Dynamic and Stochastic Search Equilibrium

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Abstract

I study the business cycle properties of wage posting models with random search, for which the distributions of employment and wages play a nontrivial role for the equilibrium path. In fact, the main result of this paper is that *the distribution of firms is one of the most important elements to understand business cycle fluctuations in the labor market.* The distribution of firms (1) determines which shocks are relevant for the labor market, (2) implies that wage rigidity does not significantly amplify shocks, and (3) puts discipline on the relative value of the flow opportunity cost of employment. To assess these type of models quantitatively, I propose a new algorithm that finds the steady state and computes transitional dynamics rapidly. Hence, integrating wage posting models with random search to larger models becomes possible (and easy) with this new algorithm.

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1 Introduction

Wage posting models emerged as an appealing way of studying wage dispersion and growth, as firms post high wages to prevent their employees from taking other job offers and to poach more workers from other companies. Since the wage posting model of Burdett and Mortensen (1998), significant progress has been achieved in this literature. Moscarini and Postel-Vinay (2013, 2016) prove the existence and uniqueness of a rank-preserving equilibrium in the context of a model with random search in which firms post and commit to state-contingent wage contracts. Menzio and Shi (2011) propose a model with directed search and random match quality in which agents' decision rules do not depend on the distribution of employment in equilibrium, simplifying the simulation of the model. Coles and Mortensen (2011) develop a model in which firms cannot commit to wage contracts, but the effect of offering a high wage on firms' reputations works as a commitment device. They also show that under some assumptions, the equilibrium path depends only on the level of employment and not on its distribution.

In spite of the significant theoretical progress in this literature, quantifying the cyclical properties of these types of models has remained a challenging task.[1] In this paper, I study the business cycle properties of wage posting models with random search by proposing a new algorithm that computes the steady state and transitional dynamics of the model rapidly. The goal is to study a class of models in which the distribution of employment and wages matter for agents' decision rules and the aggregate equilibrium path. These models not only provide a richer description of the economy but also enable us to study in deeper detail distributional dynamics facts documented, for example, by Haltiwanger, Hyatt, and McEntarfer (2015) and Moscarini and Postel-Vinay (2012) in an economy with a well-defined distribution.

In fact, the main message from this paper is that *the distribution of firms is one of the most important elements to understand the business cycle fluctuations in the labor market.* For example, the distribution of firms determines which shocks are relevant for the volatility in the labor market. In addition, the distribution of firms indicates that wage rigidity generates little amplification of shocks in the labor market, and it provides information about the relative value of the flow opportunity cost of employment (FOCE).[2].

I present a model that extends the work by Moscarini and Postel-Vinay (2013) [MPV13]. I introduce capital and a strictly concave utility function for households, and I assume imperfect substitution between jobs. There is a continuum of firms that produce a homogeneous good that is sold in a competitive market to the household that can be used for consumption or capital

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[1]For example, Moscarini and Postel-Vinay (2016) write *"In a series of articles (...) we explore, both theoretically and empirically, the business cycle implications of the wage posting paradigm. Progress in this direction has been stunted by technical difficulties in finding equilibrium where the law of one price fails"* (p. 136).

[2]FOCE is the forgone value of unemployment benefits plus the forgone value of non-working activities in terms of consumption.

accumulation. *A priori*, the only difference among firms is their permanent log-TFP level. Firms open new vacancies and face a convex recruiting cost function that depends either on the amount of vacancies or hires. Following the work of MPV13, I initially assume that firms post and commit to state-contingent wage contracts (values of employment) to all of their workers. However, I show that that assumption results in counterfactual responses and business cycle moments for wages. For that reason, I also propose a version of this model in which firms post and commit to wages to all of their employees, rather than values of employment, and assume wage rigidity. I refer to these two versions of the model as value posting and wage posting models, respectively. However, in all cases, workers' decision to stay at or leave a job depends exclusively on the "value of employment." Based on the results of this paper:

1. I show that aggregate productivity shocks (alone) are unlikely to be an important driver of the business cycle fluctuations in the labor market. In contrast to the standard Diamond-Mortensen-Pissarides model (DMP), this result holds even for large FOCE values and in the presence of rigid wages. The reason for this result is that productivity represents a larger fraction of the value of a filled vacancy for the least-productive firms than for the most-productive firms. Hence, productivity shocks generate a larger hiring response from the least-productive firms, which are small and represent a small fraction of aggregate employment.

2. As in the standard DMP model, the FOCE value is key to determining how much shocks are amplified in the labor market. But that is where the similarities end. Large FOCE values do not significantly amplify productivity shocks and, in fact, reduce the amplification of other relevant shocks—for instance, recruiting cost shocks. The reason is that the FOCE becomes more sensitive to changes in employment as it gets bigger, preventing aggregate employment from increasing in booms. Also, in the standard DMP model the relative FOCE value is, to a large extent, a "free" parameter. However, I show that information on the distribution of firms helps determine the relative size of the FOCE in wage posting models.

3. Wage rigidity does not significantly amplify shocks in the labor market as wages represent a large fraction of the value of a filled vacancy only for the least-productivity firms, which are small and account for a small fraction of total employment.

4. Recruiting cost shocks and matching efficiency shocks are likely to be import drivers of the business cycle volatility in the labor market. Those shocks generate large volatility and good co movements in the labor market. These two shocks, in contrast to productivity shocks, tend to affect all firms homogeneously.

5. As in the standard DMP model, separation rate shocks can generate large fluctuations in the labor market but generate counter factual dynamics. For example, those shocks generate a

positive correlation between unemployment and hires.

Moscarini and Postel-Vinay (2016) [MPV16] also propose an algorithm for solving these types of models. Even though I am able to reproduce the simulated business cycle moments of MPV16 with my algorithm, my conclusions are significantly different. In particular, my findings seem at odds with the claim that value posting models (with productivity shocks alone) can generate good business cycle moments in the labor market. Thanks to the tractability of my algorithm, I am able to show that the large volatility that results from the MPV16 calibration comes exclusively from the assumption that productivity shocks are correlated with changes in the separation rate. I show that in the absence of that correlation, (1) labor market volatility is insignificant, and (2) the movements in the separation rate generate the same counterfactual dynamics that I present in this paper—for example, the positive correlation between unemployment and hires.

As mentioned before, to assess quantitatively the performance of this model, I propose a new algorithm that computes transitional dynamics in seconds and may be of interest in its own right. This new method consists, mainly, of three steps: First, given a distribution of idiosyncratic productivities, I find the exact solution for a finite number of points, and approximate the remaining points by interpolating. Second, I compute the deterministic steady state by iterating on a guess for the vacancy decision rule or value of employment. Third, I take a numerical first-order approximation around the deterministic steady state, as proposed by Reiter (2009). While I see the MPV16 algorithm as a useful method that enables researchers to answer particular questions that my algorithm may not, the method proposed in this paper has some particular (and powerful) advances over MPV16. My method does not suffer from the *curse of dimensionality*. I can include as many shocks and state variables as I want without increasing significantly the computational burden, allowing researchers to study other frictions and sources of aggregate fluctuations. Hence, one can easily integrate wage posting models with random search in an even more general framework (such as a medium scale New-Keynesian model). Also, because of the nature of my algorithm, stochastic simulations and impulse response functions are an easy and useful exercise to implement.

The rest of this paper is organized as follows: Section 2 presents the model. Section 3 defines and characterizes the equilibrium of the model. I describe the computational method in Section 4, present the calibration of my model in Section 5, and present the results in Section 6. Then, I compare my results with MPV16 in Section 7. Finally, Section 8 concludes and discusses some caveats related to my results.

2 Theoretical Framework

The model presented in this section is a generalization of the wage posting model presented in MPV13. I introduce capital and a strictly concave utility function for households. I also allow for imperfect substitution between jobs, which will help me explain the observed wage dispersion in the economy.

2.1 Model Overview

There are two types of agents in this economy: households and firms. There is a representative household in the economy made up of a continuum of workers. The household derives utility from consumption and leisure, discounts future utility at rate β, supplies capital and labor to firms, and owns all firms in the economy. Capital is supplied in a perfectly competitive market at the capital rental rate r_t and depreciates at rate δ_k, while labor supply is subject to search frictions. I assume complete consumption insurance, which implies that workers seek to maximize income for the household. A worker can be employed or unemployed at each point in time. Unemployed workers receive unemployment compensation b and are matched with a firm with probability q_t. Employed workers are separated from their job with exogenous probability δ_{nt}, in which case they must spend at least one period in unemployment before they can be matched with another firm. Employed workers can search on the job. An employed worker is matched with another firm with probability $\bar{i} \cdot q_t$, where $\bar{i}$ is the search intensity of employed workers relative to unemployed workers and is fixed. However, employed workers only change jobs if they find a firm that offers a greater or equal value of employment.

There is a continuum of firms indexed by j with mass normalized to 1. All firms produce a homogeneous good that is sold in a competitive market to the household and can be used for consumption or capital accumulation. *A priori*, the only difference among firms is their permanent log-TFP level, which is denoted by a_j and is distributed across firms according to a continuous pdf f over the interval $(-\infty, \infty)$. Without loss of generality, I assume that a_j is increasing in j ($a_x \geq a_y$ for all $x \geq y$), and to save on notation $f_j = f(a_j)$. Firms produce with capital k_{jt} and labor n_{jt}, and firms' output is denoted by $y_{jt} = e^{a_j + a_t} k_{jt}^{\alpha} n_{jt}^{1-\alpha}$, where a_t stands for aggregate log-TFP, which is common to all firms and follows an exogenous process. At the beginning of each period, firms rent capital, open new vacancies (v_{jt}), and post a (net) employment value for the next period ($W_{jt+1} \geq 0$).[3] A vacancy is matched with a worker with probability $\tilde{q}_t$. If a vacancy is matched with an unemployed worker, in equilibrium, the vacancy is filled with probability 1. However, if a vacancy is matched with a worker employed at firm y, the vacancy is filled only if $W_{jt+1} \geq W_{yt+1}$. As is standard, new hires (filled vacancies) become productive in the subsequent

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[3]In other words, value W_{jt+1} is net of the unemployment value.

period. At the beginning of each period, after the realization of all shocks, only firms with a non-negative value will be active. As will be shown below, firms with productivity lower than $\underline{a}_t$ will exit the market or remain inactive, as the presented discounted value of their profits and, as a consequence, their values are negative. Hence, each period, employment will be destroyed at firms with a low productivity level.

The total number of matches in the economy $m(v_t, s_t)$ is an increasing function in the total number of vacancies (v_t) and the total number of job searchers $(s = u + (1 - \delta_{nt})\bar{i}n_t)$, where $u_t = 1 - n_t$ is the number of unemployed workers. Following the literature, $m(v_t, s_t)$ is assumed to be homogeneous of degree 1. Hence, $q_t = m(\theta_t, 1)$ and $\tilde{q}_t = m(1, \theta_t^{-1})$, where $\theta_t = v_t/s_t$ is labor market tightness.

The timing of the model each period is as follows: (1) aggregate shocks are realized; (2) firms enter/exit the market; (3) firms rent capital, post vacancies and employment offers; (4) production takes place, and factors are paid; (5) the household makes a consumption decision; (6) a fraction δ_{nt} of employed workers lose their jobs, and a fraction q_t of unemployed workers find new jobs; (7) a fraction $(1 - \delta_{nt})\bar{i}q_tG_{jt}$ of employed workers leave firm j to join another firm, where G_{jt} is the probability of firm j's employees being matched with a firm that offers a higher employment value.

2.2 Household

Consumption and savings decisions are made at the household level to maximize the life time utility function

$$\mathbb{U}(k_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi\frac{\tilde{n}_t^{1+\eta}}{1+\eta} + \beta E_t\left[\mathbb{U}(k_{t+1}, n_{t+1})\right] \tag{1}$$

subject to the budget constraint (2) and the aggregation of labor (3):

$$c_t + k_{t+1} \leq (r_t + 1 - \delta_k)k_t + \int w_{jt}n_{jt}dj + \int \pi_{jt}dj + b \cdot u_t - T_t \tag{2}$$

$$\tilde{n}_t = \left(\int n_{jt}^{1+\xi}dj\right)^{\frac{1}{1+\xi}} \tag{3}$$

where c is consumption, k is capital, w_j is the wage paid by firm j, and π_j stands for firm j's profits. $u = \int_0^1(1 - n_j)dj$ is the total number of unemployed workers, and b is unemployment compensation, which is financed by *lump sum* taxes $(T = b \cdot u)$. Parameter ξ in (3) governs the elasticity of substitution between n_x and n_y for all $x \neq y$.[4] Hence, the optimality condition for

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[4]We can interpret this parameter as follows: firms and workers are located uniformly on a circle, and firms hire workers who are closer to them. As firms increase in size, they have to attract workers who are farther away, which implies that workers have to spend more time commuting, which reduces their utility level.

consumption is given by

$$c_t^{-\sigma} = \beta E_t \left[(1 - \delta_k + r_{t+1}) c_{t+1}^{-\sigma} \right]. \tag{4}$$

A worker can be employed or unemployed at each point in time. Unemployed workers receive unemployment compensation b and are matched with a firm with probability q_t. Meanwhile, employed workers are separated from their job with exogenous probability δ_{nt}, in which case they have to spend at least one period in unemployment before they can be matched with another firm. I assume that employed workers can search on the job and are matched with another firm with probability $\bar{i}q_t$. However, employed workers only change jobs if they find a firm that offers a greater or equal value of employment than their current employer. Hence, the net value of employment at firm j is given by:

$$W_{jt} = w_{jt} - z_{jt} + E_t \Big[Q_t \Big((1 - \delta_{nt})(1 - \bar{i}q_t G_{jt}) W_{jt+1}$$
$$+ (1 - \delta_{nt})\bar{i}q_t \int_{W_{jt+1}}^{\infty} W f_{Wt}^v dW - q_t \int_0^\infty W f_{Wt}^v dW \Big) \Big], \tag{5}$$

where employment offers are distributed according to a continuous pdf f_{Wt}^v over the interval $W \in [0, \infty)$, z_{jt} is the FOCE for firm j, G_{jt} is the probability of receiving a better employment value than W_{jt+1}, and Q_t is the stochastic discount factor between t and $t+1$:

$$z_{jt} = b + \Psi \frac{\tilde{n}_t^{\eta - \xi}}{c_t^{-\sigma}} n_{jt}^{\xi} \tag{6}$$

$$G_{jt} = \int_{W_{jt+1}}^{\infty} f_{Wt}^v dW \tag{7}$$

$$Q_t = \beta E_t \left[\frac{\partial \frac{\mathbb{U}_{t+1}}{\partial c_{t+1}}}{\frac{\partial \mathbb{U}_t}{\partial c_t}} \right] \tag{8}$$

