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High-Frequency Estimates of the Natural Real Rate and Inflation Expectations

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Abstract

We propose a new method of estimating the natural real rate and long-horizon inflation expectations, using nonlinear regressions of survey-based measures of short-term nominal interest rates and inflation expectations on U.S. Treasury yields. We find that the natural real rate was relatively stable during the 1990s and early 2000s, but declined steadily after the global financial crisis, before dropping more sharply to around 0 percent during the recent COVID-19 pandemic. Long-horizon inflation expectations declined steadily during the 1990s and have since been relatively stable at close to 2 percent. According to our method, the declines in both the natural real rate and long-horizon inflation expectations are clearly statistically significant. Our estimates are available at whatever frequency we observe bond yields, making them ideal for intraday event-study analysis—for example, we show that the natural real rate and long-horizon inflation expectations are not affected by temporary shocks to the stance of monetary policy.

Keywords: Natural real rate, nonlinear regression, term structure model.

JEL: E43, G12.

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1 Introduction

The natural real rate is an important concept in monetary economics. We can think of it as being "the real interest rate consistent with output equalling its natural rate and stable inflation" (Laubach and Williams (2003)) that "provides a neutral benchmark to calibrate the stance of monetary policy" (Christensen and Rudebusch (2019)). But the natural real rate cannot be observed directly and it is notoriously difficult to estimate. We propose a novel approach that combines information in surveys of professional forecasters about short-term nominal interest rates and inflation with information from the yields on U.S. Treasury securities. Our approach avoids some of the important drawbacks of previous approaches.

Most recent related studies have adopted one of two broad approaches to estimating the natural real rate. First, following Laubach and Williams (2003), various studies infer the natural real rate from the dynamics of observed macroeconomic variables. They assume an IS equation in which real activity reacts to the "interest rate gap" between the observed real interest rate and the unobserved natural real rate. They further assume that the natural real rate follows a random walk and that the interest rate gap is expected to close in the long run. Thus, we can also think of the natural real rate as a long-run concept, since it corresponds to the long-horizon expectation of the short-term real interest rate. Closely related studies include Holston, Laubach, and Williams (2017), Lewis and Vazquez-Grande (2017), and Kiley (2020). Second, studies such as Johannsen and Mertens (2020) and Christensen and Rudebusch (2019) measure long-horizon expectations of the short-term real rate using reduced-form time-series models of interest rates; these studies also interpret the model-implied long-horizon expectations as a proxy for the natural real rate.[1] Both broad approaches have limitations that cause the resulting estimates of natural real rates to be extremely uncertain: The structural macroeconomic models rely primarily on the correlation

[1] In addition, various studies estimate long-horizon expectations of short-term nominal interest rates using time-series methods (the many examples include Adrian, Crump, and Moench (2013), Bauer and Rudebusch (2020), and Kim and Orphanides (2012)).

between economic activity and real interest rates; there is a long literature observing that this correlation is empirically weak (see, for example, Taylor (1999)). And the more reduced-form time-series models rely primarily on pinning down the long-run conditional mean of the real interest rate from the time-series behavior of interest rates, which is known to be extremely challenging (see, for example, Kim and Orphanides (2012) and Wright (2014)).

Our approach avoids these difficulties by looking directly at the real interest rate expectations of professional forecasters, as measured by the Blue Chip Economic Indicators and Blue Chip Financial Forecasts surveys. Survey-based measures of expectations have their own limitations—we focus on two in particular. First, there is a longstanding literature observing that surveys likely measure expectations with errors (see, for example, Nordhaus (1987)). Second, surveys are only observed infrequently, which means that we cannot cleanly observe the response of the natural real rate to news. To address these two shortcomings, we estimate nonlinear regressions of survey-based measures of 5-to-10-year-ahead expectations of the short-term nominal interest rate and Consumer Price Index inflation on the yields on U.S. Treasury securities. The difference between the fitted values from the regressions for short-term nominal interest rates and inflation can be interpreted as a measure of the natural real rate, which is adjusted for measurement error and is available at whatever frequency we observe Treasury yields.

We find that the natural real rate has declined from around 2.5 percent in January 1990 to below 0 percent in December 2020. For much of the sample, the natural real rate fluctuated around an average level of about 2 percent, before declining steadily in the decade following the global financial crisis. It then dropped more sharply in 2020, following the onset of the COVID-19 pandemic. We also find that long-horizon inflation expectations have declined since 1990—from about 4 percent to about 2 percent at the end of our sample. However, this decline primarily took place during the 1990s; since then long-horizon inflation expectations have been relatively stable.

Two key assumptions underpin our approach, as well as also other studies that estimate the natural real rate using Treasury yields, such as Christensen and Rudebusch (2019). First, we assume that all of the information that matters for determining expectations of future interest rates and inflation is contained in the current term structure of yields on U.S. Treasury securities—that is, that expectations are "spanned" by the current yield curve. This spanning assumption is entirely standard in much of the term structure literature and means that the part of surveys that is unexplained by Treasury yields can be interpreted as measurement error.[2]

Second, we assume that the 5-to-10-year-ahead expectation of the short-term real interest rate is a reasonable proxy for the natural real rate—that is, that it is a sufficiently long-horizon expectation that it is not affected by transitory shocks, such as temporary changes to the stance of monetary policy. Similarly, we would not expect temporary monetary policy shocks to affect long-horizon inflation expectations if those expectations are well-anchored. To shed light on the validity of this assumption, we explore the relationship between monetary policy shocks and our estimates of the natural real rate and long-horizon inflation expectations. Our model is uniquely positioned for this analysis due to its computational efficiency: macroeconomic models depend on infrequently published macroeconomic data, while term structure models require estimation of computationally intensive time series models; our regression framework enables us to estimate these rates in a fraction of a second at whatever frequency we observe bond yields. Using standard event study methods and a data set of high-frequency estimates from 2004 to 2020, we find evidence supporting our assumption that the natural real rate is not affected by monetary policy shocks. We also find evidence that long-horizon inflation expectations are not affected by monetary policy shocks.

Our regression-based approach is closely related to no-arbitrage term structure models (ATSMs) that are augmented with information from surveys, as in Kim and Orphanides (2012) and D'Amico, Kim, and Wei (2018) but crucially allows for nonlinearities in the

[2]Evidence that is supportive of this spanning hypothesis is provided by Bauer and Rudebusch (2017).

relationship between surveys and yields. We find that a linear version of our regression model delivers almost identical results to such a survey-augmented ATSM. The intuition is simple: while ATSMs also incorporate information about the observed time-series dynamics of interest rates, this information is only weakly informative about future interest rates (as shown previously by Kim and Orphanides (2012), among others). And we show nonlinearities are necessary to capture the broad movements in natural real rates and long-horizon inflation expectations. Moreover, our approach can also deliver results at a tiny fraction of the complexity and computational cost of a term structure model.

The remainder of the paper proceeds as follows. In Section 2, we explain our nonlinear regression-based approach. In Section 3, we report our central findings from an application of this approach to Blue Chip surveys regressed on U.S. Treasury yields. In Section 4, we explore the relationship between monetary policy shocks and our estimates of the natural real rate and long-horizon inflation expectations. In Section 5, we shed further light on the relationship between our approach and no-arbitrage term structure models. In Section 6, we offer some concluding remarks.

2 Nonlinear Regression Model

In this section, we explain our nonlinear regression-based approach to estimating the natural real rate and long-horizon inflation expectations. Suppose that we observe a noisy, survey-based proxy for the 5-to-10-year ahead short-term nominal interest rate ($s_t^{(60,120)}$) and inflation. We estimate a model of the form

$$s_t^{(60,120)} = g(\mathbf{x}_t) + u_t, \tag{1}$$

where $\mathbf{x}_t$ is an $n_x \times 1$ vector of regressors, $g(\mathbf{x}_t)$ is an unknown scalar valued function of $\mathbf{x}_t$, and u_t is a measurement error. The fitted value from equation (1), $\widehat{g}(\mathbf{x}_t)$, can be interpreted of a time-t estimate of the expected short-term nominal rate from 5 to 10 years ahead. We estimate an exactly analogous regression for inflation expectations and take the difference between the fitted expected nominal rate and fitted expected inflation as a proxy for the natural real rate.

