Balancing Before and After: Treasury Market Reform Proposals and the Connections Between Ex-Ante and Ex-Post Liquidity Tools

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Balancing Before and After:
Treasury Market Reform Proposals and the
Connections Between Ex-Ante and Ex-Post Liquidity Tools

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Introduction

Over recent years, issues involving liquidity provision and liquidity regulation have been at the forefront of both policy discussions and policy actions. Most recently, the onset of the pandemic in the spring of 2020 triggered a worldwide panic that sparked intense demands for liquidity across multiple markets and classes of institutions. A defining element of the initial stages of the “dash for cash” in the spring of 2020 was the severe deterioration of trading conditions in the Treasury market. That degradation in Treasury market functioning, in turn, spurred knockon effects in broader financial markets that amplified the economic effects of the pandemic and contributed to the steep downturn in the global economy. In the event, the Federal Reserve along with other central banks and fiscal authorities acted very quickly and forcefully to address these developments. Among many steps taken to address the panic, the Federal Reserve aggressively conducted open market operations, rapidly purchasing massive volumes of Treasury securities and agency MBS to provide liquidity. These actions were critical in addressing liquidity pressures, stemming the panic, and ultimately restoring smooth market functioning.

In the aftermath of the crisis, a series of important papers including Duffie (2020), Liang and Parkinson (2020), and reports by the Brookings Institution (2021), the G-30 (2021), the Interagency Working Group (2021), and the Securities Industry and Financial Markets Association (2021a, b, c) reviewed Treasury market developments in 2020 with an eye toward proposals for structural changes that could enhance the resiliency of the Treasury market to future shocks. In general, the proposals could be classified as including steps to strengthen the resilience of the “ex-ante” infrastructure in place supporting Treasury market functioning and also steps to bolster the “ex-post” tools available to address liquidity shocks when they arise. Examples of proposals in the first group include calls for improved liquidity regulation or fees to strengthen the incentives for financial firms to more fully internalize the social costs of liquidity risks. A key example of ex-post tools is the possibility of providing a lender of last resort for a broad class of participants in Treasury markets.

In some respects, the Treasury market might seem an unlikely candidate for such reform proposals. The market is very large, and trading is generally very active and dominated by sophisticated investors. And the underlying financial assets—claims on the U.S. government—are essentially risk-free and also largely free of the sorts of informational asymmetries and other idiosyncratic risks that often arise in private credit markets. In ordinary circumstances, the Treasury market is often viewed as a close real-world approximation to the platonic ideal of an “efficient market.” But as the Brookings Institution, G-30, and Interagency Working Group reports emphasize, efficient functioning of the Treasury market is central to the smooth functioning of financial markets more broadly. So any breakdowns in the Treasury market can have negative spillovers or externalities far beyond the boundary of the Treasury market per se. Both the Brookings Institution and G-30 reports point to the potential for such negative externalities as a rationale both for government intervention in the Treasury market during periods of distress and also for public sector involvement in establishing regulations and prudential measures that may help to strengthen the infrastructure of the Treasury market and lessen its vulnerabilities to shocks.

With this background in mind, the analysis below examines some of the Treasury market reform proposals that have been put forward within the context of a simple model that incorporates a role for external effects and a distinction between ex-ante and ex-post liquidity tools. The basic
framework is closely patterned after the discussion in Stein (2013), Tarullo (2014) and Brunnermeir and Cheridito (2019), and many of the results and conclusions echo themes in the expanding literature on liquidity regulation and lender of last resort, Rochet and Vives (2004), Ennis and Keister (2009), Carlson et. al. (2015), Diamond and Kashyap (2016). The model developed here is a simple variation on the venerable Poole (1968) model. Financial firms finance a combination of illiquid and liquid assets with stable and volatile liabilities. Volatile liabilities are an inexpensive form of funding but expose the firm to a potentially large ex-post funding shortfall. That risk leads the firm to hold relatively low yielding liquid assets that can be sold on short notice to meet an unusually large funding need. As in Stein (2013), Brunnermeir and Cheridito (2019) and the Brookings Institution and G-30 reports, a funding shortfall creates external costs that private firms would not take fully into account in their liquidity risk management decisions. A financial stability planner then would seek to take actions such as imposing liquidity regulations or upfront liquidity insurance fees or establishing a lender of last resort to help mitigate these external costs. The model has an element of the time consistency problem noted by Tucker (2014) in connection with the provision of the lender of last resort; ex-ante the financial stability planner might wish to establish a relatively high cost for access to a lender of last resort. However, once a liquidity deficiency has occurred, the financial stability planner would want to provide liquidity at only a modest penalty relative to the risk-free rate. Some combination of liquidity regulations and upfront liquidity insurance fees can help to address this tension. Similar to a proposal in the Brookings Institution report, the model points to a mandatory ex-ante fee for liquidity insurance as a potentially very useful tool in encouraging appropriate liquidity risk management on the part of financial firms. The model also points to some simple relationships that define optimal liquidity policies across different classes of financial institutions; the rules embed principles that are similar in spirit to those discussed in Metrick and Tarullo (2021).

The discussion below proceeds as follows. Section 1 presents the basic stylized model. Section 2 discusses the introduction of a hypothetical “financial stability planner” and the determination of socially optimal outcomes from the perspective of the planner. Section 3 discusses the actions the financial stability planner could take in the form of upfront fees and liquidity requirements to shape the incentives for financial firms to internalize the social costs of their asset-liability management decisions. In addition, the section discusses the introduction of a lender of last resort that can lower the potential ex-post costs of a liquidity deficiency. Section 4 calibrates the model in a plausible way and uses the calibrated model to explore some of the connections between the ex-ante tools—liquidity regulation and fees—and the ex-post tool—a lender of last resort. Section 5 discusses some potential policy implications of the model and section 6 concludes.

1. A Stylized Model
The model developed in this paper focuses on the connections between ex-post liquidity provision and ex-ante liquidity regulation. The framework considers the problem of a financial firm that can finance its operations with expensive term liabilities or relatively inexpensive volatile liabilities. And on the other side of the balance sheet, the firm can invest in relatively high yielding illiquid assets or in relatively low yielding liquid assets that may be sold quickly (or borrowed against) to meet a potential liquidity need. The firm chooses the shares of volatile and term liabilities together with illiquid and liquid assets in an initial period. Thereafter, the
firm’s volatile liabilities are subject to a shock that may result in a runoff of these liabilities. If that case, the firm draws down its liquid assets to meet the runoff of liabilities. If its liquid assets are not sufficient to address the runoff, the firm is subject to a “penalty” that reflects a cost of obtaining funding on short notice.

The firm’s decision about the extent to which it relies on volatile liabilities in its funding profile depends importantly on several key factors—the cost of volatile liabilities relative to the cost of term liabilities, the variance of the liquidity shocks, the spread between the rate on illiquid assets and liquid assets, and the penalty for a liquidity deficiency. As one would expect, the firm chooses to rely more heavily on volatile liabilities when there is a significant advantage in the interest cost of volatile liabilities relative to term liabilities. Conversely, all else equal, higher levels of the variance of liquidity shocks and the penalty for a liquidity deficiency reduce the desired share of volatile liabilities. Similarly, a wider spread between the rate on illiquid assets relative to that on liquid assets depresses the optimal level of volatile liabilities because the opportunity cost of holding liquid assets to guard against liquidity deficiencies is higher.

Many of these same factors drive the firm’s decisions regarding its desired liquidity coverage ratio (LCR), defined here as the ratio of liquid assets to volatile liabilities. The desired LCR represents the extent to which the firm wishes to insure against adverse outcomes in which a liquidity shock results in a penalty for a deficiency. Again, as one would expect, the desired LCR is driven by the spread between the returns on illiquid and liquid assets, the penalty for a deficiency, and the variance of shocks to volatile liabilities. A high penalty for a liquidity deficiency or a high variance of liquidity shocks induces the firm to “take out more insurance” against liquidity risk by boosting its LCR. Conversely, a wider spread between the return on illiquid assets and the return on liquid assets depresses the LCR; in this case, the firm is willing to run a higher risk of incurring a liquidity deficiency by reducing low yielding liquid assets and investing more heavily in higher yielding illiquid assets.

