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Macroeconomic Effects of Capital Tax Rate Changes*

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Abstract

We study aggregate, distributional, and welfare effects of a permanent reduction in the capital tax rate in a quantitative model with capital-skill complementarity and household heterogeneity. Such a tax reform leads to expansionary long-run aggregate output and investment effects, but those are coupled with increases in wage, consumption, and income inequality. The tax reform is not self-financing and its effects depend crucially on whether the government cuts lump-sum transfers or raises distortionary labor or consumption tax rates for financing. The former results in a larger aggregate expansion, but at the expense of a greater rise in inequality. As a result, the latter is relatively more beneficial for unskilled households. We find that the tax reform, when the consumption tax rate adjusts, leads to a Pareto improvement in terms of life-time welfare. For transition dynamics, monetary policy, in addition to the fiscal adjustments, matters. In particular, if monetary policy inflates away a portion of the public debt, the economy can avoid the short-run contraction that would arise otherwise.

JEL Classification: E62, E63, E52, E58, E31

Keywords: Capital tax rate; Distortionary financing; Capital-skill complementarity; Inequality; Welfare implications

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1 Introduction

The macroeconomic and distributional effects of permanent capital tax cuts have recently become a subject of widespread discussion, spurred by the recent U.S. tax reform that reduced the corporate tax rate from 35% to 21%. Several questions have been raised. What are the long-run and the short-run effects on output and investment? What are the distributional consequences in terms of wage and consumption inequality? What are the welfare implications? Will such a large tax cut be self-financing? If not, what are the positive and normative implications of alternative ways to finance the reform? How does the monetary policy response matter for the short-run effects of a capital tax cut?

Given the nature of these questions, it is useful to pursue an analysis through the lens of a quantitative dynamic model that can capture both aggregate and distributional effects. Moreover, since the tax reform is large-scale, it is imperative to consider general equilibrium effects as well as the sources of financing. This paper therefore addresses these questions using a quantitative, dynamic general equilibrium model with skill and consumption heterogeneity.[1] We present the long-run equilibrium, as well as full (nonlinear) transition dynamics and welfare evaluation.

Compared to existing studies on the effects of capital tax changes, our analysis is integrative in terms of the model used to address them. We consider several relevant features, such as (equipment) capital-skill complementarity, household heterogeneity, different types of long-run fiscal adjustments, as well as short-run monetary and fiscal policy interactions, in a unified framework. In terms of research questions, this paper can also be distinguished from the existing studies in that we pay special attention to the source of financing. We show how the government finances capital tax cuts–that is, how the resulting increase in public debt is ultimately retired–has important aggregate, distributional, and welfare consequences in the long-run as well as during the transition periods.

The paper starts with a long-run analysis. We show analytically in a simplified model and numerically in the quantitative model with capital-skill complementarity and household heterogeneity that capital tax cuts, as expected, have expansionary long-run aggregate effects on the economy. For instance, with a permanent reduction of the capital tax rate from 35% to 21%, output in the new steady state, compared to the initial steady state, is greater by 4.24%, structure investment by 20.24%, and equipment investment by 6.27%, in our baseline calibration. The mechanism for aggregate effects is well understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising demand for capital by firms. This stimulates investment and capital accumulation. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Finally, given the increase in the factors of production, output expands.

[1] In particular, unskilled households in our model cannot smooth consumption over time.

This aggregate expansion however, is coupled with worsening wage and consumption inequality in our model. For instance, skilled wages increase by 4.66% while unskilled wages increases by only 0.56%, driven by capital-skill complementarity. In addition, there is a rise in consumption inequality. The major reason is that the aggregate effects above are obtained in our baseline scenario where the government finances the capital tax cuts by cutting back lump-sum transfers on the unskilled household.[2] In this scenario, consumption of the unskilled falls with the capital tax rate reform, by 3.86%. In contrast, consumption of the skilled rises by 4.82%.

Importantly, in this baseline scenario, although aggregate labor hours increase as in a standard representative-agent framework, skilled hours and unskilled hours move in opposite directions. A capital tax rate cut financed by appropriate transfer adjustments leads to an increase in unskilled hours not only due to the aforementioned standard mechanism that works through the labor demand channel, but also due to the wealth effects that encourage the unskilled households to supply more hours. By contrast, the wealth effects on labor supply and the standard labor demand channel work in opposite directions for the skilled households: the former now discourages the skilled households to supply hours whereas the latter continues to work as a force to increase skilled hours. In the baseline case, the former dominates the latter. Consequently, there is a slight decrease in skilled hours. Notice that because of these two-way wealth effects, the rise in consumption inequality will lead to a further increase in wage inequality. Thus, in our model with heterogeneous households, consumption inequality and wage inequality interact and reinforce each other to amplify the distributional effects.[3]

The baseline scenario above, while it may serve as a useful benchmark, is unrealistic because in our calibration, the lump-sum transfer cuts in fact imply imposing lump-sum *taxes* on the unskilled households. We thus consider two other financing options in which the government relies on distortionary labor or consumption taxes. The three financing schemes under consideration–the lump-sum transfer adjustment, the labor tax rate adjustment, and the consumption tax rate adjustment–generally produce different effects on aggregate output because each scheme influences workers' labor supply decisions differently. First, in our baseline scenario above, as mentioned, lump-sum transfer cuts generate a (negative) wealth effect on the unskilled labor supply, which boosts unskilled hours and in turn, contributes to greater aggregate output (again, in addition to the standard mechanism through firms' demand for labor). In comparison, a rise in the labor or consumption tax rate decreases the effective wage rate (as is well-understood) and additionally,

[2]Regarding the distribution of unearned income, in our baseline case, skilled households receive profits from firms and unskilled households receive transfers from the government. We however, parameterize our model in a way such that other distribution possibilities can be easily explored. We provide some sensitivity analysis.

[3]This additional feedback channel is absent in a representative-household version of our model. In addition, household heterogeneity amplifies the aggregate effects, too. The reason is that the wealth-effect-induced increase in unskilled hours is only partially offset by the wealth-effect-induced decrease in skilled hours in this baseline scenario. We do not show this result for brevity, but is available upon request.

weakens the wealth effect for the unskilled household. These two mechanisms work together to generate a smaller aggregate expansion under the distortionary tax adjustments. We nevertheless find that aggregate expansion is greater under the consumption tax rate adjustment than under the the labor tax adjustment. We show this result again both analytically in a simplified model and numerically in the quantitative model.

Specifically, in our baseline calibration, a permanent reduction of the capital tax rate from 35% to 21% requires an increase in the labor tax rate from 22.7% to 25.4%.[4] Then, output in the new steady state, compared to the initial steady state, is greater by 2.08%, equipment investment by 17.75%, and structures investment by 4.81%. The reason for the smaller boost in aggregate variables, compared to the lump-sum transfer adjustment case, is a fall in hours of both the skilled and the unskilled. Importantly, the behavior of unskilled hours is different even qualitatively, compared to the lump-sum transfer adjustment case, for two reasons. First, the *after-tax* wage rate for unskilled hours in fact declines. Second, the wealth effect on unskilled labor supply is not as strong because the massive reduction in transfer income for the unskilled household is absent. Overall, the decrease in skilled and unskilled hours dampens the expansionary aggregate effect of capital tax cuts.

On distributional implications, both wage and consumption inequality increase, as they did before with the transfer adjustment case. In comparison to the transfer adjustment case however, these distributional effects are smaller. The main reason is that the burden from the labor tax increase is shared by both types of households, whereas transfer reduction only affects the unskilled. Therefore, consumption of the unskilled does not fall as much while consumption of the skilled does not increase as much now. The reduced consumption inequality in turn diminishes the role of the wealth effect on labor supply, leading to a smaller increase in the skill premium.

For consumption tax rate adjustment, the third source that can finance the reduction in the capital tax rate from 35% to 21%, consumption tax rates have to increase from 1.29% to 3.49%. Generally, the effects are qualitatively similar to those in the labor tax rate adjustment case. However, quantitatively, the aggregate effects are bigger compared to labor tax rate adjustment, as labor supply gets distorted less.[5] One important qualitative difference, compared to both transfer and labor tax rate adjustment, is in consumption of the unskilled households, which increases slightly in the long-run. This is driven by the larger output effects when the consumption tax rate adjusts and has important implications for our welfare results.

We then move to an analysis of transition dynamics as the economy evolves from the initial steady-state to the new steady-state. During the transition, the economy experiences a decline in

[4]We keep debt-GDP ratio the same between the initial and the new steady-state. Debt-GDP ratio, however, is allowed to deviate from the steady-state level along the transition path, when we study short-run effects.

[5]This happens as income and substitution effects offset under consumption tax rate increase.

not only consumption of both types of households, but also output. This holds even if lump-sum transfers finance the capital tax rate cut.[6] Consumption and output falls are more severe under the distortionary tax rate adjustments. The short-run contraction may be viewed as another side effect of a permanent capital tax cut besides the increase in inequality in the long-run that we highlighted above.

Another important aspect of transition dynamics that we highlight is on the need to analyze monetary and fiscal policy adjustments jointly. This is because the short-run effects depend critically on the monetary policy response. In particular, when the government has access only to distortionary labor taxes, we consider a case where the central bank accommodates inflation so that nominal government debt is partially inflated away along the transition after the capital tax rate cut. In this interesting scenario, the government does not raise labor tax rates as much, and the rise of inflation in the short-run completely negates any short-run contraction in output as well as consumption.

Next, while our paper does not study optimal policy, we analyze welfare consequences of the permanent capital tax rate cut, given the various financing possibilities we consider. We show that long-term social or *aggregate* welfare gains contrast with short-term (but, still prolonged) aggregate welfare losses, regardless of how the capital tax rate cut is financed. Moreover, the same result mostly holds at the household level as well: Even the skilled household, who always benefits from the capital tax reform in the long-run, suffers significant short-run welfare losses except for the case when lump-sum transfers adjust. The unskilled households experience welfare losses in the short-run under all financing options.

Now focusing on the (long-term) life-time welfare, we show that the skilled gains at the expense of the unskilled under transfer adjustment, and thus the capital tax reform is not Pareto improving. The same result holds when labor tax rate adjusts. In fact, financing a capital tax cut by lump-sum transfer adjustment, while leading to a higher level of aggregate output, is not necessarily better than financing it by labor tax adjustment. Intuitively, when some agents' income relies relatively more on transfers (the unskilled households), a reduction in transfer to offset tax revenue losses from capital tax cuts can decrease their welfare. In contrast, labor tax adjustment works better for the unskilled as the labor tax increase burden is shared by both types, whereas transfer reduction only affects the unskilled.

Finally, to underscore yet again the importance of considering and modeling different sources of financing the capital tax reform, we show that compared to the labor tax rate adjustment, the welfare results are different for consumption tax rate adjustment. In this case, there is in fact a Pareto improvement as not only the skilled households, but also the unskilled households, gain

[6] The short-run fall in output is a result of investment adjustment cost, nominal rigidities, and our empirical monetary policy rule.

in terms of lifetime welfare.[7] The main reason for this result is that compared to labor tax rate adjustment, consumption of the unskilled goes up in the long-run, as we discussed above, due to lower distortions on labor supply, leading to a larger aggregate output effect.

Related literature This paper is related to several strands of the literature, some of which have been developed without much interaction with each other. While we focus mostly on a positive analysis, our paper is related to classic normative analysis of Chamley (1986) and Judd (1985), which was re-addressed recently in Straub and Werning (2020).[8] We do not analyze optimal monetary and fiscal policy issues, but do compute welfare implications given the capital tax rate cut and various financing rules we consider. While doing so, we find that increasing the consumption tax rate to finance the capital tax rate cut leads to a Pareto improvement in our heterogeneous household model.

Our analysis of the central bank allowing inflation to directly facilitate debt stabilization, through passive monetary policy, when the government has access to only distortionary taxes relates this paper to the literature on monetary and fiscal policy interactions – in particular, the normative analysis in Sims (2001). We implement this scenario using a rules-based positive description of interest rate policy, as in Leeper (1991), Sims (1994), and Woodford (1994) for instance.[9] Relatedly, our work is also motivated by the study of effects of government spending and how that depends on the monetary policy response, as highlighted recently by Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), and Leeper, Traum and Walker (2017).

In terms of analyzing the long-run effects of changes in the capital tax rate in an equilibrium macroeconomic model, our paper is close to Trabandt and Uhlig (2011) and the more recent work of Barro and Furman (2018) that analyzes the U.S. tax reform. Compared to this literature, one key difference is that our baseline model features capital-skill complementarity, following Krusell et al. (2000), as well as household heterogeneity, such that both wage and consumption inequality issues can be analyzed. Trabandt and Uhlig's (2011) main focus is on the important issue of the presence of Laffer curves for labor and capital tax, under either transfer or government spending adjustment. We show, both analytically and numerically, how the macroeconomic effects of a given capital tax rate change are different depending on whether non-distortionary or distortionary sources of

[7] While life-time welfare is higher for the unskilled households, there is welfare loss in the short- and medium-run, as we mentioned above.

[8] The literature on optimal capital taxation in a dynamic setting is extensive. Earlier work typically finds significant welfare gains from eliminating capital income taxes in representative-household infinite-horizon frameworks. More recent studies move away from the standard setup, featuring heterogeneous households and overlapping generations, and find optimality of non-zero capital taxes (e.g. Aiyagari 1995, Erosa and Gervais 2002, and Conesa, Kitao and Krueger 2009.)

[9] In this case, the central bank does not follow the Taylor principle. Bhattarai, Lee and Park (2014) and Ascari, Florio and Gobbi (2020) analytically characterize the effects of such a case in a model with sticky prices. Canzoneri, Cumby and Diba (2010), Leeper and Leith (2016), and Cochrane (2019) provide excellent surveys of the literature.

government financing are available as well as how different types of distortionary financing can have qualitatively different welfare implications. In addition, we study transition dynamics in detail, highlighting that it is imperative to model monetary and fiscal policy adjustments jointly for determining short-run effects, and explore aggregate, distributional, and welfare implications taking dynamics fully into account.

Barro and Furman's (2018) recent important contribution studies macroeconomic implications of a given capital tax rate change, like we do, in a model with more details of the tax code and five types of capital. Our baseline model is simpler in that respect, but features endogenous labor supply such that distortionary sources of capital tax reform financing can be explored carefully. Moreover, as mentioned above, we also study wage and consumption inequality implications, transition dynamics, and welfare properties. Furthermore, our model with endogenous labor supply and household heterogeneity allows us to highlight how wage inequality and consumption inequality interact and reinforce each other, thereby amplifying distributional effects.

Another closely related paper is Domeij and Heathcote (2004). Similar to our study, they explicitly take into account both transition dynamics and steady-state change after a tax reform in the welfare analysis, and show that a capital tax rate reduction is not Pareto improving. We also find such a result in this paper for labor tax rate adjustment. For consumption tax rate adjustment however, we do find that capital tax rate reduction is Pareto improving in our baseline calibration. Domeij and Heathcote's (2004) model, in addition, abstracts from capital-skill complementarity. The more recent work of Slavík and Yazici (2019) uses a model with capital-skill complementarity and analyzes the effects of a tax reform that eliminates tax differentials between equipment and structure capital.

Besides the different research questions we aim to address, our paper is different from these two contributions in modeling choices. Their models feature a richer form of household heterogeneity, as in Aiyagari (1994), while our model is more stylized in that dimension, focusing on a particular type of heterogeneity as in the tradition of the Two Agent New Keynesian (TANK) literature. Our analysis thus misses potentially important implications of a realistic wealth distribution. It however, allows us to include a richer set of model elements that enable us to conduct a more realistic analysis of transition dynamics. Moreover, we do find new results related to distortionary financing using consumption taxes. Furthermore, our empirically motivated specifications of monetary and fiscal policy, coupled with model elements that matter for transition, allow us to consider positive and welfare implications of monetary and fiscal policy interactions.

As mentioned above, the way we introduce household heterogeneity connects our paper to the growing TANK literature. This literature has analyzed extensively various issues on monetary policy (e.g. Bilbiie 2008, Bhattarai, Lee and Park 2015, Cúrdia and Woodford 2016, and Debortoli and Galí 2017.) On the fiscal side, Galí, López-Salido and Vallés (2007), Bilbiie, Monacelli and Perotti

(2013) and Eggertsson and Krugman (2012) have considered the effects of government spending and (lump-sum) transfers. Much of the literature, however, abstracts from capital accumulation (with the exception of Galí, López-Salido and Vallés 2007), thereby precluding an analysis of capital income taxes. Existing papers also do not consider several sources of distortionary financing. Moreover, our model importantly also features capital-skill complementarity, which allows us to analyze the effect of a policy change on the wage distribution.[10]

There is by now a fairly large dynamic stochastic general equilibrium modeling literature that assesses the effects of distortionary tax rate changes and of fiscal policy generally. For instance, among others, Forni, Monteforte and Sessa (2009) study transmission of various fiscal policies, including government spending and transfer changes in a quantitative model. Sims and Wolff (2018) additionally study state-dependent effects of tax rate changes. These papers often study effects of transitory and small changes in the tax rate while our main focus is on the long-run effects of a permanent reduction in the capital tax rate under various sources of financing, and then on an analysis of full (nonlinear) transition dynamics following a fairly large reduction. Additionally, we provide analytical results on comparison of different sources of financing that help illustrate the key mechanisms on the long-run effects, while in the quantitative part, we use a model that can assess distributional consequences.

While we are motivated by the particular recent U.S. episode of a permanent tax rate change, generally, our paper is influenced also by a large literature that empirically assesses the macroeconomic effects of tax policy. In particular, various identification strategies, such as narrative (Romer and Romer 2010) and statistical (Blanchard and Perotti 2002, Mountford and Uhlig 2009) have been used to assess equilibrium effects of tax changes. Relatedly, House and Shapiro (2008) study a particular case of change in investment tax incentive. The effects on aggregate variables that we find using a calibrated equilibrium model is consistent with this work, although these papers have generally focused either explicitly on temporary tax policies or do not explicitly separate out permanent changes from transitory ones. Perhaps most importantly, we also use our model to assess distributional and welfare effects following a permanent capital tax rate cut.

2 Model

We now present the baseline model, which is a quantitative equilibrium framework augmented with two types of workers (skilled and unskilled) and two types of capital (structures and equipment). We introduce equipment capital-skill complementarity following Krusell et al. (2000), and a skill premium arises endogenously in the model. Moreover, the households are heterogeneous:

[10] On a technical side, the existing literature typically focuses on linear dynamics around a steady state. We present exact, nonlinear transition dynamics in a TANK model with capital-skill complementarity.

the skilled household makes optimal consumption/savings decisions while the unskilled household is “hand-to-mouth”. This framework allows us to study both aggregate as well as wage and consumption inequality implications of a capital tax rate change in a unified way. The model also features adjustment costs in investment, variable capacity utilization, and nominal pricing frictions to enable a realistic study of transition dynamics. Pricing frictions additionally allow an analysis of the role of monetary policy for the transition dynamics.

2.1 Private sector

We start by describing the maximization problems of the private sector.

2.1.1 Households

There are two types of households who supply skilled labor (type s) and unskilled labor (type u), respectively. The measure of type-i household for $i \in \{s, u\}$ is denoted by N^i. The skilled households are “Ricardian”, and their problem is to

$$\max_{\left\{C_t^s, H_t^s, B_t^s, I_{b,t}^s, I_{e,t}^s, \hat{K}_{b,t+1}^s, \hat{K}_{e,t+1}^s, u_{e,t}^s, u_{b,t}^s\right\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t^s, H_t^s\right) \right\}$$

subject to a sequence of flow budget constraints

$$\begin{aligned}
&\left(1+\tau_t^C\right) P_t C_t^s + P_t I_{b,t}^s + P_t I_{e,t}^s + B_t^s \\
&\qquad = \left(1-\tau_t^H\right) W_t^s H_t^s + R_{t-1} B_{t-1}^s + \lambda_b \tau_t^K P_t I_{b,t}^s + \lambda_e \tau_t^K P_t I_{e,t}^s \\
&\qquad\quad + \left(1-\tau_t^K\right) R_t^{K,b} u_{b,t}^s \hat{K}_{b,t}^s + \left(1-\tau_t^K\right) R_t^{K,e} u_{e,t}^s \hat{K}_{e,t}^s \\
&\qquad\quad - P_t \left(1-\lambda_b \tau_t^K\right) \mathcal{A}_b\left(u_{b,t}^s\right) \hat{K}_{b,t}^s - \frac{P_t}{q_t}\left(1-\lambda_e \tau_t^K\right) \mathcal{A}_e\left(u_{e,t}^s\right) \hat{K}_{e,t}^s + P_t \frac{\chi_\Phi^s}{N^s} \Phi_t + P_t \frac{\chi_S^s}{N^s} S_t,
\end{aligned}$$

where E_t is the mathematical expectation operator, C_t^s is consumption, H_t^s is hours, and $I_{b,t}^s$ and $I_{e,t}^s$ are investment in the capital stock of structures and equipment denoted by $\hat{K}_{b,t}^s$ and $\hat{K}_{e,t}^s$, respectively. Similarly, $K_{b,t}^s \equiv u_{b,t}^s \hat{K}_{b,t}^s$ and $K_{e,t}^s \equiv u_{e,t}^s \hat{K}_{e,t}^s$ are the effective structure and equipment capital and $u_{b,t}^s$ and $u_{e,t}^s$ are the respective variable capacity utilization rates. $\mathcal{A}_b(u_{b,t}^s)$ and $\mathcal{A}_e(u_{e,t}^s)$ are the costs of variable capital utilization.

The skilled Ricardian households trade nominal risk-less one-period government bonds B_t^s. They are paid a fraction χ_Φ^s of the aggregate profits Φ_t from the firms and a fraction χ_S^s of the aggregate lump-sum transfers S_t from the government. The aggregate price level is P_t, W_t^s is the nominal wage for skilled households, R_t is the nominal one-period interest rate, and $R_t^{K,b}$ and $R_t^{K,e}$ are the rental rate of capital structures and equipment, respectively. The government levies taxes on consumption, labor income, and capital income with tax rates τ_t^C, τ_t^H, and τ_t^K, respectively. The parameters λ_b and λ_e are the rates of expensing of capital investment in structures and equipment, respectively. The discount factor is β.

The evolutions of the two types of capital stock are described by

$$\hat{K}^s_{b,t+1} = (1-d_b)\,\hat{K}^s_{b,t} + \left(1 - \mathcal{S}\left(\frac{I^s_{b,t}}{I^s_{b,t-1}}\right)\right) I^s_{b,t},$$
$$\hat{K}^s_{e,t+1} = (1-d_e)\,\hat{K}^s_{e,t} + \left(1 - \mathcal{S}\left(\frac{I^s_{e,t}}{I^s_{e,t-1}}\right)\right) I^s_{e,t} q_t,$$

where q_t is the relative price between investment in capital structures and equipment, and d_b and d_e are the rates of depreciation of the capital stock invested in structures and equipment, respectively.[11] $\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)$ represent investment adjust cost.

The unskilled households are "hand-to-mouth" (HTM), and their problem is to

$$\max_{\{C^u_t, H^u_t\}} \quad U\left(C^u_t, H^u_t\right)$$

subject to a flow budget constraint

$$\left(1+\tau^C_t\right) P_t C^u_t = \left(1-\tau^H_t\right) W^u_t H^u_t + P_t \frac{\chi^u_\Phi}{N^u} \Phi_t + P_t \frac{\chi^u_S}{N^u} S_t,$$

where C^u_t is their consumption and H^u_t is hours. The unskilled HTM households are paid a fraction χ^u_Φ of the aggregate profits Φ_t from the firms and a fraction χ^u_S of the aggregate lump-sum transfers S_t from the government. W^u_t is the nominal wage for unskilled households.

The period utility $U(C_t, H_t)$, investment adjustment cost $\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)$, and variable capacity utilization cost $\mathcal{A}(u_t)$ have standard properties, which are detailed later.

2.1.2 Firms

The model has final goods firms and intermediate goods firms. Perfectly competitive final goods firms produce aggregate output Y_t by combining a continuum of differentiated intermediate goods, indexed by $i \in [0,1]$, using the CES aggregator given by $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution between intermediate goods. The corresponding optimal price index P_t for the final good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta}\, di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate good i and the optimal demand for good i is $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$. The final good is used for private and government consumption as well as investment in capital structures and equipment.