To estimate equation (1), we use the local linear estimator of the function $g(\mathbf{x}_t)$ for period t,

$$\widehat{g}(\mathbf{x}_t) = \mathbf{e}_1' \left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1} \mathbf{X}'\mathbf{W}\mathbf{s}, \tag{2}$$

where $s = \left[s_{s_1}^{(n)}, s_{s_2}^{(n)}, ..., s_{s_T}^{(n)}\right]'$ is a $T \times 1$ vector of survey observations for periods $s_1, s_2, ..., s_T$, $\mathbf{e}_1$ is a $n_x \times 1$ vector with the first element equal to 1 and the remaining elements equal to zero,

$$\mathbf{X} = \begin{bmatrix} 1 & (\mathbf{x}_{s_1} - \mathbf{x}_t)' \\ 1 & (\mathbf{x}_{s_2} - \mathbf{x}_t)' \\ ... & ... \\ 1 & (\mathbf{x}_{s_T} - \mathbf{x}_t)' \end{bmatrix}, \tag{3}$$

and $\mathbf{W}$ is a $s_T \times s_T$ weighting matrix. We use the local linear regression weighting matrix

$$\mathbf{W} = \begin{bmatrix} \prod_{d=1}^{n_x} K\left(\frac{x_{s_1,d} - x_{t,d}}{b_d}\right) & 0 & ... & 0 \\ 0 & \prod_{d=1}^{n_x} K\left(\frac{x_{s_2,d} - x_{t,d}}{b_d}\right) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \prod_{d=1}^{n_x} K\left(\frac{x_{s_T,d} - x_{t,d}}{b_d}\right) \end{bmatrix}. \tag{4}$$

Thus, the weightings are determined by the Gaussian kernel function $K(.)$ and the n_x bandwidth parameters $\{b_d\}_{d=1}^{n_x}$, where $b_d \geq 0$ for all d. Intuitively, very small values of b_d mean that the weighted regression only assigns material weight to the near neighbors of $x_{t,d}$, while very large values assign roughly equal weights across the sample. In the case where $b_d \to \infty$ for all d, the regression approaches the linear ordinary least squares estimator (in Section 3.4, we discuss the importance of allowing for a nonlinear relationship between yields and surveys).

We choose the values of the bandwidth parameters b_d optimally, using a standard leave-one-out cross validation technique. Specifically, we choose b_d^* according to

$$b_d^* = \arg \min_{\{b_d\}_{d=1}^{nx}} \sum_{i=s_1}^{s_T} \left(\widehat{u}_i \middle| \mathcal{I}_i \right)^2, \tag{5}$$

where $\widehat{u}_i^{(n)} \Big| \mathcal{I}_i$ is the "out-of-sample" squared residual from equation (1), that is, it is estimated using all observations except for s_i.

3 Estimates of the Natural Real Rate and Long-Horizon Inflation Expectations

We now turn to our application and results. In Section 3.1, we describe our core data set. In Section 3.2, we present our estimates of the natural real rate and long-horizon inflation expectations. In Section 3.3, we compare our estimates with those from other prominent studies that estimate the natural real rate. In Section 3.4, we demonstrate the importance of allowing for nonlinearities in the relationship between bond yields and survey-based measures of expectations. In Section 3.5, we discuss an important advantage of our estimates, which is that they are available at a higher frequency than previous estimates of the natural real rate.

3.1 Data

We use survey-based measures taken from the Blue Chip Economic Indicators and Blue Chip Financial Forecasts surveys published by Wolters Kluwer Legal and Regulatory Solutions U.S. as the dependent variable in our regressions. These surveys ask panels of business economists for their expectations of various economic and financial variables, including the 3-month U.S. Treasury bill yield, which we use as a proxy for the short-term nominal interest rate, and the year-on-year rate of inflation measured by the Consumer Price Index. We use the mean forecast across respondents to the survey over a sample from January 1990 to December 2020. Although both Blue Chip surveys are conducted monthly, we only use the two editions each year for each survey where the forecast horizon extends beyond the next couple of years; in recent years, these "long-range" versions of the Economic Indicators surveys have been conducted in March and October, and of the Financial Forecasts surveys in June and December. One complication is that the surveys do not ask directly for expectations over periods of fixed length (such as the next 10 years), but instead ask for the average expectation over a calendar period, such as a particular calendar quarter or year. We therefore construct estimates of the average expected Treasury bill yield and inflation rate over the next 5 and 10 years using the linear interpolation method described in Appendix A.

Our regressors are estimates of yields on zero-coupon bonds with maturities of 6 months, 5 years, and 10 years, that is, $\mathbf{x}_t = \left[y_t^{(6)}, y_t^{(60)}, y_t^{(120)}\right]'$, where $y_t^{(n)}$ is the n-month zero-coupon yield. The zero-coupon yields are based on a smoothed yield curve fitted to the prices of off-the-run nominal U.S. Treasury securities, as in Gürkaynak, Sack, and Wright (2007).[3] We pick three points on the yield curve because a large literature dating back at least to Litterman and Scheinkman (1991) has shown that three linearly independent combinations of yields span almost all of the information in the cross section of yields.

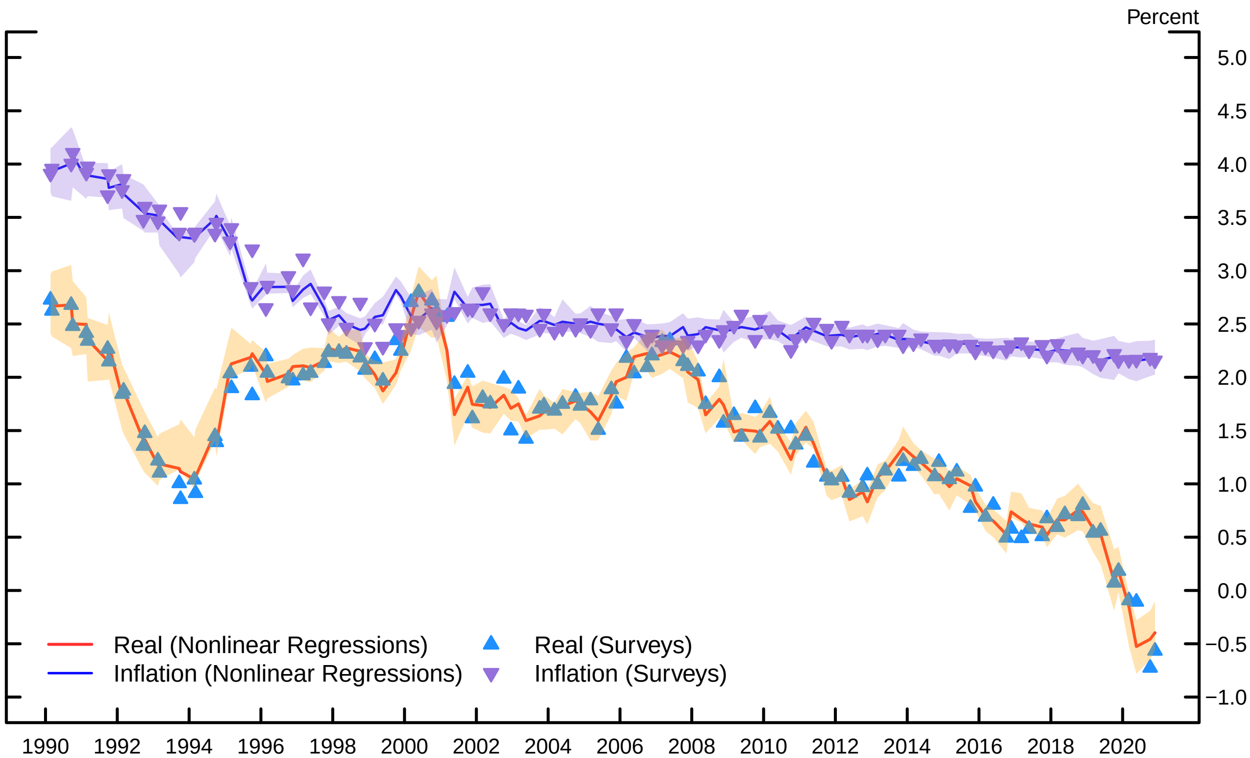
3.2 Results

Figure 1 shows resulting estimates of the natural real rate (the red line) and the average expected inflation rate (the blue line) between 5 and 10 years ahead. The natural real rate declines from about 2.5 percent in January 1990 to a little below 0 percent in December 2020. However, the decline was not steady over the sample: The natural real rate fluctuated around a constant mean of about 2 percent until the global financial crisis in 2008. Since 2008, it has declined fairly steadily, but with a sharper drop in early 2020 following the

[3] Updates of the data set are available at https://www.federalreserve.gov/data/nominal-yield-curve.htm.

onset of the COVID-19 pandemic. Long-horizon inflation expectations have also declined over the sample. Between 1990 and about 1998 they declined fairly steadily from about 4 percent to a little above 2 percent. Since the turn of the century, long-horizon inflation expectations have stabilized at about 2 percent, although there is tentative evidence that they have declined very gradually since about 2014.

