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Collective Moral Hazard and the Interbank Market

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Abstract

The concentration of risk within financial system is considered to be a source of systemic instability. We propose a theory to explain the structure of the financial system and show how it alters the risk taking incentives of financial institutions. We build a model of portfolio choice and endogenous contracts in which the government optimally intervenes during crises. By issuing financial claims to other institutions, relatively risky institutions endogenously become large and interconnected. This structure enables institutions to share the risk of systemic crisis in a privately optimal way, but channels funds to relatively risky investments and creates incentives even for smaller institutions to take excessive risks. Constrained efficiency can be implemented with macroprudential regulation designed to limit the interconnectedness of risky institutions.

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1. INTRODUCTION

A salient feature of the financial systems of advanced economies is the predominance of a few large financial institutions who are highly interconnected with many smaller institutions. This structure, sometimes referred to as core-periphery or hub-and-spoke, is a source of systemic instability as it concentrates resources in systemically important financial institutions (henceforth SIFIs), leaving the rest of the system vulnerable to their failure. Indeed, this concentrated structure is widely seen as a contributing factor for financial crises of late, and has received considerable attention from policymakers as a result. What leads to such a structure to arise in the first place? Why do financial institutions concentrate risk in a manner that generates systemic instability, rather than spreading risk across the financial system?

Addressing these questions requires an understanding of the interaction between the portfolio choices and risk sharing incentives of financial institutions in a setting with heterogeneous agents and bilateral exposures. Modeling these elements jointly poses methodological challenges, and as a result the literature has typically sought to make progress on one dimension or another. The literature on collective moral hazard (e.g. Davila and Walther (2020), Keister (2016), Bornstein and Lorenzoni (2018), Farhi and Tirole (2012), Acharya and Yorulmazer (2007)) has shed important light on how government intervention affects the portfolio choices of financial institutions, but has stopped short of explaining the structural features of the financial system, or takes as given the existence of institutions which are 'too big to fail'. By contrast, the literature on endogenous network formation has broken new ground in this regard but often takes financial contracts to be exogenous, or does not consider how this structure affects agents' portfolio choices.

In this paper, we study the interaction between the portfolio choices of financial institutions and their risk sharing incentives. To this end, we construct a parsimonious model of portfolio choice and systemic crisis with two key ingredients: a government which optimally intervenes during crises and is subject to limited commitment; and an endogenous interbank market through which financial institutions can exchange endogenous financial contracts. Our framework is tractable enough to provide an analytical characterization of equilibria and welfare.

Our paper makes three contributions. First, we offer a new theory of the structure of the financial system.[1] In our setting, the concentrated structure that emerges in equilibrium enables financial institutions to share the risk of systemic crisis in a privately optimal way. During a crisis, the government optimally bails out any institution which is sufficiently large or interconnected

[1] Undoubtedly, there are a multitude of factors which likely play a role in shaping the structure of the financial system, including economies of scale, regulation, and technology, to name a few. We set aside these relevant considerations to focus on the role of risk sharing and systemic crises.

in equilibrium. Interconnected SIFIs arise endogenously because they are the only private agents that can insure other financial institutions against systemic crises, since their liabilities are implicitly guaranteed by the government. Risk sharing between these SIFIs and non-SIFIs generates a core-periphery structure in financial markets. Despite its parsimony, the predictions of our model are therefore consistent with this and other important qualitative features of the data, discussed in section 1.2.

Our second contribution is to show that the financial structure that emerges in equilibrium alters the risk taking behavior of financial institutions in two ways. First, the interbank market channels funds to investment opportunities with high upside risk. As a result, the institutions which become large and interconnected (SIFIs) are relatively risky. Second, the implicit insurance offered by a SIFI's liabilities enables smaller, peripheral institutions to take excessive risks, even though they do not directly benefit from bailouts themselves. In this manner, the excessive risk taking that characterizes SIFIs propagates to other financial institutions by means of interbank financial markets. Thus, our theory shows that the systemic instability generated by the financial system is more severe than previously understood.

Third, we show that this equilibrium structure of the financial system is suboptimal from a welfare point of view. We characterize the optimal regulatory policy required to implement constrained efficiency and show that it is macroprudential in nature and discourages interbank lending to institutions with risky portfolios. Moreover, we show that restrictions on bank leverage are insufficient to implement constrained efficiency. We thus provide a rationale for some post-crisis regulations designed to limit size and interconnectedness, but show that they may be inadequate in important respects.

Our theory is guided in part by important qualitative features of data, which we discuss in section 1.2. Various financial markets are highly concentrated and feature a pronounced core-periphery structure in which the small number of large and interconnected institutions at the core hold riskier assets. Moreover, these SIFIs typically benefit from an implicit government guarantee which lowers the cost of their liabilities. This lower cost is consistent with the evidence that, historically, the creditors and counterparties of systemically important financial institutions (SIFIs) have been among the main beneficiaries of government bailouts, despite not being bailed out directly themselves.[2] These empirical observations suggest an important role for implicit guarantees for understanding the structure of the financial system.

Relative to the literature on collective moral hazard (e.g. Davila and Walther (2020), Farhi and Tirole (2012), Acharya and Yorulmazer (2007)), the key new element in our model is an interbank

[2] For example, according to the Financial Crisis Inquiry Commission report into the 2008 financial crisis, of the $182 billion bailout funds issued to AIG from the US government, about half was passed on to its major creditors and other counterparties. See United States (2011), page 377.

market through which financial institutions can exchange endogenous financial contracts. We view this very broadly as capturing the various markets through which financial institutions interact, such as derivatives, insurance, equity, syndicated loan, corporate debt, deposit, or money markets. We nevertheless adopt the term 'interbank market' for ease of exposition.

We consider an environment with three dates (0, 1, and 2) in which a risk averse household owns many financial institutions, which we call banks. Banks have access to two projects at date 0 which convert physical capital into a consumption good after one period: a 'prudent' project which is common to all banks, and a risky project which is specific to each bank. While the prudent project has a risk-free return, the returns to the risky project are subject to an aggregate shock at date 1. All risky projects are excessively risky from a social perspective: any bank's risky project offers the same expected return as the prudent project, but with higher risk. We assume banks are heterogeneous with respect to the projects to which they have access. Specifically, each bank differs in how exposed its risky project is to the aggregate shock. At date 1, after the resolution of uncertainty, banks have access to a risk-free continuation project which pays out at date 2. In addition, the household owns outside,'traditional' firms who make less productive use of capital, similar to Lorenzoni (2008).

To invest in either project at date 0, a bank can raise funds from two sources: the household and other banks. Banks can borrow funds from the household subject to a limited commitment problem, through the use of a state-contingent debt contract similar to Lorenzoni (2008). Because of limited commitment, the privately optimal state-contingent debt contract limits the borrowers' ability to efficiently allocate funds across states of the world, as banks may not be able to fully insure ex ante or raise funds ex post against losses from risky investments.

In addition to borrowing from the household, banks can raise funds from one another in an interbank financial market – meant broadly to capture any of the financial markets through which financial institutions exchange risk – by entering into bilateral interbank financial contracts at date 0. We assume the contracting environment between any two banks is free of commitment or enforcement problems, so that interbank contracts are complete and channel funds to the investment opportunities which maximize the joint surplus of the borrower and lender.[3]

At date 0, each bank decides how to allocate its portfolio across the available investment projects (prudent and risky) and financial claims issued by other banks on the interbank market. Assets are priced by the stochastic discount factor of the risk averse household and accordingly reflect a risk premium.

At date 1, the aggregate shock to risky projects is realized and financial claims are settled.

[3]While our broad results would hold with incomplete interbank contracts, the assumption that these contracts are complete is useful to highlight that any constrained inefficiency of private risk sharing between banks does not derive from imperfections in interbank financial markets.

Banks can then trade physical capital in a spot market in order to invest in the continuation project. In the bad state of the world, a bank holding risky assets incurs losses which it must finance by selling physical capital. If the aggregate losses of the banking sector as a whole are sufficiently large – that is, if banks' collective exposure to risky assets is sufficiently high – then the economy enters into a crisis in which banks are forced to fire sell their capital holdings to the less productive traditional sector.

Next, we introduce a benevolent government which seeks to maximize household welfare using taxes and transfers which are available only after the aggregate shock at date 1 is realized. The government takes agents' optimizing behavior as given and faces information frictions which imply bailouts can be only imperfectly targeted across banks. In a crisis, the government always finds it optimal to bail out banks in order to prevent the inefficiencies associated with the fire sale of capital to the traditional sector. The government bails out only the most critical banks so as not to incentivize excessive risk taking by other banks. The government cannot commit to a policy which is suboptimal ex post. Moreover, since a bailout occurs only when many banks are failing at the same time, the bailout policy introduces a strategic complementarity in banks' date 0 portfolio choices.

As a result of the strategic complementarity, there are two subgame perfect Nash equilibria. In the 'prudent equilibrium', all banks undertake prudent investments, and so crises and bailouts never occur in equilibrium. In the 'risky equilibrium', both risk sharing and risk taking are constrained inefficient. The interbank market channels funds to the investment opportunities with the highest upside risk. As a result, the banks with relatively risky investment opportunities become excessively large and interconnected, such that they benefit from an implicit government guarantee on their assets. The safety provided by the implicit guarantee drives a wedge between the private and social value of financial claims issued by risky banks. In turn, these SIFIs invest in their risky project, indirectly exposing all other banks to precisely the riskiest projects in the economy through their holdings of SIFI liabilities.

The core-periphery structure of interbank market plays a crucial role in insuring non-SIFI banks against systemic crises. When a bank holds a risky asset, it bears crisis risk – the risk that it incurs a loss during a crisis, precisely when the the stochastic discount factor is highest. Banks are unwilling to hold excessively risky assets in the absence of some form of insurance against this risk. The government provides such insurance in the form of bailouts, but only to SIFIs.

While smaller, peripheral banks do not directly benefit from government guarantees, they benefit indirectly through the interbank market by investing in the liabilities of SIFIs. In a crisis, a SIFI forgoes some of the bailout funds it receives from the government to pay its claimholders a higher rate of return than what it earns on its own assets. This insures claimholders against losses from the SIFI's investments during crises, making risky assets appear safer from the perspective of each individual bank. The insurance value provided by SIFI liabilities is reflected in a lower risk premium, consistent with empirical evidence. As a result of this insurance, even smaller banks who do not directly benefit from the government guarantee may take excessive risks.

To elucidate the role of the interbank market, we analyze two benchmark variants of the model. In the first, we consider a special case in which there is no interbank market and show that there is never excessive risk taking in equilibrium. Without an interbank market in which banks can share the proceeds of bailouts widely, the government's optimal bailout policy is sufficient to eliminate the risky equilibrium. In the second variant of the model, we vary the degree of household risk aversion and show that, under risk neutrality, banks never undertake excessive risk and SIFIs never arise in equilibrium. Indeed, it is the insurance provided by SIFI claims that makes excessively risky investments worthwhile for other banks.

The risky equilibrium is associated with strictly lower household welfare due to excessive consumption volatility. The source of the constrained inefficiency is a soft budget constraint externality, common to models with strategic complementarities, in which agents do not internalize how their collective exposure to the aggregate shock reduces household consumption in the bad state through the lump-sum taxes needed to finance bailouts.[4]

A regulator can address the constrained inefficiency through ex ante intervention in banks' date 0 portfolio decisions through the use of tax incentives or quantity restrictions in interbank financial markets which distort portfolio choices away from claims issued by banks with risky portfolios, in order to prevent these banks from become excessively large and interconnected. We thus provide a rationale for macroprudential policies designed to reduce the interconnectedness of large and risky institutions. Importantly, we show that the focus on limiting bank leverage, while helpful, may be inadequate, and that greater attention should be devoted to reducing interconnectedness and size directly.

1.1. Related literature

Our paper relates most closely to the collective moral hazard literature, particularly Farhi and Tirole (2012), Davila and Walther (2020), Keister (2016), and Acharya and Yorulmazer (2007), who show that bailouts may create inefficient incentives for risk taking through leverage, maturity mismatch, or by undertaking similar projects. Other papers highlighting the significance of time-inconsistency in government interventions are Freixas (1999), Chari and Kehoe (2016), Holmstrom and Tirole (1998), Nosal and Ordonez (2016), Schneider and Tornell (2004), Dell'Ariccia and Ratnovski (2019), and Morrison and Walther (2018).

[4]For instance, see Farhi and Tirole (2012).

Relative to this literature, we examine collective moral hazard on a different margin of agents' portfolio decisions: their risk sharing incentives. The key new element in our framework is an interbank market through which banks can exchange endogenous financial contracts. We show that interconnected SIFIs arise endogenously in this market due to banks' inefficient risk sharing incentives, and that this may lead even peripheral, non-SIFI banks to take excessive risk. In addition, our stylized model generates empirically relevant features of interbank financial markets.

Our paper is also related to a growing literature on ex ante inefficiencies and macroprudential policy, particularly Lorenzoni (2008), Davila and Korinek (2017), Bianchi (2016), and Bianchi and Mendoza (2018). In our model, the government's optimal bailout policy completely eliminates inefficiencies related to pecuniary externalities, but replaces them with an inefficiency deriving from the effect of strategic complementarities on the hardness of the household budget constraint. In Bornstein and Lorenzoni (2018), ex post government intervention does not lead to inefficient ex ante incentives. This is not the case in our setting because ex post intervention alone cannot fully eliminate the inefficiencies associate with excessive risk taking.

Several papers examine optimal policy for regulating systemically important financial institutions, such as Freixas, Parigi, and Rochet (2000), Freixas and Rochet (2013), and Davila and Walther (2020). While these papers take as given that large banks exist, SIFIs emerge endogenously in our model as the result of private risk sharing arrangements. We also show that the inefficiencies associated SIFIs are intimately linked with the risk sharing incentives of all banks.

There is a growing literature which analyzes the endogenous formation of financial networks, including Acemoglu et al. (2014), Chang (2019), Di Maggio and Tahbaz-Salehi (2014), Elliot et al. (2014), Elliot et al. (2018), Erol (2018), Kanik (2019), Leitner (2005), Shu (2019). Our model includes an element of endogenous network formation which depends on a strategic complementarity in banks' portfolio choices. Moreover, to solve for equilibrium, we do not need to keep track of the full structure of the underlying network; solving for a few features of the network is sufficient to characterize allocations. As a result, our model is tractable enough to yield analytical characterizations of equilibria and welfare.

The rest of the paper is organized as follows. We first summarize motivating empirical evidence. Then we introduce the model and characterize the optimizing decisions of private agents. We then introduce a government lacking commitment at date 0 and characterize the ex post efficient policy of transfers. After solving for general equilibrium, we setup the problem of a constrained planner and characterize the inefficiencies in the competitive equilibrium. Finally, we analyze how macroprudential policies can implement constrained efficiency.

1.2 Motivating empirical evidence

Here, we present a brief review of the empirical evidence on the structure of interbank markets, with a focus on three ‘stylized facts’. The overall picture painted by these facts is one of a highly concentrated financial system in which a small number of large and interconnected institutions hold riskier assets, and benefit from an implicit government guarantee which lowers the cost of their liabilities.

The first stylized fact is that interbank financial markets typically exhibit a strong core-periphery structure, in which a few highly interconnected institutions at the core interact with the many sparsely connected institutions in the periphery. This has been shown for a wide range of markets including inter-dealer markets for corporate bonds, over-the-counter derivatives markets, interbank markets, and fed funds markets.[5]

The second fact is that these large and interconnected financial institutions often benefit from an implicit government guarantee of their assets or liabilities. Moreover, this guarantee lowers their costs of funding on deposit or wholesale funding markets, and lowers their cost of insurance via credit default swaps or put options on equity prices.[6]

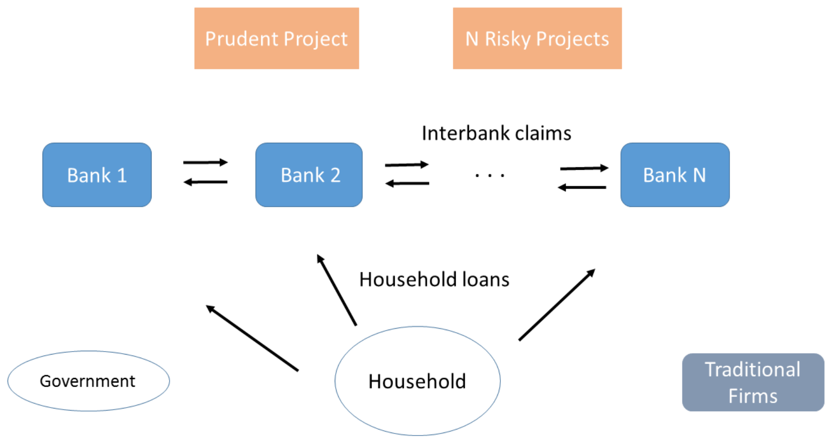
The third fact is that these large and interconnected institutions often make riskier investments than those in the periphery. Afonso et al. (2015), and several papers cited therein, show that the anticipation of government support is associated with increased risk taking. Moreover, Elliott et al. (2019) provide evidence that banks who are more interconnected also undertake more correlated risks.