Monopolistically competitive intermediate goods firms, indexed by i, produce output using a

[11] As we describe in detail later, this relative price is exogenous to ensure balanced growth in the model.

CRS production function $F(.)$

$$Y_t(i) = F\left(A_t, K_{b,t}(i), K_{e,t}(i), L_{s,t}(i), L_{u,t}(i)\right), \tag{2.1}$$

where A_t is an exogenous stochastic process that represents technological progress, with its gross growth rate given by $a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}$.[12] As we describe in detail later, we follow Krusell et al. (2000) in functional form assumptions on $F(.)$, a nested CES formulation, and parameterizations of the elasticities of substitution across factors, such that it features (equipment) capital-skill complementarity. Firms rent the two types of capital and hire the two types of labor in perfectly competitive factor markets.

Intermediate goods firms face nominal rigidity. As in Calvo (1983), a firm resets its price optimally with probability $1-\alpha_P$ every period. Firms that do not optimize adjust their price according to the indexation rule $P_t(i) = P_{t-1}(i)\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}$, where γ_P measures the extent of dynamic indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$.

Optimizing firms choose a common price P_t^* to solve their problem

$$\max_{\{P_t^*, Y_{t+k}(i), H_{t+k}(i), K_{t+k}(i)\}} E_t\left\{\sum_{k=0}^{\infty}\left(\alpha_p\beta\right)^k \frac{\Lambda_{t+k}}{\Lambda_t} P_{t+k}\Phi_{t+k}(i)\right\},$$

subject to (2.1), where Λ_t is the marginal utility of nominal income of the skilled household[13] and flow profit $\Phi_t(i)$ is given by

$$\Phi_{t+k}(i) = \frac{P_t^*}{P_{t+k}} X_{P,t,k} Y_{t,t+k}(i) - \frac{W_{t+k}^u}{P_{t+k}} L_{u,t+k}(i) - \frac{W_{t+k}^s}{P_{t+k}} L_{s,t+k}(i) - \frac{R_{t+k}^{K,b}}{P_{t+k}} K_{b,t+k}(i) - \frac{R_{t+k}^{K,e}}{P_{t+k}} K_{e,t+k}(i),$$

where

$$X_{P,t,k} = \begin{cases} (\pi_t\pi_{t+1}\cdots\pi_{t+k-1})^{\gamma_P}\,\bar{\pi}^{(1-\gamma_P)k}, & k \geq 1 \\ 1 & k = 0 \end{cases}$$

and

$$Y_{t,t+k}(i) = \left(\frac{P_t^* X_{P,t,k}}{P_{t+k}}\right)^{-\theta} Y_{t+k}.$$

Note that there is an endogenous skill premium in the model, which we define as the wage of skilled labor relative to that of unskilled labor, $\frac{W_t^s}{W_t^u}$. Given the CRS production function and the assumption of perfectly competitive factor markets, the factor prices are equal to marginal products

[12] Steady-state of a variable x is denoted by $\overline{x}$ throughout. As we discuss later, we restrict preferences and technology such that the model is consistent with balanced growth.

[13] While we have written the model generally above in terms of profit distribution to households, we assume that firm profits go to the skilled households in our baseline parameterization. Therefore, the firms use the stochastic discount factor of the skilled household in their dynamic profit maximization problem. For long-run analysis, as we will assume that the marginal utility is equated across households (only) in steady-state, it is irrelevant which stochastic discount factor is used by the firms.

of each factor multiplied by firms' marginal costs. Moreover, as we show in detail later, if capital-skill complementarity exists, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. It is also increasing in the ratio of unskilled to skilled labor.

2.2 Government

We now describe the constraint on the government and determination of monetary and fiscal policy.

2.2.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is

$$\frac{B_t}{P_tY_t}+\tau_t^C\frac{C_t}{Y_t}+\tau_t^H\left(\frac{W_t^s}{P_tY_t}L_{s,t}+\frac{W_t^u}{P_tY_t}L_{u,t}\right)+\tau_t^K\sum_{i\in\{b,e\}}\left(\frac{R_t^{K,i}}{P_tY_t}K_{i,t}-\lambda_i\left(I_{i,t}+\mathcal{AC}_{i,t}\right)\right)$$
$$=R_{t-1}\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\frac{1}{\pi_t}\frac{Y_{t-1}}{Y_t}+\frac{G_t}{Y_t}+\frac{S_t}{Y_t},$$

where $B_t=N^sB_t^s$, $S_t=\sum_{i\in\{s,u\}}N^iS_t^i$, $S_t^i=\frac{\chi_S^i}{N^i}S_t$, $\mathcal{AC}_{b,t}=N^s\mathcal{A}_b(u_{b,t}^s)\hat{K}_{b,t}^s$, $\mathcal{AC}_{e,t}=N^s\mathcal{A}_e(u_{e,t}^s)\hat{K}_{e,t}^s$, and G_t is government spending on the final good.[14]

2.2.2 Monetary policy

Monetary policy is given by a feedback rule as in Coibion and Gorodnichenko (2011),

$$\frac{R_t}{\bar{R}}=\left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_1^R}\left[\frac{R_{t-2}}{\bar{R}}\right]^{\rho_2^R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}}\left(\frac{Y_t}{Y_t^n}\right)^{\phi_x}\right]^{\left(1-\rho_1^R-\rho_2^R\right)}, \tag{2.2}$$

where ρ_1^R and ρ_2^R govern interest rate smoothing, $\phi_\pi\geq 0$ is the feedback parameter on inflation, Y_t^n is the natural (that is, flexible price) level of output, $\phi_{\Delta y}$ is the feedback parameter on output growth, ϕ_x is the feedback parameter on output gap, and $\bar{R}$ is the steady-state value of R_t. For large enough feedback coefficients (a combination of ϕ_π, $\phi_{\Delta y}$, and ϕ_x), the Taylor principle is satisfied.[15] We will also consider a case, described in more detail next, where the Taylor principle is not satisfied, and inflation response will play a direct role in government debt stabilization along the transition.

[14] We introduce government spending in the model for a realistic calibration. As we discuss later, government spending-to-GDP ratio is held fixed throughout in the model.

[15] In the textbook linearized sticky price model, this condition is $\phi_\pi>1$. In the model here, as there are several endogenous propagation mechanisms and an empirically grounded interest rate rule, such a condition has to be determined numerically, although $\phi_\pi>1$ is often a good benchmark.

2.2.3 Fiscal policy

We consider a one-time permanent change in the capital tax rate τ_t^K in period 0, when the economy is in the initial steady-state. In order to isolate the effects of the capital tax rate cut, $\frac{G_t}{Y_t}$ is kept unchanged from its initial steady-state value in all periods. The debt-to-GDP ratio, $\frac{B_t}{P_t Y_t}$, may deviate from the initial steady-state in the short-run but will converge back to the initial steady-state in the long-run, through appropriate changes in fiscal instruments. We consider the following four policy adjustments. First, only lump-sum transfers adjust following a feedback rule similar to the monetary policy rule specification in Coibion and Gorodnichenko (2011),

$$\begin{aligned}\frac{S_t}{Y_t}-\frac{\bar{S}_{new}}{\bar{Y}_{new}} =& \rho_1^S\left(\frac{S_{t-1}}{Y_{t-1}}-\frac{\bar{S}_{new}}{\bar{Y}_{new}}\right)+\rho_2^S\left(\frac{S_{t-2}}{Y_{t-2}}-\frac{\bar{S}_{new}}{\bar{Y}_{new}}\right)\\ &+\left(1-\rho_1^S-\rho_2^S\right)\left\{\psi_B^S\left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}}-\overline{\frac{B}{PY}}\right)+\psi_{\Delta y}^S\left(\frac{Y_t}{Y_{t-1}}\right)+\psi_x^S\left(\frac{Y_t}{Y_t^n}\right)\right\},\end{aligned} \tag{2.3}$$

where $0 \leq \rho_1^S+\rho_2^S < 1$ governs labor tax rate smoothing, $\psi_B^S \geq 0$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^S$ is the feedback parameter on output growth, ψ_x^S is the feedback parameter on the output gap, $\bar{S}_{new}$ is the new steady-state value of S_t, and $\overline{\frac{B}{PY}}$ is the (initial and new) steady-state value of $\frac{B_t}{P_t Y_t}$. A large enough feedback coefficient on debt (high ψ_B^S) ensures that fiscal policy leads to stationary debt dynamics.

Second, only labor tax rates τ_t^H adjust following a feedback rule similar to the monetary policy rule specification in Coibion and Gorodnichenko (2011),

$$\begin{aligned}\tau_t^H-\bar{\tau}_{new}^H =& \rho_1^H\left(\tau_{t-1}^H-\bar{\tau}_{new}^H\right)+\rho_2^H\left(\tau_{t-2}^H-\bar{\tau}_{new}^H\right)\\ &+\left(1-\rho_1^H-\rho_2^H\right)\left\{\psi_B^H\left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}}-\overline{\frac{B}{PY}}\right)+\psi_{\Delta y}^H\left(\frac{Y_t}{Y_{t-1}}\right)+\psi_x^H\left(\frac{Y_t}{Y_t^n}\right)\right\},\end{aligned} \tag{2.4}$$

where $0 \leq \rho_1^H+\rho_2^H < 1$ governs labor tax rate smoothing, $\psi_B^H \geq 0$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^H$ is the feedback parameter on output growth, ψ_x^H is the feedback parameter on output gap, $\bar{\tau}_{new}^H$ is the new steady-state value of τ_t^H.[16] A large enough feedback coefficient on debt (high ψ_B^H) ensures that fiscal policy leads to stationary debt dynamics.[17]

[16]Feedback rules for fiscal policy were estimated in an early contribution by Bohn (1998). Bhattarai, Lee and Park (2016) estimate slightly simpler versions than above in a general equilibrium model with lump-sum taxes. Note importantly that in (2.4), distortionary tax rates adjust smoothly during the transition. This is motivated by the theoretical analysis of Barro (1979), but also, by our empirical estimates of tax rules that take this form. For completeness and comparison, we will also consider a case where labor tax rates adjust as necessary to ensure a constant debt-to-GDP ratio throughout the transition.

[17]In the textbook linearized model with one source of taxes, this condition is $\psi_B^H > \beta^{-1}-1$. In the model here, we solve for non-linear dynamics and additionally, as there are several sources of taxes and an empirically grounded tax rule, such a condition has to be determined numerically.

Third, only consumption tax rates τ_t^C adjust following the simple feedback rule

$$\begin{aligned}\tau_t^C - \bar{\tau}_{new}^C =& \rho_1^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \rho_2^C \left(\tau_{t-2}^C - \bar{\tau}_{new}^C\right) \\ &+ \left(1 - \rho_1^C - \rho_2^C\right) \left\{ \psi_B^C \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^C \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^C \left(\frac{Y_t}{Y_t^n} \right) \right\},\end{aligned} \tag{2.5}$$

where $0 \leq \rho_1^C + \rho_2^C < 1$ governs consumption tax rate smoothing, $\psi_B^C \geq 0$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^C$ is the feedback parameter on output growth, ψ_x^C is the feedback parameter on the output gap, and $\bar{\tau}_{new}^C$ is the new steady-state value of τ_t^C. A large enough feedback coefficient on debt (high ψ_B^C) ensures that fiscal policy leads to stationary debt dynamics.

For transition dynamics, the behavior of the monetary authority generally matters. In the three fiscal policies described above, the monetary policy rule (2.2) satisfies the Taylor principle, which thereby, implies that inflation plays no direct role in government debt stabilization. Moreover, given our restrictions that ψ_B^S, ψ_B^H and ψ_B^C are high enough, taxes respond strongly enough to ensure that debt dynamics are mean-reverting.

We consider a fourth case to highlight the role of monetary policy response to inflation for transition dynamics. In this case, labor taxes adjust, but not sufficiently, as the tax rule response coefficients are not large enough, and inflation partly plays a direct role in government debt stabilization, as the monetary rule response coefficients are not large enough. The monetary and labor tax rules are still given by (2.2) and (2.4), but now with these appropriate restrictions on the feedback parameters. Thus, in this fourth case, we allow debt stabilization, (only) along the transition, to occur partly through distortionary labor taxes and partly through inflation.[18]

2.3 Equilibrium and functional forms

Equilibrium definition is standard, given the maximization problems of the private sector and the monetary and fiscal policy rules described above. Goods, asset, and factor markets clear in equilibrium. The economy features balanced growth. As we describe below, we use standard assumptions on preferences that ensure balanced growth. Moreover, since our production function features two types of capital and capital-skill complementarity, we impose an additional assumption on the growth rate of q_t, the exogenous relative price between investment in capital structures and equipment. Generally, we normalize variables growing along the balanced growth path by the level of technology. Fiscal variables, as mentioned above, are normalized by output. We use the notation, for instance, $\tilde{Y}_t \equiv \frac{Y_t}{\gamma^t}$ and $\tilde{b}_t \equiv \frac{B_t}{P_t Y_t}$ to denote these stationary variables where γ is the growth rate of output. We also use the notation T_t^C, T_t^H, and T_t^K to denote (real) consumption, labor, and cap-

[18]An analogous consumption tax rule, with low enough response to debt, generates similar results and is thus omitted here. Moreover, while we consider these various fiscal/monetary adjustment scenarios to investigate how both positive and normative results depend on policy choices, our analysis is not in the Ramsey policy tradition.

ital tax revenues. Nominal variables are denoted in real terms in small case letters, for instance, $w_t = \frac{W_t}{P_t}$. All the equilibrium conditions are derived and given in detail in the Appendix A.1.

We use the following functional forms for preferences and technology

$$U(C_t^i, H_t^i) \equiv \log C_t^i - \bar{\omega}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi},$$

$$F\left(A_t, K_{b,t}, K_{e,t}, L_{s,t}, L_{u,t}\right) \equiv A_t \left(K_{b,t}\right)^{\alpha} \left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda \left(K_{e,t}\right)^{\rho} + (1-\lambda)\left(L_{s,t}\right)^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}},$$

and standard functional forms for the investment adjustment and variable capacity utilization costs

$$\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\xi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2, \ \mathcal{A}(u_t) \equiv \chi_1 (u_t - 1) + \frac{\chi_2^2}{2}(u_t - 1)^2.$$

The utility function is standard and consistent with balanced growth. The production function $F(\cdot)$ is a nested CES structure used in Krusell et al. (2000). This implies that equipment capital and skilled labor have the same elasticity of substitution against unskilled labor, given by $1/(1-\sigma)$. The elasticity of substitution between equipment capital and skilled labor is $1/(1-\rho)$. Capital-skill complementarity exists when $\sigma > \rho$. The parameters μ and λ govern income shares. Note that when $\rho \to 0$, the production function reduces to a standard Cobb-Douglas formulation, which we will use for analytical results. Suppose that the gross growth rate of A_t is $\bar{a} = \gamma^{1-\alpha}$. We then assume that the exogenous relative price between consumption (structures) and equipment investment, q_t, grows at rate $\gamma_q = 1/\gamma$, which leads to balanced growth in the model. It follows that all growing variables except A_t and equipment capital grow at rate γ.[19]

3 Long-run results

This section presents our results on the long-run effects of changes in the capital tax rate. Our key focus is on the source of financing the capital tax rate cut. We consider the three different fiscal policy adjustments presented above in Section 2.2.3, all of which ensure that the government debt-to-GDP ratio is at the same level in the long-run. The first is by (non-distortionary) transfer adjustment, which we take as the starting point. We then look at how a distortionary adjustment of labor tax rate and consumption tax rate alters the results.

[19] King, Plosser and Rebelo (2002) describes the required restrictions on preferences and technology in the standard neoclassical model. Balanced growth with capital-skill complementarity in the production function was shown in Maliar and Maliar (2011), who pointed out the need to have an exogenous path for relative price between consumption (structures) and equipment and restrictions on the growth rate.

3.1 Analytical results of a simplified model

We start with analytical results that help clarify the mechanisms regarding the long-run aggregate effects under the different ways of financing. For this, we simplify our model such that it converges to a textbook business cycle model. In particular, we first assume $\rho \to 0$ to get a nested version of the model with a Cobb-Douglas production function. It is also assumed that the two share parameters are zero, $\mu = \alpha = 0$, and that the fraction of skilled households is 1, $N^S = 1$. In this case, the model now has one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$ and a standard Cobb-Douglas production function that implies a unit elasticity of substitution between capital and labor. In the analytical results below, we then drop subscripts e and s for variables. For simplicity, there is no expensing of the tax rate.

The mechanism for the aggregate effects of a capital tax rate cut when non-distortionary transfers adjust in such a textbook model is well-understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising firms' demand for capital. This stimulates investment, and the capital-to-labor ratio increases as a result. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Given the increase in the factors of production, output increases, which also raises consumption unless the steady-state ratio of government spending-to-GDP is unrealistically very high.[20]

How do these effects change when distortionary labor or consumption tax rates increase to finance the capital tax reform? Proposition 3.1 below presents the results.

Proposition 3.1. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$ denote a new capital tax rate with a small change $\Delta\left(\bar{\tau}^K\right)$. Also, denote the new labor and consumption tax rate required to finance the capital tax rate change by $\bar{\tau}^H_{new} = \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right)$ and $\bar{\tau}^C_{new} = \bar{\tau}^C + \Delta\left(\bar{\tau}^C\right)$, respectively, with respective changes $\Delta\left(\bar{\tau}^H\right)$ and $\Delta\left(\bar{\tau}^C\right)$. The new steady-state values of a variable $\bar{X}_{new}$ in transfer adjustment case, in labor tax rate adjustment case, and in consumption tax rate adjustment case are denoted by $\bar{X}^T_{new}$, $\bar{X}^L_{new}$ and $\bar{X}^C_{new}$, respectively. For $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$*

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^L_{new}}\right) = -\Theta^T_L \Delta\left(\bar{\tau}^K\right) = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\Delta\left(\bar{\tau}^H\right),$$

where $\Theta^T_L = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\frac{\lambda}{1-\lambda}\left(1 + \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C\right) > 0$, and

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^C_{new}}\right) = -\Theta^T_C \Delta\left(\bar{\tau}^K\right) = \frac{1}{1+\varphi}\left(\frac{1}{1+\bar{\tau}^C}\right)\Delta\left(\bar{\tau}^C\right),$$

[20] In such a case, government consumption or investment crowds out private consumption. See Appendix B.3, Proposition B.1 for details, which formally presents the results discussed above.

where $\Theta_C^T > 0$ *if* $\bar{\tilde{G}} < 1 - \lambda \frac{\theta-1}{\theta} \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)} \left(1 - \bar{\tau}^K_{new}\right)$. *Moreover,*

$$\ln\left(\frac{\bar{X}^L_{new}}{\bar{X}^C_{new}}\right) = \Theta_C^L \Delta\left(\bar{\tau}^K\right) = \frac{1}{1+\varphi}\left(\left(\frac{1}{1+\bar{\tau}^C}\right)\Delta\left(\bar{\tau}^C\right) - \left(\frac{1}{1-\bar{\tau}^H}\right)\Delta\left(\bar{\tau}^H\right)\right),$$

where $\Theta_C^L > 0$ *if* $\bar{\tilde{G}} < 1 - \lambda \frac{\theta-1}{\theta} \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)} \left(1 - \bar{\tau}^K_{new}\right) - (1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}$. *If follows that under the mild condition on government spending, for a small change* $\Delta\left(\bar{\tau}^K\right) < 0$*,*

$$\bar{X}^L_{new} < \bar{X}^C_{new} < \bar{X}^T_{new}.$$

Proof. See Appendix B.5. □

Proposition 3.1 implies that, generally, when the capital tax rate is cut, output, capital, investment, consumption and hours increase by more in the transfer adjustment case than in either the labor tax rate adjustment case or the consumption tax adjustment case. Moreover, generally, the consumption tax rate adjustment leads to bigger expansionary effects than the labor tax rate adjustment.

Compared to the case of lump-sum transfer adjustment, the macroeconomic effects are smaller because of distortions created by the labor or consumption tax rate increases that affect labor supply negatively. Moreover, Proposition 3.1 states that the differences in the changes in output, investment, consumption, and hours across the three financing options are given by the same amount. This constant difference depends intuitively and precisely on the labor supply parameter for a given change in the tax rates. A higher Frisch elasticity ($\frac{1}{\varphi}$) makes workers more responsive to labor tax rates or consumption tax rates, thereby generating greater distortions, which in turn, magnifies the difference. The three fiscal adjustments produce the same outcomes when labor supply is completely inelastic ($\frac{1}{\varphi} = 0$). Moreover, as is intuitive, higher is the initial level of the labor tax rate, bigger is the difference. Thus, for the same change in the labor tax rate, if the initial labor tax rate is higher, the increase in output, investment, consumption, and hours will be relatively smaller.

When it comes to comparing the labor tax rate to the consumption tax rate adjustment, the latter is less distortionary for labor hours, thereby generating bigger aggregate effects. In fact, with $\beta = 1$, steady-state hours do not depend at all on the consumption tax rate, but depend negatively on the labor tax rate.[21] This interesting finding will play a critical role when we later conduct a welfare analysis in Section 5.

[21] See Appendix B.5 for details. This result arises because of the complete offset of substitution and income effects. Moreover, if there is no capital tax rate in the model, then the restriction that $\beta = 1$ is not needed. This case however, is not relevant for our paper as the focus is on the response of the economy to changes in the capital tax rate.

3.2 Numerical results of the quantitative model

We now present the numerical results employing our baseline quantitative model with heterogeneity and skill premium.

3.2.1 Parameterization

The frequency of the model is a quarter. Table 1 contains numerical values we use for the parameters that are relevant for long-run effects. The parameterization is standard, and we provide detailed justification and references in Table 1. As given above, we use separable preferences that imply log utility in consumption and then calibrate a modest, unit Frisch elasticity of labor supply ($\frac{1}{\varphi}$= 1) based on Smets and Wouters (2007). For the production function elasticity of substitution parameters, we use the estimates in Krusell et al. (2000) ($\sigma = 0.401, \rho = -0.495$). This parameterization implies (equipment) capital-skill complementarity. We also follow Krusell et al. (2000) in matching the income share of structure ($\alpha = 0.117$) as well as the depreciation rates of the two types of capital. For the income share of equipment, we pick parameter value to get a steady-state aggregate labor income share of 0.56, which is consistent with estimates in Elsby, Hobijn and Şahin (2013) and Ohanian, Orak and Shen (2021).

We use the Monthly Outgoing Rotation Group (MORG) of the US Census Bureau's Current Population Survey data to calculate both the fraction of unskilled households and the skill premium. The skill premium is defined as the ratio of the hourly wage of workers with 14 or more years of schooling to the hourly wage of workers with less than 14 years of schooling. The share of workers with 14 or more years of schooling is 0.505 and the skill premium for median wage is 55%. We then set the income share of unskilled households ($\mu = 0.359$) to match the skill premium observed in data.

Additionally, across various fiscal adjustment scenarios and preference and technology functions specifications, we normalize hours for skilled labor to be 0.330 and hours for unskilled labor to be 0.307 in the initial steady-state by appropriately adjusting the scaling parameters $\bar{\omega}^s$ and $\bar{\omega}^u$. We follow the calibration of Lindquist (2004) for this choice of steady-state hours as well as the fraction of skilled labor (N^s=0.5).

