Macroeconomic Effects of Large-Scale Asset Purchases: New Evidence

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Abstract

We provide new evidence on the macroeconomic effects of the Federal Reserve’s large-scale asset purchases (LSAPs), using a structural VAR with survey-based measures of the LSAP policy stance and instruments constructed from high-frequency yield changes. We estimate that, at the peak, a $500 billion LSAP shock raises output and the price level by about 1.2 percent and 0.8 percent, respectively, while reducing the unemployment rate by 0.5 percentage points. These results are robust to considering possible central bank information effects and allowing for an endogenous switch between the interest rate and the balance sheet tool at the effective lower bound.

Keywords: Unconventional monetary policy; quantitative easing (QE); structural VAR; external instruments; macroeconomic effects; survey expectations.

JEL classification: E44, E52, E58.

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1 Introduction

What are the effects of large-scale asset purchases (LSAPs) on unemployment and inflation? More than 10 years after the Federal Reserve and the Bank of England initiated large expansions of their balance sheets, and more than two decades after the Bank of Japan initiated Quantitative Easing (QE), a great deal of controversy remains around the effectiveness of this policy tool in helping central banks achieve their macroeconomic objectives. This paper provides new evidence on the efficacy of LSAPs by using previously underutilized survey data and newly developed identifying assumptions in a structural VAR (SVAR). In doing so, we focus on the important but underappreciated role of the formation of the public’s understanding of systematic responses of the Federal Reserve’s balance sheet policies to economic conditions.

There are three key challenges in assessing the effects of balance sheet policies. The first one is the classic problem of distinguishing exogenous innovations from endogenous responses. One could argue that balance sheet policies were entirely the endogenous response of policymakers to unprecedented economic challenges at a time when policy rates were at their effective lower bound (ELB). Nonetheless, some of these policy responses may have come as a surprise to the public. The key identification issue is therefore to properly measure the surprise component from private agents’ perspective. For this purpose we build on recent studies using proxy SVARs with external instruments, such as Gertler and Karadi (2015) and Caldara and Herbst (2019). Different from those studies, we focus on the period since the Great Financial Crisis (GFC), during much of which the ELB was binding and the balance sheet policy was the main policy tool.

The second challenge is how to properly measure the balance sheet policy stance. Previous studies have used the size of the central bank’s balance sheet or the amount of announced asset purchases. However, such measures would fail to capture any effects on the economy from evolving expectations about future asset purchases through the forward-looking behavior of economic agents. Capturing those expectations is important because, although the first LSAP announcement came largely as a surprise, subsequent announcements were frequently
well anticipated. In this paper, we propose a survey-based measure of the expected size of
the Fed’s balance sheet over the coming year, which allows us to capture the expansionary
effects of both announced and expected asset purchases.\textsuperscript{1} As discussed later, the divergence
between expected and actual Fed holdings also helps explain why long-term yields rose in
some periods despite ongoing purchases, an observation sometimes cited as evidence that
LSAPs are ineffective.

The final challenge is that central banks used QE in conjunction with other tools, such
as forward guidance (FG) about the future path of policy rates. On several occasions, the
Federal Reserve and other central banks combined announcements about future balance sheet
policies with revised forward guidance about the future path of policy rates. This feature
makes it difficult to disentangle the effects of these two policies, and we present results based
on different identification strategies. In this regard, we are building on studies by Gürkaynak
et al. (2005) and Swanson (2021) who document the multiple dimensions of monetary policy
announcements. Those studies measure an unconventional policy shock directly by its effects
on yields at various maturities. By contrast, we relate such policy shocks back to the size and
maturity composition of an asset purchase program, while using the asset price responses as
instruments.

We find that asset purchase shocks have significant stimulative effects on the macroe-
conomy. In particular, a surprise $500 billion asset purchase is estimated to raise the level
of industrial production by about 2 percent and the level of the CPI by about 0.8 percent,
while reducing the unemployment rate by about 0.5 percentage points, at the peak, although
uncertainties surrounding those estimates are large. The shock operates both through asset
prices and through bank lending. In contrast to previous studies that considered the effects
of asset purchase \textit{programs}, our VAR analysis suggests that the bulk of the asset purchase
programs was endogenous, systematic policy responses to dire macroeconomic conditions,
rather than policy shocks. Reflecting the limited persistence of the latter (as opposed to
the systematic component of the asset purchases), the decline in Treasury yields and term

\textsuperscript{1} Using a forward-looking measure can also help alleviate concerns about non-invertibility of small-scale
VARs due to investor foresight (see recent discussions by Miranda-Agrippino and Ricco (2023) and Forni
et al. (2019)).
premiums appear to be short-lived, while the decline in excess bond premiums and the rise in equity prices and bank loans appear more persistent. We use the model to decompose the Fed’s various asset purchase programs into endogenous responses and exogenous shocks. For example, shutting down the LSAP shocks—the unexpected component—associated with QE3 would have kept the level of CPI 71 basis points lower and the unemployment rate 68 basis points higher at the end of 2015.

We conduct extensive robustness analyses, including alternative ways to calculate the impulse responses, alternative VAR specifications and sample periods, controlling for possible central bank information effects, and taking into account changes in the average maturity of the Fed’s asset holdings. In addition, we show that the results are broadly similar if we allow the central bank to switch endogenously between the short rate tool and the asset purchase tool at the ELB, where the switch is controlled by a latent shadow rate.

Our paper contributes to the literature on the efficacy of unconventional monetary policy measures, recently reviewed by Kuttner (2018) and Rossi (2021). Most studies of central bank asset purchases focus on their effects on financial markets. By contrast, studies of the macroeconomic effects of LSAPs are far fewer and their findings less conclusive. Most of those studies take a two-step approach, combining event study estimates of asset price responses to LSAP announcements with specific macroeconomic models or a structural VAR to assess the macro effects. A few studies estimate the macro effects in one step. For example, Gambacorta et al. (2014) and Weale and Wieladek (2016) examine how realized changes in the size of the central bank’s balance sheet or announced central bank purchases affect the macroeconomy, but do not consider the effects of expected future asset purchases. Stéphane Lhuissier (2021) construct a survey-based measure of unexpected ECB asset purchases and use it as an instrument to identify asset purchase shocks in a SVAR, but do not include survey-based forecasts among the VAR variables. The paper closest to ours is Hesse et al. (2018), who explored using surveys as an alternative approach to assess the macro

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2 See, for example, Gagnon et al. (2011), D’Amico and King (2013) and Li and Wei (2013) for the U.S., Joyce et al. (2011) for the U.K., and Andrade et al. (2016) and Eser et al. (2019) for the Euro Area.

3 See, for example, Chung et al. (2012) and Engen et al. (2015) for the former, and Baumeister and Benati (2013), Liu et al. (2019), and Eberly et al. (2020) for the latter.
effects of asset purchases. Due to data availability, they only examine survey forecasts for
the QE3 program between June 2012 and October 2014, a sample much shorter than ours
and, as discussed later, associated with much smaller divergence between the expected and
actual Fed holdings. Those studies typically use a combination of zero and sign restrictions
to identify the shocks, while we utilize information from high-frequency financial market
responses to monetary policy news to sharpen the identification.

Our paper is also related to the growing literature on identifying structural shocks in a
VAR using high-frequency asset price changes as external instruments. After being intro-
duced by Mertens and Ravn (2013) and Stock (2008), this approach has quickly become one
of the workhorse methods in SVAR studies of monetary policy shocks (Gertler and Karadi
(2015) and Caldara and Herbst (2019)). The more recent literature considers multiple types
of monetary policy shocks, drawing on evidence that asset price movements around central
bank announcements has a factor structure with more than one dimensions (Gürkaynak et al.
(2005) and Swanson (2021)). For example, Rogers et al. (2018) uses this approach to study
the effects of Federal Reserve target, forward guidance and asset purchase shocks on global
financial markets, while Eberly et al. (2020) and Swanson (2023) study the effects on the
macroeconomy. Relative to those studies, our contribution is to introduce a quantity-based
measure of the LSAP policy stance, which allows us to directly link asset purchases to the
macroeconomy without having to rely on external estimates of the effect of LSAP shocks on
long-term interest rates.

2 Methodology and data

2.1 Structural VAR with external instruments

Consider a reduced-form VAR with $L$ lags:

$$y_t = B_0 + B_1y_{t-1} + \ldots + B_Ly_{t-L} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma)$$

(1)
where $y_t$ and $u_t$ are $n$-by-1 column vectors containing the VAR variables and reduced-form residuals, respectively,

The reduced-form residuals are related to the structural shocks $\epsilon_t$ by

\[ u_t = C\epsilon_t, \tag{2} \]

where $C$ is an $n$-by-$n$ matrix that satisfies $CC' = \Sigma$ and $\epsilon_t \sim \mathcal{N}(0, I)$ is a column vector of length $n$. The goal is to estimate the columns in $C$ that correspond to the shocks we are trying to identify.

The use of external instruments help put additional restrictions on $C$. Let $z_t$ be a column vector of $m$ instruments where $m < n$. We assume that $z_t$ is only correlated with the first $m$ structural shocks, $\epsilon_{1:m,t} = [\epsilon_{1t}, \ldots, \epsilon_{mt}]$, but has zero correlation with the remaining structural shocks. In other words we have the following “relevance” and “exclusion” restrictions:

\[ \det(E[z_t \epsilon_{1:m,t}]) \neq 0. \tag{3} \]
\[ E[z_t \epsilon_i] = 0, \quad i = m + 1 \ldots n. \tag{4} \]

Under these assumptions, a matrix $C$ that satisfies $CC' = \Sigma$ and the exclusion restriction always exists. In addition, such a matrix identifies the two blocks of shocks, the instrumented shocks, $\{\epsilon_{1t}, \ldots, \epsilon_{mt}\}$, and the remaining shocks, $\{\epsilon_{(m+1)t}, \ldots, \epsilon_{nt}\}$, both uniquely up to a rotation.\footnote{The proofs can be found in Online Appendix OA.1.}

Following Gertler and Karadi (2015), we use as instruments high-frequency changes in money market futures rates and Treasury yields over short windows around releases of Federal Open Market Committee (FOMC) statements. Those asset price movements are valid instruments because changes over such narrow windows would reflect predominantly surprises associated with the corresponding monetary policy events, plus a small amount of noise that is unlikely to be related to any other structural shocks.

In most parts of the paper, we focus on identifying one shock using one instrument, with the identifying restriction that all other shocks in the VAR are uncorrelated with this
instrument. In this case, the shock is determined up to a scale parameter and we only need to choose a convention to determine the sign and size of the shock. In a few places, we attempt to disentangle two monetary policy shocks—related to forward guidance and asset purchases—using two orthogonal factors extracted from the yield responses as instruments. In those cases, we need to impose one more restriction in addition to determining the signs and sizes of the shocks. We explore two alternatives in Section 3.2.

2.2 Sample choice, VAR variables, and estimation strategy

Our baseline sample goes from January 1990 to December 2015. By comparison, Gertler and Karadi (2015) and many others use a longer sample starting in July 1979. We choose a later starting point due to concerns that the economy likely experienced structural changes over the long span since 1979, and including the early years in the sample may bias our estimates of the effect of monetary policy on inflation. We report in Section 4.3 results using two alternative samples.

Our baseline VAR has five monthly variables, including two monetary policy indicators—the 10-year Treasury yield and a survey-based LSAP measure—and three other variables used in Gertler and Karadi (2015)—the logarithm of the consumer price index (CPI), the logarithm of industrial production (IP), and the Gilchrist and Zakrajšek (2012) excess bond premium (EBP). Our baseline LSAP measure is the one-year-ahead expected size of the Federal Reserve's balance sheet constructed from median responses to the Survey of Primary Dealers, conducted by the Federal Reserve Bank of New York (FRBNY) eight times a year about a week before each FOMC meeting.

As an alternative we also construct a purchase-based LSAP measure, calculated as the median expectations of cumulative asset purchases by the Federal Reserve through one year from the survey dates. Compared with the balance-sheet based measure, the purchase-

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5 For example, there is evidence that the slope of the Phillips curve flattened significantly from the mid-1970s to the early 1990s (see Blanchard et al. (2015) among others).

6 The size of the balance sheet refers to the total par value of security holdings in the Federal Reserve’s System Open Market Account (SOMA). All securities that the Federal Reserve bought through the various asset purchase programs between 2008 and 2014 were managed on this account.
based measure ignore previous or expected future asset redemptions. As the Federal Reserve redeemed only a tiny fraction of its assets between 2008 and 2015, the difference between these two measures is small and using either measure leads to similar results.

A few considerations go into constructing these LSAP measures. First, no survey questions were asked about the Federal Reserve’s balance sheet before April 2009 or between December 2009 and August 2010. For these periods we assume that investors did not expect additional purchases and SOMA was expected to remain at its present levels, an assumption consistent with market narratives at the time. Second, the survey questions changed over time, varying between asking about specific purchase programs and about the size of the Fed’s balance sheet, and we combine the questions to construct a constant-horizon one-year-ahead forecast of SOMA holdings. Finally, the surveys are conducted at the FOMC frequency and we convert them to a monthly series by first constructing a daily series and then calculating the monthly averages.

Figure 1 shows the two LSAP measures along with the actual size of the Federal Reserve’s balance sheet. The survey-based LSAP measures evolve distinctly from the actual path of SOMA. For example, during the latter portion of QE1 and more notably during the first few months of QE2, the expected future SOMA size declined even as the Fed’s balance sheet was growing rapidly. This divergence between expected and actual Fed asset holdings, together with improving economic conditions potentially resulting from the LSAP policy, helps explain why long-term yields rose over these periods despite ongoing purchases, one observation sometimes cited as evidence that LSAPs are ineffective. A different pattern of divergence can be observed during QE3, when the Fed’s actual balance sheet initially lagged

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7 In particular, the survey asks about cumulative purchases over the next 5 to 8 quarters from April 2009 to November 2009, about the total size of LSAP2 from September 2010 to April 2011, and about SOMA size at various horizons over the next five years from January 2011 to December 2014. Online Appendix OA.2 describes the data and the construction of the LSAP measures in more detail.

