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When It Rains It Pours: Cascading Uncertainty Shocks☆

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Abstract

We empirically document that serial uncertainty shocks are (1) common in the data and (2) have an increasingly stronger impact on the macroeconomy. In other words, a series of bad (positive) uncertainty shocks exacerbates the economic decline significantly. From a theoretical perspective, these findings are puzzling: existing benchmark models do not deliver the observed amplification. We show analytically that a state dependent precautionary motive with respect to uncertainty shocks is required. Our derivations suggest that the state dependent precautionary motive only shows up at fourth order approximations or higher. Fundamentally, in DSGE models solved with perturbations, agents have always possessed a state dependent precautionary motive but typical solution methods were hiding this fact. Future studies need to consider solving the model via fourth (or higher) order perturbation in order to avoid understating the effect of uncertainty shocks that occur in succession.

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JEL Classification: C63; C68; E37

1 Introduction

Uncertainty shocks are often thought of as fleeting events. A one time increase or decrease in stochastic volatility, for example, is a widely used tool that economists employ to generate dynamic responses in theoretical models.[1] However, a careful analysis of previous economic crises should prompt us to rethink that paradigm. For instance, the Euro-zone debt crisis lasted for years and sent uncertainty shocks through the global economy as nations grappled with solvency and fiscal concerns. Each ripple was frequently met with even greater uncertainty in the form of austere policy responses that tended to exacerbate the dour economic outlook. More recently, successive shocks to trade policy uncertainty unfolded in 2018 and 2019, as tariff threats and retaliations ricocheted back and forth between the United States and China. Finally, the recent downturn is also unlikely to be an isolated uncertainty event. With no potential vaccine likely until 2021 at the earliest, the aftershocks following the original infectious propagation are raising alarms for both researchers and government officials alike. As uncertainty layers on top of existing uncertainty with shocks occurring in rapid succession, many are left to wonder how this will shape the economic recovery. With these historical episodes in mind, our goal in this project is to document and shed light on the importance and ramifications of successive shocks to uncertainty.

Economic uncertainty has shown to be a significant determinant of investment dynamics (Bloom, Bond and van Reenen, 2007), a driver of firm production declines and recoveries over time (Bloom, 2009), and a source of business cycle variations (Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry, 2018) in general. Using survey data, Bachmann, Elstner and Sims (2013) shows that both in the U.S. and in Europe, ex ante forecast disagreements about business conditions is a strong predictor of lower production. Moreover, Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015) show that, both empirically and theoretically, an increase in fiscal policy uncertainty is associated with an adverse effect on economic activity. As another example, Basu and Bundick (2017) demonstrate that demand uncertainty is the primary cause of large declines in output and investment in the data.

Nevertheless, how these uncertainty shocks interact with each other sequentially is

[1] For the purpose of exposition, words like "uncertainty shocks" and "volatility shocks" are used interchangeably. What we have in mind is second-moment innovations to the underlying state process.

not well known. It is reasonable to conjecture that in the midst of a downturn along with its build-up, positive shocks to uncertainty can occur in succession, causing consecutive negative movements in economic aggregates. Whether these serial positive uncertainty shocks have a diminishing effect, a neutral effect, or an amplifying effect is unclear. In other words, if two positive uncertainty shocks arrive in back-to-back periods, will the economic impact of the second shock be less powerful, the same, or more powerful than the first realized shock?

In this paper, we study the effect of serial positive uncertainty shocks on variables such as output, investment, inflation, and the stock market. Furthermore, we also investigate the non-linear scaling effect of large versus small uncertainty shocks. We empirically document that serial positive uncertainty shocks have a cascading effect such that the later realizations are more powerful and longer lasting than the earlier ones. In a dynamic general equilibrium model, the cascading impact of uncertainty shocks can not be generated by a third order approximation. This phenomenon can only be generated by a solution solved under fourth order perturbation or higher. To our knowledge, this is the first study to document this finding.

We observe that serial uncertainty shocks are fairly common in the data. As an illustrative example, Panel (a) of Figure 1, Figure 2, and Figure 3 plot the realized market volatility, the financial uncertainty index proposed by Ludvigson, Ma and Ng (2015), and the economic policy uncertainty (EPU) developed by Baker, Bloom and Davis (2016), respectively. Panel (b) in these figures plots the time series of the standardized shocks. Panel (c) and (d) highlight the events with two or three consecutive positive shocks, respectively. We note several episodes characterized by sequences of positive shocks. Focusing on Panel 2(c) and 2(d) we observe that these events are not confined to the two largest financial uncertainty shock recorded in 1987:Q4 (Black Monday) and in 2008:Q3 during the financial crisis, and are indeed spread out over the entire sample.

[**Insert Figure 1, Figure 2, and Figure 3 about here**]

Given the series of uncertainty shocks constructed from the data, we employ smoothed local projection (SLP) popularized by Barnichon and Brownlees (2018) to examine the conditional response of the economy produced by consecutive positive shocks to uncertainty. The idea is straightforward: using an indicator variable to denote whether a given positive shock is preceded by one or more positive shocks, we regress economic aggregates on the shock itself as well as the interaction term between the shock and the indicator variable. The coefficient loading on the interaction term, or the state multiplier, is only valid if the indicator variable is turned to 1 and can be interpreted as the additional reaction due to the fact that the shock is not the first realization in a sequence of positive shocks.

Our baseline empirical specification employs monthly data of industrial production, the nominal short rate, inflation, and the stock market valuation. The estimation results are striking. For all uncertainty shock measures, the estimated state multipliers are negative in general across prediction horizons. The resulting conditional impulse responses show much larger declines in all four economic variables compared with their unconditional responses in the data. To highlight the magnitude, industrial production drops by between 1% to 2% as opposed to around 0.2% to 0.5% at the maximum if a positive shock is the second shock in a row. For the nominal interest rate, the contrast is roughly -1% vs. -0.2% within two years of the shock. Similar implications can be found for inflation and stock market valuation.

Furthermore, the cascading effect of serial uncertainty shocks also prolongs the negative economic impact in the data. The impulse responses show that the maximal drop in economic activity, as measured by industrial production, for a second consecutive positive shock occurs at about 6 quarters post-realization. Whereas the economy recovers to the steady state within 3 years after an unconditional positive shock, the recovery following a second consecutive positive shock drags beyond that horizon.

In the theoretical section, we show that - within a standard New-Keynesian framework - demand uncertainty shocks have rather different implications on the economy depending on what precedes them (history dependency) and magnitude (violation of shape invariance). We focus on demand uncertainty because it has been shown to generate a significant impact on economic aggregates in dynamic equilibrium models, see Basu and Bundick (2017).[2] In the model, a positive shock to demand uncertainty lowers output, investment, inflation, and stock market valuation. A second consecutive positive shock amplifies the declines in these endogenous variables and causes them to fall further. The cascading effect is similar to what is documented

[2]Bretscher, Hsu and Tamoni (2020) show that productivity uncertainty in conjunction with time-varying risk aversion can also produce similarly large effects on aggregate economic variables from the supply side.

empirically.

Importantly, one needs at least a fourth-order perturbation solution of the model to study the non-linear effects (i.e., history and size dependency) of uncertainty shocks. Indeed, our analytical derivation shows that households exhibit a state dependent precautionary motive with respect to uncertainty at the fourth order approximation thanks to the nonzero loading on the quadratic stochastic volatility term in the perturbation solution. This loading is not present at the third order approximation. Intuitively, the greater precautionary motive that generates this amplification is hidden by the third order approximation, just as movements in risk premiums are hidden by the second order approximation. The economic implications are straightforward: Agents respond to additional increases in uncertainty with even larger declines in demand, which translates into lower output along with each of its components in the demand-driven New Keynesian model.

Our results have important and widespread implications for the analysis of the macroeconomy. Since the seminal contribution of Sims (1980), researchers in macroeconomics often compute dynamic multipliers of interest, such as impulse responses and forecast-error variance decomposition, by specifying a vector autoregression (VAR). Impulse responses (and variance decomposition) are important statistics in their own right: they provide the empirical regularities that substantiate theoretical models of the economy and are therefore a natural empirical objective.

However, the impulse responses implied by linear models, such as VARs, are characterized by a few (arguably restrictive) properties: (1) shape invariance, responses to shocks of different magnitudes are scaled versions of one another; and (2) history independence, the shape of the responses is independent of the local conditional history.[3]

Our results show that the macroeconomic responses implied by (suitably solved) standard New Keynesian model indeed manifest violation of these properties typical of a linear system, and call for a more flexible approach like that proposed by Jorda (2005).

[3] VARs also impose symmetry, i.e., responses to positive and negative shocks are mirror images of each other. We do not investigate (violations of) symmetry in this work.

2 Literature review

Our work is related to the growing literature on uncertainty shocks, which started with the seminal contribution by Bloom (2009). From an empirical perspective, the literature has shown – using alternative measures – that a one-period rise in uncertainty can cause a significant fall in economic activity (e.g., Baker et al., 2016; Bloom, 2009; Jurado et al., 2015; Rossi and Sekhposyan, 2015; Caldara and Iacoviello, 2018). From a theoretical point of view, the literature has provided mixed evidence on the quantitative relevance of uncertainty. With standard business cycle models, the effects of uncertainty shocks tend to be economically insignificant (e.g., Born and Pfeifer, 2013; Bachmann et al., 2013). However, an important strand of literature has shown that uncertainty can be amplified in the presence of non-linearities such as the zero lower bound (Fernández-Villaverde et al., 2015; Basu and Bundick, 2017), of frictions in the financial sector (Christiano et al., 2014) and in the labour market (Leduc and Liu (2016); see also denHaan et al. (2020) for a reappraisal of the role of uncertainty shocks in search and matching models). We contribute to this literature by showing (empirically and theoretically) the non-linear, amplifying effects induced by a *sequence* of uncertainty shocks.

In a series of important contributions, Barnichon and Matthes (2018) and Barnichon and Matthes (2020) propose a novel methodology to investigate the asymmetric and state dependent effects of macroeconomic shocks; their empirical focus is mostly on how the effect of monetary and government spending shocks depends on their sign. We contribute to this literature by showing that the effects of shock may also depend on whether they happen to be isolated or multi-period (consecutive) shocks. We also provide evidence that the size of the changes in uncertainty (large vs small shocks) can be a potential source of non-linear dynamics.

To the best of our knowledge, our analysis of the (non-linear) effects of consecutive shocks is in its infancy. The closest in spirit is the work by Borovicka and Hansen (2014, Section 2.4) who builds upon Hansen and Scheinkman (2009) and proposes a methodology to compute multi-period perturbation of cash flows and prices.

Recently, in response to the COVID-19, a literature is emerging on the effect of large and/or multi-period shocks. Most notably, Ludvigson et al. (2020) uses linear VAR methods to investigate the compounding effect of prolonged shocks like the

COVID-19,[4] and Primiceri and Tambalotti (2020) propose a novel approach that "synthesizes" a Coronavirus shock from typical disturbances that have historically driven macroeconomic fluctuations, and that tilts its propagation to account for alternative scenarios about the evolution of the pandemic. In line with these papers, our hope is to contribute to an active debate about new reduced-form as well as structural tools that could shed light on the effects and propagation of shocks, and to inform about the appropriate policy response.

Similar to Ludvigson et al. (2020) we are interested in understanding the effect of successive multi-period shocks, with particular emphasis on second-moment ones. We view our analysis as complementary to that of Primiceri and Tambalotti (2020). These authors study the propagation of a single (unprecedented in its type and scale) shock and show that it can lead to a severe and prolonged recession. We put forward the hypothesis that such severe and prolonged patterns may also emerge naturally as a response to consecutive uncertainty shocks. Thus, whereas Primiceri and Tambalotti (2020) work under the assumption of a single shock that hits in March 2020 and then propagates, our analysis suggests to describe the unfolding of the pandemic as a sequence of shocks.[5]

We also contribute to the literature on perturbation methods (Judd, 1998), and their use to compute generalized impulse response function (Koop et al., 1996) to uncertainty shocks.[6] Schmitt-Grohe and Uribe (2004) show that in a second-order expansion of the model solution the presence of uncertainty affects only the constant term of the decision rules. Andreasen (2012) show that a third order perturbation solution is needed for rare disasters and stochastic volatility to influence the level of risk premia in DSGE models. Fernández-Villaverde et al. (2011) and Fernández-Villaverde et al. (2015) show that one has to consider at least a third-order Taylor expansion of the solution to study the dynamic implications of a volatility increase

[4] While Ludvigson et al. (2020) use linear VAR methods for the prolonged shocks, when it comes to non-linearities induced by large shocks, they employ local projections, as we do. A similar approach has been adopted by Foerster (2014). In contrast, our usage of local projections to investigate the non-linearities induced by consecutive shocks is new.

[5] These two alternative approaches are acknowledged in footnote 2 of Primiceri and Tambalotti (2020).

[6] Another important literature study the effect of risk aversion in models solved using perturbation methods. E.g., Binsbergen et al. (2012) and Caldara et al. (2012) show that, when a second-order expansion is employed to solve models with Epstein-Zin-Weil preferences, the parameter of relative risk aversion only enters the solution through a constant term in the policy rule. This term can be interpreted as the precautionary behavior toward risk.

while keeping the level of the variable constant. Recently, de Groot (2019) shows that a fourth order approximation is required for the standard deviation of the stochastic volatility (a.k.a. vol. of vol.) to affect the stochastic steady state through the risk correction or constant term. We contribute to this literature by showing that a fourth-order Taylor expansion of the solution is also needed to study the non-linear effects of a sequence of volatility shocks. Moreover, within a simplified model, we show exactly which terms are crucial in the fourth order perturbation solution to generate the cascading effect of serial uncertainty shocks. Specifically, we are the first to document a non-zero term that loads on quadratic variation in the stochastic volatility process, which generates amplification, state dependence, as well as non-linearity with respect to the size of the shocks. Finally, we show that the third order approximation only loads linearly on the stochastic volatility process in a simplified model.

2.1 Previous episodes of cascading uncertainty

This section provides background on several historical episodes which exhibited successive cascading shocks to uncertainty. As evidenced by the discussion below, shocks to uncertainty will often invite additional shocks, whether it be uncertainty regarding policy responses, uncertainty generated by other economic actors (e.g. ratings agencies, labor unions, other countries), or uncertainty regarding the duration of the initial shock to uncertainty. The examples discussed below represent just a fraction of the episodes we observe in the data.

Gold Crisis of 1967 - 1968. A series of positive shocks to uncertainty unfolded surrounding the Gold Crisis of 1967 and 1968 along with the Vietnam War.[7] The Gold Crisis was sparked by a surprise devaluation of the British Sterling, which touched off a frenzy of volatility in global financial markets. This required a response by the United States to uphold its commitment to the existing price of gold, which could be achieved by shoring up its balance of payments position through reductions in expenditures and increases in taxes (in effect generating heightened fiscal policy uncertainty).

Likewise, the Tet Offensive in January 1968 increased the possibility of greater

[7] See Collins (1996) for more details.

spending on the Vietnam War, amplifying uncertainty about future inflation and a potential response by monetary policy while exacerbating the balance of payments. This ended up weakening the dollar and amplifying the rush to gold, forcing the United States to change its gold cover ratio, which set in motion the decline and ultimate termination of the Bretton Woods system in the years ahead while sowing the seeds of the recession that began the following year.

Recession of 1973. In addition to a basic supply shock, the Arab oil embargo of 1973 contributed a series of uncertainty shocks that lasted through 1974 in the midst of a global stock market crash that began earlier in the year.[8] The embargo generated global uncertainty about oil supplies, with no hint of a potential end-date in sight, as OPEC treated importing countries differently based on subjective favorability ratings.

The resulting shortfalls in gasoline and increased production costs helped to amplify uncertainty about future inflation in the United States, which generated higher interest rates and increased uncertainty about how monetary policy would balance the competing forces of higher unemployment and inflation. The United States government responded by instituting price controls and regulations, which exacerbated the crisis by creating greater scarcity and more rationing. The end result was a recession that extended through 1975, with the oil embargo contributing a series of shocks that lasted for multiple quarters.

1998 Asian and Russian Financial Crisis. A cascade of shocks to uncertainty rained down in the build up to the Asian and Russian Financial Crisis. Thailand's devaluation of its currency in 1997 triggered a currency and financial crisis that spread to Malaysia, Indonesia, the Philippines. Further shocks to uncertainty arrived in the months that followed, as South Korea, Hong Kong, and China's elevated exposure to these countries set off dramatic declines in currency and financial markets, which required intervention by the IMF. Part of the IMF's terms for assistance included massive budget cuts and tax increases which amplified uncertainty about the path of fiscal policy in these countries.

The ongoing crisis spread to Russia in 1998, as the price of oil dramatically declined along with demand. Massive selling of rubles by investors forced Russia to devalue its currency and default on its debts, which amplified uncertainty about the soundness

[8] See `https://history.state.gov/milestones/1969-1976/oil-embargo` for more details.

of the U.S. banking system, which had significant exposure. Brazil was also impacted by the crisis as it was forced to cut spending and raise taxes to try to maintain the value of its currency. The buildup of these events culminated with its spread to the United States as Long-Term Capital Management required a bailout from the Federal Reserve.[9] In addition, monetary policy cut rates on multiple occasions to head off increased volatility and uncertainty as U.S. markets declined close to 20% while experiencing some of the largest single-day declines in history.

2010-12 Euro-zone debt crisis. The Euro-zone debt crisis propagated and sent shockwaves of uncertainty through the global economy for several years. What started as a Greek debt crisis in the aftermath of the Great Financial Crisis, spread to many of its neighbors in the years that followed. This typically caused a series of policy reactions that exacerbated uncertainty about the future path of the economy.

