

**Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.**

What Can We Learn from Idiosyncratic Wage Changes?

Cynthia L. Doniger

2021-055

Please cite this paper as:

Doniger, Cynthia L. (2021). "What Can We Learn from Idiosyncratic Wage Changes?," Finance and Economics Discussion Series 2021-055. Washington: Board of Governors of the Federal Reserve System, <https://doi.org/10.17016/FEDS.2021.055>.

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What Can We Learn from Idiosyncratic Wage Changes?

Cynthia L. Doniger
Federal Reserve Board

August 4, 2021

Abstract

I document six facts about wage changes. First, most pay revisions occur at yearly frequency, but a small proportion occur at idiosyncratic times. Second, idiosyncratic pay changes are larger and more dispersed than year-end pay changes and resemble more pay changes occurring at job-to-job transitions. Third, idiosyncratic pay changes are more common for workers with less experience and, fourth, in firms higher on the job-ladder. Fifth, industries in which the incidence of idiosyncratic raises have risen have experienced greater declines in labor share. Sixth, industries in which more firms report willingness to negotiate wages have greater concentrations of idiosyncratic revisions. An on-the-job search model with heterogeneous wage contracts can rationalize these facts.

JEL CLASSIFICATIONS:

E24: WAGES

E25: WAGE • WAGE DISTRIBUTION • LABOR SHARE • WAGE SHARE

J31: WAGE DIFFERENTIALS

J33: COMPENSATION AND PAYMENT METHODS

M52: COMPENSATION METHODS AND THEIR EFFECTS

M55: LABOR CONTRACTING DEVICES

KEYWORDS: LABOR CONTRACTS, IDIOSYNCRATIC WAGE CHANGES, LABOR SHARE, JOB LADDER

Email: Cynthia.L.Doniger@frb.gov. I thank Wendy Edelberg, Valerie Rammey, Jon Steinsson, Emi Nakamura, and Benjamin Schoefer for insightful questions; Aditya Aladangady, Pierre Cahuc, Jan Eeckhout, Leo Kaas, Rasmus Lentz, Seth Murray, Dmitriy Stolyarov, Heiko Stüber, and Phillip vom Berge for comments; Sarah Baker and Carter Bryson for excellent research assistance; and Chris Black, Mark Carey, Daniela Hochfellner, Egon Zakrajsek, and the managers of the data enclaves without whose support this research would not be possible. I also thank participants at the ifo conference on Macroeconomics and Survey Data; the Ensuring Economic and Employment Stability conference on New Developments in the Macroeconomics of Labor Markets and Macroeconomics; the Annual Meetings of the Search and Matching Network, Society for Economic Dynamics, Society of Labor Economists, and Allied Social Sciences Associations; and seminars at the Federal Reserve Board, Institute for Employment Research (IAB), Census Bureau, University of Illinois Urbana-Champaign, University of Wisconsin, and University of California, Berkeley.

This study uses the weakly anonymous Sample of Integrated Labour Market Biographies (1975-2014) and the IAB Establishment Panel (1993-2014). Data access was provided via on-site use at the Research Data Centre of the German Federal Employment Agency at the IAB in Nuremberg, Germany; Ann Arbor, MI; Cambridge, MA; Los Angeles, CA; Berkeley, CA; and Princeton, NJ; and remotely via JoSuA.

The views expressed in this paper solely reflect those of the author and not necessarily those of the Federal Reserve Board, the Federal Reserve System as a whole, nor of anyone else associated with the Federal Reserve System.

1 Introduction

The timing of pay changes is important for our understanding of the macroeconomy. Many questions lead to and from attempts to decipher the covariance between pay changes, employment, and macroeconomic shocks and their lags. Findings of missing or mild pro-cyclicality of wages motivate theories of labor contracting that deliver wage rigidity, and these, along with availability and data quality, motivate the treatment of micro data on wages. This paper breaks from the beaten path and documents hitherto unstudied features of pay changes. In this fresh look, I find facts that corroborate a view of wage setting and labor market dynamics that—despite originating in the 1990’s—has only recently entered mainstream macroeconomics. In addition, I offer a nuance to this model that improves the fit to the facts and offers a new interpretation of secular trends in the past several decades and their relation to cyclical dynamics.

This paper asks “What can we learn from idiosyncratic pay changes?” This question is novel in the context of a literature that has focused primarily on cyclical covariances using data at fixed frequencies, in part because of to data constraints. The novel view of the data taken here—facilitated by administrative data that allow for observation of unconventional features—uncovers facts that are not easily reconciled with prevailing views of wage rigidity. Instead, they are consistent with a model from a class that has more recently entered the mainstream discussion of business cycles.

I document six facts about idiosyncratic pay changes. First, most pay revisions occur at yearly frequency, but a small proportion occur at idiosyncratic times. Second, idiosyncratic pay changes are larger and more dispersed than year-end pay changes and resemble more pay changes occurring at job-to-job transitions. Third, idiosyncratic pay changes are more common for workers with less experience and, fourth, in firms higher on the job-ladder. Fifth, industries in which the incidence of idiosyncratic raises have risen have experienced greater declines in labor share. Sixth, industries in which more firms report willingness to negotiate wages have greater concentrations of idiosyncratic revisions.

These facts complement but also contradict a small subset of the macro literature that has documented that the majority of wage changes are synchronized.¹ In particular, this literature has interpreted synchronization as evidence of a staggered contracting model a la [Taylor \(1980\)](#). However, in contradiction of this hypothesis, I document that idiosyncratic wage changes are more, rather than less, dispersed than synchronized changes.² Meanwhile, the [Taylor \(1980\)](#) view offers little guidance in interpreting facts three through six.

¹[Lünnemann and Winttr \(2010\)](#); [Sigurdsson and Sigurdardottir \(2011\)](#); [Le Bihan, Montornès, and Heckel \(2012\)](#); [Barattieri, Basu, and Gottschalk \(2014\)](#); [Murray \(2019\)](#) and [Grigsby, Hurst, and Yildirmaz \(2021\)](#)

²[Grigsby et al. \(2021\)](#) document a similar pattern in the ADP data.

This contradiction and the additional facts motivate exploration of alternative models of wage setting and inter-firm competition for employees. To this end, I describe an on-the-job search model with a mix of wage contracting protocols (a la [Doniger \(2015\)](#)). The model is in a class with newer theories of labor market cyclically that are recently entering the mainstream ([Robin, 2011](#); [Moscarini and Postel-Vinay, 2019](#); [Faccini and Melosi, 2019](#)). In this view, pro-cyclical wage growth (as it exists) is driven by inter-firm competition for already employed workers, rather than the threat point of nonemployed workers. This competition delivers the correlation in the timing of idiosyncratic wages with respect to the job-cycle and between incidence of idiosyncratic pay changes and proxies for an employer’s ranking on the job-ladder.

Meanwhile, a key difference between my model and those just mentioned is the presence of multiple contracting mechanisms. This feature allows my model to speak to the secular trends in the data that I have documented—in particular the rise in the incidence of idiosyncratic pay changes over time and their covariance with compensation of employees and opens additional novel possibilities. While my model is specified as a steady state, embedding my contracting structure within the macroeconomic model of [Moscarini and Postel-Vinay \(2019\)](#) or [Faccini and Melosi \(2019\)](#) would allow for secularly increasing divergence in the timing price and wage inflation.³

The facts I document are revealed by the relatively unique structure of the data I use—the Sample of Integrated Labour Market Biographies (SIAB) made available by the Institute for Employment Research (IAB) of the German Federal Employment Agency. Under the provisions of the law that mandates data collection, employers are required to report the exact dates of employment for all employees at least once per year. Reports contain basic demographics and, importantly, the daily wage. The typical ongoing employment spell is reported once yearly, on December 31, in accordance with the law. However, I document that a small minority of ongoing spells feature an additional, midyear report. After removing additional reports associated with changes in employment law, industry-wide union wage agreements, and employee-initiated changes in health insurance (which induce employers to submit an additional report), I document the six properties of midyear pay changes noted earlier.

I then propose an on-the-job search model capable of conforming with all of the documented properties. The key feature of the model is a choice between a fixed and a renegotiable wage contract. When the renegotiable contract is costly to the firm, only the most productive utilize it.⁴ The multi-contract assumption allows the model to be flexible on key

³Such a model is left for future work.

⁴Segmentation seems somewhat obvious given the assumed cost structure, but, in fact, requires that I

dimensions. First, in the nested models the incidence of pay changes either zero, by assumption, or closely tied to the rate of job-to-job mobility. Allowing for a mixture of contracts admits a more flexible relation between the incidence of pay changes and job-to-job mobility. Second, varying the share of firms using the renegotiable contract delivers comparative statics with respect to compensation that conform with the data.

Finally, to bring the findings to bear on the macro-economic questions central to this strain of the literature, I turn to U.S. data. Using data from the Survey of Income and Program Participation (SIPP), I identify pay changes using an application of the Bai-Perron (1998) filter as proposed by Barattieri et al. (2014). Specifically, I extend their treatment of the data to cover the 1996-2008 panels (data covering 1996 to 2014). I then categorize pay changes as “synchronized” and “idiosyncratic”. Specifically, a pay change is synchronized if that respondent receives at least one other pay change in the same calendar month during the survey period. I then show that while the incidence of synchronized pay changes responds to the unemployment rate, as theories in the vein of Taylor (1980) suggest, only idiosyncratic pay changes respond to the aggregate incidence of job-to-job mobility, as theories in the vein of Moscarini and Postel-Vinay (2019) suggest. In light of the fact that idiosyncratic pay changes are substantially larger than synchronized changes—corroborated in U.S. data by Grigsby et al. (2021)—this finding suggests that wage cyclically is driven by cyclical fluctuation in the intensity of direct competition between monopsonistic employers rather than by the improving outside option of all workers as a whole.

Section 2 describes the SIAB data. Section 3 lays out the six facts. Section 4 presents the model. Section 5 describes the SIPP and results. Section 6 concludes.

2 High frequency data on wages and employment

The primary dataset I use is the SIAB. The SIAB is a 2 percent random sample of Germans with employment liable to social security or recipients of social insurance made available by IAB. I restrict the sample to workers with full-time employment.⁵ These data are based on mandatory, yearly reports filed by employers for each public sector employee. Reports contain the exact dates of employment and the average daily wages during the employment period. The data also contain basic worker demographics such as age, sex, occupation, and schooling, as well as firm-level characteristics such as industry, age, size, moments of the within-firm wage distribution, in-flows, and out-flows. As is common in the literature, I

show that the benefit to the firm of such a contract is increasing in productivity and that, on the margin, changes in contract type do not alter the allocation of employees across firms.

⁵The data also contain rich information on job seekers and those with officially regulated part time employment that is not used in this paper.

restrict my sample to prime-age workers (ages 20 to 60).

An unusual feature of these data is the high frequency of reporting. This quality is particularly important for this work because it enables tracking the worker’s employment status to the exact day. Employers report each employee at least once each year, typically on new years. Thus, the typical record records a sub-part of a multi-year employment relationship. Such a record reports start and end dates on January 1 and December 31, average daily wages over that period, and the aforementioned employee and employer characteristics. Further, when employment relationships begin and end the exact start and end dates are recorded. This practice results in an employment record covering a sub-part of a year and enables the identification of separations to unemployment and job-to-job moves.⁶ In addition, for a small proportion of employees with ongoing spells, the employer has submit multiple contiguous reports within a single year.⁷ I call a pair of continuous employment reports with a single employer within a year a “reregistration.” I focus on the subset of reregistrations that include a pay change on impact, do not coincide with legal reforms or a large number of reregistrations within the same industry (indicative of a change in the industry contract), and are not triggered by an employee-initiated change in health insurance coverage.⁸ I refer to these as “idiosyncratic pay changes”.

