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Bubbles and Stagnation *

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Abstract

This paper studies the consequences of asset bubbles for economies that are vulnerable to persistent stagnation. Stagnation is the result of a shortage of assets that creates an oversupply of savings and puts downward pressure on the level of interest rates. Once the zero lower bound on the nominal interest rate binds, the real rate cannot fully adjust downward, forcing output to fall instead. In such context, bubbles are useful as they expand the supply of assets, absorb excess savings and raise the natural interest rate – the real rate that is compatible with full employment – crowding in consumption and raising welfare. While safe bubbles are more likely to expand economic activity, riskier bubbles command a risk premium that, in equilibrium, lowers the real interest rate. A lower rate loosens borrowing constraints, potentially improving welfare when financing conditions are especially tight. Finally, fiscal policy that promises a bail-out transfer in case of a bubble collapse can support an existing bubble and improve welfare.

JEL classification: E31, E32, E43, E44, G11

Keywords: bubbles, secular stagnation, liquidity traps

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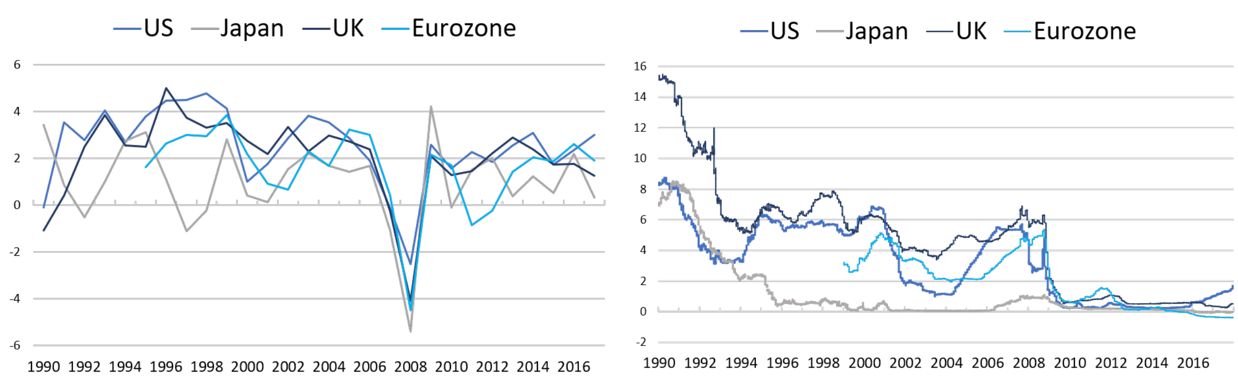
†Board of Governors of the Federal Reserve System.

1 Introduction

The aftermath of the Great Recession has been characterized by sluggish growth and interest rates very close to the zero lower bound (figure 1). The remarkably slow recoveries from the financial crisis generated fears of a new *Lost Decade* and motivated the revival of the secular stagnation hypothesis.[1] The idea behind this hypothesis is that structural factors can create a chronic excess of savings relative to the demand for new investments, depressing interest rates, output and growth. More than a decade after the crisis, the secular stagnation debate remains alive, sustained by plausible concerns of low rates and low growth in the industrialized world for the foresseable future (Rachel and Summers, 2019).

Figure 1: GDP growth and interest rates in advanced economies

(a): Real GDP growth, 1990-2017 (b): Short term interest rates, 1990-2017

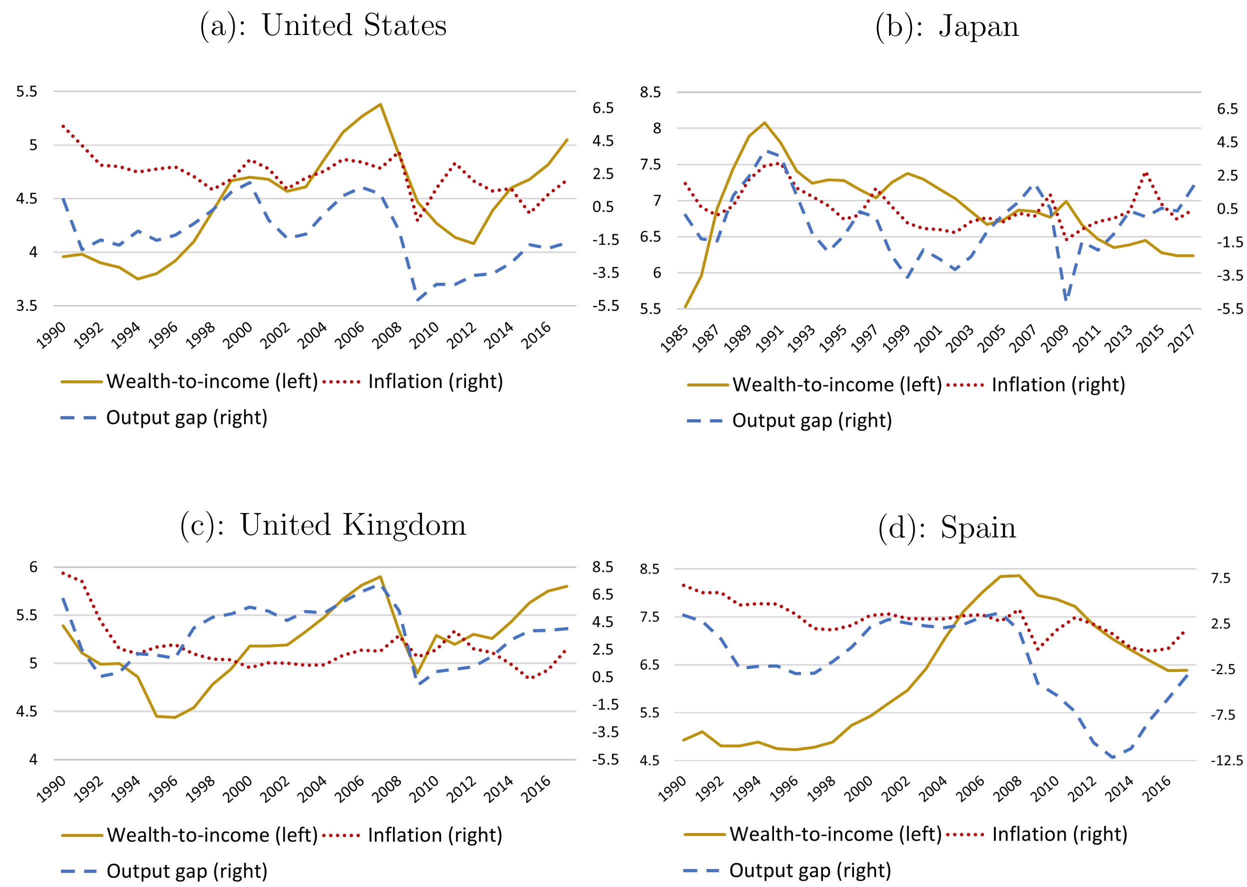


Source: National Income and Product Accounts (NIPA), Organisation for Economic Cooperation and Development (OECD), Federal Reserve Economic Data (FRED).

