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Jin-Wook Chang, R. Matthew Darst

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Moldy Lemons and Market Shutdowns*

Jin-Wook Chang[†] R. Matthew Darst[‡]

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Abstract

This paper studies competitive market shutdowns due to adverse selection, where sellers post nonexclusive menus of contracts. We first show that the presence of the worst type of agents (moldy lemons) causes markets to fail only if their mass is sufficiently large. We then show that a small mass of moldy lemons can lead to a large cascade of exits when buyers possess outside options. Finally, we show that more precise information about agents' types makes markets more prone to exit cascades. The model is general and does not rely on institution details or structure, and thus can be applied to many different markets and context.

JEL Classification: D52, D53, D82, E44, G32
Keywords: asymmetric information, market unraveling, non-exclusive contracting

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[†]Federal Reserve Board of Governors: jin-wook.chang@frb.gov, 1801 K St, NW, Washington D.C. 20006

[‡]Federal Reserve Board of Governors: matt.darst@frb.gov, 1801 K St, NW, Washington D.C. 20006

1 Introduction

How can a market that functions well in normal times suddenly collapse under stress? Throughout history—from banking panics in the national banking era to the dry-up of asset-backed securities markets (for example, see Figures 1 and 2) and freezing of interbank markets in the Global Financial Crisis—one common answer is adverse selection (Calomiris and Gorton, 1991; Mishkin, 1999; Ivashina and Scharfstein, 2010; Covitz et al., 2013; Foley-Fisher et al., 2020). Although there have been many studies on market shutdowns with adverse selection, theoretical models that generate shut downs due to small changes in underlying conditions usually assume exclusive contracting between agents. However, in practice, most markets are characterized by nonexclusive contracting where agents are generally free to trade and contract simultaneously with multiple counterparties (for example, collateralized debt and loan obligations (CDOs and CLOs), derivatives, over-the-counter, capital, and insurance markets). The inability to monitor a counterparty's trades was certainly a major factor behind the 2021 collapse of hedge fund Archaegos and the bail out of AIG in 2008 due to its credit derivatives positions. Therefore, the goal of this paper is to derive general principles under which small changes in underlying fundamentals, which we coin the entry of moldy lemons, cause markets subject to adverse selection with non-exclusive contracting to fail.

Our main results can be stated as follows: The presence of the worst type of agent (a moldy lemon) causes trade to unravel only if their mass is sufficiently large. By contrast, when agents have outside options or reservation utilities, a small mass of moldy lemons can lead to a large cascade of exits. Moreover, we show two additional conditions on the susceptibility of markets to unraveling: (1) as the number of types in the economy increases and (2) when shocks are isolated to market trades and do not affect outside options. These results have policy implications for interventions in markets subject to adverse selection, which we discuss later in the paper.

We derive these results in a general and flexible model of perfectly competitive trade subject to adverse selection that allows for non-exclusive contracting between buyers and sellers developed by Dubey and Geanakoplos (2019) and Attar et al. (2021). These non-exclusive contracting models impose little structure on prices and quantities and abstract away from complicated model structures or restrictions backed by institutional details (e.g. information production in Dang et al. (2020) or dynamics of collateral and reputation in Chari et al. (2014)) that are generally needed to generate market shutdowns through small changes in underlying fundamentals; all we require is a standard single-crossing property on preferences and a monotonicity condition on costs. Together, these conditions imply that types more willing to trade larger amounts are more costly to serve. Hence, there is weak-adverse selection.

Following Attar et al. (2021), we characterize a market shut down or unraveling of trade when an active market becomes inactive and "entry-proof." An inactive market is one for which the no-trade point dominates trade in the market; markets are entry-proof when the willingness of each agent to trade at the no-trade point does not exceed the cost to serve all types that will enter the market. Finally, the cost to serve the market is given by the upper-tail conditional expectation of unit cost of all agents who are expected to trade in the market. We extend the environment of Attar et al. (2021) by considering an agent that is the worst type among all possible types—*e.g.,* probability of loss for this type is $p = 1$ in Hendren (2013). We call this agent a "moldy lemon." In an investment or trading environment, moldy lemons are agents whose project or asset produces nothing with near certainty. Enlarging the support of types to include moldy lemons can encompass an aggregate (macro) shock to the economy for which the presence of moldy lemons affects every agent in the market. Our main research question thus becomes: Does a shift in underlying fundamentals that create a small mass of moldy lemons cause the market to shutdown?

Before turning to the impact of the entry of moldy lemons, it is useful to understand how allocations in the general non-exclusive contracting economy of Attar et al. (2021) are characterized. Allocations in this environment are recursive layers along a convex-market tariff. Along the first layer, the market price is equal to the expected cost to serve all types and the quantity traded equals the demand of the lowest (best in suppliers' perspective) type. Along the second layer, the lowest type does not trade, and the price is equal to the expected cost of serving all types except the first type. Along this layer, the quantity each agent—other than the lowest type—trades is the residual demand of the second type. This structure continues until the final layer meets the residual demand of the highest type for whom sellers break even to serve. Hence, all types except the lowest type generally combine layers along the market tariff to arrive at an aggregate level of trade, which is prohibited by assumption in exclusive contracting environments.

Our first result stated in Theorem 1 is that moldy lemons change the equilibrium quantities traded in a smooth fashion. This implies that markets shut down *iff* the mass of moldy lemons is sufficiently large. The intuition comes from noticing that the first agent to exit the market is the best type. When the best type does not trade, the level of trade among all remaining types must fall and the price must rise. However, lower trade in equilibrium *raises* the marginal utility of participation among all remaining agents. In other words, although the exit of a good-type raises the market price because the average quality of the remain-

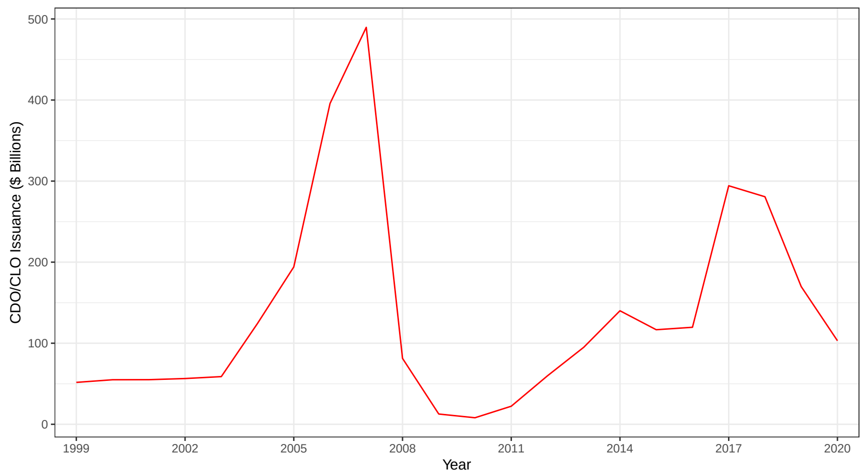


Figure 1: Issuance of Collateralized Debt Obligations and Collateralized Loan Obligations

Source: Securities Industry and Financial Markets Association.

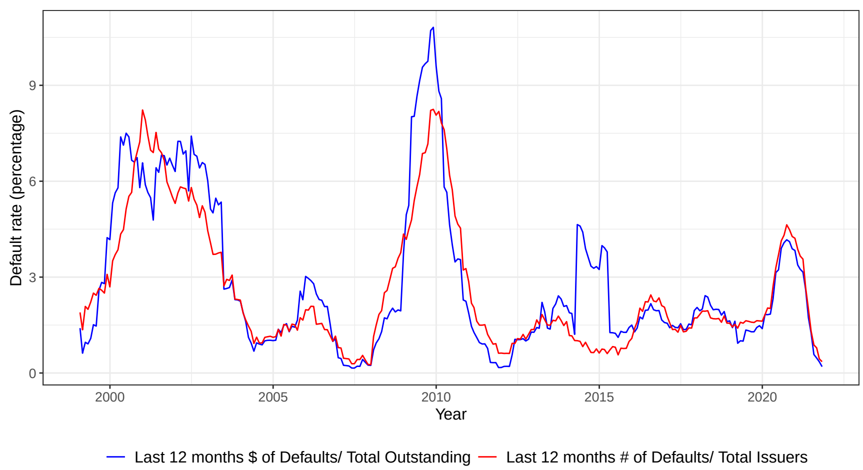


Figure 2: U.S. leveraged loan default rate

Note: CDO and CLO issuance plummets in times of crises, but the default rate of the underlying securities–leveraged loans–modestly increase.
Source: S&P Global Market Intelligence.

ing pool is worse, each agent's marginal rate of substitution between quantity and price *increases*. Therefore, all remaining agents' incentives to trade actively in the market remain at least as strong as they previously were. The only way to generate a large cascade of exits beyond the best types is if the initial entry mass of moldy lemons is sufficiently large, a result reminiscent of Akerlof (1970) and Azevedo and Gottlieb (2017) under exclusive contracts.

We then turn to understand the more relevant question: Under what circumstances can a normal functioning market collapse due to a *small* mass of moldy lemons entering. We extend the model by endowing agents with an outside option to market participation. These outside options may represent payoffs obtainable to agents outside of the market.[1] Alternatively, they can represent fixed participation costs. For example, an agent may enjoy a higher utility by staying out of the market, because entering the market may take time, effort, and cost. Moreover, it could capture the time and effort required to search for a counterparty or supplier, the effort and monetary cost of negotiating and verifying contract terms, or paying other brokerage or settlement fees.[2]

The second result stated in Theorem 2 states that a small mass of moldy lemons can generate a large cascade of exits and market shut downs when agents have outside options. The intuition comes from the fact that outside options provide agents a level of utility against which trade in the market is compared. Hence, the marginal utility characterization of the baseline model is replaced by a total utility representation. In particular, the exit of the best type of agent has negative spillover effects on the remaining agents, because the lower quantity of trade reduces overall utility and the higher market price of remaining contracts together makes the outside option more appealing despite higher marginal rates of substitution. In this case, the exit of a single good type can cause the next best agent to exit the market because the relative value of their outside option increases. Therefore, further exits can happen, and the rest of the agents may suffer even more utility loss and so on.

Theorems 1 and 2 also suggest that some of the results in Hendren (2013, 2014) do not necessarily apply to nonexclusive contracting economies. In particular, Hendren (2014) shows that in the exclusive contracting settings of either Akerlof (1970) or Rothschild and Stiglitz (1976), the presence of a type whose loss probability is equal to 1–a moldy lemon–causes the market to break down either via Akerlof pricing or failure to find a competitive equilibrium as in Rothschild and Stiglitz (1976). Theorem 1 in our paper shows that under non-exclusive contracting, moldy lemons are *not sufficient* to generate a complete market breakdown.

We show an important comparative static result that relates the degree of asymmetric information to exit cascades. Economies with more uncertainty regarding the underlying types of the agents are less vulnerable to exit cascades than economies that feature more known types. More specifically, economies that feature what we call more *coarse partitions* of types—weighted averages of types—can be interpreted as having more uncertainty about the true type of each agent in the partition. Economies with more coarse partitions are

[1] For example, an outside option is an agent's reservation value in search and matching models.