8 We assign to each day the forecast from the survey for the upcoming FOMC meeting. This procedure assumes that the survey forecasts follow a step function with changes occurring only on FOMC dates. This assumption has the benefit of aligning the timing of the response of the LSAP measures with that of the monthly financial variables and the monthly instruments for monetary policy surprises, to be discussed in the next section. We also experimented with an alternative assumption under which the survey forecasts follow a linear path between survey dates, and the results are similar.
the expectations but the two subsequently converged as asset purchases came to an end.\footnote{This is the period examined by Hesse et al. (2018).}

We estimate the VAR as follows. Prior to 2008, there were no expectations that the FOMC would use LSAPs as an active policy tool. We therefore treat expected SOMA holdings as exogenous during that period and estimate the LSAP equation using only data after December 2008. We estimate all other equations in the VAR over the full sample. We identify the LSAP and forward guidance shocks using VAR residuals and instruments for December 2008 and later.\footnote{Online Appendix OA.3 discusses the estimation process in more detail.}

### 2.3 Instruments for monetary policy shocks

We construct instruments from interest rate changes around monetary policy events in three steps. First, we follow Gürkaynak et al. (2005) and compile a list of important policy events. We extend their event set to include not just FOMC statements, but also selected speeches and several FOMC-related news articles that significantly affected market expectations about upcoming asset purchases at the time.\footnote{See Online Appendix OA.4 for the full list of events. The few news articles included in the list attracted significant attention from investors and typically refer to policymakers’ speeches or comments made earlier during the day of publication. Removing the news articles from the list does not change the results appreciably.}

We also examine robustness to including additional events such as FOMC minutes and the Chair’s press conferences.\footnote{For FOMC statements and minutes, we use a window from 5 minutes before to 25 minutes after each statement. For FOMC speeches, the Chair’s press conference and news articles, we use a window from 5 minutes before to 55 minutes after the start of the speech or the publication of the article.}

We use changes in eight money market futures rates and Treasury yields: the current-month and two-month-ahead federal funds futures rates, the second, third and fourth quarterly Eurodollar futures rates, and yields on two-, five- and ten-year on-the-run Treasury notes.\footnote{Following Gertler and Karadi (2015), we modify the current-month federal funds futures rates to better capture the expected federal funds rates after the upcoming FOMC meetings, taking into account the fact that the federal funds futures contracts settle on the average funds rate over the entire month.}

This dataset spans the period from July 1991 to December 2015.

Each of the three monetary policy shocks discussed in the literature—target, FG, and LSAP—could in principle affect the entire term structure, albeit in different ways. The target...
shock has the largest effect at the very front end, while the FG shock has a hump-shaped effect on the yield curve peaking at 2- to 5-year maturities (Gürkaynak et al. (2005)). By contrast, the yield effects of LSAP announcements peak at maturities around 10 years (Gagnon et al. (2011)). Therefore, in the second step, we extract three orthogonal factors from the cross section of intraday yield changes that correspond to the three types of monetary policy shocks, following the method proposed by Swanson (2021).\textsuperscript{14} The usage of factors serves two purposes: First, it reduces the dimensionality of instruments to match the number of shocks; and second, it increases the correlation between the instruments and the shocks they instrument for and helps sharpen the identification.

The target factor we construct has the largest effect on the front-month federal futures rate and the 3-month Treasury bill yield, and the FG factor has a hump-shaped effect peaking around 2 years, while the effect of the LSAP factor rises monotonically up to the 10-year maturity. These are consistent with previous literature that forward guidance operates at the intermediate horizon while LSAPs operate at much longer maturities.\textsuperscript{15} Over the full 1991-2015 sample, the target factor and the FG factor each account for more than 40 percent of the variation in event-study yield changes, while the LSAP factor explains only 10 percent. The LSAP factor is much more important over the post-crisis period, while the importance of the target factor is greatly diminished. Together the three factors explain about 96 percent of event study changes in yields in all three samples.

In the last step, we convert the orthogonalized factors on event days into monthly series following the procedure in Gertler and Karadi (2015), by accumulating event-day factors over time and calculating the monthly averages. This procedure gives more weight to events

\textsuperscript{14} This method consists of first computing the first three principal components from intraday interest rate changes and then rotating them into target, FG, and LSAP factors by imposing the restrictions that (i) both the FG and LSAP factors have zero effect on the current-month federal funds futures rate and (ii) the magnitude of the LSAP factor over the pre-crisis period (prior to the QE announcement at the November 2008 FOMC meeting) is minimized. We scale the factors to have unit variances and sign them to have positive loadings on the front-month futures and 2- and 10-year yields, respectively. These restrictions uniquely determine the three factors.

\textsuperscript{15} In the Online Appendix, Figure OA-3 shows the loadings of these factors on rates of various maturities, and Table OA-2 reports the percentage of intraday yield changes explained by those factors over different time periods.
that occur earlier in the month and allows events that occur later in the month to affect
the instruments in the current and the following months. This procedure also matches
the way the LSAP indicator and the financial variables in our VAR are measured, which are
daily averages within the month and thus affected more by events that occur earlier in the
month.

Table 1 tests the strength of the FG and LSAP instruments over the post-crisis sample
by regressing first-stage VAR residuals for the two policy indicators on the two instruments.
The first two columns look at one indicator/instrument pair at a time, for which case Stock
et al. (2002) recommended a threshold of 10 for the F test. The last two columns regress
the residuals on both instruments jointly. In this case, we use the minimal eigenvalue of
the concentration matrix proposed by Cragg and Donald (1993) and Stock and Yogo (2005),
which can be viewed as a matrix analog of the F test, to quantify the strength of the
instruments. The F-statistics from the univariate regressions suggest that the LSAP factor
is a strong instrument for the LSAP indicator, while the strength of the FG factor for the
10-year yield is weaker. Most results in the paper will therefore focus on using the LSAP
factor to identify LSAP shocks.17

3 Baseline estimates

3.1 The macro effects of the LSAP shock

We first estimate the baseline model using one single instrument—the LSAP factor. In
doing so, we impose the identifying assumption that the identified shock is the only one
correlated with the LSAP instrument; therefore, any signaling effects on the future path of
the short rate from the LSAP policy would be combined with the direct effects of LSAP
shocks rather than being attributed to a separate contemporaneous FG shock. We use two

16 Reversing these two steps to first construct monthly event study surprises and then extract the three
factors gives nearly identical results.
17 The only exception is when we discuss forward guidance in Section 3.2. Note that the FG factor remains
weak if we use the 2- or the 5-year yield instead of the 10-year (See Table OA-5 in the Online Appendix).
lags for the VAR, based on conventional information criteria.\footnote{The Aikake Information Criterion (AIC) is maximized with two lags, based on either post-crisis residuals from the baseline VAR or residuals from re-estimating the VAR using post-crisis observations only.}

We simulate the VAR 10,000 times using resampled (with replacement) residuals and instruments, and report the 68 percent confidence intervals for the impulse responses. Previous studies of SVARs with external instruments typically use wild bootstraps to generate confidence bands for impulse responses.\footnote{See, e.g., Mertens and Ravn (2013) and Gertler and Karadi (2015).} Recently, Jentsch and Lunsford (2019) show that the wild bootstrap approach is not valid in such a setting and would generate confidence intervals that are too narrow, which is also the case in our setup.\footnote{This can be seen in Figure OA-4 in the Online Appendix.} Our methodology is equivalent to the moving block bootstrap suggested by Jentsch and Lunsford (2019) with a block length of 1.\footnote{Online Appendix OA.7 discusses the different methods to construct confidence intervals.}

Figure 2 plots the impulse responses, with the shock scaled to raise the expected SOMA holdings by $500 billion on impact. The shock dissipates rapidly over the first year.\footnote{In Gambacorta et al. (2014) find an even faster decay in LSAP shocks based on the actual size of the central bank’s balance sheet, with the shock fading out after 3 months.} The transitory nature of the shock reflects the historical feature of LSAPs: As shown in Figure 1, the Federal Reserve consistently expanded its asset holdings between 2009 and 2015, and the model sees the LSAP measure as endogenously responding to economic conditions and quickly converging to its overall expanding path following any unexpected shocks. Both industrial production and the price level rise following the shock. The effect on the log IP peaks around 3 years after the shock at about 220 basis points. The effect on the CPI is more subdued, peaking at around 80 basis points. The EBP declines in response to the shock and stays significantly lower for about two years. The ten-year Treasury yield falls about 40 basis points on impact and is about 15 basis points lower on average over the first year.

Table 2 compares the peak effects estimated here to those from the literature, where the LSAP shock is normalized to be comparable across studies.\footnote{The price response is from the baseline model reported above. The output response is from a VAR when log real GDP is added to the baseline model. The inflation and unemployment rate responses are from a}
magnitude of the shock, our estimate of the peak effect on the 10-year yield (Column 4) is larger than those from Gertler and Karadi (2013) and Chen et al. (2011) using DSGE models, but similar to those from Carlstrom et al. (2017) and the two SVAR-based studies that report such estimates. The peak output effect in Column (5) is close to those from Chung et al. (2012) using the FRB/US model and a yield-based shock, Weale and Wieladek (2016) using a SVAR, and Carlstrom et al. (2017) using an estimated DSGE model, but larger than the other estimates. Our estimate of the peak price level response in Column (6), on the other hand, is in the middle of the wide range of the estimates and comparable to that from Chung et al. (2012) and Hesse et al. (2018). The relative peak response of output to that of the price level (Column 9), which can be roughly thought of as the inverse of the slope of the Phillips curve, is estimated to be around 2.5 in our baseline model, within the range of previous estimates for LSAP shocks. Finally, the peak effect on the unemployment rate (Column 8) is also in line with previous estimates, all using yield-based shocks. One caveat in this comparison is that all DSGE models cited here treat the entire QE programs as unanticipated, while this study—as well as the other structural VAR-based studies—look at the surprise component that appears to die out faster than the endogenous component.

The estimated LSAP structural shock has a correlation of about -48% with the LSAP factor, which, together with the evidence shown in Table 1, suggests that the LSAP factor is a reasonably strong instrument for asset purchase policy shocks between 2008 and 2015. Figure 3 plots the estimated shock as well as the instrument, with vertical lines corresponding to the eight major announcements listed in Table 3. Some of those announcements—e.g. March 2009 and September 2013—are estimated to result in large accommodative monetary policy shocks, while for others—e.g. November 2010 and December 2012—the estimated shocks are close to zero or even contractionary. This heterogeneity reflects the fact that those announcements were anticipated to varying degrees by investors.

VAR with the log CPI and log IP in the baseline model replaced by the 3-month core CPI inflation and the unemployment rate. Table OA-3 in the Online Appendix shows the original estimates reported by the various papers, including additional papers studying conventional policy rate shocks only.

Recall that the LSAP shock is estimated as the linear combination of VAR residuals that has the maximum correlation with the LSAP instrument.
3.2 The macro effects of the forward guidance shock

In this section, we experiment with identifying two monetary policy shocks using both LSAP and FG factors as instruments. As discussed earlier, we need one additional restriction to separately identify the two shocks, in addition to choosing the scales and the signs of the shocks. We examine two such restrictions.

Under the first restriction, the FG shock is assumed to have a zero correlation with the LSAP instrument; as a result, the LSAP shock remains the only shock correlated with the LSAP instrument and its estimates and properties are identical to those discussed in the previous section. The FG shock is then identified as the only remaining shock with a nonzero correlation with the FG factor. Figure 4 plots the resulting impulse responses to a FG shock, scaled to raise the 10-year yield by 25 basis points on impact. The responses of all the other variables in the VAR are very imprecisely estimated. The point estimates indicate that both the log CPI and the log IP decline, while the EBP rises, after the shock, as can be expected following a surprise monetary policy tightening. The expected level of SOMA holdings increases on impact but only slightly.

One interpretation of the wide confidence intervals in Figure 4 is that forward guidance played a relatively minor role over this period and thus its effect is not well identified. Another interpretation is that as the ELB introduces restrictions on rates as far out as two years, FG shocks might be affecting the yield curve in different ways before and after the GFC, posing measurement challenges to the FG instrument.

We also explore a second identification restriction that the FG shock has no contemporaneous effect on expected SOMA holdings. However, we already see from Figure 4 that the expected level of SOMA holdings responds only slightly to the FG shock. Therefore, the results under this restriction look nearly identical to those under the first restriction and thus are not reported.
4 Robustness

4.1 Calculating impulse responses by local projections

Jordà (2005) advocates calculating impulse responses using local projections by regressing future target variables on measures of structural shocks one horizon at a time, controlling for other lagged explanatory variables. Impulse responses calculated this way do not rely on the structural parameters of the VAR model and hence are less susceptible to model misspecifications which could get compounded as the horizon lengthens.

We estimate the following regressions for each horizon $h$:

$$y_{t+h} = A^h_0 + A^h_1 y_{t-1} + A^h_2 y_{t-2} + A^h_e e_t + \eta_{t,h}, \tag{5}$$

where $y_t$ is the vector of VAR variables, $e_t$ is a vector of estimated structural shocks, and $A^h_i, i \in \{0, 1, 2, e\}$ are coefficient vectors or matrices. The matrices $\{A^h_e\}$ over different horizons $h$ determine the impulse response to the shock.

Figure 5 shows that the impulse responses calculated using local projections (right-hand panels) are more jagged than those from the SVAR (left-hand panels), but follow broadly similar patterns.

4.2 Alternative macro variables and policy indicators

Alternative macro variables. We consider two alternative sets of macro variables. First, we verify that replacing log CPI by log core CPI or log PCE does not change the results.\(^{25}\) Second, we replace log IP and log CPI by 3-month core CPI inflation and unemployment, two variables directly related to the Federal Reserve’s dual mandate. As shown in Figure 6, core inflation rises 49 basis points on impact while unemployment rate declines gradually over time, peaking at about -54 basis points 2 to 3 years after the shock. The impact on core inflation is transitory and disappears after 12 months, consistent with price level responses roughly leveling off after 12 months.