For instance, Italy's sovereign bond yields were skyrocketing as investor confidence waned on the country's solvency, with the second highest debt-GDP ratio in Europe. This sparked an effort by fiscal policy to introduce spending cuts and tax increases to calm markets. Adding to the increased economic uncertainty, large union strikes were organized to protest the fiscal policy and this virtually shut down the country. An austerity package was eventually passed in the months that followed but despite this, ratings agencies continued to further downgrade Italy's credit rating while characterizing the outlook as negative.[10]

Similar events unfolded in Spain, Portugal, Ireland and Cyprus, as fiscal policy responses to the increased financial uncertainty were often met with further downgrades in the country's debt ratings, which only added to the uncertainty in the economic environment. Overall, the unease about possible spillovers from fiscal and financial strains in the euro area was cited over the course of numerous releases of the FOMC minutes during this time period as concerns persistently weighed on the U.S. economic outlook.

2018-19 Trade Policy Uncertainty. Successive shocks to trade policy uncertainty were fully evident throughout 2018 and 2019. Tariffs were put in place by the United States on China in early 2018, and threats of further tariffs and retaliations

[9] See https://www.federalreservehistory.org/essays/asian_financial_crisis for additional details.

[10] See https://www.bbc.com/news/business-13856580 for a full time-line of events.

continued in the months that followed. For instance, the months of April, June, July, August, September all had important tariff announcements and subsequent retaliations that added considerable uncertainty to the U.S. economic outlook.[11]

After a brief truce, the back-and-forth continued in 2019 and further tariffs were put in place by the United States that were met with more retaliations from China. The escalation grew so much that financial markets started to price in serious downside risks. Additionally, uncertainty about the path of monetary policy grew and culminated with several interest rate cuts in a bid to offset the risks associated with the ongoing trade uncertainty. Overall, the trade war between the United States and China contributed a series of successive shocks to uncertainty that had adverse effects on economic activity.

3 Empirical Analysis

In this section we investigate the effects of consecutive uncertainty shocks on macroeconomic and financial outcomes. We are particularly interested in uncovering the presence, if any, of non-linearities in the response associated with multi-period shocks. We start describing our data; we then turn to the empirical methodology, and finally describe the results.

3.1 Data

We use three types of uncertainty proxies which have been widely used in applied research: the market realized volatility, the financial uncertainty index (U^F) constructed by Ludvigson, Ma and Ng (2015), and the economic policy uncertainty (EPU) developed by Baker, Bloom and Davis (2016). Realized volatility is computed as $\widehat{RV} \equiv 100 \times \sqrt{\frac{252}{21} \sum_{i=1}^{21} r_{t,i}^2}$ with $r_{i,t}$ denoting the daily stock returns of the S&P 500 index from the CRSP database.[12] All these uncertainty series are available at monthly frequency.

[11]See https://www.reuters.com/article/us-usa-trade-china-timeline/timeline-key-dates-in-the-u-s-china-trade-war-idUSKBN1ZE1AA for a detailed account of events.

[12]Essentially we estimate the diffusive term of the stock price process via the realized volatility estimator. We agree with Berger et al. (2019) on the fact that realized equity volatility is not a perfect measure of uncertainty. This is why we used the word "uncertainty proxies". With this caveat in mind, henceforth, we often refer to these series as uncertainties without further qualification.

Ultimately, we want to compute impulse response functions for a set of variables of interest conditional on the realization of n consecutive positive shocks to a given uncertainty measure. To this end, we need to construct the shocks. Since our goal here is not to come up with a new identification scheme, we rely on methods that have been proposed in the literature to extract the shocks. Specifically, for realized volatility we follow Berger et al. (2019) and obtain realized volatility shocks from a linear predictive regression of next month (log) realized volatility on lagged (log) RV, and option-implied volatility. Importantly, Berger et al. (2019) show that despite the fact that other macro and financial time series (such as industrial production and the default spread) on their own can help predict future volatility, their forecasting power is subsumed by current realized and option-implied volatility. The sample period for RV shocks is constrained by the availability of the option-implied volatility. We use the S&P 500 implied volatility (VIX) extended to February 1983 and kindly provided by Ian Dew-Becker.

For the financial uncertainty index, we employ the identification scheme proposed by Ludvigson, Ma and Ng (2015) and extract shocks from a trivariate system that includes an index of macro uncertainty, a measure of real economic activity, and the financial uncertainty index.[13] The financial uncertainty index is available for a long time-span beginning in July 1960. We focus on financial uncertainty rather than macroeconomic uncertainty or real economic uncertainty (all three are proposed in the same paper) because financial market uncertainty is shown to be the most exogenous of the three uncertainty measures in terms of driving business cycle variations.

For EPU, we employ a VAR model as in the original paper by Baker, Bloom and Davis (2016). Specifically, we fit a VAR to monthly U.S. data from January 1985 to December 2019. To recover orthogonal shocks, we use a Cholesky decomposition with the following ordering: the EPU index, the log of the S&P500 index, the federal funds rate, log employment, and log industrial production. Our baseline VAR specification includes three lags of all variables as in the original specification by Baker, Bloom and Davis (2016).

Panels (a) and (b) in Figures 1, 2, and 3 show the uncertainty series and the time series of its (standardized) shocks for the three proxies at hand. For all proxies we generally see that uncertainty tends to be countercyclical, rising during NBER

[13] Identification is achieved by combining "synthetic external variables" with restrictions based on economic reasoning.

recessions, and that these increases are associated with positive shocks and, thus, unexpected.[14] We also observe some differences, particularly when comparing RV and U^F to EPU. Indeed, we observe a period of high EPU uncertainty toward the end of our sample, that is paired with low (by historical standard) financial volatility (whether proxied by RV or U^F). This phenomenon has been called the "volatility-uncertainty disconnect" and it has been thoroughly investigated by Ait-Sahalia et al. (2020). In our analysis we indeed acknowledge that the three proxies capture potentially different facets of uncertainty. However, our hope is to establish the effect of consecutive shocks as a general phenomenon that is not tided to the specific type of uncertainty, being it political or financial.

Next we focus on Panel (c) and (d). These panels display the episodes with two and three consecutive shocks. As a general remark we observe several episodes with two consecutive shocks (specifically, we obtain 43, 81, and 40 events for RV, U^F, and EPU; these numbers correspond to a frequency of about 10% independently of the proxy considered). We also observe that the second shock can be larger or smaller than the previous one. In our analysis we do not distinguish whether the consecutive positive shocks are increasing or decreasing. Importantly, the discussion of previous episodes provided in section 2.1 matches well with the extracted series of shocks, particularly for EPU. Finally, we also observe occurences of multi-period and large shocks. For example, for RV (as in panel (c) in Figure 1) we see four instances of consecutive shocks where one of the shock is larger than 2 standard deviations (the large shocks occurred in May 2010, August 2015, February 2018, and October 2018). Since we do not want to confound the effect of multi-period shocks with the effect of large ones, we remove from the analysis those events where the previous shock was larger than two standard deviations. For RV this happens in two occasions, which leaves us with 41 events comprising two consecutive positive shocks (and the previous shock is not large).

After the occurrence of two consecutive positive shocks, it is still possible to observe another positive shock which generates a sequence of three consecutive shocks. In other words events with two consecutive shocks are a subset of periods with three

[14]RV and U^F also spike frequently outside of recessions, the most notable being the 1987 stock market crash. Furthermore, U^F is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than RV.

shocks.[15] These three consecutive shocks events are shown in Panel (d). We count 30 events for realized volatility, 49 events for the financial uncertainty index, and 13 events for EPU. Adjusting for the length of the sample period, these numbers correspond to a frequency of 7% for RV and U^F and 3% for EPU. Thus, measures related to financial volatility or uncertainty are more susceptible to longer stream of positive shocks relative to measure of economic policy uncertainty. This is contrary to the similar behavior in terms of frequency for the case of two consecutive shocks.

Overall these figures make clear that consecutive volatility and uncertainty shocks are abundant in the data.

3.2 Response to Serial Uncertainty Shocks

To estimate the response to consecutive shocks, we rely on the smoothed version of Jorda (2005) local projections developed by Barnichon and Brownlees (2018). The Smooth Local Projections (SLP) strikes a balance between the efficiency of Vector Autoregressions (VAR) and the robustness (to model misspecification) of the Local Projections (LP) approach. In practice, SLP consists in estimating LP under the assumption that the impulse response is a smooth function of the forecast horizon. Specifically, we estimate an h-step ahead predictive regressions,

$$y_{t+h} = \alpha_h + \left(\beta_{0,h} + \beta_{1,h} I_{\{\varepsilon_{unc,t-L}>0 \& \ldots \& \varepsilon_{unc,t-1}>0\}}\right) \varepsilon_{unc,t} + \sum_{i=1}^{p} \gamma_{i,h} w_{t-i} + u_{t+h} \tag{1}$$

where h ranges from 0 to H and p is the number of lags used for the control variables, w_t. y_{t+h} is the h period ahead realization of the macroeconomic or financial variable of interest. $I_{\{\varepsilon_{unc,t-L}>0 \& \ldots \& \varepsilon_{unc,t-1}>0\}}$ denotes an indicator that takes value of one if each one of the previous L shocks, $\{\varepsilon_{unc,t-L}, \ldots, \varepsilon_{unc,t-1}\}$, has been positive. This indicator allows us to compute the response of y_{t+h} to a positive shock at t, $\varepsilon_{unc,t}$, conditional on having L previous consecutive positive shocks. Overall, to capture state dependence, the response of y_{t+h} to uncertainty at time t is a function, $\beta_{0,h} + \beta_{1,h} I_{\{\varepsilon_{unc,t-L}>0 \& \ldots \& \varepsilon_{unc,t-1}>0\}}$, of the previous occurrence of a positive shock. In what follows, the $\beta_{1,h}$ coefficient capturing the amplification due to a cascade of shocks

[15] Consider three periods, $t = 1, 2, 3$. If we observe positive shocks at $t = 1$ and $t = 2$, we categorize this as an event with two-period shocks: $[1, 1, \cdot]$. If in the third period we do not observe a positive shock then we have a pure event comprised of two consecutive shocks: $[1, 1, 0]$. If instead we observe a positive shock in the third period, we categorize $[1, 1, 1]$ as a two-period shocks event from the perspective of $t = 2$, and a three-period shocks event from the perspective of $t = 3$.

is called the *state multiplier*. We are interested in knowing whether an uncertainty shock has a larger effect on, e.g., output if the previous shock was positive too. Interestingly, a linear system like an AR(1) delivers $\beta_{1,h} = 0$ since conditioning on the previous realization of shocks does not alter the response function.

Before turning to the empirical analysis, two remarks are in order. First, we impose the constraint that the shock at $\varepsilon_{unc,t-L-1} \leq 0$ so that we effectively consider the response to $L+1$ consecutive, positive shocks (for $L = 1$, the run $\{\varepsilon_{unc,t-2} = 0, \varepsilon_{unc,t-1} = 1, \varepsilon_{unc,t} = 1\}$ is a valid one; $\{\varepsilon_{unc,t-2} = 1, \varepsilon_{unc,t-1} = 1, \varepsilon_{unc,t} = 1\}$ is not). Second, we eliminate a run (i.e., set the indicator to zero) if the run of L positive shocks preceding the one at time t contains a large (≥ 2 standard deviations) shock. Hence our analysis is not contaminated by the effect of rare and large events.[16]

For our empirical application, we employ industrial production, the short-rate, inflation as proxied by the personal consumption expenditures (PCE) chain-weighted price indices, and the Standard & Poor's 500 Stock Price Index as dependent variables.[17] The control vector includes eight lags of the dependent variable, along with eight lags of the short rate to proxy for the monetary policy stance and the state of the economy.

To start, we focus on the case of two consecutive positive shock ($L = 1$ in eq. (1)). The top row in Figure 4 shows the SLP estimates for the case of market realized volatility. Each subplot depicts the impulse response of a specific dependent variable to two consecutive positive shocks ($I_{\{\varepsilon_{unc,t-1}>0\}} = 1$; line with circles), contrasted with the unconditional response (labelled average; dashed line) where we do not condition on previous shocks being positive ($I_{\{\varepsilon_{unc,t-1}>0\}} = 0$). The bottom row shows the estimates of the state multipliers $\beta_{1,h}$ over horizon h (c.f., equation (1)) along with the 90% confidence interval. The state multiplier estimates are negative for all four dependent variables. This means that the negative effect of a realized uncertainty shock is more pronounced conditional on the shock being the second in a sequence of two positive ones.

In line with this observation, the responses of industrial production and the short

[16]In general, we find that our results strengthen if we keep the large shocks in our empirical analysis.

[17]Being a chain-weighted index, the PCE is subject to less substitution bias than the consumer price index (CPI). The inflation data are seasonally adjusted. The macro data are available from the Federal Reserve Economic Database (FRED) and have FRED mnemonics INDPRO, PCECTPI, FEDFUNDS, respectively. The SP500 is available at Yahoo Finance (GSPC ticker).

rate are substantially more negative after a sequence of consecutive shocks has occurred. For example, industrial production has dropped by as much as 2% at an horizon of 18 months relative to 0.5% for the average case.[18] Not only the magnitude of the response is larger, but also the recovery is slower after multi-period shocks have occurred. The response of industrial production to two consecutive shocks is still in a negative territory even after 3 years; on the other hand, the average response would suggest the shock has been completely absorbed with industrial production returning to its average level. Turning to inflation we observe a larger drop on impact after two consecutive shocks. The gap between the (average and conditional) responses closes at about 24 months. Finally, we also see an amplification effect in the response of the stock market following two consecutive shocks; similar to inflation, the conditional and unconditional response difference is large and significant up to 18 months, after which the gap closes as the horizon lengthens.

Figure 5 shows the responses of market realized volatility to a sequence of three consecutive positive shocks ($L = 2$ in equation (1); $I_{\{\varepsilon_{unc,t-1}>0, \varepsilon_{unc,t-2}>0\}} = 1$). In general the pattern of responses and state multipliers is similar to that reported in Figure 4 for the case of two consecutive shocks. Specifically, the state multipliers are negative and the conditional impulse responses of the dependent variables are orders of magnitude larger compared to their average responses. For example, industrial production in subplot (a) decreases by about 2% 18 months after the realization of a third consecutive positive uncertainty shock, whereas the unconditional decline in output due to a positive uncertainty shock is about 0.5% at the same horizon. The most notable difference between two and three consecutive shocks is a more persistent drop in the stock market. Perhaps not surprisingly, this is accompanied by more statistical uncertainty on the state multiplier since the inference is now relying on a lower number of events. In the interest of space we do not report responses to three consecutive shocks for alternative measures.[19]

[Insert Figures 4 and 5 about here]

Next we discuss the response to two consecutive shocks for the financial uncertainty index and EPU, see Figures 6 and 7 respectively. The responses to the financial

[18] For industrial production in subplot (e), the $\beta_{1,h}$ estimate of around −1.5 at 18 months corresponds to the additional decline in output relative to the unconditional response of about −0.5.

[19] The limited number of events - 3% of the total sample size - would prevent in any case the estimation of three-period consecuive shocks for EPU.

uncertainty index are overall quite similar to those reported for market volatility. After two consecutive shocks, the drop in industrial production is quite large at 2% and more persistent than in the unconditional case. Short rate, inflation, and stock market valuation dynamics are also in line with each other across market volatility and financial uncertainty shocks. However, comparing the responses of EPU to those of market volatility and the financial uncertainty index, we observe few differences. First, although we continue to observe a larger drop for industrial production, the short rate, and inflation in the case of consecutive shocks, the gap between conditional and unconditional responses disappears after 3 years. Second, we observe a more muted response for the stock market in the case of EPU.

[**Insert Figures 6 and 7 about here**]

Finally, we study the behavior of variables that are available at quarterly frequency, such as output and investment. The shock at quarterly frequency is obtained as the sum of the shocks occurring within the quarter. The aggregation process reduces greatly the number of events that are available. For example, in the case of EPU, at monthly frequency we have 40 events characterized as two consecutive positive shocks. On the other hand, at quarterly frequency we have only 14 of those. To increase power, we splice EPU with its historical series to obtain a final time series that spans January 1961 to December 2019, which gives rise to 24 events. For similar reasons, we focus on the financial uncertainty index rather than realized volatility, since the former is available over a longer time span and gives rise to more events (furthermore, the responses to RV in Figure 4 and to U^F in Figure 6 were similar). Figure 8 display the results. The two leftmost (rightmost) columns report results for the financial uncertainty index (EPU).

SLP results for output largely confirm the analysis of industrial production: economic activity drops substantially and the recovery is slower than in the unconditional case. For financial uncertainty, we see that output and investment steadily drop until they reach their maximal decline (around 8 quarters) of 1.2% in (a), and 2.5% in (b), respectively. In the case of EPU, the decrease in investment in subplot (d) is more immediate: 1.0% on impact, and the difference between the conditional and unconditional responses shrinking monotonically with the horizon.

[**Insert Figure 8 about here**]

Taking stock of the analysis across different measures of volatility and uncertainty, different frequencies and dependent variables, we conclude that the disparities between the conditional response and the average response in these IRF plots are easy to see. Sequential positive shocks to second moment exacerbate economic declines substantially. To the best of our knowledge, this is the first documentation of the cascading effect of uncertainty shocks in the data.