Figure 1: Estimates of the Long-Horizon Expectations of the Short-term Real Interest Rate and Inflation
The chart reports estimates of average expected short-term interest rates and inflation from 5 to 10 years ahead. The markers show the raw estimates implied by the Blue Chip surveys, while the lines show the corresponding estimates from our regressions. The real rate expectations are computed as the difference between the nominal rate and inflation expectations, with 95 percent confidence intervals estimated using a residual bootstrap. The survey-based measures are derived from Blue Chip Economic Indicators and Blue Chip Financial Forecasters data published by Wolters Kluwer Legal and Regulatory Solutions U.S.



The light blue and purple markers show the corresponding measures of the natural real rate and long-horizon inflation expectations, respectively, based on the raw (interpolated) 5-to-10-year ahead surveys. Our regression-based measures track the broad moves in the surveys, without fitting every fluctuation in the survey exactly. Of course, because true expectations are unobserved, it is hard to say for sure whether we are appropriately accounting for measurement error in the surveys. However, it seems intuitively encouraging that our model does not match what proved to be a very short-lived decrease in the raw survey measure of the natural real rate in 1996 and a series of reversals in the early 2000s.

Figure 1 also plots 95 percent confidence intervals computed using a residual bootstrap.[4] At first glance, the confidence intervals may appear surprisingly tight given the nature of the difficulty in estimating long-horizon expectations of interest rates, with widths in the order of only $\frac{1}{2}$ percentage point. The confidence intervals are generally slightly wider earlier in the sample than later in the sample, because there are relatively few observations when the regressor yields were high—as tended to be the case early in the sample. Of course the reason for the generally tight confidence intervals is that it is possible to explain most of the variation in surveys using yields. Providing surveys are unbiased, we can be fairly certain that the decline in long-horizon expectations of short-term real interest rates and inflation over the sample is statistically significant.

3.3 Comparison with Other Estimates

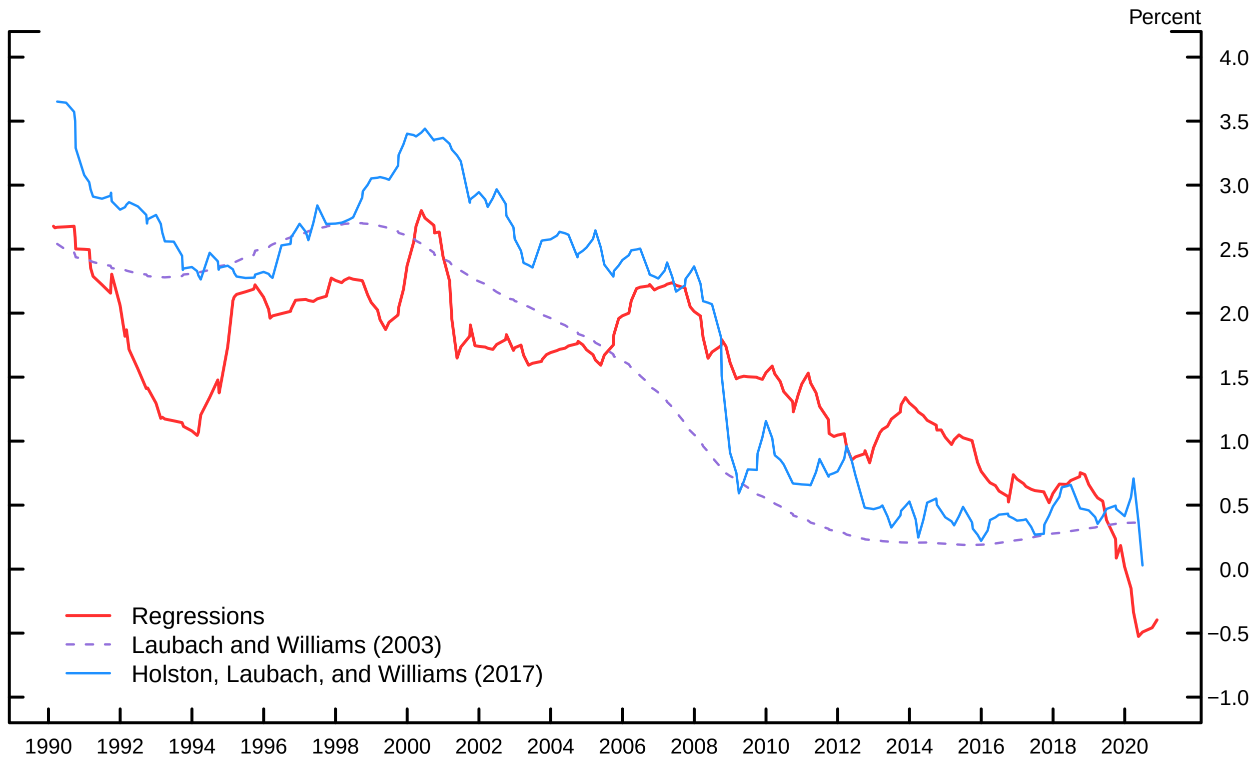
How do our estimates of the natural real compare with those from other studies? As discussed above, perhaps the most popular approach for estimating the natural real rate in recent years has been the approach of Laubach and Williams (2003). In Figure 2, we report our regression-based estimate in the red line, alongside the Laubach and Williams (2003) estimates (the purple line) and the estimates from the similar model of Holston et al. (2017).[5]

[4] To estimate the confidence intervals around our estimates, we employ a standard bootstrapping procedure utilizing residual resampling. At each iteration of the bootstrap, we first randomly sample residuals with replacement from the observed results for each survey date, separately for the nominal and inflation models. Adding the resampled residuals to the model-implied fitted values gives us a new bootstrapped set of surveys. Then, for each regression, we re-optimize the bandwidth parameters for the bootstrapped sample and estimate the models. We take the difference between the fitted values from the bootstrapped nominal and inflation regressions for the real model results. The 95 percent confidence intervals report the range between the 2.5^{th} and 97.5^{th} percentiles of the estimates from the bootstrap samples.

[5] We report estimates of the natural real rate from these studies that are smoothed, that is, which estimate the natural real rate at each point in time using data for the full sample. That is closest in spirit to our regressions, which use full-sample information.

Broadly speaking, the three estimates have some notable similarities: all three decline from between 2 percent and 3 percent in the early 1990s to between -0.5 percent and 0.5 percent in December 2020. The models disagree to some extent over the timing of the fall, with the estimates from the macroeconomic models declining somewhat earlier than our estimates. One potential explanation for this timing difference could be that the expectations of the forecasters responding to the Blue Chip survey may have been influenced by the estimates from these prominent macroeconomic models.

Figure 2: Estimates of the Natural Real Rate: Comparison with Other Estimates
The chart reports estimates of the natural real rate: our regression-based estimator and two alternative estimates from macroeconomic models.



3.4 Importance of Nonlinearities

To shed light on how much the nonlinearities in equation (1) matter for our ability to match surveys, we also estimate a linear version of the model, that is,