Consistent with these three features of the data, our model will endogenously feature a core-periphery structure in the interbank market in which large, interconnected banks at the core benefit from an implicit government subsidy and undertake riskier investments. In addition, the liabilities of these SIFIs will command a lower risk premium, reflecting the insurance value provided by the implicit government guarantee.

[5]For example, see Di Maggio et al. (2015) for evidence of a core-periphery structure in the inter-dealer market for corporate bonds, Peltonen et al. (2014) and Vuillemey and Breton (2013) for over-the-counter derivatives markets, Boss et al. (2004), Chang et al. (2008), Craig and von Peter (2014), and van Lelyveld and in ’t Veld (2014) for interbank markets, and Afonso and Lagos (2015), Allen and Saunders (1986), Bech and Atalay (2010) for the fed funds market.

[6]See Kelly et al. (2016) for evidence of the size of implicit government guarantees from out of the money put options, and Veronesi and Zingales (2010) from data on credit default swaps for the largest firms from 2008 Paulson plan, and Lucas and McDonald (2006) and Lucas (2019) for the size of guarantees government-sponsored enterprises. See also O’Hara and Shaw (1990), Baker and McArthur (2009), and Demirguc-Kunt and Huizinga (2013).

Figure 1: Model Environment



2. MODEL

There are three periods: dates 0, 1, and 2. All uncertainty is resolved at date 1. There are four types of agents: a representative household, banks, traditional firms, and later we introduce a government. The household owns N banks, where I denotes the set of banks. Each representative bank i consists of a continuum of identical, atomistic banks. There are two goods, a consumption good and a capital good. The consumption good can be costlessly converted one-for-one into the capital good at any date. Capital can be converted into the consumption good only via investment projects, which are available only to investors. Banks pay out dividends to the household only at date 2.

Figure 1 illustrates the general environment. The risk averse representative household owns banks and traditional firms, the latter of which always makes less productive use of capital. Each bank has access to a prudent (risk-free) project and a risky project, which are subject to an aggregate shock at date 1. Banks are heterogeneous in how exposed their risky projects are to the aggregate shock. Banks can raise funds to invest in these projects from the household, via an optimal state-dependent debt contract, or from one another via optimal bilateral interbank financial contracts. Finally, we will later introduce a government. We now discuss each agent in more detail.

2.1. Household

The representative, risk averse household gets utility from consuming the consumption good according to $u(\cdot)$, where $u(\cdot)$ is twice-differentiable, $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $u(\cdot)$ satisfies the

Inada conditions. Each period the household is endowed with e units of the consumption good. At date 1 (after uncertainty is resolved), the household also has access to a riskless storage technology, which we call bond B_1.[7] At date 0, the household is offered a state-contingent financial contract $\left(d_0^i, \left\{d_1^i(s), d_2^i(s)\right\}_s\right)$ by each bank i, which consists of a loan d_0^i from the household to bank i at date 0, and a set of state-contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ from the bank back to the household at dates 1 and 2, where states are indexed by s. Let f_0^i be an indicator function taking a value of 1 if the household accepts bank i's contract. In addition, the household faces lump-sum taxes T_t in each period t.

Appendix 1 specifies the household's problem in more detail. The household solves a consumption-saving and portfolio allocation problem, taking as given the financial contracts that banks offer, to maximize expected utility $E\left[u(c_0)+u(c_1(s))+u(c_2(s))\right]$, subject to each period's budget constraints. The first-order conditions for f_0^i and the date 1 bond holdings B_1 are

$$u'(c_0)d_0^i \geq E\left[u'\left(c_1(s)\right)d_1^i(s)+u'\left(c_2(s)\right)d_2^i(s)\right] \tag{1}$$

$$u'\left(c_1(s)\right)=u'\left(c_2(s)\right). \tag{2}$$

Condition (1) implies that the household accepts bank i's contract only if the expected discounted return promised by the contract exceeds the marginal utility of consumption at date 0. Condition (2) equates marginal utility across dates 1 and, in any state.

2.2. Investment projects and aggregate risk

At date 0, banks have access to investment projects which convert capital at date 0 into the consumption good at date 1. Each bank has access to a prudent project, which is common to all banks, and a risky project which is specific to each bank. We assume that a bank cannot directly invest in another bank's risky project; rather, each bank can directly invest in the prudent project or their own risky project.[8]

The prudent project yields a risk-free return at date 1 of $R_C > 0$ units of the consumption good for each unit of capital invested in the project. On the other hand, each bank's risky project yields a risky return $R_A^i(s) > 0$ at date 1, which varies across states of the world s and across banks i.

[7] The date 1 risk-free bond is not necessary for the model's results, but improves tractability by allowing the household to completely smooth consumption ex post between dates 1 and 2.

[8] This assumption captures the notion that there may be limitations in a bank's ability to replicate the business model or investment opportunities of other banks, due to considerations related to banks' business models, geographic exposures, regulatory constraints, etc. However, as we will see, banks in the model will be able to generate exposure to each others' risky projects by trading financial claims.

Our assumptions will imply that risky projects are *excessively* risky: their expected returns do not sufficiently compensate investors for the risk they entail. We elaborate on this below.

The only source of risk in the economy is an aggregate shock $R_A(s)$ to the returns on all banks' risky projects at date 1. The aggregate shock can take a high value or a low value $s \in H,\ L$, where $R_A(H) > R_A(L) > 0$ and $E[R_A(s)] = R_C$. More precisely, the return on bank i's risky project at date 1 is given by

$$R_A^i(s) = \rho^i R_A(s) - \mu^i. \tag{3}$$

μ^i is a constant which we assume to be $\mu^i = R_C\left(\rho^i - 1\right)$. This constant simply adjusts the return of i's risky project to ensure that all risky projects have an expected return of $E\left[R_A^i(s)\right] = R_C$ (the return on the prudent project). Thus, each bank's risky project entails more risk than the prudent project, but does not compensate the investor for that risk. By construction, risky projects are therefore 'excessively risky' from a social perspective.

Banks are heterogeneous in the riskiness of projects to which they have access, through the parameters $\rho^i > 1$. The parameter ρ^i determines how exposed the returns of bank i's risky project are to the aggregate shock. A bank with a higher ρ^i is riskier in the sense that the risky project to which it has access has a higher variance. One may interpret ρ^i as capturing factors inherent to bank i's business model which expose it to aggregate risk.[9] The parameters ρ^i are thus a reduced-form way to capture the heterogeneity in the inherent 'riskiness' of banks.

2.3. Market for physical capital

The spot market for capital features the potential for fire sales due the presence of the less productive traditional firms.[10] At date 0, each bank decides how much capital k_0^i to invest in projects and how to allocate its capital across the prudent and risky projects. The fraction of its portfolio bank i invests in the prudent project is denoted ω^i, with the $1-\omega^i$ being invested in i's risky project.

Turning to date 1, bank i must pay a unit maintenance cost $\gamma < 1$ on its capital holdings k_0^i for the capital to remain productive at date 1. At date 1, each bank i has access to a riskless investment, which we call the *continuation project*, which transforms one unit of capital into one unit of the consumption good at date 2. Each household also owns a so-called *traditional firm* which has access to a less productive, but riskless investment technology at date 1 only. An investment of $k_1^T(s)$ of capital in a traditional firm at date 1 produces $F(k_1^T(s))$ units of the consumption good at

[9] In practice, this could arise from any behavior which increases the bank's portfolio returns in good states of the world but magnifies losses in bad states, such as maturity mismatch, leverage, reliance on wholesale funding, having runnable liabilities, or simply having access to projects or assets with a higher market beta.

[10] We introduce these less productive traditional firms into the model to capture some notion of capital misallocation.

date 2, where $F(\cdot)$ is strictly concave, and $F'(\cdot)$ is bounded from above by 1 and below by $\underline{q} < 1$. Since $F(\cdot)$ is strictly concave and $F'(0) < 1$, traditional firms make less productive use of capital at date 1 than investment banks, for any level of investment.

Assumption 1: We assume that $F(0) = 0$, $F'(\cdot) > 0$, $F'(0) < 1$, and $F'(\cdot) \geq \underline{q}$, where $\gamma < \underline{q} < 1$.

Let $q(s)$ denote the price of capital in date 1 state s, which is determined in a competitive market. Because the consumption good can be costlessly converted one-for-one into the capital good at any date, $q(s) \leq 1$ by arbitrage.[11] At date 1 state s, traditional firms choose their date 1 capital holdings $k_1^T(s)$ to maximize $F(k_1^T(s)) - q(s)k_1^T(s)$. If $q(s) \geq 1$, the manager optimally chooses $k_1^T(s) = 0$, whereas if $q(s) < 1$, then $k_1^T(s)$ is chosen to satisfy the first order condition[12]

$$F'(k_1^T(s)) = q(s). \tag{4}$$

Finally, we allow for the possibility of government transfers to banks at date 1. Let $g^i(s, k_0^i, \omega^i)$ denote a subsidy to i's date 1 return on its capital, as a function of the state of the world and bank i's date 0 portfolio.

2.4. Financial markets

We allow agents to share resources at dates 0 and 1 in two ways. First, the household can save in the consumption good by lending it to banks at date 0. By lending to banks through the use of an optimal state-contingent debt contract, the household obtains a set of claims on banks' portfolios returns. However, a limited commitment problem between the household and banks constrains the allocation of funds between these agents – this is the key friction in the model.

In addition to borrowing from the household, we allow banks to borrow from one another in a financial market which opens at date 0, which we refer to as the interbank market.[13] In the interbank market, banks can trade claims on each other's portfolio returns in the form of state-contingent financial contracts. An interbank financial contract issued by bank j to bank i is a debt

[11] We assume that the economy begins with 0 units of the capital goods at date 0, which pins down the date 0 price of capital at 1, while the price of capital at date 2 is 0.

[12] More precisely, traditional firms' objective function is to maximize the household's utility $E_0\left[m_2(s)\left(F(k_1^T(s)) - q(s)k_1^T(s)\right)\right]$. But since traditional firms only buy capital at date 1 after uncertainty is resolved, the household's stochastic discount factor does not affect its investment decision, and its objective simplifies to maximizing $F(k_1^T(s)) - q(s)k_1^T(s)$. The first order condition follows from the assumptions that $F(\cdot)$ is strictly concave and $F'(0) = 1$.

[13] We view the interbank market broadly as capturing a wide range of financial markets through which financial institutions can gain exposure to one another, such as corporate debt, commercial paper, repo, derivatives, insurance, wholesale funding, or equity markets.

contract which specifies a date 0 loan ℓ^{ij} from i to j, and a state-contingent repayment $r^{ij}(s)$ at date 1 from j to i, per unit of ℓ^{ij}.[14]

2.5. Banks

Bank budget constraints at date 0 At date 0, each bank is endowed with n units of the consumption good. It can borrow from the household by offering the household a financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}_s\right)$ which consists of a date 0 loan d_0^i from the household to bank i, and a set of state-contingent returns $\left\{b_1^i(s), b_2^i(s)\right\}_s$ at dates 1 and 2 from the bank back to the household.[15] In addition, the bank may choose to raise funds from other banks at date 0 by offering another bank an interbank financial contract. An interbank financial contract between bank j and bank i specifies the date 0 initial investment ℓ^{ji} from j to i in units of capital, and a set of state-contingent repayments $r^{ji}(s)$ at date 1 from i back to j, which are chosen optimally.

At date 0, bank i can use its internal funds and debt to finance capital holdings of size k_0^i and make loans ℓ^{ij} to other banks j, subject to a date 0 budget constraint given by

$$k_0^i + \sum_j \ell^{ij} \leq n + d_0^i + \sum_j \ell^{ji}. \tag{5}$$

Given its capital holdings, the bank also decides how to allocate its capital across the prudent project versus its risky project. Let $\omega^i \varepsilon [0,1]$ denote the fraction of bank i's capital holdings k_0^i invested in the prudent project; then $1-\omega^i$ is the fraction of i's capital invested in its risky project. Let $\boldsymbol{\omega}^{\mathbf{i}}$ denote the one-by-two vector $[\omega^i \;\; 1-\omega^i]$ and let $\mathbf{R}^{\mathbf{i}}(s)$ denote the two-by-one vector $[R_C \;\; R_A^i(s)]^T$, so that the date 1 return on bank i's projects are given by the scalar $\boldsymbol{\omega}^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)$.

Bank budget constraints at date 1 At date 1, bank i must pay the unit maintenance cost $\gamma < 1$ on its capital holdings, where $R_C \geq \gamma$. Once the state of the world is realized at date 1, bank i's date 1 funds are given by the sum of its portfolio returns $\boldsymbol{\omega}^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)k_0^i$, the value of its capital holdings $q(s)k_0^i$, and any government transfers $g^i(s,\omega^i)k_0^i$ net of its capital maintenance costs and its debt repayment to the household and other banks. The bank then chooses how much capital $k_1^i(s)$ to hold and invest in the continuation project at date 1 subject to its date 1 budget constraint in state s.

[14] The network structure of interbank financial claims will be pinned down by the set of interbank financial contracts which emerge in equilibrium.

[15] More precisely, the contract defines state-contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ at dates 1 and 2 from the bank back to the household. To simplify the notation, we redefine the contract in terms of returns $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{n+d_0^i}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$, which scale these repayments by the bank's total net liabilities at dates 0 and 1, respectively.

Let $\theta_k^i\left(s,\omega^i,g^i\right)$ denote bank i's rate of return on it's own physical capital holdings in state s, given the allocation ω^i of its capital across projects and any government g^i subsidy to i. $\theta_\ell^i(s,j)$ denotes the rate of return on bank i's loan to bank j in state s, given the contract $\left\{r^{ij}(s)\right\}_s$.

$$\theta_k^i\left(s,\omega^i,g^i\right)\equiv q(s)+\omega^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)-\gamma-b_1^i(s)+g^i(s,\omega^i) \tag{6}$$

$$\theta_\ell^i(s,j)\equiv r^{ij}(s)-b_1^i(s) \tag{7}$$

Define the total rate of return on bank i's assets at date 1 in state s as $\theta^i(s)\equiv\frac{\theta_k^i(s,\omega^i,g^i)k_0^i+\sum_j\theta_\ell^i(s,j)\ell^{ij}}{k_0^i+\sum_j\ell^{ij}}$. We can write bank i's date 1 budget constraint in state s as

$$q(s)k_1^i(s)\leq\theta_k^i\left(s,\omega^i,g^i\right)k_0^i+\sum_j\theta_\ell^i(s,j)\ell^{ij}-\sum_h\left(r^{hi}(s)-b_1^i(s)\right)\ell^{hi}+b_2^i(s)k_1^i(s) \tag{8}$$

Finally, in period 2, investment bank i pays dividends $\pi_2^i(s)$ back to the household, which are determined by bank i's final profits at date 2 net of debt repayments to the household.

$$\pi_2^i(s)=k_1^i(s)-d_2^i(s) \tag{9}$$

2.5.1. Contracting environment between the household and banks

At date 0, each bank i may offer the household a contract which specifies an initial loan d_0^i from the household and a set of state-contingent returns $\left\{b_1^i(s),b_2^i(s)\right\}_s$ to the household at dates 1 and 2. We assume that both the household and banks have a limited ability to commit to honoring the contract at dates 1 and 2. Namely, at dates 1 and 2, the bank chooses whether to honor the contract or not. If the bank does not pay, it makes the household a take-it-or-leave-it offer regarding the date 1 and 2 payments. If the household refuses the offer, the bank is liquidated. The liquidation value of a bank depends on its date 0 portfolio choice and the price of capital in state. The contracting environment and liquidation value of banks are spelled out in further detail in appendix 2.

This limited commitment problem imposes no-default constraints on the optimal contracts which ensure that agents never default in equilibrium. The no-default constraints are given by

$$0\leq b_1^i(s)\leq q(s)-\gamma \tag{10}$$

$$0\leq b_2^i(s)\leq\Gamma. \tag{11}$$

To entice the household to accept the contract, bank i's contract must satisfy a participation constraint, which is the household's optimality condition (1). An additional set of assumptions will be useful.

Assumption 2: We assume that
a) $\gamma - \Gamma - R_A^L > 0$, $R_C + \Gamma \geq 1$, and $R_C - \gamma + \Gamma < \rho^i \left(R_C - R_A(L)\right)$ for $\underline{\rho} \equiv \min_i \left\{\rho^i\right\}$.
b) $1 - \Gamma > F'(0)$
c) $\left(F'(k_1^T(s)) - \Gamma\right) k_1^T(s)$ is increasing in $k_1^T(s)$.

Assumption 2(a) makes puts bounds on size of project returns relative to costs; assumption 2(b) ensures that the banks always make more productive use of capital than traditional firms at date 1; and assumption 2(c) will help us rule out multiple equilibria in the date 1 market for capital in any given state.

2.5.2. Contracting environment between banks

Each bank i may raise funds from any other bank h by offering an interbank financial contract at date 0. Recall that an interbank financial contract between bank h and bank i consists of a date 0 investment ℓ^{hi} of capital from h to i, and a set of state-contingent repayments at date 1 given by $r^{hi}(s)\ell^{hi}$ plus the value of the capital that h initially invested in i, given by $q(s)\ell^{hi}$.