At the same time, the last thirty years have been marked by recurrent episodes of large fluctuations in asset prices, often associated with "bubbles" due to an apparent disconnect between asset prices and fundamentals. Figure 2 is suggestive of such phenomena by plotting the aggregate wealth-to-income ratio in four advanced economies over the past three decades (solid line). In the United States, two bubbly episodes stand out as large appreciations followed by sudden collapses in net worth, reflecting the movements in stock and real estate prices that collapsed around 2000 and 2007, respectively. Similarly, the Japanese economy observed a rapid rise in stock and real estate prices in the late 1980s, which are commonly interpreted as bubbles that eventually collapsed in the early 1990s. More recently, the UK and Spain experienced a rapid appreciation of asset prices during the early 2000s, but an

[1]The Lost Decade became a popular term to describe the decade of economic stagnation in Japan that followed the Japanese asset price bubble's collapse in late 1991 and early 1992.

Figure 2: Wealth, inflation and the output gap



The solid line depicts the wealth-to-income ratio in the United States, Japan, the United Kingdom and Spain from 1990 to 2017 (World Inequality Database). The dashed line represents estimates of the output gap for the same period made available by the OECD. The dotted line describes the behavior of inflation (FRED).

important share of these price gains was quickly undone around 2007, coinciding with the bursting of real estate bubbles.

Motivated by the events of the past few decades, economists have speculated about the implications of asset bubbles for economies in stagnation, raising important questions about the macroeconomic consequences of bubbly episodes. Lawrence Summers and Paul Krugman have provided some of the most notable commentaries offering a view that is nicely summarized by Krugman:"*...how can you reconcile repeated bubbles with an economy showing no sign of inflationary pressures? Summers's answer is that we may be an economy that needs bubbles just to achieve something near full employment – that in the absence of bubbles the economy has a negative natural rate of interest*".[2] Despite the importance of the

[2] Quote from Krugman (2013b). See Summers (2013) and Krugman (2013a) for other contributions to the public debate on bubbles and the secular stagnation.

public debate on bubbles and the secular stagnation, most of the discussion has taken place in informal outlets, without the support of a framework to clearly identify the mechanisms at work. The goal of this paper is to provide a model to formally address the questions raised and inform the discussion on bubbles and stagnation.

The stagnation environment, inspired by Eggertsson, Mehrotra and Robbins (2019), results from the combination of a zero lower bound (ZLB) on the nominal interest rate and a financial friction that restricts the supply of non-bubble assets, creating an excess supply of savings relative to demand. Away from the ZLB, the excess savings can be eliminated by an appropriate reduction in the nominal (and real) rate that raises the demand for funds. However, at the ZLB, the nominal interest rate cannot be lowered further and equilibrium is, instead, restored through a fall in output that reduces desired savings. In a departure from their framework, I introduce a bubble asset, with no fundamental value, that households can purchase with the sole expectation of its future resale. The focus is on how this type of asset interacts with, and potentially alters, the stagnation equilibrium.

Four broad insights emerge from the analysis. First, bubbles have different implications in periods of stagnation and periods of full employment. In a stagnation, when aggregate demand is chronically low and the economy operates below full capacity, bubbles can be expansionary and raise welfare. In this case, bubbles absorb resources that would otherwise be wasted and crowd in aggregate consumption. Instead, outside of the stagnation environment, bubbles crowd out private lending and reduce welfare by tightening borrowing constraints. Second, bubbles make stagnation less likely, as they raise the natural rate of interest to levels compatible with the economy operating at potential. Third, risky bubbles that occasionally collapse are less effective in avoiding stagnation than safe bubbles. Nonetheless, some bubble risk may be useful in the presence of financial frictions. Riskier bubbles command a risk premium which, in equilibrium, lowers the interest rate. A lower rate loosens borrowing constraints and improves the allocation of resources when financing conditions are tight. Finally, fiscal policy shares with bubbles the ability to mitigate asset shortages through public debt issuance, and levying of taxes and transfers, but the welfare implications may differ depending on the specific design of fiscal policy (e.g., who pays taxes) and the risk characteristics of a bubble. What is more, fiscal policy can be a complement to an existing bubble, and not just a substitute. For instance, the government can raise the size of an existing bubble by promising a future transfer to its owners in case it collapses, making it effectively less risky ex-ante. Such complementarity can be welfare-improving because the size of a bubble need not be optimal in an equilibrium without government intervention.

The framework developed in this paper contains the key ingredients of the secular stagnation literature, on the one hand, and the literature on rational bubbles on the other. I briefly describe them next. The economy is inhabited by overlapping generations of young, middle-aged and old households. Individuals can borrow and lend to each other but financial frictions impose an upper limit on the demand for funds. This affects young agents who issue debt to middle-aged households in order to consume at early stages of life. Middle-aged households, in turn, save a part of their income to consume at old age. When financing conditions are tight, the desired savings of the middle-aged exceed the young's ability to borrow, which triggers a fall in the natural interest rate. In the absence of bubbles, the existing scarcity of assets generates a shortfall of aggregate demand that cannot be compensated by a fall in prices due to nominal rigidities and the ZLB. In this case, equilibrium is restored with a fall in labor demand and output. However, if a bubble is created, it can prevent output from falling below potential. Essentially, bubbles allow old households to compensate for the low consumption of the young, who face tight borrowing constraints. By selling bubbles to the middle-aged, the old can absorb the excess savings generated by the tight borrowing conditions, making it no longer necessary for output to fall for an equilibrium to be reached. Relative to the fundamental stagnation regime, bubbles raise output and inflation, consistent with the empirical behavior of the output gap (dashed line) and inflation (dotted line) during the bubbly episodes depicted in figure 2.

After presenting the main intuitions in the context of safe bubbles, I consider the possibility that bubbles collapse and discuss the implications of risky bubbles. Similar to a safe bubble, a risky bubble raises the natural interest rate, making stagnation less likely. However, a riskier bubble is smaller and commands a risk premium which, in equilibrium, lowers the interest rate. At the ZLB, a lower interest rate requires a higher level of inflation for an equilibrium to exist, but such level may not be feasible when it is too high relative to the central bank's inflation target. In that case, no bubble equilibria exist and the economy settles at a fundamental equilibrium, possibly a stagnation trap if the ZLB binds. While safe bubbles are more likely to avoid stagnation, the lower interest rates associated with risky bubbles loosen borrowing constraints, which can improve welfare when financing constraints are particularly tight. The final section of the paper explores potential interactions between fiscal policy and bubbles in a secular stagnation. Three points are worth highlighting. First, fiscal policy affects the feasibility and size of bubbles through the choice of taxation, transfers and public debt. A bubble can only coexist with fiscal policy if some asset shortage persists and the natural interest rate is sufficiently depressed, otherwise it would not be sustainable. Second, fiscal policy shares with bubbles the ability to mitigate a shortage of assets through a reduction in asset demand (e.g., by taxing the savers) and/or an increase in the asset supply (e.g., through public debt issuance). In this sense, fiscal policy and bubbles are substitutes in a stagnation. However, the ability of fiscal policy to move the economy away from a stagnation equilibrium hinges on the specific composition of fiscal transfers, taxes, and debt, which determines the impact on the natural rate of interest, aggregate consumption and its distribution across generations, which need not be the same as that of a bubble. Third, there is a potential role for fiscal policy to complement an existing bubble, even if it cannot directly affect its probability of bursting. For instance, the government may improve the size of a risky bubble that is *too* small by promising a future transfer to its owners in case that it collapses. This policy effectively reduces the risk associated with a bubble and has the potential to improve welfare whenever the expansionary effects of a larger bubble on output outweigh the potentially tighter borrowing constraints.