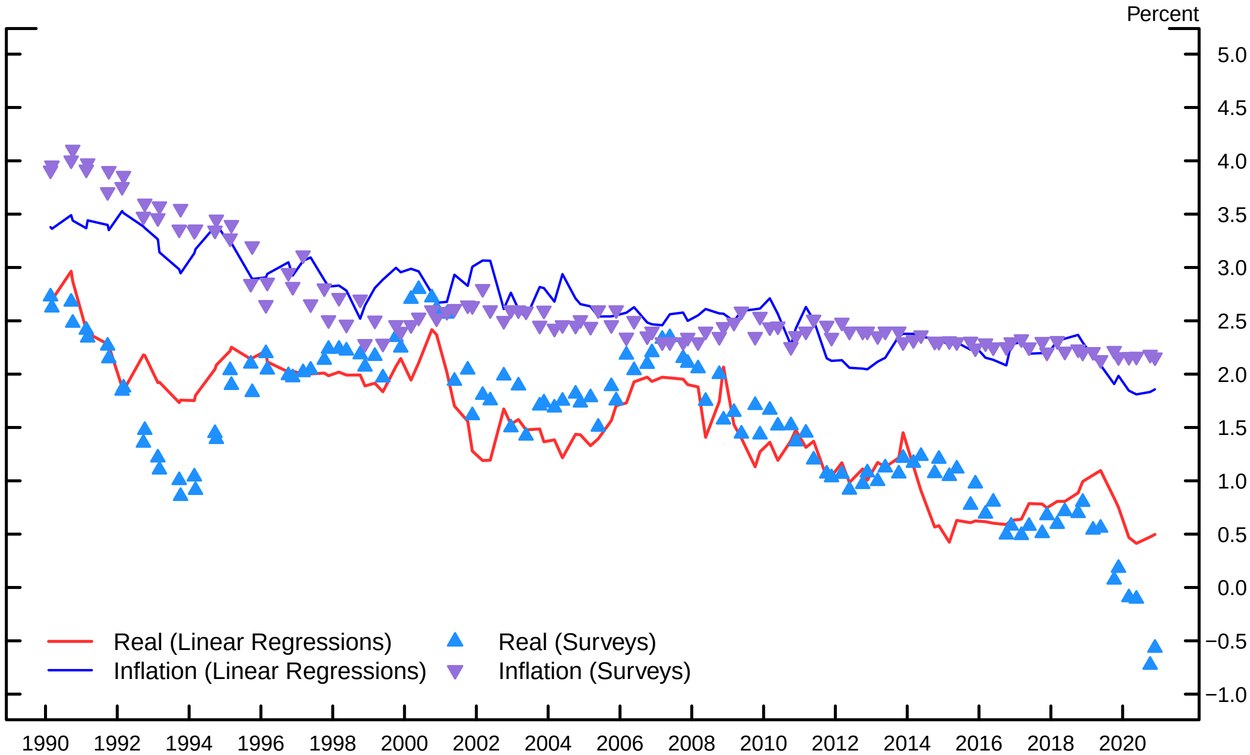
$$s_t^{(60,120)} = g_0 + \mathbf{g}_x' \mathbf{x}_t + u_t. \tag{6}$$

Figure 3 shows how well this linear model matches the raw surveys. The red and blue lines again show the results for the natural real rate and long-horizon inflation expectations, respectively, while the pale blue and purple markers show the corresponding raw survey-based measures. The estimates from the linear model decline over the sample, but the model is clearly insufficiently flexible to match some of the broad movements in the surveys. In particular, the model is not able to generate the observed pattern in long-horizon inflation expectations, for which the raw surveys decline steadily in the 1990s before levelling off over the remainder of the sample. Similarly, the linear model is not able to capture the extent of the decline in the natural real rate since the global financial crisis.

3.5 Daily Estimates

An important advantage of our estimates of the natural real rates is that they are available at whatever frequency we observe Treasury yields, whereas previous estimates are typically only available at a monthly or quarterly frequency. This feature of our estimates has the potential to make them particularly useful for policymakers and practitioners who are interpreting higher-frequency movements in Treasury yields. We would intuitively expect the natural real rate to be slow-moving, with daily changes that are less volatile than changes in Treasury yields. Similarly, if long-term inflation expectations are well anchored we would also expect them not to change materially from day to day. Because financial market prices can fluctuate from day to day, when we compute our estimates at a daily frequency we also find day-to-day variation. However, we confirm the intuitive result that they are much less

Figure 3: The Importance of Nonlinearities: Estimates of Long-Horizon Expectations of the Short-Term Real Interest Rate and Inflation
The chart reports estimates of average expected short-term interest rates and inflation from 5 to 10 years ahead. The markers show the raw estimates implied by the Blue Chip surveys, while the lines show the corresponding estimates from linear versions of our regressions. The real rate expectations are computed as the difference between the nominal rate and inflation expectations. The survey-based measures are derived from Blue Chip Economic Indicators and Blue Chip Financial Forecasters data published by Wolters Kluwer Legal and Regulatory Solutions U.S.

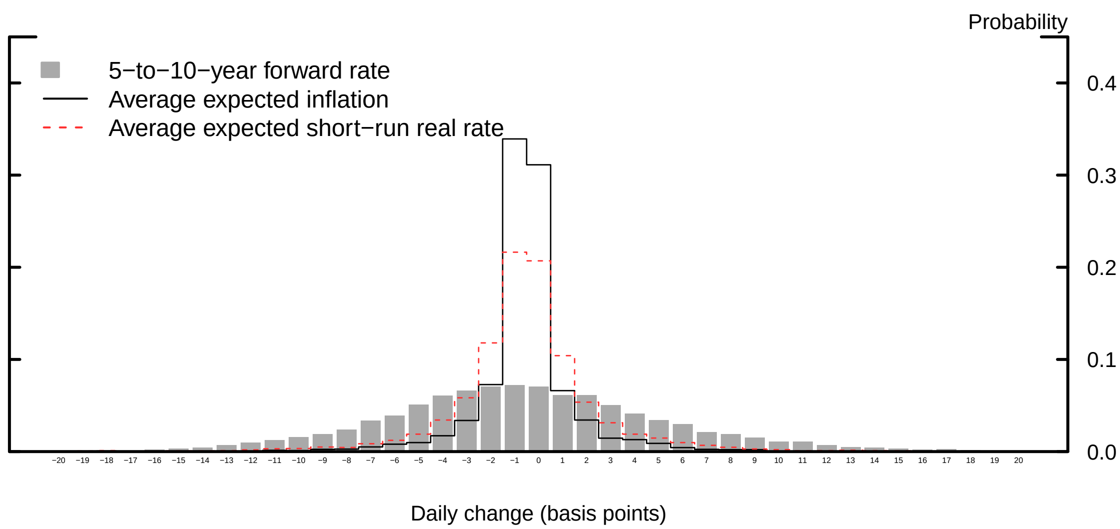


volatile than financial market prices.

Figure 4 illustrates this result. The gray bars show the empirical distribution of daily changes in the 5-to-10-year nominal forward rate. As is standard, we can think of this forward rate as reflecting the sum of the average expected short-term real, the average expected inflation rate, and an additional term premium component that compensates investors for risks. Most of the variation in the forward rate appears to be accounted for by changes in term premiums: the variability of daily changes in the expected real rate (the distribution shown by the red dashed line) and expected inflation (the black line) is much less than the variability of the raw forward rates. The large majority of daily changes in long-term inflation expectations are between -1 basis point and 1 basis point. The distribution of daily changes in the long-horizon expectations of the short-term real rate is slightly more dispersed, although a clear majority of daily changes are between -2 basis points and 2 basis points. Thus, the daily estimates confirm our prior that long-horizon expectations are not subject to frequency large daily changes.

Figure 4: Histogram of Daily Changes in Long-Horizon Expectations of Real Interest Rates and Inflation Real Rate

The chart plots histograms of daily changes in the 5-to-10-year forward rate implied by nominal Treasury yields alongside similar histograms of daily changes in our estimates of 5-to-10-year expectations of the short-term real interest rate and inflation. The probability for each bin on the x-axis refers to the proportion of daily changes falling in the 1-basis-point interval above the axis label (for example, the bin labeled "0" refers to the range from 0 basis points to 1 basis point). A very small proportion of daily changes fall outside the plotted range.



4 Effect of Monetary Policy Shocks on Natural Real Rates and Long-Horizon Inflation Expectations

As discussed above, a key assumption underpinning our approach to estimating the natural real rate is that 5-to-10-year ahead real rate expectations are not affected by short-lived disturbances such as temporary changes in the stance of monetary policy. Unfortunately, assessing whether this assumption holds in practice is usually challenging because previous estimates are only available at a monthly or quarterly frequency. However, the fact that our method can produce estimates at whatever frequency we observe Treasury yields means that we can identify the effects of monetary policy shocks over narrow windows using standard event study methods. Similarly, we can also consider whether long-horizon inflation expectations respond to what should be transitory monetary policy shocks. In Section 4.1, we explain the data set and methodology we employ to conduct event studies, while in Section 4.2, we report our results.

4.1 Event Study Method

To measure whether our estimates the natural real rate are significantly affected by monetary policy shocks, we estimate a regression of the change in our estimates on measures of monetary policy shocks:

$$\Delta r^*_{t,t+h} = \beta_0 + \beta_1 \Delta z_{t,t+h} + \varepsilon_{t,t+h}, \tag{7}$$

where $\Delta r^*_{t,t+h}$ is the change in our measure of the natural real rate over a narrow window between time t and time $t+h$, $\Delta z_{t,t+h}$ is our proxy for the monetary policy shock, and $\varepsilon_{t,t+h}$ is an error term. If the slope coefficient β_1 is significantly different from zero then we would conclude that the natural real rate is affected by monetary policy shocks. This is an entirely standard event study method, following Gürkaynak, Sack, and Swanson (2005), among others. We estimate exactly analogous regressions for long-term inflation expectations.

