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The Fed's Discount Window in "Normal" Times

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Abstract

We study new transaction-level data of discount window borrowing in the U.S. from 2010-17, merged with quarterly data on bank financial conditions (balance sheet and revenue). The objective is to improve our understanding of the reasons why banks use the discount window during periods outside financial crises. We also provide a model of the decision of banks to borrow at the window, which is helpful for interpreting the data. We find that decisions to gain access and to borrow at the discount window are meaningfully correlated with some relevant banks' characteristics and the composition of banks' balance sheets. Banks choose simultaneously to obtain access to the discount window and hold more cash-like liquidity as a proportion of assets. Yet, conditional on access, larger and less liquid banks tend to borrow more from the discount window. In general, our findings suggest that banks adapt their operations in conjunction with their use of the discount window in "normal" times.

JEL CLASSIFICATION: E52, E58, G28
KEYWORDS: Banking, Federal Reserve, Central Bank, Liquidity

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1 Introduction

The discount window is a long-standing and important institution in U.S. financial markets. The idea behind having the central bank acting as the lender of last resort during a financial crisis has been around for more than two hundred years (Humphrey (1989)). While the role of the central bank as a backup source of funding has been particularly important during financial crises, it is the case that the U.S. discount window is open *at all times*, not just during crises. It is also regularly used during those "normal" times. Having the discount window open involves explicit (operational) and implicit (incentives) costs. Then, it is natural to ask the following questions: What is the role of the discount window during normal times? What is the discount window helping with at those times? Who borrows from the discount window outside of crises and why? We address these questions in this paper.

Discount window lending in the U.S. is thought to play a dual role during normal times. In one role, it is intended to induce a ceiling on interbank market interest rates, allowing the Fed to maintain control of short-term rates as part of the process of monetary policy implementation (Ennis and Weinberg (2016)). In its other role, the discount window is considered a vehicle for providing short-term liquidity insurance to eligible depository institutions. Both objectives are related insofar as interbank market rates reflect, at times, liquidity events experienced by subgroups of banks in the system.

In both cases, market frictions play a role in the justification of central bank lending. If frictions are not significant, then open market operations should be sufficient to achieve the objectives of the central bank (Goodfriend and King (1988)). It is possible that frictions manifest themselves with different strengths depending on general conditions in financial markets. So, in the midst of a financial crisis, banks may face stronger impediments to finding suitable counterparties and, as a result, trading may be particularly impaired. During calmer times, however, the relevant frictions may be less acute and, possibly, somewhat different in nature.

At the same time, there are (explicit and implicit) costs associated with having a discount window open at all times. Aside from the obvious operational costs, the central bank is often involved in monitoring potential counterparties to be able to discern in a timely manner when illiquidity is (and is not) directly tied to insolvency. Aside from these explicit costs, there are also potential moral hazard costs linked to the availability of central bank funding, as this backup support changes the incentives of banks to manage their liquidity and credit risk (Ennis and Price (2015)).

Using assessments of these costs and benefits based on very limited data, many observers over the years have suggested the possibility of not having a discount window available at all times. Friedman (1960) wrote that "rediscounting should be eliminated" after reviewing possible pros and cons and concluding that the latter clearly outweigh the former. Schwartz (1992) also provides a particularly pointed indictment of discount window lending, based on the argument that, at times, large amounts of discount window funding have gone to banks not experiencing a temporary liquidity shortfall, but rather to those suffering an insolvency problem. In a related argument, Goodfriend and King (1988) object to discount window lending based on the idea that it is not essential for monetary policy and, when relevant, it contains an element of credit allocation that is best left to the fiscal authority. More recently, Selgin (2017) has proposed a complete overhaul of discount window practices based on similar arguments.

On the other side of the debate, Clouse (2000), for example, argues that the discount window is an important backup source of liquidity, filling a gap for occasional liquidity shortfalls. In the same line of thinking, Fischer (2016) also unambiguously supports the idea that the discount window has a role in promoting stability in the U.S. financial system, not only during major financial crises, but more generally, at all times.

Another, more practical argument for keeping the discount window open in the period between crises is to have it "ready to go" when a crisis happens. However, it may be possible to re-activate a facility in a timely manner at the onset of a crisis. If a lending facility targeting banks can be closed down when markets and the economy are working smoothly and then re-opened (if necessary) when a crisis first hits, an independent evaluation of the costs and benefits exclusively outside of crises is indeed the appropriate approach.

To be able to undertake such an evaluation, it is essential to understand better the motivations of borrowers who tap the discount window during normal times. With that in mind, we search for and identify systematic patterns in the data that inform us about the type of banks that use the discount window and the financial conditions of those banks at the time of borrowing. Knowing these patterns should allow us to make better judgements related to the essentiality (or lack of thereof) of a permanent discount window. It can also help us design substitute arrangements to address more directly specific problems when they exist.

Until recently, public information about activity at the discount window was very limited. Traditionally, the Federal Reserve published discount window lending only at an aggregate level and at a weekly frequency. One of the justifications for providing limited information has been the fear that the disclosure of information could impact the effectiveness of the facility (Kleymenova (2016)). Indeed, in March 2020, the Fed made further changes to its weekly reporting to reduce the amount of discount window information available at high frequency. The view supporting such move is that banks might become reluctant to access the discount window if they perceive that the information will be made public and subsequently interpreted in an adverse manner by potential counterparties. This type of stigma effect is often discussed by policymakers (Bernanke (2008)) and has received some attention in the theoretical and empirical literature.[1]

A competing view and a common reaction to the events of the 2008 financial crisis is that transparency is particularly important when it comes to the administration of government lending programs. In response to such demand for extra information, starting in July 2010, the Dodd-Frank Act requires the Fed to make public more detailed information about individual loans taken at the discount window. In a compromise that reflects the concerns associated with excessive disclosure of information, the transaction-level data is released with, approximately, a two-year delay. The availability of this new, more detailed information provides an opportunity to take a closer look at the actual borrowing that occurs at the discount window. We take that opportunity in this paper using the first seven years of data.

We focus on the period between July 2010 and December 2017, a period of relative calm in U.S. financial markets.[2] One important factor that characterizes our sample

[1] See, for example, Klee (2019), Ennis and Weinberg (2013), Armantier et al. (2015), Gauthier et al. (2015), Ennis (2019), Ennis and Price (2020), and the citations therein.

[2] The role of the discount window during crises has been more extensively studied in recent years. For example, Berger et al. (2017), Gauthier et al. (2015), Li, Milne, and Qiu (2020), Gilbert et al. (2012), and Gerlach and Beyhaghi (2020) study empirically discount window lending in the U.S. during the financial crisis. Klee (2019) and Armantier et al. (2015) focus more narrowly on discount window stigma, also during the financial crisis. Drechsler et al. (2016) study empirically discount window lending in Europe during the financial and sovereign debt crises. The study by Gerlach and Beyhaghi (2020) includes the period under consideration here, but the focus of that paper is on the signal value of discount window activity about the financial conditions of the bank and, in particular, their probability of failure. Here, instead, we focus on the determinants of discount window activity.

period is that the banking system was operating in an environment with ample reserves (Carpenter et al. (2015), Ennis and Wolman (2015)). This was unprecedented and a significant change from pre-crisis conditions (i.e., the previous period of "normal times"). In principle, abundant reserves would tend to reduce the chances of banks experiencing liquidity shocks that push them to borrow from the discount window. This tendency notwithstanding, we observe non-trivial amounts of borrowing at the discount window during the period under consideration (Ackon and Ennis (2017)).[3]

As a preliminary step, we report in Section 2 some broad correlations between bank characteristics and the use of the discount window. We find that larger banks are more likely to borrow from the discount window–that is, as a group, the percent of large banks that borrow is relatively high–even though most of the borrowing is done by smaller banks, which are more numerous. Borrowers tend to hold less reserves and more illiquid asset portfolios. On the liability side, borrowers rely more on short term funding (such as repurchase agreements). In this way, based on the composition of their liabilities, borrower banks look more similar to larger banks, while on the asset side similarities are less evident. Borrowers also seem to have more risky assets that tend to lower their risk-based capital ratios. Again, this is something that one can observe in larger banks. In general, then, discount window borrowers share some characteristics with larger banks, although less than 30% of the banks borrowing at the discount window are larger than $1 billion in assets. Accounting for these broad patterns is relevant as a preliminary step, but the confounding of size with other various characteristics quickly makes clear that a multivariate analysis is needed to untangle the origin of such patterns.

Before moving to a more thorough empirical analysis, we present in Section 3 a model of the decision of a bank that, under some circumstances, values having access to the discount window and borrows from it. The model is intended as a framework to guide our thinking when interpreting the various patterns uncovered in the data. The decision of the bank in our model is similar to that studied recently in Ennis (2018) and Afonso, Armenter, and Lester (2019) and is in the tradition of Poole (1968) and the extensive literature that sprang after that seminal paper.[4] Relative to the

[3]Large quantities of excess reserves in the system imply that many banks may be close to indifference when it comes to holding the marginal unit of reserves. Hence, holding patterns in the cross section of banks may be harder to identify. However, as our model will illustrate, for those banks that actually borrow from the discount window, their holdings of reserves are likely more tightly linked to other financial decisions.

[4] See, for example, Ennis and Keister (2008) and, more recently, Bianchi and Bigio (2014), Armenter

previous literature, and given our interest in aiding the interpretation of the discount window data, our model involves a more complete description of the bank's balance sheet and a more flexible interpretation of the trading possibilities of the bank facing a liquidity or payment shock.

In the model, when the interest rate charged at the discount window is higher than the rate in the interbank market, as is generally the case in the U.S., and the bank has access to the interbank market, it will not use the discount window. However, in some situations, depending on the timing of a shock, the bank my not have ready access to the discount window when the shock occurs. In such a case, the bank will follow a "pecking order" to cover its liquidity need, using first its holdings of reserves, then discount window borrowing and, finally, if the shock is large enough to exhaust the bank's collateral pledged at the discount window, then the bank will incur a more expensive overdraft in its account at the central bank. This pecking order, in turn, determines the way the bank will choose ex ante its level of reserves and other components of its balance sheet. The model also illustrates how the structure of the distribution of the shock influences the ex-ante choices of the bank with respect to its level of reserves and how such level interacts with the shock to determine the probability of the bank actually borrowing from the discount window. To close this section, we investigate the bank's decision to gain access to the discount window and how that decision is connected to the decision to hold bank reserves. This will help us in the interpretation of some of the empirical results that relate to how banks that have access to the discount window behave in different ways relative to those that do not.

Section 4 presents the multivariate empirical analysis and our main empirical results. We start our analysis by looking at the probability that a bank borrows from the discount window, conditional on having access to it (that is, having taken the necessary actions – such as pledging collateral – to have ready access to the discount window if the need arises). We establish conclusively that banks holding less bank reserves (as a proportion of assets) are more likely to tap the discount window for funding. This finding survives when we control for the bank's balance sheet composition, size measured by assets, and a number of other bank characteristics that, in principle, could matter. Furthermore, the finding is robust to accounting for possible endogeneity in reserve holdings.

and Lester (2017), Berentsen, Kraenzlin, and Mueller (2018).

We also confirm that larger banks are more likely to borrow, as discussed in Section 2. This is the case even after controlling for various observable bank characteristics. One possibility that may explain this finding is that larger banks have a more sophisticated liquidity-management process that integrates the discount window more effectively. In line with this idea, we find that size matters most when we restrict the sample to banks below $1 billion in total assets. That sample presumably includes highly heterogeneous banks, with very different levels of sophistication, which asset size is helping to capture.

In terms of balance sheet composition, banks with more illiquid and riskier asset portfolios are more likely to borrow from the discount window, even after controlling for bank size. Similarly, those banks relying more heavily on less stable funding sources also find themselves more likely to need to borrow from the discount window. Interestingly, even after controlling for size and other bank characteristics, banks in certain Federal Reserve districts appear more likely to use the discount window than those in other districts. This seems to suggest that there may be a certain degree of differential treatment with respect to discount window usage across Fed districts.

We also investigate in some detail how the decision to obtain access to the discount window interacts with the choice of balance sheet composition. While in general banks that have access to the discount window hold less reserves (as a proportion of assets), the magnitude of this effect is relatively small – and, in fact, adding controls tends to attenuate it. Of course, there is possible endogeneity in the choice of reserves and access to the discount window. When we try to control for that endogeneity using a treatment-effects model, we see that in fact the impact on reserves from gaining access to the discount window goes in the opposite direction: banks that gain access tend to also increase their holdings of reserves. This is not what we expected based on the model in Section 3 and suggests that there still may be important unobserved factors driving both decisions.

Taken together, our empirical results suggest that banks' decisions to gain access to the discount window, and their consequent exposure to borrowing from it during "normal" times, result from a deliberate liquidity-management effort undertaken by each bank. Banks are intentional with respect to their use of the discount window and adapt their balance sheet accordingly.

We close the paper with a short Section 5, where we briefly discuss how we read and understand our findings more broadly and, then, conclude.

2 The data

Our dataset comes from a combination of various regulatory and central bank data-collecting efforts. The primary source is detailed information on daily borrowings at the discount window, available on the Federal Reserve Board's public website.[5] The data include information on the name of the borrower, the size and duration of the discount window loan, the type of loan (primary credit, secondary credit, or seasonal credit), and the borrower's Federal Reserve district. Also available is information on the types and amounts of collateral the borrower has posted at the discount window for borrowing purposes. The universe is all discount window loans for each quarter from July 2010 to December 2017.

We pair the discount window data with information on bank balance sheets. We use quarterly Call Report filings for all depository institution eligible to borrow at the discount window, which includes commercial banks (form FFIEC 031/041), foreign banking organizations (FFIEC 002), and credit unions (NCUA 5300). These reports provide information on balance sheet items, including various assets and liabilities, as well as some off-balance sheet items, such as unused loan commitments.

2.1 An overview of the data

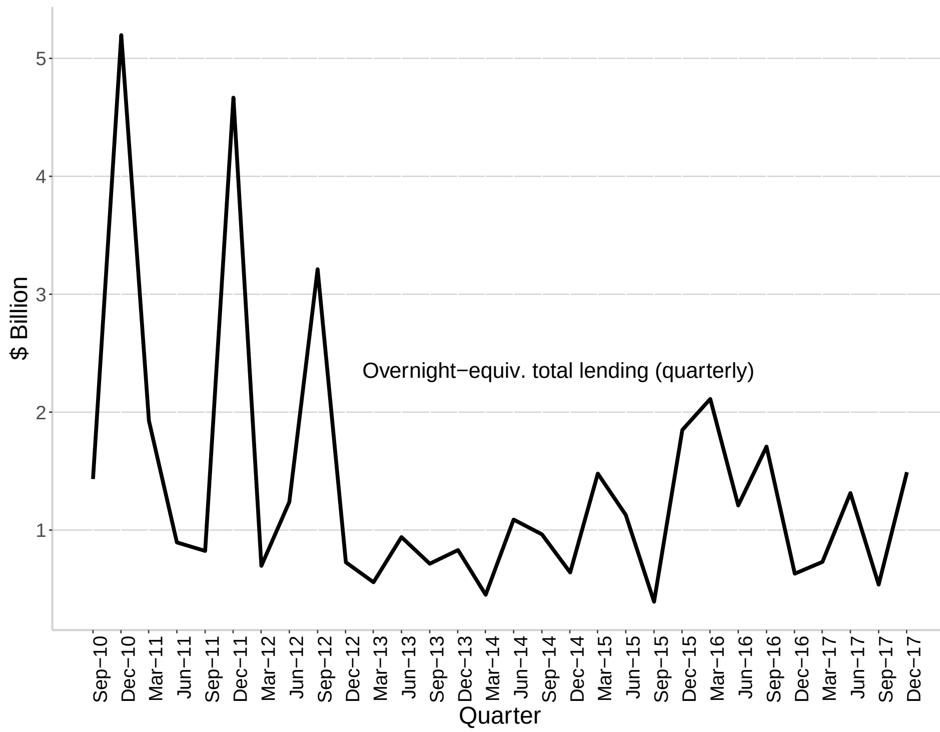
There are three discount window programs: primary credit, secondary credit, and seasonal credit. Primary and secondary credit are the main programs through which the Federal Reserve provides back-up, short-term funding to depository institutions. These two programs conform closely with the traditional lender-of-last-resort view of central bank liquidity provision. By contrast, the seasonal credit program is aimed at satisfying seasonal fluctuations in the funding needs of particular institutions.

The primary credit program is a standing facility (no questions asked) available to depository institutions in sound financial condition. Primary credit loans carry a penalty interest rate, which was 50 basis points higher than the rate of interest paid on reserves during our sample period. Secondary credit is available (subject to the discretion of each Reserve Bank) to depository institutions not eligible to borrow from the primary credit program. The penalty rate in the secondary credit program is 50 basis points higher than the rate paid for primary credit. Most lending in the

[5] Refer to https://www.federalreserve.gov/regreform/discount-window.htm.

primary and secondary credit programs is overnight.[6]

Figure 1: Total lending – Primary credit (quarterly)



We focus mainly on primary credit, as it is the program most often considered and used. Figure 1 plots the total amount of lending done in the primary credit program each quarter between July 2010 and December 2017. Because some loans have longer maturities than overnight, we calculate overnight-equivalent amounts that are then added up to a quarterly frequency.[7] While discount window lending is an order of magnitude smaller during normal times than in crisis periods, it is still a meaningful amount – in many quarters during the sample period more than a billion dollars in loans were made in the primary credit program.

In terms of the number of loans, between July 2010 and December 2017, there

[6] By contrast, seasonal credit is not provided at a penalty interest rate and instead is offered at a floating market rate based on the average of the federal funds rate and the rate on three-month CDs. The interest rate is reset every two weeks and applies to all outstanding seasonal credit loans. Moreover, seasonal credit is generally longer term than overnight. Refer to https://www.frbdiscountwindow.org/ for more details.