2.3 Firms

Firms produce with capital and labor, and their output can be used for consumption or for capital accumulation. At the beginning of each period, firms rent capital in a perfectly competitive market at rate r_t, open v_{jt} new vacancies, and post and commit to an employment value (W_{jt+1}) based on which workers decide to accept employment at firm j. As is standard, each firm is subjected to an equal treatment constraint and has to pay the same wage to all of its employees. Hence, a worker employed at firm j will move to firm y if and only if $W_{yt+1} \geq W_{jt+1}$. Since negative employment offers are never accepted, $W_{jt+1} \geq 0$ for all j. As a consequence, unemployed workers and continuing workers who are not contacted by any other firm always accept an employment offer in equilibrium. Vacancies are filled with probability $\tilde{q}_{jt}$, which will be defined below. I assume

that firms face either a vacancy cost function $\kappa \frac{v_{jt}^{1+\chi}}{1+\chi}$ or a hiring cost function $\kappa \frac{(\tilde{q}_{jt} v_{jt})^{1+\chi}}{1+\chi}$. These functions can be written as a general function: $\kappa \frac{\left(\tilde{q}_{jt}^{I_h} v_{jt}\right)^{1+\chi}}{1+\chi}$, where I_h is an indicator function equal to 1 if firms face a hiring cost function and 0 otherwise. Hence, the problem for firm j is given by:

$$\Pi_{jt}\left(n_{jt}, \bar{W}\right) = \max_{v_{jt}, k_{jt}, W_{jt+1}} \pi_{jt} + E_t\left[Q_t \Pi_{jt+1}\left(n_{jt+1}, W_{jt+1}\right)\right] \tag{9}$$

s.t.

$$\pi_{jt} = y_{jt} - w_{jt} n_{jt} - r_t k_{jt} - \kappa \frac{\left(\tilde{q}_{jt}^{I_h} v_{jt}\right)^{1+\chi}}{1+\chi} \tag{10}$$

$$y_{jt} = e^{a_j + a_t} k_{jt}^{\alpha} n_{jt}^{1-\alpha} \tag{11}$$

$$n_{jt+1} = (1-\delta_n)(1-\bar{i} q_t G_{jt}) n_{jt} + \tilde{q}_{jt} v_{jt} \tag{12}$$

$$W_{jt} = w_{jt} - z_{jt} + E_t\Big[Q_t\Big((1-\delta_{nt})(1-\bar{i} q_t G_{jt}) W_{jt+1}$$
$$+ (1-\delta_{nt})\bar{i} q_t \int_{W_{jt+1}}^{\infty} W f_{Wt}^v dW - q_t \int_0^{\infty} W f_{Wt}^v dW\Big)\Big] \tag{13}$$

$$G_{jt} = \int_{W_{jt+1}}^{\infty} f_{Wt}^v dW \tag{14}$$

$$\tilde{q}_{jt} = \tilde{q}_t\left(\frac{u_t}{s_t} + \bar{i}(1-\delta_{nt}) \int_0^{W_{jt+1}} f_{Wt}^n dW\right) \tag{15}$$

$$W_{jt} \geq \bar{W} \tag{16}$$

$$W_{jt+1} \geq 0 \tag{17}$$

where $\Pi_{jt}\left(n_{jt}, \bar{W}\right)$ is the value of a firm with log-TFP productivity equal to a_j, employment equal to n_{jt}, and a state continent wage contract equal to $\bar{W}$. f_{Wt}^v and f_{Wt}^n denote the density functions of wage offers and employment over the net value of employment, respectively. Letting $f(a(W))$ denote the density of firms offering an employment value equal to W, and $v(W)$ and $n(W)$ the vacancy and employment decisions of those firms:

$$f_{Wt}^v = \frac{v(W)_t}{v_t} \frac{f(a(W))}{\frac{\partial W}{\partial a}} \tag{18}$$

$$f_{Wt}^n = \frac{n(W)_t}{s_t} \frac{f(a(W))}{\frac{\partial W}{\partial a}} \tag{19}$$

where $\frac{\partial W(a)}{\partial a}$ is the derivative of the value of employment decision rule with respect to a_j. Hence,

the optimality conditions for capital, vacancies, and employment values are given by:

$$r_t = \alpha e^{a_j + a_t} \left(\frac{k_{jt}}{n_{jt}} \right)^{\alpha - 1} \tag{20}$$

$$\kappa \left(\tilde{q}_{jt}^{I_h} v_{jt} \right)^\chi = E_t \left[Q_t \tilde{q}_{jt}^{1 - I_h} J_{jt+1} \right] \tag{21}$$

$$E_t \left[Q_t h_{jt} \right] \geq E_t \{ Q_t J_{jt+1} (1 - \delta_{nt}) \bar{i} \left[q_t f_{jt}^v n_{jt} + (1 - I_h) \cdot \tilde{q}_t f_{jt}^n v_{jt} \right] \} \tag{22}$$

where p_{jt} is labor productivity, h_{jt} is hires, and J_{jt} is the value of a filled vacancy:

$$p_{jt} = (1 - \alpha) e^{a_j + a_t} \left(\frac{k_{jt}}{n_{jt}} \right)^\alpha \tag{23}$$

$$h_{jt} = \tilde{q}_{jt} v_{jt} \tag{24}$$

$$J_{jt} = p_{jt} - w_{jt} + E_t \left[Q_t (1 - \delta_{nt})(1 - \bar{i} q_t G_{jt}) J_{jt+1} \right] \tag{25}$$

And to save on notation:

$$f_{jt}^v = \frac{v \left(W_{jt+1} \right)_t}{v_t} \frac{f \left(a \left(W_{jt+1} \right) \right)}{\frac{\partial W_{jt+1}}{\partial a_j}} \tag{26}$$

$$f_{jt}^n = \frac{n \left(W_{jt+1} \right)_t}{s_t} \frac{f \left(a \left(W_{jt+1} \right) \right)}{\frac{\partial W_{jt+1}}{\partial a_j}} \tag{27}$$

The job creation condition (21) is standard in the literature and needs no additional explanation. However, notice that large firms implicitly internalize the higher utility cost that they induce in their workers. In other words, notice that you could add to equation (21) term $\frac{\partial \Pi_{jt}}{\partial W_{jt+1}} \frac{\partial W_{jt+1}}{\partial z_{jt+1}} \frac{\partial z_{jt+1}}{\partial n_{jt+1}} \frac{\partial n_{jt+1}}{\partial v_{jt}}$, which is the increase in a firm's wages due to its larger size (larger z_{jt+1}). Based on the FOC with respect to W_{jt+1}, we know that this term is equal to 0.

The wage decision (22) equates the marginal cost of an additional increase in W_{jt+1} (left-hand side) with the marginal benefit (right-hand side). On the one hand, a 1 unit increase in W_{jt+1} implies increasing future wages by 1 unit. But, given that current employees were promised $W_{jt} = \bar{W}$, current wages can decrease so W_{jt} remains constant. Hence, an increase in future wages by 1 unit increases payroll only by h_{jt} (new hires). On the other hand, an increase in W_{jt+1} allows firm j to retain a larger fraction of its employees (the first term on the right-hand side) and to poach more workers from other firms (the second term on the right-hand side).

Firm entry and exit Notice that for low values of a_j, firms' value Π_{jt} becomes negative for any value of employment. Given that the production technology exhibits constant returns to scale, firms with a productivity lower than $\underline{a}_t$ will exit or remain inactive, while firms with a productivity

greater than or equal to $\underline{a}_t$ will enter or remain active. $\underline{a}_t$ is such that:

$$\underline{a}_t = \min_{a_j} \{a_j \quad | \quad J_{jt} \geq 0\} \tag{28}$$

Notice $\underline{a}_t > -\infty$, meaning that $e^{\underline{a}_t} > 0$ even if $b = \Psi = 0$. Hence, even if you assume no unemployment benefits or a utility cost of employment, there will be a finite threshold below which firms will exit the market. The reason is that unemployed workers give up the possibility to find a better job by accepting an employment offer, which not all firms can compensate for. To see this clearly, assume that the economy is in steady state. For inactive firms $W = 0$, $G_j = 1$ and $\tilde{q}_j = \tilde{q}$. Hence, using the value of employment, wages have to be greater than or equal to $\underline{w}$ where:

$$\underline{w} = b + \beta q \left[1 - (1 - \delta_n)\bar{i}\right] \int_0^\infty W f_W^v dW > 0 \tag{29}$$

Hence, in steady state, for a firm to be active, $p_j - \underline{w} \geq 0$, which implies that a firm will be active as long as:

$$a_j \geq (1 - \alpha) log \left(\frac{\underline{w}}{(1 - \alpha)(\beta \alpha)^{\frac{\alpha}{1-\alpha}}}\right) - a. \tag{30}$$

Even if $b = 0$, $\underline{w} > 0$ because workers give up the possibility to find a better job by taking an offer at a low-productivity firm (the second term in equation (29)). Hence, firms should compensate workers for that opportunity cost, which only firms with a minimum level of productivity can pay for.

2.4 Aggregate Resource

Notice that total income in this economy is used for consumption, capital accumulation, and vacancy posting costs. Hence, the aggregate resource constraint is given by:

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t + \kappa \int \frac{\left(\tilde{q}_{jt}^{I_h} v_{jt}\right)^{1+\chi}}{1 + \chi} dj, \tag{31}$$

where it is straight forward to define aggregate production as $y_t = \int y_{jt} dj$

3 Equilibrium

Definition 1. ***Competitive Search Equilibrium with Value Posting.*** *A competitive search equilibrium with value posting is a sequence of prices $\{r_t, w_t\}$, quantities $\{y_t, c_t, k_t, u_t, n_t\}$, probabilities $\{q_t, \tilde{q}_t\}$, and functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ on productivity a_j, firm size n_{jt} and W_{jt}, such*

that given exogenous variables, an initial stock of capital and initial distributions of employment and employment values: (i) The household optimizes, taken as given prices and exogenous shocks. Consumption satisfies the optimality condition (4). (ii) Taking as given the exogenous variables, $\{r_t\}$, and all other firms' strategies (i.e., employment, wage, and vacancies), firms optimize. Functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ solve equations (12), (21), (22), and (25), and prices satisfy equations (5) and (20). (iii) Probabilities evolve according to $q_t = m(\theta_t, 1)$ and $\tilde{q}_t = m(1, \theta_t^{-1})$. (iv) Markets clear: the aggregate resource constraint holds.

It is worth noticing that a competitive search equilibrium with value posting does not establish any properties regarding functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$. For example, this definition allows an equilibrium in which firms' productivity a_j is not perfectly correlated with its employment size or employment value offer, which makes the problem intractable. However, MPV13 defined and proved the existence of a particular class of equilibrium: a rank-preserving equilibrium, which I will focus on throughout this paper. Intuitively, a rank-preserving equilibrium establishes that the most-productive firms are larger and offer higher employment values at all times.

Definition 2. *Rank-preserving competitive search equilibrium:* *A rank-preserving competitive search equilibrium is a competitive search equilibrium in which function n_{jt+1}, W_{jt+1} are increasing in a_j and n_{jt}.*

As will be explained in the simulation section, and because of the counterfactual behavior of wages with value posting, I propose a modification to the baseline model. I assume that firms post wages instead of employment values to retain workers and poach workers from other firms . Also, I introduce wage rigidity by assuming that firms face quadratic cost of wage adjustment *a la* Rotemberg (1982). Posting wages instead of values of employment is unlikely to increase the volatility in the labor market, as the agents' decision rules will continue to depend on the net value of employment. These extensions are straight forward, and the details can be found in Appendix C. In the case of wage posting with wage rigidity, the optimality condition for wages and aggregate resource constraint are given by:

$$E_t\left[Q_t n_{jt+1}\right] + \phi\left(\frac{w_{jt+1}}{w_{jt}} - 1\right)\frac{1}{w_{jt}} - E_t\left[Q_t\phi\left(\frac{w_{jt+2}}{w_{jt+1}} - 1\right)\frac{w_{jt+2}}{w_{jt+1}^2}\right]$$

$$\geq \quad E_t\{Q_t J_{jt+1}(1 - \delta_{nt})\bar{i}_t\left[q_t f_{jt}^v n_{jt} + (1 - I_h) \cdot \tilde{q}_t f_{jt}^n v_{jt}\right]\} \tag{32}$$

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t + \kappa\int\frac{\left(\tilde{q}_{jt}^{I_h} v_{jt}\right)^{1+\chi}}{1+\chi}dj + \int\frac{\phi}{2}\left(\frac{w_{jt+1}}{w_{jt}} - 1\right)^2 dj \tag{33}$$

where ϕ is the Rotemberg parameter and determines how sticky wages are. When firms commit to a higher wage in the future, they retain a larger fraction of their workforce and poach more workers

(right-hand side of equation (81)), but their payroll will increase by the size of their new workforce (the first term on the left-hand side), and they will have to pay a wage adjustment cost (the last two terms on the left-hand side). When firms post employment values, the cost is proportional to the increase (and not the level) of the firm's workforce because firms can change their current wages to keep the current value of employment constant. Hence, in an economy with wage posting and wage rigidity, the equilibrium is given by the following:

Definition 3. *Competitive Search Equilibrium with Wage Posting. A competitive search equilibrium with wage posting is a sequence of prices $\{r_t, w_t\}$, quantities $\{y_t, c_t, k_t, u_t, n_t\}$, probabilities $\{q_t, \tilde{q}_t\}$, and functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ on productivity a_j, firm size n_{jt} and w_{jt}, such that given exogenous variables, an initial stock of capital and initial distributions of employment and wages: (i) The household optimizes, taken as given prices and exogenous shocks. Consumption satisfies the optimality condition (4). (ii) Taking as given the exogenous variables, $\{r_t\}$, and all other firms strategies (i.e. employment, wage, and vacancies), firms optimize. Functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ solve equations (5), (12), (21), and (25); and prices satisfy equations (20) and (81). (iii) Probabilities evolve according to $q_t = m(\theta_t, 1)$ and $\tilde{q}_t = m(1, \theta_t^{-1})$. (iv) Markets clear: the aggregate resource constraint holds.*

4 Computation

In this section, I propose a computational algorithm for this class of models that is easy and fast to implement.[5] The main idea is based on the Reiter (2009) method, which solves heterogeneous agents' models by numerically approximating the equilibrium dynamics around the deterministic steady state.