I define three types of yearly spells: job stayer, job-to-job mover, and job-loser. To do so, I begin with the sample of workers employed in the first two weeks of January. I then identify the highest paying incumbent employer during this period. Job stayers subsequently have a year long relation with this employer that may include gaps so long as they are less than 15 days. Job-to-job movers transit from this initial employer to another with a gap less than 15 days. Meanwhile job-losers experience a gap of 15 days or more. Noate that while stayers, movers and losers are mutually exclusive each has an intersection with idiosyncratic

⁶Following the literature, I define the latter as occurring whenever a worker has records at two different employers with 15 days or less elapsing between and the former occurring whenever more than 15 days elapse.

⁷Analogous to job-to-job moves I define “contiguous” as two reports from the same employer with 15 days or less elapsing between.

⁸Specifically, I focus on idiosyncratic pay changes that do not coincide with more than 15 percent of workers within the same industry. In Germany many labor unions are organized by industrial sector. This restriction excludes changes in industrial labor contracts that affect many workers simultaneously. Additionally, I exclude idiosyncratic pay changes on July 1, 1990 (reunification); July 1, 1991 (statutory change to East German contribution limits); April 1, 1999 (first inclusion of marginal jobs); and April 1, 2003 (implementation of the Hartz III reform) as these events trigger a high volume of reregistrations. Finally, from 1999 onward the administrative data include reporting on health insurance coverage. Before 1996 Germans had little discretion over their health insurance coverage. After, Germans are able to make adjustments to their health insurance coverage if they meet certain requirements and from 1999 onward changes in health insurance coverage are required to be reported and trigger idiosyncratic reregistrations. Changes in health insurance coverage account for roughly half of idiosyncratic reregistrations after 1999. I exclude them from the main analysis and examine their properties separately in appendix A.

pay change recipients. In particular, so long as the pay change occurs before the move or job loss a mover or loser may be counted as a pay-change recipient as well.

An issue with the data is the presence of top-coding. Administrative data record the daily wage up until a maximum contribution limit, which varies by year and region (East/West).⁹ Top-coding results in under counting of idiosyncratic pay changes because the data do not reveal if idiosyncratic reregistrations of top-coded individuals were concurrent with pay changes. To adjust for this, I impute the share of reregistrations of top-coded workers that contain a pay change or a pay increase. Specifically, I estimate the share of workers in the top 5 percent of non-censored earners that receive an idiosyncratic pay change or raise by year, industry and region. I then randomly assign a pay change or pay increase to a corresponding fraction of top-coded workers within the same year, industry and region. In the following analysis, I note when imputed pay changes are included in the analysis and when they are excluded.

In addition to these data, I use data on unionization, labor share, and import exposure from the IAB, the EU KLEMS project (European Union level analysis of capital (K), labour (L), energy (E), materials (M), and service (S) inputs), and the Organisation for Economic Co-operation and Development (OECD) and World Input-Output Database (WIOD). Details of these data are discussed in the sections where they are used.

3 Six facts about idiosyncratic raises

3.1 Rare but increasingly common.

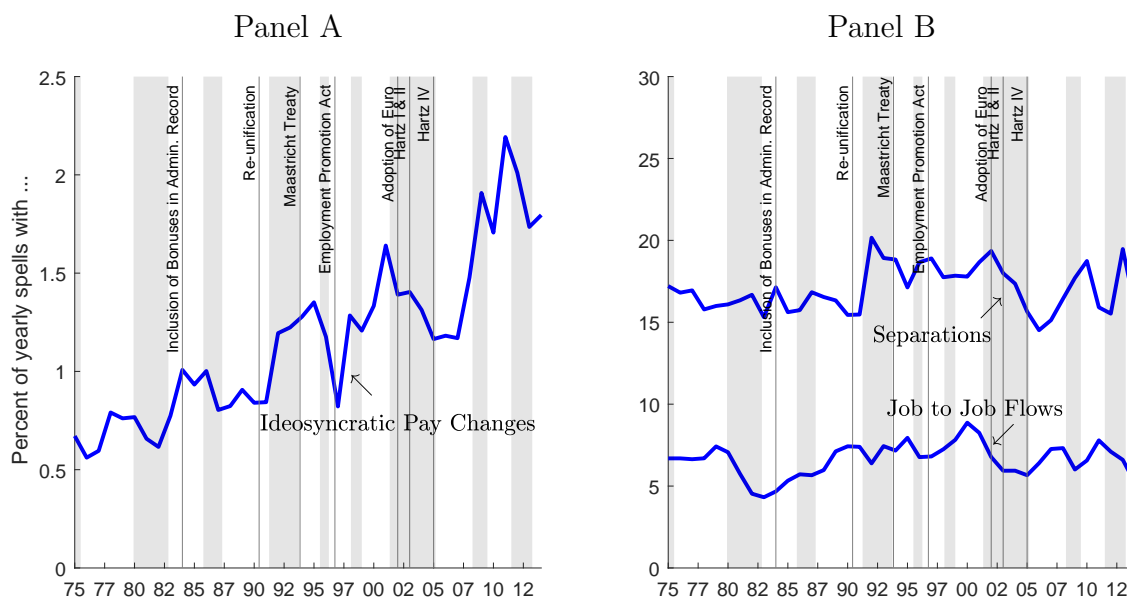
Figure 1 plots the percent of spells in progress in the first two weeks of January of each year since 1975 that exhibit an idiosyncratic pay change, separation to nonemployment, and/or job-to-job transition before year end. Idiosyncratic pay changes are defined earlier as inclusive of imputed pay changes for top-coded individuals. As previously noted, separations and job-to-job transitions are distinguished by the elapsed time in nonemployment. Apart from the expected cyclical patterns, separations and job-to-job flows in Germany are remarkably stable over the horizon.¹⁰

In contrast, a smaller and growing proportion of yearly employment spells contain an idiosyncratic pay change. Such idiosyncratic changes occur in less than 0.75 percent of

⁹Top-coding affects less than 10 percent of observations.

¹⁰This stability stands in sharp contrast to the documented decline in dynamism in the United States (Fallick and Fleischman (2004); Davis, Faberman, and Haltiwanger (2012); Davis and Haltiwanger (2014)) Although recently Fujita, Postel-Vinay, and Moscarini (2019) offer nuance to this view.

Figure 1: Idiosyncratic Raises are Rare but Increasingly Common



Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

yearly spells at the series start in 1975. By 2014, the last of my data, they occur in nearly 1.75 percent of spells. With the exception of the early 1990's recession, which closely followed reunification, the secular rise in rates of idiosyncratic pay-changes regresses during recessions, suggesting that these pay changes respond to similar labor market dynamics as transitions.

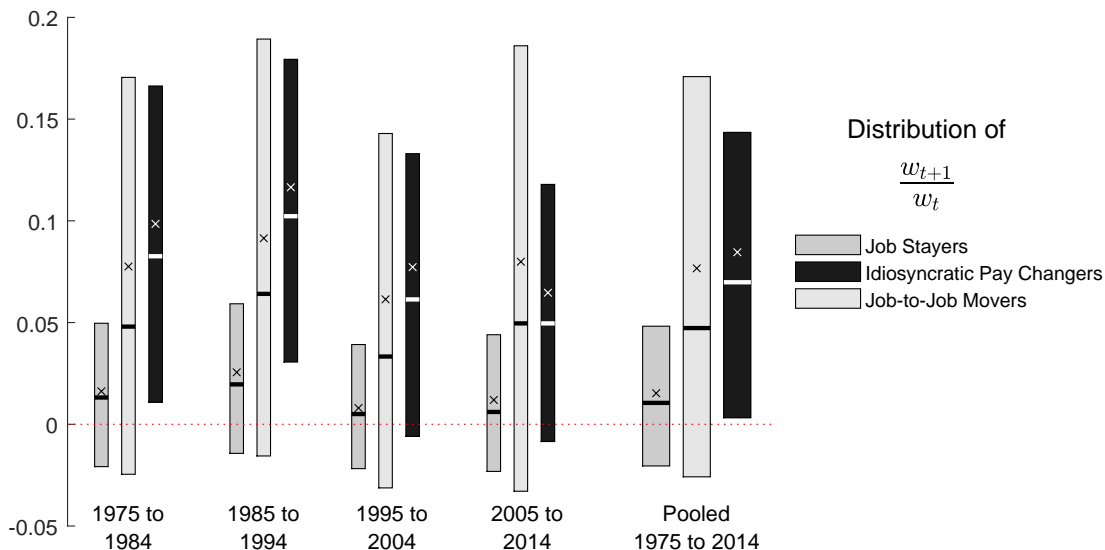
3.2 Large and dispersed

Idiosyncratic wage changes have large and lasting effects on wages. Figure 2 records the mean, median, and interquartile ranges of year-over-year pay changes following years that contain idiosyncratic pay changes by decade starting with 1975 to 1984. Idiosyncratic pay changes are large and dispersed.

To compare these changes with those of workers with other labor market histories figure 2 also records the year-over-year pay changes of job stayers without idiosyncratic pay changes, job stayers with idiosyncratic pay changes, and job-to-job movers by decade starting with 1975 to 1984. Workers with idiosyncratic pay changes have year-over-year earnings growth far in excess of other job stayers. Indeed, the mean, median and distribution of year-over-year wage growth for these workers resembles that of job-to-job movers more closely than that of other job stayers.¹¹

¹¹Potentially, part of these phenomena are driven by the typical timing of “Christmas” bonuses. From

Figure 2: Idiosyncratic Pay Changes Are Large and Dispersed



Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

3.3 Covariance with life-cycle, experience, job-cycle, and tenure

In figure 3 panels A to D plot the relationship between age, experience, length of job-cycle, and tenure relative to job-to-job mobility and idiosyncratic pay changes.¹²¹³¹⁴ As has been well documented in a long literature beginning with [Topel and Ward \(1992\)](#), job-to-job mobility declines in each measure of seniority. While the decline with age and experience is close to linear (dashed red line in panels A and B, right axis) the more typically observed pattern in which the rate at which the hazard of job-to-job mobility decreases with time slows, producing a convex relationship, is evident for the job-cycle length and tenure (dashed red