[7] In these calculations, a loan of $100 million for two days is equivalent to two overnight loans of $100 million each. This transformation is necessary to properly account for varying maturities across loans in the computation of aggregates.

were 17,802 discount window loans made in the primary credit program and 754 loans made in the secondary credit program (Table 1). Many of those loans were for relatively small amounts and were initiated by the borrowing institutions in order to test the processes and systems involved in executing the transaction (Ackon and Ennis (2017)).

Table 1: Primary and secondary credit, July 2010 to December 2017

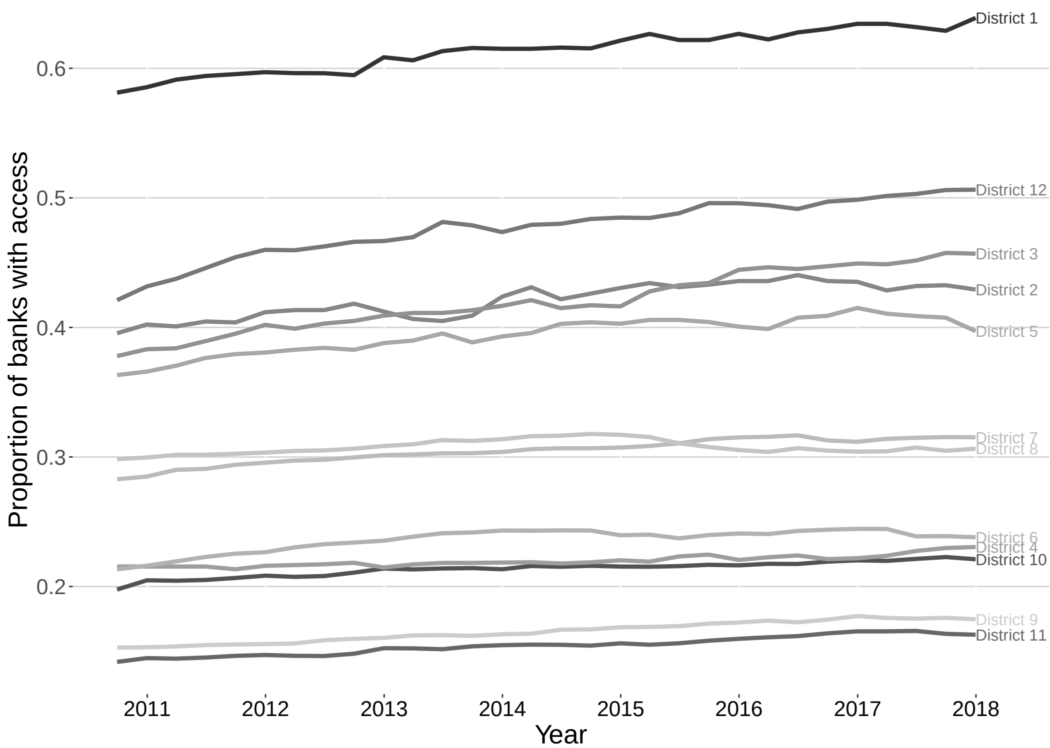
	All loans	Loans greater than or equal to $1 million					
			Amount in $ millions				
	N	N	Mean	Median	Min	Max	Std Dev
Primary credit	17,802	5,063	6.96	2.1	1	1,017	24.86
Secondary credit	754	27	4.19	2	1.2	17	4.25

The data do not include information on which loans are tests and which ones are not. To account for this fact and study banks' lending behavior uncontaminated from other administrative decisions, we focus our on loans greater than or equal to $1 million. Even though this subsample of 5,063 loans is much smaller than the full sample, we think that only loans of relatively larger size would reflect a deliberate economic decision on the part of the borrower.[8]

We still use the information on smaller loans and testing in meaningful ways. The information allows us to differentiate between institutions that have (immediate) access to the discount window and those that may not. To have access to the discount window, institutions need to submit several documents, including a lending agreement, and pledge collateral at their respective Reserve Banks. Since we do not have information on which institutions have the needed arrangements in place to borrow at the discount window, we use as a proxy for access whether the bank in question has taken a loan (test or otherwise) during our sample period. The behavior of banks that have access to the discount window can be expected to differ from the behavior of banks without access, and we will investigate this issue theoretically and empirically below.

[8] Smaller loans are not likely to receive much managerial scrutiny, either because the loan is just a test loan or because it is too small to warrant much evaluation – just as an example, a $1 million overnight loan at an interest rate 100 basis points higher than the alternative rate produces a total extra cost of approximately $30, i.e., a very small amount.

Figure 2: Proportion of domestic banks in each district with access to the discount window



As is evident from Figure 2, the prevalence of access to the discount window varies across Federal Reserve districts. The figure plots the proportion of domestic banks in each district that have access to the discount window (measured with the proxy described above). These proportions for District 1 (Boston) and District 12 (San Francisco), for example, are very different than those for districts 9 (Minneapolis) and 11 (Dallas). This variation, among other things, could reflect idiosyncrasies on the approach to lending of different Reserve Banks. We will use this variation as a source of exogenous variation (at the bank level) in our empirical strategy.

The discount window data include borrower information that allows us to split the sample into institutions of different types, such as domestic banks, credit unions, and foreign banking organizations (FBOs). As shown in Table 2, while most of the borrowing (defined as loans greater than or equal to $1 million) was done by small domestic banks (defined as those with less than $1 billion in assets), the percentage of institutions of each type that borrowed from the discount window is the highest for large domestic banks (32 percent), followed by large credit unions (30 percent) and large FBOs (27 percent). Interestingly, small credit unions are the least likely to borrow from the discount window.

Table 2: Discount window borrowing by type and size, July 2010-December 2017

	Number of banks	Borrow at least once		Borrow at least five times	
		Number	Percent of total	Number	Percent of total
Smaller banks					
Domestic banks	7,176	503	7	358	5
Credit unions	7,362	149	2	86	1
Foreign banks	163	24	17	11	7
Larger banks					
Domestic banks	598	190	32	128	21
Credit unions	165	49	30	30	18
Foreign banks	75	20	27	11	15

Note: This table provides summary statistics on balance sheet items for banks in the estimation sample. "Borrowers" are defined as domestic banks that file Call Reports, and that take out a discount window loan at least once for $1 million or more over the sample period. "Large banks" are defined as banks with at least $1 billion in assets at some point during the sample period. We eliminate banks with missing or negative assets, negative cash-to-asset ratios, loan-to-asset ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported posted discount window collateral greater than total assets. Eliminating these banks leads to a smaller sample size than used in Table 1.

To get a sense of intensity of use, we also compute the number of banks that borrowed at least five times during our sample period. Of the roughly 600 larger banks in our sample, 128 borrowed at least five times during the period, or 21 percent of the total. This figure is only 5 percent for smaller domestic banks. Taken together, these statistics suggest that larger domestic banks are more likely to borrow repeatedly from the discount window relative to smaller domestic banks.

Due to regulatory reporting rules, more information is available for domestic banks. For this reason, in general, we will narrow the attention to this important group. As we discussed, Table 2 suggests that larger domestic banks are more likely to borrow. At the same time, larger banks tend to have business models that can produce a differential approach to discount window borrowing. In Table 3, we report the average balance-sheet composition of domestic banks that borrowed at least one time

Table 3: Balance sheet ratios – Domestic banks

	Borrowers		Non-borrowers		Large banks	
	Mean	Median	Mean	Median	Mean	Median
Percent of assets						
Reserves	0.036	0.026	0.048	0.037	0.045	0.028
CRE loans	0.018	0.012	0.011	0.004	0.025	0.012
C&I loans	0.097	0.08	0.078	0.066	0.104	0.082
Treasury securities	0.004	0	0.006	0	0.007	0
Short-term loans to total loans	0.298	0.28	0.28	0.267	0.318	0.298
Percent of liabilities						
Transaction deposits	0.198	0.153	0.271	0.276	0.113	0.092
Fed funds borrowed	0.003	0	0.002	0	0.004	0
Repo borrowings	0.016	0.002	0.007	0	0.023	0.012
FHLB advances	0.050	0.036	0.034	0.014	0.056	0.037
Other relevant indicators						
Log(assets)	13.437	13.268	12.085	11.970	15.219	14.826
Tier-1 ratio	0.152	0.138	0.173	0.151	0.150	0.131
Unused commitments to assets	0.130	0.122	0.089	0.078	0.158	0.144
ROA—annual	0.035	0.034	0.034	0.034	0.031	0.032
Number of observations	683		6,946		585	

Note: This table provides summary statistics on balance sheet items for banks in the estimation sample. "Borrowers" are defined as domestic banks that file Call Reports, and that take out a discount window loan at least once for $1 million or more over the sample period. "Non-borrowers" are defined as banks that file Call Reports and do not take out a discount window loan over the sample period. "Large banks" are defined as banks with at least $1 billion in assets at some point during the sample period; these can be either borrowers or non-borrowers. Summary statistics are calculated as means and medians of bank averages over the sample period. We eliminate banks with missing or negative assets, negative cash-to-asset ratios, loan-to-asset ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported posted discount window collateral greater than total assets. Eliminating these banks leads to a smaller sample size than in Table 1.

from the discount window. We use the table to compare them with non-borrowing banks in the sample and with all large domestic banks in the sample (with more than $1 billion in assets), regardless of whether they were borrowers or non-borrowers.

On the asset side, relative to the average for non-borrowers, discount window borrowers seem to have lower shares of assets in reserves, higher shares of C&I and CRE loans, and lower shares of Treasury securities. Some of these patterns are also observed for larger banks and could be (partly) behind the higher incidence of borrowing on the part of larger banks. Overall, the shares suggest that borrowing banks have less liquid asset holdings than non-borrowers. Of particular interest is the fact that borrowers seem to hold less reserves than both non-borrowers and large banks, which suggests that size is not what explains the fact that borrowers hold less reserves.

On the liability side, borrowing banks hold lower shares of liabilities in transaction deposits, although not quite as low as the largest banks. Fed funds and repo borrowings tend to be small in general, and the preponderance of larger borrowing banks probably boosts the calculated shares of these for borrowing banks. Banks that borrow at the discount window also tend to borrow at FHLBs; differences in shares are particularly notable when looking at the difference in medians between those for borrowers and non-borrowers. Based on these patterns, it is hard to rule out that the difference between borrowers and non-borrowers on the liability side of the balance sheet is not just driven by the difference in size.

Similarly, borrowers have more unused commitments compared with non-borrowers. This pattern may be driven by larger banks: Unused commitments at larger banks are higher than those at both borrowers and non-borrowers, and given the difference between mean and median, this pattern may be driven by a few banks with a very high proportion of unused commitments relative to assets.

Also in the bottom panel of Table 3, we see that there are no significant differences in return on assets across the different subsamples. Borrowers do appear to have lower Tier-1 capital ratios relative to non-borrowers. Larger banks also have lower capital ratios, so the difference could be driven mainly by size; the median large bank has a Tier-1 capital ratio that is 2 percentage points below that of the median non-borrower.

One of the main findings in Drechsler et al. (2016) is that banks that borrowed from the discount window in Europe during the sovereign debt crisis in 2011-12 held less capital and more risky assets. In principle, this could be a pattern that arises mainly during crises. While we consider our sample representative of "normal" times in the financial system, Table 4 suggests that U.S. domestic banks present a similar pattern even outside of crises.

In Table 4, we again display information on the tier-1 capital ratio but also include other ratios to investigate bank risk and borrowing behavior more closely. As before, we see that borrower banks tend to have lower tier-1 capital ratios (15.2 percent) relative to non-borrowers (17.3 percent) but similar to large banks (15 percent). The lower ratios may be explained by higher risk-weighted assets (RWA) for a given level of total assets (i.e., more risky assets) or by lower levels of capital. The bottom two rows suggest risky assets drive the difference. Borrowers tend to have more risky assets that translate into higher levels of risk-weighted assets relative to un-weighted assets–this ratio hovers around 71 percent for borrowers and large banks but is 3 percentage points lower for the median non-borrower. By contrast, tier-1 capital to total assets is notably similar across all three categories of banks.

Table 4: Domestic banks–capital ratios, DW borrowers and non-borrowers

	Borrowers		Non-borrowers		Large banks	
	Mean	Median	Mean	Median	Mean	Median
Tier-1 capital ratio	0.152	0.138	0.173	0.151	0.150	0.131
Risk-weighted over total assets	0.701	0.710	0.652	0.667	0.699	0.705
Tier-1 capital to total assets	0.102	0.097	0.105	0.099	0.099	0.094
Number of banks	683		6,946		585	

In general, it seems fair to say that in terms of the composition of assets and liabilities, capital ratios and profitability, borrowers are different from non-borrowers, and also, they do not just replicate larger banks. Furthermore, the propensity of banks to rely on the discount window seems to depend on the Federal Reserve district where they are located. This suggests that there could be systematic selection among the banks that borrow from the discount window.

In summary, it is evident that multiple factors simultaneously influence discount window activity, which makes a multivariate approach a promising avenue of inquiry. This is the approach that we take in Section 4.

2.2 Interest rates

Before we move on to study the theoretical framework, it is helpful to briefly discuss the configuration of interest rates most relevant for understanding the period in question in the U.S. As we will see, portfolio decisions depend on the relative level of the various interest rates confronted by banks. During our sample period, interest rates exhibit a pattern that will allow us to narrow the discussion of the theoretical and empirical possibilities. For this reason, we review such pattern here.

The interest rate on overnight overdrafts at the Federal Reserve was set at a penalty rate equal to the interest rate at the discount window primary credit program plus 4 percentage points (annual rate). There was also a minimum fee, and the rate was adjusted upward after multiple days of running an overdraft.

The interest rate at the discount window primary credit program was 50 basis points higher than the top of the target range for the policy interest rate (i.e., the effective federal funds rate). During our sample period, the interest rate on reserves was set equal to the top of the target range and hence 50 basis points lower than the primary credit rate. The secondary credit interest rate, in turn, is generally set at 50 basis points higher than the primary credit rate. Furthermore, under some circumstances, the interest rate in the secondary credit program can be adjusted to even higher levels if the Reserve Bank deems it appropriate.

3 A theoretical framework

In this section, we introduce a framework to help with the interpretation of our empirical strategy and results. Basically, the framework describes the decisions of a bank that is exposed to shocks and needs to make adjustments to its balance sheet in response to those shocks. Under some conditions, but not always, the optimal response of the bank is to borrow from the discount window. The framework is not intended to be descriptive but rather an illustration of the mechanisms that can generate the patterns observed in the data.

3.1 The model

Consider the problem of a bank that can make loans (l), hold liquid and illiquid securities (s^L and s^I, respectively) and reserves (f) and is able to fund those assets by attracting deposits (d) and engaging in other borrowing (b), and by holding equity capital (k). The bank also has an administrative resource cost $\chi(l)$ from lending an amount l in loans.

After choosing the initial portfolio of assets and the structure of its liabilities, the bank is exposed to various shocks that can alter certain components of its balance sheet.[9] For example, the bank may experience an outflow of borrowed money (b), or a valuable client may choose to draw down a line of credit with the bank that changes the bank's lending (l). To confront the funding needs that result from those shocks, the bank may use its reserves, liquidate some of its securities holdings, or borrow from the interbank market (b^{FF}) or the central bank (via a discount window loan b^{DW} or with an overnight overdraft ν).

The framework is general in the sense that it allows us to think of different shocks as potentially reflecting access (or lack of thereof) to different markets that the bank can use to adjust its balance sheet in response to those shocks. In particular, the bank may be able to trade in the securities market, or in the fed funds market, or in no market at all, depending on the timing of the shock and the time-sensitivity of the required adjustment.

For example, if a source of borrowed funds disappears late in the day, a bank's only alternative may be to use owned reserves or to borrow from the central bank (through the discount window or with an overnight overdraft) to cover certain payments needs (as in Poole (1968)). Some shocks, however, may give the bank more time to adjust, in which case the bank may be able to liquidate short-term securities or borrow in the interbank market, for example.

Denote by ϵ the vector of shocks that a bank experiences. Initially, the bank chooses loans, securities, reserves, deposits, other borrowing, and capital, subject to the balance sheet constraint:

$$l + s^L + s^I + f = d + b + k, \tag{1}$$

with all variables restricted to be positive. After these decisions are made, the bank is exposed to the shocks ϵ which (possibly) impact the values of l, d, and/or b. We

[9] The problem of the bank is similar to the one presented in Ennis (2018), but modified to consider a situation where the bank experiences liquidity shocks that need to be accommodated with reserves, other holdings of liquid assets, or short-term borrowing from the interbank market or the central bank.

denote by $l(\epsilon)$, $d(\epsilon)$, and $b(\epsilon)$ the value of these variables, respectively, after the shocks. In response to a shock, the bank may be able to adjust its reserves and securities holdings. We denote by $f(\epsilon)$, $s^L(\epsilon)$, and $s^I(\epsilon)$ the ex-post value (after the adjustment) of these variables. Furthermore, the bank may borrow in the interbank market ($b^{FF}(\epsilon)$), at the discount window ($b^{DW}(\epsilon)$), or may run an overnight overdraft on its account at the central bank ($\nu(\epsilon)$). All these decisions together must satisfy the following "flow" constraint:

$$(l(\epsilon) - l) + (d - d(\epsilon)) + (b - b(\epsilon)) = \\ (f - f(\epsilon)) + (s^L - s^L(\epsilon)) + \sigma(s^I - s^I(\epsilon)) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + \nu(\epsilon), \quad (2)$$

where the parameter σ is the liquidation value per unit of illiquid securities.

If a variable is not affected by the shocks or is not adjusted (potentially due to the presence of market frictions and the timing of the shocks), then its ex-post value equals its ex-ante value. For example, if total loans are not affected by the shock and cannot be adjusted in a timely manner in response to the shock (because, say, they are longer-term commitments), then $l(\epsilon) = l$. Similarly, if the shock ϵ is such that, due to its timing, it does not allow the bank to adjust its securities holdings, then $s^L(\epsilon) = s^L$ and $s^I(\epsilon) = s^I$. That is, the bank's securities holdings after the shock are the same as before the shock. One way to interpret such a situation is that the shock is realized after securities markets are closed, or activity in these markets is so reduced that no significant trading can be executed effectively.

