

**Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.**

**Housing Supply and Affordability: Evidence from Rents, Housing
Consumption and Household Location**

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2020-044

Please cite this paper as:

Molloy, Raven, Charles G. Nathanson, and Andrew Paciorek (2020). "Housing Supply and Affordability: Evidence from Rents, Housing Consumption and Household Location," Finance and Economics Discussion Series 2020-044. Washington: Board of Governors of the Federal Reserve System, <https://doi.org/10.17016/FEDS.2020.044>.

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Housing Supply and Affordability: Evidence from Rents, Housing Consumption and Household Location*

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March 2020

We examine how housing supply constraints affect housing affordability, which we define as the quality-adjusted price of housing services. In our dynamic model, supply constraints increase the price of housing services by only half as much as the purchase price of a home, since the purchase price responds to expected future increases in rent as well as contemporaneous rent increases. Households respond to changes in the price of housing services by altering their housing consumption and location choices, but only by a small amount. We evaluate these predictions using common measures of supply constraints on housing outcomes and data from US metropolitan areas from 1980 to 2016. We find sizeable effects of supply constraints on house prices, but modest-to-negligible effects on rent, lot size, structure consumption, location choice within metropolitan areas, sorting across metropolitan areas, and housing expenditures. We conclude that housing supply constraints distort housing affordability, and therefore housing consumption and location decisions, by less than their estimated effects on house prices suggest.

* All errors are our own. The views we express herein are not necessarily those of the Board of Governors or others within the Federal Reserve System. We thank seminar and conference participants at Kellogg, UBC-Sauder, George Washington, the DC Urban Day, the Urban Institute, and the Urban Economics Association for helpful comments. Charles Nathanson thanks the Guthrie Center for Real Estate Research for financial support.

1. Introduction

A large and growing literature has documented a strong connection between housing supply constraints and house prices.¹ Less work has analyzed how these effects map into changes in housing affordability.² One reason for this gap is that housing affordability is defined in many different ways in the academic and policy realms. We define housing affordability as the quality-adjusted price of housing services—i.e. rent—a definition founded on the idea that the price of housing services is more relevant for household welfare than the purchase price of a home. The price of housing services is also a meaningful metric because it affects housing consumption and location decisions through the household’s welfare optimization problem. These outcomes are important dimensions of household wellbeing and are the subject of much attention from policymakers. Thus, to obtain a comprehensive view of the effects of housing supply constraints on housing affordability and households, this paper examines the effects of these constraints on rents, housing consumption, and household location.

Our analysis begins with a dynamic model in which households choose a level of housing services and whether to live in an unregulated city or in a city with supply constraints that explicitly limit how fast the city can grow. Developers combine structure and lots to supply housing services given local constraints and household demand. Supply constraints raise rent (the spot price of housing services) by reducing the supply of housing relative to demand. However, the increase in rent is smaller than the increase in the purchase price of homes because supply constraints increase expected growth in future rent as well as the current level of rent.³ In a calibration exercise, we find that the effects on rent are about half of the effects on the purchase price of housing. In response to the higher price of housing services, households with a given income choose to live on smaller lots, and fewer households choose the constrained city. Other housing outcomes depend on whether the constraint limits the city growth by land area or by population, and on parameters of the housing production function

¹ See Glaeser and Gyourko (2003), Quigley and Raphael (2005), Ihlanfeldt (2007), Zabel and Dalton (2011), Hilber and Vermeulen (2016), and Albouy and Ehrlich (2018), among others.

² A few studies have found some correlation between regulation and affordability as measured by rent relative to median metropolitan area income (Somerville and Mayer 2003, Pendall 2000). Glaeser and Gyourko (2003) argue that housing affordability should be assessed by the level of house prices relative to construction costs, and show that metropolitan areas with longer permitting times more regulated metropolitan areas have a larger fraction of homes with prices substantially greater than construction costs. Albouy and Ehrlich (2018) estimate the effect of regulations on metropolitan amenities and construction productivity and find that the total effect of regulations on social welfare is negative.

³ Gyourko, Mayer and Sinai (2013) also develop a model in which an inelastic supply of housing raises house prices more than rent, although they do not derive the effects of supply constraints on rent. While their model of consumer choice is static, ours is dynamic, giving us a richer framework to estimate the quantitative effects on rents relative to prices.

and the household utility function. Changes in these outcomes are substantially smaller than the effect on the purchase price of housing.

Next, we empirically evaluate the model's predictions using variation across metropolitan areas in two measures of housing supply constraints that are standard in the literature. As a measure of land availability, we use geographic constraints calculated by Saiz (2010), which he derives from the fraction of land on a steep slope or covered by water. As a measure of regulations that restrict the growth of the housing stock, we use the Wharton Residential Land Use Regulation Index, which is composed of a range of types of regulations from a survey of local government officials that was conducted in 2006 (Gyourko, Saiz and Summers 2008).

Importantly, regulations do not arise randomly across areas, and the regulatory environment is likely correlated with characteristics of an area that affect the housing market outcomes in which we are interested (Davidoff 2016). Geographic constraints also might be correlated with omitted variables that affect housing outcomes. We address this endogeneity problem in three ways. The first is to focus on changes in housing market outcomes from 1980 to 2016, in order to difference out local characteristics that are relatively unchanging over time. We do not have data on regulatory constraints in 1980, so we assume that metropolitan areas with stricter regulations in the early 2000s experienced larger increases in the severity of regulation during this period. Section 3 provides evidence for this assumption. We also assume that geographic constraints became more binding over this period, which is consistent with the increasing density in metropolitan areas. Our second approach to mitigate endogeneity is to control for time-varying factors that might also be correlated with supply constraints and housing outcomes. Our third approach is to drop metropolitan areas that experienced persistently low demand over our sample period, as these locations are likely different from growing metros along many unmeasurable dimensions.

We begin our empirical analysis by estimating the effects of supply constraints on house prices and rent using property-level data from the 1980 Census and the 2012-2016 American Community Survey (ACS). Consistent with the model's predictions, the effects of the supply constraints on rent are about half the estimated effects on house prices. Moreover, the estimated effects on rent are small in absolute magnitude. For example, a metro with regulation 2 standard deviations tighter than average experienced 7 percentage point stronger rent growth in total over this 35-year period, which works out to less than $\frac{1}{4}$ percentage point faster growth per year. A few other studies have noted that rents tend to be less correlated with housing supply regulations than house prices (Malpezzi 1996, Malpezzi and Green 1999, Green 1999, Xing, Hartzell and Godschalk 2006), but they do not explain why this occurs or link the results to implications for housing affordability. Gyourko, Mayer and Sinai (2013) show that metropolitan areas with a tight housing supply and strong demand have higher ratios of prices

to rent, but they do not look at the role of supply constraints separately from demand, nor do they examine the effects on rent directly.

Next, we examine the effects of supply constraints on a variety of housing consumption decisions: unit size, lot size, structure type, number of rooms and household size. We obtain the first two outcomes from property tax records and the last three outcomes from the Census and ACS data. We find small effects of supply constraints on all of these outcomes, and the standard errors are generally small enough that we can reject large negative effects.

Turning to effects on household location choices, in the Census and ACS data we find that regulatory constraints lead to lower growth in the housing stock and a small amount of sorting by income and education. These results explain very little of the aggregate amount of sorting by income and education across metros that has occurred between 1980 and 2016. Geographic constraints reduce the number of housing units but do not appear to cause any sorting by income or education across areas. The amount of sorting by income that we find in our analysis is materially less than the amount found by Gyourko, Mayer and Sinai (2013), likely because they examine sorting into areas that have both a constricted supply and strong demand, whereas we focus solely on supply constraints.

Finally, we estimate the effects of housing supply constraints on housing expenditures. These expenditures combine effects on housing costs with consumption and location decisions. Consistent with the model, we find that both constraints raise housing expenditures by less than the estimated effects on house prices. They also have only small effects on the fraction of households spending more than 30 percent of their income on housing, an indicator of affordability favored by housing policy analysts.

Broadly speaking, our empirical results accord with the predictions of the model, in that we find much smaller effects of regulation on rent than on house prices and can reject large household adjustments along most dimensions. One interesting exception is the effect of geographic constraints on lot size. The model predicts that most households remain in the city and occupy houses on much smaller lots, while spending no more on housing. In contrast, in the data, we find that lot sizes do not shrink in response to geographic constraints but that the number of households choosing to live in the city decreases by more than expected. Moreover, the data suggest that household expenditures rise somewhat in more geographically constrained areas. These results are consistent with the possibility that minimum lot sizes and other constraints prevent households from adjusting their land consumption as much as they would prefer, pointing to a potentially important interaction between geographic and other

constraints. Banzhaf and Mangum (2019) also find evidence that structural and regulatory constraints create frictions in housing consumption.

One issue that our model does not address is location choice within a metro area. We might observe little adjustment along the dimensions of housing consumption or metro-level sorting because households instead offset higher housing costs by choosing to live in neighborhoods with relatively low land prices, such as those with long commutes. We look for evidence of this possibility by examining housing construction by Census tract from 1980 to 2016. We measure neighborhood amenities with distance to the metro central business district, average commute time, school quality and crime. We find no evidence that regulatory constraints have shifted housing demand to neighborhoods with lower amenities. Specifically, while we find some evidence that geographic constraints have increased the housing stock in areas with lower school quality and higher crime rates, they also appear to have led to a shift in construction to locations closer to the CBD with shorter commute times. On net, we find little support for the idea that household location choice within a metro has responded to supply constraints in such a way as to obscure or offset large effects of supply constraints on the price of housing services.

In summary, we find that the effects of supply constraints on the price and quantity of housing services are substantially smaller than their effects on house prices. Because the consumption of housing services has a clearer, more direct effect on welfare than homeownership, our results suggest that the housing consumption and affordability distortions from supply constraints are much smaller than the effect on prices.⁴

2. Model

2.1. Environment and equilibrium

There are two cities, R (for “regulated”) and F (for “free”). Time runs continuously from $t = 0$. The economy consists of N_t households, each living in one of these two cities. The utility of household i is

$$U_i = \int_{t_i}^{\infty} e^{-rt} \log(a_{i,j,t} v(c_{i,t}, h_{i,t})) dt,$$

⁴ By making it harder to buy (rather than rent) a house, supply constraints may have welfare costs from which we abstract in this paper. Entrepreneurs use housing wealth as collateral for small business ventures (Adelino, Schoar, and Severino 2015; Kerr, Kerr, and Nanda 2019). Homeownership may act as a forced saving mechanism, helping households achieve higher future consumption (Ghent 2015; Schlafmann 2016). In addition, homeowners invest more than renters in social capital (DiPasquale and Glaeser 1999), and their children obtain higher test scores and are more likely to graduate from high school (Haurin, Parcel and Haurin 2002; Aaronson 2000).

where t_i is the time the household is born, r is the discount rate, $a_{i,j}$ is its taste for city j , $c_{i,t}$ is non-housing consumption, and $h_{i,t}$ is housing consumption. Flow utility from non-housing and housing consumption is Cobb-Douglas: $v(c_{i,t}, h_{i,t}) = c_{i,t}^{1-\alpha} h_{i,t}^\alpha$, where $\alpha \in (0,1)$. The household receives income y_i that is constant over time. The distribution of income across households has a probability density function f .

Households differ in their city tastes:

$$a_{i,j} = a_j e^{\frac{\epsilon_{i,j}}{\beta}},$$

where $\epsilon_{i,R}$ and $\epsilon_{i,F}$ vary as independent standard extreme value distributions and $\beta > 0$.⁵ We assume that city tastes are independent from household income.

Each household is part of a “dynasty,” a collection of households with identical income and city tastes. At a given time, the dynasties contain the same number of households, and the number of households grows at a rate g . Each dynasty chooses cities and consumption levels for its households to maximize the sum of their utility. The dynasty can borrow against the future income of its households at a constant rate r , yielding the budget constraint

$$\int_0^\infty e^{-rt} \sum_{i \in d} (c_{i,t} + p_{j_{i,t},t}(h_{i,t})) dt \leq \int_0^\infty e^{-rt} \sum_{i \in d} y_i dt,$$

where $p_{j,t}(h)$ is the rental price of h units of housing in city j at time t , and the price of non-housing consumption equals one.⁶ Although an artificial modeling device, the dynasty allows us to model population growth in a tractable manner, and can be thought of as representing bequests between generations.

Competitive developers supply housing in each city using two inputs: land, l , and tradeable capital, q . Epple, Gordon, and Sieg (2010), Ahlfeldt and McMillen (2014), and Combes, Duranton, and Gobillon (2016) find that a constant returns to scale, Cobb-Douglas function of these two inputs approximates the production process for housing very well. Thus, we assume the following production function in our model:

$$h(l, q) = l^{1-\gamma} q^\gamma,$$

where $0 < \gamma < 1$. To abstract from issues of durability, we allow developers to supply housing services in frictionless spot markets. The marginal flow cost of structure is k^q and the marginal flow cost of land assembly is k^l . These costs are identical across cities and constant over time.

⁵ McFadden (1973) demonstrates the useful properties of extreme value distributions in the context of logit choice models.

⁶ Dynasties rent housing in spot markets, which is equivalent to buying and selling ownership claims to housing without transaction costs.

In city F , the number of housing units and area of land used are unconstrained. In city R , regulators unexpectedly impose one of two restrictions on developers for all $t > 0$:

- The total number of separate housing units cannot grow at a rate greater than g^n .
- The total land used for housing cannot grow at a rate greater than g^l .

These rules come at the end of time 0, after developers and dynasties have made their initial decisions. The first restriction limits the speed at which developers may supply new housing, so it corresponds to delays in the permitting process as well as regulations such as permit limits that restrict the amount of new construction. Because each household lives in a separate housing unit, this regulation also restricts the growth rate of the city's population. In contrast, the second restriction limits the geographic expansion of the city, so it corresponds to geographic constraints on housing supply. It could also reflect some regulatory restrictions, such as open space requirements.

Developers must obtain a permit to supply a housing unit at time t . The endogenous permit price is $x_{j,t}$, with $x_{F,t} = 0$ due to the absence of regulations in city F . Unpermitted land available for development in city j trades among developers at an endogenous spot price $p_{j,t}^l$, which also equals zero in F .⁷ Developer cost minimization pins down the rental price of housing:

$$p_{j,t}(h) = x_{j,t} + \gamma^{-\gamma}(1 - \gamma)^{\gamma-1}(p_{j,t}^l + k^l)^{1-\gamma}(k^q)^\gamma h.$$

The price to buy housing outright equals the expected net present value of future rents:

$$p_{j,t}^{own}(h) = E_t \int_t^\infty e^{-r(t'-t)} p_{j,t'}(h) dt'.$$

Equilibrium consists of prices $p_{j,t}^l$, $x_{j,t}$, and $p_{j,t}(h)$ such that dynasties maximize utility subject to their beliefs and budget constraints, developers maximize profits while obeying the regulations in R , and the housing market clears in each city. At $t = 0$, dynasties expect prices that hold in an equilibrium without any regulation, while they expect the prices that hold in the regulated equilibrium for $t > 0$. Appendix A.4 characterizes equilibrium at $t = 0$.

2.2. Equilibrium effects of population constraints

To isolate the effect of the population constraint, we set $g^n < g$ so that the constraint binds, while assuming that g^l is sufficiently large so that the price of land in R equals zero. Proposition 1 describes household city choices given the path of permit prices (all proofs appear in the appendix).

⁷ The land price in F equals zero because no alternative use for land exists (such as farming). Our results remain identical if the opportunity cost of land is positive, as long as this cost remains constant and equal in the two cities. Adding this cost to the model is equivalent to increasing k^l .

Proposition 1 (sorting). *If $a_{R,i} < a_{F,i}$, household i always lives in F . If $a_{R,i} \geq a_{F,i}$, household i lives in R only while $t \leq t_i^*$, which solves*

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) = \frac{x_{R,t_i^*}}{y_i - \bar{x}(x_{R,t_i^*})}$$

where $\bar{x}(x) \equiv \int_{(t|x_{R,t} \leq x)} (r-g)e^{-(r-g)t} x_{R,t} dt$.

According to the proposition, households with a greater taste for R live there until the permit price becomes too high. This threshold price is larger when the relative taste for R is greater or the household's income is higher. Because the threshold rises in income, regulation skews the city R income distribution to the right, inducing outmigration of poorer households.