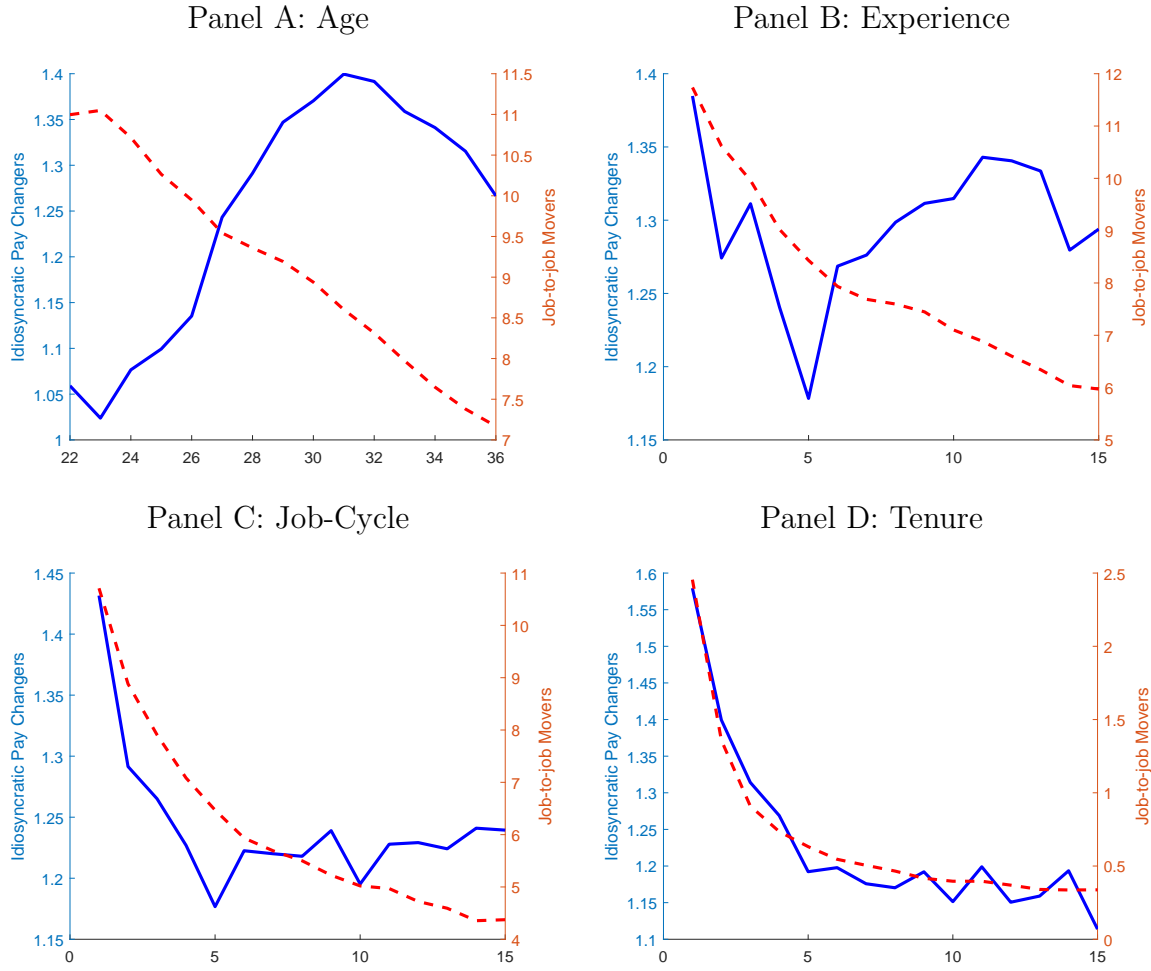
1984 onward, these are averaged into daily wages in the spell in which they occur. Because most idiosyncratic pay changes and job-to-job moves occur before December, the average daily wage in the initial year may be artificially depressed. To address this potential bias, I compute analogues to the the figure 2 using the starting wage in the year before the year in which the idiosyncratic pay changes or job-to-job mobility is observed. Results are reported in figure 10 and yield broadly the same conclusions.

¹²The typical measure of experience would be Age-Schooling-6, where schooling is measured in years; however, schooling is measured coarsely (four categories) in the SIAB. Instead, I measure experience as years since a workers first job recorded in the administrative records. To account for left-censoring, I omit workers who first appear in 1975 (the year records begin for West Germans) or in 1993 (the year records begin for East Germans).

¹³Job cycle is measured as years since the last episode of non-employment lasting more than 15 days or years since first appearance (whichever is shorter). As with experience (previous footnote), I omit job-cycles beginning in 1975 or 1993 to address left-censoring.

¹⁴As with experience and job cycle (previous two footnotes), I omit tenures beginning in 1975 or 1993 to address left-censoring.

Figure 3: Variation over the Life and Job Cycles



Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

line in panels C and D, right axis). For both job cycle and tenure the data are close to consistent with an exponential hazard model.

Turning to idiosyncratic pay changes, the data show an increasing hazard in age and an initially decreasing but later increasing hazard in experience (solid blue line in panels A and B, left axis). For job-cycle and tenure, a pattern similar to what has been documented here and in the literature for job-to-job mobility emerges. Idiosyncratic pay changes are most common for workers early in the job-cycle and at low tenures. As job-cycle and tenure increase the hazard of idiosyncratic pay change steadily decreases (solid blue line in panels C and D, left axis).

The job-to-job mobility data suggest the presence of a job ladder that contains a common component. (In other words, most workers rank firms similarly.) Consistent with a job ladder

theory, the ladder is most cleanly revealed by the decline in job-to-job mobility as job cycle length and tenure increase. The decline in idiosyncratic pay changes as these measures of seniority increase suggests that competition between firms on the job ladder is at least one of the mechanisms leading to pay changes. Meanwhile, the opposing pattern with respect to age and, to a lesser degree, experience, illustrates that the relation documented for job cycle and tenure are not a spurious reflection of life-cycle events (for example, family formation, fertility, and so forth).

Taken together, these facts suggest a job ladder model in which both allocations and wages depend on workers’ labor market histories (see, for example, the sequential auction model proposed by [Postel-Vinay and Robin 2002b,a](#)). In a model of this type, on-the-job contacts between workers and new firms generate job-to-job mobility when workers would be more productive working for the new firm. In addition, on-the-job contacts produce pay increases when worker would be less productive with the new firm than the incumbent but they would be more productive working for the new firm than their best-to-date outside option. Further, embedding such a model in a business cycle—as in [Robin \(2011\)](#) or [Moscarini and Postel-Vinay \(2019\)](#)—delivers the cyclical covariation in idiosyncratic pay changes and job-to-job mobility observed in figure 1.

3.4 Offered by firms higher on the job ladder

I pursue the job ladder hypothesis further. To evaluate the covariation between firm characteristics and job-to-job mobility and idiosyncratic pay changes, I exploit the quasi-link between employer and employee between the SIAB and objectees measured in the IAB Establishment History Panel (BHP). Specifically, while the SIAB is a panel of workers and not a linked employer employee data set, the data include particular firm-level covariates that the job ladder literature considers to be correlated with a firms rank: size and moments of the within-firm wage distribution.¹⁵¹⁶

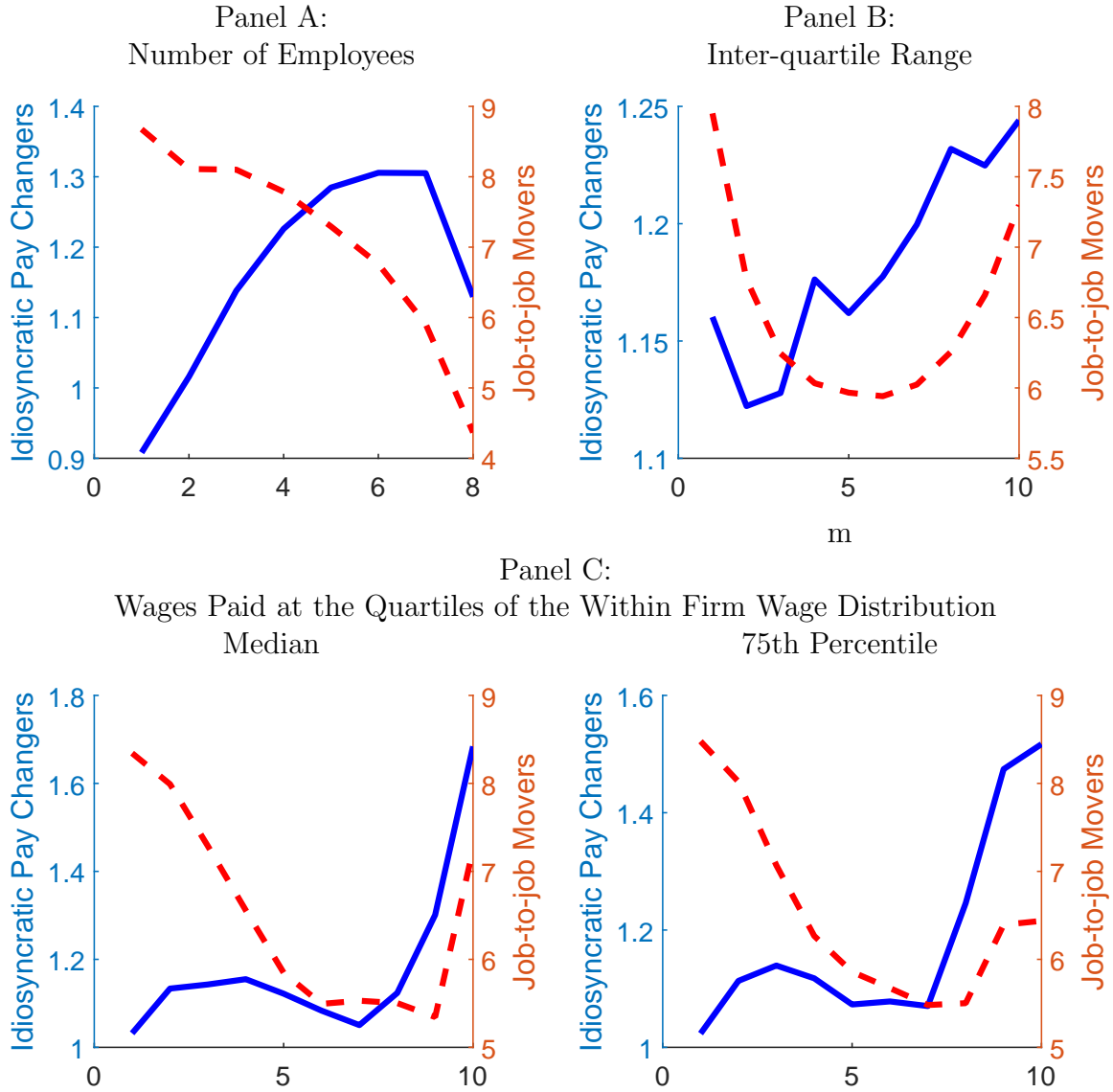
Figure 4 plots the share of yearly job spells including a job-to-job transition or an idiosyncratic pay change by firm size bins (panel A), inter-quartile range of the within firm wage-distribution (panel B), median wages (panel C), and the 75th percentile (panel D).¹⁷

¹⁵Inflows and outflows are also observed, but unfortunately, coarsely and without information about employees prior or future job status, which, makes it impossible to construct metrics suggested by the literature, (for example the poaching index proposed by [Lentz and Bagger 2009](#)).

¹⁶Note that, due to censoring, the mean wage is imputed while the median is observed. Thus, I focus on the median.

¹⁷Increasing inter-quartile range is associated with higher rank on the job-ladder under the sequential auction contract studied in [Postel-Vinay and Robin \(2002b,a\)](#) and the literature that follows.

Figure 4: Variation with Firm Characteristics



Source: Sample of Integrated Labor Market Biographies (SIAB), IAB Establishment History Panel (BPH), author's calculations.

Despite the coarseness of the measures, the pattern predicted by a job-ladder model is evident in the generally declining incidence of job-to-job transitions as the proxy for firm quality increases (red lines, right axis). Meanwhile, the incidence of idiosyncratic pay changes is rising (blue lines, left-axis). This finding is, again, consistent with the predictions of a job-ladder model with wages set under a sequential auction protocol.

Table 1: Survey Evidence

| <i>Dependent Variable:</i> | Share Reporting Negotiation | | |
|--------------------------------------|--------------------------------|-----------------|------------------|
| | (I) | (II) | (III) |
| Incidence of IPC | 9.83 (6.14) | 9.70 (6.19) | |
| Incidence of JTJ | | 1.04 (1.04) | |
| Incidence of Sep. | | -0.88 (0.66) | |
| Incidence of IPC relative to JTJ | | | 0.48** (0.21) |
| Incidence of JTJ relative to Sep. | | | 0.47** (0.22) |
| Constant | 0.19 (0.12) | 0.27* (0.15) | 0.028 (0.14) |
| R-Squared | 0.11 | 0.19 | 0.25 |
| Observations | 23 | 23 | 23 |

Source: Sample of Integrated Labor Market Biographies (SIAB), Brenzel (2014), author's calculations.