Discount window lending in the U.S. is collateralized. For simplicity, we assume here that only securities can be used as collateral.[10] Discount window borrowing, then, has to satisfy the following collateral constraint:

$$b^{DW}(\epsilon) \leq s^L(\epsilon) + \theta s^I(\epsilon), \quad (3)$$

where θ is the haircut applied on illiquid securities, while liquid securities have no haircut. When σ is expected to be less than one, the value of θ will likely also be set at a level lower than one to appropriately reflect the liquidation risk.

We also assume that, after the shock, the bank cannot sell more than the amount of

[10] In the U.S., banks can pledge many other types of assets as collateral at the discount window. Extending the model to accommodate broader classes of collateral is not hard but notationally cumbersome.

securities it is holding (no shorting of securities is allowed). Finally, there are natural non-negativity constraints on reserves, discount window borrowing, and overnight overdrafts:

$$f(\epsilon) \geq 0, \quad b^{DW}(\epsilon) \geq 0, \quad \nu(\epsilon) \geq 0. \tag{4}$$

The bank takes as given the interest rates paid on deposits, r_L, interbank loans, r_{FF}, and other borrowings, r_B, the rates earned on loans, r_L, securities, r_{SL} and r_{SI}, and the cost of capital, r_K. Also, the bank takes as given the interest rates fixed by the central bank: the interest on reserves rate, r_{IOR}, the discount window rate, r_{DW}, and the rate charged on overnight overdrafts, r_ν.

Given all those rates, the bank chooses the initial values of l, s^L, s^I, f, d, b, and k. The bank also chooses the functions $f(\epsilon)$, $s^L(\epsilon)$, and $s^I(\epsilon)$ subject to the feasibility constraints imposed by the timing of trade and the possibility that some markets are no longer available at the time that particular shocks are realized. Finally, the bank also chooses $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $\nu(\epsilon)$. The objective of the bank is to maximize the following profit function:

$$\begin{aligned} E[(1+r_L)l(\epsilon) + (1+r_{SL})s^L(\epsilon) + (1+r_{SI})s^I(\epsilon) + (1+r_{IOR})f(\epsilon) \\ -(1+r_D)d(\epsilon) - (1+r_B)b(\epsilon) - (1+r_K)k - \chi(l) \\ -(1+r_{FF})b^{FF}(\epsilon) - (1+r_{DW})b^{DW}(\epsilon) - (1+r_\nu)\nu(\epsilon)], \end{aligned} \tag{5}$$

subject to constraints (1), (2), (3), and (4). Note that in general $b(\epsilon)$ will be a function of the initial value of other borrowings chosen by the bank, b, modified by the impact of the shock on this variable. This is the case, in principle, for loans and deposits, as well.

To understand the decisions of the bank, we start with the ex-post adjustment that the bank optimally makes in response to a shock. Then, we study the ex-ante decisions on reserves holdings and other variables given the optimal ex-post response previously analyzed.

3.2 Ex-post response to shocks

Consider a bank that has chosen the level of loans (l), deposits (d), securities holdings (liquid and illiquid, s^L and s^I), and capital (k). After the shock ϵ, the bank liquidity needs are $\Delta(\epsilon)$ given by

$$\Delta(\epsilon) \equiv (l(\epsilon) - l) + (d - d(\epsilon)) + (b - b(\epsilon)).$$

To simplify the exposition, assume that the timing of the shock is such that the bank is not able to adjust securities after the shock. Then, using equation (2), we have that

$$\Delta(\epsilon) = f - f(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + \nu(\epsilon), \tag{6}$$

which tells us that the bank will use reserves, borrowings (from the interbank market or the discount window), and/or overnight overdrafts to cover its ex-post liquidity needs.

The relevant portion of the payoff function (5) for the bank in the ex-post decision-making process is given by

$$(1 + r_{IOR})f(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_{\nu})\nu(\epsilon), \tag{7}$$

with the bank still subject to constraints (3) and (4), the collateral and non-negativity constraints, respectively. The bank needs to choose $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $\nu(\epsilon)$ to maximize objective (7) given that $f(\epsilon)$ satisfies (6).

In terms of the relevant configurations of interest rates to consider, it is standard to have that $r_{IOR} < r_{DW} < r_{\nu}$. That is, the central bank's lending rate is higher than the deposit rate, and overnight overdrafts carry a penalty over borrowing at the discount window. With respect to the interbank market, given the simplified nature of the model, it makes sense to restrict attention to $r_{IOR} \leq r_{FF} < r_{DW}$. If $r_{FF} < r_{IOR}$, it would be profitable for any bank (facing no other balance sheet costs, as assumed here) to borrow in the interbank market to hold reserves and earn interest on reserves paid by the central bank. Since all banks would want to do the same, such configuration of interest rates would be inconsistent with the clearing of the interbank market.[11]

[11] For a recent paper where balance sheet costs are explicitly modeled, and hence the interbank rate can be below the interest on reserves, see Afonso, Armenter, and Lester (2019).

3.2.1 Access to interbank markets

The ex-post funding decisions of the bank will depend crucially on the funding alternatives open at the time of receiving the liquidity shock. In particular, if the bank still has access to the interbank market when the shock occurs, then the discount window will not be used, as the following proposition demonstrates.

Proposition 1 (Interbank market access) *If the bank has access to the interbank market when the shock ϵ occurs, and $r_{IOR} \leq r_{FF} < r_{DW} < r_\nu$, then $b^{DW}(\epsilon) = 0$ and $\nu(\epsilon) = 0$. Furthermore, if $r_{IOR} < r_{FF}$, then $b^{FF}(\epsilon) = \Delta(\epsilon) - f$.*

Note that $b^{FF}(\epsilon)$ may be positive or negative, depending on the relative size of $\Delta(\epsilon)$ compared with the ex-ante level of reserves held by the bank, f. When $r_{IOR} < r_{FF}$, the bank will borrow or lend (respectively) in the interbank market the reserves that it needs to end the period with no holdings of reserves (i.e., so as to have $f(\epsilon) = 0$). If instead $r_{IOR} = r_{FF}$, then whenever $\Delta(\epsilon) < f$, the bank may choose to finish the period with a positive level of (*excess*) reserves (i.e., so as to have $f(\epsilon) > 0$).

The case when $r_{FF} = r_{DW}$ is less relevant in practice and hence not discussed in the proposition. This would be a situation where the system as a whole is systematically "short" on reserves and some banks have to borrow at the discount window to balance aggregate supply with aggregate demand. While there have been times when this situation was relevant (see, for example, Kasriel and Morris (1982)), the recent history in the U.S. is inconsistent with such general configuration of rates. It is, of course, still possible for certain banks at certain times to face interest rates in the interbank market that are higher than the discount window rate. This could happen, for example, if segmentation in the market created market power on the lending side (Bech and Klee (2011)). The situation in that case is equivalent to the case when the bank does not have access to the interbank market altogether, which we study next.

3.2.2 No access to interbank markets

Proposition (1) describes the case when the bank can (effectively) use the interbank market as a source of funding to accommodate a given shock. Other shocks may find the bank with no access to the interbank market either because the shock occurs late in the day, when the interbank market is no longer active, or because the bank's usual counterparties are not able to accommodate its liquidity demand, and the bank is not able to find other suitable trading partners on short notice. In that case, some discount window borrowing may be optimal as the next proposition demonstrates.

Evidently, when $s^L + \theta s^I = 0$, the bank has no available collateral to borrow at the discount window and, as a consequence, $b^{DW}(\epsilon) = 0$ regardless of the shock. In this case, whenever $\Delta(\epsilon) > f$, the bank incurs an overnight overdraft $\nu(\epsilon) = \Delta(\epsilon) - f$. To focus on the more interesting case when discount window borrowing can happen, assume that $s^L + \theta s^I > 0$.

Proposition 2 (No access to interbank market. The pecking order) *Assume that $s^L + \theta s^I > 0$. If the bank has no access to the interbank market when the shock $\Delta(\epsilon)$ happens, and $r_{IOR} < r_{DW} < r_\nu$, then:*

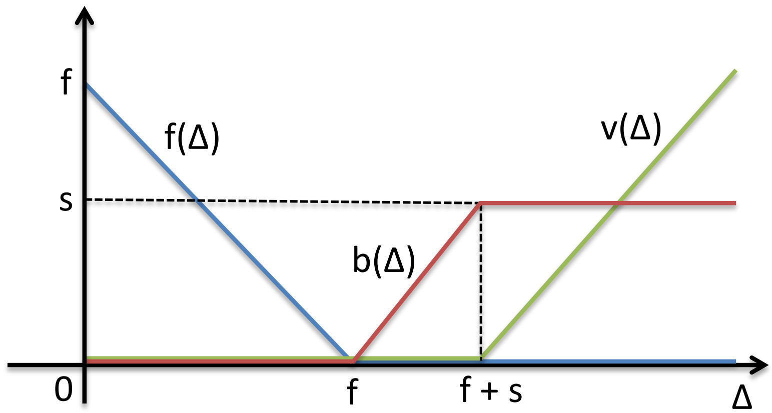
- *when $f \geq \Delta(\epsilon)$ we have that $b^{DW}(\epsilon) = 0$ and $\nu(\epsilon) = 0$;*
- *when $f < \Delta(\epsilon)$ we have that $b^{DW}(\epsilon) > 0$ and $f(\epsilon) = 0$. Furthermore,*
    - *if $\Delta(\epsilon) - f \leq s^L + \theta s^I$ then $\nu(\epsilon) = 0$,*
    - *if $\Delta(\epsilon) - f > s^L + \theta s^I$ then $\nu(\epsilon) > 0$.*

In a sense, the bank follows *a pecking order* for funding the liquidity shock, given that it has no access to the interbank market. If the shock is relatively small, the bank uses its holdings of reserves to cover the shock. For larger shocks, when the ex-ante stock of reserves held by the bank is not enough, the bank borrows from the discount window. In such case, the bank may or may not need to also run an overnight overdraft, depending on whether the collateral pledged at the central bank is sufficient to back the required discount window loan.

Figure (3) illustrates the pecking order. To simplify notation, we denote with s the total value of available collateral for the bank (i.e., $s \equiv s^L + \theta s^I$). On the horizontal axis, we measure the size of the shock Δ. When Δ is smaller than ex-ante reserves f, the bank adjusts its reserves holdings down to accommodate the shock. No central-bank credit is used in this case, and ex-post reserves are given by $f(\Delta) = f - \Delta$. When Δ is greater than the level of ex-ante reserves f, discount window borrowing $b(\Delta)$ is positive and ex-post reserves are zero. Finally, if the shock is greater than ex-ante reserves plus the discount window borrowing capacity of the bank, given by its available collateral s, then the bank incurs a positive overnight overdraft $\nu(\Delta)$ (i.e., a negative balance in its account at the central bank).

It is important to note that some of the structural parameters (such as, for example, the variances of the distributions of possible shocks) that influence the likelihood

Figure 3: The pecking order



of observing a bank borrowing from the discount window also determine the bank's ex-ante choice of the level of reserves (and other components of its balance sheet). We turn to this issue next.

3.3 Ex-ante balance sheet decisions

When the bank is choosing the composition of its balance sheet, it anticipates that it will be exposed to shocks and that, depending on those shocks, it may have different alternatives to address the resulting liquidity needs (including borrowing from the discount window). In this section, we study the ex-ante portfolio decision of banks. To simplify the analysis, consider a situation when a bank has already decided the amount of loans, other borrowed money, and capital in its portfolio and now has to decide the amount of reserves, securities, and deposits to hold. For concreteness, we also assume that the shocks affect only other borrowed money. We discuss the more general case in Section 3.5.

At such point in the decision process, the problem of the bank is to choose reserves (f), securities (s^L and s^I) and deposits (d) to maximize

$$\widehat{V} \equiv E_\epsilon[(1+r_{SL})s^L + (1+r_{SI})s^I + (1+r_{IOR})f(\epsilon) - (1+r_D)d + (1+r_B)\Delta(\epsilon) \\ -(1+r_{FF})b^{FF}(\epsilon) - (1+r_{DW})b^{DW}(\epsilon) - (1+r_\nu)\nu(\epsilon)], \quad (8)$$

subject to (1), (2), (3), (4), and $f > 0$. Using (2) we have that

$$f(\epsilon) = f - \Delta(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + \nu(\epsilon),$$

which can be substituted into the objective function (8). Here, the functions $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $\nu(\epsilon)$ are ex post optimal (as described in the previous section).

The idea is to show how the choice of reserves (and securities) depends on the distribution of shocks and the ability of banks to use different sources of funding to accommodate those shocks. To this end, we use a simple example that clearly captures the tradeoffs involved. In this example, the structure of the shock-distribution is as follows:

$$\Delta(\epsilon) = \begin{cases} \Delta_0 = 0 & \text{with prob. } 1 - p_1 - p_2, \\ \Delta_1 & \text{with prob. } p_1, \\ \Delta_2 & \text{with prob. } p_2, \end{cases}$$

with $0 < \Delta_1 < \Delta_2$.

Furthermore, after the shock happens, with probability q the bank is able to access the interbank market and with probability $1 - q$ the bank can only cover a liquidity shortfall via the central bank (in the form of a discount window loan or an overnight overdraft). We denote with the subscript A the value of a variable when the bank has access to the interbank market and with the subscript N when the bank has no access to it.

Given our maintained assumptions on interest rates ($r_{FF} < r_{DW} < r_\nu$), when the bank has access to the interbank market after the shock, it will neither borrow at the discount window nor run an overnight overdraft. That is, $b_A^{DW} = 0$ and $\nu_A = 0$ regardless of the size of the shock. This is the case because it is cheaper for the bank to borrow in the interbank market at rate r_{FF} than to borrow from the central bank at rates r_{DW} or r_ν.

When the bank has no access to the interbank market (with probability $1 - q$), whether it borrows from the central bank depends on its chosen level of securities and reserves. This choice, of course, depends on the cost of funding (in this simple case, the interest rate on deposits). We denote by b_{iN}^{DW} the amount borrowed at the discount window when the shock equals Δ_i with $i = 0, 1, 2$.

We define three threshold levels for the interest rate on deposits (i.e., the bank's funding cost) as follows:

$$\begin{aligned}
r^{T1} &= qr_{FF} + (1-q)r_{IOR} \\
r^{T2} &= qr_{FF} + (1-q)[(1-p_2)r_{IOR} + p_2 r_{DW}] \\
r^{T3} &= qr_{FF} + (1-q)[(1-p_1-p_2)r_{IOR} + (p_1+p_2)r_{DW}].
\end{aligned}$$

One way to think about these thresholds is that they represent the value for the bank of holding an extra unit of reserves, depending on whether the bank needs to borrow from the discount window in response to the different realization of the liquidity shock Δ. The bank will compare such value with the cost of obtaining an extra unit of reserves ex ante, which is given by r_D here.[12]

So, for example, if the bank is holding reserves sufficient to cover all possible realizations of the liquidity shock, then with probability q the bank will be able to lend out leftover reserves in the interbank market. With probability $1-q$, however, the bank will have no access to the interbank market and will keep those leftover reserves, remunerated at the level of the interest on reserves. This situation generates the interest rate r^{T1}, and if the interest rate on deposits is higher than this threshold rate, then the bank would have no incentives to hold such a high level of reserves.

Notice that if the interest rate on deposits r_D is below the threshold rate r^{T1}, then the bank would benefit from increasing deposits and reserves indefinitely. This situation would not be compatible with equilibrium, so we proceed by considering only situations where $r_D \geq r^{T1}$. Interestingly, if $r_D = r_{FF} = r_{IOR}$, then r_D equals r^{T1} and the bank will choose to hold sufficient reserves to cover all possible shocks and possibly a significant level of *excess reserves*. While this is a situation that appears relevant for many banks in the U.S. during the past several years, it is inconsistent with observing discount window lending. For this reason, we proceed by studying the model under the assumption that $r_D > r^{T1}$.

Note also that if the rates of return on securities (r_{SL} and r_{SI}) are equal to the rate of interest on deposits, then the bank will hold enough securities to avoid overnight overdrafts. We consider this case first and later explain what happens when the rate

[12] In principle, the cost of funding an extra unit of reserves is the cost of the marginal liabilities created by the bank to obtain those reserves. Here, we have simplified the timing so that deposits always play the role of marginal liability for the bank. Importantly, the assumption here is that the bank does not need to increase capital as its balance sheet grows – the capital constraint is not binding. If it were binding, then the marginal cost of funding would include a capital charge (as in Ennis (2018)).

of return on securities is lower than the deposit rate and the collateral constraint at the discount window is occasionally binding.

Depending on the level of the interest rate on deposits, the bank will decide to hold reserves to cover, partially or fully, the different possible realizations of the liquidity shock. The following proposition describes such decision.

Proposition 3 (Ex-ante decisions. No overdrafts.) *When $r_{IOR} \leq r_{FF} < r_{DW} < r_{\nu}$ and $r_{SI} = r_{SL} = r_D$, we have that:*

- ∘ *if $r^{T1} < r_D < r^{T2}$ then $f = \Delta_2$, and if $r_D = r^{T2}$ then $\Delta_1 \leq f < \Delta_2$,*
- ∘ *if $r^{T2} < r_D < r^{T3}$ then $f = \Delta_1$, and if $r_D = r^{T3}$ then $0 \leq f < \Delta_1$,*
- ∘ *if $r^{T3} < r_D$ then $f = 0$.*

Furthermore,

- *when the bank can access the interbank market, $b_{iA}^{DW} = 0$ and $\nu_{iA} = 0$ for $i = 0, 1, 2$; and*
- *when the bank cannot access the interbank market, $b_{iN}^{DW} = max\ \{0, \Delta_i - f\}$ and $\nu_{iN} = 0$ for $i = 1, 2$.*

Finally, $s_L + \theta s_I \geq max\ \{b_{iN}^{DW}\}_{i=1,2}$.