The main technical challenge is to compute the steady state, which is a complicated task for a given productivity distribution. Notice that if you impose a distribution for a_j, you have to find the equilibrium decision rules for employment, vacancies, and employment values (n_{jt+1}, v_{jt}, and W_{jt+1}) to compute the distributions of employment value offers (f_{Wt}^v) and employment (f_{Wt}^n). However, in equilibrium, W_{jt}, n_{jt} and v_{jt} are functions of f_{Wt}^v and f_{Wt}^n. Hence, in order to find the equilibrium, you have to iterate on multiple infinite dimensional elements. Additionally, we know that distributions f_{Wt}^v and f_{Wt}^n are atomless, which poses an additional challenge to the computation. If you assume discrete distributions, by the equilibrium properties, firms in an atom will want to deviate to gain a larger fraction of workers (Proposition 1 of MPV2013).

Given these challenges, I follow these steps to compute the equilibrium of the model: First, iven a distribution for a_j, I find the exact solution for n_a points, and approximate the remaining

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[5]I found that if you assume $\xi = 0$, the computation time is reduced to a few seconds.

points by interpolating. Second, epending on the value of ξ, I compute the deterministic steady state by iterating on a guess for the vacancy decision rule (if $\xi = 0$) or value of employment (if $\xi > 0$). Third, I take a numerical first-order approximation around the deterministic steady state.[6]

4.1 Preliminaries

In what follows, I assume that the steady state values for $\{u, q, \tilde{q}, \tilde{\Psi}\}$ are known, where $\tilde{\Psi} = \Psi \frac{\tilde{n}^{\eta - \xi}}{c^{-\sigma}}$. I make use of parameters $\{\kappa, \delta_n, \bar{m}\}$ to target those steady-state moments, and I assume that the remaining parameters are known.[7] For simplicity, I assume that $a = 0$ in steady state and that the idiosyncratic distribution a_j is continuous, which does not have to be the case. Given that this distribution, as well as other equilibrium elements, is an infinite dimensional element, I will find the exact solution for n_a points and approximate all other points by interpolating. In particular, I will find the exact solution for the grid $a^G = \{a_1, a_2 ... a_{n_a}\}$ and will approximate the solution for all other points. *It is important to notice that $f(a_j)$ will be equal to the value of the density function evaluated at a_j and NOT a mass point approximated by, for example, the Tauchen method.* This is particularly important given the equilibrium result that f^v_{Wt} and f^n_{Wt} are atomless.

4.2 Computing the Deterministic Steady State

First notice that in steady state, $r = 1/\beta - (1 - \delta_k)$. Hence, given a value for q, $\tilde{q}$, and u in steady state: $\delta_n = qu/(1 - u)$, $s = u + (1 - \delta_n)\bar{i}n$, $\theta = \frac{q}{\tilde{q}}$ and $v = \theta s$. Also, because the production technology exhibits constant returns to scale $p_j = (1 - \alpha) \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} (e^{a_j})^{\frac{1}{1-\alpha}}$.

Now, depending on the value for ξ, the computation of the steady state could be more efficient by iterating on a guess for the vacancy decision (v_j) rule or the net value of employment (W_j). Under these model assumptions, I have found that iterating on the net value of employment is accurate, but could be *"slow"*. On the other hand, iterating on the vacancy decision rule could be fast but less accurate. Let me describe both approximations first. Then, I will discuss both approaches.

Iterating on the vacancy decision rule (ideal if $\xi = 0$): With this approach, the computation of the steady state could take just a couple of seconds and could be very accurate if $\xi = 0$. However, the accuracy of this approach could deteriorate significantly for positive values of ξ. The steps are as follows:

1. Given a guess for the vacancy decision rule v_j, make use of the rank preserving equilibrium definition to compute the values of G_j

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[6]Even though I work with a first-order approximation in this paper, it is straightforward to take an nth approximation.

[7]This is not a restrictive assumption at all. Researchers can iterate on these moments to target some parameters.

2. Find the job filling rate by iterating on:

$$\tilde{q}_j^{\{i+1\}} = \frac{\tilde{q}}{s}\left(u + \bar{i}(1-\delta_n)\int_{-\infty}^{a_j}\left(\frac{n}{\Omega^{\{i\}}}\right)\frac{v_x\tilde{q}_x^{\{i\}}}{[1-(1-\delta_n)(1-\bar{i}qG_x)]}f(x)dx\right) \tag{34}$$

$$\Omega^{\{i\}} = \int_{-\infty}^{\infty}\frac{v_x\tilde{q}_x^{\{i\}}}{[1-(1-\delta_n)(1-\bar{i}qG_x)]}f(x)dx \tag{35}$$

where $\Omega^{\{i\}}$ converges to n, and term $\frac{n}{\Omega}$ guarantees that total employment is equal to n in equilibrium.

3. Find the implied decision rule for employment values (W_j), based on which you can compute the value of a filled vacancy (J_j). For this step:

   (a) Given a guess for the decision rule of value of employment W_j, compute the conditional and unconditional expected value of employment and the productivity threshold $\underline{a}$:[8]

$$\bar{W} = \int_{-\infty}^{\infty}W_x\frac{v_x}{v}f(x)dx \tag{36}$$

$$\hat{W}_j = \int_{a_j}^{\infty}W_x\frac{v_x}{v}f(x)dx \tag{37}$$

$$\underline{a} = (1-\alpha)log\left(\frac{b + \beta q\left[1-(1-\delta_n)\bar{i}\right]\bar{W}}{(1-\alpha)(\beta\alpha)^{\frac{\alpha}{1-\alpha}}}\right) \tag{38}$$

   (b) Find the level of wages and value of a filled vacancy:

$$w_j = b + \tilde{\Psi}n_j^\xi + W_j(1 - \beta(1-\delta_n)(1-\bar{i}qG_j)) - \beta(1-\delta_n)\bar{i}q\hat{W}_j + \beta q\bar{W} \tag{39}$$

$$J_j = \frac{p_j - w_j}{1 - \beta(1-\delta_n)(1-\bar{i}qG_j)} \tag{40}$$

   The level of employment can be computed based on the job filling rate as:

$$n_j = \frac{\frac{\tilde{q}_j v_j}{[1-(1-\delta_n)(1-\bar{i}qG_j)]}}{n^{-1}\int_{-\infty}^{\infty}\frac{\tilde{q}_x v_x}{[1-(1-\delta_n)(1-\bar{i}qG_x)]}f(x)dx} \tag{41}$$

   where the denominator is a transformation of κ.

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[8]Knowing the value of the productivity threshold helps to improve the accuracy of the calculation because we know that $v(\underline{a}) = J(\underline{a}) = n((\underline{a})) = 0$.

(c) Compute $\frac{\partial W_j}{\partial a_j}$ using the optimality condition for the values of employment[9]:

$$\frac{\partial W_j}{\partial a_j} = \frac{2^{1-I_h} max\left\{J_j, 0\right\}(1-\delta_n)\bar{i}q\frac{v_j}{v}f(a_j)}{(1-(1-\delta_n)(1-\bar{i}qG_j))} \tag{42}$$

(d) Find the new guess for the values of employment decision rule:

$$W_j^{\{1\}} = \int_{-\infty}^{a_j} \frac{\partial W_x}{\partial a_x} f(x)dx \tag{43}$$

(e) If the maximum difference between $W_j^{\{1\}}$ and W_j is smaller than your predetermined tolerance level, continue to the next step. Otherwise, update your guess as follows:

$$W_j = s^w W_j + (1-s^w)W_j^{\{1\}}; \qquad 0 < s^w < 1 \tag{44}$$

and go back to step (3a), where s^w is a smoothing parameter.

4. Find the new guess for vacancies using the optimality condition for vacancies:

$$v_j^{\{1\}} = \frac{\frac{1}{\tilde{q}_j^{I_h}}\left(\beta\tilde{q}_j^{1-I_h} max\left\{J_j, 0\right\}\right)^{\frac{1}{\chi}}}{v^{-1}\int_{-\infty}^{\infty}\frac{1}{\tilde{q}_x^{I_h}}\left(\beta\tilde{q}_x^{1-I_h} max\left\{J_x, 0\right\}\right)^{\frac{1}{\chi}}f(x)dx} \tag{45}$$

where the denominator is a transformation of κ and guarantees that vacancies add up to v.

5. If the maximum difference between $v_j^{\{1\}}$ and v_j is smaller than your predetermined tolerance level, continue to the next step. Otherwise, update your guess for the vacancy decision rule as follows:

$$v_j = s^v v_j + (1-s^v)v_j^{\{1\}}; \qquad 0 < s^v < 1 \tag{46}$$

and go back to step 1, where s^v is a smoothing parameter.

Once you compute the steady state equilibrium, notice that the value of κ could be recovered by $\kappa = \frac{\beta\tilde{q}_j^{1-I_h}J_j}{(\tilde{q}_j^{I_h}v_j)^\chi}$ for any $a_j > \underline{a}$.

Iterating on the value of employment decision rule (ideal if $\xi > 0$): With this approach, the computation of the steady state could be very accurate regardless of the value for ξ. However,

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[9]In the case of wage posting, this equation should be replaced based on the optimal condition for wage offers.

this approach could be *"slower"* than iterating on the vacancy decision rule. The steps on this approach are the following:

1. Given a guess for the decision rule of value of employment W_j, compute the equilibrium distribution of vacancies and job filling rate. For this step:

   (a) Given a guess for the vacancy decision rule v_j, make use of the rank preserving equilibrium definition to compute the conditional and unconditional expected value of employment and the productivity threshold:

   $$\bar{W} = \int_{-\infty}^{\infty} W_x \frac{v_x}{v} f(x) dx \tag{47}$$

   $$\hat{W}_j = \int_{a_j}^{\infty} W_x \frac{v_x}{v} f(x) dx \tag{48}$$

   $$\underline{a} = (1-\alpha)log\left(\frac{b + \beta q\left[1 - (1-\delta_n)\bar{i}\right]\bar{W}}{(1-\alpha)(\beta\alpha)^{\frac{\alpha}{1-\alpha}}}\right) \tag{49}$$

   (b) Compute the values of G_j

   (c) Find the job filling rate by iterating on:

   $$\tilde{q}_j^{\{i+1\}} = \frac{\tilde{q}}{s}\left(u + \bar{i}(1-\delta_n)\int_{-\infty}^{a_j}\left(\frac{n}{\Omega^{\{i\}}}\right)\frac{v_x\tilde{q}_x^{\{i\}}}{[1-(1-\delta_n)(1-\bar{i}qG_x)]}f(x)dx\right) \tag{50}$$

   $$\Omega^{\{i\}} = \int_{-\infty}^{\infty}\frac{v_x\tilde{q}_x^{\{i\}}}{[1-(1-\delta_n)(1-\bar{i}qG_x)]}f(x)dx \tag{51}$$

   where $\Omega^{\{i\}}$ converges to n, and term $\frac{n}{\Omega}$ guarantees that total employment is equal to n in equilibrium.

   (d) Find the new guess for equilibrium vacancies decision rule, given a decision rule for W_j and a job filling rate $\tilde{q}_j$, by solving for each j:

   $$n_j = \frac{\tilde{q}_j v_j^{\{1\}}}{[1-(1-\delta_n)(1-\bar{i}qG_j)]} \tag{52}$$

   $$w_j = b + \tilde{\Psi}n_j^\xi + W_j(1 - \beta(1-\delta_n)(1-\bar{i}qG_j)) - \beta(1-\delta_n)\bar{i}q\hat{W}_j + \beta q\bar{W} \tag{53}$$

   $$J_j = \frac{p_j - w_j}{1 - \beta(1-\delta_n)(1-\bar{i}qG_j)} \tag{54}$$

   $$v_j^{\{1\}} = \tilde{\kappa}\frac{\frac{1}{\tilde{q}_j^{I_h}}\left(\beta\tilde{q}_j^{1-I_h}max\left\{J_j, 0\right\}\right)^{\frac{1}{\chi}}}{v^{-1}\int_{-\infty}^{\infty}\frac{1}{\tilde{q}_x^{I_h}}\left(\beta\tilde{q}_x^{1-I_h}max\left\{J_x, 0\right\}\right)^{\frac{1}{\chi}}f(x)dx} \tag{55}$$

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where $\tilde{\kappa}$ is a transformation of κ that is set such that vacancies add up to v. A key feature of this step (1d) is that we fix the decision rule of all other firms by fixing G_j, $\tilde{q}_j$, $\bar{W}$ and $\hat{W}_j$. Hence, the value for $v_y^{\{1\}}$ is independent of the value for $v_x^{\{1\}}$ in this step.

(e) If the maximum difference between $v_j^{\{1\}}$ and v_j is smaller than your predetermined tolerance level, continue to the next step. Otherwise, update your guess for the vacancy decision rule as follows:

$$v_j = s^v v_j + (1 - s^v)v_j^{\{1\}}; \qquad 0 < s^v < 1 \tag{56}$$

and go back to step (1a), where s^v is a smoothing parameter.

2. Compute $\frac{\partial W_j}{\partial a_j}$ using the optimality condition for the values of employment[10]::

$$\frac{\partial W_j}{\partial a_j} = \frac{2^{1-I_h} max\{J_j, 0\}(1 - \delta_n)\bar{i}q\frac{v_j}{v}f(a_j)}{(1 - (1 - \delta_n)(1 - \bar{i}qG_j))} \tag{57}$$

3. Find the new guess for the values of employment decision rule:

$$W_j^{\{1\}} = \int_{-\infty}^{a_j} \frac{\partial W_x}{\partial a_x} f(x)dx \tag{58}$$

4. If the maximum difference between $W_j^{\{1\}}$ and W_j is smaller than your predetermined tolerance level, continue to the next step. Otherwise, update your guess as follows:

$$W_j = s^w W_j + (1 - s^w)W_j^{\{1\}}; \qquad 0 < s^w < 1 \tag{59}$$

and go back to step 1, where s^w is a smoothing parameter.

Once you compute the steady-state equilibrium, notice that the value of κ could be recovered by $\kappa = \frac{\beta\tilde{q}_j^{1-I_h}J_j}{(\tilde{q}_j^{I_h}v_j)^\chi}$ for any $a_j > \underline{a}$.