In equilibrium, the number of households choosing R must equal the maximal number that city R allows. To calculate the former, we compute the number of households at t whose relative taste for R exceeds the right side of the equation in Proposition 1 for $x_i^* = x_t$. The latter comes from growing the initial population (appearing in Appendix A.2) by g^n . Equating these gives

$$e^{-(g-g^n)t} = \int_{\bar{x}(x_{R,t})}^{\infty} \frac{a_F^\beta + a_R^\beta}{a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}(x_{R,t})}\right) + a_R^\beta} f(y) dy.$$

This equation pins down $x_{R,t}$. In particular, $x_{R,t}$ must strictly increase over time, reflecting the increasingly binding nature of the population constraint. The proof of proposition 2 provides a formal argument, but it is easy to see because the left side decreases in t while the right side decreases in $x_{R,t}$. The increasing nature of the permit price means that regulation increases prices more than rents:

Proposition 2 (prices versus rents). *The permit price, $x_{R,t}$, strictly increases in t . The effect of regulation on rents,*

$$\frac{p_{R,t}(h)}{p_{R,0}(h)} - 1 = \frac{x_{R,t}}{\gamma^{-\gamma}(1-\gamma)^{\gamma-1}(k^l)^{1-\gamma}(k^q)^\gamma h}$$

is therefore less than the effect of regulation on ownership prices,

$$\frac{p_{R,t}^{own}(h)}{p_{R,0}^{own}(h)} - 1 = \frac{\int_t^\infty r e^{-r(t'-t)} x_{R,t'} dt'}{\gamma^{-\gamma}(1-\gamma)^{\gamma-1}(k^l)^{1-\gamma}(k^q)^\gamma h}$$

for all positive t and h .

Each household living in R subtracts some constant amount from its flow income to pay the permit price. This deduction corresponds to \bar{x} in Proposition 1. The remaining income goes toward structure, lot, and non-housing consumption. Due to Cobb-Douglas preferences and

production, the shares of remaining income going to these purposes are $\alpha\gamma$, $\alpha(1 - \gamma)$, and $1 - \alpha$, respectively. Proposition 3 formalizes this argument.

Proposition 3 (housing characteristics). *Structure and lot sizes for household i in R are*

$$\begin{aligned} q_i^* &= \alpha\gamma(k^q)^{-1}(y_i - \bar{x}(x_i^*)) \\ l_i^* &= \alpha(1 - \gamma)(k^l)^{-1}(y_i - \bar{x}(x_i^*)). \end{aligned}$$

Both $E(q_i^ | y_i)$ and $E(l_i^* | y_i)$ strictly increase in y_i at each t , where the averages are over $a_{R,i}$ and $a_{F,i}$.*

Proposition 3 establishes two offsetting effects of regulation on housing characteristics. Holding income constant, regulation unambiguously decreases structure and lot sizes by increasing $\bar{x}(x_i^*)$. This mechanism is an income effect: the permit price makes households poorer, leading them to consume less housing. Offsetting the income effect is a sorting effect. Holding the characteristics conditional on income constant, the sorting of poor households out of city R drives up average characteristics in R because these characteristics increase in income. The net effect of regulation on the average structure and lot size in city R is ambiguous.

2.3. Equilibrium effects of geographic constraints

To isolate the effect of geographic constraints, we set $g^l < g$ so that the constraint binds, while assuming that g^n is sufficiently large so that the permit price in R equals zero. Proposition 4 describes household city choices given the path of permit prices.

Proposition 4 (sorting). *Household i lives in R only if*

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) \geq \alpha(1 - \gamma) \log\left(1 + \frac{p_{R,t}^l}{k^l}\right)$$

and lives in F when this inequality does not hold.

As with population constraints, geographic constraints lead some households with a higher taste for R to live in F . But different from the population constraints, this outmigration is independent of household income because of Cobb-Douglas preferences and production. The housing characteristics for households in R clarify this point:

Proposition 5 (housing characteristics). *Structure and lot sizes for household i in R are*

$$\begin{aligned} q_i^* &= \alpha\gamma(k^q)^{-1}y_i \\ l_i^* &= \alpha(1 - \gamma)(p_{R,t}^l + k^l)^{-1}y_i. \end{aligned}$$

By driving up the marginal cost of assembled land ($p_{R,t}^l + k^l$), geographic constraints lead to smaller lot sizes. The proportional decrease in lot size is the same for all income groups, and coincides with the term on the right side of the inequality in Proposition 4. This result holds because of Cobb-Douglas preferences and production. Another important modeling choice is

the absence of minimum lot size requirements. With a minimum lot size, the geographic constraint would act as a fixed cost for poor households whose lot size is constrained to be the minimum. Structure size also does not depend on geographic constraints because of Cobb-Douglas preferences and production, assumptions that we will relax below.

To solve for the equilibrium price of land, we equate the total lot sizes of households choosing R with the maximal size that city R allows. The former comes from Propositions 4 and 5, while the latter comes from growing the initial city land size (appearing in Appendix A.4) by g^l . This equation yields a closed-form solution for the land price:

$$p_{R,t}^l = k^l \left(\left(1 + \frac{(e^{(g-g^l)t} - 1) \alpha_R^\beta}{a_F^\beta} \right)^{\frac{1}{\beta\alpha(1-\gamma)}} - 1 \right),$$

which strictly increases over time. Using this formula, we prove our final proposition:

Proposition 6 (prices versus rents). *The effect of geographic constraints on rents,*

$$\frac{p_{R,t}(h)}{p_{R,0}(h)} = \left(1 + \frac{p_{R,t}^l}{k^l} \right)^{1-\gamma},$$

is less than the effect of regulation on ownership prices,

$$\frac{p_{R,t}^{own}(h)}{p_{R,0}^{own}(h)} = \int_t^\infty r e^{-r(t'-t)} \left(1 + \frac{p_{R,t'}^l}{k^l} \right)^{1-\gamma} dt',$$

for all positive t and h .

2.4. Calibration

We calibrate the model to quantify the effects of supply constraints on rents, housing expenditures, housing characteristics, and incomes given the effect of constraints on ownership prices. To perform this exercise, we need values for the various model parameters. The appendix gives details on how we quantitatively solve the model given parameter values.

We use a discount rate of $r = 0.05$. We set the income distribution, f , to a lognormal with mean \$50,000 and log standard deviation 0.96, which is the mean of the standard deviations of positive log household income in the 1980 and 2016 U.S. Census data samples. We take $\alpha = 0.25$ from Davis and Ortalo-Magné (2011), who find that this share of income is spent on rent in many cities from 1980 to 2000. We set β , which governs the distributions of preferences for R and F , equal to three, a value that is within the range estimated by Diamond (2016) by computing the elasticity of cross-city migration with respect to changes in wages and rents. We set $\gamma = 2/3$, share of construction expenditure on structure that Albouy and Ehrlich (2018) estimate. The ratio a_R/a_F pins down the initial relative size of city R . We set it to 1 so that the

cities have identical populations absent regulations in R . The unconstrained growth rate of the number of households, g , equals 0.01, reflecting average annual population growth in the U.S. between 1980 and 2016.

The final parameters are g^n and g^l , which describe the supply constraints. We choose these parameters so that each constraint raises the ownership price of a constant-quality house (at the median of the quality distribution at time zero) by 10% over 30 years. This magnitude is convenient because in our empirical estimates below, we find that a one standard deviation tighter constraint is associated with about 10 percent faster price growth over a roughly 30-year period. This methodology gives us values of $g^n = 0.0092$ and $g^l = 0.0093$.

We can assess our assumptions for the increase in supply constraints using empirical evidence on permitting time found in two surveys conducted by researchers at the Wharton School of Business. Both surveys asked local government officials about the length of time typically required for a building permit application to be approved. The first survey was conducted in the 1980s (Linneman, Summers, Brooks and Buist 1990), and the second survey was conducted in 2006 (Gyourko, Saiz and Summers 2008). Table 1 shows that permitting time increased by 3 to 4 times between the 1980s and 2006, depending on the type of permit. In the simulation, we can think of $t = 0$ as mapping to 1980 in the data. Therefore, 2006 corresponds to $t = 26$. In our simulation, the permit price at $t = 26$ is four times higher than it was at $t = 8$. In other words, our choice of parameters leads to a quadrupling of the supply constraints between 1988 and 2006, which is consistent with what we find in the data.

Table 2 reports changes in outcomes given this assumed price increase. The case of population constraints appears in column (1), while the results under land area constraints are in column (2). Under both supply constraints, the rent of the initial median unit rises far less than prices—by only about half. In other words, about half of the effect on ownership prices comes from anticipation of future rent increases that the supply constraints will continue to cause. Figure 1 illustrates this result by showing the evolution of prices and rents in response to the population constraint. Initially, rent is unchanged because the population constraint only affects future growth. But prices jump by about 4 percent in response to anticipated future rent increases. Over time, prices and rents rise by similar amounts, so that the net increase in prices remains larger. Although the differential between prices and rents becomes a smaller fraction of rent as time goes on, it is still quite substantial after 30 years. Results are similar for the geographic constraint, not shown. Propositions 2 and 6 prove that the effect on rents is smaller than the effect on ownership prices, while Table 2 and Figure 1 illustrate that this difference is meaningful.⁸

⁸ In some models, convergence between prices and rents occurs much faster than in our model. See Rappaport (2004) for an example. In that case, convergence occurs more quickly because it is modeling the response to a one-time shock to the level of labor demand. By contrast, in our model, regulation constrains the

Holding income constant, population constraints decrease structure and lot sizes by 1.7%. To compute this number, we calculate the drop for each household in R after 30 years and then take the average across households. The 1.7% decrease in structure and lot sizes is nearly an order of magnitude less than the increase in ownership prices and is significantly less than the increase in rents, in part because housing begins as only 25 percent of households' budgets, and households pay for the permit price by cutting back on both housing and non-housing consumption.

Population constraints reduce the number of households that choose to live in the regulated city by about 3½ percent. And because poorer households are more likely to leave the regulated city, these constraints raise the median income in the city by 3.0%.

Geographic constraints reduce lot sizes by 18.8%, which is nearly double the effect on house prices. This type of constraint has no effect on structure size, housing expenditure shares, or median city income. The lack of adjustment along these dimensions results from our assumptions that preferences and the housing production function are Cobb-Douglas, so that an increase in the unit price of land leads only to less land consumption and some out-migration. Also important is the lack of a minimum lot size, which would lead to adjustments along other margins by limiting the decrease in lot sizes.

Although much of the literature has found that Cobb-Douglas functions are good approximations for both utility and production, the remaining columns of Table 1 explore the case of geographic constraints while relaxing the Cobb-Douglas assumptions. We instead use constant elasticity of substitution (CES) preferences or production, for which Cobb-Douglas is a special case. The appendix solves this more general model. Column (3) reports results when preferences are CES, in which case we take the elasticity of substitution between housing and non-housing consumption from Albouy, Ehrlich, and Liu (2016). In column (4), we also use a CES production function, taking the elasticity of substitution between land and structure from Albouy and Ehrlich (2018). In both cases, we keep the initial expenditure share on structure and housing the same as in the baseline calibration.

With CES preferences, households cut lot consumption by 13.9%, still a large number but less extreme than before. By contrast, structure sizes actually increase because, with a Cobb-Douglas production function, structure costs must scale with total housing costs. When the housing production function also is CES, lot sizes only fall by 6.6% and structure sizes now fall slightly. CES production makes structure and lots strong complements, meaning that developers cut structure sizes in response to the increase in land prices. In summary, lot sizes

population growth rate by a greater amount over time, so it takes longer for rents to catch up to the increase in prices.

shrink markedly in these CES extensions but not by as much as in the Cobb-Douglas case. Effects on other outcomes remain small.

To clarify the role of the discount rate, Appendix Table 1 reproduces Table 2 using a very high discount rate ($r = 0.1$) and a very low discount rate ($r = 0.02$). Under the high discount rate, rents rise between 7 and 8 percent in response to the supply constraints, while they rise only about 3 percent under the low discount rate. Structure and lot sizes fall more sharply under the low discount rate, however, because households save more in anticipation of future increases in rents. Therefore, our baseline result of a small effect of supply constraints on rents grows stronger with a smaller discount rate, whereas the small effect of supply constraints on real outcomes grows stronger with a larger discount rate.

3. Empirical Strategy and Data

3.1. Identification

Our goal is to estimate the effect of housing supply constraints on housing affordability, as measured by rent, and on households' housing consumption and location decisions. We identify these effects by comparing outcomes across metropolitan areas in the US. Because of the large amount of heterogeneity in regulatory and geographic environments across locations, cross-metro analysis provides a good environment in which to look for its effects. One major empirical challenge, however, is that housing supply regulations correlate with many other aspects of local housing and labor markets that also affect the outcomes that we are interested in (Davidoff 2016). Therefore, we cannot simply regress our outcomes of interest on regulatory variables and expect to identify a causal effect.

We address this issue in three ways. The first way is to focus on changes in our outcomes of interest over time. This strategy allows us to abstract from other factors that might be correlated with regulation and housing characteristics, but are unchanging over time. For example, structure costs might vary across locations due to the availability of various types of construction materials. Or preferences over housing versus other consumption might differ across locations. The second way is to control for some time-varying factors that might be correlated with regulation and housing outcomes—specifically variables that reflect local productivity growth and amenities. The third way is to exclude metropolitan areas with low housing demand from our analysis, since housing markets in these areas likely differ from other areas in many unobservable ways that might be correlated with our outcomes of interest. We write this identification strategy as:

$$Y_{imt} = \delta_m + \delta_t + \beta_z Z_{mt} + \beta_x X_{imt} + \epsilon_{imt},$$

where Y_{imt} is an outcome for household i in metro m at time t , δ_m is a metro dummy, δ_t is a time dummy, Z_{mt} is a vector of supply constraints in metro m at time t , and X_{imt} is a vector of controls. The coefficient of interest is β_z .

We do not have detailed data on how supply constraints have changed over time, so we cannot include these changes directly in our analysis.⁹ Instead, we assume that locations with tighter constraints in the 2000s experienced a greater tightening of constraints over the past four decades. This assumption is based on evidence that supply constraints were much less binding four decades ago. For example, in a sample of 402 California cities, Jackson (2016) finds that most regulations that affect the size, location, or density of the housing stock were established after 1985. In a study of communities in the Greater Boston area, Glaeser and Ward (2009) show that most cluster zoning regulations, wetlands bylaws, and septic system requirements were adopted in the 1980s or later. While subdivision requirements were more common than these other regulations in the 1970s, nearly half of the communities in their sample adopted subdivision requirements after 1980. Massachusetts and California are well-known to be among the more highly-regulated states, so it is unlikely that housing supply regulations became widespread in other states before reaching these two. In addition, Ganong and Shoag (2017) report that the fraction of state appellate court cases that contain the phrase “land use” increased by about 60 percent from 1980 to 2010.¹⁰

The topography of the land changes quite slowly over time, so one might question how geographic constraints might become more binding over time. Cosman, Davidoff and Williams (2018) develop a model to show that in an expanding city, it is the marginal supply of land at the edge of the city that affects prices, not the average supply of land throughout the city. They argue that the marginal supply of land at the edge of the city does not decrease over time since the boundary of the city shifts out. However, in some metropolitan areas like San Francisco the terrain becomes more mountainous toward the edge of the metro, so the constraints become more binding as the metro grows toward these constraints. Moreover, prior research has found infill development to be fairly common in many metropolitan areas (Brueckner and Rosenthal 2009; Burchfield, Overman, Puga and Turner 2006; Hilber, Rouendal and Vermeulen 2018; and McDonald and McMillen 2000). As housing demand in a city increases and more homes get built, less land will be available in the interior of the city for further new construction. To demonstrate the importance of infill development, Figure 2 shows how housing unit density in

⁹ Although Table 2 compares results of the 2006 Wharton survey to an earlier survey conducted in the 1980s, this comparison is only sensible for a single survey question. More generally, the other sets of survey questions are not readily comparable across the two surveys. Moreover, only 60 metro areas are observed in both surveys and we would like to use a wider set of metro areas in our analysis.

¹⁰ The incidence of court cases related to land use began increasing in 1960, illustrating that some regulations were binding in some locations prior to 1980.

the central parts of metropolitan areas has changed from 1980 to 2016.¹¹ In 1980, about two-thirds of metropolitan areas had an average density of less than 40 units per square kilometer in their central counties. By 2016, only about one-third of metros had an average density this low in their central counties. Thus, there has been a substantial amount of residential construction in the interior of metropolitan areas, and so we think it is reasonable to assume that the supply of land throughout the city matters for determining the supply of housing.