3.3 Contrasting impacts over some historical episodes

In this section, we compare the SLP approach to the standard linear framework in terms of the overall effects of uncertainty shocks on macroeconomic aggregates. We focus on two episodes that were discussed in detail in section 2.1.

The first episode we focus on is related to the uncertainty shocks that hit in the middle of 1998. As discussed in section 2.1, the Asian and Russian Financial Crisis resulted in a series of shocks to uncertainty that culminated with an intervention by the Federal Reserve. For the approach based on EPU, the maximum effect on industrial production is about -0.5% based on the linear approach that does not condition on previous positive shocks. In contrast, for the SLP approach, the maximum impact of uncertainty on industrial production is close to -2.5%. Likewise, for the approach based on realized variance, the linear approach implies a 1.5% decline in industrial production after 12 months whereas our approach suggests a 3.25% decline over a similar time frame.

Another example includes the series of trade policy uncertainty shocks that unfolded near the end of 2018. For the approach based on EPU, the maximum effect after 12 months is close to 4.5%, whereas the linear setup would suggest about a 1% decline. Likewise, the realized variance approach that does not condition on previous positive shocks to uncertainty suggests about a 2.75% decline in industrial production after 12 months, whereas the SLP approach implies a 4.5% decline over a similar period of time. Summing it up, the real-world examples in this section suggest that successive shocks to uncertainty can have relatively larger impacts that are both statistically and economically significant.

3.4 Response to Large Shocks

We provide evidence that non-linear dynamics in macroeconomic outcomes emerge also as a response to large, positive uncertainty shocks. We highlight that the analysis in this subsection is similar to, and confirms to a large extent, the one reported in Foerster (2014) who documents that large increases in VIX tend to decrease activity by a larger degree than small changes.[20] Differently from Foerster (2014) who uses changes in VIX to proxy for uncertainty shocks, we compute shocks to various proxies for uncertainty as described in Section 3.1.

The empirical model is

$$y_{t+h} = \alpha_h + \left(\beta_{0,h} + \beta_{1,h} I_{\{\varepsilon_{unc,t}>2\}}\right) \varepsilon_{unc,t} + \sum_{i=1}^{p} \gamma_{i,h} w_{t-i} + u_{t+h} \tag{2}$$

where the indicator variable takes the value of one in correspondence of large shocks. The cutoff value determining an increase as large is 2 standard deviation.[21] Thus, the effect of a small uncertainty shock is $\beta_{0,h}$; the effect of increases in uncertainty by more than 2 standard deviation is $\beta_{0,h} + \beta_{1,h}$. If the estimation implies that $\beta_{1,h}$ equals zero, then large increases in uncertainty affect activity in the same manner as small ones, as in linear models. Among the controls w_{t-i} we add the level of the stock market to the list of variables in considered in (1).

Figure 9 displays the effects of large changes in uncertainty on economic activity. Each column refers to an uncertainty proxy. To make the comparison with consecutive shocks easy, the scale of y-axis is the same as in Figure 4, 6, and 7.

Focusing on realized market volatility and the financial uncertainty index (columns 1 and 2) we observe that big shocks induce a larger decrease in economic activity. The state multiplier tends to be large and significant for about six months, after which it returns to zero. In general the response is smaller in magnitude, and less persistent relative to the effect induced by consecutive shocks. This is particularly the case of EPU (column 3).

[20] Foerster (2014) also show that large decreases have no statistically significant effect. Therefore we focus only on large increases.

[21] Foerster (2014) uses 1 standard deviations.

4 The Model

In this section, we present our baseline model. We focus on demand side uncertainty rather than supply side uncertainty. It is well known that supply side (productivity) uncertainty can have a weaker impact on macroeconomic dynamics in general equilibrium models. The reason is that capital becomes a hedge when productivity uncertainty is high. Higher productivity uncertainty means higher expected productivity in the future for any convex function. This fact combined with the hedging motive typically entails an increase in investment upon the realization of a shock to uncertainty. Basu and Bundick (2017) show that demand side uncertainty, on the other hand, can easily generate declines in investment and economic activity when uncertainty is high. For this reason, we focus on a model with demand side uncertainty to illustrate the cascading effect of uncertainty we observe in the data.[22]

4.1 Households

The representative household exhibits Epstein-Zin recursive utility. As in Basu and Bundick (2017), there is a preference shock with stochastic volatility which affects the consumption dynamics. Let X_t denote the preference shock, C_t is aggregate consumption, N_t is aggregate labor supply, and A_t is the permanent component of labor productivity. The value function the representative household is seeking to maximize is:

$$V_t = \left[X_t \left(C_t{}^{\omega} (1 - N_t)^{1-\omega} A_t^{1-\omega} \right)^{1-\psi} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}}, \tag{3}$$

while been subjected to the budget constraint:

$$C_t + Q_t B_t(t+1) + \frac{P_t^E}{P_t} S_{t+1} = W_t N_t + B_{t-1}(t) + \left(\frac{P_t^E}{P_t} + \frac{D_t^E}{P_t} \right) S_t.$$

where β is the time discount factor, γ is relative risk aversion, ω is consumption share in the utility function, and ψ is the inverse of intertemporal elasticity of substitution (IES). $B_t(t+1)$ denotes the quantity of a one-period to maturity zero coupon bond issued at time t and maturing at time $t+1$, and Q_t is the price of the bond at issuance.

[22] To be clear, the amplification that we show for demand side uncertainty also occurs with supply side uncertainty.

S_t is the amount of equity held at time t, and P_t^E is its price while D_t^E is the dividend paid on the asset. W_t is wage income per unit of labor supply. Lastly, P_t is the price level of the final consumption good at time t.

4.2 Intermediate Goods Producers

There is a continuum of intermediate good producers in the economy. Each firm employs labor $N_{t,j}$ from households and rent capital $K_{t-1,j}$ to produce good $Y_{t,j}$. The optimization problem from firm j is:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} M_{t,t+n} \left(\frac{D_{t+n,j}}{P_{t+n}} \right),$$

where $M_{t,t+1}$ is the stochastic discount factor calculated from the representative household's optimization problem in Eq. (3).

Firm j follows a Cobb-Douglas production function of the following form:

$$Y_{t,j} = (A_t u_{t,j} K_{t-1,j})^{\kappa} (Z_t N_{t,j})^{1-\kappa},$$

where κ is the capital share, and $u_{t,j}$ is the capacity utilization rate. A_t and Z_t are permanent and transitory components of productivity, respectively. The main difference from Basu and Bundick (2017) is that we add growth, generated by the presence of A_t, to our model.

The capital accumulation equation with quadratic adjustment cost is standard:

$$K_{t,j} = \left[1 - \delta(u_{t,j}) - \frac{\phi_K}{2} \left(\frac{I_{t,j}}{K_{t-1,j}} - \delta \right)^2 \right] K_{t-1,j} + I_{t,j},$$

in which depreciation as a function of utilization can be expressed as:

$$\delta(u_{t,j}) = \delta + \delta_1 (u_{t,j} - 1) + \left(\frac{\delta_2}{2} \right) (u_{t,j} - 1)^2.$$

The steady state capacity utilization is set to 1.

Finally, firm j's dividend, can be computed as revenue subtract labor cost subtract investment minus price adjustment cost. In real terms:

$$\frac{D_{t,j}}{P_t} = \frac{P_{t,j}}{P_t} Y_{t,j} - \frac{W_t}{P_t} N_{t,j} - I_{t,j} - \frac{\phi_P}{2} \left(\frac{P_{t,j}}{\Pi P_{t-1,j}} - 1 \right)^2 Y_t.$$

Notice $P_{t,j}$ is the price firm j charges per unit of output $Y_{t,j}$, whereas P_t is the prevailing price level in the economy. ϕ_P is the price adjustment cost parameter. Π is the steady state of aggregate inflation in the economy. Y_t is aggregate demand, which is defined in the section below.

4.3 Final Goods Producers

There is a representative final goods producers which aggregates intermediate goods into the final good using a constant elasticity of substitution (CES) technology. The aggregator can be expressed as:

$$Y_t = \left[\int_0^1 Y_{t,j}^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)},$$

where θ is the elasticity across intermediate goods.

Given the profit maximization problem of the final goods producer, we can derive the demand function for intermediate goods produced by firm j, which is standard for New-Keynesian models.

$$Y_{t,j} = \left[\frac{P_{t,j}}{P_t} \right]^{-\theta} Y_t,$$

Assuming the market for final goods is perfectly competitive, this implies the final goods firm earns zero profit in equilibrium. As a result, the aggregate price index is then

$$P_t = \left[\int_0^1 P_{t,j}^{1-\theta} dj \right]^{1/(1-\theta)}.$$

4.4 Monetary Policy

There is a central bank conducting monetary policy according to the Taylor rule. In particular, the central bank adjusts the nominal short term interest rate according to deviations of inflation from its steady state and output growth. Furthermore, there is interest rate smoothing in the Taylor rule such that today's nominal short rate depends on its level in the last period.

The Taylor rule can be written as:

$$r_t = r_{ss} + \rho_r(r_{t-1} - r_{ss}) + (1 - \rho_r)[\rho_\pi(\pi_t - \pi_{ss}) + \rho_y(y_t - y_{t-1})],$$

where $r_t = \log(R_t)$, $\pi_t = \log(\Pi_t)$, and $y_t = log(Y_t)$. Subscript ss denotes the steady state of a given variable. R_t is the one-period risk-free interest rate. Π_t is inflation defined as $\frac{P_t}{P_{t-1}}$. In terms of parameters, ρ_r is the degree of interest rate smoothing, ρ_π is the Taylor coefficient, and ρ_y governs the central bank's response to output growth.

4.5 Equilibrium Characteristics

The model admits a symmetric equilibrium. This means the intermediate goods producers can be thought of as a representative firm since they make the same choices on inputs and prices. Therefore, $N_{t,j} = N_t$, $K_{t,j} = K_t$, $u_{t,j} = u_t$, and $P_{t,j} = P_t$.

4.6 Defining the Shocks

There are four exogenous processes driving the model: preference (X_t), transitory technology (Z_t), permanent technology (A_t), and cost-push shocks. Transitory and permanent technology, respectively, evolve according to the processes below:

$$\begin{aligned} z_{t+1} &= \rho_z z_t + e^{\sigma_{z,t+1}} \varepsilon_{z,t+1} \\ \sigma_{z,t+1} &= (1 - \rho_{\sigma_z})\theta_{\sigma_z} + \rho_{\sigma_z}\sigma_{z,t} + \sigma_{\sigma_z}\varepsilon_{\sigma_z,t+1}, \end{aligned}$$

and

$$\Delta a_{t+1} = (1 - \rho_{\Delta a})\theta_{\Delta a} + \rho_{\Delta a}\Delta a_t + e^{\sigma_{\Delta a}}\varepsilon_{\Delta a,t+1}$$

where $z_t = \log(Z_t)$ and $\Delta a_t = \log(A_t/A_{t-1})$. σ_z is the conditional volatility of the transitory technology processes. Notice the conditional volatility enters the state variables through an exponential function to ensure they are always positive.

Similarly, the demand shock, X_t, is driven by a first order autoregressive process with stochastic volatility:

$$
\begin{aligned}
X_{t+1} &= (1-\rho_X)\theta_X + \rho_X X_t + e^{\sigma_{X,t+1}}\varepsilon_{X,t+1} \\
\sigma_{X,t+1} &= (1-\rho_{\sigma_X})\theta_{\sigma_X} + \rho_{\sigma_X}\sigma_{X,t} + \sigma_{\sigma_X}\varepsilon_{\sigma_X,t+1}.
\end{aligned}
$$

The cost-push shock enters as a shock to the price-markup for intermediate goods producers:

$$
log(\theta_t^*) = log(\theta) + \sigma_\theta \epsilon_t^\theta
$$

where θ represents the elasticity across goods parameter and $\sigma_\theta = 0.15$. Price markup shocks tend to be an important source of fluctuations for the variance decomposition with respect to inflation, as demonstrated by Smets and Wouters (2007). Indeed, the inclusion of this shock modestly increases the inflation volatility for the purposes of our calibration. Importantly, however, cost-push shocks play no role with respect to the amplifying impact of successive shocks to uncertainty.

Next, we discuss the solution method and calibration of the baseline model.

5 Solution Method and Impulse Response Functions

5.1 Solution with Simplified Model

We make use of higher-order perturbation techniques to solve the model.[23] In particular, we solve the model to fourth order to illustrate the cascading effect. Here, we use a simple model to demonstrate why fourth order perturbation matters in the context of serial uncertainty shocks. Suppose the representative agent has CRRA utilities. Also, under market clearing conditions there is one unit of stock outstanding and consumption (C_t) equals the dividend payout of the stock; thus, we can write down

[23] Caldara, Fernández-Villaverde, Rubio-Ramírez and Yao (2012) show that perturbation methods for DSGE models with stochastic volatility and recursive preferences are comparable, in terms of accuracy, to global solution methods such as Chebyshev polynomials and value function iteration, while being computationally more efficient.

an equilibrium model of the price-consumption ratio with the following equations:

$$\begin{aligned} M_{t,t+1} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \\ \frac{P_t}{C_t} &= \mathbb{E}_t\left[M_{t,t+1}\left(\frac{P_{t+1}+C_{t+1}}{C_t}\right)\right], \\ z_t &= v(x_{t-1})\epsilon_{z,t}, \\ x_t &= (1-\rho_x)x+\rho_x x_{t-1}+\omega\epsilon_{x,t}, \end{aligned}$$

where $M_{t,t+1}$ is the stochastic discount factor, $\frac{P_t}{C_t}$ is the price-consumption ratio or Y_t, z_t is log consumption growth ($\log\left(\frac{C_t}{C_{t-1}}\right)$), and x_t is stocastic volatility of the log consumption growth process. ω is the volatility-of-volatility of the log consumption growth process. For ease of exposition, we assume consumption growth is zero in steady state and there is no persistence.

After rearranging terms and expressing all variables in logs (small caps.), the Euler equation and the price-consumption ratio can be rewritten as:

$$\begin{aligned} m_{t,t+1} &= log(\beta)-\gamma z_{t+1}, \\ e^{y_t} &= \mathbb{E}_t\left[e^{m_{t,t+1}}\left(e^{y_{t+1}+z_{t+1}}+e^{z_{t+1}}\right)\right]. \end{aligned}$$

The equilibrium solution to the optimization problem is of the following form:

$$y_t=\mathbb{E}_t f(z_{t+1}),$$

and after substitutions:

$$y_t=\mathbb{E}_t f(v((1-\rho_x)x+\rho_x x_{t-1}+\omega\Lambda\epsilon_{x,t})\Lambda\epsilon_{z,t+1}),$$

where functions $f(\bullet)$ and $v(\bullet)$ are continuous and at least four times differentiable. Λ is the perturbation parameter, which is evaluated at 0 in the steady state following the Taylor expansion of the policy function f. In particular, (x_{t-1},Λ) are evaluated at $(x,0)$ respectively.

To find the terms in the Taylor series, we start by taking derivatives of y_t with respect to the state variables. For ease of exposition, we will only focus on the third and fourth order terms in the main text. The full derivation with respect to all terms can be found in the Appendix A. For stochastic volatility to matter in the policy function, the coefficient loading associated with x_{t-1} has to be non-zero. The only term that survives at the third order is:

$$\left.\frac{\partial^3 y_t}{\partial \Lambda^2 \partial x_{t-1}}\right|_{0,x,0} = \mathbb{E}_t[2f''(0)v(x)v'(x)\rho_x\epsilon_z^2 + 2f'(0)v''(x)\rho_x\omega\epsilon_x\epsilon_z],$$
$$= 2f''(0)v(x)v'(x)\rho_x,$$

where derivatives of $f(\bullet)$ are denoted by $f'(\bullet)$ and the second equality uses the fact that $\mathbb{E}_t[\epsilon_z^2] = 1$, $\mathbb{E}_t[\epsilon_x] = 0$, $\mathbb{E}_t[\epsilon_z] = 0$, as well as $corr(\epsilon_x, \epsilon_z) = 0$. Interestingly, note that the above term loads linearly on $(x_{t-1} - x)$. Furthermore, $\frac{\partial^3 y_t}{\partial \Lambda \partial x_{t-1}^2}$ turns out to be zero. This is a well known result from Fernández-Villaverde and Rubio-Ramírez (2010).

At the fourth order, in addition to the term above, we have another non-zero term:

$$\left.\frac{\partial^4 y_t}{\partial \Lambda^2 \partial x_{t-1}^2}\right|_{0,x,0} = \mathbb{E}_t[2f''(0)v'(x)^2\rho_x^2\epsilon_z^2 + 2f''(0)v(x)v''(x)\rho_x^2\epsilon_z^2],$$
$$= 2f''(0)\rho_x^2[v'(x)^2 + v(x)v''(x)].$$

This is the only fourth-order term associated with stochastic volatility that survives as $\frac{\partial^4 y_t}{\partial \Lambda \partial x_{t-1}^3}$ and $\frac{\partial^4 y_t}{\partial \Lambda^3 \partial x_{t-1}}$ are proven to be zeros when evaluated at the steady state $(0, x, 0)$. It is therefore the crucial term that helps explain why the fourth order approximation has a non-linear relationship with x_t. To see this allow us to write y_t in its fourth order perturbation form below

$$y_t = y_{ss} + \overbrace{\frac{\partial^4 y_t}{\partial \Lambda^4}}^{\text{risk correction}} + \overbrace{\frac{\partial^3 y_t}{\partial \Lambda^2 \partial x_{t-1}} \cdot (x_{t-1} - x)}^{\text{linear term}} + \overbrace{\frac{\partial^4 y_t}{\partial \Lambda^2 \partial x_{t-1}^2} \cdot (x_{t-1} - x)(x_{t-1} - x)}^{\text{quadratic term}}$$

where the first term is the risk correction (discussed more below), the second term is linearly related to x_{t-1}, and the latter term loads on quadratic variation in x_{t-1}. The latter term explains why the fourth order approximation has amplification with respect to uncertainty shocks. The third order approximation only loads on the second term and therefore has a purely linear relationship with respect to x_{t-1} i.e. no state-dependence or amplification with respect to uncertainty.