[2] This interpretation is similar to entry fees studied by Bisin and Gottardi (1999, 2003).

less likely to feature exit cascades for a given mass of moldy lemons. The reason is that relative weight of each partition is larger when multiple times comprise it than the weight of each type alone. Thus, a mass of mold lemons sufficient to generate an exit cascade in an economy with fully disaggregated types may be insufficient to generate the same amount of exit with a more coarse partition. For example, the cost increase due to moldy lemons that triggers a cascade in which agent 1 exiting causes agent 2 to exit may not be sufficient to cause both agents 1 and 2 to exit as a group when agent 2 only exits only because of the spillover effect associated with agent 1's exit. Therefore, economies in which there is more precise information or certainty about each agent's type are more likely to feature exit cascades. This result provides a theoretical underpinning to the information production models of Gorton and Ordoñez (2019); Dang et al. (2020); Gorton and Ordoñez (2020) in which market failure arise endogenously due to the incentives to produce private information.

A numerical example confirms the formal results of Theorems 1 and 2 and our comparative static. The example also shows that markets are more prone to shut down as the number of types in the economy grow large and each type is more similar. In particular, if the masses of adjacent types are combined into a single type with the same average probability of a bad state as the original partition, then the introduction of moldy lemons is less likely to cause a cascade of exits. Therefore, markets are more vulnerable to shutdowns when there are many different types, even when aggregate risk or uncertainty remain the same. The reason is due to Theorem 2: the sufficient mass of new moldy lemons needed to trigger market shutdowns increases as the relative mass of existing agents increases. In other words, if both the best type and the second best type do not know whether they are the best type or not, the initial trigger of cascade by the best type is less likely to occur.

The final result shows how aggregate shocks can impact the proclivity for moldy lemons to induce market shut downs. For example, consider a uniform increase in the cost of serving agents (i.e. probability of default) for all types. If outside options remain unchanged, then markets are more prone to shutdowns due to moldy lemons. The reason is that higher cost among all types raises the upper-tail conditional unit cost and leads to higher contract prices in the market. High prices for trade in the market then raise the relative value of the outside option, which generates a cascade of exits. By contrast, if the outside option is not fixed and also varies with the aggregate shock, then moldy lemons are less likely to cause market shutdowns. The reason is that the fall in the utility-level associated with the shock to the outside option can dominate the fall in utility of the consumption bundle obtained via market trades. This happens when the utility difference between the no-trade and market consumption bundle rises as the marginal rate of substitution increases, which makes it less likely that an agent exits.

Overall, our results suggest a parsimonious yet realistic way of generating sudden market shutdowns without imposing additional structure or institutional details on the model. Thus, our model is widely applicable to many different markets and contexts, and provides simple insights on the properties of market shutdowns.

The paper proceeds as follows: Section presents the non-exclusive contracting framework of Attar et al. (2021), and introduces the critical concept of entry-proofness. Section 3 introduces moldy lemons and states Theorem 1. Section 4 introduces outside options and states Theorem 2. Section 5 provides a numerical example and simulations to show the impact of moldy lemons and outside options in the non-exclusive contracting environment.

Relation to the Literature. Our paper relates to the literature studying non-exclusive contracting in economies with adverse selection pioneered by (e.g. Pauly, 1974; Jaynes, 1978; Hellwig, 1988; Glosten, 1994). Allocations in these non-exclusive contracting environments are recursive; agents trade in layers consisting of multiple contracts.[3] More recent results extend to generalized settings with divisible goods, general preferences, and multiple types (e.g Attar et al., 2011, 2014, 2021; Dubey and Geanakoplos, 2019). Equilibrium always exists and is unique, which may include the no-trade equilibrium. Among these, our paper builds directly upon Attar et al. (2021) and Dubey and Geanakoplos (2019) but with a different focus; we study the conditions under which small changes in the distribution of agents causes markets to unravel.

Our focus on the unraveling of competitive equilibrium due to adverse selection dates back to Akerlof (1970) and Rothschild and Stiglitz (1976), and is recently generalized by Hendren (2013, 2014) and Azevedo and Gottlieb (2017). Unlike these papers, our model features non-exclusive contracting. Like Akerlof (1970), markets unravel when the cost to serve the market exceeds the marginal willingness of each agent to trade. With non-exclusive contracting, active markets are robust to small changes in underlying fundamentals and this sort of unraveling does not generally occur. Hendren (2014) shows that equilibrium in insurance economies with exclusive contracting must unravel either via Akerlof-pricing or failure to satisfy competitive-Nash equilibrium a la Rothschild-Stigliz when the distribution of types either contains a continuous interval, or the type whose accident probability equals 1. We show that this result is partially sensitive to competitive contracting as in Azevedo and Gottlieb (2017) and exclusive contracting environment. In general, entry of a type with accident probability equal to 1 does not cause markets to unravel with non-exclusive

[3] Bisin and Gottardi (1999, 2003) show that some form of non-linear pricing is needed to make compatible price taking behavior with asymmetric information.

contracting.

Recent applications of non-exclusive contracting under adverse selection have been applied to security design (e.g. Asriyan and Vanasco, 2021), and asset markets with heterogeneously informed buyers (e.g. Kurlat, 2016). One common feature of non-exclusivity is that neither security nor asset markets more broadly are fully separating; there is always some form of cross-subsidization among markets. Our model inherits semi-pooling in equilibrium, but our focus is on how small changes in underlying fundamentals cause markets to unravel. Auster et al. (2021) study a form of non-exclusivity in search with adverse selection. Workers can apply to as many jobs as possible, but ultimately sell their labor to a single firm. Fully separating equilibria are precluded because high types always send some applications to low-wage-offering firms to hedge against remaining unemployed.

Philippon and Skreta (2012) show that the failure of the price mechanism and market unraveling justify public interventions during liquidity or credit freezes. A key insight in their framework is that interventions impact the set of agents that choose to participate in government programs, which in turn impacts trade in the market. In a nonexclusive contracting framework, policies that increase entry cost prevent market unraveling only if the policy can discriminate among types; otherwise, a uniform cost increase makes markets more prone to unraveling because the best types exit first, which raises the cost of trade for all remaining types.

2 Model

The model builds on the recent contributions of Attar et al. (2011, 2014, 2021) and Dubey and Geanakoplos (2002, 2019). We briefly lay out the specifics of the environment and state the relevant theorems in Attar et al. (2021) that aid in our analysis of the conditions under which markets shut down as worse types enter the market—*i.e., moldy lemons.*[4]

The demand side of the market consists of privately informed agents with a finite number of types–indexed by $i \in I \equiv \{1, .., n\}$ with a strictly positive measure of each type, m_i. Utility for each type is given by $u_i(q, t)$ and assumed to be continuous, quasi-concave in the arguments and strictly decreasing in t. Generically, q represents the quantity of a good that is consumed and t is the transfer required to obtain the good. An important feature of asymmetric information models is that privately informed types are ordered by a single-

[4] We refer the reader to their paper for detailed proofs.

crossing property (Milgrom and Shannon, 1994), $\forall i < j, \ q < q', \ t, \ t'$:

$$u_i(q,t) \leq u_i(q',t') \Rightarrow u_j(q,t) < u_j(q',t').$$

Single-crossing implies that a higher type is at least as willing as a lower type to trade an additional unit of the good for an additional transfer.[5] More generally, we can define a marginal rate of substitution without assuming differentiable utility functions. Let $\tau_i(q,t)$ be the supremum set of prices, p, such that utility increase by an (optimally chosen) additional quantity q' exceeds the associated fall in utility that corresponds to the higher transfer pq',

$$\tau_i(q,t) \equiv \sup \left\{ p : u_i(q,t) < \max_{q' \geq 0} u_i(q+q', t+pq') \right\}.$$

Hence, $\tau_i(q,t)$ is the slope of the indifference curve at an additional quantity, $q' > q$, and can be considered as a (pseudo-)marginal rate of substitution for agent i at consumption bundle (q,t). An important assumption for our analysis of "moldy lemons" to come is that, absent a transfer, a strictly positive endowment of q lowers agents' marginal rate of substitution:

Assumption 1 $\tau_i(q,0) \leq \tau_i(0,0)$, $\forall i, q > 0$.[6]

By way of concrete examples, the model translates into the insurance economy of Rothschild and Stiglitz (1976), where i indicates an agent's risk of loss, q is the amount of insurance purchased, and t is the insurance premium. In a credit economy, i indexes borrower default probability, q is the loan quantity demanded, and t is the gross loan promise made to the lender. We will later use this framework for our numerical analysis.

The supply of contracts in the economy is a linear technology provided at a unit cost, $c_i > 0$, for each type i. Assume c_i is increasing in i. Adverse selection occurs if c_i is increasing in type. That is, higher types wish to trade more than low types, but the cost of servicing these types is higher. For each type, define by $\bar{c}_i$ the expected unit cost of serving all types $j \geq i$ (the upper-tail conditional expected cost) given that higher types will be willing to

[5] Here we are using strict single-crossing condition rather than weak single-crossing, which allows for equality. This is because we want to focus on the strict notion of market breakdown in light of Corollary 1 in Attar et al. (2021).

[6] The same assumption is also used in Attar et al. (2021) in their analysis.

trade any contract offered to type i. Formally,

$$\bar{c}_i \equiv E[c_j | j \geq i] = \frac{\sum_{j \geq i} m_j c_j}{\sum_{j \geq i} m_j} \tag{1}$$

Hence, for any $j < i$, $c_j < \bar{c}_i$. A contract is defined by the pair (q, t) for $q \geq 0$. Suppliers of contracts are competitive risk-neutral, expected profit maximizers.

2.1 The Concept of Entry-proof, Inactive Markets

In this framework, Attar et al. (2021), first state the conditions under which any inactive market, (q, t), is resilient to competitive entry, hence "entry-proof." The key is to first consider the no-trade contract, $(0, 0)$, as any agent's outside option. Then, a market is entry proof iff, for any menu of contracts an entrant offers, the buyer's best response earns the entrant zero expected profit. That is, the entry-proof condition is given by

Condition EP: $\tau_i(0, 0) \leq \bar{c}_i \qquad \forall i.$

Condition EP simply says that there will be no trade in a market when the cost of offering a contract for agent i, given that all other types higher than i must also be served, exceeds type i's marginal utility of not trading. Theorem 1 in Attar et al. (2021) states that Condition EP is necessary and sufficient for markets to be inactive.