Motivated by the evidence that supply constraints were much less binding in 1980, we compare observations from 1980 ($t = old$) to observations in the 2010s ($t = recent$). Given that $Z_{m,old} = 0$, we may rewrite the above specification as

$$Y_{imt} = \delta_m + \delta_{t=recent} + \beta_z 1_{t=recent} Z_m + \beta_x 1_{t=recent} X_{im} + \epsilon_{imt},$$

where Z_m equals the average value of the supply constraint and X_{im} equals the average values of the controls that we use to proxy for changes in local productivity and amenities.

Our specification identifies β_z when $E(\epsilon_{imt} | Z_m, X_{im}) = 0$. The controls must explain all of the changes in the outcomes over time within metros that correlate with the growth in supply constraints but are not caused by the supply constraints. The controls that we think are most important are proxies for productivity growth and changes in the value of local amenities. Metros that have witnessed growth in regulatory constraints have also seen higher productivity growth (Saiz 2010; Davidoff 2016), which could increase household income and alter equilibrium housing characteristics. Similarly, many supply-constrained metros are in locales commonly viewed as having desirable amenities. The amenity premium may have increased over time, perhaps because the aggregate population has become wealthier. We will discuss the variables that we use as proxies for changes in local productivity and amenities below.

Our third approach to addressing the endogeneity of supply constraints is to exclude low-demand metropolitan areas from our analysis. These areas have quite different housing market dynamics from growing areas, and they are different along many unobservable dimensions as well as observable dimensions. Moreover, it is unlikely that supply constraints would bind in these areas. Following Gyourko, Mayer and Sinai (2013) and Charles, Hurst, and Notowidigdo (2018), we calculate ex-post housing demand in each metro area as the sum of the percent change in the number of housing units and the percent change in house value from 1980 to 2016.¹² Low-demand areas are those in the bottom quartile of the demand distribution, and

¹¹ In the 2013 designation of which counties are in metropolitan areas, the Census Bureau identifies some counties in each metropolitan area as “central”. We use this indicator to define central counties and limit our analysis to metropolitan areas for which not all counties are designated as central.

¹² Data are from the 1980 Census and 2016 American Community Survey. We take published data by county and aggregate to the 2013 metro area definitions. House value is calculated as the housing unit-weighted average of county median values.

are dropped from our analysis. Figure 3 plots growth in the housing stock against growth in house values over this period and shows the dropped metro areas in blue.

In all specifications, we weight the observations so that our results reflect the effect for the average household or housing unit in the US. This choice means that large metro areas will have larger weight than small metros. In Section 5.1, we discuss alternative results in which we weight each metropolitan area in our sample equally.

3.2. Data on supply constraints

As a proxy for constraints that reduce the future growth rate of the housing stock, we use an index of the strictness of housing supply regulation based on the Wharton Residential Land Use Regulation survey (Gyourko, Saiz and Summers 2008). In 2006, these researchers sent a survey to local government officials asking a range of questions about the types of residential land use regulation currently used and the political process through which land use regulations are formed. They combine the answers to the questions into a single index of regulatory stringency which is available for 259 metropolitan areas. The index is normalized to have a mean of zero and a standard deviation equal to one.

As a proxy for the supply of buildable land, we use data on geographic constraints.¹³ Specifically, we use the data underlying Saiz's (2010) estimates of the fraction of land that is unavailable for construction because it is on a steep slope or covered by water.¹⁴ This measure is also normalized to have a mean of zero and a standard deviation equal to one. The regulation index and the index of geographic constraints constitute our two components of Z_m .

Not only are the estimated effects of geographic constraints interesting in their own right, but they are helpful to include in our analysis for better identification of the effects of regulation. For example, Saiz (2010) shows that stricter housing supply regulations developed in areas with tighter geographic constraints. Also, the mountains and bodies of water that make it more difficult to build are frequently seen as positive amenities, and an increase in the desirability of these amenities from the 1970s to the 2000s may have raised housing demand in areas with tight geographic constraints (Cosman, Davidoff and Williams 2018). Consequently,

¹³ One of the components of the regulation index is related to open space requirements, which one could view as a constraint on the supply of buildable land. However, it is so strongly correlated with other components of the index that we do not believe it is possible to use it to identify the effects of land supply separately from other types of regulation.

¹⁴ Saiz (2010) calculates these constraints for a radius of 50 kilometers around the center of each of 100 metropolitan areas. We alter this calculation slightly by calculating the fraction of unavailable land for all of the land area in the metropolitan area, which allows us to compute geographic constraints for a larger set of metropolitan areas.

while the identification strategy is not as clear for the geographic constraints as it is for the regulatory constraints, we would want to include the geographic constraints anyway in order to more clearly identify the effect of regulation.

3.3. Data on outcomes

To examine affordability directly, we use data on rent and property value from the 1980 Census and the 2012-2016 American Community Survey (ACS).¹⁵ Specifically, we use the variable reflecting gross rent, which adds utilities costs to contract rent in cases when utilities are not already included, to ensure comparability across units. We assign a value of $t = old$ to all responses in the 1980 Census and a value of $t = new$ to all responses in the 2012-2016 ACS.

Our housing consumption outcomes come from two different sources of property-level data. The first is a 2014 cross-section of tax assessor data collected by CoreLogic. Tax assessors record a variety of property characteristics for the purpose of assessing property values and determining property taxes. This dataset covers the vast majority of single-family housing in the US, although important variables are missing or have unreasonable values in a non-trivial number of cases.¹⁶ Importantly for our purposes, we can obtain information on the square footage of the housing unit, the square footage of the lot, and the construction year of the property. We use the natural logarithm of unit size and lot size as outcomes. We assign a value of $t = old$ to any house built between 1960 and 1980 and a value of $t = new$ to any house built on or after 2000.¹⁷ We drop units built before 1960 or between 1980 and 2000 from the analysis.

While the tax assessor data provide the most comprehensive data on housing unit characteristics with coverage across all metropolitan areas in the US, two drawbacks of the data are worth discussing. The first is that we only observe housing characteristics as they were in 2014. To the extent that some homes built in the 1960s and 1970s have been renovated, their 2014 characteristics do not reflect the characteristics at the time the homes were built. The second drawback is that the data only cover single-family homes. To the extent that household demand can switch between single-family and multifamily units in response to price changes, these data may not capture all of the effects in which we are interested.

¹⁵ Data obtained from the IPUMS USA (Ruggles et al. 2019). To harmonize the definition of metropolitan area over these two samples, we construct a cross-walk from four-digit metropolitan delineations based on 1990 OMB definitions (IPUMS variable METAREAD) to the 2013 OMB delineations (IPUMS variable MET2013).

¹⁶ For computational reasons, we use a 25 percent random sample with 19 million usable observations. Thus the full dataset, with the same restrictions, would have about 75 million observations.

¹⁷ To prevent our results from being driven by outliers with very high values, we drop housing units larger than 10,000 square feet and units with lots larger than 175,000 square feet (about 4 acres). We also drop units with extremely small recorded lots (less than 2000 square feet) and units with very high ratios of floor area to lot size.

To address these drawbacks, we return to the Census/ACS data and examine several additional outcomes. The first outcome is an indicator for whether a property is a single-family structure, which is helpful because the property tax records only cover single-family homes and households could shift to multifamily units to reduce lot or structure consumption. The second outcome that we examine is the number of rooms in the home as a proxy for the structure size. Since single-family homes tend to have more rooms than apartments and we analyze the effect on single-family status separately, we limit the analysis of number of rooms to single-family homes. The third outcome that we examine is the number of adults (defined as age 22 or older) per household, since a larger household implies that each individual consumes less structure per person.

In order to examine the effects of housing supply constraints on sorting across metropolitan areas, we aggregate the Census/ACS data to the metro level and set the outcome of interest as the change in a metropolitan area characteristic from 1980 to 2012-2016. The first set of characteristics that we examine are the fraction of people in each decile of the national distribution of income. Then, because annual income may not always reflect a person's permanent income, we also look at two proxies for permanent income: education and occupation. Specifically, we examine the fraction of the population age 25 and older with at least 4 years of college and the fraction of the population with a high occupation score. The occupation score is created by the Census Bureau using median incomes by detailed occupation category using the 1950 Census.

Finally, we examine the effects of the supply constraints on housing expenditures. While our model clearly demonstrates that housing expenditures are not a good measure of affordability because they reflect household choices as well as the cost of housing services, we think these results provide a nice way to combine the effects on housing costs, housing consumption and location decisions. We also examine effects on the ratio of expenditures to household income, a metric that frequently appears in analyses of housing affordability.

3.4. Data on controls

Our empirical specification includes three proxies for productivity growth and two proxies for local amenities. Our first proxy for productivity growth is the share of the population age 25 or older with at least 4 years of college in 1980, obtained from the 1980 Census.

Our second proxy for productivity growth is the share of employment in industries that experienced fast wage growth from 1990 to 2016. To calculate this share, we first calculate wage growth from 1990 to 2016 by industry using the annual files of the Quarterly Census of Employment and Wages (QCEW). Wages are defined as total annual wages divided by total annual employment. We define industries using 3-digit NAICS codes, which gives us 96 industry categories. Although we would prefer to calculate wage growth from 1980 to 2016, the QCEW

data by NAICS industry are not available prior to 1990.¹⁸ We define high wage growth industries as those in the top decile of wage growth and calculate the fraction of employment in 1990 in those industries.

Our third proxy for productivity growth is an estimate of predicted employment growth following Bartik (1991) and many subsequent papers in labor and urban economics. To create this measure, we use County Business Pattern data and industry code concordances from Eckert et al. (2020).¹⁹ We interact national industry-level employment growth rates with the 1980 shares of employment in those industries in particular CBSAs to get a plausibly exogenous shock to local employment stemming only from national trends at the industry level.²⁰

Our first proxy for local amenities is average January temperature. The value of nice weather seems to have increased since the 1970s (Glaeser and Gyourko, 2003) and many supply-constrained metros are in warm locales such as California. This weather premium may have affected land prices, and hence housing characteristics, in ways we do not want to attribute to supply constraints. We obtain average January temperature by weather station from 1981 to 2010 from the National Oceanic and Atmospheric Administration. We average the station-level data by county, then take a weighted average across counties within each metropolitan area using county land area as weights.

Our second proxy for local amenities is the share of seasonally vacant housing from the 1980 Census. Demand for seasonal housing has grown over time with the ageing of the population and rising incomes, and seasonal housing tends to be in high-amenity areas that also may have tighter topographic or regulatory constraints. To make coefficients comparable across variables, we standardize all five of the controls to have a mean equal to zero and standard deviation equal to one.

The variables that we include as controls are all positively and significantly correlated with our estimate of growth in local housing demand (Appendix Table 2). In section 5.2 we show that our results are robust to controlling for observed demand growth instead of these proxies for productivity and amenities.

¹⁸ We could use data by 1-digit SIC code to extend our analysis back to 1980. However, doing so would give us only about 10 industry categories, and we think having more detailed industry definitions is more valuable than having a longer time period.

¹⁹ The Census Bureau replaces many values in the County Business Pattern employment data with ranges to avoid disclosure of information on particular firms. Eckert et al. (2020) impute precise values for the suppressed cells using linear programming methods. They also provide concordances to map between SIC and NAICS codes, as well as between different vintages of NAICS.

²⁰ To calculate the “national” growth rate for industry 1 that we apply to city A, e.g., we strip out city A’s own employment in industry 1, to guarantee that the “national” shock is not driven by city A itself. In practice, this procedure makes very little difference for our resulting instrument.

Beyond the metro-level controls for productivity and amenities, two other sets of controls bear mentioning. For the specifications with rent and house value as the dependent variable, we control for all available property characteristics: building age, number of rooms, number of bedrooms and a single-family indicator.²¹ We include these controls because we are interested in constant-quality rent and price effects.

For specifications that examine housing characteristics as an outcome, we need to control for household income. As shown by the model, doing so accounts for the effects of supply constraints on sorting across metros, isolating the effects on the choices made by households of a given income level. The specific method of controlling for household income depends on the outcome data we are using. When we are using the Census and ACS data, we include indicators for the household's decile in the national distribution of household income. We allow for this flexible specification of income in case housing consumption choices are not a linear function of income. When we are using the tax assessor data, we include indicators for the Census tract's decile in the national income distribution, using median Census tract income from the 2011-2015 ACS for recently built homes and tract income from the 1980 Census for older homes.²² For both sets of outcomes, we interact the income decile indicators with the recent indicator to allow for the effect of income on housing outcomes to have changed over time.

4. Results

4.1. *Effects on Housing Affordability*

We start by examining the effects of housing supply constraints on real house prices and rents. The first column of Table 3 reports the estimated effects of our two supply constraints on single-family house values in the Census/ACS data. In this table, and in all subsequent analysis that uses housing unit or property-level data, we cluster standard errors by metro area since the supply constraints are observed at the metro level. A metropolitan area with regulations that are one standard deviation tighter than average experienced a 0.09 log point (about 10 percent) stronger house price appreciation over our sample period. The estimated effect of geographic constraints is similar. Results are similar when we measure house prices using a repeat-sales price index instead of owner-reported house values in the Census/ACS (not shown).²³

²¹ Specifically, we include indicators for decade of year built, indicators for each value of number of bedrooms, and indicators for each value of number of rooms.

²² Specifically, for older homes, we use 1980 Census data imputed to 2010 Census tracts from Logan, Xu and Stults (2014).

²³ We use the repeat-sales index for single-family detached homes published by CoreLogic, converting the monthly index to annual averages and comparing 1980 to 2016. The estimated effect of regulation is almost

It may be surprising that the estimated effect of regulation on house prices is not larger. House prices nearly tripled in real terms in New York, Boston and San Francisco over this period, and our estimated coefficients suggest that regulation can explain less than one sixth of the price increases in these highly-regulated regulated metros. And including the effect of geographic constraints still leaves more than two thirds of price growth in these cities unexplained. However, it is important to keep in mind that housing supply regulations are often tighter in areas with strong demand (Davidoff 2016), and it is quite possible that price growth in these areas reflects strong demand as well as tight supply. Indeed, our estimated effects are larger when we don't include controls for local demand or geographic constraints (see Appendix Table 3). In addition, the effect of regulation appears to be nonlinear, with larger effects on prices for very tight constraints. For example, if we estimate separate effects for each quartile of the distribution of regulation, we find that house value increased by 0.19 log point more in the top quartile of regulation than in the bottom quartile (see Appendix Table 4). Even in this specification, though, regulation can explain less than one fifth of the price growth in New York, Boston and San Francisco.

While our estimated price effects are smaller than casual observation might suggest, they are in line with other research that has tried to estimate the causal effect of regulations on house prices. In an analysis of local planning authorities in England, Hilber and Vermeulen (2016) find that a one standard deviation decrease in regulation is associated with 14 percentage point lower cumulative house price growth from 1974 to 2008. And Zabel and Dalton (2011) find that imposing a 1-acre minimum lot size leads to at most a 20 percent increase in house prices in Massachusetts towns.²⁴ Effects of this magnitude are not small in a qualitative sense. They are just too small to explain much of the outsized house price growth in in many major US metropolitan areas.

The second column of Table 3 reports the estimated effects of supply constraints on the rent of single-family homes. We start with single-family rentals because these structures are more similar to the structures used to estimate the effects on house prices. For each supply constraint, the estimated effect on rent growth is less than half of the estimated effect on house price growth. The third column of Table 3 reports the estimated effects on rent in a sample of all rental homes. The estimated effects on rent are still much smaller than the estimated effect on prices. These results are especially striking because the average increase in

exactly the same as in the Census/ACS data, at 0.09. The estimated effect of geographic constraints is somewhat larger, at 0.23.

²⁴ Other research has found larger effects of supply constraints on house prices by looking at a combination of supply and demand. For example, Gyourko, Mayer and Sinai (2013) find that metro areas with both strong demand and a constrained supply experienced a 0.44 log point larger (real) house price increase from 1970 to 2000. Glaeser, Gyourko and Saks (2006) find that a 1-standard deviation increase in local productivity leads to an increase in house prices that is four times larger in a highly-regulated area compared with a less-regulated area.

real rent over this period was about the same as the average increase in real house prices, as shown by the coefficients on the 2012-2016 indicators. Thus, consistent with the model, we find that supply constraints increase rent by much less than house prices.

We can compare the effects on prices and rents directly by estimating the effects of supply constraints on the price-to-rent ratio. To do so, we estimate average value and rent by metro area, calculate the ratios of mean price to mean rent for each metro, and regress the change in this price-to-rent ratio on our measures of supply constraints and controls for metro-level productivity and amenities. To be consistent with the micro-level estimates, we weight each metro by the sum of the household-level weights in that metro. Appendix Table 5 shows that each supply constraint has led to larger increases in the price-to-rent ratio. We find the same result when we limit the sample to single-family homes, for which prices and rents are more comparable. We also find the same results when we calculate the ratio of median value to median rent instead of average value to average rent. In sum, these specifications also show that supply constraints have much smaller effects on rents than on house prices.

