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Capacity Choice, Monetary Trade, and the Cost of Inflation*

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Abstract

Firms often make production decisions before meeting a buyer. We incorporate this often-overlooked fact into an otherwise standard monetary search model and show that it has important implications for the set of equilibria, efficiency, and the cost of inflation. Our model features a strategic complementarity between the buyers' ex ante choice of money balances and sellers' ex ante choice of productive capacity. When resale value of unsold inventories is high, sellers carry excess capacity and the equilibrium is unique. But, when resale value is low, there is a continuum of equilibria, all of which are inefficient and welfare-ranked. Effects of inflation are highly nonlinear. When inflation is high, the buyer's money holdings bind, and inflation therefore reduces trade through a standard real-balance channel. When inflation is low, the seller's capacity constraint binds, real balances have no effect at the margin, and inflation has no effect on output or welfare.

Keywords: Search, Money, New Monetarism, Inflation.
JEL codes: D43; E31; E40.

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1 Introduction

Both buyers and sellers in decentralized markets need to make irreversible ex ante investments prior to meeting trading partners. Buyers invest ex ante in carrying liquid balances, subject to the cost of the inflation tax, which limit their ability to purchase ex post. This investment results in a well-studied holdup problem (see, e.g. Lagos and Wright (2005)). Sellers typically also make costly ex ante choices that limit their ex post productive capacity: for example, retailers must decide in advance how much inventory to stock. Much of the literature on decentralized monetary exchange (with some important exceptions discussed below) abstracts from this capacity dimension, and assumes that decentralized trade involves production on demand. In this paper, we relax this assumption by introducing costly capacity choices into an otherwise standard monetary search framework, and study the two-sided holdup problem that ensues. We show that this plausible and natural modification has important implications not only for uniqueness and efficiency of equilibrium, but also for the effects of inflation on output and welfare.

In our framework, buyers face an inflation cost of carrying money balances, and sellers face a real cost of carrying inventories. At the point of sale, a buyer's money balances constrain his ability to buy, and the seller's capacity constrains her ability to sell; in addition, unsold inventories may have some scrap or resale value to the seller unless they fully depreciate, just as buyers' unspent money balances have residual value. The model generates a strategic complementarity: when choosing money balances, buyers takes into account sellers' capacity, and vice versa. Our first key result is that this strategic complementarity generates multiple equilibria as long as the resale value of unsold goods is not too high. Intuitively, if all sellers carry low capacity, then buyers have an incentive to carry low money balances, and vice versa. This results in a continuum of equilibria differing in capacity and real balances, which, moreover, are welfare-ranked. This multiplicity disappears only when resale values are high, in which case sellers set inventories, on the margin, with an eye for resale, without regard for buyers' money balances in the retail market.

The second main result concerns the effects of inflation. As implied by the above, if the resale value of unsold goods is not too high, buyers and sellers coordinate in their choices of money balances and inventories. For high values of inflation, this means that buyers carry low real balances, and the sellers' best response is to carry little inventory even though the marginal cost of additional inventory falls below its productive benefit. In this case, a further rise in inflation lowers the quantity of real balances, and hence inventories. Inflation has the standard detrimental effect on real balances, output, and welfare. However, for low inflation rates, it is capacity, not money holdings, that constraints the amount traded in decentralized meetings. In this case, buyers carry less money than would be implied by equating the marginal utility of consumption to the marginal inflation cost of carrying money balances. The equilibrium quantity is therefore pinned down by the seller's problem, which equates the marginal cost of additional inventory to the marginal benefit, a real decision independent of inflation. Thus, for low inflation rates, marginal changes in inflation have no effect on allocations. In essence, the model with endogenous capacity behaves like the benchmark monetary model for high inflation rates; at low inflation rates, it behaves as an indivisible-good model in which inflation has no effect on output or welfare. The threshold for inflation at which inflation starts to have an effect depends on the resale value of unsold goods.

1.1 Relationship to literature

The analysis in this paper is closely related to the literature on coordination-induced indeterminacy of equilibria in monetary search models, starting with Green and Zhou (1998) and applied to the Lagos and Wright (2005) framework by Jean *et al.* (2010), Rabinovich (2017), and Baughman and Rabinovich (2019). Sellers post prices for indivisible goods, while buyers simultaneously choose money balances, producing a coordination problem. This coordination problem produces a continuum of welfare-equivalent single-price equilibria in Jean *et al.* (2010), and a large set of dispersed-price equilibria in Baughman and Rabinovich (2019). In the present paper, there is an analogous coordination problem: buyers choose money balances while sellers choose productive capacity. We show that this can likewise result in a continuum of equilibria with different money and capacity levels. In contrast to the aforementioned literature, this occurs even though goods are perfectly divisible and sellers do not commit to posted prices ex ante. This reaffirms the robustness of indeterminacy of monetary equilibria stemming from a coordination problem between ex ante choices. There are two substantive differences between our result here and those of Jean *et al.* (2010) and Baughman and Rabinovich (2019). First, in the Jean *et al.* (2010) any two pure-strategy equilibria have the same level of welfare, as they differ only in the split of the surplus between the seller and the buyer; in contrast, in our environment, equilibria differ in *quantities*; in fact, they are welfare-ranked, as explained in section 6. Second, unlike the price posting environment Jean *et al.* (2010), our environment does not allow for mixed-strategy equilibria and hence does not allow for price dispersion (Lemma 1). This is perhaps surprising in light of the similarity between the two coordination problems. We explain the difference in section 8.1.

Our model, and in particular the result on the inflation-output relationship, is also closely related to the work by Han *et al.* (2016) on monetary search with indivisible goods. That paper shows that indivisibility breaks the link between inflation and the quantity traded. Regardless of the inflation rate, buyers bring the lowest amount of money sufficient to buy the (indivisible) quantity supplied by the seller. This happens unless inflation is above a threshold, in which case trade simply shuts down. In our model, quantity is divisible but there is an upper bound, determined endogenously by the seller's capacity choice. If inflation is low enough, capacity rather than money balances is the binding constraint; this means that on the margin, bringing slightly more money does not increase the quantity traded, much like in the indivisible-goods environment. As a result, our model behaves like the standard Rocheteau and Wright (2005) model for high inflation rates, but reproduces the result of the indivisible-goods model of Han *et al.* (2016) for low inflation rates. This also suggests that the intuition of Han *et al.* (2016) is perhaps more robust than previously recognized: the good does not need to be literally indivisible to obfuscate the allocative effect of inflation.

Our result on the inflation-output relationship also bears a similarity to the findings of Rocheteau (2012) and Hu and Zhang (2019). They employ mechanism design to identify the optimal trading mechanism and find that, for low inflation rates, the traded quantity is the first-best independently of inflation. This occurs because the optimal trading mechanism adjusts the seller's surplus in response to inflation so that the seller is still willing to produce the efficient quantity. In our framework a flat inflation-output relationship can occur under a familiar bargaining mechanism, and is instead driven by capacity constraints. Moreover, such a flat inflation-output relationship does not indicate that an efficient quantity is being traded, as the coordination problem still leads to the existence of inefficient equilibria.