To the best of our knowledge, this study is the first to identify this fourth-order term and show its importance with respect to successive shocks to uncertainty. Note that the volatility-of-volatility, ω, enters via the x_{t-1} as defined above. The larger the ω, the more x_{t-1} deviates from its steady state value and the greater the non-linearities with respect to uncertainty shocks.

Moreover, as shown by de Groot (2019), the volatility-of-volatility ω term also shows up in the risk-adjustment term (associated with Λ) at the fourth order in the Taylor expansion.

$$\begin{aligned}\left.\frac{\partial^4 y_t}{\partial \Lambda^4}\right|_{0,x,0} &= \mathbb{E}_t[f''''(0)v(x)^4\epsilon_z^4 + 12f''(0)v'(x)^2\omega^2\epsilon_x^2\epsilon_z^2 + 12f''(0)v(x)v''(x)\omega^2\epsilon_x^2\epsilon_z^2], \\ &= f''''(0)v(x)^4 + 12\omega^2[f''(0)v'(x)^2 + f''(0)v(x)v''(x)],\end{aligned}$$

whereas $\frac{\partial^3 y_t}{\partial \Lambda^3} = 0$ when evaluated at the steady state. The presence of this term at the fourth order can potentially generate additional non-linearities in the perturbation solution, especially with respect to uncertainty shocks. This will typically be the case when there are endogenous state variables such as capital, as is the case with our baseline model. Effectively, the model is solved further away from the non-stochastic steady state when fourth order perturbation is applied due to the distortion provided by this term containing ω. This will act to further amplify non-linearities with respect to uncertainty shocks because the quadratic term $(k_t - k_{ss})(k_t - k_{ss})$ will be further from zero thanks to the risk correction.

5.2 Impulse Response Functions

To compute the dynamic response of the model to uncertainty shocks, we follow Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011) and define the Impulse Response Functions (IRFs) at time $t+n$ after a shock u_t as

$$\begin{aligned}IRF_n(u_t, \Omega_{t-1}) = E\left[Y_{t+n} \mid u_t, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+1}^{fut} = \{0, \ldots\}\right] \\ - E\left[Y_{t+n} \mid 0, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+1}^{fut} = \{0, \ldots\}\right],\end{aligned} \tag{4}$$

where Ω_{t-1} is the past history of shocks, and Ω_{t+1}^{fut} denote the future realization of shocks. In words, we condition on future shocks by setting them to 0 when generating their IRFs and start the IRFs at the ergodic mean in the absence of shocks (EMAS).[24]

[24] An alternative is to compute the Generalized Impulse Response Functions (GIRFs) at the ergodic mean as suggested by Koop, Pesaran and Potter (1996). For robustness, we have also constructed responses based on this procedure and our results do not change.

Our interest lies in studying the responses of the economic model after sequences of shocks. To this end, we also compute the response of the economy after two and three consecutive *positive* shocks, namely

$$\begin{aligned}
IRF_n(u_{t+1}, u_t, \Omega_{t-1}) &= E\left[Y_{t+n} \mid u_{t+1}, u_t, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+2}^{fut} = \{0, \ldots\}\right] \\
&\quad - E\left[Y_{t+n} \mid 0, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+1}^{fut} = \{0, \ldots\}\right] , \\
IRF_n(u_{t+2}, u_{t+1}, u_t, \Omega_{t-1}) &= E\left[Y_{t+n} \mid u_{t+2}, u_{t+1}, u_t, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+3}^{fut} = \{0, \ldots\}\right] \\
&\quad - E\left[Y_{t+n} \mid 0, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{t+1}^{fut} = \{0, \ldots\}\right] ,
\end{aligned}$$

Our approach compares the difference in paths depending on how many previous uncertainty shocks there are. Practically, we compute and display

- $IRF_n(u_t, \Omega_{t-1})$,
- $IRF_n(u_{t+1}, u_t, \Omega_{t-1}) - IRF_n(u_t, \Omega_{t-1})$ (the incremental contribution of two consecutive positive shocks relative to one shock),
- $IRF_n(u_{t+2}, u_{t+1}, u_t, \Omega_{t-1}) - IRF_n(u_{t+1}, u_t, \Omega_{t-1})$ (the incremental contribution of three consecutive positive shocks relative to two shocks).

Appendix B presents the pseudo-code used to construct these responses to serial shocks. Under linearity, one should observe (delayed) replica of the response to a single-period shock (history independence).

Finally we study the response to large shocks. In this case we compute $IRF_n(1 \times u_t, \Omega_{t-1})$, $IRF_n(2 \times u_t, \Omega_{t-1})$, and $IRF_n(3 \times u_t, \Omega_{t-1})$. We then compare these paths by dividing by the magnitude of the shock e.g. $IRF_n(1 \times u_t, \Omega_{t-1})$ vs $\frac{IRF_n(2 \times u_t, \Omega_{t-1})}{2}$ vs $\frac{IRF_n(3 \times u_t, \Omega_{t-1})}{3}$. If the responses are linear, the paths will perfectly overlap (shape invariance).

5.3 Model Calibration

We calibrate the model to capture key quarterly U.S. macroeconomic moments over the past 50 years. The key moments are associated with output, consumption, investment, wages, and labor. For the preference parameters, we set the effective risk aversion to 10, which is often cited as the upper bound for levels of risk aversion. We set the intertemporal elasticity of substitution close to but less than one at 0.995, consistent with the findings in the meta-study of Havránek (2015). The subjective discount factor is set to match a real-risk free rate of around 3%. The consumption share parameter is set so the agent is working a third of the time. The depreciation rate and capital utilization rates are set to match the values from Christiano et al. (2005). We choose the price adjustment cost parameter to be consistent with prices resetting on average about every three quarters and the price elasticity parameter across intermediate goods is set to 13. The leverage parameter is set to ensure a volatile equity return, as in the data. The monetary policy parameters are relatively standard with weights of 1.5 on inflation, 0.5 on output growth, and 0.5 on the inertia coefficient.

[**Insert Table 1 about here**]

The remaining parameters are used to match the moments of interest. The capital adjustment cost parameter helps pin down the volatility of investment. The persistence of the transitory technology shock and preference shock are set to ensure autocorrelations as in the data while also being consistent with the correlations with respect to output. Both the preference shock parameters and technology shock parameters are set to be consistent with a one-standard deviation increase in uncertainty being associated with a 10-20 basis point initial decline in output, as in the data. These moments are key to pin down for our simulations as we determine the magnitude of responses to consecutive and different-sized shocks.

We also note that the fourth order approximation requires a relatively modest effective risk aversion of about 10 in order to generate an equity premium of around 4.5%. An identical parameterization solved at the third order approximation generates an equity risk premium that is closer to 1%. This difference in the risk premium is consistent with findings from de Groot (2019). For the purposes of our comparisons that follow, we use identical parameterizations to contrast the third and fourth order approximated impulse response functions. However, in untabulated results, we find that re-calibrating the third order approximation to be consistent with the data moments does not in any way change our conclusions that the third order approximation yields no amplification or nonlinearities with respect to uncertainty shocks.

[**Insert Table 2 about here**]

6 Analysis of Uncertainty Shocks in General Equilibrium

In this section, we study the impact of serial uncertainty shocks as well as large uncertainty shocks in the benchmark model. To be specific, we define uncertainty shock in the model as the innovation $\varepsilon_{\sigma_X,t}$, which perturbs the conditional volatility of household preference (demand). As shown by Basu and Bundick (2017), demand uncertain generates large and effective drops in economic aggregates under price stickiness.

6.1 Cascade of Uncertainty Shocks

Figure 10 displays the responses of output, investment, hours worked, inflation, and the short rate following 1, 2 and 3 *consecutive* one standard deviation *positive* shocks to uncertainty in the model. We employ the fourth order perturbation method to solve the model and construct the IRFs discussed in Section 5. Subplot 10(f) displays the sequence of uncertainty shocks that drive model dynamics. Higher uncertainty leads to declines in macroeconomic variables across the board, consistent with what one would expect in the literature. More importantly, we find that consecutive positive shocks to uncertainty exacerbates the drop in macroeconomic variables as evidenced by the difference between the solid-blue lines and the dashed-green lines in Figure 10. Using output in subplot 10(a) for example, a single one standard deviation positive shock to uncertainty lowers output by roughly 0.15% (solid-blue). On the other hand, the third one standard deviation positive shock to uncertainty following two previous consecutive shocks lowers output, in addition to the existing decline, by more than 0.3% (dashed-green).

[Insert Figures 10 and 11 about here]

Recall that the IRFs in Figure 10 are calculated following the pseudo-code in Appendix B in order to demonstrate the *incremental* impact of consecutive uncertainty shocks to the macroeconomy. What we observe in the model under fourth order perturbation is that consecutive positive uncertainty shocks build on top of each other and amplify the endogenous responses of the economy. Relatively speaking, the second consecutive positive uncertainty shock drives the economic decline to a greater extent than the first positive uncertainty shock, and the third consecutive positive shock has more impact than the second one, etc.

The implication produced by the model reflects what we see in the data. Recall that in Figure 8 (quarterly frequency, consistent with the model), we show that the second positive shock to uncertainty following an initial positive shock causes output, investment, and inflation to fall dramatically. Relative to the decline due to the initial positive shock, the magnitude of the maximal drop is typically greater by a factor of at least 2. Figure 10 shows the model solved by fourth order perturbation can capture the power of consecutive positive uncertainty shocks by comparing the blue line (first shock) and the red dotted line (second shock). On the other hand, the model seems to fall short on picking up the prolonged response of endogenous variables to the second or third consecutive positive shock as it is in the data. Again, in Figure 8, we find that it takes anywhere around six to fifteen quarters for output and investment to reach the maximal decline due to a second consecutive shock. In the model, however, the maximal decline is reached as soon as the positive shock is realized. Furthermore, the magnitude of declines in the model fall short relative to declines documented in the data. This is a potential point for future investigation.

Figure 11 presents the same impulse responses of the endogenous variables when the model is solved under third order perturbation. Comparing it to Figure 10 it is clear that no amplification effect is at work when the model is solved at third order. The impact of each of the shocks in a sequence of consecutive positive shocks is exactly the same: there is no path-dependency. As shown in our analytical derivation in Section 5.1 for the simplified model, households exhibit a state dependent precautionary motive with respect to uncertainty at the fourth order approximation thanks to the nonzero loading on the quadratic stochastic volatility term in the perturbation solution. Furthermore, the coefficient loading on this interaction at the fourth order is nontrivial such that it generates a noticeably larger response in the endogenous variables. The implication of this finding is that models solved with perturbation under first to third order can potentially understate the impact of sequential positive uncertainty shocks, which we have shown are fairly common in the data.

Intuitively, the greater precautionary motive that generates this amplification is hidden by the third order approximation, just as movements in risk premiums are hidden by the second order approximation. The economic implications are straightforward: Agents respond to additional increases in uncertainty with even larger declines in demand, which translates into lower output along with each of its components in the demand-driven New Keynesian model.

6.2 Large Shocks

The final theoretical finding we document is on the effect of large uncertainty shocks. Specifically, we examine the impulse responses of macroeconomic aggregates following 1 and 2 standard deviations positive shocks to uncertainty. Typically, in DSGE models featuring stochastic volatility, third order perturbation is a common solution method which results in linearly scalable impulse responses to volatility shocks. This implies, for example, the model-implied decline in the variables of interest (e.g, output, investment, etc.) following a two standard deviation positive uncertainty shocks is simply twice the decline following a one standard deviation positive shock. We show here that this linear scalability of model-implied impulse responses disappears under fourth order perturbation.

Figure 12 displays the responses of output, investment, hours worked, inflation, and the (nominal) short rate after a single positive 1 standard deviation volatility shock and a single (not serial) positive 2 standard deviation shock (bottom right panel) when the model is solved at fourth order. We then rescale the 2 standard deviation responses by dividing them by 2 before plotting for comparison purposes. The difference between the baseline case of a small shock (solid blue line) and the case of a rescaled large shock (solid red line) is large. Doubling the size of the positive uncertainty shock actually more than doubles the reaction within the model. For instance, output growth in subplot 12(a) decreases by about 0.1% following a single 1 standard deviation shock, which is less than half of the drop in output growth observed when a 2 standard deviation shock is applied. At the same time, the rebound in output growth two periods after the realization of the large positive shock is also more robust relative to the small shock scenario in subplot (a). This phenomenon holds in all endogenous variables we examine.

[Insert Figures 12 and 13 about here]

Similar to the main result discussed in Section 6.1 for serial positive uncertainty shocks, we see that large shocks intensify the macroeconomic response in a nonlinear fashion when fourth order perturbation is employed to solve the model. Not surprisingly, the scalability reverts back to linear when third order perturbation is used. Figure 13 plots the impulse responses under that scenario, and the rescaled IRFs following a 2 standard deviation positive shock align exactly with those in the baseline case.

6.3 SLP with Simulated Data

Figure 14 displays impulse response functions from model simulated data. Specifically, we (1) solve the model to the fourth-order; (2) simulate 200 observations of output, investment, consumption, short-term rate, inflation and stock market prices (this is a length comparable to our quarterly sample in Figure 8); (3) run the regression specified in equation (1); (4) repeat this experiment 1000 times and report median and 90% confidence intervals. Figure 14 shows that our regression model is able to pick up the non-linearity in the model-simulated data. Indeed, we see that, in response to the second consecutive positive uncertainty shocks (circle-dash lines), output, investment, consumption, and the short-rate drop more relative to the unconditional case (dash lines). On impact, the decline in output, investment, and consumption roughly doubles for the second consecutive positive shock relative to the average positive shock. The difference in impulse responses is large up to 4 quarters after the shock hits. This is consistent with subplots (d)-(f) and (j), which show state multipliers that are negative and significant up to the four-quarter horizon (the sole exception being investment). For inflation, the drop is small on impact; however the effect of consecutive shocks builds up with time, with the two responses that differ by more than four times at about four quarters out. Interestingly, the responses depicted in Figure 14 align quite well with the theoretical ones obtained from the model. We view this as evidence that our simple procedure based on local projection is able to pick up the non-linearities induced by multi-period shocks.

[Insert Figure 14 about here]

Although a model solved to the fourth order generates non-linear effects in response to consecutive shocks, we guard against the fact that to proper mirror the dynamics in the data a richer model economy is required. E.g., in the data, the difference between conditional and unconditional response is generally the largest at about four quarters – see the response in figure 8(a) and 8(b). In the model, the difference is instead the largest on impact. Also, the recovery in the data is often slower than what observed in the model. This paper has documented that multi-period shocks matter in the data and that a perturbation solution of at least fourth order is required to study the cascading effect of shocks quantitatively. Future research should investigate which features in the model can help producing a better alignment between empirical and theoretical IRFs, by generating responses to consecutive shocks that build over time and slowly recover as in the data.

6.4 Alternative Parameterizations and Shocks

In this section, we briefly note that the amplification discussed above is not contingent on a specific parameterization. Any parameterization that generates meaningful economic responses to uncertainty at the third order will also have amplifying effects at the fourth order. For instance, reducing the risk aversion to become the inverse of the elasticity of intertemporal (IES) substitution (or reducing the IES to be the inverse of the risk aversion parameter) will not only kill the amplification that we observe at fourth order, but also kill the responses to uncertainty in general (results not shown). Likewise, in other results not shown, moving to a flexible price framework by reducing price adjustment costs to zero continues to exhibit amplification and non-linearities at the fourth order, but investment moves in the wrong direction compared to the data.

Upon careful checking of each of the parameters in the model, we were unable to find a parameter space that keeps the responses to uncertainty at the third order approximation but removes the amplification effects at the fourth order. This suggests to us that our findings for amplification are not necessarily model-specific or parameter-specific, but instead represent a general implication of not solving the model at a higher order of approximation.[25] At a more fundamental level, agents in the standard framework have always had a state dependent precautionary motive with respect to uncertainty, but the existing solution methods were hiding this fact.

In addition, the same patterns can be found for the shocks to uncertainty with respect to the transitory productivity process. The main difference is that investment moves in the wrong direction compared to what we observe in the data i.e. investment goes up. This can be seen in Appendix Figure C.1, which shows the fourth order responses to successive shocks to uncertainty and compares them to the responses at the third order approximation. Likewise, the non-linearity with respect to the size of the shocks also goes through, as shown in Appendix Figure C.2.

[25] We've also confirmed that our results do not change when moving to the fifth order approximation.

7 Conclusion

We document the existence and impact of serial uncertainty shocks in the post-war data. Given that positive uncertainty shocks lead to a drop in economic activity, we find that consecutive positive uncertainty shocks greatly exacerbates the decline. In particular, we show that the second uncertainty shock in a series of positive shocks generates an output decline that is at least twice as large and three times longer, compared with the decline caused by the initial positive shock. Similar outcomes are observed for investment, inflation, and stock market valuation.