We sketch the proof here since the arguments are useful in our extensions. The single-crossing condition implies that an entrant offering an arbitrary menu of contracts will end up trading (q_i, t_i) with type i and $q_j \geq q_i$ with all agents $j \geq i$. The expected profit to the entrant of this menu is $\sum_i m_i [t_i - c_i q_i]$. Using summation by parts, the expected profit can be written in terms of layers $(q_i - q_{i-1})$ and $(t_i - t_{i-1})$: $\sum_i \left(\sum_{j \geq i} m_j\right) [t_i - t_{i-1} - \bar{c}_i(q_i - q_{i-1})]$, where $(q_0, t_0) \equiv (0, 0)$. Moreover, it must be the case that agent i is willing to trade the additional layer on top of the original set of contracts that yielded (q_{i-1}, t_{i-1}), if the entrant's offer is accepted. That is, the marginal rate of substitution of agent i at the original allocation times the new layer must exceed its cost: $\tau_i(q_{i-1}, t_{i-1})(q_i - q_{i-1}) > t_i - t_{i-1}$. In addition, the entrant cannot make a loss on each type given the expected cost, $\bar{c}_i$, to serve all types $j > i$ that will also accept the contract. Therefore, $t_i - t_{i-1} - \bar{c}_i(q_i - q_{i-1}) \geq 0$. Single-crossing implies that $q_i \geq q_{i-1}$, so combining the two previous inequalities, entry will be non-profitable when $\tau_i(q_{i-1}, t_{i-1}) \leq \bar{c}_i$. Then, using the fact that type $i-1$ prefers their optimal trade to no trade, we have $\tau_{i-1}(q_{i-1}, t_{i-1}) < \tau_{i-1}(0, 0)$, and so does type i. Invoking the assumption that, absent transfers, agents' marginal rate of substitution is weakly decreasing in quantities, $\tau_i(\underline{q}_i, 0) \leq \tau_i(0, 0)$, where $\underline{q}_i \in [0, q_{i-1}]$ is the quantity that makes agent i indifferent between (q_{i-1}, t_{i-1}) and $(\underline{q}_i, 0)$. Finally, by quasi-concavity of preferences, $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(\underline{q}_i, 0)$, we have the desired result $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(\underline{q}_i, 0) \leq \tau_i(0, 0) \leq \bar{c}_i$.

Under a weak single-crossing condition, a market breakdown in Attar et al. (2021) is characterized by the situation for which a non-null set of contracts yields strictly negative expected profits, so there exists a best response for the buyer such that entry on an inactive market is unprofitable. This notion of market breakdown can be more strict as in many papers in the literature such that a market breakdown is a situation in which any menu of contracts that strictly attracts at least some agents yields a strictly negative expected profit, even if the buyer's best response is most favorable to the entrant. A Corollary to Theorem 1 is that this notion of market breakdown occurs if and only if **Condition EP** is satisfied when the preferences are strictly convex and strict single-crossing holds, and this is the notion of the *market shutdown* that we will use in our analysis.

2.2 The Concept of Entry-proof, Active Markets

The main question we are after is, Under what conditions does the entry of worse type buyers or a worsening of the buyer type distribution cause active markets to break down? In order to answer this question, we first ask when active markets are entry-proof in the sense that entry of a supplier is unprofitable. Armed with the answer to the latter question based on Attar et al. (2021), we can answer the former.

Trade in the market is non-exclusive in the sense that no agent can be stricken from trading with multiple firms or suppliers. Therefore, we must define the market tariff, or the minimum aggregate transfer that is made across active markets to obtain aggregate consumption, q. Define the market tariff by $T(q)$. We will assume that $T(q)$ is convex and the domain is a compact interval with lower bound equal to 0. Then, all agents choose q_i to maximize $u_i(q_i, T(q_i))$. An allocation, $(q_i, T(q_i))_{i \in I}$ is *implemented* by the market tariff, T, if $q_i = \arg\max_q u_i(q, T(q))$. This allocation is *budget feasible* if suppliers make non-negative expected profits at the market tariff

$$\sum_i m_i \left[T(q_i) - c_i q_i\right] \geq 0. \tag{2}$$

The assumption that trade is non-exclusive means that for an active market to be entry proof, no agent can combine any menu of potential new contracts in the market with trade along the existing market tariff, T, where the entrant makes positive expected profits. Exclusive trading implies that the menu of contracts offered in a specific market yield at most zero expected profits. Under non-exclusivity, the entrant must deal with the fact that agents can combine the new offer with any existing offers in the market. Therefore, the entrant faces types with indirect utility functions of trading a proposed new contract (q', t') in addition to the existing allocation $(q_i, T(q))$:

$$u_i^T(q', t') \equiv \max \{u_i(q + q', T(q) + t' : q)\} \tag{3}$$

An agent's individual rationality constraint from an entrant's perspective is determined by the indirect utility of not trading the proposed contract on top of the existing market tariff, $u_i^T(0,0)$. We can define the marginal rate of substitution along the indirect utility functions as above by $\tau_i^T(q', t')$.[7] As shown by Attar et al. (2021), the indirect utility functions will also satisfy single-crossing because the primitives satisfying the same condition. In order to apply Theorem 1 of Attar et al. (2021) to the indirect marginal rates of substitution, one needs to assume that for all types, i, and all transfers, t, the indirect marginal rates of substitution are nonincreasing in q, which is slightly stronger than Assumption 1.

Assumption 2 *For all i and t, $\tau_i(q, t)$ is nonincreasing in q.*

Intuitively, this assumption implies that a higher quantity always reduces each agent's willingness to pay for any additional quantity. For the given market tariff, T, and the allocation, $(q_i, T(q))$, we can define $\tau_i^T(0,0)$ as the supremum of the set of prices p for the indirect utility function, $u_i^T(0,0)$—that is,

$$u_i(q_i, T(q_i)) = u_i^T(0,0) < \max \left\{u_i^T(q', pq') : q'\right\} = \max \{u_i(q + q', T(q) + pq' : q, q')\}.$$

With this, we can invoke the necessity and sufficiency result of **Condition EP** to state that a market tariff is entry-proof iff:

$$\forall\, i, \quad \tau_i^T(0,0) \leq \bar{c}_i. \tag{4}$$

This condition states that an active market is entry-proof if and only if the cost required to enter the market exceeds the willingness of each agent to trade the contract on top of

[7] This is possible because: 1) the maximizers in (3) are continuous from Berge's Maximization Theorem; 2) the market tariff, T, is convex; 3) the utility functions, $u_i(q,t)$, are weakly quasi-concave in (q,t) and strictly decreasing in t; 4) and, hence, the indirect utility functions u_i^T are weakly quasi-concave in (q,t) and strictly decreasing in t.

the allocation they may already obtain. For each agent, the utility they receive from their market trades must be at least as large as trading the proposed new contract given the cost required to serve the market:

$$\text{for each } i, \quad u_i(q_i, T(q_i)) \geq \max\left\{u_i(q + q', T(q) + \bar{c}_i q' : q, q')\right\}. \tag{5}$$

By convention of letting $q_0 \equiv 0$, setting $q' = q_i - q$, and applying condition (5) for each layer, $q \in [q_{i-1}, q_i]$, we see that the implied market tariff necessary to induce entry must be at least as large as the pre-entry tariff:

$$\text{for each } i \text{ and } q \in [q_{i-1}, q_i], \quad T(q_i) \leq T(q) + \bar{c}_i(q_i - q). \tag{6}$$

For a given $q = q_{i-1}$, we have $T(q_i) \leq T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1})$. Using the budget-feasibility condition from (2), and re-writing it in terms of layers, we have

$$\sum_i \left(\sum_{j \geq i} m_j\right) [T(q_i) - T(q_{i-1}) - \bar{c}_i(q_i - q_{i-1})] \geq 0. \tag{7}$$

Therefore, it must be the case that the inequalities in $T(q_i) \leq T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1})$ hold as equalities:

$$T(q_i) = T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1}). \tag{8}$$

It must also be true that the allocation, $u_i(q_i, T(q_i))$, implied by the new layer, $q_i - q_{i-1}$ maximizes the utility of all the agents that choose it, given that the new market tariff must rise to serve all agents. Hence, for each i,

$$u_i(q_i, T(q_i)) = \max\left\{u_i(q_{i-1} + q', T(q_{i-1}) + \bar{c}_i q' : q')\right\}. \tag{9}$$

Finally, because the tariff is convex and satisfies (6) and (9), it must be affine with slope $\bar{c}_i$ over the interval $[q_{i-1}, q_i]$.

With this, Attar et al. (2021) state **Theorem 2**—An allocation $(q_i, T(q_i))_{i \in I}$ is budget-feasible and implemented by an entry-proof convex market tariff, T, with domain $[0, q_n]$ if and only if

1. $(q_0, T(q_0)) \equiv (0, 0)$,

2. $q_i - q_{i-1} \in \arg\max \{u_i(q_{i-1} + q', T(q_{i-1}) + \bar{c}_i q' : q')\}$ for each i,

3. $q_{i-1} < q_i \Rightarrow T$ is affine with slope $\bar{c}_i$ over $[q_i, q_{i-1}]$ for each i.

To sum up, an affine convex market tariff is entry proof as long as the upper-tail conditional cost of entry exceeds the marginal willingness of all agents to trade the additional layer on top of their pre-entry allocation. This market tariff consists of layers of trade with unit prices $\bar{c}_i$ that trace out a polygon with an upward kink at each $q^*_{i+1} \geq q^*_i$ for each $i \in I$.

As discussed in Attar et al. (2021), uniqueness of an entry-proof convex market tariff also follows if the solution to each agent's maximization problem is unique. This is guaranteed if the agents' preferences are strictly convex, which we assume to be the case. The problem of multiplicity arises only if the marginal rate of substitution of some type i equals $\bar{c}_i$ over a whole interval of quantities, which is not a generic phenomenon (Attar et al., 2021).

3 Moldy Lemons

We now ask what happens to trades in markets and the market tariff as increasingly worse types of agents enter the market. What we have in mind are situations for which firms anticipate a very high cost of serving some types of agents who may generate losses with near certainty. For example, the COVID-19 pandemic led to a surge in defaults.

To fix ideas, assume that a new agent of type $n+1$ (a "moldy lemon") enters the market with mass m_{n+1}. The cost to serve this agent is $c_{n+1} > c_n$. Given that the unit cost of serving agent $n+1$ is strictly greater than the unit cost of serving any other agent, the upper-tail conditional expected cost of serving all agents must also rise. Specifically, the new upper-tail conditional expected cost for any agent i is given by

$$\tilde{\bar{c}}_i \equiv \frac{\sum_{j \geq i} m_j c_j + m_{n+1} c_{n+1}}{\sum_{j \geq i} m_j + m_{n+1}}, \tag{10}$$

and $\tilde{\bar{c}}_i > \bar{c}_i$. Then, it is clear that any market that was inactive *ex ante* remains inactive *ex post* agent $n+1$ entering, because $\tau_i(0,0) \leq \bar{c}_i < \tilde{\bar{c}}_i$. This is quite natural in the sense that making the average quality of the pool worse will never lead to the opening of new markets.