As discussed above, our approach for estimating the natural real rate and inflation expectations framework relies on estimates of zero-coupon yields derived from the prices of off-the-run U.S. Treasury securities. These off-the-run zero-coupon yields have the advantage of being available at a daily frequency over an extended historical period, but they are not available at a sufficiently high intraday frequency to conduct reliable event studies. To create a higher-frequency proxy for off-the-run zero-coupon yields, we estimate separate linear regressions of the 6-month, 5-year, and 10-year zero-coupon yields on 2-, 5-, and 10-year on-the-run yields, which are available to us at a much higher intraday frequency. We calculate these on-the-run yields using data from the BrokerTec exchange for January 2004 to December 2020; the data are processed in the same way as in Fleming, Mizrach, and Nguyen (2018) and Fleming and Nguyen (2018).[6] We use the fitted values from these regressions as proxies for the zero-coupon yields. We choose the 2-, 5-, and 10-year yields as regressors because these are the only maturities that are available throughout our sample. Regressing zero-coupon yields on these on-the-run yields likely provides a very accurate proxy for intraday zero-coupon yields, because the R^2 statistics for the daily zero-coupon yields are 0.985 or higher.

To calculate the monetary policy shocks for use in equation (7), we employ the standard methods described in Gürkaynak et al. (2005): We calculate changes in financial market prices around FOMC statement releases over our sample. We consider two alternative measures of monetary policy shocks: First, we use the change in the market-implied expectation of the federal funds rate after the fourth future Federal Open Market Committee (FOMC) meeting that occurred around FOMC statement releases between January 2004 and December 2020.[7] However, while this is a fairly standard measure of monetary policy surprises, it may be distorted during much of our sample because short-term interest rates were close

[6] We use mid quotes from the top level of the BrokerTec order book. Our results do not change materially if we instead use realized transaction prices, or if we use ask or bid yields.

[7] Our results are not materially affected by using rate expectations following other FOMC meetings (we consider the first through sixth future meetings).

to their effective lower bound. We therefore also consider an alternative measure based on the change in the S&P 500 equity index.[8] For both measures of monetary policy shocks, we calculate price changes using the same windows reported by Gürkaynak et al. (2005): a "tight" window from 10 minutes before the FOMC statement release to 20 minutes after, and a "wide" window from 15 minutes before the FOMC statement release to 45 minutes after.

4.2 Results

We now turn to the results from event studies using intraday data. Results from estimating equation (7) for the natural real rate and inflation expectations are presented in Figures 5 and 6, respectively. We find no significant relationship between our estimates and monetary policy shocks. These results hold both for monetary policy shocks measured by changes in fed funds futures and equity indices. The only minor caveat is that our results for the natural real rate are sensitive to the large outlier corresponding to the March 18, 2009 FOMC meeting, when the FOMC expanded its Large Scale Asset Purchase program to cover long-term U.S. Treasury securities during the global financial crisis. Following that announcement, our estimate of the natural real rate dropped by about 0.35 percentage points, although it partly recovered over the remainder of the trading day, as shown in Figure 7. Removing that observation yields a relationship between our natural real rate estimates and monetary policy shocks represented by S&P 500 index changes[9] that is statistically significant at the 5 percent level for both the tight and wide window; an accommodative monetary policy shock, as reflected in an increase in equity prices, is associated with an increase in the natural real rate. However, even here, the size of the relationship is economically trivial: a one percentage point increase in the S&P 500 index is associated with an increase in the

[8] Another alternative would be to consider the change in a long-term Treasury yield. We prefer not to do this is because we use long-term yields inputs to the regressions used to estimate the natural real rate and long-horizon inflation expectations. However, unreported results show that out main results also hold if we use changes in 10-year yields as the measure of the monetary policy shock.

[9] We use S&P 500 index data observed once every five minutes from Bloomberg Finance LP to calculate index changes.

natural real rate of less than one basis point, for both the tight and wide windows. Thus, we conclude that there is no evidence that we should be concerned that our proxy for the natural real rate is being contaminated by the effects of short-lived changes in the stance of monetary policy.

Figure 5: Relationship between Monetary Policy Shocks and the Natural Real Rate
The chart plots changes in financial market prices around FOMC statement releases on the x-axes against changes in the natural real rate on the y-axes. The left column (MP4) shows results for federal funds rate expectations following the fourth future FOMC meeting, as measured by federal funds futures prices. The right column (SP500) shows results for the S&P 500 equity index. The top row shows results for a tight event window from 10 minutes before the statement release to 20 minutes after, and the bottom row shows results for a wide event window from 15 minutes before the statement release to 45 minutes after. Each black dot represents one FOMC meeting, with the March 18, 2009 meeting picked out in red. The blue line shows a linear regression line, with the gray shaded area showing the corresponding 95 percent confidence interval. The estimated regression equation and the p-value of the slope coefficient for the regression equation are reported in the text inset.

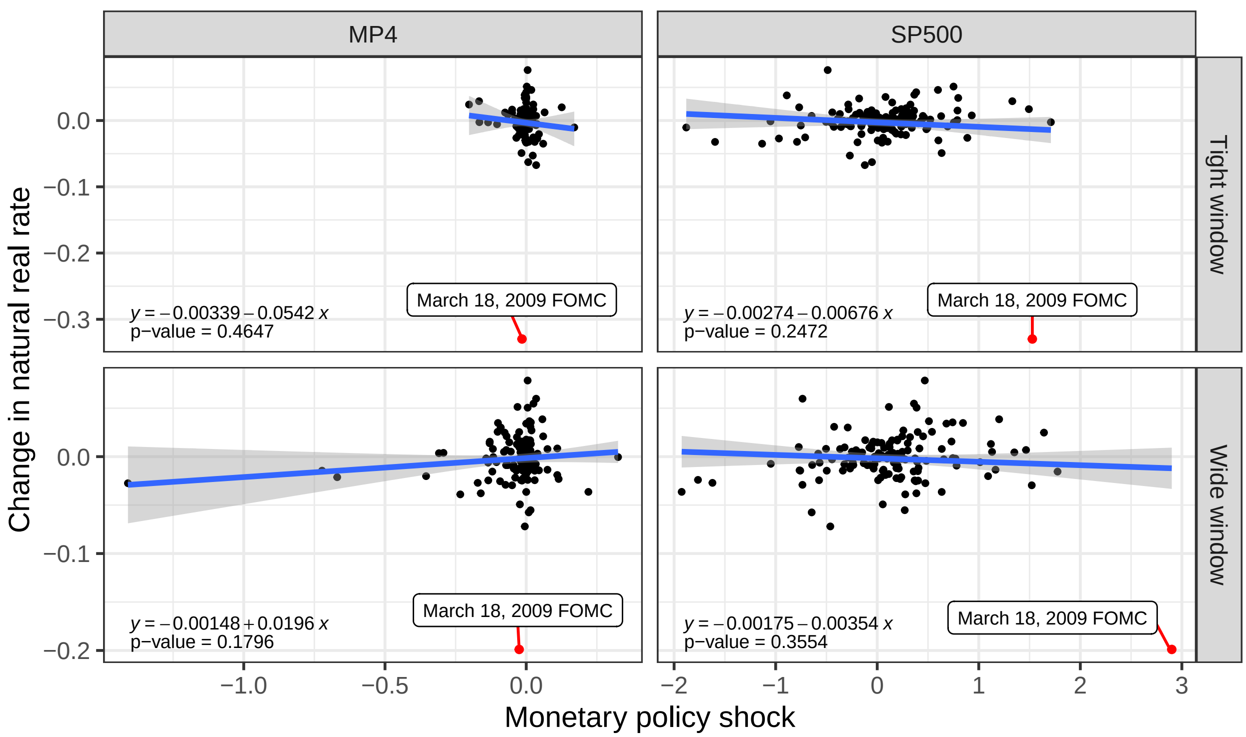


Figure 6: Relationship between Monetary Policy Shocks and Long-Horizon Inflation Expectations

The chart plots changes in financial market prices around FOMC statement releases on the x-axes against changes in the long-horizon inflation expectation on the y-axes. The left column (MP4) shows results for federal funds rate expectations following the fourth future FOMC meeting, as measured by federal funds futures prices. The right column (SP500) shows results for the S&P 500 equity index. The top row shows results for a tight event window from 10 minutes before the statement release to 20 minutes after, and the bottom row shows results for a wide event window from 15 minutes before the statement release to 45 minutes after. Each black dot represents one FOMC meeting, with the March 18, 2009 meeting picked out in red. The blue line shows a linear regression line, with the gray shaded area showing the corresponding 95 percent confidence interval. The estimated regression equation and the p-value of the slope coefficient for the regression equation are reported in the text inset.