*Note: ** and * indicate significance at the 5 and 10 percent level. Observations weighted by industry employment.*

3.5 Covariance with survey-reported willingness to renegotiate

While survey-based longitudinal data on the composition of wage contracts does not exist, I am able to test the relationship between the idiosyncratic pay changes and cross-sectional survey data on the negotiability of wage contracts using data collected in the early 2010's. In the 2011 German Job Vacancy Survey conducted by the IAB, employers were asked “Did you negotiate with the applicant about remuneration (basic salary and further components if applicable)?” Brenzel, Gartner, and Schnabel (2014) document substantial variation across industries in the probability of an affirmative response.

In table 1, column (I) records the relationship, at the industry level, between the share of affirmative survey responses—as reported by Brenzel et al. (2014)—and the incidence of idiosyncratic pay changes. The relationship has the expected sign, but is not statistically significant. Column (II) includes the incidence of job-to-job mobility and of separations. Each has the expected sign, but again, no statistical significance. Finally, column (III) transforms the data to test the predictions of a job ladder model in which some but not all

wages set under sequential auction as in [Doniger \(2015\)](#). This model is described in greater detail in section 4.

In a job-ladder model in which all wages are set under sequential auction, the ratio of the hazard of on-the-job contact with a new firm to the separation hazard determines the expected number of jobs per job cycle. A higher value implies a higher incidence of direct competition among firms for already employed workers. Thus, the expected relationship with the surveyed incidence of negotiations is positive. In addition, in a job-ladder model in which only the firms higher on the ladder set wages under sequential auction, this pattern is reinforced. Industries in which the expected number of jobs per cycle is larger have more workers sorted toward the higher end of the job ladder, which is where the model predicts wage renegotiations occur. Indeed, the coefficient on this ratio is positive and statistically significant, as predicted.

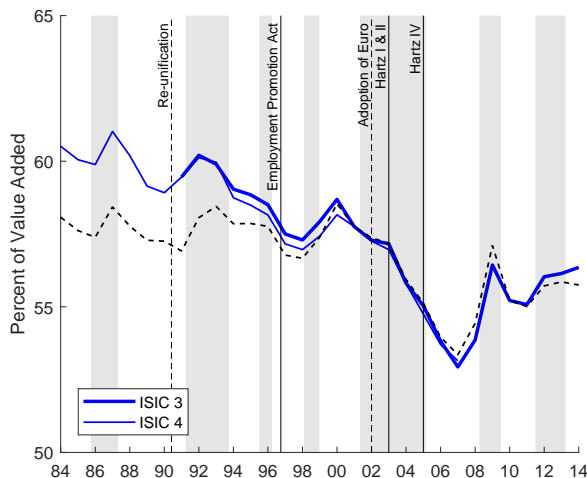
In addition, in a model in which all wages are set under sequential auction, the incidence of pay changes and of job-to-job mobility is tightly linked. The first depends on the evolution of the best-to-date job offer and the second depends on the evolution of the best-to-date outside option, both of which are simple functions of the on-the-job job offer arrival rate. Thus, if all wages in all industries are set under sequential auction, one would expect the ratio of idiosyncratic pay changes to job-to-job mobility to be roughly constant across industries and, therefore, to have no predictive power. Admitting a choice of wage setting mechanisms to the model, as in [Doniger \(2015\)](#), admits the possibility of variation in the ratio of idiosyncratic pay changes to job-to-job mobility across industries and predicts a positive relation between this ratio and the incidence of wage negotiations. Again, the coefficient on this ratio is positive and statistically significant, as predicted.

3.6 Covariance with Secular Trends in Payroll Share

A natural question, given the unique ability to look into history with the SIAB data, is if secular variation in labor contracting is correlated with other secular trends. Here I exploit heterogeneity across industries to explore the relationship between trends in the incidence of re-negotiable contracts and payroll share.¹⁸ Recent papers [Elsby et al. \(2013\)](#) and [Karabarbounis and Neiman \(2014\)](#) have highlighted a secular decline in labor share in the United States and internationally, respectively. Germany is among the countries documented by

¹⁸Following [Elsby, Hobijn, and Şahin \(2013\)](#), I focus on the ratio of compensation of employees to total value added to avoid issues stemming from imputing the contribution of sole proprietor income to labor versus capital share. In addition, the model that I discuss in the following section pertains to an employer employee relation, not a sole proprietor.

Figure 5: Payroll Share



Source: EU KLEMS project, author's calculations.

Note: black dashed line holds the industrial composition fixed at the 2010 mix.

Karabarbounis and Neiman (2014), and the solid blue line in figure 5 documents Germany's declining labor share. The data come from EUKLEMS, which synchronize national accounts data and definitions for a large set of countries. From 1984 to 2014, payroll share declined 5 percentage points.

The trend in payroll share is in part driven by changes in industry composition over this period. To address, I adjust the data to hold constant the shares of employment in each industry at the 2010 share. The dotted black line in figure 5 plots the adjusted series. It is readily apparent that much of the early trend is accounted for by composition. In contrast, starting in the late 1990s, trend in payroll share transitions to a within-industry phenomenon. From 1984 to 2000, composition-adjusted payroll share holds steady at approximately 58 percent. Thereafter, it falls nearly 5 percent to a trough in 2007 before partially recovering 2.3 percent by 2014.

A host of theories have been put forth to explain the remaining trend.¹⁹ Here, I document a relationship between within-industry changes in payroll share and the incidence of idiosyncratic pay changes relative to job-to-job transitions. As discussed in the previous subsection, if one has in mind a job ladder model in which some firms post wages while others renegotiate according to a sequential auction, variation across industries in this ratio

¹⁹Elsby et al. (2013) Karabarbounis and Neiman (2014) Autor, Dorn, Katz, Patterson, and Reenen (2017) Autor, Dorn, Katz, Patterson, and Van Reenen (2020) Gouin-Bonenfant (2018) Edmond, Midrigan, and Xu (2015) Eggertsson, Robbins, and Wold (2018) Kehrig and Vincent (2018)

Table 2: Determinants of Payroll Share

| <i>Dependent Variable: Payroll Share</i> | Levels | | | | | | Stacked 5-year Differences |
|--|-----------|--------------------|---------------------|---------------------|----------------------|---------------------|----------------------------------|
| | (I) | (II) | (III) | (IV) | (V) | (VI) | |
| | IPC / JTJ | -0.117* (0.060) | -0.131** (0.057) | -0.090** (0.037) | -0.092*** (0.032) | -0.121** (0.056) | |
| Year FE | X | X | X | X | X | X | |
| Industry FE | X | X | X | X | X | X | |
| JTJ/separations | | X | | | | X | |
| Demographics ^a | | | X | | | X | |
| Unionization ^b | | | | X | | X | |
| Import Exposure ^d | | | | | X | X | |
| R-Squared | 0.162 | 0.181 | 0.314 | 0.305 | 0.193 | 0.406 | 0.308 |
| Observations | 544 | 544 | 544 | 544 | 544 | 544 | 560 |

Source: Sample of Integrated Labor Market Biographies (SIAB), EU KLEMS project, OECD and WIOD input-output tables, IAB Establishment Panel (BH), author's calculations.

*Note: ***, ** and * indicate significance at the 1, 5 and 10 percent level. Observations weighted by industry employment. Standard errors clustered at the industry level.*

^a *Share West German, sex composition, establishment age (5-year bins), employee age (5-year bins).*

^b *Quadratic fit in time to concentration of union types by industry. Author's calculations based on the IAB Establishment Panel (BH).*

^d *Percentage difference between total domestic requirements and total requirements constructed from the OECD and WIOD input-output tables. Following the methodology of Elsby et. al (2013).*

is correlated with the share of firms setting wages according to sequential auction.

I focus on 1999 and onward because, during this period, shifts in payroll share occurred within industry. To test document the relationship between idiosyncratic raises and payroll share, I run the following industry-level regression:

$$labor\ share_{i,t} = \beta \left\{ \frac{IPC}{JTJ}_{i,t} \right\} + year\ fixed\ effects + industry\ fixed\ effects + \varepsilon_{i,t}$$

where $\frac{IPC}{JTJ}_{i,t}$ is the ratio of idiosyncratic pay changes to job-to-job transitions in an industry \times year. Regressions are weighted according to the average value added over the period 1999 to 2014 and standard errors are clustered at the industry level. I consider the 34 broad industries in the International Standard Industrial Classification of All Economic Activities 4. Table 5 presents the result. Interpreting the magnitude of the coefficients requires theoretical structure, which I provide in the following section. However, across specifications, increasing the incidence of an idiosyncratic raise relative to job-to-job mobility decreases payroll share

with statistical significance ranging between 5 and 10 percent. Alternative specifications include industry level demographics as well as measures of competing theories—notably, declining unionization and increasing trade exposure.²⁰

4 Model

4.1 Environment

The model assumes the basic structure of a random-search job ladder model à la [Burdett and Mortensen \(1998\)](#) and [Postel-Vinay and Robin \(2002b\)](#), both of which are nested cases. Identical workers search on and off the job for employers. Employers produce using a constant returns to labor technology, p , which is distributed across employers according to a differentiable distribution, $\Gamma(p)$. Search is random and balanced, meaning that the probability of drawing a job offer from an employer with technology p corresponds to its weight in the distribution $\Gamma(p)$. Balanced matching is equivalent to assuming that all firms, regardless of their steady state size and profitability, maintain exactly one vacancy at all times and workers have equal probability of being matched with any of these vacancies. For workers, job offers arrive according to exogenous Poisson processes with hazards $\lambda_e < \lambda_u$ on and off the job. Separation also occurs according to an exogenous Poisson process with parameter δ .

I diverge from the nested models by allowing employers to choose between a non-negotiable wage contract, as in the posted-wage model of [Burdett and Mortensen \(1998\)](#), or a contract that can be re-negotiated as the worker’s outside option evolves through further search, as in the sequential Bertrand auction of [Postel-Vinay and Robin \(2002b\)](#).

Definition 1. *An equilibrium in this environment is such that*

- *given the opportunity, workers accept employment in the firm offering the highest value contract.*

²⁰**Unionization:** To study the effects of unionization I construct estimates of industry level concentration of the various union types extant in Germany during this time—industry wide, plant level, works council and “other” representation—as well as the share without representation of any kind, using the IAB Establishment Panel. Because of small cell sizes for some industry-years estimates are as industry specific flexible time trends. These are then merged to industry level aggregates constructed from the SIAB. Regression coefficients, not shown, suggest that an industry wide agreement or a works council is positively correlated with payroll share as compared to no collective representation. Plant level and “other” representation have no effect. **Import Exposure:** Following [Elsby et al. \(2013\)](#) I measure import exposure as the ratio of final value added accounted for by imported inputs, by industry. To construct the measure, I use the OECD input-output tables. Coefficients, not shown, suggest that a 1 percentage point increase in import exposure decreased payroll share by approximately 0.3 percentage points, implying that over the horizon considered variation in import exposure accounts for approximately 2 percentage points of the decline in payroll share.

- firms employ using the contract and wage schedule set under said contract that yields the greatest rents in steady state.
- labor supply to a firm of type p is pinned down in steady state by the equilibration of flows of workers in from unemployment and other firms and out to unemployment and other firms.

4.2 Equilibrium

Proposition 1. *When the cost of maintaining a vacancy featuring a renegotiable contracts exceeds that of a vacancy for a non-renegotiable contract the equilibrium is **segmented** and only more-productive employers re-negotiate.*

I derive the wage equations and employment distributions and present formal proof of segmentation in appendix B of this paper. They can also be found in Doniger (2015). Intuition rests on three (sets of) observations.