Using a standard DSGE model, we investigate the impact of serial uncertainty shocks to macroeconomic aggregates. We find that consecutive positive shocks to demand uncertainty generates steep declines in endogenous economic variables under fourth order perturbation. The cascading effect of serial shocks is such that the second positive shock is more impactful than the first one, and the third positive shock is more impactful than the second one, and so on. Moreover, we also find large positive uncertainty shocks intensify the economic declines in a non-linear fashion when fourth order perturbation is employed.

Our findings are generally robust to the source of uncertainty: they are preserved when uncertainty is switched from demand shocks to transitory productivity shocks. Finally, fourth order perturbation is crucial to our results as the findings outlined above disappear when third order perturbation is used.

Overall, our results suggest that the existing paradigm is understating the true effects of uncertainty. Uncertainty shocks that happen in succession may have a much larger impact. Our theoretical models, when appropriately solved, concur with this empirical evidence. With the ability to now solve with the fourth order approximation in the newest version of Dynare, we view the exploration of the non-linear and state-dependent effects of uncertainty as an area of great interest for future research.

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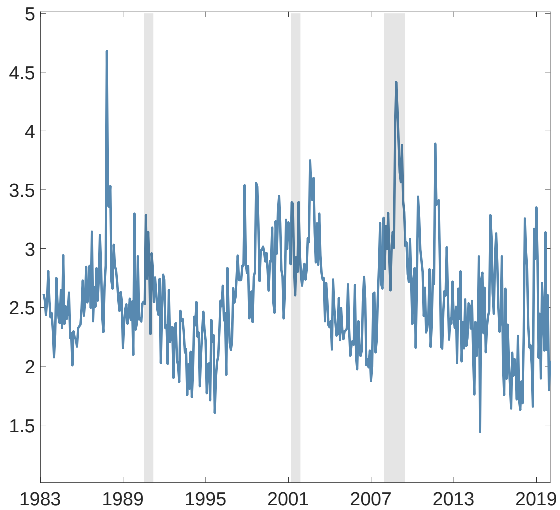
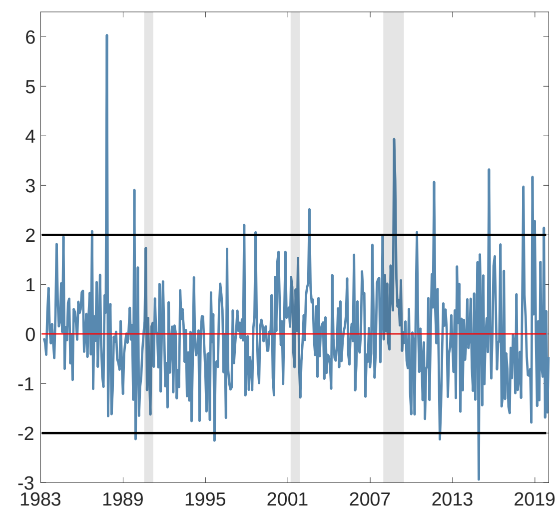
Figures

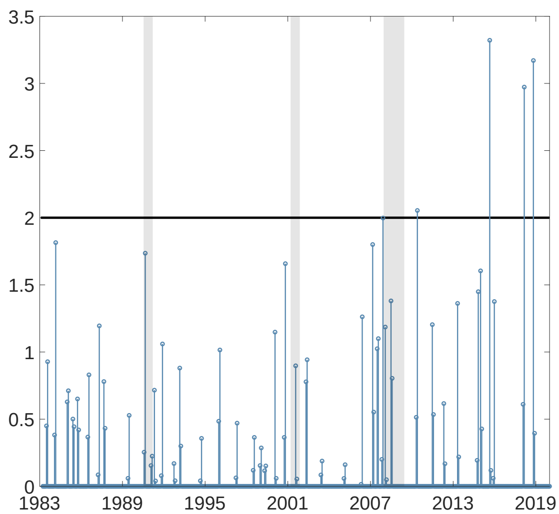
Stock market volatility and estimated uncertainty shocks.



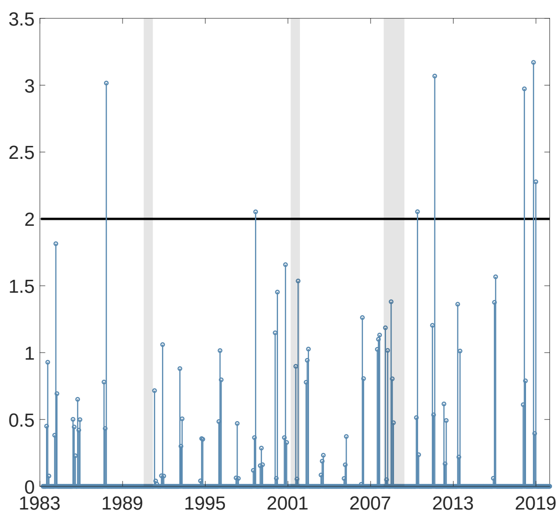
(a) Log market volatility.



(b) Shock to market volatility.



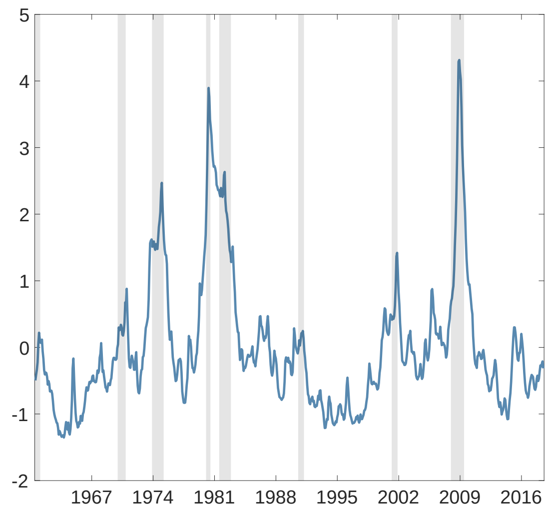
(c) Two Consecutive Shocks.



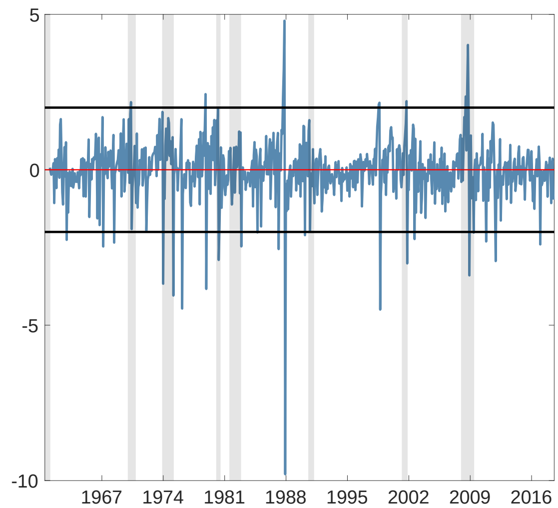
(d) Three Consecutive Shocks.

Figure 1: Panel (a) plots the time series of the log of realized volatility (computed as the square root of market realized variance). Panel (b) shows the time series of shocks (in standardized units). The shocks are obtained from a regression of future realized volatility on current realized and option-implied volatility as in Berger, Dew-Becker and Giglio (2019). Panels (c) and (d) display the time series of, respectively, two and three consecutive shocks.

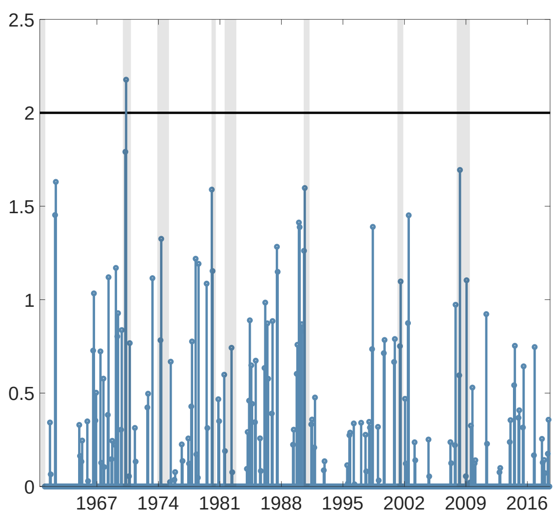
Financial uncertainty and estimated uncertainty shocks.



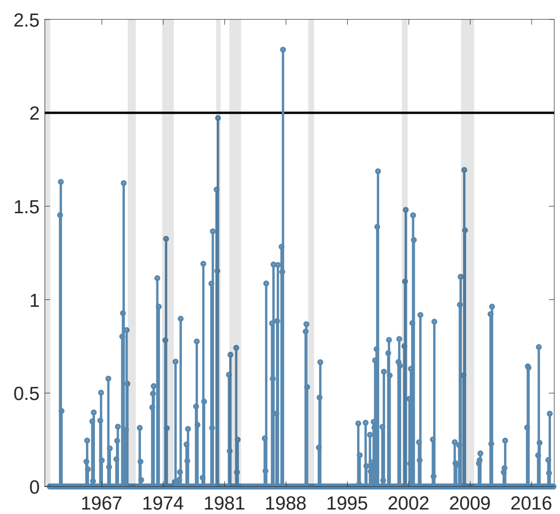
(a) Aggregate Financial Uncertainty.



(b) Shock to Financial Uncertainty.



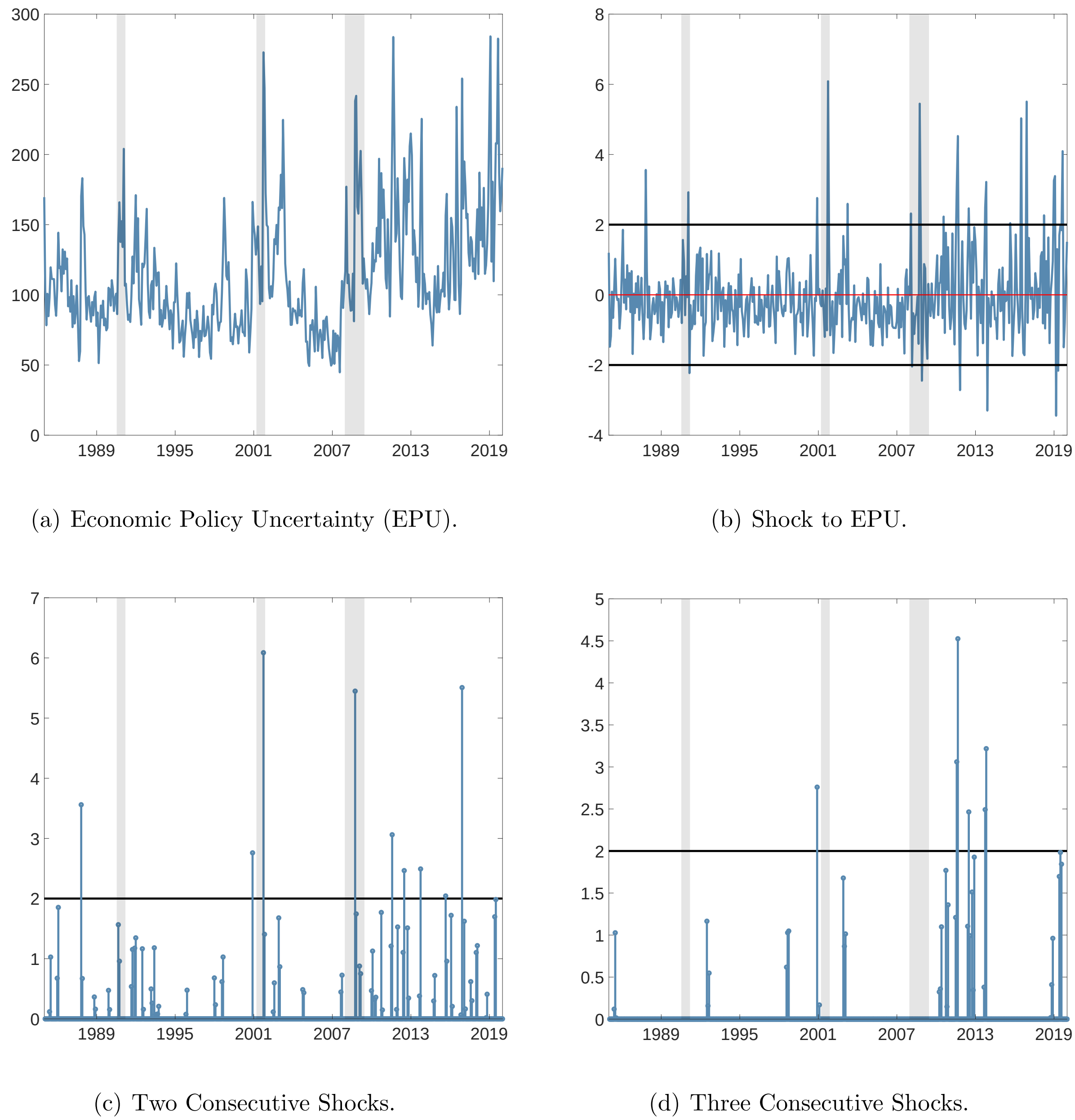
(c) Two Consecutive Shocks.



(d) Three Consecutive Shocks.

Figure 2: Panel (a) plots the time series of financial uncertainty (expressed in standardized units) from Ludvigson, Ma and Ng (2015). Panel (b) shows the time series of shocks from a SVAR System with three variables, namely industrial production, macro and financial uncertainties using the identification scheme described in Ludvigson, Ma and Ng (2015). Panels (c) and (d) display the time series of, respectively, two and four consecutive shocks.

Economic policy uncertainty and estimated uncertainty shocks.



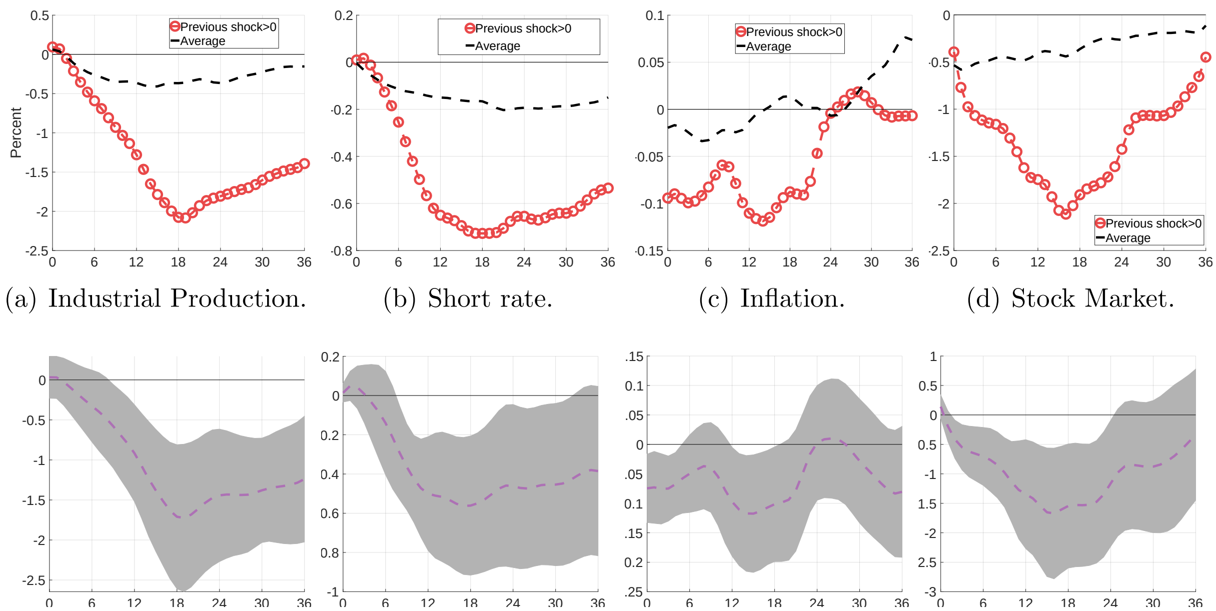
(a) Economic Policy Uncertainty (EPU).

(b) Shock to EPU.

(c) Two Consecutive Shocks.

(d) Three Consecutive Shocks.

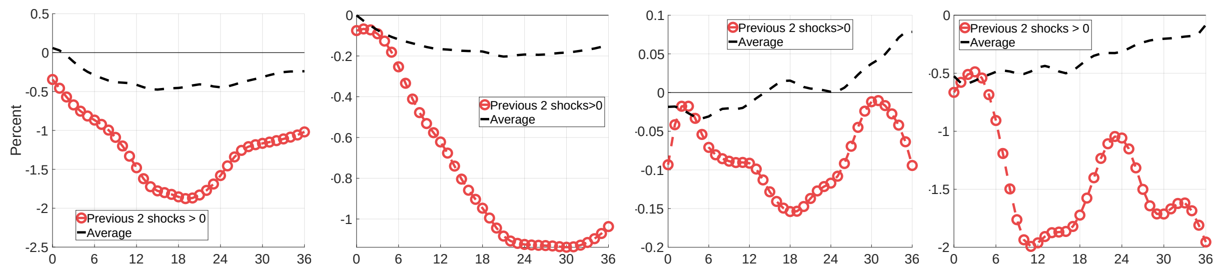
Figure 3: Panel (a) plots the time series of the economic policy uncertainty (EPU) from Baker, Bloom and Davis (2016). Panel (b) shows the time series of shocks (in standardized units) from an AR(1) fitted on the EPU series. Panels (c) and (d) display the time series of, respectively, two and three consecutive shocks.



