currently not constrained by its effective lower bound (ELB), the possibility that the policy rate will become constrained in the future lowers today's inflation by creating tail risk in future inflation and thus reducing expected inflation. In an empirically rich model calibrated to match key features of the U.S. economy, we find that the tail risk induced by the ELB causes inflation to undershoot the target rate of 2 percent by as much as 50 basis points at the economy's risky steady state. Our model suggests that achieving the inflation target may be more difficult now than before the Great Recession, if the likely decline in long-run neutral rates has led households and firms to revise up their estimate of the frequency of future ELB events.

JEL: E32, E52

Keywords: Deflationary Bias, Disinflation, Inflation Targeting, Risky Steady State, Tail Risk, Effective Lower Bound.

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1 Introduction

It is well known that when the effective lower bound (ELB) constraint on the policy rate is binding, it becomes more difficult for the central bank to stabilize inflation at its objective, as it cannot lower its policy rate further in response to adverse shocks. However, even when the current policy rate is not constrained by the ELB, the possibility that the policy rate may be constrained by the ELB in the future—which we will refer to as the ELB risk—can pose a challenge for the central bank's stabilization policy. Such a challenge can arise because, if the private sector agents are forward-looking, they may factor in the ELB risk when making economic decisions today.

This paper examines the implications of such ELB risk for the task of the central bank to meet its inflation objective. We do so by contrasting the risky steady state with the deterministic steady state in an empirically rich sticky-price model with an occasionally binding ELB constraint on nominal interest rates. The risky steady state is the "point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date" (Coeurdacier, Rey, and Winant, 2011). The risky steady state is an important object in dynamic macroeconomic models: This is the point around which the economy fluctuates, the point where the economy eventually converges to when all headwinds and tailwinds dissipate. Thus, a wedge between the rate of inflation in the risky steady state and the rate of inflation in the deterministic steady state—the latter corresponds to the inflation target in the policy rule—can be seen as a failure of central banks to meet their inflation objective.

We first use a stylized New Keynesian model to illustrate how, and why, the risky steady state differs from the deterministic steady state. Monetary policy is assumed to be governed by a simple interest-rate feedback rule of the type widely used by central banks to help them gauge the appropriate policy stance (Yellen, 2017; Powell, 2018). We show that, under this standard policy rule, inflation and the policy rate are lower, and output is higher, at the risky steady state than at the deterministic steady state. This result obtains because the lower bound constraint on interest rates makes the distribution of firm's marginal costs of production asymmetric; the decline in marginal costs caused by a large negative shock is larger than the increase caused by a positive shock of the same magnitude. As a result, the ELB constraint reduces expected marginal costs for forward-looking firms, leading them to lower their prices even when the policy rate is not currently constrained.[1] Reflecting the lower inflation rate at the risky steady state, the policy rate is lower at the risky steady state than at the deterministic one. In equilibrium, the ex-ante real interest rate is lower at the risky steady than at the deterministic steady state, and the output gap is positive as a result. These qualitative results are consistent with those in Adam and Billi (2007) and Nakov (2008) on how the ELB risk affects the economy near the ELB constraint under

[1]ELB risk ceteris paribus also reduces expected consumption of forward-looking households, leading them to lower their consumption expenditures.

optimal discretionary policy.

We then turn to the main exercise of our paper, which is to explore the quantitative importance of the wedge between the risky and deterministic steady states in an empirically rich DSGE model calibrated to match key features of the U.S. economy. We find that, under the standard monetary policy rule, the wedge between the deterministic and risky steady states is nontrivial in our calibrated empirical model. Inflation is a bit more than 30 basis points lower than the target rate of 2 percent at the risky steady state, with about 25 basis points attributable to the ELB constraint as opposed to other nonlinear features of the model. Output is 0.3 percentage points higher at the risky steady state than at the deterministic steady state. The risky steady state policy rate is 2.9 percent—about 80 basis points lower than the deterministic steady state policy rate—and is broadly in line with the median projection of the long-run federal funds rate in the latest Summary of Economic Projections by FOMC participants. The magnitude of the wedge depends importantly on the frequency of hitting the ELB, which in turn depends importantly on the level of the long-run equilibrium policy rate. Under an alternative plausible assumption about the long-run level of the policy rate, then the inflation wedge between the deterministic and risky steady states can exceed 60 basis points, with the ELB risk contributing about 50 basis points to the overall inflation wedge.

The observation that inflation falls below the inflation target in the policy rule at the risky steady state is different from the well known fact that the average rate of inflation falls below the target rate in the model with the ELB constraint. The decline in inflation arising from a contractionary shock can be exacerbated when the policy rate is at the ELB, while the rise in inflation arising from an expansionary shock is tempered by a corresponding increase in the policy rate. As a result, the distribution of inflation is negatively skewed and the average inflation falls below the median. This fact is intuitive and has been well recognized in the profession for a long time (Coenen, Orphanides, and Wieland, 2004; Reifschneider and Williams, 2000). The risky steady state inflation is different from the average inflation; it is the rate of inflation that would prevail at the economy's steady state when agents are aware of risks. It is worth mentioning that the average inflation falls below the target even in perfect foresight models or backward-looking models where the inflation rate eventually converges to its target. On the other hand, for the risky steady state inflation to fall below the inflation target, it is crucial that price-setters are forward-looking and take tail risk in future marginal costs into account in their pricing decisions.

In the final part of the paper, we explore some implications of ELB risk for the design of monetary policy rules. Specifically, we show that one way to eliminate the wedge between the deterministic and risky steady states of inflation is to lower the intercept term of the interest-rate feedback rule. We refer to this augmented rule as the risk-adjusted policy rule. In our empirical model, the intercept of the standard policy rule is 3.75 percent while the intercept of the risk-adjusted policy rule that allows the central bank to achieve its 2 percent inflation target in the risky steady state is only 3.24 percent.[2] While our risk-adjusted monetary policy rule is mathematically equivalent to assigning a value different from the central bank's inflation objective to the inflation targeting parameter in the standard monetary policy rule, we argue that our proposed policy rule has the benefit that the inflation target parameter in our rule retains the structural interpretation as the central bank's inflation objective.[3]

The policy rates in some advanced economies are still at the ELB, and whether inflation rates will eventually return to the central bank's inflation objective after liftoff remains to be seen. The first part of our analysis can be read as a cautionary tale of a potential systematic policy mistake central banks could make if they do not appropriately adjust their strategies in light of ELB risk. In the United States, inflation was stubbornly below the target rate of 2 percent for at least two years after the policy rate liftoff in December 2015. At the same time, the unemployment rate moved below most estimates of its natural rate. Hence, initial economic dynamics after the liftoff seem to be consistent with the model featuring the standard policy rule. However, more recently, the rate of inflation has moved close to the target rate. Thus, current inflation dynamics in the U.S. economy seems to be more consistent with the model with the risk-adjusted monetary policy rule.

Throughout the paper, we focus on a rational expectations equilibrium where the economy fluctuates around a positive level of nominal interest rates so that if all uncertainty were *permanently* resolved the economy would converge to a deterministic steady state where the ELB constraint is not binding and inflation is at target. It is well known that accounting for the ELB can give rise to two deterministic steady states and equilibrium multiplicity (Benhabib, Schmitt-Grohe, and Uribe, 2001). In particular, rational expectations equilibria may exist where the ELB constraint is binding either permanently or at least in most states of nature (Armenter, 2018). It is also possible to construct sunspot equilibria where a sunspot shock can move the economy from a regime with an occasionally binding ELB constraint to one with a binding ELB constraint in most states of nature. The effects of policy interventions and changes in policy regimes on macroeconomic outcomes in these sunspot equilibria may differ from those in the fundamental equilibrium that is at the core of our analysis (see, e.g. Mertens and Ravn, 2014; Bilbiie, 2018; Coyle and Nakata, 2019; Nakata and Schmidt, 2019b).

Our choice of equilibrium is supported by some empirical evidence. Aruoba, Cuba-Borda, and Schorfheide (2018) estimate a New Keynesian model with a lower bound, a set of fundamental shocks, and a sunspot shock that captures shifts from a regime where the economy fluctuates around a strictly positive nominal interest rate to a regime where the lower bound constraint is binding in most states. For the United States, they do not find evidence that she moved to the latter regime in the aftermath of the Great Recession. Since our quantitative

[2]While we focus on the risky steady state, the risk-adjusted monetary policy rule also mitigates the deviations of inflation from target in other states. For alternative approaches to mitigate the deflationary bias problem, please see Nakata and Schmidt (2019a) and Bianchi, Melosi, and Rottner (2019).

[3]In section 4.5, we also consider two unconventional monetary policies—"lower-for-longer" forward guidance and negative interest rate policy.

analysis focuses on the U.S. economy, we align our choice of equilibrium with the empirical results in Aruoba, Cuba-Borda, and Schorfheide (2018). Nevertheless, we document analytically the possibility of equilibrium multiplicity for a variant of our stylized model. For this model, we show that in any equilibrium with an occasionally binding ELB constraint—that is, with ELB risk—the risky steady state of inflation is below the inflation target.

The question of how the possibility of returning to the ELB affects the economy has remained largely unexplored. The majority of the literature adopts the assumption that the economy will eventually return to an absorbing state where the policy rate is permanently away from the ELB constraint, and analyzes the dynamics of the economy, and the effects of various policies, when the policy rate is *at* the ELB (Eggertsson and Woodford, 2003; Christiano, Eichenbaum, and Rebelo, 2011). While an increasing number of studies have recently departed from the assumption of an absorbing state, the focus of these studies is mostly on how differently the economy behaves at the ELB versus away from the ELB, instead of how the ELB risk affects the economy away from the ELB.[4,5] With the longer-run equilibrium real rates expected to stay lower going forward, the question of how the possibility of returning to the ELB affects the economy is as relevant as ever.[6]

Our paper builds on the work by Adam and Billi (2007), Nakov (2008), and Evans, Fisher, Gourio, and Krane (2015) who noted that the possibility of returning to the ELB has consequences for the economy even when the policy rate is currently away from the ELB. Our work differs from these papers in two substantive ways. First, while they pointed out the anticipation effect of returning to the ELB on the economy when the policy rate is *near* the ELB and the economy is away from the steady state, our work shows that the possibility of returning to the ELB has consequences for the economy even when the policy rate is well above the ELB and the economy is at the steady state. Second, while they studied the effects of the ELB risk in a stylized model, we quantify the magnitude of the effects of the ELB risk in an empirically rich, calibrated model.

Our paper is closely related to Nakata and Schmidt (2019a) and Seneca (2018). Nakata and Schmidt (2019a) analytically show that ELB risk confronts discretionary central banks with a trade-off between inflation and output gap stabilization when the ELB constraint is not binding that manifests itself in a systematic undershooting of the inflation target. This so-called deflationary bias can be reduced and welfare be improved by appointing an inflation-conservative central banker. Unlike Nakata and Schmidt (2019a), we are silent

[4]For example, Gavin, Keen, Richter, and Throckmorton (2015) and Keen, Richter, and Throckmorton (2016) ask how differently technology and anticipated monetary policy shocks affect the economy when the policy rate is constrained than when it is not, respectively. Schmidt (2013) and Nakata (2016) ask how differently the government should conduct fiscal policy when the policy rate is at the ELB than when it is not.

[5]As discussed in Section 4, in many existing models with an occasionally binding ELB constraint, the probability of being at the ELB is small, typically comfortably below 10 percent—often below 5 percent. As a result, the anticipation effects described in these papers are weak.

[6]According to the Federal Reserve Bank of New Yorks Survey of Primary Dealers from July 2019, the median respondent attached a 35 percent probability to the event that the federal funds rate returns to the ELB between July 2019 and the end of 2021.

about normative implications of the ELB risk. Instead, the main goal of our paper is to document the quantitative relevance of ELB risk using an empirically rich model. Like our paper, Seneca (2018) highlights the importance of ELB risk for the dynamics of the economy when the policy rate is currently not constrained. We examine the quantitative importance of ELB risk in an empirically rich model, whereas he examines the consequences of time-varying ELB risk—induced by exogenous time-variations in the variance of the demand shock—in a stylized model.

Our paper shares the same spirit with Kiley and Roberts (2017) in that both papers aim to understand the implications of the ELB constraint for the dynamics of the economy and monetary policy. However, our paper differs from Kiley and Roberts (2017) in a fundamental way: we focus on the anticipation effects of hitting the ELB in the future on the steady state allocations—and how large those might be in an empirically rich model—whereas Kiley and Roberts (2017) abstract from the anticipation effects of the ELB risk by solving the model with the perfect-foresight assumption. In particular, in the model of Kiley and Roberts (2017), there is no undershooting of the inflation target at the steady state, even though the distribution of inflation is asymmetric and average inflation is below the 2 percent target. In our model, the distribution of inflation is asymmetric and average inflation is below the target as in their model. However, unlike in their model, inflation in our model fluctuates around a steady state below the 2 percent target.

This seemingly subtle difference—our focus on the steady state inflation and the focus on the average inflation by Kiley and Roberts (2017)—is relevant in thinking about inflation dynamics when the policy rate is away from the ELB. Perfect-foresight models, like the model of Kiley and Roberts (2017), can explain why inflation averages below 2 percent over a long period of time including the period in which the ELB constraint binds. However, it cannot explain why inflation persistently falls below the 2 percent objective when the policy rate is away from the ELB, the situation the U.S. economy has found itself in over the past few years. Almost all estimates of the ELB frequency we are aware of suggest that the probability of being at the ELB is less than half. That is, the economy spends more than half of the time away from the ELB. Thus, understanding the dynamics of the economy away from the ELB—and how that might be affected by the possibility of returning to the ELB in the future—is of first-order importance.

Our focus on the anticipation effects of the ELB also differentiates our work from papers examining different aspects of the model with the ELB in perfect-foresight environment setups, including Coibion, Gorodnichenko, and Wieland (2012)—who focus on the implication of the ELB for optimal inflation target—and Guerrieri and Iacoviello (2017)—who estimate a model with a borrowing constraint and the ELB constraint—among others.

Finally, our paper is related to other papers which also work with models with the ELB without abstracting from uncertainty—especially those which work with empirically rich models with ELB (Gust, Herbst, López-Salido, and Smith (2017), Plante, Richter, and Throckmorton (2018), Hirose and Sunakawa (2016), among others). Our paper differs from these papers in two important ways. First, we focus on the anticipation effect of the ELB risk when the policy rate is away from the ELB, whereas the existing papers focus on other aspects of the model (for example, Gust, Herbst, López-Salido, and Smith (2017) on the dynamics of the economy at the ELB; Plante, Richter, and Throckmorton (2018) on endogenous uncertainty at the ELB, Hirose and Sunakawa (2017) on the natural rate of interest at the ELB).

Second, the existing models in this category—those papers working with fully nonlinear models with uncertainty—typically predict low probabilities of being at the ELB because these models are intended to fit the data including the 1980s and 1990s when the long-run equilibrium interest rate was higher than it is now and it is expected to be in the future.[7] We calibrate our model using the data starting in the mid 1990s and the baseline ELB probability is 16 percent. We consider parameter values under which the probability is even higher. Thus, our calibration is suited for understanding how large the anticipation effect of ELB risk might be in the future. All in all, we see our empirical model as a valuable complement to the existing fully-nonlinear DSGE models with uncertainty.

The rest of the paper is organized as follows. After a brief review of the concept of the risky steady state in Section 2, Section 3 analyzes the risky steady state in a stylized New Keynesian economy. Section 4 quantifies the wedge between the deterministic and risky steady states in an empirically rich DSGE model. Section 5 discusses a simple modification to the monetary policy rule to achieve the inflation objective at the risky steady state. Section 6 discusses empirical relevance of our main results, whereas Section 7 discusses some additional thoughts and results. Section 8 concludes.

2 The Risky Steady State: Definition

The risky steady state is defined generically as follows.

Let Γ_t and S_t denote vectors of endogenous and exogenous variables, respectively, in the model under investigation. Let $f(\cdot,\cdot)$ denote a vector of policy functions mapping the values of endogenous variables in the previous period and today's realizations of exogenous variables into the values of endogenous variables today.[8] That is,

$$\Gamma_t = f(\Gamma_{t-1}, S_t) \tag{1}$$

The risky steady state of the economy, Γ_{RSS}, is given by a vector satisfying the following

[7]For example, about 4 percent for Gust, Herbst, López-Salido, and Smith (2017), about 5 percent in Richter and Throckmorton (2016) and 9.7 percent for Hirose and Sunakawa (2016). As discussed in Richter and Throckmorton (2015), when the shock variance is too high, the equilibrium ceases to exist once uncertainty is correctly accounted for in solving these models, making it difficult to achieve a high ELB frequency.

[8]Note that the policy function does not need to depend on the entire set of the endogenous variables in the prior period. It may not depend on any endogenous variables in the prior period at all, as in the stylized model presented in the next section.

condition.

$$\Gamma_{RSS} = f(\Gamma_{RSS}, S_{SS}) \tag{2}$$

where S_{SS} denotes the steady state of S_t.[9] That is, the risky steady state is where the economy will eventually converge as the exogenous variables settle at their steady state. In this risky steady state, the agents are aware that shocks to the exogenous variables can occur, but the current realizations of those shocks are zero. On the other hand, the deterministic steady state of the economy, Γ_{DSS}, is defined as follows:

$$\Gamma_{DSS} = f_{PF}(\Gamma_{DSS}, S_{SS}) \tag{3}$$

where $f_{PF}(\cdot, \cdot)$ denotes the vector of policy functions obtained under the perfect foresight assumption.

In principle, the deterministic steady state does not have to be unique. Indeed, as discussed in Section 1, it is well known that accounting for the ELB constraint can induce multiplicity of steady state equilibria in models that would otherwise have a unique deterministic steady state. In the next section, we show that such multiplicity can also arise in the context of the risky steady state.

3 The Risky Steady State in a Stylized Model with the ELB

3.1 Model

We start by characterizing the risky steady state in a stylized New Keynesian model. Since the model is standard, we only present its equilibrium conditions here. The details of the model are described in Appendix A.

$$C_t^{-\chi_c} = \beta \delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1}, \tag{4}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c}, \tag{5}$$

$$\frac{Y_t}{C_t^{\chi_c}} \left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta) - \theta w_t\right] = \beta \delta_t \mathrm{E_t} \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi\left(\Pi_{t+1} - 1\right)\Pi_{t+1}, \tag{6}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\Pi_t - 1\right]^2 Y_t, \tag{7}$$

$$Y_t = N_t, \tag{8}$$

$$R_t = \max\left[R_{ELB}, \quad \frac{\Pi^{targ}}{\beta}\left(\frac{\Pi_t}{\Pi^{targ}}\right)^{\phi_\pi}\right], \tag{9}$$

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_{\delta,t}, \tag{10}$$

C_t, N_t, Y_t, w_t, Π_t, and R_t are consumption, labor supply, output, real wage, inflation, and

[9] There is no distinction between deterministic and risky steady states for S_t because S_t is exogenous.

the policy rate, respectively. δ_t is an exogenous shock to the household's discount rate, and follows an AR(1) process with mean one, as shown in equation (10). The innovation to the discount rate shock process, $\epsilon_{\delta,t}$, is normally distributed with a standard deviation of σ_ϵ. Equation (4) is the consumption Euler equation, equation (5) is the intratemporal optimality condition of the household, Equation (6) is the optimality condition of the intermediate good producing firms relating today's inflation to real marginal cost today and expected inflation tomorrow (forward-looking Phillips Curve), equation (7) is the aggregate resource constraint capturing the resource cost of price adjustment, and equation (8) is the aggregate production function. Equation (9) is the interest-rate feedback rule where R_{ELB} is the lower bound on the gross nominal interest rate and Π^{targ} is the inflation target parameter.

A recursive equilibrium of this stylized economy is given by a set of policy functions for $\{C(\cdot),\ N(\cdot),\ Y(\cdot),\ w(\cdot),\ \Pi(\cdot),\ R(\cdot)\}$ satisfying the equilibrium conditions described above. As discussed in Section 1, we focus on a rational expectations equilibrium that fluctuates around a strictly positive nominal interest rate level. The model is solved with a global solution method described in detail in Appendix B. Table 1 lists the parameter values used for this exercise.