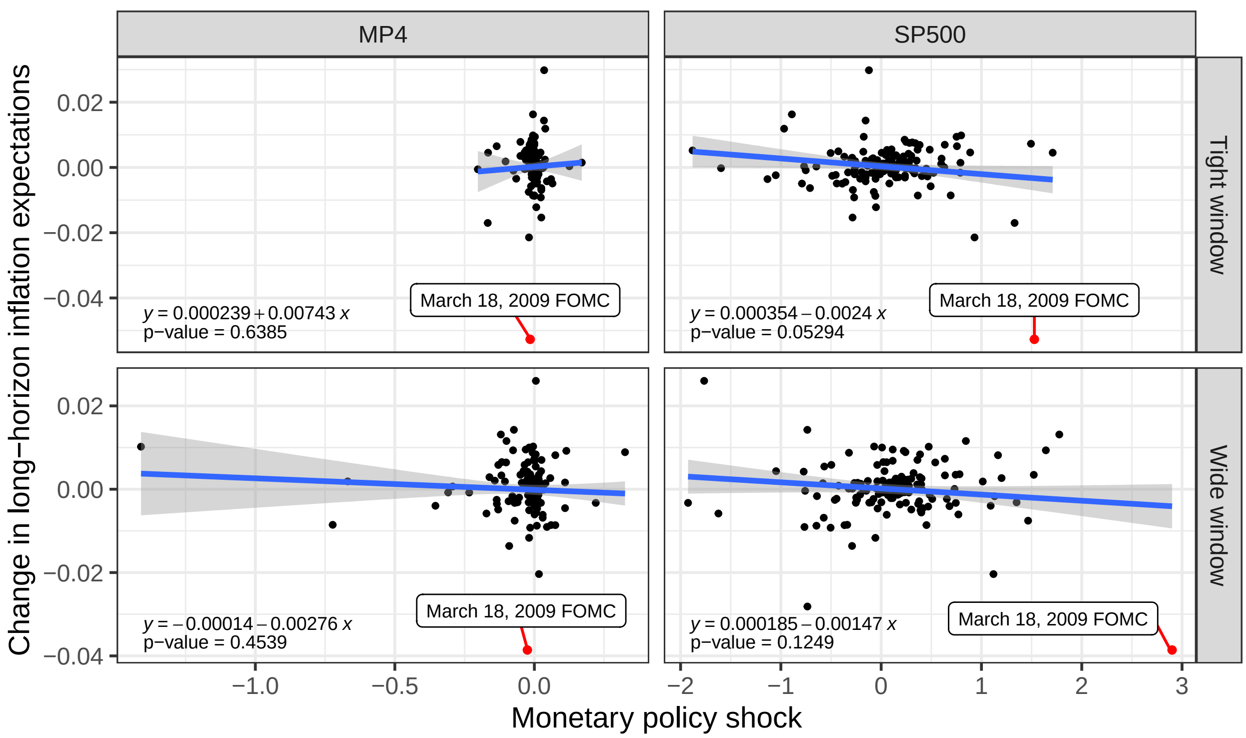
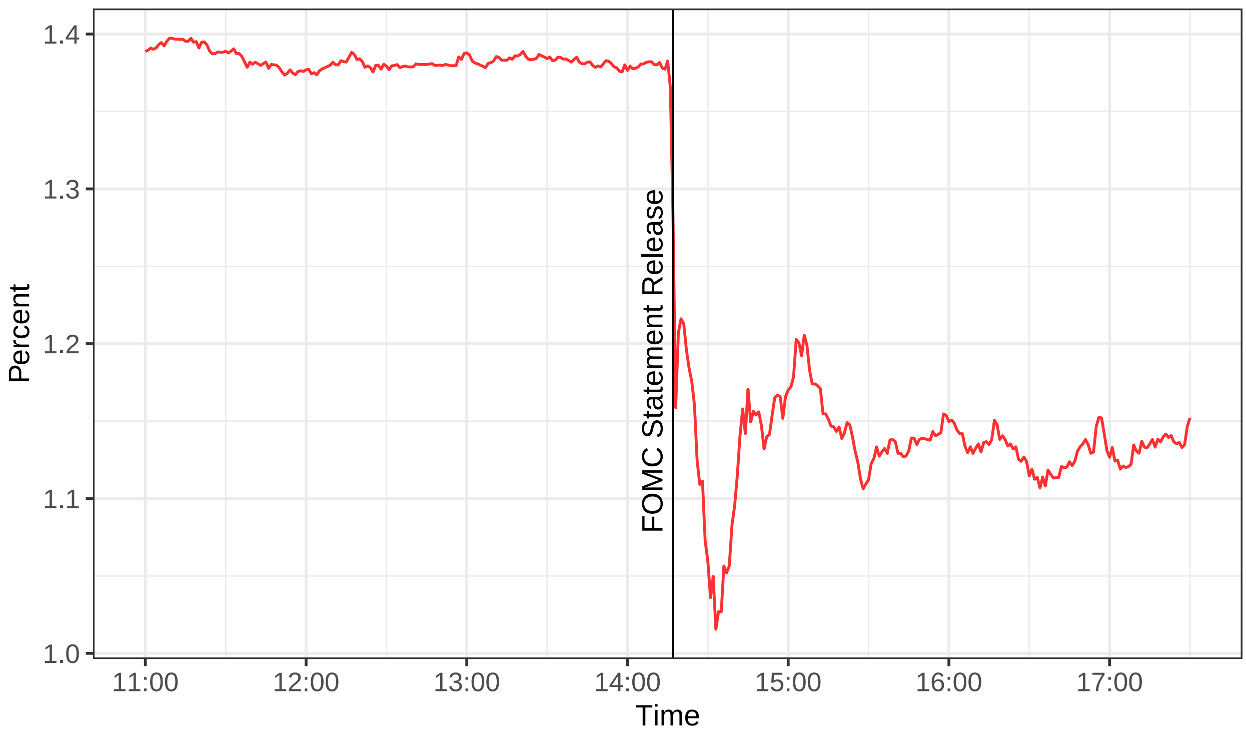


Figure 7: Natural Real Rate on March 18, 2009



5 Comparison with Affine Term Structure Models

In this section, we shed further light on the relationship between our estimates and those obtained from no-arbitrage term structure models. In Section 5.1 we show that term structure models estimated using only the history of interest rates deliver highly uncertain and implausible estimates of 5-to-10-year ahead interest rate expectations. However, in Section 5.2, we show that augmenting the term structure model with information from surveys delivers essentially the same results as linear versions of our regressions—but at considerable additional computational cost.

5.1 Time-Series Estimates: No-Arbitrage Term Structure Models

As discussed above, an alternative approach to estimating financial market participants' expectations of the short-term interest rates and inflation is to use an econometric model that describes the time-series properties of interest rates or inflation. A standard approach is to use the dynamic no-arbitrage ATSM of Duffee (2002). The many examples include Christensen, Diebold, and Rudebusch (2011), Adrian et al. (2013), and Abrahams, Adrian, Crump, Moench, and Yu (2016). Focusing on the case of nominal interest rates, in an ATSM the short-term interest rate (r_t) takes the form

$$r_t = \delta_0 + \boldsymbol{\delta}_x' \mathbf{x}_t, \tag{8}$$

where $\mathbf{x}_t$ is an $n_x \times 1$ vector of bond yields observed at month t, δ_0 is a scalar and $\boldsymbol{\delta}_x$ is an $n_x \times 1$ vector. The factors follow the first-order vector autoregression