Figure (4) summarizes the results from the proposition. The most interesting situation occurs when $r_D \in (r^{T2}, r^{T3}]$ because then, if the shock is large (equal to Δ_2) and the bank has no access to the interbank market, it borrows from the discount window even when holding a positive amount of reserves ex ante. For other interest rate values, either the bank never borrows from the discount window or it chooses to hold no reserves and hence borrows from the discount window whenever it receives a liquidity shock and has no access to the interbank market.

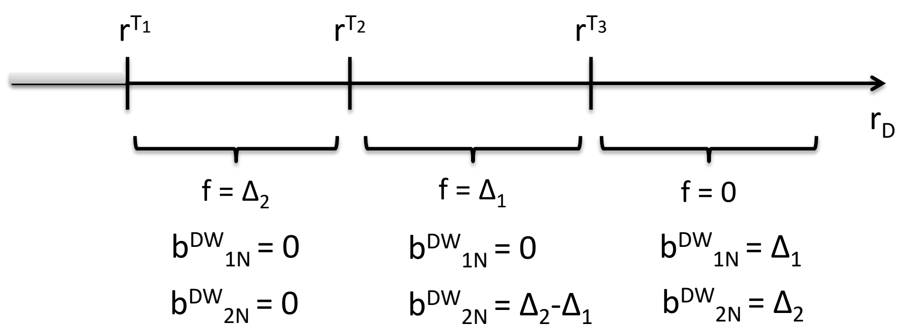
The proposition suggests a negative association between the level of reserves and discount window borrowing, given that $b_{iN}^{DW} = \max\{0, \Delta_i - f\}$. In other words, for a given shock process, higher levels of reserves holdings tend to be associated with lower discount window lending. However, this negative association may weaken when looking at a cross section of banks facing different shock processes. We illustrate this situation in the following corollary.

Corollary 4 (Ex-ante effects of heterogeneous shock-distributions.) *Consider two banks, 1 and 2, facing two different shock processes, with $\Delta^2(\epsilon) = \rho\Delta^1(\epsilon)$ and*

$\rho > 1$ so that bank 2 experiences larger shocks than bank 1. When the conditions in Proposition (3) are satisfied and $r_D \in (r^{T2}, r^{T3})$, bank 2 will hold (ex ante) higher levels of reserves and borrow more (ex post) from the discount window.

As we saw in Proposition (3), when $r_D \in (r^{T2}, r^{T3})$, bank i will hold reserves $f_i = \Delta_1^i$ and will borrow at the discount window $b_i^{DW} = \Delta_2^i - \Delta_1^i$ when the shock $\Delta^i(\epsilon)$ equals Δ_2^i. As a result, bank 2 will hold higher reserves, since $\Delta_1^2 > \Delta_1^1$, and will borrow more from the discount window since $\Delta_2^2 - \Delta_1^2 > \Delta_2^1 - \Delta_1^1$.

Figure 4: Interest rate thresholds



The proportionality factor ρ is, of course, not necessary for the result. It is assumed here for convenience.[13] The corollary highlights the importance of recognizing the endogeneity of reserves holdings. Conditional on a shock process, higher reserves imply that a bank is more able to accommodate those shocks without tapping the discount window. However, banks exposed to larger liquidity shocks may choose to hold higher levels of reserves and, at the same time, may need to borrow more (and more often) from the discount window. While the first logic indicates a negative relationship between reserves and discount window borrowing, the second can generate a positive relationship (as the corollary illustrates).

3.3.1 The impact of discount window access

The ex-ante level of reserves (and other components of the balance sheet) also depends on the ability of banks to access the discount window. If the bank is not able to access the discount window (because it has not made the necessary arrangements,

[13] As long as Bank 2 faces a shock process such that Δ_1 and $\Delta_2 - \Delta_1$ are both larger quantities than for Bank 1, then it will hold higher reserves and borrow more from the discount window.

for example) then the threshold values for the interest rate on deposits change. In particular, the rate charged on overnight overdrafts substitutes for the discount window rate in the formula for the thresholds; we denote this new set of thresholds with a "prime" and we have:

$$
\begin{aligned}
r^{T1'} &= r^{T1} \\
r^{T2'} &= q(1+r_{FF}) + (1-q)[(1-p_2)(1+r_{IOR}) + p_2(1+r_\nu)] \\
r^{T3'} &= q(1+r_{FF}) + (1-q)[(1-p_1-p_2)(1+r_{IOR}) + (p_1+p_2)(1+r_\nu)].
\end{aligned}
$$

This change in thresholds occurs because a bank that is short on reserves will need to incur an overnight overdraft when it has no access to the discount window. Given these thresholds, the bank's ex-ante choice of reserves is given by the following proposition.

Proposition 5 (<u>Ex-ante decisions. No discount window access.</u>) *When $r_{IOR} \leq r_{FF} < r_{DW} < r_\nu$ and the bank has no access to the discount window, we have that:*

- *if $r^{T1'} < r_D < r^{T2'}$ then $f = \Delta_2$, and if $r_D = r^{T2'}$ then $\Delta_1 \leq f < \Delta_2$,*
- *if $r^{T2'} < r_D < r^{T3'}$ then $f = \Delta_1$, and if $r_D = r^{T3'}$ then $0 \leq f < \Delta_1$,*
- *if $r^{T3'} < r_D$ then $f = 0$.*

Furthermore, $\nu_{iA} = 0$ for $i = 0, 1, 2$ $\nu_{iN} = max\ \{0, \Delta_i - f\}$ and for $i = 1, 2$.

The parallels between propositions (3) and (5) highlight the fact that during "normal" times, the discount window operates, in part, as an alternative central bank funding source that is less costly than overnight overdrafts.[14] It is also the case that for certain combinations of rates of return and funding costs, a bank with no access to the discount window will tend to hold higher levels of reserves than a similar bank that has access to the discount window, as the following corollary demonstrates.

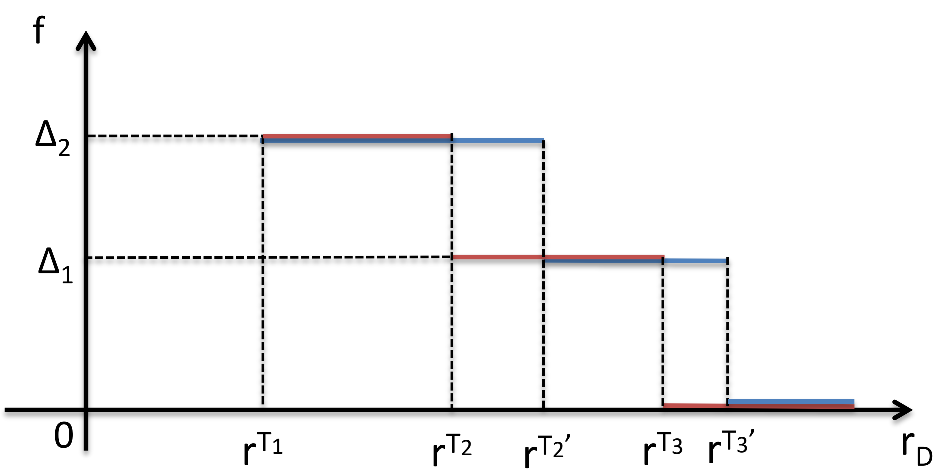
Corollary 6 (<u>Ex-ante effects of discount window access.</u>) *Consider two banks, one with access to the discount window and one without it. Both banks face the same funding cost r_D. When the conditions in propositions (3) and (5) are satisfied and $r^{T2} < r_D < r^{T2'}$ or $r^{T3} < r_D < r^{T3'}$, the bank without access to the discount window holds more reserves than the bank with access the discount window.*

[14] In fact, a common discussion in policy circles is the possibility of automatically transforming any shortfall in a bank's account at the central bank into a discount window loan, as long as the bank has the appropriate amount of collateral pledged (see, for example, Nelson (2019)).

The basic logic behind this result is simple. When a bank has no access to the discount window, if the shock exhausts its reserves, then it has to incur an overdraft with the central bank, which is more expensive than a discount window loan ($r_{DW} < r_\nu$). For this reason, given the interest rates, the bank with no access to the discount window has more incentive to hold reserves.

More specifically, in the context of the model, this logic is captured by the fact that the relevant interest rate thresholds for a bank with access to the discount window are lower than the thresholds for the bank without access. As we see in Figure 5, when funding costs are between the two values of a given threshold, the bank with access to the discount window (red lines) chooses a lower level of reserves than the bank without access (blue lines). For example, when $r^{T2} < r_D < r^{T2'}$, the bank with access to the discount window will choose reserves equal to Δ_1, and the bank without access to the discount window will choose reserves equal to Δ_2.

Figure 5: Endogenous reserves with and without discount window access



Of course, in principle, economic reasons may be driving a bank to make the necessary arrangements to be able to access the discount window. In that sense, "access" could be partly determined by, for example, the distributions of shocks faced by the bank, as was the case with the level of reserves (see corollary 4). For this reason, the relationship between reserves and access in a cross section of heterogeneous banks can be difficult to disentangle.

3.3.2 Binding collateral

The ability of a bank to use the discount window also depends on the amount of collateral available to the bank. In this model, we have simplified collateral to be represented only by securities holdings. The ex-ante decision by the bank to hold securities, then, determines the amount of collateral held by the bank and its ability to use the discount window. When the return on securities (r_{SL} and/or r_{SI}) is equal to the funding cost (r_D), the bank will hold enough securities so that the collateral constraint is never binding. This is the case we have discussed so far (see Proposition 3).

When the collateral constraint is binding for some realizations of the shock, the value of holding an extra unit of securities would be equal to the rate of return on that security plus the shadow value of relaxing the collateral constraint, which we denote by $\lambda_{CC} \geq 0$. For the bank to hold both types of securities in its portfolio, the following two conditions must hold:

$$1 + r_{SL} + \lambda_{CC} = 1 + r_D$$
$$1 + r_{SI} + \theta\lambda_{CC} = 1 + r_D.$$

Since we are assuming that illiquid securities are subject to a haircut in the collateral pool ($\theta < 1$), the bank will only hold both kinds of securities if the illiquid securities have a higher rate of return than the liquid ones.[15]

Additionally, the shadow value of relaxing the collateral constraint depends on the level of reserves chosen by the bank. For example, if the funding rate is low enough so that the bank is choosing $f = \Delta_2$, then $\lambda_{CC} = 0$, and the bank will hold no collateral whenever the return on securities is below the funding cost.

Furthermore, if the return on securities is low enough, the bank will hold no securities regardless of its level of reserves, and the choice of reserves is equivalent to the case when the bank has no access to the discount window (as in Proposition (5)). For intermediate values of the rate of return on securities, the bank simultaneously chooses reserves and securities to minimize the costs associated with funding the liquidity needs originated in the $\Delta(\epsilon)$ shocks. The general direction of this relationship

[15] When the structure of shocks is such that for some realizations of the shocks the bank can liquidate securities to obtain the needed extra funding, then the decision to hold securities is also driven by those considerations.

is that, for a given shock process, a bank with higher reserves can afford to hold less collateral. But, as with Corollary (4), when banks differ in their exposure to shocks, the cross-sectional heterogeneity may tend to attenuate these basic patterns.

3.4 The discount window access decision

In general, the decision to gain access to the discount window also responds to basic cost-benefit evaluations. If the costs of gaining access to the discount window were zero, banks exposed to liquidity shocks would choose to have access (or be indifferent about it). However, there are some costs that a bank incurs when gaining access to the discount window, stemming from, for example, setting up the systems and collateral-pledging processes. The endogeneity coming from such cost-benefit evaluation creates selection across banks in terms of those that have and do not have ready access to the discount window. We investigate here the nature of this endogenous selection in the context of the model.

Suppose that bank i has a cost c_i^{DW} from gaining access to the discount window. Going back to expression (8), we have that bank i will choose to gain access to the discount window whenever

$$\widehat{V}_{iA} - c_i^{DW} \geq \widehat{V}_{iN},$$

where $\widehat{V}_{iA}$ is the optimized value of $\widehat{V}_i$ in expression (8) (when the bank has access to the discount window) and $\widehat{V}_{iN}$ is the optimized value of $\widehat{V}_i$ when the bank has no access to the discount window, as in Section 3.3.1. It is important here to realize that the choice of reserves (and securities) depends on whether the bank obtains access to the discount window. Corollary 6 illustrates this point.

Continuing with our leading example, we know from propositions 3 and 5 that the choice of reserves depends on the level of the interest rate on deposits relative to other relevant interest rates. For example, when $r_D \in (r^{T1}, r^{T2})$ we have that the bank would set the level of reserves to equal Δ_2 regardless of whether it has access to the discount window. In fact, the bank will not need to use the discount window or overnight overdraft in such case. When the cost $c_i^{DW} > 0$, the bank will choose not to pay it and hence will not have ready access to the discount window.

A more interesting case ensues when $r_D \in (r^{T2}, r^{T2'})$. In this case, a bank with access to the discount window would set its level of reserves equal to Δ_1, and a bank without access to the discount window would instead choose reserves equal to Δ_2 (see

Figure 5). This, in turn, implies that the bank without access will not need central bank funding, while the bank with access to the discount window will borrow from the central bank when the shock is large (equal to Δ_2). Based on these patterns, after some algebra, it can be shown that the bank will choose to have access to the discount window if

$$(r_D - r^{T2})(\Delta_2 - \Delta_1) \geq c_i^{DW}. \tag{9}$$

The interpretation of this condition is simple. When a bank chooses to not have access to the discount window, it chooses to hold $\Delta_2 - \Delta_1$ extra reserves at a cost of r_D. A bank that chooses to have access, instead, would decide to hold lower reserves but would have to borrow from the interbank market or the discount window (by and amount $\Delta_2 - \Delta_1$) according to the probability that the interbank market is open (q), or not $(1 - q)$, at the time when the bank needs the funds (this average funding cost is exactly reflected in the formula for r^{T2}). When the differential funding cost associated with the two alternatives is greater than the cost of obtaining access, the bank will choose to gain access.

A second interesting case is when $r_D \in (r^{T2'}, r^{T3})$. In this case, both the bank with access to the discount window and the one without it would choose the same level of reserves Δ_1. In that way, both banks would experience the same liquidity needs with the same probabilities. In particular, when the shock equals Δ_2 and the bank does not have access to interbank markets, it will need to seek funding from the central bank. Having access to the discount window lowers the cost of that funding (by avoiding a more expensive overdraft). Hence, whenever

$$(1 - q)p_2(r_\nu - r_{DW})(\Delta_2 - \Delta_1) \geq c_i^{DW}, \tag{10}$$

bank i will choose to obtain access to the discount window by paying the cost c_i^{DW}.

The rest of the cases (for higher values of r_D) are similar, with the bank saving in funding costs by lowering the cost of central bank liquidity in some contingencies (when the shock is large) even if in some cases the choice of lower reserve levels increases exposure to that liquidity risk.

With the set of interest rates taken as given by the bank, equations (9) and (10) (and their counterparts for the other cases) capture the factors that determine whether a bank will choose to gain access to the discount window. For example, banks with lower c^{DW} are more likely to choose to gain access to the discount window. These differences in cost may originate, for example, on a differential treatment of discount window requirements across Federal Reserve districts in the U.S.

Furthermore, banks facing different shock processes will, in principle, make different access decisions. The difference in shock process is reflected not only on the support of possible values of Δ, but also on the probabilities over those values and the probability that the bank could face the shock when the interbank market is closed (these probabilities determine r^{T2} in equation (9)). When looking at a cross section of banks, as we do in our empirical section, this heterogeneity in cost and structure of shocks is likely to play a role.

For a given structure of the shock, the model suggests that a bank choosing to obtain access to the discount window would choose to hold no more reserves than a bank not choosing to have such access. However, note that the decision to gain access is driven (among other things) by the differential between values of Δ (the variability on the size of the shock) while the level of reserves chosen by the bank depends on the *level* of the different values of Δ (the size of the shocks). For this reason, when banks are heterogeneous over the shock process they face, some of those banks can be choosing to gain access to the discount window and also hold relatively high levels of reserves compared with other banks that face smaller and less variable shocks and choose to not have access to the discount window. Even though we try to control for differences in the structure of the shock across banks in our empirical section, some residual (unobservable) heterogeneity, in combination with this logic, may help to explain some of our results.

3.5 Generalized implications

In the general version of the model, the shock can affect the amount of loans and deposits, in addition to the change in borrowed money discussed above. Also, depending on the timing of the shock, the bank may be able to liquidate securities to accommodate a shock. So, the bank can use reserves, central bank borrowing, and/or sales of securities to respond to changes in its liquidity needs. And, the distribution of shocks can have a more complex structure than the example studied here, including continuum support and mean and variance heterogeneity.

More generally, then, we can think that bank i at time t is facing general liquidity risk, which is proxied by a variable ψ_{it} and is, in turn, a function of the bank's size, balance sheet composition, and other factors. That is, we have

$$\psi_{it} = \psi(A_{it}, \mathbf{p}_{it}, ...),$$

where A_{it} is total assets of bank i at time t (a measure of size) and $\mathbf{p}_{it}$ is a vector of portfolio ratios capturing the bank's exposure to liquidity risks and market access to credit.

Discount window activity for bank i at time t, then, is a function of its liquidity risk, its holdings of reserves R_{it}, and other factors, such as its Federal Reserve district (denoted by $D_{d(i)t}$). So, we have

$$DW_{it} = DW(\psi_{it}, R_{it}, D_{d(i)t}, ...).$$

As it was clear from the model, reserves holdings are in turn also a function of the bank's liquidity risk and whether the bank has access to the discount window (denoted with the indicator variable I^{DW}). That is

$$R_{it} = R(\psi_{it}, I_i^{DW}, ...)$$

Finally, the decision to incur the costs to have access to the discount window responds, in principle, to the ex-ante perceptions of risks and relative costs of obtaining funding faced by the bank:

$$I_i^{DW} = A(\psi_{it}, D_{d(i)t}, ...)$$

This is the representation of the generalized framework that we will use to approach our empirical investigations in the next section.