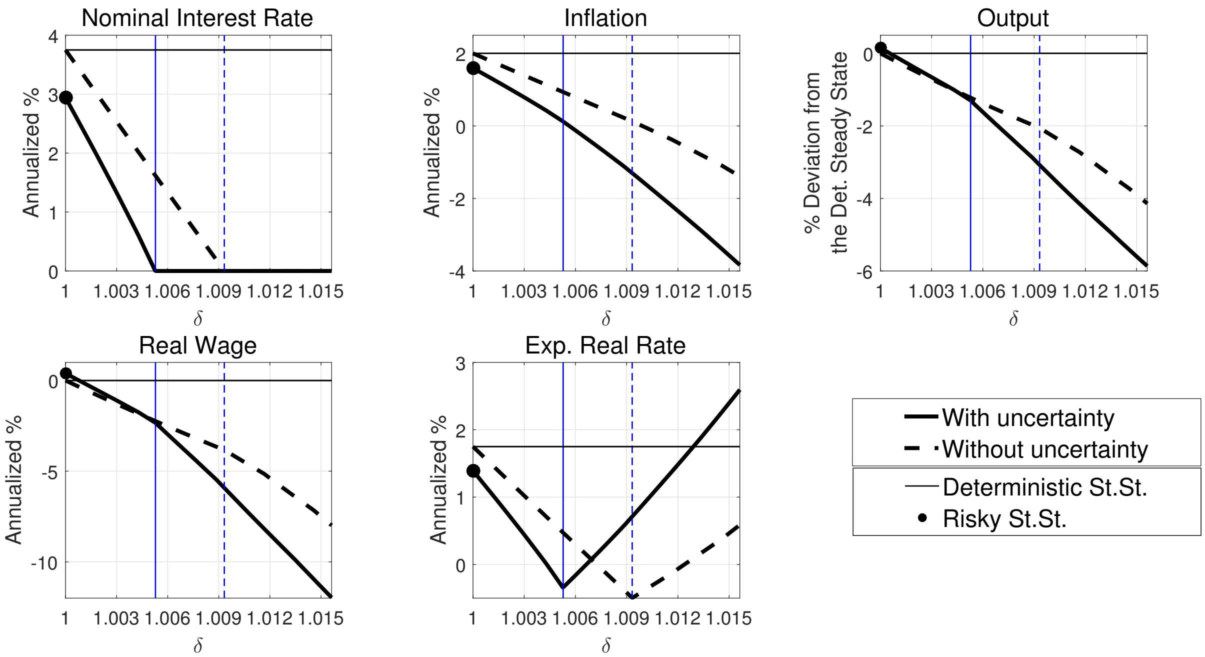
Table 1: Parameter Values for the Stylized Model

Parameter	Description	Parameter Value
β	Discount rate	$\frac{1}{1+0.004365}$
χ_c	Inverse intertemporal elasticity of substitution for C_t	1
χ_n	Inverse labor supply elasticity	1
θ	Elasticity of substitution among intermediate goods	6
φ	Price adjustment cost	200
$400(\Pi^{targ}-1)$	(Annualized) target rate of inflation	2
ϕ_π	Coefficient on inflation in the Taylor rule	1.5
R_{ELB}	Effective lower bound	1
ρ	AR(1) coefficient for the discount factor shock	0.8
σ_δ	Standard deviation of shocks to the discount factor	$\frac{0.38}{100}$
	*Implied prob. that the policy rate is at the lower bound	20%

3.2 Dynamics and the risky steady state

Before analyzing the risky steady state of the model, it is useful first to look at the dynamics of the model. Solid black lines in Figure 1 show the policy functions for the policy rate, inflation, output, and the expected real interest rate. Dashed black lines show the policy function of the model obtained under the assumption of perfect foresight. Under the perfect foresight case, the agents in the model attach zero probability to the event that the policy rate will return to the ELB when the policy rate is currently away from the ELB. Under both versions of the model, an increase in the discount rate makes households want to save more for tomorrow and spend less today. Thus, as δ increases, output, inflation, and the policy rate decline. When δ is large and the policy rate is at the ELB, an additional increase in the discount rate leads to larger declines in inflation and output than when δ is small and the policy rate is not at the ELB, as the adverse effects of the increase in δ are not countered by a corresponding reduction in the policy rate.

Figure 1: Policy Functions from the Stylized Model



*The dashed black lines ("Without uncertainty" case) show policy functions obtained under the perfect foresight assumption (i.e., $\sigma_\epsilon = 0$).

When the policy rate is at the ELB, the presence of uncertainty reduces inflation and output. This is captured by the fact that the solid lines are below the dashed lines for inflation and output in the figure. The non-neutrality of uncertainty is driven by the ELB constraint. If the economy is buffeted by a sufficiently large expansionary shock, then the policy rate will adjust to offset some of the resulting increase in real wages. If the economy is hit by a contractionary shock, regardless of the size of the shock, the policy rate will stay at the ELB and the resulting decline in real wages will not be tempered. Due to this asymmetry, an increase in uncertainty reduces the expected real wage, which in turn reduces inflation as price-setters are forward-looking and thus inflation today depends on the expected real wage. With the policy rate constrained at the ELB, a reduction in inflation leads to an increase in the expected real rate, pushing down consumption and output today. These adverse effects of uncertainty at the ELB are studied in detail in Nakata (2017).

When the policy rate is away from the ELB, the presence of uncertainty reduces inflation and the policy rate, but increases output. If the economy is hit by a sufficiently large contractionary shock, the policy rate will hit the ELB and the resulting decline in real wages will not be tempered. If the economy is hit by an expansionary shock, regardless of the size of the shock, the policy rate will adjust to partially offset the resulting increase in real wages. Thus, the presence of uncertainty, by generating the possibility that the policy rate will return to the ELB, reduces the expected real wage and thus today's inflation. When the policy rate is away from the ELB, its movement is governed by the Taylor rule. Since the Taylor principle is satisfied (i.e., the coefficient of inflation is larger than one), the reduction in inflation comes with a larger reduction in the policy rate. As a result, the expected real rate is lower, and thus consumption and output are higher, with uncertainty than without uncertainty.

Table 2: The Risky Steady State in the Stylized Model

	Inflation	Output*	Policy rate
Deterministic steady state	2	0	3.75
Risky steady state	1.59	0.15	2.93
(Wedge)	(−0.41)	(0.15)	(−0.82)
Risky steady state w/o the ELB	1.98	−0.04	3.71
(Wedge)	(−0.02)	(−0.04)	(−0.04)

*Output is expressed as a percentage deviation from the determistic steady state.

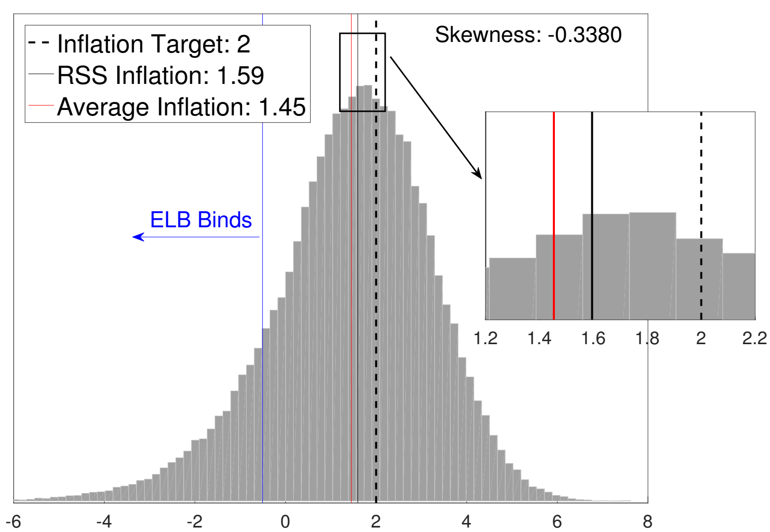
While these effects are stronger the closer the policy rate is to the ELB, they remain nontrivial even at the economy's risky steady state. In the stylized model of this section, in which the policy functions do not depend on any of the model's endogenous variables from the previous period, the risky steady state is given by the vector of the policy functions evaluated at $\delta = 1$. That is, inflation, output, and the policy rate at the risky steady state are given by the intersection of the policy functions for these variables and the left vertical axes. As shown in Table 2, inflation and output are 41 basis points lower and 0.15 percentage points higher at the risky steady state than at the deterministic steady state, respectively. The risky steady state policy rate is 82 basis points lower than its deterministic counterpart.

In our model, the ELB constraint is not the only source of nonlinearity. Our specifications of the utility function and the price adjustment cost also make the model nonlinear, and thus explain some of the wedge between the deterministic and risky steady states. To understand the extent to which these other nonlinear features matter, Table 2 also reports the risky steady state in the version of the model without the ELB constraint. Overall, the differences between the deterministic and risky steady states would be small were it not for the ELB constraint. Inflation and the policy rate at the risky steady state are only 2 and 4 basis points below those at the deterministic steady state, respectively. Output at the risky steady state is about 4 basis points below that at the deterministic steady state. Thus, the majority of the overall wedge between the deterministic and risky steady states is attributed to the nonlinearity induced by the ELB constraint, as opposed to other nonlinear features of the model.

3.3 The risky steady state and the average

It is important to recognize that the risky steady state is different from the average. Let's take inflation as an example. The risky steady state inflation is the point around which inflation fluctuates and coincides with the median of its unconditional distribution in the model without any endogenous state variables like the one analyzed here. On the other hand, the average inflation depends on the inflation rate in all states of the economy. Provided that the probability of being at the ELB is sufficiently large, the unconditional distribution of inflation is negatively skewed and therefore the risky steady state inflation is higher than the average inflation, as depicted in Figure 2. The observation that the ELB constraint pushes down the average inflation below the median by making the distribution of inflation negatively skewed is intuitive and has been well recongnized for a long time (Coenen, Orphanides, and Wieland, 2004; Reifschneider and Williams, 2000). This observation holds true even when price-setters form expectations in a backward-looking manner and thus there is no anticipation effect due to the ELB risk. The result that the ELB risk lowers the median of the distribution below the target is less intuitive and requires that price-setters are forward-looking in forming their expectations.

Figure 2: Unconditional Distribution of Inflation in the Stylized Model



*RSS stands for the risky steady state.

3.4 The risk-adjusted Fisher relation

One way to understand the discrepancy between deterministic and risky steady states is to examine the effect of the ELB risk on the Fisher relation. Let R_{DSS} and Π_{DSS} be the deterministic steady state policy rate and inflation. In the deterministic environment, the consumption Euler equation evaluated at the steady state becomes

$$R_{DSS} = \frac{\Pi_{DSS}}{\beta} \tag{11}$$

after dropping the expectation operator from the consumption Euler equation and eliminating the deterministic steady-state consumption from both sides of the equation. This relation is often referred to as the Fisher relation.

In the stochastic environment, the consumption Euler equation evaluated at the (risky) steady state can be written as

$$R_{RSS} = \frac{\Pi_{RSS}}{\beta} \cdot \frac{1}{\mathrm{E}_{RSS}[\left(\frac{C_{RSS}}{C_{t+1}}\right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}}]} \tag{12}$$

where R_{RSS}, Π_{RSS}, and C_{RSS} are the risky steady-state policy rate, inflation, and consumption. $\mathrm{E}_{RSS}[\cdot]$ is the conditional expectation operator when the economy is at the risky steady state today. In the stylized model with one shock and without any endogenous state variables, $\mathrm{E}_{RSS}[\cdot] := E_t[\cdot|\delta_t = 1]$. We will refer to Equation (12) as the *risk-adjusted Fisher relation*. Relative to the standard Fisher relation, there is an adjustment term that reflects the discrepancy between today's economic conditions and the expected economic conditions next period. This term captures the effect of the tail risk in future economic conditions induced by the ELB constraint on household expectations. Notice that the adjustment term is less than one,

$$\frac{1}{\mathrm{E}_{RSS}[\left(\frac{C_{RSS}}{C_{t+1}}\right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}}]} < 1, \tag{13}$$

because of the fat tail on the lower end of the distributions of future inflation and consumption induced by the ELB constraint.

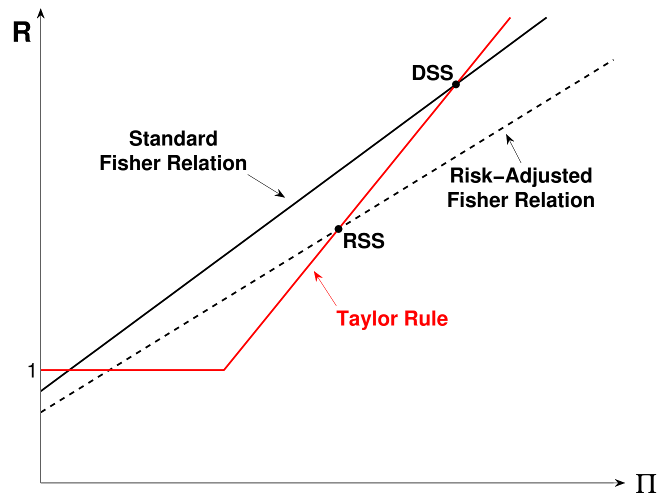
Figure 3 plots the standard Fisher relation (11), the risk-adjusted Fisher relation (12), and the Taylor rule (9). Consider, first, the deterministic case. Deterministic steady state equilibria are represented by the intersections of the standard Fisher relation and the Taylor rule. Due to the kink in the Taylor rule induced by the lower bound constraint there exist two deterministic steady state equilibria: One where the lower bound constraint is slack and inflation is at target and another one where the lower bound constraint is binding and inflation is below target.

Now consider the stochastic case. In equilibrium, The risky steady state of a rational-expectations equilibrium is given by the intersection of the line representing the risk-adjusted Fisher relation and the line representing the Taylor rule. Since the risk-adjustment term is less than one, the risk-adjusted Fisher relation is flatter than the standard Fisher relation. There are two intersections with the Taylor rule: One where the lower bound constraint is binding and one where the lower bound constraint is slack. The two risky steady states are associated with two distinct rational expectations equilibria. As discussed before, we focus on the equilibrium where the policy rate is above the lower bound in the risky steady state.[10]

[10]The intersection of the standard Fisher relation and the Taylor-rule equation in a region where the Taylor-rule equation is flat at the ELB is the deflationary steady state explored by Benhabib, Schmitt-Grohe, and Uribe (2001). Comparing that intersection with the intersection of the risk-adjusted Fisher relation and the Taylor-rule equation in the ELB region indicates that, in a deflationary rational expectations equilibrium, inflation is higher at the risky steady state than at the deterministic steady state. We have confirmed the validity of this feature in a semi-loglinear model with a three-state discount rate shock. This property of the

In the region where the policy rate is positive, the line representing the risk-adjusted Fisher relation crosses the line representing the Taylor rule at a point below the line for the standard Fisher relation crosses it., as shown in Figure 3. Thus, inflation and the policy rate are lower at the risky steady state than at the deterministic steady state.

Figure 3: The Risk-Adjusted Fisher Relation and the Taylor Rule



†DSS stands for "deterministic steady state," and RSS stands for "risky steady state."

3.5 An analytical example

Even though the model we have used so far is stylized, we still had to rely on numerical analysis to expose the discrepancy between deterministic and risky steady states that is induced by ELB risk. We close this section by providing some analytical results on the deflationary bias at the risky steady state in a semi-loglinearized version of the model.

Aggregate private sector behavior and monetary policy of the semi-loglinearized model are described by the following three equations

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa y_t \tag{14}$$

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\chi_c} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right) \tag{15}$$

$$i_t = \max\left[0, r^n + \pi^{targ} + \phi_\pi \left(\pi_t - \pi^{targ}\right)\right], \tag{16}$$

where π_t and y_t denote the rate of inflation and output expressed in percentage deviations from the deterministic steady state with stable prices, respectively, i_t denotes the level of the policy rate, and r_t^n denotes the natural real rate of interest which is a function of the discount factor shock. To be able to obtain analytical results, we assume that r_t^n is uniformly

deflationary equilibrium is further explored in Coyle, Nakata, and Schmidt (2019).

distributed, $r_t^n \sim U(r^n - \epsilon, r^n + \epsilon)$, with $r^n = \frac{1}{\beta} - 1$, and $\epsilon \geq 0$. The slope of the log-linearized Phillips curve is a function of the structural model parameters $\kappa = \frac{\theta(\chi_c + \chi_n)}{\varphi}$. Finally, we assume that the monetary policy parameters satisfy $\phi_\pi > 1$ and $\pi^{targ} \geq 0$. Note that this model has two deterministic steady state equilibria. In the intended steady state equilibrium, $\pi_{DSS} = \pi^{targ}$ and the lower bound constraint is slack. In the unintended steady state equilibrium, $\pi_{DSS} = -r^n$ and the lower bound constraint is binding.

We now proceed in two steps. We first show that in any equilibrium of the stochastic model where the lower bound is an occasionally binding constraint, unconditional expected inflation is below the inflation target. We then show that the inflation rate at the risky steady state of any such equilibrium is below the inflation target, and, therefore, below the intended deterministic steady state of inflation.[11]

Using (14) to substitute out (expected) output in (15), we obtain

$$\pi_t = (1 + \kappa\chi_c^{-1})\mathrm{E}\pi_{t+1} - \kappa\chi_c^{-1}(i_t - r_t^n), \tag{17}$$

where we made use of the fact that $\mathrm{E}_t\pi_{t+1} = \mathrm{E}\pi_{t+1}$. Next, substituting (17) into (16), one obtains an equation that relates the policy rate to expected inflation and the natural real rate shock

$$i_t = \max\left[0, \frac{r^n}{1+\kappa\chi_c^{-1}\phi_\pi} + \frac{\kappa\chi_c^{-1}\phi_\pi}{1+\kappa\chi_c^{-1}\phi_\pi} r_t^n + \frac{1-\phi_\pi}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} + \frac{\phi_\pi(1+\kappa\chi_c^{-1})}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi_{t+1}\right] \tag{18}$$

From (17) and (18), it follows that

$$\pi_t = \begin{cases} (1+\kappa\chi_c^{-1})\mathrm{E}\pi_{t+1} + \kappa\chi_c^{-1} r_t^n, \\ \qquad \text{if } r_t^n < -\frac{r^n}{\kappa\chi_c^{-1}\phi_\pi} - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} \\ \frac{1+\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi_{t+1} + \frac{\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}(r_t^n - r^n) - \frac{\kappa\chi_c^{-1}(1-\phi_\pi)}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}, \\ \qquad \text{if } r_t^n \geq -\frac{r^n}{\kappa\chi_c^{-1}\phi_\pi} - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}. \end{cases}$$

Let μ_t be the probability that the lower bound constraint is binding in period t. Then,

$$\mu_t = \begin{cases} 1, \quad \text{if } -\frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi} r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} \geq \epsilon \\ \frac{1}{2\epsilon}\left(\epsilon - \frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi} r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}\right), \\ \qquad \text{if } -\epsilon < -\frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi} r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} < \epsilon \\ 0, \quad \text{if } -\frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi} r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} \leq -\epsilon. \end{cases}$$

Taking unconditional expectations of π_t, we have

[11] Mertens and Williams (2018) show how to solve for expected inflation in a log-linearized New Keynesian model with a lower bound on nominal interest rates and a uniformly distributed price markup shock. We follow their approach to solve for expected inflation. Unlike us, they consider an optimizing central bank with a zero inflation target acting under discretion, and they do not characterize the risky steady state.

$$
\mathrm{E}\pi_t = \begin{cases} \left(1+\kappa\chi_c^{-1}\right)\mathrm{E}\pi_{t+1} + \kappa\chi_c^{-1}r^n, & \text{if } \mu_t = 1 \\ -\frac{1}{4\epsilon}\frac{(\kappa\chi_c^{-1})^2\phi_\pi}{1+\kappa\chi_c^{-1}\phi_\pi}\left(\epsilon - \frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi_{t+1} - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}\right)^2 & \\ \quad + \frac{1+\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi_{t+1} - \frac{\kappa\chi_c^{-1}(1-\phi_\pi)}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}, & \text{if } 0 < \mu_t < 1 \\ \frac{1+\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi_{t+1} - \frac{\kappa\chi_c^{-1}(1-\phi_\pi)}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}, & \text{if } \mu_t = 0. \end{cases}
$$

In equilibrium, $\mathrm{E}\pi_t = \mathrm{E}\pi_{t+1}$. In any equilibrium in which the probability of a binding lower bound constraint is one, $\mathrm{E}\pi = -r^n$. That is, expected inflation is equal to the unintended deterministic steady state. In any equilibrium, in which the probability of a binding lower bound constraint is zero, $\mathrm{E}\pi = \pi^{targ}$. That is, expected inflation is equal to the intended deterministic steady state. In both cases, actual inflation fluctuates symmetrically around the respective deterministic steady state. We are, however, interested in equilibria where the lower bound constraint is *occasionally* binding. For the remainder, we thus focus on equilibria where $0 < \mu_t < 1$. According to the above functional relationship between $\mathrm{E}\pi_{t+1}$ and $\mathrm{E}\pi_t$, there can be up to two such equilibria. Rearranging terms, in these equilibria

$$
\kappa\chi_c^{-1}\left(\phi_\pi - 1\right)\left(\mathrm{E}\pi - \pi^{targ}\right) = -\frac{1}{4\epsilon}(\kappa\chi_c^{-1})^2\phi_\pi\left(\epsilon - \frac{1+\kappa\chi_c^{-1}\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}r^n - \frac{1+\kappa\chi_c^{-1}}{\kappa\chi_c^{-1}}\mathrm{E}\pi - \frac{1-\phi_\pi}{\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}\right)^2
$$

Hence, given $\phi_\pi > 1$, any equilibrium where the lower bound constraint is occasionally binding features below-target unconditional inflation expectations, $\mathrm{E}\pi < \pi^{targ}$.[12] If two such equilibria exist, unconditional inflation expectations in one of them will be strictly higher than in the other. This is the one that we focus on in our quantitative analysis.