The steady state of the fiscal variables such as the debt-to-GDP ratio, the government spending-to-GDP ratio, and the taxes-to-GDP ratio, is matched to their respective long-run values in the data. Appendix C describes this data in detail. We then calibrate the steady-state markup to obtain a 35% capital tax rate initially. The implied initial levels of labor tax rate and consumption tax rate are 12.8% and 0.9% respectively. For the effective expensing rates of the two types of capital, we use the estimates in Barro and Furman (2018), which imply lower expensing of structure investment. We assume that the profit shares for skilled labor (χ^s_Φ) is 1 and the transfers share for unskilled

Table 1: Calibration for long-run analysis

	Value	Description	References
Households			
β	0.9975	Time preference	Smets and Wouters (2007)
φ	1.0	Inverse of Frisch elasticity of labor supply	Smets and Wouters (2007)
$\bar{\omega}^s$	4.67	Labor supply disutility parameters for skilled households	Steady-state $\bar{H}^s = 0.33$
$\bar{\omega}^u$	9.90	Labor supply disutility parameters for unskilled households	Steady-state $\bar{H}^u = 0.31$
N^s	0.5	Fraction of skilled labor	Data (See Appendix C)
d_e	0.031	Equipment capital depreciation	Krusell et al. (2000)
d_b	0.014	Structures capital depreciation	Krusell et al. (2000)
Firms			
σ	0.401	Elasticity of substitution between unskilled labor and equipment	Krusell et al. (2000)
ρ	-0.495	Elasticity of substitution between skilled labor and equipment	Krusell et al. (2000)
α	0.117	Structures capital Income share	Krusell et al. (2000)
λ	0.35	Equipment capital income share	Steady-state labor share: 56% (Ohanian, Orak and Shen, 2021)
μ	0.359	Unskilled labor income share	Steady-state skill premium:55% Data (See Appendix C)
γ	1.0054	Long-run growth rate of output	Smets and Wouters (2007)
$\bar{\pi}$	1.0078	Steady-state inflation rate	Smets and Wouters (2007)
θ	4.0	Elasticity of substitution between goods	Steady-state markup: 33% (Hall, 2018)
q_0	0.95	Relative price of structure to equipment capital	Maliar and Maliar (2011)
Government			
$\bar{\bar{b}}$	0.363	Steady-state debt to GDP ratio	Data (See Appendix C)
$\bar{\bar{G}}$	0.161	Steady-state government spending to GDP ratio	Data (See Appendix C)
$\bar{\bar{T}}^C$	0.009	Steady-state consumption tax revenue to GDP ratio	Data (See Appendix C)
$\bar{\bar{T}}^H$	0.128	Steady-state labor tax revenue to GDP ratio	Data (See Appendix C)
χ^s_Φ	1	Fraction of profit distribution to skilled worker	Assigned
χ^s_S	0	Fraction of transfer to skilled worker	Assigned
λ_b	0.338	Effective expensing rate of structure investment	Barro and Furman (2018)
λ_e	0.812	Effective expensing rate of equipment investment	Barro and Furman (2018)

labor (χ^u_S) is 1.[22]

We present detailed sensitivity analysis of our baseline parameterization in Section 6.

3.2.2 Numerical results

The results for long-run effects under various financing policies are in Figure 1. While our focus is on a reduction of the capital tax rate from 35% to 21%, which are clearly shown with colored dots in the Figure, we show the entire range of tax rate changes for completeness.

Let us first look at the case of transfer adjustment presented by the blue lines in Figure 1. A decrease in the capital tax rate reduces total (tax) revenues-to-GDP ratio (driven by decrease in capital tax revenue-to-GDP ratio). The reduction in tax revenues is then financed by a decline in

[22] We discuss how results might change with alternate assumptions later in a sensitivity analysis in Section 6.

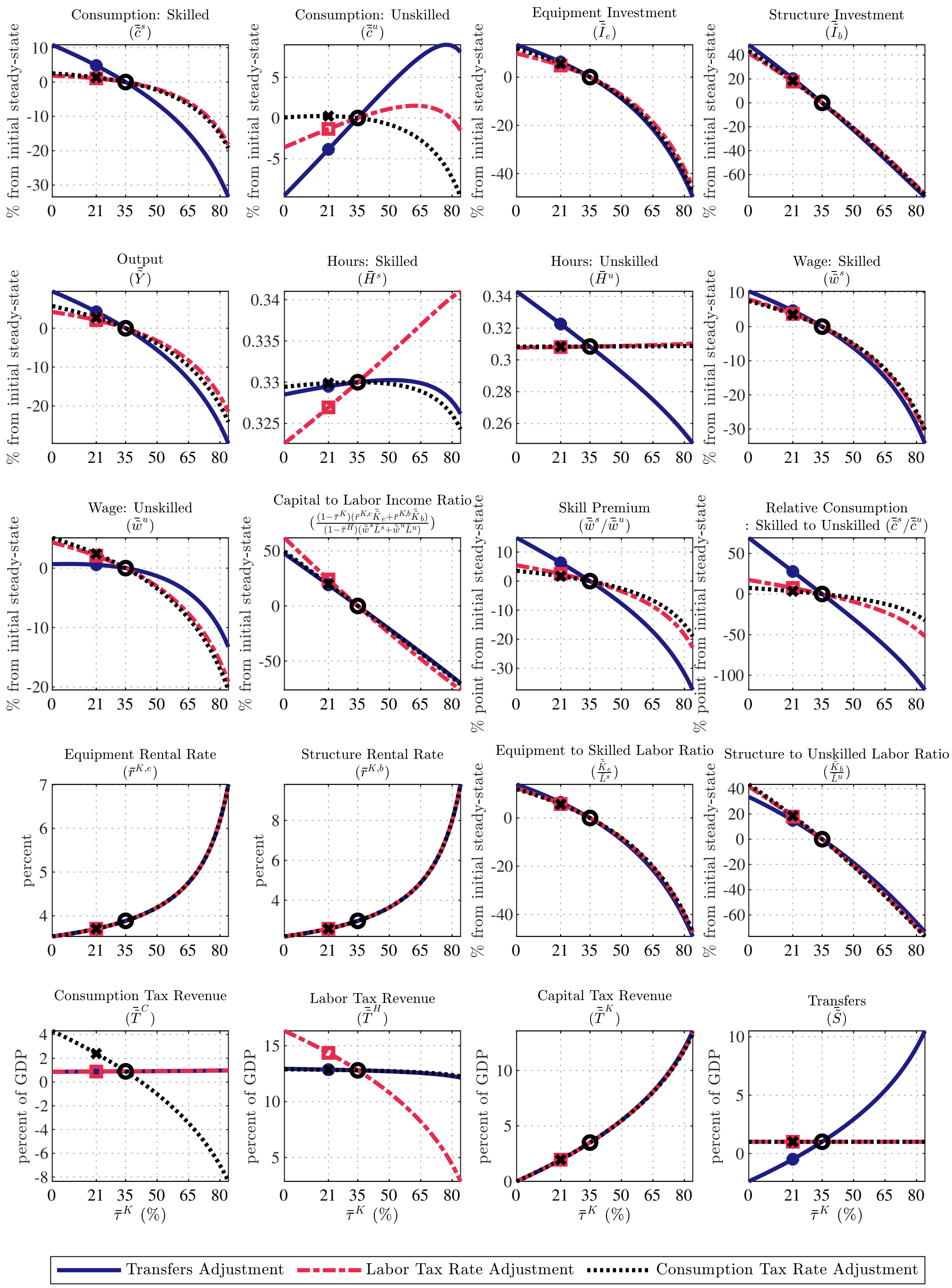

Figure 1: Long-run effects of permanent capital tax rate changes

transfers-to-GDP ratio from 1.0% to -0.49% as shown in the last panel of the Figure 1.[23] This need to engage in lump-sum tax to finance the capital tax reform is unrealistic, and motivates our analysis of distortionary financing later.

In terms of aggregate implications, for a reduction of the capital tax rate from 35% to 21%, output increases by 4.24% relative to the initial steady state, structure investment by 20.24%, and equipment investment by 6.27%.[24] These effects are illustrated respectively in the fifth, fourth, and third panels of Figure 1. The aggregate effects are driven by the same mechanism as in the simple model we discussed earlier, where the rental rate of capital falls, which leads to a boost in the capital-labor ratio and investment.

Let us now look at distributional implications. First, the skill premium or wage inequality rises following a capital tax rate cut as skilled wages increase by more than unskilled wages. In particular, the former increases by 4.66% while the latter increases by only 0.56%. Accordingly, the skill premium goes up by 6.32% points (shown in the eleventh panel of the Figure 1).[25] Second, income inequality, measured by the the ratio of after-tax capital-to-labor income (shown in the tenth panel), unambiguously increases—although both types of income increase. Third, these rises in wage and income inequality, coupled with the fall in transfers which are all distributed to the unskilled in our baseline calibration, result in an increase in consumption inequality. Specifically, consumption of the unskilled falls by 3.86% while consumption of the skilled rises by 4.82%. As a result, the relative consumption of the skilled vs. the unskilled, increases in the long-run by 27.7% (shown in the twelfth panel). Notice that the rise in consumption inequality generates differential wealth effects on labor supply. Consequently, skilled hours decreases slightly while unskilled hours increases (shown in the sixth and seventh panels).[26] The increases in wage, consumption, and income inequality can be considered as caveats to the effectiveness of the capital tax rate cut in our model, even when lump-sum transfers are allowed to finance the tax cut.

To understand the mechanisms behind these increases in inequality, we can express the skill

[23] Note that this result is obtained not only because output (i.e. the denominator) increases. In fact, the total tax revenues also decline. In particular, there is a significant decrease in capital tax revenues (about 44% decline relative to the initial steady state), which is only partially offset by an increase in consumption and labor tax revenues. The government therefore finances such a deficit by taking resources away from the household: transfers decline by roughly 149% of the initial steady-state. There is a "Laffer curve" for capital tax revenues but the capital tax revenue starts to decline at very high and empirically irrelevant range, such as above 90% in our baseline calibration.

[24] For comparison, Barro and Furman (2018) predict that the long-run increase in output will be 3.1% for a permanent capital tax rate cut from 38% to 26%, assuming that the employment-to-population ratio is fixed. Unlike their analysis, we model an endogenous labor supply decision, which is especially important when considering distortionary financing.

[25] In Appendix B.2, we show analytically how the skill premium increases with the capital tax rate cut in our model.

[26] Additionally, we find that structure investment increases by more than equipment investment. Quantitatively, the major reason for this finding is the lower expensing rate on structure investment in our calibration. Qualitatively, a role is also played by the fact that in the production function, the elasticity of substitution between equipment investment and skilled hours makes them complements.

premium in our model as

$$\frac{W_t^s}{W_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\lambda\left(\frac{K_{e,t}}{L_{s,t}}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma-\rho}{\rho}}\left(\frac{L_{u,t}}{L_{s,t}}\right)^{1-\sigma}.$$

Thus, if a capital-skill complementarity ($\sigma > \rho$) exists, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. This mechanism—which is operative even in a model with a representative household or perfect consumption insurance—drives our result on the skill premium. On top of this well-known mechanism, in our model with heterogeneous households, the rise in consumption inequality affects labor hours in a way such that wage inequality increases even further. To see this, first notice that the skill premium is increasing in the ratio of unskilled-to-skilled labor in the expression. As stated above, the unskilled-to-skilled labor ratio increases in response to a capital tax cut because the rise in consumption inequality produces differential wealth effects on labor supply. Furthermore, the decrease in skilled hours raises the equipment capital-to-skilled labor ratio, which in turn generates a further increase in wage inequality. The two sources of heterogeneity we introduce into an otherwise textbook macroeconomic model (skill heterogeneity and household heterogeneity) thus interact in economically meaningful ways to generate new distributional effects.

Now, we contrast the results when labor tax rate, instead of transfers, adjusts to finance the capital tax rate cut. These results are illustrated by the red lines in Figure 1. In this case, labor tax rates, in the long-run, have to increase from 22.7% to 25.4% to finance the reduction of the capital tax rate from 35% to 21%. While the capital tax cut continues to be expansionary, the increase in output and investment is now less under labor tax rate adjustment—as is consistent with what we showed in Proposition 3.1 in the simplified model. In particular, for the baseline experiment of a reduction of the capital tax rate from 35% to 21%, output increases by 2.08%, equipment investment by 17.75%, and structures investment by 4.81%. The reason for the smaller boost in aggregate variables is fall in hours of both the skilled and the unskilled. There is now a smaller increase in the after-tax wages for skilled workers, which leads to a bigger decrease in labor hours in the long-run, from 0.330 to 0.327. For unskilled workers, there is in fact a decline of the after-tax wages, by 1.45%, which leads to a smaller increase in labor hours in the long-run. The labor supply response of the unskilled is qualitatively different from the lump-sum transfer adjustment case, as now the wealth effect on labor supply due to decrease in transfers is no longer in operation. Overall, the decrease in skilled and unskilled hours dampens the expansionary effect of capital tax cuts on output and investment.

Turning to distributional effects, both wage and consumption inequality increase, as they did in the transfer adjustment case. In comparison to the transfer adjustment case however, these distributional effects are smaller. The main reason is that the burden from the labor tax increase is shared by both types of households, whereas transfer reduction only affects the unskilled. Therefore, consumption of the unskilled does not fall as much while consumption of the skilled does not increase as much now. The reduced consumption inequality in turn diminishes the role of the wealth effect on labor supply, leading to a smaller increase in the skill premium.[27]

We finally analyze the case when consumption tax rate increases in the long-run to finance the capital tax rate cut. In this case, to finance the reduction of the capital tax rate from 35% to 21%, consumption tax rates have to increase from 1.29% to 3.49%. Generally, as we emphasized before in Proposition 3.1 for the simple model, the effects are qualitatively similar to labor tax rate adjustment, with the main distortion again coming in labor supply decisions, which leads to a smaller expansionary effect on output compared to transfer adjustment.

One important qualitative difference compared to both transfer and labor tax rate adjustment can be seen in consumption of the unskilled household, which now increases slightly (shown in the second panel of Figure 1). As in the labor tax rate adjustment case, the burden on consumption tax rate increase is shared by both types of households, and thus the unskilled household benefits compared to transfer adjustment case. Compared to labor tax rate adjustment, consumption tax rate adjustment is more beneficial for the unskilled as the aggregate output effects are bigger. This numerical result is consistent with what we showed in Proposition 3.1 in the simplified model in which consumption taxes are relatively less distortionary for labor supply. As transfers-to-GDP is fixed in the long-run, a greater output expansion implies that transfers (in level) will increase by more now, which allows the unskilled to sustain a higher level of consumption in the long-run.

4 Transition dynamics

We now discuss transition dynamics associated with a permanent capital tax rate cut, from 35% to 21%, by tracing out the evolution of the economy from the initial steady-state to the new steady-state. Studying transition dynamics is important as we find that it typically takes a quite long time, around 156 quarters, for consumption to converge to a new steady-state following a permanent reduction in the capital tax rate. This allows us in particular to analyze short-run effects, which are the focus here.

As in the long-run analysis in the previous section, we present the baseline model under different policy adjustments. Compared to the long-run analysis, we pay special attention to the role of monetary policy, which can be potentially important due to nominal rigidities in the short-run. A new theme we highlight thus in this section is how a joint analysis of monetary and fiscal policy is important to understand the short-run effects of a permanent capital tax rate change.

[27] Finally, our measure of income inequality also continues to increases, but by more here compared to transfer adjustment, as after-tax labor income now decreases.

Table 2: Calibration for transition dynamics

	Value	Description	References
Households			
ξ	4.0	Investment adjustment cost	Smets and Wouters (2007)
$\frac{\mathcal{A}''}{\mathcal{A}'}$	0.85	Elasticity of cost of capital utilization	Smets and Wouters (2007)
Firms			
α_P	0.65	Calvo sticky price parameter	Smets and Wouters (2007)
γ_P	0.22	Degree of price indexation	Smets and Wouters (2007)
Government(Fiscal/Monetary Policy): Transfer or Labor Tax Rate Adjustment			
ρ_1^R	1.12	Interest rate smoothing parameter lag 1	Coibion and Gorodnichenko (2011)
ρ_2^R	-0.18	Interest rate smoothing parameter lag 2	Coibion and Gorodnichenko (2011)
ϕ_π	1.58	Inflation feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ϕ_x	0.11	Output gap feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
$\phi_{\Delta y}$	2.21	Output growth feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ρ_1^H	0.869	Transfer/Labor tax rate smoothing parameter lag 1	Estimated (See Appendix D)
ρ_2^H	0.0	Transfer/Labor tax rate smoothing parameter lag 2	Estimated (See Appendix D)
ψ_B^H	0.111	Transfer/Labor tax rate response to debt	Estimated (See Appendix D)
$\psi_{\Delta y}^H$	0.831	Transfer/Labor tax rate response to output growth	Estimated (See Appendix D)
ψ_x^H	0.0	Transfer/Labor tax rate response to output gap	Estimated (See Appendix D)
Government(Fiscal/Monetary Policy): Labor Tax Rate and Inflation Adjustment			
ρ_1^R	1.12	Interest rate smoothing parameter lag 1	Coibion and Gorodnichenko (2011)
ρ_2^R	-0.18	Interest rate smoothing parameter lag 2	Coibion and Gorodnichenko (2011)
ϕ_π	0.95	Inflation feedback parameter under Taylor rule	Assigned
ϕ_x	0.0	Output gap feedback parameter under Taylor rule	Assigned
$\phi_{\Delta y}$	0.0	Output growth feedback parameter under Taylor rule	Assigned
ρ_1^H	0.785	Labor tax rate smoothing parameter lag 1	Estimated (See Appendix D)
ρ_2^H	0.107	Labor tax rate smoothing parameter lag 2	Estimated (See Appendix D)
ψ_B^H	0.001	Labor tax rate response to debt	Estimated (See Appendix D)
$\psi_{\Delta y}^H$	1.821	Labor tax rate response to output growth	Estimated (See Appendix D)
ψ_x^H	0.0	Labor tax rate response to output gap	Estimated (See Appendix D)

4.1 Parameterization

For transition dynamics, the parameterization of policy rules, investment adjustment costs, nominal rigidities, and capacity utilization costs matters. The parameterization is in Table 2. We use estimates from Smets and Wouters (2007) for investment adjustment costs, capacity utilization costs, the probability of resetting prices, and degree of inflation indexation.

We now discuss how we parameterize the policy rules, which govern the associated fiscal and monetary adjustments along the transition. We use estimates from Coibion and Gorodnichenko (2011) for monetary policy rule parameters from the post-Volcker period (1983-2002), in which the Taylor principle is satisfied. For the transfer and tax rule parameters, we use our estimates of the policy rule (2.4) using US data. The details of the estimation are presented in Appendix D and

Table F.1 shows the estimation results.

We point out here that the transfer and tax rule parameters we use for the case where transfer and taxes respond sufficiently to debt are based on estimates obtained using post-Volcker period data for the same period as in Coibion and Gorodnichenko (2011) (1983-2002). This combination, using data and estimates for the exact time period, then describes the regime where labor tax rates adjust to ensure stationary debt dynamics while monetary policy stabilizes inflation. Next, the tax rule parameters we use for the case where taxes do not respond sufficiently to debt, while inflation plays a role in debt stabilization in the short-run, are based on estimates obtained using data from a later period (2001-2019). In this latter case, the monetary policy rule does not satisfy the Taylor principle and we parameterize it accordingly.[28] In this regime, inflation plays a partial, but direct role, in debt stabilization.

4.2 Four different fiscal/monetary adjustments

We now consider four different fiscal/monetary policy adjustments, as described in Section 2.2.3. In particular, a new policy response that we consider here is one where inflation plays a partial, but direct, role in debt stabilization.

4.2.1 Lump-sum transfer adjustment

Once again, the starting point is the case of transfer adjustment, shown by the blue lines in Figure 2.[29] What makes the short-run distinct from the long-run is that in principle, capital tax cuts can now generate a contractionary aggregate effect during the transition periods.

The results can be best understood as depicting transition dynamics when the capital stock is initially below the new steady-state. As mentioned before, a reduction in the capital tax rate leads to a decrease in the rental rate of capital, thereby facilitating capital accumulation via more investment. In the short-run, to finance this increase of investment, consumption of skilled declines initially. Moreover, there is a large decline along the transition in consumption of the unskilled. This is because of the large dynamic decline in transfers. Given this postponement of consumption, combined with sticky prices, output also falls temporarily, before rising towards the high new steady-state. The temporary contraction in output is a result of sticky prices and investment adjustment costs, which renders output (partially) demand-determined and markups countercyclical in

[28] In particular, we use $\phi_\pi < 1$, $\phi_{\Delta y} = 0$, and $\phi_x = 0$. We do not formally estimate the monetary policy rule for this sub-period due to the binding ZLB for much of the sample and the lack of enough observations. Passive monetary policy arguably characterizes this period well. Our results are robust to the precise parameterization of ϕ_π as long as it is below 1, as we show in Section 6.

[29] The dashed straight lines in the Figure 2 represent the new steady state relative to the initial steady state.

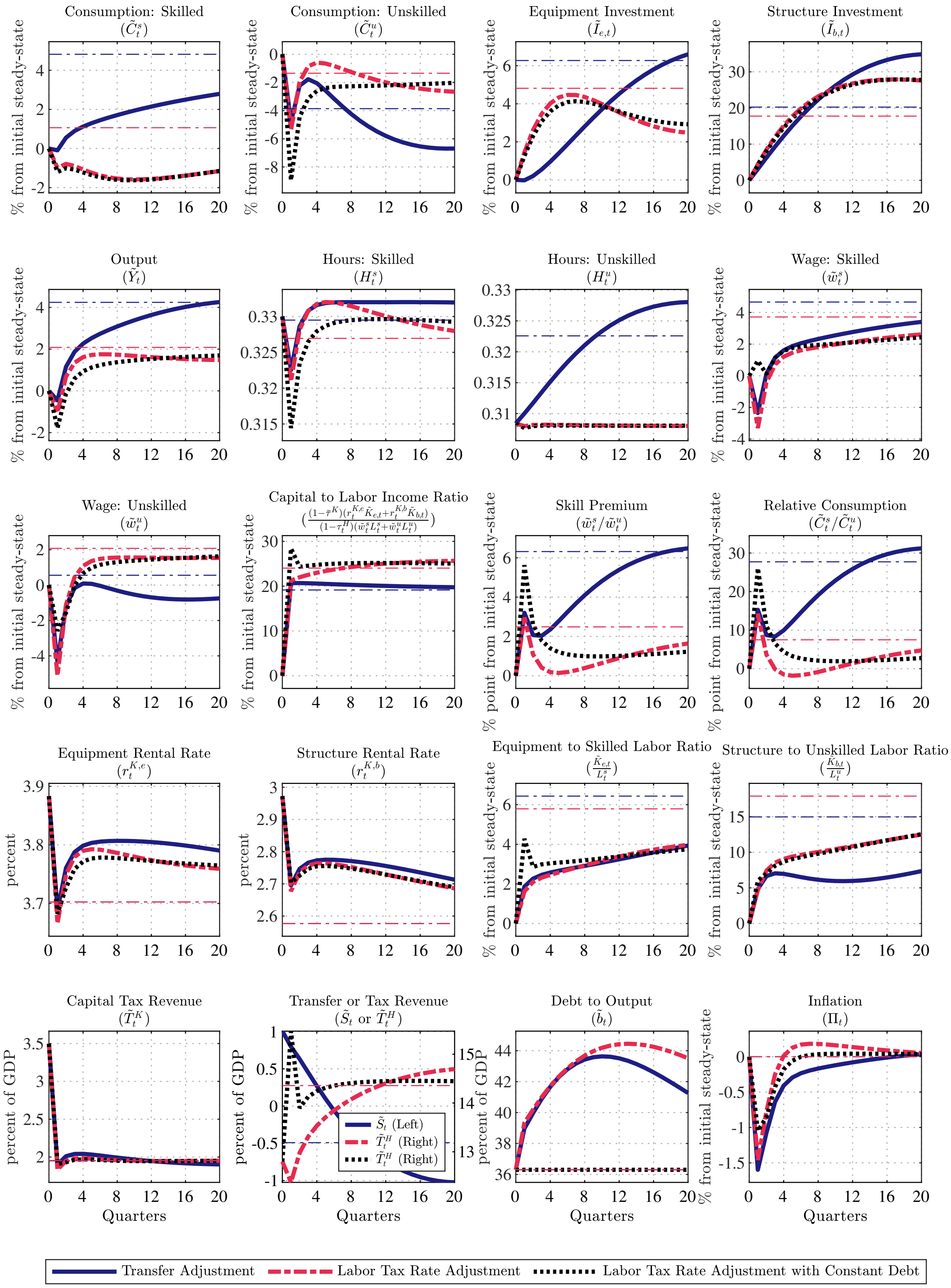

Figure 2: Transition dynamics of a permanent capital tax rate decrease under alternate financing

the model.[30] The temporary fall in output (which is coupled with increased capital stock), in turn leads to fall in hours of the skilled. For the unskilled workers however, because of the effects on marginal utility, they work more along the transition. This increased labor supply by the unskilled mitigates the fall in output that would occur otherwise.

Inflation also declines. It is determined by forward looking behavior of firms, and thus depends on current and future real marginal costs. As real marginal costs are a function of wages and capital rental rate, the path of inflation roughly follows that of these factor prices. The decrease in wages is driven by both supply and demand forces. The drop in consumption and the rise in marginal utility of consumption raise the supply of hours for a given wage rate, which plays an important role for unskilled labor response. On the other hand, labor demand declines as firms produce a smaller amount of output as discussed above.