4 Empirical analysis

Our empirical approach is motivated by the ideas brought to the forefront by the simple model we just discussed. At the same time, we intend to be more general to accommodate as much as possible the multiple dimensions of the complex problem that determines when a bank borrows from the discount window.

Throughout the section, we restrict attention to discount window loans at the primary credit facility.[16] For the most part, we also only consider loans by domestic banks, although in one of our robustness tables, we include data for foreign-related institutions.

4.1 Incidence of borrowing, conditional on access

Our first specification relates the probability of borrowing at the discount window to a bank's holdings of reserves and other balance sheet characteristics. To limit the effect of endogeneity, we restrict our sample to banks that are observed as having obtained discount window access. In particular, as we showed in the model developed above, discount window borrowing and reserves may potentially be co-determined. As such, there is some risk of unobserved heterogeneity or endogeneity that could potentially bias our parameter estimates. At the same time, banks must obtain discount window access well in advance of any borrowing decision. By limiting our sample to banks that have already obtained access, we condition on some of this unobserved heterogeneity, minimize the effect on our parameter estimates, and interpret our results accordingly.

We estimate the following equation on the sample of banks that have discount window access:

$$DW_{it} = \alpha_{dw} + \gamma_{dw} R_{it} + \beta_{dw} \psi_{it} + \zeta_{it}, \tag{11}$$

where DW_{it} equals 1 if bank i borrows from the discount window in quarter t, R_{it} is reserves as a share of bank assets, and ψ_{it} refers to the set of control variables that proxy for liquidity risk described above.[17]

We have three subsets of control variables. The first subset includes assets, liabilities, and other balance sheet characteristics of the bank. For assets, we explore the effects of liquidity, risk, and duration of bank assets on discount window borrowing and include balance sheet measures of Treasury securities holdings, C&I and CRE loan holdings, and short versus longer-term loans. For liabilities, we gauge the effect

[16] Secondary credit loans are not very common and often involve very special circumstances. For a discussion of the specifics behind some recent secondary credit loans, see Ennis, Ho, and Tobin (2019).

[17] We eliminate banks with missing or negative assets, negative cash-to-asset ratios, loan-to-asset ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported discount window collateral greater than total assets. We also eliminate trust companies that file the Call Report, as their balance sheet structure is notably different from that of a typical bank.

of core funding, money market activity, and other borrowings, and incorporate measures of transactions deposits, federal funds and repo borrowing, and FHLB advances. For balance sheet size and capital, we examine proxies for business models on discount window borrowing and include (the log of) total assets, unused commitments to assets, the tier-1 capital ratio, and return on assets.

The second set of controls captures the bank's Federal Reserve district; as shown above, there appear to be some systematic differences in discount window behavior associated with this variable. Finally, the third set of controls capture changes in conditions over time and, in principle, secular trends: first, we consider changes in the macroeconomic environment as proxied by the quarter-over-quarter change in real GDP; and, second, we consider short-term shifts in Federal Reserve balance sheet composition as represented by the quarterly standard deviation in the level of balances held in the Treasury General Account (σ_{TGA}).

We assume that ζ_{it} is normally distributed and estimate the model using an unbalanced panel probit estimator. We use standard procedures for clustering standard errors by bank to control for potential heteroskedasticity and correlation of errors across observations.

In Table 5, we report the panel probit results when we estimate equation (11) on the sample of banks that either execute a test loan or borrow at the discount window at some point in our sample. As described above, we take such activity as indicative that the bank is familiar with and ready to possibly tap the discount window; in short, the bank has access to the discount window.[18] We have five results.

First, conditional on discount window access, banks are more likely to borrow from the discount window if they hold fewer reserves. The first row of column (1) presents the marginal effect of reserves to assets on borrowing from the discount window. Across all specifications, the estimated coefficients on reserves to assets imply that, at the mean of the distribution of the reserves-to-assets ratio, for a one standard deviation decrease in the reserves to assets ratio (roughy 4 percentage points), the probability of borrowing at the discount window increases by a little less than 1 percentage point.[19] The overall probability of borrowing at the discount window for

[18] Banks taking test loans and other smaller loans (of less than $1 million) are not coded as having borrowed at the discount window but are coded as having discount window access.

[19] The standard deviation of the reserves-to-assets ratio is roughly 4 percentage points. Four percentage points multiplied by -0.231, the coefficient reported on the reserves-to-assets ratio in column (1), is a little less than 1 percentage point.

this subsample of banks is 3 percent; thus, this shift in probability is notable.

Second, banks with illiquid balance sheets are more likely to borrow from the discount window. Column (2) presents results from including asset measures in the specification, and column (3) from including liability measures. On the asset side, banks that hold more illiquid, riskier loans, such as C&I loans, are more likely to borrow from the discount window. Analogously, banks that use less stable funding, such as repo funding or FHLB advances, are more likely to borrow at the discount window, as well.[20] The magnitude of these effects is economically meaningful: a one standard deviation increase in the share of C&I loans to assets or a one standard deviation decrease in transaction deposits to liabilities each boosts the probability of borrowing at the discount window by roughly 50 basis points.[21]

Third, larger, more profitable banks are more likely to borrow at the discount window. As reported in column (4), conditional on access and at the mean of the distribution of asset holdings, for every one standard deviation increase in assets (from about $450 million in assets to $1.8 billion in assets), the probability of borrowing at the discount window climbs by roughly 1 percentage point.[22] Likewise, for every one standard deviation increase in return on assets, the probability of borrowing at the discount window moves up about 30 basis points.[23] These results potentially reflect unobserved effects of business models on the probability of borrowing at the discount window. In particular, over our sample period, although banks that have riskier and more illiquid portfolios were more likely to borrow at the discount window, more profitable and larger banks were also more likely to do so.

Fourth, there are differences in borrowing behavior across Federal Reserve Districts. Specifically, borrowing banks are more likely to be in District 12.[24] There are differences in bank characteristics across Federal Reserve Districts; for example, some Districts have smaller banks than other Districts, and some Districts have more

[20] See Ashcraft, Bech, and Frame (2010) for a discussion of the role of FHLB advances during the 2008 financial crisis and its interaction with the discount window.

[21] For the banks in the estimation sample, the standard deviation of the share of C&I loans on bank balance sheets is roughly 0.08, and the standard deviation for transaction deposits is 0.23. Multiplying these standard deviations by the coefficients reported in columns (2) and (3) is about 0.0047 and 0.0066, respectively.

[22] This is calculated as (log($1800 million) - log($450 million))*0.009, where 0.009 is the (rounded) coefficient on log(assets) reported in column (4).

[23] In the estimation sample, one standard deviation in ROA is roughly 0.0088. Multiplying this by the coefficient on ROA reported in column (4) is 0.00295.

[24] We evaluated individual coefficients for each District. Most of these coefficients were statistically or economically insignificant; we include District indicators for those that are significant.

Table 5: Borrowing—conditional on access

Dependent variable	Borrowed ($DW_{it} = 1$)				
	(1)	(2)	(3)	(4)	(5)
Asset composition					
Reserves to assets	-0.231***	-0.233***	-0.189***	-0.205***	-0.184***
	(0.0373)	(0.0384)	(0.0355)	(0.0355)	(0.0357)
CRE to assets		-0.0128			-0.0398
		(0.0270)			(0.0262)
C&I to assets		0.0543**			0.0375
		(0.0194)			(0.0203)
Treasury securities to assets		-0.0222			-0.0188
		(0.0598)			(0.0618)
Short-term to total loans		0.0153			0.0114
		(0.00864)			(0.00879)
Liability composition					
Transaction deposits to liabilities			-0.0289**		-0.00294
			(0.0107)		(0.0110)
Fed funds borrowed to liabilities			0.141		0.137
			(0.0856)		(0.0834)
Repos to liabilities			0.159***		0.134**
			(0.0449)		(0.0501)
FHLB advances to liabilities			0.103***		0.117***
			(0.0224)		(0.0224)
Balance sheet size and capital					
Log(assets)				0.00858***	0.00720***
				(0.00110)	(0.00122)
Unused commitments to assets				0.0247	0.0229
				(0.0153)	(0.0162)
Tier-1 capital to risk weighted assets				-0.00351	0.00858
				(0.0264)	(0.0255)
ROA				0.334**	0.358***
				(0.115)	(0.105)
Other controls					
District 1	-0.00651	-0.00339	-0.0126*	-0.00876	-0.0108*
	(0.00532)	(0.00536)	(0.00550)	(0.00517)	(0.00546)
District 12	0.0246***	0.0228***	0.0256***	0.0178***	0.0198***
	(0.00454)	(0.00459)	(0.00456)	(0.00444)	(0.00453)
ΔGDP	-0.000338	-0.000340	-0.000290	-0.000466	-0.000407
	(0.000372)	(0.000370)	(0.000373)	(0.000371)	(0.000372)
σ_{TGA}	-0.000161***	-0.000161***	-0.000137***	-0.000199***	-0.000169***
	(0.0000400)	(0.0000399)	(0.0000389)	(0.0000409)	(0.0000402)
Number of observations	56,041	56,041	56,041	56,041	56,041
Number of banks	2,027	2,027	2,027	2,027	2,027
pseudo R^2	0.010	0.012	0.016	0.017	0.022

Cluster-robust standard errors are in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. The dependent variable is an indicator that equals one if a bank borrowed at the discount window in a quarter. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model.

foreign branches. That said, we expect that our balance sheet controls adequately account for these types of differences. In Section 4.3, we use this observation of differences across Districts to investigate discount window access decisions more generally.

Finally, during these "normal" times, changes in general macroeconomic conditions do not appear to affect the decision to borrow at the discount window, while changes in the Federal Reserve's balance sheet do influence this decision. Specifically, quarterly changes in real GDP have little significant effect on the likelihood of discount window borrowing. At the same time, a more volatile TGA over a quarter implies a lower likelihood of borrowing. The volatility of the TGA increased notably as Federal Reserve balances climbed, as it was no longer critical for the TGA to remain at a steady level to ensure monetary control. Possibly reflecting this correlation, the estimated coefficient appears to suggest that borrowing is less frequent when reserves in the system are ample.

The final rows of the table provide goodness of fit statistics. Overall, the variation in discount window borrowing explained by our specification is not large. The pseudo-R^2 statistic ranges from 1 percent to 2 percent, depending on the specification.

4.2 Robustness

4.2.1 Alternative definitions of borrowing

We distinguish decisions to borrow from the discount window from decisions to test access by using a rule of thumb for the loan size of $1 million. As explained above, the extra funding cost of an overnight loan of this size during our sample period was only about $15 (given the 50-basis-point premium on the discount window interest rate). Because this funding cost is modest, the decision to borrow such low amounts may not respond strongly to the financial motivations that we aim to identify. Furthermore, given the modest cost, it may be the case that our test threshold is too low – banks may execute a larger test loan or may test more than once per quarter.[25]

To address this possibility, we re-evaluate the specifications in Table 5 with a higher definition of a borrowing threshold: A bank is defined as borrowing at the discount window in an individual quarter if the total amount borrowed exceeds $10 million. This threshold captures the possibility that banks execute more than one

[25] For these reasons, we believe that the risk of choosing too low threshold versus too high threshold is asymmetric; smaller loans, regardless of whether they are tests, are unlikely to make a critical difference in our study.

loan per quarter or that test loans are larger. The overnight funding costs of $10 million are around $300, still a moderate amount.

Results are displayed in Table 6. Overall, the results are a sharper version of Table 5: qualitatively similar with larger marginal effects. For example, with our baseline threshold, a one standard deviation (about 4 percentage points) increase in the level of reserves to assets leads to a little less than a 1 percentage point decline in the probability of borrowing. With the higher borrowing threshold, the effect is about twice as large, and a one standard deviation increase in the level of reserves to assets leads to a 2 percentage point fall in the probability of borrowing. The statistical significance of some coefficients related to asset and liability composition also moves up; for example, as reported in column (5), the coefficients on CRE to assets, C&I to assets, and short-term to total loans are significant in this specification, as are the share of fed funds borrowed to liabilities. At the same time, the marginal effect of the coefficient on asset size is somewhat smaller, perhaps reflecting the higher threshold.

As indicated by the pseudo R^2 statistics, this model fits the data somewhat better than the more conservative threshold, with the new model explaining around 18 percent of the variation in the probability of borrowing. Even so, we choose the more conservative threshold in what follows, as these estimates could be considered a lower bound on economical and statistical significance.

4.2.2 Heterogeneity on the extensive margin

The decision to borrow at the discount window may vary across broad classes of banks. For example, for any given portfolio allocation, smaller banks may respond differently to their funding needs and the tapping of the discount window, as they may have limited access to alternative funding sources relative to larger banks. In addition, foreign branches may have a different approach to discount window borrowing relative to domestic banks, perhaps relying instead on the parent for funding.

Against this backdrop, we next examine whether factors that determine borrowing vary according to bank size or domicile (foreign or domestic), as well as the intensive margin of discount window borrowing. The results, from separate estimates of equation (11) for each group of banks or dependent variable, are presented in Table 7.

We begin with heterogeneity of the borrowing decision according to bank size. Three items stand out. First, our previous result that borrowing banks hold lower

Table 6: Borrowing–conditional on access, higher threshold

Dependent variable	Borrowed ($DW_{it} = 1$ and $loan_{it} > \$10,000,000$)				
	(1)	(2)	(3)	(4)	(5)
Reserves to assets	-0.369***	-0.379***	-0.329***	-0.353***	-0.334***
	(0.0520)	(0.0530)	(0.0514)	(0.0522)	(0.0531)
CRE to assets		-0.0710			-0.0915*
		(0.0455)			(0.0445)
C&I to assets		0.0727*			0.0719*
		(0.0327)			(0.0340)
Treasury securities to assets		0.00148			0.0166
		(0.0927)			(0.0908)
Short-term to total loans		0.0327*			0.0326*
		(0.0147)			(0.0150)
Liability composition					
Transaction deposits to liabilities			-0.0148		0.00406
			(0.0174)		(0.0178)
Fed funds borrowed to liabilities			0.320*		0.308*
			(0.142)		(0.142)
Repos to liabilities			0.229**		0.217*
			(0.0857)		(0.0895)
FHLB advances to liabilities			0.187***		0.207***
			(0.0366)		(0.0370)
Balance sheet size and capital					
Log(assets)				0.00747***	0.00548*
				(0.00225)	(0.00240)
Unused commitments to total assets				0.00553	-0.00282
				(0.0376)	(0.0372)
Tier-1 capital ratio				-0.00966	0.0127
				(0.0398)	(0.0398)
ROA				0.340	0.379
				(0.231)	(0.219)
Other controls					
District 1	-0.0359***	-0.0302***	-0.0444***	-0.0379***	-0.0394***
	(0.00881)	(0.00887)	(0.00910)	(0.00884)	(0.00914)
District 12	0.0450***	0.0429***	0.0479***	0.0395***	0.0421***
	(0.00725)	(0.00733)	(0.00732)	(0.00745)	(0.00748)
ΔGDP	-0.000425	-0.000373	-0.000385	-0.000515	-0.000401
	(0.000564)	(0.000561)	(0.000564)	(0.000564)	(0.000562)
σ_{TGA}	-0.000303***	-0.000283***	-0.000280***	-0.000329***	-0.000278***
	(0.0000568)	(0.0000562)	(0.0000563)	(0.0000571)	(0.0000560)
Number of banks	2,027	2,027	2,027	2,027	2,027
Number of observations	56,041	56,041	56,041	56,041	56,041
pseudo R-sq	0.175	0.176	0.178	0.176	0.179

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. The dependent variable is an indicator that equals one if a bank borrowed at least $10 million on an overnight equivalent basis from the discount window in a quarter. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Standard errors (shown in parentheses) are clustered at the bank level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

shares of reserves remains robust to bank size. That said, the results are not of similar economic magnitude across bank size, as the elasticity with respect to reserves holdings is higher for larger banks than for smaller ones. Our estimated coefficients imply that a one standard deviation increase in the share of reserves to assets leads to a 2.2 percentage point decline in the probability of borrowing from the discount window for larger banks but only a 60-basis-point decline for smaller ones. Taken together, we believe that these results suggest some unobserved heterogeneity in borrowing behavior. One possible source of this heterogeneity is "sophistication." That is, larger banks may have more sophisticated asset and liability management, permitting larger swings in asset and liability composition, including in the share of reserve balances, for any given shock.

Second, turning to the second line of Table 7, total assets do not predict borrowing for larger banks, but do for smaller ones. It is likely the case that business models with respect to discount window borrowing for the set of larger banks are similar and not based on overall bank size, while those for smaller banks can be more diverse. Together, these results suggest that the "larger" small banks tend to have borrowing behavior closer to the larger banks and are more likely to borrow at the discount window and that there is a group of "smaller" small banks that are not as likely to borrow. That said, our small-banks sample is defined as those banks with $1 billion or less in assets, which in other contexts, suggests these institutions share more in common with each other than with the global or otherwise more sophisticated banks in our large-banks bucket. Third, foreign branch decisions to borrow at the discount window do not appear to be driven by the same factors as those associated with domestic bank borrowing. Although the sample size is relatively small (fewer than 100 institutions), neither the share of reserves nor other balance sheet factors significantly predict foreign branch discount window borrowing. This observation supports our choice to concentrate on domestic institutions in the majority of our analysis.

4.2.3 Collateral and the extensive margin of borrowing

The Federal Reserve requires that all discount window loans be fully collateralized, and collateral usually needs to be posted at the Federal Reserve well in advance of obtaining a discount window loan. Consequently, all observed borrowing banks also have collateral posted at the Federal Reserve. At the same time, the amount of collateral relative to assets can differ across borrowing banks. Here, we explore whether the amount of posted collateral significantly predicts borrowing at the discount window.