Related Literature. This paper relates to three broad strands of the literature. First, it contributes to the extensive work on liquidity traps.[3] It is closest to Eggertsson et al. (2019) who argue that liquidity traps can last indefinitely, possibly triggered by different mechanisms including a deleveraging shock, a fall in population growth, an increase in income inequality or a drop in the relative price of investment. Although my model shares similarities with theirs, I use this framework to analyze a natural question that is largely ignored in the paper: how can asset bubbles affect the equilibrium of an economy that can fall permanently into a stagnation trap? Similarly, Benhabib, Schmitt-Grohe and Uribe (2001) and Benigno and Fornaro (2017) feature the possibility of permanent liquidity traps driven by self-fulfilling expectations, but they exclude from the analysis the possibility of asset bubbles.

Second, this work relates to the literature on asset shortages and their macroeconomic implications. Closely related is Caballero and Farhi (2017), in which a strong scarcity of safe assets coupled with a binding ZLB generates a permanent fall in output and employment ("safety trap"). One implication of their analysis is that while safe assets and bubbles improve a "liquidity trap", risky bubbles are dominated by safe assets for improving a safety trap. In this paper, I argue that while safe bubbles are more likely to avoid stagnation, some risk may be desirable because it relaxes financing constraints.

Finally, this paper relates to the literature on rational asset bubbles that goes back to Samuelson (1958) and Tirole (1985). It closely relates to the theoretical models of bubbles due to borrowing constraints, where this type of friction allows an intrinsically worthless asset to trade at a positive price. Some examples include Farhi and Tirole (2012), Caballero and Krishnamurthy (2006) and Martin and Ventura (2012a, 2016), in which bubbles affect investment and production decisions. In my model, instead, bubbles act as aggregate demand shifters as they interact with the frictions that generate the secular stagnation:

[3] Eggertsson and Woodford (2003), Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) are just a few examples that feature the possibility of zero lower bound episodes.

tight borrowing constraints, a ZLB on the nominal interest rate and nominal wage rigidities. Closely related is Biswas, Hanson and Phan (2020) who study the aggregate implications of bubbles in a setting that features a ZLB and nominal wage rigidities. However, similar to the previous examples, the paper develops an investment-driven demand for bubbles and their macroeconomic effects operate primarily through their impact on investment and production.

The rest of the paper is organized as follows. Section 2 outlines a model of secular stagnation with bubbles. Section 3 characterizes the equilibria of the model when bubbles are deterministic, while section 4 considers the possibility of stochastic bubble destruction. Section 5 explores the interactions between fiscal policy and bubbles in a secular stagnation. Finally, section 6 concludes.

2 A model of secular stagnation and bubbles

Consider an economy populated by overlapping generations of young, middle-aged and old households that live for three periods. Time is discrete and infinite, and is denoted by $t \in \{0, ..., \infty\}$. The output of the economy, Y_t, is produced by perfectly competitive firms that use labor, L_t, to produce $Y_t = L_t^\alpha$. The firms' labor demand is determined by the maximization of profits, $Z_t \equiv P_t Y_t - W_t L_t$, and satisfies

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \quad , \tag{1}$$

where P_t and W_t denote the price level and the nominal wage, respectively. The households born in period t seek to maximize expected utility

$$\mathbb{E}_t\big\{log(C_t^y) + \beta log(C_{t+1}^m) + \beta^2 log(C_{t+2}^o)\big\},$$

where C_t^i, $i = y, m, o$ denotes consumption when young, middle-aged and old. In the first period, young households have no income and borrow to consume. In turn, middle-aged households earn income from wages and profits, a fraction of which they save for retirement when old. For simplicity, I assume that the labor supplied by the middle-aged is inelastic and equal to $\bar{L}$.

Households can trade three types of assets. First, they can borrow or lend via a one period risk-free bond, A_t^i, with return r_t, subject to the exogenous borrowing limit D_t. Second, households have access to one period nominal debt whose return, i_t, is controlled by the government.[4] Finally, households can trade a bubble asset, B_t, which grows at rate

[4]I assume that it trades in zero net supply. Microfoundations for the explicit introduction of money include cash-in-advance constraints or money in the utility function.

r_t^b. The bubble has no fundamental value but rational agents may decide to purchase it if they expect to resell it in the future. Moreover, bubbles may collapse randomly: if there is a positive bubble at time t, its value may collapse in the following period with probability λ, i.e., $P(B_{t+1} = 0|B_t > 0) = \lambda$, where $\lambda \in [0, 1]$. Given the structure described, each household born at time t faces the following budget constraints:

$$C_t^y = -A_t^y \tag{2}$$

$$C_{t+1}^m = \frac{W_{t+1}}{P_{t+1}} L_{t+1} + \frac{Z_{t+1}}{P_{t+1}} + (1 + r_t)A_t^y - A_{t+1}^m - B_{t+1} \tag{3}$$

$$C_{t+2}^o = A_{t+1}^m(1 + r_{t+1}) + B_{t+2} \tag{4}$$

$$B_{t+2} = \begin{cases} B_{t+1}(1 + r_{t+1}^b) & \text{w.p. } \lambda \\ 0 & \text{w.p. } 1 - \lambda \end{cases} \tag{5}$$

$$-(1 + r_t)A_t^i \leq D_t \tag{6}$$

Equation (2) corresponds to the budget constraint of the young household who consumes the total amount borrowed. Equation (3) reflects the budget constraint of the middle-aged agent who receives income from labor and profits. Moreover, they repay the amount borrowed when young, lend to the current young and buy the bubble. At old age, the household simply consumes the (gross) return to their savings, as stated in equations (4) and (5). Finally, inequality (6) reflects the existence of a financial friction that imposes an upper bound on the amount that can be repaid by each household. In what follows, I assume that (6) binds for the young, so that they borrow as much as possible. I am, thus, considering an economy in which young agents would like to borrow more in order to consume, but financing frictions impose a limit to the amount of credit that they can obtain. Therefore,

$$C_t^y = -A_t^y = \frac{D_t}{1 + r_t} \tag{7}$$

In the spirit of Eggertsson et al. (2019), I introduce downward nominal wage rigidities. Suppose that in any given period t the household would never accept a wage W_t lower than $\gamma W_{t-1} + (1 - \gamma)P_t \alpha \bar{L}^{\alpha-1}$. The degree of price rigidity is governed by γ: the case $\gamma = 0$ corresponds to a perfectly flexible wage, while $\gamma = 1$ implies a perfectly downward rigid wage. In the latter case, households at time t would not accept a wage lower than what it was at $t - 1$. Finally, monetary policy is conducted by a central bank that sets the nominal interest rate, i_t, such that expected inflation equals the (exogenous) target π^*, unless it is constrained by the zero lower bound. This policy corresponds to the limit $\phi_\pi \to \infty$ of a standard Taylor rule.[5]

2.1 Equilibrium

For a given exogenous process $\{D_t\}$ and an initial bubble B_0, an equilibrium is defined as a collection of $\{C_t^y, C_t^m, C_t^o, A_t^y, A_t^m, B_t, L_t, Y_t, Z_t\}$ and prices $\{P_t, W_t, r_t, r_t^b, i_t\}$ that solve the optimization problems of households and firms, and are consistent with the monetary policy rule. I focus on steady state equilibria and start by deriving aggregate demand and aggregate supply.