Not only are the estimated effects on rent small relative to the effect on prices, they are small in absolute magnitude. For example, based on column 3 in Table 3 a metro area with regulation 2 standard deviations tighter than average experienced only 0.07 log point larger rent increases from 1980 to 2016, which works out to less than $\frac{1}{4}$ percentage point faster growth per year. By contrast, the average increase in (real) rent among all metros in this sample from 1980 to 2016 was 0.49 log points—7 times larger. The fourth column of Table 3 reports the estimated effects of supply constraints on the rent of 2-bedroom apartments, which is a structure type commonly occupied by low-income households. While the magnitudes are a bit larger for this sample than for the sample of all rental units, they still suggest that metropolitan areas with 2 standard deviation tighter regulatory constraints than average experienced only a 0.09 log point larger increase in real rent growth over this period. Even when we allow for the effect of regulation to be non-linear, metropolitan areas in the top quartile of the distribution of regulation experienced only 0.08 to 0.12 larger rent growth over this period (Appendix Table 4). Thus, we find that supply constraints have only reduced housing affordability by a modest amount over this period.

One immediate question that may come to mind is whether our measures of supply constraints may be poor proxies for true supply constraints, which would cause us to underestimate the effects on prices and rent. While there is surely some degree of measurement error in these measures, they are commonly used in academic research and have been shown to be correlated with the elasticity of housing supply (Saiz 2010).²⁵ In addition, it is

²⁵ To date, the papers introducing the regulatory index and the geographic constraint measure have been cited in 170 and 347 published journal articles, respectively.

reassuring that our estimated effects on house prices are qualitatively similar to other estimates in the literature. There is much less research on effects of supply constraints on rent, but Howard and Liebersohn (2019) also find that the elasticity of housing supply can explain little of the variation in rent growth across metropolitan areas from 2000 to 2018.²⁶

Another question that may come to mind is whether rent control might prevent rents from responding to supply constraints as much as prices. We obtain a list of jurisdictions with rent control in 2014 from Landlord.com and drop metropolitan areas with any jurisdictions that have rent control.²⁷ The estimated effects on rent in this sample remain about half of the estimated effect on house prices, indicating that rent control cannot explain the differential between these two outcomes (see Appendix Table 6).

A third concern with our analysis is that the rents paid by tenants may not reflect market rents if the tenants have occupied the unit for a long time. We address this issue by limiting our sample to households where the household head moved in within the previous 5 years. Again, we find estimated effects on rent that are only half as big as the estimated effects on house prices (see again Appendix Table 6).

4.2. Effects on Housing Consumption

One might be skeptical of our ability to directly examine the effects of supply constraints on the price of housing services because this concept is impossible to observe for owner-occupied housing, which makes up roughly two thirds of all housing units. The model illustrates how an increase in the price of housing services should cause households with a given income to reduce their consumption of housing services.²⁸ In the case of population constraints, this showed up as small decreases in structure size and lot size. In the case of land area constraints, this showed up as a fairly sizeable decrease in lot size, with small changes in structure size depending on the elasticities of substitution between lots and structure and between housing and non-housing consumption. We now test these predictions by examining the empirical effects of housing supply constraints on direct measures of housing consumption.

²⁶ Their data are similar to ours in that they use an elasticity of housing supply derived from the two supply constraint measures that we use. However, their empirical methodology is not subject to downward bias from measurement error.

²⁷ http://www.landlord.com/rent_control_laws_by_state.htm. There are 13 metropolitan areas with rent control according to this definition. We treat the metro areas of Washington DC and Riverside CA as having rent control, even though most jurisdictions in these metros do not have rent control. Results are similar if we treat these two metros as not having rent control.

²⁸ Even if the model were to allow households to be homeowners, it is still the price of housing services that would affect consumption decisions because it is the price of housing services that would appear in the household's budget constraint.

The first two columns of table 4 report the estimated effects on lot size of single-family homes and on a single-family indicator. Regulatory constraints do not appear to reduce lot size; the coefficients are positive but insignificantly different from zero. A 95 percent confidence interval around the estimated effect on lot size would encompass a negative effect in line with the model's prediction, but we can reject that regulation has a large negative effect on lot size and structure type. The estimated effects of geographic constraints are negative, but also of a fairly small magnitude. We can reject that a 1 standard deviation increase in geographic constraints reduces lot size by the amounts predicted by the model under Cobb-Douglas assumptions or with only CES preferences (columns 2 and 3 in Table 2). That said, we cannot reject that a 1 standard deviation increase in geographic constraints reduces lot size 6 percent, the effect predicted by the model with CES preferences and a CES housing production function (column 4 in Table 2).

Geographic constraints also appear to lead to a small decrease in the fraction of single-family homes, implying a reduction in a household's consumption of land. Because the durability of housing may prevent the housing stock from adapting to changes in housing demand, the third column shows results for the single-family indicator in a sample restricted to recently-built homes.²⁹ In this sample, the effect of geographic constraints is more negative than in the full sample, implying that a 1 standard deviation tighter constraint would reduce the fraction of single-family homes by about 3 percentage points. It is difficult to compare this estimate to the model's predicted effect, but it seems fairly small, in that it is only about one third of the standard deviation across metros of changes in the single-family fraction of newly-built homes.

Table 5 shows results for various measures of structure consumption. The coefficients on the recent indicator in the first two columns indicate that single-family homes have grown substantially larger over time, especially in terms of square footage. But square footage is unrelated to either constraint, while both constraints appear to reduce the number of rooms in single-family homes by only a small amount. When we restrict the sample to recently-built homes, the effect of geographic constraints on rooms becomes a little more negative, but the effect of regulation goes to zero.

Another way that households might reduce their consumption of housing would be to add more adults to their household, effectively decreasing the amount of structure consumed per person. Consistent with this hypothesis, column 4 shows that supply constraints increase the number of adults per household.³⁰ The final column of Table 5 reports results where the

²⁹ Defined as built in 1970 or later for the 1980 sample, and built since 2000 for the 2016 sample.

³⁰ In this specification, we control for income by dividing household income by the number of adults in the household and then calculating their decile in the national distribution. We do not control for total household income because it is mechanically related to the number of adults living there. In unreported analysis, we have

dependent variable is the number of rooms per adult, combining the effects on household size and the effects on the number of rooms. It shows that both supply constraints reduce the number of rooms per adult. The estimated effect of regulation of is the same magnitude as implied by the model (negative 1½ percent), while the estimated effect of geographic constraints is somewhat larger than the model prediction: We estimate a 3 percent decrease in rooms per adult, whereas the model with CES production and utility predicted a 1½ percent decrease, and the fully Cobb-Douglas model predicted no change at all.³¹ Recall that our estimated effects of geographic constraints on land consumption are somewhat smaller than predicted by the model. It is possible that people find it easier to adjust structure consumption than land consumption, perhaps because minimum lot sizes prevent the desired decreases in land consumption.³²

In summary, we do find some evidence that households reduce their land and structure consumption in response to housing supply constraints. But the effects are small, consistent with the model and with these constraints only having a small effect on the price of housing services.

4.3. Effects on Sorting Across Metropolitan Areas

Next, we examine the effects of housing supply constraints on sorting across metropolitan areas. The model predicted that population growth would be lower in areas with greater supply constraints. Prior research has generally found regulatory constraints to reduce growth in the housing stock (Mayer and Somerville 2000, Saks 2008, Jackson 2016). There has been less research on the effects of geographic constraints on local housing or population growth, and the research on the effects of regulation generally does not control for geographic constraints. Consequently, we start by estimating effects on the housing stock using our data and identification strategy.

Table 6 reports the results of regressing the change in a metro's housing stock from 1980 to 2016 on our two supply constraints, controlling for metro area productivity and amenities. We find significantly negative effects of both constraints. A 1 standard deviation increase in regulatory constraints reduces the housing stock by about 6 percent, while a 1 standard deviation increase in geographic constraints reduces the housing stock by about 10 percent. Both effects are larger than predicted by the model, possibly because the durability of housing

confirmed that demographic changes are not driving these results by estimating regressions at the individual level and controlling for age and sex.

³¹ Results are similar when we include all housing units instead of only single-family homes.

³² Although the Wharton Survey does include a measure of minimum lot size restrictions, it only indicates the presence of minimum lot size restrictions of 1 acre or more, so it does not capture the smaller minimum lot size restrictions that exist in many metros (Gyourko and Molloy 2015)

and other constraints prevent people from adjusting along other dimensions as much as they would like.

Next, we turn to how supply constraints affect the types of people who choose to live in the area. The model predicted that people with more income would be more likely to stay in areas with population constraints, while it predicted no effect of land supply constraints on sorting because we assumed that an individual's taste for the regulated city is uncorrelated with income. If instead we were to assume that income is positively correlated with changes in the taste for the regulated city—say because the regulated city has amenities that have become more valued by richer people—then we would expect land supply constraints also to cause sorting by income.

We look for evidence of income-based sorting using data from the Census and ACS on income. Specifically, we calculate the fraction of individuals in a metropolitan area that are in each decile of the national distribution of income. An increase in the fraction of individuals in the upper deciles would be consistent with richer people sorting into that metropolitan area. Therefore, we regress the change in the fraction of individuals in a decile on the supply constraints and metro-level controls for productivity and amenities.

Figure 4 plots the coefficient estimates for each decile. The results are consistent with a mild amount of sorting in response to regulatory constraints (panel A), as these constraints have led to larger shares of individuals in the top two deciles and a smaller share of individuals in the middle of the income distribution (specifically, the 4th to 7th deciles). But these effects are not large, as a 1 standard deviation greater regulatory constraint is associated with only about 0.6 percentage point more of the population being in each of the top two deciles. Similarly, we find that a 1 standard deviation increase in regulatory constraints is associated with only a 3 percent increase in average income and a 2 percent increase in real median income (Table 6). These small magnitudes are consistent with the magnitudes implied by the model.

Panel B of Figure 4 and Table 6 show no evidence of income sorting across metropolitan areas in response to geographic constraints, consistent with a model in which preferences for local amenities are uncorrelated with income.

Next we look at effects on sorting by education and occupation. Regulation is associated with a small and marginally significant increase in the fraction of highly-educated adults—see the second column of table 6—but the estimated effect on the fraction of people in high-income occupations is small and insignificantly different from zero. As with the income results, geographic constraints are unrelated to these measures of permanent income.

To get a sense of the magnitudes of these effects, consider the metropolitan area of San Francisco, which has an appreciable amount of regulation and experienced large increases in its

fractions of high-income and highly educated residents from 1980 to 2016. Our estimated coefficients imply that regulatory constraints can only explain one eighth of the increase in the share of residents in the top decile of the income distribution and one twentieth of the increase in the share of highly educated residents.

4.4. Effects on Sorting Within Metropolitan Areas

Another set of outcomes related to location choice that we examine is location within metropolitan areas. Our model did not differentiate across locations within metropolitan areas, so it does not make any predictions for this type of sorting. However, it is easy to imagine that households might also adjust to higher land prices by choosing to live in a relatively cheaper neighborhood within the metro area.

We assess this possibility by examining whether new housing units are more likely to be located in less-desirable neighborhoods in metropolitan areas with tighter housing supply constraints. Neighborhood desirability is measured using four separate neighborhood characteristics (where neighborhoods are defined as Census tracts): log distance to the central business district (CBD), log average commute time, crime, and school quality. The center of the metropolitan area comes from Holian and Kahn (2015).³³ Commute time is measured in the 2011-2015 ACS. School quality data are obtained from Location Inc., and are derived by adjusting local test score data across states using nationwide test scores to make scores comparable across school districts. Crime rate data are also obtained from Location Inc., and are calculated by assigning crimes reported by all law enforcement agencies in the U.S. to Census tracts using a proprietary model. The education and crime variables are standardized to have mean zero and standard deviation one.

We estimate the effect of supply constraints on location choice within the metro in the CoreLogic property tax data by regressing each of the four neighborhood characteristics on an indicator for whether the home was built post-2000 and an interaction of this indicator with each supply constraint. The regression controls for metropolitan area fixed effects, neighborhood income, and metro-level productivity and amenities, also interacted with the “post-2000” indicator. This specification thus reveals whether homes built post-2000 were more likely to be in lower-amenity neighborhoods if they are in more supply-constrained metro areas, relative to the distribution of housing units in the 1960s and 1970s.

³³ Holian and Kahn (2015) use the location returned when entering the central city name in Google Earth, which they found to be qualitatively “quite reasonable in all cases”. The data are available for download at <http://mattholian.blogspot.com/2013/05/central-business-district-geocodes.html>.

Table 7 reports the results. Perhaps not surprisingly, recently built homes are more likely to be located farther from the CBD, and in areas with longer average commutes, better schools, and less crime. With the exception of school quality (“education index”), we find no statistically significant or economically meaningful interactions between these effects and regulatory constraints. That is, new homes in more regulated metro areas are built in neighborhoods that are a similar distance to jobs as new homes in less regulated metro areas, and in neighborhoods with similar crime levels. They are built in neighborhoods with worse schools than new homes in less regulated metros.

Geographic constraints seem to have more consistently significant effects on location choice. Relative to less constrained metros, those with greater geographic constraints tend to have newer homes closer to the CBD and with lower commute times, as well as in areas with lower school quality and higher crime. The effects on distance and commute time are the opposite of what we would expect if geographic constraints push construction into lower-value neighborhoods, since land prices tend to be higher closer to the CBD.³⁴ While these interactions with geographic constraints are significant, they are economically small, apart from the effect on school quality. Even in the tail of the distribution—for example, two standard deviations above the mean on the index of geographic constraints—new homes are still more likely than existing homes to be built farther from the CBD and in neighborhoods with longer commutes and lower crime.

One important caveat to this analysis is that we do not observe variation in supply constraints across neighborhoods. If supply constraints were tighter in less-desirable neighborhoods, households would be less likely to choose these neighborhoods, possibly offsetting the effect that we expected. On the other hand, research has found that regulations are more likely to be found in wealthier areas with more desirable amenities (Davidoff 2016). This would create an additional mechanism by which regulation would push households into less desirable neighborhoods.

4.5. Effects on Housing Expenditures

Finally, we estimate the effects of the supply constraints on housing expenditures using the Census/ACS data at the household level. Expenditures are measured as rent for renter households, and monthly payments for owner-occupied households (which includes mortgage interest, property taxes and homeowners’ insurance). Other than including data on owner expenditures, the main difference between this specification and the specification estimating effects on quality-adjusted rent (reported in Table 3) is that we do not control for housing unit

³⁴ We find generally similar results when we look for these effects by regressing tract-level population growth on the four neighborhood quality measures and interact these measures with our supply constraints.

characteristics. Thus we are estimating effects on both rental and owner expenditures, including changes in quality, instead of constant-quality rent or house values.

Table 8 reports the coefficient estimates. A one standard deviation tighter degree of regulation is associated with 4 percent higher expenditures, an effect similar to (although a bit smaller than) the 7 percent increase predicted by our model. A one standard deviation tighter degree of geographic constraint is associated with 5 percent higher expenditures. The Cobb-Douglas version of the model predicted that housing expenditures would not respond to a decrease in land supply, but allowing for an inelastic substitution between housing and non-housing causes the model to predict a 2 percent increase in housing expenditures. As we discussed above, it is possible that other factors, such as minimum lot sizes or density restrictions, prevent people from offsetting the effects of a tighter land supply with less land consumption by as much as they would like, which would lead to higher housing expenditures.

The table also reports results where the dependent variable is an indicator for whether a household spends more than 30 percent of their income on housing, a common measure of housing “cost burden” in the affordability literature (Anthony 2018, Bieri and Dawkins 2019). Both constraints increase the fraction of “cost burdened” households by a small amount, but even the effects of a 2 standard deviation tighter constraint are much smaller than the increase in “cost burdened” households over our sample period. This pattern holds among both owner-occupiers and renters, as indicated by the two rightmost columns.

We end this section with an examination of how the effects on housing expenditures vary with household income. Concerns about housing affordability are most relevant for lower income households, so it would be helpful to know if housing supply constraints have different effects for households at different points in the income distribution. Toward this end, we regress log expenditures and an indicator for expenditures exceeding 30 percent of income on our supply constraints, running separate regressions for each decile of the national income distribution.