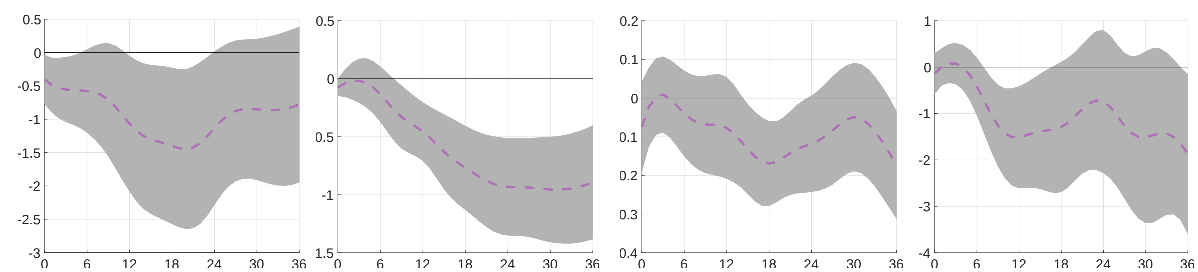
(a) Industrial Production. (b) Short rate. (c) Inflation. (d) Stock Market.

(e) State Multiplier. (f) State Multiplier. (g) State Multiplier. (h) State Multiplier.

FIGURE 4: **Impulse Response Function to Two Consecutive Positive Uncertainty Shocks**: First row plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for two consecutive positive shocks for output, short rate, inflation and the stock market. Second row plots the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty shocks using log market realized variance from Berger, Dew-Becker and Giglio (2019). We have 43 runs with two consecutive positive shocks, and we eliminate two runs due to large shocks.

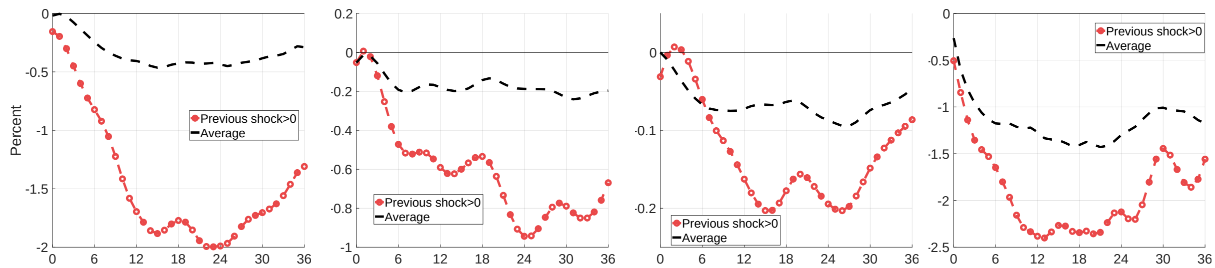


(a) Industrial Production. (b) Short rate. (c) Inflation. (d) Stock Market.

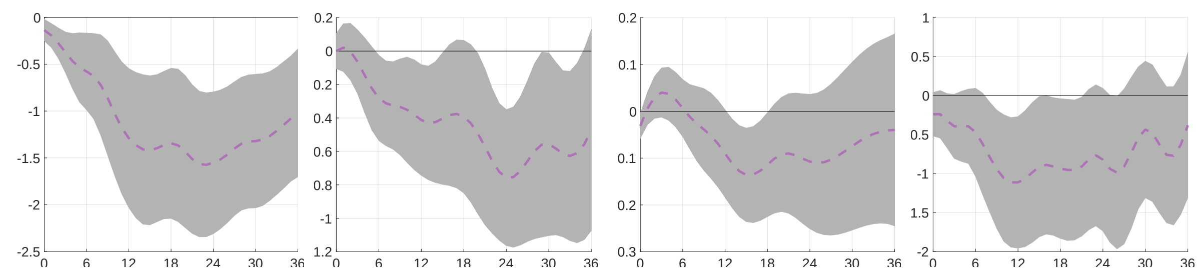


(e) State Multiplier. (f) State Multiplier. (g) State Multiplier. (h) State Multiplier.

Figure 5: **Impulse Response Function to Three Consecutive Positive Uncertainty Shocks**: First row plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for two consecutive positive shocks for output, short rate, inflation and the stock market. Second row plots the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty shocks using log market realized variance from Berger, Dew-Becker and Giglio (2019). We have 30 runs with three consecutive positive shocks, and we eliminate four runs due to large shocks.



(a) Industrial Production. (b) Short rate. (c) Inflation. (d) Stock Market.



(e) State Multiplier. (f) State Multiplier. (g) State Multiplier. (h) State Multiplier.

FIGURE 6: **Impulse Response Function to Two Consecutive Positive Uncertainty Shocks**: First row plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for two consecutive positive shocks for output, investment, inflation and the stock market. Second row plots the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty shocks as in Ludvigson, Ma and Ng (2015). We have 81 runs with two consecutive positive shocks, and we eliminate one run due to large shocks.

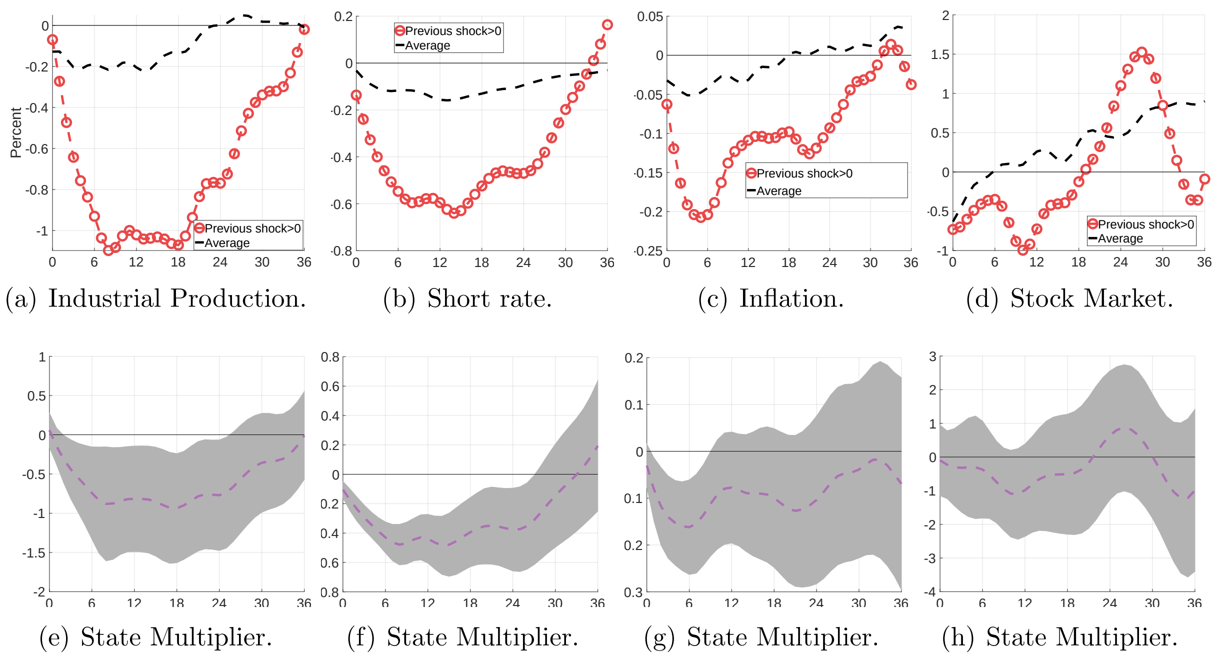
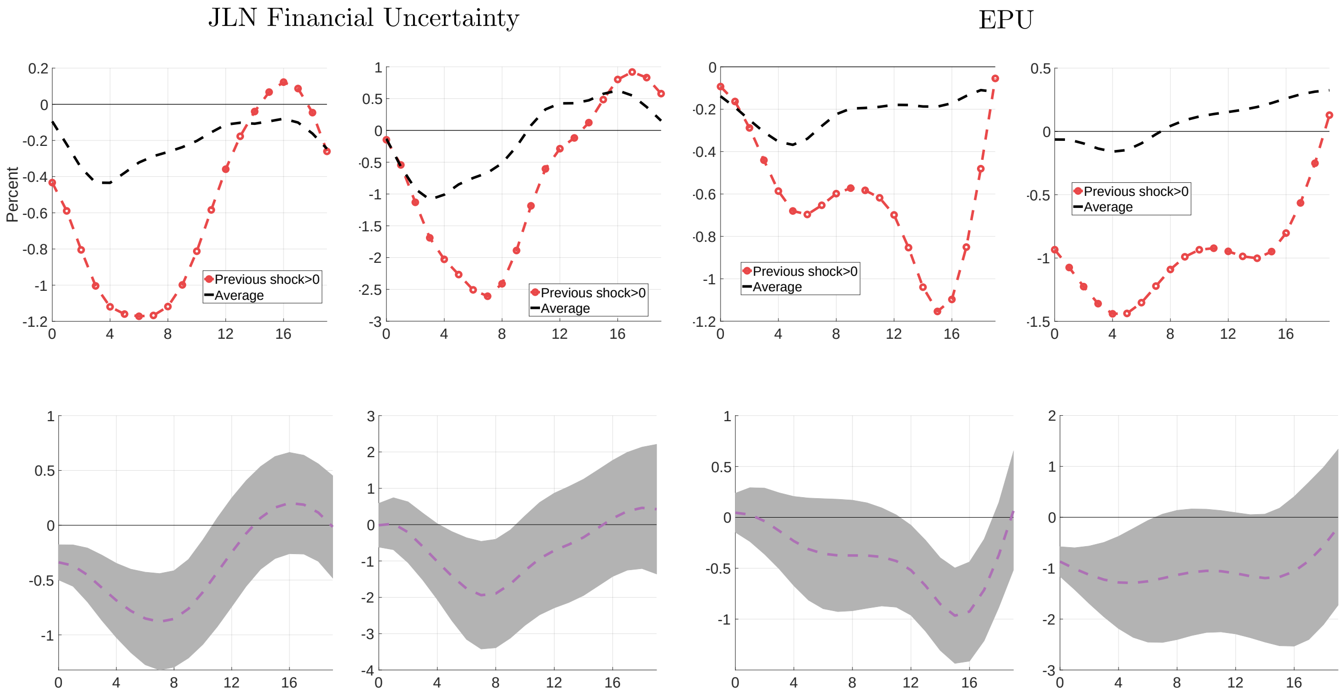
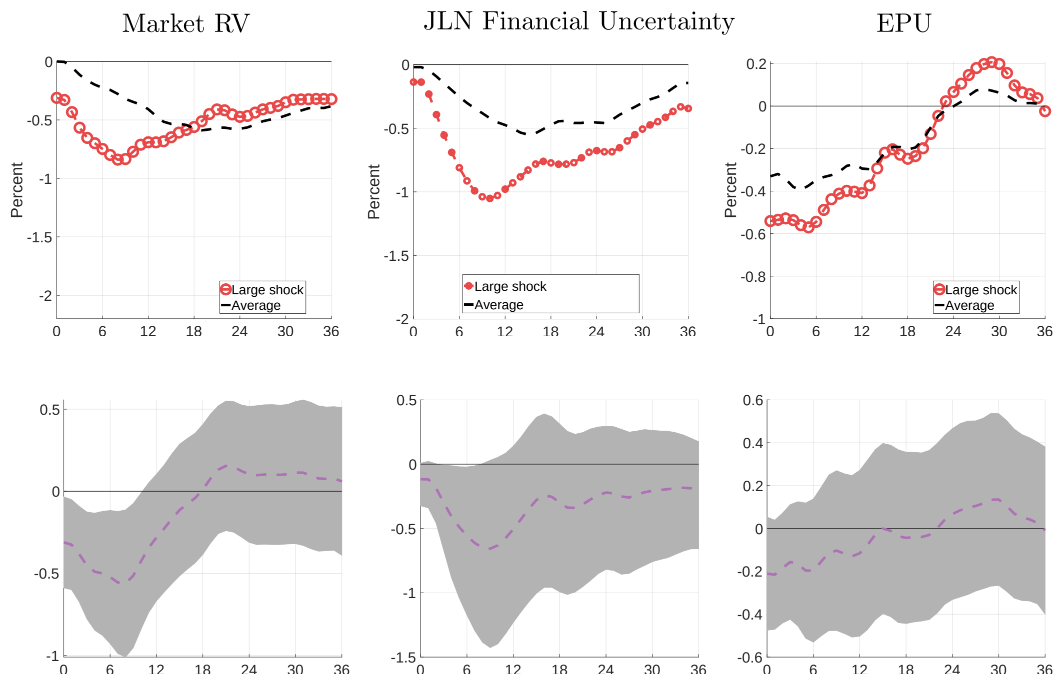


FIGURE 7: **Impulse Response Function to Two Consecutive Positive Uncertainty Shocks**: First row plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for two consecutive positive shocks for output, investment, inflation and the stock market. Second row plots the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty shocks using EPU from Baker, Bloom and Davis (2016). We have 40 runs with two consecutive positive shocks, and we eliminate four runs due to large shocks.



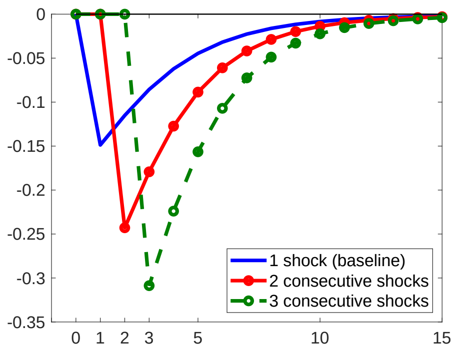
(a) Output. (b) Investment. (c) Output. (d) Investment.

FIGURE 8: **State-dependent IR to Two Consecutive Uncertainty Shocks (Quarterly Data)**: Row 1 plots the empirical state-dependent impulse responses for output and investment. Row 2 plots the associated state multipliers with 90% confidence intervals (shaded areas). In the two leftmost panels, (a) - (b), we measure uncertainty using the financial uncertainty series by Ludvigson et al. (2015). In this case we have 31 runs with two consecutive positive shocks, and we eliminate five runs due to large shocks. In the following (rightmost) panels, (c) - (d), we measure uncertainty shocks using EPU from Baker, Bloom and Davis (2016). In this case we have 27 runs with two consecutive positive shocks, and we eliminate three runs due to large shocks.

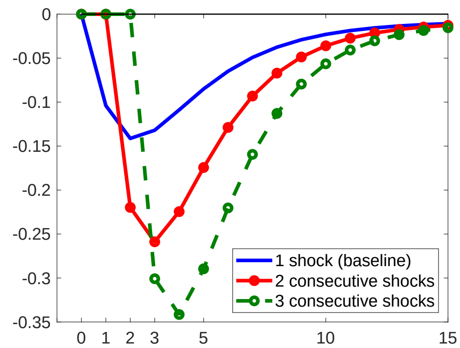


(a) Output. (b) Output. (c) Investment.

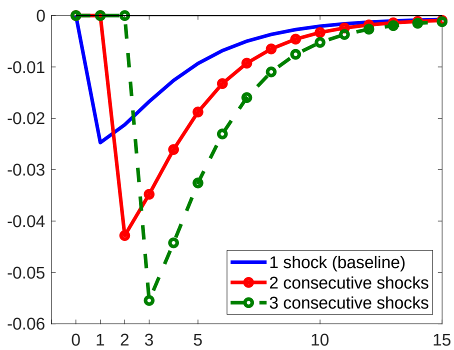
FIGURE 9: **State-dependent IR to Large (i.e., > 2 Std. Deviation) Uncertainty Shocks**: Columns 1, 2, 3 plot the empirical state-dependent impulse responses for (uncertainty shocks using) log market realized variance from Berger, Dew-Becker and Giglio (2019), the financial uncertainty series by Ludvigson et al. (2015), and the EPU series from Baker, Bloom and Davis (2016). Row 1 plots the impulse responses for output. Row 2 plots the associated state multipliers with 90% confidence intervals (shaded areas). We have 15 large positive shocks in column 1, 11 in column 2 and 20 in column 3.



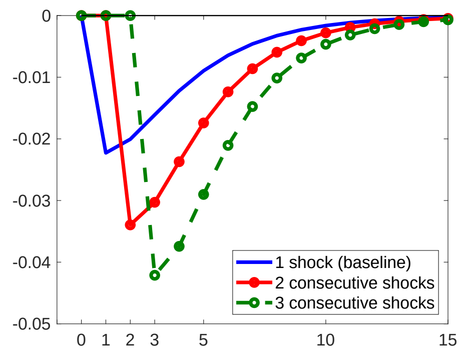
(a) Output 4th-order.



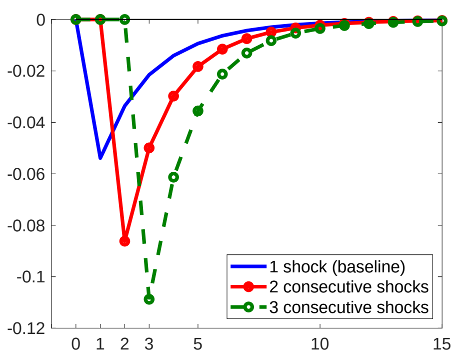
(b) Investment 4th-order.



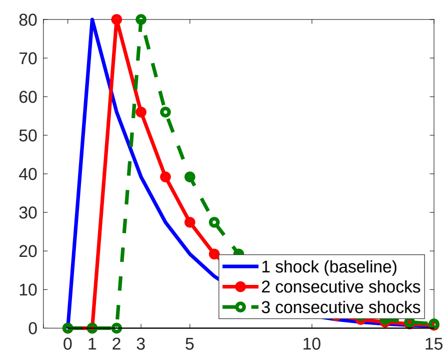
(c) Hours 4th-order.