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \mathbf{v}_t, \tag{9}$$

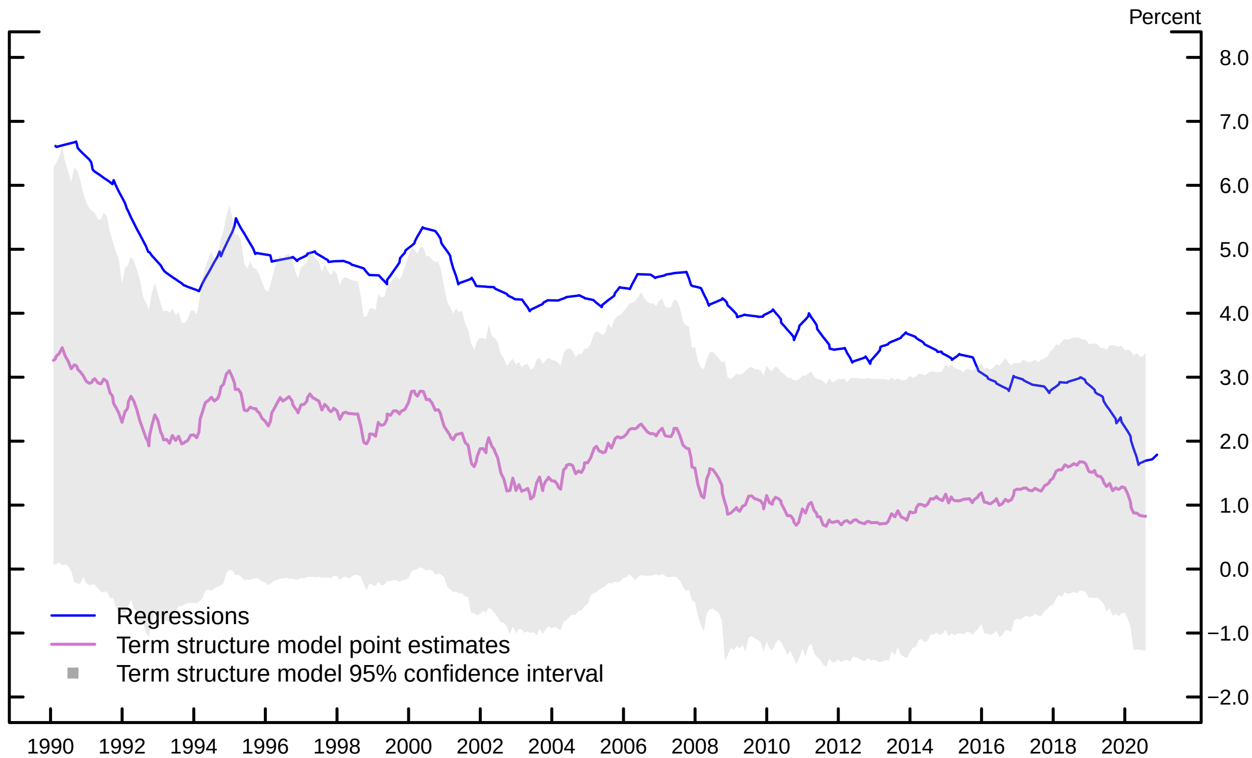
where $\boldsymbol{\mu}$ is an $n_x \times 1$ vector, $\boldsymbol{\Phi}$ is an $n_x \times n_x$ matrix, and $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}\boldsymbol{\Sigma}')$ is an $n_x \times 1$ vector of shocks. Thus, the conditional expectation of the average short-term real interest from 5 to 10 years ahead is an affine function of $\mathbf{x}_t$. Estimation can take place using standard maximum likelihood techniques. Appendix B provides additional details about the model.

We estimate the above model using end-month Treasury yields with maturities of 6 months, 5 years, and 10 years, that is, $\mathbf{x}_t = \left[y_t^{(6)}, y_t^{(60)}, y_t^{(120)} \right]'$, over a sample period from January 1990 to July 2020. The pink line in Figure 8 shows the model-implied fitted value of 5-to-10-year ahead short-term nominal rate expectations, with the gray shading showing the corresponding 95 percent confidence interval, computed using a bootstrap procedure.[10]

[10] Specifically, we perform 1,000 draws of samples of yields with the same number of months as our estimation sample. We use a residual block bootstrap with a block length of 60 months and construct yields using equation (9). For each simulated sample, we estimate the model parameters separately.

We highlight two important results: First, the point estimate of the expected short rate is substantially lower than our regression-based estimate, shown in the blue line; while surveys may measure expectations with some error, it seems implausible to us that surveys would be this far away from true expectations. Second, the parameter uncertainty for the term structure model is extremely wide, such that the 95 percent confidence interval for 5-to-10-year-ahead expectations cover a region about four percentage points wide. Thus, we cannot with material confidence reject the hypothesis that the short rate expectation was constant over the sample.

Figure 8: Estimates of Long-Horizon Nominal Interest Rate Expectations: Comparison with Standard ATSM
The chart reports estimates of the average expected short-term nominal interest rate from 5 to 10 years ahead. The blue line shows estimates based on our regressions. The pink line shows estimates from a standard ATSM, with the gray shaded area showing the corresponding 95 percent confidence interval computed using a block bootstrap.



5.2 Survey-Augmented Term Structure Models

The reason that estimates of the natural real rate from a standard ATSM are so uncertain is well-known: bond yields are extremely persistent, so it is hard to pin down the parameters that determine their time-series dynamics ($\boldsymbol{\mu}$ and $\boldsymbol{\Phi}$ in equation (9)) using just a few decades' worth of data. Kim and Orphanides (2012) therefore propose to incorporate additional information from survey expectations within an ATSM to help pin down those time-series parameters; D'Amico et al. (2018) apply the same technique to a model that also decomposes nominal interest rates into real and inflation components.

We can incorporate our survey-based measure of 5-to-10-year ahead short-term real rate expectations into the above ATSM in the same way as in Kim and Orphanides (2012). We assume that our survey-based measure of the expected average short-term nominal interest rate from 5 to 10 years ahead ($s_t^{(60,120)}$) measures expectations with independent Normally-distributed measurement errors, that is

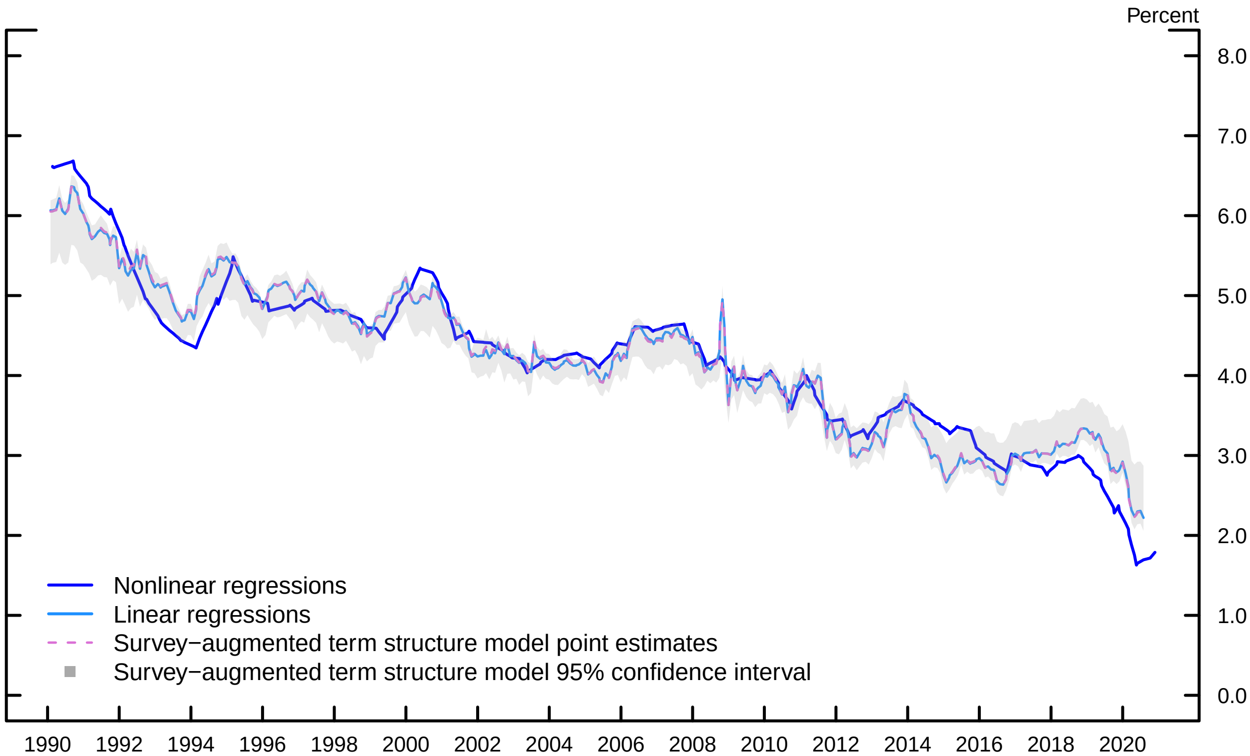
$$s_t^{(60,120)} = \gamma_0 + \boldsymbol{\gamma}_x' \mathbf{x}_t + w_t, \tag{10}$$

where the scalar γ_0 and an $n_x \times 1$ vector $\boldsymbol{\gamma}_x$ are determined by the structural parameters in equations (8), and (9), $w_t \sim \mathcal{N}(0, r_s)$, and $\mathbb{E}[w_t \mathbf{v}_t] = \mathbf{0}$. Estimation can still proceed by maximum likelihood. Appendix B provides further details.