What about active markets? Does the arrival of agent $n+1$ lead any active markets to shut down? Recall that an active-market is subject to entry if the marginal willingness to trade for all agents is greater than the upper-tail conditional cost to serve them. Then, if a market was active and $\tau_i^{\tilde{T}}(0,0) > \tilde{\bar{c}}_i$ for some i, the market will remain active with a new equilibrium level of aggregate trade given by quantities $(\tilde{q}_i)_{i\in N\cup\{n+1\}}$. Intuitively, active markets must be characterized by terms of trade that do not make agents worse off compared with not trading. If there is at least one type of agent that is lower than type-$n+1$ willing to trade in a market given that the cost will rise, then the market will not close despite the presence of type $n+1$. More interestingly, type $n+1$ causes the aggregate quantity of trade to decrease for all types $\tilde{q}_i < q_i$, $\forall i$, and the slope of the market tariff to rise along all segments of the polygon that trace out the new market tariff, $\tilde{T}(\tilde{q})$. Finally, active markets cease to remain active only when $\tau_i^{\tilde{T}}(0,0) \leq \tilde{\bar{c}}_i$, $\forall i$. From (10), moldy lemons lead to market breakdowns only when their mass, m_{n+1} is sufficiently large relative to the rest of the agents.

Formally, we have the following:

Theorem 1 (Moldy Lemons)

Suppose $n+1$ type (moldy lemons) with mass m_{n+1} and $c_{n+1} > c_n$ enters the market. Then,

1. *if a market was inactive (market shutdown), then the market remains inactive under the new equilibrium.*
2. *if a market was active and $\tau_i^{\tilde{T}}(0,0) > \tilde{\bar{c}}_i$ for some i, then the market remains active under the new equilibrium with quantities $(\tilde{q}_i)_{i\in N\cup\{n+1\}}$ and the following statements are true:*

   (i) $\forall i$, $\tilde{q}_i \leq q_i$

   (ii) $\forall i$, if $\tilde{q}_{i-1} < \tilde{q}_i$, the new slope for affine tariff $\tilde{T}$ is steeper as $\tilde{\bar{c}}_i > \bar{c}_i$ over $[\tilde{q}_{i-1}, \tilde{q}_i]$.

3. *if a market was active and $\tau_i(0,0) \leq \tilde{\bar{c}}_i$ for each i, then the market shuts down.*

Proof. Statements 1 and 3 are almost trivial as explained in the discussion before the theorem. We show that statement 2 holds by mathematical induction.

First, consider type 1. Suppose that agent 1 was trading a positive quantity $q_1 > 0$, without loss of generality.

Case 1. Suppose that type 1 agent exits the market after the entry of moldy lemons. Then, the quantity demanded trivially decreases as $\tilde{q}_1 = 0 < q_1$, and the market tariff increases as $\tilde{T}(q) \geq \tilde{\bar{c}}_2 q > \bar{c}_2 q > \bar{c}_1 q$ for any q in $[0, q_1]$ and $[q_1, q_2]$. Since $(q_2, T(q_2))$ was the type 2 agent's best response, $\tau_2(q_2, T(q_2)) \leq \bar{c}_2 < \tilde{\bar{c}}_2$. Therefore, the new optimal quantity for type 2, $\tilde{q}_2$ should be less than q_2 because $\tau_2(q,t)$ is nonincreasing in q for all t.

Case 2. Suppose that type 1 agent still trades a positive quantity in the market. The market tariff trivially increases to $\tilde{T}(q) = \tilde{\bar{c}}_1 q$ for any q in $[0, \tilde{q}_1]$ as in the previous case. Since $(q_1, T(q_1))$ was the type 1 agent's best response, $\tau_1(q_1, T(q_1)) \leq \bar{c}_1 < \tilde{\bar{c}}_1$. Therefore, the new optimal quantity for type 1, $\tilde{q}_1$ should be less than q_1 since $\tau_1(q, t)$ is nonincreasing in q for all t. Given that, type 2 agent will face a higher total tariff for the same quantity as

$$\tilde{T}(q_2) \geq \tilde{\bar{c}}_1 \tilde{q}_1 + \tilde{\bar{c}}_2 (q_2 - \tilde{q}_1) > T(q_2) = \bar{c}_1 q_1 + \bar{c}_2 (q_2 - q_1),$$

because $\tilde{\bar{c}}_j > \bar{c}_j$ for $j = 1, 2$ and $\tilde{q}_1 \leq q_1$. Therefore, the new optimal quantity for type 2 becomes $\tilde{q}_2$ that is less than q_2 as in the previous case.

Now consider an arbitrary type $i > 2$. Suppose that the inductive hypothesis holds up to $i - 1$, so $\tilde{q}_j \leq q_j$ and the slope of the new market tariff $\tilde{T}$ became steeper as $\tilde{c}_j$ over $[\tilde{q}_{j-1}, \tilde{q}_j]$ for any $j \leq i - 1$. Then,

$$\begin{aligned} \tilde{T}(q_i) &\geq \sum_{j<i} \tilde{\bar{c}}_j \left(\tilde{q}_j - \tilde{q}_{j-1}\right) + \tilde{\bar{c}}_i (q_i - \tilde{q}_{i-1}) \\ &> \sum_{j<i} \bar{c}_j \left(q_j - q_{j-1}\right) + \bar{c}_i (q_i - q_{i-1}) = T(q_i), \end{aligned}$$

with the convention $\tilde{q}_0 \equiv 0$. Again, since $(q_i, T(q_i))$ was the type i agent's best response, $\tau_i(q_i, T(q_i)) \leq \bar{c}_i < \tilde{\bar{c}}_i$ and the total tariff for the same quantity actually increases further. Therefore, the new optimal quantity for type i, $\tilde{q}_i$ should be less than q_i.

Thus, by mathematical induction, $\tilde{q}_i \leq q_i$ holds for any i and the new slope for the affine market tariff $\tilde{T}$ becomes steeper in each and every interval of the new optimal quantity layers. ∎

The observation that markets shut down with non-exclusive contracting only if there a is sufficient mass of bad types is not terribly surprising. Azevedo and Gottlieb (2017) show a similar result under an exclusive contracting environment. However, we are extending their results to a formal result under the non-exclusive contracting environment.

The next set of questions we address are which layers are most susceptible to exit of agents and under what conditions do exits become pervasive with multiple inactive layers? The single-crossing condition implies that higher types have higher willingness to trade at higher prices. Hence, as worse types enter the market and increase the market tariff along each layer, the lowest types exit the market first. This is easiest to see for the case where the first type $i = 1$ is just indifferent to trading given the upper-tail conditional cost of serving all greater types $j > 1$ where $\tau_i^{\tilde{T}}(0, 0) = \bar{c}_i < \tilde{\bar{c}}_i$, while other agents $j > 1$ could still have

$\tau_j^{\tilde{T}}(0,0) > \tilde{\tilde{c}}_j$. In this case, type 1 is no longer willing to trade at the new market tariff, $\tilde{T}(\tilde{q})$, given the additional cost required to make the tariff budget-feasible while all types $j > 1$ remain active in the market. Hence, only the first layer of trade, q_1, becomes inactive.

Proposition 1 *For a fixed c_{n+1}, only the first agents start exiting the market in the equilibrium as m_{n+1} increases.*

Proof.

First, the marginal rate of substitution, $\tau_i(q,t)$, is nonincreasing in q. Second, utility, $u_i(q,t)$, is continuous and decreasing in t. Combining this with strict single-crossing, we have $u_{i-1}(0,0) \leq u_{i-1}(\tilde{q}_{i-1}, \tilde{T}(\tilde{q}_{i-1})) \Rightarrow u_i(0,0) < u_i(\tilde{q}_{i-1}, \tilde{T}(\tilde{q}_{i-1}))$. By continuity, there exists m_{n+1}^i such that $\tilde{T}$ makes $u_{i-1}(0,0) > \max_q u_{i-1}(q, \tilde{T}(q))$ and $u_i(0,0) < \max_q u_i(q, \tilde{T}(q))$. If $i-1$ exits, then any type $j < i-1$ also exits. ■

We conclude that the arrival of moldy lemons (worse than worst types) has a negative, but marginal effect on active markets. The reason is that when the first type drops out and the first layer of trade, $(q_1, \tilde{T}(q_1))$, is removed from the market, the marginal value of each remaining layer for each agent goes up relative to the alternative of no trade at $(0,0)$. In other words, $\tau_i(0, \tilde{T}(0)) \geq \tau_i(q_1, \tilde{T}(q_1))$, and the spillovers from the exit of an agent would only increase the incentives to enter the market rather than decreasing it. This force keeps the remaining markets active. On the one hand, the EP condition is very robust in the sense that a complete unraveling with non-exclusive contracting requires a tautological extremely large mass of bad types. On the other hand, there is a gap between the model and the real world phenomena that exhibit a sudden collapse of the markets after hitting the tipping point of the severity of adverse selection (Calomiris and Gorton, 1991; Covitz et al., 2013; Mishkin, 1999; Ivashina and Scharfstein, 2010; Foley-Fisher et al., 2020).

4 Outside Options and Market Shutdown

We now show that a natural contracting friction, outside options, can generate additional market shut downs when a small mass of moldy lemons enters the market. An outside option may be thought of as an alternative to entering the market, which requires either a contractual barrier or a cost of entry. For example, signing a contract or searching for the right supplier may require significant time and effort. Alternatively, it may represent an external market into which agents can enter and secure a certain level of utility, or reservation utility. What is important is that agents have some utility outside of trade in the market that is higher than the no-trade contract $u_i(0,0)$ for each i. Denote the utility from the outside options (and not entering the market) as γ_i for type i.

Type i now compares the set of market contracts to their outside option, $\gamma_i > u_i(0,0)$, rather than the null action, $u_i(0,0)$. Thus, the individual rationality condition for i becomes

$$V_i(T) = \max\left\{\gamma_i, \max_{q\geq 0} u_i(q, T(q))\right\} \tag{11}$$

for a given market tariff T. We assume the following condition:

$$u_i(q,t) \geq \gamma_i \Rightarrow u_j(q,t) > \gamma_j \qquad \forall i < j,\ \forall q > 0,\ \forall t. \tag{12}$$

This condition implies that if agent i prefers a contract (q,t) with a positive quantity to the outside option, then agent j also prefers the contract to the outside option, which is in line with the idea of single-crossing property. We discuss the role and micro-foundation of this assumption in subsection 4.2.

Also, agents who enter the market optimize their utility for the given market tariff just as in the baseline model without outside options. Therefore, the arguments in Theorem 2 of Attar et al. (2021) for budget feasible, entry-proof market tariffs also hold with outside options. In particular, the entry-proof convex market tariff is T with the slope of $\bar{c}_i$ for each layer $[q_{i-1}, q_i]$ for each i with the convention of $q_0 = 0$. Therefore, the same argument used in Proposition 1 holds for the new setup.

Proposition 2 *In any active market equilibrium, there is a cutoff $\theta \in N \cup \{0\}$ such that any agents with type less than or equal to θ exit the market and any agents with type greater than θ remain in the market.*

In other words, agent 1 will be the first to exit the market in any active market equilibrium. Further exits will be in the sequential order of agent type as 2, 3, and so on. This is because the new equilibrium tariff after an agent's exit makes the optimization problem of the next lowest agent isomorphic to the optimization problem of the exiting agent.