Finally, there are only a few papers in the literature that, like ours, incorporate an ex ante production choice into a monetary search model, in contrast to the majority of the literature that has assumed production on demand.[1] Notable contributions incorporating ex ante production into monetary search include Dutu and Julien (2008), Masters (2013), Anbarci *et al.* (2019), and Lebeau (2019). We see our work as complementary to those papers. The innovations in Dutu and Julien (2008) and Masters (2013) differ in focus from ours: Dutu and Julien (2008) consider the question of equilibrium existence in a competitive search model with indivisible money, while Masters (2013) focuses on the interaction between inflation and private information about product quality. Of particular relevance for our analysis are the insights in Anbarci *et al.* (2019) and Lebeau (2019), who show that Nash bargaining in economies with ex ante production can lead to stark unraveling results. Our paper's environment differs from Anbarci *et al.* (2019) and Lebeau (2019) by assuming proportional (Kalai) bargaining, which has the important feature that each agent's surplus is monotonically increasing in their ex ante investment, be that capacity or money balances.[2] As we discuss in section 8.2, this bargaining protocol leads to *economically significant* differences in results.

The paper is organized as follows. Section 2 describes the model environment. Section 3 characterizes the efficient allocation. Section 4 defines the equilibrium. Section 5 contains our main result characterizing the equilibrium set. Section 6 discusses the welfare properties of equilibria. Section 7 discusses the effects of inflation in this economy, in particular the result that inflation does not affect output within a certain parameter range. Section 8 discusses the coordination problem that leads to multiplicity, as well as the role of the bargaining protocol assumed here. Section 9 concludes.

[1] Models with ex ante capacity choice, like ours, are distinct, from e.g. Aruoba *et al.* (2011), which incorporates ex ante investment in productive capital. The latter affects the marginal cost of production, whereas ex ante capacity choice in our model instead places an upper bound on traded quantity. The results are quite different in the two cases.

[2] See Aruoba *et al.* (2007) for the first application of this insight to monetary theory.

2 Environment

The model environment closely follows the random matching monetary economy considered by Rocheteau and Wright (2005), with the addition of an ex ante capacity choice for sellers. Time is discrete and the time horizon is infinite. Each period is divided into two sub-periods: a decentralized market (DM) with random matching and bargaining, followed by a centralized market (CM) with Walrasian trade. Two different goods are produced and consumed in the two submarkets. The economy consists of a unit measure each of buyers and sellers, both with quasilinear preferences and discount factor $\beta \in (0, 1)$. Buyers consume in the DM and can either produce or consume in the CM. They have utility

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(q_t\right) + x_t \right], \tag{1}$$

where q_t is the quantity of the DM good consumed, and x_t is the net quantity of the CM good consumed (with negative values denoting production). The periodic utility function u satisfies $u' > 0, u'' < 0$. Sellers have utility

$$\sum_{t=0}^{\infty} \beta^t \left[-\kappa\left(y_t\right) + \gamma\left(y_t - q_t\right) - c\left(q_t\right) + x_t \right], \tag{2}$$

where y_t is the seller's productive capacity carried into the DM in period t, q_t is the quantity of the DM good produced, x_t is the net quantity of the CM good consumed. In other words, in every DM, the seller makes a choice of productive capacity before meeting a buyer at cost $\kappa(\cdot)$. Then, at the point of sale in the DM, sellers produce at cost $c(\cdot)$ subject to the constraint $q_t \leq y_t$. Any goods unsold in the DM, $y_t - q_t$, yield $\gamma \geq 0$ CM goods per unit to the seller in the subsequent CM; we will refer to γ as the *resale* value. Other than this possibility of resale across sub-periods, both goods are non-storable. The cost functions are assumed to satisfy $\kappa' > 0, \kappa'' > 0$, and $c' > 0, c'' > 0$. Note that if $\kappa(y) \equiv 0$ and $\gamma = 0$, this model reduces to the now-standard monetary search model, in which production is on demand.

The medium of exchange in this economy is fiat money, whose supply is augmented at rate $\pi > \beta - 1$ via lump-sum transfers to buyers in the CM. Define the Fisherian nominal rate as $\iota = (1 + \pi)/\beta - 1$. Search in the DM is random: buyers and sellers match bilaterally with probability $\sigma \in (0, 1)$ and, in a match, set the terms of trade via proportional (Kalai) bargaining. The terms of trade specify the quantity the seller produces for the buyer, and the transfer of money from the buyer to the seller. Note that the transfer of money cannot exceed the buyer's money holdings, and production cannot exceed the seller's capacity.

3 Efficient allocation

We first consider the allocation chosen by a utilitarian social planner.[3] This social planner maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[-\kappa\left(y_t\right) + \gamma y_t + \sigma\left(u\left(q_t\right) - c\left(q_t\right) - \gamma q_t\right)\right] \tag{3}$$

facing the constraints $q_t \leq y_t$. It is clear that the solution to this stationary problem amounts to choosing $y_t = y$ and $q_t = q$ to solve

$$\max_{y,q} -\kappa\left(y\right) + \gamma y + \sigma\left(u\left(q\right) - c\left(q\right) - \gamma q\right) \tag{4}$$

subject to the capacity constraint $q \leq y$.

Throughout the paper, we assume parameters to be such that the capacity constraint binds under the socially efficient allocation. Let us define the quantity y^s to be the solution to

$$\kappa'\left(y^s\right) = \gamma. \tag{5}$$

This is the capacity level that would have been chosen by a seller who carries goods, on the margin, for the purpose of collecting their resale value γ. Also, define the ex post optimal quantity q^* as the quantity satisfying

$$u'\left(q^*\right) = \gamma + c'\left(q^*\right). \tag{6}$$

This is the quantity that would be socially optimal to exchange in the DM if the constraint $q \leq y$ does not bind. The capacity constraint binds under the efficient allocation if and only if $y^s < q^*$. This amounts to assuming, all else equal, that γ is not too large.

Assumption 1 *The quantities y^s and q^* defined by (5) and (6) satisfy $y^s < q^*$.*

Under assumption 1, in the first best allocation sellers carry no excess capacity and we have $y^{FB} = q^{FB}$, where q^{FB} is given by

$$\kappa'\left(q^{FB}\right) = \gamma + \sigma\left(u'\left(q^{FB}\right) - c'\left(q^{FB}\right) - \gamma\right) \tag{7}$$

Note that q^{FB} satisfies $y^s < q^{FB} < q^*$. The ex ante socially optimal capacity is lower than the ex post (unconstrained) quantity because of the cost κ of accumulating capacity.

Assumption 1 is necessary for the problem to be an interesting one. If it were violated, the social planner would simply set $y = y^s$ and $q = q^*$; moreover, it will be clear below that, in the decentralized equilibrium, sellers would then choose $y = y^s$ as well and the capacity constraint in the DM would never bind. In other words, assumption 1 is required

[3] Owing to quasilinear preferences, only DM quantities y_t and q_t are relevant for welfare-ranking allocations. The CM resource constraint simply states that net aggregate CM consumption is 0.

for feedback effects between money balances and capacity choice, and otherwise the model reduces to Rocheteau and Wright (2005).