Figure 5 shows the results. The estimated effects on expenditures (panel A) are fairly similar across the income distribution, but are somewhat smaller for the bottom-most deciles. The effects on the indicator for having high housing expenditures relative to income (panel B) are more hump-shaped, with the largest effects of in the middle of the income distribution. For households in the top decile and the bottom deciles, supply constraints appear to have no effect on the probability of spending a large fraction of income on housing. A large majority of households in the lowest income deciles already spend more than 30 percent of their income on housing, so there may be less margin for constraints to push households over the threshold. In addition, it is possible that housing affordability programs are helping to reduce the effects of supply constraints for these households. Meanwhile, because the effects on expenditures are

fairly similar for middle- and high-income households, they are a smaller share of income as income rises, and it is less likely that the increase in expenditures pushes high-income households over the 30 percent threshold. It is worth noting that even for the household income categories with the largest effects, a 2 standard deviation increase in supply constraints can explain less than half of the increase in the average fraction of these households spending more than 30 percent of their income on housing.

5. Robustness

In this section, we assess the robustness of our main results to a number of alternate specifications.

5.1. Weighting

Our baseline analysis weights households to be representative of the US population, so that our estimates reflect the average effects for the average household in the US. However, because some metropolitan areas are much larger than others, this choice means that larger metros have much more weight in our results. Panel A of Table 9 reports results that weight each metro area equally. The estimated effects of both constraints on house value are still positive and significant, but smaller in magnitude than in our baseline. The effects on rent are similar to the baseline. The combination of these results implies that supply constraints raise the price-to-rent ratio more in larger metros, which could be the case if supply constraints raise expected rent growth more in larger metros. Nevertheless, even in the average metro, supply constraints boost house value more than rent. The housing consumption and location outcomes are fairly similar to the baseline results—we still find zero to small negative effects on housing consumption, and a small positive effect of regulatory constraints on the fraction of households in the top decile of the national income distribution (the baseline results for the fraction of households in the top of the national income distribution are shown as the rightmost dots in Figure 4).

5.2 Controlling for ex-post demand

Our baseline specification controlled for ex-ante proxies for local housing demand. If these variables are not sufficiently correlated with local demand factors that are also related to housing market outcomes, then our results could be biased. We address this concern by replacing these proxies for demand with our measure of ex-post demand described in section 3.1. Panel B of Table 9 reports the results; the estimates are very close to the baseline across the board.

5.3 Interactions with demand

Our empirical analysis focuses on trying to estimate the effects of supply constraints on the changes in average outcomes over time. This approach is consistent with our model, which predicts changes in outcomes in a regulated city compared to an unregulated city, holding demand constant. Other research on effects of housing supply constraints has examined how these constraints alter the effects of a local demand shock (Glaeser, Gyourko and Saks 2006, Hilber and Vermeulen 2015, Saks 2008). Focusing on this interaction can provide an alternative means of identification, since absent supply differences, a demand shock of a given magnitude would be expected to have the same effect in any location.

We first assess the relevance of interactions of supply constraints with demand shocks by estimating the effect of an increase in local demand in each separate quartile of the distribution of supply constraints.³⁵ The effects on house value, rent, and median income are shown in Figure 6. As expected, demand appears to have larger effects on house prices in metros with tighter supply constraints, although the gradient is more clear for regulation than for geographic constraints (the top panels). By contrast, effects on rent are more similar across quartiles for both constraints (the middle panels). Turning to sorting by income (the bottom panels), it again appears that there is a positive gradient for regulation but not for geographic constraints. In other words, an increase in demand leads to more positive income sorting in metro areas with more regulation, but not in metros with more geographic constraints.³⁶ In sum, while it seems clear that an increase in demand results in larger house price increases and more income-based sorting in more supply-constrained areas, effects of demand on other housing outcomes do not appear systematically related to the tightness of supply constraints.

Our measure of demand reflects all sources of local demand, and thus it is difficult to interpret this measure as an exogenous demand shock. To be even more consistent with the research that has examined effects of demand shocks, we calculate predicted demand using our five proxies for productivity growth and changes in amenities. In this case, predicted demand appears to have larger effects on house prices and on rents in metro areas in the top half of the distributions of supply constraints.

5.4 Ex-post measure of supply constraints

Finally, we return to the question about whether our estimated effects of supply constraints may be biased downward due to measurement error. There is surely some degree of measurement error in the regulatory variable since it does not capture all forms of regulation, and because it is derived from a survey of local planning officials. And while the two types of

³⁵ In order to be able to interpret the coefficient on the demand variable as the effect of an increase in demand, we drop the controls for income from the housing consumption regressions.

³⁶ We find no consistent patterns in the differential responses to demand of other outcomes—including the fraction of single-family homes, rooms per adult, structure size, or house size—across quartiles of either supply constraint.

geographic constraint—the slope of the land and the amount of area under water—may be measured fairly precisely, there are surely other geographic constraints not captured by this measure. A different way to measure supply constraints would be to look at ex-post growth in house prices relative to the housing stock. For example, Gyourko, Mayer and Sinai (2013) identify “superstar metros” as metro areas with a combination of tight supply and strong demand using ex-post changes in house values and the housing stock. Following their approach, we create an indicator for tight supply and strong demand, defined as having demand in above the median and supply above the 90th percentile. Demand is defined as above, and supply is measured as the difference between the growth in house prices and the growth in the housing stock. One important drawback to this approach is that most metro areas with tight supply by this measure also have strong demand, so it is not possible to credibly identify the effect of supply constraints separately from the effect of supply constraints combined with strong demand. In order to mitigate the influence of demand on the estimated coefficient of the Superstar indicator, these specifications also control for ex-post demand separately.

Panel D of Table 9 reports the estimated effects. Superstar metros had 0.4 log point larger house price growth than other metros, similar to the magnitudes reported in Gyourko, Mayer and Sinai (2013). Consistent with the model and with the empirical estimates reported above, the effects of Superstar status on rent, housing consumption, and household location are substantially smaller than the effects on house prices. In fact, the effects on rooms per adult are in line with the model’s predicted effects given a supply constraint that raises house prices by the estimated effect of the Superstar indicator, while the effects on rent and sorting are somewhat smaller than what the model would predict (see Appendix Table 7).³⁷ The one material departure of these results from the model’s predictions is the estimated effect of Superstar status on lot consumption. Superstar metros have experienced 0.15 log point *larger* growth in lot size, whereas the model expected lot size to decrease. It is possible that Superstar metros tend to have regulations that increase required lot sizes.³⁸ In sum, measuring supply constraints using ex-post housing market outcomes does not change our finding that rent, housing consumption and household location decisions are much less responsive to supply constraints than house prices.

³⁷ With regard to sorting, the model predicts a 15 percent increase in median income, whereas we estimate a 5 percent increase (not shown).

³⁸ This regression also shows that metro areas with higher demand tend to have smaller lots. This result may be surprising if demand increases income, and richer households want to consume more land. We interpret this result as suggesting that in most areas, strong local demand causes in-migration and that the influx of people causes lot sizes to fall, as not all new construction occurs on the metro’s periphery. Also, an increase in demand could cause an increase in household formation, and new households will consume less land than more established households, which tend to have older and richer members.

Not only do these results speak to the effects of supply constraints, but they also present striking new evidence on the long-run evolution of housing markets in metropolitan areas with a combination of strong demand and tight supply. Many of these metropolitan areas, such as San Francisco and New York, are notorious for having very high rents and a lack of “affordable” housing. We estimate that Superstar metros experienced only 12 percentage point higher rent growth than other metros between 1980 and 2016, a small effect relative to the average rent increase of 76 percent in this sample. Our results are inconsistent with the belief that these locations are experiencing much larger increases in housing unit size. That said, these locations have experienced larger increases in household size and in the fraction of high-income residents.

6. Conclusion

We have shown both theoretically and empirically that housing supply constraints have a smaller effect on housing affordability than on the purchase price of housing. We also find that supply constraints have only limited effects on housing consumption and location decisions. Our results may seem surprising in light of the strong cross-sectional correlation between supply constraints and rents. Indeed, in our sample, the cross-sectional correlation between supply constraints and rents is three times larger than our panel estimates using controls for productivity growth and amenities. But locations with tight supply constraints tended to have had higher rent even back in 1980, so the changes in rent over time are not as strongly correlated with supply constraints as the current levels. Controlling for measurable differences in demand further reduces the estimated effects of supply constraints, suggesting that supply constraints are also correlated with strong housing demand.

One should not conclude from our analysis that housing affordability is not a problem in supply-constrained metropolitan areas. Rather, our results suggest that the supply constraints alone have not been the driving force behind high rents. Why are our estimated causal effects so much smaller than the effects suggested by the cross-sectional correlation between rent and supply constraints? One possibility is that our measures of supply constraints are not good proxies for true supply constraints. That said, the two constraints that we use the most commonly-used measures in the literature, and we still find relatively small effects when using an alternate measure of supply based on ex-post housing market outcomes.

A second possibility is that supply constraints were at least somewhat binding even back in 1980, in which case we have underestimated the true effects of these constraints. Other research has documented the existence of some regulations prior to 1980—for example, Ganong and Shoag (2017) document the appearance of the words “land use” in state court cases as far back as 1950—and some geographic constraints were surely binding back then. But as we have shown, many constraints did become much stronger between 1980 and the 2000s.

A third possibility is that strong rent growth since 1980 is due largely to increases in demand. It is not possible for prices and rents to increase if supply is entirely unconstrained. However, supply is constrained along some dimensions in most locations. For example, nearly all local governments have zoning regulations that separate residential land uses from other uses. The durability of existing structures can make land assembly challenging, especially in neighborhoods close to the urban center. Thus, demand might have sharply increased rent in many metropolitan areas in the US, even ones with relatively less restrictive supply constraints. Future research should examine the sources of rising demand in US metropolitan areas and the connection with housing affordability.

Beyond the results for rent and housing affordability, our research reveals interesting implications of housing supply constraints for housing consumption. We find much smaller reductions in housing unit size and lot size than expected. People adjust to a higher price of housing services by living in larger households rather than by living in smaller homes. An inability to reduce land consumption as much as desired appears to have reduced the number of households living in geographically constrained metropolitan areas. Further work studying why structure sizes and lot size are so unresponsive to housing supply constraints would be fruitful.

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Table 1
Months from Application to Permit Issuance for SF construction

Percentile	Application for Rezoning		Application for Subdivision	
	< 50 units	≥ 50 units	< 50 units	≥ 50 units
	1980s			
10 th	1	2	1	1
50 th	2	2	2	2
90 th	3	3	3	2
	2006			
10 th	3.9	4.8	3.6	3.8
50 th	6.4	8.0	5.6	6.8
90 th	10.7	13.0	9.0	10.7
	Change from 1980s to 2006			
10 th	1.7	2.4	2.2	2.6
50 th	4.7	5.8	4.1	5.2
90 th	8.0	10.2	6.7	8.7

Note. Sample includes the 60 metropolitan areas that appear in both surveys. Data from the 1980s are from a survey conducted by Linneman, Summers, Brooks and Buist (1990) and data from 2006 are from a survey conducted by Gyourko, Saiz and Summers (2008).

Table 2
Reponses to Supply Constraints that Raise Prices 10% over 30 Years
(Model Simulation, %)

	Population constraints	Land area constraints		
	(1)	(2)	(3)	(4)
Quality-adjusted rent (median)	6.0	5.9	5.9	5.7
Housing expenditure, holding income constant	7.3	0.0	2.1	2.1
Structure size, holding income constant	-1.7	0.0	2.1	-1.4
Lot size, holding income constant	-1.7	-15.8	-13.9	-6.6
Structure size, city average	1.6	0.0	2.1	-1.4
Lot size, city average	1.6	-15.8	-13.9	-6.6
Median city income	3.0	0.0	0.0	0.0
Population	-3.6	-2.1	-2.2	-2.1
Housing services consumption	-2.1	-7.6	-5.6	-5.4
<i>Assumptions</i>				
Housing/non-housing substitution elasticity	-	1	0.5	0.5
Lot/structure substitution elasticity	-	1	1	0.33

Table 3
Effect of Housing Supply Constraints on House Prices and Rent

	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Rent) All homes	Ln(Rent) 2-Bed Apt.
2012-2016 Indicator	0.496 (0.024)	0.501 (0.015)	0.489 (0.014)	0.458 (0.016)
Indicator interacted with:				
Regulatory constraints	0.091 (0.023)	0.040 (0.012)	0.034 (0.011)	0.043 (0.013)
Geographic constraints	0.110 (0.022)	0.018 (0.011)	0.042 (0.011)	0.046 (0.013)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Number of metro areas	133	133	133	133
Number of observations	2.2 million	0.38 million	1.2 million	0.35 million

Note. Standard errors are clustered by metropolitan area. Supply constraints are standardized to have a mean equal to zero and standard deviation equal to one. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Value and rent are expressed relative to the price index for personal consumption expenditures. Controls for productivity and amenities are the variables listed in Appendix Table 2 interacted with the “recent” indicator. Observations are weighted to be nationally representative of the housing stock using the household weight provided by the Census Bureau.

Table 4
Effect of Housing Supply Constraints on Housing Lot Consumption

	Ln(Lot Size) SF Homes	SF Indicator All Homes	SF Indicator Recently-Built Homes
“Recent” Indicator	-0.150 (0.030)	-0.012 (0.005)	0.053 (0.011)
Indicator interacted with:			
Regulatory constraints	0.022 (0.022)	0.001 (0.004)	-0.002 (0.008)
Geographic constraints	-0.041 (0.023)	-0.015 (0.005)	-0.029 (0.006)
Controls for Income	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes
Outcome Data	CoreLogic	Census/ACS	Census/ACS
Number of Observations	4.4 million	3.7 million	0.65 million

Note: Standard errors are clustered by metropolitan area. Supply constraints are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for deciles in the national distribution of income and interactions of these indicators with the “recent” indicator. When the outcome uses CoreLogic data, income is median household income by Census tract. When the outcome uses Census/ACS data, income is property-level household income. Controls for productivity and amenities are the variables listed in Appendix Table 2 interacted with the “recent” indicator.

Table 5
Effect of Housing Supply Constraints on Housing Structure Consumption

	Ln(Unit Size) SF Homes	Ln(Rooms) SF Homes	Ln(Rooms) SF, Recently- Built	Ln(Adults per Household) SF Homes	Ln(Rooms Per Adult) SF Homes
“Recent” Indicator	0.291 (0.011)	0.048 (0.004)	0.028 (0.006)	-0.024 (0.005)	0.066 (0.005)
Indicator interacted with:					
Regulatory constraints	-0.001 (0.008)	-0.009 (0.003)	-0.003 (0.005)	0.007 (0.004)	-0.016 (0.004)
Geographic constraints	-0.007 (0.008)	-0.013 (0.003)	-0.023 (0.004)	0.023 (0.004)	-0.033 (0.004)
Controls for Income	Yes	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes	Yes
Outcome Data	CoreLogic	Census/ACS	Census/ACS	Census/ACS	Census/ACS
Number of Observations	4.4 million	2.7 million	0.48 million	2.7 million	2.7 million

Note: Standard errors are clustered by metropolitan area. Supply constraints are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for deciles in the national distribution of income and interactions of these indicators with the “recent” indicator. When the outcome uses CoreLogic data, income is median household income by Census tract. When the outcome uses Census/ACS data, income is property-level household income. Controls for productivity and amenities are the variables listed in Appendix Table 2 interacted with the “recent” indicator.

Table 6
Effect of Housing Supply Constraints on Changes in Metropolitan Area Housing Stock
and Population Characteristics 1980 to 2016

	Ln(Housing Stock)	Fraction 4+ Years College	Fraction High Occupation Score	Ln(Average Real Income)	Ln(Median Real Income)
Constant	0.517 (0.031)	0.138 (0.005)	0.037 (0.002)	0.411 (0.013)	0.306 (0.015)
Regulatory constraints	-0.058 (0.026)	0.007 (0.004)	0.001 (0.002)	0.028 (0.011)	0.020 (0.012)
Geographic constraints	-0.106 (0.023)	0.003 (0.003)	-0.002 (0.001)	0.004 (0.009)	-0.018 (0.011)
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes	Yes
Number of observations	133	133	133	133	133

Note. The housing stock includes single-family and multifamily units. High occupation score is defined as above the 90th percentile of the national distribution of occupation scores in the same year. Metro areas with tight constraints are those in the top third of the distribution of constraints. Controls for productivity and amenities are reported in Appendix Table 2. Observations are weighted by the average number of housing units in 1980 and 2016.