(d) Inflation 4th-order.

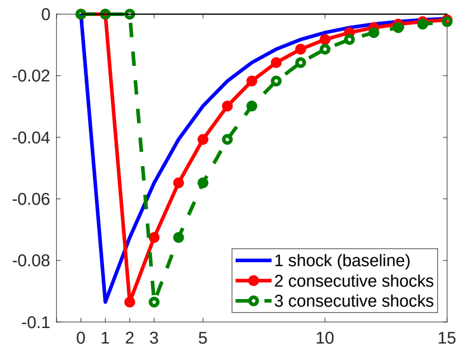


(e) Short Rate 4th-order.

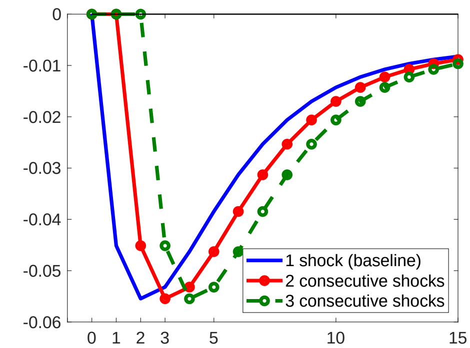


(f) Volatility

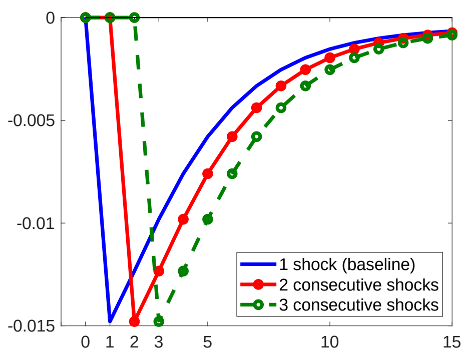
FIGURE 10: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model 4th-order**: This figure plots the impulse responses for consecutive, positive shocks to demand uncertainty. Impulse responses are for one standard deviation shocks when the model is approximated to the fourth order.



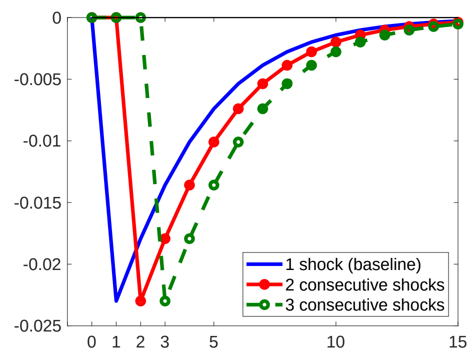
(a) Output 3rd-order.



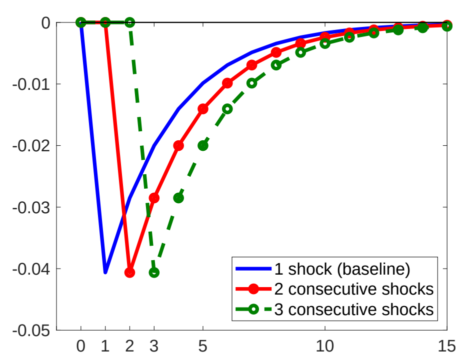
(b) Investment 3rd-order.



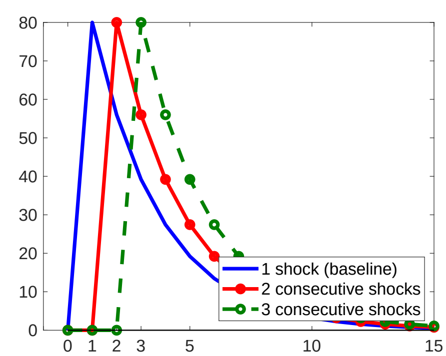
(c) Hours 3rd-order.



(d) Inflation 3rd-order.

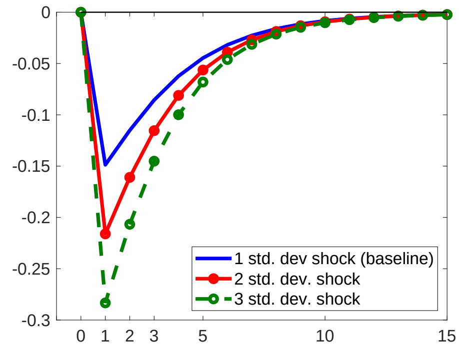


(e) Short Rate 3rd-order.

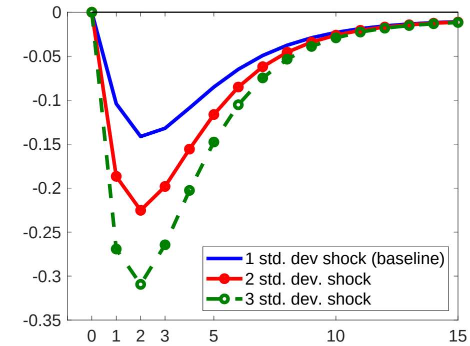


(f) Volatility

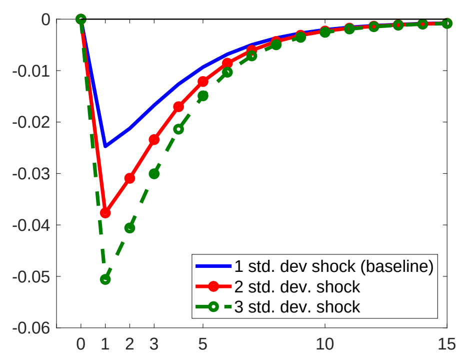
FIGURE 11: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model 3rd-order**: This figure plots the impulse responses for consecutive, positive shocks to demand uncertainty. Impulse responses are for one standard deviation shocks when the model is approximated to the third order.



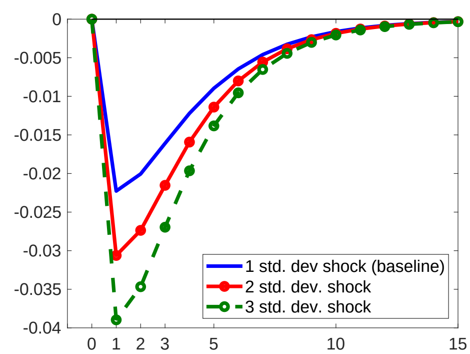
(a) Output 4th-order.



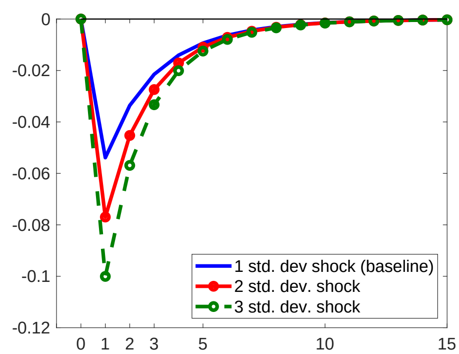
(b) Investment 4th-order.



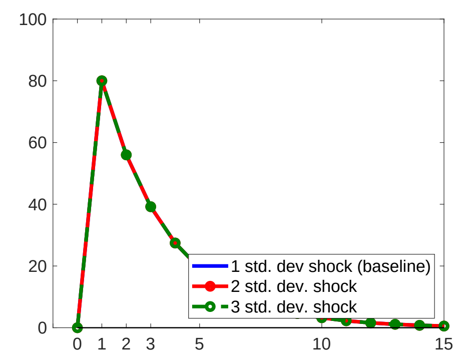
(c) Hours 4th-order.



(d) Inflation 4th-order.

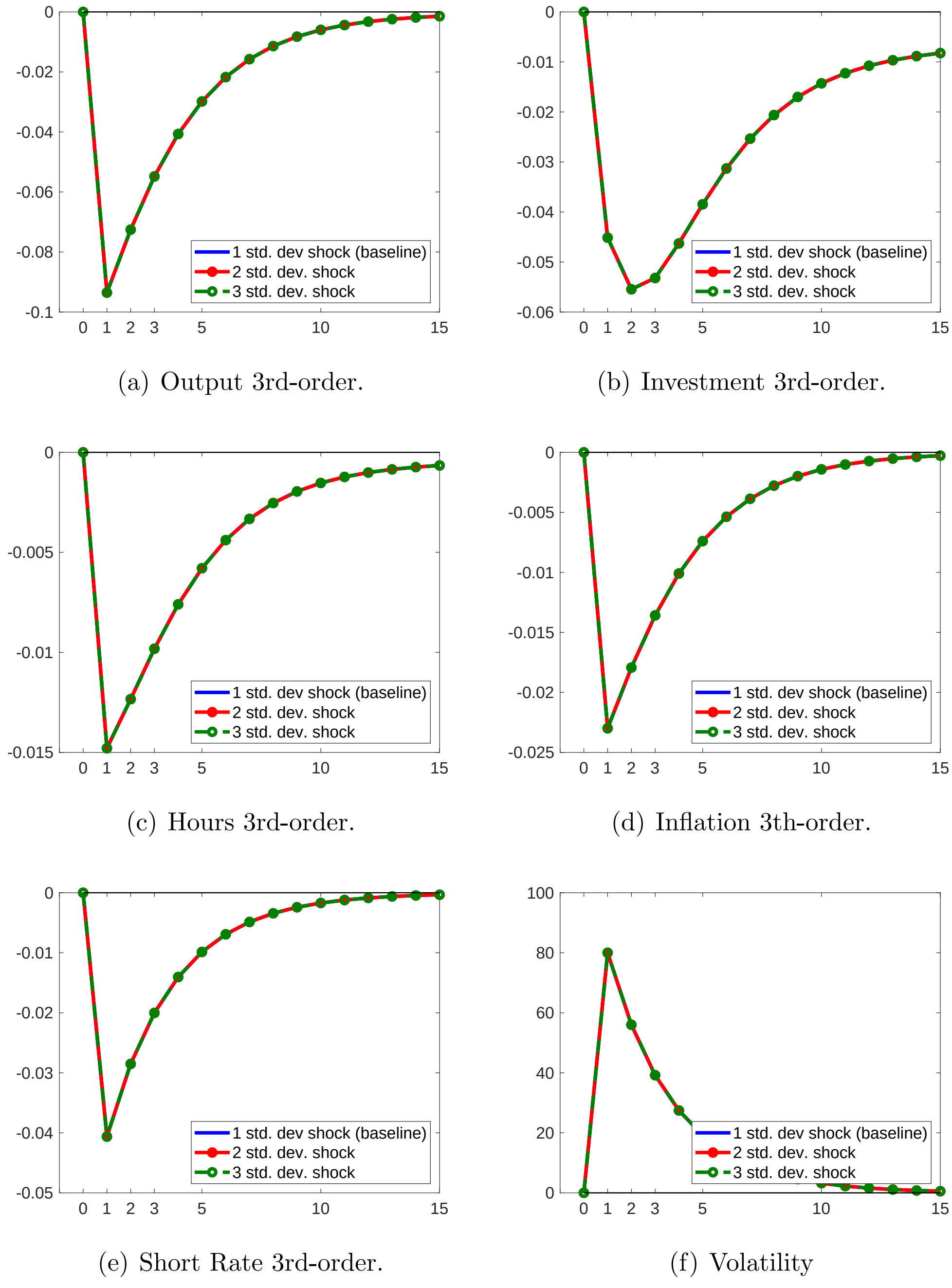


(e) Short Rate 4th-order.



(f) Volatility

FIGURE 12: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model 4th-order**: This figure plots the impulse responses for a one standard deviation positive shock, a two-standard deviation positive shock (divided by two), and a three standard deviation positive shock (divided by three) to demand uncertainty. The model solution is approximated to the fourth order.



(a) Output 3rd-order.

(b) Investment 3rd-order.

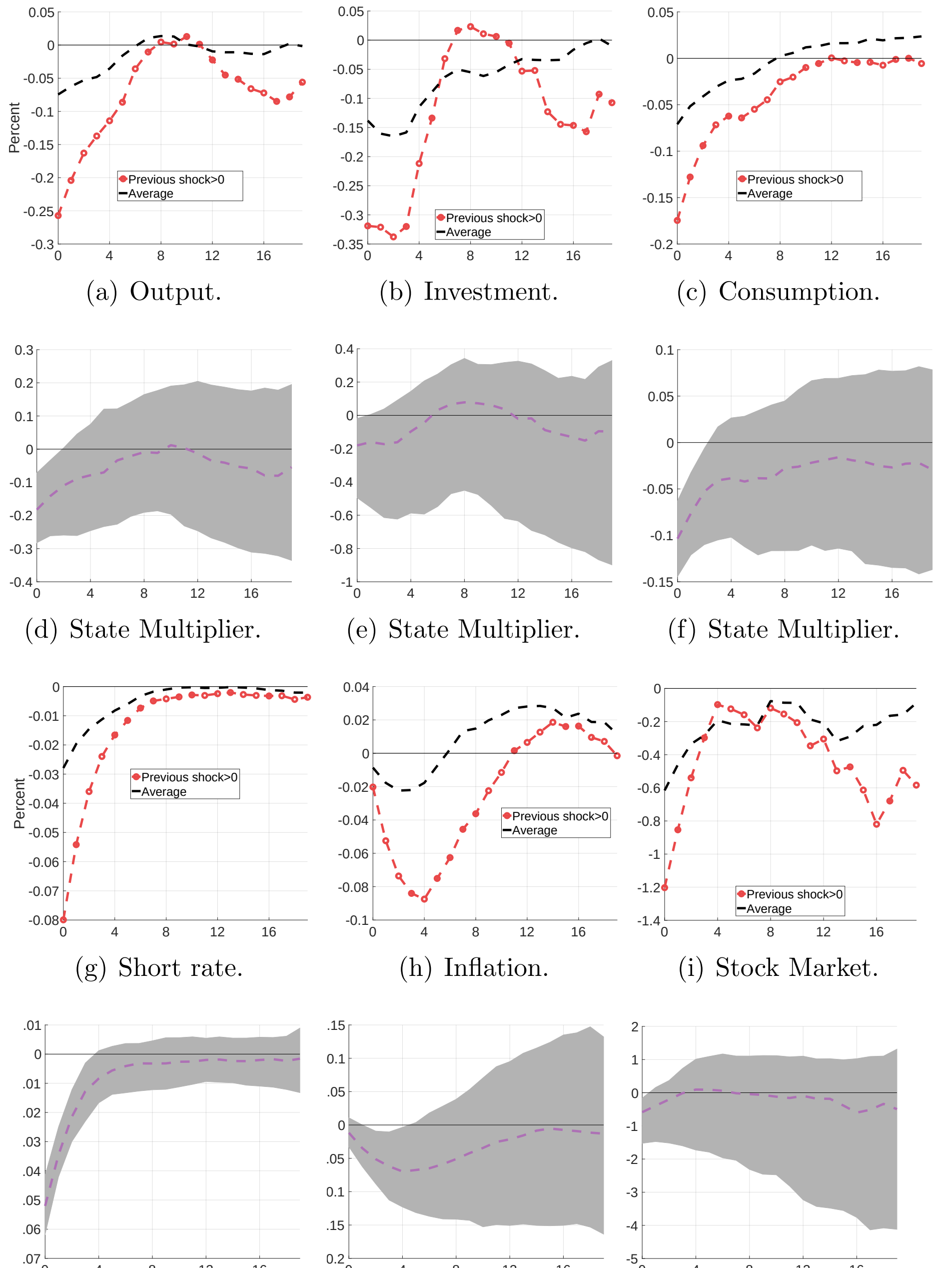
(c) Hours 3rd-order.

(d) Inflation 3th-order.

(e) Short Rate 3rd-order.

(f) Volatility

FIGURE 13: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model 3th-order**: This figure plots the impulse responses for a one standard deviation positive shock, a two-standard deviation positive shock (divided by two), and a three standard deviation positive shock (divided by three) to demand uncertainty. The model solution is approximated to the third order.



(a) Output. (b) Investment. (c) Consumption.

(d) State Multiplier. (e) State Multiplier. (f) State Multiplier.

(g) Short rate. (h) Inflation. (i) Stock Market.

(j) State Multiplier. (k) State Multiplier. (l) State Multiplier.

FIGURE 14: **Model-implied Impulse Response Function to Two Consecutive Positive Uncertainty Shocks**: First and third rows plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for two consecutive positive shocks for output, investment, consumption, short rate, inflation and the stock market. The second and fourth rows plots the associated state multipliers with 68% confidence intervals (shaded areas).

Tables

TABLE 1: **Model Parameters**: This table reports the calibrated parameters for the baseline model. The parameters are organized in 4 subgroups that relate them to preferences, monetary policy, firms or shocks.

Preferences:			*Firms:*		
β	time discount parameter	0.992	κ	capital share	0.333
$\gamma \cdot \eta$	effective risk aversion	10	δ	capital discount rate	0.025
ψ	inverse of IES	0.995	δ_1	capital discount rate	0.031
ω	consumption share	0.35	δ_2	capital discount rate	0.0003
Shocks:			Φ_K	capital adjustment cost parameter	4.3
ρ_z	AR(1) transitory technology	0.995	Φ_P	price rigidity parameter	60
$log(\bar{\sigma_z})$	steady-state vol. transitory tech.	-6.502	θ	elasticity across goods	13
ρ_{σ_z}	AR(1) vol. of vol. transitory tech.	0.75	ν_u	leverage ratio	0.91
σ_{σ_z}	steady-state vol. of vol. transitory tech.	0.91	*Monetary Policy:*		
ρ_a	AR(1) vol. preference	0.935	Π	steady-state inflation	1.01
$log(\bar{\sigma_a})$	steady-state vol. preference	-6.724	ρ_r	AR(1) short rate	0.50
ρ_{σ_a}	AR(1) vol. of vol. preference	0.70	ρ_π	TR coefficient inflation gap	1.500
σ_{σ_a}	steady-state vol. of vol. preference	0.80	ρ_y	TR coefficient output growth	0.500
$\sigma_\Delta a$	steady-state vol. permanent tech	0.0043			
$\rho_{\sigma_\Delta a}$	AR(1) permanent tech.	0.5			
$\mu_{\Delta a}$	growth rate of permanent tech.	0.005			
σ_{θ_μ}	steady-state vol. cost-push shock	0.15			

TABLE 2: **Empirical and Model-Based Unconditional Moments.** This table reports the mean, standard deviations, and correlations for observable variables in the baseline model. The sample period for the data is 1970Q1 to 2016Q4. All data, except inflation, are in logs, HP-filtered, and multiplied by 100 to express them in percentage deviation from trend. Model moments calculations are based on 100 simulations over 250 periods with a burn-in of 50 periods.