First, balanced matching, assumed in both nested models, implies that all firms maintain an equal mass of vacancies and the relative size of firms is determined ex-post by the probabilities of hiring and retention conditional on these equal masses of vacancies. I assume balanced matching for tractability and comparability with the nested models but acknowledge that alternative assumptions may better fit the distribution of firm sizes. Balanced matching implies that relative cost of the two wage contracts is independent of equilibrium firm size.

Second, under segmentation, labor flows are efficient, meaning that no worker rejects an offer from an employer that is more productive than their incumbent. To see this, note that if $q < p$ are the productivity of two firms then the posited segmentation implies that either q and p both do not negotiate, both negotiate, or q does not negotiate while p does. In the first two cases Burdett and Mortensen (1998) (resp. Postel-Vinay and Robin (2002b)) demonstrate that optimal posted (resp. renegotiated) wages imply that the worker will flow from q to p if given the chance. Define the non-negotiable wage offered by an employer with labor productivity p as $w^n(p)$ and the re-negotiable wage as $w^r(p, q)$ where q is the labor productivity of the worker's best-to-date outside option. Since $w^n(q) < q < p$ there exists a profitable wage offer from p that will trigger the worker to flow from q to p .²¹ Thus, under optimally set wage offers, the worker always flows from q to p in equilibrium.

Further, let \tilde{p} be the most productive firm that does not negotiate and consider the impact on worker flows if this firm deviates to renegotiating. As discussed just previously,

²¹Indeed, as noted in the following $w^r(p, q) < w^n(q)$.

this firm can still hire from all firms with $q < \check{p}$. Meanwhile under the posited segmentation, all firms with $p > \check{p}$ renegotiate and therefore workers always flow from \check{p} to p if given the opportunity. Thus, the marginal firm may change its wage bill but not its size by changing its wage setting rule.

Third, in appendix B, I show that optimally set wages have the following properties:

$$\begin{aligned}
 (3.1) \quad \frac{dw^n(p)}{dp} &> 0, & (3.2) \quad \frac{dw^r(q, p)}{dq} &> 0, \\
 (3.3) \quad \frac{dw^r(q, p)}{dp} &< 0, \text{ and} & (3.4) \quad w^r(p, q) &< w^n(q) \forall q \leq \check{p} < p.
 \end{aligned}$$

Properties (3.1) and (3.2) are intuitive. The first, property (3.1), states that under a non-negotiable contract, wages are increasing in an incumbent’s productivity. [Burdett and Mortensen \(1998\)](#) and [Bontemps, Robin, and van den Berg \(2000\)](#) document this property in their wage-posting models, which are nested in the present model.²² The intuition follows from noting that, conditional on selecting the nonnegotiable contract, a more productive employer is willing to pay more for an employee because the output it forgoes if it is rejected is larger. That this property carries through to the segmented equilibrium follows from noting that labor flows are constrained efficient under segmentation. The second, property (3.2), states that under a renegotiable contract wages are increasing in a worker’s best-to-date outside option. [Postel-Vinay and Robin \(2002b\)](#) prove this in their sequential Bertrand auction which is also nested in the present model.²³ The intuition follows by noting that a larger wage offer is required to beat out a more competitive offer from a more productive employer. That this holds even if that employer’s offer is non-negotiable follows from property (3.1).

Property (3.3), which states that more-productive employers that use the renegotiable contract employ workers at lower wages conditional on their best-to-date outside options, appears counterintuitive. However, by noting two things this can be made intuitive. First, the cost-minimizing wage offer from the renegotiating employer is a wage equal to the worker’s current reservation given the worker’s current best-to-date outside offer. A lower wage offer would not be accepted and, because the employer can renegotiate wages, the prospect of future competition provides no incentive to offer a higher wage. Second, as in [Postel-Vinay and Robin \(2002b\)](#), the option value of search—and the resulting pay increases with this and future incumbents—is an amenity associated with the job offer specifying a renegotiable

²²An economy identical to that considered by [Bontemps et al. \(2000\)](#) is recovered when the relative cost of maintaining a renegotiable vacancy is sufficiently large.

²³An economy identical to that considered by [Postel-Vinay and Robin \(2002b\)](#) is recovered when the relative cost of maintaining a renegotiable contract falls weakly below zero.

wage and is thus priced into the wage paid at hiring. Because the option value stems from the expectation of future wages and a more productive employer will place higher wage bids in the future, employment in a high productivity employer now locks in a longer tail of the expected wage distribution in the future. In other words, the option value of search is increasing in the incumbent’s productivity whenever the incumbent employs under a renegotiable contract. This mechanism gives rise to property (3.3).

Finally, property (3.4), which states that an employer that renegotiates can hire from a employer that does not renegotiate at a wage cut, also appears counterintuitive. However, the logic also stems from comparing the option value of search in a nonrenegotiable contract at a q -productivity employer which does not renegotiate, with that in a renegotiable contract at a p -productivity employer which renegotiates. Because the renegotiating employer will increase the wage offer in an attempt to retain the worker, the renegotiating employer offers a larger option value of search. As in the logic of property (3.3), the value of expected this competition is priced into the wage that induces transition from the q -productivity employer to the p -productivity employer and results in the noted wage cut.

Constrained efficient labor flows and properties (3.1) through (3.4) can be used to establish that the benefit of switching from not negotiating to renegotiating is increasing in the firms productivity. Meanwhile the assumptions of balanced matching and relative vacancy costs imply that the relative costs of the two contracts are independent of productivity. This is sufficient to show single crossing and the existence of a segmented equilibrium.²⁴

4.3 Discussion

As discussed in the preceding section, this model is consistent with facts 2 through 5. Admission of dual contracting methods allows for the trend observed in the incidence of idiosyncratic pay changes observed in fact 1. This fact suggests a decrease over time in the costs associated with renegotiable contracts. This implication is consistent with a backdrop of weakening unionization in Germany through this period; however, table 5 column (IV) shows statistically important independent variation between the ratio of idiosyncratic pay changes to job-to-job mobility and unionization, as measured in the IAB Establishment Panel.²⁵

Turning to fact 6, the model delivers the following comparative static:

²⁴Uniqueness requires that the difference in wage bills \check{p} under non-negotiating and negotiating for the firm is monotonically increasing in \check{p} . This is guaranteed if the rate of job-to-job mobility relative to separations is sufficiently small. See appendix ?? for details.

²⁵Note, the data do not reveal if a particular worker is paid under any of the possible types of union wage agreements present in Germany during this period.

Proposition 2. *All else being equal, increasing the share of employers offering the re-negotiable contract reduces labor share.*

As before, proof can be found in appendix B. Here I provide intuition.

Note that three forces that act upon the compensation of employees when the composition of contracts shifts. First, some marginal employer switches from the non-negotiable contract to the renegotiable contract. From property (3.4), we know that this employer reduces its average wage bill. Second, countervailing this force, wages necessary to hire from this marginal employer increase. These first two forces offset perfectly, because the promise of these higher wages is exactly what the marginal employer prices in as an amenity when it switches to the renegotiable contract. Third, renegotiating employers yield no rent at hiring, because they can adjust in the face of future competition, while non-negotiating employers offer wages above the history dependent reservation wage of a prospective employee because they cannot adjust later to ward off future competition. Indeed, nonnegotiable wages are optimally set to balance the lost rent against the hazard of attrition (Burdett and Mortensen, 1998; Bontemps et al., 2000). As the share of re-negotiating employers rises the result is that the rent yielded at hiring by all firms falls.²⁶

To see that falling compensation of employees implies falling labor share in the present context requires specifying an accounting framework that includes a treatment of rents and the costs of the labor contracts. There are two reasonable options. The first is to treat contracting costs as an intermediate input. This assumption gives rise to

$$labor\ share = \frac{compensation\ of\ employees}{output - contracting\ costs}.$$

The second imagines that the microfoundation of contracting costs are payment employees in a human resources department. In this case

$$labor\ share = \frac{compensation\ of\ employees + contracting\ costs}{output - contracting\ costs}.$$

It is straightforward to show that declining compensation of employees implies falling labor share under either accounting framework.

²⁶Indeed, it falls to zero as the game approaches Postel-Vinay and Robin (2002b) making that game subject to the Diamond (1971) Paradox. Doniger (2015) discusses how contract heterogeneity serves to break the Diamond (1971) Paradox, restoring a relation between wage dispersion and search incentives among the unemployed. The problem of the Diamond (1971) Paradox could also be alleviated by introducing bargaining power on the part of workers as in Cahuc, Postel-Vinay, and Robin (2006). As compared with Cahuc et al. (2006), which offers a reduced form incentive for search on the part of the unemployed, the present model provides a microfounded incentive that leaves observable tracks in the data. The incidence of idiosyncratic pay changes relative to job-to-job transitions.

5 Cyclical behavior

To return to the broader context regarding how these facts fit into the macroeconomics literature on the timing of wage changes, these facts clearly stretch beyond the explanatory capacity of the popular and classic Taylor (1980) model of wage adjustments in the secular and crosssectional views. While the model described here is specified in steady state, models in its class have recently been proposed to offer a novel view into the relation among competition for workers, wage growth, and inflation (Moscarini and Postel-Vinay, 2019; Faccini and Melosi, 2019). Each of these models embeds one of the steady state models nested here into a dynamic economy. To speak to these developments, I return to data; however, I now turn to U.S. data—in particular the SIPP—because cyclical facts are most commonly documented for the United States and the recessions experienced in Germany following reunification and during and after the Hartz reforms do not lend themselves to typical business cycle analyses pioneered in the United States.

5.1 Idiosyncratic pay changes in the SIPP

The SIPP is composed of a series of short panels querying, among other things, labor market outcomes such as employment histories and wages. Panels running from 1996 to 2008 (spanning years 1996 to 2014) have the same design, facilitating pooled analysis of the data. In these years, data are collected every three months and retrospective questions fill in wage and employment histories in the intervening two months. Barattieri et al. (2014) propose an algorithm for identifying persistent wage change from the survey data. Simply described, the algorithm conducts a Bai-Perron 1998 test against the null hypothesis of constant nominal wages for each respondents wage history. The advantage of the algorithm over the raw wage data is that it addresses measurement errors due to survey design or response errors, which are assumed to be transitory. Barattieri et al. (2014) consider only the SIPP 1996 panel, but the commonality of structure through to the 2008 panel allows me to apply their algorithm forward.

A key finding of Barattieri et al. (2014) and others in the literature is a spike in the hazard for a wage change at intervals of 12 months. This finding guides my search for idiosyncratic changes. Having identified persistent pay changes, I identify the modal month in which a pay change occurs for each individual. I then identify idiosyncratic changes as pay changes occurring in other months.²⁷

²⁷Thus, if a respondent has only one identified pay change it is counted as idiosyncratic.

Table 3: Cyclicalty.

| | (I) | (II) | (III) |
|----------------------------|--------------------|--------------------|--------------------|
| LHS= | Any | Synchronized | Idiosyncratic |
| | Pay Change | Pay Change | Pay Change |
| unemployment rate | -0.81*** (0.08) | -0.27*** (0.05) | -0.53*** (0.06) |
| job-to-job transition rate | 0.23 (0.17) | -0.05 (0.11) | 0.27** (0.14) |
| R-squared | 0.20 | 0.24 | 0.21 |
| Observations | 652,901 | 652,901 | 652,901 |
| Jobs | 124,048 | 124,048 | 124,048 |

Source: SIPP Panels 96-08 and authors's calculations. Pay changes identified using the Barattieri, Basu, & Gottschalk (2014) algorithm. Pay changes are categorized as synchronized if a SIPP respondent has more than one pay change in that same calendar month.