To simplify the exposition, we assume there are no incentive or enforcement problems or other frictions between banks. Because banks never have an incentive to default on interbank claims, there are no no-default conditions that needed to be imposed on the interbank contract. The interbank contract between banks h and i simply has to satisfy a bank participation constraint to incentivize bank h to lend to bank i.

$$u^{hi}\left(\ell^{hi}, \left\{r^{hi}(s)\right\}_s\right) \geq \bar{u}^h \tag{12}$$

The value $\bar{u}^h$ is the equilibrium marginal benefit to bank h of investing in its next best alternative , while u^{hi} is the value of bank h if it accepts bank i's contract, while $\bar{u}^h$ is bank h's reservation value – i.e. the value of bank h if it invests in its next best alternative (either lending to another bank, or investing in a project on its own behalf). Therefore, the participation constraint says that bank i must choose a set of state contingent returns to bank h which yields a benefit at least equal to h's outside option. The exact form of this constraint will be derived later from each bank's first order condition for lending to another bank.

2.6. Banks' optimizing behavior

We can now put these elements together to solve each bank's optimization problem. At date 0, each investor i chooses the financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ with the household, the financial contract $\left\{\ell^{ji}, r^{ji}(s)\right\}_s$ with each other investor j, how much to lend to other investors $\left\{\ell^{ij}\right\}_j$, investment levels k_0^i,$k_1^i(s)$, and portfolio allocation ω^i across projects, to maximize the value of its investment bank $E_0\left[m_2(s)\left(1-b_2^i(s)\right)k_1^i(s)\right]$. Here, $m_2(s)$ denotes the stochastic discount factor at date 2 given state s, and reflects the risk aversion of the household. This problem is subject to budget constraints (5) and (8), no-default constraints for the household contract (10) and (11), the household participation constraint (1), the other banks' participation constraints for each j (12), and non-negativity constraints on capital holdings and interbank loans $k_0^i,\, k_1^i(s),\, \ell^{ij} \geq 0\ \forall\, j$.

The full optimization problem and its solution are given in detail in appendix 3. In what follows, we characterize banks' optimizing behavior.

2.6.1. Date 0 portfolio choice

At date 0, bank i decides how to allocate its funds across its available investment opportunities: claims issued by other banks, or capital to be invested in the prudent project or its risky project. In deciding which assets to hold at date 0, the bank compares the expected discounted value of each asset. Let $\theta_a^i(s)$ denote the state-dependent return to i from investing in some asset a (a project or an interbank claim). Because each bank is owned by a risk averse household, the bank discounts the returns by the household's stochastic discount factor at date 1.[16] Therefore, an asset's value can be decomposed into the expected discounted return plus a risk premium component.

$$\underbrace{E\left[m_1(s)\theta_a^i(s)\right]}_{value\ of\ asset} = \underbrace{E\left[m_1(s)\right]E\left[\theta_a^i(s)\right]}_{expected\ discounted\ return} + \underbrace{Cov\left(m_1(s), \theta_a^i(s)\right)}_{risk\ adjustment}$$

Bank i prefers to invest in asset a rather than another asset b if and only the expected discounted returns to investing in a exceeds that of b.

$$E\left[m_1(s)\theta_a^i(s)\right] \geq E\left[m_1(s)\theta_b^i(s)\right]. \tag{13}$$

In what follows, we lay out a few results regarding the investment behavior of banks that will help characterize equilibria later on. Proposition 1 shows that each bank is always at a corner solution in its date 0 portfolio allocation decision.

Proposition 1: *Corner solutions in portfolio choice*

Each bank's portfolio allocation choice is a corner solution. Namely, for any bank i,

a) Either $k_0^i = 0$ and $\ell^{ij} > 0$ for some j; *or* $k_0^i > 0$ and $\ell^{ij} = 0$ for all j.

b) If $k_0^i > 0$, then either $\omega^i = 0$ or $\omega^i = 1$.

[16] We will show in general equilibrium that the Lagrange multiplier $z_1(s)$ on a bank's date 1 budget constraint is equal to the household's stochastic discount factor $m_1(s)$.

Proof: For a formal proof, see appendix 12.

Part (a) of Proposition 1 says that bank i either invests in capital on its own behalf and does not lend funds to any other bank, or the bank lends funds to at least one other bank and does not invest in capital on its own behalf. Part (b) says that if the bank chooses to invest in capital on its own behalf, then it either invests all of its capital in the risky project or it invests all of its capital in the prudent project.[17] Proposition 1 follows from the linearity of the bank's portfolio allocation problem, which arises from the constant returns-to-scale of each bank's investment technologies the absence of idiosyncratic risk, which implies that there is no diversification benefit from investing in different assets.

Intermediaries and investing banks A corollary of this proposition is that, in equilibrium, all banks can be divided into two groups: banks who invest all of their funds in a project in their own behalf, and banks who forgo their own projects in order to intermediate funds to those investing banks. Henceforth, we refer to banks who invest as *investing banks*, and those banks who intermediate funds between the household and investing banks as *intermediaries*. J and L are defined as the sets of investing banks and intermediaries, respectively. Because (15) holds for all investing banks, investing banks all hold identical portfolios in equilibrium.

Banks endogenously sort themselves into these groups in general equilibrium based on the investment opportunities available to them. In equilibrium, the only banks who invest in a project are those with access to the projects with the highest expected discounted returns $E\left[m_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$. Define this set of banks by W.[18] Therefore, the set of investing banks J is given by $J = W$. All other banks become intermediaries who forgo their own investment projects in favor of intermediating funds between the household and these investing banks (or other intermediaries).[19]

2.6.2. Choice of which interbank contracts to offer

The analysis above implies that any bank h's participation constraint (12) takes the form

$$E\left[z_1(s)\theta_\ell^h(s,i)\right] \geq E\left[z_1(s)\bar{o}^h(s)\right] \tag{14}$$

where $\bar{o}^h$ is the bank h's opportunity cost of funds, which is determined in general equilibrium. Thus, bank h accepts an interbank contract offered by bank i if and only if the value of contract exceeds that of h's opportunity cost of funds. This opportunity cost of funds will depend on all of

[17]Without loss of generality, we ignore the knife-edge cases in which one of these conditions holds with equality, in which case interior choices may also be optimal.

[18]More precisely, $W \equiv \left\{w \mid E\left[z_1(s)\theta_k^w\left(s,\omega^w,g^w\right)\right] \geq E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]\ \forall\, i \in I\right\}$.

[19]The set of intermediary banks is given by the complement of J.

its investment opportunities (including interbank contracts offered by other banks) and government transfers.

2.6.3. Optimal contract with household Proposition 1 characterizes the optimal contract between banks and the household. Intuitively, the optimal contract provides some risk sharing between banks and the household, whereby the state-contingent return $b_1^i(s)$ paid to the household is high if the bank's portfolio return is high. However, this risk sharing is limited by the no-default constraints. Moreover, since each representative bank i consists of a continuum of atomistic banks, we have that $z_1^i(s)$ is the same across all banks.[20]

Proposition 2: *Optimal financial contracts with household*

Given a vector of equilibrium prices, an individually optimal financial contract satisfies the conditions for each s: $b_2^i(s) = \Gamma$, $b_1^i(s) = 0$ if $z_0^i < \frac{z_1^i(s)}{m_2(s)}$, $b_1^i(s) \in [0, q(s) - \gamma]$ if $z_0^i < \frac{z_1^i(s)}{m_2(s)}$, and $b_1^i(s) = q(s) - \gamma$ if $z_0^i > \frac{z_1^i(s)}{m_2(s)}$, where $\frac{z_1^i(s)}{m_1(s)} = \frac{1-\Gamma}{q(s)-\Gamma}$, and $z_0^i = \frac{\max\left\{E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right],\ \max_j\left\{E\left[z_1^i(s)\theta_\ell^i(s,j)\right]\right\}\right\}}{1-E\left[m_1(s)b_1^i(s)\right]}$.

Proof: See appendix 4.

2.7. Privately optimal interbank financial contracts

We now characterize interbank financial contracts in partial equilibrium. To obtain funds on the interbank market, banks must essentially compete with one another for funds by offering contracts with the most favorable terms. We suppose that the market for interbank funds at date 0 is perfectly competitive.[21] Given perfect competition, interbank contracts are pinned down by the opportunity cost of banks' funds in each state.[22] The proposition below states characterizes interbank contracts as a function of banks' collective choices $\left\{\omega^i\right\}_{i\in I}$, which will be determined in general equilibrium. To put it briefly, interbank contracts are pinned down by the opportunity cost of funds of the investing banks $w \in W$.

[20] Since banks are atomistic, their investment decisions the market variables determined in general equilibrium, such as the date 1 price of capital $q(s)$. As a result, $z_1^i(s)$ is the same across all banks i.

[21] We suppose that each bank i consists of a continuum of identical atomistic banks, and that there is free entry at date 0. We model competition between banks as a static game between these atomistic banks banks who offer a contract $\left\{\ell^{hi}, r^{hi}(s)\right\}_s$ to a prospective investor h at date 0. Bank h then evaluates each offered contract based on its risk-return profile and accepts that which has the highest expected discounted returns. We solve for the Nash equilibrium of this game and summarize the key results here.

[22] All banks will have the same opportunity cost of funds in equilibrium, implying that there is only one contract to solve for. This follows from the constant returns-to-scale of all banks' projects and the fact that interbank financial contracts are frictionless.

Proposition 3: *Interbank financial contracts in partial equilibrium*

For each intermediary bank $h \in L$, interbank contracts are pinned down by the return on capital $\theta_k^w(s, \omega^w, g^w)$ of investing banks $w \in W$, so that $\theta_k^w(s, \omega^w, g^w) = \theta_\ell^h(s)$ for all states of the world. Moreover, the state-contingent return paid by this contract s is given by the unit return on bank w's investment project $r(s) = \omega^{\mathbf{w}}\mathbf{R}^{\mathbf{w}}(s) + q(s) - \gamma + g^w(s, \omega^w)$.

Proof: See appendix 5.

To understand intuitively how we arrive at Proposition 3, recall that bank h only accepts a contract offered by bank i if the present discounted value of the contract exceeds h's opportunity cost of funds. Because there is only aggregate risk in the economy, a bank w with access to the most privately valuable investment project can always design an interbank contract which incentivizes the prospective lender to accept. In this way, these banks w can out-compete all others for funds on the interbank market.[23]

The optimal contract outlined in Proposition 3 is effectively an equity contract in which an investor purchases a claim on the portfolio returns of the issuing bank, where the return on the claim perfectly reflects the portfolio risks of the issuer. Interbank risk sharing is therefore efficient in a partial equilibrium sense: the interbank market channels funds to the most privately valuable assets. However, we will see that in general equilibrium, the private value of risky assets can differ from their social value.

2.8. Investment at date 1

In order to evaluate the date 1 spot market for capital, we first characterize aggregate net investment in capital at date 1. Define $K_0 \equiv \sum_i k_0^i$ and $K_1(s) \equiv \sum_i k_1^i(s)$ to be the aggregate capital holdings of the banking sector at dates 0 and 1, respectively. In appendix 6, we show that banks' net aggregate investment in capital at date 1 is given by

$$K_1(s) - K_0 = K_0 \left[\frac{\theta_k^w(s, \omega^w, g^w)}{q(s) - \Gamma} - 1 \right]. \tag{15}$$

Equation (15) says that the banking sector's net aggregate investment is given by the aggregate rate of return on capital holdings at date 1, discounted by the cost of capital at date 1. At date 1, the

[23] The results in Proposition 3 imply that we do not have to keep track of the full structure of the underlying network of interbank claims in general equilibrium in order to solve for the allocation of resources and welfare, a feature which greatly improves the model's tractability. The equilibrium network structure of interbank claims matters only insofar as it determines which banks are investing banks in equilibrium.

aggregate rate of return on banks' date 0 capital holdings K_0 is given by the rate of return earned by bank w's assets $\theta_k^w(s, \omega^w, g^w)$. Since banks do not pay out dividends at date 1, this return is invested in capital at date 1. The cost of capital at date 1 is given by the spot price $q(s)$ net of the date 2 repayment to the household $b_2^i = \Gamma$.

2.8.1. Date 1 spot market for capital

We now analyze in partial equilibrium the date 1 spot market for capital. The market for capital features two possible regimes at date 1: *normal times* or a *crisis*. Normal times are characterized by a net positive investment in capital by the banking sector as a whole. This occurs only if the aggregate losses of the banking sector are not too high. In the good state, banks' portfolio returns are high and so they increase their investment in capital. In the bad state, capital is reallocated from banks facing net losses to those not, but otherwise remains entirely within the banking sector.

In contrast, a crisis is characterized by a net sale of capital from the banking sector to traditional firms due to a fire sale externality. This occurs in the bad state when the banking sector's aggregate losses are high – therefore, the precondition for a crisis to occur in the bad state is for the banking sector to have large holdings of risky assets ex ante. In the bad state at date 1, the banking sector is facing a net aggregate loss which needs to be financed. Because there are insufficient funds in the banking sector to cover banks' aggregate losses, banks are forced to liquidate their capital holdings to the traditional sector at a fire sale price. These results are summarized in the proposition below.

Proposition 4: *Date 1 market for capital*

A) In equilibrium we have

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

B) Moreover, $K_1(s) - K_0 < 0$ if and only if $s = L$ and $\omega^w = 0$.

Proof: See appendix 7.

2.8.2 Date 1 bond market clearing The date 1 market for bonds clears when supply equals demand. The demand is derived from the household's date 1 budget constraint: $D_B(s) = e_1 + \sum_i f_0^i d_1^i(s) - T_1 - c_1(s)$. Then the supply of bonds $B_1(s)$ adjusts to clear the market.

2.9. Government's problem

We now introduce a benevolent government which seeks to maximize household welfare using unit transfers $g^i(s,\omega^i)$ to each bank, which are financed by lump-sum taxes $T_1(s)$ on the household at date 1.[24] We analyze the government's optimal policy at date 1 after the resolution of uncertainty; the government therefore takes all date 0 variables as given.[25] The government chooses these taxes and transfers to maximize household welfare subject to a budget constraint each period, which at date 1 is given by

$$\sum_i k_0^i g^i(s,\omega^i) = T_1(s). \tag{16}$$

The government fully internalizes how its actions at date 1 affect those of private agents, and therefore takes the equilibrium conditions which determine agents' date 1 and date 2 choices as constraints when solving its problem.

We assume that the government faces two types of frictions, which we discuss in greater detail in appendix 9A. First, we rule out counterfactual situations in which the government bails out banks even in the absence of a crisis. Moreover, we assume that the government cannot bail out a bank unless it can verify that the bank is facing a net loss.[26] Assumptions of this type are common in models of bailouts, and prevent the government from using transfers as a way to enable banks to circumvent their financial constraints and increase investment in non-crisis states (for example, see Acharya and Yorulmazer (2007)).

Second, we assume that the government and banks have asymmetric information about the returns on banks' portfolios, which will imply that government transfers can be only imperfectly targeted across banks at date 1. More precisely, the government can only verify a bank's returns on its own investments, but cannot verify a bank's returns from its interbank claims.[27] See appendix 9A for more discussion.

The proposition below characterizes the solution to the government's problem, which we derive formally in appendix 8.

[24] The assumption that bailouts are financed by lump-sum taxes rather than government bonds, and the preclusion of date 2 transfers are without loss of generality. This is because date 1 transfers are sufficient to maximize the government's objective, and because government bonds $B_1(s)$ are supplied perfectly elastically and therefore adjust in equilibrium to facilitate perfect consumption smoothing across dates 1 and 2, irrespective of the size of lump-sum taxes.

[25] We rule out taxes and transfers at date 0 in order to exclude macroprudential policy for now, allowing us to isolate the effects of ex post interventions on the equilibrium. We will solve for the jointly optimal macroprudential and ex post interventions later.

[26] These assumptions can be interpreted as a stand-in for the political constraints that governments face when considering direct transfers to the private sector, or for the the distortionary effect of government intervention or the taxes required to finance bailouts. For instance, in 2008, the US Treasury faced considerable political opposition and pressure from the press against the fiscal measures it had proposed for the rescue of the financial sector.

[27] This assumption is similar to that in Farhi and Tirole (2012) in which the government has an imperfect ability to verify bank losses, and can be interpreted as capturing the greater difficulty that bank regulators and supervisors often have in verifying a financial institution's losses from off-balance sheet exposures, which are often complex and opaque in practice and are frequently associated with interbank financial claims.

Proposition 5: *Government's ex post optimal bailout policy*

A) The total size of the optimal bailout is given by $G(s,\omega^w)$:

$$G(s,\omega^w) = \begin{cases} K_0\left(q(s) - \Gamma - R_A^w(s)\right) & for\ \ s = L\ and\ \omega^w = 0 \\ 0 & otherwise \end{cases}.$$

B) The bailout is distributed arbitrarily across the set of investing banks W. Each bank $i \in W$ receives $g^i(s,\omega^w)k_0^i$, where $\sum_{i\in W} g^i(s,\omega^w)k_0^i = G(s,\omega^w)$. Banks outside of W do not receive a bailout, so that $g^i(s,\omega^w) = 0$ for all $i \notin W$.