1.1 Basic Model Setup

The basic model is a variation on Poole (1968) in which a financial firm chooses optimal asset and liability compositions ex-ante taking into account the effect of those choices on the likelihood and severity of a “liquidity deficiency” after the realization of a “liquidity shock.” The firm maximizes the expected value of profits given by:

\[
E(\pi) = r_l l + \int_u^{\infty} r_b \tilde{b} g(u) - r_v \int_{-\infty}^{\infty} \tilde{v} g(u) - r_t t - \left( \frac{\theta}{2} \right) (v - \mu(1 - k))^2 + (r_b + \theta) \int_{-\infty}^{\infty} \tilde{b} g(u)
\]

where

\[l = \text{share of illiquid assets}\]
\[b = \text{share of liquid assets}\]
\[v = \text{ex-ante share of volatile liabilities}\]

Note that the “LCR” as defined here is the ratio of liquid assets relative to the ex-ante level of volatile liabilities, \(b/v\), and that ratio is less than 1. The liquidity coverage ratio in existing regulations is defined as the ratio of high quality liquid assets to “net cash outflows” and that ratio must be greater than 1. Net cash outflows, however, are defined as the portion of liabilities that would be expected to runoff in a stress scenario, \(\beta v\). So the “liquidity coverage ratio” as that term is used in regulation would correspond to \(b/(\beta v)\) in the model.
\[ t = \text{ex ante share of stable liabilities} \]
\[ \tilde{b} = b + \tilde{v} - v = \text{ex post liquid assets} \]
\[ \tilde{v} = ve^{\sigma_u - \frac{1}{2} \sigma^2} = \text{ex post volatile liabilities} \]

and

\[ n_l = \text{rate on illiquid assets} \]
\[ r_b = \text{rate on liquid assets} \]
\[ r_v = \text{rate on volatile liabilities} \]
\[ n_t = \text{rate on stable liabilities} \]
\[ r_b + \theta = \text{penalty for liquidity deficiency} \]

Here \( u \) is a shock drawn from the normal distribution with mean zero and unit variance. With this specification, ex-post volatile liabilities are always positive, and the expectation of ex-post volatile liabilities is equal to the ex-ante level of volatile liabilities.

\[ \tilde{v} = ve^{\sigma_u - \frac{1}{2} \sigma^2} > 0 \text{ and } E(\tilde{v}) = v \]

The threshold level of a shock that results in the firm exhausting its liquid assets ex-post is given by:

\[ u^* = (\log \left( 1 - \frac{b}{v} \right) + \frac{1}{2} \sigma^2) / \sigma \]

And the balance sheet constraints of the firm are given by

\[ l + b = 1 \]
\[ v + t + k = 1 \]
\[ \left( \frac{\delta}{2} \right) (v - \mu(1 - k))^2 = \text{term capturing g preferences for liability shares} \]

With this structure, the expected value of profits is given by:

\[ E(\pi) = n_l l + r_b b - r_d v - n_t t + \theta ((b - v)G(u^*) + vG(u^* - \sigma)) - \left( \frac{\delta}{2} \right) (v - \mu(1 - k))^2 \]

As discussed in more detail in the appendix, the first order conditions that describe the optimal choices of all the asset and liability shares are given by:

\[ n_l = r_b + \theta G(u^*) \quad (1) \]
\[ n_t = r_v + \theta (G(u^*) - G(u^* - \sigma)) + \delta (v - \mu(1 - k)) \quad (2) \]

These two expressions capture the basic tradeoffs at the margin in determining optimal asset and liability shares. Equation (1) indicates that the firm will balance the return from an extra dollar of illiquid assets with the return from an extra dollar of liquid balances. The latter includes both
the explicit return in holding liquid balances plus the implicit return in helping to avoid liquidity deficiencies. Similarly, equation (2) captures the basic tradeoff in shifting a dollar of volatile liabilities to a dollar of term liabilities. The left-hand side of the expression is the all-in marginal cost of increasing term liabilities and the right-hand side is the corresponding all-in marginal cost of increasing volatile liabilities.

Equations (1) and (2) can be combined with the balance sheet constraints to derive the reduced form expressions for \( v, b, t \) and \( l \) as:

\[
\begin{align*}
v & = \mu(1-k) + \frac{(r_l-r_v)}{\delta} + \left( \frac{G(u^*)}{G(u^* - \sigma)} \right) \\
b & = v \cdot \varphi = v \cdot \left( 1 - e^{\sigma u^* - \left( \frac{1}{2} \right) \sigma^2} \right) \\
t & = (1-k)(1-\mu) - \frac{(r_l-r_v)}{\delta} + \left( \frac{G(u^*)}{G(u^* - \sigma)} \right) \\
l & = 1 - b = 1 - v \varphi \\
u^* & = G^{-1}\left( \frac{(r_l-r_b)}{\theta} \right)
\end{align*}
\]

Together, equations (4) and (7) could be viewed as describing a “liquidity demand” curve with a scale factor equal to the level of volatile liabilities, \( v \), and a term, \( \varphi \), capturing the relevant price terms including the opportunity cost of holding liquid assets and the penalty for a liquidity deficiency. Equation (4) implies that the liquidity coverage ratio, \( \varphi = b/v \), is greater than zero when \( u^* < \left( \frac{1}{2} \right) \sigma \) and that implies \( \theta > (r_l - r_b)/G\left( \frac{1}{2} \sigma \right) \). All the analysis below assumes that this condition holds so that private firms have an incentive to hold positive levels of liquidity against their volatile liabilities. If the penalty for a deficiency did not meet this condition, the financial firm would have an incentive to hold infinitely negative liquid assets (e.g., borrow at the risk-free rate) in order to finance higher yielding illiquid assets.

1.2 Comparative Statics

This section considers the effects of changes in selected parameters on the key variables of interest. Given the balance sheet constraints, the comparative statics for illiquid assets, \( l \), and stable liabilities, \( t \), are equal in magnitude and with the opposite sign of those for liquid assets \( b \) and volatile liabilities \( v \). As a result, the discussion below focuses mostly on the effects of changes in parameters on the optimal choices for \( b \) and \( v \).

1.21 Optimal Liquidity Coverage

The basic relationship given by \( r_l = r_b + \theta G(u^*) \) in equation (1) captures the firm’s decision at the margin in choosing between holding a dollar more in liquid assets and a dollar less in illiquid assets. At the margin, the loss in revenue from a dollar reduction in illiquid assets must be matched by the incremental all-in return in holding a dollar more in liquid assets where the all-in return includes both the pecuniary interest earnings on liquid assets as well as the implicit return in reducing the expected cost of a liquidity deficiency. One implication of this relationship is that \( u^* \) is determined by the exogenous factors \( r_l, r_b, \) and \( \theta \) through the ratio \( (r_l - r_b)/\theta \). And since the ratio of liquid assets to volatile liabilities, \( b/v \), and hereafter referred to as the liquidity
coverage ratio, is a function of \( u^* \) in equation (4), these same exogenous parameters pin down this ratio as well. All else equal, an increase in \((r_l - r_b)/\theta\) is associated with a lower desired liquidity coverage ratio. For a solution to exist, it must be the case that \((r_l - r_b)/\theta < 1\); that is, the penalty associated with a liquidity deficiency is larger than the extra return that may be earned by investing in illiquid assets. Moreover, as noted above, economically sensible solutions to equation (1) involving positive ratios of liquid assets to volatile liabilities further require that \(\theta > (r_l - r_b)/G(\sigma^2)\). If this were not the case, the firm would choose to borrow at the rate \(r_b\), use the proceeds to invest in illiquid assets, and pay the expected penalty on liquidity deficiencies.

### 1.22 Deficiency Penalty

As shown in the appendix, an increase in the penalty for a liquidity deficiency pushes the volatile liability share down. The sign of the effect on the liquid balance share could be positive or negative depending on other parameters. On the one hand, the decline in the volatile liability share tends to depress the desired liquid asset share. On the other hand, the higher cost of a liquidity deficiency results in a higher desired liquidity coverage ratio. The sign of the net effect of these scale and substitution effects on the level of the liquid asset share depends on other parameters of the model.