Aggregate Demand. Aggregate demand can be derived as the total consumption of all generations as follows. First, recall that the young simply consume the amount borrowed, A_t^y. Equilibrium in the bond market requires that the supply of bonds from the young equalizes the demand for bonds by the middle-aged, i.e., $-A_t^y = A_t^m$. Therefore, the young consume

$$C_t^y = A_t^m \tag{9}$$

In turn, the optimal asset holdings of the middle-aged households, $\{A_t^m,\ B_t\}$, and consumption, C_t^m, can be obtained from their optimization problem: they decide how much to consume or save at time t, and how to allocate their savings between the existing vehicles. Solving this problem requires the following optimality conditions

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1+r_t}{C_{t+1}^o} \tag{10}$$

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1}{C_{t+1}^o}(1+i_t)\frac{P_t}{P_{t+1}} \tag{11}$$

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1+r_t^b}{C_{t+1}^o} \tag{12}$$

[5]That is, this policy can be derived as the limit $\phi_\pi \to \infty$ of the Taylor rule

$$1 + i_t = \max\left\{1, (1+i^*)\left(\frac{\mathbb{E}_t \Pi_{t+1}}{\Pi^*}\right)^{\phi_\pi}\right\} \quad , \ \phi_\pi > 1 \tag{8}$$

where $\mathbb{E}_t$ is the expectations operator conditional on information available at time t, $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$, Π^* is the (exogenous) inflation target and i^* is the nominal interest rate target that is consistent with the inflation target.

Equations (10) and (11) imply the usual Fisherian relationship:

$$1 + r_t = (1 + i_t)\mathbb{E}_t \frac{P_t}{P_{t+1}} \tag{13}$$

Moreover, one can rewrite conditions (10) and (12) using the budget constraints (3) and (4), and the law of motion of the bubble (5):

$$\frac{1}{C_t^m} = \beta(1 + r_t)\left[\frac{\lambda}{A_t^m(1 + r_t)} + \frac{1 - \lambda}{A_t^m(1 + r_t) + B_t(1 + r_t^b)}\right] \tag{14}$$

$$\frac{1}{C_t^m} = \beta(1 + r_t^b)\left[\frac{1 - \lambda}{A_t^m(1 + r_t) + B_t(1 + r_t^b)}\right], \tag{15}$$

Equations (14) and (15) imply that, for households to hold bonds and bubbles, they must be indifferent between the two assets. If the growth rate of the bubble were lower, holding a bubble would not be sufficiently attractive and agents would only buy bonds. On the other hand, if it was larger, the middle-aged – the ones who supply savings in the model – would like to borrow to purchase the bubble and this cannot be an equilibrium either. In addition, the bubble must be feasible which requires enough funds to purchase it. For simplicity, I assume no growth and therefore a positive growth rate of the bubble is ruled out (i.e. a bubble is only sustainable in the long run for $r^b \leq 0$). In general, though, it is only required that the bubble does not grow faster than the economy's resources which allows for a positive bubble growth rate if income is also growing. Finally, old households do not make any choice at time t, and simply consume

$$C_t^o = A_{t-1}^m(1 + r_{t-1}) + B_t. \tag{16}$$

Steady state aggregate demand is, then, given by $Y^d = C^y + C^m + C^o$, satisfying (9)-(16). An explicit expression for Y^d can be derived for the special case of deterministic bubbles, which is done in the next section. Before, I derive aggregate supply.

Aggregate supply. Production can be either at full capacity or below full capacity, depending on the prevailing real wage. The latter, in turn, depends on the degree of nominal rigidities and inflation. To see this, note that for $\Pi \geq 1$, the bound on the nominal wage does not bind and the real wage is consistent with full employment $\bar{L}$. Output is given by

$$Y = \bar{L}^\alpha \equiv Y^f \qquad \text{for } \Pi \geq 1. \tag{17}$$

In this case, production is fully determined by the supply of labor $\bar{L}$. Instead, for $\Pi < 1$, the wage norm binds. In this case, the real wage is given by $\omega = \frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{1-\gamma\Pi^{-1}}$, and aggregate supply is given by

$$Y = \left(\frac{1-\gamma}{1-\gamma\Pi^{-1}}\right)^{\frac{\alpha}{\alpha-1}} Y^f \qquad \text{for } \Pi < 1. \tag{18}$$

Production increases with inflation because of wage stickiness. As inflation rises, real wages decrease and firms demand more labor.

Using the demand and the supply relationships derived in this section, the following sections characterize the model's equilibria. Section 3 considers the case of deterministic bubbles, and section 4 extends the analysis to the case of stochastic bubbles that randomly collapse. The focus is on the conditions that generate a secular stagnation, and how bubbles interact with it.

3 Deterministic bubbles

Consider first the case of deterministic bubbles, i.e., $\lambda = 0$. This section characterizes the equilibria of the model for two alternatives: (1) a perfectly flexible economy and (2) an economy with nominal rigidities.

It is useful to start by deriving an expression for aggregate demand, Y^d, under the special case of deterministic bubbles. To do this, impose $\lambda = 0$ in (14) and (15), which implies

$$A_t^m + B_t = \frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) \tag{19}$$

where $Y_t^m \equiv \frac{W_{t+1}}{P_{t+1}} L_{t+1} + \frac{Z_{t+1}}{P_{t+1}}$. That is, middle-aged households save a constant fraction of their disposable income and use it to purchase bonds and bubbles. The remaining fraction is used to consume:

$$C_t^m = \frac{1}{1+\beta}(Y_t^m - D_{t-1}) \tag{20}$$

Combining equations (9), (16), (19) and (20) yields the following expression for the steady state aggregate demand

$$Y^d = D + \frac{1+\beta}{\beta}\left(\frac{D}{1+r} + B\right). \tag{21}$$

All else equal, aggregate demand falls with the interest rate r and rises with the size of the bubble B. The following discussion characterizes the model's equilibria by imposing that aggregate demand and aggregate supply are equalized.