Proof: See appendix 8.

Part (A) of the proposition establishes the size of the aggregate bailout to the banking sector, whereas part (B) establishes how these funds are distributed across banks. First consider part (A). The optimal aggregate bailout $G(s,\omega^w)$ is the minimum aggregate transfer to the banking sector to ensure $K_1(s) = K_0$, i.e. that all capital remains within the banking sector rather than being fire-sold to the traditional sector. Combining this optimal policy with the expression (15) for net aggregate investment in capital at date 1 shows that the optimal bailout puts a floor on the aggregate return on aggregate capital K_0 equal to $q(s) - \Gamma$. Therefore, capital is never misallocated and we always have $q(s) = 1$ and $k_1^T(s) = 0$ in equilibrium.[28]

Part (B) shows that the government distributes the bailout arbitrarily across investing banks.[29] How exactly the bailout is distributed across investing banks is indeterminate and allocatively irrelevant. Put differently, our results are robust to any such distribution that the government might choose.[30] While we sketch a proof of this in appendix 8, the intuition is that the perfect risk sharing that occurs between banks via the interbank market implies that, in general equilibrium, the bailout is perfectly shared across all banks regardless of the government's choice of how to initially disburse it across investing banks.[31]

The government bailout effectively serves as an implicit, free put option on risky assets, conditional on the banking sector as a whole having sufficiently great exposure to the aggregate shock. Note that the optimal bailout policy features a kink: a bailout occurs if and only the banking sec-

[28]This is sufficient to maximize household welfare ex post since at date 1, given date 0 variables, the only inefficiency which reduces welfare is the potential misallocation of capital which occurs during a crisis.

[29]The government does not bail out intermediary banks since it cannot verify their losses, as described above. See appendix 9A for a discussion of this assumption.

[30]This means that, in general, the government is not necessarily expected to bail out only the riskiest or most interconnected bank ex ante.

[31]We show in section 4, that in a version of the model without an interbank market, this distribution matters. Indeed, the government's optimal policy is then to disburse the bailout to only a small subset of investing banks, so as not to incentivize other banks to take excessive risks. In that setting, this policy is sufficient to eliminate excessive risk taking in equilibrium.

tor's aggregate losses at date 1 are sufficiently large to cause a crisis. This kink will introduce a strategic complementarity in banks' date 0 investment decisions.

In general, the government's optimal bailout policy at date 1, characterized above, is not the same as that which the government would choose at date 0. However, the government cannot credibly commit ex ante at date 0 to implementing a bailout policy which is suboptimal ex post at date 1. As a result, this lack of commitment generates a time-consistency constraint which is at the heart of the collective moral hazard literature.

Alternative government policies In appendix 9B, we discuss the implications of possible alternative government policies, including the notion of randomized bailouts put forth in Nosal and Ordonez (2016).

3. GENERAL EQUILIBRIUM

In the preceding sections, we characterized the equilibrium conditions for all date 1 and date 2 variables as a function of banks' date 0 portfolio choices. It remains to jointly determine the set of investing banks W and their date 0 investment choices $\omega^w \ \forall w \in W$ in general equilibrium. The government's optimal bailout policy introduces a strategic complementarity in banks' date 0 investment actions: a bank's expected discounted payoff to investing in an asset at date 0 depends on the portfolio choices of all other banks due to the possibility of a bailout.[32] In this section, we characterize each bank's best response functions and solve for the subgame perfect Nash equilibria.

An equilibrium is given by a vector of prices $q(s)$, financial contracts $d_0^i,\ d_1^i(s),\ d_2^i(s)_{si}$, portfolio and investment decisions for the investment banks k_0^i, ω^i, $\{\ k_1^i(s)\}_{s_i}$, consumption and investment decisions for the household $c_0,\ c_1(s),\ c_2(s)_s$, $\{k_1^T(s)\}_s$, $\{f_0^i\}_i$, bonds $B_1(s)_s$, and bailout policy and lump-sum taxes $\left\{\left\{g_1^i(s)\right\}_i, \{T_1(s))\}\right\}_s$, such that the household's, banks', traditional firm's, and government's behavior is optimal given their constraints, and capital and goods markets clear in all periods and states.

[32]This is due to a kink in the government's optimal policy outlined in Proposition 5: a bailout is positive if and only if aggregate banking losses are large enough to cause a crisis.

3.1. Bank best response functions at date 0

Given the government's optimal policy, we now characterize each investing bank i's best response function for its date 0 portfolio choices. We begin with bank i's choice ω^i – the fraction of its capital that bank i invests in the prudent project as opposed to its risky project. Recall from (13) that bank i chooses $\omega^i = 0$ if and only if the discounted value of returns from investing in the risky project exceed those of the prudent project. In equilibrium, this condition reduces to

$$E\left[m_1(s)\right]R_C < E\left[m_1(s)\left(R_A^i(s)+g^w(s)\right)\right] \tag{17}$$

where the return $R_A^i(s)+g^w(s)$ on i's risky project in state s consists of the fundamental return $R_A^i(s)$ and the government subsidy to investing banks $g^w(s)$.[33]

We can split the investment decision of a bank into two cases: when conditions for a bailout to occur in the bad state of the world are satisfied or not, where these conditions are defined in Proposition 4. First, supposed that the conditions for a bailout in the bad state do not hold. Then $g^w(s) = 0$, and banks are forced to fully internalize the riskiness of risky projects. Since the risky project is excessively risky, (17) does not hold and banks choose $\omega^i = 1$ to invest only in the prudent project.

Now suppose that the conditions for bailout to occur during a crisis hold. In this case, the government's optimal bailout policy in Proposition 5 implies that (17) holds, and so banks choose $\omega^i = 0$ to invest only in their risky projects. The implicit put option on the risky asset provided by the government places a floor on the risky asset's return. Therefore, we can summarize bank i's best response function for ω^i as follows.

$$\omega^i\left(\{\omega^w\}_{w\varepsilon W}\right) = \begin{cases} 1 & if\ g^w(L,\omega^w) = 0 \\ 0 & otherwise \end{cases} \tag{18}$$

We now turn to a bank's choice of whether to invest in an interbank claim or invests in a project. In equilibrium, bank i chooses to invest in a project if and only its expected discounted return from investing in a project is at least as great as that of other investing banks $w \in W$.

$$E\left[m_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right] \geq E\left[m_1(s)\theta_k^w\left(s,\omega^w,g^w\right)\right] \tag{19}$$

Moreover, given the definition of $\theta_k^w(s,\omega^w,g^w)$ and the government's optimal bailout policy, this condition is satisfied if and only if $\omega^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s) = \omega^{\mathbf{w}}\mathbf{R}^{\mathbf{w}}(s)$. Therefore, bank i is an investing bank in equilibrium if and only if its portfolio returns are equal to that of other investing banks. This pins

[33] Recall that, because of perfect interbank risk sharing, regardless of how the government initially distributes bailout funds across investing banks, all ultimately receive the same unit subsidy $g^w(s)$ in equilibrium.

down the set W of investing banks.

3.2. Subgame perfect Nash equilibria

We first use the banks' best response functions to show that there are two equilibria which vary by banks' date 0 portfolio decisions ω^i, both of which are characterized by herding behavior in date 0 investment. The following lemma shows that there is an equilibrium in which all banks adopt only the prudent investment, and an equilibrium in which all investors adopt only the risky investment.

Lemma 1: *Two subgame perfect Nash equilibria*

There are two subgame perfect Nash equilibria: a 'prudent' equilibrium in which all banks invest in only safe assets (the prudent project or interbank claims on the prudent project), and a 'risky' equilibrium in which all banks invest only in the riskiest assets. Given equilibrium conditions established so far, each equilibrium can be fully described by the investment choice of investing banks ω^w and the set of investing banks W.

Prudent equilibrium: $\omega^w = 1 \quad \forall w \in W$ and the set W is non-empty.

Risky equilibrium: $\omega^w = 0 \quad \forall w \in W$ and $w \in W \iff \rho^w = \bar{\rho}$.

Proof: See appendix 10.

3.2.1. Prudent equilibrium

In the prudent equilibrium, all banks minimize their portfolio exposures to the aggregate shock by investing only in safe assets, i.e. the prudent project on interbank claims which yield the same return as the prudent project in each state. As a result, in this equilibrium, neither crises nor bailouts ever occur in equilibrium. Moreover, no bank is systemically important from the government's perspective. As a result, no bank has incentive to deviate from investing in safe assets at date 0 to investing in a risky project, since any losses incurred during in the bad state would be fully borne by the bank. Moreover, in the prudent equilibrium, which banks are in the set W of investing banks in equilibrium is both indeterminate and inconsequential for output and welfare (beyond that W is non-empty).[34]

[34] To see why, recall that Proposition 3 established that equilibrium interbank contracts are effectively equity claims on the issuer which pay the return on the portfolio of the issuer. Since investing banks invest only in the prudent project in the prudent equilibrium, all interbank claims pay the prudent return in all states. Therefore, each bank is indifferent in the prudent equilibrium between investing in the prudent project and investing any interbank claim, and so the structure of interbank financial claims $\left\{\left\{\ell^{ij}\right\}_j\right\}_i$ are indeterminate. Furthermore, the structure of these claims is allocatively irrelevant in the prudent equilibrium, as it has no bearing on aggregate investment or consumption at any date or state.

3.2.2. Risky equilibrium

In the risky equilibrium, on the other hand, all banks maximize their exposure to the aggregate shock. The set of investing banks W is given by only those banks who have access to risky projects with the greatest exposure to the aggregate shock, i.e. W consists only of all banks i for whom $\rho^i = \bar{\rho} \equiv \max_j \left\{\rho^j\right\}$. These banks invest exclusively in their risky projects. In turn, they finance these investments by issuing claims on these risky investments which are held by all other banks in the economy. In this way, all other banks become exposed to the riskiest project available by forgoing their own projects in favor of buying claims on the riskiest banks' portfolios.[35]

The risky equilibrium thus features a concentration of funds and capital at date 0 in the riskiest banks in the economy. These banks endogenously become highly interconnected with the rest of the banking sector through interbank financial contracts. From the perspective of the government, these risky banks are 'systemically important' in that they are too big and too interconnected to fail. Namely, in the bad state, since losses incurred by these banks on their risky assets are sufficiently large to cause a crisis, the government always bails them out.

The government guarantee enjoyed by a SIFI w has two effects on its portfolio: the guarantee not only increases the expected return of its assets $E\left[\theta_k^w(s,\omega^w,g^w)\right]$ by putting a floor on their losses in the bad state, but it also lowers the risk premium of its assets by increasing the covariance $Cov\left(m_1(s),\theta_k^w(s,\omega^w,g^w)\right)$ between the portfolio return of the SIFI and the household's stochastic discount factor. Since interbank claims issued by a SIFI are essentially claims on the total portfolio return of the SIFI so that $\theta_k^w(s,\omega^w,g^w) = \theta_\ell^i(s,w)$, it follows that the implicit guarantee reduces the risk premium on these claims (i.e. it increases $Cov\left(m_1(s),\theta_\ell^i(s,w)\right)$), making interbank claims issued by SIFIs relatively safe assets with a relatively high expected return. The safety provided by interbank claims issued by SIFIs is what gives rise to the risk sharing motive on the interbank market.

3.2.3. Risk sharing in the risky equilibrium

The systemically important banks that emerge in the risky equilibrium play an important role in allocating risk across the interbank market. In the risky equilibrium, some banks (the SIFI) benefit directly from the government guarantee while the rest do not. Because the government guarantee reduces the risk premium $Cov\left(m_1(s),\theta_k^w(s,\omega^w,g^w)\right)$ associated with the SIFI's portfolio, there is scope for banks to share risk with the SIFI by exchanging interbank contracts at date 0 in a way which reduces the covariance between each bank's return and the stochastic discount factor. Indeed, the interbank contracts that are traded in the risky equilibrium facilitate precisely this kind of risk sharing between the SIFI and non-SIFI banks.

[35] Given our characterization of equilibrium interbank contracts in Proposition 3, the structure ℓ^{ij} for all i and j of interbank claims are neither determinate nor allocatively relevant beyond describing which banks are in set W.

When a SIFI w sells a financial claim to another bank i, in the bad state of the world, the SIFI pays to bank i not only the low return $R_A^w(L)$ earned on its risky investment, but it also forgoes some of bailout funds transferred by gov $g^w(L,\omega^w)K_0$ and pays bank i a higher amount, so that bank i earns a return of $R_A^w(L)+g^w(L,\omega^w)$ instead of just $R_A^w(L)$. This partially insures the holder of the claim, bank i, against losses from the SIFI's investments during crises. This insurance motive is reflected in the lower risk premium of financial claims issued by the SIFI.

Corollary 1: *Interbank risk sharing in the risky equilibrium*

The interbank market in the risky equilibrium features risk sharing between the SIFI and non-SIFI banks. Consider a financial claim issued by SIFI w to a non-SIFI bank i. In a crisis state of the world, the SIFI gives up some of the bailout funds it receives from the government and pays a return of $R_A^w(L)+g^w(L,\omega^w)$ to bank i, which exceeds the return $R_A^w(L)$ it received on its own investments. This amounts to insurance provided by the SIFI to bank i. This insurance reduces the date 0 riskiness of bank i's portfolio, increasing $Cov\left(m_1(s),\theta_\ell^i(s,w)\right)$. Therefore, by buying a claim issued by the SIFI on the interbank market, bank i's portfolio becomes relatively safer and yields a higher expected return.

This risk sharing motive for investing in interbank claims issued by the SIFI is reflected in the low risk premium of these claims. In this way, the SIFI acts as an intermediary insurer whereby it benefits directly from the government guarantee, and effectively insures other banks against losses during a crisis through the interbank market. The safety offered by SIFI liabilities makes risky assets more attractive to all banks.

To summarize, the structure of the interbank market in the risky equilibrium affects risk taking in two ways: it channels funds to the riskiest projects, and it provides incentives for peripheral banks to hold excessively risky assets, even though they do not benefit directly from an implicit guarantee. Importantly, in our parsimonious environment, the emergence of SIFIs is necessary and sufficient to support excessive risk taking in equilibrium. This is because SIFIs are the only private agents that can provide insurance against crisis risk, as they are only agents who benefit directly from gov guarantee. Indeed, this insurance is necessary to incentivize non-SIFI banks to hold excessively risky assets. We explore these points further in the two benchmark versions of the model next.

3.3. Welfare-ranking the equilibria

Let ex ante welfare in the prudent and risky equilibria respectively be denoted by $\bar{\Phi}$ and $\tilde{\Phi}$, so that $\bar{\Phi}\equiv u(\bar{c}_0)+u(\bar{c}_1)+u(\bar{c}_2)$ and $\tilde{\Phi}\equiv u(\tilde{c}_0)+E\left[u(\tilde{c}_1(s))\right]+E\left[u(\tilde{c}_2(s))\right]$. It is straightforward to show that household welfare is strictly greater in the prudent equilibrium, $\tilde{\Phi}<\bar{\Phi}$. The risky equilibrium is associated with lower welfare because the household's consumption is more volatile. This simply reflects that aggregate output at date 1 is more exposed to aggregate risk in the risky equilibrium (without having a higher mean), and is therefore second-order stochastically dominated by aggregate output in the prudent equilibrium.

4. TWO BENCHMARK ECONOMIES

To further elucidate the role of the interbank market in facilitating risk sharing and creating the collective excessive risk taking, we analyze two variants of the model.

Benchmark 1: Model without interbank market In the first benchmark, we consider a special case of the baseline model in which we shut down the market for interbank financial claims. In this setting, there is a unique subgame perfect Nash equilibrium in which all banks undertake only prudent investments. The reason for this is that, to support risk taking in equilibrium, the insurance benefits of government guarantees need to be shared widely across banks – otherwise not enough banks will be exposed to the aggregate shock to trigger a crisis and bailout in the bad state.

Without an interbank market to facilitate risk sharing between banks, the sole benefactors of a bailout are those that the government bails out directly. By concentrating the bailout on small number of investing banks, the government is able to force the majority of banks to internalize the riskiness of their investments, eliminating the risky equilibrium.[36] Thus, the interbank market is the means by which banks can ensure that the benefits of implicit guarantee are shared widely enough to support collective investment in risky assets.

Benchmark 2: Varying the degree of risk aversion In the second benchmark, we illustrate how the risk sharing between SIFIs and non-SIFIs *per se* leads to excessive risk taking. (This benchmark is analyzed in detail in appendix 13.) We modify the model so that only the risk sharing role of the interbank market affects banks portfolio choices, and then perform a comparative static exercise in which we vary the degree of risk aversion of the household.