We can now proceed with the second step and show that in any equilibrium where the lower bound constraint is occasionally binding, the risky steady state of inflation is below the inflation target, and, therefore, below the intended deterministic steady state of inflation. At the risky steady state, the policy rate is given by

$$
i_{RSS} = \max\left[0, r^n + \frac{1-\phi_\pi}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ} + \frac{\phi_\pi(1+\kappa\chi_c^{-1})}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi\right] \tag{19}
$$

Substituting (19) into (17), evaluated at the risky steady state, one obtains

$$
\pi_{RSS} = \begin{cases} \left(1+\kappa\chi_c^{-1}\right)\mathrm{E}\pi + \kappa\chi_c^{-1}r^n, & \text{if } \mathrm{E}\pi \le -\frac{1+\kappa\chi_c^{-1}\phi_\pi}{\phi_\pi\left(1+\kappa\chi_c^{-1}\right)}r^n - \frac{1-\phi_\pi}{\phi_\pi\left(1+\kappa\chi_c^{-1}\right)}\pi^{targ} \\ \frac{1+\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}\mathrm{E}\pi + \frac{\kappa\chi_c^{-1}(\phi_\pi-1)}{1+\kappa\chi_c^{-1}\phi_\pi}\pi^{targ}, & \text{if } \mathrm{E}\pi > -\frac{1+\kappa\chi_c^{-1}\phi_\pi}{\phi_\pi\left(1+\kappa\chi_c^{-1}\right)}r^n - \frac{1-\phi_\pi}{\phi_\pi\left(1+\kappa\chi_c^{-1}\right)}\pi^{targ}. \end{cases} \tag{20}
$$

Consider, first, the case where the lower bound constraint is binding in the risky steady state

[12]The closed-form solutions for expected inflation and inflation are rather complicated and not of particular interest for what follows.

$$
\begin{aligned}
\pi_{RSS} &= \left(1+\kappa\chi_c^{-1}\right)\mathrm{E}\pi + \kappa\chi_c^{-1} r^n \\
&< -\frac{1}{\phi_\pi} r^n - \frac{1-\phi_\pi}{\phi_\pi}\pi^{targ},
\end{aligned}
$$

and, hence, $\phi_\pi\left(\pi_{RSS} - \pi^{targ}\right) < -(r^n + \pi^{targ}) < 0$. That is, the risky steady state of inflation is below the target. Next, consider the case where the lower bound constraint is not binding in the risky steady state. Rearranging terms, we have

$$
\pi_{RSS} - \pi^{targ} = \frac{1+\kappa\chi_c^{-1}}{1+\kappa\chi_c^{-1}\phi_\pi}\left(\mathrm{E}\pi - \pi^{targ}\right)
$$

From $\mathrm{E}\pi < \pi^{targ}$ follows $\pi_{RSS} < \pi^{targ}$. Hence, even when the policy rate is not constrained by the lower bound at the risky steady state, risky steady state inflation will be below its target.

4 The Risky Steady State in an Empirical Model with the ELB

We now quantify the magnitude of the wedge between the deterministic and risky steady states in an empirically rich model calibrated to match key features of the U.S. economy.

4.1 Model

Our empirical model adds four additional features as well as two additional shocks on top of the stylized New Keynesian model of the previous section. The four additional features are (i) a non-stationary productivity process, (ii) consumption habits, (iii) sticky wages, and (iv) an interest rate smoothing term in the interest-rate feedback rule. The two additional shocks are a productivity shock and a monetary policy shock. Since these features are standard, we relegate the detailed description of them to Appendix C and only show the equilibrium conditions of the model here. Let $\tilde{Y}_t = \frac{Y_t}{A_t}$, $\tilde{C}_t = \frac{C_t}{A_t}$, $\tilde{w}_t = \frac{w_t}{A_t}$, and $\tilde{\lambda}_t = \frac{\lambda_t}{A_t^{-\chi_c}}$ be the stationary representations of output, consumption, real wage, and marginal utility of consumption, respectively, where A_t is a non-stationary productivity path. The stationary equilibrium is characterized by the following system of equations:

$$
\tilde{\lambda}_t = \frac{\beta}{a^{\chi_c}}\delta_t R_t \mathrm{E}_t \tilde{\lambda}_{t+1}\left(\Pi_{t+1}^p\right)^{-1}\exp(-\chi_c a_{t+1}), \tag{21}
$$

$$
\tilde{\lambda}_t = (\tilde{C}_t - \frac{\zeta}{a}\tilde{C}_{t-1}\exp(-a_t))^{-\chi_c}, \tag{22}
$$

$$\frac{N_t \tilde{w}_t}{\tilde{\lambda}_t^{-1}} \left[\varphi_w \left(\frac{\Pi_t^w}{\bar{\Pi}^w} - 1 \right) \frac{\Pi_t^w}{\bar{\Pi}^w} - (1 - \theta^w) - \theta^w \frac{N_t^{\chi_n}}{\tilde{\lambda}_t \tilde{w}_t} \right] \\ = \frac{\beta \varphi_w}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{N_{t+1} \tilde{w}_{t+1}}{\lambda_{t+1}^{-1}} \left(\frac{\Pi_{t+1}^w}{\bar{\Pi}^w} - 1 \right) \frac{\Pi_{t+1}^w}{\bar{\Pi}^w} \exp\left((1 - \chi_c) \, a_{t+1} \right), \tag{23}$$

$$\Pi_t^w = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \Pi_t^p \exp\left(a_t \right), \tag{24}$$

$$\frac{\tilde{Y}_t}{\tilde{\lambda}_t^{-1}} \left[\varphi_p \left(\frac{\Pi_t^p}{\bar{\Pi}^p} - 1 \right) \frac{\Pi_t^p}{\bar{\Pi}^p} - (1 - \theta^p) - \theta^p \tilde{w}_t \right] \\ = \frac{\beta \varphi_p}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}} \left(\frac{\Pi_{t+1}^p}{\bar{\Pi}^p} - 1 \right) \frac{\Pi_{t+1}^p}{\bar{\Pi}^p} \exp\left((1 - \chi_c) \, a_{t+1} \right), \tag{25}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\varphi_p}{2} \left[\frac{\Pi_t^p}{\bar{\Pi}^p} - 1 \right]^2 \tilde{Y}_t + \frac{\varphi_w}{2} \left[\frac{\Pi_t^w}{\bar{\Pi}^w} - 1 \right]^2 \tilde{w}_t N_t, \tag{26}$$

$$\tilde{Y}_t = N_t, \tag{27}$$

and

$$R_t = \max\left[R_{ELB}, R_t^* \right], \tag{28}$$

where

$$R_t^* = \bar{R} \left(\frac{R_{t-1}^*}{\bar{R}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}} \right)^{(1-\rho_r)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}} \right)^{(1-\rho_r)\phi_y} \exp\left(\epsilon_{R,t} \right), \tag{29}$$

and the following process for the discount rate and the technology growth:

$$(\delta_t - 1) = \rho_\delta (\delta_{t-1} - 1) + \epsilon_{\delta,t}, \tag{30}$$

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t, \tag{31}$$

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}. \tag{32}$$

$\epsilon_{\delta,t}$, $\epsilon_{a,t}$, and $\epsilon_{R,t}$ are normally distributed with mean zero and standard deviation of $\sigma_{\epsilon,\delta}$, $\sigma_{\epsilon,a}$, and $\sigma_{\epsilon,R}$, respectively. While the discount rate shock and the technology shock follow AR(1) processes, the monetary policy shock is i.i.d., a common assumption in the literature. ζ is the degree of consumption habits in the household's utility function and a is the trend growth rate of productivity. φ_p and φ_w are the price and wage adjustment costs. ρ_R is the weight on the lagged shadow policy rate in the truncated interest-rate feedback rule. $\bar{\Pi}^p$ and $\bar{\Pi}^w$ are price and wage inflation rates in the determistic steady state, and they are equal to Π^{targ}. In the truncated interest-rate feedback rule, $\bar{R}$ is the intercept of the policy rule and is given by the deterministic steady state policy rate. That is,

$$\bar{R} = \frac{a^{\chi_c} \Pi^{targ}}{\beta} \tag{33}$$

This specification of the intercept term of the policy rule is universal in the literature on

New Keynesian DSGE models. In Section 5, we consider an alternative specification of the intercept term that allows the central bank to achieve its inflation objective at the risky steady state. $\bar{Y}$ is the level of output (normalized by A_t) at the deterministic steady state and is a function of the structural parameters. $\tilde{Y}_t/\bar{Y}$ is the deviation of the stationarized output from its deterministic steady state and will be referred to as the output gap in this paper.

4.2 Calibration

We calibrate our model to match key features of the output gap, inflation, and the policy rate in the U.S. since the mid 1990s, which are shown in Figure 4. We focus on this relatively recent past for two reasons. First, long-run inflation expectations were low and stable during this period. As shown in Figure 5, the median of CPI inflation forecasts 5-10 years ahead in the Survey of Professional Forecasters, a commonly used measure of long-run inflation expectations, declined to 2.5 percent in the second half of the 1990s and has been relatively stable since then, except for recent small declines during the ELB episode.[13] Second, the ELB was either a concern or a binding constraint to the Federal Reserve during this period. The concern for the ELB surged in the U.S. in the second half of the 1990s when the Bank of Japan lowered the policy rate to the lower bound for the first time in the Post WWII history among major advanced economies.[14]

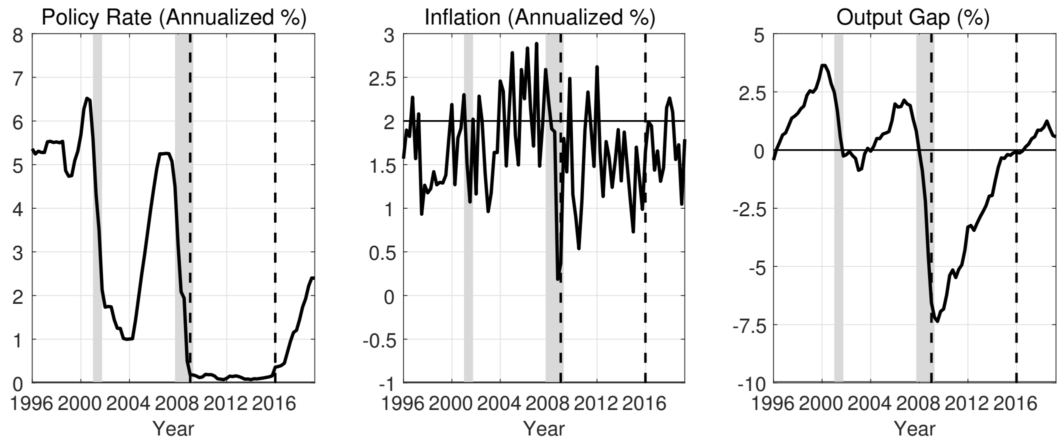
We set the time discount rate to 0.99875 so that the contribution of the discount rate to the deterministic steady state real rate is 50 basis points. We set the target rate of inflation in the interest-rate feedback rule to 2 percent as this is the FOMC's official target rate of inflation. In our baseline calibration, we set the trend growth rate of productivity to 1.25 percent so that the policy rate is 3.75 percent at the economy's deterministic steady state. Later in this section, we will consider alternative values for this productivity parameter, which imply alternative policy rates at the deterministic steady state.

In the household utility function, the degree of consumption habits, the inverse Frisch labor elasticity, and the inverse intertemporal elasticity of substitution are set to 0.5, 1 and 1, respectively. These are all within the range of standard values found the literature. Following Erceg and Lindé (2014), the parameters governing the steady-state markups for intermediate goods and the intermediate labor inputs are set to 11 and 4 and the parameters governing the price adjustment costs for prices and wages to 1000 and 300. In a hypothetical log-linear environment, these values would correspond to 90 and 85 percent probabilities that prices and wages cannot adjust each quarter in the Calvo version of the model, respectively. High

[13] The long-run inflation expectations measured by PCE inflation are available only from 2007. The average differential between CPI and PCE inflation rates over the past two decades is about 50 basis points. Thus, the stability of CPI inflation expectations at 2.5 percent can be interpreted as the stability of PCE inflation expectations at 2 percent.

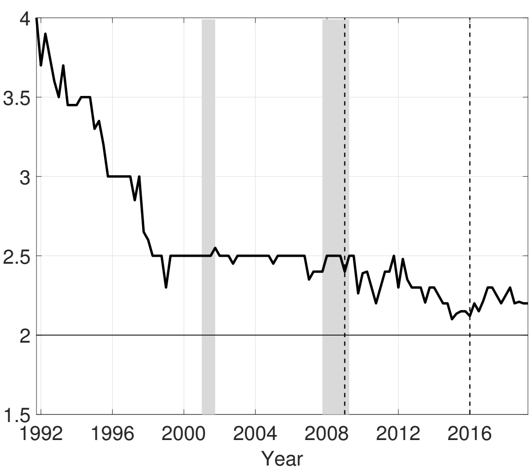
[14] Some of the earliest research on the ELB were initiated within the Federal Reserve System in this period. See, for example, Clouse, Henderson, Orphanides, Small, and Tinsley (2003), Reifschneider and Williams (2000), and Wolman (1998).

Figure 4: Policy Rate, Inflation, and Output Gap†



†The measure of the output gap is based on the public version of the FRB/US model. The inflation rate is computed as the annualized quarterly percentage change (log difference) in the personal consumption expenditure core price index (St. Louis Fed's FRED). The quarterly average of the (annualized) federal funds rate is used as the measure for the policy rate (St. Louis Fed's FRED). Dashed vertical lines mark the beginning and the end of the ELB era. Horizontal lines represent target values for the respective variables.

Figure 5: Long-Run Inflation Expectations†



†Source: Federal Reserve Board, Survey of Professional Forecasters, accessed August 2019, https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/. Dashed vertical lines mark the beginning and the end of the ELB era.

degrees of stickiness in prices and wages help the model to capture the moderate decline in inflation in the data while the federal funds rate was constrained at the ELB.

The coefficients on inflation and the output gap in the interest-rate feedback rule are set to 3 and 0.25. The coefficient on the output gap, 0.25, is standard. The coefficient on inflation is a bit higher compared to the values commonly used in the literature. A higher coefficient serves two purposes. First, it reduces the volatility of inflation relative to the volatility of the output gap. Second, a higher value makes the existence of the equilibrium more likely.[15]

[15]Richter and Throckmorton (2015) show that the model with occasionally binding ELB constraints may not have minimum-state-variable solutions when this coefficient is low even if the Taylor principle is satisfied.

Table 3: Parameter Values for the Empirical Model

Parameter	Description	Parameter Value
β	Discount rate	0.99875
a	Trend growth rate of productivity	$\frac{1.25}{400}$
ζ	Degree of consumption habits	0.5
χ_c	Inverse intertemporal elasticity of substitution for C_t	1
χ_n	Inverse labor supply elasticity	1
θ_p	Elasticity of substitution among intermediate goods	11
θ_w	Elasticity of substitution among intermediate labor inputs	4
φ_p	Price adjustment cost	1000
φ_w	Wage adjustment cost	300
Interest-rate feedback rule		
$400(\Pi^{targ} - 1)$	(Annualized) target rate of inflation	2
ρ_R	Interest-rate smoothing parameter in the Taylor rule	0.8
ϕ_π	Coefficient on inflation in the Taylor rule	3
ϕ_y	Coefficient on the output gap in the Taylor rule	0.25
$400(R_{ELB} - 1)$	(Annualized) effective lower bound	0.13
Shocks		
ρ_d	AR(1) coefficient for the discount factor shock	0.85
$\sigma_{\epsilon,\delta}$	Standard deviation of shocks to the discount factor	$\frac{0.62}{100}$
ρ_a	AR(1) coefficient for the technology shock	0.9
$\sigma_{\epsilon,a}$	Standard deviation of innovations to the technology shock	$\frac{0.1}{100}$
$\sigma_{\epsilon,r}$	Standard deviation of the monetary policy shock	$\frac{0.19}{100}$

Erceg and Lindé (2014) argue that an inflation coefficient of this magnitude is consistent with an IV-type regression estimate of this coefficient based on a recent sample. The interest rate smoothing parameter for the policy rule is set to 0.8. This high degree of interest rate smoothing helps in increasing the expected duration of the lower bound episodes, improving the model's implication in this dimension. The ELB on the policy rate is set to 0.13 percent, the average of the annualized federal funds rate during the recent ELB episode (from 2009:Q1 to 2015:Q4).

The persistence parameters of the discount rate shock and the technology shock are set to 0.85 and 0.9, respectively. As discussed earlier, the monetary policy shock is assumed to be i.i.d. The standard deviation of the monetary policy shock is set to the standard deviation of the residuals in the interest-rate feedback rule computed using the U.S. data before the federal funds rate hit the ELB ($\sigma_r = \frac{0.19}{100}$). The standard deviations of the discount factor shock and the technology shock are chosen so that (i) the volatility of the policy rate from the model is consistent with that in the data and (ii) the TFP shock accounts for about 10 percent of the standard deviation of output.

Table 4 shows the key statistics for the output gap, inflation and the policy rate in the model and in the data. The measure of the output gap is based on the estimate of potential output from the FRB/US model. As for the measure of inflation, we use core PCE Price Index inflation.

The standard deviation of the output gap in the model is 2.7, which is the same as

Table 4: Key Moments

Moment	Variable	Model	Data† (1996Q1–2019Q2)
St.Dev.(·)	Output gap	2.7	2.7
	Inflation	0.4	0.5
	Policy rate	2.3	2.2
E(X|ELB)	Output gap	−2.7	−3.6
	Inflation	1.2	1.5
	Policy rate	0.13	0.13
ELB	Frequency	16.0%	30.0%
	Expected/Actual Duration	5.7 quarters	28 quarters

†The measure of the output gap is based on the public version of the FRB/US model. Inflation rate is computed as the annualized quarterly percentage change (log difference) in the personal consumption expenditure core price index. The quarterly average of the (annualized) federal funds rate is used as the measure for the policy rate.

the sample standard deviation from the data. The conditional mean of the output gap at the ELB in the model is -2.7 percent, which is is a bit higher than the estimate from the data. The standard deviation of inflation in the model is 0.4 percent, which is in line with what's observed in the data, while the ELB conditional mean of inflation in the model is 1.2 percent, which is somewhat lower than what's observed in the data. The model-implied unconditional probability of being at the ELB and the expected ELB duration are 16.0 percent and 5.7 quarters, respectively. While these numbers are substantially higher than those in other existing models with occasionally binding ELB constraints, they are substantially lower than the empirical counterparts over the past two decades in the U.S.[16,17] In particular, the duration of the recent ELB experience is seen by the model as surprisingly long. Consistent with this interpretation, the data on liftoff expectations shows that market participants have underestimated how long the policy rate will be kept at the ELB throughout the recent ELB episode, as described in Appendix E.