4 Equilibrium

We focus on steady states throughout.[4] In general, equilibrium is characterized by a distribution $F(y)$ of capacity across sellers and a distribution $G(z)$ of real money balances across buyers (although we will show below that these distributions in equilibrium must be degenerate). Terms of trade are set by Kalai bargaining, with the buyer's bargaining weight given by $\phi \in (0,1)$. In other words, a buyer with money holdings z and a seller with capacity y choose quantity q and a money transfer d to maximize

$$u(q) - d \tag{8}$$

subject to the constraint

$$u(q) - d = \phi\,(u(q) - c(q) - \gamma q) \tag{9}$$

as well as the feasibility constraints $0 \le d \le z$, $0 \le q \le y$. The constraint (9) can be rearranged to read $d = g(q)$, where $g(q) \equiv (1-\phi)\,u(q) + \phi\,(\gamma + c(q))$ is the transfer of money required to purchase quantity q. Define $h(d) = g^{-1}(d)$ to be the quantity affordable with d real balances. Then the solution to the above bargaining problem is the pair $q(y,z), d(y,z)$ given by

$$q(y,z), d(y,z) = \begin{cases} q^*, g(q^*) & \text{if } q^* \le \min\{y, h(z)\} \\ y, g(y) & \text{if } y \le \min\{q^*, h(z)\} \\ h(z), z & \text{if } z \le \min\{g(y), g(q^*)\}. \end{cases} \tag{10}$$

If neither constraint binds, the seller produces q^* and receives $g(q^*)$. If the money constraint binds, the buyer pays all his money z and receives quantity $h(z)$; if the capacity constraint binds, then the seller sells all his capacity y and receives $g(y)$.

Having obtained the bargaining solution $(q(y,z), d(y,z))$, we can write the total surplus from trade

$$S(y,z) = u(q(y,z)) - c(q(y,z))$$

from which we immediately obtain the buyer's surplus

$$u(q(y,z)) - d(y,z) = \phi S(y,z),$$

[4] Appendix A provides the expressions for the buyers' and seller's value functions leading to the equations in this section.

and the seller's surplus

$$d(y,z) - c(q(y,z)) = (1-\phi) S(y,z).$$

The value to a buyer of carrying z units of real balances into the DM, net of the inflation cost, can therefore be written as

$$\nu^b(z) = -\iota z + \sigma\phi \int S(\tilde{y}, z)\, dF(\tilde{y}). \tag{11}$$

Analogously, the value to a seller of holding capacity y going into the DM is given by

$$\nu^s(y) = -\kappa(y) + \left[\gamma y + \sigma(1-\phi) \int S(y, \tilde{z})\, dG(\tilde{z})\right]. \tag{12}$$

Definition 1 *An equilibrium consists of distributions F and G and values $\bar{\nu}^b \geq 0$ and $\bar{\nu}^s \geq 0$ such that the following hold:*

1. *$\nu^b(z) \leq \bar{\nu}^b$ for all $z \geq 0$, with equality for all z in the support of G;*
2. *$\nu^s(y) \leq \bar{\nu}^s$ for all $y \geq 0$, with equality for all y in the support of F.*

The equilibrium is *monetary* if buyers carry strictly positive money balances z with a positive probability.

5 Equilibrium Characterization

Our first result is that all equilibria are in pure strategies.[5]

Lemma 1 *In all equilibria, the distributions F and G are degenerate.*

Proof. See Appendix B. ■

This result simplifies the description of equilibrium considerably. Every equilibrium is characterized by a level of capacity y and a level of real balances z such that y is the seller's best response to all buyers carrying real balances z, and z is the buyer's best response to all sellers carrying capacity y. Since real balances z correspond one-for-one to a quantity $q = h(z)$ affordable (conditional on capacity not binding) with those real balances, choosing z is equivalent to choosing q, and we can describe the equilibrium by the pair (y, q), which will turn out to be convenient below.

[5] This is far from obvious. In fact, apparently similar environments that exhibit equilibrium indeterminacy from a coordination problem, such as Jean *et al.* (2010), *do* allow for mixed-strategy equilibria; see the analysis in Baughman and Rabinovich (2019). We explain the differences in some detail in Section 8.1.

Consider first the buyer's optimal choice of q given that all sellers are carrying y. This optimal q maximizes

$$\nu^b\left(g\left(q\right)\right)=\begin{cases}-\iota g\left(q\right)+\sigma\phi\left(u\left(q^*\right)-c\left(q^*\right)-\gamma q^*\right) & \text{if } q^*\leq\min\{y,q\}\\ -\iota g\left(q\right)+\sigma\phi\left(u\left(y\right)-c\left(y\right)-\gamma y\right) & \text{if } y\leq\min\{q^*,q\}\\ -\iota g\left(q\right)+\sigma\phi\left(u\left(q\right)-c\left(q\right)-\gamma q\right) & \text{if } q\leq\min\{y,q^*\}.\end{cases}\tag{13}$$

Now, define q^b as the solution to $\max_q\left\{-\iota g\left(q\right)+\sigma\phi\left(u\left(q\right)-c\left(q\right)-\gamma q\right)\right\}$, which is given by

$$\iota=\sigma\phi\frac{u'\left(q^b\right)-c'\left(q^b\right)-\gamma}{\left(1-\phi\right)u'\left(q^b\right)+\phi\left(c'\left(q^b\right)+\gamma\right)}.\tag{14}$$

In other words, $g\left(q^b\right)$ is the level of real balances that would have been chosen by a buyer expecting the seller's capacity constraint not to bind. It transpires that $q>q^b$ is never a best response. Furthermore, $q^b\leq q^*$, with equality if and only if $\iota=0$. The buyer's best response function maximizing (13) is then given by

$$\hat{q}\left(y\right)=\min\left\{y,q^b\right\}.\tag{15}$$

The buyer carries enough real balances to buy q^b if the seller's capacity is sufficient to supply q^b, since the seller's capacity constraint does not bind in this case; otherwise, the best response to capacity y is to carry balances sufficient to afford $q=y$.

Consider next the seller's optimal choice of y given that all buyers are carrying real balances $g\left(q\right)$. This optimal y maximizes

$$\nu^s\left(y\right)=\begin{cases}-\kappa\left(y\right)+\gamma y+\sigma\left(1-\phi\right)\left(u\left(q\right)-c\left(q\right)-\gamma q\right) & \text{if } y>q\\ -\kappa\left(y\right)+\gamma y+\sigma\left(1-\phi\right)\left(u\left(y\right)-c\left(y\right)-\gamma y\right) & \text{if } y\leq q\end{cases}\tag{16}$$

where we have already used the fact that $q\leq q^*$ in any equilibrium, by (15). Recall that y^s given by (5) is the capacity level that would have been chosen by a seller who carries goods on the margin for the purpose of collecting their resale value γ. Inspection of (16) reveals that capacity below y^s is never a best response. Next, define q^s to be the solution to

$$\kappa'\left(q^s\right)=\gamma+\sigma\left(1-\phi\right)\left(u'\left(q^s\right)-c'\left(q^s\right)-\gamma\right)\tag{17}$$

In other words, this is the capacity level that would have been chosen by a seller who carries goods on the margin for the purpose of selling them to the buyer, given that the buyer's money holdings constraint does not bind. The seller's best response function maximizing (16) is then given by

$$\hat{y}\left(q\right)=\max\left\{\min\left\{q,q^s\right\},y^s\right\}.\tag{18}$$

A pair (y,q) is an equilibrium if and only if $q=\hat{q}(y)$ and $y=\hat{y}(q)$. Taken together, (15) and

(18) imply that equilibria take one of two forms: either $y > q$ or $y = q$. In the first case, sellers carry excess capacity, money and capacity are determined independently, and buyers' money balances constrain DM trade. In the second case, buyers and sellers coordinate, with buyers bringing balances just sufficient to afford the sellers' capacity, $z = g(y)$, and vice versa.[6] Which form the equilibrium takes depends on parameter values, most crucially γ and ι. In what follows, we will characterize the equilibrium set for any combination of these two parameters.