### 1.23 Volatility

As noted in the appendix, for any case in which the initial desired liquidity coverage ratio is positive, an increase in the volatility parameter produces an increase in the desired liquidity coverage ratio. Intuitively, an increase in the volatility parameter results in a higher risk of a liquidity deficiency and firms respond to this higher risk by boosting their liquidity coverage. In addition, firms respond to the increased risk of volatile liability shocks by reducing the volatile liability share of funding. The net effect of an increase in volatility on the share of liquid balances then incorporates both scale and substitution effects that work in opposite directions. On the one hand, an increase in volatility tends to depress the share of volatile funding resulting in a corresponding decline in the need to maintain liquid assets. On the other hand, the higher risk of a liquidity deficiency tends to encourage firms to substitute out of illiquid assets and toward liquid assets so as to boost their liquidity coverage ratio. The net effect of these two effects depends on the parameterization of the model.

### 1.24 Liability Preference Parameters

An increase in the target share for volatile liabilities \(\mu(1 - k)\) pushes volatile liabilities higher. An increase in volatile liabilities is associated with an increase in liquid assets with no effect on the desired liquidity coverage ratio. Similarly, the cost of deviating from the target share for volatile liabilities, \(\delta\), also does not affect the liquidity coverage ratio. If other parameters are such that the optimal level of volatile liabilities is below the target share, then an increase in this cost parameter pushes up the level of volatile liabilities and the level of liquid assets and vice versa.
1.25 Capital Ratio
An increase in the capital ratio depresses both the volatile liability share and term liability share. The reduction in the volatile liability share feeds through to a small decline in the share of liquid assets and a corresponding small increase in the share of illiquid assets.

2. Liquidity from the Financial Stability Perspective
A basic problem confronting policymakers is that private financial firms operating to maximize profits may make choices that do not fully internalize the external social costs of a liquidity deficiency. Similar to the discussion in Stein (2013) and Brunnermeir and Cheridito (2019), this section considers the problem faced by a hypothetical financial stability planner that seeks to promote the efficiency of the financial sector while also minimizing the systemic risks associated with a liquidity deficiency. The financial stability planner’s objective function is assumed to take the form of a weighted sum of the profits of the representative financial firm and a separate term that represents the social cost of liquidity deficiencies. The social cost term is intended to capture the externality associated with the liquidity risks assumed by a representative individual firm.

The solution to the financial stability planner’s problem is analogous to that for the profit maximizing firm except that the financial stability planner places more weight on the potential costs of a liquidity deficiency. As a result, the social planner generally chooses a lower volatile liability share and a higher LCR than the optimal choices for a private firm. These effects are larger the larger is the weight attached to the external cost factor.

2.1 The Financial Stability Planner’s Problem
The objective function for the financial stability planner maximizes the profits of the representative firm but also takes account of the external effects associated with a liquidity deficiency.

\[ E(\pi_{FS}) = r_l l + \int_{\omega}^{\infty} r_b \bar{b} g(u) - r_v \int_{\omega}^{\infty} \bar{v} g(u) - r_t t - \left( \frac{\delta}{2} \right) (v - (1 - k)\mu)^2 + (r_b + \theta (1 + \omega)) \int_{\omega}^{\infty} \bar{b} g(u) \] (8)

In this set up, \( \omega \) determines the magnitude of the externality associated with the liquidity shortfall of the representative financial firm. The specification implies that the social costs of a liquidity deficiency are a function of the private costs through the term \( \theta (1 + \omega) \). So if the private costs of a deficiency are very high, the social costs are that much higher.

2.11 The First Best Outcome: Financial Stability Planner Chooses the Optimum
The solution for the financial stability planner is very similar to that for the private firm except that the solutions now include the factor \( (1 + \omega) \).

\[ r_l = r_b + \theta (1 + \omega) G(u^*) \] (9)
\[ r_t = r_v + \theta (1 + \omega) (G(u^* - G(u^* - \sigma)) + \delta (v^* - \mu (1 - k)) \] (10)

In effect, the financial stability planner would make choices that incorporate a higher marginal cost associated with a liquidity deficiency than would be the case for a private financial firm. As a result, the financial stability planner holds more liquid assets, lower volatile liabilities, more
term liabilities, and fewer illiquid assets. As above, these first order conditions can be combined with the balance sheet constraints to generate reduced form expressions for all the asset and liability shares.

\[
v^{**} = \mu (1-k) + \frac{(r_t - r_v)}{\delta} - \left(\frac{\theta(1+\omega)}{\delta}\right)(G(u^{**}) - G(u^{**} - \sigma))
\] (11)

\[
b^{**} = v^{**} \cdot \varphi = v^{**} \cdot \left(1 - e^{\sigma u^{**} - \left(\frac{1}{2}\right)\sigma^2}\right)
\] (12)

\[
t^{**} = (1-k)(1-\mu) - \frac{(r_t - r_v)}{\delta} + \left(\frac{\theta(1+\omega)}{\delta}\right)(G(u^{**}) - G(u^{**} - \sigma))
\] (13)

\[
l^{**} = 1 - b^{**} = 1 - \varphi v^{**}
\] (14)

\[
u^{**} = G^{-1}\left(\frac{r_t - r_b}{\theta(1+\omega)}\right)
\] (15)

The first best outcome sets the liquidity coverage ratio according to \((r_t - r_b) = \theta(1 + \omega)G(u^{**})\). The comparative statics of an increase in the systemic risk factor are analogous to those for an increase in the private cost of a liquidity deficiency. An increase in \(\omega\) tends to depress the socially optimal volatile liability share and boost the share of term liabilities. The liquidity coverage ratio increases but the net effect on the level of the liquid asset share is ambiguous.

### 2.12 Heterogeneity

The discussion above focuses on a situation in which there is a “representative financial firm.” A more realistic assumption is that the financial system is populated by a broad range of financial institutions varying by type and size and other characteristics. In this case, it might be reasonable to assume that a liquidity deficiency for each type of financial firm may imply different degrees of social costs. So, for example, a liquidity deficiency for a large global bank active across many markets could have very high social costs while those for smaller or more specialized institutions could have smaller social costs. In this case, the financial stability planner could be viewed as maximizing an objective function that includes the profits and social costs for each group indexed by \(j\) given by:

\[
E(\pi_{FS}) = \sum_j \left\{ r_t l_j + \int_{u_j}^{\infty} r_b \delta_j g(u) - r_t \int_{\infty}^{\infty} \delta_j g(u) - r_t l_j - \left(\frac{\delta}{2}\right) (v_j - (1-k)\mu_j)^2 + (r_b + \theta(1+\omega)) \int_{\infty}^{\infty} \delta_j g(u) \right\}
\]

The first order conditions for the optimal choices of the financial stability planner with this specification for each class of firms are analogous to those above. Analogous to equation (9) above, we have:

\[
\frac{r_t - r_b}{\theta(1+\omega_j)} = G(u_j^{**})
\] (16)

or

\[
(1 + \omega_j)G(u_j^{**}) = (1 + \omega_k)G(u_k^{**})
\] (17)

Not surprisingly, if the social cost of a liquidity deficiency for firms in group \(j\), \(\omega_j\), is much larger than the social cost of a liquidity deficiency for group \(k\), \(\omega_k\), the social planner would
choose a liquidity coverage ratio for group $j$ that results in a much smaller probability of a liquidity deficiency for group $j$ than the probability of a liquidity deficiency for firms in group $k$. One could think of that relationship as similar in spirit to the “congruence” principle discussed in Metrick and Tarullo (2021). That is, the financial stability planner should seek to equalize the probability of a liquidity deficiency across sectors weighted by the social cost of a liquidity deficiency in each sector.

3. What’s a Financial Stability Planner to Do?
While a financial stability planner might wish to be able to choose optimal asset and liability shares for all financial firms, a more realistic case is that the planner must try to establish incentives or regulations or take other steps that indirectly move private sector outcomes toward the socially optimal outcome noted above. As discussed in the Brookings Institution and G-30 reports, there are a number of approaches the planner could consider along these lines including establishing liquidity regulations that directly limit the extent of liquidity risk that can be assumed by financial firms, a mandatory upfront fee designed to provide incentives for financial firms to internalize the social costs of their asset and liability management decisions, and providing a lender of last resort to mitigate social costs in the event of a systemic liquidity shock. The discussion below considers each of these options in turn within the context of the model developed here.