4.1 Cascade of Exits

In this subsection, we present our main result that a market shutdown results from a cascade of exits. The main mechanism for the market shutdown is based on the spillover effects of exits across agents when outside options are introduced. We show that the form of spillovers needed to generate a market shutdown do not exist in the model without outside options.

First, consider the model without outside options (the model of Attar et al. (2021)). The exit of low (cost) type agents has two effects: 1) it raises the average tariff for all remaining types, and 2) it lowers the total quantity purchased in the market. For example, suppose that agent 1 exits the market. Then, agent 2 has to pay a higher tariff to consume the same quantity as before, $\bar{c}_2 q_2 > \bar{c}_2(q_2 - q_1) + \bar{c}_1 q_1$. However, agent 2's indirect marginal rate of substitution along the new market tariff becomes $\tau_i^T(q,t) = \tau_i(q,t)$ because agent 2 cannot trade the contract $(q_1, T(q_1))$ that agent 1 was trading. Then, by Assumption 2, agent 2 is even more likely to enter/remain in the market ceteris paribus. Although the exit of agent 1 will decrease the total utility of agent 2, it does not increase agent 2's likelihood of exit. Agents enter/remain in the market based on their marginal rate of substitution and not on their total utility. Hence, one type's exit weakly increases the incentive for all other types to remain active in the market despite higher prices and lower quantities.

Outside options overturn the incentive to remain active in the market, and exits trigger additional exits. The reason is that agents decide to enter/remain in the market based on the total utility they achieve through market trades in addition to their marginal rate of substitution. Put simply, agents opt out of the market when the maximum utility they obtain through market trades falls below the utility they obtain through their outside option. Hence, when good types exit, which raises market prices and lowers equilibrium trade quantities, it can trigger additional exits among remaining higher types according to the same logic as the original exit. Therefore, the existence of outside options can create a cascade of exits and generate a larger decline of quantities traded compared with the baseline model without outside options.

Suppose the moldy lemons, type $n+1$ with $c_{n+1} > c_n$, enter the market. Without loss of generality, we assume that all agents enter the market in the equilibrium before the moldy lemons joined. As in the previous section, the new upper-tail conditional expected cost is given by (10). Denote equilibrium quantities by $\tilde{q}_i$ for each i that enters the market where

$$\tilde{q}_i - \tilde{q}_{i-1} \in \arg\max_q \left\{ u_i \left(\tilde{q}_{i-1} + q, \tilde{T}(\tilde{q}_{i-1}) + \tilde{\bar{c}}_i q \right) : q \right\},$$

and $\tilde{T}(q)$ is the equilibrium affine tariff with slope $\tilde{\bar{c}}_i$ over $[\tilde{q}_{i-1}, \tilde{q}_i]$. In addition, set $\tilde{q}_0 = \tilde{q}_j = 0$ for any j who exits the market. Suppose further, without loss of generality, that at least agent 1 exits the market after the entry of moldy lemons, which occurs when **Condition IT** holds:

$$\max_{q\geq 0} u_1(q, \tilde{\tilde{c}}_1 q) \leq \gamma_1. \tag{13}$$

Note that we do not need to check Condition EP, $\tau_1(0,0) \leq \tilde{\tilde{c}}_1$, because each agent prefers the outside option over the no-trade contract as $\gamma > u_i(0,0)$ for any i. We can extend this intuitive condition to a condition that any agent with type j that is below i exits the market. We denote such a condition, **Condition ML(i)–Moldy Lemons**, as:

$$\max_{q\geq 0} u_j\left(q, \tilde{\tilde{c}}_j q\right) \leq \gamma_j, \qquad \forall j < i, \tag{14}$$

where agents up to type i exit the market. Condition ML(i) is less strict than Condition EP because there is an additional case that prevents entry. Therefore, agents who did not exit the market under Condition EP may exit under Condition ML(i). We formally show that Condition ML(i) is necessary and sufficient to generate a cascade of exits up to agent i.

Theorem 2 (Cascade of Exits) *Any agent j such that $j < i$ exits the market in equilibrium if and only if Condition ML(i) is satisfied.*

Proof. The proof of sufficiency is straightforward as Condition ML(i) prevents entry of agents with type $j < i$ for the entry-proof market tariffs.

Now consider the proof of necessity. By Proposition 2, we check the lowest agent's entry decision for each candidate equilibrium. Consider an equilibrium in which agent 1 also enters the market. Then, the corresponding market tariff will be

$$T^0(q) \equiv \sum_{i\in N} \tilde{\tilde{c}}_i(q - q_{i-1})\mathbf{1}\left\{q \in [q_{i-1}, q_i]\right\},$$

which is based on the same quantities $\{q_i\}_{i\in N}$ as in the equilibrium before the moldy lemons entered. Under Condition IT, type 1 agent exits the market and the updated market tariff is

$$T^1(q) \equiv \sum_{i\in N} \tilde{\tilde{c}}_i(q - q^1_{i-1})\mathbf{1}\left\{q \in [q^1_{i-1}, q^1_i]\right\},$$

where $q^1_i - q^1_{i-1} \in \arg\max\left\{u_i(q^1_{i-1} + q, T^1(q^1_{i-1}) + \tilde{\tilde{c}}_i q : q\right\}$ with $q^1_1 = 0$. The exit of agent 1 lowers the utility of each agent because the first layer over the interval $[0, q_1]$ with the lowest tariff, $\tilde{\tilde{c}}_1$, disappears. Also, the same arguments in the proof of Theorem 1 hold as all the remaining agents are participating in the market with the higher average cost of service due to moldy lemons, so $q_i^1 \leq q_i$ for any $i \in N \backslash \{1\}$. Thus, agents $i > 2$ trading the next available lowest cost interval, $[q_1, q_2]$, suffer further utility declines even without the exit of agent 2. If the new market tariff T^1 induces agent 2 to exit, then it must be the case that

$$\max_{q \geq 0} u_2 \left(q, T^1(q)\right) = \max_{q \geq 0} u_2 \left(q, \tilde{\tilde{c}}_2 q\right) \leq \gamma_2,$$

and exiting the market gives higher utility to agent 2.

Now extend the argument recursively to finish the proof by mathematical induction. For an arbitrary $k < i$, suppose that under any candidate equilibrium, agents up to k exit the market. Then, the new market tariff becomes

$$T^k(q) \equiv \sum_{i \in N} \tilde{\tilde{c}}_i (q - q_{i-1}^k) \mathbf{1} \left\{ q \in [q_{i-1}^k, q_i^k] \right\},$$

where $q_i^k - q_{i-1}^k \in \arg\max \left\{ u_i(q_{i-1}^k + q, T^k(q_{i-1}^k) + \tilde{\tilde{c}}_i q : q \right\}$ with $q_1^1 = q_2^2 = \cdots = q_k^k = 0$. If agent $k+1$ exits the market under T^k, then

$$\max_{q \geq 0} u_{k+1} \left(q, T^k(q)\right) = \max_{q \geq 0} u_{k+1} \left(q, \tilde{\tilde{c}}_{k+1} q\right) \leq \gamma_{k+1}$$

should hold, because agent $k+1$ will still enter the market otherwise. If the above inequality does not hold, then the equilibrium of active markets is determined starting from $q_{k+1} > 0$ and the initial assumption is violated. Therefore, Condition ML(i) is necessary. ■

Outside options have two roles. First, outside options generate discontinuous jumps in market quantities, q_i, after the entry of moldy lemons. A small mass of moldy lemons m_{n+1} will trigger type i agent to exit with strictly positive quantity q_i when i's utility is close to the utility from the outside option as $u_i(q_i, T(q_i)) \approx \gamma_i$. In contrast, quantities always change in a smooth fashion when outside options are absent. Second, outside options generate negative spillovers resulting from exits that trigger additional exits, causing a discontinuous cascade of exits.[8] Unlike the marginal rate of substitution, which only increases as other agents exit, total utility decreases when other agents exit. Therefore, the utility level from market trades

[8]This mechanism is similar to the worsening adverse selection after a price increase in Stiglitz and Weiss (1981).

can fall below the utility of outside options, γ_i.

A natural question to ask is the following: How sensitive are the exit-cascades to the assumptions about the partition of types considered? Are exit cascades more or less likely when there are large masses of fewer types of agents or smaller masses of more types of agents? Let $\{I, m, u, c, \gamma\}$ represent an economy where $m = \{m_i\}_{i\in I}$, $u = \{u_i\}_{i\in I}$, $c = \{c_i\}_{i\in I}$, and $\gamma = \{\gamma_i\}_{i\in I}$. Consider another economy, $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, that is a *coarser partition* of $\{I, m, u, c, \gamma\}$ if the following holds:

1. $\hat{I} \subset I$.

2. If $i \in \hat{I}$ and $i+1 \in \hat{I}$, then $\hat{m}_i = m_i$, $\hat{u}_i = u_i$, and $\hat{c}_i = c_i$.

3. If $i \in \hat{I}$ and $i+1, \ldots, i+k \notin \hat{I}$, while $i+k+1 \in \hat{I}$, where $k \geq 1$, then agent $i \in \hat{I}$ *includes* agents $i, i+1, \ldots, i+k$ and $\hat{m}_i = \sum_{l=0}^{k} m_{i+l}$, $\hat{u}_i(q,t) = \dfrac{\sum_{l=0}^{k} m_{i+l} u_{i+l}(q,t)}{\sum_{l=0}^{k} m_{i+l}}$, $\hat{c}_i = \dfrac{\sum_{l=0}^{k} m_{i+l} c_{i+l}}{\sum_{l=0}^{k} m_{i+l}}$, and $\hat{\gamma}_i = \dfrac{\sum_{l=0}^{k} m_{i+l} \gamma_{i+l}}{\sum_{l=0}^{k} m_{i+l}}$.

The above definition implies that a coarser partition of an economy groups adjacent types of agents into one type of agent. The mass of the new type of agent is equal to the sum of all masses for each type in the group, and the servicing cost and outside option values are the weighted average cost and outside option values, respectively. The utility of this new agent is the weighted average utility across different types. For example, for $I = \{1, 2, 3, 4, 5\}$ and $\hat{I} = \{1, 2, 4\}$, $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ is a coarser partition of $\{I, m, u, c, \gamma\}$, if

$$\begin{aligned}
\hat{m} &= \{m_1, m_2 + m_3, m_4 + m_5\} \\
\hat{u} &= \left\{u_1, \frac{m_2 u_2 + m_3 u_3}{m_2 + m_3}, \frac{m_4 u_4 + m_5 u_5}{m_4 + m_5}\right\} \\
\hat{c} &= \left\{c_1, \frac{m_2 c_2 + m_3 c_3}{m_2 + m_3}, \frac{m_4 c_4 + m_5 c_5}{m_4 + m_5}\right\} \\
\hat{\gamma} &= \left\{\gamma_1, \frac{m_2 \gamma_2 + m_3 \gamma_3}{m_2 + m_3}, \frac{m_4 \gamma_4 + m_5 \gamma_5}{m_4 + m_5}\right\}.
\end{aligned}$$

Note that the upper-tail conditional expected cost remains the same.