[16]In most existing models with an occasionally binding ELB constraint, the probability of being at the ELB is comfortably less than 10 percent—often less than 5 percent—and the expected ELB duration is less than one year. A few exceptions are Nakata (2017) and Hirose and Sunakawa (2017). In Nakata (2017), the probability of being at the ELB is 14.1 percent and the expected ELB duration is 8.6 quarters. In Hirose and Sunakawa (2017), they are 11.8 percent and 4.3 quarters.

[17]More generally, since we have only one ELB episode in the U.S. recently, the probability of being at the ELB in the data is very sensitive to the starting date of the sample period considered. In particular, the earlier the starting date, the lower the frequency. While the choice of our reference sample can be justified by the fact that it focuses on an episode where long-run inflation expectations and long-run real interest rates have become markedly lower than has previously been the case, there is arguably some arbitrariness in this choice. We report the ELB frequency for this sample in the table just as a reference, without insisting that the value is a reasonable approximation to the "true" frequency in the U.S. economy. Also, note that because of the likely decline in the long-run neutral rate in recent years, the ELB is likely to be binding more frequently in the future than in the past.

4.3 Main results

Table 5 shows the risky and deterministic steady state values of inflation, the output gap, and the policy rate from our empirical model. For this model, the risky steady state is computed by simulating the model for a long period while setting the realization of the exogenous disturbances to zero. All (stationarized) endogenous variables eventually converge in that simulation, and that point of convergence is the risky state of the economy. By construction, the deterministic steady state of inflation is given by the target rate of inflation and the output gap is zero at the deterministic steady state. As explained earlier, parameter values (β, χ_c and a) are chosen so that the deterministic steady state of the policy rate is 3.75 percent.

Table 5: The Risky Steady State in the Empirical Model

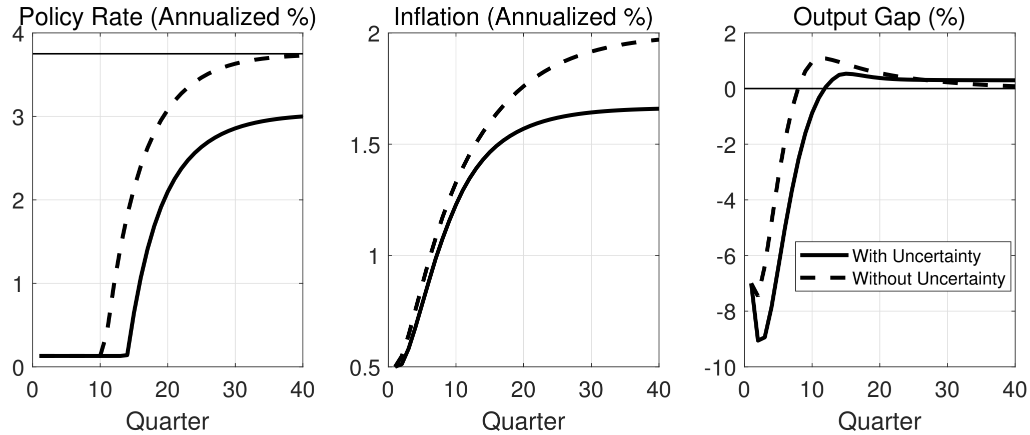
	Inflation	Output gap	Policy rate
Deterministic steady state	2	0	3.75
Risky steady state	1.66	0.29	2.94
(Wedge)	(−0.34)	(0.29)	(−0.81)
Risky steady state w/o the ELB	1.91	0.01	3.48
(Wedge)	(−0.09)	(0.01)	(−0.27)

Consistent with our earlier analyses based on a stylized model, inflation and the policy rate are lower, and the output gap is higher, at the risky steady state than at the deterministic steady state. Inflation falls 34 basis points below the target rate of inflation at the risky steady state. This is large given the small standard deviation of inflation. The policy rate at the risky steady state falls 81 basis points below its deterministic counterpart and is 2.94 percent. The risky steady state policy rate of 2.94 percent is in line with the average of the median projections of the long-run federal funds rate in the Summary of Economic Projections over the past few years. Finally, the output wedge between the deterministic and risky steady states is small, with the output gap standing at 0.29 percentage point at the risky steady state.

As explained in the previous section, the discrepancy between the deterministic and risky steady states is not only driven by the lower bound constraint on policy rates, but is also affected by other nonlinear features of the model. To isolate the effects of the lower bound constraint, the fourth line of Table 5 shows the risky steady state of the model without the lower bound constraint. Inflation, the output gap, and the policy rate are 1.91, 0.01, and 3.48 percent, respectively. Thus, most of the wedge between the deterministic and risky steady states in the model with the ELB constraint is attributed to the nonlinearity associated with the ELB constraint, as opposed to other nonlinear features of the model. For inflation, the ELB risk accounts for 25 basis points of the overall steady state deflationary bias.

To visualize the difference between the deterministic and risk steady state in our economy, Figure 6 contrasts the paths of the model's key variables from our empirical model (shown

Figure 6: The Effect of the ELB Risk: A Recession Scenario



by solid black lines) to those from a perfect foresight version of our model that abstracts from uncertainty (shown by dashed black lines). In each version of the model, the size of the initial shock is set so that the inflation rate and the output gap at time 1 are 0.5 percent and -7 percent, respectively. Under the perfect foresight version of the model, the model's endogenous variables eventually converge to the determistic steady state. The policy rate leaves the ELB after 10 quarters and gradually returns to its deterministic steady state of 3.75 percent. Inflation increases monotonically and converges to 2 percent, whereas the output gap converges to zero. In our model that correctly takes into account the effect of uncertainty on the private sector's decision making, the policy rate leaves the ELB after 15 quarters and eventually converges to its risky steady state of 2.94 percent. Inflation monotonically increases, but never reaches the central bank's inflation objective of 2 percent, whereas the output gap eventually converges to a small positive value.

4.4 Long-run interest rates

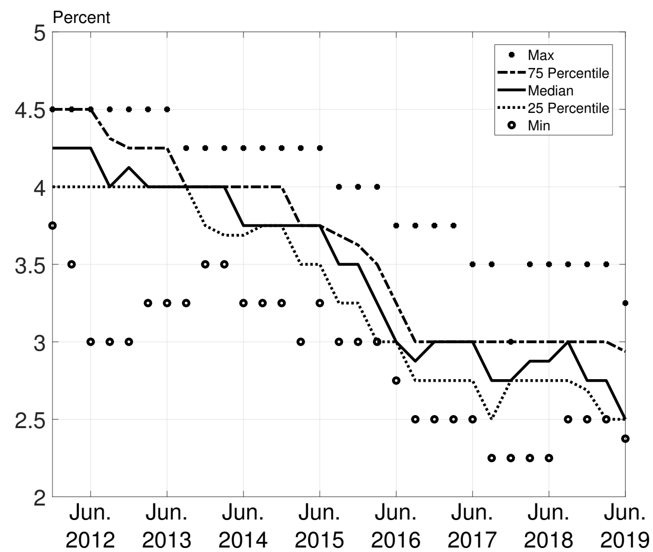
There are substantial uncertainties surrounding the level of the long-run real equilibrium interest rate on short-term risk free assets. Many economists recently have argued that various structural factors—including a lower trend growth rate of productivity, demographic trends, and global factors—have contributed to a persistent downward trend in long-run equilibrium interest rates of risk free assets.[18] A lower long-run equilibrium interest rate means that the probability of hitting the ELB is higher, which ceteris paribus increases the magnitude of the undershooting of the inflation target at the risky steady state.

The high degree of uncertainty regarding the long-run equilibrium interest rate in the United States is rflected in U.S. policymakers' long-run projections for the federal funds rate in the Summary of Economic Projections (SEPs) released four times a year. According to Figure 7, at any given point in time, there is a wide range of views regarding the long-run

[18]See, for example, Hamilton, Harris, Hatzius, and West (2015) and Rachel and Smith (2015).

level of the federal funds rate. In the June-2019 SEPs, the lowest and highest projections are 2.38 percent and 3.25 percent, respectively. The projected rates have come down quite a bit over the past six years since the beginning of SEPs. The median projection was 4.25 percent in early 2012, versus 2.5 in the June-2019 SEPs.

Figure 7: The Evolution of the Long-Run Federal Funds Rate Projections



Source: Data come from the Summary of Economic Projections (https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm or https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm).

Figure 8 shows how sensitive the risky steady state of our empirical model is to alternative assumptions about the deterministic steady state interest rate. In this exercise, we vary the long-run deterministic steady state policy rate by varying the trend growth rate. As shown in the top-left panel, the probability of the policy rate being at the ELB increases as the deterministic steady state policy rate declines. With the deterministic steady state policy rate at 3.4 percent—at the left edge of the panel—the probability of being at the ELB is 29 percent.[19] A higher probability of being at the ELB increases the wedge between the deterministic and risky steady states. With the deterministic steady state policy rate at 3.4 percent, the risky steady state inflation, output, and the policy rates are 1.35 percent, 0.65 percent, and 2.05 percent.[20]

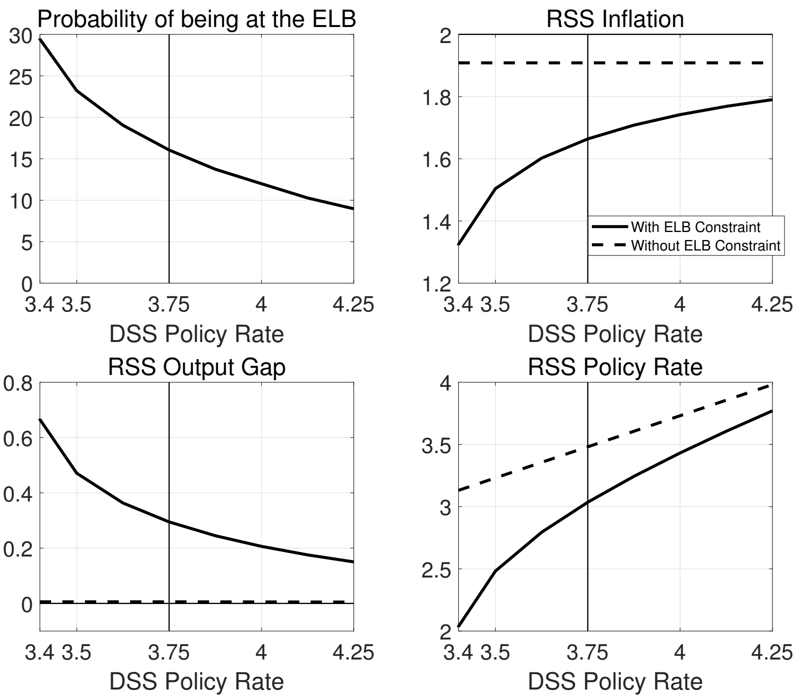
The lower the deterministic steady state of the policy rate, the larger the fraction of the overall wedge between the risky and deterministic steady states of output and inflation explained by the ELB risk. This follows from the fact that risky steady state output and inflation are an increasing function of the deterministic steady state policy rate in the model with the ELB constraint, whereas they are insensitive to the policy rate in the model without ELB constraint. For inflation, the ELB risk accounts for 55 basis points of the overall steady

[19]This probability is comparable to the estimate in Kiley and Roberts (2017).

[20]Note that an increase in the output gap does not necessarily mean an increase in the level of output because output measures are stationarized by the trend growth rate.

state deflationary bias of 65 basis points when the deterministic steady state policy rate is 3.4 percent.[21]

Figure 8: Long-Run Interest Rates and the Risky Steady State†



†DSS stands for "deterministic steady state," and RSS stands for "risky steady state." Solid vertical lines mark the deterministic steady state policy rate in the baseline calibration (3.75 percent).

4.5 Unconventional monetary policies

In this subsection, we analyze how our quantitative results are affected by two unconventional monetary policies that have been used by several central banks in the aftermath of the Global Financial Crisis. The first is forward guidance to keep the policy rate lower for longer than would be warranted by future economic conditions alone. The second is negative interest rate policy.

4.5.1 "Lower-for-longer" forward guidance

In our model, "lower-for-longer" forward guidance is captured by the policy inertia parameter, ρ_R. Unlike an interest-rate feedback rule that responds to the lagged actual policy rate, our policy rule responds to the lagged shadow policy rate—see equation (29)—and thus makes the period for which the policy rate is kept at the ELB beyond the point in time

[21]Hamilton, Harris, Hatzius, and West (2015) argue that any value between 0 and 2 percent is a plausible value for the long-run real rate. Thus, the long-run nominal rate of 2.05 percent—or equivalently, the long-run real rate of 0.05 percent—in this example is within their plausible range.

where current economic conditions would call for an increase in the policy rate depend on the severity of the previous economic downturn.[22]

In Figure 9, we show the dynamics of the economy in the recession scenario considered in Figure 6 for three alternative values of ρ_R—0.85, 0.9 and 0.95—as well as those under the baseline value of 0.8. According to the figure, a higher value of the policy inertia parameter is associated with higher inflation and output at the ELB as well as a shorter ELB duration.[23] Even though a higher policy inertia parameter would lead to a longer ELB duration if the path of inflation and output were unchanged, a longer ELB duration mitigates the declines in inflation and output at the ELB through expectations, which puts downward pressure on the ELB duration. In equilibrium, the ELB duration is shorter with a higher policy inertia parameter. It is interesting to note that, if the policy inertia parameter is sufficiently high, inflation is above 2 percent even when the policy rate is at the ELB.

Figure 9: A Recession Scenario with Forward Guidance

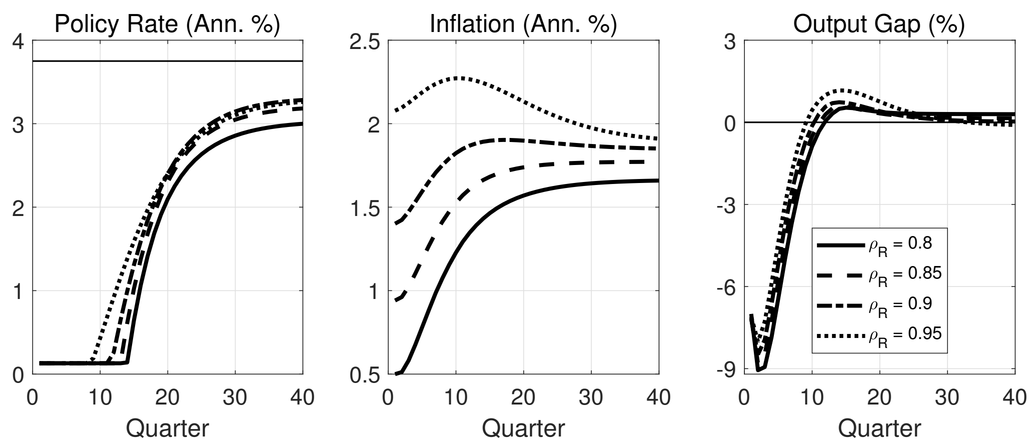
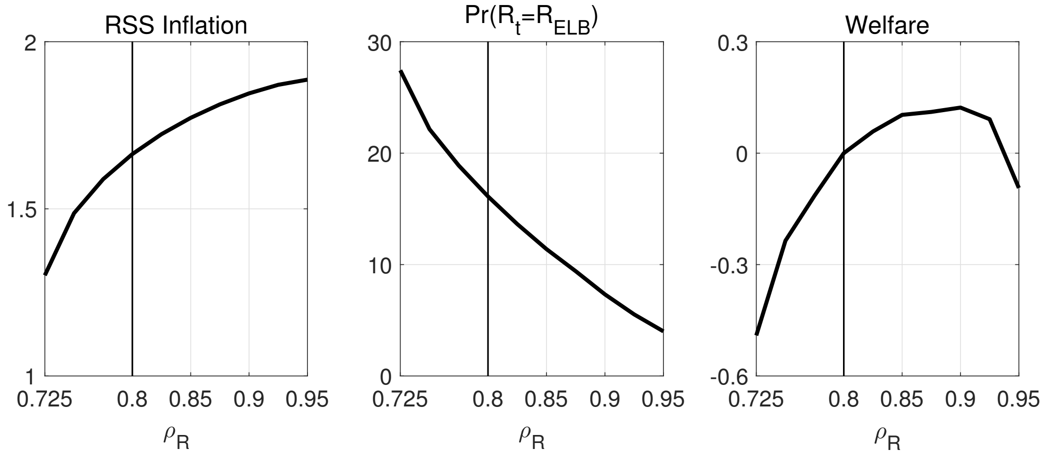


Figure 10 shows how the risky steady state inflation, the probability of being at the ELB, and welfare depend on the policy inertia parameter. The risky steady state inflation is higher with a higher policy inertia. The probability of being at the ELB is lower because a higher risky steady state inflation is associated with a higher risky steady state policy rate and also because a higher policy inertia lowers the volatility of the policy rate. With better stabilization outcomes both at and away from the ELB, welfare is higher with a higher policy inertia parameter, but only up to a point, as shown in the right panel of Figure 10. If the policy inertia is sufficiently high, the economy suffers from too high inflation, and a further increase in the policy inertia parameter lowers welfare.

[22] See Gust, Herbst, López-Salido, and Smith (2017), Hills and Nakata (2018), and Nakata and Schmidt (2019c) for more details.

[23] Bilbiie (2018) shows that when the policy rate is temporary at the ELB due to a confidence shock, "lower-for-longer" forward guidance reduces output and inflation at the ELB.

Figure 10: The Risky Steady State Inflation, Probability of Being at the ELB, and Welfare: Forward Guidance



4.5.2 Negative interest rate policy

In our baseline calibration, we imposed a lower bound of 0.13 (annualized) percent based on the mid-point of the target range for the federal funds rate in place from December 2008 to December 2015 in the U.S. However, other central banks lowered the policy rate into negative territory.

Figure 11: A Recession Scenario with Negative Interest Rate Policy

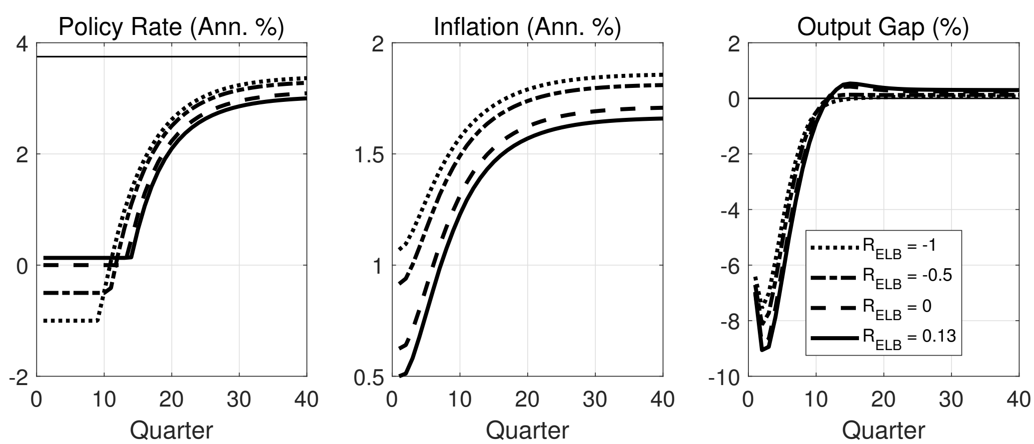
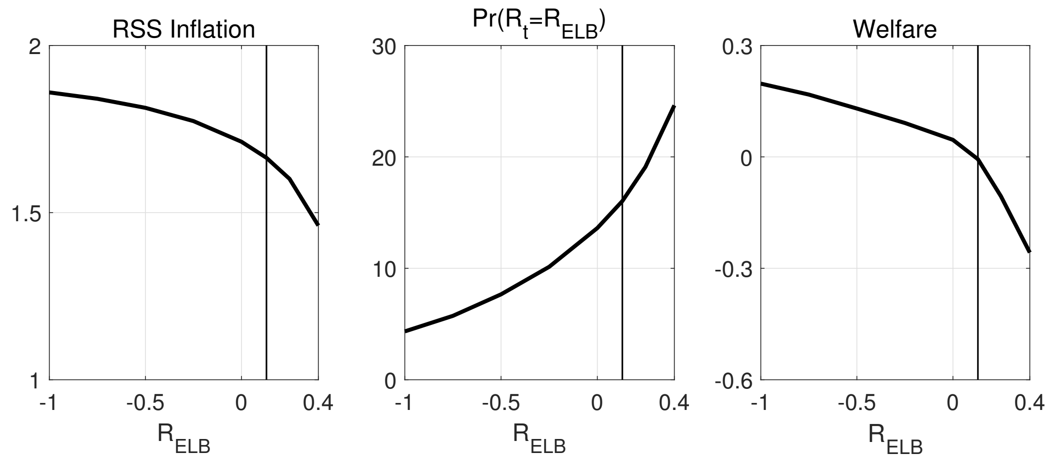


Figure 11 shows the dynamics of the economy in the same recession scenario considered earlier under three alternative values of the lower bound—0, -0.5 and -1—as well as under the baseline value. Not surprisingly, the declines in output and inflation at the ELB are smaller the lower the ELB, because there is more scope for the central bank to lower the policy rate if the ELB is lower.[24] The better stabilization outcomes at the ELB mitigate the deflationary bias problem away from the ELB through expectations. As shown in the left panel of Figure 12, the risky steady state inflation is higher if the ELB is lower. Consistent

[24]Cuba-Borda and Singh (2019) shows that when the policy rate is at the ELB due to a confidence shock, lowering the ELB reduces output and inflation at the ELB.

with higher risky steady state inflation, the probability of being at the ELB is lower when the ELB is lower. Finally, a lower ELB is associated with higher welfare because a lower ELB improves allocations both at and away from the ELB. In our model, there is no cost of lowering the policy rate into negative territory. As a result, welfare monotonically increases as we decrease the ELB.[25] This feature is in stark contrast to "lower-for-longer" forward guidance policy in which a further increase in the policy inertia parameter lowers welfare when the policy inertia parameter is sufficiently high.