3.1 Flexible wages: $\gamma = 0$

When there are no price rigidities, the nominal wage can always adjust so that the labor market clears at full employment. Production is fixed at $Y^f = \bar{L}^\alpha$ which can be thought of as an endowment. The equilibrium can be found by plugging Y^f into equation (21), which imposes that aggregate demand equals aggregate supply:

$$(1+r) = \frac{D}{\frac{\beta}{1+\beta}(Y^f - D) - B} \tag{22}$$

The equilibrium interest rate falls with the discount factor β and rises with the debt limit D and the size of the bubble B. A larger β lowers r because more patient households save a larger fraction of income for old age. Instead, a higher D pushes the interest rate up because it raises the demand for funds by the young households. Finally, a larger B raises r by reducing the supply of loans from the middle-aged who allocate a larger share of their savings to the bubble instead of bonds. This crowding-out effect is intuitive as bubbles compete with bonds as a saving vehicle. And when can a positive bubble be sustained in equilibrium? Given that the bubble must grow at the interest rate, its law of motion can be expressed as

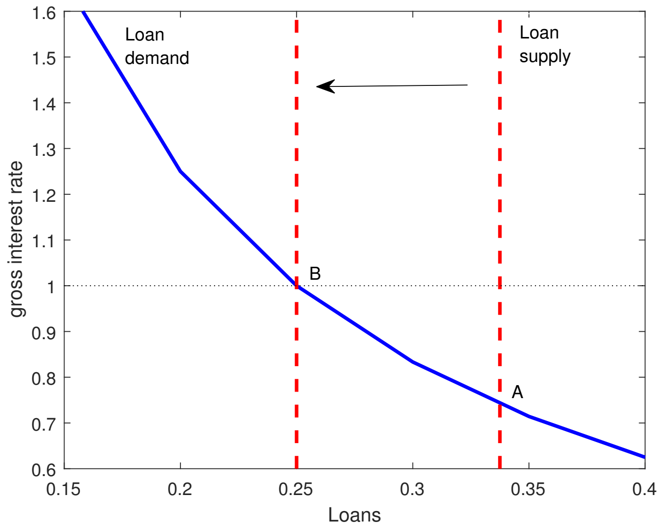
$$B_{t+1} = \frac{D_t}{\frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) - B_t} B_t \tag{23}$$

It follows that the economy features two steady state equilibria: one in which there is no bubble and one with a positive bubble. To see this, impose $B_{t+1} = B_t \equiv B^*$ in (23) which yields two solutions: $B^* = 0$ and $B^* = \frac{\beta}{1+\beta}(Y - D) - D$.[6]

Consider first the fundamental equilibrium, which is analogous to the endowment economy of Eggertsson et al. (2019). It can be fully characterized by (22) with $B = 0$ and, in general, it depends on the parameters D and β. Graphically, this equilibrium is illustrated in figure 3. The demand for savings by the young generation, equal to $\frac{D}{1+r}$, corresponds to the downward sloping solid line. The vertical dashed lines depict the supply of savings from the middle-aged implied by equation (19). Point A in figure 3 depicts the equilibrium in the bond market for one possible parametrization. In this example, the interest rate that equalizes the demand and the supply of savings is negative. Intuitively, this is an economy in which bond supply (borrowing from the young) is strongly limited by the exogenous constraint D. At positive rates, middle-aged savings exceed the amount that the young agents can borrow. The interest rate needs to fall below zero so that the demand and the supply of savings are equalized.

[6]Note that a positive bubble is only possible if $\frac{\beta}{1+\beta}(Y - D) > D$, which implies that the savings of the middle-aged exceed the amount of debt that the young can repay.

Figure 3: Equilibrium in the bond market



Consider, now, the bubble steady state. Relative to the fundamental regime, the supply of loans shifts to the left, as depicted in figure 3, as the middle-aged now allocate some of their savings to the bubble asset. This raises the interest rate which, in the bubble steady state, is equal to $r^* = 0$ (point B).[7] Note that, in this economy, output is fixed at full employment and must all be consumed. As such, bubbles do not affect aggregate consumption. However, they promote a reallocation of consumption across generations throught their effect on the equilibrium interest rate. The young consume more at the fundamental equilibrium as a lower interest rate relaxes their financing constraint, allowing them to borrow more. On the contrary, a lower interest rate means lower returns at old age, which reduces old age consumption. The roles reverse at the bubble equilibrium, where a higher interest rate implies lower consumption for the young and larger returns for the old. Given the preferences of the middle-aged, who consume a constant fraction of their disposable income independently of the prevailing interest rate, bubbles do not affect this generation's consumption level.

What are, then, the welfare implications of asset bubbles? In this endowment economy, bubbles reduce expected lifetime utility and, thus, welfare. Essentially, output and aggregate consumption are fixed at potential, but different bubble sizes reallocate consumption between young agents and old. In particular, a larger bubble raises the real interest rate, which tightens the borrowing constraint of the young households who are forced to consume less. The old households compensate for the fall in consumption of the young by selling the bubble to the middle-aged agents and consuming the proceedings. However, this reallocation

[7] The equilibrium interest rate is obtained by plugging $B^* = \frac{\beta}{1+\beta}(Y - D) - D$ into equation (22).

of consumption is associated with a fall in expected lifetime utility as shown in appendix A.1. Intuitively, this is an environment in which the optimal consumption profile requires a negative real interest rate due to tight financing constraints. In this context, bubbles are sustainable and, in equilibrium, they crowd-out private lending and raise the real interest rate. This forces young agents to postpone consumption to old age in a way that is not optimal.

Nominal rigidities, the ZLB, output and bubbles. The zero lower bound on the nominal interest rate imposes a restriction on the rate of inflation. In particular, the requirement that $(1 + i) \geq 1$ implies that $\Pi \geq (1 + r)^{-1}$ for an equilibrium with constant inflation to exist.[8] At positive real rates, this restriction has little empirical relevance as it requires a level of inflation well below central banks' common targets. However, if the real interest rate is negative, this condition implies a positive inflation rate, possibly above the common 2% target followed by many central banks. As the real interest rate falls, steady state inflation needs to be higher for an equilibrium to exist. But what if the central bank is unwilling to raise inflation enough? The result may be a permanent drop in output if there are some price rigidities. This is the message of Eggertsson et al. (2019) who show that if the central bank refuses to accept enough inflation, the economy can settle at a secular stagnation equilibrium with some unemployment. And what is the role of bubbles in this setting? For a given level of output, bubbles raise the real interest rate by reducing the supply of savings in the bond market, thereby relaxing the requirement imposed on inflation by the zero lower bound. As I introduce nominal wage rigidities in the model, I show that bubbles can prevent output from falling and that, contrary to the flexible wage economy, bubbles can improve welfare in this setting. I turn to this case next.

3.2 Nominal wage rigidities: $\gamma \in (0, 1)$

Assume now that nominal wages are downwardly rigid, that is, $\gamma \in (0, 1)$. Similar to the perfectly flexible setting, equilibrium is determined by the intersection of aggregate supply and aggregate demand, as described in section 2. However, the presence of wage stickyness implies that the real wage may not always be consistent with full employment. In this case, production is determined by the demand for labor.

As before, agents can always coordinate on the bubbleless steady state, in which case the economy lands at a fundamental equilibrium. And what kind of equilibria may emerge in the absence of bubbles? As dicussed next, the economy may land in a "good" equilibrium

[8] Note that when $i = 0$, $(1 + r) = \Pi^{-1}$.

with full employment or in a "bad" stagnation equilibrium, depending on whether there is a shortage of assets. To illustrate this point, let us compare two economies that are identical except for the degree of borrowing frictions determined by D. Consider first an environment in which D is *high*, such that the natural interest rate is positive and high enough to be consistent with full employment.[9] The central bank is unconstrained, $i > 0$, inflation is determined by the inflation target, $\Pi = \Pi^* > 1$, and the economy operates at full capacity, $Y = Y^f$. There is no asset shortage, and bubbles are not sustainable in the long run since the requirement that they grow at the rate of interest, $r^b = r$, implies that any exisiting bubble would eventually outgrow income.