By assumption 1, $y^s < q^s$, and, moreover, one can show that both q^s and y^s – defined by (5) and (17) – increase in γ. In turn, the quantity q^b defined by (14) decreases in both γ and ι. These quantities, q^b, q^s and y^s bound equilibrium q as a function of the parameters γ and ι.

Define $\gamma_1(\iota)$ as the unique solution to

$$q^b\left(\gamma_1, \iota\right) = y^s\left(\gamma_1\right). \tag{19}$$

If $\gamma > \gamma_1(\iota)$, then $y^s > q^b$, so (18) implies $y = y^s$ whereupon (15) implies $q = q^b$. That is, when resale value is high, sellers choose to carry excess capacity and so buyers need not consider capacity when choosing money balances.

Alternately, $\gamma < \gamma_1(\iota)$ implies that $y^s < q^b$ so all capacity y is sold in equilibrium, $y = q$. Such capacity must satisfy $y \leq \min\left\{q^s, q^b\right\}$. Which upper bound obtains depends on the ordering of q^s and q^b. Define $\gamma_0\left(\iota\right)$ as the unique solution to

$$q^b\left(\gamma_0, \iota\right) = q^s\left(\gamma_0\right). \tag{20}$$

If $\gamma > \gamma_0(\iota)$, then $q^s > q^b$, buyers' money holdings limit trade, and $q = y \leq q^b$. Otherwise, for $\gamma < \gamma_0(\iota)$, sellers' capacity constraint limits trade, and $q = y \leq q^s$.

Turning to the bounds γ_1 and γ_0, the following can be shown: From expressions (5)-(14), when $\iota > 0$, $\gamma_1\left(\iota\right) > \gamma_0\left(\iota\right)$ whereas at $\iota = 0$ we have $\gamma_1\left(0\right) = \gamma_0\left(0\right)$. From Assumption (1), one derives $\gamma < \gamma_1\left(0\right)$. Finally, both $\gamma_1\left(\iota\right)$ and $\gamma_0\left(\iota\right)$ are decreasing in ι. These observations are depicted graphically in Figures 1 and 2.

Figure 1 illustrates the resulting equilibrium set when inflation is sufficiently high, such that $\gamma_0\left(\iota\right) < 0$. Figure 2 illustrates the alternative possibility that inflation is low, and as a result q^b and q^s intersect at a positive γ_0. Note that multiple equilibria occur whenever $y^s < \min\left\{q^s, q^b\right\}$, since in that case the buyer's best response to capacity y is $q = y$, and vice versa.

The above discussion leads to the following characterization of the set of monetary equilibria for each combination of γ and ι.

Proposition 2 *The equilibrium set for any γ and ι is characterized by the following:*

[6]The case where buyers carry excess money holdings is ruled out by $\iota > 0$. This could be relaxed if fiat money is replaced by a dividend-bearing asset.

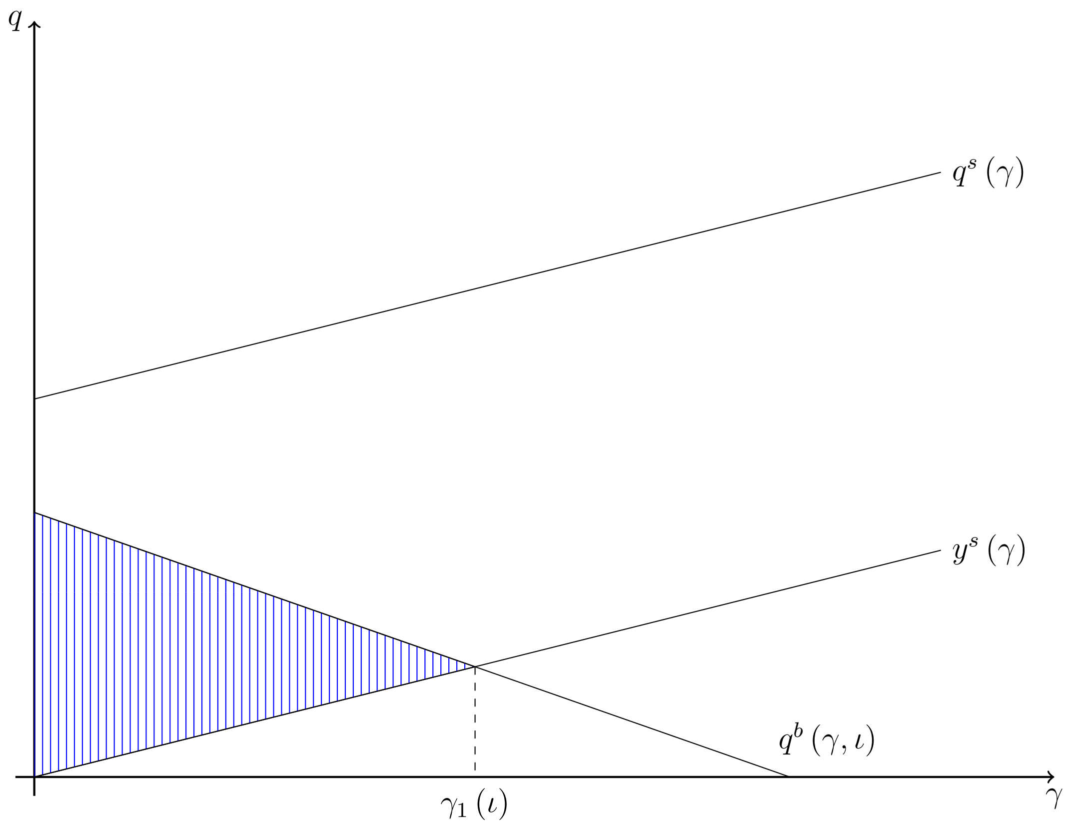

Figure 1: An example where inflation is high and $\gamma_0(\iota) < 0$. The shaded area indicates multiple equilibria for each γ.