Figure 12: The Risky Steady State Inflation, Probability of Being at the ELB, and Welfare: Negative Interest Rate Policy



Note: The horizontal axis is expressed in the unit of annualized percent.

5 The risk-adjusted monetary policy rule

If the ELB risk makes the task of the central bank in meeting its inflation objective more difficult, a natural question is what, if anything, the central bank can do to counteract the ELB risk. In the previous section, we have shown that unconventional monetary policies—"lower-for-longer" forward guidance and negative interest rate policy—that improve stabilization outcomes at the ELB can mitigate the deflationary bias away from the ELB through expectations. These two policies improve welfare, but fall short of achieving 2 percent inflation at the risky steady state. In this section, we propose a simple modification of the standard interest rate rule (28)-(29)—the *risk-adjusted* monetary policy rule—that allows the central bank to achieve its inflation objective at the risky steady state.[26]

[25]As recently argued by Brunnermeier and Koby (2019) and Eggertsson, Juelsrud, Summers, and Wold (2019), if the lower bound is sufficiently negative, a further reduction in the policy rate can be contractionary. Because our model abstracts from any costs a negative policy rate may have on the economy, the effect of lowering the ELB in our model can be seen as the upper bound of what the central bank can actually achieve by lowering the policy rate into negative territory in reality.

[26]We interpret the central bank's inflation objective as specifying the desired rate of inflation at the risky steady state, as opposed to the desired unconditional average rate of inflation. See Appendix F for an extensive discussion on this interpretation.

5.1 The proposed rule

Let the *intercept-adjusted* monetary policy rule be given by (28) and

$$R_t^* = S_R \frac{a^{\chi_c} \Pi^{targ}}{\beta} \left(\frac{R_{t-1}^*}{S_R \frac{a^{\chi_c} \Pi^{targ}}{\beta}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}} \right)^{(1-\rho_R)\phi_y}, \tag{34}$$

where S_R that appears in front of $\frac{a^{\chi_c}\Pi^{targ}}{\beta}$ is the intercept adjustment term and $S_R \frac{a^{\chi_c}\Pi^{targ}}{\beta}$ is the adjusted intercept. We call S_R an intercept-adjustment term because this term would show up as the intercept in the linearized version of the policy rule.

A key feature of equation (34) is that the presence of the intercept-adjustment parameter breaks the standard link between the intercept and the model's structural parameters. When the standard monetary policy rule is specified in the context of structural models, the intercept is a function of the model's structural parameters—a, β, χ_c, and Π^{targ} in our model—as seen in equation (33). Thus, under the standard monetary policy rule, one needs to adjust at least one of the structural parameters of the model to change the intercept. Under the intercept-adjusted policy rule, the intercept is a free parameter of the model.

Our proposal is to use the intercept-adjusted policy rule with the value for S_R chosen so that the risky steady state inflation and the inflation target parameter coincide. We will refer to the intercept-adjusted policy rule with S_R so chosen as the *risk-adjusted* policy rule. The size of the intercept adjustment that equates the risky steady state of inflation to the inflation target parameter depends on the specifics of the model and needs to be found numerically.

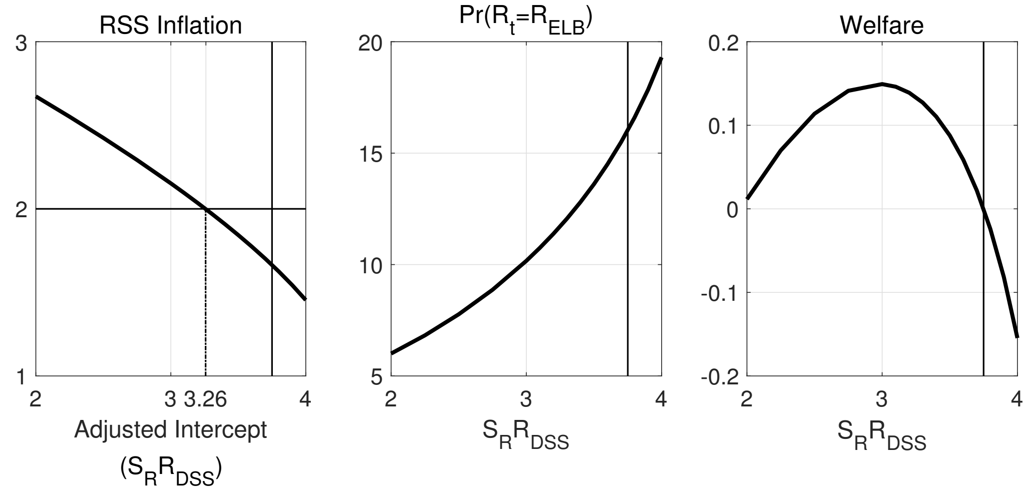
The left panel in Figure 13 shows how the risky steady state of inflation changes with the adjusted intercept in the empirical model of the previous section. Since the risky steady state of inflation is below 2 percent under the standard policy rule, the necessary adjustment would be to lower the intercept, as a lower intercept implies, all else being equal, a more accommodative policy stance, generating upward pressures on inflation. According to the panel, lowering the intercept by about 50 basis point (from 3.75 percent to 3.26 percent) leads to the risky steady state inflation of 2 percent.

Table 6 reports the risky steady state of the empirical model under the risk-adjusted monetary policy rule. By construction, under the risk-adjusted policy rule, the risky steady state of inflation is 2 percent. Even though the intercept in the policy rule is lower in the risk-adjusted policy rule than in the standard policy rule, the risky steady state policy rate is higher—3.45 percent (versus 2.94 percent in the model with the standard monetary policy rule)—reflecting a higher risky steady state inflation.[27]

The risky steady state output gap is positive, but it is a bit lower under the risk-adjusted policy rule than under the standard policy rule. The lower risky steady state output gap is explained by the lower frequency of being at the ELB (11.8 percent under the risk-adjusted

[27] If the long-run natural rate of interest is interpreted as the level of the policy rate that is consistent with the achievement of the central bank's inflation objective, then the ELB risk can be seen as pushing down the long-run natural rate of interest.

Figure 13: The Risky Steady State Inflation, Probability of Being at the ELB, and Welfare: The Risk-Adjusted Monetary Policy Rule



Notes: The figure shows how the risky steady state inflation varies with the adjusted intercept. Both the risky steady state inflation and the adjusted intercept are expressed in annualized percent.

rule versus 16.2 percent under the standard rule), which in turn is explained by the higher risky steady state policy rate. This positive relationship between the intercept adjustment term and the probability of being at the ELB can be also seen in the middle panel of Figure 13. Note that the risky steady state policy rate is higher than the intercept of the policy rule (3.45 percent versus 3.26 percent) because inflation is at the target and the output gap is positive at the risky steady state.

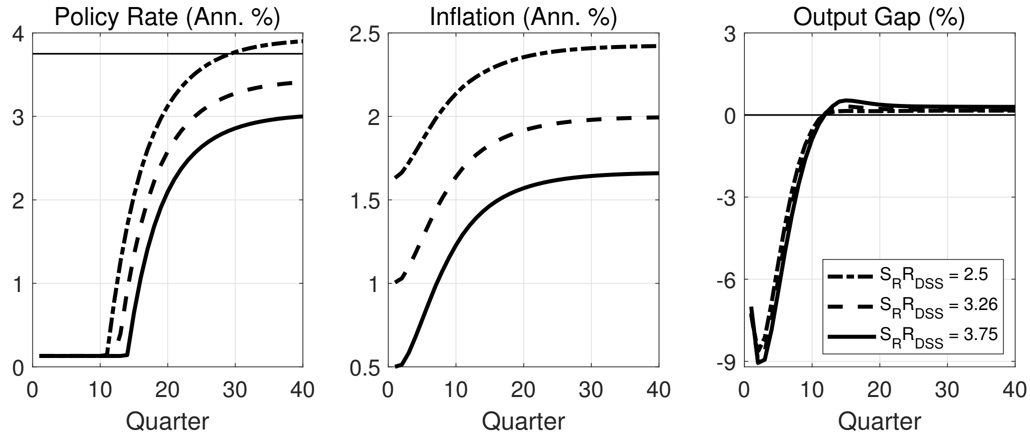
Table 6: The Risky Steady State with the Risk-Adjusted Monetary Policy Rule

	Inflation	Output Gap	Policy Rate
Risky steady state	2	0.19	3.45
Adjusted Intercept			3.26
Prob($R_t = R_{ELB}$)			11.8%

Figure 14 shows the dynamics of the economy in the same recession scenario as in the previous section under three alternative values of the intercept ($S_R R_{DSS} =$ (2.5, 3.26, 3.75)) to understand the implications of adopting the risk-adjusted monetary policy rule at the ELB. According to the figure, the higher inflation away from the ELB associated with a lower intercept adjustment term mitigates the declines in output and inflation through expectations.[28] The smaller declines in inflation and output at the ELB in turn are associated with a shorter ELB duration.

[28] As discussed shortly, lowering the intercept of the policy rule is mathematically equivalent to increasing the inflation target parameter. As discussed in Cuba-Borda and Singh (2019), an increase in the inflation target does not affect the allocations in the deflationary equilibrium, provided that there is no possibility of switching to the target regime. If there is a positive probability of switching to the target regime, Nakata and Schmidt (2019b) and Coyle and Nakata (2019) show that an increase in the inflation target worsens allocations at the ELB.

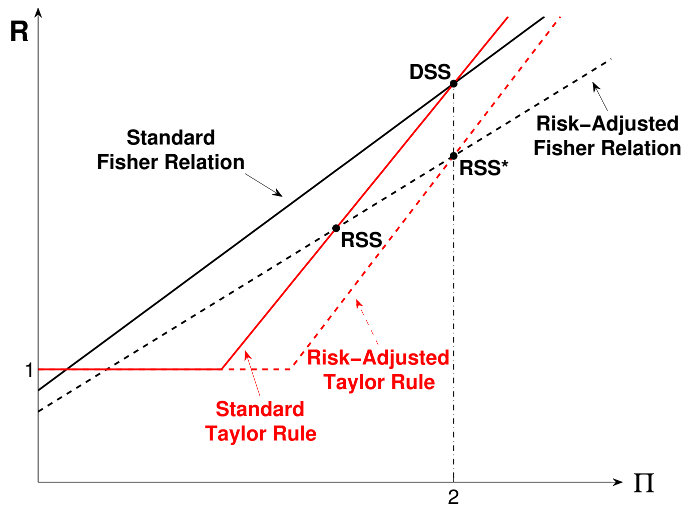
Figure 14: A Recession Scenario with the Risk-Adjusted Monetary Policy Rule



Turning to the right panel of Figure 13, welfare increases as we lower the intercept of the policy rule from the baseline value—indicated by the thin vertical line. However, if the intercept is sufficiently low, a further decrease in the intercept lowers welfare. As discussed in the next subsection, lowering the intercept of the policy rule is mathematically equivalent to increasing the inflation target. As a result, in terms of welfare, lowering the intercept involves involves costs and benefits similar to those associated with increasing the inflation target. On the one hand, a lower intercept is associated with more inefficient outcomes in normal times because inflation is too high. On the other hand, a lower intercept is associated with a lower ELB frequency and better stabilization outcomes at the ELB. The intercept adjustment that maximizes welfare is lower than the level that induces the risky steady state inflation of 2 percent, which is 3.26 percent.

To better understand how our proposed policy rule works, it is useful to graphically illustrate how the intercept adjustment affects the risky steady state of inflation and the policy rate. Figure 15 plots the standard Fisher relation, the risk-adjusted Fisher relation, the standard policy rule, and the risk-adjusted policy rule. The risky (deterministic) steady state of the model with the standard policy rule is given by the intersection of the risk-adjusted (standard) Fisher relation and the standard policy rule. The risky steady state of the model with the risk-adjusted policy rule is given by the intersection of the risk-adjusted Fisher relation and the risk-adjusted policy rule. According to the figure, and as discussed in Section 3.4, the presence of risk lowers the line representing the Fisher relation, pushing down the steady state inflation rate in the absence of an adjustment in the policy rule. The risk adjustment in the policy rule lowers the level of the policy rate corresponding to a given inflation rate, pushing up the risky steady state inflation back to the level consistent with the inflation target parameter.

Figure 15: The Risk-Adjusted Fisher Relation and the Risk-Adjusted Monetary Policy Rule



†DSS stands for "deterministic steady state," and RSS stands for "risky steady state."

5.2 Comparison with the inflation target parameter adjustment

The intercept adjustment in equation (34) is mathematically equivalent to an adjustment in the value assigned to the inflation target parameter in the standard policy rule. Let Π^{CB} be the value of the central bank's inflation objective. The standard policy rule with an adjusted value for the inflation target parameter is given by

$$R_t^* = \frac{a^{\chi_c} S_\Pi \Pi^{CB}}{\beta} \left(\frac{R_{t-1}^*}{\frac{a^{\chi_c} S_\Pi \Pi^{CB}}{\beta}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{S_\Pi \Pi^{CB}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{\tilde{Y}}} \right)^{(1-\rho_R)\phi_y},$$

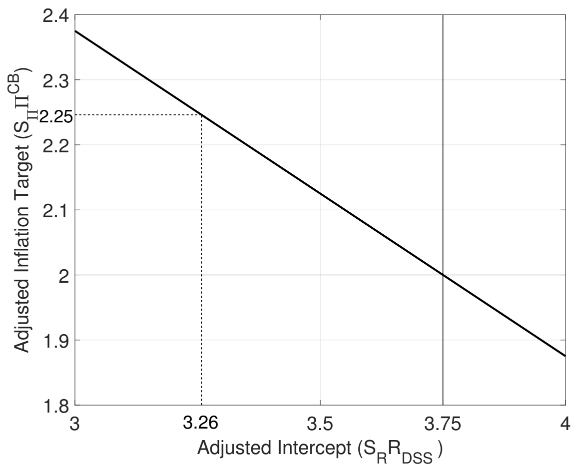
where S_Π is the inflation target adjustment term and $S_\Pi \Pi^{CB}$ is the adjusted value assigned to the inflation target parameter. This approach has a precedent. With the interpretation that the model's unconditional average of inflation corresponds to the central bank's inflation objective, Reifschneider and Williams (2000) proposed an upward adjustment in the value assigned to the inflation target parameter in the standard policy rule so that the model's unconditional average of inflation is 2 percent, which is the Federal Reserve's inflation objective.

To see the equivalence between the intercept-adjusted policy rule with $\Pi^{targ} = \Pi^{CB}$ and the standard policy rule with an adjusted value assigned to the inflation target parameter ($\Pi^{targ} = S_\Pi \Pi^{CB}$), notice that there is a one-to-one mapping between the intercept-adjustment term and the inflation target adjustment term:

$$S_R = S_\Pi^{1-\phi_\pi}. \tag{35}$$

When the Taylor principle is satisfied (i.e., $\phi_\pi > 1$), this relationship implies that an adjustment to lower the intercept (i.e., a lower S_R) is equivalent to an adjustment to increase the inflation target parameter (i.e., a higher S_{Π}). Figure 16 shows this mapping between the adjusted intercept and the adjusted inflation target parameter in our model.

Figure 16: Mapping Between the Adjusted Intercept and the Adjusted Inflation Target Parameter



Note: The figure shows the mapping between the adjusted intercept and the adjusted inflation target parameter. Both the adjusted intercept and the adjusted inflation target parameter are expressed in annualized percent.

An advantage of the risk-adjusted policy rule over the standard policy rule with an adjusted value assigned to the inflation target parameter is that the former allows for a simple structural interpretation of the inflation target parameter. In the risk-adjusted policy rule, the inflation target parameter can be naturally interpreted as the central bank's inflation objective because the risky steady state of inflation coincides with the value of the inflation target parameter. In the standard policy rule, the risky steady state of inflation is different from the value assigned to the inflation target parameter unless the model is linear, complicating the interpretation of the inflation target parameter. In our model, assigning the value of 2.25 percent to the inflation target parameter achieves the risky steady state inflation of 2 percent, as seen in Figure 16. However, in this setup, the value assigned to the inflation target parameter is simply a number that allows the model to generate the risky steady state of inflation so that it is consistent with the central bank's inflation objective, and does not have any structural interpretation.

The desirability of being able to interpret the inflation target parameter as the central bank's inflation objective depends on the purpose of the analysis. In many estimation exercises using U.S. data, the estimated value for the inflation target parameter can substantially differ from 2 percent, depending on the sample used for estimation. The discrepancy between the estimated value and the Federal Reserve's target rate of 2 percent may not pose any issue if the goal of estimation is only to fit the data and understand the past.

However, if the goal of the analysis is to think about the implications of alternative policy rules for the economic outlook and if such analyses are used to inform policymakers, it is typically useful—from the perspective of communication between modelers and policymakers—if the inflation target parameter in the policy rule can be simply interpreted as the central bank's inflation objective. For example, in the EDO Model—short for Estimated Dynamic Optimization-based Model—which is used at the Board of Governors of the Federal Reserve System for various policy analyses, the inflation target parameter in the policy rule is set to 2 percent, the Federal Reserve's target rate of inflation, to allow for a simple interpretation of the inflation target parameter (Chung, Kiley, and Laforte, 2010).[29] Similarly, in the New Area-Wide Model of the Euro Area, the DSGE model used for policy analyses at the European Central Bank, the inflation target parameter is interpreted as the monetary authority's long-run inflation objective and is set to 1.9 percent, which is "consistent with the ECB's quantitative definition of price stability of inflation below, but close to 2 percent" (Christoffel, Coenen, and Warne, 2008).[30]

6 Empirical relevance

In this section, we discuss why the steady-state undershooting of the inflation target arising from the anticipation effects of the ELB risk in our model might be empirically relevant. Our main result on the deflationary bias depends on a number of assumptions, in particular that (i) that firms attach a positive probability to the event that the ELB will bind again in the future, (ii) that this ELB risk lowers their inflation expectations, and (iii) that lower inflation expectations lead firms to lower their prices today. We are not aware of any direct evidence on each of these three assumptions. However, available measures of ELB risk and long-run inflation expectations are supportive for the first two assumptions. There is little evidence regarding the third assumption. Finally, we demonstrate that U.S. policymakers are concerned about the possibility that long-run inflation expectations might be slipping down.