3.1 Liquidity Regulation
One approach the financial stability planner might take in influencing private sector outcomes is to specify minimums for the relationships among different components of the balance sheet. For example, a required liquidity coverage ratio might be viewed as specifying a minimum for the ratio of liquid assets to volatile liabilities. In the model above, an LCR requirement might be specified as $b \geq \beta_{LCR} v$ with the parameter $\beta_{LCR}$ representing the fraction of volatile liabilities that would be expected to runoff in a stress scenario. The case in which the required LCR ratio is binding then adds one constraint to the basic optimization problem discussed above. The first order conditions are given by:

\[ r_l + \tau_A = 0 \]
\[ r_b + \theta G(u^*) + \tau_{LCR} + \tau_A = 0 \]
\[ -r_l + \tau_L = 0 \]
\[ -r_v - \theta \left( G(u^*) - G(u^* - \sigma) \right) - \delta(v - \mu(1 - k)) + \tau_L - \tau_{LCR} \beta_{LCR} = 0 \]

Where

\[ \tau_{LCR} = \text{Lagrange multiplier associated with LCR constraint} \]

These equations again give rise to two behavioral relationships that define the optimal choices given by:

\[ r_l = r_b + \theta G(u^*) + \tau_{LCR} \]

(18)
\[ r_t = r_v + \theta(G(u^*) - G(u^* - \sigma)) + \delta(v - \mu(1 - k)) + \tau_{LCR}\beta_{LCR} \]  
\[ u^* = \frac{\log(1 - b/v) + \frac{1}{2}\sigma^2}{\sigma} = \frac{\log(1 - \beta_{LCR}) + \frac{1}{2}\sigma^2}{\sigma} \]  
Equations (18) and (19) are nearly identical to the basic optimality conditions (1) and (2) except that when the required LCR constraint is binding, the associated Lagrange multiplier, \( \tau_{LCR} \), is positive. In this case, with the liquidity coverage ratio pinned at the required ratio, \( \beta_{LCR} \), equation (18) determines the value of this multiplier as a function of the exogenous variables. And with that in hand, equation (19) then characterizes the optimal choices for the volatile funding share. The solutions in this case are given by the equations below.

\[ v = \mu(1 - k) - \frac{\theta(G(u^*) - G(u^* - \sigma))}{\delta} - \beta_{LCR}(r_t - r_b - \theta G(u^*))/\delta + (r_t - r_v)/\delta \]  
\[ b = \beta_{LCR}v \]  
\[ l = 1 - \beta_{LCR}v \]  
\[ t = 1 - k - v \]

By construction, a binding LCR requirement drives up the ratio of liquid assets to volatile liabilities and drives down volatile liabilities as a share of total assets. The net effect on the level of liquid balances is ambiguous.

### 3.2 Mandatory Insurance Fee

As discussed in the Brookings Institution and G-30 reports, the planner could impose a mandatory ex-ante fee schedule that reflects the expected social costs of a liquidity deficiency as an alternative to liquidity regulation. In some respects, this fee schedule would be similar to the risk-based schedule of deposit insurance fees employed by the FDIC.\(^3\)

Under this fee-based approach, the financial stability planner would impose a fee schedule that mimics the social cost of a liquidity deficiency. The upfront fee schedule would specify the fee as the expected social cost associated with a liquidity deficiency as a function of volatile deposits and liquid assets:

\[ fee = \theta\omega((v - b)G(u^*) - vG(u^* - \sigma)) \]  

In this case, the financial stability planner would post a fee schedule that would map liquid asset and volatile liability shares to a fee as described by equation (25). Private firms would then choose desired liquid asset and volatile liability shares recognizing the implications for their “liquidity insurance assessment.” Note that equation (25) can be written as:

\[ fee = v \cdot (s\omega) \cdot (1 - \frac{G(u^* - \sigma)}{G(u^*)} - \frac{b}{v}) \]

\(^3\) FDIC assessments also incorporate a scale variable—total assets—and a per-unit charge based on the risk profile of individual banks. See the excellent review of the evolution of the FDIC’s deposit insurance assessments in FDIC (2020).
Not surprisingly, the total fee thus depends on a scale factor captured by the level of the volatile liability share, \( v \), and then a “per-unit” fee that is a function of the spread between the rates on liquid and illiquid assets, \( s \), the social cost parameter, \( \omega \), and ex-ante liquidity coverage ratio, \( b/v \). The term \( 1 - G(u^* - \sigma)/G(u^*) \) represents the expected value of the share of volatile liabilities that runoff conditional on a liquidity deficiency. The liquidity coverage ratio \( b/v \) represents the maximum share of volatile deposits that can runoff without incurring a deficiency. Figure 1 displays what the per-unit fee schedule looks like as a function of the liquidity ratio \( b/v \). As the firm’s liquidity ratio increases, the per-unit fee falls, and the total fee assessed also declines to zero. Changes in the social cost parameter, \( \omega \), shift the per unit fee schedule up or down, and especially so at low levels of the liquidity ratio.

In this model, there is a close connection between a mandatory fee schedule and a required liquidity ratio. In both cases, the private firm is induced to hold more liquid assets relative to its volatile liabilities than would otherwise be the case. In the case of the mandatory fee, the costs are both direct in the form of the fee and also indirect in the form of sacrificed income resulting from holding more low yielding liquid assets and relying on more high-cost term liabilities than would otherwise be the case. In the case of a liquidity coverage ratio requirement, the private firm is again induced to hold more liquid assets than would otherwise be the case and there is a corresponding reduction in the firm’s profits.

### 3.3 Lender of Last Resort

Another step the financial stability planner might consider would be to establish a lender of last resort facility that would, in effect, lower the ex-post cost of a liquidity deficiency. The Brookings Institution and G-30 reports, for example, recommended the creation of a lender of last resort for dealers and other participants in the Treasury market in the form of a “standing repo facility.”

In the context of the model developed above, a standing repo facility could be viewed as a mechanism by which the financial planner can reduce the ex-post private cost, \( \theta \), associated with a liquidity deficiency. All else equal, a reduction in the private cost \( \theta \) to a lower value \( \theta' \) also reduces the external costs of a liquidity deficiency through the term \( \theta \omega \). One might think of this as crudely capturing the intuition that a lender of last resort can help reduce run risks and other types of contagion that contribute to the social costs of a liquidity deficiency. In the model, absent any other policy steps aimed at mitigating liquidity risks, the reduced private cost of a liquidity deficiency \( \theta' \) would still need to satisfy the basic condition noted above \( \theta' > (\gamma - \gamma_b)/G(^\frac{1}{2}\sigma) \). One could think of this as a form of Bagehot’s celebrated principle that a lender of last resort should stand ready to lend freely in the event a liquidity shortfall but at a penalty rate. Here the rate on borrowing through the lender of last resort would be equal to the risk-free rate plus \( \theta' \), and that rate would be above the return on the illiquid asset. At the same time, the

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4 The Federal Open Market Committee (FOMC) moved a step in that direction in July of 2021 with the creation of a standing repo facility for primary dealers and depository institutions. See the announcement at Federal Reserve Board - Statement Regarding Repurchase Agreement Arrangements.
penalty rate on borrowing from the lender of last resort is below the cost the institution would otherwise incur in the absence of a lender of last resort.

As noted, the reduction in the private costs of a liquidity deficiency has a direct effect in reducing the external social costs of a deficiency. However, in the model, the introduction of a lender of last resort has important indirect effects as well that underscore the tension between policies that aim to mitigate the costs of ex-post outcomes and the ex-ante behavior of private financial institutions. As noted above, a reduction in $\theta$ tends to encourage private firms to rely more heavily on volatile funding and to reduce the extent of their liquidity coverage.

The implications of this adverse incentive effect depend on the parameterization of the model. As shown in the appendix, the value of the objective function for private firms unambiguously declines with an increase in the penalty for a deficiency. Conversely, all else equal, an increase in the penalty for a deficiency reduces the ex-ante social cost associated with a deficiency. In the model, this effect stems from the strong response of private firms to an increase in the penalty for a liquidity deficiency. Thus, even though the direct effect of an increase in the penalty for a deficiency on external costs is positive, the induced increased in the liquidity coverage ratio of private firms is enough to reduce the overall external costs of a deficiency. The net effect on the financial stability planner’s objective function depends on the relative strength of the effect on the profits of financial firms and the effect on the external costs of a liquidity deficiency.