We make a few assumptions in our construction of our variable of interest, bank-level collateral to assets. These assumptions are necessary because our observation of collateral posted at the discount window is imperfect. Specifically, we observe posted collateral only when the bank executes a discount window loan but not at any other time. This creates two issues. The first is that if we only use those observations, our data would be sparse, and estimation of the models would be difficult. The second is that we do not observe collateral posted for any bank that does not execute a loan over our sample period.

We can use another attribute of bank behavior at the discount window to address the first issue. In particular, collateral posted at the discount window is sticky – banks infrequently change the amount and composition of collateral posted. As a result, even though we observe collateral only infrequently, this information may be sufficient to gauge the effect of collateral posted on discount window activity. For quarterly observations when a bank does not borrow from the discount window, we substitute the bank's maximum amount of collateral posted over the sample period as a proxy. We then scale this amount of collateral by reported bank assets in the corresponding quarter. The second issue is more problematic; no easy solution exists. As such, our coefficient estimates should be interpreted with care, as we are aware of some potential bias that results from our imperfect observation of collateral.

The fourth column of Table 7 explores the implications of collateral held at the discount window. The results suggest that collateral-constrained banks are more likely to borrow at the discount window. Specifically, the coefficient on collateral posted as a share of assets indicates that for every one standard deviation decrease in collateral as a share of assets, the probability of borrowing at the discount window climbs by roughly 1.8 percentage points, an economically meaningful amount. Banks may be more likely to borrow because they do not have sufficient high-quality collateral to access alternative funding sources or do not have access to alternative funding sources altogether. As a result, they post the collateral on hand at the discount window and use it relatively more intensively than banks that have access to a range of funding sources.

4.2.4 The intensive margin of borrowing

We examine also the intensive margin of discount window borrowing by looking at the share of a bank's liabilities accounted for by discount window borrowings. Our dependent variable is then the amount borrowed in a quarter, as a ratio to total liabilities, which indicates the share of the balance sheet a bank funds with discount window borrowing.[26]

Column 5 of Table 7 displays our results. Smaller banks with fewer reserve balances tend to fund a greater share of their balance sheet with discount window loans. That said, the effects are not economically large: For every one standard deviation decline in reserves as a share of total assets, the discount window funding share increases by about 4 basis points. To put this in perspective, the standard deviation of the share of reserves in total assets is about 4 percentage points, and one standard deviation of collateral as a share of total assets is roughly 7 percentage points. Taken together, while the amount borrowed as a share of liabilities does appear to be significantly correlated with some balance sheet measures, it seems more economically relevant to explore factors that determine the probability of borrowing instead of focusing on factors that determine the intensity of use.

[26]The amount borrowed is calculated as the aggregate amount of daily borrowings outstanding over a quarter. We include weekends in the daily calculation.

Table 7: Borrowing—Robustness, conditional on access

	Extensive margin				Intensive margin
Dependent variable	Borrowed ($DW_{it} = 1$)				Amount borrowed to liabilities
	Heterogeneity			Discount window	Funding
	Large banks	Small banks	Foreign branches	collateral	share
	(1)	(2)	(3)	(4)	(5)
Reserves to assets	-0.409***	-0.131***	-0.0215	-0.197***	-0.0105***
	(0.0933)	(0.0352)	(0.0473)	(0.0388)	(0.00257)
Log(assets)	-0.00287	0.0129***	-0.00125	0.00713***	-0.000659***
	(0.00333)	(0.00208)	(0.00698)	(0.00146)	(0.000153)
District 1	-0.0141	-0.0115*		-0.0119	-0.0000481
	(0.0136)	(0.00544)		(0.00652)	(0.0000254)
District 12	0.0362**	0.0146**		0.0281***	-0.00000201
	(0.0112)	(0.00446)		(0.00585)	(0.00000212)
Total collateral to assets				-0.229***	
				(0.0444)	
Constant					0.00905***
					(0.00275)
Balance sheet controls?	Y	Y	Y	Y	Y
Collateral type controls?	N	N	N	Y	N
Other controls?	Y	Y	Y	Y	Y
Number of observations	12,447	43,594	2,116	56,041	56,040
Number of banks	568	1,675	72	2,027	2,027
(pseudo) R^2	0.001	0.020	0.014	0.033	0.003

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window (first four columns) and from a random effects panel regression model of the effects of selected bank characteristics on the amount of borrowing at the discount window. All samples are restricted to those banks that executed at least a test loan. Columns (1), (2) and (3) evaluate the probit model on large bank, small bank, and foreign branch subsamples, respectively. Column (4) evaluates the probit model on all bank types. The dependent variable is an indicator that equals one if a bank borrowed at the discount window in a quarter. Column (5) evaluates the regression model on all bank types. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Standard errors (shown in parentheses) are clustered at the bank level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

4.2.5 Collateral composition

Our final exercise builds on the results presented in Table 7 and takes a closer look at the collateral posted at the discount window. Because the collateral information is observed only when a bank borrows or tests, the collateral data are sparse relative to the balance sheet data. As a result, for this specification, we collapse our sample to isolate variation by bank in the collateral posted at the discount window.

Table 8 presents the results. The dependent variable equals one if the bank borrows at any time in our sample, where borrowing is defined at the lower threshold used in Table 5. The independent variables are averages of nonzero values of the share of each category of collateral expressed as a share of overall collateral.

The first column looks at broad categories of collateral. Overall, a one-half percentage point, or one standard deviation increase in the share of loan collateral – the least-liquid type – is associated with a 6 percentage point higher probability of borrowing at the discount window. In effect, this result confirms the insights suggested by our model, which indicate that banks with less ability to liquidate assets are more likely to borrow at the discount window.

Turning to the individual collateral types displayed in columns (2) through (4), again we see results consistent with the model. Those banks that post liquid securities are less likely to borrow; those banks that post loans are more likely. One question is which effect dominates. As shown in column (5), when all collateral types are included, the variation in loan collateral appears to be a relatively stronger predictor of borrowing behavior than does the posting of liquid securities.

The final two columns display information that indicate our results are reasonably robust. In particular, when we include collateral posted as a share of assets as an additional control variable (the same variable included as a control in Table 7), the economic and statistical significance remains similar. One curious point is that the sign of total collateral to assets flips at the bank level relative to that shown in the previous table, where banks that post large amounts of collateral relative to assets are less likely to borrow in the panel context but more likely to borrow in the cross section. There are two possible explanations for this. The most likely is that there is significant collinearity of collateral and asset allocations, which leads the coefficient on total collateral to assets to be negative in Table 7. In fact, specifications (not shown) at the bank level suggest imprecisely estimated coefficients when both the balance sheet and collateral variables are included. Another reason, albeit less likely, is that

Table 8: Borrowing and collateral

	Dependent variable: Borrowed ($DW_i = 1$)						
	Across categories	Within categories				Share of assets	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Liquid securities	-0.0126					0.00286	
	(0.0357)					(0.0355)	
Treasury securities		-0.158***			-0.0567		-0.0410
		(0.0270)			(0.0408)		(0.0407)
Municipal securities		-0.0873**			0.0185		0.0304
		(0.0298)			(0.0439)		(0.0434)
Agency MBS		-0.0782**			0.0170		0.0300
		(0.0276)			(0.0407)		(0.0404)
Illiquid securities	0.0750					0.0765	
	(0.0560)					(0.0554)	
Private-label MBS			0.245		0.313		0.319
			(0.184)		(0.178)		(0.175)
International securities			-0.223		-0.216		-0.223
			(0.197)		(0.192)		(0.194)
Corporate securities			-0.0259		0.0383		0.0448
			(0.0639)		(0.0683)		(0.0677)
Asset-backed securities			0.264		0.304		0.262
			(0.174)		(0.172)		(0.174)
Loans	0.124**					0.104*	
	(0.0402)					(0.0404)	
CRE loans				0.113***	0.109*		0.0938*
				(0.0310)	(0.0461)		(0.0460)
C&I loans				0.159***	0.156***		0.132**
				(0.0295)	(0.0464)		(0.0468)
Consumer loans				0.153**	0.149*		0.123*
				(0.0479)	(0.0608)		(0.0620)
Residential mortgages				0.0312	0.0266		0.0223
				(0.0588)	(0.0683)		(0.0682)
Total collateral to assets						0.504*	0.468*
						(0.213)	(0.207)
Number of observations	2,027	2,027	2,027	2,027	2,027	2,027	2,027
Number of banks	2,027	2,027	2,027	2,027	2,027	2,027	2,027
(pseudo) R^2	0.0180	0.0164	0.0020	0.0188	0.0237	0.0249	0.0297

Note: This table provides estimates from a probit model of the marginal effects of selected collateral types on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. Observations are at the bank level. The dependent variable equals 1 if a bank borrowed at the discount window at any time during the sample. Collateral variables are bank-level averages of nonzero values. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Robust standard errors are shown in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

the dependent variables in the specifications differ. Borrowing at any one point in time may not be determined by the same factors as borrowing overall. Other results to be discussed in the following sections indicate that borrowing and access results in panel settings are similar to those in the cross section, pointing to the collinearity of the balance sheet and collateral variables as the likely culprit.

Another thing to keep in mind is that the amount of variation explained by the specifications in Table 8 is not large, ranging only from 2 to 3 percent of overall variation, as measured by the pseudo R^2 statistics. In what follows, we build on these results to gain a broader understanding of factors that predict discount window borrowing.

4.2.6 Endogeneity

Our previous empirical exercises avoid the tricky issue of endogeneity. The implicit assumption underlying the exercises is that by limiting our sample to banks that have access to the discount window, we minimize potential simultaneity bias from joint decisions of borrowing from the discount window and balance sheet characteristics. However, discount window borrowing could depend on unobserved factors – for example, sophistication of balance sheet management – that are also correlated with balance sheet composition. Here, we explore the possibility that holdings of reserves could be correlated with these unobserved factors.

We estimate a series of instrumental-variable specifications to evaluate the potential endogeneity of reserves and borrowing behavior. The sets of instruments for R_{it} we use recognize the possibility of some heterogeneity across bank sizes. Consistent with this, we split our sample into size classes by asset holdings to form our instruments – large banks and small banks. We focus our analysis on domestic banks.

Our first instrument is the bank's reserves as a share of total reserves held by the bank's size class as of slightly more than one year prior to the start of our sample (June 30, 2009). For example, if Bank A is a small bank, the instrument is Bank A's reserve balances as a share of reserves held by small banks as a group. If Bank B is a large bank, the instrument is constructed analogously, using total reserves of that group. As shown in Table 7, there are important differences across size classes in the use of the discount window. At the same time, the factors influencing an individual bank's decision to hold reserves are likely not the same as those that determine the share of reserves it holds with respect to its size class. Consequently, this instrument should be correlated with the choice of an individual bank's holdings of reserves as a share of its assets, but independent of any unobserved factors that could lead the bank to borrow at the discount window. Moreover, using an instrument with an as-of date well before the start of the sample minimizes simultaneity bias.[27]

Our second instrument is the four-quarter change in the share of total (system-wide) reserve balances in total bank credit. The logic of this instrument is straightforward. From the perspective of any individual bank, the change in the aggregate level of reserves is exogenous, as the quantity of reserves available to the banking system is largely determined by actions taken by the central bank. In addition, the change in the share of reserves in total bank credit is also largely exogenous, as the ability of any one bank to affect the overall share is limited. At the same time, there may be differences in how larger banks and smaller banks react to changes in aggregate reserves to assets. As such, we allow for the coefficient on this instrument to differ according to whether a bank is large or small.

We use a maximum-likelihood approach to evaluate a probit model with instrumental variables. In addition, we use a cluster-bootstrap procedure to calculate parameter estimates, marginal effects, and associated standard errors.[28] Other test statistics use standard cluster-robust techniques.

Table 9 displays the results. The sample consists of banks with discount window access. The upper section of the table reports first-stage regression results. The first set of columns (columns (1), (2), and (3)) provides estimates without balance sheet controls, while the second set (columns (4), (5), and (6)) includes them. In each column, the dependent variable is a bank's reserve balances as a share of the bank's total assets in a given quarter. The instrument in the first row is a bank's reserve balances as a share of total reserves held by that size of bank in the third quarter of 2009. The instruments in the second and third rows are the lagged four-quarter change in the aggregate reserves-to-bank assets ratio interacted with bank size-group dummies. In each set of columns, the first two columns use the instruments separately, the third uses all instruments together. All regressions include time controls (GDP changes and TGA variability) and indicators for Districts 1 and 12. Overall, the results are comparable across specifications and suggest that a bank's holdings of reserves to assets are positively correlated with its reserves holdings as a share of

[27]This formulation is in the spirit of a Bartik-like instrument; for a recent description, refer to Goldsmith-Pinkham, Sorkin, and Swift (2020).

[28]We use 500 replications for our bootstrap procedure.

Table 9: Borrowing – Endogenous reserves, conditional on access

	(1)	(2)	(3)	(4)	(5)	(6)
First stage—Dependent variable: Reserves to assets						
1. $\frac{R_{i,t=2009Q3}}{R_{j,t=2009Q3}}$	7.472***		7.480***	6.531***		6.525***
	(0.473)		(0.473)	(0.491)		(0.491)
2. $\frac{\Delta R_{t-4}}{\Delta A_{t-4}}, large$		0.0474*	0.0185		0.0664***	0.0734***
		(0.0237)	(0.0187)		(0.0187)	(0.0166)
3. $\frac{\Delta R_{t-4}}{\Delta A_{t-4}}, small$		0.0302**	0.0403***		0.046***	0.033***
		(0.0110)	(0.00889)		(0.009)	(0.008)
Second stage—Dependent variable: Borrowed ($DW_{it} = 1$)						
4. Reserves to assets	-0.170***	-0.167***	-0.170***	-0.099***	-0.114***	-0.099***
	(0.0237)	(0.0234)	(0.0236)	(0.0218)	(0.0214)	(0.0218)
Balance sheet controls?	N	N	N	Y	Y	Y
Other controls?	Y	Y	Y	Y	Y	Y
Specification tests						
Wald test of exogeneity						
χ^2 stat	0.81	0.04	0.85	3.5	0.47	3.68
p-value	0.37	0.83	0.36	0.06	0.49	0.05
Weak instrument test						
F-stat	9.11	13.45	12.09	6.63	23.51	16.50
p-value	0.0026	0	0	0.01	0	0
Hansen *J* test						
χ^2 stat	n/a	0.97	0.99	n/a	1.13	2.24
p-value		0.32	0.61		0.29	0.33
Number of observations	56,041	56,041	56,041	56,041	56,041	56,041
Number of banks	2,027	2,027	2,027	2,027	2,027	2,027

Notes: This table provides cluster-bootstrap estimates from an instrumental-variable probit model to control for the possible endogeneity of the reserves-to-assets ratio. Sample is all domestic banks that have borrowed at the discount window or have executed a test loan. Instruments for the first-stage regression include the ratio of the bank's reserves to that of its size group as of 2009Q3, and the four-quarter change in the share of reserve balances in total bank credit, allowing the coefficient to differ across size groups. Marginal effects presented in the second-stage results. The specifications include balance sheet and other controls shown in table 5. Bootstrapped standard errors clustered by bank are in parentheses. Bootstrap-robust specification tests are shown in the lower rows of the table.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

size-group reserves, and positively correlated with aggregate changes in the reserves-to-assets ratio.

The middle section of the table reports the second-stage results. The reported parameter is the marginal effect of the reserves-to-assets ratio on the probability of borrowing at the discount window. Relative to the marginal effects reported in Table 5, these marginal effects are slightly smaller. Reading across the columns, a one standard deviation increase in the reserves-to-assets ratio depresses the probability of borrowing at the discount window by roughly 50 basis points to 1 percentage point, depending on the specification.[29] Even with the change in magnitude, the coefficients remain economically meaningful, pointing to a significant link between borrowing and reserves.

The lower panel of the table reports specification tests. Test statistics indicating whether the (bank-level) reserves-to-assets ratio is endogenous are mixed; some specifications suggest that this ratio is endogenous, others do not. In general, the first-stage F-statistics exceed typical thresholds, indicating that we do not use weak instruments. Furthermore, our instruments typically satisfy overidentifying restrictions tests. Taken together, these results indicate reasonable specifications with acceptable instruments.

4.3 Discount window access and holdings of reserves

Our second question concerns the joint decision of gaining access to the discount window and holdings of reserves. In the previous section, we showed that discount window borrowing and reserves may be weakly co-determined – the results are mixed across specifications. Even so, the results point to a significant relationship between reserves and borrowing. But, before banks borrow at the discount window, they must post collateral and obtain access. In what follows, we explore the access decision, with specific focus on whether access is endogenously determined with holdings of reserves.

To study access to the discount window, we need to include all banks in our analysis, not just those with discount window access. During our sample period, many banks (with and without discount window access) operated with significant levels of excess reserves. As a result, the level of reserves may not be directly linked

[29] The standard deviation of the reserves-to-assets ratio in the estimation sample is 0.046. Multiplying this number by the coefficients displayed in row 4 yields changes between 46 and 78 basis points.

to the financial variables that we used to control for heterogeneity across banks. Many banks were indifferent between holding extra units of reserves and not holding them; for that reason, the level of reserves might have been influenced by multiple intangible factors. This was less of a problem for banks that actually borrowed from the discount window since those banks are unlikely to be in possession of significant levels of excess reserves. The empirical methods we use below are partly motivated by the need to deal with extra variability in reserves over assets across the full sample of banks.

4.3.1 Reduced form

To set the stage, we investigate reduced-form correlations between reserves holdings and discount window access using a panel estimator approach without controlling for endogeneity. The results are reported in Table 10. The dependent variable is a bank's reserves-to-assets ratio measured on a quarterly frequency. Control variables include a zero-one indicator of whether a bank has discount window access, defined as having executed at least a test loan at some point in our sample. Other control variables include the balance sheet measures used to explore the borrowing decision above, as well as the time effects controls. Standard errors are clustered at the bank level.