Proposition 3 *Let $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ be a coarser partition of $\{I, m, u, c, \gamma\}$ with $i \in \hat{I}$ and $i+1 \notin \hat{I}$. Then, there exists a moldy lemon mass, m_{n+1}, such that i does not exit in $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, while i exits in $\{I, m, u, c, \gamma\}$.*

Proof. First, we show that if the entry of a mass of moldy lemons m_{n+1} causes the new type $i \in \hat{I}$ to exit in the coarser partition $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, then agent i exits in the original economy $\{I, m, u, c, \gamma\}$. Suppose that $i \in \hat{I}$, $i+1, \ldots, i+k \notin \hat{I}$, and $i+k+1 \in \hat{I}$, i exits the market in the economy $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$. Thus,

$$\max_{q \geq 0} \left[\frac{m_i u_i(q, \tilde{\bar{c}}_i q) + \cdots + m_{i+k} u_{i+k}(q, \tilde{\bar{c}}_i q)}{m_i + \cdots + m_{i+k}} \right] \leq \hat{\gamma} = \frac{m_i \gamma_i + \cdots + m_{i+k} \gamma_{i+k}}{m_i + \cdots + m_{i+k}}$$

holds. Suppose the contrary that i does not exit the market in the economy $\{I, m, u, c, \gamma\}$. Then,

$$\max_{q \geq 0} u_i(q, \tilde{\bar{c}}_i q) = u_i(q_i, \tilde{\bar{c}}_i q_i) > \gamma_i.$$

Thus, for each l, $u_{i+l}(q_i, \tilde{\bar{c}}_i q_i) > \gamma_{i+l}$ holds by (12), where $1 \leq l \leq k$. However, this implies the weighted average of utilities would exceed the weighted average of outside options. In other words,

$$\max_{q \geq 0} \left[\frac{m_i u_i(q, \tilde{\bar{c}}_i q) + \cdots + m_{i+k} u_{i+k}(q, \tilde{\bar{c}}_i q)}{m_i + \cdots + m_{i+k}} \right] \geq \frac{m_i u_i(q_i, \tilde{\bar{c}}_i q_i) + \cdots + m_{i+k} u_{i+k}(q_i, \tilde{\bar{c}}_i q_i)}{m_i + \cdots + m_{i+k}} > \hat{\gamma},$$

which is a contradiction.

Now we show the converse that agent i, who exits the market in the economy $\{I, m, u, c, \gamma\}$ due to the entry of moldy lemons with mass m_{n+1}, may not exit in the coarser partition $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ with the same mass of moldy lemons entering the market. Suppose that $i \in \hat{I}$, $i+1, i+2, \ldots, i+k \notin \hat{I}$, and $i+k+1 \in \hat{I}$. Agent i exits the market, if

$$\max_{q \geq 0} u_i(q, \tilde{\bar{c}}_i q) \leq \gamma_i. \tag{15}$$

By continuity of the utility function, maximand (by Berge's maximum theorem), and upper-tail conditional expected cost, the left-hand side of (15) is continuously decreasing in the moldy lemon mass m_{n+1}. Then, there exists $\underline{m}$ such that when $m_{n+1} = \underline{m}$,

$$\max_{q \geq 0} u_i(q, \tilde{\bar{c}}_i q) = u_i(q_i, \tilde{\bar{c}}_i q_i) = \gamma_i.$$

For each l, $u_{i+l}(q_i, \tilde{\bar{c}}_i q_i) > \gamma_{i+l}$ holds by (12), where $l \geq 1$. Again, the utility of $i \in \hat{I}$, the weighted average of utilities, become

$$\max_{q\geq 0}\left[\frac{m_i u_i(q,\tilde{\bar{c}}_i q)+\cdots+m_{i+k}u_{i+k}(q,\tilde{\bar{c}}_i q)}{m_i+\cdots+m_{i+k}}\right]\geq \frac{m_i u_i(q_i,\tilde{\bar{c}}_i q_i)+\cdots+m_{i+k}u_{i+k}(q_i,\tilde{\bar{c}}_i q_i)}{m_i+\cdots+m_{i+k}}$$
$$>\hat{\gamma}=\frac{m_i\gamma_i+\cdots+m_{i+k}\gamma_{i+k}}{m_i+\cdots+m_{i+k}},$$

and type $i\in\hat{I}$ agent does not exit. Hence, there exists a moldy lemon mass $m_{n+1}=\underline{m}$, which makes i to exit in $\{I,m,u,c,\gamma\}$ but not in $\{\hat{I},\hat{m},\hat{u},\hat{c},\hat{\gamma}\}$. ■

This result shows that markets are less vulnerable to freezes and exit cascades when the partition of types in the economy shrinks. At the other extreme, exit cascades are more likely when the distribution of types is close to a continuum. The reason is that the sufficient mass of new moldy lemons needed to trigger market shutdowns increases as the relative mass of exiting agents increases.

A corollary of Proposition 3 is that markets can be more vulnerable to exits as the number of types in the economy grows large despite no change in underlying aggregate risk or uncertainty. The result emphasizes why having multiple types in the model is important for generating large swings in trade with a small mass of moldy lemons. Thus, a model with only two types, while tractable and perhaps sufficient to highlight certain forces, does not correctly capture the vulnerability of the market to exit cascades more generally.

4.2 Discussion of Outside Options

Though sufficient, the condition (12) is not necessary for exits to generate additional exits due to negative spillovers. The only additional complexity if the condition does not hold is that the pattern of exits could become more complicated as there might be no cutoff type, θ, that partitions agents into those who exit and remain. In particular, some intermediate valued agent may remain in the market if the agent's outside option utility is very low.

We introduce two different interpretations of outside options to justify our assumption. The first interpretation is a fixed entry cost. Suppose that each agent must pay a fixed entry cost given by $\xi>0$ if they choose to enter the market. Upon entry, the agent trades a positive quantity, q, while paying the market tariff of $T(q)$. The outside option is not paying the entry cost of ξ, or a negative payment with zero trade quantity: $u_i(0,-\xi)$. Then, by continuity and Assumption 2, there exists $\underline{q}_i$ such that

$$u_i(\underline{q}_i,0)=u_i(0,-\xi)=\gamma_i$$

for any i. Also, by the single-crossing property,

$$u_i(\underline{q}_i, 0) \geq u_i(0, -\xi) \Rightarrow u_j(\underline{q}_i, 0) > u_j(0, -\xi),$$

for any $j > i$. Therefore, $\underline{q}_i$ is decreasing in the index i, and

$$u_i(q, t) \geq \gamma_i \Rightarrow u_j(q, t) > \gamma_j, \qquad \forall i < j,$$

by the single-crossing property.

The second way to interpret outside options is to consider the opportunity cost of agents entering a separate market that requires costly verification of agent's type.[9] In this market, agents pay a fixed cost of κ to trade, and the market can verify each agents type. Therefore, agent 1 may be happy to pay κ and get the lowest price c_1 for the quantity q_1, whereas agent n would not be happy to pay κ and pay the highest price c_n. Thus, whenever agent $j > i$ exits, agent i should also exit as

$$u_j(q, t) \leq \gamma_j \Rightarrow u_i(q, t) < \gamma_i \qquad \forall i < j,\ \forall q > 0,\ \forall t,$$

which implies that (12) holds as it is the contraposition of (12).

4.3 Implications and Broader Discussion

The model of moldy lemons with outside options generates an important feature of the market—a small mass of moldy lemons can generate a sudden market shutdown. This property not only resembles the movements in the financial markets in the real world, but also has an important policy implication: relatively inexpensive policy interventions can prevent sudden and costly market collapses. Policy needs only to prevent the small mass of moldy lemons from contaminating the market and moderately increasing the overall supply cost. For example, if the social planner lowers the market tariff with a total subsidy of $(\tilde{\bar{c}}_1 - \bar{c}_1) q_1 \sum_{i \in I} m_i$, then it is sufficient to prevent the exit of type 1 and the cascade of exits afterwards.

[9] The secondary market structure of agency mortgage-backed securities (MBS) is a good example. A majority of MBS are traded in the to-be-announced (TBA) market, which pools heterogeneous MBS into a few liquid TBA contracts but induces adverse selection. At the same time, traders can trade high-value MBS outside the TBA market in a much less liquid specified-pool (SP) market by specifying the individual CUSIP, but traders pay higher trading cost in the SP market (Huh and Kim, 2021).

This policy would be desirable as long as the potential welfare losses, $\sum_{i \in I} (u_i(q_i, T(q_i)) - \gamma_i)$[10], on top of the spillovers to other markets is higher than the intervention cost.[11] Therefore, our model provides a simple yet important reason to support market functioning even with lemons to prevent a more widespread market breakdown.

The model does not rely on detailed market structure or other types of complex interactions of the agents. Therefore, the model could be applied to various contexts and markets with adverse selection to provide insights on how adverse selection problems can cause partial or full market shutdowns through a variety of changes within a given setting. For example, as we show below, if the degree of adverse selection in market relative to the outside option becomes stronger, then the market is more vulnerable to sudden market shutdowns.

Our result that economies with more types are more vulnerable to exit cascades provides a general theoretical underpinning to the information production literature (see for example, Gorton and Ordoñez (2019, 2020); Dang et al. (2020)). In particular, one interpretation is that economies with more types grouped together have less precise or opaque information about each individual type. This interpretation is apt for models where an agent's type is stochastic as in production economies or asset holdings where agents maximize over the expected value of their types. Our model shows that if more precise information about each agent's type results in more recognizable agents in the economy and more asymmetric information between buyers and sellers, then the market is more unstable. This is in line with Dang et al. (2020) who show that information production can lead to a collapse in the market. Our result shows a similar phenomenon with a more simple, static model.

The cascade of exits is determined not only by the degree of adverse selection, but also by the outside options of the agents. If entry is very costly—for example, because of high entry or regulatory costs due to heavy usage of balance sheets—the outside option of agents not entering the market could be more profitable. Then, there will be more exits in the market. A moderate reduction of such costs (or reduction of the opportunity cost) could drastically change the allocation by preventing the chain of exits and sudden collapse of the quantities traded in the equilibrium.

[10] Since suppliers are competitive, they break even in any equilibrium even under complete breakdown. Therefore, the measure of social welfare is simply the sum of utilities across all types of agents (buyers).

[11] There are still many complicated issues related to the optimal interventions such as changing incentives under the new rules (or mechanisms) as discussed in Philippon and Skreta (2012).

5 Numerical Analysis

With the insights on market breakdown in the model with outside options, we analyze the model further numerically. In particular, we bring the model into the context of insurance market with binary loss model of Rothschild and Stiglitz (1976) and Hendren (2014) following the idea of Dubey and Geanakoplos (2019). With the numerical model, we first compare the results between the baseline model of Attar et al. (2021) without outside options and our model with outside options. Then, we perform comparative statics to obtain more implications of moldy lemons and market shutdowns.