4. Model Calibration and Policy Options

To more clearly draw out some of the connections among the different policy options available to the financial stability planner, it’s helpful to work with a version of the basic model that is parameterized in a plausible way. This section describes an illustrative calibration of the model parameters and discusses some implications of the calibrated model for the policy options discussed above—mandatory upfront fees, liquidity regulations, and provision of a lender of last resort.

4.1 Calibration

To calibrate the model, we search for parameter values that (i) result in a liquid asset share that matches that for the banking industry in 2006Q4 in the absence of liquidity regulations; (ii) results in a liquid asset share that matches that for the banking industry in 2019Q4 after the implementation of the LCR regulations and (iii) would lead a financial stability planner to

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5 The discussion here focuses on versions of the model with an interior solution for the private firm and financial stability planner. This assumption rules out the possibility that the financial stability planner can simply set the private cost of a liquidity deficiency to zero. In that very favorable case, all private and external costs of a liquidity deficiency would be eliminated, and the firm and financial stability planner would both seek only to maximize profits. The firm would hold no liquid assets and the level volatile liabilities would be determined only by the preference parameter and the spread between rates on term and volatile liabilities. Costless liquidity deficiencies would occur 50 percent of the time.
impose an LCR requirement similar in stringency to that in the LCR regulations.\(^6\) Regarding the first two criteria, the share of liquid assets—Treasury securities plus reserve balances—stood at 0.58 percent and 10.4 percent in 2006Q4 and 2019Q4, respectively.\(^7\)

The key parameter settings shown in Table 1 below meet all these criteria.

<table>
<thead>
<tr>
<th>(r_l - r_h)</th>
<th>(\sigma)</th>
<th>(\mu)</th>
<th>(\theta)</th>
<th>(\omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0020</td>
<td>0.144</td>
<td>0.32</td>
<td>0.0042</td>
<td>172</td>
</tr>
</tbody>
</table>

As shown in equations (4) and (7), the solution for the optimal asset shares and the desired liquidity coverage ratio in the model depend on the spread between the rates on illiquid assets and liquid assets, column 1. For illustrative purposes, this spread is set at 20 basis points.\(^8\)

The most important parameters in the model are the private and social costs of a liquidity deficiency, \(\theta\) and \(\omega\), respectively, the volatility parameter, \(\sigma\), and the target volatile liability share of total liabilities, \(\mu\). In the model, \(\theta\), \(\sigma\), and \(\mu\), are key factors driving holdings of liquid assets and reliance on volatile liabilities in the absence of liquidity regulations.

For the calibration, we assume that the liquidity stress scenario implicit in the LCR regulations is associated with a liquidity shock that would be expected to occur only once in every 30-year period. For this exercise, each “period” is assumed to be 30 days long, consistent with the window for calculating outflows under the LCR regulations. With a single period spanning 30 days, we assume a “year” in the model is associated with 12 such periods, so that the probability of a shock that would be expected to occur once every 30 years is equal to \(\frac{1}{12 \times 30} = 0.0028\). The theoretical probability of a deficiency, \(G(u^*)\) along with equation (20) above then defines the linkage between this assumed probability of the stress scenario, the volatility parameter, and the LCR constraint in the model, \(\beta_{LCR}\). We can then search over the parameters \(\theta\), \(\sigma\), and \(\mu\) to find a solution for which the unconstrained value for the liquid asset share in equation (4) matches the 2006Q4 liquid asset share, and the LCR constrained value of the liquid asset given in equation

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\(^6\) For details regarding the LCR regulations, see the press release discussing the final rule adopted by U.S. regulatory agencies and associated documents at [Federal Reserve Board - Federal banking regulators finalize liquidity coverage ratio](https://www.federalreserve.gov/newsevents/press/monetary/20130630a.htm).

\(^7\) Data on Treasury holdings and total assets for commercial banks are taken from the balance sheet summaries in the FDIC’s Quarterly Bank Performance reports at [FDIC: Quarterly Banking Profile](https://www2.fdic.gov/qbp/). Reserve balance data for dates corresponding the FDIC data are taken from the Federal Reserve’s H.4.1 release.

\(^8\) The calibration here is purely illustrative. Indeed, developing a strong empirical basis for the settings of all the model parameters is challenging and some settings here involve considerable judgment. Regarding the interest rate spread in column 1, the illiquid and liquid assets in the model are both risk-free, with the illiquid asset defined as perfectly illiquid and the liquid asset defined as perfectly liquid. A benchmark one could appeal to in rationalizing this choice would be the spread between the yield on a short-dated Treasury security and the yield on a short-dated REFCO security. Both are full faith and credit obligations of the U.S. government, but Treasuries are far more liquid than REFCO securities. A typical REFCO-Treasury spread at short maturities is often on the order of 20 basis points or more. The spread between the rates on liabilities affects the optimal choice of the volatile liability share but otherwise has a relatively limited role in the model. For simplicity, we also set this spread equal to 20 basis points. The penalty, \(\delta\), for deviations of the volatile liability share from the target share is set at 1. The capital ratio is set at 5 percent, consistent the Tier1 leverage ratio requirement in place for large banks.
(22) matches the 2019Q4 liquid asset share. The values of those parameter settings are shown in columns 2-4 of Table 1.

With the parameter settings as described in Table 1, the assumed probability of a liquidity deficiency under the LCR regulations maps to an LCR requirement of $\bar{\beta}_{LCR} = 0.35$ by equation (22). This value can be interpreted as the stress runoff rate for volatile liabilities consistent with the LCR requirement in the model. The actual runoff rates for more volatile funding categories in the LCR regulations are similar to this value. For example, the LCR runoff rate for uninsured nonoperational deposits is 40 percent.

Finally, to obtain a value for the social cost parameter, $\omega$, we benchmark against the policymaker preferences that are implicit in the LCR regulations. For any given value of the social cost parameter, $\omega$, there is an associated value of the LCR requirement $\beta_{LCR}(\omega)$ that maximizes the financial stability planner’s objective function given the asset and liability management choices of financial firms. 9 We then solve for the value of $\omega$ so that $\beta_{LCR}(\omega) = \bar{\beta}_{LCR} = 0.35$. This is the value of $\omega$ that would lead the financial stability planner to impose an LCR requirement comparable in stringency to that embedded in the LCR regulations. Based on this methodology, the implied value for the social cost parameter $\omega$ is set at 172, column 5 of the table. To put that more sharply, this calibration suggests that while a financial institution might perceive the private cost of a liquidity deficiency at the margin to be only about 42 basis points, the marginal social cost of a liquidity deficiency with this calibration is more than 70 percent—an enormous difference.

4.2 Policy Options

With the parameter settings described above, the value of the social objective function based on private sector decisions is an increasing function of the penalty for a liquidity deficiency. As noted above, the external cost of a liquidity deficiency in this model declines with an increase in the penalty for a liquidity deficiency. And with a very high weight attached to the external cost through the parameter $\omega$, the decline in the external cost associated with an increase in the cost of a liquidity deficiency outweighs the loss the financial stability planner assigns to the decline in the profitability of financial firms. As a result, the financial stability planner faces the time consistency problem noted by Ennis and Keister (2009) and Tucker (2014) and discussed above. Before the realization of the liquidity shock, the financial stability planner would find it optimal to substantially increase the penalty for a liquidity deficiency faced by private firms relative to the perceived cost of a deficiency of 42 basis points. In the model, imposing a credible and very punitive ex-post penalty for a deficiency would lead private firms to hold more liquid assets, greatly reducing the probability of a liquidity deficiency and reducing the ex-ante expected external costs associated with a liquidity deficiency. However, the high ex-post penalty option would not be optimal for the financial stability planner ex-post. In the event of a liquidity deficiency, the financial stability planner would strongly prefer to reduce the penalty for a liquidity deficiency to the lowest value possible. Ex-post, that would both lower the direct private costs of the deficiency for the affected private firm and also the external social costs

9 It turns out that the relationship is given implicitly by equation (9) above with the threshold value $u^{**}$ defined by the LCR requirement.
associated with a liquidity deficiency. Recognizing the incentives for the financial stability planner to forgo this high penalty after the fact, private financial firms might not take actions ex-ante to lower the risk of a liquidity deficiency as intended.