In terms of inequality, Figure 2 shows that the skill premium and consumption inequality increase in the short-run and slowly converge to the new steady state. The capital-to-labor income ratio also increases in the short-run, above the new long-run level.

Moreover, the long-run positive effects of capital tax cuts come at the expense of short-run decline of labor tax revenue—even under lump-sum transfer adjustments. Furthermore, the decrease in labor income requires a larger adjustment of transfers. Transfers fall sharply and in fact, have to go negative after a few periods (due to the smoothing feature of our transfer policy). This needs to engage in lump-sum taxes is arguably unrealistic, and motivates our study of distortionary financing next.

4.2.2 Labor tax rate adjustment

Next, we analyze the case of labor tax rate increase, which is shown by the red lines in Figure 2. Here, labor tax rate evolves according to the tax rate rule, (Equation 2.4), given in Section 2.2.3. Overall, model dynamics are qualitatively similar to those in the transfer adjustment case. We still see capital accumulation, achieved by increased investment and postponement of consumption, which in turn also causes output to fall with sticky prices.

Quantitatively, however, the drop in consumption and output is larger in this case compared to the lump-sum transfer adjustment case. As in the lump-sum transfer adjustment case, delayed consumption decreases hours by lowering firm's labor demand. In addition, increased labor tax rate decreases hours even further by discouraging workers from supplying labor. Consequently, hours in equilibrium fall by more, of both the skilled and the unskilled. In fact, in contrast to the transfer adjustment case, unskilled hours now decreases. This is due to the same mechanism as in the long-run we discussed above. The wealth effect on labor supply is relatively weak for the

[30] The monetary policy rule specification also plays a role quantitatively.

unskilled because there is no reduction in transfers and the burden of labor tax rate increases is shared by both types of households. Therefore, the substitution effects dominate. Overall, the fall in hours of both types in turn amplifies the short-run contraction in consumption and output.

In terms of distributional implications, consumption inequality increase is less pronounced compared to the transfer adjustment case. This fiscal adjustment is relatively more beneficial for the unskilled as it does not feature a decline in transfers.[31] The labor tax increase burden is shared by both types, whereas transfer reduction only affects the unskilled.

Note that the dynamics associated with labor tax rate adjustment are fairly close, especially in the very short-run, compared to lump-sum transfer adjustment. This is driven by our use of an empirically driven tax rule where tax rates adjust smoothly, allowing higher-than-normal debt-to-GDP ratio. To highlight this, in Figure 2 we additionally consider a case where labor tax rates adjust as necessary to maintain a constant debt-to-GDP ratio throughout the transition. In that case, the short-run contraction is clearly more severe compared to lump-sum transfer adjustment, driven by more rapid increases in the labor tax rates that has a strong negative effect on labor hours—especially of the skilled households.

4.2.3 Labor tax rate and inflation adjustment

The results are quite different, even qualitatively, in the case where labor tax rate increases, but not sufficiently, and inflation partly plays a role in government debt stabilization, as described in Section 2.2.3.[32] Figure 3 shows these results, where for comparison we also show the pure labor tax rate adjustment case discussed just above.

The main difference now, compared to the pure labor tax adjustment analysis, is that there is a short-run burst of inflation to help stabilize debt. This increase in inflation, as the model has nominal rigidities, helps counteract the short-run contractionary effects. Output, consumption, investment, hours, and wages, in fact, all increase in the short-run and the differences are quite pronounced for all variables other than investment. After 8 quarters or so, the transition dynamics become very similar to the pure labor tax rate adjustment case. Interestingly, debt-to-GDP ratio, in sharp contrast to other fiscal adjustment cases, decreases for extended periods due to the rise in output and the price level.

On the distribution side, a new result emerges. As we discussed above, labor tax rate adjustment, compared to transfer adjustment, is relatively more beneficial for the unskilled as it does not feature a decline in transfers and the labor tax increase burden is shared by both types. Generat-

[31] Labor tax rates change smoothly here, as do transfers.

[32] Note that in this case, the monetary policy rule (2.2) does not satisfy the Taylor principle, which is coupled with a weak response of the tax rate in the tax rule (2.4). Clearly, we can analyze a similar fiscal adjustment case where inflation plays a role in debt stabilization even with lump-sum transfer adjustment. When non-distortionary sources of revenue is possible, allowing inflation to play a role in debt stabilization might not be a very insightful experiment.

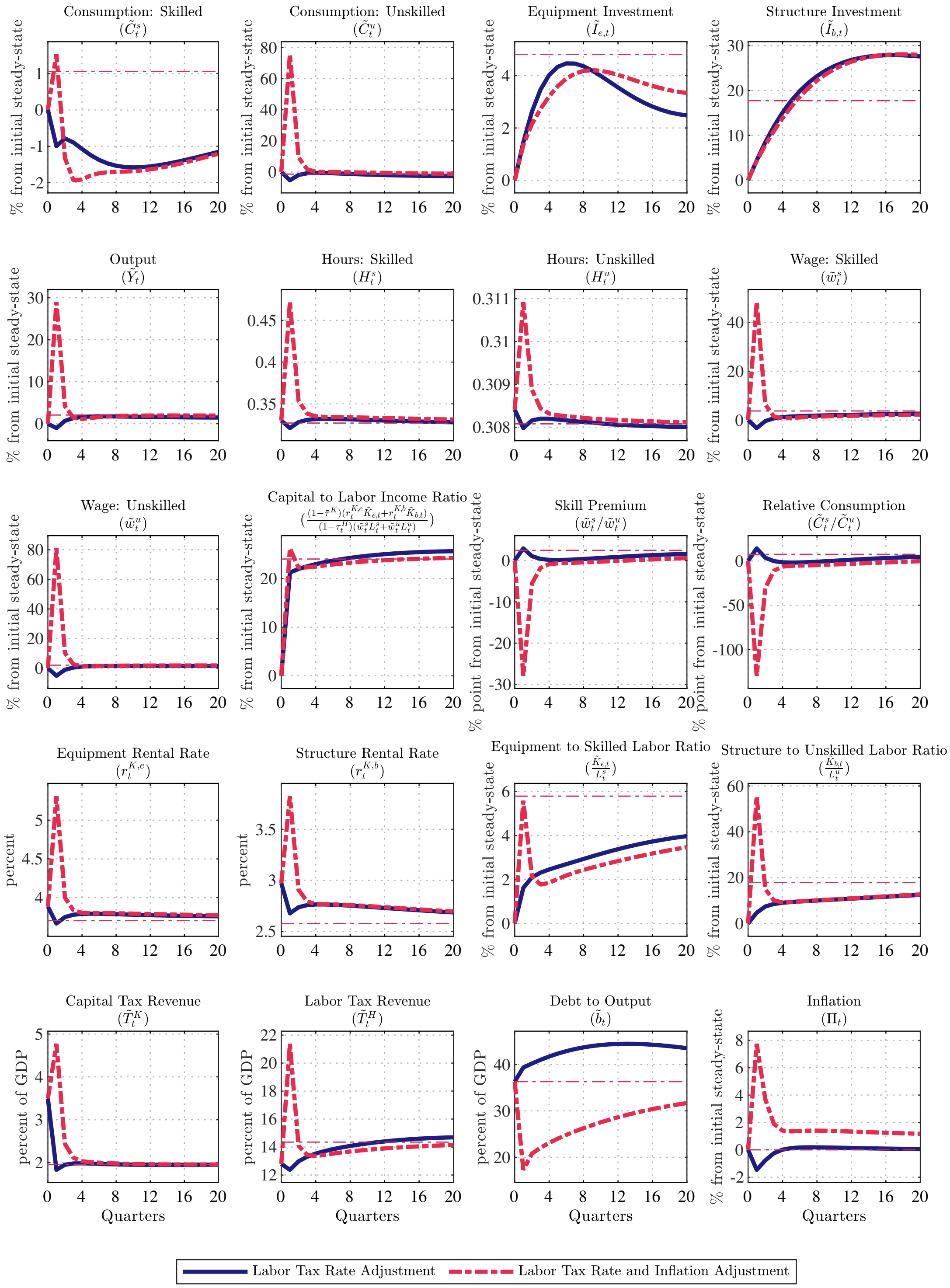

Figure 3: Transition dynamics of a permanent capital tax rate decrease under labor tax rate and inflation adjustment

ing inflation, on top of that, favors the unskilled even more as government debt, whose real value deteriorates during the transition, is owned only by the skilled. Thus, while both the skilled and unskilled household's consumption rises due to a smaller increase in the labor tax rate with rising inflation, the latter rises relatively more. The decrease in consumption inequality in turn leads to a decline in skill premium in the short-run due to the wealth effect on labor supply. These results on consumption and wage inequality are in contrast with those obtained under other types of fiscal adjustments.

4.2.4 Consumption tax rate adjustment

For completeness, we next study transition dynamics for the case of consumption tax rate adjustment. We show the results in Appendix Figure F.1 in Appendix F, where we use the same policy rule parameters as for the labor tax rate adjustment. The transition dynamics associated with the labor tax rate and consumption tax rate adjustment are very similar.

5 Welfare implications

While the focus of this paper is not necessarily on normative policy issues, we can nevertheless evaluate welfare implications of the permanent capital tax rate cut from 35% to 21%. Our results in the previous sections suggest that a reduction of the capital tax rate has different welfare implications depending on time horizon, household types, and policy adjustments. In this section, we formally calculate a measure of welfare gain that can be achieved through a permanent capital tax cut, taking into account transition dynamics as well as the long-run effect.

5.1 Welfare measure

Our measure of welfare gain for type-i agent, $\mu^i_{k,t}$, is implicitly defined by the sequence of values satisfying

$$\sum_{j=0}^{t} \beta^j U\left(\tilde{C}^i_j, H^i_j\right) = \sum_{j=0}^{t} \beta^j U\left(\left(1+\mu^i_{k,t}\right)\bar{\tilde{C}}^i_j, \bar{H}^i_j\right),$$

where $\left\{\tilde{C}^i_j, H^i_j\right\}$ and $\left\{\bar{\tilde{C}}^i_j, \bar{H}^i_j\right\}$ are respectively the time path of type-i agent's normalized consumption and hours with and without a capital tax cut under the various fiscal/monetary adjustments (indexed by k) we have considered above. We denote by $\mu^i_{T,t}$ the case of transfer adjustment, by $\mu^i_{H,t}$ the case of labor tax rate adjustment, and by $\mu^i_{C,t}$ the case of consumption tax rate adjustment. Thus, $\mu^i_{k,t}$ measures welfare gains from period 0, when the tax reform initiates, till (arbitrary) period t, in units of a percentage of the level of normalized initial consumption.[33] The lifetime (total) welfare gain is then measured by $\lim_{t\to\infty}\mu^{i}_{k,t}$, which is of interest in the business cycle literature (Lucas 1987).

5.2 Welfare results

Panel (a) of Figure 4 shows welfare results for a reduction in the capital tax rate from 35% to 21% when transfers adjust. It is clear that in this case, the tax reform does not lead to a Pareto improvement: the skilled gain at the expense of the unskilled because the latter type, as we pointed out earlier, consumes less, which in turn also forces the agent to work more through wealth effects on labor supply. Specifically, the lifetime welfare gains under transfer adjustment amounts to 4.59% of the initial consumption level for the skilled and *negative* 8.24% for the unskilled.[34] Turning to the short-run, the skilled worker's welfare never become negative while in contrast, for the unskilled, $\mu^{u}_{T,t}$ never become positive.

The tax reform not leading to a Pareto improvement is also true under labor tax rate adjustment, as shown in panel (b) of Figure 4. Moreover, when labor tax rates adjust, compared to transfer adjustment, while welfare gains are smaller for the skilled in the long-run and $\mu_{H,t}$ for the skilled becomes positive only 80 quarters after the onset of the capital tax reform, welfare losses are in fact smaller for the unskilled. Labor tax adjustment works better for the unskilled as the labor tax increase burden is shared by both types, whereas transfer reduction only affects the unskilled. We discussed the same mechanism for the consumption effects in the long-run in Section 3.2.2. This finding implies that lump-sum transfer adjustment, while leading to a higher level of aggregate output, is not necessarily a better policy response than labor tax rate adjustment in our model.

Compared to the labor tax rate adjustment, the results are different for consumption tax rate adjustment, as shown in panel (b) of Figure 4. In this case, the capital tax reform leads to a Pareto improvement as the unskilled workers also gain in terms of lifetime welfare. The main reason for this result is that the consumption of the unskilled goes up in the long-run, as we discussed in Section 3.2.2, due to a larger aggregate output effect compared to labor tax rate adjustment. While life-time welfare is higher for the unskilled workers, there is welfare loss in the short-run: $\mu_{C,t}$ for the unskilled becomes positive only 264 quarters after the onset of the capital tax reform.

[33] It thus measures welfare gains at the point when the agents are t quarters old.

[34] The lifetime welfare gains are presented by the dotted lines in the Figure 4.

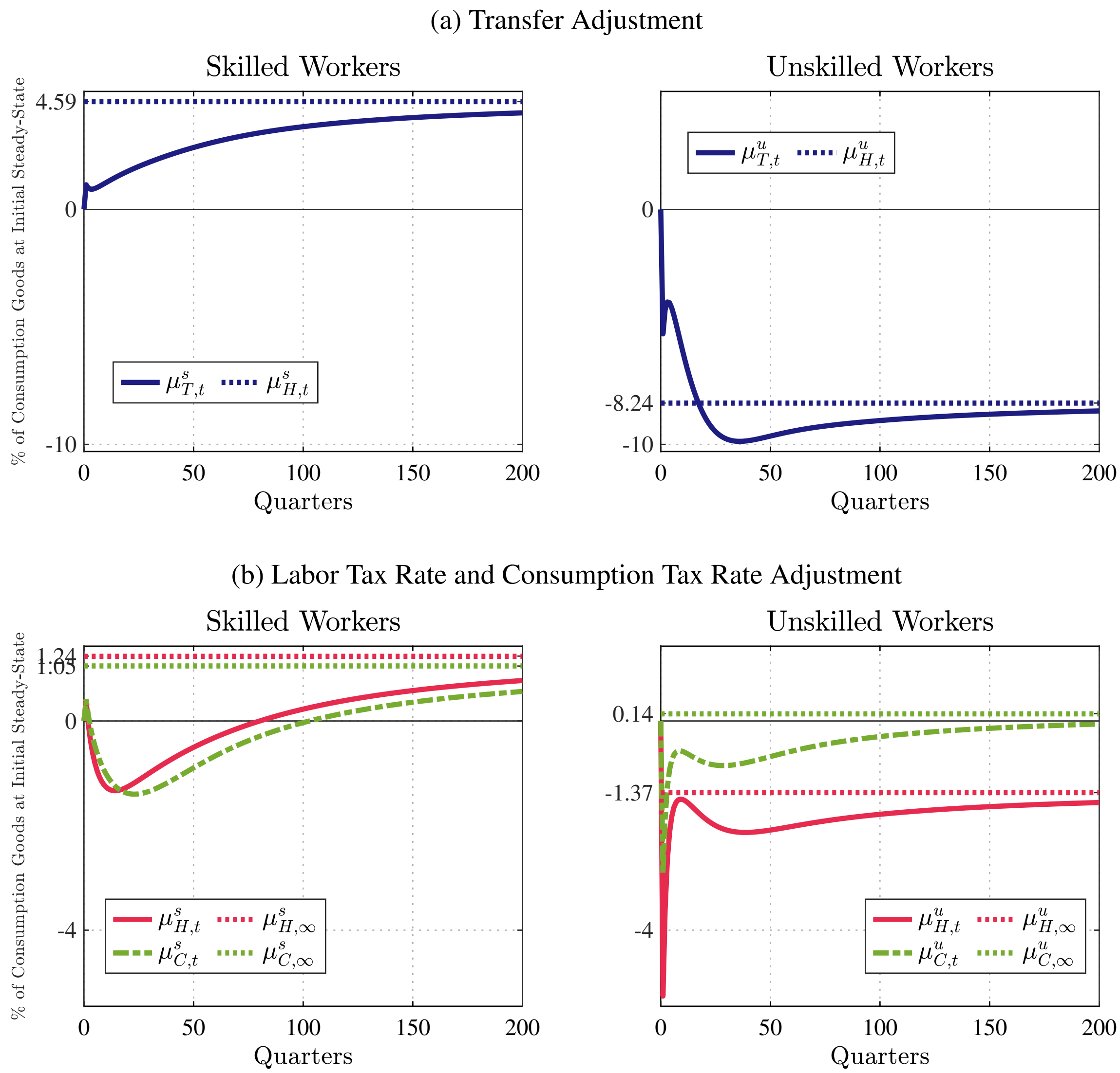

Figure 4: Welfare implications of a permanent capital tax rate decrease

6 Sensitivity analysis

Before concluding the paper, we present some important sensitivity analyses. All the results from this section are described in detail in Appendix E and we discuss them succinctly below.

6.1 Long-run results

We start with long-run effects. First, we present comparative statics results with respect to Frisch elasticity of labor supply. This is an important parameter, given that different sources of financing imply different labor supply responses. We show how a higher Frisch elasticity leads to larger output effects under transfer adjustments while the reverse holds under labor tax adjustments. Next, we compare the two fiscal adjustment cases varying the values of Frisch elasticity. Consistent with Proposition 3.1, the difference between the two cases is bigger for a higher Frisch elasticity.

As there are heterogeneous agents in our model, clearly the assumptions made on how profits and transfers are distributed across the two types of households makes a non-trivial difference for distributional variables. We show long-run results under various combinations of these distributions. For instance, if the skilled workers receive both the profits and (cut in) transfers, it leads to a decline in consumption inequality, in sharp contrast to the baseline case. The results also show that aggregate effects on output and investment however, are relatively similar across the various possibilities for profits and transfer distributions.

Finally, we do a sensitivity analysis on the equipment capital share parameter, λ. Our calibration strategy for this parameter is to match the labor share and now we vary the targeted labor share to both higher and lower values than the baseline. We find that the results are robust qualitatively overall and for aggregate variables, the quantitative differences are small. For consumption, there are some quantitative difference, as a smaller λ is more beneficial for the unskilled.

6.2 Transition dynamics

We next turn to the short-run analysis. We compare the transition dynamics under the labor tax and inflation adjustment finance scheme for different inflation feedback parameters in the Taylor rule (the inflation feedback parameter has to be below 1 in this regime). Our results are very robust. The differences across the parameterizations show up most clearly in inflation and debt dynamics, with a stronger Taylor rule coefficient in fact leading to a bigger effect on inflation dynamically. [35]

We then consider transition dynamics under transfer adjustment with two different rules for profit and transfer distributions, one the baseline and the other where the skilled workers get both the profits and (cut in) transfers. Again, as in the long-run, the differences are less prominent in output effects, but show up more prominently in distributional variables. For instance, consumption of the unskilled falls for a short-period only, whereas consumption of the skilled falls persistently, thereby leading consumption inequality to actually fall after a few periods. Moreover, the same dynamic pattern holds for wage inequality, which falls after a few periods.

6.3 Welfare results

We end with welfare results under various values of Frisch elasticity of labor supply. We show that our main finding that transfer and labor tax adjustments do not lead to a Pareto improvement, but consumption tax adjustment in fact does, is robust to both a higher and lower Frisch elasticity than our baseline value. Next, we conduct a sensitivity analysis on the equipment capital share

[35] This is consistent with the analytical results for the simple sticky price model in Bhattarai, Lee and Park (2014).

parameter, λ, and again find that our main finding that transfer and labor tax adjustments do not lead to a Pareto improvement, but consumption tax adjustment in fact does, is robust.

7 Conclusion

We study aggregate, distributional, and welfare effects of a permanent reduction in the capital tax rate in a quantitative equilibrium model with capital-skill complementarity and household heterogeneity. Such a tax reform leads to expansionary long-run aggregate output and investment effects, but those are coupled with increases in wage, consumption, and income inequality. The expansionary aggregate effects in the long-run are smaller when distortionary labor or consumption tax rates have to increase to finance the capital tax rate cut.

We study transition dynamics and show that there are contractionary effects in the short-run, with a fall not just in consumption of both the skilled and unskilled, but also aggregate output, and which is in turn coupled with increases in both wage and consumption inequality. The short-run contraction in consumption and output are more severe under distortionary tax rate adjustment. Importantly, we show that joint modeling of monetary and fiscal policy response is critical for analyzing short-run effects. In particular, when the government has access only to distortionary labor taxes, we consider the central bank directly accommodating inflation to facilitate government debt stabilization along the transition. In this interesting scenario, the government does not raise labor tax rates as much, and the rise of inflation in the short-run completely negates any short-run contraction in output as well as consumption.

Finally, we show that the capital tax cut has different welfare implications for each type of household depending on time horizon and policy adjustments. In the long-run, the tax reform increases the life-time utility of the skilled households under all the financing schemes considered, whereas it decreases the life-time utility of the unskilled households under lump-sum transfer and labor tax rate adjustments. The tax reform benefits the skilled households the most when transfers adjust, whereas the unskilled households prefer distortionary financing to avoid a significant reduction in transfer incomes. Importantly, we find that increasing consumption tax rate leads to a Pareto improvement. In the short-run, distortionary financing leads to short-term welfare losses for both the skilled and the unskilled.

We could extend our analysis in future work in a couple of directions. Our analysis of the short-run and the long-run suggests that the proposed tax reform will have heterogeneous effects on different generations. Thus, exploring generational heterogeneity is a particularly interesting avenue for future research. Introducing firm heterogeneity and financing constraints, in addition to household and skill heterogeneity we already incorporate, might also be an interesting avenue for research.

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Appendix (not for publication)

A Model with household heterogeneity

A.1 Stationary equilibrium

A.1.1 Notations

A balanced growth path can be achieved if $\frac{A_t}{A_{t-1}} = \bar{a} = \gamma^{1-\alpha}$and $\frac{q_t}{q_{t-1}} = \gamma_q = \frac{1}{\gamma}$, i.e. $\gamma_q \gamma = 1$. In this case, the growth rate of output $\frac{Y_t}{Y_{t-1}} = \gamma$.

$$\text{Quantities}: \tilde{C}_t^s = \frac{C_t^s}{\gamma^t}, \tilde{C}_t^u = \frac{C_t^u}{\gamma^t}, \tilde{I}_{b,t}^s = \frac{I_{b,t}^s}{\gamma^t}, \tilde{I}_{e,t}^s = \frac{I_{e,t}^s}{\gamma^t}, \tilde{K}_{b,t}^s = \frac{K_{b,t}^s}{\gamma^t}, \tilde{K}_{e,t}^s = \frac{K_{e,t}^s}{\left(\gamma_q\gamma\right)^t}$$

$$\tilde{Y}_t = \frac{Y_t}{\gamma^t}, \tilde{K}_{b,t} = \frac{K_{b,t}}{\gamma^t}, \tilde{K}_{e,t} = \frac{K_{e,t}}{\left(\gamma_q\gamma\right)^t}, \tilde{\hat{K}}_{b,t} = \frac{\hat{K}_{b,t}}{\gamma^t}, \tilde{\hat{K}}_{e,t} = \frac{\hat{K}_{e,t}}{\left(\gamma_q\gamma\right)^t}$$

$$\text{Prices}: \tilde{w}_t^s = \frac{W_t^s}{P_t\gamma^t}, \tilde{w}_t^u = \frac{W_t^u}{P_t\gamma^t}, r_t^{K,b} = \frac{R_t^{K,b}}{P_t}, r_t^{K,e} = \frac{R_t^{K,e}}{P_t\gamma^t}$$

$$\tilde{\Phi}_t = \frac{\Phi_t}{\gamma^t}, \tilde{\Phi}_t^i = \frac{\Phi_t^i}{\gamma^t}, \pi_t = \frac{P_t}{P_{t-1}}, \tilde{p}_t^* = \frac{P_t^*}{P_t}, mc_t = \frac{MC_t}{P_t}$$

$$\tilde{Z}_{1,t} = \frac{Z_{1,t}}{(P_t)^{\theta+1}\gamma^t}, \tilde{Z}_{2,t} = \frac{Z_{2,t}}{(P_t)^{\theta}\gamma^t}$$

$$\text{Fiscal variables}: \tilde{b}_t = \frac{B_t}{P_tY_t}, \tilde{G}_t = \frac{G_t}{Y_t}, \tilde{T}_t^C = \frac{T_t^C}{Y_t}, \tilde{T}_t^H = \frac{T_t^H}{Y_t}$$

$$\tilde{T}_t^{K,b} = \frac{T_t^{K,b}}{Y_t}, \tilde{T}_t^{K,e} = \frac{T_t^{K,e}}{Y_t}, \tilde{S}_t = \frac{S_t}{Y_t}, \tilde{S}_t^i = \frac{S_t^i}{Y_t}$$

$$\text{Multipliers}: \tilde{\Lambda}_t^i = \gamma^t P_t \Lambda_t^i, \tilde{\Psi}_{b,t}^i = \gamma^t \Psi_{b,t}^i, \tilde{\Psi}_{e,t}^i = \Psi_{e,t}^i$$

A.1.2 Stationary equilibrium conditions

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output. Given nonlinear equilibrium conditions, we detrended variables to specify stationary equilibrium conditions. All the notations are the same as before. We now state all the stationary equilibrium equations under incomplete markets.