Remarks about the two approaches As I mentioned before, iterating on the vacancy decision rule can be less accurate when $\xi > 0$. The reason is that the vacancy decision rule is very sensitive around the productivity threshold ($\underline{a}$). Hence, finding the equilibrium rules for firms with a productivity level close to $\underline{a}$ could be very difficult. To see this clearly, suppose that you are trying to find the vacancy decision rule for a firm whose productivity is slightly greater than $\underline{a}$. That firm is certainly small in equilibrium. However if you guess a slightly larger size, based on

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[10]In the case of wage posting, this equation should be replaced based on the optimal condition for wage offers.

equations (39) and (40), you might erroneously think that the wage is too high for that firm to be active. That belief will lead you to increase the vacancy decision rule for all other firms and, as less firms are active, lower the expected values of employment. Nonetheless, small declines in $\bar{W}$ and the vacancy decision rule can make you think, again, that the low productivity firm is active (and larger) because of the same equations (39) and (40). In contrast, the approximation to the vacancy decision rule is smoother by iterating on the value of employment (W_j) because the decision rules of all other firms are constant in step 1d.[11] However, iterating on W_j requires more rounds of iterations to make sure that all these elements are consistent with the equilibrium of the model.

Table 1: Parameter Values

Externally Calibrated

| Parameter | Value Posting Model | | | Wage Posting Model | | Description |
	Vacancy	Hiring	MPV16	Vacancy	Hiring	
σ	1.00	1.00	0	1.00	1.00	Intertemporal elasticity of substitution
η	1.50	1.50	0	1.50	1.50	Inverse of Frisch elasticity
α	0.33	0.33	0	0.33	0.33	Output Elasticity of labor
β	0.996	0.996	0.996	0.996	0.996	Discount factor.
δ_k	0.0087	0.0087	1.00	0.0087	0.0087	Capital depreciation rate.
l	0.50	0.50	1.00	0.50	0.50	Matching function parameter.

Internally Calibrated

| Parameter | Value Posting Model | | | Wage Posting Model | | Description |
	Vacancy	Hiring	MPV16	Vacancy	Hiring	
δ_h	0.016	0.016	0.016	0.016	0.016	Exogenous separation rate.
Ψ	0.146	0.185	0.000	0.162	0.195	Disutility of labor parameter.
b	6.750	6.750	0.000	6.901	6.388	Unemployment benefits.
$\bar{i}$	0.350	0.350	0.130	0.356	0.356	Search intensity of employed workers.
ξ	0.500	0.400	0.000	0.462	0.539	Elasticity of substitution between jobs.
χ	1.500	1.500	49.000	7.760	1.692	Convexity of cost function.

Note: This table summarizes the parameterization of the model. Details are reported in Section 5. Model "*Vacancy*" refers to my model in which firms face a vacancy cost function ($I_h = 0$). Model "*Hiring*" refers to my model in which firms face a hiring cost function ($I_h = 1$). Model MPV16 is my baseline model in which firms face a hiring cost function and calibrated to match the same moments as in MPV16.

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[11]Those decision rules are constant by fixing G_j, $\tilde{q}_j$, $\hat{W}_j$, and $\bar{W}$.

4.3 Approximation around the Deterministic Steady State

Notice that the equilibrium of the economy is described by a system of nonlinear equations given by the Euler equation (4); the wage rate (5); the law of motion for employment (12); the decision rules for capital (20), vacancies (21), and employment values (22); the value of a filled vacancy (25); the job-finding probability ($q_t = m(\theta_t, 1)$); the vacancy contact rate ($\tilde{q}_t = m(1, \theta_t^{-1})$); the unemployment rate ($u_t = 1 - E_t$); the productivity threshold (28); the aggregate resource constraint (31); and the aggregate stock of capital ($k_t = \int k_{jt} dj$). This system of equation can be written as:

$$E_t \mathcal{F}\left(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, \overrightarrow{\epsilon}_{t+1}, \overrightarrow{\epsilon}_t\right) = 0 \tag{60}$$

where $\overrightarrow{X}_t$ and $\overrightarrow{\epsilon}_t$ are the vectors of endogenous and exogenous variables of the model, respectively, and can be summarized by:

$$\overrightarrow{X}_t = \begin{bmatrix} vec(n_{jt}), & vec(W_{jt}), & k_t, & vec(w_{jt}), & vec(v_{jt}), & vec(J_{jt}), & (k_{jt}), \end{bmatrix}$$
$$\begin{bmatrix} u_t, & r_t, & q_t, & \tilde{q}_t, & c_t, & \underline{a}_t \end{bmatrix} \tag{61}$$
$$\overrightarrow{\epsilon}_t = [a_t, \quad \delta_{nt}] \tag{62}$$

Following Reiter (2009), the system of equations (60) can be linearized numerically around the deterministic steady state to get:

$$E_t \mathcal{F}_{\overrightarrow{X}_{t+1}} \partial \overrightarrow{X}_{t+1} + \mathcal{F}_{\overrightarrow{X}_t} \partial \overrightarrow{X}_t + \mathcal{F}_{\overrightarrow{\epsilon}_{t+1}} \partial \overrightarrow{\epsilon}_{t+1} + \mathcal{F}_{\overrightarrow{\epsilon}_t} \partial \overrightarrow{\epsilon}_t = 0, \tag{63}$$

where $\mathcal{F}_x$ is the partial derivative of $\mathcal{F}$ with respect to x. This system of linear equations can be solved using a standard method. In this paper, I use the method proposed by Klein (2000).

5 Calibration

I calibrate my model to a monthly frequency. I set the discount factor to $\beta = 1.04^{-1}$ per year, the inverse of the Frisch elasticity (η) is set to 1.5, and the coefficient of relative risk aversion (σ) is set to 1. The capital depreciation rate is set to 10% per year. The output elasticity of labor (α) is set to 0.33. I assume that a_j has a truncated Pareto distribution and calibrate the distribution parameters to target a standard deviation, among active firms, for a_j and $log(p_j)$ of 0.5 and 0.8, respectively, following the empirical findings of Decker et al. (2020).

I set the exogenous separation rate in steady state to target an unemployment rate of 5.5% and a job finding rate (q) equal to 0.27, which implies that $\delta_n = 0.016$. The search intensity of employed workers is calibrated to match a fraction of job changers equal to 2% in steady state

(Fallick and Fleischman, 2004). I calibrate κ such that the vacancy contact rate is equal to $\tilde{q} = 0.9$, which also implies that in equilibrium $v = \frac{q}{\tilde{q}}s$. I assume a Cobb-Douglas matching function of the form: $m = \bar{m}s^l v^{1-l}$. I set l to 0.5, and $\bar{m}$ is given by $\bar{m} = q\theta^{-l}$.

Table 2: Log-Wage Distribution in Steady State

| | Decile of Distribution | | | | | | | | | Std |
	10	20	30	40	50	60	70	80	90	
value posting model (vacancy)	-0.445	-0.201	-0.064	0.028	0.097	0.153	0.200	0.242	0.278	0.298
value posting model (hiring)	-0.422	-0.223	-0.091	0.006	0.083	0.146	0.199	0.244	0.285	0.282
wage posting model (vacancy)	-0.667	-0.396	-0.191	-0.027	0.109	0.224	0.322	0.406	0.476	0.432
wage posting model (hiring)	-0.652	-0.353	-0.151	0.002	0.123	0.224	0.309	0.382	0.446	0.432
Data	-0.499	-0.322	-0.198	-0.094	0.004	0.103	0.209	0.335	0.514	0.433

Note: This table reports data and model-generated moments of the distribution of log-wages. To compute data moments, a Mincer equation with time fixed effects is estimated using the CPS microdata from January 1994 to December 2015. Then, using the residuals from this Mincer equation, these statistics are computed for each month. The data numbers presented in this table are the sample means of those moments. Models "hiring" refer to my baseline model in which firm face a hiring cost function. Similarly, models "Vacancy" refer to my baseline model in which firms post values of employment.

I calibrate ξ to t the standard deviation of log wages in steady state to 0.43, which is consistent with the Current Population Survey (CPS) microdata. Parameter χ, which governs the curvature of the hiring or vacancy cost function, is calibrated to match the average firm size based on data from the Business Dynamics Statistics. Based on Chodorow-Reich and Karabarbounis (2016), the unemployment benefit is calibrated such that b represents 6% of total output per worker, and Ψ is calibrated to target an average ratio $\frac{z_j}{p_j}$ of 0.71 across firms.

Now, it remains to define and calibrate the exogenous variables in this model. Given the goal of understanding the amplification and propagation of shocks in this model, I assume the following list of shocks: (1) aggregate productivity shocks (a), (2) separation rate shocks (δ_n), (3) preference shocks (β), (4) labor supply shocks (Ψ), (5) matching efficiency shocks (m), and (6) vacancy/hiring costs (κ). For simplicity, I assume that these shocks are uncorrelated and that each shock follows an AR(1) process. I use the capital-utilization adjusted TFP and the employment-to-unemployment transition rate to calibrate the exogenous processes for productivity and separation.[12] The exogenous process for matching efficiency, preference, and labor supply are calibrated based on Furlanetto and Groshenny (2016), Gertler, Sala, and Trigari (2009), and Lubik (2009), respectively. Finally, I assume that κ follows an AR(1) process in logs with auto correlation equal to 0.9 and standard deviation equal to 0.01. Table 1 presents the model parameters.

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[12]I fit an AR(1) process on the HP-filtered series (with a smoothing parameter equal to 10^5).

Table 3: Employment Size Distribution in Steady State

Average Size	1 to 4	5 to 9	10 to 19	20 to 49	50 to 99	100 to 249	250+	All
value posting model (vacancy)	1.000	7.290	19.878	34.191	43.875	47.960	50.317	8.981
value posting model (hiring)	1.000	8.046	21.467	40.100	54.923	61.705	65.744	10.299
wage posting model (vacancy)	1.000	5.154	14.087	37.160	72.201	96.686	113.171	9.720
wage posting model (hiring)	1.000	6.538	18.531	39.008	57.796	67.019	72.651	9.720
Data	1.000	2.990	6.150	13.810	31.370	68.620	740.340	9.720

Note: This table reports data and model-generated moments for the employment-size distribution across firms in the United States. Average employment size is relative to the smallest firms (1 to 4 employees). Firm size is defined as the number of employees per firm. Average size is computed as the total number of employees over the total number of firms. Data source is Business Dynamics Statistics. Models "hiring" refer to my baseline model in which firm face a hiring cost function. Similarly, models "Vacancy" refer to my baseline model in which firms post values of employment.

6   Results

6.1   Data

In this section, I present the quantitative predictions of the model, which are judged against quarterly U.S. data from 1994 to 2015.[13] I use data for output, labor productivity, unemployment, vacancies, hires, employment transition rates, and wages. Since the model generates artificial monthly series, I take the quarterly average of these simulated data.

Output is real output in the nonfarm business sector (GDPC1), labor productivity is measured as real output per hour in the nonfarm business sector (OPHNFB), and aggregate TFP is measured by the utilization adjusted TFP from the San Francisco FED. Unemployment is total number of unemployed workers (UNEMPLOY). Vacancies are measured by the composite help-wanted index computed by Barnichon (2010). Using the CPS microdata, I construct monthly series for transition rates from Employment to Unemployment (EUr), Unemployment to Employment (EUr), and Employment to Employment (EEr). Based on the CPS, total hires (h) is constructed as the sum of flows from Non Employment-to-Employment and Employment-to-Employment. Also, to assess the transitional dynamics generated by this model, I construct average hourly log-wages for all workers (w^a), controlling for individual characteristics.[14] I aggregate these monthly series to a quarterly frequency by taking a simple average of the quarter's months and seasonally adjust these series using the X-13 filter. Following the literature, I detrend all series in logs using the HP filter with a smoothing parameter equal to 10^5. Figures 3 plots these series in log-level (solid black line) along with their HP trends (dashed black lines).

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[13]The relatively short length of this sample period is due to the fact that job-to-job transitions can only be computed in the CPS data since 1994.

[14]Details about wages can be found in Appendix A.

Table 4: Business Cycle Moments. Standard deviation.

			value posting model (vacancy)							
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	0.3	0.6	0.4	0.3	0.0	0.7	1.0	1.0	1.0	0.9
Separation	26.6	13.3	23.3	5.6	36.3	17.8	1.8	1.0	0.6	0.0
Preference	25.8	17.6	14.9	29.1	0.0	29.8	12.8	1.0	0.5	0.0
Labor supply	1.0	2.4	4.1	1.2	0.0	6.7	3.6	1.0	1.0	0.0
Recruiting	102.2	224.0	36.0	116.3	0.0	55.4	120.7	1.0	6.4	0.0
Matching efficiency	1.6	3.6	8.9	1.9	0.0	15.3	3.7	1.0	0.9	0.0

			value posting model (hiring)							
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	1.0	2.1	0.5	1.1	0.0	0.7	0.9	1.0	0.9	0.9
Separation	20.4	9.7	6.6	5.5	17.1	2.4	1.9	1.0	0.4	0.0
Preference	11.1	58.4	19.3	12.2	0.0	36.8	15.0	1.0	0.4	0.0
Labor supply	3.4	7.6	2.4	4.0	0.0	2.9	4.6	1.0	0.9	0.0
Recruiting	55.3	120.4	23.8	62.6	0.0	30.2	14.2	1.0	2.2	0.0

			wage posting model (vacancy)							
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	0.2	0.6	0.1	0.3	0.0	0.2	0.5	1.0	1.0	0.9
Separation	12.2	0.9	6.4	0.2	13.8	3.6	3.3	1.0	0.6	0.0
Preference	29.5	9.0	16.8	32.9	0.0	36.6	5.8	1.0	2.1	0.0
Labor supply	1.0	2.6	0.2	1.3	0.0	1.0	6.2	1.0	1.0	0.0
Recruiting	10.3	25.1	3.1	12.9	0.0	10.2	14.2	1.0	1.5	0.0
Matching efficiency	80.1	179.6	44.2	93.0	0.0	69.1	422.5	1.0	4.8	0.0

			wage posting model (hiring)							
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	1.2	2.5	0.6	1.3	0.0	0.8	0.6	1.0	0.9	0.9
Separation	16.8	12.4	3.1	6.4	11.8	1.1	3.3	1.0	0.5	0.0
Preference	35.2	21.9	4.5	38.8	0.0	16.5	2.3	1.0	1.1	0.0
Labor supply	3.5	7.7	2.2	4.0	0.0	2.7	5.5	1.0	0.9	0.0
Recruiting	75.4	162.6	33.2	84.5	0.0	43.8	23.3	1.0	3.4	0.0
Data	9.1	8.3	1.5	6.3	4.9	3.4	0.7	1.0	0.8	0.7

Notes and source: Statistics for the U.S. economy are based on the following. u: Unemployment level (UNEMPLOY). v: Help-wanted index (Barnichon, 2010). h: total hires. UEr: Unemployment-to-employment transition rate. EUr: Employment-to-unemployment transition rate. w^a: Average wage in the economy. y: Real output in the nonfarm business sector (GDPC1). p: Real output per-hour in the non-farm business sector (OPHNFB). a: Utilization adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000. "Hiring" refers to a calibrated model in which firms face a hiring cost function. "Vacancy" refers to a calibrated model in which firms face a vacancy cost function. Wages are flexible in all models.