\(^{25}\) See Figure OA-8 in the Online Appendix.
Alternative policy indicators. We consider several alternative LSAP policy indicators, including the expected SOMA holdings-to-GDP ratio; the expected cumulative purchases from the primary dealer survey, either in level or scaled by the GDP; and the size of actual SOMA holdings. The LSAP instrument explains a larger portion of variations in survey-based measures than those in the actual SOMA holdings, and the associated F statistics appear much larger. Among the survey-based measures, the F-stats are slightly larger for the holdings-based measures than for the purchase-based measures, consistent with the view that it is not expected purchases but expected holdings that matter for yields. We also explore alternative FG policy indicators but find all of them to lack strength.\footnote{See Online Appendix OA.8 for more discussion about instrument strength.}

Figure 7 plots the impulse responses to the LSAP shock using the 18-month ahead expected federal funds rate and expected SOMA-to-GDP ratio, both from the primary dealer surveys, as the policy indicators. The results are qualitatively similar to those from our baseline (Figure 2), although the responses of the log IP and the log CPI appear more transitory.\footnote{The impulse responses to the FG shock from this SVAR, shown in Figure OA-9 in the Online Appendix, remain imprecisely estimated as in the baseline (Figure 4).}

4.3 Alternative sample periods

We examine two alternative samples. The first one (“start 1979”) extends back to July 1979, a starting point used by Gertler and Karadi (2015) and many others. The second one (“post-crisis”) focuses on the post-crisis period from December 2008 to December 2015.

Figure 8 compares the estimated impulse responses across samples. Using a longer sample starting in 1979 leaves the results qualitatively similar to the baseline estimates, although the estimated responses by log IP and log CPI are smaller. Compared with the baseline, the response by log CPI is moderated less than that by log IP, which is consistent with the evidence that the Phillips curve was steeper in the 1980s than in recent years. By contrast, when only post-crisis data are used in the estimation, the 10-year response quickly reverses its sign, while log IP and log CPI show more positive near-term responses that quickly
dissipate.

4.4 Accounting for potential central bank information effects

Echoing earlier work by Romer and Romer (2000, 2004), a recent literature shows that high-frequency asset price changes around monetary policy announcements may reflect market reactions not only to exogenous shocks to monetary policy, but also to new information revealed through the announcements about central banks’ private forecasts about the economy. Two approaches have been proposed in the literature to control for such a central bank information effect, either by imposing sign restrictions on the responses of a range of asset prices (Jarociński and Karadi (2020) and Cieslak and Schrimpf (2019)) or using central banks’ internal forecasts to remove the effect of central banks’ private information (Miranda-Agrippino and Ricco (2021)).

An examination of the responses of Blue Chip survey forecasts suggests that the LSAP shock we identify does not suffer from a notable bias associated with a Fed information effect. Figure 9 shows that survey forecasts of real GDP growth, inflation and the 3-month short rate all rise, while those of the unemployment rate decline, in response to an easing LSAP shock, as one would expect following unexpected monetary policy easing. We also experiment with dropping the few event days when the 2-year yield and the S&P 500 moved in the same direction and by more than one standard deviations, and the results are little changed.

Nonetheless, it is still possible that the results we document are attenuated by the central bank information effect. We therefore follow Miranda-Agrippino and Ricco (2021) and regress all event study changes on past Greenbook/Tealbook forecasts to remove possible responses to the Fed’s private information and use the residuals to construct the monthly

\[\text{28 There is a debate whether the empirical findings are due to private central bank information, as argued in the papers cited above, or due to investor misperception of the central bank’s reaction functions (Cieslak (2018) and Bauer and Swanson (2023)) or risk aversion shocks associated with central bank announcements (Kroencke et al. (2021) and Cieslak and Schrimpf (2019)). We examine the robustness of our findings to a potential central bank information effect, but do not separately consider the central bank reaction function effect or risk aversion shocks.}\]
factors. As in Miranda-Agrippino and Ricco (2021), we then fit an AR(12) model to the monthly factors and remove all autoregressive components to control for slow absorption of information by market participants.\textsuperscript{29} The biggest change is in April 2009 following the QE1 Treasury purchase announcement. The methodology attributes part of the market response to perceptions of a more negative economic outlook as signaled by the unprecedented policy action.

We re-estimate the model using instruments with the information effect removed and reports the results in Figure 10. Compared with the baseline model, the impulse responses become stronger, especially for macroeconomic variables, suggesting the existence of moderate central bank information effects, consistent with the findings by Miranda-Agrippino and Ricco (2023).\textsuperscript{30}

4.5 Adjusting for the duration of asset holdings

The analysis so far does not take into account the duration of asset holdings by the Federal Reserve. However, previous studies emphasize the duration channel of asset purchases, whereby central bank purchases of longer-maturity securities depress longer-term yields by removing duration risks from private hands (see Gagnon et al. (2011), Vayanos and Vila (2021), and Li and Wei (2013)). The maturity extension program (MEP), in particular, consists of simultaneous sales of shorter-maturity Treasury securities and purchases of longer-maturity ones of the same amount, and was designed to push down long-term yields by lengthening the maturity of Fed holdings while keeping the dollar amount of holdings constant. Taking maturity or duration into consideration is challenging, however, as the Surveys of Primary Dealers typically do not ask about the expected average maturity or duration of the Fed’s asset holdings, with the only exception in September 2011, right before the MEP announcement.\textsuperscript{31}

\textsuperscript{29} The original and the cleaned monthly LSAP factors are plotted in Figure OA-10 in the Online Appendix.

\textsuperscript{30} The impulse responses to the FG shock remain imprecisely estimated even after we clean the FG instrument using the same method.

\textsuperscript{31} That survey asked respondents about the size and length of any future program to extend the maturity of SOMA holdings, the maturity ranges of securities that would be purchased and sold under such a program, as well as the odds they attached to such a program. The median response indicates a 75% probability of a
We conduct a preliminary analysis by constructing a duration-weighted measure of expected SOMA holdings as follows. First, we use the September 2011 survey and the announced MEP program details to duration-adjust the expected SOMA holdings measure between August 2011 and December 2012. Second, we make some simplifying but reasonable assumptions about how investors expect the duration of SOMA holdings to evolve outside the MEP period.\textsuperscript{32}

Figure 11 shows that the duration-weighted measure of expected SOMA holdings leads the duration-weighted measure of actual SOMA holdings by roughly one year, as expected.\textsuperscript{33} The duration-weighted measure of expected SOMA holdings increased sharply over the months leading up to the MEP, as the anticipation of such a program started to build after the end of QE2. The measure increased further over the course of the MEP, as the MEP continued past the originally announced end-date and investors viewed additional asset purchases—QE3—as becoming increasingly likely. In contrast, the baseline measure ignores the duration-lengthening effects of both expected and actual MEP, staying flat between QE2 and the MEP and rising more slowly than the duration-weighted measure during the MEP.

Figure 12 shows that using the duration-weighted LSAP measure leads to only modest changes in the estimated impulse responses. The estimated structural shocks, shown in Figure 13, are also close to those in the baseline, with two exceptions. First, the announcement of the MEP in September 2011 (line 4) is interpreted as a slightly expansionary shock using the duration-adjusted measure but as contractionary using the baseline measure. Similarly, the extension of the MEP announced in June 2012 (line 5) is interpreted as expansionary based on the duration-weighted measure but as a near-zero-shock event based on the baseline measure. With the duration-weighted measure, the model’s interpretation of these events

\textsuperscript{32} Before the MEP, we assume that investors expect the reinvestment program to maintain a constant duration of SOMA Treasury holdings. Right after the conclusion of the MEP, we incorporate an expected shortening of SOMA duration due to the depletion of short-maturity securities and the associated lack of reinvestment opportunities. Throughout the period we assume that SOMA MBS duration is expected to stay at its post-crisis sample average. Online Appendix OA.9 describes our procedures in more detail.

\textsuperscript{33} One notable exception is around the taper tantrum in late 2013, when the actual measure unexpectedly increased due to a lengthening of MBS duration as interest rates rose notably (Figure OA-11).
is more consistent with the event-study-based instrument variable which shows negative (expansionary) values at those events (Figure 3).

5 Transmission channels

To explore how the LSAP shocks get transmitted through the economy, Figure 14 plots the impulse responses to a $500 billion LSAP shock of additional financial and macro variables, when they are added to the baseline VAR one at a time. The 5-year yield declines by about 30 basis points on impact, about 10 basis points less than the 10-year yield, and quickly reverts back to the pre-shock level. Lower term premiums account for all of the decline in yields at the 10-year maturity and about three quarters at the 5-year maturity.

The effects on yields and term premiums documented here are more persistent than what is found by Wright (2012) using a daily structural VAR and by Greenlaw et al. (2018) using an expanded event study dataset. The LSAP shock also has persistent effects on credit spreads, such as the EBP in the baseline model and the mortgage and corporate bond spreads over Treasury yields shown here, and on equity prices. The LSAP shock boosts C&I lending by about 2.5 percentage and real GDP by about 1.2 percent, while reducing the unemployment rate by about 60 basis points out to 3 years after the shock. Consistent with the response of log IP within the baseline VAR, a broader measure of economic activity constructed by the Chicago Fed—the Chicago Fed National Activity Index (CFNAI)—rises in response to the shock. As noted earlier, the LSAP shock also raises one-year-ahead Blue Chip survey forecasts of real GDP growth, inflation and the short rate, and reduces the unemployment rate forecasts.

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34 The unemployment rate response reported here is slightly larger than the 54 basis points reported elsewhere in the paper, which is based on a VAR including the 3-month core CPI inflation and the unemployment rate, as discussed in section 4.2.
6 Systematic LSAP responses vs LSAP shocks

6.1 A model-based decomposition

Our discussions so far focus on the surprise component of monetary policy actions. However, as we shall see, most of the increase in expected SOMA holdings over this period is not driven by LSAP shocks but by endogenous policy responses to the economy. We now look at both components jointly.

The three left panels of Figure 15 show model predictions of the future path of expected SOMA holdings in December 2008 and six months before the QE2 and QE3 announcement, respectively, against the actual path. The right panels decompose the forecast errors into monetary policy vs non-monetary-policy shocks using our baseline VAR. Panel A1 shows that by December 2008, the VAR already predicts a significant increase in SOMA holdings in the next couple of years, reflecting expectations of a further slowdown in the economy and the associated monetary policy response, though not the additional purchases associated with QE2 and QE3. Panel A2 shows that most of the forecast errors for the period after 2011 are due to unexpected weakness in the economy and the associated endogenous monetary policy responses, rather than more-accommodative-than-expected monetary policy.

Panels B1 shows that by April 2010, the model already partially predicts the upcoming announcement of QE2 in November 2010. The steeper rise in the actual path and the subsequent decline in late 2010 reflect the FOMC’s decision to reinvest all maturing securities in August 2010 and the small-than-expected size of QE2. Panel B2 shows that non-monetary-policy shocks explain a smaller portion of the forecast errors over the remainder of the sample period than in Panel A2.

Finally, Panel C1 shows that QE3 is also partially expected six months ahead of its announcement in September 2012.\textsuperscript{35} The “taper tantrum” over the summer of 2013 is not visible in Panel C2, as survey forecasts of SOMA holdings did not materially change over this period; however, the decision not to taper in late 2013 was viewed as a dovish surprise.

\textsuperscript{35} QE3 was announced in two stages, with MBS purchases announced in September 2012 followed by purchases of Treasury securities announced in December that year.
In addition, the MEP program discussed in Section 4.5 likely had expansionary effects on the economy but is not captured by the baseline LSAP measure used here.

6.2 QE3: a case study

We use QE3 as a case study to illustrate how to assess the effect of an asset purchase program that exceeds investor expectations. We construct a counterfactual scenario setting all LSAP shocks to zero in and after September 2012. This removes the expansionary LSAP shocks associated with two notable events: the announcement of MBS purchases in September 2012 and the decision not to immediately taper QE3 in September 2013.

The first panel of Figure 16 shows that, as can be expected from Figure 15, shutting down LSAP shocks does not flatten the path of expected SOMA holdings, as the VAR still predicts additional asset purchases in response to the worsening economic outlook. Overall, removing the LSAP shocks has a modestly contractionary impact on the path of expected SOMA holdings, with expected SOMA holdings about $160 billion lower in the last quarter of 2015. The rest of Figure 16 shows that the slightly less accommodative LSAP policy would have raised the ten-year yield and the EBP by 4 and 14 basis points, respectively, on average between September 2012 and the end of 2015, and reduced the levels of CPI and IP by 0.7 and 2.4 percents, respectively, by the end of 2015.

Adding other variables to the model following the same approach as in Section 5, we find that the unemployment rate would be hovering around 5.6 percent, rather than declining to 5 percent, and real GDP would be about 1 percent lower by the end of 2015. Other financial variables such as mortgage and corporate bond spreads under the counterfactual also exhibit more restrictive conditions, although the differences approach zero by the end of 2015 as the actual and counterfactual LSAP measures converge.\(^{36}\)

\(^{36}\) Figure OA-12 in the Online Appendix reports results for these additional variables.
7 Endogenous switch in monetary policy tools due to an occasionally binding ELB constraint

In our analysis so far, we treat the switch in the main monetary policy tools in late 2008 as an exogenous event. In reality, the switch arguably occurred endogenously as policymakers tried to employ new tools to respond to challenging economic conditions when the federal funds rate fell to the ELB. Recently, empirical techniques have been proposed in the literature to capture endogenous switches in monetary policy tools (e.g. Mavroeidis (2021) and Aruoba et al. (2021)). These studies assume the existence of a latent or “shadow” short rate that follows a certain monetary policy rule and can turn negative; when that occurs, the central bank switches from adjusting the short rate to unconventional monetary policy tools.