To address this type of time consistency problem, the financial stability planner could impose an upfront fee schedule sufficient to induce private sector firms to manage liquidity risk in a socially responsible way. Based on the parameterization above, that schedule would look as shown in Figure 1 with a quite high per-unit fee on the order of 4 percent of volatile deposits for firms with a low liquid asset to volatile liability ratio. However, the per-unit fee asymptotes to zero as the firm’s ex-ante ratio of liquid assets to volatile liabilities increases. Faced with such a fee schedule, private firms would be induced to hold liquid assets sufficient to address a wide range of liquidity shocks and, as a result, the fee they would incur would be very low. If the planner can impose this type of upfront fee schedule, there would also be a benefit in establishing a lender of last resort to lower the ex-post cost of a liquidity deficiency. In effect, the planner can have the best of both worlds—an upfront fee that encourages socially appropriate ex-ante decision-making on the part of financial firms and a lender of last resort that imposes only a modest penalty for a liquidity deficiency ex-post. Moreover, the introduction of a lender of last resort that reduces the private cost for a liquidity deficiency would also lower the external social cost of a liquidity deficiency. As a result, the optimal fee schedule would shift lower with the introduction of a lender of last resort. For example, in the calibrated model, if the private penalty for a liquidity deficiency is cut from 42 basis points to 10 basis points by the introduction of a lender of last resort, the associated per-unit fee schedule would shift substantially lower as shown in Figure 2.

Imposing an ex-ante liquidity requirement is another way the financial stability planner can overcome the time consistency problem. In the model, imposing an LCR requirement with $\bar{\beta}_{LCR} = 0.35$ would move the private sector outcome very close to the fully socially optimal outcome conditional on the assumed private cost for a liquidity deficiency of 42 basis points. With the liquidity requirement in place, the financial stability planner could introduce a lender of last resort that lowers the cost of an ex-post liquidity deficiency without greatly distorting the ex-ante asset and liability management decisions of financial firms. Lowering the ex-post private cost of a liquidity deficiency in this case could also allow the financial stability planner to relax the optimal liquidity requirement. As in the example of the fee schedule above, with the private penalty for a liquidity deficiency cut from 42 basis points to 10 basis points following the introduction of a lender of last resort, the socially optimal liquidity coverage ratio in the model would drop from 0.35 to about 0.29. In the model, that reduction implies that the “self-insurance” required of financial firms in the form of liquid assets would drop from a level sufficient to cover volatile liability runoffs that would be expected occur only every 30 years to a lower level that would only cover runoffs that would be expected to occur once every 6 years.

5. Possible Policy Implications

The analysis above points to some possible policy implications but first some fair labelling. There is no shortage of shortcomings in the analysis above, and alternative modeling frameworks
Figure 1: Optimal Per Unit Fee Schedule

Figure 2: Effect of Change in Penalty for Deficiency on Optimal Per Unit Fee Schedules
could easily deliver qualitatively different results. Many aspects of the model, importantly
including all the relevant interest rates and the volatility parameter for volatile liabilities, are
simply treated as exogenous variables rather than arising from investor behavior and the
interaction of demand and supply. That assumption is quite important and potentially quite
limiting in that one would expect investor behavior and the configuration of rates to change in
response to various policy choices. Finally, liquidity risk management at large financial firms is
vastly more complicated than depicted in the simple model. Financial firms may face various
types of “off balance sheet” liquidity risks stemming from unexpected customer demands for
credit, draws on committed lines of credit, margin calls, and many other factors. These types of
liquidity risks may also have external effects and are not addressed in the model. All that said,
the framework has at least some compensating virtues in delivering simple results that are well
aligned with the intuition underlying many aspects of the current approach to liquidity risk
regulation and supervision in the United States. Moreover, many of the core themes highlighted
in the framework here are similar to those in more sophisticated setups. With that as preamble,
we venture a few observations below that may be relevant for policy.

At the broadest level, the model suggests that a financial stability planner could effectively
reduce the social cost of liquidity deficiencies through a comprehensive balanced liquidity risk
management (BLRM) program that would include mandatory fees or liquidity regulations
coupled with a lender of last resort with the latter structured so that usage entails only a modest
penalty. All firms whose activities pose significant potential external costs would fall under the
BLRM. All of the liquidity risks associated with the activities of these firms would be precisely
quantified, and the liquid assets held by the encompassed set of financial firms would be
perfectly safe and perfectly liquid in all states of the world. The ex-ante tools—mandatory
upfront fees or liquidity requirements—would be tailored to the systemic risk profile of the
covered firms. And the BLRM would be best implemented as a package. Indeed, in the model,
implementing only a lender of last resort without the accompanying ex-ante risk mitigants would
be counterproductive.

When it comes to establishing a structure comparable to the notional BLRM described above, far
easier said than done! An enormous gulf of complexity lies between this simple vision of a
balanced program and many practical realities.

Some key issues are connected with the most basic aspect the model—the “liquid asset.” The
critical characteristic of the liquid assets in the model is that they can in fact be liquidated
instantly and costlessly in all states of the world to address a runoff of volatile liabilities. In the
case of Treasury securities held as liquid assets, the degradation of Treasury market liquidity
observed in March 2020 and in prior episodes of severe market stress as well raises some serious
questions about whether this criterion is likely to be met in practice. The notional BLRM would
resolve this issue by requiring that all HQLA maintained by firms covered under the program
would include only perfectly safe and perfectly liquid assets. HQLA maintained by depositories,
for example, would be entirely in the form of balances maintained at the Federal Reserve—the
one real world financial asset that meets the perfectly safe and perfectly liquid standard. For
entities not eligible to maintain accounts at the Federal Reserve, one possibility is that the
government could consider establishing a special type of money market mutual fund in which firms could maintain their regulatory liquidity buffers. This role for the government in providing the safe and liquid asset would be similar in spirit to that discussed in Holmstrom and Tirole (1998). Maintaining HQLA only in the form of reserves or another equivalently liquid government obligation would ensure that the liquid assets held by financial firms to guard against liquidity risks can serve their intended purpose in all states of the world. Moreover, in the case of a systemic liquidity shock, reliance on these types of special liquid assets in prudential requirements would avoid scenarios in which the simultaneous efforts of financial firms to sell illiquid Treasury securities in a “dash for cash” compounds the initial market stress. Short of such wholesale changes to the existing structure of HQLA portfolios, the model would point toward a role for differentiating among different classes of assets in HQLA portfolios based on their effective liquidity during periods of distress.

Another basic issue is that systemic risk in the analysis above is directly related to a financial firm’s reliance on volatile funding and the potential for a sizable funding runoff to exceed available liquid assets. In the context of the Treasury market, this source of risk may be underrecognized by existing regulations. For example, under the LCR regulation, financing potentially illiquid Treasury securities in short-term repo markets is viewed as posing essentially no liquidity risk. Repo financing obligations backed by Treasury securities have a zero runoff rate and thus do not contribute to the “net cash outflow” under the LCR regulations while the securities financed are “encumbered” and thus do not count as HQLA. The result is that the risk that a firm may lose access to secured funding markets during a period of distress and find itself facing difficulty in selling its Treasury holdings or acquiring other financing to make up the shortfall is not recognized by the LCR regulations. The import of this underrecognized risk is likely to be most significant for firms with a heavy reliance on short-term repo financing. For example, short-term repo financing of securities inventories is central to the business model of many dealers and other types of nonbank financial firms. Under the notional BLRM described above, the risks associated with financing of potentially illiquid Treasury securities with short-term repo would be recognized, and financial firms would be required to hold truly liquid assets in the form of reserves or especially liquid government obligations to mitigate those risks.

Recognizing the possibility that Treasury markets may become illiquid during periods of severe distress, the Brookings Institution and G-30 reports both point to the potential benefits of an official sector lender of last resort that would effectively guarantee that financial firms could rely

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10 In some respects, this could be analogous to the so-called G-Fund operated by the federal government in connection with civil service Thrift Savings Plan. The G-Fund obligations mature each day and the assets of the fund are held in the form of special government-account series Treasury securities. In the case of a “G-Fund” for financial firms, a decision by a financial firm to draw down its balances at the government-operated fund would require the Treasury to issue short-term marketable debt or draw down its account at the Federal Reserve.