Table 5: Business Cycle Moments. Correlation with unemployment.

	u	v	h	UEr	EUr	EEr	w^a	y	p
value posting model (vacancy)									
Productivity	1.00	-0.89	-0.22	-0.90	0.03	-0.35	-0.97	-0.95	-0.95
Separation	1.00	1.00	1.00	1.00	0.93	1.00	-0.24	-0.97	0.91
Preference	1.00	0.98	1.00	-0.90	-0.02	0.99	0.10	-0.98	0.94
Labor supply	1.00	-0.77	-0.11	-0.79	0.01	-0.31	0.01	-0.61	-0.57
Recruiting	1.00	-0.88	-0.91	-0.89	-0.02	-0.86	0.09	0.41	0.99
Matching efficiency	1.00	-0.87	-0.24	-0.88	-0.03	-0.37	-0.22	-0.91	-0.89
value posting model (hiring)									
Productivity	1.00	-0.93	-0.71	-0.94	0.04	-0.89	-0.95	-0.90	-0.89
Separation	1.00	-0.92	0.99	-0.94	0.86	0.90	-0.58	-0.91	0.41
Preference	1.00	0.29	0.57	-0.93	-0.03	0.62	0.06	-0.96	-0.73
Labor supply	1.00	-0.87	-0.51	-0.88	0.01	-0.77	0.08	-0.69	-0.56
Recruiting	1.00	-0.89	-0.79	-0.90	0.00	-1.00	0.28	-1.00	1.00
wage posting model (vacancy)									
Productivity	1.00	-0.82	-0.97	-0.83	0.03	-0.50	-0.86	-0.97	-0.97
Separation	1.00	0.89	0.97	-0.29	0.91	0.60	-0.75	-0.77	-0.12
Preference	1.00	0.60	0.92	-0.92	-0.02	0.88	0.55	0.05	0.88
Labor supply	1.00	-0.64	-0.81	-0.65	0.01	-0.05	0.95	-0.22	-0.16
Recruiting	1.00	-0.75	-0.39	-0.76	0.00	0.02	0.97	0.69	0.87
Matching efficiency	1.00	-0.85	-0.93	-0.86	-0.01	-0.95	0.99	0.09	0.98
wage posting model (hiring)									
Productivity	1.00	-0.93	-0.78	-0.94	0.04	-0.94	-0.92	-0.91	-0.89
Separation	1.00	-0.90	0.95	-0.93	0.81	-0.49	-0.88	-0.84	0.03
Preference	1.00	-0.96	0.93	-0.94	-0.01	0.71	-0.99	-0.97	0.97
Labor supply	1.00	-0.86	-0.63	-0.87	0.01	-0.90	0.62	-0.65	-0.51
Recruiting	1.00	-0.91	-0.83	-0.91	0.03	-1.00	0.95	-0.98	1.00
Data	1.00	-0.93	-0.40	-0.97	0.92	-0.84	-0.04	-0.76	0.42

Notes and source: Statistics for the U.S. economy are based on the following. u: Unemployment level (UNEMPLOY). v: Help-wanted index (Barnichon, 2010). h: total hires. UEr: Unemployment-to-employment transition rate. EUr: Employment-to-unemployment transition rate. w^a: Average wage in the economy. y: Real output in the nonfarm business sector (GDPC1). p: Real output per-hour in the non-farm business sector (OPHNFB). a: Utilization adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000. "Hiring" refers to a calibrated model in which firms face a hiring cost function. "Vacancy" refers to a calibrated model in which firms face a vacancy cost function. Wages are flexible in all models.

6.2 Steady State

Compared to the observed wage distribution in the economy, wage posting models do a good job of generating wage dispersion. But value posting models struggle to generate high wages. Table 2 presents the distribution of log-wages in the economy (based on CPS microdata) and the implied moments by the models in steady state. In particular, this table presents the deciles of the log-wage distribution as a fraction of the mean.[15] In terms of the firm-size distribution, all models generate a flatter distribution than in the data. The firm-size distribution for value posting models is flatter than for wage posting models, and that is the reason for the lack of high wages in Table 2. As shown in Table 3, in the data, large firms are, on average, 740 times larger than small firms. Compared to the data, all models predict smaller firms at the right tail of the distribution and larger firms everywhere else.[16]

6.3 Business Cycle Moments

Table 4 presents the standard deviation of the variables of interest relative to the standard deviation of output for each model and each shock. For this exercise, I assumed flexible wages ($\phi = 0$). As in the standard DMP model, productivity shocks generate little amplification in the labor market quantities. For example, the unemployment rate is, at most, as volatile as output, despite a FOCE value equal to 72% of the average labor productivity. I explore the small amplification of productivity shocks further in the next subsections. But productivity shocks are little amplified even for large values of the FOCE because productivity shocks are more important for the least-productive firms, which are small and account for a small fraction of total employment.

Table 4 suggests that matching efficiency shocks ($\bar{m}$) and recruiting cost shocks (κ) are likely to be important drivers of the business cycle in the labor market. Those shocks generate a large amplification and result in good co-movements in the labor market (Table 5). As in the standard DMP model, separation rate shocks are able to generate large fluctuations in the labor market. But those shocks generate counterfactual dynamics. Depending on the recruiting cost function, the correlation between unemployment and vacancies (in response to separation rate shocks) could be negative as in the data, but the correlation of unemployment with hires is always positive, as shown in Table 5. Also, in most cases, the correlation between unemployment and job-to-job transitions is also positive. In Appendix B, I explain in more detail that the average duration of a match increases in response to a decline in the separation rate shock, making firms increase

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[15]To compute the data moments, I run a Mincer equation with time fixed effects using the CPS microdata from January 1994 to December 2015. Then, using the residuals from this Mincer equation, I compute the deciles of the distribution for each month. The data numbers presented in table 2 are the sample means of those deciles. Appendix A presents details.

[16]Firm size is relative to the average size of the smallest firms in the economy (those with one to four employees).

their wages to retain a larger fraction of their workers. For that reason, hires and the job-to-job transition rate tend to decline in response to a negative separation rate shock.

Table 4 also highlights one issue with value posting models: the average-wage volatility is significantly larger than the data. As briefly discussed in previous sections, values-of-employment offers increase in response to positive aggregate shocks. Therefore, current wages can go down to keep the value-of-employment constant for job-stayers. That wage dynamic is counterfactual and is shown in Figure 1, which plots the impulse response functions of average wages to a positive productivity shock in these four models. Note that average wages decline in value posting models on impact. That decline is not present in wage posting models, which was the reason for introducing wage posting and wage rigidity in Section 3. Not surprisingly, the quarterly autocorrelation of wages in value posting models is very low relative the data, regardless of the shock (Table 6). However, in wage posting models, the quarterly autocorrelation of wages is more in line with the empirical evidence.

Table 6: Business Cycle Moments. Wage autocorrelation

	Productivity	Separation	Preference	Labor supply	Recruiting	Matching efficiency
value posting model (vacancy)	0.782	0.490	0.206	0.178	-0.127	0.741
value posting model (hiring)	0.788	0.661	0.363	0.293	-0.084	
wage posting model (vacancy)	0.930	0.977	0.703	0.867	0.952	0.973
wage posting model (hiring)	0.948	0.977	0.714	0.851	0.973	

Note: This table presents the autocorrelation for the average wage in the economy. "Hiring" refers to a calibrated model in which firms face a hiring cost function. "Vacancy" refers to a calibrated model in which firms face a vacancy cost function. Wages are flexible in all models.

6.3.1 Wage Rigidity

Wage rigidity has emerged as a popular mechanism to boost the unemployment volatility in the standard search and matching model. However, as shown in Table 7, wage rigidity generates little or insignificant amplification in wage posting models. In particular, Table 7 presents the business cycle volatility of the wage posting model with a hiring cost function for three different values for ϕ: 0, 100, and 500. The reason behind this result is the same reason for the small amplification of productivity shocks: as with productivity, wages only represent a large fraction of the value of a filled vacancy for low productivity firms, which are small and, more importantly, account for a small fraction of total employment in the data. Figures 2a and 2b plot the ratio of wages and productivity, respectively, to the value of a filled vacancy for all active firms in the economy.

These figure indicate that wages and productivity are proportionally more important for the least-productive firms. Hence, a 1 percentage point increase in aggregate productivity translates into larger percentage changes in the value of a filled vacancy for less productive firms, even with rigid wages. But those firms have a limited impact on aggregate dynamics because they are small and represent a small fraction of employment.

6.3.2 FOCE

In the standard DMP model, the relative size of the FOCE determines how much shocks are amplified (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017). In wage posting models, even though larger values of the FOCE tend to increase the volatility of labor market variables, the size of that value does not significantly amplify shocks, as shown in Table 8.

Figure 1: Average Wage Response to a 1% Increase in Aggregate TFP.
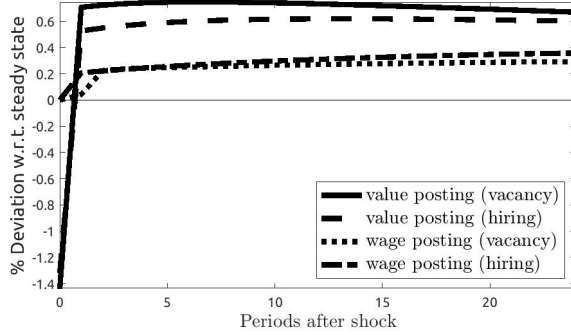
Interestingly, recruiting cost shocks are more amplified for *small* values of the FOCE, while productivity shocks are more amplified for *large* values of the FOCE.

As discussed before, productivity shocks results in proportionally larger changes in the value of a filled vacancy (J_{jt}) for the least-productive firms than for the most-productive firms. Hence, those least-productive firms try to expand employment the most in response to those shocks. In contrast, recruiting cost shocks tend to affect all firms homogeneously as seen from the job creation condition (21).

In response to a decline in the recruiting cost, all firms expand employment, making the FOCE increase and preventing the value of a filled vacancy from increasing. That negative effect is larger for large values of the FOCE.[17] Hence, recruiting cost shocks are more amplified when the value

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[17]Taking the derivative of z_j with respect to n_j:

$$\hat{z}_j = \epsilon \left(1 - \frac{b}{p_j} \frac{p_j}{z_j}\right) \hat{h}_j \tag{64}$$

where $\hat{x}$ represents percentage deviation with respect to the steady state. Based on the evidence of Chodorow-Reich and Karabarbounis (2016), unemployment benefits represent 6% of labor productivity. Hence, the term in front of $\hat{n}_j$ becomes bigger for large values of the FOCE.

Table 7: Business Cycle Moments. Standard deviation. Wage posting model with hiring cost and wage rigidity.

| | wage posting model (hiring). $\phi=0$ | | | | | | | | | |
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	1.2	2.5	0.6	1.3	0.0	0.8	0.6	1.0	0.9	0.9
Separation	16.8	12.4	3.1	6.4	11.8	1.1	3.3	1.0	0.5	0.0
Preference	35.2	21.9	4.5	38.8	0.0	16.5	2.3	1.0	1.1	0.0
Labor supply	3.5	7.7	2.2	4.0	0.0	2.7	5.5	1.0	0.9	0.0
Recruiting	75.4	162.6	33.2	84.5	0.0	43.8	23.3	1.0	3.4	0.0

| | wage posting model (hiring). $\phi=100$ | | | | | | | | | |
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	1.2	2.6	0.6	1.3	0.0	0.8	0.6	1.0	0.9	0.9
Separation	16.8	12.4	3.1	6.4	11.8	1.1	3.3	1.0	0.5	0.0
Preference	35.3	21.8	4.5	38.8	0.0	16.5	2.3	1.0	1.1	0.0
Labor supply	3.4	7.6	2.2	3.9	0.0	2.7	5.2	1.0	0.9	0.0
Recruiting	75.5	162.7	33.2	84.5	0.0	43.8	22.7	1.0	3.4	0.0

| | wage posting model (hiring). $\phi=500$ | | | | | | | | | |
	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity	1.3	3.4	0.8	1.7	0.0	0.8	0.6	1.0	1.0	0.9
Separation	16.8	12.4	3.1	6.4	11.8	1.1	3.3	1.0	0.5	0.0
Preference	35.3	21.7	4.5	38.9	0.0	16.4	2.3	1.0	1.1	0.0
Labor supply	3.3	7.3	2.1	3.8	0.0	2.6	4.7	1.0	0.9	0.0
Recruiting	75.6	162.3	33.0	84.4	0.0	43.9	22.0	1.0	3.4	0.0
Data	9.1	8.3	1.5	6.3	4.9	3.4	0.7	1.0	0.8	0.7

Notes and source: Statistics for the U.S. economy are based on the following. u: Unemployment level (UNEMPLOY). v: Help-wanted index (Barnichon, 2010). h: total hires. UEr: Unemployment-to-employment transition rate. EUr: Employment-to-unemployment transition rate. w^a: Average wage in the economy. y: Real output in the nonfarm business sector (GDPC1). p: Real output per-hour in the non-farm business sector (OPHNFB). a: Utilization adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000. "Hiring" refers to a model in which firms face a hiring cost function. ϕ is the degree of wage rigidity per equation (81).

of the FOCE is small.