Table 7
Effect of Housing Supply Constraints on Neighborhood Choice

	Ln(Distance to Metro Center)	Ln(Average Commute Time)	Education Index	Crime Index
“Recent” Indicator	0.325 (0.035)	0.082 (0.009)	0.109 (0.038)	-0.389 (0.049)
Indicator interacted with:				
Regulatory Constraints	-0.014 (0.026)	0.005 (0.005)	-0.082 (0.026)	0.031 (0.032)
Geographic Constraints	-0.076 (0.019)	-0.018 (0.005)	-0.115 (0.021)	0.154 (0.026)
Controls for Income	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	CoreLogic	CoreLogic	CoreLogic	CoreLogic
Number of Observations	4.4 million	4.4 million	4.4 million	4.4 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. The education index and crime index are also standardized. Controls for income are median household income by Census tract interacted with decade indicators. Controls for productivity and amenities are the variables listed in Appendix Table 2 interacted with the “recent” indicator.

Table 8
Effect of Housing Supply Constraints on Housing Expenditures

	Ln(Real Expenditure)			Expenditure > 30% of Income		
	Full Sample	Owners	Renters	Full Sample	Owners	Renters
“Recent” Indicator	0.314 (0.012)	0.259 (0.014)	0.381 (0.013)	0.246 (0.015)	0.200 (0.007)	0.307 (0.024)
Indicator interacted with:						
Regulatory constraints	0.040 (0.010)	0.041 (0.013)	0.030 (0.009)	0.022 (0.005)	0.023 (0.005)	0.019 (0.005)
Geographic constraints	0.048 (0.010)	0.063 (0.013)	0.025 (0.009)	0.019 (0.004)	0.021 (0.004)	0.015 (0.005)
Controls for Income	Yes	Yes	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	3.6 mil.	2.4 mil.	1.2 mil.	3.6 mil.	2.4 mil.	1.2 mil.

Note. Standard errors are clustered by metropolitan area. Supply constraints are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for the household’s decile in the national distribution of household income and interactions of these indicators with the “recent” indicator. Controls for productivity and amenities are the variables listed in Appendix Table 2 interacted with the “recent” indicator.

Table 9
Robustness to Alternate Specifications

	Ln(Value) SF Homes	Ln(Rent) SF Homes	Ln(Lot Size) SF Homes	SF Indicator	Ln(Unit Size) SF Homes	Ln(Rooms Per Adult)	Fraction in Top Decile Income Dist.
Panel A: Weighting Each Metro Equally							
“Recent” Indicator	0.513 (0.016)	0.502 (0.011)	-0.138 (0.040)	-0.012 (0.005)	0.261 (0.013)	0.056 (0.005)	-0.003 (0.002)
Indicator interacted with:							
Regulatory const.	0.063 (0.017)	0.045 (0.010)	0.013 (0.027)	0.004 (0.003)	0.011 (0.007)	-0.013 (0.003)	0.006 (0.002)
Geographic const.	0.069 (0.016)	0.010 (0.010)	-0.015 (0.022)	-0.006 (0.003)	-0.001 (0.007)	-0.020 (0.003)	-0.002 (0.002)
Panel B: Controlling for Ex-Post Demand							
“Recent” Indicator	0.478 (0.025)	0.517 (0.012)	-0.125 (0.031)	-0.014 (0.005)	0.289 (0.011)	0.053 (0.006)	-0.006 (0.003)
Indicator interacted with:							
Regulatory const.	0.089 (0.030)	0.037 (0.014)	0.038 (0.019)	0.007 (0.005)	-0.003 (0.008)	-0.020 (0.004)	0.007 (0.003)
Geographic const.	0.095 (0.025)	0.029 (0.012)	-0.039 (0.020)	-0.018 (0.005)	-0.007 (0.008)	-0.037 (0.006)	0.001 (0.002)
Ex-Post Demand	0.097 (0.041)	0.036 (0.017)	-0.105 (0.022)	-0.004 (0.010)	0.012 (0.011)	-0.008 (0.007)	0.009 (0.003)
Panel C: Interactions with Demand							
“Recent” Indicator	0.467 (0.023)	0.516 (0.012)	-0.127 (0.030)	-0.014 (0.005)	0.292 (0.011)	0.055 (0.007)	-0.008 (0.003)
Indicator interacted with:							
Regulatory const.	0.076 (0.032)	0.049 (0.013)	0.022 (0.025)	0.006 (0.006)	-0.001 (0.010)	-0.023 (0.005)	0.005 (0.003)
Reg. x demand	0.037 (0.041)	-0.022 (0.019)	0.031 (0.024)	0.000 (0.009)	-0.009 (0.012)	0.005 (0.006)	0.006 (0.004)
Geographic const.	0.060 (0.032)	0.027 (0.015)	-0.027 (0.022)	-0.016 (0.006)	-0.005 (0.008)	-0.030 (0.009)	-0.007 (0.003)
Geog. x demand	0.058 (0.042)	0.000 (0.019)	-0.018 (0.025)	-0.003 (0.010)	-0.008 (0.011)	-0.010 (0.010)	0.013 (0.004)
Ex-Post Demand	0.086 (0.040)	0.047 (0.022)	-0.133 (0.023)	-0.004 (0.010)	0.010 (0.011)	-0.012 (0.007)	0.015 (0.004)
Panel D: Measuring Supply Constraints using Superstar Indicator							
“Recent” Indicator	0.413 (0.018)	0.503 (0.013)	-0.123 (0.033)	-0.013 (0.007)	0.287 (0.011)	0.062 (0.007)	-0.009 (0.002)
Indicator interacted with:							
Superstar indicator	0.391 (0.029)	0.119 (0.026)	0.145 (0.062)	0.004 (0.017)	0.035 (0.026)	-0.067 (0.019)	0.024 (0.004)
Ex-Post Demand	0.110 (0.027)	0.052 (0.017)	-0.112 (0.023)	-0.007 (0.010)	0.009 (0.012)	-0.020 (0.009)	0.013 (0.003)

Note. Details for specifications shown in columns (1) and (2) can be found in Table 3. Details for columns (3) and (4) can be found in Table 4. Details for columns (5) and (6) can be found in Table 5. Details for column 7 can be found in Figure 4.

Figure 1
Increases in House Prices and Rent in Response to an Unanticipated Population Constraint

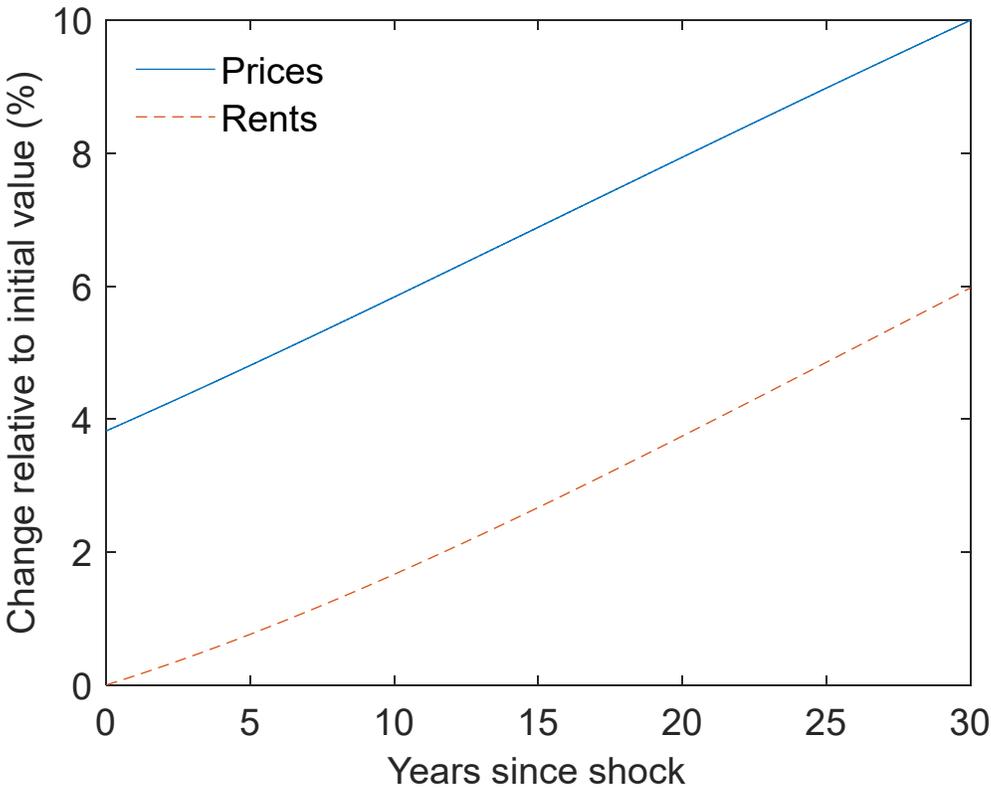
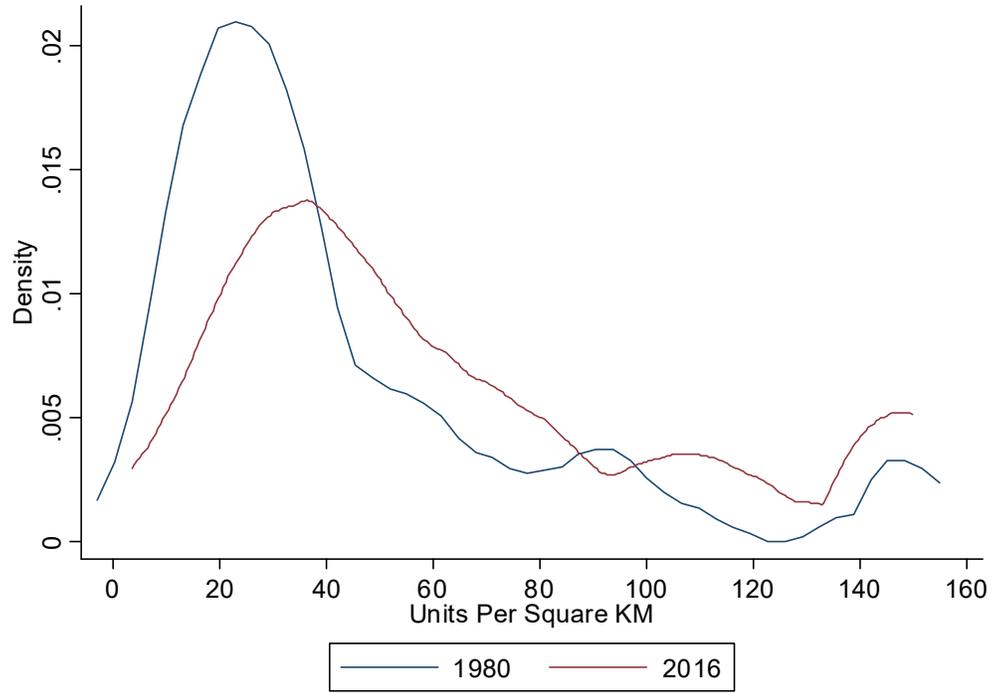
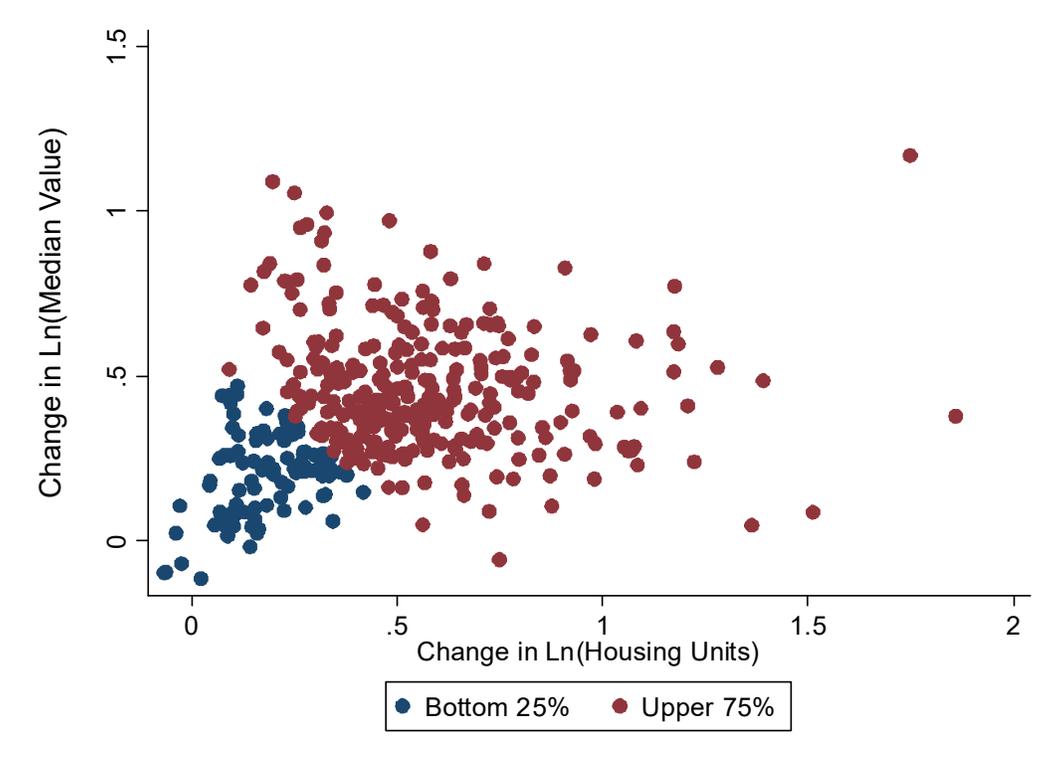


Figure 2
Distribution of Housing Unit Density
Among Central Parts of Metropolitan Areas



Note. The figure shows the distribution of housing units per square kilometer across metropolitan areas in 1980 and 2016. In each metropolitan area, density is calculated only among counties that are designated as “central” according to the 2013 OMB delineation. The sample is restricted to metropolitan areas for which not all counties are designated as central.

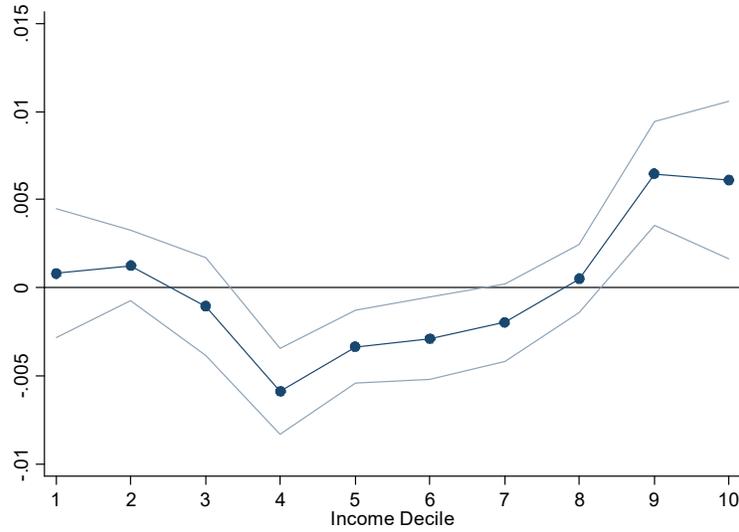
Figure 3
Identification of Low-Demand Areas Based on Growth in Housing Stock and House Value 1980-2016



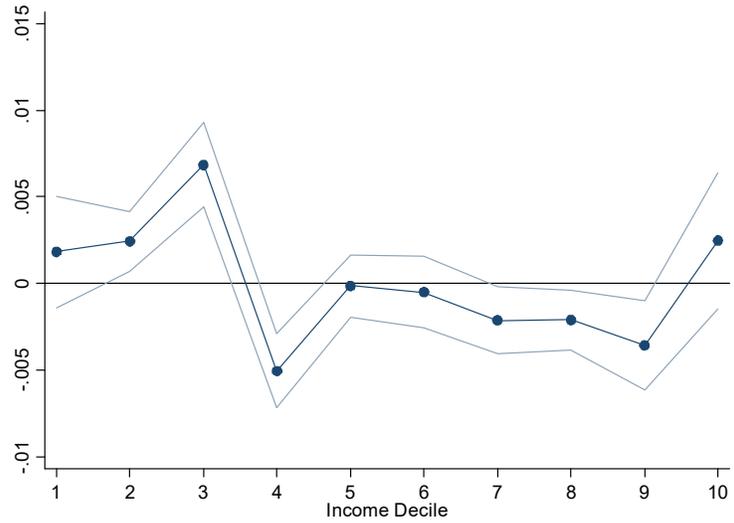
Note. Housing units include single-family and multifamily units. Median value is expressed relative to the price index for personal consumption expenditures.