Looking across the columns, it appears that banks with discount window access tend to have lower reserves holdings as shares of total assets. That said, the magnitude of the estimated parameter suggests the effect is small. Without controlling for various balance sheet attributes, as shown in the first column, obtaining access depresses the share of the balance sheet in reserves by a little less than a percentage point. This effect attenuates across columns as balance sheet controls are included in the specification. In our preferred specification, reported in column (5), access does not appear to significantly predict reserves as a share of balance sheet assets.

Turning to the effect of balance sheet attributes on the reserves-to-assets ratio, our estimates suggest that, in general, balance sheets tilted toward more liquid components are associated, also, with higher reserves to assets. On the asset side, as shown in columns (2) and (5), more illiquid loans (CRE and C&I loans) predict lower reserves, while shorter-term loans predict higher reserves. A notable exception is Treasury securities to assets: higher Treasury securities implies lower reserves. This finding points to the substitutability of reserves and Treasury securities in liquidity management or regulatory ratios.

On the liability side, the results in columns (3) and (5) go also in the same di-

Table 10: Reserves and access—Exogenous

	Dependent variable: Reserves to assets				
	(1)	(2)	(3)	(4)	(5)
DW Access	-0.00898***	-0.00694***	-0.00511***	-0.00284*	-0.000503
	(0.00103)	(0.00105)	(0.00101)	(0.00141)	(0.00133)
Asset composition					
CRE to assets		-0.0532***			-0.0626***
		(0.00804)			(0.00822)
C&I to assets		-0.165***			-0.110***
		(0.00845)			(0.00805)
Treasury securities to assets		-0.0753***			-0.119***
		(0.0179)			(0.0181)
Short-term to total loans		0.0364***			0.0437***
		(0.00314)			(0.00304)
Liability composition					
Transaction deposits to liabilities			0.0474***		0.0474***
			(0.00377)		(0.00374)
Fed funds borrowed to liabilities			-0.0996***		-0.0837**
			(0.0284)		(0.0271)
Repos to liabilities			-0.0117		-0.0528***
			(0.0147)		(0.0149)
FHLB advances to liabilities			-0.101***		-0.0893***
			(0.00688)		(0.00635)
Balance sheet size and capital					
Log(assets)				-0.000713	0.00190*
				(0.000944)	(0.000879)
Tier-1 risk-weighted capital ratio				0.144***	0.128***
				(0.00965)	(0.00925)
Unused commitments to assets				-0.0604***	-0.0620***
				(0.00690)	(0.00707)
ROA				-1.140***	-1.115***
				(0.216)	(0.214)
Number of observations	196,489	196,489	196,489	196,489	196,489
Number of banks	7,629	7,629	7,629	7,629	7,629
R^2					
Over time (within)	0.0023	0.0337	0.0216	0.0649	0.1047
Across banks (between)	0.0090	0.0121	0.0646	0.0671	0.1106
Overall	0.0065	0.0145	0.0590	0.0634	0.1052

Notes: This table provides estimates from a random effects panel regression model of the effects of selected bank characteristics on the share of assets held in reserve balances. Sample includes all domestic banks with positive assets, although it eliminates some outlier observations. Regression includes an intercept, controls for the change in nominal GDP and the volatility of the TGA. Standard errors (shown in parentheses) are clustered at the bank level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

rection. In particular, higher transaction deposits to liabilities are associated with a higher reserves-to-assets ratio. Moreover, dependence on short-term wholesale funding suggests lower reserves; for example, more repo or fed funds borrowings are associated with lower reserves.

Separately, and in line with the evidence reviewed in Section 2, larger banks hold more reserves as a share of assets; this supports our previous decisions to allow some heterogeneity according to bank size in our specifications. Banks with higher capital ratios hold proportionally more reserves, consistent with the formulation of these ratios, although profitability appears to take a hit.

The last few rows of the table display R^2 statistics. The specification with asset composition appears to explain more of the variation over time; other specifications appear to explain variation across banks. Overall, the specifications explain roughly 5 and 10 percent of the variation in reserves to assets across banks and over time, a reasonable proportion for panel data.

4.3.2 Endogeneity

Our next exercise explores the potential endogeneity of the discount window access decision and the reserves decision. In particular, we evaluate a treatment-effects model that allows for correlation between unobserved factors that affect reserve balance holdings and discount window access. In this case, the "treatment" is discount window access, and the "outcome" is reserves to assets.

We follow the literature on evaluating treatment effects using non-experimental data to derive our estimates of the effect of access to the discount window on reserves holdings.[30] We choose a control function approach. Intuitively, this approach includes terms in the specification to "control" for omitted variables or correlations between endogenous variables and error terms. In our setting, this approach is appealing because we lack an event or other exogenous policy change that could be used to identify the effect of discount window access on reserves.

Again, we need a suitable instrument. The instrumental variable should be correlated with the choice to gain access to the discount window but uncorrelated (except through the discount window) to the outcome of interest. We posit that the observed probability of discount window access in a particular District is suitable for this purpose. This variable is plotted in Figure 2 and was discussed in Section 2. As is clear

[30] Wooldridge (2010), chapter 21, and Wooldridge (2015) give an overview.

from the figure, there is variability across Fed Districts in the propensity to access the discount window. Such variability is likely uncorrelated with (or at least predetermined with respect to) individual banks' holdings of reserve balances. Moreover, we assume that no individual bank has the ability to affect overall average District-level discount window access.

Our estimation procedure follows four steps. First, we estimate the probability of obtaining discount window access,

$$I_i^{DW} = \alpha_I + \beta_I D_{d(i)t} + \gamma_I \psi_{it} + \mu_{it}, \tag{12}$$

where I_i^{DW} equals 1 if a bank has discount window access at any time in the sample, and $D_{d(i)t}$ is the observed probability for banks in district $d(i)$ (the district where bank i is located) of having access to the discount window in quarter t. The District variable serves as our instrument to identify the exogenous portion of discount window access. We evaluate the specification using pooled probit, which assumes the error term μ_{it} is normally distributed. Standard errors are clustered at the bank level.

Second, we calculate the predicted probability of a bank accessing the discount window, $\widehat{\Phi}_{it}$, as well as the predicted probability density, $\widehat{\phi}_{it}$. We use these to form the control function terms $\frac{\widehat{\phi}_{it}}{\widehat{\Phi}_{it}}$ and $\frac{\widehat{\phi}_{it}}{1-\widehat{\Phi}_{it}}$.

Third, we construct differences in means for the balance sheet control variables interacted with the indicator for access, $I_i^{DW}(\psi_{it} - \bar{\psi})$. These terms control for any unobserved systematic differences across banks that access the discount window versus those that do not. Importantly, these terms can be interpreted as a type of "Chamberlain" fixed effect, as they control for unobserved factors that can influence the share of assets held as reserves.

Fourth, we use ordinary least squares to estimate the parameters of the following regression line:

$$R_{it} = \alpha_R + \beta_R \psi_{it} + \gamma_R I_i^{DW} + \zeta_R I_i^{DW}(\psi_{it} - \bar{\psi}) + \sigma_1 I_i^{DW} \frac{\widehat{\phi}_{it}}{\widehat{\Phi}_{it}} + \sigma_0 (1 - I_i^{DW}) \frac{\widehat{\phi}_{it}}{1 - \widehat{\Phi}_{it}} + \epsilon_i. \tag{13}$$

The first two terms encompass the intercept and our usual balance sheet controls. If we think discount window access is endogenous, we are concerned that the coefficients β_R are biased because they may be correlated with an unobserved factor influencing discount window access. Our control function approach is intended to correct for bias in the estimation of these coefficients.

In addition, our control function approach aims to provide an unbiased estimate of the average treatment effect, the coefficient on the third term in the specification. Importantly, this average treatment effect is across all banks, both those that have access to the discount window and those that do not. That said, we can use this (unbiased) coefficient to calculate separately the treatment effect for those banks that gain access to the discount window.

The fourth, fifth, and sixth terms are the control variables described above. Statistical significance of the coefficients on these terms suggest endogeneity of discount window access and reserves holdings. For example, statistical significance of the ζ_R coefficients suggest systematic differences in observed factors between banks that access the discount window and banks that do not. In addition, significant coefficients on the next two terms, σ_1 and σ_0, indicate statistically significant effects of unobservable factors that influence the decision to gain access to the discount window.

We evaluate equations (12) and (13) on a sample of domestic banks that file Call Reports, only eliminating banks based on outliers for selected balance sheet measures.[31]

Table 11 displays the results. The top part of the table provides the first-stage estimates and diagnostic statistics for the validity of our instrument. Not surprisingly, as shown in line (1), higher mean District access predicts individual bank discount window access. This result suggests that "friendliness" toward access to the discount window in a given District reduces the overall cost of gaining access. The effect is not one-to-one, however, particularly once the full complement of balance sheet controls are included. The coefficient in column (5) suggests that for every 1 percentage point increase in average access to the discount window for a bank's district, the probability that an individual bank gains access increases by roughly 60 basis points. Moreover, as indicated by the pseudo-R^2 statistics reported in line (2), there is still significant variation in the decision to access the discount window, which we will explore below. Line (3) reports cluster-robust weak instrument F-test results. Across all specifications, test statistics exceed typical significance thresholds.

[31] We eliminate banks with missing or negative assets, negative cash-to-assets ratios, loan-to-assets ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported posted discount window collateral greater than total assets. We also eliminate trust companies that file the Call Report, as their balance sheet structure is notably different from that of a typical bank.

The second-stage results are displayed in the lower part of the table. Lines (4), (5), and (6) show the selection results. Overall, the selection terms show that banks that gain access hold lower reserves than banks that do not, and gaining access and holding reserves are not independent choices. Lines (4) and (5) report coefficients on the selection terms for banks with access and banks without access, respectively. The magnitude of the coefficients on the selection terms indicate that, on average, banks that have discount window access hold lower levels of reserves (as shown in column (5), by 2 percentage points on average as a share of the balance sheet) than those that do not (by 5 percentage points). And, the statistical significance of these coefficients indicate selection bias. Moreover, as shown in line (6), these selection differences are statistically significant. Not reported are the Chamberlain-style fixed effects discussed above; in general, these fixed effects are jointly statistically significantly different from zero.

Lines (7), (8), and (9) display the access results. Line (7) indicates that if a bank does choose to access the discount window, it also increases its reserves. This is true for banks that ultimately choose to access the discount window (line 8) and *hypothetically* for those that do not (line 9). However, those banks that ultimately choose to access the discount window increase reserves more than banks that do not make that choice, by roughly 1 percentage point.[32] The results are generally robust across different specifications. The bottom part of the table shows which controls are included in each specification; the columns in Table 11 correspond to the general portfolio specifications reported in Table 5. The specifications reported in columns (2) and (3) suggest that selection on unobserved factors is more significant when asset controls are not included. At the same time, as shown in line (11), the amount of variation explained by the specification increases substantially with all portfolio controls included. Taken together, these results indicate that the full specification reported in column (5) most likely gives us the best picture overall. Separately, results that eliminate the time dimension of the data and rely solely on cross-sectional variation for identification are similar; these estimates are reported in Table A1.

[32]The 1 percentage point is the difference between the coefficients reported in lines 8 and 9. Line 10 is the Z-score test statistic; the p-values indicate that these differences are generally statistically significant.

Table 11: Reserves to assets and access, panel

	(1)	(2)	(3)	(4)	(5)
First stage: Dependent variable – Discount window access ($I_{it}^{DW} = 1$)					
(1) Mean District access	0.922***	0.910***	0.777***	0.591***	0.573***
	(0.0417)	(0.0425)	(0.0435)	(0.0423)	(0.0440)
(2) pseudo R^2	0.0502	0.0593	0.0705	0.1347	0.1362
(3) Weak instrument F-test	427.78	402.90	289.26	185.45	163.16
p-value	0	0	0	0	0
Second stage: Dependent variable—Reserves to assets					
Selection and DW access					
Selection					
(4) Selection, DW access	0.000662	-0.00529	-0.0128***	0.00744	-0.0171***
	(0.00328)	(0.00338)	(0.00371)	(0.00501)	(0.00499)
(5) Selection, no DW access	0.00314	0.0121*	0.0274***	0.0374***	0.0544***
	(0.00451)	(0.00501)	(0.00520)	(0.00575)	(0.00573)
(6) χ^2 stat for selection terms	0.51	8.24	40.53	45.67	105.86
p-value	0.77	0.016	0.00	0.00	0.00
DW Access					
(7) Avg treatment effect	-0.00734	0.00458	0.0244***	0.00575	0.0454***
	(0.00427)	(0.00462)	(0.00508)	(0.00704)	(0.00678)
(8) Avg treatment effect on treated	-0.007***	0.005***	0.0252***	0.0145***	0.052***
	(1.35e-06)	(0.0001)	(0.0001)	(0.0004)	(0.0003)
(9) Avg treatment effect on untreated	-0.007***	0.005***	0.0242***	0.0021***	0.043***
	(9.44e-07)	(0.00004)	(0.00007)	(0.00023)	(0.0002)
(10) Z-score for difference	-7.45	1.26	7.29	26.46	23.17
p-value	0	0.208	0	0	0
Asset controls?	N	Y	N	N	Y
Liability controls?	N	N	Y	N	Y
Balance sheet controls?	N	N	N	Y	Y
Time controls?	Y	Y	Y	Y	Y
Number of observations	196,489	196,489	196,489	196,489	196,489
Number of banks	7,629	7,629	7,629	7,629	7,629
(11) R^2	0.0067	0.0379	0.0720	0.0875	0.1593

Note: This table provides second-stage estimates from a control function panel model that explores the effects of selected bank characteristics on the reserves-to-assets ratio. The dependent variable is the share of bank assets held in reserve balances. An observation is a bank-quarter. Bootstrapped standard errors clustered by bank are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.3.3 Discussion and robustness

How do we interpret the three results reported in Table 11? At first glance, high District access predicting individual bank access, banks with access holding lower reserves than banks without access, and all banks increasing their holdings of reserves when gaining discount window access appear inconsistent with one another. In particular, it could be surprising that banks with lower reserves-to-assets ratios both gain access and increase reserves simultaneously.

A few reminders can clarify the results. The first reminder is that the results are consistent with some of the predictions of the model discussed in Section 3. Specifically, the model suggests that there may be systematic differences between the distribution of shocks of banks that choose to gain access versus banks that do not. As a result, these systematic differences can lead to the result that reserves and access increase simultaneously. Even though our balance sheet measures and our instrumental-variable approach attempt to control for some of these systematic differences, it is possible that unobserved heterogeneity is still partly driving some results.

The second reminder is that there were substantial changes in the level and distribution of reserves over our sample period. At the time, the Federal Reserve was expanding the size of its balance sheet at a pace well above that of currency growth (which until that point, was a reasonable proxy for the growth rate of Fed assets over most of the post-war years). As a result, explaining banks' strategies for holding reserves (as a share of assets) over this period is a tall order.

With these challenges in mind, we undertake two exercises to explore the robustness of our baseline results. In the first exercise, we use the observation that in the aggregate, large banks disproportionately absorbed the increase in total reserves, while small banks generally held their reserves steady. We, then, split the sample into large and smaller banks, as we did in Section 4.2. As displayed in columns (1) through (3) of Table 12, the positive coefficient for access reported in Table 11 appears to be driven by smaller bank access behavior. The positive sign, magnitude, and significance of the coefficient for smaller banks is in line with that for the overall sample. By contrast, while there is a positive correlation between access and reserves for large banks, the effect is not statistically significant. Given the dramatic changes in the Federal Reserve's balance sheet, there is likely a range of other factors outside of discount window access driving reserves holdings for larger banks.

Table 12: Reserves to assets and access, robustness

	Heterogeneity			Alternative access measure	
	All	Large banks	Small banks	All	Large banks
	(1)	(2)	(3)	(4)	(5)
DW access	0.0454***	0.0405	0.0425***	0.0456***	0.0378
	(0.00172)	(0.0328)	(0.00670)	(0.00169)	(0.0341)
Selection, access	-0.0171***	-0.0283	-0.0162***	-0.0160***	-0.0291
	(0.00118)	(0.0256)	(0.00467)	(0.00115)	(0.0240)
Selection, no access	0.0544***	0.0277	0.0539***	0.0566***	0.0252
	(0.00192)	(0.0276)	(0.00603)	(0.00188)	(0.0287)
Asset controls?	Y	Y	Y	Y	Y
Liability controls?	Y	Y	Y	Y	Y
Balance sheet controls?	Y	Y	Y	Y	Y
Time controls?	Y	Y	Y	Y	Y
Number of observations	196,489	19,671	176,818	196,489	19,671
Number of banks	7,629	971	7,044	7,629	971
R^2	0.159	0.275	0.161	0.161	0.276

Note: This table provides second-stage estimates from a control function panel model that explores the effects of selected bank characteristics on the reserves-to-assets ratio. The dependent variable is the share of bank assets held in reserve balances. An observation is a bank-quarter. Bootstrapped standard errors clustered by bank are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In our second exercise, we test whether the relationship between discount window access and reserves is constant across the distribution of reserves holdings. In particular, we create subsamples by splitting the sample by quartiles of reserves to assets. We then re-evaluate our empirical specification on each sample separately. The idea behind this exercise is based on the expectation that those banks holding lower levels of reserves over assets (and, hence, less excess reserves) respond more systematically to gaining access at the discount window.

Table 13 reports our results. As shown in the first row, the coefficient on access is consistently positive across quartiles. However, the magnitude and significance varies. For example, while access is associated with a 20-basis-point increase in reserves to assets at the lower end of the distribution, it is associated with a much larger 4 percentage point increase in the upper quartile. This magnitude shift is also true with the selection terms. Still, the results suggest that those banks that choose to have discount window access also increase their levels of reserves to assets.