The benefit of allowing endogenous switches in monetary policy tools comes at a cost of requiring more restrictive assumptions about the continuity between the conventional and unconventional monetary policy regimes. For example, this approach implies that unconventional monetary policy tools affect the economy the same way as the short rate does, though possibly less effectively. However, asset purchases might affect the economy via different or additional channels relative to changes in short-term interest rates, for example by reducing pressures on commercial banks’ balance sheets and relaxing the financial constraints they face (e.g. Boehl et al. (2020)). In addition, the introduction of asset purchases as a monetary policy tool was not widely expected before the GFC and hence may not be well approximated as a fully-anticipated switch to a different but recurring regime. Finally, asset purchases are now considered a permanent part of central banks’ toolkit and can be employed in conjunction with other tools, rather than merely as a temporary substitute to deal with the ELB on short-term interest rates. For these reasons, we prefer the SVAR setup with a direct measure of the balance sheet policy—one-year-ahead expected SOMA holdings—as it places less restrictions on how unconventional policy tools affect the economy and how they interact with the conventional tools.

Despite these concerns, in this section we employ a modified version of the censored and kinked SVAR (CKSVAR) model developed by Mavroeidis (2021). That model allows
the shadow rate to be censored from below but still have an effect on the economy at the ELB ("censored"), partially capturing the usage of asset purchases and other unconventional monetary policy tools. The effects of the shadow rates are allowed to differ at and away from the ELB ("kinked"), reflecting the possibility that unconventional policy tools cannot fully overcome the ELB. If the ELB is never reached, this model reduces to the our baseline VAR.  

One additional benefit of the CKSVAR model is that it allows us to extend the sample period. After raising the federal funds rate off the ELB in December 2015, the Federal Reserve retained significant asset holdings but stopped using asset purchases as the primary policy tool. As a result, the dynamics of SOMA holdings and expectation about their future path likely changed around that time, making it challenging to describe the periods before and after December 2015 with a single linear model. The CKSVAR model can overcome this challenge by allowing the expected SOMA holdings to depend differently on the federal funds rate before and after the lift-off. This setup allows us to estimate the CKSVAR model over a longer sample, adding the period after the policy rate lift-off in December 2015 through June 2019.

We first estimate a CKSVAR model with five variables, adding the federal funds rate to our baseline set while removing the one-year-ahead expected SOMA holdings from it. We calculate the impulse responses to a LSAP shock normalized to have the same effect on the 10-year Treasury yield as in our baseline estimation. We then add back the one-year-ahead expected SOMA holdings and estimate a CKSVAR model with six variables. We extend the methodology of Mavroeidis (2021) to identify the shocks using the same set of external instruments as in our baseline model.

Due to the nonlinearity of the CKSVAR model, the impulse responses will in general depend on the initial state of the economy. As an illustration, we examine impulse responses for July 2012, about the midpoint of the period when the fed funds rate was at the ELB. Figure 17 shows that the impulse responses from these two CKSVAR models are slightly

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37 More details about the CKSVAR model can be found in Online Appendix OA.11.
38 One debatable implicit assumption here is that the system follows the same dynamics after the lift-off as before the GFC.
weaker than our baseline results but consistent in signs. Figure 18 uses the 6-variable CKSVAR to conduct the same exercise as in Figure 15 for the “quantitative tightening” (QT) period from Oct 2017 to the end of the sample, when the Federal Reserve actively shrank its balance sheet by not reinvesting maturing securities into new securities holdings. The left panel shows that the decline in SOMA holdings is faster and more pronounced than what the model predicts in September 2017, right before the start of the QT. The right panel shows that the forecast errors are mostly attributed to a tighter-than-expected monetary policy, rather than more favorable economic conditions.

8 Conclusions

This paper provides new evidence on the effects of the Federal Reserve’s LSAP programs on both financial markets and the macroeconomy, using information on primary dealers’ forecasts of the Federal Reserve’s asset holdings, among other variables, collected before each FOMC meeting. The evidence suggests most of the expansion in the Federal Reserve’s balance sheet during the ELB period between 2008 and 2015 can be attributed to endogenous responses to changing economic conditions, rather than discretionary shocks. We find that unexpected expansions in the Federal Reserve’s asset holdings had significant expansionary effects on the macroeconomy, with real activity and inflation rising and unemployment declining notably following the shock. The policy accommodation appears to be transmitted to the economy both through financial markets—including Treasury yields, credit spreads and equity prices—and through bank lending. The effects on Treasury yields and term premiums appear to be longer-lived than previously documented, while the effects on credit spreads especially and bank lending also appear persistent. These results appear fairly robust to alternative econometric methodologies, using alternative policy indicators, and allowing an endogenous shift in the Federal Reserve’s policy tools. That said, the model is less successful in identifying the effect of forward guidance shocks, a task we leave to future explorations.

39 Additional discussions of this model, including the estimated shadow rates, can be found in Online Appendix OA.11.
References


Figures and Tables

![Figure 1: LSAP Measures, Actual Size of SOMA, and Long-Term Yield](image)

The blue solid line shows the expected size of SOMA, one year into the future. The blue dashed line shows the actual path of SOMA, which roughly aligns with the blue solid line shifted by one year to the right. The red line shows the expected cumulative asset purchases over the following year. The initial difference between the expected SOMA size and the expected cumulative purchase reflects the size of SOMA holdings when purchases began. The black dotted line plots the 10-year Treasury yield. The grey shaded areas mark the time periods of QE1, QE2, and QE3. The blue dashed line lies slightly below the blue solid line and moves around a little even after the end of QE3, the latter mainly because MBS reinvestments are not reflected in actual SOMA holdings prior to settlement.
Figure 2: Impulse Responses to an LSAP Shock

The solid lines plot the impulse responses to a $500 billion shock to one-year-ahead expected SOMA holdings. The shaded areas represent 68 percent confidence intervals estimated by bootstrap with replacement.
Figure 3: Monetary Policy Shocks and the LSAP Factor.

The top panel shows the estimated LSAP structural shock from the VAR, while the bottom panel shows the LSAP instrument extracted from high-frequency yield changes around FOMC events. Note that in the top panel positive values represent expansionary shocks, while in the bottom panel positive values represent positive changes in yields, which would correspond to contractionary shocks. The numbered vertical lines indicate important LSAP-related announcements listed in Table 3.
Figure 4: Impulse Responses to FG Shocks

The solid lines plot the impulse responses to a rate path shock that raises the ten-year yield by 25 basis points on impact. The shaded areas represent 68 percent confidence intervals estimated using bootstrap.
Figure 5: Impulse Responses to LSAP Shocks: VAR vs Local Projections

The solid lines plot impulse responses of the variables in the baseline VAR to a LSAP shock that raises the expected SOMA holdings by $500 billion on impact, with impulse responses calculated either using the VAR (left panel) or using local projections with estimated structural shocks from the VAR (right panel). The shaded areas represent 68 percent confidence intervals (bootstrap for VAR and HAC OLS standard errors for local projection).
Figure 6: Impulse Responses to LSAP Shocks (Alternative Macro Variables)

The solid lines plot the impulse responses to a $500 billion shock to 1-year-ahead expected SOMA holdings, from a SVAR estimated with log IP and log CPI replaced by the unemployment rate and 3-month core CPI inflation. The shaded areas represent 68 percent confidence intervals estimated using bootstrap methods.
Figure 7: Impulse Responses to LSAP Shocks (Alternative Policy Indicators)

The solid lines plot the impulse responses to a one-percentage-point positive shock to 1-year-ahead expected SOMA-to-GDP ratio, from a SVAR with the 10-year Treasury yield and the expected SOMA holdings replaced by 18-month ahead expected federal funds rate and 1-year-ahead expected SOMA-to-GDP ratio. The shaded areas represent 68 percent confidence intervals estimated using bootstrap with replacement.
Figure 8: Impulse Responses to LSAP Shocks (Alternative Samples)

Blue lines and bands are impulse responses and confidence bands to an LSAP shock that increases expected SOMA size by $500 billion on impact estimated with the post-crisis sample ("post-crisis"), starting from December 2008. Green and red lines are impulse responses estimated with a longer sample, starting from July 1979 ("start 1979"), and the baseline sample, respectively.
Figure 9: Impulse Responses of Survey Forecasts to LSAP Shocks

The solid lines plot the impulse responses to a $500 billion shock to expected SOMA holdings. The shaded areas represent 68 percent confidence intervals estimated using bootstrap methods. ‘Fct’ is a shorthand for forecast.
Figure 10: Impulse Responses to LSAP Shocks (Removing Fed Information Effect)

The solid blue lines plot the impulse responses to a $500 billion shock to expected SOMA holdings using instruments that are cleaned for potential central bank information effect, and the blue shaded areas represent the associated 68 percent confidence intervals. The solid red lines and the dashed red lines plot the impulse responses from the baseline model and the associated 68 percent confidence intervals, respectively.
Figure 11: Duration-Weighted One-Year-Ahead Expected SOMA Asset Holdings

All duration-weighted measures are divided by 5.4 years, the average duration of SOMA over the sample period. The blue line shows the expected duration-weighted size of SOMA, one year into the future. The black line shows the baseline LSAP measure, which is not weighted by duration. The blue dashed line shows the actual path of duration-weighted SOMA, which roughly aligns with the blue line shifted by one year to the right. The grey shaded areas mark the time periods when QE1, QE2, and QE3 were ongoing. The blue shaded area mark the time period when the MEP was ongoing, which partially overlapped with QE3, indicated by a darker shade.
Figure 12: Impulse Responses to LSAP Shocks (Duration-Weighted)

The blue lines and blue shaded regions are impulse responses and 68 percent confidence bands to an LSAP shock that increases one-year-ahead duration-weighted SOMA size by $500 billion × 5.4 years on impact, where 5.4 years is the average duration of SOMA holdings over the sample period. The red lines show impulse responses from the baseline estimation for comparison.
Figure 13: LSAP structural shocks with duration-weighted SOMA holdings

The blue line shows the series of LSAP structural shocks from the VAR estimated with the duration-weighted expected SOMA holdings. The red line shows the same series estimated with the baseline expected SOMA holdings, which is not weighted by duration. Numbered vertical lines indicate the dates for important LSAP-related announcements, as described in Table 3; in particular, lines 4 and 5 mark the announcement of the MEP and that of its extension, respectively.
Figure 14: Transmission of LSAP Shocks

The solid lines plot the IRFs to a $500 billion shock to expected SOMA holdings of various financial and macro variables, when they are added to the baseline VAR one at a time (except for the 10-year yield, which is already in the baseline VAR). The variables include the 5- and 10-year Treasury yields, the 5- and 10-year Kim and Wright (2005) term premium estimates, the 30-year mortgage spread, the 10-year BBB-rated and high yield corporate bond spreads, the log S&P 500 index, log C&I loans, log real GDP, the civilian unemployment rate, and the Chicago Fed National Activity Index (“CFNAI”). The shaded areas represent 68 percent confidence intervals estimated using bootstrap with replacement.
Figure 15: Actual and Expected LSAP Measures at Different Points in Time

The black lines in the left-hand panels plot the actual path of the 1-year-ahead SOMA holdings measure, while the blue lines plot the paths expected at the beginning of December 2008 (Panel A1) and six months prior to the announcements of QE2 (Panel B1) and QE3 (Panel C1), respectively. Panels A2, B2 and C2 decompose the differences between the actual and expected paths in Panels A1, B1 and C1, respectively, into the cumulative contributions of monetary policy and non-monetary-policy shocks. The grey shaded areas mark the time periods when QE1, QE2, and QE3 were ongoing.
Figure 16: QE3 Counterfactual Analysis

The top left panel plots the actual (blue) and the counterfactual (red) LSAP measure. The top middle panel plots the estimated (blue) and the counterfactual (red) LSAP shocks. The other panels plot the actual (blue) and counterfactual (red) paths for the other VAR variables. Prior to September 2012, the two paths are identical in all panels.
The red (“5-var”) lines plot the impulse responses from the CKSVAR model estimated without one-year ahead expected SOMA holdings using the longer sample extended through June 2019. The green (“6-var”) lines plot the impulse responses estimated with one-year ahead expected SOMA holdings using the same longer sample. The impulse responses are obtained using the baseline LSAP instrument, with initial conditions as those for July 2012. The red lines are scaled to match the initial response of the 10-yr yield for the baseline model and the green lines to have an initial $500 billion impact on SOMA one-year ahead. The solid blue lines and blue shaded areas show the impulse responses and 68 percent confidence intervals from the baseline specification for comparison.
The black line in the left panel plots the actual path of the 1-year-ahead SOMA holdings measure, while the blue line plots the path expected right before (one month prior to) the beginning of QT. The right panel decomposes the difference between the actual and expected paths into the cumulative contributions of monetary policy and non-monetary-policy shocks. The decomposition is only approximate due to the non-linearity associated with the ELB, but the discrepancy is small because the fed funds rate was sufficiently away from the ELB—above 1 percent—when QT started and the simulated paths generally do not reach the ELB quickly enough—if at all—to create a significant discrepancy within the simulation horizon. The grey shaded areas mark the time periods when QE1, QE2, QE3, and QT were ongoing.
Table 1: First-Stage Instrument Strength Test: Post-crisis Sample

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This table reports the first-stage instrument strength tests of the baseline model. The first two columns report results from univariate regressions where the VAR residual for each policy indicator is regressed on the corresponding instrument alone. The last two columns report results from bivariate regressions where the residuals are regressed on both instruments. The symbols ***, ** and * denote significance at the 1, 5 and 10 percent levels, respectively.
Table 2: Selected Estimates of Peak Yield and Macro Effects of QE Shocks

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<td></td>
<td></td>
<td>0.59</td>
<td>0.65</td>
<td>-0.49</td>
<td></td>
</tr>
<tr>
<td>Liu et al. (2019)</td>
<td>SVAR$^1$</td>
<td>1965M4-2011M3</td>
<td></td>
<td></td>
<td>0.78</td>
<td>0.78</td>
<td>-0.23</td>
<td></td>
</tr>
<tr>
<td>Eberly et al. (2020)</td>
<td>SVAR$^2$</td>
<td>1990M1-2019M2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-Step Estimates; Purchase-Based Shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gambacorta et al. (2014)</td>
<td>SVAR$^1$</td>
<td>2008M1-2011M6</td>
<td></td>
<td></td>
<td>-38</td>
<td>1.45</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>Weale and Wieladek (2016)</td>
<td>SVAR$^1$</td>
<td>2009M3-2014M5</td>
<td></td>
<td></td>
<td>-38</td>
<td>1.45</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>Hesse et al. (2018)</td>
<td>SVAR$^1$</td>
<td>2008M11-2014M10</td>
<td></td>
<td></td>
<td>-25</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Gertler and Karadi (2013)</td>
<td>DSGE</td>
<td>n/a (calibrated)</td>
<td></td>
<td></td>
<td>-8</td>
<td>0.92</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Chen et al. (2011)</td>
<td>DSGE</td>
<td>1959Q3-2009Q3</td>
<td></td>
<td></td>
<td>-8</td>
<td>0.08</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>Carlstrom et al. (2017)</td>
<td>DSGE</td>
<td>1962Q1-2008Q4</td>
<td></td>
<td></td>
<td>-32</td>
<td>1.25</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

This table reports selected estimates of the peak macro effects of QE shocks in the literature. Shocks are normalized to be surprise purchases of $500 billion or 2.5% GDP in one-step estimations with purchase-based shocks and a surprise 40-basis-point decline in the long rate in two-step estimations using yield-based shocks. For Gambacorta et al. (2014), the purchase shock of 3% of the central bank's total assets (in log) is converted to about $500 billion purchases based on the average Fed balance sheet over that paper's sample period of 2008Q1-2011Q2.