Under risk neutrality, each bank i's best response function is to always invest in prudent assets. Since agents do not value risk sharing, the insurance provided by claims on SIFIs has no value. As a result, no bank ever undertakes a risky investment in equilibrium. As the household's risk aversion increases, the safety offered by an interbank claim issued by a SIFI is valued more highly,

[36] In the absence of an interbank market, how the government distributes the bailout across investing banks is now relevant for agents' behavior. The government's optimal policy is therefore a modified version of that in Proposition 5 in which the bailout is concentrated on only a small subset of investing banks, to discourage the remainder from undertaking risky projects ex ante.

increasing the safety premium $Cov\left(m_1(s), \theta^i_\ell(s,w)\right)$ commanded by these claims. If the insurance value of these claims is sufficiently high, then other banks forgo their prudent projects in favor of investing in these claims. Protected by the government guarantee, SIFIs in turn invest in risky projects. Hence, banks undertake excessive risks only when the insurance provided by these SIFI claims is sufficiently high.

5. SOCIAL PLANNER'S PROBLEM

In this section, we characterize the constrained efficient allocation. Consider the problem of a social planner who seeks to maximize household welfare and faces the same constraints as private agents and the government. The planner's problem is to choose consumption and investment plans for all agents, the allocation of funds across banks, and taxes and transfers, subject to the limited commitment problem between banks and the household, and the constraint that the allocation of capital at date 1 is determined in a spot market.[37] Moreover, the planner must respect the inability of the government to credibly commit at date 0 to policies which are suboptimal at date 1, which implies the planner faces the same time consistency constraint faced by the government discussed in section on the government's problem.

Therefore, the only ways in which the planner's problem differs from those of private agents is that the planner internalizes the effect of contracts and portfolio choices on the price of capital, and also internalizes the effect of government transfers on the softness of the household budget constraint via taxes. We formalize and solve the full planner's problem in appendix 11.

5.1. Social planner's solution

In appendix 11, we show that the planner's optimal transfer policy at date 1 in the planner's solution coincides with the government's optimal bailout policy in Proposition 5, which simply reflects that this bailout policy eliminates the inefficiencies related to the fire sale externality.[38]

[37] The planner's choices are subject to banks' participation constraints for interbank claims, asymmetric information between the household and banks which prevents households from contracting directly on banks' portfolio choices and forces them to contract on banks' ex post returns instead.

[38] The planner's optimality conditions imply that, as in the competitive equilibrium, we always have $b^i_1(s) = 1-\gamma$ and $b^i_2(s) = \Gamma$. Therefore, the household contract is constrained efficient. Henceforth, we impose this result on the planner's optimality conditions.

There are three margins through which the planner's optimality conditions differ from those of private agents in the competitive equilibrium: the aggregate leverage (or date 0 investment) of the banking sector, the exposure of investing banks to the aggregate shock, and the risk sharing arrangements of banks. The planner internalizes the effect of each margin on the likelihood and size of a bailout, and the effects that a bailout has on the softness of the household budget constraint through lump-sum taxes, which are required to finance any bailout. In addition, the planner internalizes the network effects of interbank lending on each bank's date 0 investment. We characterize the constrained efficient allocation below; the full planner's solution is analyzed in detail in appendix 11.

5.1.1. Social valuation of assets

We discussed in section 2.6.1. how private agents value assets based on the standard asset pricing condition. From the planner's first order conditions, we can see the value to the planner of an asset with state-dependent returns $R(s)$ is given by

$$\underbrace{E\left[m_1(s)R(s)\right]}_{\textit{private valuation}} - \underbrace{E\left[m_1(s)\frac{\partial T_1(s)}{\partial \omega^i}\right]}_{\textit{social cost}}. \tag{20}$$

Hence, the social value of an asset adjusts the value to private agents for the social cost of investing in the asset. In turn, this social cost reflects how investing in the asset tightens the household budget constraint in the bad state through the lump-sum taxes needed to finance a government bailout.

In the competitive economy, the prudent equilibrium is constrained efficient – the allocation lines up with that of the planner. On the other hand, relative to the constrained efficient allocation, the risky competitive equilibrium features three margins of inefficiency: over-borrowing, excessive risk taking, and constrained inefficient risk sharing, the last of which is the focus of our paper.

5.2. Constrained inefficient risk sharing

In this section, we show that risk sharing is constrained inefficient in the risky equilibrium. The planner cares about interbank risk sharing to the extent that it influences the allocation of risk across heterogeneous banks at date 0.[39] Recall that we defined the total rate of return on bank i's portfolio by $\theta^i(s)$, given after equation (7). The ratio $\frac{\theta^i(H)}{\theta^i(L)}$ of its portfolio returns in each state corresponds the extent to which bank i's portfolio of assets is exposed to the aggregate shock. (More precisely,

[39]The planner does not care about interbank risk sharing beyond its implications for date 0 portfolio choices. Namely, the ex post distribution of net worth across banks is irrelevant for output and welfare due to the constant returns-to-scale of the continuation project and the fact that all uncertainty is resolved at the beginning of date 1

$\frac{\theta^i(H)}{\theta^i(L)} = 1$ if and only if bank i has no net exposure to the aggregate shock, while this ratio diverges from 1 as this net exposure increases.) Let M and m denote the 'riskiest' and 'safest' banks, i.e. the banks with the greatest and least net exposure to the aggregate shock, respectively.[40] It follows that $\frac{\theta^M(H)}{\theta^M(L)} \geq \frac{\theta^m(H)}{\theta^m(L)}$.

Condition for efficient risk sharing The condition for constrained efficient risk sharing is that all banks have the same exposure to the aggregate shock as the safest bank. More formally, taking as given each bank i's choice of risk taking ω^i, this condition is

$$\forall i \in I \qquad \frac{\theta^i(H,\omega^i)}{\theta^i(L,\omega^i)} = \frac{\theta^m(H,\omega^m)}{\theta^m(L,\omega^m)} \geq 1. \tag{21}$$

Intuitively, constrained efficiency requires that each bank use interbank contracts to minimize its exposure to the aggregate shock. Moreover, the planner would have banks undertake only prudent investments, implying that under constrained efficiency, $\frac{\theta^i(H)}{\theta^i(L)} = 1$ for all i. This condition is satisfied in the prudent equilibrium.

In the risky equilibrium, by contrast, private risk sharing is constrained inefficient. Indeed, there is a drastic divergence between the risk sharing behavior of banks and that desired by the planner: private risk sharing arrangements in the risky equilibrium *maximize* banks' exposure to the aggregate shock, rather than minimizing it as in the planner's solution. The lemma below conveys this stark result.

Lemma 2: *Constrained inefficient risk sharing in the risky competitive equilibrium*

Fix each bank i's portfolio choice of ω^i. Then risk sharing in the risky equilibrium is characterized by

$$\forall i \in I \qquad \frac{\theta^i(H,\omega^i)}{\theta^i(L,\omega^i)} = \frac{\theta^M(H,\omega^M)}{\theta^M(L,\omega^M)} \tag{22}$$

where $\frac{\theta^M(H,\omega^M)}{\theta^M(L,\omega^M)} > \frac{\theta^m(H,\omega^m)}{\theta^m(L,\omega^m)}$.

Moreover, in the risky equilibrium each investing bank undertakes only risky projects ($\omega^i = 0$), implying that $\frac{\theta^i(H)}{\theta^i(L)} > 1$ for all i.

Importantly, the constrained inefficiency of interbank risk sharing does not derive from any imperfections in interbank financial markets. Rather, the source of this constrained inefficiency is the strategic complementarity in banks' portfolio choices, which can collectively incentivize banks to become exposed to the aggregate shock, leading to a government bailout in the bad state of the world. A bailout is ultimately funded by lump sum taxes on the household, which reduces

[40] These banks are define by $M \equiv \left\{ i : \max_{j\in I} \frac{\theta^j(H)}{\theta^j(L)} \right\}$ and $m \equiv \left\{ i : \min_{j\in I} \frac{\theta^j(H)}{\theta^j(L)} \right\}$.

household consumption in the bad state. Ex ante, private agents do not internalize how their exposure to the aggregate shock, whether through their holdings of risky interbank claims or from investing in their own risky projects, affects the budget constraint of the household in the bad state of the world. As a result, this externality generates a welfare loss in the form of excessively high consumption volatility ex ante.

Moreover, it is precisely the concentration of resources in the riskiest banks that allows the banking sector as a whole to maximize its exposure to the aggregate shock. By issuing claims to the rest of the banking sector and effectively acting as an intermediary between the government and other banks, the SIFIs ensure that the insurance benefits of government guarantees on its assets are shared. This smooths the returns of all banks across states, incentivizing them to invest in risky assets in the first place.

5.3. Over-borrowing and excessive risk-taking

In addition to inefficient risk sharing, the risky equilibrium features over-borrowing and excessive risk taking at date 0. Over-borrowing is reminiscent of Lorenzoni (2008), and arises because agents do not internalize how aggregate leverage and investment affect the softness of the household budget constraint in a crisis through the taxes needed to finance bailout.[41] Excessive risk taking is a feature shared with other papers in the literature on collective moral hazard, including Farhi and Tirole (2012) and Acharya and Yorulmazer (2007).

6. OPTIMAL MACROPRUDENTIAL POLICY

We now discuss how the constrained efficient allocation can be implemented using portfolio taxes and interventions in interbank financial markets at date 0.

6.1. Regulation of the interbank market

A regulator can implement constrained efficiency of risk sharing through appropriate taxes on holdings of claims on banks with risky portfolios, in order to prevent these banks from becoming excessively interconnected or large at date 0. Proposition 6 characterizes the taxes on interbank claims which are necessary and sufficient to implement constrained efficiency of risk sharing.

[41] An important difference with Lorenzoni (2008) is that while excessive leverage in that paper derives from a pecuniary externality, the optimal bailout policy in our economy completely eliminates this source of inefficiency. In its stead, the bailout policy introduces the 'soft budget constraint' externality described above.

Proposition 6: *Optimal taxes on interbank claims*

Taxes τ_ℓ^{ij} on each bank i's holdings ℓ^{ij} of claims on each bank j's portfolio can implement constrained efficiency of interbank risk sharing, where τ_ℓ^{ij} is given by

$$\tau_\ell^{ij}\left(s, g^j(s,\omega^j)\right) = E\left[u'\left(c_1(s)\right) g^j(s,\omega^j)\right] - \frac{E\left[m_1(s)\theta^j\left(s,\omega^j,g^j\right)\right]}{1-(1-\gamma)E\left[m_1(s)\right]}. \tag{23}$$

These taxes distort private portfolio choices away from investing in claims issued by banks with risky portfolios, to prevent these banks from become too interconnected and large to fail from the perspective of the government. The first term of τ_ℓ^{ij} reflects the welfare costs of bank i holding a claim on j's portfolio from increasing the bailout to bank j. The second term is the shadow value of bank j's funds at date 0.[42]

Three features of these taxes on interbank claims are worth emphasizing. First, the tax on i for investing in a claim issued by j is increasing in the riskiness of issuer j of the claim, captured by the exposure of j to the aggregate shock through the dependence of g^j on ω^j. Second, the full set of taxes $\left\{\left\{\tau_\ell^{ij}\right\}_j\right\}_i$ between all bank pairs addresses the indirect exposures of each bank to the aggregate shock through the network effects of their higher order interbank linkages.[43] Third, the taxes are *macroprudential* in nature, and depend on the aggregate exposure of the banking sector as a whole in general equilibrium.[44]

These results rationalize the use of macroprudential tools designed to reduce the systemic consequences of interconnectedness in the financial system.[45] However, the many policies that have thus far been proposed or implemented may be insufficient in some important respects to implement constrained efficiency. Unlike the taxes given in (23), the proposed policies often do not take into account the full network of higher-order exposures to risky assets, nor are they generally macroprudential in nature.

[42]This term adjusts the tax in a way which captures the benefit of interbank claims which relax the constraints of banks with prudent portfolios.

[43]The set of taxes $\left\{\left\{\tau_\ell^{ij}\right\}_j\right\}_i$ fully addresses the consequences of any higher-order network exposures of each bank to the aggregate shock – i.e. the indirect exposure of bank j to the aggregate shock through the higher-order exposures of its counterparties' counterparties.

[44]These taxes are non-zero only when aggregate exposures of the banking sector as a whole to the aggregate shock are sufficiently large to trigger a crisis and a bailout in the bad state, since otherwise $g^j(s) = 0$ for all s.

[45]An example of such regulations are the limitations on counterparty exposures of banks, which were proposed by the Basel III framework of the Basel Committee on Banking Supervision and have been adopted by bank regulators in several advanced economies.

6.2. Regulation of risk taking and aggregate leverage

Turning to the risk taking margin of banks' portfolio choice, excessive risk taking can be eliminated using a date 0 tax τ_ω^i on banks' investments in their risky projects, while a tax τ_k^i on investment at date 0 – or equivalently, a tax on borrowing from the household – can implement constrained efficiency in aggregate investment or leverage.

$$\tau_\omega^i\left(s, g^i(s,\omega^i)\right) = \tau_k^i\left(s, g^i(s,\omega^i)\right) = E\left[u'\left(c_1(s)\right) g^i(s,\omega^i)\right] \geq 0 \tag{24}$$

These taxes reflect the welfare cost that crises and government bailouts impose on the household in certain states. In this setting, we do not need all three taxes outlined above in order to implement the constrained efficient allocation. In particular, the taxes on either risk sharing or risk taking alone are sufficient to implement constrained efficiency.

6.3. Practical considerations

In our setting, interventions in leverage or investment alone are never sufficient to implement constrained efficiency. This highlights that the constrained inefficiencies in our setting derive from banks' exposures on their asset side of the balance sheet to risky investments. Leverage only exacerbates these inefficiencies to the extent that they already exist in equilibrium.

In order to address the inefficiencies associated SIFIs, policy must address banks' risk sharing incentives in interbank financial markets rather than simply curtailing the risk taking behavior of SIFIs themselves. Nevertheless, the policies required to fully implement the constrained efficient allocation in this setting are likely to be unfeasible in practice due to the nature and amount of information required of a regulator.

7. CONCLUSION

We analyzed the efficiency of risk sharing between banks in a setting where the government has limited commitment. We have shown implicit government guarantees, which are ex post efficient, generate strategic complementarities in banks' portfolio choices. In the risky equilibrium, implicit guarantees distort private risk sharing incentives of banks in a way which concentrates resources in the riskiest banks, who in turn become excessively interconnected. By issuing claims on their portfolios, these intermediate the insurance benefits of implicit guarantees to the banking sector as a whole. This smooths the returns of banks across states, facilitating collective risk taking. Thus, the constrained inefficiency in private risk sharing arrangements transforms a risk-shifting problem of an individual bank into a collective risk shifting problem involving the banking sector as a whole. In this way, the inefficient risk sharing arrangements which emerge in interbank financial markets exacerbates the inefficiencies associated with the collective moral hazard problem, generating a welfare loss from excessive consumption volatility. We characterized optimal macroprudential policy and provided a rationale for macroprudential interventions interbank financial markets which disincentivizes the interconnectedness of large and risky institutions.

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APPENDICES

APPENDIX 1: Household optimization problem

At date 0, the household solves a consumption-saving and portfolio allocation problem, given the financial contracts available to it. Namely, it chooses consumption at each date and in each state $\{\{c_t(s)\}_s\}_t$, and how to allocate its date 0 savings across investment banks i, described by weights the indicator functions f_0^i which take the value of 1 if the household accepts bank i's contract and 0 otherwise. Given d_0^i, the total amount of funds the household invests in bank i is given by $f_0^i d_0^i$, and aggregate date 0 saving is then $\Sigma_i f_0^i d_0^i$.[46]We further assume that banks cannot commit at date 0 to investing in particular projects at date 1. Therefore, the household has no information on which projects each bank will invest in at date 1. As a result, the household chooses f_0^i based only on the contract $\left(d_0^i, \left\{d_1^i(s), d_2^i(s)\right\}_s\right)$ offered by each bank.

At date 1, the household also chooses its date 1 bond holdings to maximize expected utility subject to its budget constraint each period.

$$\max_{\{\{c_t(s)\}_s\}_t, \left\{f_0^i\right\}_i, \{B_1(s)\}_s} E\left[u(c_0) + u(c_1(s)) + u(c_2(s))\right]$$

$$c_0 + \Sigma_i f_0^i d_0^i \leq e_0 - T_0 \tag{25}$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1 - q(s) k_1^T(s) \tag{26}$$

$$c_2(s) \leq e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) - T_2 \tag{27}$$

Here, $\Pi_2(s)$ is the date 2 profits of all banks and traditional firms in state s, and T are lump-sum taxes. Let $e_0 = e_1 = e_2$. Also assume that e is sufficiently large that non-negativity constraints for c_0,c_1, and c_2 are never binding. The first-order conditions for f_0^i and the date 1 bond holdings are

$$u'(c_0) d_0^i \geq E\left[u'(c_1(s))\, d_1^i(s) + u'(c_2(s))\, d_2^i(s)\right] \tag{28}$$

$$u'(c_1(s)) = u'(c_2(s)) \tag{29}$$

[46]The household's problem is equivalent to a consumption CAPM in which the household simultaneously solves a consumption-savings and portfolio allocation problem, in which it chooses total savings and the share of savings allocated to each bank i.