Figure 9 shows the results for the survey-augmented term structure model, corresponding to those in Figure 8. The point estimate from the survey-augmented term structure model comes reasonably close to tracking our regression-based estimate, and hence the raw surveys, suggesting that we should consider it far more plausible than the estimate from a standard term structure model. Moreover, the 95 percent confidence interval is now much narrower because the surveys help to pin down the parameters that determine the time-series properties of interest rates with much greater precision.

How should we think about the differences between our regression-based approach and

Figure 9: Estimates of Long-Horizon Nominal Interest Rate Expectations: Comparison with Survey-Augmented ATSM
The chart reports estimates of the average expected short-term nominal interest rate from 5 to 10 years ahead. The dark blue line shows estimates based on our regressions. The pink line shows estimates from a survey-augmented ATSM, with the gray shaded area showing the corresponding 95 percent confidence interval computed using a block bootstrap. The pale blue line shows an estimate from a linear regression of surveys on yields (this line is usually hidden behind the pink line).



a term structure model approach? There are four differences between the two approaches. First, our regressions use yields to fit contemporaneous survey-based expectations, whereas the term structure model needs to trade-off the fit to contemporaneous surveys with the need to match the observed time-series properties of interest rates. Second, a term structure model imposes cross-equation restrictions in order to rule out the possibility of arbitrage profits. Third, the term structure model imposes a distributional assumption on the measurement error w_t. And fourth, our regression relaxes the assumption of linearity; as discussed above, equation (10) can be interpreted as the limiting case of equation (1), in which the functional mapping from yields to expected future short rates is linear.

In order to shed light on the importance of these differences, the pale blue line in Figure 9 plots an estimate from a linear version of the regression in equation (1). The estimates from the term structure model and the linear regression are virtually identical, which suggests that the only difference between our regressions and the survey-augmented term structure model that matters is allowing for nonlinearities. The reasons why the other differences between the approaches are unimportant are straightforward to understand. It is well-known that imposing no-arbitrage restrictions makes little difference to conditional expectations of yields (see, for example, Duffee (2011)). And it is also well-known that there is little information about future interest rates in their past behavior; indeed, that was exactly the rationale of Kim and Orphanides (2012) for including surveys in term structure models in the first place. Thus, the value of term structure models for gaining insights into long-horizon interest rate expectations seems fairly limited: if we omit surveys from term structure models we obtain estimates that are highly uncertain and potentially implausible, and if we include surveys we end up with a model that is unnecessarily complicated, because we could just as well regress long-horizon surveys on yields. If all we are interested in is long-horizon expectations of interest rates, why go to the trouble of estimating a term structure model when we can just estimate a linear regression of surveys on yields?

6 Conclusion

We construct new estimates of the natural real rate and long-horizon inflation expectations derived from nonlinear regressions of survey forecasts of interest rates and inflation on U.S. Treasury yields. Our estimates provide an alternative to previous estimates from studies that use structural macroeconomic and reduced-form time-series models, and avoid some of the drawbacks of those approaches. We additionally provide evidence that the natural real rate and long-horizon inflation expectations are not affected by temporary shocks to the stance of monetary policy. Finally, we show that our estimates are closely related to those from dynamic term structure models that are augmented with survey forecasts, but have the advantage of allowing for greater flexibility in the relationship between yields and short rate expectations, as well as being far easier to compute.

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Appendix A: Constructing Constant-Horizon Surveys

We use the biannual long-horizon Blue Chip Economic Indicators and Blue Chip Financial Forecasts surveys of 3-month Treasury bill yield and CPI inflation expectations. In recent years, the long-horizon Financial Forecasts have been published in June and December and the Economic Indicators in March and October, although the precise timings have varied somewhat over the sample. Blue Chip does not routinely provide the dates on which the surveys were submitted by respondents, only the dates the results were published. For greater precision, we estimate the survey dates based on recent practice regarding publication lags. In the case of long-horizon Blue Chip Financial Forecasts, the surveys are published on the first of the month. For these, we label the survey date as the previous Wednesday before the last business day of the month. If the last business day of the previous month is a Thursday or Friday, we take the Wednesday of the prior week. Some adjustments are made for the January survey due to the holiday season: the previous Wednesday from Christmas Day is calculated. Blue Chip Economic Indicators, on the other hand, are published on the tenth of the month. For these, we simply take the date three business days before the tenth.

The surveys ask respondents for their average expectations of the Treasury bill yield and inflation over various calendar periods, starting with the current calendar quarter and ending with a period ending with a period covering the sixth to eleventh calendar years ahead. We start by computing the mean expectations across survey respondents for each of these reference periods. We then interpolate the average expectations for these calendar periods to estimate expectations over constant horizons of 5 and 10 years, assuming that the expected value at the mid-point of the reference period is equal to the average value over the reference period. In some editions of the survey, the first reference point covered by the survey has already started by the time the survey is taken; in this case, we construct a purely forward-looking expectation by accounting for the realized Treasury bill yield or inflation since the beginning of the reference period. In some other editions of the survey, the first reference period has not started by the time the survey is taken; in this case, we assume the Treasury bill yield is expected to remain constant at its most recent value until the start of the first reference period, and that the expectation for inflation over the period until the first reference period is the same as the average expectation in the first reference period.

Our interpolation also requires us to make an assumption about the expected value of the Treasury bill yield and inflation at the end of the final reference period. We assume that this expectation is the same as the average expectation for the final reference period.

We use the interpolated 5- and 10-year expectations to calculate survey respondents' expectations of the average Treasury bill yield and inflation over a period from 5 to 10 years ahead. We finally take the difference between the Treasury bill expectation and inflation expectation to represent respondents' expectations of 5-to-10-year ahead real rate expectations, that is, the natural real rate.

Appendix B: Affine Term Structure Model

In addition to equations (8) and (9), we also assume that $\mathbf{x}_t$ follows a first-order vector autoregression under the equivalent risk-neutral probability measure (denoted $\mathbb{Q}$):

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \mathbf{v}_t^{\mathbb{Q}} \tag{11}$$

where $\boldsymbol{\mu}^{\mathbb{Q}}$ is an $n_x \times 1$ vector, $\boldsymbol{\Phi}^{\mathbb{Q}}$ is an $n_x \times n_x$ matrix, and $\mathbf{v}_t^{\mathbb{Q}} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}\boldsymbol{\Sigma}')$ is an $n_x \times 1$ vector of shocks. Under the assumption of no arbitrage, we can show that the yield on an n-period bond ($y_t^{(n)}$) is given by

$$y_t^{(n)} = -\frac{1}{n}\left(a_n + \mathbf{b}_n'\mathbf{x}_t\right), \tag{12}$$

where

$$a_n = a_{n-1} + \mathbf{b}'_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} + \frac{1}{2}\mathbf{b}'_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{n-1} - \delta_0 \text{ and} \tag{13}$$

$$\mathbf{b}'_n = \mathbf{b}'_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} - \boldsymbol{\delta}_x. \tag{14}$$

See, for example, Joslin, Singleton, and Zhu (2011) for further details. To ensure parameter identification, we impose the normalization restrictions as in Joslin et al. (2011). We assume that the 6-month, 5-year, and 10-year yields are measured without error. Maximum likelihood estimation the proceeds straightforwardly as in Joslin et al. (2011).

The conditional expectation of the average short-term interest from 5 to 10 years ahead is given by

$$\mathbb{E}_t\left[\frac{1}{60}\sum_{i=60}^{119} r_{t+i}\right] = \left\{\begin{array}{c} \delta_0 + \boldsymbol{\delta}'_x\left(\mathbf{I}-\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\mu} \\ +\frac{1}{60}\boldsymbol{\delta}'_x\left(\mathbf{I}-\boldsymbol{\Phi}\right)^{-2}\left(\boldsymbol{\Phi}^{120}-\boldsymbol{\Phi}^{60}\right)\boldsymbol{\mu} \\ +\frac{1}{60}\boldsymbol{\delta}'_x\left(\mathbf{I}-\boldsymbol{\Phi}\right)^{-1}\left(\boldsymbol{\Phi}^{60}-\boldsymbol{\Phi}^{120}\right)\mathbf{x}_t \end{array}\right\}. \tag{15}$$

To augment the standard ATSM with a long-horizon survey expectation, we follow Kim and Orphanides (2012) and assume that the average expected short-term interest rate from 5 to 10 years ahead measures expectations with an error w_t, as in 10, where the coefficients γ_0 and $\boldsymbol{\gamma}_x$ are given by equation (15). Thus, there is one additional parameter to be estimated by maximum likelihood (r_s).