- Production function:(Let $A_0 = 1$)

$$\tilde{Y}_t^A = \tilde{K}_{b,t}^{\alpha}\left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

- Aggregate output

$$\tilde{Y}_t^A = \tilde{Y}_t\Xi_t$$

- Cost minimization

$$r_t^{K,b} = \alpha mc_t \frac{\tilde{Y}_t^A}{\tilde{K}_{b,t}}$$

$$r_t^{K,e} = (1-\alpha) mc_t \frac{\tilde{Y}_t^A}{\tilde{K}_{e,t}} \left(\frac{(1-\mu)\left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{\lambda \tilde{K}_{e,t}^{\rho}}{\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}} \right)$$

$$\tilde{w}_t^u = (1-\alpha) mc_t \frac{\tilde{Y}_t^A}{L_{u,t}} \left(\frac{\mu L_{u,t}^{\sigma}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}} \right)$$

$$\tilde{w}_t^s = (1-\alpha) mc_t \frac{\tilde{Y}_t^A}{L_{s,t}} \left(\frac{(1-\mu)\left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{(1-\lambda) L_{s,t}^{\rho}}{\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}} \right)$$

- Skill-premium

$$\frac{\tilde{w}_t^s}{\tilde{w}_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{\tilde{K}_{e,t}}{L_{s,t}} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}}{L_{s,t}} \right)^{1-\sigma}$$

- Firms' maximization:

$$\tilde{Z}_{1,t} = mc_t \tilde{Y}_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{-\theta} E_t \left\{ \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{Z}_{1,t+1} (\pi_{t+1})^{\theta} \right\} \tag{A.1}$$

$$\tilde{Z}_{2,t} = \tilde{Y}_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{1-\theta} E_t \left\{ \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{Z}_{2,t+1} (\pi_{t+1})^{\theta-1} \right\} \tag{A.2}$$

- Price dispersion

$$\Xi_t = (1-\alpha_P)(\tilde{p}_t^*)^{-\theta} + \alpha_P \pi_t^{\theta} \left(\pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right)^{-\theta} \Xi_{t-1} \tag{A.3}$$

where

$$\tilde{p}_t^* = \frac{\theta}{\theta-1} \frac{\tilde{Z}_{1,t}}{\tilde{Z}_{2,t}}.$$

- Aggregate price index

$$\pi_t^{1-\theta} = (1-\alpha_P)(\pi_t \tilde{p}_t^*)^{1-\theta} + \alpha_P \left(\pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right)^{1-\theta}$$

- Profit

$$\tilde{\Phi}_t = \tilde{Y}_t - \tilde{w}_t^u L_{u,t} - \tilde{w}_t^s L_{s,t} - r_t^{K,b} \tilde{K}_{b,t} - r_t^{K,e} \tilde{K}_{e,t}$$

- Hand-to-mouth households

$$\tilde{w}_t^u \frac{1-\tau_t^H}{1+\tau_t^C} = \bar{\omega}^u \tilde{C}_t^u \left(H_t^u\right)^{\varphi}$$
$$\left(1+\tau_t^C\right)\tilde{C}_t^u = \left(1-\tau_t^H\right)\tilde{w}_t^u H_t^u + \tilde{\Phi}_t^u + \tilde{S}_t^u \tilde{Y}_t$$

- Skilled households
  - Marginal utilities:

$$\tilde{\Lambda}_t^s \left(1+\tau_t^C\right) = \frac{1}{\tilde{C}_t^s}$$
$$\tilde{\Lambda}_t^s \left(1-\tau_t^H\right)\tilde{w}_t^s = \bar{\omega}^s \left(H_t^s\right)^{\varphi}$$

  - Capacity utilization costs

$$\mathcal{A}_b\left(u_{e,t}\right) = \chi_{b,1}\left(u_{b,t}-1\right) + \frac{\chi_{b,2}^2}{2}\left(u_{b,t}-1\right)^2$$

$$\mathcal{A}_e\left(u_{e,t}\right) = \chi_{e,1}\left(u_{e,t}-1\right) + \frac{\chi_{e,2}^2}{2}\left(u_{e,t}-1\right)^2$$

  - FOCs and Capital Accumulation

$$\tilde{w}_t^s \frac{1-\tau_t^H}{1+\tau_t^C} = \bar{\omega}^s \tilde{C}_t^s \left(H_t^s\right)^{\varphi}$$

$$\tilde{\Lambda}_t^s = \frac{\beta}{\gamma} R_t E_t \left\{\tilde{\Lambda}_{t+1}^s \frac{1}{\pi_{t+1}}\right\} \tag{A.4}$$

$$\tilde{\Psi}_{b,t}^s = \frac{\beta}{\gamma} E_t \left\{(1-d_b)\tilde{\Psi}_{b,t+1}^s + \left[\left(1-\tau_{t+1}^K\right) r_{t+1}^{K,b} u_{b,t+1} - \left(1-\lambda_b \tau_{t+1}^K\right)\mathcal{A}_b\left(u_{b,t+1}\right)\right]\tilde{\Lambda}_{t+1}^s\right\} \tag{A.5}$$

$$\begin{aligned}\left(1-\lambda_b\tau_t^K\right)\tilde{\Lambda}_t^s = {} & \tilde{\Psi}_{b,t}^s \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) - \mathcal{S}'\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right)\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) \\ & + \frac{\beta}{\gamma} E_t \left\{\tilde{\Psi}_{b,t+1}^s \left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)\right\}\end{aligned}$$

$$\tilde{\Psi}_{e,t}^s = \beta E_t \left\{(1-d_e)\tilde{\Psi}_{e,t+1}^s + \left[\left(1-\tau_{t+1}^K\right) r_{t+1}^{K,e} u_{e,t+1} - \frac{1}{q_0}\left(1-\lambda_e \tau_{t+1}^K\right)\mathcal{A}_e\left(u_{e,t+1}\right)\right]\tilde{\Lambda}_{t+1}^s\right\} \tag{A.6}$$

$$\begin{aligned}\left(1-\lambda_e\tau_t^K\right)\frac{1}{q_0}\tilde{\Lambda}_t^s = {} & \tilde{\Psi}_{e,t}^s \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right) - \mathcal{S}'\left(\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right)\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right) \\ & + \frac{\beta}{\gamma} E_t \left\{\tilde{\Psi}_{e,t+1}^s \left(\frac{\tilde{I}_{e,t+1}^s}{\tilde{I}_{e,t}^s}\gamma\right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{e,t+1}^s}{\tilde{I}_{e,t}^s}\gamma\right)\right\}\end{aligned}$$

$$\gamma\tilde{\bar{K}}^s_{b,t+1} = (1-d_b)\,\tilde{\bar{K}}^s_{b,t} + \left(1-\mathcal{S}\left(\frac{\tilde{I}^s_{b,t}}{\tilde{I}^s_{b,t-1}}\gamma\right)\right)\tilde{I}^s_{b,t}$$

$$\tilde{\bar{K}}^s_{e,t+1} = (1-d_e)\,\tilde{\bar{K}}^s_{e,t} + \left(1-\mathcal{S}\left(\frac{\tilde{I}^s_{e,t}}{\tilde{I}^s_{e,t-1}}\gamma\right)\right)\tilde{I}^s_{e,t}q_0$$

$$\left(1-\tau^K_t\right)r^{K,b}_t = \left(1-\lambda_b\tau^K_t\right)\mathcal{A}'_b\left(u_{b,t}\right)$$

$$\left(1-\tau^K_t\right)r^{K,e}_t = \frac{1}{q_0}\left(1-\lambda_e\tau^K_t\right)\mathcal{A}'_e\left(u_{e,t}\right)$$

- Resource constraint

$$\begin{aligned}\left(1-\tilde{G}_t\right)\tilde{Y}_t &= N^s\left(\tilde{C}^s_t + \tilde{I}^s_{b,t} + \tilde{I}^s_{e,t} + \mathcal{A}_b\left(u_{b,t}\right)\tilde{\bar{K}}^s_{b,t} + \frac{1}{q_0}\mathcal{A}_e\left(u_{e,t}\right)\tilde{\bar{K}}^s_{e,t}\right) + N^u\tilde{C}^u_t \\ &= \tilde{C}_t + \tilde{I}_{b,t} + \tilde{I}_{e,t} + \mathcal{A}_b\left(u_{b,t}\right)\tilde{\bar{K}}_{b,t} + \frac{1}{q_0}\mathcal{A}_e\left(u_{e,t}\right)\tilde{\bar{K}}_{e,t} \qquad \text{(A.7)}\end{aligned}$$

- Market clearing

$$\tilde{C}_t = N^s\tilde{C}^s_t + N^u\tilde{C}^u_t$$

$$L_{u,t} = N^uH^u_t,\ L_{s,t} = N^sH^s_t$$

$$\tilde{K}_{b,t} = N^s\tilde{K}^s_{b,t},\ \tilde{K}_{e,t} = N^s\tilde{K}^s_{e,t}$$

$$\tilde{I}_{b,t} = N^s\tilde{I}^s_{b,t},\ \tilde{I}_{e,t} = N^s\tilde{I}^s_{e,t}$$

$$\tilde{K}^s_{b,t} = u^s_{b,t}\tilde{\bar{K}}^s_{b,t},\ \tilde{K}^s_{e,t} = u^s_{e,t}\tilde{\bar{K}}^s_{e,t}$$

- Government budget constraint

$$\tilde{b}_t + \tilde{T}^C_t + \tilde{T}^H_t + \tilde{T}^{K,b}_t + \tilde{T}^{K,e}_t = R_{t-1}\tilde{b}_{t-1}\frac{1}{\pi_t\gamma}\frac{\tilde{Y}_{t-1}}{\tilde{Y}_t} + \tilde{G}_t + \tilde{S}_t \qquad \text{(A.8)}$$

where

$$\tilde{T}^C_t = \tau^C\frac{\tilde{C}_t}{\tilde{Y}_t},\ \tilde{T}^H_t = \tau^H_t\sum_{i\in s,u}\left(\tilde{w}^i_t\frac{L_{i,t}}{\tilde{Y}_t}\right),$$

$$\tilde{T}^{K,b}_t = \tau^K_t\left(r^{K,b}_t\frac{\tilde{\bar{K}}_{b,t}}{\tilde{Y}_t}u_{b,t} - \lambda_b\left(\frac{\tilde{I}_{b,t}}{\tilde{Y}_t} + \mathcal{A}_b\left(u_{b,t}\right)\frac{\tilde{\bar{K}}_{b,t}}{\tilde{Y}_t}\right)\right),$$

$$\tilde{T}^{K,e}_t = \tau^K_t\left(r^{K,e}_t\frac{\tilde{\bar{K}}_{e,t}}{\tilde{Y}_t}u_{e,t} - \lambda_e\left(\frac{\tilde{I}_{e,t}}{\tilde{Y}_t} + \frac{1}{q_0}\mathcal{A}_e\left(u_{e,t}\right)\frac{\tilde{\bar{K}}_{e,t}}{\tilde{Y}_t}\right)\right)$$

- Fiscal Policy Rules:

$$\frac{S_t}{Y_t} - \frac{\bar{S}_{new}}{\bar{Y}_{new}} = \rho_1^S \left(\frac{S_{t-1}}{Y_{t-1}} - \frac{\bar{S}_{new}}{\bar{Y}_{new}}\right) + \rho_2^S \left(\frac{S_{t-2}}{Y_{t-2}} - \frac{\bar{S}_{new}}{\bar{Y}_{new}}\right) \tag{A.9}$$

$$+ \left(1 - \rho_1^S - \rho_2^S\right) \left\{ \psi_B^S \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^S \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^S \left(\frac{Y_t}{Y_t^n} \right) \right\},$$

$$\tau_t^H - \bar{\tau}_{new}^H = \rho_1^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H \left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right) \tag{A.10}$$

$$+ \left(1 - \rho_1^H - \rho_2^H\right) \left\{ \psi_B^H \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^H \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^H \left(\frac{Y_t}{Y_t^n} \right) \right\},$$

$$\tau_t^C - \bar{\tau}_{new}^C = \rho_1^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \rho_2^C \left(\tau_{t-2}^C - \bar{\tau}_{new}^C\right) = \tag{A.11}$$

$$+ \left(1 - \rho_1^C - \rho_2^C\right) \left\{ \psi_B^C \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^C \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^C \left(\frac{Y_t}{Y_t^n} \right) \right\}$$

$$\tau_t^K = \begin{cases} \bar{\tau}^K & \text{if } t = 0 \\ \bar{\tau}_{New}^K & \text{if } t > 0 \end{cases} \tag{A.12}$$

$$\tilde{G}_t = \bar{\tilde{G}} \tag{A.13}$$

- Monetary policy

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_1^R} \left[\frac{R_{t-2}}{\bar{R}}\right]^{\rho_2^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_x} \right]^{\left(1-\rho_1^R-\rho_2^R\right)} \tag{A.14}$$

- Government debt to GDP (or transfers):

$$\tilde{S}_t = \bar{\tilde{S}} \text{ if adjusting labor tax rate} \tag{A.15}$$

A.2 Steady state

Recall that in steady-state, $\mathcal{S}(\gamma) = \mathcal{S}'(\gamma) = 0$:

From (A.1)–(A.3), we get

$$\bar{mc} = \frac{\theta - 1}{\theta}.$$

From skilled workers' optimal capital investment decisions, we get

$$\bar{r}^{K,b} = \left(\frac{\gamma}{\beta} - (1 - d_b)\right) \frac{1 - \lambda_b \bar{\tau}^K}{1 - \bar{\tau}^K}$$

$$\bar{r}^{K,e} = \frac{1}{q_0} \left(\frac{1}{\beta} - (1 - d_e)\right) \frac{1 - \lambda_e \bar{\tau}^K}{1 - \bar{\tau}^K}$$

From the production function, we get

$$\frac{\bar{\tilde{Y}}^A}{\bar{L}_s} = \frac{\bar{\tilde{Y}}}{\bar{L}_s} = \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^\alpha \left[\mu \left(\frac{\bar{L}_u}{\bar{L}_s}\right)^\sigma + (1 - \mu) \left(\lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^\rho + (1 - \lambda) \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1-\alpha}{\sigma}}$$

and

$$\bar{\Xi} = 1.$$

From firms' optimality conditions, we get

$$\bar{\tilde{w}}^s = (1-\lambda)(1-\mu)(1-\alpha)\bar{mc}\left(\frac{\bar{r}^{K,b}}{\alpha\bar{mc}}\right)^{1-\frac{\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}$$

$$\bar{\tilde{w}}^u\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{1-\sigma} = \mu(1-\alpha)\bar{mc}\left(\frac{\bar{r}^{K,b}}{\alpha\bar{mc}}\right)^{\frac{1-\alpha-\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma}$$

$$\bar{r}^{K,e}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{1-\rho} = \lambda(1-\mu)(1-\alpha)\bar{mc}\left(\frac{\bar{r}^{K,b}}{\alpha\bar{mc}}\right)^{1-\frac{\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}$$

$$\left(\frac{\bar{r}^{K,b}}{\alpha\bar{mc}}\right) = \left(\frac{\tilde{K}_b}{\bar{L}_s}\right)^{\alpha-1}\left[\mu\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma}+(1-\mu)\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

Also, from skilled-household's budget constraint, we have budget constraint

$$\begin{aligned}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}^s}{\bar{\tilde{Y}}} &= \left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)\bar{\tilde{w}}^s\frac{\bar{H}^s}{\bar{\tilde{Y}}} \\ &+\left(\left(1-\bar{\tau}^K\right)\bar{r}^{K,b}-\left(1-\lambda_b\bar{\tau}^K\right)(\gamma-(1-d_b))\right)\frac{\bar{\tilde{\hat{K}}}^s_b}{\bar{\tilde{Y}}} \\ &+\left(\left(1-\bar{\tau}^K\right)\bar{r}^{K,e}-\left(1-\lambda_e\bar{\tau}^K\right)\frac{d_e}{q_0}\right)\frac{\bar{\tilde{\hat{K}}}^s_e}{\bar{\tilde{Y}}} \\ &+\left(\frac{1}{\beta}-1\right)\bar{\tilde{b}}^s+\frac{\chi^s_\Phi}{N^s}\frac{\tilde{\Phi}_t}{\bar{\tilde{Y}}}+\frac{\chi^s_S}{N^s}\tilde{S}_t\end{aligned}$$

where the profit is

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1-\bar{\tilde{w}}^u\frac{\bar{L}_u}{\bar{\tilde{Y}}}-\bar{\tilde{w}}^s\frac{\bar{L}_s}{\bar{\tilde{Y}}}-\bar{r}^{K,b}\frac{\bar{\tilde{K}}_b}{\bar{\tilde{Y}}}-\bar{r}^{K,e}\frac{\bar{\tilde{K}}_e}{\bar{\tilde{Y}}}$$

and the transfer is

$$\bar{\tilde{S}} = \bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\left(\sum_{i\in\{s,u\}}\lambda^i_{\tau^H}\bar{\tilde{w}}^i\frac{\bar{L}_i}{\bar{\tilde{Y}}}\right)+\bar{\tau}^K\bar{r}^{K,b}\left(\frac{\bar{\tilde{\hat{K}}}_b}{\bar{\tilde{Y}}}-\lambda_b\frac{\bar{\tilde{I}}_b}{\bar{\tilde{Y}}}\right)+\bar{\tau}^K\bar{r}^{K,e}\left(\frac{\bar{\tilde{\hat{K}}}_e}{\bar{\tilde{Y}}}-\lambda_e\frac{\bar{\tilde{I}}_e}{\bar{\tilde{Y}}}\right)-\left(\left(\frac{1}{\beta}-1\right)\bar{\tilde{b}}+\bar{\tilde{G}}\right).$$

From (A.5) and (A.6) for both types of households, we get

$$\begin{aligned}(\gamma-(1-d_b))\frac{\bar{\tilde{\hat{K}}}^s_b}{\bar{H}^s} &= \frac{\bar{\tilde{I}}^s_b}{\bar{H}^s} \\ d_e\frac{\bar{\tilde{\hat{K}}}^s_e}{\bar{H}^s} &= \frac{\bar{\tilde{I}}^s_e}{\bar{H}^s}q_0 \\ (\gamma-(1-d_b))\frac{\bar{\tilde{\hat{K}}}^u_b}{\bar{H}^u} &= \frac{\bar{\tilde{I}}^u_b}{\bar{H}^u} \\ d_e\frac{\bar{\tilde{\hat{K}}}^u_e}{\bar{H}^u} &= \frac{\bar{\tilde{I}}^u_e}{\bar{H}^u}q_0\end{aligned}$$

From household's intra-temporal Euler equations, we have

$$\bar{\tilde{w}}^u \frac{1-\bar{\tau}^H}{1+\bar{\tau}^C} = \bar{\omega}^u \bar{\tilde{C}}^u \left(\bar{H}^u\right)^{\varphi}$$
$$\left(1+\bar{\tau}^C\right)\bar{\tilde{C}}^u = \left(1-\bar{\tau}^H\right)\bar{\tilde{w}}^u \bar{H}^u + \bar{\tilde{\Phi}}^u + \bar{\tilde{S}}^u \bar{\tilde{Y}}$$

$$\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}\bar{\tilde{w}}^s = \bar{\omega}^s \bar{\tilde{C}}^s \left(\bar{H}^s\right)^{\varphi}$$

$$\Lambda_{s,u} = \frac{\bar{\tilde{\Lambda}}^s}{\bar{\tilde{\Lambda}}^u} = \frac{\bar{\tilde{C}}^u}{\bar{\tilde{C}}^s}$$

$$\bar{\tilde{C}}^s = \left(\frac{1}{N^u \Lambda_{s,u} + N^s}\right)\bar{\tilde{C}}$$

$$\bar{\tilde{C}}^u = \left(\frac{\Lambda_{s,u}}{N^u \Lambda_{s,u} + N^s}\right)\bar{\tilde{C}}$$

From the market clearing conditions, we have

$$\frac{\bar{\tilde{K}}_b}{\bar{L}_s} = \frac{\bar{\tilde{K}}_b^s}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s}\frac{\bar{\tilde{K}}_b^u}{\bar{H}^u}$$
$$\frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \frac{\bar{\tilde{K}}_e^s}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s}\frac{\bar{\tilde{K}}_e^u}{\bar{H}^u}$$

From (A.7), we get

$$\frac{\bar{\tilde{C}}}{\bar{L}_s} + (\gamma - (1-d_b))\frac{\bar{\tilde{K}}_b}{\bar{L}_s} + \frac{d_e}{q_0}\frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \left(1-\tilde{G}\right)\frac{\bar{\tilde{Y}}}{L_s}$$

The nominal interest rate is obtained from Euler equation (A.4)

$$\bar{R} = \frac{\gamma\bar{\pi}}{\beta}.$$

We fix steady-state hours for skilled labor $\bar{H}^s = 0.33$ by assuming the skilled works 40 hours per week and $\bar{H}^u = 0.93 * \bar{H}^s$(Skilled workers work 7% more than low-skilled worker).

B Additional Analytical Results and Proofs of Propositions

B.1 Steady-state equilibrium equations for a nested version of the model

We assume $\mu = 0$, $\alpha = 0$, and $\rho \to 0$ to get a nested version of the model with a Cobb-Douglas production function. In this case, to have balanced growth, the growth rate of output is the same with the growth rate of technology,$\gamma = \bar{a}$, and the growth rate of relative price (γ_q) is 1. Let $q_0 = 1$. Also, let the fraction of skilled workers $N^S = 1$ and set $\chi^s_\Phi = 1$ and $\chi^s_S = 1$ for profit and transfers distributions. Then, in this economy, we have one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$. We derive steady-state equilibrium equations for this nested version of the model and drop subscripts e and s.