5.1 Model Setup and Equilibrium

Agents have initial endowment $(e, 0)$ for (e_g, e_b), where e_g and e_b represent good state and bad state endowment, respectively. They will consume $(x_g, x_b) = (e, 0)$ under autarky, where x_g and x_b represent good state and bad state consumption, respectively. Suppose that agents have constant relative risk aversion (CRRA) and agent i's utility function is

$$v_i(x_g, x_b) = p_i \log(1 + x_b) + (1 - p_i) \log(1 + x_g),$$

where p_i is the probability of agent i faces a loss and receives bad state endowment. The marginal rate of substitution for agent i is

$$\tau_i(x_g, x_b) \equiv \frac{\dfrac{\partial v_i(x_g, x_b)}{\partial x_b}}{\dfrac{\partial v_i(x_g, x_b)}{\partial x_g}} = \frac{p_i}{1 - p_i} \frac{1 + x_g}{1 + x_b}.$$

Following the assumption of competitive suppliers in the main model, the service cost for suppliers is $c_i = p_i$, so they need p_i amount of x_g to insure $1 - p_i$ amount of x_b to agent i. Under nonexclusive contracts, suppliers require the upper-tail conditional expected cost,

$$\bar{c}_i = \sum_{j \geq i} \frac{m_j p_j}{\sum_{j \geq i} m_j},$$

where m_i is the relative mass of agent i.

Denote the additional utility level of taking the outside option on top of utility under no trade, $v_i(e, 0)$, as γ for each i. Agents will compare the level of utility they can get from their optimal consumption bundle in the market to the level of utility they can get from the outside option and decide whether to enter or exit the market. Utilizing the results on the general model, the Condition ML(i) for this model is

$$v_i(x_g^i, x_b^i) \leq \gamma_i = v_i(e, 0) + \gamma,$$

where $v_i(e, 0) + \gamma$ is decreasing in i. If the above inequality holds, then any agent $j \leq i$ exits the market and we set $x_b^j = 0$.

5.2 Effect of Outside Options and Moldy Lemons

We verify the main result of this paper with the numerical model. Figure 3 shows equilibrium of the baseline model in the top panel and equilibrium of the model with outside options in the bottom panel. For each panel, the horizontal axis represents the good state consumption x_g and the vertical axis represents the bad state consumption x_b with the 45-degree line depicted as dotted lines. Each colored curve represents the consumption possibility frontier for a given mass of moldy lemons in the market. Different shapes of dots represent each agent's optimal consumption bundle for the given mass of moldy lemons and other agents' consumption in the equilibrium.

First, the results show that x_b decreases monotonically with the increase in the mass of moldy lemons, verifying Theorem 1. As more moldy lemons enter the market, the upper-tail conditional expected cost $\tilde{\tilde{c}}_i$ increases, depressing the consumption possibility frontier for every agent in the market. With the higher cost, agents purchase less (insurance) contracts.

Second, the results show a market shutdown (collapse) in the model with outside options, which does not exist in the model without outside options. Even though both models show decrease in overall quantity of contract purchases, the model without outside options shows a smooth contraction of quantities traded over the increase in the mass of moldy lemons. Thus, even when the moldy lemons mass is 0.4, all agents still enter the market and trade in positive amounts. In contrast, the model with outside option shows the exit of agents and total market shutdown at the moldy lemons mass of 0.3. Thus, the numerical exercise shows how the existence of outside options can generate market shutdowns even for a small mass of moldy lemons.[12]

[12]The definition of market shutdown here is the exit of all agents except for moldy lemons.

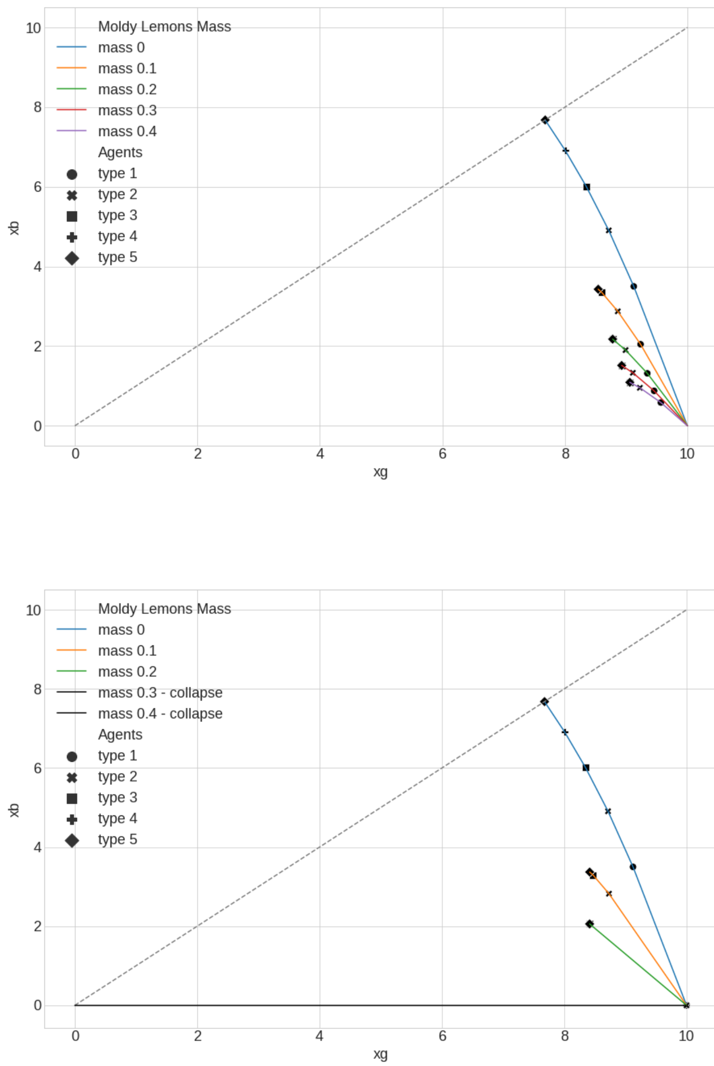
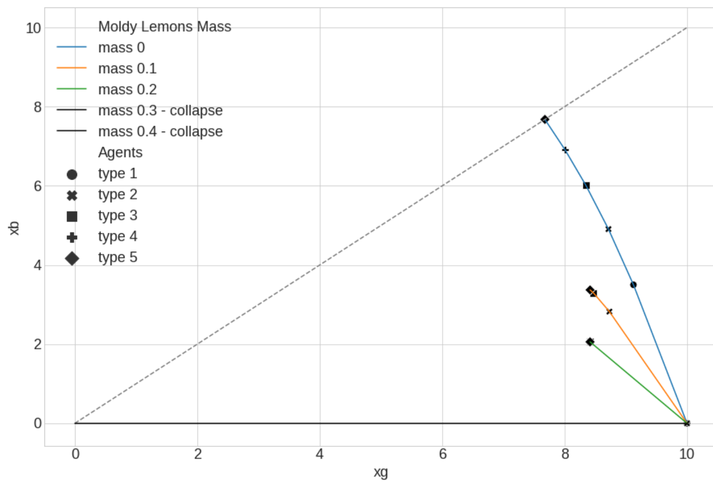


Figure 3: Consumption bundles of models without and with outside options

Note: Each curve represents consumption possibility frontier and consumption bundles of each agent represented by different shape of dot for a different mass of moldy lemons.

5.3 Coarse Partition of Types and Moldy Lemons

Using the model with outside options, we analyze comparative statics of Proposition 3 on the division of types depicted in Figure 4. In particular, we group a couple of adjacent types of agents into a one type of agent with the average cost of servicing them (average probability of bad state), with the mass as the sum of mass for each type. The top panel of Figure 4 is the baseline case as shown in the bottom panel of Figure 3, while the bottom panel of



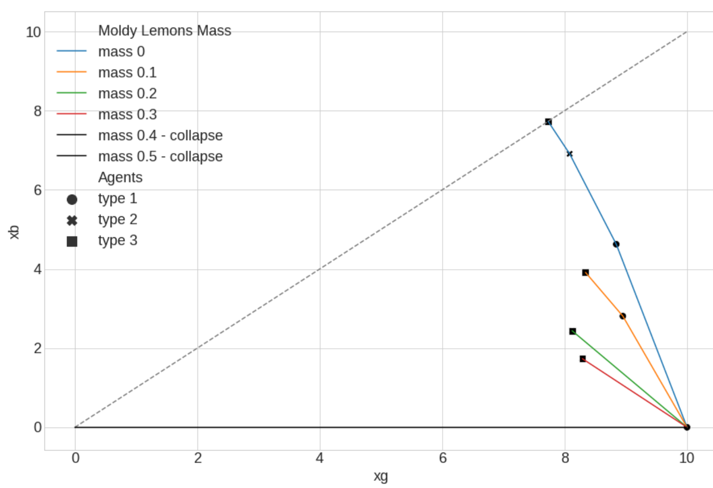


Figure 4: Consumption bundles of models with different partition of types

Note: Each curve represents consumption possibility frontier and consumption bundles of each agent represented by different shape of dot for a different mass of moldy lemons.

Figure 4 is the case with less division of types of agents. In particular, we combine type 1 and 2 together to create a new type 1 agent with probability and mass as

$$\hat{p}_1 = \frac{m_1 p_1 + m_2 p_2}{m_1 + m_2}$$
$$\hat{m}_1 = m_1 + m_2.$$

Similarly, we also combine type 3 and 4 in the baseline case into a new type 2 agent as

$$\hat{p}_2 = \frac{m_3 p_3 + m_4 p_4}{m_3 + m_4}$$
$$\hat{m}_2 = m_3 + m_4,$$

while simply renaming the previous agent 5 as agent 3. Therefore, the market-wide uncertainty and service cost remain the same as before.

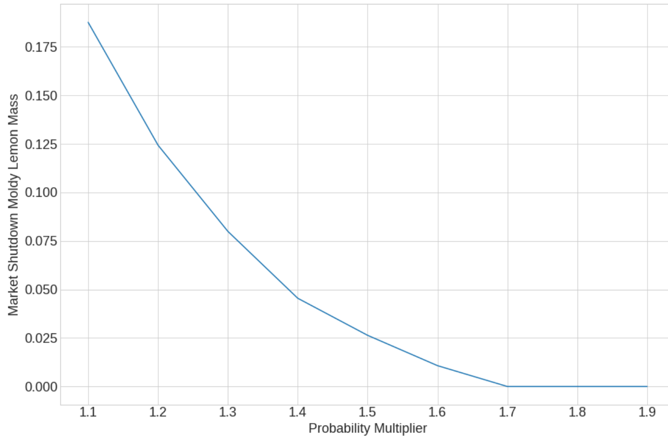
The numerical results show that the decrease in the partition of types makes the equilibrium less vulnerable to moldy lemons. The example with fewer types shows fewer exits across varying masses of moldy lemons. Also, full market shutdown does not happen even when the moldy lemons mass is 0.3, in which the market collapses under the baseline case. The results imply that if more agents are unsure about their true type, the market is less likely to experience market shutdown as the exit cascades is less likely.