1. *Suppose* $\gamma > \gamma_1(\iota)$. *The equilibrium is unique. In this equilibrium, capacity equals* $y^s(\gamma)$, *and the quantity traded in a DM meeting is* $q^b(\gamma,\iota) < y^s(\gamma)$. *In other words, sellers carry excess capacity into the DM, and the quantity traded is pinned down by the buyers' money demand.*

2. *Suppose* $\max\{0, \gamma_0(\iota)\} < \gamma < \gamma_1(\iota)$. *In this case,* $y^s < q^b < q^s$. *There is a continuum of equilibria: for every* $q \in \left[y^s, q^b\right]$, *there is an equilibrium in which both capacity and quantity traded equals* q. *In other words, sellers carry just enough capacity to meet buyers' money demand.*

3. *Suppose* $0 < \gamma < \gamma_0(\iota)$. *In this case,* $y^s < q^s < q^b$. *There is a continuum of equilibria: for every* $q \in [y^s, q^s]$, *there is an equilibrium in which both capacity and quantity traded equals* q. *In other words, buyers carry just enough real balances to meet sellers' capacity.*

This main result is illustrated graphically in Figure 3. The equilibrium configuration depends on the sizes of the resale value γ of sellers' inventories and the inflation cost of buyers' money balances, captured by ι. Suppose that either the resale value γ of unsold goods or the nominal interest rate ι is sufficiently high. Then sellers carry goods, on the margin,

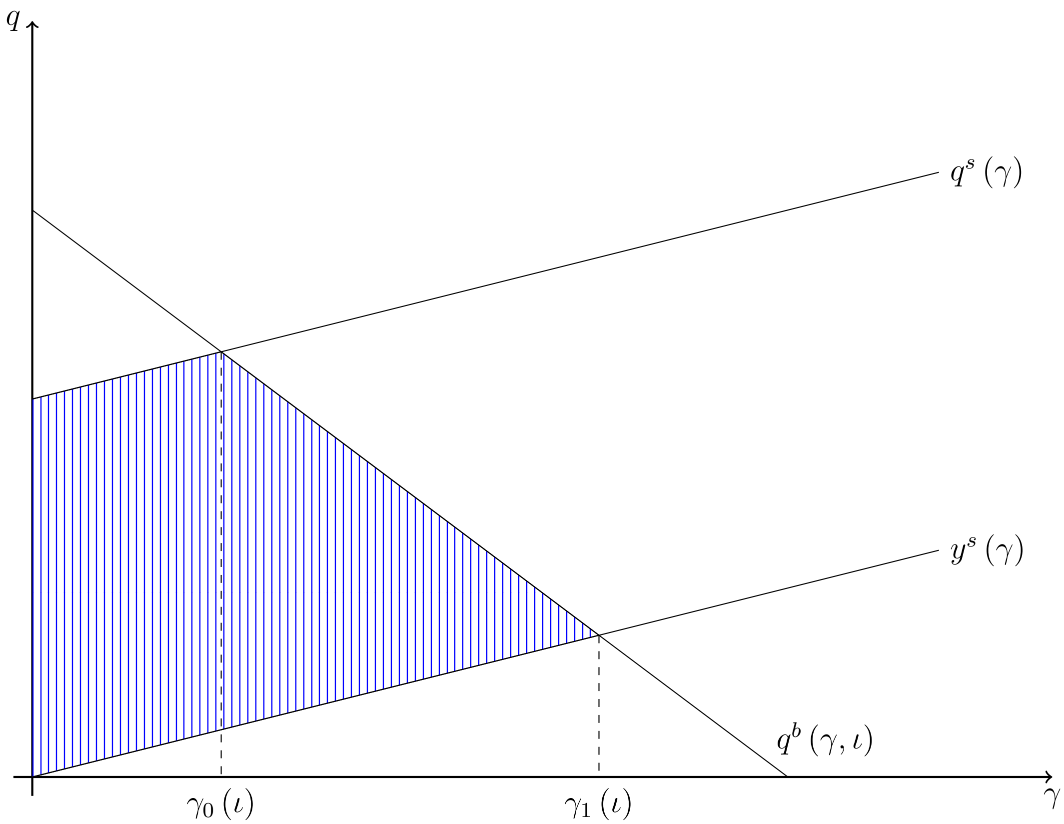

Figure 2: An example where inflation is low and $\gamma_0(\iota) > 0$. The shaded area indicates multiple equilibria for each γ.

for the purpose of reselling them in the following period. In this case, the endogeneity of capacity choice becomes irrelevant for the amount traded in the DM; the latter is pinned down exclusively by money demand, and the model essentially behaves like a Rocheteau and Wright (2005) monetary model with a *non-binding* capacity constraint on sellers. In this case, the equilibrium is unique, and in this equilibrium capacity and money holdings are pinned down independently of one another. This is represented by the region denoted Case 1 in Figure 3.

Alternately, for moderate values of resale value and inflation sellers carry goods, on the margin, for the purpose of selling them in the DM. Hence, sellers are concerned with buyers' ability to pay and a coordination problem arises, leading to multiple equilibria. Equilibrium features money balances equal to $z = g(y)$, where y is the seller's capacity; either the buyer's or the seller's ex ante first-order condition then holds with slack. Consider any q such that $y^s < q < \min\left\{q^s, q^b\right\}$. Suppose that buyers carry real balances $g(q)$. A seller can then do no better than carry capacity q, even though $\kappa'(q) < \gamma + \sigma(u'(q) - c'(q) - \gamma)$. Conversely, given that sellers are all carrying that capacity q, a buyer has no incentive to carry real balances above $g(q)$, despite their low inflation cost. These scenarios are represented by the regions labeled Case 2 and and Case 3 in Figure 3 corresponding to the cases where q^b or q^s

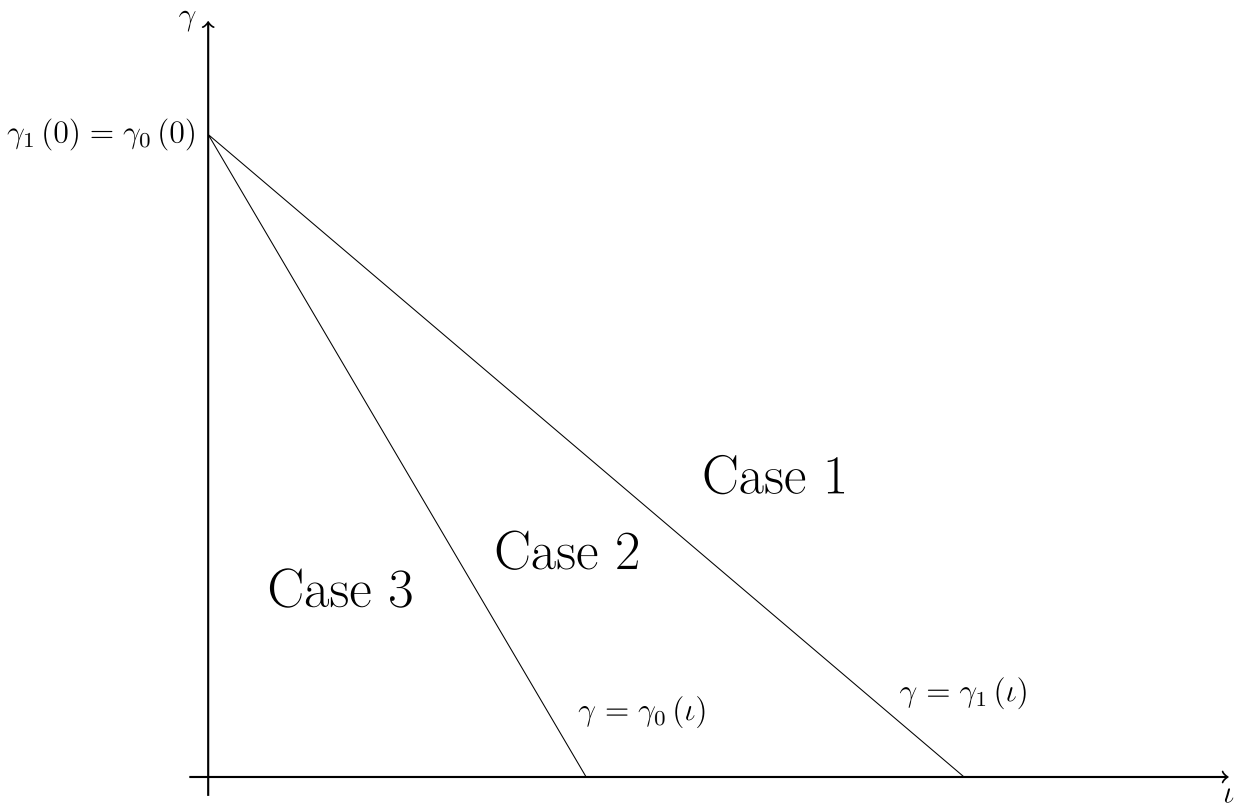

Figure 3: Equilibrium configurations depending on γ and ι.

limit the equilibrium set, respectively.