1 - identified with zero and sign restrictions; 2 - identified with external instruments.
Table 3: Major LSAP Announcements.

<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Announcement content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3/18/2009</td>
<td>$1.15 trillion Treasury and MBS purchases.</td>
</tr>
<tr>
<td>2</td>
<td>8/10/2010</td>
<td>Reinvesting all maturing securities.</td>
</tr>
<tr>
<td>4</td>
<td>9/21/2011</td>
<td>Maturity extension purchases of Treasury securities.</td>
</tr>
<tr>
<td>5</td>
<td>6/20/2012</td>
<td>Continuing maturity extension.</td>
</tr>
<tr>
<td>6</td>
<td>9/13/2012</td>
<td>$40 billion MBS purchase per month.</td>
</tr>
<tr>
<td>7</td>
<td>12/12/2012</td>
<td>$45 billion purchases of Treasury securities per month.</td>
</tr>
<tr>
<td>8</td>
<td>9/18/2013</td>
<td>No immediate tapering of ongoing asset purchase.</td>
</tr>
</tbody>
</table>
We show that the exclusion restriction in Section 2.1 determines the matrix $C$ uniquely up to a restricted rotation. More specifically, if a matrix $C_0$ satisfies the exclusion restriction, then a different matrix $C$ satisfies the same restriction if and only if

$$C = C_0 \begin{bmatrix} R_m & 0 \\ 0 & R_{n-m} \end{bmatrix},$$

where $R_m$ and $R_{n-m}$ are $m$-by-$m$ and $(n-m)$-by-$(n-m)$ orthonormal matrices, respectively. Additional restrictions are needed to pin down $R_m$ and $R_{n-m}$ to uniquely identify the matrix $C$.

There exists $C_1$ such that $C_1 C_1' = \Sigma$. Define $e_t$ by $u_t = C_1 e_t$. We have that

$$E[z_t e_t'] = E[z_t u_t'](C^{-1}_1)' .$$

Since the rank of $E[z_t e_t']$ is $m$—the dimension of $z_t$—there exists an orthonormal matrix $R$ such that the last $n-m$ columns of $E[z_t e_t'] R = E[z_t u_t']((C_1 R)^{-1})'$ are zero. Define $C = C_1 R$, and $C$ satisfies the exclusion restriction.

Let $S$ be an $n$-by-$n$ orthonormal matrix such that only its top-left $m$-by-$m$ submatrix (denoted $S_1$) and bottom-right $(n-m)$-by-$(n-m)$ submatrix (denoted $S_2$) are nonzeros. Let $C_{eq} = CS$. The last $n-m$ columns of $E[z_t u_t'](C_{eq}^{-1})'$ are simply the last $n-m$ columns of $E[z_t u_t'](C^{-1})'$ post-multiplied by $S_2$ and are all zeros, hence $C_{eq}$ satisfies the exclusion restriction.

Conversely, suppose that $C_{eq}$ is a matrix that satisfies $C_{eq} C_{eq}' = \Sigma$ and the exclusion restriction. Then, $C_{eq} C_{eq}' = CC'$. Pre-multiplying this equation by $C^{-1}$ and post-multiplying it by $(C_{eq}^{-1})'$ lead to $C^{-1} C_{eq} = C'(C_{eq}^{-1})' = [(C^{-1} C_{eq})^{-1}]'$, hence $C^{-1} C_{eq}$ is orthonormal. Denoting $R = C^{-1} C_{eq}$, we have $C_{eq} = CR$. Because $C_{eq}$ satisfies the exclusion restriction, the last $(n-m)$ columns of $E[z_t u_t'](C_{eq}^{-1})'$ are all zeros. Because $C_{eq} = CR$, these columns are equal to $E[z_t u_t'](C^{-1})'$ post-multiplied by the last $n-m$ columns of $R$. Since the first $m$ columns of $E[z_t u_t'](C^{-1})'$ are
nonsingular, the top-right $m$-by-$(n - m)$ submatrix of $R$ has to be all zeros. By a similar argument for $C = C_{eq} R'$, the top-right $m$-by-$(n - m)$ submatrix of $R'$—i.e. the bottom-left $(n - m)$-by-$m$ submatrix of $R$—are all zeros. Therefore $R$ has the block form described by Equation (OA-1).

**OA.2 Data sources**

**Macroeconomic variables, Treasury yields, and the EBP.** We download the macroeconomic and Treasury yield data from the FRED database. For the baseline model, we use the monthly consumer price index for all urban consumers and for all items, seasonally adjusted (series name in FRED: CPIAUCSL) for CPI; monthly industrial production index, seasonally adjusted (series name: INDPRO) for IP; and monthly 10-year constant-maturity Treasury rate (series name: GS10) for the 10-year yield. The actual size of the Federal Reserve’s SOMA portfolio is from the weekly H.4.1. releases by the Federal Reserve, which can also be downloaded from FRED. We download the EBP from the Federal Reserve Board’s website.

**Baseline survey-based LSAP measure.** To create the survey-based LSAP measure, we used median responses to the following questions from the Survey of the Primary Dealers conducted by the FRBNY, along with other data:  

- **October 2008–March 2009:** There were no survey questions about future expected balance sheet size. Instead, we construct the expected SOMA size using FRBNY’s SOMA portfolio data and the size and pace of asset purchase announced at the October 2008 FOMC meeting. To account for maturing holdings, we note that no Treasury securities in SOMA at the time will mature within a year. For agency securities, we know exactly how much will roll off the balance sheet within one year. For mortgage-backed securities (MBS), the roll-off rate, determined by prepayment rate, is uncertain. We assume a constant monthly rate (1.5 percent) of reduction in par value, consistent with the observed roll-off of Federal Reserve’s MBS holdings between January 2009 and March 2010.

- **April 2009–March 2010:** We use the current size of the Fed’s balance sheet and the size and pace of purchases announced at the March 2009 FOMC meeting. In addition, we use median expectations of purchases in excess of announced

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2 See the website of the FRBNY.
amount from the Survey of Primary Dealers, which asks dealers about the expected cumulative purchases by the end of the next several quarters. We also adjust for expected asset roll-off within a year.

- **April 2010–August 2010**: QE1 purchases ended in March 2010. We take the current size of the Fed’s balance sheet and adjust down by expected roll-off of agency securities and MBS. Dealers expect no redemption of Treasury securities in all surveys during this period. In the survey prior to the August 2010 FOMC meeting, dealers expect continuing MBS redemption. At the August 2010 FOMC meeting, reinvestment plan is announced, with all maturing securities reinvested into Treasury securities. Therefore, no roll-off adjustment is applied after the August 2010 survey.

- **September 2010**: The survey asks about the probability of additional asset purchases and the expected size and time horizon. We use these information to calculate a median estimate of the expected asset purchases by the end of 2010 and add it to the existing holdings of $2,054 billion.\(^3\)

- **November and December 2010**: We use the median responses to the question about the expected size of asset purchases over the “intermediate horizon” combined with the size of existing SOMA holdings of $2,054 billion. Note that the $600 billion purchases announced at the November meeting is smaller than expected prior to the meeting, and the expectation in the December survey decreases accordingly.

- **January 2011–September 2012**: We use responses to the questions about the expected size of SOMA at the end of the current year and the next year, linearly interpolate to calculate the expected size in one year, and take the median across dealers. There were four surveys in which this question did not appear. For them use the last available survey but adjust interpolation weights based on the time of the survey.

- **October 2012–January 2013**: The surveys ask about expected change in the size of SOMA holdings by the end of the next few half-years. As previously, we interpolate to calculate the expected change in one year and add to the existing size of SOMA.

\(^3\) The number on existing holdings comes from a technical report released at the August 2010 FOMC meeting and is very close to the actual size of SOMA holdings at the September meeting. The size of SOMA holdings has been released to the public at a weekly frequency.
- **March and May 2013**: The surveys only ask about expected changes in SOMA by the end of Q2 2013. We assume a full continuation of ongoing purchase programs ($45 billion of Treasury securities purchases per month, announced at the December 2012 FOMC meeting, and $40 billion of MBS purchases per month, announced at the September 2012 meeting). Responses to other survey questions are broadly consistent with our assumption of full continuation. First, dealers show very little expectation for any changes in the pace of purchases to be announced at the next three meetings. Second, the median expected end dates of the current purchase programs stay at least one year away from the survey dates.

- **June 2013–June 2014**: The surveys ask about the expected pace of purchases following each of the next 8 FOMC meetings, which give us the median expected purchase path for a whole year. We then add those estimates to the existing size of SOMA.

- **July 2014–January 2015**: All but one surveys ask about expected changes in the SOMA size following the next few quarters and half-years until at least 2017. For those surveys, we interpolate between two appropriate horizons to calculate expected SOMA size change in one year and add to the current size of SOMA. No questions about SOMA holdings were asked in the December 2014 survey, and we use responses from the November 2014 survey but adjust the interpolation weights accordingly. In December 2014, dealers expressed very little expectation of a rate liftoff within a year, so the continuation of reinvestment, which is implied by the November 2014 survey responses, was likely expected in December 2014 survey as well. Note that since November 2014, there had been no ongoing purchases and all maturing securities were fully reinvested.

- **March 2015–December 2015**: No direct questions about expected balance sheet size were asked in the surveys. However, in all surveys dealers’ median expected end of reinvestment were about a year away, so we simply use the current size of SOMA.

**Survey-based short rate expectations measure.** We also construct measures of 12- and 18-month-ahead short rate expectation using the Survey of the Primary Dealers. Over the sample period, the survey consistently includes questions about
policy rate expectation, so the construction of rate expectation is relatively straightforward. Generally, the survey asks about the expected policy rate at the end of individual quarters and, from time to time, half-years and years, and we linearly interpolate the median of those responses to calculate the expected policy rate in 12 and 18 months. Sometimes the responses are in terms of 25-basis-point ranges, in which case we use the midpoints of the ranges.

Survey-based, duration-adjusted LSAP measure. For the duration-weighted exercise in Section 4.5, we download the monthly average duration of Treasury and MBS holdings from the data appendix to the FRBNY’s annual reports on domestic open market operations. The universe of outstanding Treasury securities is constructed from publicly available records of Treasury auction results. Snapshots of all security holdings in the Federal Reserve’s SOMA portfolio are available on a weekly basis.

OA.3 Estimation strategy

OA.3.1 OLS estimation of the VAR

The baseline model contains five variables in the order of the LSAP measure, the 10-year yield, log CPI, log IP, and the EBP. Recall that we have the following reduced-form VAR:

\[ y_t = B_0 + B_1 y_{t-1} + B_2 y_{t-2} + u_t. \]  

We set the LSAP measure to zero before December 2008 and subtract the size of the SOMA right before the beginning of the first LSAP program from the LSAP measure from December 2008 onward. This ensures that the increase in the LSAP measure around December 2008 captures only expected increase in SOMA holdings due to LSAP programs.

The coefficients on the equations that determine the four non-policy variables (the bottom four rows of \( B_0, B_1 \) and \( B_2 \)) are calculated by OLS over the entire sample, as those equations are not directly affected by the restrictions on the LSAP measure. We only use observations in and after December 2008 when estimating the equation that determines the LSAP measure (the first row of \( B_0, B_1 \) and \( B_2 \)).

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4. See the website of the FRBNY. Data files for 2012 and 2016 reports fully cover our sample period.

5. See the website of TreasuryDirect.

6. See the website of the FRBNY.
We use residuals from the post-December-2008 period when estimating the variance-covariance matrix $\Sigma$. We apply post-December-2008 instruments to post-December-2008 residuals to identify shocks of interest.

**OA.3.2 MLE estimation of the VAR**

In this section, we formally take into account the possibility that the variance-covariance matrix of the residuals differs between pre- and post-GFC periods and hence different weights should be assigned to observations from those two periods. Specifically, we assume

$$u_{-1t} \sim \mathcal{N}(0, \Sigma_E).$$  \hspace{1cm} (OA-3)

in the early period, $t = 1, ..., T_E$ and

$$u_t \sim \mathcal{N}(0, \Sigma_L)$$  \hspace{1cm} (OA-4)

in the late period, $t = T_E + 1, ..., T$, where $u_{-1t}$ denotes a sub-vector of $u_t$ that excludes its first element, corresponding to one-year-ahead SOMA holdings, and $\Sigma_E$ and $\Sigma_L$ are $(n-1)$-by-$(n-1)$ and $n$-by-$n$ positive definite matrices, respectively.

The OLS estimator calculates the coefficients in the two policy equations using different sample periods, without accounting for the differences between $\Sigma_E$ and $\Sigma_L$. Therefore, the OLS is not equivalent to the MLE in the current setup.