Consider now an environment in which the borrowing constraint is tight, i.e., D is *low*. As a result of the tighter financing conditions, the supply of bonds by the young is more restricted. The middle-aged households, who want to save for old age, face an increased scarcity of assets that drives down the natural interest rate. Outside of the ZLB, a fall in the supply of assets can be compensated by a fall in the nominal (and real) interest rate, which relaxes financing conditions and stimulates the consumption of young households. At the ZLB, however, the interest rate cannot fall any further to stimulate demand and, instead, equilibrium in the bond market is restored endogenously through a drop in output. When the income of the middle-aged falls, asset demand also drops, which reequilibrates the market. The economy settles at a stagnation trap featuring permanent unemployment, $Y < Y^f$, and a binding ZLB, $i = 0$.[10] Low employment puts downward pressure on wages and prices, so that the stagnation equilibrium features deflation ($\Pi < 1$). Stagnation is, thus, a result of a severe shortage of assets coupled with a binding ZLB and nominal rigidities. When the ZLB becomes binding, the adjustment no longer happens through the interest rate but, instead, through output. As discussed next, the key role of bubbles is to avoid the need for a drop in output by expanding the supply of assets and, in this way, avoiding stagnation.

To focus on the role of bubbles in a secular stagnation, consider an economy whose unique fundamental equilibrium is the stagnation trap, with $Y < Y^f$ and $\Pi < 1$. Are there any bubble equilibria and, if so, how are they characterized? As discussed in section 3.1, if a positive bubble exists in equilibrium, it must grow at the real interest rate, i.e., $r = r^b$. Moreover, in steady state, $B_{t+1} = B_t = B^*$, implying that the return on the bubble, r^b, is equal to 0. It, thus, follows that any bubble equilibrium of the model is characterized by $r = 0$. It is straightforward to see that there are two equilibria consistent with a positive bubble that differ in nominal terms. Using the Fisher equation (13) and the monetary policy rule, there are two values for steady state inflation that are consistent with $r = 0$, which are

[9]See appendix A.2, figure 4, for a graphical representation and further details.

[10]The stagnation equilibrium is depicted as point B in figure 4 of appendix A.2.

given by

$$\Pi = \begin{cases} \Pi^* & \text{if } \ i > 0 \\ 1 & \text{if } \ i = 0 \end{cases} \tag{24}$$

In addition to the stagnation (fundamental) steady state, there are now two additional equilibria, expressed in (24), that feature a positive bubble, full employment and a zero interest rate, $r^* = 0$. The two equilibria simply differ in the nominal interest rate and the inflation rate that are consistent with the central bank's policy rule.

At the stagnation equilibrium, financing frictions impose an upper limit on the demand for funds. At this point, the demand for assets by the middle-aged households exceeds the amount of assets that the economy is able to produce (debt issued by the young). The ZLB prevents the interest rate from adjusting so that the supply of assets (and the consumption) of the young is restored. Without any other asset in the economy, equilibrium is reestablished through a drop in output that, by reducing income, decreases the demand for bonds by the middle-aged. The key role played by bubbles is to avoid the drop in output by reequilibrating the demand and the supply of liquidity through an increase in asset supply. Once the old are allowed to sell bubbles to the middle-aged, the supply of assets rises, equalizing its demand at full employment. The old are, thus, compensating for the fall in the consumption of the young that follows a tightening of their financing constraint. As they provide additional liquidity, bubbles raise aggregate demand and wealth, sustaining higher levels of consumption and employment.[11] As shown in appendix A.2, all generations are unambiguously better off at the bubble equilibria since all agents consume more relative to the stagnation equilibrium. In the presence of nominal rigidities, safe bubbles improve welfare by allowing the economy to avoid stagnation and increasing the consumption of all generations. In this sense, bubbles can be desirable in times of stagnation.

[11]The model implies that, relative to stagnation, bubbles raise aggregate wealth relative to income, measured by the ration $\frac{B}{Y}$, which is consistent with the behavior of the wealth-to-income ratios depicted in figure 2. Moreover, the implied behavior of inflation and output during bubbly episodes is consistent with evidence provided in the literature: Martin and Ventura (2012b) regarding the positive comovement between output (as well as consumption) and bubbles; Bordo, Dueker and Wheelock (2007), Christiano, Ilut, Motto and Rostagno (2010) and Ikeda (forthcoming) which suggest that inflation tends to be moderate during stock market booms. In the model, bubble equilibria are consistent with "moderate" inflation: compared to the stagnation steady state, bubble equilibria are associated with higher inflation, but the latter is still equal to or below the target, as reflected in equation (24).

4 Stochastic bubbles

The previous section discussed the implications of bubbly episodes for economies in stagnation under the assumption that bubbles are safe assets. In practice, however, bubbly episodes are generally associated with large and volatile movements in asset prices that are difficult to predict. In this section, I consider the possibility of stochastic bubble destruction and discuss the implications of this type of risk for the steady-state equilibria. I focus on the equilibria *before* bubble risk is realized.

4.1 Equilibrium

Consider an economy that is at steady state with a positive bubble, which may collapse in any future period with probability $\lambda \in (0, 1]$. The Euler equations (14) and (15) give rise to the following expression for the steady-state risk premium

$$\frac{r^b - r}{1 + r} = \frac{\lambda}{1 - \lambda} \frac{B(1 + r^b) + D}{D} \tag{25}$$

That is, stochastic bubbles command a risk premium that is greater when the bubble is riskier (greater λ) or when it is larger relative to D. In a steady-state equilibrium, the net return on the bubble conditional on no collapse is equal to zero, $r^b = 0$, so that (25) implies a decreasing relation between the riskless rate and the bubble: a larger bubble raises the risk premium which implies a lower risk-free rate.[12]

On the other hand, the optimality condition (15) and bond market clearing imply an increasing relation between the interest rate r and the bubble B,

$$\left[1 + \frac{\beta(1 - \lambda)}{1 + r}\right] D + \left[1 + \beta(1 - \lambda)\right] B = \beta(1 - \lambda)(Y - D) \tag{26}$$

Intuitively, a higher interest rate lowers the value of other assets, which requires a larger bubble to absorb the supply of savings for any given income Y.

The equilibrium interest rate r and the equilibrium bubble B can be found by combining equations (25) and (26), which yield

$$1 + r = \frac{D}{\frac{\beta}{1+\beta} \lambda(Y - D) + D} \tag{27}$$

[12]The fact that risky bubbles command a positive risk premium is a steady state result. Outside of the bubble steady state, the return on the bubble need not be larger than that of the safe bond. For instance, the realized (net) return on a risky bubble that bursts falls below the return of the safe asset and is equal to -1.

$$B = (1-\lambda)\frac{\beta}{1+\beta}(Y - D) - D \tag{28}$$

In equilibrium, the size of the steady state bubble shrinks with its riskiness: the greater the risk of collapse, the larger is the bubble risk premium and the lower is the riskless rate. This raises the value of the non-bubble assets and reduces the size of the equilibrium bubble.