11 See the discussion of HQLA and assumed runoff rates for secured financing arrangements in the Basel Committee on Banking Supervision documents LCR - Liquidity Coverage Ratio (bis.org).

12 For the largest institutions, supervisory programs such as the Comprehensive Liquidity Analysis and Review (CLAR) process would consider the ability of a firm to monetize “liquid” assets in a stress scenario.

13 Financial firms recognize some of the risks associated with secured financing. See, for example, the excellent overview of the approach to maintaining “global core liquid assets” at Goldman Sachs as outlined in the presentation for investors at PowerPoint Presentation (goldmansachs.com).
on their holdings of Treasury securities as a source of liquidity even during periods of distress. Both reports note the possibility that providing this type of lender of last resort could have unintended consequences by undercutting the incentives for financial firms to appropriately manage their liquidity risks. Using the model developed in this paper, these types of effects can be quite pronounced. Particularly if the social costs of a liquidity deficiency are very high, the model suggests that providing a lender of last resort in isolation without accompanying steps to strengthen incentives for appropriate ex-ante liquidity risk management such as mandatory upfront fees or liquidity requirements could end up increasing expected social costs. On the other hand, the model suggests that if the official sector can first establish a regime with mandatory fees of the type discussed above or can impose well designed liquidity requirements, providing a lender of last resort that lowers the ex-post costs of a liquidity deficiency is likely to lower both expected private and expected social costs.

Finally, all financial institutions whose activities generate potential systemic risks—banks, nonbank subsidiaries of bank holding companies, and nonbanks—would be covered under the BLRM. The model results focused on heterogeneity across financial firms highlight a simple principle that the socially optimal risk of a liquidity deficiency across firms is inversely proportional to the marginal social cost of a liquidity deficiency across firms. Based on that principle, a financial stability planner would choose higher liquidity coverage ratios and other risk mitigants for firms that pose relatively high social costs in the event of a liquidity deficiency. Of course, that principle informs the more stringent liquidity regulations in place for larger and more systemically important firms, for example. As noted by Metrick and Tarullo (2021), a corollary of this simple principle implies the financial stability planner should seek to impose regulations or other risk mitigants that result in similar risks of a liquidity deficiency for firms with similar social costs in the event of a liquidity deficiency. Under the hypothetical BLRM, the financial stability planner would triage all firms covered by the program to assess the systemic liquidity risk associated with each firm’s activities. The ex-ante liquidity risk tools would then be calibrated appropriately across all firms based on that assessment.

6. Conclusion
The framework developed in this paper helps to draw out some of the potential connections between ex-ante liquidity risk management tools such as liquidity requirements or mandatory fees and ex-post liquidity tools such as a lender of last resort. A central message of this analysis echoes the points raised by many other authors suggesting that policy actions that expand the lender of last resort function so as to better address periods of financial distress are likely to be most effective when accompanied by regulations or other mechanisms that encourage socially responsible ex-ante liquidity risk management on the part of financial firms. Similar to proposals in the Brookings Institution report, a mandatory fee schedule emerges as a potentially very useful tool in this regard. The structure of the fee schedule depends on both the scale of volatile liabilities and the extent of “liquidity coverage” maintained to cover potential funding shortfalls. The framework also provides some potentially useful benchmarks in evaluating the distribution of liquidity risks across different classes of financial firms.
As alluded to above, the analysis here really only scratches the surface of liquidity risk management and public policy. The model developed above focuses on one dimension of liquidity risk—the private and external costs associated with volatile liabilities and the potential for a liquidity deficiency. Other aspects of liquidity risk management broadly defined are also likely to be quite important from a financial stability planner’s perspective—perhaps most notably, the preparedness and willingness of financial firms to provide liquidity to others during a period of distress. These kinds of issues raise challenging questions about how to create appropriate incentives for private financial firms to provide liquidity and participate in markets when a withdrawal from risk-taking might seem the most prudent course of action from the perspective of an individual firm.
Appendix: Basic Model Details

A.1 Basic Model

The basic model is a variation on that developed by Poole (1968) in which a financial firm chooses optimal asset and liability compositions ex-ante taking into account the effect of those choices on the likelihood and severity of a “liquidity deficiency” after the realization of a “liquidity shock.” The firm maximizes the expected value of profits given by:

\[ E(\pi) = r_l + \int_{u}^{\infty} r_b \tilde{b}(u) - r_v \int_{-\infty}^{\infty} \tilde{v}(u) - r_t - \left( \frac{b}{2} \right) (v - \mu(1-k))^2 + (r_b + \theta) \int_{-\infty}^{u^*} \tilde{b}(u) \]

where

- \( l \) = share of illiquid assets
- \( b \) = share of liquid assets
- \( v \) = exante share of volatile liabilities
- \( t \) = exante share of stable liabilities
- \( \tilde{b} = b + \tilde{v} - v \) = expost liquid assets
- \( \tilde{v} = ve^{(\frac{1}{2})\sigma^2} \) = expost volatile liabilities

and

- \( r_l \) = rate on illiquid assets
- \( r_b \) = rate on liquid assets
- \( r_v \) = rate on volatile liabilities
- \( r_t \) = rate on stable liabilities
- \( r_b + \theta \) = penalty for liquidity deficiency

Here \( u \) is a shock drawn from the normal distribution with mean zero and unit variance. With this specification, ex-post volatile liabilities \( \tilde{v} \) are always positive and the expected value of ex-post volatile liabilities is equal to the ex-ante level of volatile liabilities.

\[ \tilde{v} = ve^{(\frac{1}{2})\sigma^2} > 0 \text{ and } E(\tilde{v}) = v \]

The threshold level of a shock results in the firm exhausting its liquid assets ex-post is given by:

\[ u^* = (\log \left( \frac{1 - b}{v} \right) + \frac{1}{2} \sigma^2) / \sigma \]

And the balance sheet constraints of the firm are given by

\[ l + b = 1 \]
\[ v + t + k = 1 \]
\[ \left( \frac{\delta}{2} \right) (v - \mu(1 - k))^2 = \text{term capturing preferences for liability shares} \]

The expected value of the objective function is then given by:
\[ E(\pi) = \eta_l + r_b b - r_v t + \theta((b - v)G(u^*) + vG(u^* - \sigma)) - \left( \frac{\delta}{2} \right) (v - \mu(1 - k))^2 \tag{a0} \]

And the first order conditions are given by:
\[ r_l - \tau_A = 0 \]
\[ r_b + \theta G(u^*) - \tau_A = 0 \]
\[ -r_v - \theta (G(u^*) - G(u^* - \sigma)) - \delta (v - \mu(1 - k)) - \tau_L = 0 \]

Where \( \tau_A \) and \( \tau_L \) are the Lagrange multipliers associated with the constraints on asset shares and liabilities shares, respectively.

These first order conditions can be combined to generate expressions that describe the optimal choices of all the asset and liability shares.
\[ r_l = r_b + \theta G(u^*) \tag{a1} \]
\[ r_v = r_t + \theta (G(u^*) - G(u^* - \sigma)) + \delta (v - \mu(1 - k)) \tag{a2} \]

These two expressions capture the basic tradeoffs at the margin in determining optimal asset and liability shares. Equation (a1) indicates that the firm will balance the return from an extra dollar of illiquid assets with the return from an extra dollar of liquid balances. The latter includes both the explicit return in holding liquid balances plus the implicit return in helping to avoid liquidity deficiencies. Similarly, equation (a2) captures the basic tradeoff in shifting a dollar of liquid volatile liabilities to a dollar of term liabilities. The left hand side of the expression is the all-in marginal cost of increasing term liabilities and the right hand side is the corresponding all-in marginal cost of increasing volatile liabilities.