When the FOCE is large, on the one hand, the value of a filled vacancy increases proportionally more in response to productivity shocks. On the other hand, the subsequent increase in employment makes the FOCE increase, preventing the value of a filled vacancy from rising even more. On

Figure 2: Wages and Idiosyncratic Productivity as a Fraction of Filled Vacancy Value

(a) Wages   (b) Productivity
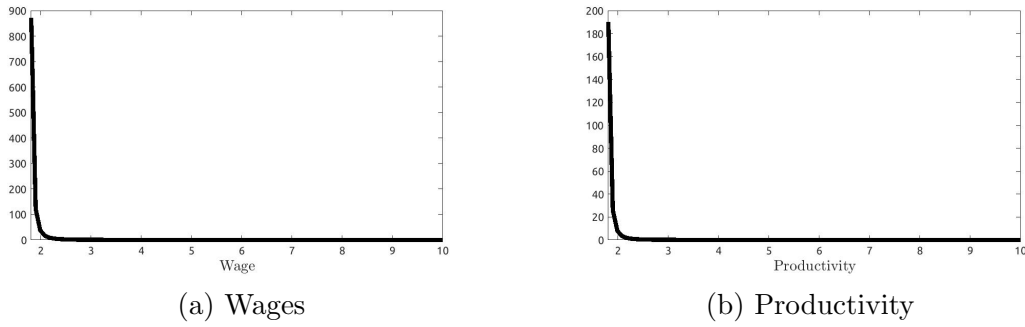
net, as shown in Table 8, larger values of the FOCE tend to amplify the responses to productivity shocks.

Hence, we can see that large values of the FOCE will tend to make some shocks even more (or less) important at explaining the business cycle. However, in contrast to the standard DMP model, the relative size of the outside option has an important implication on the simulated firm-size distribution: In wage posting models, *the larger the value of the FOCE, the flatter the firm size distribution* with respect to the firm-productivity distribution, as shown in Table 9. For large FOCE values ($FOCE = z_j = b + \Psi\tilde{n}^{\eta-\epsilon}c^{\sigma}n_j^{\epsilon}$), the FOCE elasticity with respect to employment increases more for the most-productive firms, which prevents those firms from being large. Even though the firm-size distribution is always flatter in the model than in the data in Table 9, large values for the FOCE seem at odds with the firm-size distribution in the data. It is worth noting that the firm-productivity distribution is the same in each case and was calibrated to target empirical evidence related to the distribution of log-TFP and log-labor productivity. Given my firm-productivity distribution calibration, the model demands low values of the FOCE to generate a steep firm-size distribution, which reinforces the observation that recruiting cost shocks and matching efficiency shocks are likely drivers of the business cycle volatility in the labor market.

As illustrated in Table 10, if the firm-productivity distribution was very steep, the model would demand high values for the FOCE. Table 10 presents the simulated firm-size distribution for an alternative and steeper calibration of the firm-productivity distribution. However, in the alternative calibration of Table 10, the standard deviation of log-TFP and log-labor productivity are around 0.16 and 0.25, respectively, which are below the empirical evidence of Decker et al. (2020).[18]

Hence, in wage posting model, the FOCE value could be disciplined by data on the distribution of firms: firm-productivity and firm-size distribution. In the standard DMP model, the value of the FOCE is, to a large extent, a *"free"* parameter.

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[18]In this alternative calibration the Pareto parameter is equal to 6. In my baseline calibration that parameter was 0.125.

Table 8: Business Cycle Moments. Standard deviation. Wage posting model with hiring cost. Different FOCE values.

	u	v	h	UEr	EUr	EEr	w^a	y	p	a
wage posting model (hiring). z=0.2										
Productivity	0.674	1.419	0.375	0.737	0.000	0.530	0.699	1.000	0.966	0.925
Separation	14.606	11.192	1.792	5.580	9.767	2.293	4.447	1.000	0.618	0.000
Preference	30.052	21.592	1.998	33.108	0.000	10.449	7.035	1.000	0.901	0.000
Labor supply	0.944	1.955	0.586	1.020	0.000	0.852	3.103	1.000	0.946	0.000
Recruiting	97.943	207.486	40.632	108.009	0.000	55.118	120.011	1.000	4.941	0.000
wage posting model (hiring). z=0.6										
Productivity	0.979	2.076	0.515	1.076	0.000	0.706	0.617	1.000	0.949	0.911
Separation	15.912	12.124	2.526	6.162	10.827	1.386	3.549	1.000	0.551	0.000
Preference	33.618	22.430	3.411	36.977	0.000	14.122	2.937	1.000	1.040	0.000
Labor supply	2.317	5.115	1.505	2.652	0.000	1.927	5.125	1.000	0.903	0.000
Recruiting	83.852	179.768	36.417	93.475	0.000	48.500	34.192	1.000	3.889	0.000
wage posting model (hiring). z=0.72										
Productivity	1.184	2.557	0.607	1.321	0.000	0.798	0.612	1.000	0.941	0.904
Separation	16.770	12.422	3.107	6.424	11.753	1.053	3.282	1.000	0.514	0.000
Preference	35.261	21.812	4.529	38.778	0.000	16.493	2.304	1.000	1.115	0.000
Labor supply	3.411	7.627	2.154	3.948	0.000	2.670	5.177	1.000	0.882	0.000
Recruiting	75.479	162.663	33.163	84.540	0.000	43.847	22.682	1.000	3.424	0.000
Data	9.092	8.316	1.519	6.326	4.932	3.391	0.723	1.000	0.754	0.669

Notes and source: Statistics for the U.S. economy are based on the following. *u:* Unemployment level (UNEMPLOY). *v:* Help-wanted index (Barnichon, 2010). *h:* total hires. *UEr:* Unemployment-to-employment transition rate. *EUr:* Employment-to-unemployment transition rate. w^a: Average wage in the economy. *y:* Real output in the nonfarm business sector (GDPC1). *p:* Real output per-hour in the non-farm business sector (OPHNFB). *a:* Utilization adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000. "Hiring" refers to a calibrated model in which firms face a hiring cost function. "Vacancy" refers to a calibrated model in which firms face a vacancy cost function. Wages are flexible in all models.

6.4 What Have We Learned? The Distribution of Firms Matters... a Lot

Based on the results of this section, we can conclude that there are some similarities and differences between wage posting models and the standard DMP model. The main similarity: Separation rate shocks can generate large fluctuations in the labor market but generate counterfactual dynamics

Table 9: Employment Size Distribution in Steady State. Wage posting moel with hiring cost. Different values of the FOCE.

Average Size	1 to 4	5 to 9	10 to 19	20 to 49	50 to 99	100 to 249	250+	All
z = 0.20	1.000	5.175	15.655	39.811	68.146	83.543	93.029	9.720
z = 0.40	1.000	5.475	16.306	39.400	65.673	80.558	90.232	9.720
z = 0.60	1.000	6.067	17.631	39.349	61.041	72.266	79.270	9.720
z = 0.72	1.000	6.538	18.531	39.008	57.796	67.019	72.651	9.720
z = 0.85	1.000	7.278	19.657	38.127	53.507	60.688	64.993	9.720
z = 0.95	1.000	8.243	20.714	36.665	49.029	54.648	57.990	9.720
Data	1.000	2.990	6.150	13.810	31.370	68.620	740.340	9.720

Note: This table reports data and model-generated moments for the employment-size distribution across firms in the United States. Average employment size is relative to the smallest firms (1 to 4 employees). Firm size is defined as the number of employees per firm. Average size is computed as the total number of employees over the total number of firms. Data source is Business Dynamics Statistics. Model-generated moments are for a wage-posting model with a hiring cost function and flexible wages.

and correlations.

More important, the main difference between wage posting models and the standard DMP model is that *the distribution of firms, which is absent in the standard DMP model, is one of the most important elements for the amplification and propagation of shocks.* To see this conclusion more clearly, let me go over some of the main results in this section. First, productivity shocks are unlikely to generate significant fluctuations in the labor market even in the presence of wage rigidity or for high values of the FOCE. Productivity shocks result in proportionally large changes in the value of filled vacancy only for low-productivity firms, which are small and account for a small fraction of total employment in the economy. Second, wage rigidity generates insignificant business cycle amplification because wages are an important part of the value of filled vacancy only for low-productivity firms, which, again, are small and account for a small fraction of total employment. Third, the relative size of the FOCE governs the amplification of shocks in the labor market, as in the standard DMP model. For example, large (small) values of the FOCE amplify productivity (recruiting cost) shocks. However, while the relative size of the FOCE is a free parameter in the standard DMP model, the distribution of firms (productivity and size) disciplines that parameter in wage posting models. Based on my calibration strategy, the size of the outside options should be small, indicating the productivity shocks are unlikely to be an important source of business cycle volatility in the labor market, consistent with the recent empirical evidence by Angeletos, Collard, and Dellas (2020). In contrast, recruiting cost shocks and matching efficiency shocks seem more likely to be relevant at explaining the volatility in the labor market.

Table 10: Employment Size Distribution in Steady State. Wage posting model with hiring cost. Different FOCE values and alternative productivity distribution

Average Size	1 to 4	5 to 9	10 to 19	20 to 49	50 to 99	100 to 249	250+	All
$z = 0.20$	1.000	3.418	8.369	23.322	60.570	126.599	379.545	9.720
$z = 0.30$	1.000	3.385	8.160	22.320	57.681	122.886	410.565	9.720
$z = 0.40$	1.000	3.351	7.971	21.528	55.596	120.332	434.375	9.720
$z = 0.50$	1.000	3.317	7.796	20.829	53.810	118.139	455.348	9.720
$z = 0.60$	1.000	3.285	7.633	20.217	52.331	116.468	473.249	9.720
$z = 0.70$	1.000	3.259	7.505	19.781	51.423	115.892	484.770	9.720
$z = 0.80$	1.000	3.248	7.457	19.691	51.614	117.583	483.377	9.720
$z = 0.90$	1.000	3.228	7.429	19.973	53.923	125.755	459.688	9.720
Data	1.000	2.990	6.150	13.810	31.370	68.620	740.340	9.720

Note: This table reports data and model-generated moments for the employment-size distribution across firms in the United States. Average employment size is relative to the smallest firms (1 to 4 employees). Firm size if defined as the number of employees per firm. Average size is computed as the total number of employees over the total number of firms. Data source is Business Dynamics Statistics. Model-generated moments are for a wage-posting model with a hiring cost function and flexible wages. The productivity distribution used for this table is steeper than in my baseline calibration. The pareto parameter used for this table was 6.

7   Comparison with MPV16

Moscarini and Postel-Vinay (2016) also proposed an algorithm for solving wage posting models when firms face a hiring cost function and other specific assumptions. I would like to close this paper by showing that my algorithm is able to reproduce (qualitatively) the same results as MPV16 when the model is calibrated to match the same targets.

The results of this paper seem at odds with the conclusions of MPV16 particularly that value posting models (only with productivity shocks) are capable of generating good business cycle moments (volatility and correlations) in labor market quantities. I show that the large volatility in labor market quantities generated by MPV16 was purely due to the negative correlation between productivity shocks and the separation rate. Also, I show that the reported counterfactual dynamics generated by separation rate shocks in Section 6.3 also arise in the MPV16 simulation. Hence, the results of this section reinforce my conclusions.

Calibration: In the main text, MPV16 assumed a truncated Pareto distribution between 1 and 10 with a shape parameter equal to 2.5, a linear utility function in consumption, no capital, a zero FOCE value, and a matching function that is linear in vacancies. These assumptions and model parameters are mapped into the model outline in this paper. Table 1 (under column MPV16) lists the parameter values that make both models comparable. To get as close as possible to

their calibration, I assume a correlation between TFP and separation rate shocks to target their simulated correlation between labor productivity and UEr (-0.885). This number is higher than the data, but targeting this correlation seems like the right way to compare these models. Similarly, I calibrate the standard deviation of the exogenous shocks to match the simulated volatilities of the labor productivity and the separation rate reported in MPV16.

Results: Table 11 reproduces Table 3 in MPV16 and adds model-generated moments based on the algorithm proposed in this paper. In table 11, rows (D) report data moments, rows (MPV) present the simulated moments reported by MPV16, and rows (MJ) report the simulated moments generated by the algorithm presented in this paper. Based on Table 11, I conclude that my algorithm is able to reproduce qualitatively the same moments as the algorithm of MPV16.

Table 11: Business Cycle Moments in MPV16.

		U rate	UE rate	EU rate	V/U	ALP
U rate	(D)	0.216				
	(MPV)	0.201				
	(MJ)	0.186				
UE ratev	(D)	-0.974	0.121			
	(MPV)	-0.987	0.130			
	(MJ)	-0.797	0.120			
EU rate	(D)	0.889	-0.887	0.144		
	(MPV)	0.783	-0.682	0.115		
	(MJ)	0.573	-0.501	0.117		
V/U	(D)	-0.978	0.972	-0.912	0.366	
	(MPV)	-0.998	0.994	-0.752	0.279	
	(MJ)	-0.988	0.864	-0.573	0.240	
ALP	(D)	0.108	-0.017	-0.275	-0.011	0.013
	(MPV)	-0.715	0.627	-0.885	0.686	0.014
	(MJ)	-0.599	0.607	-0.853	0.617	0.014

Note: Rows (D) and (MPV) of this Table reproduce Table 3 of Moscarini and Postel-Vinay (2016). Rows (MJ) are the simulated business cycle moments generated by the algorithm presented in this paper when the model is calibrated to match the same moments as in MVP16. Elements on the main diagonal are standard deviations, and all other numbers are correlations. Data and simulated moments are based on log deviation from an HP trend with a smoothing parameter equal to 1,600.

For completeness, Table 12 in Appendix D presents additional business cycle moments generated under this calibration. Based on these tables, and as noticed throughout this paper, TFP shocks tend to generate small responses in labor market quantities unless they are correlated with the separation rate, which can generate large responses in the labor market but makes total hiring countercyclical. Based on the Appendix tables, we can also see that the wage volatility for the average wage in the economy (w^a) is less persistent and much more volatile than in the data, mainly because of the large and counterfactual wage responses on impact. However, in contrast to my baseline calibration, the job-to-job transition rate is pro-cyclical under this particular calibration. To understand this result, notice that the total hiring volatility is small (Table 12), which is due to the large value for χ. Notice that log-linearizing the optimality condition for the hiring decision rule in this model, we get:

$$\hat{h}_{jt} = \frac{1}{\chi}\hat{J}_{jt+1}, \tag{65}$$

where $\hat{x}$ denotes log-deviation of x with respect to the steady state. As discussed in Appendix B, J_j tends to decline in response to a decrease in the separation rate. But even if most of the J_j responses were positive, given that $\chi = 50$ under this calibration, the hiring response will tend to be small. As a consequence, it is not surprising that total hiring volatility is small for large values of χ. Then, if job-to-job transitions are going up, it is because there is a recomposition of hiring in the economy: The most-productive firms are hiring more, relative to the low-productivity firms. Therefore, if χ was smaller, the decline in total hiring would be larger, which would make job-to-job flows go down even if there is a recomposition of hiring in the economy. Hence, the apparent success of this model to generate a positive correlation between unemployment and EEr is only possible by generating a counterfactually small volatility in total hiring.