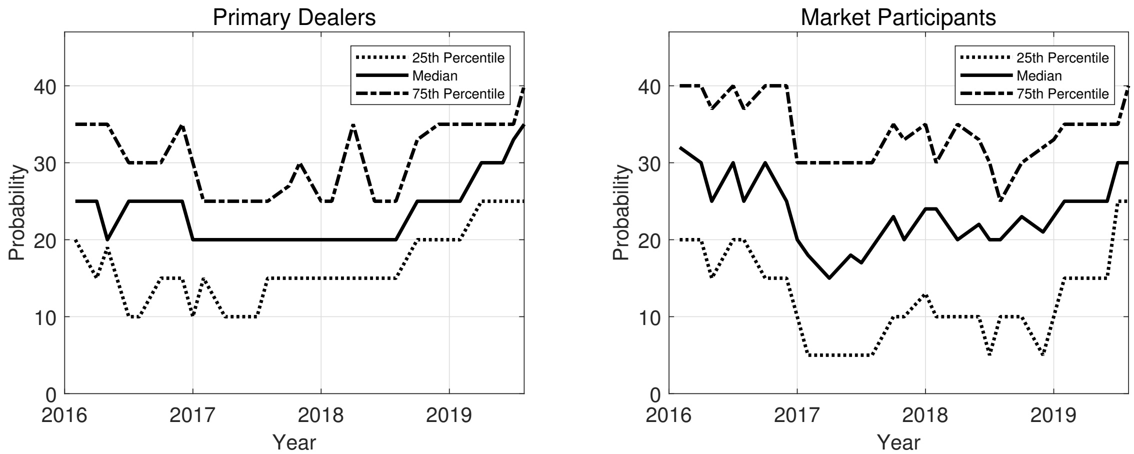
6.1 On the ELB frequency

Figure 17 shows the evolution of subjective measures of the medium-term ELB risk—defined as the probability of returning to the ELB at least once over the next 3 years or so—in the Survey of Primary Dealers (SPD) and the Survey of Market Participants (SMD).

[29] Chung, Kiley, and Laforte (2010) explain their calibration choice by saying, "some important determinants of steady-state behavior were calibrated to yield growth rates of GDP and associated price indexes that corresponded to "conventional" wisdom in policy circles, even though slight deviations from such values would have been preferred (in a "statistically significant" way) to our calibrated values."

[30] These models are used for policy analyses in the context of the ELB on nominal interest rates (see, for instance, Chung, Laforte, Reifschneider, and Williams (2012) and Coenen and Warne (2014)). Due to the computational difficulty of globally solving large-scale DSGE models, the models are currently solved and simulated using solution methods that rely on the assumption that certainty equivalence holds ("perfect foresight" assumption). As a result of this assumption, the risky steady state coincides with the deterministic steady state in these applications.

Figure 17: Medium-term ELB Risk



Note: Data come from the Survey of Primary Dealers (https://www.newyorkfed.org/markets/primarydealer_survey_questions) and the Survey of Market Participants (https://www.newyorkfed.org/markets/survey_market_participants).

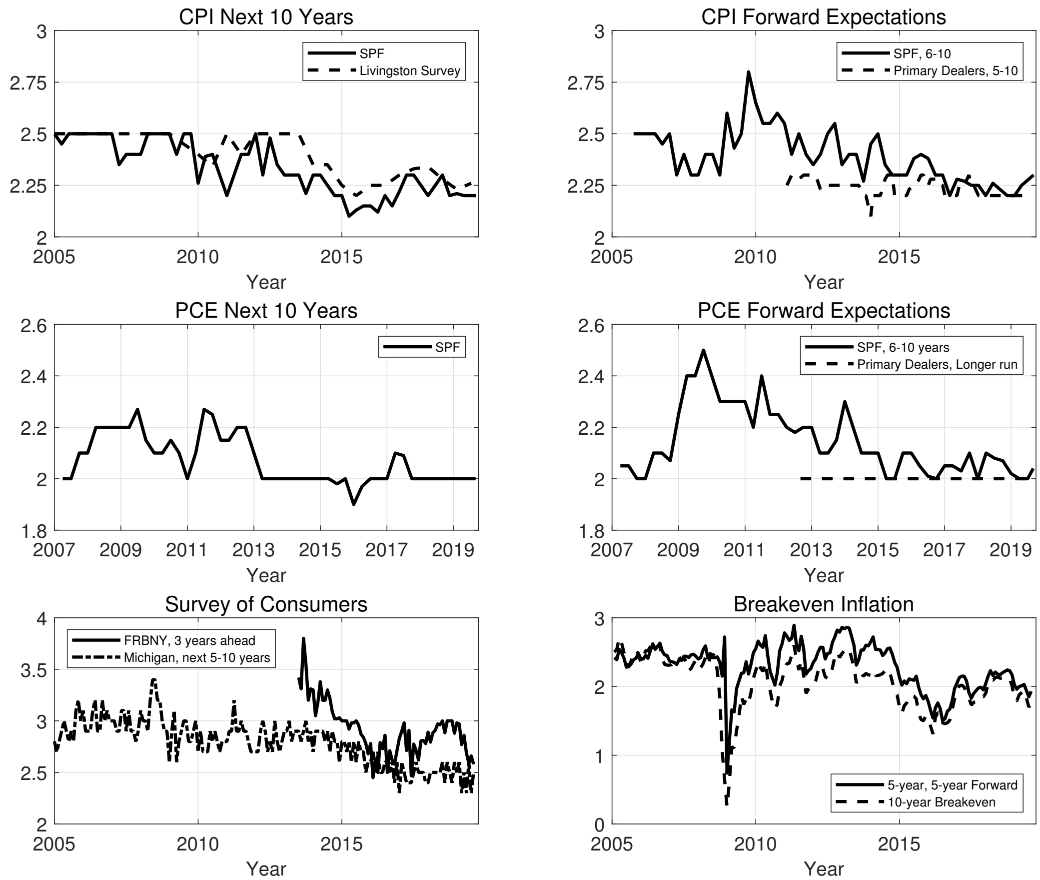
According to the figure, the respondents in these surveys have attached non-trivial probabilities of returning to the ELB since the liftoff that occurred in December 2015. Right after the liftoff, the probabilities were high, reflecting the fact that the federal funds rate was only slightly above the ELB. The SPD measure of ELB risk came down to 20 percent in early 2017, stayed at that level until mid 2018. The SMD measure of ELB risk hovered around 20 percent in 2017 and 2018. Both measures have edged up a bit since later 2018. As of July 2019, the median probabilities for the SPD and the SMD are 35 percent and 30 percent, respectively.

What matters in our model is the probability that firms attach to the event of returning to the ELB in the future. Unfortunately, we are not aware of any survey asking firms about their subjective assessment of the probability of returning to the ELB constraint in the future. Nonetheless, the very high probabilities that financial market participants attach to the event of returning to the ELB are supportive of the general idea that economic agents are cognizant of the fact that the ELB constraint can bind again.

6.2 On long-run inflation expectations

Figure 18 shows various measures of long-run inflation expectations in the recent past. Top-left and top-right panels show the evolution of long-run CPI inflation expectations from the Survey of Professional Forecasters (SPF), the SPD, and the Livingston Survey. Middle-left and middle-right panels show the evolution of long-run PCE inflation expectations from the SPF and the SPD. The bottom-left panel shows the inflation expectations over the next three years from the Survey of Consumer Expectations as well as the inflation expectations over the next 5-10 years from the University of Michigan's Survey of Consumers. Finally, the bottom-right panel shows two measures of breakeven inflation rates.

Figure 18: Long-Run Inflation Expectations



Source: The Survey of Professional Forecasters (SPF) data come from FRB Philadelphia (https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters), as does the Livingston Survey (https://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey/historical-data). The primary dealer series are taken from FRB New York's Survey of Primary Dealers (https://www.newyorkfed.org/markets/primarydealer˙survey˙questions). The FRB New York series for inflation expectations three years ahead comes from the FRB NY Survey of Consumer Expectations (https://www.newyorkfed.org/microeconomics/sce). The series from the University of Michigan Survey of Consumers is available here (http://www.sca.isr.umich.edu/). The breakeven inflation data come from FRED. All sources accessed on August 15, 2019.

According to the figure, over the last several years, many—albeit not all—measures have drifted down and they are currently at the lower ends of their historical ranges. The declines in these measures coincided with the decline in the estimates of the long-run equilibrium real rate, as shown earlier in Figure 7. This pattern is consistent with our model's prediction because a lower long-run equilibrium real rate implies a higher ELB frequency, as shown in Figure 8.

The extent to which lower long-run inflation expectations are pulling down inflation is an empirical question. To our knowledge, there are few research papers investigating the causal relationship between inflation expectations and firm's pricing decisions and the evidence is mixed. Using a survey of Italian firms, Coibion, Gorodnichenko, and Ropele (2019) show that higher inflation expectations lead firms to raise their prices. Using a survey of New Zealand firms, Coibion, Gorodnichenko, and Kumar (2018) show that higher inflation expectations do not necessarily lead them to change prices in a statistically or economically significant way.[31] Also, even if inflation expectations affect the behavior of households and firms, the effects may not be as strong as standard rational-expectations models suggest.

6.3 Policymakers' concerns

While there is uncertainty about whether long-run inflation expectations are currently below 2 percent, and if so, whether below-target inflation expectations are pushing downward pressure on inflation today, these possibilities have been real concerns for U.S. monetary policymakers who have been struggling to raise inflation to their objective of 2 percent. While there are many hypotheses about the cause of the persistent inflation undershoot, some policymakers have stated that a higher ELB risk might be causing long-run inflation expectations to be below 2 percent and that such below-target long-run inflation expectations in turn are causing inflation to stay below the target, sometimes citing the previous version of our paper.[32] For example, Williams (2019) states:

> *"Even in times when policy is not constrained, the expectation of below-target inflation in the future affects current decisions, putting additional downward pressure on inflation. In other words, monetary policy is always swimming upstream, fighting a current of too-low inflation expectations that interferes with achieving the target inflation rate."*

Our paper directly speaks to this concern by U.S. policymakers on the inflation undershooting by showing that a moderate degree of ELB risk can quantitatively explain a nontrivial degree of the inflation undershoot when the federal funds rate is away from the ELB.

7 Additional results and discussion

7.1 On the importance of other nonlinear features

As we saw in Section 4, the effect of nonlinearities unrelated to the ELB on the risky steady state inflation can be nontrivial.

Even before the ELB literature blossomed, many papers studied the implication of various nonlinearities on the dynamics of the economy using New Keynesian models (for example,

[31] There are more research papers analyzing whether inflation expectations affect the behaviors of households, but the evidence is mixed here as well. See, for example, Ichiue and Nishiguchi (2015), D'Acunto, Hoang, and Weber (2018), Malmendier and Nagel (2016), Armantier, de Bruin, Topa, Klaauw, and Zafar (2015), Abe and Ueno (2015), Coibion, Gorodnichenko, Kumar, and Pedemonte (2018), and Duca, Kenny, and Reuter (2018) for papers finding that inflation expectations affect the behaviors of households. See Bachmann, Berg, and Sims (2015) and Burke and Ozdagli (2013) for papers finding that inflation expectations do not affect the behaviors of households.

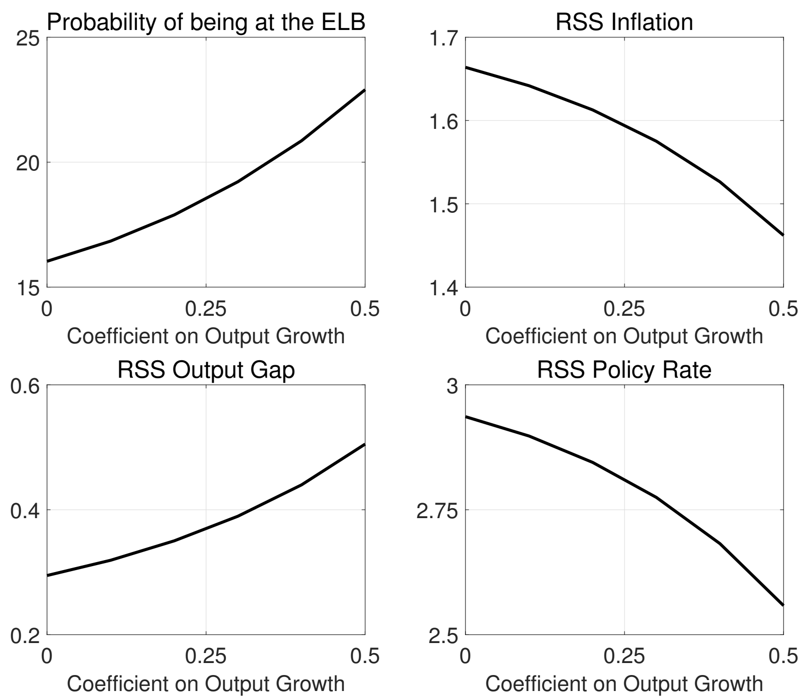
[32] See, for example, Brainard (2018), Evans (2019), and Williams (2019).

downward nominal wage rigidities by Kim and Ruge-Murcia (2011), stochastic volatilities by Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), and an Epstein-Zin preference with a high risk-aversion by Rudebusch and Swanson (2012), among many others). These papers typically use higher-order perturbation methods to solve the model, which can capture the steady-state implication of uncertainty. Nonetheless, to our knowledge, no one has pointed out that the risky steady state inflation can nontrivially differ from the inflation target, likely because these papers are focused on the implications of various nonlinearities for other aspects of the model, as opposed to the risky steady state inflation (Kim and Ruge-Murcia (2011) on asymmetric economic dynamics, Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) on the effect of fiscal volatility shock, Rudebusch and Swanson (2012) on term-premiums implications).

While our paper focuses on the implication of the ELB constraint for the risky steady state, it would be useful to study how these other realistic sources of nonlinearity affect the risky steady state inflation.

7.2 Output growth in the interest-rate rule

Figure 19: Coefficient on Output Growth and the Risky Steady State



In this section, we analyze the implication of introducing an output growth term to the interest-rate feedback rule, a common feature of many estimated DSGE models (see, for example, Smets and Wouters (2007) and Gust, Herbst, López-Salido, and Smith (2017)). Figure 19 shows how the risky steady state inflation and output gap—as well as the ELB frequency—depend on the coefficient on the output growth term. According to the figure, the larger the coefficient, the lower (higher) the risky steady state inflation (output gap). This in turn is driven by the fact that a higher coefficient leads to worse allocations at the ELB.

This result is consistent with Nakata, Schmidt, and Yoo (2018), who show speed-limit policies (policies in which the policy rate reacts positively to output growth) worsen allocations at and away from the ELB in the context of the optimal discretionary policymaker. In the aftermath of a deep recession, a speed-limit policy prescribes a higher policy rate path, because output growth is positive when the economy is recovering. However, such a higher policy rate path—the opposite of what the optimal commitment policy prescribes—lowers output and inflation at the ELB through expectations. And, lower inflation at the ELB exacerbates the deflationary bias away from the ELB through expectations.

8 Conclusion

In this paper, we have examined the implications of ELB risk—the possibility that the policy rate will be constrained by ELB in the future—for the economy when the policy rate is currently not constrained. Using an empirically rich DSGE model calibrated to capture key features of the U.S. economy since the mid 1990s, we have shown that the ELB risk causes inflation to fall below the target rate of 2 percent by about 25 basis points at the risky steady state. This deflationary bias induced by ELB risk at the risky steady state can be as much as 50 basis points under alternative plausible assumptions about the long-run growth rate of the economy. Our analysis suggests that achieving the inflation target may be more difficult now than before the Great Recession, if the recent recognition of lower long-run equilibrium real rates has led households and firms to increase their assessment of future ELB risk: prescribing policies that accommodate or adjust to such risk may therefore be the natural step moving forward.

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For Online Publication: Technical Appendix

A Details of the Stylized Model

This section describes a stylized DSGE model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and government policies.

A.1 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_t, N_t, B_t} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s\Big] \Big[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n}\Big] \tag{A.1}$$

subject to the budget constraint

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t \tag{A.2}$$

or equivalently

$$C_t + \frac{B_t}{R_t P_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + \Phi_t \tag{A.3}$$

where C_t is consumption, N_t is the labor supply, P_t is the price of the consumption good, W_t (w_t) is the nominal (real) wage, Φ_t is the profit share (dividends) of the household from the intermediate goods producers, B_t is a one-period risk free bond that pays one unit of money at period t+1, and R_t^{-1} is the price of the bond.

The discount rate at time t is given by $\beta\delta_t$ where δ_t is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. This shock follows an AR(1) process:

$$\delta_t - 1 = \rho(\delta_{t-1} - 1) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon) \tag{A.4}$$

This increase in δ_t is a preference imposed by the household to increase the relative valuation of future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

A.2 Firms

There is a final good producer and a continuum of intermediate goods producers indexed by $i \in [0, 1]$. The final good producer purchases the intermediate goods $Y_{i,t}$ at the intermediate price $P_{i,t}$ and aggregates them using CES technology to produce and sell the final good

Y_t to the household and government at price P_t. Its problem is then summarized as

$$\max_{Y_{i,t}, i\in[0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{A.5}$$

subject to the CES production function

$$Y_t = \Big[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di\Big]^{\frac{\theta}{\theta-1}}. \tag{A.6}$$

Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{i,t} = N_{i,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[33] by setting the price of its own good. We can assume that each firm receives a production subsidy τ so that the economy is fully efficient in the steady state.[34] In our baseline, however, we set $\tau = 0$. Price changes are subject to quadratic adjustment costs.

$$\max_{P_{i,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1}\Big[\prod_{s=0}^{t-1}\delta_s\Big]\lambda_t\Big[P_{i,t}Y_{i,t} - (1-\tau)W_t N_{i,t} - P_t\frac{\varphi}{2}[\frac{P_{i,t}}{P_{i,t-1}} - 1]^2 Y_t\Big] \tag{A.7}$$

such that

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t.^{35} \tag{A.8}$$

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\Big[\prod_{s=0}^{t-1}\delta_s\Big]\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

A.3 Government policies

It is assumed that the monetary authority determines nominal interest rates according to a Taylor rule

$$R_t = \max\left[R_{ELB}, \quad \frac{\Pi^{targ}}{\beta}\left(\frac{\Pi_t}{\Pi^{targ}}\right)^{\phi_\pi}\right] \tag{A.9}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. This equation will be modified in order to do an extensive sensitivity analysis of policy inertia and other rule specifications.

[33]Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level P_t which we take care of further along in the document.

[34]$(\theta-1) = (1-\tau)\theta$ which implies zero profits in the zero inflation steady state. In a welfare analysis, this would extract any inflation bias from the second-order approximated objective welfare function. τ therefore represents the size of a steady state distortion (see Chapter 5 Appendix, Galí (2008)).

[35]This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \Big[\int_0^1 P_{i,t}^{1-\theta} di\Big]^{\frac{1}{1-\theta}}$.

A.4 Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi}{2}\Big[\frac{P_{i,t}}{P_{i,t-1}} - 1\Big]^2 Y_t di \tag{A.10}$$

$$N_t = \int_0^1 N_{i,t} di \tag{A.11}$$

and

$$B_t = 0. \tag{A.12}$$

A.5 Recursive equilibrium

Given P_0 and a two-state Markov shock process establishing δ_t and γ_t, an equilibrium consists of allocations $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}, G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{i,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that (i) given the determined prices and policies, allocations solve the problem of the household, (ii) $P_{i,t}$ solves the problem of firm i, (iii) R_t follows a specified rule, and (iv) all markets clear.

Combining all of the results from (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$C_t^{-\chi_c} = \beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1} \tag{A.13}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{A.14}$$

$$\frac{Y_t}{C_t^{\chi_c}}\left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta) - \theta(1-\tau)w_t\right] = \beta\delta_t \mathrm{E_t}\frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\varphi\left(\Pi_{t+1} - 1\right)\Pi_{t+1} \tag{A.15}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\Pi_t - 1\right]^2 Y_t \tag{A.16}$$

$$Y_t = N_t \tag{A.17}$$

$$R_t = \max\left[R_{ELB}, \quad \frac{\Pi^{targ}}{\beta}\left(\frac{\Pi_t}{\Pi^{targ}}\right)^{\phi_\pi}\right] \tag{A.18}$$

Equation A.13 is the consumption Euler equation, Equation A.14 is the intratemporal optimality condition of the household, Equation A.15 is the optimal condition of the intermediate good producing firms (forward-looking Phillips Curve) relating today's inflation to real marginal cost today and expected inflation tomorrow, Equation A.16 is the aggregate resource constraint capturing the resource cost of price adjustment, and Equation A.17 is the aggregate production function. Equation A.18 is the interest-rate feedback rule.