Figure 4

Panel A: Effect of Regulatory Constraints on the Fraction of People in Each Income Decile



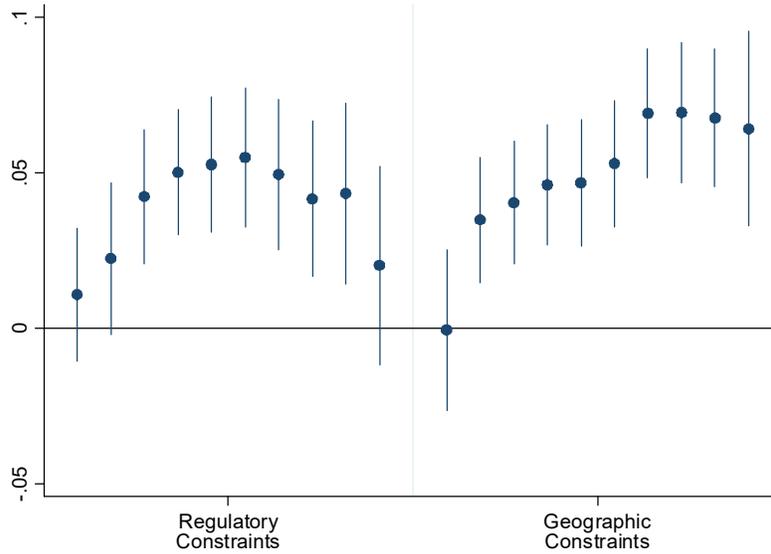
Panel B: Effect of Geographic Constraints on the Fraction of People in Each Income Decile



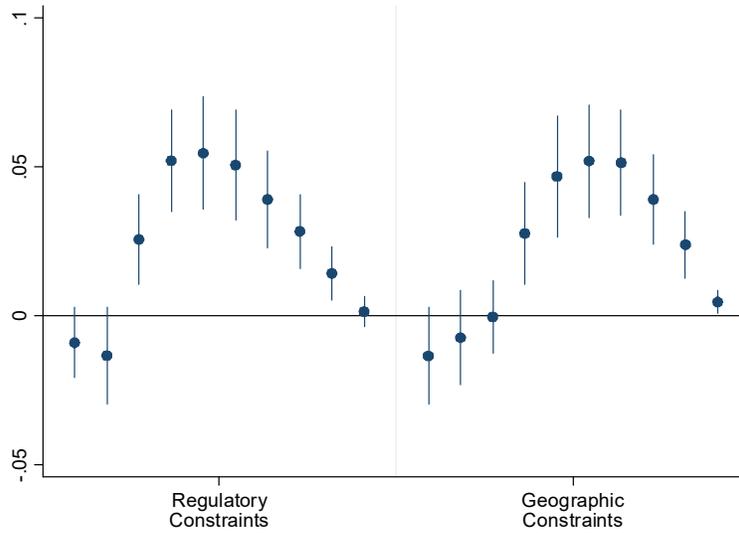
Note. The chart shows the estimated effects of a supply constraint on the change in the fraction of people in each decile of the national income distribution from 1980 to 2016. Regressions control for the variables listed in Appendix Table 2. Regressions are weighted using the average number of housing units in 1980 and 2016.

Figure 5

Panel A: Effect of Supply Constraints on Ln(Real Housing Expenditures) by Decile of Household Income

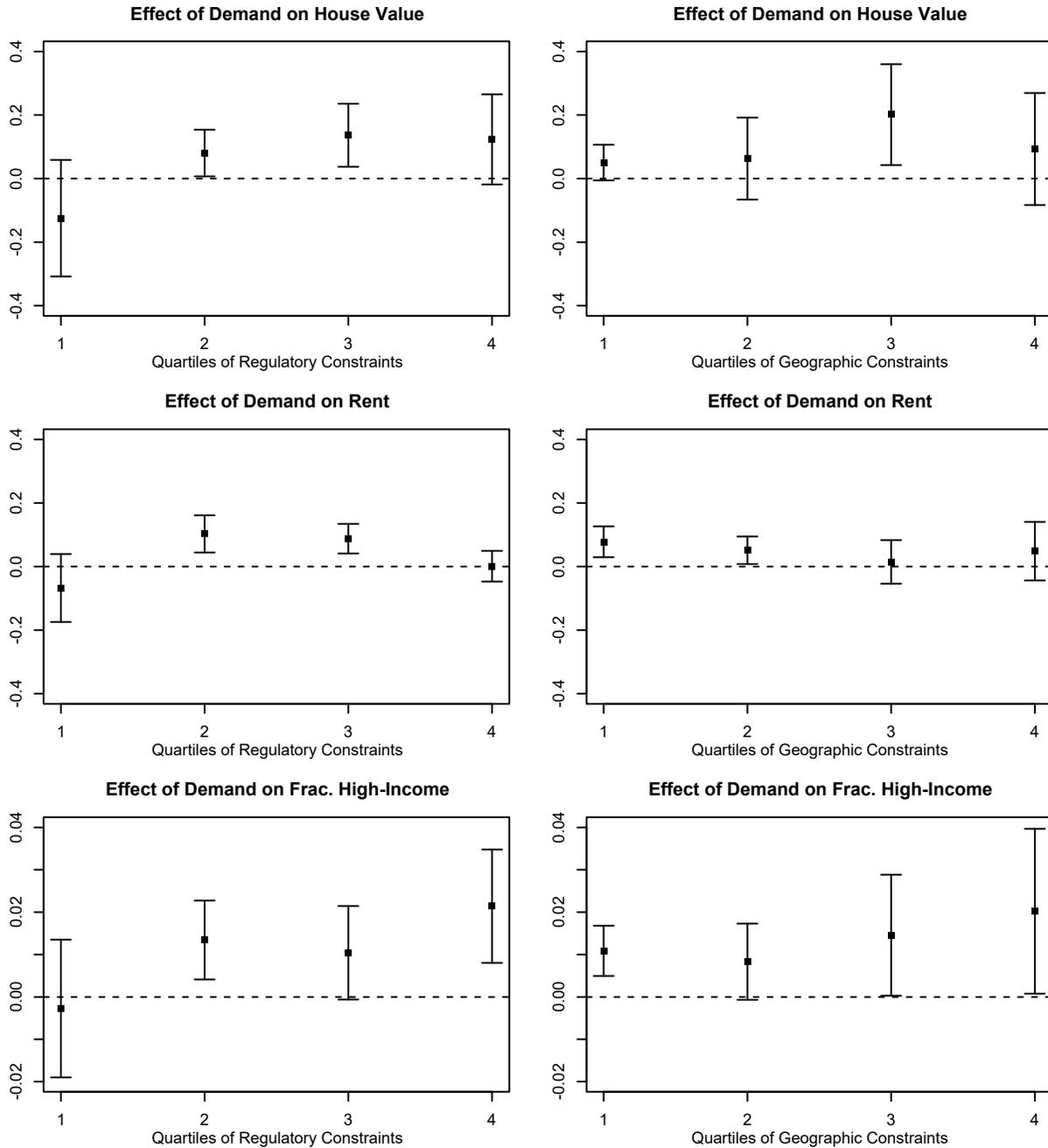


Panel B: Effect of Supply Constraints on Housing Expenditure Shares by Decile of Household Income



Note. The dots show coefficient estimates from regressions using the same specification as shown in Table 7, except that regressions are estimated separately for households in each decile in the national distribution of household income.

Figure 6
Effect of Demand on House Value, Rent, and Fraction High-Income, by Constraint Quartiles



Note. The squares show the effect of a 1 percent increase in local demand, estimated separately for each quartile of regulation (left-hand panels) and geographic constraints (right-hand panels). Regressions also control for a linear function of the other supply constraint (i.e., the regression estimated on the lowest quartile of regulatory constraint controls for a linear function of geographic constraints). Price and rent regressions also control for housing unit characteristics.”

Appendix Table 1
Reponses to Supply Constraints that Raise Prices 10% over 30 Years
(Model Simulation, %)

	Population constraints	Land area constraints		
	(1)	(2)	(3)	(4)
<i>Panel A: low discount rate (r = 0.02)</i>				
Quality-adjusted rent (median)	3.2	3.6	3.6	3.4
Housing expenditure, holding income constant	1.5	0.0	1.3	1.3
Structure size, holding income constant	-3.5	0.0	1.3	-0.9
Lot size, holding income constant	-3.5	-10.2	-8.9	-4.1
Structure size, city average	-0.5	0.0	1.3	-0.9
Lot size, city average	-0.5	-10.2	-8.9	-4.1
Median city income	1.6	0.0	0.0	0.0
Population	-1.9	-1.3	-1.3	-1.3
Housing services consumption	-2.4	-4.8	-3.5	-3.3
<i>Panel B: high discount rate (r = 0.1)</i>				
Quality-adjusted rent (median)	7.9	7.4	7.4	7.3
Housing expenditure, holding income constant	10.6	0.0	2.7	2.7
Structure size, holding income constant	-1.0	0.0	2.7	-1.8
Lot size, holding income constant	-1.0	-19.3	-17.1	-8.2
Structure size, city average	2.6	0.0	2.7	-1.8
Lot size, city average	2.6	-19.3	-17.1	-8.2
Median city income	4.0	0.0	0.0	0.0
Population	-4.8	-2.7	-2.7	-2.7
Housing services consumption	-2.3	-9.4	-7.0	-6.9

Note: the *Assumptions* panel of Table 1 specifies the parameters in columns (2) through (4).

Appendix Table 2
Determinants of Growth in Local Housing Demand

	$\Delta \ln(\text{Demand})$ 1980 to 2016
Share of high wage growth industries in 1990	0.17 (0.05)
Share of population with 4+ years college in 1980	0.29 (0.07)
Predicted employment growth 1980 to 2016	0.29 (0.07)
Average January temperature 1980-2010	0.25 (0.04)
Share of seasonal housing units in 1980	0.60 (0.12)
Adjusted R ²	0.43
Number of observations	244

Note. All variables are scaled to have a mean equal to zero and standard deviation equal to one. Observations are weighted by the number of housing units in 1980.

Appendix Table 3
Illustration of Identification Strategy: Effects on Ln(Real House Value)

	2016	Panel: 1980 and 2016			
Regulatory Constraints	0.328 (0.071)	--	--	--	--
2012-2016 Indicator	--	0.454 (0.023)	0.453 (0.024)	0.429 (0.024)	0.496 (0.024)
Indicator interacted with:					
Regulatory constraints	--	0.177 (0.029)	0.155 (0.031)	0.116 (0.025)	0.091 (0.023)
Geographic constraints	--	--	0.074 (0.027)	0.062 (0.024)	0.110 (0.022)
Share with 4+ years college	--	--	--	0.021 (0.032)	-0.032 (0.031)
Share in high wage growth industries	--	--	--	0.048 (0.027)	0.045 (0.025)
Predicted employment growth 1980-2016	--	--	--	0.054 (0.044)	0.039 (0.032)
January temperature	--	--	--	0.012 (0.029)	-0.055 (0.028)
Share seasonal housing units	--	--	--	0.068 (0.055)	-0.022 (0.046)
Exclude low-demand areas?	No	No	No	No	Yes
Number of metro areas	177	177	176	176	133
Number of observations	2.5 mil.	2.7 mil.	2.7 mil.	2.7 mil.	2.2 mil.

Note. Observations are owner-occupied single-family housing units. House value is deflated with the price index for personal consumption expenditures. Standard errors in columns 2 to 5 are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. All columns control for indicators for decade built, indicators for number of rooms and indicators for number of bedrooms.

Appendix Table 4
Effect of Housing Supply Constraints on House Prices and Rent

	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Rent) All homes	Ln(Rent) 2-Bed Apt.
2012-2016 Indicator	0.310 (0.049)	0.413 (0.033)	0.368 (0.046)	0.313 (0.059)
Indicator interacted with:				
2 nd quartile regulation	0.066 (0.061)	0.044 (0.038)	0.054 (0.049)	0.070 (0.060)
3 rd quartile regulation	-0.032 (0.067)	0.015 (0.037)	0.051 (0.051)	0.058 (0.060)
4 th quartile regulation	0.188 (0.072)	0.076 (0.037)	0.095 (0.051)	0.123 (0.062)
2 nd quartile geographic constraints	0.072 (0.053)	0.064 (0.025)	0.046 (0.032)	0.057 (0.040)
3 rd quartile geographic constraints	0.161 (0.055)	0.056 (0.040)	0.083 (0.038)	0.091 (0.048)
4 th quartile geographic constraints	0.258 (0.058)	0.090 (0.037)	0.117 (0.042)	0.132 (0.050)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Number of metro areas	133	133	133	133
Number of observations	2.2 million	0.38 million	1.2 million	0.35 million

Note. Standard errors are clustered by metropolitan area. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Value and rent are expressed relative to the price index for personal consumption expenditures. Controls for productivity and amenities are the variables listed in Appendix Table 1 interacted with the “recent” indicator. Observations are weighted to be nationally representative of the housing stock using the household weight provided by the Census Bureau.

Appendix Table 5
Effect of Housing Supply Constraints on the Change in Ln(Price-to-Rent Ratios) 1980 to 2016

	All Homes		Single-Family Homes	
	Ratio of Averages	Ratio of Medians	Ratio of Averages	Ratio of Medians
Constant	0.108 (0.017)	0.039 (0.018)	0.073 (0.019)	-0.032 (0.021)
Indicator interacted with:				
Regulatory constraints	0.047 (0.014)	0.056 (0.015)	0.042 (0.015)	0.046 (0.018)
Geographic constraints	0.048 (0.012)	0.059 (0.013)	0.072 (0.014)	0.099 (0.015)
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Number of observations	133	133	133	133

Note. Controls for productivity and amenities are the variables listed in Appendix Table 1. Observations are weighted to be nationally representative of the housing stock using the household weights provided by the Census Bureau.

Appendix Table 6
Effect of Housing Supply Constraints on House Prices and Rent
Selected Subsamples

	Metro Areas Without Rent Control		Household Head Moved In Within Past 5 Years	
	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Value) SF Homes	Ln(Rent) SF homes
2012-2016 Indicator	0.518 (0.022)	0.513 (0.016)	0.419 (0.025)	0.473 (0.016)
Indicator interacted with:				
Regulatory constraints	0.105 (0.027)	0.050 (0.012)	0.084 (0.024)	0.041 (0.012)
Geographic constraints	0.067 (0.018)	0.009 (0.010)	0.105 (0.021)	0.031 (0.011)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Controls for metro productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Number of metro areas	124	124	133	133
Number of observations	1.7 million	0.29 million	0.49 million	0.27 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Controls for metro productivity and amenities are the variables shown in Appendix Table 1 interacted with the “recent” indicator. Metropolitan areas with rent control are identified from http://www.landlord.com/rent_control_laws_by_state.htm.

Appendix Table 7
Reponses to Supply Constraints that Raise Prices 48 Percent over 30 Years
(Model Simulation, %)

	Population constraints	Land area constraints		
	(1)	(2)	(3)	(4)
Quality-adjusted rent (median)	27.9	26.6	25.9	24.4
Housing expenditure, holding income constant	28.7	0.0	8.9	8.4
Structure size, holding income constant	-5.5	0.0	8.9	-4.4
Lot size, holding income constant	-5.5	-50.7	-45.4	-21.3
Structure size, city average	8.2	0.0	8.9	-4.4
Lot size, city average	8.2	-50.7	-45.4	-21.3
Median city income	15.1	0.0	0.0	0.0
Population	-16.1	-8.8	-9.0	-8.5
Housing services consumption	-9.3	-28.0	-21.3	-20.2
<i>Assumptions</i>				
Housing/non-housing substitution elasticity	-	1	0.5	0.5
Lot/structure substitution elasticity	-	1	1	0.33

Note. The model simulation is calibrated to match the effect of Superstar status on house prices, which is an effect on $\ln(\text{house price})$ of 0.39 as reported in Table 9.

A. Appendix

A.1. CES specification

In the most general form of the model, both preferences and the housing production function take the CES functional form:

$$v(c_{i,t}, h_{i,t}) = \left((1 - \alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho \right)^{\frac{1}{\rho}}$$

and

$$h(q, l) = (\gamma q^\sigma + (1 - \gamma)l^\sigma)^{\frac{1}{\sigma}},$$

where $\rho, \sigma \leq 1$. When either parameter approaches zero, the corresponding CES function limits to the Cobb-Douglas specification in the text. We first solve the model under this generality and reduce to the Cobb-Douglas case to prove the results in the text.