Equations (a1) and (a2) can be combined with the balance sheet constraints to derive the reduced form expressions for \( v, b, t, \) and \( l \) as:
\[ v = \mu(1 - k) + \frac{(r_l - r_v)}{\delta} - \frac{\theta}{\delta} (G(u^*) - G(u^* - \sigma)) \tag{a3} \]
\[ b = v \cdot \varphi = v \left( 1 - e^{\sigma u^* - \frac{(1)}{2} \sigma^2} \right) \tag{a4} \]
\[ t = (1 - k)(1 - \mu) - \frac{(r_l - r_v)}{\delta} + \frac{\theta}{\delta} (G(u^*) - G(u^* - \sigma)) \tag{a5} \]
\[ l = 1 - b = 1 - v \varphi \tag{a6} \]

Where
\( u^* = G^{-1}\left(\frac{r_l-r_b}{\theta}\right) \) \hspace{1cm} (a7)

Solutions in which the financial firm chooses a positive level of liquid assets imply:

\[ b > 0 \text{ requires } u^* < \left(\frac{1}{2}\right) \sigma \text{ and that implies } \theta > \frac{(r_l - r_b)}{G\left(\frac{1}{2} \sigma\right)} \] \hspace{1cm} (a8)

The condition in (a7) thus implies a lower bound on the penalty for an ex-post liquidity deficiency that will provide incentives for the firm to hold liquid assets.

The expected value of the penalty for liquidity deficiencies is given by:

\[ C = \theta \left( (b - v)G(u^*) + vG(u^* - \sigma) \right) = \theta G(u^*)v \left( \frac{G(u^* - \sigma)}{\theta(u^*)} - \frac{g(u^* - \sigma)}{g(u^*)} \right) = sv \left( \frac{G(u^* - \sigma)}{\theta(u^*)} - \frac{g(u^* - \sigma)}{g(u^*)} \right) < 0 \] \hspace{1cm} (a9)

By the envelope theorem, the derivative of the expected profits for the financial firm with respect to an increase in the penalty rate is given by:

\[ \frac{\partial E(\pi)}{\partial \theta} = \frac{C}{\theta} \]

### A.2 Comparative Statics

Here we concentrate on the comparative statics with respect to a penalty for a deficiency and the volatility parameter.

#### A.2.1 Penalty for a Deficiency

The comparative statics for asset and liability choices of the private financial firm with respect to the penalty for a deficiency are given by:

\[ v_\theta = -\left(\frac{1}{\sigma}\right) (G(u^*) - G(u^* - \sigma)) - \left(\frac{\theta}{\delta}\right) (g(u^*) - g(u^* - \sigma)) u^*_\theta \]

\[ u^*_\theta = -\frac{G(u^*)}{\theta g(u^*)} < 0 \]

And with \( u^* < \left(\frac{1}{2}\right) \sigma \), this implies that the volatile liability share declines with an increase in the penalty for a deficiency.

\[ v_\theta < 0 \]

The effect on the liquidity coverage ratio \( \varphi = b/v \) is given by:

\[ \varphi_\theta = -\sigma e^{\sigma u^* - \left(\frac{1}{2}\right) \sigma^2} u^*_\theta = \sigma \left( e^{\sigma u^* - \left(\frac{1}{2}\right) \sigma^2} \right) \frac{G(u^*)}{\theta g(u^*)} > 0 \]

Given the signs for \( v_\theta \) and \( \varphi_\theta \), the effect on the liquid asset share \( b \) is ambiguous in general.

\[ b_\theta = v_\theta \varphi + \nu \varphi_\theta \]

#### A.2.2 Volatility Parameter

The effect of a marginal increase in the volatility parameter on volatile deposits is given by:
\[ v_\sigma = -\left( \frac{\theta}{\sigma} \right) g(u^* - \sigma) < 0 \]

The volatility parameter does not directly affect the threshold cutoff value so
\[ u^*_\sigma = 0 \]

The facts imply that the derivative of the liquidity coverage ratio with respect to the volatility parameter is given by:
\[ \varphi_\sigma = -(u^* - \sigma)e^{\sigma u^*-\left(\frac{1}{2}\right)\sigma^2} > 0 \]

And this term is positive given that \( u^* < \sigma/2 \). The effect of an increase in the volatility parameter on the liquid asset share is given by:
\[ b_\sigma = v_\sigma \varphi + v \varphi_\sigma \]

That effect can be negative or positive depending the relative magnitudes of \( v_\sigma \) and \( \varphi_\sigma \).

A.3. Financial Stability Planer’s Objective Function

The financial stability planner’s objective function is the sum of the expected profits for the representative firm plus the external effects:
\[ V = E(\pi) + \omega C \]

Where C is defined above as in (a9) and \( E(\pi) \) is defined as noted in (a0).

The derivative of the financial stability planner’s objective function with respect to \( \theta \) with the endogenous variables evaluated at the optimal values chosen by private firms is given by:
\[ \frac{\partial V}{\partial \theta} = \frac{C}{\theta} + \omega C_\theta \]

(a10)

The net effect of an increase in the penalty for a deficiency on the financial stability planner’s objective function depends on both the direct effect on the profits of the representative financial firm in the first term and the marginal effect on the external costs given by the second term. An increase in the penalty for a liquidity deficiency unambiguously lowers the expected profits of the financial firm and the first term is negative. The effect on the external costs depends on the sign of the derivative \( C_\theta \) given by:
\[ C_\theta = sv_\theta \left( \frac{G(u^* - \sigma)}{G(u^*)} - \frac{g(u^* - \sigma)}{g(u^*)} \right) + svu^*_\theta \frac{G(u^* - \sigma)}{G(u^*)} \left( Q(u^* - \sigma) - Q(u^*) - \sigma \frac{Q(u^* - \sigma)}{Q(u^*)} \right) \]

Where \( Q(\cdot) \) is the inverse Mills ratio. The derivative of the inverse Mills ratio is bounded by 0 and 1 so:
\[ 0 > Q'(u^*) > -1 \]

Using this fact, we have:
\[ Q(u^*) - Q(u^* - \sigma) = \int_{u^* - \sigma}^{u^*} Q'(u)du > \int_{u^* - \sigma}^{u^*} -du = -\sigma \text{ so that } Q(u^* - \sigma) - Q(u^*) < \sigma \]
The inverse Mills ratio is a declining function so:

\[
\frac{Q(u^* - \sigma)}{Q(u^*)} > 1
\]

As a result, the second term in large brackets is negative and given that \(u^*_\theta < 0\), the entire second term is positive. The first term is also positive so \(C_\theta > 0\).

Thus, the external effect of a liquidity deficiency becomes smaller in magnitude (less negative) as the penalty for a deficiency increases. This implies that there is a potential tradeoff for the financial stability planner in contemplating alternative settings of the penalty for a deficiency. A higher penalty lowers the expected profits of the representative financial firm (the first term in (a9)) but also lowers the expected external costs of a liquidity deficiency (the second term in (a9)). The net effect of these two factors depends on the relative weight of external costs and private profits in the financial stability planner’s objective function and that, in turn, is heavily influenced by social cost parameter \(\omega\).

A.4 Binding LCR Requirement

As noted in the main text, the solutions in the case of a binding LCR requirement are given by:

\[
\begin{align*}
v &= \mu (1 - k) - \frac{\theta G(u^*) - G(u^* - \sigma)}{\delta} - \beta_{LCR} (r_l - r_b - \theta G(u^*)) / \delta + (r_t - r_v) / \delta \quad \text{(a11)} \\
b &= \beta_{LCR} v \\
l &= 1 - \beta_{LCR} v \\
t &= 1 - k - v \quad \text{(a13)}
\end{align*}
\]

With

\[
u^* = (\log \left( \frac{1 - b}{v} \right) + \frac{1}{2} \sigma^2) / \sigma = (\log(1 - \beta_{LCR}) + \frac{1}{2} \sigma^2) / \sigma \quad \text{(a15)}
\]

The response of volatile asset share to an increase in \(\beta_{LCR}\) is given by:

\[
v_{\beta_{LCR}} = -\frac{r_l - r_b - \theta G(u^*)}{\delta} < 0
\]

Here the indirect effects of the change in \(\beta_{LCR}\) working through the threshold value \(u^*\) in (a10) end up canceling out.

By equation (a12) we have:

\[
b_{\beta_{LCR}} = v + \beta_{LCR} v_{\beta_{LCR}}
\]

The sign of this effect is ambiguous in general.
References

Carlson, Mark, Burcu Duygan-Bump and William Nelson (2015), “Why do We Need both Liquidity Regulations and a Lender of Last Resort,” FEDS;