Turn next to the distinction between Case 2 and Case 3. Note that the coordination problem arising in either case leads to an interval of possible equilibrium quantities. The lower bound of this interval is y^s: since a seller can always collect at least the scrap value of unsold inventories, carrying less than y^s is strictly dominated. The upper bound, $\min\left\{q^s, q^b\right\}$, depends on which constraint binds - capacity or real balances - which depends, in turn, on the relative size of γ and ι. In Case 2, inflation is moderately high, and therefore $q^b < q^s$, the money balances constraint binds, and the capacity first-order condition holds with slack. In this case, the upper bound q^b on the equilibrium set is pinned down by real balances. On the other hand, in Case 3, inflation is sufficiently low, the capacity constraint binds, the first-order condition for money demand holds with slack, and the upper bound q^s on the equilibrium set is pinned down by capacity. As implied by Proposition 2, such a case occurs if $\gamma_0(\iota) > 0$, i.e. if inflation is sufficiently low that $q^s(\gamma) < q^b(\gamma, \iota)$ at $\gamma = 0$. Importantly, if this is the case, the equilibrium set does not depend on inflation at all. We return to this point below.

6 Welfare

The existence of multiple equilibria raises the question of how they compare in terms of welfare. Because of quasilinear preferences, DM utility is sufficient to welfare-rank allocations: steady state aggregate welfare is equal to

$$\mathcal{W} = \frac{1}{1-\beta}\left[-\kappa\left(y\right) + \gamma y + \sigma\left(u\left(q\right) - c\left(q\right) - \gamma q\right)\right], \tag{21}$$

where y is the seller's capacity and q is the amount traded in DM meetings. As explained in Section 3, welfare is maximized at $y = q = q^{FB}$ given by (7).

To start, we compare the equilibrium allocation to the efficient allocation. On the one hand, equation (17) implies that $q^s < q^{FB}$ unless $\phi = 0$ (and, recall, $q^s > y^s$ always). On the other hand, when $\phi = 0$, equation (14) implies that $q^b = 0$ unless $\iota = 0$, in which case buyers are indifferent over their money holdings. Combining these implies

Corollary 3 *The efficient allocation is attainable in equilibrium only if sellers have all bargaining power, $\phi = 0$, and the central bank runs the Friedman rule, $\iota = 0$.*

Specifically, suppose, first, that ι and γ are such that $\gamma > \gamma_1\left(\iota\right)$; that is, the equilibrium falls into Case 1 of Proposition 2. In this case, the unique equilibrium has capacity $y = y^s$ given by $\gamma = \kappa'\left(y^s\right)$ and $q = q^b < y^s$. Therefore, $q < q^{FB}$ because $y^s < q^{FB}$ by Assumption (1). In other words, capacity carried into the DM is inefficiently low and the amount sold in a meeting is even lower. Intuitively, inflation is so high that buyers carry very low real balances. Knowing this, sellers carry goods, on the margin, for the purpose of getting the resale value. Hence, by assumption, they underproduce capacity, although this capacity does not constrain DM trade.

Now, consider the case when $\gamma < \gamma_1\left(\iota\right)$, corresponding to Cases 2 and 3 of Proposition 2. In this case, there is an interval of equilibria, whose upper bound is $\min\left\{q^b, q^s\right\} < q^{FB}$, and in every equilibrium we have $y = q$. Since $-\kappa\left(q\right) + \gamma q + \sigma\left(u\left(q\right) - c\left(q\right) - \gamma q\right)$ is concave, we conclude that in all equilibria the quantity is inefficiently low, and furthermore, the equilibria are welfare-ranked: an equilibrium with a higher q has higher welfare. Intuitively, in every equilibrium, sellers carry goods, on the margin, for the purpose of selling them in the DM; however, either the sellers' capacity production or the buyers' money demand suffers from a holdup problem. Note that the highest-welfare equilibrium is $q = \min\left\{q^b, q^s\right\}$.

7 Effects of Inflation

A crucial implication of the above analysis is that the effects of inflation are highly nonlinear. Since the model has multiple equilibria, we focus the discussion on the "best" equilibrium $q = \min\left\{q^b, q^s\right\}$, but the discussion is almost identical when considering the effects of inflation on the equilibrium *set*, since the worst equilibrium $q = y^s$ is always independent of inflation.

Consider first the case $\gamma > \gamma_1(\iota)$. This is the case in which equilibrium is unique, capacity equals y^s, and quantity traded equals $q^b < y^s$. The seller's capacity constraint does not bind, the buyers' choice of money balances equates the marginal inflation cost to the marginal benefit, and seller capacity has, on the margin, no bearing on this choice. The model then behaves like the model without capacity constraints analyzed by Rocheteau and Wright (2005) and Aruoba *et al.* (2007), in which inflation reduces the traded quantity q through the standard real-balance channel; capacity is determined independently. Note that this occurs when both resale value and inflation are sufficiently high.

Consider next the case $\max\{0, \gamma_0(\iota)\} < \gamma < \gamma_1(\iota)$. In this case, sellers carry just enough capacity to meet buyers' money demand: $y = q$ in every equilibrium and the interval of admissible q is $\left[y^s, q^b(\iota)\right]$. Note that the best equilibrium is now $y = q = q^b(\iota)$, which is a decreasing function of inflation. In other words, as in Case 1, inflation still reduces the quantity traded through the classic real-balance effect; the difference is that now this reduction in q affects capacity as well, since sellers carry goods on the margin for the purpose of selling them in the DM. In other words, inflation lowers the traded quantity by reducing demand for real balances, and sellers' ex ante production likewise falls to meet this reduction in real balances.

Now, contrast this to the case $0 < \gamma < \gamma_0(\iota)$. In this case, buyers carry just enough real balances to meet sellers' capacity: the interval of equilibrium q is $[y^s, q^s]$, so that the best equilibrium $q = q^s$ is *independent* of inflation. Intuitively, suppose that inflation is low, and so is the resale value of the goods. Then, sellers carry goods on the margin for the purpose of trading them in the DM, and the buyers' optimality condition for real balances holds with slack: the amount of real balances carried by the buyer is limited by the seller's low capacity, not by the inflation cost. In this case, inflation does not affect equilibrium allocations at all: the Phillips curve is flat.