Note: All regressions control for job fixed effects and cluster standard errors at the job. Standard errors, clustered at the individual, in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

5.2 Idiosyncratic pay changes and U.S. business cycles

Table 3 presents the covariation between the incidence of pay changes, synchronized pay changes, and idiosyncratic pay changes with measures of cyclical variation in the labor market. Column (I) documents a negative relationship between pay changes (pooling synchronized and idiosyncratic pay changes) and the unemployment rate and a positive, but statistically insignificant, relationship with the rate of job-to-job mobility. Columns (II) and (III) separately consider synchronized and idiosyncratic pay changes. Both exhibit negative co-movement with the unemployment rate, but, interestingly, the sign of the relation with job-to-job mobility differs between the two. For synchronized pay changes the coefficient is negative, but economically and statistically insignificant. Meanwhile, idiosyncratic pay changes have a positive and statistically significant co-movement with job-to-job mobility.²⁸

This finding is particularly interesting in light of the empirical evidence provided in Moscarini and Postel-Vinay (2016, 2017). These papers document that wage growth for SIPP respondents is positively related to job-to-job mobility even if the respondents themselves are not job-to-job movers. Moscarini and Postel-Vinay's 2016; 2017 fact, in part, motivates

²⁸A similar pattern can be casually observed for the German data from figure 1, omitting the Hartz recession.

the macroeconomic models proposed in [Moscarini and Postel-Vinay \(2019\)](#) and [Faccini and Melosi \(2019\)](#). In both, increased job-to-job mobility is correlated with increased wages of job-stayers through their employers' response to increased on-the-job contacts between employees and other firms. However, the mechanism driving wage growth is different, and the differences have non-trivial implications for the relative timing of the driving macroeconomic shock, growth in remitted wages, and price inflation. In [Faccini and Melosi \(2019\)](#) wages are set under wage posting. An implication of this assumption is that the entire wage schedule is revised at the time of the macroeconomic shock, making the shock, wage growth (as measured in both allocative and remitted wages), and inflation coincident.²⁹ Meanwhile, in [Moscarini and Postel-Vinay \(2019\)](#) wages are set under sequential auction. An implication of this assumption is that additional wage growth for job-stayers when the contact rate between firms and employees increases accrues in large part only after such a contact. This finding makes the timing of the shock and inflation coincident while wage growth as measured by remitted wages lags because firms anticipate increased allocative labor costs even though they do not immediately appear in remitted wages.

The evidence present in this section supports a wage setting convention similar to that embedded in the [Moscarini and Postel-Vinay \(2019\)](#) model. Meanwhile, the model present in section 4 provides a flexible intermediate capable of accounting for secular trends in the relative timing of the shock and wage and price reactions.³⁰

6 Conclusion

This paper provides a novel look at wage-change data and uncovers new stylized facts. In particular, I focus on idiosyncratic pay changes rather than the synchronized pay changes that have been taken as evidence of the [Taylor \(1980\)](#) hypothesis of infrequently but synchronously renegotiated wage contracts. In contrast to the literature, the new facts that I document are inconsistent with the [Taylor \(1980\)](#) mechanism and instead point toward microfoundations for wage changes that have recently been introduced into the macroeconomics main-stream in particular, a job ladder model with in which wages are renegotiable in the event of an innovation in a worker's alternative employment prospects. I describe a steady state version of such a model due to [Doniger \(2015\)](#) and discuss the ways it is consistent with the novel facts about wage changes. Turning back to the relationship between the model and the macroeconomic questions of interest to the broader literature, I document

²⁹See [Basu and House \(2016\)](#) for a discussion of the differences between allocative and remitted wage measurements.

³⁰Such analysis is left for future work.

that idiosyncratic pay changes covary cyclically with job-to-job mobility while synchronized wages changes do not. This finding suggests that a renegotiable contract is needed to match important business cycle features of the labor market.

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A Data Appendix

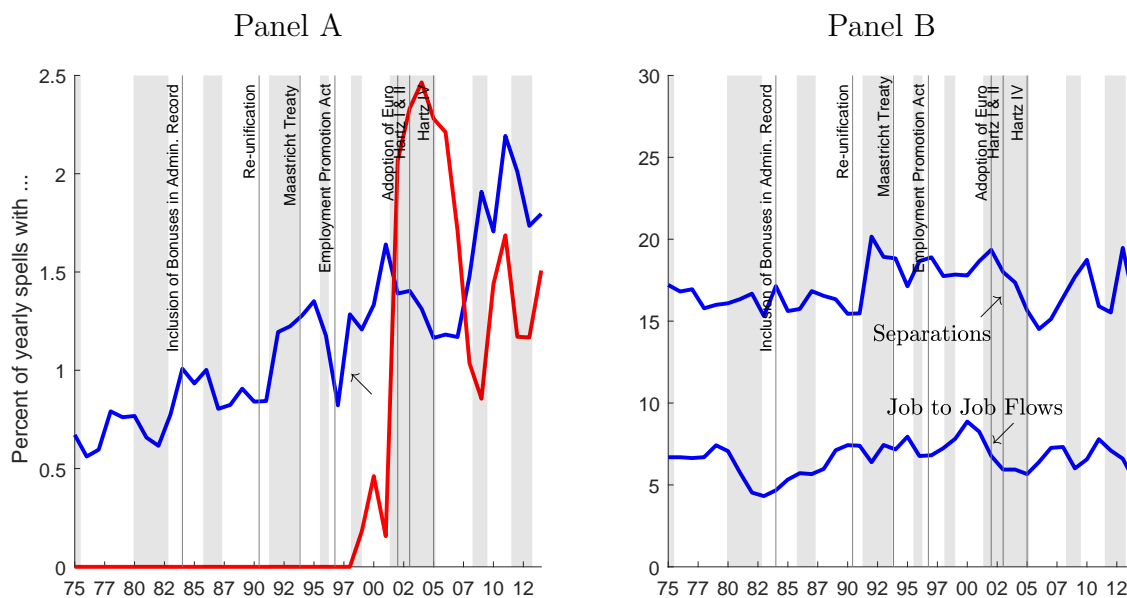
A.1 Health-care induced registrations

Figures 6 through 9 document the behavior of change of health care induced reregistrations in the lenses of section 3. From 1998 such changes are recorded as a reason for reregistration in the administrative record. Figure 6 documents a spate of health care changes during and after the Hartz reforms, after which such changes fall in frequency, affecting 1-1.5 percent of spells. Figure 7 documents that such changes follow a distribution between that of job stayers with no reregistration and that of idiosyncratic pay change recipients but resembles more the distribution of ideosyncratic pay change recipients. In contrast, figure 8 reveals a starkly different pattern than figure 3, with heath care changes growing in likelihood with all measures of increased seniority. Finally, 9 reveals more limited covariation with firm characteristics.

A.2 Christmas Bonus

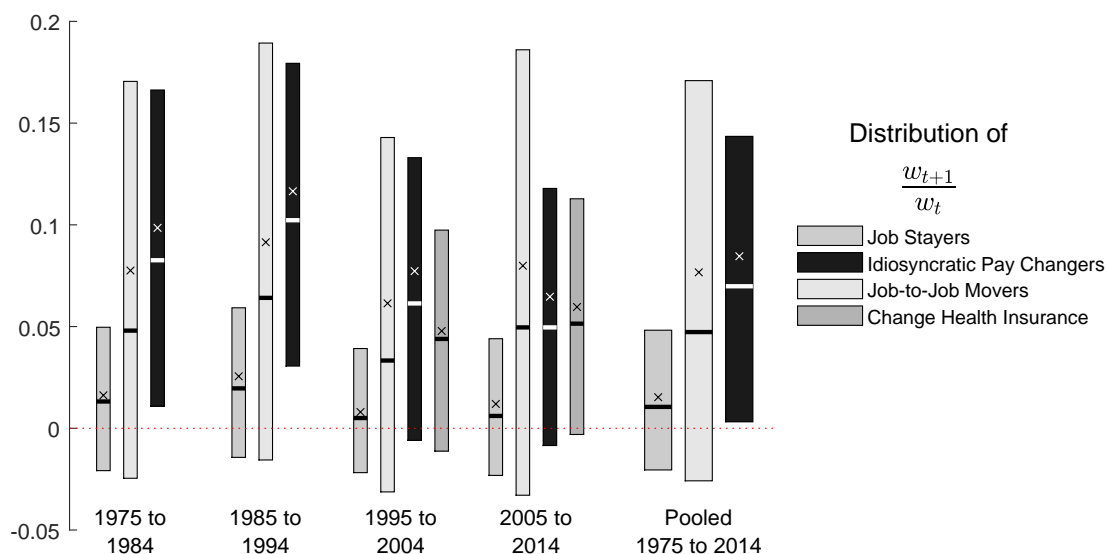
Typically bonus pay in Germany is distributed at year end as a “Christmas Bonus.” After 1984 such bonuses are included in the yearly pay reported in the IAB source data and are averaged into the average daily wage for the episode. It is possible, therefore, that the pattern recorded in Section 3.2 spuriously arises from comparing average daily wages from an episode that includes only the beginning of a year, up until the point when an idiosyncratic pay change or job ending occurred to average daily wages from a completed year (the year after).

Figure 6: Idiosyncratic Raises are Rare but Increasingly Common



Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

Figure 7: Idiosyncratic Pay Changes Are Large and Dispersed



Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

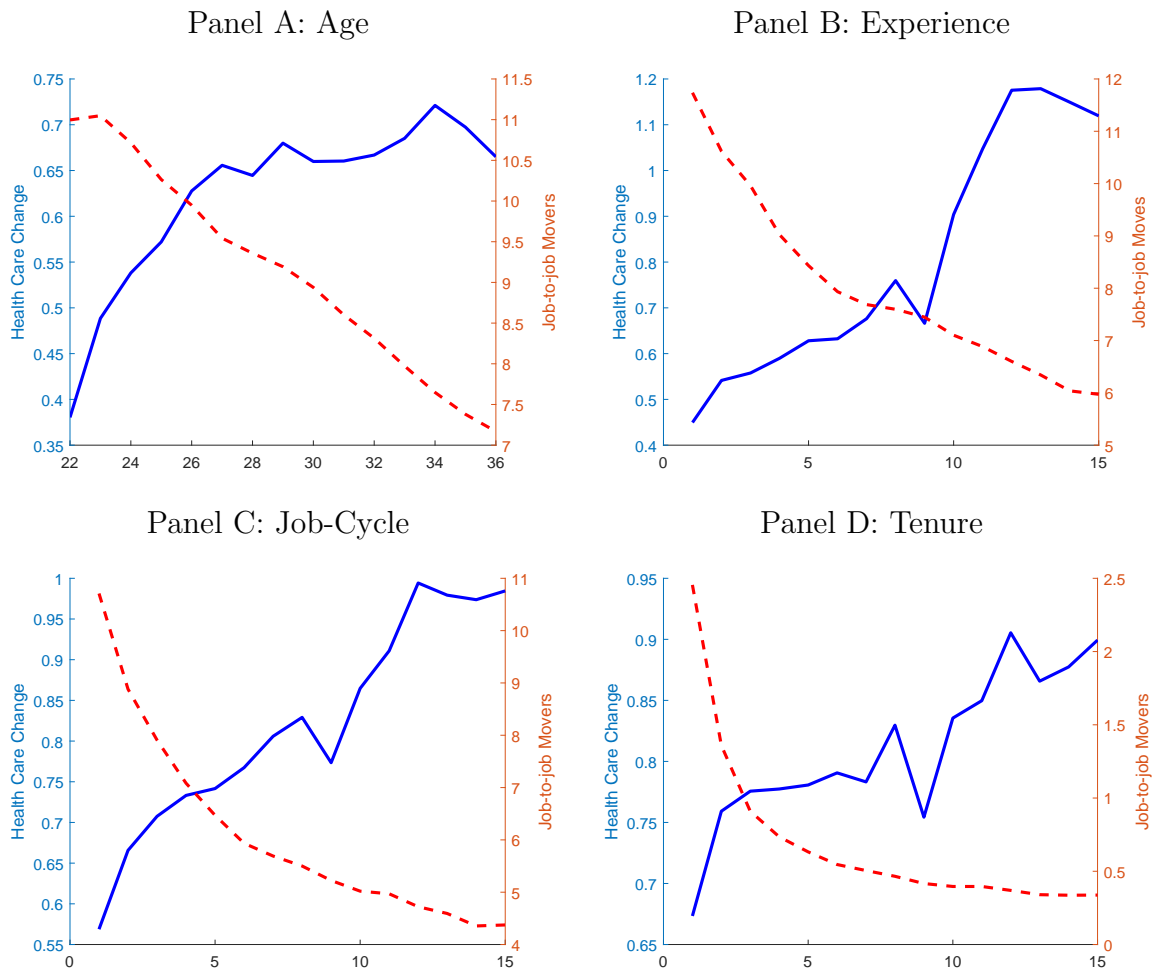
Here, I make an alternative calculation which compares the average daily wage from the year before the year of the pay or job change to the year after. Specifically, figure 10 plots the mean and interquartile range of