Macro Variables

	Model			Data		
	SD	AR(1)	Cor(.,yt)	SD	AR(1)	Cor(.,yt)
Targeted:						
Output	1.85	0.71	1.00	1.54	0.87	1.00
Consumption	1.24	0.70	0.63	1.27	0.89	0.88
Investment	7.00	0.62	0.81	7.07	0.85	0.92
Inflation	0.54	0.23	0.04	0.61	0.89	0.11
Non-Targeted:						
Wages	1.37	0.69	0.88	1.13	0.78	-0.29
Hours	1.22	0.55	0.67	1.94	0.93	0.87

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A Analytical Fourth Order Perturbation Derivation

The following is the symbolic differentiation that we use to show which terms are non-zero with respect to the stochastic volatility process, x. The stochastic volatility enters the model through an exponential function, which for ease of exposition will be denoted as v in the derivations below. Also note that since v is of the exponential form, we will denote v and its derivatives as equivalent.

Additionally, recall that the term y represents the price-consumption ratio, *lambda* reflects the perturbation parameter, *ez* is the level shock to consumption growth, *ex* is the shock to stochastic volatility, w is the volatility-of-volatility, and F represents the function such that $y_t = \mathbb{E}_t f(z_{t+1})$ and

$$y_t = \mathbb{E}_t f\left(v\left(\left(1-\rho_x\right)x_{ss} + \rho_x x_{t-1} + \omega\Lambda\epsilon_{x,t}\right)\Lambda\epsilon_{z,t+1}\right)$$

Note that $\mathbb{E}_t[\epsilon_z^2] = 1$, $\mathbb{E}_t[\epsilon_x] = 0$, $\mathbb{E}_t[\epsilon_z] = 0$, as well as $corr(\epsilon_x, \epsilon_z) = 0$ and $\Lambda = 0$ in steady state. Below, terms denoted in red are equal to zero, while terms denoted in blue are non-zero.

```
dy_dlambda =
  F'(0)*(v*ez + v*ex*w*lambda*ez)


dy_dx =
  ez*lambda*v*rho*F'(0)


dy^2_dx^2 =
  F''(0)*ez^2*lambda^2*v^2*rho^2   +
  F'(0)*ez*lambda*v*rho^2


dy^2_dxdlambda =
  ez*v*rho*F'(0)  +
  ez*lambda*v*rho*F''(0)*(ez*v + ex*ez*lambda*v*w)  +
  ex*ez*lambda*v*rho*w*F'(0)


dy^2_dlambda^2 =
  F'(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)   +
  F''(0)*(ez^2*v^2)   +
  F''(0)*(2*ex*ez^2*lambda*v^2*w)   +
  F''(0)*(ex^2*ez^2*lambda^2*v^2*w^2)
```

```
dy^3_dx^3 =
  3*ez^2*lambda^2*v^2*rho^3*F''(0)  +
  ez^3*lambda^3*v^3*rho^3*F'''(0)  +
  ez*lambda*v*rho^3*F'(0)

dy^3_dx^2dlambda =
  ez*v*rho^2*F'(0)  +
  2*ez^2*lambda*v^2*rho^2*F''(0)  +
  ez*lambda*v*rho^2*F''(0)*(ez*v + ex*ez*lambda*v*w)  +
  ez^2*lambda^2*v^2*rho^2*F'''(0)*(ez*v + ex*ez*lambda*v*w)  +
  2*ex*ez^2*lambda^2*v^2*rho^2*w*F''(0)  +
  ex*ez*lambda*v*rho^2*w*F'(0)

dy^3_dxdlambda^2 =
  2*ez*v*rho*F''(0)*(ez*v)   +
  2*ez*v*rho*F''(0)*(ex*ez*lambda*v*w)   +
  ez*lambda*v*rho*F''(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)  +
  ez*lambda*v*rho*F'''(0)*(ez*v + ex*ez*lambda*v*w)^2  +
  2*ex*ez*v*rho*w*F'(0)  +
  ex^2*ez*lambda*v*rho*w^2*F'(0)  +
  2*ex*ez*lambda*v*rho*w*F''(0)*(ez*v + ex*ez*lambda*v*w)

dy^3_dlambda^3 =
  F'(0)*(ez*lambda*v*ex^3*w^3 + 3*ez*v*ex^2*w^2) +
  F'''(0)*(ez*v + ex*ez*lambda*v*w)^3   +
  3*F''(0)*(ez*v + ex*ez*lambda*v*w)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)

dy^4_dx\string^4 =
  7*ez^2*lambda^2*v^2*rho^4*F''(0)  +
  6*ez^3*lambda^3*v^3*rho^4*F'''(0)  +
  ez^4*lambda^4*v^4*rho^4*F''''(0)  +
  ez*lambda*v*rho^4*F'(0)

dy^4_dx^3dlambda =
  ez*v*rho^3*F'(0)  +
  3*ez^3*lambda^2*v^3*rho^3*F'''(0)  +
  6*ez^2*lambda*v^2*rho^3*F''(0)  +
  ez*lambda*v*rho^3*F''(0)*(ez*v + ex*ez*lambda*v*w)  +
```

```
  3*ez^2*lambda^2*v^2*rho^3*F'''(0)*(ez*v + ex*ez*lambda*v*w)  +
  ez^3*lambda^3*v^3*rho^3*F''''(0)*(ez*v + ex*ez*lambda*v*w)  +
  6*ex*ez^2*lambda^2*v^2*rho^3*w*F''(0)  +
  3*ex*ez^3*lambda^3*v^3*rho^3*w*F'''(0)  +
  ex*ez*lambda*v*rho^3*w*F'(0)

dy^4_dx^2dlambda^2 =
  2*ez^2*v^2*rho^2*F''(0)  +
  2*ez*v*rho^2*F''(0)*(ez*v + ex*ez*lambda*v*w) +
  ez^2*lambda^2*v^2*rho^2*F'''(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)  +
  4*ez^2*lambda*v^2*rho^2*F'''(0)*(ez*v + ex*ez*lambda*v*w)  +
  ez^2*lambda^2*v^2*rho^2*F''''(0)*(ez*v + ex*ez*lambda*v*w)^2  +
  2*ex*ez*v*rho^2*w*F'(0)  +
  ez*lambda*v*rho^2*F''(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)  +
  ez*lambda*v*rho^2*F'''(0)*(ez*v + ex*ez*lambda*v*w)^2  +
  ex^2*ez*lambda*v*rho^2*w^2*F'(0)  +
  8*ex*ez^2*lambda*v^2*rho^2*w*F''(0)  +
  4*ex^2*ez^2*lambda^2*v^2*rho^2*w^2*F''(0)  +
  4*ex*ez^2*lambda^2*v^2*rho^2*w*F'''(0)*(ez*v + ex*ez*lambda*v*w)  +
  2*ex*ez*lambda*v*rho^2*w*F''(0)*(ez*v + ex*ez*lambda*v*w)

dy^4_dxdlambda^3 =
  3*ez*v*rho*F''(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)  +
  3*ez*v*rho*F'''(0)*(ez*v + ex*ez*lambda*v*w)^2  +
  ez*lambda*v*rho*F''(0)*(ez*lambda*v*ex^3*w^3 + 3*ez*v*ex^2*w^2)  +
  3*ex^2*ez*v*rho*w^2*F'(0)  +
  ez*lambda*v*rho*F''''(0)*(ez*v + ex*ez*lambda*v*w)^3  +
  ex^3*ez*lambda*v*rho*w^3*F'(0)  +
  6*ex*ez*v*rho*w*F''(0)*(ez*v + ex*ez*lambda*v*w)  +
  3*ez*lambda*v*rho*F'''(0)*(ez*v + ex*ez*lambda*v*w)*(ez*lambda*v*ex^2*w^2 +
    2*ez*v*ex*w)  +
  3*ex*ez*lambda*v*rho*w*F''(0)*(ez*lambda*v*ex^2*w^2 + 2*ez*v*ex*w)  +
  3*ex*ez*lambda*v*rho*w*F'''(0)*(ez*v + ex*ez*lambda*v*w)^2  +
  3*ex^2*ez*lambda*v*rho*w^2*F''(0)*(ez*v + ex*ez*lambda*v*w)

dy^4_dlambda^4 =
  ez^4*m^4*F''''(0)  +
  4*ex^3*ez*m*w^3*F'(0) +
```

```
24*ex^2*ez^2*m^2*w^2*F''(0) +
12*ex*ez^3*m^3*w*F'''(0) +
24*ex^3*ez^3*lambda^2*m^3*w^3*F'''(0) +
6*ex^4*ez^3*lambda^3*m^3*w^4*F'''(0) +
4*ex*ez^4*lambda*m^4*w*F''''(0) +
6*ex^2*ez^4*lambda^2*m^4*w^2*F''''(0) +
4*ex^3*ez^4*lambda^3*m^4*w^3*F''''(0) +
ex^4*ez^4*lambda^4*m^4*w^4*F''''(0) +
ex^4*ez*lambda*m*w^4*F'(0) +
28*ex^3*ez^2*lambda*m^2*w^3*F''(0) +
30*ex^2*ez^3*lambda*m^3*w^2*F'''(0) +
7*ex^4*ez^2*lambda^2*m^2*w^4*F''(0)
```

This confirms the analysis in the main text, which shows that the only terms that survive at the third and fourth order are in $\frac{\partial^3 y_t}{\partial\Lambda^2\partial x_{t-1}}$, $\frac{\partial^4 y_t}{\partial\Lambda^2\partial x_{t-1}^2}$, and $\frac{\partial^4 y_t}{\partial\Lambda^4}$.

B Relative Contribution of IRFs to Consecutive Shocks

To obtain the impulse response to one shock (these are the solid blue lines in Figures 10 and 11 we proceed as follows.

1. We compute the EMAS as the fixed point of the approximated policy functions in the absence of shocks. To this end, we perform a (baseline) simulation (call it #0) for variables Y with all shocks in the system set to 0 for all time periods, starting at the deterministic steady state. We denote the EMAS $Y0$. The simulation must be long enough to attain convergence (in practice we use 100 periods).
2. Starting at the EMAS, we add one standard deviation to the simulated series for shock i in period 101, $e_{i,101}$; we then perform a simulation, call it #1, for variables Ys. Call the result $Y1$.

The effect of a 1 standard deviation shock is given by:

$$Y1 - Y0 \; .$$

To obtain the (incremental) contribution of two consecutive shocks relative to one shock (these are the solid red lines with circles in Figures 10 and 11 we proceed as follows.

1. Starting at the EMAS, we add *two consecutive* (one standard deviation) shocks to the simulated series for shock i, in period 101 and in period 102, $e_{i,101}$ and $e_{i,102}$; we then perform a simulation, call it #2, for variables Ys. Call the result $Y2$.

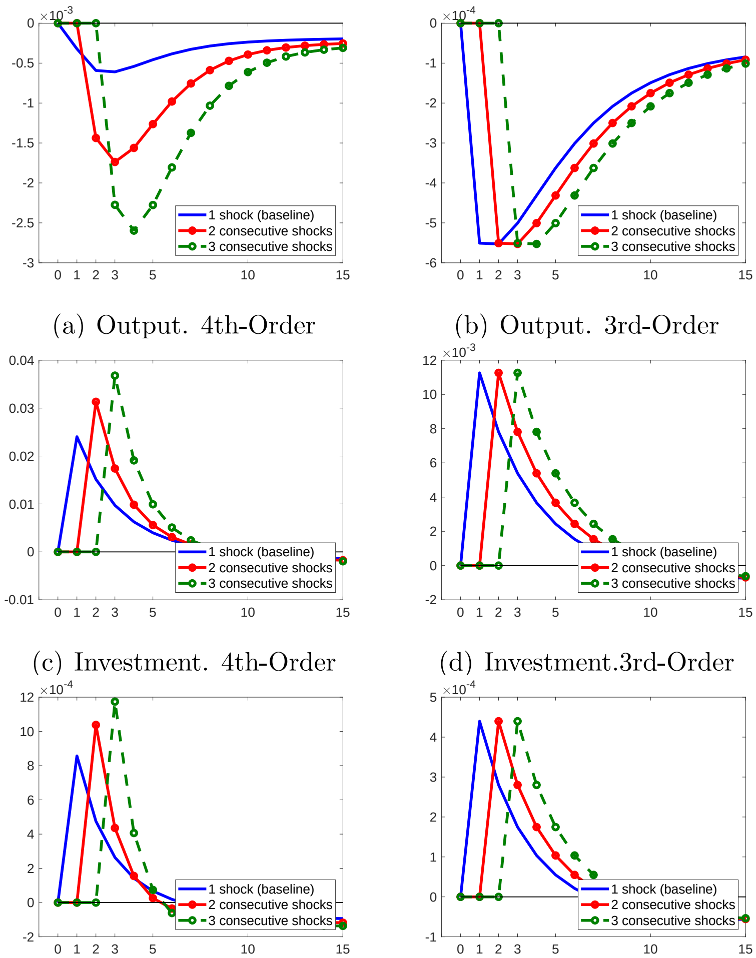
The incremental contribution of two consecutive positive shocks relative to one shock is given by:

$$Y2 - Y1 \; .$$

We proceed in an analogous way for three consecutive shocks, and compute $Y3 - Y2$ where $Y3$ is the outcome of simulation #3 with *three consecutive* (one standard deviation) shocks.

A similar procedure delivers the incremental contribution of consecutive *negative* shocks.

C Response to Uncertainty in Productivity



(a) Output. 4th-Order

(b) Output. 3rd-Order

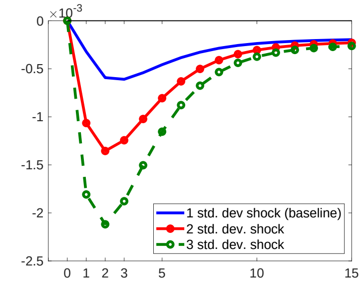
(c) Investment. 4th-Order

(d) Investment.3rd-Order

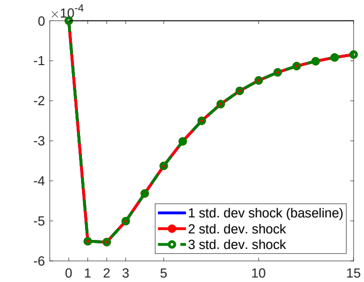
(e) Hours. 4th-Order

(f) Hours. 3rd-Order

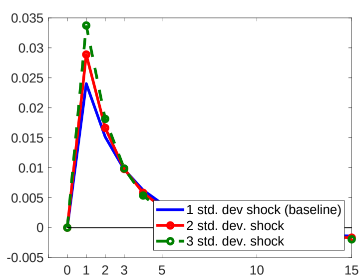
FIGURE C.1: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model**: This figure plots the impulse responses for consecutive, positive shocks to transitory productivity uncertainty. Impulse responses are for one standard deviation shocks when the model is approximated to the fourth and third order, respectively.



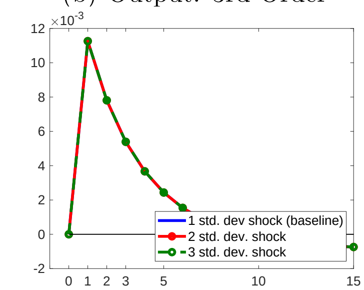
(a) Output. 4th-Order



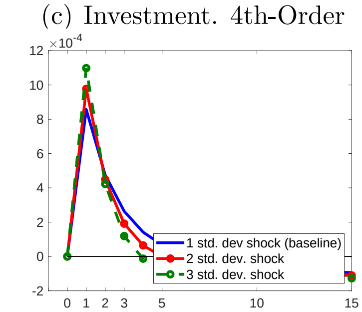
(b) Output. 3rd-Order



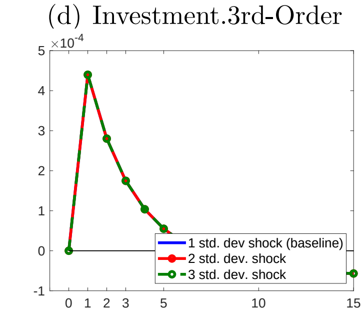
(c) Investment. 4th-Order



(d) Investment.3rd-Order



(e) Hours. 4th-Order



(f) Hours. 3rd-Order

FIGURE C.2: **Impulse Response Function to Positive Shocks in Uncertainty – NK-EZ Model**: This figure plots the impulse responses for a one standard deviation positive shock to transitory productivity uncertainty, a two-standard deviation positive shock (divided by two), and a three standard deviation positive shock (divided by three) to demand uncertainty. The model solution is approximated to the fourth and third order, respectively.