- Marginal cost

$$\bar{mc} = \frac{\theta - 1}{\theta}.$$

- Capital rental rate

$$\bar{r}^K = \frac{\frac{\bar{a}}{\beta} - (1-d)}{1-\bar{\tau}^K} \tag{B.1}$$

- Production function

$$\frac{\bar{\tilde{Y}}}{\bar{H}} = \left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda} \tag{B.2}$$

- Wages and capital-to-labor ratio

$$\frac{\bar{\tilde{K}}}{\bar{H}} = \left(\frac{\bar{r}^K}{\lambda \bar{mc}}\right)^{\frac{1}{\lambda-1}} \tag{B.3}$$

$$\begin{aligned}\bar{\tilde{w}} &= (1-\lambda)\bar{mc}\left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda} \\ &= (1-\lambda)(\lambda)^{\frac{\lambda}{1-\lambda}}(\bar{mc})^{\frac{1}{1-\lambda}}\left(\bar{r}^K\right)^{\frac{\lambda}{\lambda-1}}\end{aligned} \tag{B.4}$$

- Resource constraint

$$\frac{\bar{\tilde{C}}}{\bar{H}} = \left(1-\bar{\tilde{G}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}. \tag{B.5}$$

- Profit

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1 - \bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}} - \bar{r}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}$$

- Transfer

$$\bar{\tilde{S}} = \left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^C + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K. \tag{B.6}$$

- The consumption, labor income and capital income tax rates are respectively given as:

$$\bar{\tau}^C = \frac{\bar{\tilde{T}}^C}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}, \ \bar{\tau}^H = \frac{1}{\bar{\tilde{w}}} \frac{\bar{\tilde{T}}^H}{\frac{\bar{H}}{\bar{\tilde{Y}}}}, \ \bar{\tau}^K = \frac{1}{\bar{r}^K} \frac{\bar{\tilde{T}}^K}{\frac{\bar{K}}{\bar{\tilde{Y}}}}.$$

- Intra-temporal Euler equation

$$\bar{H} = \left(\frac{1}{\bar{\omega}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} \frac{\bar{\tilde{C}}}{\bar{H}} \frac{1}{\bar{\tilde{w}}} \right)} \right)^{\frac{1}{1+\varphi}}. \tag{B.7}$$

- Investment

$$\frac{\bar{\tilde{I}}}{\bar{H}} = \frac{\bar{\tilde{K}}}{\bar{H}} (\bar{a} - (1-d)). \tag{B.8}$$

- Nominal interest rate

$$\bar{R} = \frac{\bar{a}\bar{\pi}}{\beta}.$$

B.2 Analytical results with capital-skill complementarity

The skill premium can be expressed as follows:

$$\begin{aligned}\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u} &= \frac{(1-\lambda)(1-\mu)}{\mu} \frac{\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1}}{\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma-1}} \\ &= \frac{(1-\lambda)(1-\mu)}{\mu} \chi_c^{\frac{1-\sigma}{\sigma}} (F)^{\frac{\sigma-\rho}{\rho}\left(\frac{\varphi}{1-\sigma+\varphi}\right)}\end{aligned}$$

where $\chi_c = \left(\frac{(1-\lambda)(1-\mu)}{\mu} \frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}} \frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)} \right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0$ and $F\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) = \lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda) > 0$. Then,

$$\begin{aligned}\frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u}\right) &= (\sigma-\rho)\left(\frac{\varphi}{1-\sigma+\varphi}\right) \frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u} \frac{1}{F} \lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1} \frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\ &< 0.\end{aligned}$$

B.3 Lump-sum transfer adjustment

We start with the case where lump-sum transfers adjust to finance the capital tax rate cut. It is useful to state a mild restriction on government spending in steady-state.[36]

Assumption 1. $\bar{\tilde{G}} < 1 - \frac{\theta-1}{\theta}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^K\right) = 1 - \frac{1}{\lambda}\left(\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}\right)$ *in the initial steady-state.*

[36] This restriction is very mild, and is just to ensure that government spending in steady-state is not very high. For instance, except for a case of an unrealistically high markup, this holds for any reasonable parameterization of government spending in steady-state.

Then, we can formally show that a permanent capital tax rate cut leads to an increase in output, consumption, investment, and wages, and a decline in the rental rate of capital, as given in Proposition B.1.

Proposition B.1. *Fix $\bar{\tau}^H$ and $\bar{\bar{b}}$. With lump-sum transfer adjustment,*

1. Rental rate of capital is increasing, while capital to hours ratio, wage, hours, capital, investment, and output are decreasing in $\bar{\tau}^K$.

2. Under Assumption 1, consumption is also decreasing in $\bar{\tau}^K$.

Proof. See Appendix B.4. □

B.4 Proof of Proposition B.1

Proof. From (B.1) and (B.4), we get

$$\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} = \frac{\bar{r}^K}{1-\bar{\tau}^K} > 0$$
$$\frac{\partial \bar{\bar{w}}}{\partial \bar{\tau}^K} = -\left(\frac{\bar{\bar{w}}}{\bar{r}^K}\right)\left(\frac{\lambda}{1-\lambda}\right)\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0.$$

Let $\bar{\bar{k}} = \frac{\bar{\bar{K}}}{\bar{H}}$ and $\bar{\bar{y}} = \frac{\bar{\bar{Y}}}{\bar{H}}$. From (B.2) and (B.3), we get

$$\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} = -\frac{\bar{\bar{k}}}{\bar{r}^K}\frac{1}{1-\lambda}\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0$$
$$\frac{\partial \bar{\bar{y}}}{\partial \bar{\tau}^K} = \lambda\left(\frac{\bar{\bar{y}}}{\bar{\bar{k}}}\right)^{\frac{1}{\varepsilon}}\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} < 0$$

Combining (B.4) and (B.5) with (B.7), we rewrite the steady-state hours as

$$\bar{H} = \left(\bar{\omega}\left(\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\bar{G}}}{\bar{mc}} - \lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\right)\right)^{-\frac{1}{1+\varphi}}.$$

Then, the partial derivative with respect to capital tax rate is

$$\frac{\partial \bar{H}}{\partial \bar{\tau}^K} = -\frac{\bar{H}^{2+\varphi}}{1+\varphi}\left(\bar{\omega}\frac{\lambda}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right) < 0.$$

Now, we find the partial derivatives of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\partial \bar{\bar{K}}}{\partial \bar{\tau}^K} = \bar{H}\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} + \bar{\bar{k}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0$$
$$\frac{\partial \bar{\bar{I}}}{\partial \bar{\tau}^K} = \frac{\partial \bar{\bar{K}}}{\partial \bar{\tau}^K}(\bar{a}-(1-d)) < 0$$
$$\frac{\partial \bar{\bar{Y}}}{\partial \bar{\tau}^K} = \bar{H}\frac{\partial \bar{\bar{y}}}{\partial \bar{\tau}^K} + \bar{\bar{y}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0$$

For consumption, combining (B.2) and (B.8) with (B.5), we get

$$
\begin{aligned}
\bar{\tilde{C}} &= \left(\left(1-\bar{\tilde{G}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}\right)\bar{H} \\
&= \left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right) - \lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}.
\end{aligned}
$$

Then, the partial derivative of consumption with respect to capital tax rate is

$$
\frac{\partial\bar{\tilde{C}}}{\partial\bar{\tau}^K} = -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\left(\frac{\lambda\bar{mc}\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right) - \bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H} + \frac{\bar{\tilde{C}}}{\bar{H}}\frac{\partial\bar{H}}{\partial\bar{\tau}^K}.
$$

Under Assumption 1, we find $\frac{\partial\bar{\tilde{C}}}{\partial\bar{\tau}^K} < 0$. □

B.5 Proof of Proposition 3.1

Proof. From (B.1), (B.3), and (B.4), we get

$$
\begin{aligned}
\frac{\bar{\tilde{K}}}{\bar{H}} &= \frac{\bar{\tilde{w}}}{\bar{r}^K}\frac{\lambda}{1-\lambda} \\
&= \left(\frac{\lambda\bar{mc}}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)^{\frac{1}{1-\lambda}}.
\end{aligned}
$$

The amount of changes in capital to hours ratio to the capital tax cut is the same in both lump-sum transfers adjustment case and labor tax rate case. In a similar way, we know that output to hours ratio, investment to hours ratio, and consumption to hours ratio change by the same amount in both cases. Thus, all the magnitudes of changes in macro quantities to capital tax cuts are determined by the hours responses. Now, we compare the changes in hours to capital tax rate changes under the transfers adjustment case with the changes under the labor tax rate adjustment case. Notice that the initial steady-states are the same in both cases. Let $\bar{H}^T_{new}$ and $\bar{H}^L_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the labor tax rate adjustment case, respectively. Then, from (B.7), we get

$$
\begin{aligned}
\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H_{new}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1-\frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}
$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$\begin{aligned}\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) &= -\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\Delta\left(\bar{\tau}^K\right)\\ &= -\Theta\Delta\left(\bar{\tau}^K\right)\end{aligned}$$

where $\Theta = \frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0$. Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$\ln\left(\frac{\bar{\tilde{Y}}^T_{new}}{\bar{\tilde{Y}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^T_{new}}{\bar{\tilde{C}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^T_{new}}{\bar{\tilde{K}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^T_{new}}{\bar{\tilde{I}}^L_{new}}\right) = \ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) = -\Theta\Delta\left(\bar{\tau}^K\right).$$

Notice that $\frac{\partial \ln\left(\frac{\bar{\tilde{Y}}^T_{new}}{\bar{\tilde{Y}}^L_{new}}\right)}{\partial \bar{\tau}^H} = -\frac{\Theta}{1-\bar{\tau}^H}\Delta\left(\bar{\tau}^K\right)$ implying that the gap is increasing in the initial level of $\bar{\tau}^H$ when there is a capital tax cut.

Now, let $\bar{H}^T_{new}$ and $\bar{H}^C_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the consumption tax rate adjustment case, respectively. Then, we get

$$\begin{aligned}\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C_{new}}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}}\\ &= \left(1+\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}\right)^{\frac{1}{1+\varphi}}\\ &= \left(1-\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\frac{\tilde{\Delta}\left(\bar{\tau}^K\right)}{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\tilde{\Delta}\left(\bar{\tau}^K\right)}\right)^{\frac{1}{1+\varphi}}\end{aligned}$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}}\right) = -\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)$$

Then, for the levels of output, consumption, capital and investment, the differences are the same:

that is,

$$\ln\left(\frac{\bar{\bar{Y}}^T_{new}}{\bar{\bar{Y}}^C_{new}}\right)=\ln\left(\frac{\bar{\bar{C}}^T_{new}}{\bar{\bar{C}}^C_{new}}\right)=\ln\left(\frac{\bar{\bar{K}}^T_{new}}{\bar{\bar{K}}^C_{new}}\right)=\ln\left(\frac{\bar{\bar{I}}^T_{new}}{\bar{\bar{I}}^C_{new}}\right)=\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}}\right)$$
$$=\frac{1}{1+\varphi}\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}$$
$$=-\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\bar{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)$$
$$=-\mathcal{M}^T_C\Delta\left(\bar{\tau}^K\right).$$

Then, $\mathcal{M}^T_C=\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\bar{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)>0$ if

$$\bar{\bar{G}}<1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)$$
$$\approx 1-0.15=0.85$$

in our calibration. In this case, capital tax cut is more expansionary in the transfer adjustment case than in the consumption tax rate adjustment case.

Notice that $\frac{\partial\ln\left(\frac{\bar{\bar{Y}}^T_{new}}{\bar{\bar{Y}}^C_{new}}\right)}{\partial\bar{\tau}^C}=\frac{1}{1+\varphi}\left(\frac{1-\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{\left(1+\bar{\tau}^C\right)^2}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\bar{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)$ implying that the gap is decreasing in the initial level of $\bar{\tau}^C$ when there is a capital tax cut.

Lastly, let $\bar{H}^L_{new}$ and $\bar{H}^C_{new}$ denote the steady-state hours after the capital tax changes in the labor tax adjustment case and in the consumption tax adjustment case, respectively. Then, we get

$$\frac{\bar{H}^L_{new}}{\bar{H}^C_{new}}=\left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H_{new}}\left(1-\bar{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C_{new}}{1-\bar{\tau}^H}\left(1-\bar{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}}$$
$$=\left(\frac{1+\bar{\tau}^C_{new}}{1+\bar{\tau}^C}\frac{1-\bar{\tau}^H_{new}}{1-\bar{\tau}^H}\right)^{\frac{1}{1+\varphi}}$$
$$=\left(\left(1+\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}\right)\left(1-\frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right)\right)^{\frac{1}{1+\varphi}}$$
$$=\left(\left(1-\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1+\bar{\tau}^C}\frac{\lambda\bar{mc}}{\left(1-\bar{\bar{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\lambda}{1-\lambda}\Delta\left(\bar{\tau}^K\right)\right)\right)^{\frac{1}{1+\varphi}}$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$
\begin{aligned}
\ln\left(\frac{\bar{H}^L_{new}}{\bar{H}^C_{new}}\right) &= \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C} - \frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right) \\
&= \frac{1}{1+\varphi}\left(1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C\right)\left(\frac{1}{1-\bar{\tau}^H}\frac{\lambda}{1-\lambda} - \frac{1}{1+\bar{\tau}^C}\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}\right)\left(1-\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}\frac{(1-\lambda)\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)
\end{aligned}
$$

Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}^L_{new}}{\bar{\tilde{Y}}^C_{new}}\right) &= \ln\left(\frac{\bar{\tilde{C}}^L_{new}}{\bar{\tilde{C}}^C_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^L_{new}}{\bar{\tilde{K}}^C_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^L_{new}}{\bar{\tilde{I}}^C_{new}}\right) = \ln\left(\frac{\bar{H}^L_{new}}{\bar{H}^C_{new}}\right) = \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C} - \frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}\right)\left(1-\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}\frac{(1-\lambda)\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right) \\
&= \mathcal{M}^L_C\Delta\left(\bar{\tau}^K\right).
\end{aligned}
$$

Notice that $\mathcal{M}^L_C = \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}\right)\left(1-\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}\frac{(1-\lambda)\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right) > 0$ if

$$
\begin{aligned}
\bar{\tilde{G}} &< 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)-(1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C} \\
&\approx 1-0.4976 = 0.5023
\end{aligned}
$$

in our baseline calibration.

To see that labor tax adjustment is more distortionary than consumption tax rate adjustment case, we can combine Equations (B.5), (B.6), and (B.7) to derive

$$
\begin{aligned}
\bar{H} &= \left(\bar{\omega}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{\tilde{C}}}{\bar{H}}\frac{1}{\bar{\tilde{w}}}\right)\right)^{-\frac{1}{1+\varphi}} \\
&= \left(\bar{\omega}\left(\frac{1}{1-\bar{\tau}^H}\left(\left(1-\left(1-\frac{1}{\beta}\right)\bar{\tilde{b}}+\bar{\tilde{S}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}}-\bar{\tau}^H\bar{\tilde{w}}-\bar{\tau}^K\bar{r}^K\frac{\bar{\tilde{K}}}{\bar{H}}-\frac{\bar{\tilde{I}}}{\bar{H}}\right)\frac{1}{\bar{\tilde{w}}}\right)\right)^{-\frac{1}{1+\varphi}} \\
&= \left(\bar{\omega}\left(\frac{1}{1-\bar{\tau}^H}\frac{1}{(1-\lambda)}\left[\frac{1-\left(1-\frac{1}{\beta}\right)\bar{\tilde{b}}+\bar{\tilde{S}}}{\bar{mc}}-(1-\lambda)\bar{\tau}^H-\lambda\bar{\tau}^K-\frac{\lambda(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}
$$

When $\beta = 1$, then we get

$$\bar{H} = \left(\bar{\omega}\left(\frac{1-\bar{mc}+\bar{\tilde{S}}}{\bar{mc}}\frac{1}{(1-\lambda)(1-\bar{\tau}^H)}+1\right)\right)^{-\frac{1}{1+\varphi}}.$$

□

C Data appendix

We calibrate the steady-state fiscal variables using US quarterly data for the post-Volcker period from 1982:Q4 to 2008:Q2.

C.1 Debt and spending data

We use the following definitions for our debt and spending variables:

- Government debt = market value of privately held gross federal debt;
- Government expenditures = government consumption;

Note that we use a single price level, GDP deflator, for both variables.

The market value of privately held gross federal debt series was obtained from Federal Reserve Bank of Dallas and the government consumption data series was taken from National Income and Product Accounts (NIPA) tables.

C.2 Tax data

We follow a method originally based on Jones (2002). Additionally, we use the tax revenues of the federal government and local property taxes.

We use federal taxes on production and imports (lines 4 of NIPA Table 3.2) for consumption tax revenues. Let this be T^C.

The average personal income tax rate is computed to get both capital tax revenues and labor tax revenues. We first compute the average personal income tax rate as

$$\tau^P = \frac{IT}{W + PRI/2 + CI}$$

where IT is the personal current tax revenues (line 3 of NIPA Table 3.2), W is wage and salary accruals (line 3 of NIPA Table 1.12), PRI is proprietor's income (line 9 of NIPA Table 1.12), and CI is capital income, which is the sum of rental income (line 12 of NIPA Table 1.12), corporate profits (line 13 of NIPA Table 1.12), interest income (line 18 of NIPA Table 1.12), and $PRI/2$. We here regard half of proprietor's income as wage labor income and the other half as capital income.

Then the capital tax revenue is

$$T^K = \tau^P CI + CT + PT$$

where CT is taxes on corporate income (line 7 of NIPA Table 3.2), and PT is property taxes (line 8 of NIPA Table 3.3). In NIPA, home owners are thought of as renting their houses to themselves and thus property taxes are included as taxes on rental income or capital income. The labor tax revenue is computed

$$T^H = \tau^P (W + PRI/2) + CSI$$

where CSI is contributions for government social insurance (line 11 of NIPA Table 3.2).

C.3 Skill Premium in the CPS Data

Following Lindquist 2004, I use the Monthly Outgoing Rotation Group (MORG) of the US Census Bureau's Current Population Survey data and check the share of the skilled and the unskilled and also the skill premium. The skill premium is defined as the ratio of the hourly wage of workers with 14 or more years of schooling to the hourly wage of workers with less than 14 years of schooling. We restrict the sample with age between 20 and 70. The share of workers with 14 or more years of schooling is 0.505 and the skill premium for mean wage is 66% and the skill premium for median wage is 55%.

D Estimation of labor tax adjustment rule

The labor tax rate adjustment rule is specified as the following:

$$\begin{aligned}\tau_t^H - \bar{\tau}_{new}^H &= \rho_1^H\left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H\left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right)\\ &\quad + \left(1-\rho_1^H-\rho_2^H\right)\left\{\psi_B^H\left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right) + \psi_{\Delta Y}^H\left(\frac{Y_t}{Y_{t-1}}\right) + \psi_x^H\left(\frac{Y_t}{Y_t^n}\right)\right\}\end{aligned} \tag{D.1}$$

where Y_t^n is the natural level of output. We estimate an empirical version of this rule by OLS. We first estimate the composite coefficients $\left(1-\rho_1^H-\rho_2^H\right)\psi_B^H$, $\left(1-\rho_1^H-\rho_2^H\right)\psi_{\Delta Y}^H$, and $\left(1-\rho_1^H-\rho_2^H\right)\psi_x^H$ and then recover ψ_B^H, $\psi_{\Delta Y}^H$, and ψ_x^H using the estimate of ρ_1^H and ρ_2^H. Quarterly US data is used for estimation: tax revenues-to-output ratio, market value of government debt-to-output ratio, output growth and the gap between actual output and potential output. The rule is estimated on two sub-periods. The first sub-sample covers the period from 1983Q1 through 2002Q4 as in Coibion and Gorodnichenko (2011) and the second sub-sample covers the period from 2001Q1 through 2019Q3. For the first sub-sample, we drop the second lag of the tax revenues to ensure stationarity of the tax rule. The data is taken from FRED of the Federal Reserve Bank of St. Louis. Potential output is real potential gross domestic product estimated by US Congressional Budget Office. Table F.1 shows the estimation results.

E Sensitivity analysis

We present some additional results on sensitivity analysis and extensions.

E.1 Long-run results

We start with long-run effects. First, we present comparative statics results with respect to Frisch elasticity of labor supply. This is an important parameter, given that different source of financing imply different labor supply response. Appendix Figures F.2 and F.3 show how with transfer adjustment higher Frisch elasticity leads to larger output effects while the reverse holds for labor tax adjustment. Next, in Appendix Figures F.4 and F.5, we compare across the three fiscal adjustments for a given Frisch elasticity. Consistent with Proposition 3.1, the difference between transfer adjustment case and labor tax adjustment case is bigger for a higher Frisch elasticity.

As there are heterogeneous agents in our model, clearly the assumptions made on how profits and transfers are distributed across the two types of households makes a non-trivial difference for distributional variables. Appendix Figure F.6 shows long-run results under various combinations of these distributions. For instance, if the skilled workers get both the profits and (cut in) transfers, it leads to a decline in consumption inequality, in sharp contrast to the baseline case. The results also show that aggregate effects on output and investment however, are relatively similar across the various possibilities for profits and transfer distributions.

We also present results on the equipment capital share parameter, λ. in Appendix Figures F.7 and F.8. Our calibration strategy for this parameter is to match the labor share and now we vary the targeted labor share to both higher and lower values than baseline. We find that the results are robust qualitatively overall and for aggregate variables, the quantitative differences are small. For consumption, there are some quantitative difference, as a smaller λ is more beneficial for the unskilled.

E.2 Transition dynamics

We next move to transition dynamics. Appendix Figure F.9 compares the transition dynamics in the baseline model under the labor tax and inflation adjustment finance scheme for different inflation feedback parameters in the Taylor rule (the inflation feedback parameter has to be below 1 in this regime). Our results are very robust. The differences across the parameterizations show up most clearly in inflation and debt dynamics, with a stronger Taylor rule coefficient in fact leading to a bigger effect on inflation dynamically. This is consistent with the analytical results for the simple sticky price model in Bhattarai, Lee and Park (2014).

We then show transition dynamics under transfers adjustment with two different rules for profit and transfer distributions, one the baseline and the other where the skilled workers get both the profits and (cut in) transfers. Again, like with the long-run, Appendix Figure F.10 shows that the differences are less prominent in output effects, but show up more prominently in distributional variables. For instance, consumption of unskilled falls for a short-period only, whereas skilled consumption falls persistently, thereby leading consumption inequality to actually fall after a few periods. Moreover, the same dynamic pattern holds for wage inequality, which falls after a few periods.

E.3 Welfare results

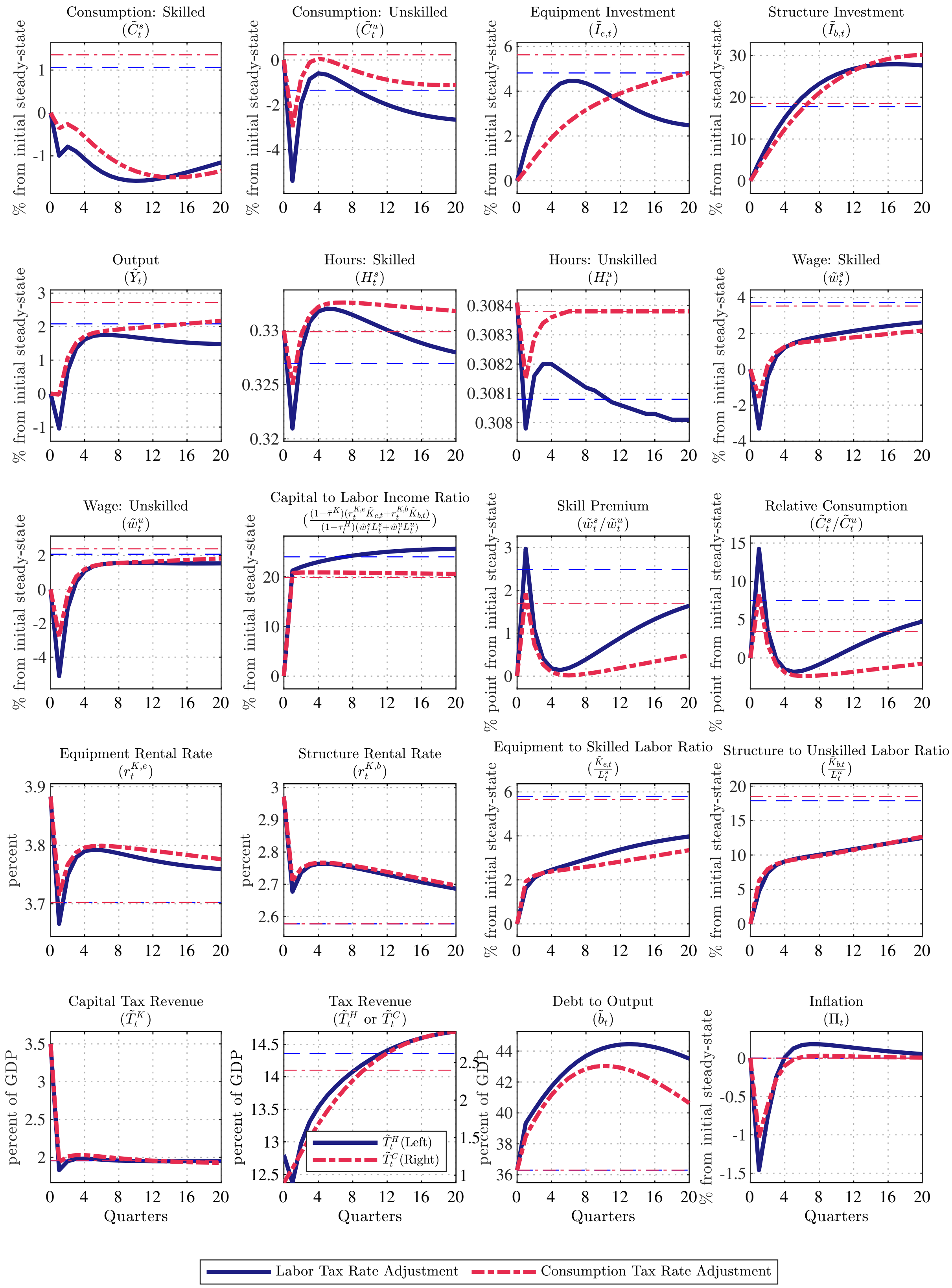
Finally, we end with welfare results under various values of Frisch elasticity of labor supply. Appendix Figure F.11 shows that our main finding that transfer and labor tax adjustment do not lead to a Pareto improvement, but consumption tax adjustment in fact does is robust to both a higher and lower Frisch elasticity than our baseline parameterization. Next, we do a sensitivity analysis on the equipment capital share parameter, λ. As shown in Appendix Figure F.12, we find that our main finding that transfer and labor tax adjustments do not lead to a Pareto improvement, but consumption tax adjustment in fact does, is robust to different values of λ.