There is a clear and important similarity between this result and the indivisible-goods setting analyzed by Han *et al.* (2016). In that model, DM goods are indivisible, hence capacity can be thought of as pegged at $q^s = 1$. Buyers, in turn, carry the lowest amount of money sufficient to to buy that quantity. Inflation has no effect on allocations, since the quantity is pinned down by the indivisible amount carried by the seller. What our analysis shows is that, with ex ante capacity choice, the model behaves similarly to a standard divisible-goods model for high inflation rates, but similarly to this alternative indivisible-goods model at low inflation rates.

Under what conditions are we likely to observe this flat inflation-output relationship? Fixing γ, the previous discussion has demonstrated this occurs at sufficiently low inflation rates. Formally, this occurs whenever $\iota < \gamma_0^{-1}(\gamma)$. Since γ_0 is a decreasing function, the the ι threshold below which inflation has no effect is decreasing in γ. In particular, a lower γ makes it more likely for the inflation-output relationship to be flat. Note that, while a low γ can be interpreted as a high rate of physical depreciation, it need not be. This may be relevant, for example, if the resale value of goods has declined due to increased rotation of product lines.

8 Discussion

In this section, we discuss the coordination problem that arises in this environment and its relationship to similar results in the literature. We also discuss the role of the bargaining protocol in generating this coordination problem.

8.1 Coordination and equilibrium indeterminacy

The fact that a goods-market coordination problem between buyers and sellers leads to equilibrium multiplicity makes the result closely related to a known result in monetary search theory shown by Jean *et al.* (2010), and extended by Rabinovich (2017) and Baughman and Rabinovich (2019).[7] In those models, the good traded in the DM is indivisible, sellers commit to posted prices, and the coordination problem is between buyers' money demand and sellers' price posting. The coordination problem arises because, if all sellers post a price p for the good, the buyers' best response is to carry the amount p of real balances, since any smaller amount would result in no trade at all, due to indivisibility. Conversely, if all the buyers carry exactly p real balances, a seller's best response is to post the price of p, since any higher price would result in no trade. As a result, there is an interval of possible prices p consistent with a pure-strategy equilibrium. That result, however, arises under quite specific assumptions: sellers commit to posted prices, and goods are indivisible, so that a meeting in which the buyer's money holdings are even slightly lower than the posted price entails no trade at all. This raises the question of whether the coordination problem identified there is robust. In our model considered here, goods are perfectly divisible and the terms of trade are freely negotiated ex post; nonetheless, a very similar equilibrium indeterminacy arises because of sellers' pre-commitment to productive capacity.

Despite this similarity between our environment and the price-posting models described above, there is also an important difference. As shown by Baughman and Rabinovich (2019), the price-posting model not only allows for a continuum of pure-strategy of equilibria with a single price, but also allows for mixed-strategy equilibria, which feature price dispersion. The reason is that a distribution of posted prices rationalized a distribution of money holdings, and vice versa: given a distribution of posted prices, a buyer is indifferent over a range of money balances, and given a distribution of buyers' money balances, a seller is indifferent over a range of posted prices. Given that earlier result, one might conjecture that a similar indifference condition could arise in our present environment with bargaining and capacity choice: a distribution of money holdings being optimal given a distribution of capacities. This conjecture turns out to be false: as shown in Lemma 1, there are no mixed-strategy equilibria in our environment. The reason is that divisibility of the good ensures that each agent has a concave optimization problem, regardless of the distribution of the other agents' choices. Intuitively, consider a buyer facing a non-degenerate distribution of seller capacities. If the buyer's real balances are lower than the capacity of some sellers, the buyer can still

[7]See also Green and Zhou(1998, 2002), as well as Zhou (2003) for related results.

trade with those sellers, and hence faces a concave problem when deciding how much money to bring. This ensures that a buyer cannot be indifferent between two levels of real balances, even if the distribution of sellers' capacity were non-degenerate. This is in contrast to the price-posting model, where, if a buyer's money holdings are less than the seller's posted price, the buyer does not trade at all with that seller; that feature of Jean *et al.* (2010) introduced a natural non-convexity into the problem that is the source of the difference from the current model.

8.2 Role of bargaining mechanism

The monotonic nature of the bargaining mechanism assumed here is important for our results, as we now discuss. The presence of irreversible investments by both buyers and sellers clearly leads to a two-sided holdup problem. As is well-recognized, the nature of the mechanism determining the terms of trade is therefore crucial for the resulting allocations that arise. As first pointed out by Aruoba *et al.* (2007) in the monetary search context, proportional (Kalai) bargaining, as assumed here, leads to a less severe holdup problem than the common assumption of Nash bargaining assumed, e.g. by Rocheteau and Wright (2005). The reason is that Nash bargaining allows the buyer's terms of trade to worsen as the buyer brings more money, and similarly for the seller's choice of capacity. The reason is that, as the buyer brings more money, Nash bargaining allows the prescribed money transfer to the seller to rise more than the amount of goods the seller supplies to the buyer.[8] As shown by Lebeau (2019), with endogenous capacity choice, Nash bargaining leads to a stark unraveling result. The buyer's best response to any level of capacity is to bring the lowest money balances sufficient to buy the seller's entire capacity. The seller's best response to that choice of money balances, however, is to bring less capacity; in fact, the seller's best response to any money balances is to bring the lowest level of capacity sufficient to exchange for those money balances. This implies that the best response functions intersect only at the point where both money balances and capacity are zero.[9]

By contrast, under the Kalai bargaining mechanism assumed here, the buyer's surplus is monotonic in his money balances, and the seller's surplus is monotonic in capacity. As a result, agents do not have the incentive to "shade down" their money holdings or capacity. The buyer's optimal money balances therefore depend on the seller's capacity only because it constrains the quantity the buyer can buy, and vice versa. This leads to a coordination

[8] To illustrate the intuition, consider the indivisible-goods setting analyzed by Han *et al.* (2016). If the buyer brings slightly more money than the lowest amount the seller is willing to accept, Nash bargaining prescribes for him to pay his entire money holdings, but the quantity provided by the seller stays constant. A similar logic is operative in a divisible-goods environment.

[9] A similar reasoning applies in Anbarci *et al.* (2019); however, if the resale value of the good is in the appropriate range, there are also mixed-strategy equilibria in which the seller randomizes between carrying the lowest capacity to exchange for the buyer's money balances, and the capacity optimal given the resale value. They show that such a mixed-strategy equilibrium has a given discrete support, and rule out the continuum of equilibria that we obtain here.

problem, which, as we have shown, results in a continuum of equilibria. It is worth pointing out that the importance of the bargaining protocol, e.g. for the welfare cost of inflation, has been recognized at least since Aruoba *et al.* (2007), but, there, the choice of Kalai vs. Nash bargaining affects the *magnitude* of the quantity traded in the unique monetary equilibrium. Our findings imply that, with capacity choice, the bargaining protocol matters qualitatively for the *set* of equilibria that can obtain.

9 Conclusion

Our analysis shows that endogenous choice of productive capacity in monetary economies naturally leads to multiple equilibria. This occurs even with perfectly divisible goods, in contrast to, e.g. Han *et al.* (2016). Unlike the price posting environment in Jean *et al.* (2010) or Baughman and Rabinovich (2019), while there is a continuum of monetary equilibria, they are in pure strategies and hence do not allow for equilibrium price or capacity dispersion. Because the equilibria differ in quantities and not just prices, they are welfare-ranked.