As mentioned in the introduction, I think that the MPV16 algorithm is a useful method that enables researchers to answer particular questions that my algorithm may not; specifically, if a researcher is interested in questions that involve nonlinearities. However, the method proposed in this paper has some particular (and powerful) advances over MPV16. My method does not suffer from the *curse of dimensionality*. I can include as many shocks and state variables as I want without increasing significantly the computational burden, allowing researchers to study other frictions and sources of aggregate fluctuations. Hence, one can easily integrate wage posting models with random search in an even more general framework (such as a medium scale New Keynesian model). Also, because of the nature of my algorithm, stochastic simulations and impulse response functions are an easy and useful exercise to implement.

8   Conclusion

Wage posting models are an appealing way of studying wage dispersion and wage growth. However, quantifying the cyclical properties of these types of models has remained a challenging task. In this paper, I study the business cycle properties of wage posting models with random search in which the distribution of employment and wages matter for agents' decision rules and the equilibrium path. I present a model that extends the work by Moscarini and Postel-Vinay (2013). I introduce capital and a strictly concave utility function for the household, and I assume imperfect substitution between jobs. A calibrated version of my model is able to deliver reasonably good steady-state moments regarding wages and, to some extent, firm-size of employment.

Based on the results of this paper, I conclude that *the distribution of firms is key to understanding business cycle fluctuations in the labor market.* The distribution of firms implies that productivity shocks are unlikely to be important drivers of the business cycle in the labor market even for high values of the FOCE. The distribution of firms implies that wage rigidity does not significantly amplify shocks in the labor market and that shocks that affect firms more homogeneously (like recruiting cost and matching efficiency shocks) are better candidates at explaining the volatility in the labor market. And, while the relative size of the FOCE determines the amplification of shocks, data on the distribution of firms can be used to calibrate the value of the FOCE.

Future research could improve the lack of firm entry and exit at different points of the distribution. The model presented in this paper allows for firm entry and exit only at the bottom of the distribution, where firms are the least productive. The main implication of this assumption is that the most-productive firms are always large and the least-productive firms are always small, which could have important implications for productivity shocks.

To evaluate this model quantitatively, I propose a new algorithm that may be of interest in its own right. The algorithm presented in this paper has powerful features that make it very useful. This method does not suffer from the curse of dimensionality, takes only a few seconds to compute the steady state and transitional dynamics, and makes it possible and easy to integrate wage posting models to even more general frameworks.

References

[1] Angeletos, George-Marios, Fabrice Collard, and Harris Dellas (2020) "Business-Cycle Anatomy." *American Economic Review* vol. 110(10), pages: 3030-3070.

[2] Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014) " Some Evidence on the Importance of Sticky Wages." *American Economic Journal: Macroeconomics* vol. 6(1),

pages: 70-101.

[3] Barnichon, Regis (2010) "Building a Composite Help-Wanted Index." *Economics Letters* vol. 109(3), pages: 175-178.

[4] Burdett, Kenneth, and Dale T. Mortensen (1998) "Wage Differentials, Employer Size, And Unemployment." *International Economic Review* vol. 39(2), pages: 257-273.

[5] Chodorow-Reich, Gabriel, and Loukas Karabarbounis (2016) "The Cyclicality of The Opportunity Cost of Employment." *Journal of Political Economy* vol. 124(6), pages: 1563-1618.

[6] Coles, Melvyn G. and Dale T. Mortensen (2011) "Equilibrium Wage and Employment Dynamics in a Model of Wage Posting without Commitment." *NBER Working Paper # 17284.*

[7] Decker, Ryan A., John Haltiwanger, Ron S. Jarmin, and Javier Miranda (2020). "Changing Business Dynamism and Productivity: Shocks versus Responsiveness." *American Economic Review*, vol. 110(12), pages: 3952-3990.

[8] Fallick, Bruce, and Charles A. Fleischman (2004) "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows" *Finance and Economics Discussion Series # 2004-34.*

[9] Furlanetto, Francesco, and Nicolas Groshenny (2016). "Mismatch shocks and unemployment during the Great Recession." *Journal of Applied Econometrics*, vol. 31(7), pages: 1197-1214.

[10] Gertler, Mark, and Antonella Trigari (2009) "Unemployment Fluctuations With Staggered Nash Wage Bargaining." *Journal of Political Economy* vol. 117(1), pages: 38-85.

[11] Gertler, Mark, Luca Sala, and Antonella Trigari (2009) "An Estimated DSGE Model with Unemployment and Staggered Nominal Wage Bargaining." *Journal of Money Credit and Bankin* vol. 40(8), pages: 1713-1764.

[12] Grigsby, John, Erik Hurst, and Ahu Yildirmaz (2019) "Aggregate Nominal Wage Adjustment: New Evidence from Administrative Payroll Data." MIMEO, University of Chicago.

[13] Haefke, Christian, Marcus Sonntag, and Thijs van Rens (2013) "Wage Rigidity And Job Creation." *Journal of Monetary Economics* vol. 60(8), pages: 887-899.

[14] Hagedorn, Marcus, and Iourii Manovskii (2008) "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* vol. 98(4), pages: 1692-1706.

[15] Hall, Robert E. (2005) "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* vol. 95(1), pages: 50-65.

[16] Hall, Robert E., and Paul R. Milgrom (2008) "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review* vol. 98(4), pages: 1653-1674.

[17] Haltiwanger, John C., Henry Hyatt, and Erika McEntarfer (2015) "Cyclical Reallocation of Workers Across Employers by Firm Size and Firm Wage." MIMEO, University of Maryland.

[18] Kahn, Shulamit (1997) "Evidence of Nominal Wage Stickiness from Microdata." *American Economic Review* vol. 87(5), pages: 993-1008.

[19] Klein, Paul (2000) "Using the Generalized Schur form to Solve a Multivariate Linear Rational Expectations Model." *Journal of Economic Dynamics and Control* vol. 24(10), pages: 1405-1423.

[20] Ljungqvist, Lars, and Thomas J. Sargent (2017) "The Fundamental Surplus." *American Economic Review* vol. 107(9), pages: 2630-2665.

[21] Lubik, Thomas (2009). "Estimating a Search and Matching Model of the Aggregate Labor Market (2009)." *FRB Richmond Economic Quarterly*, vol. 95(2), pages: 101-120.

[22] Menzio, Guido, and Shouyong Shi (2011) "Efficient Search on the Job and the Business Cycle." *Journal of Political Economy* vol. 119(3), pages: 468-510.

[23] Moscarini, Giuseppe, and Fabien Postel-Vinay (2012) "The Contribution of Large and Small Employers to Job Creation in Times of High and Low Unemployment." *American Economic Review* vol. 102(6), pages: 2509-2539.

[24] Moscarini, Giuseppe, and Fabien Postel-Vinay (2013) "Stochastic Search Equilibrium." *Review of Economic Studies* vol. 80(4), pages: 1545-1581.

[25] Moscarini, Giuseppe, and Fabien Postel-Vinay (2016) "Wage Posting and Business Cycles: A Quantitative Exploration." *Review of Economic Dynamics* vol. 19(special issue in the memory of Dale Mortensen), pages: 135-160.

[26] Reiter, Michael (2009) "Solving Heterogeneous-Agent Models by Projection and Perturbation." *Journal of Economics Dynamics and Control* vol 33 (3), pages: 649-665.

[27] Rotemberg, Julio J. (1982) "Sticky Prices in the United States." *Journal of the Political Economy* vol. 90(6), pages 1187-1211.

[28] Schmitt, John (2003) "Creating a Consistent Hourly Wage Series From The Current Population Survey's Outgoing Rotation Group, 1979-2002." Unpublished Manuscript. Center for Economic and Policy Research.

A   Wages (Details)

I use the Current Population Survey (CPS) microdata to construct wage series adjusted for workers' characteristics. The CPS is the main labor force survey for the United States, and it is the primary source of labor force statistics such as the national unemployment rate. The CPS consists of a rotating panel where households and their members are surveyed for four consecutive months, not surveyed for the following eight months, and interviewed again for another four consecutive months. The CPS includes individual information such as employment status, sex, education, race, and state. However, individual earnings and hours worked are collected only in the fourth and eighth interviews. In addition, since 1994, individuals have been asked if they still work in the same job reported in the previous month, making it possible to identify job changers.

A.1   Wage Series

I present business cycle statistics for wage series that control for individual characteristics. My empirical model is based on the following MINCER equation for the wage of individual i at time t (w_{it}):

$$log(w_{it}) = x'_{it}\beta_x + \left(\sum_{j=1}^{T} \alpha_j^a \cdot D_j + \alpha_j^{nhu} \cdot D_j \cdot D_{it,nhu} + \alpha_j^{nhc} \cdot D_j \cdot D_{it,nhc} \right) + e_{it}. \tag{66}$$

x_{it} is a vector of individual characteristics, and β_x, $\{\alpha_j^a, \alpha_j^{nhu}, \alpha_j^{nhc}\}_{j=1}^{T}$ are coefficients. D_j is a time dummy equal to 1 if $j = t$ and 0 otherwise. $D_{it,nhu}$ is a dummy variable equal to 1 if worker i spent time in unemployment during the past three months and 0 otherwise. $D_{it,nhc}$ is a dummy variable equal to 1 if worker i was previously employed at another firm during the past three months and has not been unemployed while switching jobs. Hence, the average (log) wage for all workers (w^a) is given by: $w_t^a = \alpha_t$.

The hourly wage rate is constructed by dividing weekly earnings by weekly hours. Following Schmitt (2003), top-coded weekly earnings are imputed assuming a log-normal cross-sectional distribution for earnings. Following Haefke et al. (2013), I drop hourly wage rates below the 0.25th and above the 99.75th percentiles each month. In order to uniquely identify workers in the CPS files, I use the IMPUMS-CPS ID variables CPSID and CPSIDP.[19]

Vector x_{it} includes a fourth order polynomial in experience, gender, race, marital status, state, 10 occupation dummies, and 14 industry dummies. For occupation, industry, and education, I use harmonized variables OCC1950, IND1950, and EDUC provided by IPUMS-CPS. Experience

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[19]I follow IPUMS-CPS recommendations, and I drop a few observations for which changes in sex or race are reported and for individuals whose age changes more than two years between samples

is defined as age minus years of education minus 6. Following the literature, individual i's weight is the product of the individual's weight reported by the BLS and hours worked.

Due to well known problems, it is not possible to match individuals between July and December 1985 and between June and November 1995. Hence, with the exception of the average wage for all workers, wage series have a missing value in those months. To compute business cycle statistics for these wage series, I impute the missing months using the average wage for all workers.

A.1.1 Wage Distribution

To compute the distribution of wages for Table 2, I find the deciles of the distribution of e_{it} for each month. Then, Table 2 presents the mean of those deciles.

B Separation Rate Shocks, Unemployment, Hires, and Job-to-Job Transitions

To gain some intuition about the negative correlation between unemployment and total hires and between unemployment and the job-to-job transition rate, notice that in steady state, the value of a filled vacancy (J_j) and the optimal number of new hires (h_j) at firm j are given by:

$$J_j = \frac{[1 - (1 - \delta_n)(1 - \bar{i}qG_j)]}{2^{1-I_h}(1 - \delta_n)\bar{i}qf_j^v} \tag{67}$$

$$h_j = \left(\tilde{q}_j^{1-I_h}\right)^{1+\frac{1}{\chi}} \left(\frac{\beta}{\kappa}\right)^{\frac{1}{\chi}} J_j^{\frac{1}{\chi}} \tag{68}$$

I_h is an indicator function equal to 1 if firms face a hiring cost function and 0 otherwise. Taking the total derivative with respect to a permanent change in the exogenous retention rate $(1 - \delta_n)$:

$$\partial J_j = \frac{1}{2^{1-I_h}} \left[-\frac{1}{(1 - \delta_n)^2 \bar{i}qf_j^v}\partial(1 - \delta_n) - \frac{\delta_n}{(1 - \delta_n)\bar{i}q^2 f_j^v}\partial q - \frac{\partial G_j}{f_j^v} - \frac{J_j}{f_j^v}\partial f_j^v \right] \tag{69}$$

$$\frac{\partial J_j}{\partial(1 - \delta_n)} = \frac{1}{2^{1-I_h}f_j^v} \left[-\frac{1}{(1 - \delta_n)^2 \bar{i}q} - \frac{\delta_n}{(1 - \delta_n)\bar{i}q^2}\frac{\partial q}{\partial(1 - \delta_n)} - \frac{\partial G_j}{\partial(1 - \delta_n)} - J_j\frac{\partial f_j^v}{\partial(1 - \delta_n)} \right] \tag{70}$$

Assuming that the job finding rate is pro-cyclical, based on equation (70), the value of a filled vacancy tends to go down for all firms when the separation rate goes down (increase in $(1 - \delta_n)$), as the first two terms are negative. It is possible that some firms will experience an increase in J_j if those firms face less competition for workers, meaning that the fraction of their workers that gets poached (G_j) decreases and the firm gains little from increasing its wages at the margin (decline in f_j^v). However, based on the calibration of this model, the last two terms in equation (70) seem to be small. This result means that, in response to a lower separation rate, firms tend to give up a larger fraction of the match surplus so they can retain an even larger fraction of their workers, reduce the amount of hiring, and see total profits go up. Now, by taking the total derivative of (68), the effect of a permanently lower separation rate on the hiring decision by firm j is given by:

$$\partial h_j = (1 - I_h) \cdot \left(1 + \frac{1}{\chi}\right)\left(\frac{\beta}{\kappa}\right)^{\frac{1}{\chi}}(\tilde{q}_j)^{\frac{1}{\chi}} J_j^{\frac{1}{\chi}}\partial\tilde{q}_j + \frac{1}{\chi}\left(\frac{\beta}{\kappa}\right)^{\frac{1}{\chi}}\left(\tilde{q}_j^{1-I_h}\right)^{1+\frac{1}{\chi}} J_j^{\frac{1}{\chi}-1}\partial J_j \tag{71}$$

$$\frac{\partial h_j}{\partial(1 - \delta_n)} = (1 - I_h) \cdot \left(1 + \frac{1}{\chi}\right)\left(\frac{\beta}{\kappa}\right)^{\frac{1}{\chi}}(\tilde{q}_j)^{\frac{1}{\chi}} J_j^{\frac{1}{\chi}}\frac{\partial\tilde{q}_j}{\partial(1 - \delta_n)} + $$
$$\frac{1}{\chi}\left(\frac{\beta}{\kappa}\right)^{\frac{1}{\chi}}\left(\tilde{q}_j^{1-I_h}\right)^{1+\frac{1}{\chi}} J_j^{\frac{1}{\chi}-1}\frac{\partial J_j}{\partial(1 - \delta_n)} \tag{72}$$

Based on equation (72), we can see that total hires tend to go down when the separation rate decreases. Assuming that the vacancy contact rate ($\tilde{q}$) and the ratio ($\frac{u}{s}$) are countercyclical, job finding rates ($\tilde{q}_j$) tend to decrease, even though the most-productive firms experience a lower decline. Hence, as discussed before, if firms increase their wages significantly by reducing J_j so they can retain a large fraction of their workers, hiring of new workers should fall.