F Appendix tables and figures

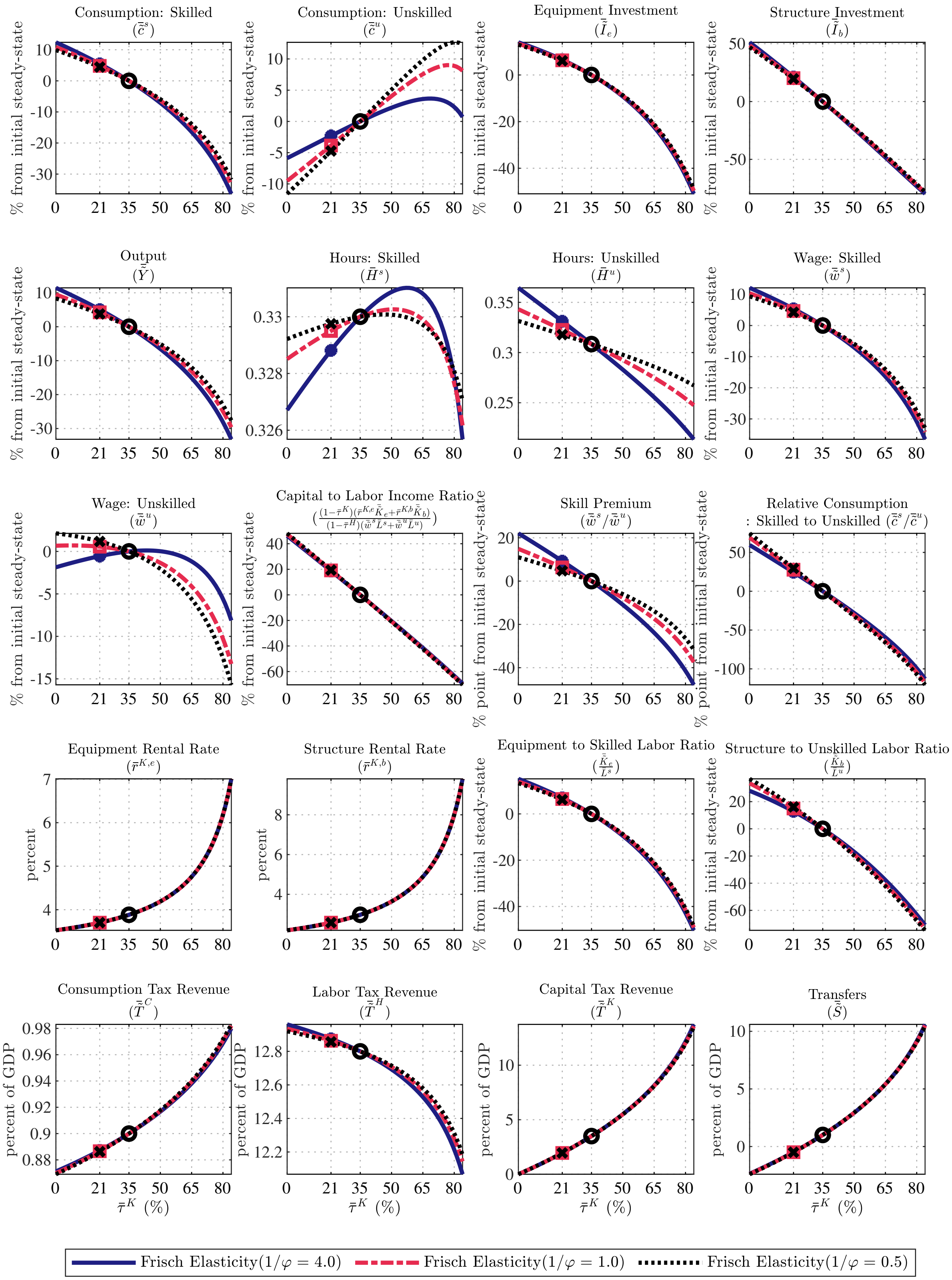
Appendix Table F.1: Estimation results for labor tax rate adjustment rules

	(1) Sample (1983Q1-2002Q4)	(2) Sample (2001Q1-2019Q3)
ρ_1^H	0.869	0.785
	(0.075)	(0.171)
ρ_2^H		0.107
		(0.168)
ψ_B^H	0.111	0.007
	(0.107)	(0.069)
$\psi_{\Delta Y}^H$	0.831	1.821
	(0.633)	(1.473)
ψ_x^H	0.032	0.040
	(0.035)	(0.099)
constant	0.001	-0.002
	(0.003)	(0.001)
R^2	0.832	0.853
Observations	79	75

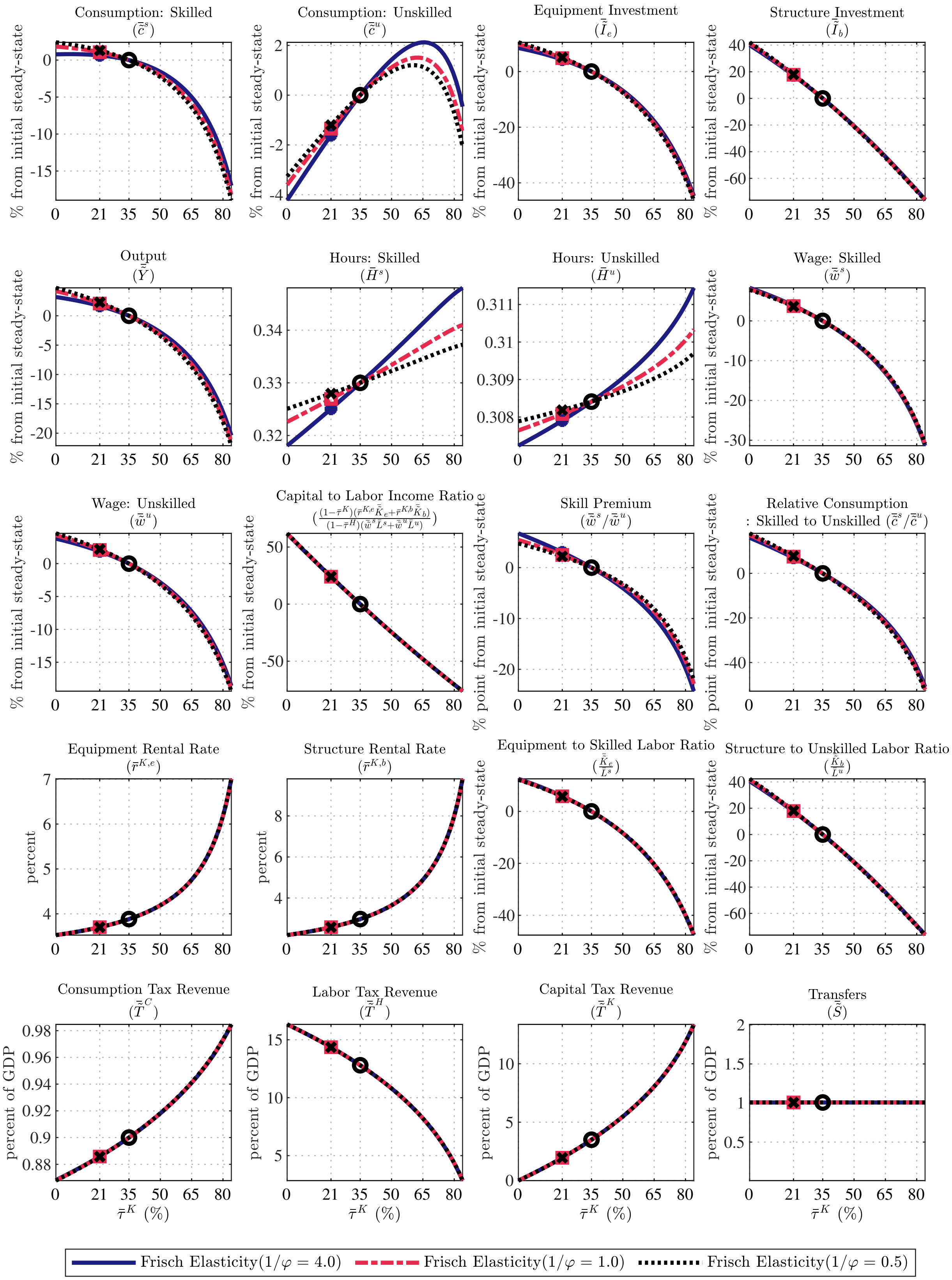
Notes: The table shows OLS estimates of the labor tax rate adjustment rule (D.1). Quarterly US data is used for estimation: tax revenues-to-output ratio, market value of government debt-to-output ratio, output growth and the gap between actual output and potential output. Column (1) shows the estimation results using the sample from from 1983Q1 through 2002Q4 and column (2) shows the estimation results using the sample from 2001Q1 through 2019Q3. For the column (1), we drop the second lag of the tax revenues to ensure stationarity of the tax rule. See Appendix D for details.



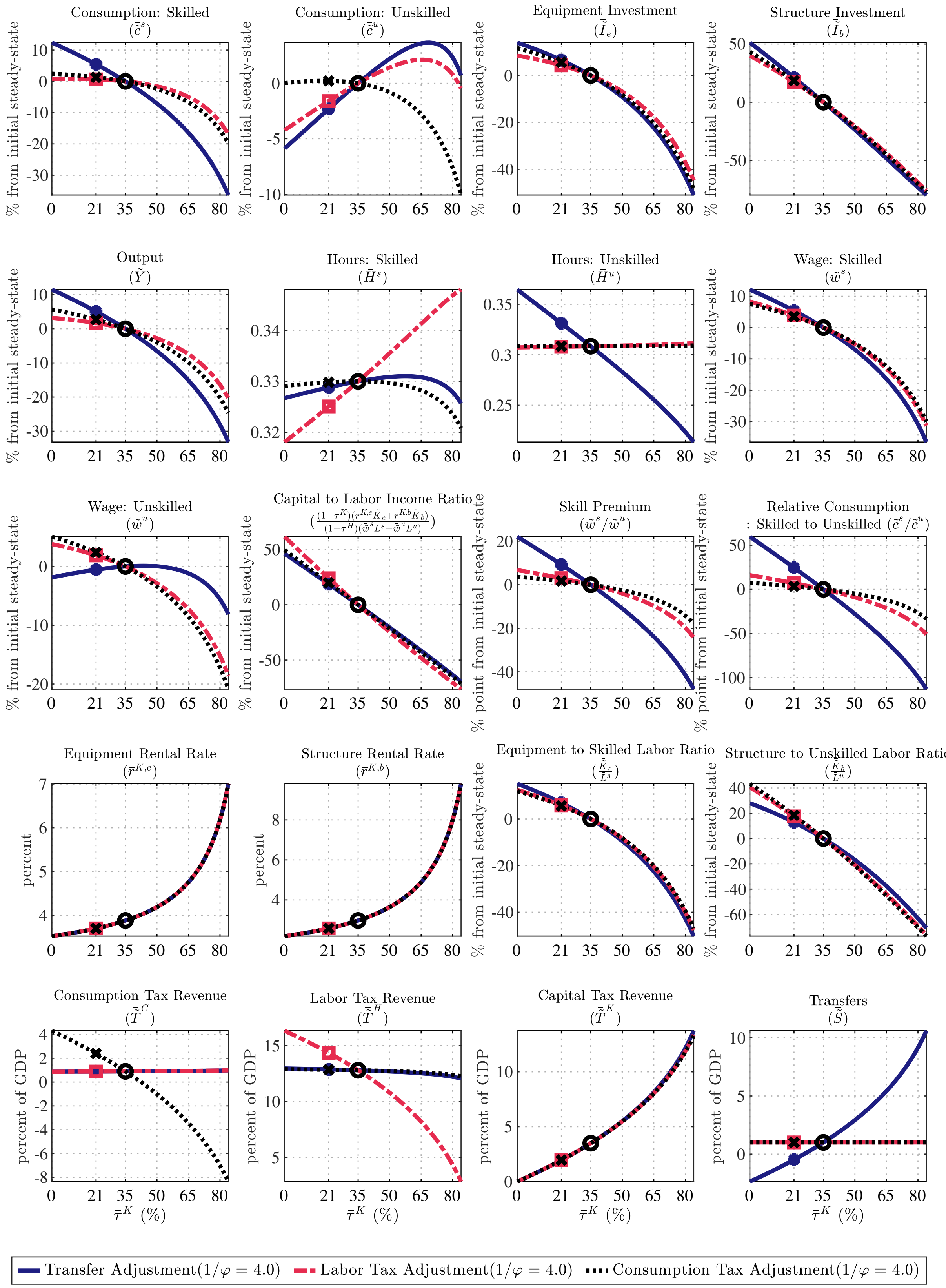
Appendix Figure F.1: Transition dynamics of a permanent capital tax rate decrease under labor tax rate and consumption tax rate adjustment



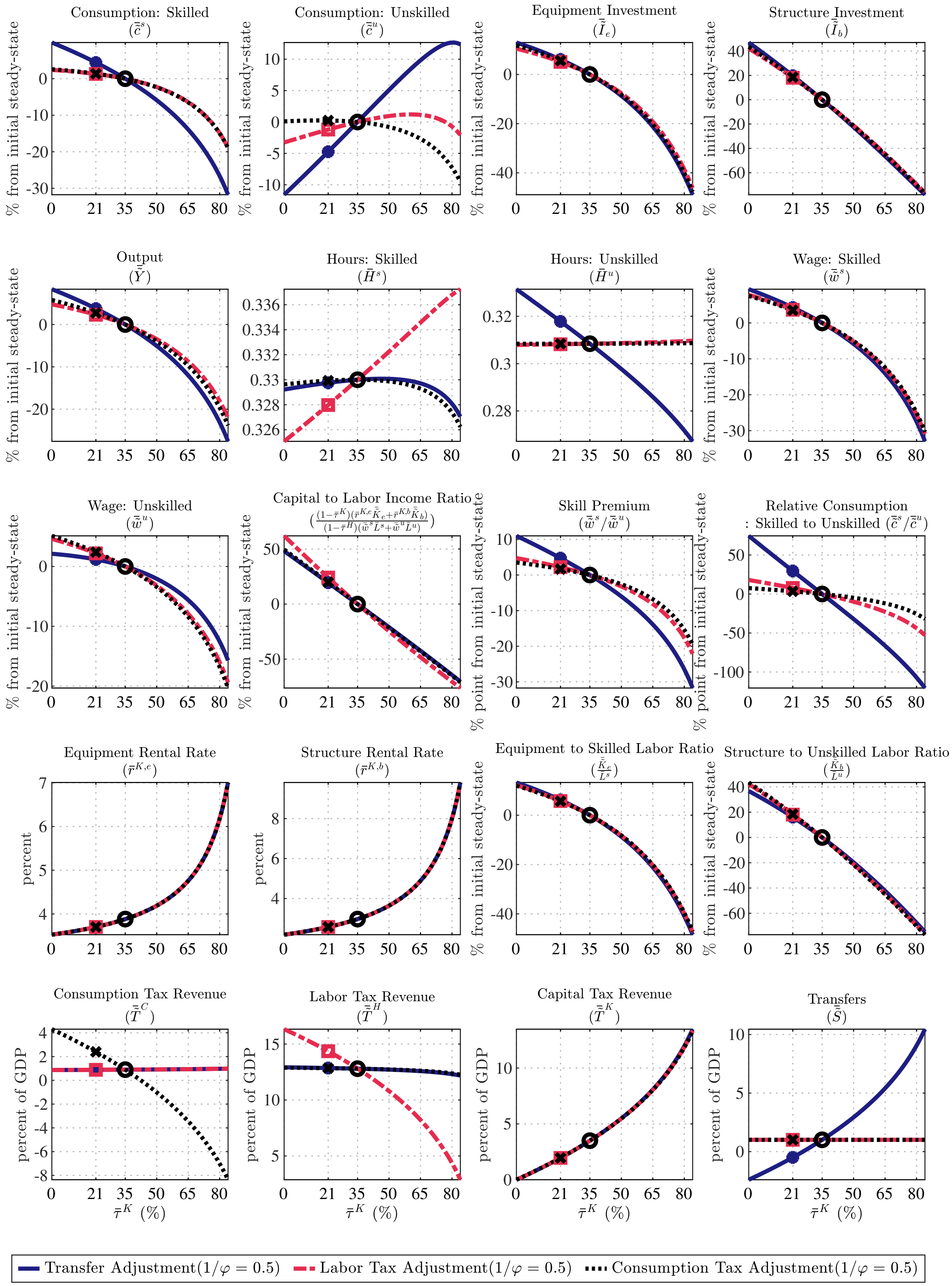
Appendix Figure F.2: Long-run effects of permanent capital tax rate changes under transfer adjustment with different Frisch elasticities



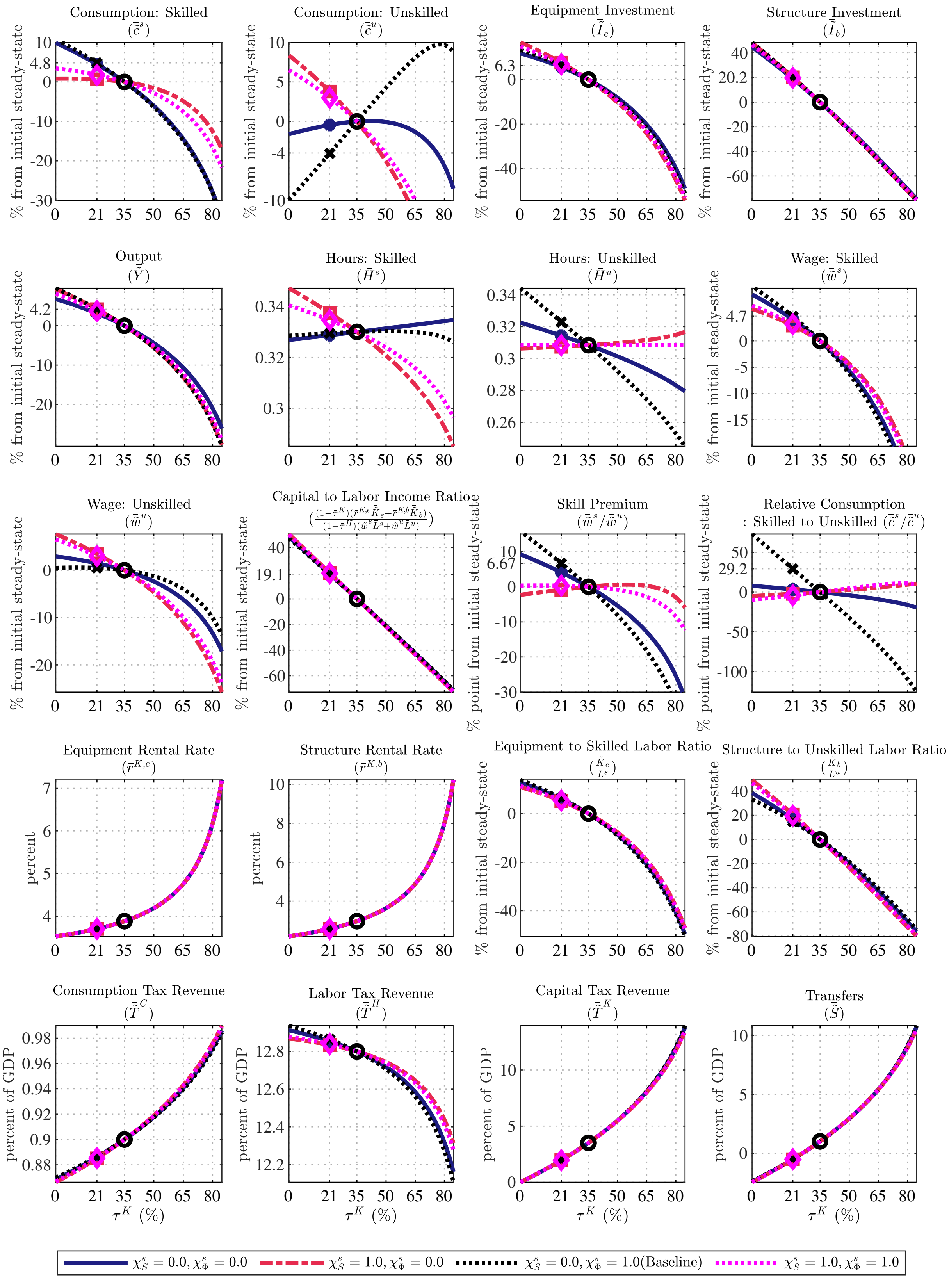
Appendix Figure F.3: Long-run effects of permanent capital tax rate changes under labor tax rate adjustment with different Frisch elasticities



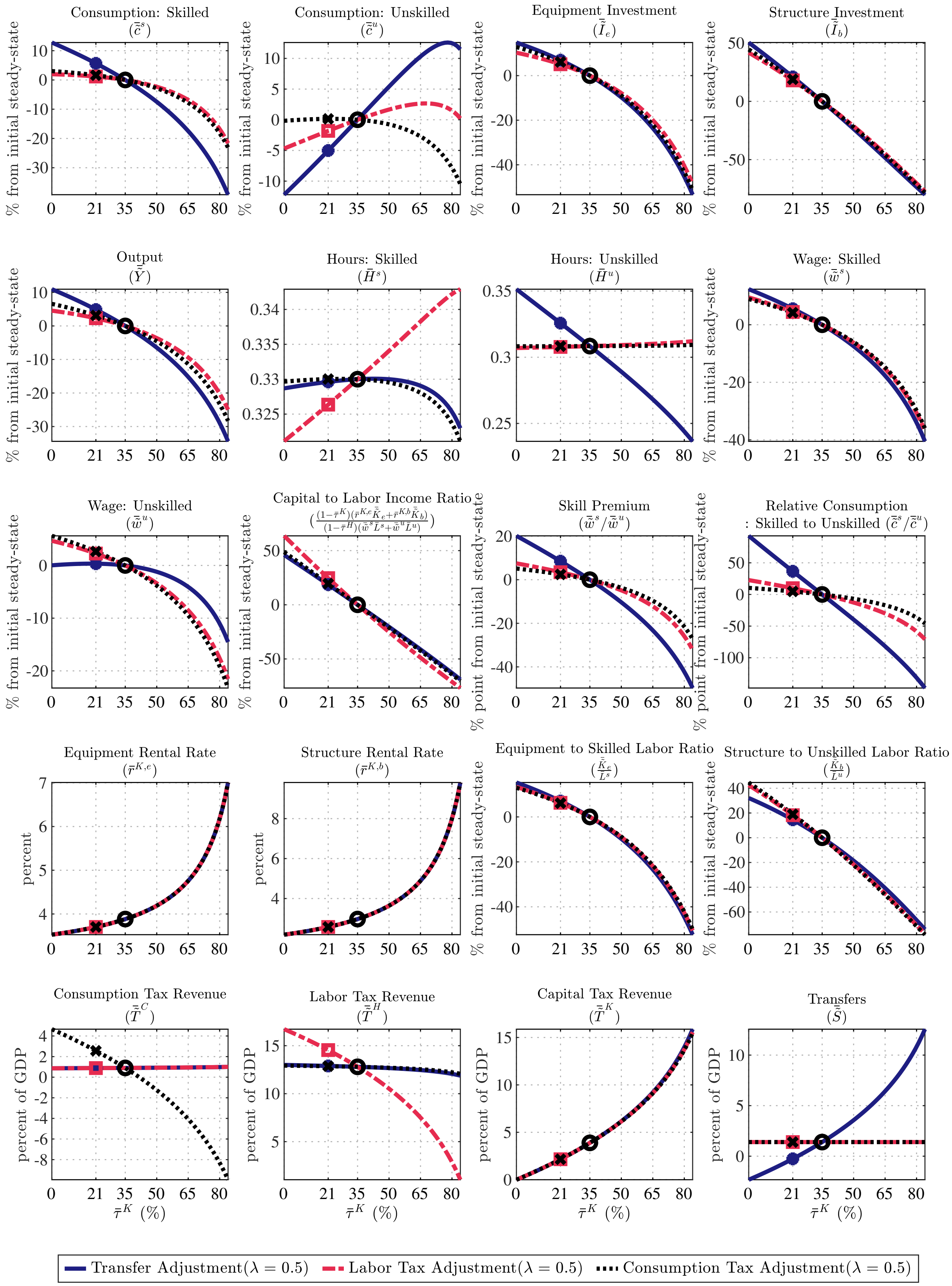
Appendix Figure F.4: Long-run effects of permanent capital tax rate changes under different fiscal adjustments $\left(\frac{1}{\varphi} = 4.0\right)$