The reason is due to Theorem 2: The sufficient mass of new moldy lemons needed to trigger market shutdowns increases as the relative mass of existing agents increases. For example, agent 1 in the baseline case might have exited under the increased cost due to moldy lemons' entry, but agent 2 might have stayed had agent 1 stayed in the market. However, the exit of agent 1 could have triggered agent 2 to exit as well. If both agents 1 and 2 are unsure about their true type, then they could both stay in the market, preventing the cascade of exits.

5.4 Aggregate Shock and Market Shutdowns

We analyze the effect of an aggregate shock to the market's vulnerability to moldy lemons. The aggregate shock will increase p_i for each agent i proportionally by multiplying β, which is the probability multiplier. As seen in the previous exercises, there is a tipping point for when a mass of moldy lemons causes a total market shutdown. Define such a value as the market shutdown moldy lemon mass denoted as m^*_{n+1}. Under this parameter, Condition ML(n) is satisfied—that is, the market shuts down.

Figure 5 depicts how the market shutdown moldy lemon mass m^*_{n+1} changes with the changes in the probability multiplier β. The top panel is the case with fixed outside options, in which the absolute level of $v_i(e, 0) + \gamma$ with v_i calculated before multiplying with β is used. Therefore, agent i's exit decision is based on comparing the utility level of the optimal consumption bundle and the outside option utility level $\gamma_i \equiv v_i(e, 0) + \gamma$. The bottom panel of Figure 5 is the case with proportional outside options, in which $v_i(e, 0)$ also changes to



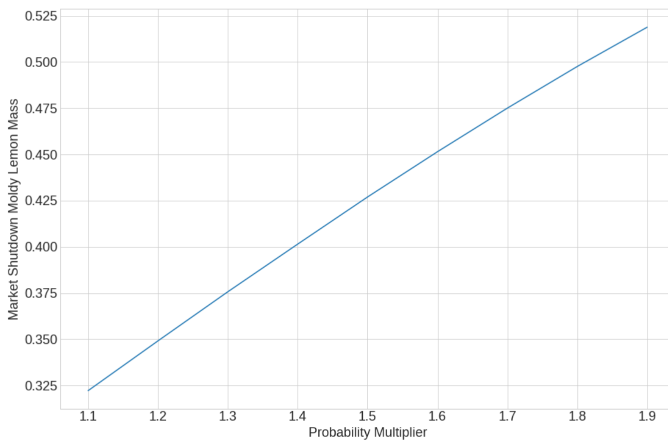


Figure 5: Market shutdown moldy lemons mass m^*_{n+1} for each probability multiplier β for models with fixed and proportional outside options

Note: Each curve represents the required mass of moldy lemons m^*_{n+1} that generate market shutdowns for a given level of aggregate increase in bad state probability.

$\tilde{v}_i(e, 0)$ following the increase in βp_i. Therefore, agent i's exit decision is based on comparing the utility level of the optimal consumption bundle and the new outside option utility level $\tilde{\gamma}_i \equiv \tilde{v}_i(e, 0) + \gamma$.

The simulation shows the opposite results of the two cases. When the outside options are fixed, increases in probabilities of bad state generate market shutdowns much easily as agents are much more likely to suffer losses in the bad state with the higher price of contracts whereas the outside option utility levels remain fixed. In contrast, if the outside option values also decrease proportionally, then the decrease in the total utility of the outside option can dominate the decrease in the total utility of the optimal consumption bundle. This is because the difference between no trade and the optimal consumption bundle increases as the marginal rate of substitution increases.

This result implies that vulnerability from moldy lemons depends on how the aggregate shocks are distributed across different markets. If a specific market is hit harder than the others, the market is more vulnerable to cascade of exits with the entry of moldy lemons. Finally, this result implies that having a policy that restricts entry by requiring additional cost (that increases relative γ) can rather make the market more vulnerable to moldy lemons. The restrictive policy itself might have a role in enhancing transparency and deterring entry of bad types, but it could also make cascade of exits more likely.

6 Conclusion

We show that the entry of a small mass of the worst type of agents (moldy lemons) can induce a cascade of exits and market shutdown. The model is based on the general framework of Attar et al. (2021), which is one of the most general and least restrictive models in the literature. The baseline model does not show a market shutdown after the entry of a small mass of moldy lemons, because the exit of agents rather increases marginal rate of substitution of the remaining agents. We extend the baseline model with a simple yet realistic extension of introducing outside options of the agents. After the entry of moldy lemons, trade quantities plunge because the exit of an agent decreases total utility of the remaining agents, which could trigger a cascade of exits. Our results suggest a parsimonious yet realistic way of generating sudden market shutdowns without imposing additional structure, belief- or sentiment-driven runs, or institutional details on the model. Thus, our model is widely applicable to many different markets and contexts. Lastly, the model provides simple insights on properties of market shutdowns as it is relatively free from other confounding features. For example, we show that economies with more precise information about the type of agents trading are more prone to exit cascades. This result provides an alternative theoretical underpinning for market failures due to information production in the recent models of Gorton and Ordoñez (2019, 2020); Dang et al. (2020).

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Appendix

A Details of Numerical Simulations

For computational tractability, we would like to represent agents' optimization problem as isomorphic optimization problem across all agents by adjusting the endowments e^i for each agent i. This can be done by exploiting the single-crossing property and any agent $i > j$ would consume as much as agent j does in equilibrium. For a given e^i, the optimal consumption bundle is

$$\begin{aligned} x_g^* &= (1-p_i)e^i - \frac{p_i - \bar{c}_i}{1-\bar{c}_i} \\ x_b^* &= \frac{p_i - \bar{c}_i}{\bar{c}_i} + \frac{(1-\bar{c}_i)p_i}{\bar{c}_i} e^i, \end{aligned}$$

for an interior solution. If it is a corner solution, then $(x_g^*, x_b^*) = \left(0, \frac{1-\bar{c}_i}{\bar{c}_i} e^i\right)$ or $(x_g^*, x_b^*) = (e^i, 0)$.

From the results in the general model, we know that agent 1 first decides on the optimal quantity q_1^* for the given price $\bar{c}_1$ and then agent 2 decides on q_2^* for the price $\bar{c}_2$ on top of purchasing q_1^*, and so forth. Agent 1's optimal consumption bundle is

$$\begin{aligned} x_g^1 &= (1-p_1)e - \frac{p_1 - \bar{c}_1}{1-\bar{c}_1} \\ x_b^1 &= \frac{p_1 - \bar{c}_1}{\bar{c}_1} + \frac{(1-\bar{c}_1)p_1}{\bar{c}_1} e, \end{aligned}$$

assuming that they have interior solutions without loss of generality. For agent 2, the same optimality condition should hold, but the budget constraint is different from the previous representation. This is because agent 2 can purchase the bundle in a cheaper price $\frac{\bar{c}_1}{1-\bar{c}_1}$ instead of $\frac{\bar{c}_2}{1-\bar{c}_2}$ up to x_b^1. Therefore, the updated budget constraint for agent 2 becomes

$$\begin{aligned} x_g^2 &= e - \frac{\bar{c}_1}{1-\bar{c}_1} x_b^1 - \frac{\bar{c}_2}{1-\bar{c}_2}(x_b^2 - x_b^1) \\ &= e + \left(\frac{\bar{c}_2}{1-\bar{c}_2} - \frac{\bar{c}_1}{1-\bar{c}_1}\right) x_b^1 - \frac{\bar{c}_2}{1-\bar{c}_2} x_b^2. \end{aligned}$$

Therefore, we simply change the endowment from e to

$$e^2 = e + \left(\frac{\bar{c}_2}{1 - \bar{c}_2} - \frac{\bar{c}_1}{1 - \bar{c}_1} \right) x_b^1$$

for the agent 2's budget constraint. Thus, the optimal consumption bundle for agent 2 is

$$x_g^2 = (1 - p_2)e^2 - \frac{p_2 - \bar{c}_2}{1 - \bar{c}_2}$$
$$x_b^2 = \frac{p_2 - \bar{c}_2}{\bar{c}_2} + \frac{(1 - \bar{c}_2)p_2}{\bar{c}_2} e^2.$$

Agent 3's problem is isomorphic to agent 2's problem except that agent 3's endowment is

$$e^3 = e + \left(\frac{\bar{c}_2}{1 - \bar{c}_2} - \frac{\bar{c}_1}{1 - \bar{c}_1} \right) x_b^1 + \left(\frac{\bar{c}_3}{1 - \bar{c}_3} - \frac{\bar{c}_2}{1 - \bar{c}_2} \right) x_b^2,$$

and agent 3's optimal consumption bundle becomes

$$x_g^3 = (1 - p_3)e^3 - \frac{p_3 - \bar{c}_3}{1 - \bar{c}_3}$$
$$x_b^3 = \frac{p_3 - \bar{c}_3}{\bar{c}_3} + \frac{(1 - \bar{c}_3)p_3}{\bar{c}_3} e^3.$$

For a general agent i, agent i's updated endowment is

$$e^i = e + \sum_{j<i} \left(\frac{\bar{c}_{j+1}}{1 - \bar{c}_{j+1}} - \frac{\bar{c}_j}{1 - \bar{c}_j} \right) x_b^j,$$

and agent i's optimal consumption bundle becomes

$$x_g^i = (1 - p_i)e^i - \frac{p_i - \bar{c}_i}{1 - \bar{c}_i}$$
$$x_b^i = \frac{p_i - \bar{c}_i}{\bar{c}_i} + \frac{(1 - \bar{c}_i)p_i}{\bar{c}_i} e^i.$$

Given these setup, the parameters of the model are as in the following Table 1.

For the numerical procedure, we check Condition ML(i) starting from $i = 1$ and updating the endowment of $e^{i+1} = e$ whenever the condition is satisfied. The algorithm repeats this until it finds the agent that does not violate the test. Then, we proceed with the rest of the agents' consumption quantities using the iterative representation.

Parameter	Description	Value
e	good state endowment	10
$(p_1, p_2, \dots, p_5)$	probability of bad state	$(0.1, 0.15, \dots, 0.3)$
$(m_1, m_2, \dots, m_5)$	mass of each type	$(0.2, 0.2, \dots, 0.2)$
γ	outside option utility level	0.0758

Table 1: Parameter values for numerical simulations

The algorithm that solves this numerical model is the following: For each i,

1. Calculate the endowment of agent i, e^i, using the previous agents' quantities.
2. Derive the optimal quantity for agent i, (x_g^i, x_b^i) under e^i and $\bar{c}_i$.
3. Compute the utility level $v_i(x_g^i, x_b^i)$ and compare that to $v_i(e, 0) + \gamma$.
4. If it is above $v_i(e, 0) + \gamma$, then the (x_g^i, x_b^i) quantity is the optimal quantity. Otherwise, set $(x_g^i, x_b^i) = (e, 0)$.
5. Move to the next agent $i + 1$ and repeat from the first step.