$$\frac{w_{t+1}}{w_{t-1}} - \frac{1}{2} \frac{w_{t+1}}{w_{t-1}} \Big|_{stay},$$

implicitly assuming that wage growth in the non-change year would have been identical to the wage growth for other non-changers. The upside to this calculation is that it excludes the year that is split by the change episode it is, on average, free from the “Christmas Bonus” bias. The downside, this skip-the-change-year analysis is selected on workers with longer tenure and—as I document in Section 3.3—incidence of both pay and job changes are negatively correlated tenure. Thus, the skip-the-change-year analysis is selected on workers with better existing match quality and remuneration conditional on that match quality and therefore, unsurprisingly, reveals smaller innovations in wages following changes.

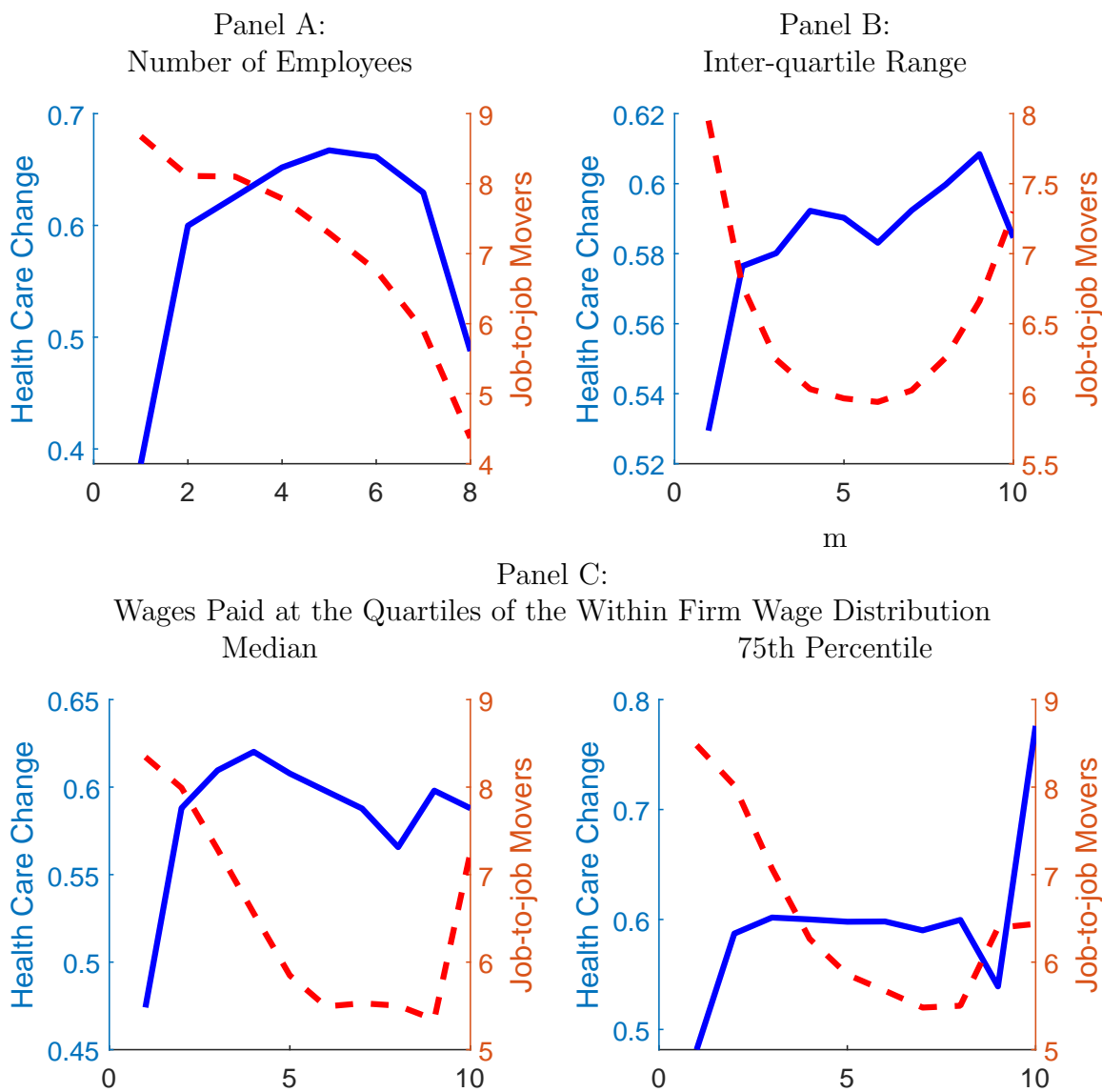
An additional comment on the “Christmas Bonus” criticism: bonuses are excluded from the data prior to 1984 yet the pattern I document is as evident, if not more so, in those years as in later decades.

Figure 8: Variation over the Life and Job Cycles



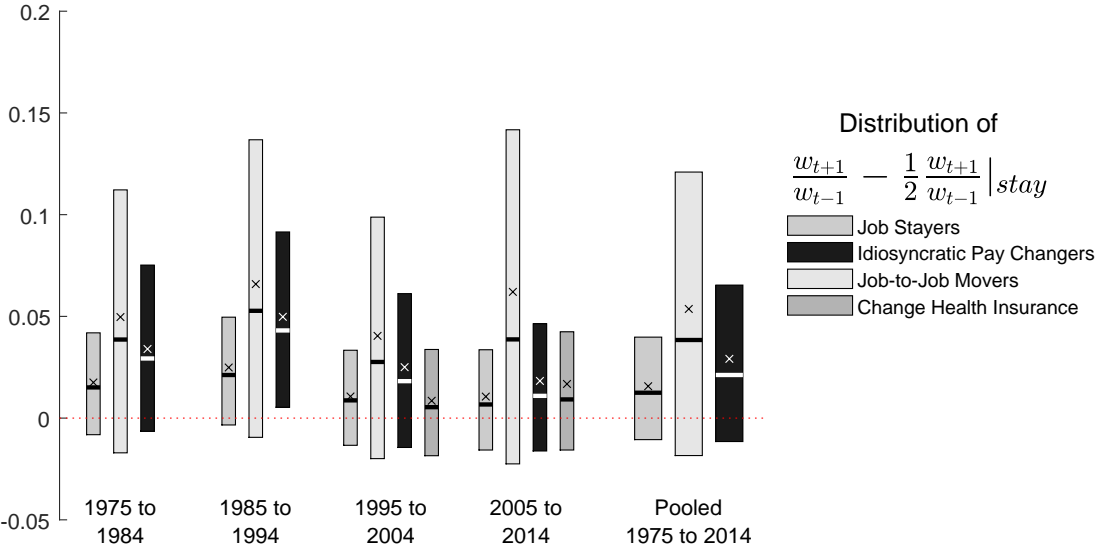
Source: *Sample of Integrated Labor Market Biographies (SIAB)*, author's calculations.

Figure 9: Variation with Firm Characteristics



Source: Sample of Integrated Labor Market Biographies (SIAB), author's calculations.

Figure 10: Idiosyncratic Pay Changes are Large And Dispersed (robustness).



Source: Sample of Integrated Labor Market Biographies (SIAB), author's calculations.

B Structural Model

B.1 *Equilibrium distributions.*

The following are standard in the literature deriving from the nested [Bontemps et al. \(2000\)](#) and [Postel-Vinay and Robin \(2002b\)](#) models. It is straightforward to show, using the method of mass balance, that the unemployment rate is

$$u = \frac{\delta}{\delta + \lambda_u}$$

and that the distribution of firm types across employed workers is

$$L(p) = \frac{\Gamma(p)}{1 + k_e \bar{\Gamma}(p)}$$

where $\bar{\Gamma}(p) = 1 - \Gamma(p)$ and $k_e = \lambda_e / \delta$ is the average number of jobs held during an employment spell. The density of $L(p)$ is:

$$\ell(p) = \frac{1 + k_e}{1 + \bar{\Gamma}(p)}$$

Finally, within a non-negotiating firm the distribution of wages is a point mass at the posted wage. Within a renegotiating firm the distribution of wages is

$$G(w|p) = \left(\frac{1 + k_e \bar{\Gamma}(p)}{1 + k_e \bar{\Gamma}(q(w, p))} \right)^2.$$

B.2 *Wage contracts.*

Noting that renegotiating employers offer the history-contingent reservation wage at all times deriving wage schedules under each contract type is also straightforward:

$$w^n(p) = p - [1 + \kappa_1 \bar{\Gamma}(p)]^2 \int_{\underline{w}}^p [1 + \kappa_1 \bar{\Gamma}(x)]^{-2} dx$$

$$w^r(q, p) = \begin{cases} w^n(q) - k_e \bar{\Gamma}(\tilde{p}) \left[\tilde{p} - w^n(q) - \int_q^{\tilde{p}} \frac{k_e [\Gamma(\tilde{p}) - \Gamma(x)]}{1 + k_e [\Gamma(\tilde{p}) - \Gamma(x)]} \frac{dw^n(x)}{dx} \right] - k_e \int_{\tilde{p}}^p \bar{\Gamma}(x) dx & \text{for } q \leq \tilde{p} \\ q - k_e \int_q^p \bar{\Gamma}(x) dx & \text{for } \tilde{p} < q. \end{cases}$$

where p is the incumbent employer's productivity, q is the productivity of the best-to-date outside offer, k_e is the ratio of the arrival rate of offers on-the-job to the separation rate, Γ is the distribution of employer types, \underline{w} is the reservation wage of a worker to take employment in a non-negotiable contract, and \tilde{p} is the least productive employer utilizing the re-negotiable contract. Comparative statics (3.1) through (3.4) follow. Existence of an invertible mapping $q(w, p)$ can be checked by noting that $w^r(q, p)$ is increasing in q for $q < \tilde{p}$ and $q > \tilde{p}$ and that $w^r(\tilde{p}^-, p) < w^r(\tilde{p}^+, p)$. Further details and value functions can be found in [Doniger \(2015\)](#).

B.3 Composition of labor contracts.

B.3.1 Existence

Current operating surplus from the proposed strategies exceed current operating surplus from each firm's best deviation.

Suppose non-negotiating is prescribed: If non-negotiating (n) is prescribed then it must be that $p < \check{p}$.

For such a p -productivity firm, current operating surplus from playing optimal wage under the prescribed wage contract, n , and the best deviation (bd) to renegotiating (r) can be written as

$$\begin{aligned}\pi^n(p) &= [p - w^n(p)]\ell(p) \\ \text{and} \\ \pi^{bd}(p) &= [p - w^r(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - w^r(q, p)]d\ell(q) - c(\check{p})\end{aligned}$$

where \check{p} is the productivity of the most productive firm that offers a posted wage less than p (for example, the most productive firm that the p -type firm can outbid by switching to SA).