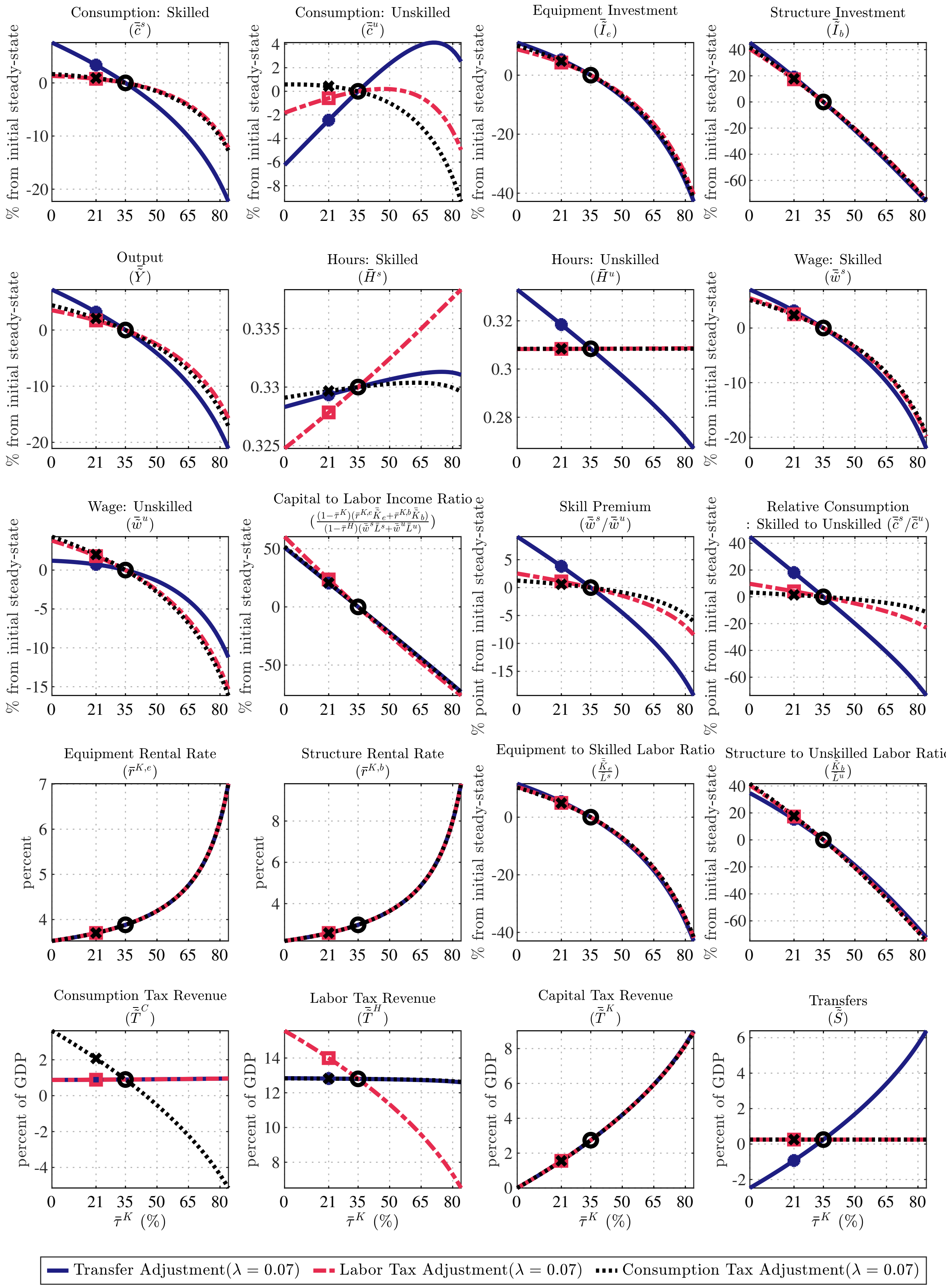
Appendix Figure F.5: Long-run effects of permanent capital tax rate changes under different fiscal adjustments $\left(\frac{1}{\varphi} = 0.5\right)$



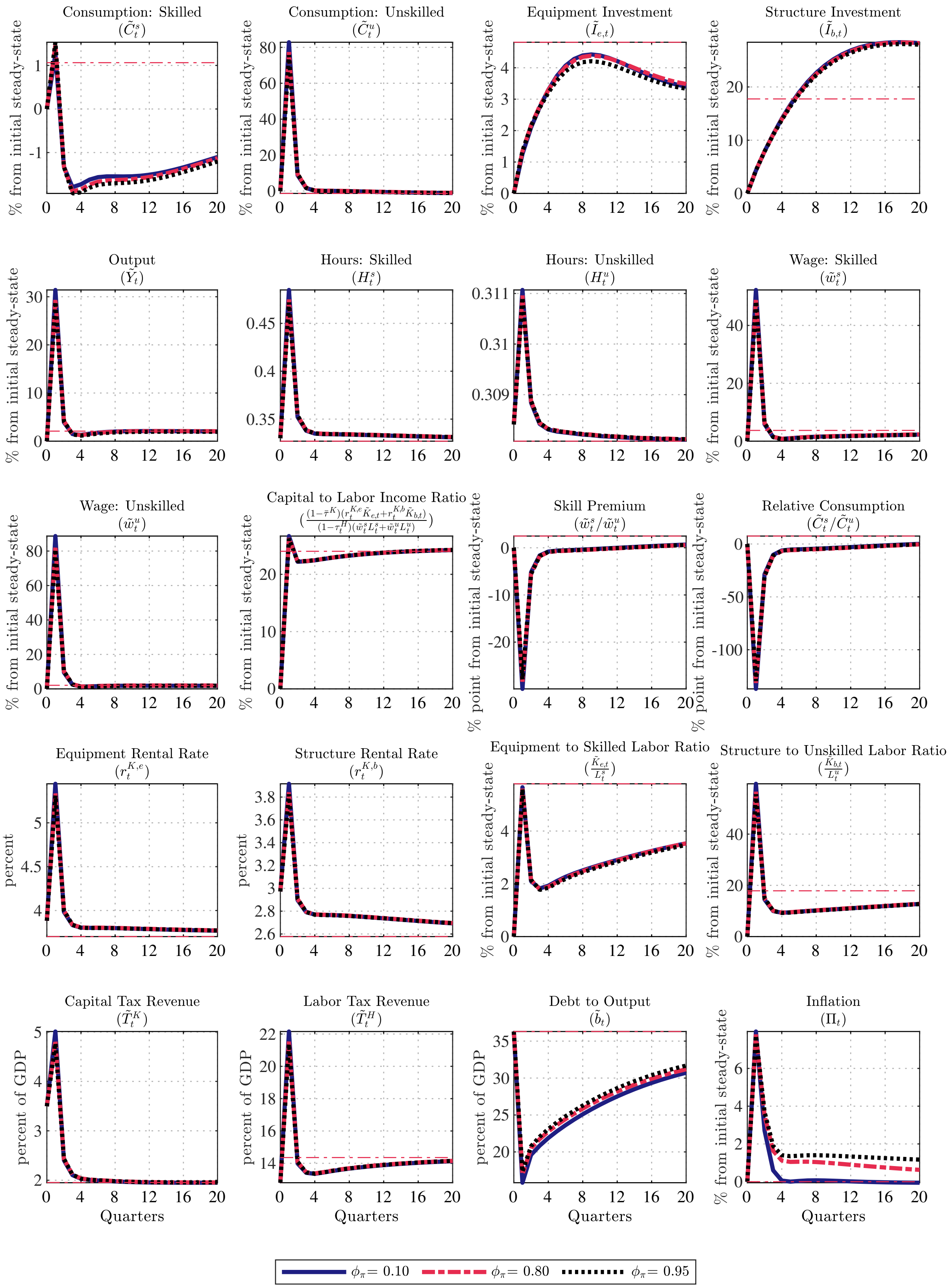
Appendix Figure F.6: Long-run effects of permanent capital tax rate changes under transfer adjustment (different profits and transfer distribution rules)



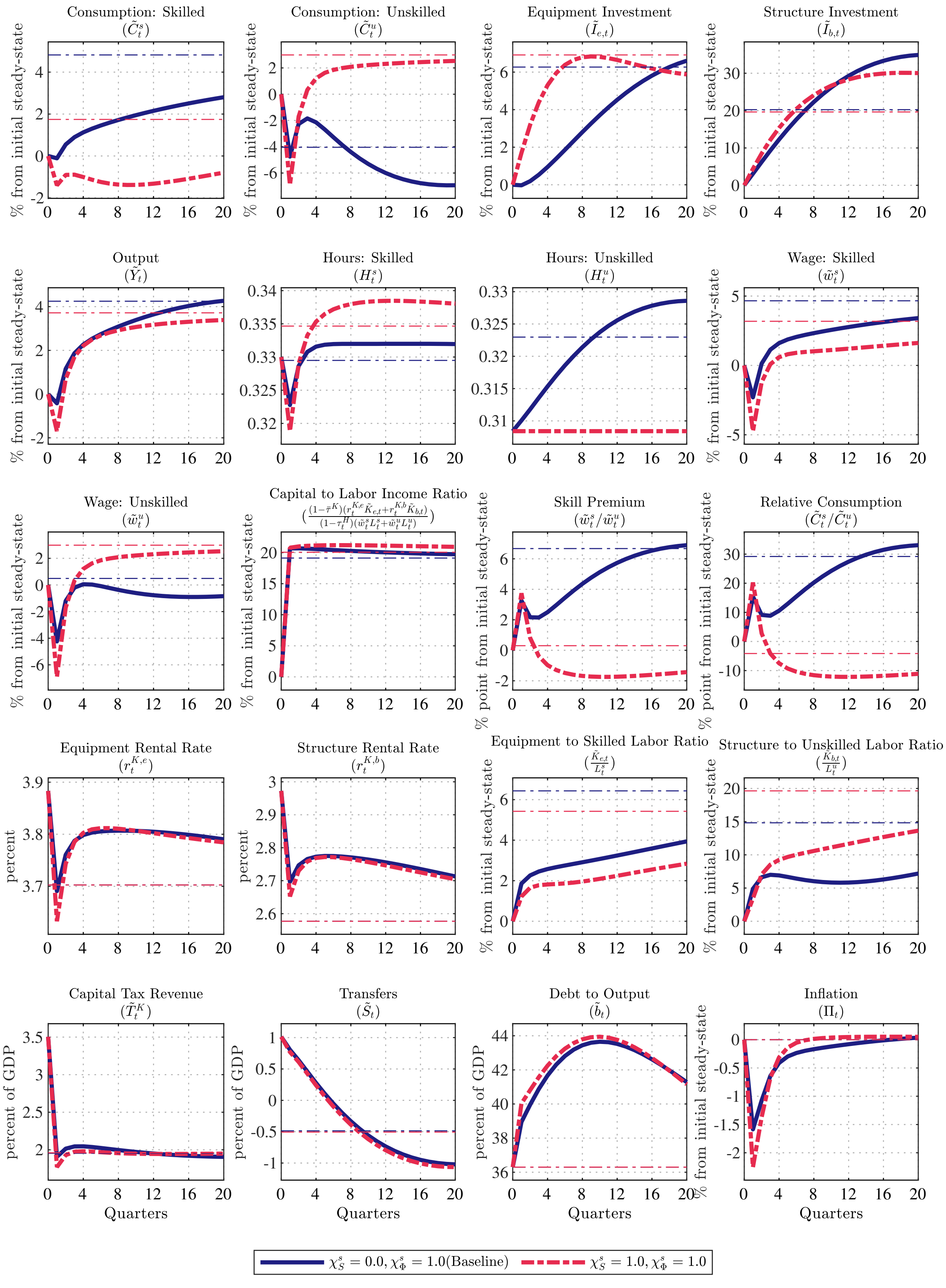
Appendix Figure F.7: Long-run effects of permanent capital tax rate changes under different fiscal adjustments ($\lambda = 0.5$, labor share = 0.53)



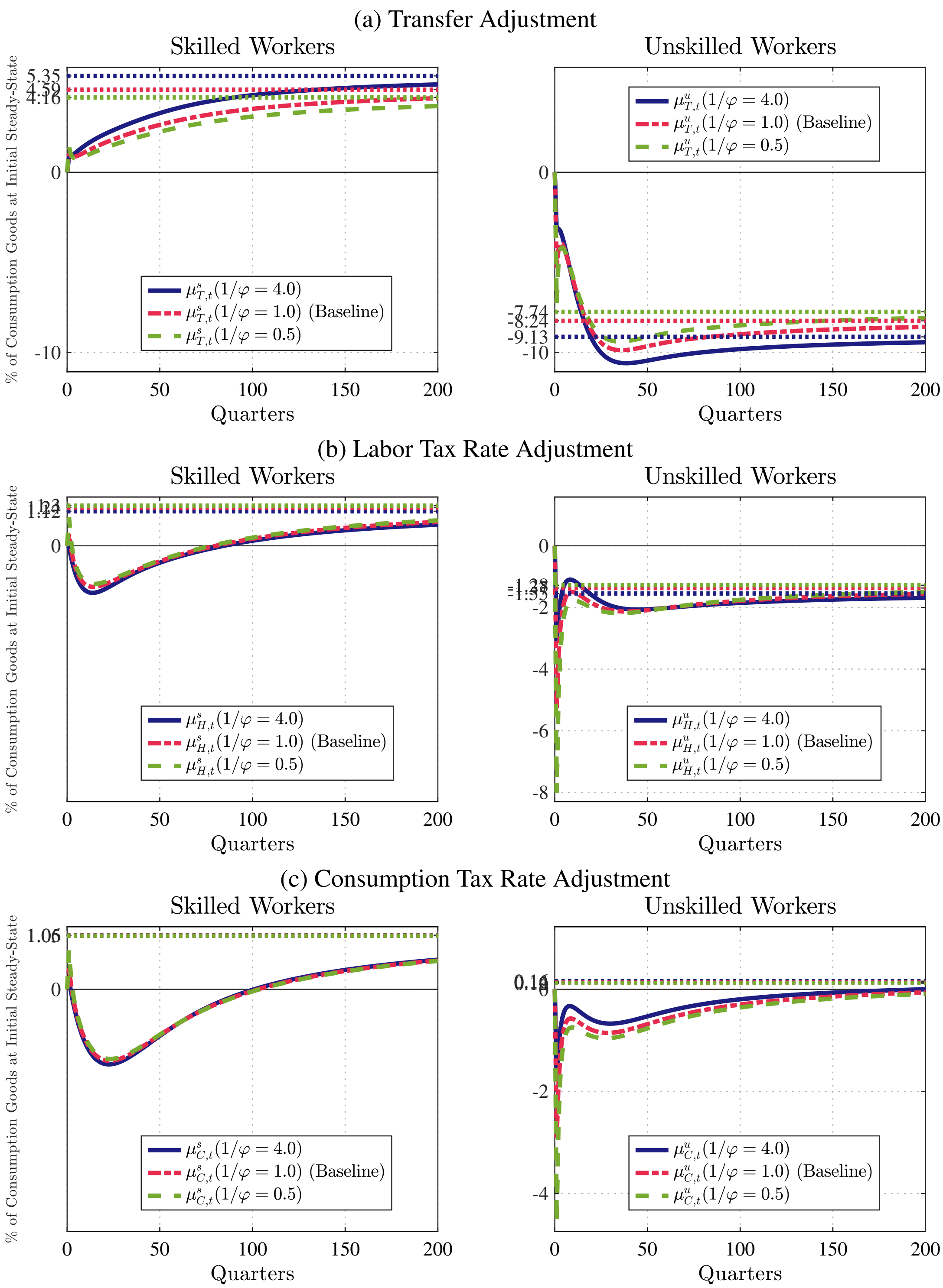
Appendix Figure F.8: Long-run effects of permanent capital tax rate changes under different fiscal adjustments ($\lambda = 0.07$, labor share = 0.63)



Appendix Figure F.9: Transition dynamics of a permanent capital tax rate decrease under labor tax rate and inflation adjustment with different inflation feedback parameters

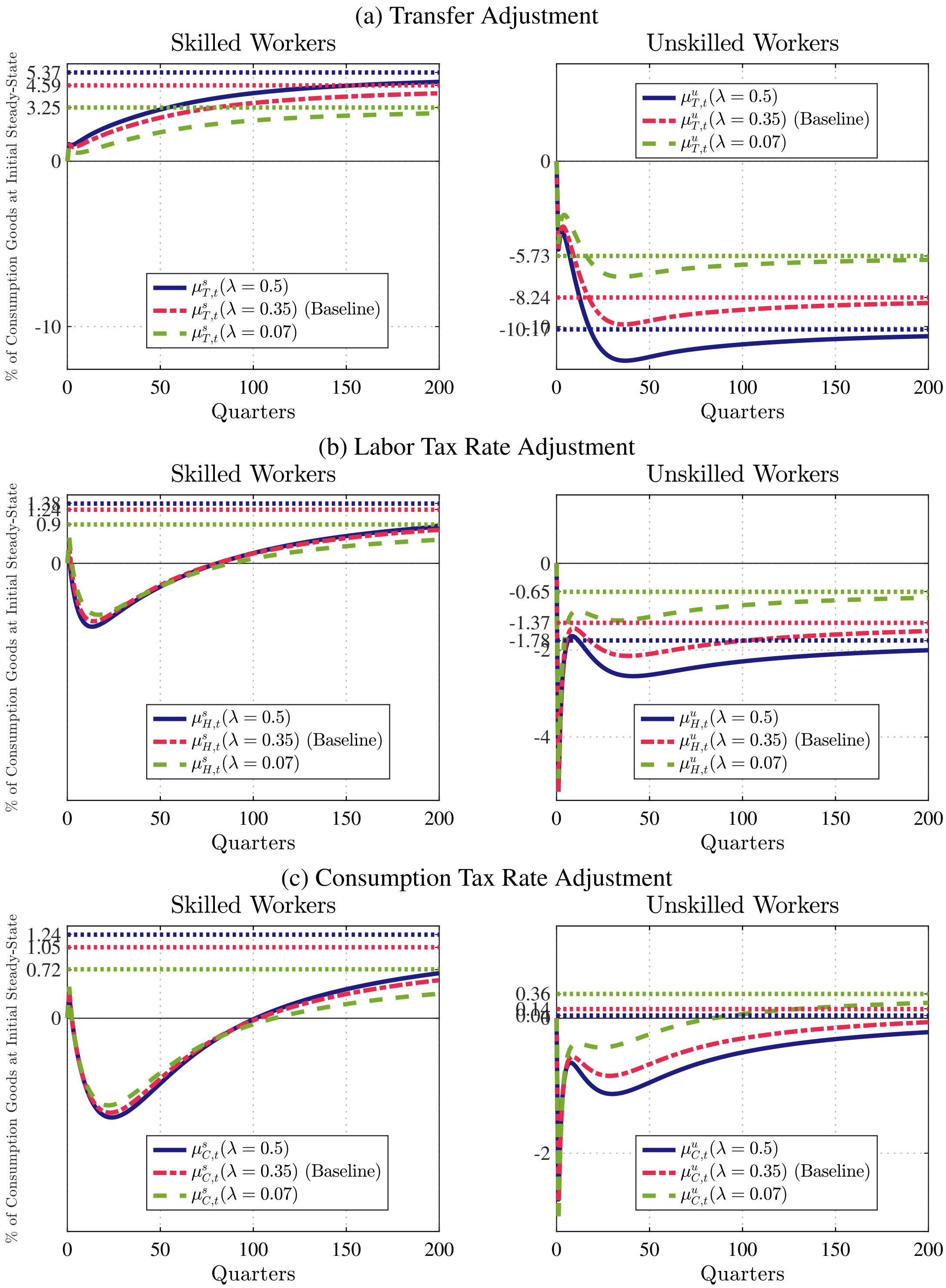


Appendix Figure F.10: Transition dynamics of a permanent capital tax rate decrease under transfer adjustment (different profits and transfer distribution rules)



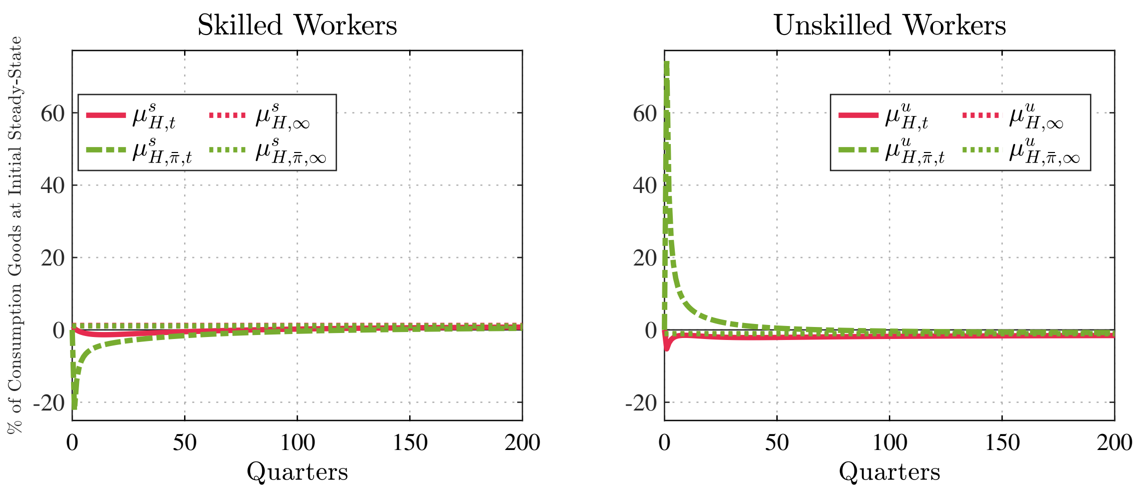
	Transfer adjustment			Labor tax adjustment			Consumption tax adjustment		
	$\frac{1}{\varphi}=4.0$	$\frac{1}{\varphi}=1.0$	$\frac{1}{\varphi}=0.5$	$\frac{1}{\varphi}=4.0$	$\frac{1}{\varphi}=1.0$	$\frac{1}{\varphi}=0.5$	$\frac{1}{\varphi}=4.0$	$\frac{1}{\varphi}=1.0$	$\frac{1}{\varphi}=0.5$
Skilled	5.352	4.589	4.156	1.118	1.236	1.301	1.056	1.052	1.050
Unskilled	-9.128	-8.242	-7.742	-1.546	-1.370	-1.276	0.157	0.136	0.124

Appendix Figure F.11: Welfare with different Frisch elasticities



	Transfer adjustment			Labor tax adjustment			Consumption tax adjustment		
	$\lambda = 0.5$	$\lambda = 0.35$	$\lambda = 0.07$	$\lambda = 0.5$	$\lambda = 0.35$	$\lambda = 0.07$	$\lambda = 0.5$	$\lambda = 0.35$	$\lambda = 0.07$
Skilled	5.368	4.589	3.248	1.376	1.236	0.895	1.238	1.052	0.725
Unskilled	-10.173	-8.242	-5.729	-1.776	-1.370	-0.655	0.039	0.136	0.36

Appendix Figure F.12: Welfare with different equipment capital income share (λ)



Appendix Figure F.13: Welfare implications under labor tax rate and inflation adjustment