4.2 Stochastic bubbles and stagnation

To understand the aggregate implications of stochastic bubbles in a secular stagnation, it is useful to analyze their impact on the natural interest rate. Similar to the deterministic case, a positive stochastic bubble raises the natural interest rate relative to the fundamental state, making the stagnation equilibrium less likely. However, the risk of collapsing affects the feasibility of bubbles and, therefore, their potential to expand economic activity. The intuition is as follows. As clear from (27) and (28), a riskier bubble is smaller and is associated with a lower natural interest rate than a safe bubble. A lower natural interest rate raises the level of inflation that is required for a steady state full employment equilibrium to exist.[13] If the inflation target is too low relative to this required level of inflation, no bubble equilibria exist. In other words, full employment equilibria become less likely as the risk of collapse rises because they require a higher level of inflation, which eventually may not be consistent with the inflation target. Potentially, if the risk of collapse is too high, no bubble equilibria exist and the economy settles at the stagnation trap.

The previous discussion suggests that stochastic bubbles are less likely to avoid stagnation than safe bubbles. In this sense, safe bubbles are preferred to stochastic bubbles. Interestingly, however, some risk may improve welfare provided it is not too high. To understand the intuition behind this result, consider the set of bubbles that are feasible and consistent with full employment. These are, thus, bubbles that are not "too risky" relative to the central bank's inflation target. The steady state consumption associated with this set of bubbles can be expressed as an explicit function of λ as follows:

$$C^y = \lambda\frac{\beta}{1+\beta}(Y^f - D) + D \tag{29}$$

$$C^m = \frac{1}{1+\beta}(Y^f - D) \tag{30}$$

[13] Recall from the discussion at the end of section 3.1 that the ZLB imposes a lower bound on the rate of inflation that is consistent with steady state equilibria.

$$C^o = (1-\lambda)\frac{\beta}{1+\beta}(Y^f - D) \tag{31}$$

The lower interest rate associated with a riskier bubble (i.e. higher λ) relaxes the borrowing constraint of the young households and allows them to consume more. Conversely, a riskier bubble reduces old age consumption due to lower returns. Given the lifetime utility implied by (29)-(31), it is easy to show that a marginal increase in bubble risk, λ, raises welfare whenever[14]

$$\lambda < \frac{1}{1+\beta^2}\Big[1 - \beta(1+\beta)\frac{D}{Y^f - D}\Big]. \tag{32}$$

Intuitively, for low values of λ, the equilibrium bubble is large and the riskless rate is high. When borrowing constraints are tight, welfare can be improved by redistributing consumption from the old savers to the young borrowers, who are consuming too little. As λ rises, the equilibrium bubble shrinks and the old consume less, while the young households borrow and consume more. This raises welfare up to the point at which λ exceeds the threshold implied by (32). For even risker bubbles, i.e., λ larger, the previous redistribution from the old to the young is no longer welfare improving. Above the threshold, the old become the ones who consume too little compared to the optimal consumption profile, such that greater risk becomes undesirable.[15]

Overall, the welfare implications of stochastic bubbles generally depend on the strength of the two effects: on the likelihood of stagnation and on the redistribution of consumption across generations. These, in turn, crucially depend on the riskiness of the bubble through its impact on the interest rate. One thing to notice is that the size of a bubble is not necessarily optimal *per se*, which raises the question of whether the government can intervene to improve it. The next section addresses this point and discusses the potential for fiscal policy to complement an existing bubble.

5 Fiscal policy and bubbles

The literature has sometimes argued that, in low interest rate environments, bubbles are similar to government debt. For example, Caballero and Farhi (2017) argue that, in a safety trap, safe bubbles mitigate a shortage of safe assets, similar to public debt. In this section,

[14]See the appendix B.1 for further details.

[15]Recall that this comparison is between bubble equilibria conditional on a given level of output. In this case, bubble risk affects the distribution of consumption across the different generations due to its impact on the riskless rate. In addition to this channel, bubble risk can also affect the level of equilibrium output, as discussed in the first part of section 4.

I further explore the comparison between fiscal policy and bubbles, and how they interact with each other in a secular stagnation. First, I discuss the role of fiscal policy in the context of safe bubbles. Subsequently, I reintroduce stochastic bubble destruction and discuss how fiscal policy may interact with risky bubbles in this setting.

5.1 Fiscal policy with safe bubbles

Fiscal policy refers to a set choices about the level and distribution of taxes, public debt and government spending subject to a government budget constraint. This section discusses how fiscal policy affects the feasibility and size of bubbles.

Let G denote government spending, which can be financed through lump-sum taxes on the different generations, T^y, T^m and T^o, or by issuing public debt A^g. The households' and the government's budget constraints can be written as follows:

$$C_t^y = -A_t^y - T_t^y \tag{33}$$

$$C_{t+1}^m = Y_t^m + (1 + r_t)A_t^y - A_{t+1}^m - B_{t+1} - A_{t+1}^g - T_{t+1}^m \tag{34}$$

$$C_{t+2}^o = A_{t+1}^m(1 + r_{t+1}) + B_{t+2} + A_{t+1}^g(1 + r_{t+1}^g) - T_{t+2}^o \tag{35}$$

$$T_t^y + T_t^m + T_t^o + A_t^g = G_t + (1 + r_{t-1}^g)A_{t-1}^g \tag{36}$$

I assume that government spending and the level of real government debt are exogenously given by $G = G^*$ and $A^g = A_*^g$. Similarly, I assume that the tax on the young is exogenous and given by $T^y = T_*^y$. The steady state demand for (private) bonds can be derived following similar steps to equation (19), and is given by

$$A^m = \frac{\beta}{1+\beta}(Y^m - D - T^m) - B - A_*^g + \frac{T^o}{(1+r)(1+\beta)} \tag{37}$$

Imposing bond market clearing yields the equilibrium interest rate:

$$1 + r = \frac{D - \dfrac{T^o}{1+\beta}}{\dfrac{\beta}{1+\beta}(Y^m - D - T^m) - B - A_*^g} \tag{38}$$

The impact of fiscal policy on the real interest rate crucially depends on how government spending is financed. All else equal, an increase in government debt or higher taxes on the middle-aged lead to a rise in the interest rate, as they reduce the demand for bonds. On the other hand, higher taxes on the old generation reduce the interest rate because middle-aged households increase desired savings in anticipation of the future taxes that they will have to pay.

In a secular stagnation, fiscal policy can be a powerful tool to expand aggregate demand and eliminate the stagnation equilibrium (Eggertsson et al., 2019). Essencially, this depends on the ability of the government to permanently reduce the shortage of assets, either by expanding their supply or by reducing their demand. In addition, fiscal policy directly affects the feasibility and size of bubbles. To see this, note that if there were no constraints on fiscal policy, the government could, in principle, issue debt and collect taxes in a way to raise the natural interest rate enough to achieve full employment. Under these circumstances, bubbles would not be sustainable in the long-run. It follows that bubbles can only coexist with fiscal policy if some asset shortage persists, for example due to (exogenous) constraints on the implementation of fiscal policy. This can also be deduced from the expression of the steady state bubble, $B^* = \frac{\beta}{1+\beta}(Y^m - D - T^m) - D - A_*^g + \frac{T^o}{1+\beta}$, which is feasible if

$$\frac{\beta}{1+\beta}(Y^m - D - T^m) + \frac{T^o}{1+\beta} > D + A_*^g \tag{39}$$

That is, a positive bubble is feasible in equilibrium if the desired savings of the middle-aged households (left-hand side) exceed the supply of non-bubble assets (right-hand side). In turn, the desired savings depend on the taxes levied on the middle-aged and old, T^m, T^o, while the supply of assets depends on the stock of public debt, A_*^g.