Equations (70) and (72) also shed light on the positive correlation between unemployment and the job-to-job transition rate. If total hiring declines in response to a decrease in the separation rate, total job-to-job transitions should fall or increase less than total employment, even if the most-productive firms reduce their hiring the least. As a consequence, the job-to-job transition rate should decline.

C   Wage Posting and Wage Rigidity

I showed that the baseline value posting model tends to predict counterfactual wage responses to aggregate shocks, which tends to increase wage volatility and reduce wage persistence. These responses seem unfeasible since wages for job stayers look sticky in the data (e.g., Kahn, 1997; Barattieri, Basu, and Gottschalk, 2014; Grigsby, Hurst, and Yildirmaz, 2019). Given that firms commit to a value of employment, once a higher employment value is offered in the future, current wages could decrease in order to keep the current employment value constant for job stayers.

Given that the Shimer puzzle gave rise to a large body of the literature studying the amplifying effects of wage rigidity (e.g., Hall, 2005; Hall and Milgrom, 2008; Gertler and Trigari, 2009), I modify the baseline model to *(1)* limit the wage responses on impact to aggregate shocks and *(2)* assess the amplifying effects of wage rigidity in this framework. To this end, I first propose a modification to the baseline model in which firms post wages instead of employment values to retain workers and poach workers from other firms. Then, I introduce wage rigidity by assuming that firms face quadratic cost of wage adjustment *a la* Rotemberg (1982).

Model: Specifically, the household problem is the same. However, firms post wages instead of employment values to attract workers, whose decisions will continue to depend on the value of employment. In addition, I assume that firms face quadratic cost of wage adjustment. Hence, we can rewrite the firm's problem as follows:

$$\Pi_{jt}\left(n_{jt},\bar{w}\right) = \max_{v_{jt},k_{jt},w_{jt+1}} \pi_{jt} + E_t\left[Q_t \Pi_{jt+1}\left(n_{jt+1},w_{jt+1}\right)\right] \tag{73}$$

$$\text{s.t.}$$

$$\pi_{jt} = y_{jt} - w_{jt}n_{jt} - r_t k_{jt} - \kappa \frac{\left(\tilde{q}_{jt}^{I_h} v_{jt}\right)^{1+\chi}}{1+\chi} - \frac{\phi}{2}\left(\frac{w_{jt+1}}{w_{jt}} - 1\right)^2 \tag{74}$$

$$y_{jt} = e^{a_j + a_t} k_{jt}^{\alpha} n_{jt}^{1-\alpha} \tag{75}$$

$$n_{jt+1} = (1-\delta_n)(1-\bar{i}q_t G_{jt})n_{jt} + \tilde{q}_{jt}v_{jt} \tag{76}$$

$$W_{jt} = w_{jt} - z_{jt} + E_t\{Q_t[(1-\delta_{nt})(1-\bar{i}q_t G_{jt})W_{jt+1}$$
$$+ (1-\delta_{nt})\bar{i}q_t \int_{W_{jt+1}}^{\infty} W f_{Wt}^v dW - q_t \int_0^{\infty} W f_{Wt}^v dW]\} \tag{77}$$

$$G_{jt} = \int_{W_{jt+1}}^{\infty} f_{Wt}^v dW \tag{78}$$

$$\tilde{q}_{jt} = \frac{\tilde{q}_t}{s_t}\left(u_t + \bar{i}(1-\delta_{nt})\int_0^{W_{jt+1}} n_{Wt}f_{Wt}^n dW\right) \tag{79}$$

$$W_{jt} \geq 0 \tag{80}$$

as before, f_{Wt}^v and f_{Wt}^n denotes the density functions of employment value offers and employment. Notice that there is a mapping between wage and employment offers given by (77). Hence, when firms post a wage w_{jt+1}, they are implicitly posting a value of employment W_{jt+1} given by (77). Therefore, given that workers only move to jobs that offer higher employment values, the relevant distributions continue to be over employment values and not over wages. It can be shown that the optimality conditions for capital and vacancies (or hires) do not change, but the optimality condition for wage offers is now given by:

$$
\begin{aligned}
E_t \left[Q_t n_{jt+1} \right] &+ \phi \left(\frac{w_{jt+1}}{w_{jt}} - 1 \right) \frac{1}{w_{jt}} - E_t \left[Q_t \phi \left(\frac{w_{jt+2}}{w_{jt+1}} - 1 \right) \frac{w_{jt+2}}{w_{jt+1}^2} \right] \\
&\geq \quad E_t \{ Q_t J_{jt+1} (1 - \delta_{nt}) \bar{i}_t \left[q_t f_{jt}^v n_{jt} + (1 - I_h) \cdot \tilde{q}_t f_{jt}^n v_{jt} \right] \}
\end{aligned}
\tag{81}
$$

When firms commit to a higher wage in the future, they retain a larger fraction of their workforce and poach more workers (right hand side of equation (81)), but their payroll will increase by the size of their new workforce (first term on the left hand side) and will have to pay a wage adjustment cost (last two terms on the left-hand side). When firms post employment values, the cost is proportional to the increase (and not the level) of the firm's workforce because firms can change their current wages to keep the current value of employment constant.

Notice that total income in this economy is now used for consumption, capital accumulation, vacancy posting costs, and wage adjustment costs. Hence, the aggregate resource constraint is now given by:

$$
y_t = c_t + k_{t+1} - (1 - \delta_k) k_t + \kappa \int \frac{\left(\tilde{q}_{jt}^{I_h} v_{jt} \right)^{1+\chi}}{1 + \chi} dj + \int \frac{\phi}{2} \left(\frac{w_{jt+1}}{w_{jt}} - 1 \right)^2 dj.
\tag{82}
$$

Definition 4. *__Competitive Search Equilibrium with Wage Posting.__ A competitive search equilibrium with wage posting is a sequence of prices $\{r_t, w_t\}$, quantities $\{y_t, c_t, k_t, u_t, n_t\}$, probabilities $\{q_t, \tilde{q}_t\}$, and functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ on productivity a_j, firm size n_{jt} and w_{jt}, such that given exogenous variables, an initial stock of capital and initial distributions of employment and wages: (i) The household optimizes, taken as given prices and exogenous shocks. Consumption satisfies the optimality condition (4). (ii) Taking as given the exogenous variables, $\{r_t\}$, and all other firms strategies (i.e. employment, wage, and vacancies), firms optimize. Functions $\{v_{jt}, W_{jt+1}, J_{jt}, n_{jt+1}\}$ solve equations (5), (12), (21), and (25); and prices satisfy equations (20) and (81). (iii) Probabilities evolve according to $q_t = m(\theta_t, 1)$ and $\tilde{q}_t = m(1, \theta_t^{-1})$. (iv) Markets clear: the aggregate resource constraint holds.*

D Additional Tables and Figures

Table 12: Business Cycle Moments. MPV16 Calibration.

Standard deviation

	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity ($\rho_{a\delta}=0$)	0.03	0.03	0.01	0.04	0.00	0.02	1.71	1.00	1.00	1.00
Separation	15.20	4.98	0.33	8.96	9.39	6.28	7.44	1.00	0.37	0.00
Productivity	8.64	2.79	0.18	5.58	5.45	3.51	3.38	1.00	0.65	0.66
Data	9.09	8.32	1.52	6.33	4.93	3.39	0.72	1.00	0.75	0.67

Correlation with Unemployment

	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity ($\rho_{a\delta}=0$)	1.00	-0.82	-0.65	-0.89	0.01	-0.99	0.78	0.67	0.67	0.67
Separation	1.00	-0.97	0.87	-0.96	0.60	-0.97	-0.20	-0.92	-0.29	-0.66
Productivity	1.00	-0.85	0.81	-0.80	0.57	-0.85	-0.35	-0.77	-0.60	-0.51
Data	1.00	-0.93	-0.40	-0.97	0.92	-0.84	-0.04	-0.76	0.42	-0.01

Autocorrelation

	u	v	h	UEr	EUr	EEr	w^a	y	p	a
Productivity ($\rho_{a\delta}=0$)	0.91	0.82	0.74	0.86	0.99	0.91	0.40	0.75	0.75	0.75
Separation	0.91	0.92	0.86	0.91	0.75	0.91	0.14	0.93	0.96	0.83
Productivity	0.90	0.91	0.86	0.90	0.75	0.91	0.30	0.85	0.75	0.75
Data	0.98	0.95	0.48	0.93	0.89	0.84	0.94	0.97	0.93	0.92

Notes and source: This table reports business cycle moments for a value posting model calibrated as in Moscarini and Postel-Vinay (2016). Standard deviation is relative to output standard deviation. Shock "Productivity" ($\rho_{a\delta}$) refers to a pure productivity shock that is not correlated with the separation. Statistics for the U.S. economy are based on the following. u: Unemployment level (UNEMPLOY). v: Help-wanted index (Barnichon, 2010). h: total hires. UEr: Unemployment-to-employment transition rate. EUr: Employment-to-unemployment transition rate. w^a: Average wage in the economy. y: Real output in the nonfarm business sector (GDPC1). p: Real output per-hour in the non-farm business sector (OPHNFB). a: Utilization adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000.

Figure 3: Data Series 1994Q1-2015Q4

(a) Unemployment

(b) Vacancies

(c) Hires

(d) UEr

(e) EUr

(f) EEr

(g) w^a

(h) Output

(i) Output per Hour

(j) TFP
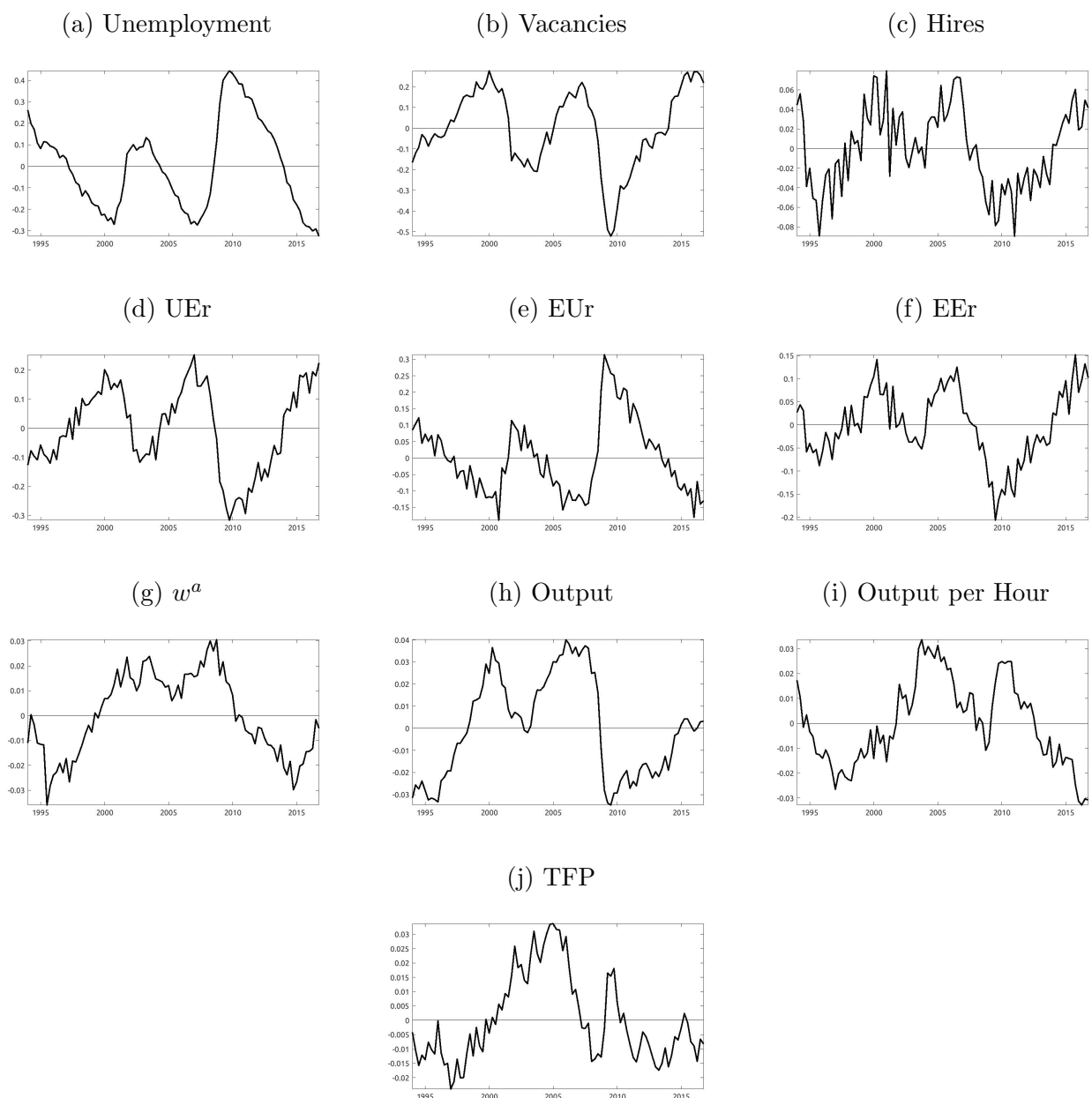
Notes and source: This figure plots data for the United States for total number of unemployed workers (UN-EMPLOY), help-wanted index (Barnichon, 2010), total hires, unemployment-to-employment transition rate, employment-to-unemployment transition rate, employment-to-employment transition rate, average wage in the economy, real output in the nonfarm business sector (GDPC1), real output per-hour in the nonfarm business sector (OPHNFB), and utilization-adjusted TFP from the San Francisco FED. Total hires, average wage, and labor transition rates are author's calculations based on the Current Population Survey (CPS). For details see sections 6.1 and A. Logged and HP-filtered series with a smoothing parameter equal to 100,000.