A.2. Developer optimization

The developer problem is

$$\min_{q,l} k^q q + (p_{j,t}^l + k^l)l + x_{j,t}$$

subject to the constraint $(\gamma q^\sigma + (1 - \gamma)l^\sigma)^{\frac{1}{\sigma}} \geq h$. Solving this gives the price of housing: $p_{j,t}(h) = x_{j,t} + m_{j,t}h$, where

$$m_{j,t} = \left((1 - \gamma)^{\frac{1}{1-\sigma}} (p_{j,t}^l + k^l)^{-\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (k^q)^{-\frac{\sigma}{1-\sigma}} \right)^{\frac{1-\sigma}{\sigma}}.$$

The minimizing structure size is

$$q_{j,t}(h) = \gamma^{-\frac{1}{\sigma}} \left(1 + \left(\frac{1 - \gamma}{\gamma} \right)^{\frac{1}{1-\sigma}} \left(\frac{k^q}{p_{j,t}^l + k^l} \right)^{\frac{\sigma}{1-\sigma}} \right)^{-\frac{1}{\sigma}} h,$$

and the minimizing lot size is

$$l_{j,t}(h) = (1 - \gamma)^{-\frac{1}{\sigma}} \left(1 + \left(\frac{\gamma}{1 - \gamma} \right)^{\frac{1}{1-\sigma}} \left(\frac{p_{j,t}^l + k^l}{k^q} \right)^{\frac{\sigma}{1-\sigma}} \right)^{-\frac{1}{\sigma}} h.$$

When $x_{j,t} = p_{j,t}^l = 0$, the budget share of structure is

$$\tilde{\gamma} = \frac{(k^l)^{\frac{\sigma}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}}}{(k^l)^{\frac{\sigma}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}} + (k^q)^{\frac{\sigma}{1-\sigma}} (1-\gamma)^{\frac{1}{1-\sigma}}}.$$

A.3. Dynasty optimization

Here we solve the dynasty problem given beliefs about future prices. We apply this solution to proofs that follow. Let λ_d denote the multiplier on the dynasty budget constraint for dynasty d . The first-order conditions for $c_{i,t}$ and $h_{i,t}$ respectively are

$$\lambda_d = \frac{(1-\alpha)c_{i,t}^{\rho-1}}{(1-\alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho}$$

and

$$\lambda_d m_{j,t} = \frac{\alpha h_{i,t}^{\rho-1}}{(1-\alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho}.$$

The solution depends only on d, j , and t , so consumption levels are identical across households in the same city and dynasty at the same time. This solution is

$$c_{d,j,t} = \frac{\lambda_d^{-1} (1-\alpha)^{\frac{1}{1-\rho}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}}}$$

and

$$h_{d,j,t} = \frac{\lambda_d^{-1} \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{1}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}}}$$

The optimized flow utility is

$$v_{d,j,t} = \lambda_d^{-1} \left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{1-\rho}{\rho}}.$$

Expenditure for households in d and j at t is $c_{d,j,t} + x_{j,t} + m_{j,t} h_{d,j,t} = x_{j,t} + \lambda_d^{-1}$. The dynastic budget constraint therefore reduces to

$$\frac{y_d}{r-g} = \frac{\lambda_d^{-1}}{r-g} + \int_0^\infty e^{-(r-g)t} s_{d,R,t} x_{R,t} dt,$$

where $s_{d,j,t}$ equals the share of households in dynasty d in city j at t . Therefore

$$\lambda_d = \left(y_d - \int_0^\infty (r-g)e^{-(r-g)t} s_{d,R,t} x_{R,t} dt \right)^{-1}.$$

Substitution into dynastic utility transforms it to

$$(r-g)^{-1} \log \left(y_d - \int_0^\infty (r-g)e^{-(r-g)t} s_{d,R,t} x_{R,t} dt \right) + \int_0^\infty e^{-(r-g)t} \sum_{j \in \{F,R\}} s_{d,j,t} \left(\log a_{d,j} + \frac{1-\rho}{\rho} \log \left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}} \right) \right) dt$$

times the number of households in the dynasty at time zero, where $a_{d,j}$ is the common taste for city j across households in dynasty d . Because $s_{d,F,t} = 1 - s_{d,R,t}$, the marginal gain from increasing $s_{d,R,t}$ is

$$\log \left(\frac{a_{d,R}}{a_{d,F}} \right) - \frac{1-\rho}{\rho} \log \left(\frac{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{F,t}^{\frac{\rho}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{R,t}^{\frac{\rho}{\rho-1}}} \right) - \frac{x_{R,t}}{y_d - (r-g) \int_0^\infty e^{-(r-g)t'} s_{d,R,t'} x_{R,t'} dt'}$$

times $e^{-(r-g)t}$. When this gain is positive, $s_{d,R,t} = 1$, when this gain is negative, $s_{d,R,t} = 0$, and when this gain equals zero, $s_{d,R,t}$ can take any value between zero and 1.

A.4. Initial equilibrium

At time zero, households believe that city R will remain unregulated. Therefore, they believe that $x_{R,t} = p_{R,t}^l = 0$ for all $t \geq 0$. As a result, $p_{R,0}(h) = m_0 h$ and $p_{R,0}^{own} = r^{-1} m_0 h$, where

$$m_0 = \left((1-\gamma)^{\frac{1}{1-\sigma}} (k^l)^{-\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (k^q)^{-\frac{\sigma}{1-\sigma}} \right)^{-\frac{1-\sigma}{\sigma}}.$$

Furthermore, $s_{d,R,t} = 1$ when $a_{d,R} > a_{d,F}$ and $s_{d,R,t} = 0$ when $a_{d,R} < a_{d,F}$. The initial population in R equals the measure of households for whom $a_{i,R} > a_{i,F}$, which reduces to $\epsilon_{i,R} - \epsilon_{i,F} > \beta(\log a_R - \log a_F)$. Given standard results about extreme value distributions, the population is

$$N_{R,0} = a_R^\beta (a_F^\beta + a_R^\beta)^{-1} N_0.$$

From Section A.2, the lot size of a house in R is $l_{R,0} h = (1 - \tilde{\gamma})(k^l)^{-1} m_0 h$, and from Section A.3, housing consumption for household i is $h_{i,0} = m_0^{-1} \tilde{\alpha} y_i$, where

$$\tilde{\alpha} = \frac{\alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}.$$

The lot size for household i in city R is $\tilde{\alpha}(1 - \tilde{\gamma})(k^l)^{-1}y_i$. The total land area of R at time zero is

$$L_{R,0} = \tilde{\alpha}(1 - \tilde{\gamma})(k^l)^{-1}a_R^\beta (a_F^\beta + a_R^\beta)^{-1} N_0 \bar{y},$$

where $\bar{y} = \int_0^\infty yf(y)dy$.

A.5. Proof of Proposition 1

By the $s_{d,R,t}$ condition from Section A.3, a household with $a_{i,R} < a_{i,F}$ always lives in F . Otherwise, given $s_{d,R,t}$, there exists a unique x_i^* such that the household lives in R if $x_{R,t} < x_i^*$ and lives in F if $x_{R,t} > x_i^*$. As a result, the household lives in R when

$$\log\left(\frac{a_{i,R}}{a_{i,F}}\right) \geq \frac{x_{R,t}}{y_i - (r - g) \int_{t|x \leq x_i^*} e^{-(r-g)t} x_{R,t} dt}.$$

This inequality is an equality when $x_{R,t} = x_i^*$, which proves the proposition. This proposition holds under the CES specification as well.

A.6. Proof of Proposition 2

As $x_{R,t}$ increases, $\bar{x}(x_{R,t})$ weakly increases because the domain of integration in the definition of \bar{x} weakly expands. The right side of the equation that we claim determines $x_{R,t}$ therefore strictly decreases in $x_{R,t}$. Because the left side of this equation strictly decreases in t , $x_{R,t}$ must strictly decrease in t , as claimed.

The equations for rent and price changes are immediate from substituting the equation for $p_{R,t}(h)$ for $t > 0$ from section 2.1 into the equation for $p_{R,0}(h)$ from appendix A.4 (taking the limit as $\sigma \rightarrow 0$). Because $r \int_t^\infty e^{-r(t'-t)} x_{R,t'} dt'$ averages $x_{R,t'}$ over the interval $[t, \infty)$, this average strictly exceeds $x_{R,t}$ because $x_{R,t'}$ increases in t' . As a result, prices rise more than rents.

A.7. Proof of Proposition 3

Using the formulas from appendix A.3 in the $\rho \rightarrow 0$ limit, we have

$$h_i^* = \alpha\gamma^\gamma(1 - \gamma)^{1-\gamma}(k^l)^{\gamma-1}(k^q)^{-\gamma}(y_i - \bar{x}(x_i^*)).$$

Due to Cobb-Douglas production, the share of the value going to structure is γ and the share going to lot is $1 - \gamma$. We substitute h_i^* into the expression for housing spot price from section 2.1 and multiply by $\gamma(k^q)^{-1}$ and $(1 - \gamma)(k^l)^{-1}$ to obtain q_i^* and l_i^* , respectively.

We now show that

$$E(y_i - \bar{x}(x_i^*) \mid y_i) < E(y_{i'} - \bar{x}(x_{i'}^*) \mid y_{i'}).$$

If $y_i < y_{i'}$ and both households are in city R at time t . Doing so proves the final statement in the proposition. By Proposition 1, households of income y_i reside in R at t only if

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) \geq \frac{x_{R,t}}{y_i - \bar{x}(x_{R,t})}.$$

Call this threshold ϕ_i . We have $\phi_i > \phi_{i'}$. If $a_{R,i}/a_{F,i} = a_{R,i'}/a_{F,i'}$, then $x_i^* < x_{i'}^*$ which means that $y_i - \bar{x}(x_i^*) < y_{i'} - \bar{x}(x_{i'}^*)$ by Proposition 1. The distribution of $\log(a_{R,i}/a_{F,i})$ and $\log(a_{R,i'}/a_{F,i'})$ is the same conditional on exceeding ϕ_i . Therefore, for such households, our claim holds. Furthermore, because $\bar{x}(x_{i'}^*)$ rises in $\log(a_{R,i'}/a_{F,i'})$, $y_{i'} - \bar{x}(x_{i'}^*)$ is larger when $\log(a_{R,i'}/a_{F,i'}) \leq \phi_i$ than when $\log(a_{R,i'}/a_{F,i'}) > \phi_i$. Therefore, the inequality we desire holds for the entire distribution.

A.8. Proof of Proposition 4

This proposition follows immediately from the $s_{d,R,t}$ condition from section A.3.

A.9. Proof of Proposition 5

Using the formulas from appendix A.3 in the $\rho \rightarrow 0$ and $\sigma \rightarrow 0$ limit, we have

$$h_i^* = \alpha\gamma^\gamma(1-\gamma)^{1-\gamma}(k^l + p_{R,t}^l)^{\gamma-1}(k^q)^{-\gamma}y_i.$$

Due to Cobb-Douglas production, the share of the value going to structure is γ and the share going to lot is $1 - \gamma$. We substitute h_i^* into the expression for housing spot price from section 2.1 and the multiply by $\gamma(k^q)^{-1}$ and $(1-\gamma)(k^l)^{-1}$ to obtain q_i^* and l_i^* , respectively.

A.10. Proof of Proposition 6

The equations for rent and price changes are immediate from substituting the equation for $p_{R,t}(h)$ for $t > 0$ from section 2.1 into the equation for $p_{R,0}(h)$ from appendix A.4. Because the price effect averages the current and future rent effects, which strictly increase over time, the price effect exceeds the rent effect.

A.11. Quantitative model solution

In the case of permit delays, we solve for $x_{R,t}$ using differential equations as follows. We define $\bar{x}_t = (r-g) \int_0^t e^{-(r-g)t'} x_{R,t'} dt'$, which equals $\bar{x}(x_{R,t})$ because $x_{R,t}$ strictly increases. Differentiation yields

$$\dot{\bar{x}}_t = (r-g)e^{-(r-g)t} x_{R,t}.$$

Because Proposition 1 holds in the general CES case, the equation from section 2.2 determining $x_{R,t}$ holds in the CES model as well. Differentiating this equation gives

$$(g - g^n)e^{-(g-g^n)t} = \int_{\bar{x}_t}^{\infty} \frac{\beta(a_F^\beta + a_R^\beta)a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) \left((y - \bar{x}_t)\dot{x}_{R,t} + x_{R,t}\dot{\bar{x}}_t\right)}{\left(a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) + a_R^\beta\right)^2 (y - \bar{x}_t)^2} f(y) dy.$$

We then substitute the equation for $\dot{\bar{x}}_t$ and solve for $\dot{x}_{R,t}$ to obtain

$$\dot{x}_{R,t} = \left(\frac{(g - g^n)e^{-(g-g^n)t}}{\beta(a_F^\beta + a_R^\beta)a_F^\beta} - x_{R,t}(r - g)e^{-(r-g)t} \right) \left(\int_{\bar{x}_t}^{\infty} \frac{\exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) f(y) dy}{\left(a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) + a_R^\beta\right)^2 (y - \bar{x}_t)} \right)^{-1}.$$

We now have two differential equations in the two unknowns $x_{R,t}$ and \bar{x}_t . The initial conditions are $x_{R,0} = 0$ and $\bar{x}_0 = 0$.

In the case of geographic constraints, we calculate $m_{R,t}$ numerically. In the Cobb-Douglas case, we use the explicit formula for $p_{R,t}^l$ appearing in section 2.3. In the CES case, we make a series of substitutions to derive differential equations pinning down this price over time from the market-clearing condition for land. Define

$$u_t = \left(\frac{m_{R,t}}{m_0} \right)^{\frac{\rho}{\rho-1}}$$

and

$$v_t = \frac{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}{(1 - \alpha)^{\frac{1}{1-\rho}} + u_t \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}.$$

From Section A.3, the measure of households choosing R is the measure of those for whom

$$\log\left(\frac{a_{d,R}}{a_{d,F}}\right) \geq \frac{1 - \rho}{\rho} \log\left(\frac{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{F,t}^{\frac{\rho}{\rho-1}}}{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{R,t}^{\frac{\rho}{\rho-1}}}\right),$$

which equals

$$s_{R,t} = \frac{a_R^\beta}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}.$$

We also define

$$w_t = \left(\frac{(1-\gamma)^{\frac{1}{1-\sigma}} (k^q)^{\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (k^l)^{\frac{\sigma}{1-\sigma}}}{(1-\gamma)^{\frac{1}{1-\sigma}} (k^q)^{\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (p_{R,t}^l + k^l)^{\frac{\sigma}{1-\sigma}}} \right)^{\frac{1}{\sigma}}$$

and

$$z_t = \left(\frac{p_{R,t}^l + k^l}{k^l} \right)^{\frac{\sigma}{1-\sigma}}.$$

Using these substitutions, we write the lot size of household i in city R as

$$l_{i,R} = \left(\frac{1-\tilde{\gamma}}{1-\gamma} \right)^{\frac{1}{\sigma}} w_t u_t^{\frac{1}{\rho}} \tilde{\alpha} v_t y_i.$$

Market-clearing then simplifies to

$$e^{-(g-g^l)t} = w_t u_t^{\frac{1}{\rho}} v_t \frac{a_R^\beta + a_F^\beta}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}.$$

This equation holds at all $t \geq 0$ and pins down $m_{R,t}$. It holds at $t = 0$ because $u_0 = v_0 = w_0 =$

1. Log differentiation yields

$$-(g-g^l) = \frac{\dot{w}_t}{w_t} + \frac{1}{\rho} \frac{\dot{u}_t}{u_t} + \frac{\dot{v}_t}{v_t} \left(1 - \frac{\frac{\beta(1-\rho)}{\rho} a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}} \right).$$

We have

$$\frac{\dot{u}_t}{u_t} = \frac{\rho}{\rho-1} \frac{\dot{m}_{R,t}}{m_{R,t}},$$

$$\frac{\dot{v}_t}{v_t} = -\dot{u}_t v_t \tilde{\alpha},$$

$$\frac{\dot{m}_{R,t}}{m_{R,t}} = \frac{\dot{p}_{R,t}^l + k^l}{p_{R,t}^l + k^l} (1-\tilde{\gamma}) w_t^\sigma,$$

$$\frac{\dot{w}_t}{w_t} = -\frac{1}{1-\sigma} \frac{\dot{p}_{R,t}^l + k^l}{p_{R,t}^l + k^l} z_t \tilde{\gamma} w_t,$$

and

$$\frac{\dot{z}_t}{z_t} = -\frac{\sigma}{1-\sigma} \frac{1}{1-\tilde{\gamma}} w_t^{-\sigma} \frac{1-\rho}{\rho} \frac{\dot{u}_t}{u_t}.$$

That gives us six differential equations in six unknowns: $u_t, v_t, w_t, z_t, \log m_{R,t}/m_0$, and $\log(1 + p_{R,t}^l/k^l)$. Solving these gives us $m_{R,t}$ at all times. From that we solve all other variables.