APPENDIX 2: Contracting environment between the household and banks

At date 0, each bank i may offer the household a contract which specifies an initial loan d_0^i from the household and a set of state-contingent repayments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ to the household at dates 1 and 2. We assume that both the household and banks have a limited ability to commit to honoring the contract at dates 1 and 2. Namely, at dates 1 and 2, the bank chooses whether to honor the contract and make payments $d_1^i(s)$ and $d_2^i(s)$ to the household. If the bank does not pay, it makes the household a take-it-or-leave-it offer regarding the date 1 and 2 payments. If the household refuses the offer, the bank is liquidated. Upon liquidation, the household cannot seize any of the bank's date 1 net returns, but can seize the bank's net capital holdings $k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji}\right]$. Notice that this implies the household cannot seize the capital holdings bank i which are owed to other banks ℓ^{ji}.[47] Therefore, the net capital holdings of bank i which can be seized by the household in the event of liquidation consist of the bank's own capital holdings $k_1^i(s)$ less the capital it owes on the interbank claims it issued $\sum_j \ell^{ji}$, plus any capital owed to it by other banks $\sum_j \ell^{ij}$. In addition, we assume the household can seize a fraction $\Gamma < 1$ of the bank's profits date 2 profits, where Γ satisfies $\Gamma A < \underline{q}$. Any profits not seized by the household is retained by the bank. (While bank profits eventually find their way to to the household in the form of dividends at date 2, this general equilibrium result is not internalized by the atomistic households).

Any assets that the household seizes can be converted to capital and invested in the date 1 project, after incurring the maintenance cost γ. Therefore, the value to the household of a liquidated bank i at date 1 is $(q(s) - \gamma)\left(k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji}\right]\right)$, and at date 2 it is $\Gamma A k_1^i(s)$. Then investor i never defaults in equilibrium if and only if the following conditions are met: in each period, the value of repayment does not exceed the liquidation value to the household of bank i.

$$d_1^i(s) + d_2^i(s) \leq (q(s) - \gamma)\left(k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji}\right]\right) \tag{30}$$

$$d_2^i(s) \leq \Gamma A k_1^i(s) \tag{31}$$

Similarly, the household can always walk away from the contract without consequence. Therefore, the household does not default in equilibrium if and only if two conditions hold.

$$0 \leq d_1^i(s) + d_2^i(s) \tag{32}$$

[47] Recall that we assumed bank i is contractually obligated to return to j its borrowed capital ℓ^{ji}. Then one can interpret this assumption as a *pari passu* clause in the debt contract in which the claims of one creditor on bank i's assets should respect those of other creditors.

$$0 \leq d_2^i(s) \tag{33}$$

We can scale the contract by the value of i's net capital holdings at dates 0 and 1 in units of the numeraire, so that the contract is denoted $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ where $b_1^i(s)$ and $b_2^i(s)$ are given by $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{n+d_0^i}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$. Then we can rewrite the no-default constraints (30)-(33) as

$$0 \leq b_1^i(s) \leq q(s) - \gamma \tag{34}$$

$$0 \leq b_2^i(s) \leq \Gamma A. \tag{35}$$

To entice the household to accept the contract, bank i's contract must satisfy a participation constraint, which is the household's optimality condition (1).

APPENDIX 3: Bank optimization problems

We can now put these elements together to solve each bank's optimization problem. At date 0, each investor i chooses the financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ with the household, the financial contract $\left\{\ell^{ji}, r^{ji}(s)\right\}_s$ with each other investor j, how much to lend to other investors $\left\{\ell^{ij}\right\}_j$, investment levels k_0^i,$k_1^i(s)$, and portfolio allocation ω^i across projects, to maximize the value of its investment bank. Here, $m_2(s)$ denotes the stochastic discount factor at date 2 given state s, and reflects the risk-aversion of the household.

$$\max \quad E_0\left[m_2(s)\left(1-b_2^i(s)\right)k_1^i(s)\right] \tag{36}$$

subject to budget constraints

$$k_0^i + \sum_j \ell^{ij} \leq n + d_0^i + \sum_j \ell^{ji} \tag{37}$$

$$q(s)k_1^i(s) \leq \theta_k^i\left(s, \omega^i, g^i\right)k_0^i + \sum_j \theta_\ell^i(s,j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s)\right)\ell^{hi} + b_2^i(s)k_1^i(s) \tag{38}$$

no-default constraints for the household contract

$$0 \leq b_1^i(s) \leq q(s) - \gamma \tag{39}$$

$$0 \leq b_2^i(s) \leq \Gamma \tag{40}$$

the household participation constraint, where we have combined the household's optimality conditions (1) and (2)

$$u'(c_0)d_0^i \geq E\left[u'(c_1(s))\,b_1^i(s)\right]\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) \tag{41}$$

and the other banks' participation constraints for each j

$$u^{ji}\left(\ell^{ji}, \left\{r^{ji}(s)\right\}_s\right) \geq \bar{u}^j \tag{42}$$

and non-negativity constraints on capital holdings and inter-bank loans.

$$k_0^i, k_1^i(s), \ell^{ij} \geq 0 \quad \forall j \tag{43}$$

Let z_0^i, $z_1^i(s)$, $\bar{\lambda}^i(s)$, $\underline{\lambda}^i(s)$, $\bar{\mu}^i(s)$, $\underline{\mu}^i(s)$, and ν^{ji} denote Lagrange multipliers on the date 0 budget constraint (37), the date 1 budget constraint (38), the upper and lower bounds on $b_1^i(s)$, the upper and lower bounds on $b_2^i(s)$, and bank j's participation constraint (42) respectively. Also, let $g'(s, k_0^i, \omega^i)$ denote the derivative of the government transfer g^i to bank i with respect to ω^i, which represents how a marginal increase in ω^i affects the bailout that i receives conditional on i being bailed out. (Importantly, this may in general depend on not only the state of the world and i's investment, but also on the investment decisions ω^j of all other banks j.) Because the household has access to a riskless bond at date 1 with gross return 1, and all uncertainty is resolved in date 1, we will have in equilibrium

$$u'\left(c_2(s)\right) = u'\left(c_1(s)\right). \tag{44}$$

The optimality conditions are then given by

$$\frac{\partial L^i}{\partial k_0^i} \leq 0 \iff z_0^i\left(\frac{1}{u'(c_0)}E\left[u'\left(c_1(s)\right)b_1^i(s)\right] - 1\right) + E\left[z_1^i(s)\theta_k^i\left(s, \omega^i, g^i\right)\right] \leq 0 \tag{45}$$

$$\frac{\partial L^i}{\partial k_1^i(s)} \leq 0 \iff m_2(s)\left(1 - b_2^i(s)\right) \leq z_1^i(s)\left(q(s) - b_2^i(s)\right) \tag{46}$$

$$\frac{\partial L^i}{\partial b_1^i(s)} \leq 0 \iff \left[\frac{u'\left(c_1(s)\right)}{u'(c_0)}z_0^i - z_1^i(s)\right]\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) \leq \lambda_1^i(s) - \lambda_0^i(s) \tag{47}$$

$$\frac{\partial L^i}{\partial b_2^i(s)} \leq 0 \iff \left[z_1^i(s) - m_2(s)\right] k_1^i(s) \leq \mu_1^i(s) - \mu_0^i(s) \tag{48}$$

$$\frac{\partial L^i}{\partial \omega^i} \leq 0 \iff E\left[z_1^i(s) k_0^i \frac{\partial \theta_k^i\left(s, \omega^i, g^i\right)}{\partial \omega^i}\right] \leq 0 \tag{49}$$

$$\frac{\partial L^i}{\partial \ell^{ij}} \leq 0 \iff E\left[z_1^i(s) \theta_\ell^i(s, j)\right] \leq z_0^i \left(1 - E\left[\frac{u'(c_1(s))}{u'(c_0)} b_1^i(s)\right]\right) \tag{50}$$

$$\frac{\partial L^i}{\partial r^{ji}(s)} \leq 0 \iff -\nu^{ji} \frac{\partial u^{ji}\left(\ell^{ji}, \left\{r^{ji}(s)\right\}_s\right)}{\partial r^{ji}(s)} \leq \pi(s) z_1^i(s) \ell^{ji} \tag{51}$$

APPENDIX 4: Optimal household contract

Notice from (46) and (48) that when the optimality condition for $k_1^i(s)$ holds, that for $b_2^i(s)$ cannot hold since $b_2^i(s) \leq \Gamma < 1$ and $q(s) \leq 1$. Therefore, given that in equilibrium the optimality condition for $k_1^i(s)$ holds, we have $z_1^i(s) \geq m_2(s)$. Although $b_2^i(s) \in [0, \Gamma]$ when $z_1^i(s) = m_2(s)$, we assume for simplicity it is at its upper bound in this situation. (This does not affect our main results.) Consequently, we always have a corner solution for $b_2^i(s)$ as it is set at its maximum.

$$b_2^i(s) = \Gamma \tag{52}$$

And since $m_2(s) > 0$ by the Inada condition of $u(\cdot)$, it follows that $z_1^i(s) > 0$, so that i's date 1 budget constraint always binds in equilibrium.

Notice from i's optimality condition for $b_1^i(s)$, the household's optimality condition for the bond and the definition of the stochastic discount factor $m_2(s) = \frac{u'(c_2(s))}{u'(c_0)}$, we can write (30) the optimality condition for $b_1^i(s)$ as

$$z_0^i m_2(s) \leq z_1^i(s) \tag{53}$$

Then $b_1^i(s)$ is set at its maximum $q(s) - \gamma$ (a corner solution) if and only if $z_0^i > \frac{z_1^i(s)}{m_2(s)} = \frac{1-\Gamma}{q(s)-\Gamma}$, at its minimum 0 (corner solution) if and only if $z_0^i < \frac{1-\Gamma}{q(s)-\Gamma}$, and is indeterminate if and only if $z_0^i = \frac{1-\Gamma}{q(s)-\Gamma}$. Proposition 1 characterizes the individually optimal financial contract in light of these conditions.

APPENDIX 5: Proof of Proposition 3

Proof: The proof relies on two results. First, perfect competition between atomistic banks implies in each state interbank contract issued from any i to h equates the return on the contract to h to the return on i's assets in each state, such that $\theta_\ell^h(s,i) = \theta^i(s)$. Second, Proposition 1 showed that each bank is always at a corner solution in its portfolio choice. This implies only one contract accepted: the contract with highest private valuation $E\left[z_1(s)\theta_\ell^h(s,i)\right]$. It follows that $\theta_\ell^h(s,i) = \theta_k^w(s,\omega^w,g^w)$, where $W \equiv \left\{w \mid E\left[z_1(s)\theta_k^w(s,\omega^w,g^w)\right] \geq E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]\ \forall\, i \in I\right\}$.

APPENDIX 6: Aggregate investment at date 1

In order to evaluate the date 1 spot market for capital, we first characterize aggregate net investment in capital at date 1. Consider net aggregate investment by all banks in state s at date 1, defined as the difference between aggregate capital holdings at date 1 and aggregate date 0 holdings of capital, $K_1(s) - K_0$, where we have defined $K_0 \equiv \sum_i k_0^i$ and $K_1(s) \equiv \sum_i k_1^i(s)$ to be the aggregate capital holdings of the banking sector at dates 0 and 1, respectively. We can write aggregate net investment in state s as

$$K_1(s) - K_0 = \sum_i \Delta^i\left(s,\omega^i,g^i\right) \tag{54}$$

where $\Delta^i\left(s,\omega^i,g^i\right) \equiv k_1^i(s) - \left[n + d_0^i\right] = k_1^i(s) - \left[k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}\right]$ denotes the difference between bank i's choice of date 1 capital $k_1^i(s)$ and its date 0 funds $n + d_0^i$ available for investment in any asset.[48]

This object can be derived from each bank i's date 1 budget constraint in state s, after imposing the partial equilibrium characterization of optimal interbank contracts given in Proposition 3 $\theta_\ell^i(s) = \theta_\ell^j(s) = \theta_k^w(s,\omega^w,g^w)$ for all i,j in the set of intermediary banks L.

$$K_1(s) - K_0 = \sum_i \Delta^i(s,\omega^w,g^w) = K_0\left[\frac{\theta_k^w(s,\omega^w,g^w)}{q(s)-\Gamma} - 1\right] \tag{55}$$

Equation (55) says that aggregate net investment in capital by the banking sector at date 1 is given by the aggregate rate of return on capital holdings at date 1, discounted by the cost of capital at date 1. At date 1, the aggregate rate of return on banks' date 0 capital holdings K_0 is given by the rate of return earned by bank w's assets $\theta_k^w(s,\omega^w,g^w)$. Since banks do not pay out dividends at date 1, this return is invested in capital at date 1. The cost of capital at date 1 is given by the

[48] To see this, first note that we can re-write aggregate date 0 holdings of capital as $\sum_i k_0^i = \sum_i\left[n + d_0^i + \sum_j\left(\ell^{ji} - \ell^{ij}\right)\right] = \sum_i\left[n + d_0^i\right] + \sum_i\sum_j\left(\ell^{ji} - \ell^{ij}\right) = \sum_i\left[n + d_0^i\right] = \sum_i\left[k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}\right]$. Given our definition of $D^i\left(s,\omega^i,g^i\right)$, it follows that aggregate net investment can be written as $\sum_i k_1^i(s) - \sum_i k_0^i = \sum_i D^i\left(s,\omega^i,g^i\right)$.

spot price $q(s)$ net of the date 2 repayment to the household $b_2^i = \Gamma$. Therefore, the aggregate net investment in new capital by the banking sector at date 1 is given by (55).

APPENDIX 7: Proof of Proposition 4

Proof of Part (A)

Recall that the assumption that the consumption good can be costlessly converted into the capital good one-for-one, but not vice versa, implies $q(s) \le 1$. This also implies that aggregate investment cannot be negative in equilibrium, i.e. $k_1^T(s) + \sum_i \left(k_1^i(s) - \chi(s) k_0^i\right) \ge 0$. If aggregate investment is strictly positive, then $q(s) = 1$ by arbitrage, and so equation (4) implies that $k_1^T(s) = 0$ since $1 = F'(0)$. If, on the other hand, aggregate investment is 0, then we have $k_1^T(s) = \sum_i \left(k_0^i - k_1^i(s)\right)$. These two cases imply that $q(s) = F'(k_1^T(s))$ and $k_1^T(s) = \max\left\{0, \sum_i k_0^i - k_1^i(s)\right\}$. Assumption 1 implies that $\gamma < \underline{q} < q(s)$. Therefore, in equilibrium, we have

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

Q.E.D.

Proof of Part (B)

Recall that the return to i's risky project is given by

$$R_A^i(s) = \rho^i R_A(s) - \mu^i. \tag{56}$$

We assumed that $\mu^i = R_C\left(\rho^i - 1\right)$, implying that we have

$$R_A^i(L) = \rho^i \left(R_A(L) - R_C\right) + R_C.$$

First we show that we have misallocation if and only if $R^w(s) < b_1^w(s) + \gamma - \Gamma A$. Suppose $b_1^w(L) = 0$, then $R^w(s) < \gamma - \Gamma A$.

$$R_A^w(L) <? \gamma - \Gamma A$$

$$\rho^i \left(R_A(L) - R_C\right) + R_C <? \gamma - \Gamma A$$

This is true by Assumption 2 that that $R_C \ge \gamma$ and $R_C + \Gamma A \ge 1$, which implies that $R_C > \gamma - \Gamma A$, and for even the smallest ρ^i, we have $R_C - (\gamma - \Gamma A) < \rho^i \left(R_C - R_A(L)\right)$.

Next, we show that misallocation does not hold for R_C, i.e.

$$R_C \geq \gamma - \Gamma A.$$

This is true by Assumption 2. Since this holds for $R_A^w(L)$ but not R_C this condition holds for any equilibrium value of $b_1(s)$.

Now we show that there is no misallocation if and only if $R^w(s) \geq q(s) - \Gamma A$. First we show that this holds for R_C.

$$R_C \geq q(s) - \Gamma A$$

This holds already by Assumption 2 that $R_C + \Gamma A \geq 1$.

We now show that this does not hold for $R_A^w(L)$.

$$R_A^w(L) <?q(s) - \Gamma A$$

$$\rho^i R_A(L) + R_C \left(\rho^i - 1\right) <?q(s) - \Gamma A$$

This is already satisfied by Assumption 2 and that $\underline{q} > \gamma$, which implies $q(s) \geq \underline{q} > \gamma$. Therefore, these conditions hold which case holds for any equilibrium value of $b_1(s)$.

Therefore, in equilibrium, $K_1(s) - K_0 < 0$ if and only if $s = L$ and $\omega^w = 0$. Q.E.D.

APPENDIX 8: Deriving the government's optimal bailout policy

Proof of Part (A):

At date 1, the government solves its problem taking date 0 variables as given. First substitute out of the household's date 1 budget constraint lump sum taxes $T_1 = K_0 g^w$ using the governments binding budget constraint.