The second key lesson from the analysis is that endogenous capacity choice – a plausible and relatively mild assumption – naturally leads to the possibility of a flat inflation-output relationship. This is a stark result in contrast to the prediction of much of classical monetary theory that ignores capacity constraints. Such a flat inflation-output relationship is more likely to be observed when inflation is low.

There are several open questions. First, a natural extension of our analysis would be replacing money with an intrinsically valued asset, as e.g. in Rocheteau and Wright (2013), Rabinovich (2017), or Han *et al.* (2019). In such an environment, the set of equilibria would depend on both the resale value of goods and the dividend value of the asset. Second, the coordination problem arising here depends on the buyers' reliance on money for trade. An extension of our model would allow credit in some matches in order to understand its implications for the set of equilibria and the effects of inflation. Third, the comparison of our results to Lebeau (2019) illustrates the importance of the bargaining protocol for the set of equilibria. This raises the question of how the results would change if more elaborate trading mechanisms are introduced, e.g. gradual bargaining as in Hu *et al.* (2019), an optimal trading mechanism as in Rocheteau (2012), or more general trading mechanisms as characterized by Gu and Wright (2016). Fourth, the model with endogenous capacity choice assumes a particular form of ex ante investment, in which it affects the upper bound on the quantity that can be supplied in a meeting. This could potentially be combined with a framework in which ex ante investment also affects the marginal cost of production, as e.g. in Aruoba *et al.* (2011). We leave these for future research.

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A Value functions

This section provides the expressions for the value functions leading to (11) and (12). We restrict attention to steady states. Let $W^b(z)$ be the CM value of a buyer entering the CM with real balances z. We can write

$$W^b(z) = \max_{x,z'} x + \beta V^b(z') \tag{22}$$

subject to

$$x + (1+\pi) z' = z. \tag{23}$$

The first-order condition for z' is $1+\pi = \beta V^b_z$, and the envelope condition gives $W^b_z = 1$. Next, the value V^b of a buyer entering the DM with z' real balances is given by

$$V^b(z') = W^b(z') + \sigma \int \left\{ u\left(q\left(\tilde{y}, z'\right)\right) + W^b\left(z' - d\left(\tilde{y}, z'\right)\right) - W^b(z') \right\} dF(\tilde{y}). \tag{24}$$

It is then straightforward to rewrite

$$W^b(z) = z + \frac{\beta}{1-\beta} \max_{z'} \nu^b(z'), \tag{25}$$

with

$$\nu^b(z') = -\iota z' + \sigma \int \left\{ u\left(q\left(\tilde{y}, z'\right)\right) - d\left(\tilde{y}, z'\right) \right\} dF(\tilde{y}). \tag{26}$$

Turning to sellers, the CM value of a seller with real balances z and unsold goods ℓ from the previous period is

$$W^s(z, \ell) = \max_{x,z'} x + \beta V^s(z') \tag{27}$$

subject to

$$x + (1+\pi) z' = z + \gamma \ell. \tag{28}$$

Similarly to the buyer's problem, this gives the first-order condition for z', $1+\pi = \beta V^s_z$, and the envelope condition for z, $W^s_z = 1$, in addition to the envelope condition for ℓ, which reads $W^s_\ell = \gamma$. The DM value of a seller with real balances z' is

$$V^s(z') = \max_y -\kappa(y) + W^s(z', y) \tag{29}$$

$$+ \sigma \int \left\{ -c\left(q\left(y, \tilde{z}\right)\right) + W^s\left(z' + d\left(y, \tilde{z}\right), y - q\left(y, \tilde{z}\right)\right) - W^s(z', y) \right\} dG(\tilde{z}). \tag{30}$$

We can rewrite

$$W^s(z, \ell) = z + \gamma \ell + \frac{\beta}{1-\beta} \max_y \nu^s(y), \tag{31}$$

with

$$\nu^s(y) = -\kappa(y) + \gamma y + \sigma \int \left\{ -c\left(q\left(y, \tilde{z}\right)\right) + d\left(y, \tilde{z}\right) \right\} dG(\tilde{z}). \tag{32}$$

Finally, consider the terms of trade in a meeting where the buyer has real balances z, and the seller has real balances z^s and capacity y. The Kalai bargaining solution consists of choosing q and d to maximize

$$W^b(z-d) - W^b(z) \tag{33}$$

subject to the constraint

$$\begin{aligned} u(q) + &W^b(z-d) - W^b(z) \\ &= \phi\left[u(q) + W^b(z-d) - W^b(z) - c(q) + W^s(z^s+d, y-q) - W^s(z^s, y)\right] \end{aligned} \tag{34}$$

as well as the feasibility constraints $d \le z$, $q \le y$. It is straightforward to show that (26), (32) and (33)-(34) are equivalent to (11), (12) and (8)-(9).

B Omitted Proofs

Proof of Lemma 1. First, we argue that capacities above q^* are strictly dominated. Observe that for $y \ge q^*$ the quantity traded is independent of capacity and hence $\frac{d\nu^s}{dy} = -\kappa'(y) + \gamma < 0$ for any $y > q^*$. Therefore, carrying capacity $y > q^*$ is strictly dominated by carrying q^*.

Next, we show that $\nu^s(y)$ is strictly concave in y for any distribution G of real balances. For notational convenience, write $\Delta(q) = u(q) - c(q) - \gamma q$. We have shown that $y \le q^*$; also, $\Delta(q)$ is increasing for $q < q^*$. Therefore, the ex post surplus from trade equals $S(y,z) = \min\{\Delta(y), \Delta(h(z))\}$ for any real balances z. Since $\Delta(y)$ is strictly concave in y, it follows that $S(y,z)$ is strictly concave in y for any z, by the concavity of the min operator. But this implies that $\nu^s(y)$, given from (12) by

$$\nu^s(y) = -\kappa(y) + \gamma y + \sigma(1-\phi)\int S(y,\tilde{z})\, dG(\tilde{z}), \tag{35}$$

is strictly concave in y, since κ is strictly convex.

Since $\nu^s(y)$ is strictly concave, the optimal y is unique for any distribution G of real balances. This means that in any equilibrium, the distribution F of capacity must be degenerate at some $y \le q^*$. But then we can rewrite the buyer's value from (11) as

$$\nu^b(z) = -\iota z + \sigma\phi \min\{\Delta(y), \Delta(h(z))\}. \tag{36}$$

Note that $\Delta(h(z))$ is strictly concave in z, since

$$\frac{d}{dz}\Delta(h(z)) = (u'(h(z)) - c'(h(z)) - \gamma)\, h'(z) \tag{37}$$

$$= \frac{u'(h(z)) - c'(h(z)) - \gamma}{u'(h(z)) - \phi(u'(h(z)) - c'(h(z)) - \gamma)}, \tag{38}$$

which is strictly decreasing in z. Therefore, $\nu^b(z)$ is likewise strictly concave in z for any y, and the utility-maximizing z is unique. As shown in the main text, the optimal z satisfies $h(z) = \min\{q^b, y\}$. ■