B Solution Method

We describe our solution method using the stylized model analyzed in the main text. The extension of the method to the empirical model is straightforward.

The problem is to find a set of policy functions, $\{C(\cdot),\ N(\cdot),\ Y(\cdot),\ w(\cdot),\ \Pi(\cdot),\ R(\cdot)\}$, that solves the following system of *functional* equations.

$$C(\delta_t)^{-\chi_c} = \beta\delta_t R(\delta_t) E_t C(\delta_{t+1})^{-\chi_c}\Pi(\delta_{t+1})^{-1} \tag{B.1}$$

$$w(\delta_t) = N(\delta_t)^{\chi_n} C(\delta_t)^{\chi_c} \tag{B.2}$$

$$\frac{N(\delta_t)}{C(\delta_t)^{\chi_c}}\Big[\varphi\Big(\Pi(\delta_t)-1\Big)\Pi(\delta_t)-(1-\theta)-\theta w(\delta_t)\Big] = \beta\delta_t E_t\frac{N(\delta_{t+1})}{C(\delta_{t+1})^{\chi_c}}\varphi\Big(\Pi(\delta_{t+1})-1\Big)\Pi(\delta_{t+1}) \tag{B.3}$$

$$Y(\delta_t) = C(\delta_t) + \frac{\varphi}{2}\Big[\Pi(\delta_t)-1\Big]^2 Y(\delta_t) \tag{B.4}$$

$$Y(\delta_t) = N(\delta_t) \tag{B.5}$$

$$R(\delta_t) = \max\left[R_{ELB}, \frac{\Pi^{targ}}{\beta}\left[\frac{\Pi(\delta_t)}{\Pi^{targ}}\right]^{\phi_\pi}\right] \tag{B.6}$$

Substituting out $w(\cdot)$ and $N(\cdot)$ using equations (B.2) and (B.5), this system can be reduced to a system of four functional equations for $C(\cdot)$, $Y(\cdot)$, $\Pi(\cdot)$, and $R(\cdot)$.

$$C(\delta_t)^{-\chi_c} = \beta\delta_t R(\delta_t) E_t C(\delta_{t+1})^{-\chi_c}\Pi(\delta_{t+1})^{-1} \tag{B.7}$$

$$\frac{Y(\delta_t)}{C(\delta_t)^{\chi_c}}\Big[\varphi\Big(\Pi(\delta_t)-1\Big)\Pi(\delta_t)-(1-\theta)-\theta Y(\delta_t)^{\chi_n}C(\delta_t)^{\chi_c}\Big] = \beta\delta_t E_t\frac{Y(\delta_{t+1})}{C(\delta_{t+1})^{\chi_c}}\varphi\Big(\Pi(\delta_{t+1})-1\Big)\Pi(\delta_{t+1}) \tag{B.8}$$

$$Y(\delta_t) = C(\delta_t) + \frac{\varphi}{2}\Big[\Pi(\delta_t)-1\Big]^2 Y(\delta_t) \tag{B.9}$$

$$R(\delta_t) = \max\left[R_{ELB}, \frac{\Pi^{targ}}{\beta}\left[\frac{\Pi(\delta_t)}{\Pi^{targ}}\right]^{\phi_\pi}\right] \tag{B.10}$$

Following the idea of Christiano and Fisher (2000), we decompose these policy functions into two parts using an indicator function: One in which the policy rate is allowed to be less than zero, and the other in which the policy rate is assumed to be zero. That is, for any variable Z,

$$Z(\cdot) = I_{\{R(\cdot)\geq 1\}}Z_{unc}(\cdot) + (1 - I_{\{R(\cdot)\geq 1\}})Z_{ELB}(\cdot). \tag{B.11}$$

The problem then becomes finding a set of a *pair* of policy functions, $\{[C_{unc}(\cdot),\ C_{ELB}(\cdot)],\ [Y_{unc}(\cdot),\ Y_{ELB}(\cdot)],\ [\Pi_{unc}(\cdot),\ \Pi_{ELB}(\cdot)],\ [R_{unc}(\cdot),\ R_{ELB}(\cdot)]\}$ that solves the system of functional equations above. This method can achieve a given level of accuracy with a considerable less number of grid points relative to the standard approach.

The time-iteration method starts by specifying a guess for the values policy functions take on a finite number of grid points. The values of the policy function that are not on any of the grid points are interpolate or extrapolated linearly. Let $X(\cdot)$ be a vector of policy functions that solves the functional equations above and let $X^{(0)}$ be the initial guess of such policy functions.[36] At the s-th iteration and at each point of the state space, we solve the system of nonlinear equations given by equations (B.7)-(B.10) to find today's consumption,

[36] For all models and all variables, we use flat functions at the deterministic steady-state values as the initial guess.

output, inflation, and the policy rate, given that $X^{(s-1)}(\cdot)$ is in place for the next period. In solving the system of nonlinear equations, we use Gaussian quadrature to evaluate the expectation terms in the consumption Euler equation and the Phillips curve, and the value of future variables not on the grid points are evaluated with linear interpolation. The system is solved numerically by using a nonlinear equation solver, `dneqnf`, provided by the IMSL Fortran Numerical Library. If the updated policy functions are sufficiently close to the guessed policy functions, then the algorithm ends. Otherwise, using the updated policy functions just obtained as the guess for the next period's policy functions, we iterate on this process until the difference between the guessed and updated policy functions is sufficiently small ($\left\|vec(X^s(\delta) - X^{s-1}(\delta))\right\|_\infty < 1\text{E-}11$ is used as the convergence criteria). The solution method can be extended to models with multiple exogenous shocks and endogenous state variables in a straightforward way.

For the stylized model, we used equally spaced 201 grid points on the interval between $[1-4\sigma_\delta, 1+4\sigma_\delta]$. For the empirical model, we used 15 grid points for the discount rate shock on the interval between $[1-4.5\sigma_\delta, 1+4.5\sigma_\delta]$, 8 grid points for the technology shock on the interval between $[1-4.5\sigma_A, 1+4.5\sigma_A]$, and 8 grid points for the monetary policy shock on the interval between $[1-3\sigma_R, 1+3\sigma_R]$. We used 9 grid points for the lagged consumption on the interval between -12 and 8 percent from the steady state (normalized) consumption, 9 grid points for the lagged real wage on the interval between -2.5 and 2 percent from the steady state (normalized) real wage, and 9 grid points on the lagged shadow policy rate on the interval between -8 and 10 annualized percent.

C Details of the Empirical Model

This section describes an extension of the stylized model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and the government.

C.1 Household markets

C.1.1 Labor packer

The labor packer buys labor $N_{h,t}$ from households at their monopolistic wage $W_{h,t}$ and resells the packaged labor N_t to intermediate goods producers at W_t. The problem can be written as

$$\max_{N_{h,t}, h\in[0,1]} W_t N_t - \int_0^1 W_{h,t} N_{h,t} df \tag{C.1}$$

subject to the following CES technology

$$N_t = \left[\int_0^1 N_{h,t}^{\frac{\theta^w - 1}{\theta^w}} dh\right]^{\frac{\theta^w}{\theta^w - 1}}. \tag{C.2}$$

The first order condition implies a labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t.^{37} \tag{C.3}$$

[37] This implies that the labor packer will set the wage of the packaged labor to $W_t = \left[\int_0^1 W_{h,t}^{1-\theta^w} dh\right]^{\frac{1}{1-\theta^w}}$.

θ^w is the wage markup parameter.

C.1.2 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_{h,t},w_{h,t},B_{h,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \left[\frac{(C_{h,t} - \zeta C_{t-1}^a)^{1-\chi_c}}{1-\chi_c} - A_t^{1-\chi_c} \frac{N_{h,t}^{1+\chi_n}}{1+\chi_n}\right] \tag{C.4}$$

subject to the budget constraint

$$P_t C_{h,t} + R_t^{-1} B_{h,t} \leq W_{h,t} N_{h,t} - W_t \frac{\varphi_w}{2} \left[\frac{W_{h,t}}{a W_{h,t-1} \left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + B_{h,t-1} + P_t \Phi_t - P_t T_t \tag{C.5}$$

or equivalently

$$C_{h,t} + \frac{B_{h,t}}{R_t P_t} \leq w_{h,t} N_{h,t} - w_t \frac{\varphi_w}{2} \left[\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + \frac{B_{h,t-1}}{P_t} + \Phi_t - T_t \tag{C.6}$$

and subject to the labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t. \tag{C.7}$$

–or equivalently

$$N_{h,t} = \left[\frac{w_{h,t}}{w_t}\right]^{-\theta^w} N_t. \tag{C.8}$$

where $C_{h,t}$ is the household's consumption, $N_{h,t}$ is the labor supplied by the household, P_t is the price of the consumption good, $W_{h,t}$ ($w_{h,t}$) is the nominal (real) wage set by the household, W_t (w_t) is the market nominal (real) wage, Φ_t is the profit share (dividends) of the household from the intermediate goods producers, $B_{h,t}$ is a one-period risk free bond that pays one unit of money at period t+1, T_t are lump-sum taxes or transfers, and R_t^{-1} is the price of the bond. C_{t-1}^a represents the aggregate consumption level from the previous period that the household takes as given. The parameter $0 \leq \zeta < 1$ measures how important these external habits are to the household. Because we are including wage indexation, measured by the parameter ι_w, we assume the household takes as given the previous period wage inflation, Π_{t-1}^w, where $\Pi_t^w = \frac{W_t}{aW_{t-1}} = \frac{w_t P_t}{a w_{t-1} P_{t-1}} = \frac{w_t}{a w_{t-1}} \Pi_t^p$.

The discount rate at time t is given by $\beta\delta_t$ where δ_t is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. δ_t is assumed to follow an AR(1) process

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_{\delta,t} \quad \forall t \geq 2 \tag{C.9}$$

and δ_1 is given. The innovation $\epsilon_{\delta,t}$ is normally distributed with mean zero and standard deviation σ_δ. It may therefore be interpreted that an increase in δ_t is a preference imposed by the household to increase the relative valuation of the future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

A_t is a non-stationary total factor productivity shock that also augments labor in the utility function in order to accommodate the necessary stationarization of the model later on. See the next section for more details on this process.

C.2 Producers

C.2.1 Final good producer

The final good producer purchases the intermediate goods $Y_{f,t}$ at the intermediate price $P_{f,t}$ and aggregates them using CES technology to produce and sell the final good Y_t to the household and government at price P_t. Its problem is then summarized as

$$\max_{Y_{f,t}, f\in[0,1]} P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} di \tag{C.10}$$

subject to the CES production function

$$Y_t = \left[\int_0^1 Y_{f,t}^{\frac{\theta^p-1}{\theta^p}} di\right]^{\frac{\theta^p}{\theta^p-1}}. \tag{C.11}$$

θ^p is the price markup parameter.

C.2.2 Intermediate goods producers

There is a continuum of intermediate goods producers indexed by $f \in [0,1]$. Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{f,t} = A_t N_{f,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[38] by setting the price of its own good. Any price changes are subject to quadratic adjustment costs. φ_p will represent an obstruction of price adjustment, the firm indexes for prices—measured by ι_p—and takes as given previous period inflation Π^p_{t-1}, and $\bar{\Pi}^p$ represents the monetary authority's inflation target.

$$\max_{P_{f,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \lambda_t \left[P_{f,t}Y_{f,t} - W_t N_{f,t} - P_t \frac{\varphi_p}{2}\left[\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p} P_{f,t-1}} - 1\right]^2 Y_t\right] \tag{C.12}$$

such that

$$Y_{f,t} = \left[\frac{P_{f,t}}{P_t}\right]^{-\theta^p} Y_t.^{39} \tag{C.13}$$

[38]NOTE: Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level P_t which we take care of further along in the document.

[39]This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\left[\prod_{s=0}^{t-1}\delta_s\right]\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

A_t represents total factor productivity which follows a random walk with drift:

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t. \tag{C.14}$$

a is the unconditional rate of growth of productivity. a_t is a productivity shock following an AR(1) process:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}. \tag{C.15}$$

where $\epsilon_{a,t}$ is normally distributed with mean zero and standard deviation σ_A. This growth factor will imply that some of the variables will acquire a unit root, meaning the model will have to be stationarized. Monetary policy will also have to accommodate this growth factor as well.

C.3 Government policies

It is assumed that the monetary authority determines nominal interest rates according to a truncated notional inertial Taylor rule augmented by a speed limit component.

$$R_t = \max\left[R_{ELB}, R_t^*\right] \tag{C.16}$$

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R}\left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi}\left(\frac{Y_t}{A_t\bar{Y}}\right)^{(1-\rho_r)\phi_y}\exp(\epsilon_{R,t}) \tag{C.17}$$

where $\Pi_t^p = \frac{P_t}{P_{t-1}}$ is the inflation rate between periods $t-1$ and t, $\bar{R} = \frac{\bar{\Pi}^p a^{\chi_c}}{\beta}$ (see the section on stationarization to see why), and $\epsilon_{R,t}$ represents white noise monetary policy shocks with mean zero and standard deviation σ_R.

C.4 Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$\begin{aligned}Y_t = C_t + \int_0^1 \frac{\varphi_p}{2}\left[\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}P_{f,t-1}} - 1\right]^2 Y_t df + ...\\ ... + \int_0^1 w_t\frac{\varphi_w}{2}\left[\frac{w_{h,t}}{aw_{h,t-1}}\frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t dh\end{aligned} \tag{C.18}$$

$$N_t = \int_0^1 N_{f,t}di \tag{C.19}$$

$$C_t^a = C_t = \int_0^1 C_{h,t}dh \tag{C.20}$$

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the final good producer will set the price of the final good $P_t = \left[\int_0^1 P_{f,t}^{1-\theta^p}di\right]^{\frac{1}{1-\theta^p}}$.

and

$$B_t = \int_0^1 B_{h,t} dh = 0. \tag{C.21}$$

C.5 An equilibrium

Given P_0 and stochastic processes for δ_t, an equilibrium consists of allocations $\{C_t, N_t, N_{f,t}, Y_t, Y_{f,t}, G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{f,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that

(i) allocations solve the problem of the household given prices and policies

$$\partial C_{h,t} : (C_{h,t} - \zeta C_{t-1}^{a})^{-\chi_c} - \lambda_t = 0 \tag{C.22}$$

$$\begin{aligned}
\partial w_{h,t} : \theta^w A_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{w_t} \left(\frac{w_{h,t}}{w_t} \right)^{-\theta^w(1+\chi_n)-1} \\
+ (1-\theta^w)\lambda_t \left(\frac{w_{h,t}}{w_t} \right)^{-\theta^w} N_t \\
-\lambda_t w_t \varphi_w \left(\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1 \right) N_t \frac{\Pi_t^p}{a w_{h,t-1} \left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} \\
+\beta \delta_t \mathrm{E}_t \lambda_{t+1} w_{t+1} \varphi_w \left(\frac{w_{h,t+1}}{a w_{h,t}} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} (\Pi_t^w)^{\iota_w}} - 1 \right) N_{t+1} \frac{w_{h,t+1}}{a w_{h,t}^2} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} (\Pi_t^w)^{\iota_w}} = 0
\end{aligned} \tag{C.23}$$

$$\partial B_{h,t} : -\frac{\lambda_t}{R_t P_t} + \beta \delta_t \mathrm{E_t} \frac{\lambda_{t+1}}{P_{t+1}} = 0 \tag{C.24}$$

(ii) $P_{f,t}$ solves the problem of firm i

By making the appropriate substitution (the intermediate goods producer's constraints in place of $Y_{f,t}$ and subsequently in for $N_{f,t}$) and by dividing each period's profits by that period's price level P_t so as to put profits in real terms (and thus make profits across periods comparable) we get the following:

$$\begin{aligned}
\partial P_{f,t} : \lambda_t \frac{Y_t}{P_t} \Big[\frac{P_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} \varphi_p \left(\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} - 1 \right) - (1-\theta^p) \left(\frac{P_{f,t}}{P_t} \right)^{-\theta^p} \\
-\theta^p \frac{w_t}{A_t} \left(\frac{P_t}{P_{f,t}} \right)^{1+\theta^p} \Big] = \beta \delta_t \mathrm{E_t} \frac{\lambda_{t+1} Y_{t+1}}{P_{t+1}} \left[P_{t+1} \varphi_p \left(\frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} (\Pi_t^p)^{\iota_p} P_{f,t}} - 1 \right) \frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} (\Pi_t^p)^{\iota_p} P_{f,t}^2} \right]
\end{aligned} \tag{C.25}$$

(iii) $P_{f,t} = P_{j,t} \quad \forall i \neq j$

$$\begin{aligned}
\frac{Y_t}{\lambda_t^{-1}} \left[\varphi_p \left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1 \right) \frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - (1-\theta^p) - \theta^p \frac{w_t}{A_t} \right] = ... \\
... = \beta \delta_t \mathrm{E_t} \frac{Y_{t+1}}{\lambda_{t+1}^{-1}} \varphi_p \left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} (\Pi_t^p)^{\iota_p}} - 1 \right) \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} (\Pi_t^p)^{\iota_p}}
\end{aligned} \tag{C.26}$$

(iv) R_t follows a specified rule

and

(v) all markets clear.