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - K_0 g^w - q(s) k_1^T(s) \tag{57}$$

Recall that we ruled out counterfactual situations in which the government bails out banks outside of a crisis. We can also substitute out total date 2 dividends to the household, which are equal to the profits of banks and traditional firms at date 2 $\pi_2^T(s) = F(k_1^T(s))$, and combine the household's date 1 and 2 budget constraints by substituting out $B_1(s)$.

$$c_1(s)+c_2(s) \leq e_2+\sum_i f_0^i d_2^i(s)+\sum_i \left(A-b_2^i(s)\right) k_1^i(s)+\pi_2^T(s)+e_1+\sum_i f_0^i d_1^i(s)-K_0 g^w-q(s)k_1^T(s) \tag{58}$$

Recall that household's optimality condition $u'(c_1(s)) = u'(c_2(s))$ implies that in equilibrium we always have $c_1(s) = c_2(s)$. Use the definitions of $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{k_0^i-\sum_h \ell^{hi}+\sum_j \ell^{ij}}$, and result that in equilibrium we have $b_1^i(s) = q(s)-\gamma$, $b_2^i(s) = \Gamma A$, and $f_0^i = 1$ for all i. Recall also that we have aggregate interbank liabilities satisfy $\sum_i \sum_h (\ell^{hi}-\ell^{ih}) = 0$, and that banks' aggregate date 1 holdings of capital are given by $K_1(s) = K_0 \frac{\theta_k^w(s,\omega^w,g^w)}{q(s)-\Gamma A}$. Finally, we plug in the two conditions characterizing the date 1 market for capital, $q(s) = F'(K_0-K_1(s))$ and $\pi_2^T(s) = F(K_0-K_1(s))$.

$$2c_1(s) \leq e_2+(q(s)-\gamma)K_0+A(1-\Gamma)K_1(s)+F(K_0-K_1(s))+e_1-K_0 g^w-q(s)(K_0-K_1(s)) \tag{59}$$

Below we characterize the bailout per unit of capital g^w, but this is equivalent to characterizing the total bailout $G = g^w K_0$. (This is because, as we show below, the distribution of bailout funds across investing banks is allocatively irrelevant.)

Given this equation, we want to find the total derivative of $c_1(s)$ with respect to g^w when the conditions for a misallocation of capital at date 1 are satisfied – namely, when $\omega^w = 0$ and $s = L$.

$$\frac{d\,2c_1(s)}{d\,g^w} = K_0\frac{d\,q(s)}{d\,g^w}+A(1-\Gamma)\frac{d\,K_1(s)}{d\,g^w}-F'()\frac{d\,K_1(s)}{d\,g^w}-K_0-K_0\frac{d\,q(s)}{d\,g^w}+q(s)\frac{d\,K_1(s)}{d\,g^w}+K_1(s)\frac{d\,q(s)}{d\,g^w}$$

$$= K_1(s)\frac{d\,q(s)}{d\,g^w}+A(1-\Gamma)\frac{d\,K_1(s)}{d\,g^w}$$

Claim: $\frac{d\,K_1(L)}{d\,g^w} > 0$

Proof:

We first show that $\frac{d}{d\,g^w}\left[\frac{\omega^w R_C+(1-\omega^w)R_A^w(s)+g^w}{q(s)-\Gamma A}\right] > 0$

$$\frac{d}{d\,g^w}\left[\frac{\omega^w R_C+(1-\omega^w)R_A^w(s)+g^w}{q(s)-\Gamma A}\right] = \frac{1}{q(s)-\Gamma A}-\frac{\left(\omega^w R_C+(1-\omega^w)R_A^w(s)+g^w\right)}{(q(s)-\Gamma A)^2}F''\frac{\partial\,k_1^T(s)}{\partial\,g^w}$$

with

$$\frac{\partial\,k_1^T(s)}{\partial\,g^w} = \frac{\partial}{\partial\,g^w}\left[K_0-K_1(s)\right] = -\frac{\partial}{\partial\,g^w}K_1(s)$$

Therefore, we have

$$\frac{dK_1(L)}{dg^w} = \frac{K_0}{\left[q(s) - \Gamma A - \frac{\left(\omega^w R_C + (1-\omega^w) R_A^w(s) + g^w\right)}{(q(s)-\Gamma A)} K_0 F''\right]}$$

For $\frac{dK_1(L)}{dg^w} > 0$ it suffices to show that

$$q(s) - \Gamma A - \frac{\left(\omega^w R_C + (1-\omega^w) R_A^w(s) + g^w\right)}{(q(s) - \Gamma A)} K_0 F'' > 0$$

which holds because $F''(\cdot) < 0$. Q.E.D.

It follows that $\frac{\partial q(s)}{\partial g^w} = -F'' \frac{dK_1(L)}{dg^w} > 0$, since $F''(\cdot) < 0$. Thus we have

$$\frac{d\,2c_1(s)}{dg^w} = K_1(s) \frac{dq(s)}{dg^w} + A(1-\Gamma) \frac{dK_1(s)}{dg^w} > 0$$

So when $\omega^w = 0$ and $s = L$, then household welfare is increasing in g^w when $k_1^T(s) > 0$. Hence, when there is a misallocation of capital at date 1, the government sets g at the minimum to ensure that capital is no misallocated to the traditional sector. This optimal choice of g^w therefore satisfies $k_1^T(s) = 0$ and is given by

$$g^i(s, \omega^w) = \begin{cases} q(s) - \Gamma A - R_A^i(s) & for\, i = w,\ s = L\, and\ \omega^i = 0 \\ 0 & otherwise \end{cases}$$

It follows that the total bailout is given by $K_0 g^i(s, \omega^w) = K_0 \left(q(s) - \Gamma - R_A^w(s)\right)$. This proves part (A) of Proposition 5.

Proof of Part (B):

With regard to part (B), first recall that the government cannot verify the losses that a bank incurs on its interbank claims. As a result, the government does not bail out any intermediaries in equilibrium.

How does the government prefer to distribute the bailout across investing banks? First note that any bailout that satisfies the conditions above will prevent a misallocation of capital ex post, regardless of how it is distributed across investing banks. Nevertheless, in principle, how the bailout is distributed across banks may affect banks' ex ante incentives. Below, we sketch a proof that equilibrium allocations are unaffected by the government's choice of how to distribute the bailout across investment banks. This relies on the characterization of general equilibrium in section 3.

Fix some rule for how the government distributes its bailout across investing banks. Perfect risk sharing between banks ensures that any bailout is perfectly shared across banks, regardless of how the government initially disburses it across investing banks. Then the rule affects neither aggregate investment at date 1 (since the aggregate bailout ensures that capital remains entirely within the banking sector) nor agent's ex ante incentives (since this is determined by the aggregate bailout and banks' risk sharing arrangements). Moreover, the arguments laid out in appendix 10 for a unique risky equilibrium then follow. Hence, this rule does not affect welfare, and is therefore indeterminate.

For the sake of example, suppose that the alternative rule of thumb is one in which the government bails out only the *least* risky investing bank. One might think that we would have a risky equilibrium in which only the *least* risky investing bank j becomes a SIFI. However, in general equilibrium, bank j would have incentive to deviate and lend to any riskier bank h. This is because interbank contracts would ensure that j gets a higher upside in the good state from bank h's riskier investment, while the downside risk is still protected by the government bailout. So then bank h would become a SIFI, while bank j becomes an intermediary bank. So our original conjecture that j is a SIFI cannot be an equilibrium. Therefore, in any equilibrium with risk taking, there is a unique investing bank, and this is always the riskiest bank, regardless of rule of thumb for how the government disperses the bailout across investing banks off-equilibrium. Then all our results regarding the risky equilibrium go through regardless of the choice of rule-of-thumb. Hence, our choice of rule-of-thumb is without loss of generality.

APPENDIX 9: Discussion of government problem

A. Discussion of the frictions faced by the government

An important assumption in the literature on collective moral hazard, and also in our model, is that bailouts cannot be perfectly targeted across banks (e.g. see Farhi and Tirole (2012)). If bailouts could be perfectly targeted to any bank in the financial system, the government could always design a transfer scheme which punishes SIFIs, thereby getting rid of the moral hazard problem (for example, by bailing out all banks except for the SIFIs). In practice, however, there are frictions which prevent the government from doing this, be it informational frictions, political constraints, etc. In the model, we impose a straightforward assumption which can capture this. While our results do not depend on the precise nature of this assumption, it is an empirically plausible and tractable way to generate imperfect targeting.

Our assumption is that it is difficult for the government to verify the losses that a bank incurs on its holdings of interbank claims. This assumption captures the fact that it is difficult for the government to identify banks' bilateral exposures during a crisis, due to the complexity of interbank markets and the fact that these markets are typically over-the-counter. Indeed, the losses that financial institutions incurred in 2008 from their (frequently off-balance-sheet) exposures to other banks on interbank markets were difficult to verify externally, and often these institutions did not themselves know the extent of these exposures in the midst of the crisis.

In the model, this assumption implies that, in general equilibrium, bailouts can be only imperfectly targeted to the SIFIs themselves, who are more central in the network and therefore hold relatively fewer interbank claims in equilibrium. Nevertheless, the results would hold under a broad class of alternative assumptions to the extent that bailouts cannot be perfectly targeted.

B. Alternative government policies

The bailout policy outlined in Proposition 5 calls for the government to transfer resources to SIFIs during a crisis, financed by lump-sum taxes on the household. Here, we consider alternative government policies and their implications.

1. *Randomized bailouts*

An alternative type of government policy is analyzed in Nosal and Ordonez (2016). In that paper, the government faces uncertainty about whether a crisis is systemic, and therefore delays intervenion to attain more information. This forces banks to internalize the riskiness of their investments to some extent, mitigating the ex ante moral hazard problem. In our setting, there is no such uncertainty; the government knows with certainty whether there is a crisis, and so this mechanism is not at play. Moreover, given the inefficiencies associated with a crisis, it would be suboptimal (and therefore not credible) for the government not to intervene during a crisis with positive probability.

Nevertheless, a government could conceivably choose to randomize *which* investing banks it bails out during a crisis. In our setting, however, the interbank contracts which emerge in equilibrium would ensure that the bailout to any individual bank would be shared more broadly. Put differently risk sharing between banks in the interbank market always ensures that the benefits of bailouts are shared perfectly. Therefore, the interank market ensures that no amount of bailout randomization can eliminate the collective moral hazard problem.

That being said, in practice, a lack of confidence in the government's ability to carry out its optimal bailout policy could mitigate risk taking ex ante.

2. *Transfer of capital from SIFIs to non-SIFIs*

One alternative policy would be for the government to simply transfer capital from SIFIs to other banks during a crisis, in a way which keeps production at the first best ex post and eliminates the risk taking incentive of SIFIs ex ante. It is important to note, however, that this would be isomorphic to a bailout of non-SIFI banks. To see why, suppose that, in a crisis, the government obtains the capital of the SIFIs (either through expropriation, or by purchasing the capital at some price) and grants it directly to non-SIFI banks. In a crisis, non-SIFI banks are also, in aggregate, facing losses. Therefore, these non-SIFI banks would be forced to liquidate these capital holdings to the traditional sector, and we would still end up with a misallocation of capital. This is because, in the bad state of the world, there are losses, incurred from risky investments, that need to be absorbed by some agents in the economy. In order to prevent a misallocation of capital, the government would need to cover losses of other banks via a transfer financed by taxing the household. This is effectively a bailout of non-SIFI banks.

However, recall from section 2.9 that the government cannot bail out banks whose losses it cannot verify. Because the government cannot verify exposures from interbank claims, it would then be infeasible for the government to bail out non-SIFI banks, as these banks are facing losses only from their holdings of interbank claims. These frictions prevent the government from perfectly targeting bailouts to non-SIFI banks. Otherwise, the government could simply design a bailout of all banks except for the SIFIs, without ever having to directly reallocate capital across banks. As we discussed above in Part (A), this does not happen in practice for various reasons.

APPENDIX 10: Proof of Lemma 1

We prove Lemma 7 by backward induction. We have already characterized banks' optimal decisions at dates 1 and 2. Given these, we also characterized each investing bank's best response function for its date 0 portfolio choice. We now prove that, given these best response functions, there exist exactly two subgame perfect Nash equilibria.

Recall that, to complete the characterization of general equilibrium, it remains to determine the investment choices ω^w of investing banks, and to determine which banks are in the set W of investing banks in equilibrium. Once these are determined jointly, the investment choices ω^i of all other banks (i.e. banks in the set $L = I/W$, who simply invest in the liabilities of investing banks) are irrelevant for the allocation.

Proof: The proof is in three parts. In all cases, we make use of the best response functions

$$\omega^i\left(\{\omega^w\}_{w\in W}\right) = \begin{cases} 1 & if\ g^w(L,\omega^w) = 0 \\ 0 & otherwise \end{cases}.$$

Claim (i): $\{\omega^w = 1 \quad \forall w \in W\}$ is an equilibrium. This is the 'prudent' equilibrium, as all banks undertake the prudent investment.

Proof: We will show that, when all investing banks in set W choose $\omega^w = 1$, then bank $w \in W$ has no incentive to deviate from $\omega^w = 1$. Suppose that all investing banks choose $\omega^w = 1$. Recall from the government's optimal bailout policy that when all investing banks are exposed to risky projects, then there is never a bailout in the low state at date 1, i.e. $g^i(s,\omega^w) = 0$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 1$.

Also, recall in that we showed in the partial equilibrium characterization of optimal interbank contracts that the set of investing banks J is given by $J = W \equiv \left\{w \mid w\ \equiv \max_{i\in M}\ E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]\right\}$. In this case when $\omega^w = 1 \quad \forall w \in W$, all banks are invested in only to prudent assets, so that that

$E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is the same for all banks i. Therefore, the structure of interbank lending in this equilibrium, and therefore the set of investing banks W, is indeterminate – in this prudent equilibrium, we can have any combination of banks investing in the prudent project on their own behalf, with rest of banks investing in their liabilities. W is non-empty, so that at least one bank invests in the prudent project in equilibrium.

Claim (ii): $\{\omega^w = 0 \quad \forall w \in W\}$ is also an equilibrium, where $w \in W \iff \rho^w = \bar{\rho}$. This is the 'risky' equilibrium, as all investing banks invest in the riskiest project available.

Proof: We will show that, when all banks set $\omega_C^j = 0$, then bank i has no incentive to deviate from $\omega_C^i = 0$. Suppose that all investing banks choose $\omega^w = 0$. Recall from the government's optimal bailout policy that when all investing banks are exposed to risky projects, then there is a bailout in the low state at date 1 given by $\hat{g}^i(s,\omega^w) = q(s) - \Gamma A - R_A^i(s)$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 0$.

Again, we showed that interbank contracts in equilibrium are such that the set of investing banks J is given by $J = W \equiv \left\{w \mid w \equiv max_{i\in M}\, E\left[z_1(s)\theta_k^i\left(s,\bar{\omega}^i,g^i\right)\right]\right\}$. Since $z_1(s) = m_1(s)$ is proportional to $u'(c_1(s))$ and in this case $\theta_k^i\left(s,\omega^i,g^i\right) = R_A^i(s) + g^i(s,\omega^i) = \rho^i R_A(s) - \mu^i + g^i(s,\omega^i)$, it is easy to show that $E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is monotonically increasing in ρ^i. This is because: (i) $u(\cdot)$ is strictly concave; (ii) the variance of $R_A^i(s)$ is increasing in ρ^i, while its mean is independent of ρ^i; and (iii) the government's optimal $g^i(s,\omega^i)$ bounds $\theta_k^i\left(s,\omega^i,g^i\right)$ from below by $1-\Gamma$. Therefore, $E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is highest for the bank with the greatest potential exposure to the aggregate shock, $\rho^i = \bar{\rho}$. Hence, $W = \{w \in W \mid \rho^w = \bar{\rho}\}$, i.e. only banks with access to the riskiest projects invest in equilibrium, while the rest of banks invest in the liabilities of these risky banks.

Claim (iii): There are no other equilibria.

Proof: Suppose for the sake of contradiction that some $\{\omega^w\}_{w\in W}$ is an equilibrium, where $\{\omega^w\}_{w\in W} \neq \{\omega^w = 1 \quad \forall w \in W\}$ and $\{\omega^w\}_{w\in W} \neq \{\omega^w = 0 \quad \forall w \in W\}$. The government's optimal bailout policy implies that, in any equilibrium, either $g^w(L,\omega^w) = 1 - \Gamma A - R_A^i(L)$ for some $w \in W$ (i.e. a crisis and bailout occurs in the bad state) or $g^w(s) = 0$ for all s (i.e. a crisis and bailout never occur). Take the latter case in which we always have $g^w(s) = 0$. Then all investing bank w's best response functions favor investing only in the prudent project by setting $\omega^w = 1$. Moreover, this is consistent with having $g^w(s) = 0$. So we must have $\{\omega^w\}_{w\varepsilon W} = \{\omega^w = 1 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium.