Simplifying,

$$\begin{aligned}\pi^{bd}(p) &= [p - w^n(\check{p}) + \underbrace{w^r(\underline{p}, \check{p}) - w^r(\underline{p}, p)}_{<0, \text{ since } \frac{dw^r(\underline{p}, p)}{dp} < 0}]\ell(\underline{p}) \\ &\quad + \int_{\underline{p}}^{\check{p}} [p - w^n(\check{p}) + \underbrace{w^r(q, \check{p}) - w^r(q, p)}_{<0, \text{ since } \frac{dw^r(q, p)}{dp} < 0}]d\ell(q) \\ &\quad - \int_{\underline{p}}^{\check{p}} \underbrace{[w^n(\check{p}) - w^r(q, \check{p})]}_{\geq 0}d\ell(q) \\ &< [p - w^n(\check{p})]\ell(\check{p}) \leq \pi^n(p).\end{aligned}$$

The last line follows from noting $w^n(p)$ was the unique profit-maximizing non-negotiable wage choice for the p -type firm.

In other words, the non-negotiating firm could increase its labor supply by deviating to renegotiating. However, the firm could also increase its labor supply by the same amount by deviating to a larger non-negotiable wage. Willingness to pay for the right to renegotiate is then strictly less than the difference between the wage bill under the deviation to renegotiating and the deviation to a higher non-negotiable wage, which in turn is strictly less than the cost of renegotiating in the posited \check{p} equilibrium.

Suppose renegotiating is prescribed: If renegotiating (r) is prescribed then it must be that $\check{p} < p$. For such a p -productivity firm, current operating surplus from playing the

prescribed r wage schedule and deviating to the best n wage are

$$\begin{aligned}\pi^r(p) &= [p - w^r(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - w^r(q, p)]d\ell(q) \\ &\quad + \int_{\check{p}}^p [p - w^r(q, p)]d\ell(q) - c(\check{p})\end{aligned}$$

and

$$\pi^{bd}(p) = [p - \dot{w}]\ell(\dot{w}).$$

Note that $\dot{w} \geq w^n(\check{p})$ since $p \geq \check{p}$. Simplifying,

$$\begin{aligned}\pi^{BD}(p) &= [p - \dot{w}]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - \dot{w}]d\ell(q) + \int_{\check{p}}^{\dot{w}} [p - \dot{w}]d\ell(q) \\ &< \underbrace{[p - w^n(\check{p})]\ell(\underline{p})}_{\leq \dot{w}} + \int_{\underline{p}}^{\check{p}} \underbrace{[p - w^n(\check{p})]d\ell(q)}_{\leq \dot{w}} + \int_{\check{p}}^{\dot{w}} \underbrace{[p - w^r(q, p)]d\ell(q)}_{< \dot{w}} \\ &< [p - w^r(\underline{p}, p) - \underbrace{w^n(\check{p}) + w^r(\underline{p}, \check{p})}_{> 0, \text{ since } \frac{dw^r(\underline{p}, p)}{dp} < 0}]\ell(\underline{p}) \\ &\quad + \int_{\underline{p}}^{\check{p}} [p - w^r(q, p) - \underbrace{w^n(\check{p}) + w^r(q, \check{p})}_{> 0, \text{ since } \frac{dw^r(q, p)}{dp} < 0}]d\ell(q) \\ &\quad + \int_{\check{p}}^p [p - w^r(q, p)]d\ell(q) < \pi^r(p).\end{aligned}$$

The best deviation to n involves a reduction in the r firm's labor supply. I can find a bound on the minimum willingness to pay for the right to r by considering only the labor supply that would arise under the *smallest possible* best deviation the r firm might select: $w^n(\check{p})$. Willingness to pay for the right to r is then larger than the difference between the wage bill under the deviation to n and the wage bill for these employees under the prescribed r contract, which in turn is strictly greater than the cost of r .

Since no firm wishes to unilaterally deviate, the pair $\{c, \check{p}\}$ form an equilibrium.

B.3.2 Uniqueness

The mapping between c and \check{p} is one-to-one if $\frac{dc}{d\check{p}} > 0$. $\forall \check{p}$ c can be written:

$$\begin{aligned}c &= \pi^r(\check{p}) - \pi^p(\check{p}) \\ &= \left[[\check{p} - w^r(\underline{p}, \check{p})]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [\check{p} - w^r(q, \check{p})]d\ell(q) \right] - [\check{p} - w^n(\check{p})]\ell(\check{p}) \\ &= [k_e \bar{\Gamma}(\check{p})[\check{p} - w^n(\check{p})]]\ell(\check{p}) + \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{1 + k_e [\bar{\Gamma}(\check{p}) - \bar{\Gamma}(q)]} \frac{dw^n(q)}{dq}\end{aligned}$$

Since integrating by parts yields

$$\int_{\underline{p}}^{\check{p}} [\check{p} - w^r(q, \check{p})] d\ell(q) = [\check{p} - w^r(\check{p}, \check{p})]\ell(\check{p}) - [\check{p} - w^r(\underline{p}, \check{p})]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} \frac{dw^n(q)}{dq}$$

and noting that

$$w^n(\check{p}) - w^r(\check{p}, \check{p}) = k_e \bar{\Gamma}(\check{p}) [\check{p} - w^n(\check{p})]$$

Differentiating gives the result:

$$\begin{aligned} \frac{dc}{d\check{p}} &= \frac{d\ell(\check{p})}{d\check{p}} [k_e \bar{\Gamma}(\check{p}) [\check{p} - w^n(\check{p})]] + \ell(\check{p}) \left[k_e \bar{\Gamma}(\check{p}) \left[1 - \frac{dw^n(\check{p})}{d\check{p}} \right] \right] - \ell(\check{p}) [k_e d\Gamma(\check{p}) [\check{p} - w^n(\check{p})]] \\ &\quad + \ell(\check{p}) (1 + k_e \bar{\Gamma}(\check{p})) \frac{dw^n(\check{p})}{d\check{p}} - k_e d\Gamma(\check{p}) \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{(1 + k_e [\Gamma(\check{p}) - \Gamma(q)])^2} \frac{dw^n(q)}{dq} \\ &= k_e \bar{\Gamma}(\check{p}) \ell(\check{p}) + k_e d\Gamma(\check{p}) [\ell(\check{p}) [\check{p} - w^n(\check{p})] - \int_{\underline{p}}^{\check{p}} \frac{[q - w^n(q)] (1 + k_e \bar{\Gamma}(p))}{(1 + k_e [\Gamma(\check{p}) - \Gamma(q)])^2} d\ell(q)] \\ &= k_e \bar{\Gamma}(\check{p}) \ell(\check{p}) - k_e d\Gamma(\check{p}) [\ell(\check{p}) [\check{p} - w^n(\check{p})] - 2 \int_{\underline{p}}^{\check{p}} \frac{\ell(q)}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} dx] \\ &= k_e \bar{\Gamma}(\check{p}) \ell(\check{p}) + k_e d\Gamma(\check{p}) \left[\int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 - k_e [\Gamma(\check{p}) - \Gamma(q)]}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} dx \right] \end{aligned}$$

Noting that:

$$\begin{aligned} \frac{d\ell(\check{q})}{d\check{q}} &= \ell(q) \frac{2k_e d\Gamma(q)}{1 + k_e \bar{\Gamma}(q)}, \text{ and} \\ \frac{dw^n(q)}{dq} &= [q - w^n(q)] \frac{2k_e d\Gamma(q)}{1 + k_e \bar{\Gamma}(q)} \end{aligned}$$

So we have that $\frac{dc}{d\check{p}} > 0$ for sufficiently small k_e . How small depends on the distribution $\Gamma(p)$. As $\Gamma(p)$ approaches a point mass k_e must approach 1; however, for disperse $\Gamma(p)$, k_e can be large. Insufficiently small k_e will result in non-monotonicity as \check{p} approaches \bar{p} .

B.4 Increasing in the share renegotiable decreases labor share.

$$\begin{aligned}
total\ wages &= \int_{\underline{p}}^{\check{p}} w^n(p)\ell(p)d\Gamma(p) \\
&\quad + \int_{\check{p}}^{\bar{p}} \left[w^r(\underline{p}, p)\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} w^r(q, p)d\ell(q) + \int_{\check{p}}^P w^r(q, p)d\ell(q) \right] d\Gamma(p) \\
&= \int_{\underline{p}}^{\check{p}} w^n(p)\ell(p)d\Gamma(p) \\
&\quad + \int_{\check{p}}^{\bar{p}} \left[w^r(\check{p}, p)\ell(\check{p}) - \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} \frac{dw^n(q)}{dq} \right. \\
&\quad \quad \left. + p\ell(p) - w^r(\check{p}, p)\ell(\check{p}) - \int_{\check{p}}^P \ell(q)(1 + k_e \bar{\Gamma}(q))dx \right] d\Gamma(p)
\end{aligned}$$

Since integrating by parts yields:

$$\begin{aligned}
\int_{\underline{p}}^{\check{p}} w^r(q, p)d\ell(q) &= w^r(\check{p}, p)\ell(\check{p}) - w^r(\underline{p}, p)\ell(p) - \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} \frac{dw^n(q)}{dq}, \text{ and} \\
\int_{\check{p}}^P w^r(q, p)d\ell(q) &= p\ell(p) - w^r(\check{p}, p)\ell(\check{p}) - \int_{\check{p}}^P \ell(q)(1 + k_e \bar{\Gamma}(q))dx
\end{aligned}$$

Differentiating:

$$\begin{aligned}
\frac{d(total\ wages)}{d\check{p}} &= [w^n(\check{p}) - w^r(\check{p}, \check{p})]\ell(\check{p})d\Gamma(p) \\
&\quad + d\Gamma(\check{p}) \int_{\underline{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{1 + k_e [\Gamma(\check{p}) - \Gamma(q)]} \frac{dw^n(q)}{dq} \\
&\quad + \int_{\check{p}}^{\bar{p}} \left[\frac{d\ell(\check{p})}{d\check{p}} [w^r(\check{p}, p) - w^r(\check{p}, p)] + \ell(q) \left[\frac{dw^r(\check{p}, p)}{d\check{p}} - \frac{dw^r(\check{p}, p)}{d\check{p}} \right] \right. \\
&\quad \quad \left. - (1 + k_e \bar{\Gamma}(\check{p}))\ell(\check{p}) \left[\frac{dw^n(\check{p})}{d\check{p}} - 1 \right] \right. \\
&\quad \quad \left. + k_e d\Gamma(\check{p}) \int_{\check{p}}^{\check{p}} \ell(q) \frac{1 + k_e \bar{\Gamma}(q)}{(1 + k_e [\Gamma(\check{p}) - \Gamma(q)])^2} \frac{dw^n(q)}{dq} \right] d\Gamma(p) \\
&= (1 + k_e) d\Gamma(\check{p}) \int_{\check{p}}^{\check{p}} \frac{1}{(1 + k_e [\Gamma(\check{p}) - \Gamma(q)])^2} \frac{dw^n(q)}{dq} \\
&> 0
\end{aligned}$$

Noting that:

$$w^r(\check{p}, p) - w^r(\check{p}, p) = (w^n(\check{p}) - \check{p})[1 + k_e \bar{\Gamma}(\check{p})], \text{ and}$$
$$\frac{d\ell(\check{q})}{d\check{q}} = \ell(q) \frac{2k_e d\Gamma(q)}{1 + k_e \bar{\Gamma}(q)}$$