To perform the MLE, it is useful to define a few stacked matrices. We define $X$ to be a $(T - L)$-by-$(nL + 1)$ matrix of stacked independent variables, whose $i$-th row of $X$ equals $[y_{i}^T \ldots y_{i+L-1}^T]$. $Y$ is a $(T - L)$-by-$n$ matrix of stacked dependent variables, whose $i$-th row equals $y_{i+L}^T$, while $U$ is a similarly defined as a $(T - L)$-by-$n$ matrix of stacked $u_t^T$. $B$ is the stacked coefficient matrix, whose first $n$ rows are $B_L^T$, the next $n$ rows are $B_{L-1}^T$, and so on, and whose last row is $B_0^T$. The VAR model can then be represented as:

$$Y = XB + U.$$  \hspace{1cm} (OA-5)

The log likelihood $l(B, \Sigma_E, \Sigma_L)$ is given by

$$l = C - \frac{T_E - L}{2} \log(|\Sigma_E|) - \frac{T - T_E}{2} \log(|\Sigma_L|)$$  \hspace{1cm} (OA-6)

$$- \frac{1}{2} vec(U_{E,-1})^T (\Sigma_E^{-1} \otimes I_{T_E - L}) vec(U_{E,-1})$$

$$- \frac{1}{2} vec(U_L)^T (\Sigma_L^{-1} \otimes I_{T - T_E}) vec(U_L),$$

where $U_{E,-1}$ is the first $T_E - L$ rows of $U$ excluding its first column and $U_L$ is the
last \(T - T_E\) rows of \(U\). \(I_i\) is an \(i\)-by-\(i\) identity matrix and \(\text{vec}(M)\) is a column vector constructed by stacking the columns of \(M\). \(C\) is a constant that can be ignored.

Next, we define an \(n\)-by-\(n\) matrix \(\Sigma_E^{-1}\) formed by adding a row of zero and a column of zero before the first row and the first column of \(\Sigma_E^{-1}\). Then,

\[
\text{vec}(U_{E, -1})^T (\Sigma_E^{-1} \otimes I_{T_E - L}) \text{vec}(U_{E, -1}) = \text{vec}(U_E)^T (\widetilde{\Sigma}_E^{-1} \otimes I_{T_E - L}) \text{vec}(U_E). \quad \text{(OA-7)}
\]

where \(U_E\) is the first \(T_E - L\) rows of \(U\). We can add back the removed column to \(U_{E, -1}\) because it will be multiplied by the zero rows and columns added to \(\Sigma_E^{-1}\).

Note that \(U_E = Y_E - X_EB\) and \(U_L = Y_L - X_LB\), where \(Y_i\) and \(X_i\) are defined in the same way as \(U_i\). Then,

\[
\frac{\partial l}{\partial \text{vec}(B)} = (\widetilde{\Sigma}_E^{-1} \otimes X_E^T) \text{vec}(Y_E) + (\Sigma_L^{-1} \otimes X_L^T) \text{vec}(Y_L)
\]

\[
- [(\widetilde{\Sigma}_E^{-1} \otimes X_E^T X_E) + (\Sigma_L^{-1} \otimes X_L^T X_L)] \text{vec}(B). \quad \text{(OA-8)}
\]

The associated first-order condition (FOC) is

\[
[(\widetilde{\Sigma}_E^{-1} \otimes X_E^T X_E) + (\Sigma_L^{-1} \otimes X_L^T X_L)] \text{vec}(B) = (\widetilde{\Sigma}_E^{-1} \otimes X_E^T) \text{vec}(Y_E) + (\Sigma_L^{-1} \otimes X_L^T) \text{vec}(Y_L). \quad \text{(OA-9)}
\]

FOCs with respect to \(\Sigma_E\) and \(\Sigma_L\) are given by the standard formulation:

\[
\Sigma_E = \frac{1}{T_E - L} U_{E, -1}^T U_{E, -1}; \quad \text{and} \quad \Sigma_L = \frac{1}{T - T_E} U_L^T U_L. \quad \text{(OA-10)}
\]

Instead of attempting a full optimization, we iterate maximizing with respect to each of \(B\), \(\Sigma_E\), and \(\Sigma_L\). The process is analogous to iterating feasible GLS or performing a simple expectation-maximization algorithm.

### OA.4 Full list of events in post-crisis sample

Table OA-1 lists the events used in constructing the instruments over the post-crisis period.
Figure OA-1: Impulse Responses to an LSAP Shock (MLE)

The solid lines plot the impulse responses to a $500 billion shock to 1-year ahead expected SOMA holdings estimated by MLE (blue) and OLS (red). The shaded areas represent 68 percent confidence intervals estimated using bootstrap methods and MLE.
The solid lines plot the impulse responses to a rate path shock that raises the ten-year yield by 25 basis points on impact estimated by MLE (blue) and OLS (red). The shaded areas represent 68 percent confidence intervals estimated using bootstrap methods and MLE.

Figure OA-2: Impulse Responses to a FG Shock (MLE)
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Event type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/25/2008</td>
<td>8:15</td>
<td>FOMC meeting (unscheduled)</td>
<td>FOMC to purchases up to $100b of agency debt and up to $500b of agency MBS.</td>
</tr>
<tr>
<td>12/1/2008</td>
<td>13:45</td>
<td>Chair speech</td>
<td>Bernanke speech at the Greater Austin Chamber of Commerce, Austin, Texas. “the Fed could purchase longer-term Treasury or agency securities on the open market in substantial quantities.”</td>
</tr>
<tr>
<td>12/16/2008</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Reduced the fed funds rate to 0-0.25%. “economic conditions ... likely to warrant exceptionally low levels of the federal funds rate for some time”</td>
</tr>
<tr>
<td>1/28/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Statement unchanged. Market commentaries suggest some investors were disappointed by the lack of concrete language regarding the possibility and timing of purchases of longer-maturity Treasury securities.</td>
</tr>
<tr>
<td>3/18/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>FOMC to keep the federal funds rate between 0 and 25 bps for “an extended period” and to purchase $750B of mortgage-backed securities, $300B of longer-term Treasuries, and $100B of agency debt</td>
</tr>
<tr>
<td>4/29/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Market commentaries before the meeting suggest some odds attached to increasing the scale of purchases at this meeting or to language in the statement suggesting that the programs would likely be expanded in the future. Commentaries also interpreted the Committee’s discussion of the economic outlook as somewhat less downbeat than in the March statement</td>
</tr>
<tr>
<td>6/24/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Market commentaries noted statement language pointing to a slowing in the pace of economic contraction as suggesting that the Committee believes the economy is in the process of bottoming out. 2-10y Treasury yields up 6-9 bps.</td>
</tr>
<tr>
<td>Date</td>
<td>Time</td>
<td>Event Type</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>-------</td>
<td>--------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>8/12/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Dropped the “up to” language and announced gradual tapering for Treasury purchases through end of October.</td>
</tr>
<tr>
<td>9/23/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Dropped the “up to” language for agency MBS purchases and announced gradual tapering for agency debt and MBS purchases through end of 2010Q1. Market commentaries focused on reiteration of the “extended period” language. Treasury yields lower by 4-9 bps.</td>
</tr>
<tr>
<td>11/4/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Changed “up to $200b” to “about $175b” for agency debt purchases</td>
</tr>
<tr>
<td>12/16/2009</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Market commentaries noted the Committee’s improved assessment of the labor market</td>
</tr>
<tr>
<td>1/27/2010</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>Market commentaries interpreted the characterization of the economic outlook as somewhat more upbeat than expected.</td>
</tr>
<tr>
<td>8/2/2010</td>
<td>19:51</td>
<td>News article</td>
<td>WSJ article “Fed Mulls Symbolic Shift” suggesting that the Federal Reserve may use proceeds from maturing agency MBS securities to purchase additional MBS or Treasuries appeared to garner significant attention from market participants.</td>
</tr>
<tr>
<td>8/10/2010</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>“Committee will keep constant the Federal Reserve’s holdings of securities at their current level by reinvesting principal payments from agency debt and agency mortgage-backed securities in longer-term Treasury securities. The Committee will continue to roll over the Federal Reserve’s holdings of Treasury securities as they mature.” Market commentaries suggest this policy change wasn’t universally expected and the assessment of the outlook was seen as more downbeat than expected.</td>
</tr>
<tr>
<td>8/27/2010</td>
<td>10:00</td>
<td>Chair speech</td>
<td>Bernanke speech at Jackson Hole</td>
</tr>
<tr>
<td>9/27/2010</td>
<td>16:10</td>
<td>News article</td>
<td>WSJ Article “Fed Mulls New Bond Approach”: “Fed officials are weighing a more open-ended, smaller-scale program that they could adjust as the recovery unfolds.”</td>
</tr>
<tr>
<td>Date</td>
<td>Time</td>
<td>Event Type</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>-------</td>
<td>------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>10/1/2010</td>
<td>8:30</td>
<td>FOMC participant speech</td>
<td>Dudley’s speech reportedly further solidified investor expectations for the FOMC to announce additional accommodative policy measures at the November meeting.</td>
</tr>
<tr>
<td>10/1/2010</td>
<td>11:44</td>
<td>FOMC participant speech</td>
<td>Evans’s speech (Bank of France conference in Rome, first appeared in news at 11:44) reportedly further solidified investor expectations for the FOMC to announce additional accommodative policy measures at the November meeting.</td>
</tr>
<tr>
<td>11/3/2010</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>QE2 announcement of $600B purchase of longer-term Treasuries. 2y yield unchanged, while 10 and 30y yields up 7 and 15 bps, reportedly because simultaneously released purchase maturity distribution reported weighted more towards short and intermediate maturities than expected.</td>
</tr>
<tr>
<td>7/12/2011</td>
<td>14:00</td>
<td>FOMC minutes</td>
<td>All but one participants agreed to “exist strategy principles” (cease reinvestment -¿ modify forward guidance -¿ raise rate -¿ sales of agency securities)</td>
</tr>
<tr>
<td>8/9/2011</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>FOMC expects to keep the federal funds rate between 0 and 25 bps “at least through mid-2013”</td>
</tr>
<tr>
<td>8/26/2011</td>
<td>10:00</td>
<td>Chair speech</td>
<td>Bernanke speech at Jackson Hole.</td>
</tr>
<tr>
<td>9/21/2011</td>
<td>14:15</td>
<td>FOMC meeting</td>
<td>FOMC will sell $400B of short-term Treasuries and use the proceeds to buy $400B of long-term Treasuries. In addition, “the Committee will now reinvest principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities.”</td>
</tr>
<tr>
<td>1/25/2012</td>
<td>12:30</td>
<td>FOMC meeting</td>
<td>FOMC expects to keep the federal funds rate between 0 and 25 bps “at least through late 2014”</td>
</tr>
<tr>
<td>6/20/2012</td>
<td>12:30</td>
<td>FOMC meeting</td>
<td>Continue the MEP through year-end and discontinue rolling over of maturing Treasury securities at auctions</td>
</tr>
<tr>
<td>Date</td>
<td>Time</td>
<td>Location</td>
<td>Event Description</td>
</tr>
<tr>
<td>------------</td>
<td>-------</td>
<td>--------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>9/13/2012</td>
<td>12:30</td>
<td>FOMC meeting</td>
<td>FOMC expects to keep the federal funds rate between 0 and 25 bps “at least through mid-2015”, and will purchase $40B of mortgage-backed securities per month for the indefinite future. 5/10y yields up 5/7 bps.</td>
</tr>
<tr>
<td>12/12/2012</td>
<td>12:30</td>
<td>FOMC meeting</td>
<td>Introduced quantitative forward guidance. will purchase $45B of longer-term Treasuries per month for the indefinite future, and that it expects to keep the federal funds rate between 0 and 25 bp at least as long as the unemployment remains above 6.5 percent and inflation expectations remain subdued.</td>
</tr>
<tr>
<td>5/10/2013</td>
<td>19:18</td>
<td>News article</td>
<td>WSJ article “Fed Maps Exit From Stimulus”</td>
</tr>
<tr>
<td>5/22/2013</td>
<td>10:00</td>
<td>Chair speech</td>
<td>Bernanke’s JEC Testimony. Comments during Q&amp;A that Fed could ‘step down’ the pace of asset purchases in the next few meetings depending on whether there is a significant improvement in the job market and a sustainable improvement in economic data. 5-10 year yields rose 5 bps following the testimony.</td>
</tr>
<tr>
<td>6/19/2013</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>Statement and SEP both viewed by market commentaries as more upbeat about economy. 5- and 10-year yields rose 7 an 6 bps following the statement and a total of 19 and 12 bps following statement and conference.</td>
</tr>
<tr>
<td>9/18/2013</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>Market commentaries suggest some investors were surprised by the decision not to taper</td>
</tr>
<tr>
<td>12/18/2013</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>FOMC to taper purchases of longer-term Treasuries and mortgage-backed securities to paces of $40B and $35B per month, respectively. accompanied by a reinforcement of the threshold-based forward guidance for the rate</td>
</tr>
</tbody>
</table>
3/19/2014 14:00 FOMC meeting Qualitative guidance. “likely will be appropriate to maintain the current target range for the federal funds rate for a considerable time after the asset purchase program ends, especially if projected inflation continues to run below the Committee’s 2 percent longer-run goal, and provided that longer-term inflation expectations remain well anchored.”

10/29/2014 14:00 FOMC meeting Concluded QE3 as expected. “likely will be appropriate to maintain the 0 to 1/4 percent target range for the federal funds rate for a considerable time following the end of its asset purchase program this month, especially if projected inflation continues to run below the Committee’s 2 percent longer-run goal, and provided that longer-term inflation expectations remain well anchored.” Emphasized the conditional nature: “However, if incoming information indicates faster progress toward the Committee’s employment and inflation objectives than the Committee now expects, then increases in the target range for the federal funds rate are likely to occur sooner than currently anticipated. Conversely, if progress proves slower than expected, then increases in the target range are likely to occur later than currently anticipated.”