The final reminder is that we have limited data, and so our access proxy is simply a proxy. We base access on observed access; presumably, many of the larger banks and some of the small ones do not execute regular test loans, and their discount window access is unobserved in our approach. Consequently, and consistent with our estimates, our results may reflect some selection bias. To explore the potential effects

Table 13: Reserves to assets, by quartile

	(1)	(2)	(3)	(4)
	Lower quartile	Interquartile range		Upper quartile
DW access	0.00215**	0.00116*	0.00122	0.0389*
	(0.000717)	(0.000513)	(0.000838)	(0.0170)
Selection, access	0.000117	-0.000549	-0.000467	-0.0147
	(0.000571)	(0.000383)	(0.000639)	(0.0117)
Selection, no access	0.00344***	0.00108*	0.00193**	0.0560***
	(0.000669)	(0.000424)	(0.000670)	(0.0113)
Constant	0.0181***	0.0272***	0.0541***	0.149***
	(0.00148)	(0.00106)	(0.00150)	(0.0200)
Asset controls?	Y	Y	Y	Y
Liability controls?	Y	Y	Y	Y
Balance sheet controls?	Y	Y	Y	Y
Time controls?	Y	Y	Y	Y
Number of observations	49,123	49,122	49,122	49,122
Number of banks	4,705	5,698	5,852	4,678
R^2	0.250	0.007	0.007	0.210

Note: This table provides second-stage estimates from a control function panel model that explores the effects of selected bank characteristics on the reserves-to-assets ratio. The dependent variable is the share of bank assets held in reserve balances. An observation is a bank-quarter. Bootstrapped standard errors clustered by bank are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

of this issue, we flagged all banks with greater than $10 billion assets as having access to the discount window and re-ran our empirical model for the overall sample and for the large-bank sample. As shown in columns (4) and (5) of Table 12, overall, the results are largely unchanged.

5 Conclusions

This paper provides new evidence on the use of the Fed's discount window in normal times. Many banks in the U.S. tap the discount window during normal times, occasionally more than once in a relatively short period of time. The reasons for borrowing from the discount window are not well-understood, and it is an open question to what extent banks can modulate their use, possibly adjusting their operations so as to not rely at all on the discount window for backup liquidity outside of crisis times.

In this paper, we show that discount window activity is tightly correlated with certain bank characteristics and portfolio decisions – the holding of bank reserves being the most critical one. We provide theoretical foundations for these correlations and investigate empirically their origins, using transactions data that only recently have become publicly available. We find that discount window access influences financial decisions. We also find that financial decisions – such as the holding of more or less reserves – influence the probability of actually borrowing from the discount window. While the estimated empirical effects are not large, it is telling that they are present even in a period of relative calm in financial markets and when the banking system as a whole is operating with large amounts of excess reserves.

The appropriateness of having a discount window open at all times has been questioned for decades. The evidence presented in this paper indicates that banks consistently adjust their behavior to influence their exposure to the need of borrowing from the discount window. This suggests that, in principle, banks would be able to cope with not having a discount window during normal times. The costs of the resulting adjustments are, of course, hard to estimate. Further research in this direction is a natural next step in the process of evaluating whether the discount window should remain open at all times.

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Appendix - Table

Table A1: Reserves and access – Endogenous, cross section

	(1)	(2)	(3)	(4)	(5)
Second stage: Dependent variable – Reserves to assets (R_{it})					
	(1)	(2)	(3)	(4)	(5)
Discount window access	-0.00355	0.00919*	0.0325***	0.0130	0.0466***
(Average treatment effect)	(0.00431)	(0.00468)	(0.00533)	(0.00680)	(0.00683)
Selection, access	-0.00181	-0.00779*	-0.0179***	-0.00182	-0.0222***
	(0.00296)	(0.00322)	(0.00350)	(0.00485)	(0.00470)
Selection, no access	0.00632	0.0173**	0.0323***	0.0332***	0.0468***
	(0.00504)	(0.00551)	(0.00557)	(0.00584)	(0.00586)
adj. R^2	0.011	0.056	0.091	0.098	0.191
χ^2 stat for selection terms	1.85	13.58	50.39	32.36	69.63
p-value	0.3962	0.00	0.00	0.00	0.00
χ^2 stat for difference in mean terms					86.36
p-value	0.3962	0.00	0.00	0.00	0.00
Asset controls?	N	Y	N	N	Y
Liability controls?	N	N	Y	N	Y
Balance sheet controls?	N	N	N	Y	Y
Time controls?	Y	Y	Y	Y	Y
Number of banks	7,629	7,629	7,629	7,629	7,629

Note: This table provides estimates from a control function cross-section model of the of the marginal effects of selected bank characteristics on the reserves-to-assets ratio. In the first-stage model, the dependent variable is an indicator that equals 1 if the bank accessed the discount window at any point in the sample. In the second-stage model, the dependent variable is the reserves-to-asset ratio. Coefficients are reported as marginal effects. An observation is a bank-quarter. Bootstrapped standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix - Proofs

Consider the following constrained maximization problem, where $\lambda_i(\epsilon)$ for $i = 1, 2$ and $\beta_i(\epsilon)$ for $i = 1, 2, 3$ are Lagrange multipliers:

$$\begin{aligned}\max\ (1 + r_{IOR})f(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_{\nu})\nu(\epsilon) \\ +\lambda_1(\epsilon)[s^L(\epsilon) + \theta s^I(\epsilon) - b^{DW}(\epsilon)] \\ +\lambda_2(\epsilon)[f - f(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + \nu(\epsilon) - \Delta(\epsilon)] \\ +\beta_1(\epsilon)f(\epsilon) + \beta_2(\epsilon)b^{DW}(\epsilon) + \beta_3(\epsilon)\nu(\epsilon).\end{aligned}$$

The first-order conditions (FOC) are:

$$\begin{aligned}(1 + r_{IOR}) - \lambda_2(\epsilon) + \beta_1(\epsilon) = 0, \\ -(1 + r_{FF}) + \lambda_2(\epsilon) = 0, \\ -(1 + r_{DW}) - \lambda_1(\epsilon) + \lambda_2(\epsilon) + \beta_2(\epsilon) = 0, \\ -(1 + r_{\nu}) + \lambda_2(\epsilon) + \beta_3(\epsilon) = 0.\end{aligned}$$

Proof of Proposition 1. Using the second FOC, we have that $\lambda_2(\epsilon) = 1 + r_{FF}$, and substituting in the fourth FOC and noting that $r_{\nu} > r_{FF}$, we have that $\beta_3(\epsilon) > 0$, and hence, by complementary slackness, $\nu(\epsilon) = 0$.

Using the third FOC and the fact that $\lambda_1(\epsilon) \geq 0$ and $r_{DW} > r_{FF}$, we have that $\beta_2(\epsilon) > 0$ and, by complementary slackness, $b^{DW}(\epsilon) = 0$.

Hence, we have that $f(\epsilon) = f + b^{FF}(\epsilon) - \Delta(\epsilon)$. Now, using the first FOC, when $r_{IOR} < r_{FF}$, we have that $\beta_1(\epsilon) > 0$ and $f(\epsilon) = 0$, which implies that $b^{FF}(\epsilon) = \Delta(\epsilon) - f$.

Proof of Proposition 2. To simplify notation, define $s \equiv s^L + \theta s^I$. Since the bank has no access to the interbank market, $b^{FF}(\epsilon) \equiv 0$ for all contingencies. As indicated in the statement of the proposition, we need to consider two cases, depending on whether the ex-ante amount of reserves held by the bank (f) is larger or smaller than the liquidity shock ($\Delta(\epsilon)$).

- We start with the case when $f \geq \Delta(\epsilon)$. Then, from the flow constraint, we have that:

$$f(\epsilon) = f + b^{DW}(\epsilon) + \nu(\epsilon) - \Delta(\epsilon),$$

and since $b^{DW}(\epsilon) \geq 0$ and $\nu(\epsilon) \geq 0$, we have that $f(\epsilon) \geq f - \Delta(\epsilon)$.
Claim 1. If $f = \Delta(\epsilon)$ then $b^{DW}(\epsilon) = 0$ and $\nu(\epsilon) = 0$. Proof: Suppose not. Since $f = \Delta(\epsilon)$, we have that $f(\epsilon) = b^{DW}(\epsilon) + \nu(\epsilon) > 0$. But then, $\beta_1(\epsilon) = 0$ and $\lambda_2(\epsilon) = 1 + r_{IOR}$, which implies that both $\beta_2(\epsilon)$ and $\beta_3(\epsilon)$ are positive and, hence, $b^{DW}(\epsilon)$ and $\nu(\epsilon)$ equal zero, which is a contradiction.■
Claim 2. If $f > \Delta(\epsilon)$ then $b^{DW}(\epsilon) = 0$ and $\nu(\epsilon) = 0$. Proof: If $f > \Delta(\epsilon)$ then $f(\epsilon) > 0$ and $\beta_1(\epsilon) = 0$ so that $\lambda_2(\epsilon) = 1 + r_{IOR} > 0$. Then, $\beta_3(\epsilon) = r_\nu - r_{IOR} > 0$ implies that $\nu(\epsilon) = 0$, and $\beta_2(\epsilon) = r_{DW} - r_{IOR} + \lambda_1(\epsilon) > 0$ implies $b^{DW}(\epsilon) = 0$.■

• When $f < \Delta(\epsilon)$ and $s > 0$, using the flow constraint, and since $f(\epsilon) \geq 0$, we have that:

$$b^{DW}(\epsilon) + \nu(\epsilon) = \Delta(\epsilon) - f + f(\epsilon) > 0,$$

which implies that either $b^{DW}(\epsilon) > 0$ or $\nu(\epsilon) > 0$ (or both).
Claim 3. If $\nu(\epsilon) > 0$ then $b^{DW}(\epsilon) > 0$. Proof: Suppose not. Suppose $\nu(\epsilon) > 0$ and $b^{DW}(\epsilon) = 0$. When $\nu(\epsilon) > 0$, we have that $\beta_3(\epsilon) = 0$ and $lambda_2(\epsilon) = 1 + r_\nu$. Also, when $b^{DW}(\epsilon) = 0$ and $s > 0$, we have that $\lambda_1(\epsilon) = 0$ and $\beta_2(\epsilon) = r_{DW} - r_\nu < 0$, which implies a contradiction (since β_2 is a Lagrange multiplier).■

Since either $\nu(\epsilon)$ or $b^{DW}(\epsilon)$ are greater than zero and when $\nu(\epsilon) > 0$, we also have that $b^{DW}(\epsilon) > 0$, then we conclude that $b^{DW}(\epsilon) > 0$.

Also, $b^{DW}(\epsilon) > 0$ implies $\beta_2(\epsilon) = 0$ and $\lambda_2(\epsilon) = 1 + r_{DW} + \lambda_1(\epsilon)$, which implies that:

$$\beta_1(\epsilon) = \lambda_2(\epsilon) - (1 + r_{IOR}) = r_{DW} - r_{IOR} + \lambda_1(\epsilon) > 0,$$

which implies that $f(\epsilon) = 0$.

We need to consider two cases now, depending on whether the collateral constraint is or is not binding.
Case 1. When $\Delta(\epsilon) - f \leq s$, given that $f(\epsilon) = 0$, from the flow constraint, we have that $b^{DW}(\epsilon) + \nu(\epsilon) = \Delta(\epsilon) - f \leq s$.
Claim 4. When $\Delta(\epsilon) - f = s$, we have that $\nu(\epsilon) = 0$. Proof: If $\Delta(\epsilon) - f = s$ then $b^{DW}(\epsilon) + \nu(\epsilon) = s$. In consequence, either $b^{DW}(\epsilon) = s$, which implies that $\nu(\epsilon) = 0$, or $b^{DW}(\epsilon) < s$, which implies $\lambda_1(\epsilon) = 0$. From the FOCs, then, we have $\lambda_2(\epsilon) = 1 + r_{DW}$ and $\beta_3(\epsilon) = r_\nu - r_{DW} > 0$, which by complementary slackness implies that $\nu(\epsilon) = 0$.■
Claim 5. When $\Delta(\epsilon) - f < s$, we have that $\nu(\epsilon) = 0$ and $b^{DW}(\epsilon) = \Delta(\epsilon) - f$. Proof: When $\Delta(\epsilon) - f < s$ then $b^{DW}(\epsilon) \leq b^{DW}(\epsilon) + \nu(\epsilon) = \Delta(\epsilon) - f < s$, so $b^{DW}(\epsilon) < s$ and $\lambda_1(\epsilon) = 0$, which implies that $\lambda_2(\epsilon) = 1 + r_{DW}$ and $\beta_3(\epsilon) = r_\nu - r_{DW} > 0$, which

by complementary slackness implies that $\nu(\epsilon) = 0$. Then, the flow constraint implies that $b^{DW}(\epsilon) = \Delta(\epsilon) - f$.■

Case 2. We consider now the case when $\Delta(\epsilon) - f > s$.

Claim 6. When $\Delta(\epsilon) - f > s$, we have that $b^{DW}(\epsilon) = s > 0$. Proof: Suppose not. Suppose $b^{DW}(\epsilon) < s$. Then, $\lambda_1(\epsilon) = 0$ and, from the FOCs, we have that $\lambda_2(\epsilon) = 1 + r_{DW}$. This, in turn, also from the FOCs, implies that $\beta_3(\epsilon) = r_\nu - r_{DW} > 0$, which by complementary slackness implies that $\nu(\epsilon) = 0$. Now using the flow constraint, we have that $b^{DW}(\epsilon) = \Delta(\epsilon) - f > s$, which is a contradiction.■

To conclude, note that given that $b^{DW}(\epsilon) = s > 0$ implies that $\beta_2(\epsilon) = 0$ then, from the FOCs, we have that $\lambda_2(\epsilon) = 1 + r_{DW} + \lambda_1(\epsilon) > 0$, and the flow constraint holds with equality – with $f(\epsilon) = 0$, which we have shown before. This implies that $\nu(\epsilon) = \Delta(\epsilon) - f - s > 0$. This completes the proof.

Proof of Proposition 3. We present here only a sketch of the proof, since many of the details are straightforward.

We focus on how the bank would choose the level of securities and reserves in Problem 8. Given the linearity of the problem, we can evaluate those decisions independently from the lending decisions.

It is easy to see that whenever $r_{SI} = r_{SL} = r_D$, the bank will choose sufficient holdings of securities for the collateral constraint to not be binding (otherwise, the shadow return on holding securities would include the value of relaxing the collateral constraint and the return on holding an extra unit of securities would be higher than the deposit rate, which represents the cost).

The choice of the ex-ante level of reserves is more complex. We need to account for the changes in the ex-post decisions that are associated with different levels of reserves under different contingencies. These decisions were characterized in propositions 1 and 2.

When the bank has access to the interbank market (as in Proposition 1), we can reduce the ex-post payoff implied by Problem 8 to:

$$(1 + r_{IOR})f(\epsilon) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon).$$

Also from Proposition 1, we know that when $r_{IOR} = r_{FF}$, we have that $f(\epsilon) - b^{FF}(\epsilon) = f - \Delta(\epsilon)$, and when $r_{IOR} < r_{FF}$, we have that $f(\epsilon) = 0$ and $-b^{FF}(\epsilon) =$

$f - \Delta(\epsilon)$. Hence, in both cases, we can further reduce the ex-post payoff to:

$$(1 + r_{FF})(f - \Delta(\epsilon)) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon).$$

This will be the payoff when the bank has access to the interbank market for all possible values of the shock. This event happens with probability q.

With probability $1 - q$, the bank has no access to the interbank market. Ex-post decisions in this case are described in Proposition 2 and can be used to reduce the ex-post payoff implied by Problem 8 to:

$$(1 + r_{IOR})f(\epsilon) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_\nu)\nu(\epsilon).$$

If the bank were to choose $f \geq \Delta_2$ then $b^{DW} = 0$ and $\nu = 0$ for all values of the shock and the ex-post payoff would be:

$$(1 + r_{IOR})(f - \Delta(\epsilon)) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon).$$

Combining these two payoffs, and given the probabilities associated with them, we can see that if $r_D < r^{T1} = qr_{FF} + (1 - q)r_{IOR}$, the bank would want to increase deposits and reserves indefinitely. Such a situation would not be consistent with equilibrium. When $r_D = r^{T1}$, the bank is indifferent between holding any quantity of reserves greater than or equal to Δ_2. This is a situation consistent with significant levels of excess reserves.

Now consider a situation where the bank chooses a level of reserves $f \in (\Delta_1, \Delta_2)$. In this case, when the bank has no access to the interbank market, if the shock is equal to Δ_2, then the bank will be short of reserves and will have to borrow from the discount window. If, instead, the shock is equal to Δ_1 or Δ_0, then the bank will have sufficient reserves to cover the shock and the leftover amount will earn the interest paid on reserves. As a consequence, the relevant portion of the expected payoff can be written as:

$$p_1(1 + r_{IOR})(f - \Delta_1) + p_0(1 + r_{IOR})f - (1 + r_D)f - p_2(1 + r_{DW})(\Delta_2 - f),$$

which, combined with the payoff when the bank has access to the interbank market (discussed above), results in a expected payoff equal to $(r^{T2} - r_D)f - X$, where X is a term independent of the choice of f.

Clearly, then, whenever $r_D \in (r^{T1}, r^{T2})$, the bank will choose to increase the level of reserves f until it reaches the value of the highest shock Δ_2 (at which point, the relevant payoff function changes to the one described in the case when $f \geq \Delta_2$).

Similarly, when the bank chooses a level of reserves lower than Δ_1, it will need to borrow from the discount window when $\Delta(\epsilon)$ equals either Δ_1 or Δ_2 and the relevant portion of the expected payoff function now equals $(r^{T1} - r_D)f - X'$. As a result, whenever $r_D \in (r^{T1}, r^{T3})$, the bank will choose $f = \Delta_1$.

Finally, when $r_D > r_{T3}$, the cost for the bank of choosing an extra unit of reserves is higher than the return even when the bank needs to borrow from the discount window after any positive shock. Hence, in such case, the bank will choose to hold no reserve. The rest of the details of the proof are straightforward.

Proof of Proposition 5 The proof of Proposition 5 follows exactly the same logic as the proof of Proposition 3. The only difference is that when the bank has no access to the interbank market and the shock is larger than the level of reserves held by the bank, then the bank has to incur an overnight overdraft – which is more expensive than a discount window loan, since $r_\nu > r_{DW}$.