Now suppose that we have a bailout in the bad state. Then the best response function of each investing bank implies all investing banks invest only in the risky their risky projects by choosing $\omega^w = 0$, which is consistent with having a bailout in the bad state. So we must have $\{\omega^w\}_{w\in W} = \{\omega^w = 0 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium either. Therefore, any equilibrium must be either the prudent equilibrium in which $\{\omega^w = 1 \quad \forall w \in W\}$, or the risky equilibrium in which $\{\omega^w = 0 \quad \forall w \in W\}$. Q.E.D.

Uniqueness of SIFI

Although the results above imply that, in the risky equilibrium, the SIFI is always the riskiest bank (i.e. the bank with the highest ρ^i), it may be instructive to reiterate why this is necessarily the case. Suppose we have an equilibrium with risk taking in which bank j is the only investing bank, where $\rho^j < \rho^h$ for some h (i.e. bank j is not the riskiest bank). Can this be an equilibrium? Given that bank j is the only investing bank, it will be bailed out in the bad state. All other banks have incentive to lend their funds to bank j in order to benefit from the bailout in the bad state. Bank j in turn invests in its risky project. Indeed, other banks may not have incentive to deviate and lend to a different bank (since it may not be bailed out) or invest in its own project. (This would indeed be the case if the government announced in advance that it would bail out the least risky investing bank.)

However, ex ante, bank j has incentive to deviate and lend all of its funds to the riskiest bank h. This is because, giving the perfect risk sharing facilitated by interbank contracts, it would benefit from a higher upside in the good state, and still benefit equally from the bailout in the bad state. Therefore, this cannot be an equilibrium. Indeed, the only risky equilibrium feature precisely bank h as the unique investing bank.

APPENDIX 11: Full planner problem

The planner's problem is to choose $c_t(s)$, f_0^i, $B_1(s)$, d_0^i, $b_1^i(s)$, $b_2^i(s)$, ℓ^{ji}, $r^{ji}(s)$, k_0^i, $k_1^i(s)$, ω^i, $T_1(s)$, and $g^i(s,\omega^i)$ for all banks i, j, all states s and all periods t to solve

$$\max \ E\left[u(c_0)+u(c_1(s))+u(c_2(s))\right]$$

s.t.

$$c_0 + \Sigma_i f_0^i d_0^i \leq e_0 - T_0 \tag{60}$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1(s) - q(s) k_1^T(s) \tag{61}$$

$$c_2(s) \le e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) \tag{62}$$

Final dividend payout (including dividend from traditional firms)

$$\Pi_2(s) = \sum_i \left(A - b_2^i(s)\right) k_1^i(s) + F(k_1^T(s))$$

budget constraints

$$k_0^i + \sum_j \ell^{ij} \le n + d_0^i + \sum_j \ell^{ji} \tag{63}$$

$$q(s)k_1^i(s) \le \theta_k^i\left(s,\omega^i,g^i\right)k_0^i + \sum_j \theta_\ell^i(s,j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s)\right)\ell^{hi} + b_2^i(s)k_1^i(s)$$

no-default constraints for the household contract

$$0 \le b_1^i(s) \le q(s) - \gamma \tag{64}$$

$$0 \le b_2^i(s) \le \Gamma A \tag{65}$$

the other firms' participation constraints for each j

$$u^{ji}\left(\ell^{ji}, \left\{r^{ji}(s)\right\}_s\right) \ge \bar{u}^j \tag{66}$$

and non-negativity constraints on capital holdings and interbank loans.

$$k_0^i, k_1^i(s), \ell^{ij} \ge 0 \quad \forall j \tag{67}$$

asset prices

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

the government's optimal bailout policy

$$k_0^w g^w(s,\omega^w) = \begin{cases} \left(q(s) - b_2^w(s)\right)\sum_i k_0^i - \sum_i \frac{q(s)-b_2^w(s)}{q(s)-b_2^i(s)}X & for\ s = L \\ 0 & otherwise \end{cases}$$

where

$$X \equiv \left(q(s)+\omega^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)-\gamma-b_1^i(s)\right)k_0^i+\sum_j \theta_\ell^i(s,j)\ell^{ij}-\sum_h\left(r^{hi}(s)-b_1^i(s)\right)\ell^{hi}$$

and the government budget constraint

$$\sum_j k_0^j g^j(s,\omega^j)+D\left(k_1^T(s)\right)=T_1(s). \tag{68}$$

Recall that the government's optimal bailout policy implies capital is never misallocated at date 1. Therefore, we have $q(s)=1, k_1^T(s)=0$. Imposing that the government budget constraint binds, replace date 1 taxes $T_1(s)$. We also replace $d_1^i(s)$ and $d_2^i(s)$ using the definitions of $b_1^i(s)$ and $b_2^i(s)$.

Notice that the planner takes the constraints of all banks i as constraints simultaneously in the Lagrangian. Hence, unlike in the competitive economy, the planner's first order conditions for ℓ^{ij} and $r^{ji}(s)$ will also capture how they affect the budget constraints of other banks j (i.e. k_0^j and $k_1^j(s)$). The planner's first order conditions are

$$\frac{\partial L'}{\partial f_0^i}\leq 0 \iff \quad E\left[u'\left(c_2(s)\right)\right]b_2^i(s)k_1^i(s)+\ldots \tag{69}$$

$$\ldots+E\left[u'\left(c_1(s)\right)\left(\left[b_1^i(s)\left(k_0^i-\sum_h \ell^{hi}+\sum_j \ell^{ij}\right)-b_2^i(s)k_1^i(s)\right]\right)\right]-E\left[u'\left(c_0\right)\right]d_0^i\leq 0$$

$$\frac{\partial L'}{\partial B_1(s)}\leq 0 \iff \quad E\left[u'\left(c_2(s)\right)\right]-E\left[u'\left(c_1(s)\right)\right]\leq 0 \tag{70}$$

$$\frac{\partial L'}{\partial d_0^i}\leq 0 \iff \quad -u'\left(c_0\right)f_0^i+z_0^i\leq 0 \tag{71}$$

$$\frac{\partial L'}{\partial k_0^i}\leq 0 \iff \quad E\left[u'\left(c_1(s)\right)f_0^ib_1^i(s)\right]-z_0^i+E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]-E\left[u'\left(c_1(s)\right)\frac{\partial T_1(s)}{\partial k_0^i}\right]\leq 0 \tag{72}$$

$$\frac{\partial L'}{\partial k_1^i(s)}\leq 0 \iff \quad -u'\left(c_1(s)\right)f_0^ib_2^i(s)+u'\left(c_2(s)\right)f_0^ib_2^i(s)+u'\left(c_2(s)\right)\left(A-b_2^i(s)\right)-z_1^i(s)\left(1-b_2^i(s)\right)\leq 0 \tag{73}$$

$$\frac{\partial L'}{\partial b_1^i(s)} \leq 0 \iff \left(u'(c_1(s)) f_0^i - z_1^i(s)\right)\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) - u'(c_1(s)) \frac{\partial T_1(s)}{\partial b_1^i(s)} \leq \bar{\lambda}_1^i(s) - \bar{\lambda}_0^i(s) \tag{74}$$

$$\frac{\partial L'}{\partial b_2^i(s)} \leq 0 \iff -u'(c_1(s)) f_0^i k_1^i(s) + u'(c_2(s)) f_0^i k_1^i(s) - \ldots \tag{75}$$

$$\ldots - u'(c_2(s)) k_1^i(s) + z_1^i(s) k_1^i(s) - u'(c_1(s)) \frac{\partial T_1(s)}{\partial b_2^i(s)} \leq \mu_1^i(s) - \mu_0^i(s)$$

$$\frac{\partial L'}{\partial \omega^i} \leq 0 \iff E\left[z_1^i(s) k_0^i \frac{\partial \theta_k^i(s, \omega^i, g^i)}{\partial \omega^i}\right] - E\left[u'(c_1(s)) \frac{\partial T_1(s)}{\partial \omega^i}\right] \leq 0 \tag{76}$$

$$\frac{\partial L'}{\partial \ell^{ij}} \leq 0 \iff E\left[u'(c_1(s)) f_0^i b_1^i(s)\right] - z_0^i + z_0^j + E\left[z_1^i(s) \theta_\ell^i(s, j)\right] - E\left[u'(c_1(s)) \frac{\partial T_1(s)}{\partial \ell^{ij}}\right] \leq 0 \tag{77}$$

$$\frac{\partial L'}{\partial r^{ji}(s)} \leq 0 \iff -z_1^i(s)\ell^{ji} + z_1^j(s)\ell^{ji} - \hat{v}^{ji} \frac{\partial u^{ji}\left(\ell^{ji}, \{r^{ji}(s)\}_s\right)}{\partial r^{ji}(s)} - u'(c_1(s)) \frac{\partial T_1(s)}{\partial r^{ji}(s)} \leq 0 \tag{78}$$

APPENDIX 12: Proof of Proposition 1

This follows from the linearity of the firm's portfolio allocation problem. Namely, the optimality conditions for the bank's portfolio allocation decisions for k_0^i,ℓ^{ij}, and ω^i do not depend on size of the firm's investment. Therefore, it immediately follows that, for each firm i, we have one of two cases. Either we are in case 1, in which there is a firm $j \neq i$ such that $E\left[z_1^i(s)\theta_\ell^i(s,j)\right] \geq E\left[z_1^i(s)\theta_\ell^i(s,h)\right]$ for all other firms h, and $E\left[z_1^i(s)\theta_\ell^i(s,j)\right] \geq E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ for any $\omega^i \in [0,1]$. In this case, the contract offered by firm j to firm i has a more favorable risk-return tradeoff that that offered to i by any other firm h. In addition, the return to lending to firm j is preferable to investing any amount in either the risky or prudent project on i's own behalf. In case 1, we have $k_0^i = 0$ and $\ell^{ij} > 0$, meaning the firm forgoes investing in its own projects in favor of lending to firm j.

The other possibility is that we are in case 2, in which there is a $\tilde{\omega}^i \in [0,1]$ such that $E\left[z_1^i(s)\theta_k^i\left(s,\tilde{\omega}^i,g^i\right)\right] \geq E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ for all $\omega^i \neq \tilde{\omega}^i$ and $E\left[z_1^i(s)\theta_k^i\left(s,\tilde{\omega}^i,g^i\right)\right] \geq E\left[z_1^i(s)\theta_\ell^i(s,h)\right]$ $\forall h$. This implies that at the optimal ω^i, the return to investing ω^i in the prudent project and $1-\omega^i$ of its capital has a more favorable risk-return profile than the returns offered by any firm's inter-firm contract. In case 2, we have $k_0^i > 0$ and $\ell^{ij} = 0$ for all j, meaning the firm does not lend to any other firm. Furthermore, since the condition for ω^i does not depend on ω^i, firm i will always be at a corner solution in its choice of ω^i, so that the optimal ω^i satisfies $\tilde{\omega}^i \in \{0,1\}$. (This is partly due to the fact that, in the government's optimization problem, we will show that g^i will be zero for $\omega^i = 1$.) Q.E.D.

APPENDIX 13: Benchmark 2: Comparative static on degree of risk aversion

How does risk sharing between the SIFI and non-SIFI banks generate excessive risk taking? In this benchmark variant of the model, we isolate the role of risk sharing *per se* in generating excessive risk taking by all banks by varying the degree of risk aversion of agents in the model.

In general, the interbank market plays two roles in the risky equilibrium. First, it directs funds at date 0 to the projects with the highest expected return. Second, as we showed in section 3.2.3., the interbank market facilitates risk sharing between SIFIs and other banks by allowing other banks to benefit from the government guarantee indirectly, thereby reducing the variance of their portfolios. This second risk sharing motive of interbank lending arises because the stochastic discount factor reflects the household's risk aversion. To elucidate this point we modify the model in this section so that only the risk sharing role of the interbank market ultimately affects banks portfolio choices. Then when capture how risk sharing incetivizes risk taking through a comparative static exercise by varying the degree of risk aversion of the household.

To do this, we modify the baseline model in three respects. First, for concreteness, we suppose that the representative household's utility feature constant relative risk aversion so that, $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$, where $0 \leq \eta \leq 1$. Second, rather than assuming that all risky projects are a mean-preserving spread of the prudent project, we now assume that $R_C > E\left[R_A^i(s)\right]$ for all i.[49] This implies that the risky projects are not only riskier than the prudent project, but also offer a lower expected return. Moreover, we assume that a stronger condition holds: $\pi(H)R_A^i(H) + \pi(L)(1-\Gamma) < R_C$. This assumption will ensure that the higher expected return on risky assets afforded by the government guarantee is not sufficient by itself to entice banks to invest in risky assets. Third, we make assumption 6 on the relative size of $\pi(L)$ relative to the degree of the household's risk aversion and the risky return which we call on below, where j is the bank with $\rho^j = \bar{\rho}$.

Assumption OA.1: $\frac{R_A^j(H)^{-\eta}\left[R_A^j(H)-R_C\right]}{R_A^j(H)^{-\eta}\left(R_A^j(H)-R_C\right)+R_A^j(L)^{-\eta}(R_C-1+\Gamma)} < \pi(L) < \frac{R_A^j(H)-R_C}{R_A^j(H)-1+\Gamma}$

[49]For this to hold, we need to assume that our assumption that $R_C \geq 1-\Gamma$ instead holds with strict inequality.

$\pi(L)$ satisfying assumption OA.1 exists in the domain (0, 1).

In this modified environment, the characterization of the date 1 spot market for capital and optimal interbank and household contracts all go through. Moreover, the government's optimal bailout policy is still characterized by Proposition 5. Therefore, to characterize the equilibrium in this version of the model, it remains to characterize banks' best response functions for their date 0 portfolio choices and interbank lending decisions. We characterize these best response functions for different degrees of the household's risk aversion η.

How does risk sharing affect portfolio choices, risk taking? Recall from section 3.5.1. that the value to bank i of an interbank claim issued by a SIFI w promising a return $\theta_\ell^i(s,w)$ is given by the sum of the expected discounted return $E[m_1(s)]\,E\left[\theta_\ell^i(s,w)\right]$ and a risk premium component given by still given by $Cov\left(m_1(s),\theta_\ell^i(s,w)\right)$. We already showed in Corollary 1 that the implicit guarantee lowers riskiness of SIFI's assets, and that the interbank market facilitates risk sharing between the SIFI and non-SIFI banks whereby banks can benefit from safety of the SIFIs interbank claims. These results apply in this modified setting as well. We now vary the degree of risk aversion of the household to show how this interbank risk sharing actually exacerbates excessive risk taking, generating collective risk shifting problem.

First suppose that $\eta = 0$, so that the household is risk neutral. In this case, the stochastic discount factor $m_1(s)$ is constant across states, and so the covariance term is 0. Agents do not value risk sharing - the variance of their portfolios is irrelevant for their portfolio choice and they care only about the expected return. Since the bailout policy $g^w(s)$ is given by by Proposition 5, our assumption above $\pi(H)R_A^w(H)+\pi(L)(1-\Gamma) < R_C$ implies that $E[R_A^w(s)+g^w(s)] < R_C$. Therefore, banks never want to invest in interbank claims issued by SIFIs, because the government guarantee does not increase the expected return on these claims sufficiently to entice banks away from prudent assets. As a result, each bank i's best response function is to always invest in prudent assets. As a result, no bank ever undertakes a risky investment in equilibrium. This is summarized in the corollary below.

Corollary: *No excessive risk taking with risk neutrality*

Under Benchmark economy 2, when the household is risk neutral ($\eta = 0$), there is never excessive risk taking in equilibrium by any bank.

Now suppose that the household is risk averse, so that $\eta > 0$. As the household's risk aversion increases, banks care more about the covariance of their portfolio returns with the stochastic discount factor, and therefore the risk premium on an interbank claim issued by the SIFI w is lower, as captured by a higher $Cov\left(m_1(s),\theta_\ell^i(s,w)\right)$. In other words, the safety offered by the SIFI's interbank claim is valued more by non-SIFI banks.

How does this affect banks' portfolio choices? Assumption OA.1 implies that $E\left[m_1(s)\left(R_A(s)+g(s)\right)\right] > E\left[m_1(s)R_C\right]$. As a result, non-SIFI banks choose to invest in claims issued by the SIFI. Therefore, the insurance value of interbank claims issued by the SIFI (together with expected discounted return) is sufficiently high to entice banks to forgo their prudent projects in favor of buying financial claims issued by the SIFI. (At same time, the SIFI invests in its risky project.) As a result, the risk sharing facilitated by the interbank market incentivizes excessive risk taking.

Corollary: *Risk sharing generates excessive risk taking by all banks*

When the household is risk averse, the insurance value of interbank claims issued by the SIFI is sufficiently high to entice non-SIFI banks to forgo their prudent investments in favor of buying claims on the SIFI's portfolio. As a result, in equilibrium, the SIFI invests in its risky project and non-SIFIs invest in financial claims issued by SIFI.

Takeaway These comparative static exercises show that, in Benchmark economy 2, risk sharing between the SIFI and non-SIFI banks in the risky equilibrium is precisely what facilitates excessive risk taking in the first place. When the insurance value of interbank claims on the SIFI are low, banks do not have incentive to invest in risky assets. Only when the insurance provided by these SIFI claims is sufficiently high do banks undertake excessive risks.