12/17/2014 14:00 FOMC meeting FOMC announces that “it can be patient in beginning to normalize the stance of monetary policy”

3/18/2015 14:00 FOMC meeting Dropped “patient” and switched to “reasonably confident that inflation will move back to its 2 percent objective over the medium term”. Indicated that ”an increase in the target range for the federal funds rate remains unlikely at the April FOMC meeting”

4/29/2015 14:00 FOMC meeting Removed the sentence about the April meeting. Rate decision now on meeting-by-meeting basis.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Meeting Type</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/28/2015</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>“In determining whether it will be appropriate to raise the target range at its next meeting, the Committee will assess progress—both realized and expected—toward its objectives of maximum employment and 2 percent inflation.”</td>
</tr>
<tr>
<td>12/16/2015</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>Lift off from the ELB. “maintaining its existing policy of reinvesting . . . until normalization of the level of the federal funds rate is well under way”</td>
</tr>
<tr>
<td>6/14/2017</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>“The Committee currently expects to begin implementing a balance sheet normalization program this year, provided that the economy evolves broadly as anticipated.” Released “Addendum to the Policy Normalization Principles and Plans” outlining the balance sheet normalization process</td>
</tr>
<tr>
<td>7/26/2017</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>“The Committee expects to begin implementing its balance sheet normalization program relatively soon, provided that the economy evolves broadly as anticipated”</td>
</tr>
<tr>
<td>9/20/2017</td>
<td>14:00</td>
<td>FOMC meeting</td>
<td>“In October, the Committee will initiate the balance sheet normalization program described in the June 2017 Addendum to the Committee’s Policy Normalization Principles and Plans.”</td>
</tr>
</tbody>
</table>
OA.5 Properties of the instruments

Table OA-2 reports the percentage of event-study yield changes explained by those factors over different samples. Figure OA-3 shows the loadings of the three factors on the event-study yield changes at different maturities.

Table OA-2: Percentage of Yield Variations Explained by Factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Target</th>
<th>FG</th>
<th>LSAP</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>44</td>
<td>42</td>
<td>10</td>
<td>96</td>
</tr>
<tr>
<td>Pre-crisis</td>
<td>50</td>
<td>42</td>
<td>5</td>
<td>96</td>
</tr>
<tr>
<td>Post-crisis</td>
<td>13</td>
<td>42</td>
<td>41</td>
<td>96</td>
</tr>
</tbody>
</table>

This table shows the percentages of variances of event study yield changes explained by the three factors over three samples: July 1991 to December 2015 ("full sample"), July 1991 to October 2008 ("pre-crisis"), and November 2008 to December 2015 ("post-crisis"). For each sub-sample, the percentage is calculated by dividing the sum of covariances between yield changes and their projections onto each factor (in tri-variate regressions) by the sum of variances of yield changes.

OA.6 Previous estimates of macro effects of LSAPs: unscaled

Table OA-3 shows the original, non-normalized estimates of the yield and macro effects of LSAPs, reported by various studies summarized in Table 2, as well as a few additional studies focusing on the effects of conventional policy rate shocks.
Figure OA-3: Yield Loadings on Target, FG, and LSAP Factors

This graph plots the loadings of the Target (in yellow), FG (in orange), and LSAP (in blue) factors on event study money market futures rate and Treasury yield changes across eight maturities ranging from 1-month to 10-year.
Table OA-3: Selected Estimates of Peak Yield and Macro Effects of Monetary Policy Shocks

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This Paper</td>
<td>SVAR²</td>
<td>1990M1-2015M12</td>
<td>$500b (2.5% GDP) expected purchase</td>
<td>-39 bps</td>
<td>1.24%</td>
<td>0.81%</td>
<td>2.5</td>
<td>0.49%</td>
<td>-0.54%</td>
</tr>
</tbody>
</table>

**QE programs/shocks: Two-Step Estimates; Yield-Based Shocks**

<table>
<thead>
<tr>
<th>Chung et al. (2012)</th>
<th>FRB/US</th>
<th>n/a</th>
<th>QE1-2 (-65 bps shock to 10y yield)</th>
<th>-65 bps</th>
<th>3%</th>
<th>1%</th>
<th>3.0</th>
<th>-0.75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engen et al. (2015)</td>
<td>FRB/US</td>
<td>n/a</td>
<td>QE1-3 and MEP (-120 bps shock to 10y term premium)</td>
<td>-120 bps</td>
<td>0.5%</td>
<td>-1.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baumeister and Benati (2013)</td>
<td>SVAR¹</td>
<td>1965Q4-2011Q4</td>
<td>QE1 (-60 bps shock to 10y-ffr spread)</td>
<td>-60 bps</td>
<td>0.9%</td>
<td>1%</td>
<td>-0.75%</td>
<td></td>
</tr>
<tr>
<td>Liu et al. (2019)</td>
<td>SVAR¹</td>
<td>1965M4-2011M3</td>
<td>-10 bps shock to 10y-ffr spread</td>
<td>-10 bps</td>
<td>0.2%</td>
<td>-0.06%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eberly et al. (2020)</td>
<td>SVAR²</td>
<td>1990M1-2019M2</td>
<td>-100 bps shock to 10y-ffr spread</td>
<td>-100 bps</td>
<td>-0.7%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**QE programs/shocks: One-Step Estimates; Purchase-Based Shocks**

<table>
<thead>
<tr>
<th>Gambacorta et al. (2014)</th>
<th>SVAR¹</th>
<th>2008M1-2011M6</th>
<th>1 SD (3%) shock to CB balance sheet</th>
<th>0.07%</th>
<th>0.02%</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weale and Wieladek (2016)</td>
<td>SVAR¹</td>
<td>2009M3-2014M5</td>
<td>1% GDP purchase shock</td>
<td>-15 bps</td>
<td>0.58%</td>
<td>0.62%</td>
</tr>
<tr>
<td>Hesse et al. (2018)</td>
<td>SVAR¹</td>
<td>2008M11-2014M10</td>
<td>1% GDP purchase shock</td>
<td>-10 bps</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Gertler and Karadi (2013)</td>
<td>DSGE</td>
<td>n/a (calibrated)</td>
<td>QE1 (6% GDP purchase shock)</td>
<td>-20 bps</td>
<td>2.2%</td>
<td>3%</td>
</tr>
<tr>
<td>Chen et al. (2011)</td>
<td>DSGE</td>
<td>1959Q3-2009Q3</td>
<td>QE2 ($600b purchase shock)</td>
<td>-10 bps</td>
<td>0.1%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Carlstrom et al. (2017)</td>
<td>DSGE</td>
<td>1962Q1-2008Q4</td>
<td>about $300b purchase shock</td>
<td>-19 bps</td>
<td>0.75%</td>
<td>0.12%</td>
</tr>
</tbody>
</table>

**Conventional policy rate shocks**

| Christiano et al. (1999) | SVAR¹ | 1965Q3-1995Q2 | 1 SD (71 bps) shock to ffr | 0.5% | 0.4% | 1.3 |
| Gertler and Karadi (2015) | SVAR² | 1979M7-2012M6 | 25 bps shock to 1-year yield | 0.4% | 0.1% | 4.0 |
| Caldara and Herbst (2019) | SVAR² | 1994M1-2007M6 | 25 bps shock to ffr | 0.4% | 0.2% | 2.0 | 0.05% |

This table reports selected estimates of macro effects of QE and conventional monetary policy shocks in the literature. Output refers to industrial production in Gertler and Karadi (2015) and Caldara and Herbst (2019) and real GDP in all other papers.

1 - identified with zero and sign restrictions; 2 - identified with external instruments
Constructing confidence intervals for the impulse responses

Previous studies of SVARs with external instruments typically use wild bootstraps (with a multiplier of 1 or -1 with equal probability) to generate confidence bands for impulse responses.\(^7\) Recently, Jentsch and Lunsford (2019) point out that the wild bootstrap approach is not asymptotically valid in such a setting, as it does not correctly replicate the variance of VAR innovations or the covariance between the VAR innovation and the instruments.\(^8\) They use simulations to show that wild bootstraps generate confidence bands that are too narrow.\(^9\) This can be seen for our model in Figure OA-4.

Jentsch and Lunsford (2019) propose an alternative approach of moving block bootstrap (MBB), which samples with replacement moving blocks of residuals at a given length.\(^{10}\) Their suggested block length is 5 times the fourth root of the sample length, which equals 15 based on the number of post-crisis observations. However, in our case, this sampling method introduces a substantial bias in the relationship between the resampled LSAP instrument and the resampled LSAP structural shock \(Ez_{1t}\hat{e}_{1t}\), partly due to the fact that the instruments and the estimated LSAP structural shocks are very large at the beginning of the post-crisis period. By construction, MBB picks observations near the beginning and the end of the sample at a lower frequency than those in the middle, which appears to have a large impact on our results because our sample is short relative to the chosen block length. Therefore, we generate bootstrap simulations by resampling with replacement, which can be thought of as MBB with a block length of 1.

Figure OA-5 compares the distribution of the mean product between the resampled LSAP instrument and the resampled LSAP structural shock, \(Ez_{1t}\hat{e}_{1t}\), which is crucial for the identification, across different sampling methods. By construction, the wild bootstrap (with multipliers of 1 and -1) fixes the value of \(Ez_{1t}\hat{e}_{1t}\) at the original estimated value, denoted by the vertical line.\(^{11}\) In contrast, MBB shifts the

---

\(^7\) See, e.g., Mertens and Ravn (2013) and Gertler and Karadi (2015).

\(^8\) In particular, when reduced-form residuals and instruments are simply multiplied by 1 or -1, the variances of the resampled residuals \((E\hat{u}_i\hat{u}_i')\) and the covariances between the resampled instruments and the resampled VAR residuals \((Ez_i\hat{u}_i')\) will be identical across simulated runs, effectively ignoring estimation uncertainties surrounding those statistics.

\(^9\) Also see the response by Mertens and Ravn (2019).

\(^{10}\) Other methods have been proposed in the literature; for example, see Montiel Olea et al. (2021).

\(^{11}\) The value will change when we re-estimate the system on VAR variables constructed from the resampled \(z_t\) and \(\hat{u}_t\), and calculate \(Ez_{1t}\hat{e}_{1t}\) with the re-estimated \(\hat{e}_{1t}\). Nonetheless, by design, the wild bootstrap will reduce the variability in the comovement between \(z_{1t}\) and \(\hat{e}_{1t}\) across simulations.
distribution substantially toward zero.

Figure OA-6 shows the impulse responses to a $500 billion LSAP shock, together with the 68% confidence bands constructed by three different bootstrap methods: resampling with replacement, as in our baseline results; MBB with a block length of 15; and the wild bootstrap (with a multiplier of 1 or -1). The pre-crisis and post-crisis residuals are resampled only among themselves, respectively. Compared with the baseline, the wild bootstrap method generates narrower confidence bands, consistent with the findings in Jentsch and Lunsford (2019). In contrast, MBB generates bands that are much wider and, somewhat surprisingly, further away from zero. To see why the latter is the case, Figure OA-7 plots the impulse responses and the associated confidence intervals to a 4.9-standard-deviation LSAP shock, selected such that it results in a $500 billion instant impact on the LSAP measure based on our SVAR estimates. By construction, the estimated impulse responses to this shock,
The black vertical line denotes the estimated mean product of the LSAP structural shock and the LSAP instrument, \( \bar{E}_{t} \),. The blue line marks the histogram of the mean product from bootstrapped samples generated by resampling with replacement (100,000 runs) across 20 uniform bins of length 0.05 between -1 and 0. The leftmost and rightmost bins show the frequencies of values below -1 and above 1, respectively. Each horizontal segment represents the probability of the calculated value falling inside each bin of length 0.05; thus the vertical location of the horizontal segments represents density times 0.05. The red line represents the histogram generated by MBB with a block length of 15 (100,000 runs), which generates a pronounced bias toward zero.

Figure OA-5: Histogram of the Mean Product
The solid lines plot the impulse responses to a $500 billion shock to 1-year ahead expected SOMA holdings. The shaded areas represent the baseline 68 percent confidence intervals estimated by bootstrap; the red and green lines represent 68 percent confidence intervals estimated by MBB (with a block length of 15) and wild bootstrap, respectively.

The blue solid lines, are identical to those in Figure OA-6. In addition, the confidence intervals using the wild bootstrap method and the MBB are much more similar than in Figure OA-6. However, as can be seen from the top left panel of Figure OA-7, the distribution of the instantaneous responses of the LSAP measure to the fixed-standard-deviation shock is shifted toward zero under MBB, possibly reflecting the fact that MBB shifts the distribution of the mean product of the LSAP shock and the LSAP instrument towards zero, as shown in Figure OA-5. As a result, when we calculate the impulse responses for each simulated path by normalizing the impulse responses to a fixed-standard-deviation shock by the instantaneous response of the LSAP measure to the same shock, the impulse responses become magnified and shift away from zero.

\[ \text{In the limit, if the correlation between the LSAP shock and the LSAP instrument goes to zero, the estimated instantaneous response of the LSAP measure will also go to zero.} \]
Figure OA-7: Comparison of Confidence Bands for a 4.9-Standard-Deviation LSAP Shock

The solid lines plot the impulse responses to a 4.9-standard-deviation LSAP shock. The shaded areas represent the 68 percent confidence intervals estimated by bootstrap; the red and green lines represent 68 percent confidence intervals estimated by MBB (with a block length of 15) and wild bootstrap, respectively.
OA.8 Robustness results

**Alternative macro variables.** Figure OA-8 reports the results replacing log CPI by log core CPI.

**Alternative policy indicators.** Table OA-4 reports the first-stage instrument strength test for the alternative LSAP policy indicators.

We also consider alternative FG indicators, including 1-, 2- or 5-year Treasury yields, or survey expectations of the federal funds rate 12 months or 18 months ahead, constructed from the same survey. The Survey of Primary Dealers used to construct our LSAP policy indicator also contains questions on dealers’ expectations about the federal funds rate at various horizons. We construct 12- and 18-month-ahead median expectations of the funds rate from the surveys, and convert them into monthly measures following the same methodology as for the LSAP measures (see Appendix OA.2 for more details). Table OA-5 reports the first-stage instrument strength test for these indicators. The F-statistic remains insignificant for all FG indicators, including the 2-year yield, on which the FG indicator loads most heavily on (Figure OA-3). Other than the 10-year Treasury yield, using the 1-year Treasury yield, the measure preferred by Gertler and Karadi (2015), as the FG indicator increases the significance of the FG factor the most. However, its use in our model is problematic because the the ELB depressed movement in yields of maturities up to 2 years (Swanson and Williams (2014)).