Combining all of the results derived from the conditions and exercises in (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t^p, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$\lambda_t = \beta\delta_t R_t \mathrm{E}_t \lambda_{t+1} \left(\Pi_{t+1}^p\right)^{-1} \tag{C.27}$$

$$\lambda_t = (C_t - \zeta C_{t-1})^{-\chi_c} \tag{C.28}$$

$$\begin{aligned} \frac{N_t}{\lambda_t^{-1}} \left[\varphi_w \left(\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1 \right) \frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - (1-\theta^w) - \theta^w \frac{A_t^{1-\chi_c} N_t^{\chi_n}}{\lambda_t w_t} \right] = ... \\ ... = \beta\delta_t \mathrm{E}_t \frac{N_{t+1}}{\lambda_{t+1}^{-1}} \varphi_w \left(\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_t^w\right)^{\iota_w}} - 1 \right) \frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_t^w\right)^{\iota_w}} \frac{w_{t+1}}{w_t} \end{aligned} \tag{C.29}$$

$$\Pi_t^w = \frac{w_t}{a w_{t-1}} \Pi_t^p \tag{C.30}$$

$$\begin{aligned} \frac{Y_t}{\lambda_t^{-1}} \left[\varphi_p \left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1 \right) \frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - (1-\theta^p) - \theta^p \frac{w_t}{A_t} \right] = ... \\ ... = \beta\delta_t \mathrm{E}_t \frac{Y_{t+1}}{\lambda_{t+1}^{-1}} \varphi_p \left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}} - 1 \right) \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}} \end{aligned} \tag{C.31}$$

$$Y_t = C_t + \frac{\varphi_p}{2} \left[\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1 \right]^2 Y_t + \frac{\varphi_w}{2} \left[\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1 \right]^2 w_t N_t \tag{C.32}$$

$$Y_t = A_t N_t \tag{C.33}$$

$$R_t = \max\left[R_{ELB}, R_t^*\right] \tag{C.34}$$

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R} \left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{Y_t}{A_t \bar{Y}}\right)^{(1-\rho_r)\phi_y} \exp(\epsilon_{R,t}) \tag{C.35}$$

and given the following processes ($\forall t \geq 2$):

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_{\delta,t} \tag{C.36}$$

and

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t. \tag{C.37}$$

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}. \tag{C.38}$$

C.6 A stationary equilibrium

Let $\tilde{Y}_t = \frac{Y_t}{A_t}$, $\tilde{C}_t = \frac{C_t}{A_t}$, $\tilde{w}_t = \frac{w_t}{A_t}$, and $\tilde{\lambda}_t = \frac{\lambda_t}{A_t^{-\chi_c}}$ be the stationary representations of output, consumption, real wage, and marginal utility of consumption respectively. The stationary symmetric equilibrium can now be characterized by the following system of equations.

$$\tilde{\lambda}_t = \frac{\beta}{a^{\chi_c}} \delta_t R_t \mathrm{E}_t \tilde{\lambda}_{t+1} \left(\Pi^p_{t+1}\right)^{-1} \exp(-\chi_c a_{t+1}) \tag{C.39}$$

$$\tilde{\lambda}_t = (\tilde{C}_t - \tilde{\zeta}\tilde{C}_{t-1}\exp(-a_t))^{-\chi_c}, \quad \tilde{\zeta} = \frac{\zeta}{a} \tag{C.40}$$

$$\frac{N_t \tilde{w}_t}{\tilde{\lambda}_t^{-1}} \left[\varphi_w \left(\frac{\Pi^w_t}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi^w_{t-1}\right)^{\iota_w}} - 1\right) \frac{\Pi^w_t}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi^w_{t-1}\right)^{\iota_w}} - (1-\theta^w) - \theta^w \frac{N_t^{\chi_n}}{\tilde{\lambda}_t \tilde{w}_t}\right] = ...$$
$$... = \frac{\beta\varphi_w}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{N_{t+1}\tilde{w}_{t+1}}{\lambda_{t+1}^{-1}} \left(\frac{\Pi^w_{t+1}}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi^w_t\right)^{\iota_w}} - 1\right) \frac{\Pi^w_{t+1}}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi^w_t\right)^{\iota_w}} \exp\left((1-\chi_c)\, a_{t+1}\right) \tag{C.41}$$

$$\Pi^w_t = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \Pi^p_t \exp\left(a_t\right) \tag{C.42}$$

$$\frac{\tilde{Y}_t}{\tilde{\lambda}_t^{-1}} \left[\varphi_p \left(\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi^p_{t-1}\right)^{\iota_p}} - 1\right) \frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi^p_{t-1}\right)^{\iota_p}} - (1-\theta^p) - \theta^p \tilde{w}_t\right] = ...$$
$$... = \frac{\beta\varphi_p}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}} \left(\frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi^p_t\right)^{\iota_p}} - 1\right) \frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi^p_t\right)^{\iota_p}} \exp\left((1-\chi_c)\, a_{t+1}\right) \tag{C.43}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\varphi_p}{2}\left[\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi^p_{t-1}\right)^{\iota_p}} - 1\right]^2 \tilde{Y}_t + \frac{\varphi_w}{2}\left[\frac{\Pi^w_t}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi^w_{t-1}\right)^{\iota_w}} - 1\right]^2 \tilde{w}_t N_t \tag{C.44}$$

$$\tilde{Y}_t = N_t \tag{C.45}$$

and

$$R_t = \max\left[R_{ELB}, R^*_t\right] \tag{C.46}$$

where

$$\frac{R^*_t}{\bar{R}} = \left(\frac{R^*_{t-1}}{\bar{R}}\right)^{\rho_R} \left(\frac{\Pi^p_t}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{(1-\rho_r)\phi_y} \exp\left(\epsilon_{R,t}\right) \tag{C.47}$$

and given the following processes ($\forall t \geq 2$):

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_{\delta,t} \tag{C.48}$$

and

D Solution Accuracy

In this section, we report the accuracy of our numerical solutions for the stylized and empirical models. Following Maliar and Maliar (2015), we evaluate these residuals functions along a simulated equilibrium path. The length of the simulation is 100,000.

D.1 Stylized model

For the stylized model, there are two residual functions, one associated with the consumption Euler equation and the other associated with the sticky-price equilibrium condition.

$$\mathrm{R}_{1,t} = \left|1 - C_t^{\chi_c}\beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1}\right| \tag{D.1}$$

$$\mathrm{R}_{2,t} = \left|(\Pi_t - 1)\,\Pi_t - \Big[\frac{(1-\theta)+\theta(1-\tau)w_t}{\varphi} - \frac{C_t^{\chi_c}}{Y_t}\beta\delta_t \mathrm{E_t}\frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\left(\Pi_{t+1}-1\right)\Pi_{t+1}\Big]\right| \tag{D.2}$$

With our log-utility specification, the residual $\mathrm{R}_{1,t}$ measures the difference between the chosen consumption today and today's consumption consistent with the optimization behavior of the household, as a percentage of the chosen consumption. The residual $\mathrm{R}_{2,t}$ is given by the difference between $(\Pi_t - 1)\Pi_t$ and the sum of the term involving today's real wages and the term involving the expectations. Given that the standard deviation of Π_t is about [0.001] ($\cong$ [40] basis points), Π_t is always close to one and thus the $(\Pi_t - 1)\Pi_t$ is roughly equal to $(\Pi_t - 1)$. $(\Pi_t - 1)$ is the deviation of inflation from the target rate of inflation. Thus, the difference between this term and the sum of the term involving today's real wages and the term involving the expectations measures how much the chosen inflation rate differs from the inflation rate consistent with the optimization behavior of firms.

Table D.1: **Solution Accuracy**

	Mean[$\log_{10}(\mathrm{R}_{k,t})$]	95th-percentile of [$\log_{10}(\mathrm{R}_{k,t})$]
Stylized Model		
$k=1$: Euler equation error	−8.5	−7.2
$k=2$: Sticky-price equation error	−9.2	−8.1
Empirical Model		
$k=1$: Euler equation error	−4.0	−3.2
$k=2$: Sticky-price equation error	−5.5	−5.1
$k=3$: Sticky-wage equation error	−5.4	−4.7

Reflecting the large number of grid points used to solve the stylized model, the approximation errors are very small. The average errors on the consumption Euler equation is $0.3*10^{-7}$ percent ($= 10^{-8.5}$) with the 95th percentile being $6.3*10^{-6}$ percent ($= 100*10^{-7.2}$). The average errors on the sticky price equation is $2.5*10^{-5}$ basis points ($= 400*100*10^{-9.2}$) with the 95th percentile being 0.0003 basis points ([$= 400*100*10^{-8.1}$]).

D.2 Empirical Model

For the empirical model, there are three residual functions of interest. As in the stylized model, the first and second residual functions are associated with the consumption Euler equation and the sticky-price equilibrium condition, respectively. The third residual function is associated with the sticky-price and sticky-wage equilibrium conditions.

$$\mathrm{R}_{1,t} = \left|1 - \frac{\beta}{a^{\chi_c}\tilde{\lambda}_t}\delta_t R_t \mathrm{E_t}\tilde{\lambda}_{t+1}\left(\Pi_{t+1}^p\right)^{-1}\right| \tag{D.3}$$

$$\mathrm{R}_{2,t} = \left|\left(\frac{\Pi_t^w}{\bar{\Pi}^w} - 1\right)\frac{\Pi_t^w}{\bar{\Pi}^w} - \Big[\frac{1-\theta^w}{\varphi_w} + \frac{\theta^w}{\varphi_w}\frac{N_t^{\chi_n}}{\tilde{\lambda}_t\tilde{w}_t} + \frac{\tilde{\lambda}_t^{-1}}{N_t\tilde{w}_t}\frac{\beta}{a^{\chi_c-1}}\delta_t\mathrm{E_t}\frac{N_{t+1}\tilde{w}_{t+1}}{\lambda_{t+1}^{-1}}\left(\frac{\Pi_{t+1}^w}{\bar{\Pi}^w} - 1\right)\frac{\Pi_{t+1}^w}{\bar{\Pi}^w}\Big]\right| \tag{D.4}$$

$$\mathrm{R}_{3,t}=\left|\left(\frac{\Pi_t^p}{\bar{\Pi}^p}-1\right)\frac{\Pi_t^p}{\bar{\Pi}^p}-\Big[\frac{(1-\theta^p)+\theta^p\tilde{w}_t}{\varphi_p}+\frac{\tilde{\lambda}_t^{-1}}{\tilde{Y}_t}\frac{\beta}{a^{\chi_c-1}}\delta_t\mathrm{E}_t\frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}}\left(\frac{\Pi_{t+1}^p}{\bar{\Pi}^p}-1\right)\frac{\Pi_{t+1}^p}{\bar{\Pi}^p}\Big]\right| \quad \text{(D.5)}$$

Table D.1 shows the average and the 95th percentile of the residuals for the three equilibrium conditions. The average size of the Euler equation errors is [0.008] percent ([$=10^{-4.1}$]) with the 95th percentile of the errors being [0.06] percent ([$=10^{-3.2}$]). These are larger than those in the stylized model, reflecting the coarser grid used in the solution of the empirical model. However, this degree of accuracy fares well in comparison with what's reported in other studies such as Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) and Maliar and Maliar (2015).

The average size of the sticky-price equation errors is 0.1 basis points ($=400*100*10^{-5.7}$) with the 95th percentile being 0.2 basis points ($=400*100*10^{-5.3}$). These are again larger than those in the stylized model, reflecting the coarser grid used in the solution of the empirical model. The sticky-wage equation errors are somewhat larger than the sticky-price equation errors. The average size is [0.3] basis points ([$=400*100*10^{-5.1}$]) with the 95th percentile being 0.8 basis points ($=400*100*10^{-4.7}$).

E Expected Time Until the Liftoff

In this section, we present the survey-based measures of the expected time until the liftoff to support the claim that the market participants consistently underestimated the duration of the lower bound episode since the federal funds rate hit the lower bound in late 2008. The surveys we examine are (i) the Blue Chip Surveys, (ii) the Survey of Professional Forecasters, and (iii) the Primary Dealers Survey.

The evidence from all three surveys is consistent with the claim that the market participants have consistently underestimated the duration of the lower bound episode. In particular, for the first two years of the lower bound episode, the market participants expected that the federal funds rate to stay at the ELB only for additional few quarters.[40]

E.1 Blue Chip Surveys

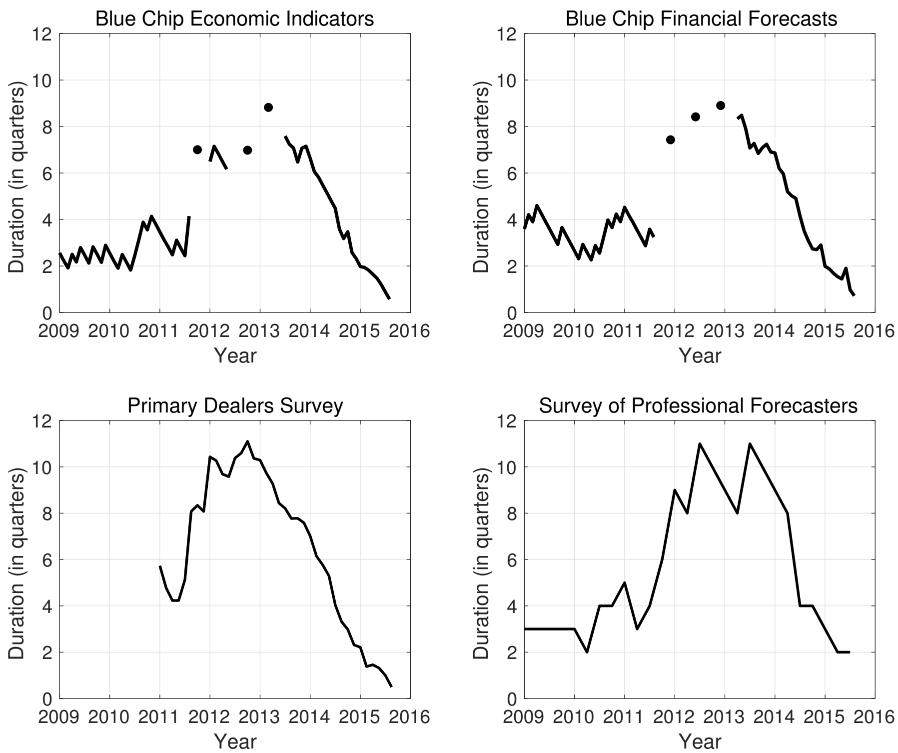
The Blue Chip Surveys consists of two monthly surveys, the Blue Chip Economic Indicators Survey and the Blue Chip Financial Forecasts Survey. These two surveys ask their participants (about 50 financial institutions for each survey) their forecast paths of various macroeconomic variables, including the 3-month Treasury Bill rate in the Economic Indicators Survey and the federal funds rate in the Financial Forecasts Survey. The near-term forecast horizon is up until the end of next calendar year and the frequency of the projection is quarterly. Thus, the forecast path of the Treasury rate or the federal funds rate can tell us the expected time until the liftoff when the participants expect the first liftoff to occur within two years.

Twice a year, the surveys ask longer-run projections of certain variables in the special question section (March and October for the Economic Indicators and June and December for the Financial Forecasts). The longer-run forecasts are in annual frequency for next 5 to 6 years. Towards the end of the lower bound episode, the Surveys also asked the participants to provide the expected liftoff date in the special questions section.

[40]While not shown, the expected duration of the lower bound episode based on the expected policy path implied by the federal funds rate futures is also consistent with this claim.

For each survey, we combine these various pieces of information in the following way to construct a series for the expected period until the liftoff. First, we use the average probability distribution over the timing of the liftoff to compute the expected time until the liftoff whenever that information is available. Second, if the probability distribution is not available, we use the information from the near-term forecasts. The time of liftoff is defined to be the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. Finally, when the policy rate is projected to stay at the ELB until the end of the near-term forecast horizon, we use the information from the long-run projections if the Survey has that information and leave the series blank when the Long-Range section is not available.

Figure E.1: Expected Time Until Liftoff†



† Data sourced from: Blue Chip Economic Indicators Survey (from January 2009 to August 2015); Blue Chip Financial Forecasts Survey (from January 2009 to August 2015); Federal Reserve Bank of New York, Primary Dealer Survey, accessed September 2015, https://www.newyorkfed.org/markets/primarydealer˙survey˙questions.html; Federal Reserve Board, Survey of Professional Forecasters, accessed September 2015, https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

The top two panels in Figure E.1 show the evolutions of the expected period until liftoff based on the Blue Chip Economic Indicators Survey and the Blue Chip Financial Forecasts Survey. According to both panels, the market participants expected the lower bound episode to be transitory in the early stage of the lower bound episode. The expectation shifted in the second half of 2011, with the expectated duration of staying at the ELB exceeding 2 years. Starting in late 2012 or early 2013, the survey respondents gradually reduced their expectation for the additional duration of the lower bound episode.

E.2 Primary Dealers Survey

The Primary Dealers Survey (the PD Survey in the remainder of the text), conducted by the Federal Reserve Bank of New York, asks primary dealers about their policy expectations eight times a year. The survey asks its participants their probability distribution over the liftoff timing (quarter or FOMC meeting). We compute the expected time until the liftoff using the average probability distribution over the liftoff timing. The results of the PD Survey are publicly available since January 2011.

The bottom-left panel of Figure E.1 shows the evolution of the expected period until the liftoff based on the PD Survey. Consistent with the measures based on the Blue Chip survey, the expected duration of the additional period of the lower bound episode increase markedly in the second half of 2011. The expected duration hovers around 10 quarters during 2012, and has declined steadily since then.

E.3 Survey of Professional Forecasters

The Survey of Professional Forecasters (SPF) is a quarterly survey of about 40 individuals in academia, financial industries, and policy institutions, administered by the Federal Reserve Bank of Philadelphia. Like the Blue Chip Surveys, the SPF asks its participants their projections of various macroeconomic variables, including the 3-month Treasury rate. For the near-term projection that extends to the end of the next calendar year, the forecasts are available in quarterly frequency. For the longer horizon, the forecast is available in annual frequency.

The bottom-right panel of Figure E.1 shows the evolution of the expected period until liftoff based on the SPF. Consistent with the Blue Chip Surveys and the Primary Dealers Survey, the SPF shows that the market anticipated the lower bound episode to last for only about one additional year until the second half of 2011. The expected duration averages about 9 quarters in 2012 and 2013. The expected duration started declining in the second half of 2013 and has come down to 2 quarters in February 2015.

F Further discussion on the risk-adjusted monetary policy rule

F.1 Interpreting the central bank's inflation objective

Throughout the paper, we interpret the central bank's inflation objective as specifying the desired level of the risky steady state inflation. Under this interpretation, the dynamics of inflation are consistent with the central bank's inflation objective of x percent if the risky steady state of inflation is x percent. Accordingly, our focus in Section 5 was on how to modify the policy rule to eliminate the wedge between the risky steady state inflation and the central bank's inflation objective. However, some have interpreted the central bank's inflation objective as specifying the average rate of inflation over a long period of time. Under this alternative interpretation, the dynamics of inflation are consistent with the central bank's inflation objective of x percent if the unconditional average of inflation is x percent. The paper by Reifschneider and Williams (2000) is a prominent example adopting this interpretation. In our model, the unconditional average of inflation is also below the target rate of 2 percent, as shown in Table 5. Thus, the need for adjusting the policy rule can also arise under this alternative interpretation of the central bank's inflation objective.

Our intercept-adjustment procedure can be easily modified if modelers take this alternative interpretation and want to set the unconditional average of inflation, as opposed to the risky steady state of inflation, to the central bank's inflation objective. Specifically, modelers would need to search for the size of the intercept adjustment such that the unconditional average of inflation, instead of the risky steady state inflation, equals the central bank's inflation objective. In our example, the intercept that achieves the unconditional average of inflation of 2 percent is lower than the intercept that achieves the risky steady state inflation of 2 percent, because the unconditional average of inflation is lower than the risky steady state inflation in the model with the standard policy rule, as shown in Table 5.

While our intercept-adjustment procedure is useful regardless of the interpretation of the central bank's inflation objective adopted by modelers, it is perhaps appropriate to explain why it is more plausible to interpret the central bank's inflation objective as specifying the desired level of the risky steady state inflation than as specifying the desired unconditional average of inflation.

The interpretation that the central bank's inflation objective specifies the desired unconditional average of inflation is inconsistent with the inflation projections by U.S. policymakers in the following sense. In the United States, both core and overall PCE inflation rates averaged non-trivially below 2 percent over the last decade or so. Thus, to achieve the average inflation rate of 2 percent over a long period of time, policymakers will need to overshoot the target rate non-trivially and persistently in the future. However, according to recent releases of the Summary of Economic Projections (SEP), U.S. policymakers expect inflation to return to 2 percent in the long-run without any overshooting.[41] In the aftermath of a recession, the inflation projection from the model with the risk-adjusted monetary policy rule monotonically increases with forecast horizon and eventually converges to the risky steady state inflation. Thus, the eventual return of the inflation projection to 2 percent without any overshooting in the SPE is consistent with the interpretation that the central bank's inflation objective specifies the desired level of the risky steady state inflation.[42]

In a similar vein, Draghi (2016) states that "In the ECB's case, our aim is to keep inflation below but close to 2 percent over the medium term. Today, this means raising inflation back towards 2 percent." Thus, even though inflation in the euro area, as measured by the HICP, averaged below 2 percent over recent years, ECB policymakers do not seem to interpret their mandate to be consistent with aiming for a transitory overshooting of inflation rates close to 2 percent.

The issue described in this section has not received much attention because linear monetary DSGE models have been predominant tools for model-based analyses of monetary policy until recently. In linear monetary DSGE models, the inflation target parameter in the policy rule coincides with the risky steady state of inflation and the unconditional average of inflation. Researchers typically set the inflation target parameter to the central bank's inflation objective, and because of this coincidence, there is less need to think about conceptual differences among these objects. Our analysis highlights the importance and difficulty of understanding conceptual differences among the central bank's objective, the inflation target parameter, and the risky steady state and the unconditional average of inflation, as nonlinear models become more widely used in the analyses of monetary policy.

[41]See, for example, the Summary of Economic Projections released on June 15, 2016 (available at `www.federalreserve.gov/monetarypolicy/files/fomcprojtabl20160615.pdf`).

[42]Note that, if the central bank tries to achieve an inflation objective at the unconditional mean, that policy is better characterized as a price level targeting.

F.2 Other applications

As discussed in the main text, the difference between the deterministic steady state and risky steady state are nontrivial in our empirical model even without the ELB constraint. The difference between the deterministic steady state and risky steady state caused by nonlinearities that are unrelated to ELB can be quite large depending on specifications of other parts of the model. Thus, our proposed intercept-adjustment will be useful in such applications even if the model abstracts from the ELB constraint.

In addition to the ELB constraint on nominal interest rates, researchers are increasingly interested in the implications of other inequality constraints in macroeconomic models. For example, since the Great Recession, the literature on financial frictions has been developing rapidly, examining the implications of occasionally binding borrowing constraints on the household or leverage constraints on banks. Our risk-adjusted policy rule is likely to be useful in models featuring these other inequality constraints.