**Accounting for potential central bank information effects.** Figure OA-10 plots the original LSAP factor and the LSAP factor after cleaning for potential central information effect.

---

13 Survey expectations of future federal funds rates are not available before the crisis, and results for those indicators are based on VARs estimated over the post-crisis period.
Figure OA-8: Impulse Responses to LSAP Shocks (Core CPI)

The solid lines plot the impulse responses to a $500 billion shock to 1-year ahead expected SOMA holdings. The shaded areas represent 68 percent confidence intervals estimated using bootstrap.
Table OA-4: Robustness to alternative LSAP indicators

<table>
<thead>
<tr>
<th>LSAP instrument</th>
<th>expected SOMA</th>
<th>expected SOMA/GDP</th>
<th>expected cumulative purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uni.</td>
<td>bi.</td>
<td>uni.</td>
</tr>
<tr>
<td>LSAP instrument</td>
<td>-0.053***</td>
<td>-0.053***</td>
<td>-0.36***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>83</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.18</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>F-statistic</td>
<td>18.0***</td>
<td>8.9***</td>
<td>19.8***</td>
</tr>
<tr>
<td>min. eigenvalue</td>
<td>0.9</td>
<td></td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LSAP instrument</th>
<th>expected cumulative purchases/GDP</th>
<th>Actual SOMA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uni.</td>
<td>bi.</td>
</tr>
<tr>
<td>LSAP instrument</td>
<td>-0.37***</td>
<td>-0.38***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>F-statistic</td>
<td>20.7***</td>
<td>10.3***</td>
</tr>
<tr>
<td>min. eigenvalue</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

This table repeats the first-stage instrument strength tests in Table 1 for various LSAP indicators, including the expected SOMA holdings in the baseline model, as well as expected SOMA-to-GDP ratio, expected asset purchases, expected asset purchases-to-GDP ratio, the ten-year yield, and the actual SOMA holdings. The two columns under each indicator reports results for univariate or bivariate regressions respectively. The symbols ***, ** and * denote significance at the 1, 5 and 10 percent levels, respectively.
Table OA-5: Robustness to alternative FG indicators

<table>
<thead>
<tr>
<th>FG instrument</th>
<th>10-year yield</th>
<th>1-year yield</th>
<th>2-year yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uni.</td>
<td>bi.</td>
<td>uni.</td>
</tr>
<tr>
<td>FG instrument</td>
<td>0.083</td>
<td>0.076</td>
<td>(0.058)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>83</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.02</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>F-statistic</td>
<td>2.1</td>
<td>2.6</td>
<td>2.9</td>
</tr>
<tr>
<td>min. eigenvalue</td>
<td>1.9</td>
<td>1.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FG instrument</th>
<th>5-year yield</th>
<th>12m ahead svy exp ffr‡</th>
<th>18m ahead svy exp ffr‡</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uni.</td>
<td>bi.</td>
<td>uni.</td>
</tr>
<tr>
<td>FG instrument</td>
<td>0.082</td>
<td>0.078</td>
<td>(0.060)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>83</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>F-statistic</td>
<td>1.9</td>
<td>1.5</td>
<td>1.7</td>
</tr>
<tr>
<td>min. eigenvalue</td>
<td>0.9</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>

This table repeats the first-stage instrument strength tests in Table 1 for various FG indicators, including the 10-year yield in the baseline model, as well as 1-, 2- and 5-year yields and survey-based measures of expected federal funds rate 12 and 18 months ahead. The two columns under each indicator reports results for univariate or bivariate regressions respectively. The symbols ***, ** and * denote significance at the 1, 5 and 10 percent levels, respectively.

‡: Survey expectations of future federal funds rates are not available before the crisis. Results for those indicators are based on VARs estimated over the post-crisis period.
Figure OA-9: Impulse Responses to FG Shocks (Alternative Indicators)

The solid lines plot the impulse responses to a 25 basis point shock to 18-month ahead expected federal funds rate. The shaded areas represent 68 percent confidence intervals estimated using bootstrap methods.
Figure OA-10: LSAP Factor: Removing Fed Information Effect

The red line is the baseline LSAP instrument used in the main analysis. The blue line removes the Fed information effect following the methodology outlined in Miranda-Agrippino and Ricco (2021). Numbered vertical lines indicate the dates for important LSAP-related announcements, as described in Table 3.
OA.9 Duration-weighted expected SOMA holdings

This section describes in more detail the construction of duration-weighted expected SOMA holdings introduced in Section 4.5.

**MEP-induced duration lengthening.** Between August 2011 and December 2012 while the MEP was ongoing, we add to our baseline SOMA holdings measure the expected size of future maturity extension operations, calculated as the announced and the survey-based expected future pace of MEP operations multiplied by the estimated duration added per dollar of operation, which varies over time but is close to 9 years on average.

After the announcement of the MEP at the September 2011 FOMC meeting, each survey asks respondents about the probability of an additional MEP being announced after the completion of the current program. To take into account expectations of an additional MEP, we construct a survey-based estimate of additional purchases under such a program, by multiplying an assumed program size of $267 billion and the median survey probability attached to such a program, and add this estimate to the remaining purchases under the announced MEP. The assumed size of $267 billion is identical to that of the MEP extension announced in June 2012 and could have been reasonably expected by the market: Assuming that the pace and maturity range of the initial MEP program remain unchanged, about $267 billion over 6 months was the maximum extension possible because the SOMA portfolio would run out of short-maturity securities to sell after that.

After the announcement of the MEP, we calculate duration added per dollar as the difference between the weighted average duration of purchases and the weighted average duration of sales based on the publicly announced maturity buckets. Prior to the announcement, we calculate it using the median maturity range from the September 2011 Survey of Primary Dealers and the results are similar if we use the announced maturity buckets instead. In addition, the average duration of purchases is calculated using information on all outstanding Treasury securities in the purchase buckets, while the average duration of sales is calculate using a snapshot of SOMA Treasury security holdings on June 1, 2011.

**Duration shortening due to MEP sales and passage of time.** Due to sales of short-maturity securities under the MEP, almost no Treasury securities in SOMA were expected to mature within one year between August 2011 and the end of 2014 and only a relatively small amount in 2015. Thus, the duration of existing SOMA
holdings is expected to shorten with the passage of time, without the usual offsetting duration-lengthening effect from reinvesting maturing securities at auctions.

To account for the absence of reinvestment within a one-year window between the beginning of the MEP and the end of 2014, we subtract an amount that equals the current par value of SOMA Treasury holdings times 1 (year) from the 1-year ahead duration-weighted SOMA holdings. Prior to the announcement of the MEP, we multiply that amount by the probability attached to an MEP program. For 2015, we multiply that amount by a factor that decreases linearly from 1 to 0 over the course of 2015. Reinvestment was expected to resume in 2016, as no securities maturing in 2016 or later were sold under the MEP, as the MEP ceased at the end of 2012 and the maximum maturity sold under the program was 3 years.

**Duration evolution before the MEP.** For the period before August 2011, we assume that investors expect the average duration of SOMA Treasury holdings to stay at the current level, a pattern consistent with what is observed during that period (Figure OA-11). We cannot apply the same assumption to MBS holdings, because the duration of MBS varies endogenously with the level of interest rates to a much greater degree than Treasury securities, due to the embedded prepayment option in MBS.\(^{14}\) Assuming a random walk process for the MBS duration will therefore introduce substantial volatility into the LSAP measure that is unrelated to the stance of monetary policy. Instead, we assume that investors expect the average duration of SOMA MBS holdings to stay constant at 3.7 years, its average level over the post-crisis period.\(^{15}\)

**Duration estimates.** Figure OA-11 plots the 1-year ahead expected duration of the Treasury and MBS holdings, respectively, together with the actual durations. Despite the rough approximation, the expected Treasury duration measure appears reasonable and tracks future duration well. For example, both in September 2011—immediately following the announcement of the MEP—and in June 2012—following the announcement of the extension of the MEP, the expected future duration is close to the actual duration observed one year later.

\(^{14}\) A lower interest rate would prompt more mortgage borrowers to prepay and hence shorten the duration of MBS, and vice versa. Any other factors that affect the prepayment speed will introduce additional volatility in MBS duration.

\(^{15}\) As a robustness check, we also estimate a version of the model assuming that investors expect the duration of SOMA MBS holdings to stay at the current level. The impulse responses are comparable to those from the baseline model when estimated over the post-crisis sample, but become explosive when estimated over the full sample.
Figure OA-11: Actual and Expected Future Durations of SOMA Treasury and MBS Holdings

The blue line shows the expected duration of SOMA Treasury holdings one year into the future implied by our construction, while the dashed blue line shows the actual duration of SOMA Treasury holdings. The red solid and dashed lines do the same for SOMA MBS holdings. The grey shaded areas mark the time periods when QE1, QE2, and QE3 were ongoing. The blue shaded area mark the time period when the MEP was ongoing, which partially overlapped with QE3, indicated by a darker shade.
OA.10 QE3 counterfactual analysis

Table OA-12 shows the difference between the counterfactual and the actual paths of various macroeconomic and financial variables, under a counterfactual of no LSAP shocks associated with QE3.

OA.11 The CKSVAR model

Model specification. The CKSVAR model, in a reduced form, can be written as follows:

\[
y_{1,t} = B_0 + \sum_{i=1}^{L} B_i y_{t-i} + \sum_{i=1}^{L} \beta_i z_{t-i} - \beta^\Pi(y_{2,t} = b)z_t + u_{1,t}; \quad (OA-12)
\]

\[
y_{2,t}^* = C_0 + \sum_{i=1}^{L} C_i y_{t-i} + \sum_{i=1}^{L} \gamma_i z_{t-i} + u_{2,t}, \quad (OA-13)
\]

where

\[
y_{2,t} = \max(y_{2,t}^*, b); \quad (OA-14)
\]

\[
z_t = \min(y_{2,t}^* - b, 0), \quad (OA-15)
\]

where the \(n\) observed variables \(y_t\) is partitioned into \((n - 1)\) variables \(y_{1,t}\) and the federal funds rate \(y_{2,t}\). The shadow rate \(y_{2,t}^*\) is unobserved if it is below the ELB \(b\), while the variable \(z_t\) captures the degree that the shadow rate falls below the ELB. The model allows the shadow rate \(y_{2,t}^*\) to be censored from below but still have an effect on the economy (“censored”), partially capturing the usage of asset purchases and other unconventional monetary policy tools. The effects of the shadow rates are allowed to differ at and away from the ELB (“kinked”), reflecting the possibility that unconventional policy tools cannot fully offset the ELB. If the ELB is never reached and \(y_{2,t} > b\) always holds, this model reduces to the simpler baseline VAR model described earlier. By contrast, if \(y_{2,t}\) occasionally hits the ELB, the shadow rate would affect the dynamics of the system through the presence of \(z_t\) in the equations.

Identification with external instruments. Due to the censoring of \(y_{2,t}^*\), we need to numerically calculate the distribution of residuals \(u_t\) using sequential importance sampling if the fed funds rate is at the ELB currently or in recent past. We apply the identifying restriction in section 2.1 on the estimated mean of \(u_t\).
Figure OA-12: Path of the Economy under the Counterfactual (Difference from the Actual).

The panels show the difference between the counterfactual and the actual paths of various macroeconomic and financial variables. As in Figure 16, the counterfactual path is generated by keeping the LSAP structural shock at zero after September 2012. CFNAI stands for the Chicago Fed National Activity Index.
Shadow rates. One variable of potential interest from the CKSVAR models is the shadow rate. The estimates from the three models discussed above are shown in Figure OA-13 with a minus sign. We also show the 1-year ahead expected SOMA holdings over the same period for comparison. The estimated shadow rates from the 5-variable CKSVAR model, the red line, show an initial steep decline near the beginning of the ELB period, consistent with the start and the expansion of QE1, but does not appear to capture later LSAP programs. The estimated shadow rate from the six-variable model, the green line, is more negative and exhibits some declines associated with QE2 and QE3, but the initial decline around the time of QE1 remains the most pronounced. The time series pattern that the shadow rate declines precipitously in the immediate aftermath of the GFC is similar to that found by Ikeda et al. (2022), but different from estimates from term structure models—eg Wu and Xia (2016), plotted as the black dashed line, and Kim and Priebsch (forthcoming)—which typically show a more gradual decline following the GFC and an acceleration around 2014.

Restricted CKSVAR models. We also estimate two restricted versions of the CKSVAR model proposed by Mavroeidis (2021). The first restricted model, the “kinked” SVAR (KSVAR), imposes the restriction that the shadow rate has no effect on other variables in the VAR. When no LSAP variable or long rates are included, this implies that monetary policy completely loses its effectiveness once the federal funds rate hits the ELB. The second restricted model, the “censored” SVAR (CSVAR), imposes that the effect of the federal funds rate on the system is the same regardless of whether it is censored or not, hence the ELB is irrelevant and monetary policy retains its full effectiveness at the ELB. We find that even after adding the one-year ahead expected SOMA measure, both restricted models are rejected based on a likelihood ratio test, with \( p \) values less than one percent, for each specification. These results are similar to those found by Ikeda et al. (2022).

References


Figure OA-13: Estimated Shadow Federal Funds Rate from CKSVAR Models

The red (“5-var CKSVAR”) and green (“6-var CKSVAR”) lines plot the negative of the estimated shadow federal funds rate from the CKSVAR model, without and with the one-year ahead expected SOMA holdings, respectively, using the longer sample, extended through June 2019. For comparison, the blue line plots the one-year ahead expected SOMA holdings and the black dashed line shows the negative of the shadow rate estimated by Wu and Xia (2016), available from [https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate](https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate).


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