5.2 Are bubbles and government debt substitutes?

As previously discussed, fiscal policy, and government debt in particular, can be used to mitigate asset shortages and raise the natural interest rate (Eggertsson et al., 2019). Similarly, we have seen that bubbles can raise the natural interest rate when the latter is depressed by structural factors. Thus, a natural question is whether bubbles and fiscal policy are substitutes in a secular stagnation.

It is useful to compare the natural interest rate associated with a bubble steady state, and the natural interest rate associated with a fundamental equilibrium with fiscal policy. From section 3, the bubble steady state is characterized by an interest rate $r^* = 0$ and a bubble of size $B = \frac{\beta}{1+\beta}(Y^f - D) - D$. Turning to the fundamental equilibrium with fiscal policy, the natural interest rate is determined by equation (38) with $Y^m = Y^f$. The latter is equal to zero if

$$\frac{\beta}{1+\beta}T^m + A_*^g - \frac{T^o}{1+\beta} = \frac{\beta}{1+\beta}(Y^f - D) - D. \tag{40}$$

That is, the two rates are identical if the net effect of fiscal policy on the natural interest rate equals the size of the equilibrium bubble in steady state. Note, however, that even if aggregate consumption and output were identical in the two equilibria, welfare need not be the same necessarily. Indeed, for welfare to be equalized, it must be that the distribution of taxes is such that the consumption of each generation is the same in the two sets of equilibria, which requires $T^y = 0$ and $T^m = -T^o$. Finally, the government budget constraint (36) must hold, which implies $T^m = \frac{\beta}{1+\beta}(Y^f - D) - D - A_*^g + \frac{G^*}{1+\beta}$.

Fiscal policy and bubbles share the ability to mitigate asset shortages and, through this channel, to expand aggregate demand and ouput. However, the overall effects depend on how they affect the level and the distribution of consumption across households. In turn, this depends on: (1) the specific design of fiscal policy (i.e., the size and the distribution of taxes, transfers and debt), and (2) the impact on the equilibrium interest rate. Therefore, even if fiscal policy and bubbles share the potential to mitigate a shortage of assets, the overall aggregate and distributional implications may differ. In addition to the previous points, it is worth noting that, while fiscal policy may eliminate the stagnation equilibrium, allowing agents to trade bubbles does not, alone, eliminate the possibility of stagnation.[16] Indeed, it is still possible that individuals coordinate on the bubbleless equilibrium, and the economy lands in a stagnation trap.

5.3 Fiscal policy with risky bubbles

This section reintroduces stochastic bubble destruction, and focuses on whether fiscal policy can *interact* with risky bubbles in a secular stagnation.[17] Recall, from section 4, that the riskiness of a bubble affects its feasibility and size. Given this observation, I now ask whether the government can design policies to affect the size of a bubble and, through this channel, improve welfare. As before, I focus on bubble equilibria before risk is realized.

It is useful to illustrate the point with a simple example. Consider an economy that is at steady state with a positive bubble. As before, the bubble may collapse at any future period with probability λ, after which it remains equal to zero forever. Now consider the following government policy: while the bubble is positive and the economy is at full employment, the government does not intervene. However, if the bubble collapses, the government promises to intervene by making a transfer to the households that had purchased the bubble

[16] See Eggertsson et al. (2019) for the result that fiscal policy can eliminate the stagnation equilibrium.
[17] I thank an anonymous referee for the suggestion.

that eventually collapsed.[18] This transfer is financed by a tax imposed on the middle-aged generation. How does this policy affect the bubble equilibrium? At time t, the households' budget constraints are given by

$$C_t^y = -A_t^y \tag{41}$$

$$C_t^m = Y_t^m + (1 + r_{t-1})A_{t-1}^y - A_t^m - B_t \tag{42}$$

$$C_t^o = A_{t-1}^m(1 + r_{t-1}) + B_t \tag{43}$$

At time $t + 1$, the households' budget constraints depend on whether the bubble collapses, and are given by

$$C_{t+1}^y = -A_{t+1}^y \tag{44}$$

$$C_{t+1}^m = \begin{cases} Y_{t+1}^m + (1 + r_t)A_t^y - A_{t+1}^m - T_{t+1}^m & \text{w.p.} \quad \lambda \\ Y_{t+1}^m + (1 + r_t)A_t^y - A_{t+1}^m - B_{t+1} & \text{w.p.} \quad 1 - \lambda \end{cases} \tag{45}$$

$$C_{t+1}^o = \begin{cases} A_t^m(1 + r_t) - T_{t+1}^o & \text{w.p.} \quad \lambda \\ A_t^m(1 + r_t) + B_{t+1} & \text{w.p.} \quad 1 - \lambda \end{cases} \tag{46}$$

As shown in appendix C, such policy gives rise to the following equilibrium bubble and bubble risk premium:

$$B = \frac{\beta}{1 + \beta} \frac{(1 - \lambda)(Y - D)(T^o - D)}{(T^o - D) - \beta\lambda T^o} - D \tag{47}$$

$$\frac{r^b - r}{1 + r} = \frac{\lambda\beta(Y - D)}{(1 + \beta)D - T^o[1 + \beta(1 - \lambda)]} \tag{48}$$

Equation (47) defines a decreasing relation between T^o and the size of the bubble, implying that a higher transfer to the old in the case that the bubble collapses, improves the size of the equilibrium bubble before that happens. Under this policy, the current middle-aged generation – the one purchasing the bubble – internalizes that, if the bubble collapses in the following period, they will receive a transfer from the government. This lowers the bubble risk premium, and raises the size of the equilibrium bubble. This example ilustrates how fiscal policy can be used to complement a bubble in a stagnation environment, even if it

[18] Assume that such government announcement is credible.

cannot directly affect the likelihood of a bubble collapse. Such complementarity may be desirable because, as discussed in section 4, bubble equilibria are not necessarily optimal, and fiscal policy can be a useful instrument to affect the size of a bubble and improve welfare.

Of course, the previous example raises several questions. Would such government announcement be credible? What would be the distributional consequences across generations? The previous discussion is not meant to be a full analysis of optimal fiscal policy in the presence of stochastic bubbles, which would exceed the scope of this paper. But it does highlight that fiscal policy can be used, not just as a substitute, but also as a complement to stochastic bubbles in a secular stagnation.

6 Concluding remarks

The model developed in this paper provides a framework to think about the role of asset bubbles in stagnant economies. Here, a severe shortage of assets depresses the interest rate and output. Low interest rates open the door for the emergence of bubbles that, in this setting, can be expansionary as they increase the supply of assets and prevent output from falling below its potential level. For this to happen, however, bubbles cannot be too risky. If the risk of collapsing is too large, bubbles are no longer feasible and they fail to expand economic activity. Although highly stylized, the model allows us to focus on the role of bubbles in the type of economies vulnerable to stagnation, and provides a formalization of the much discussed relationship between bubbles and the secular stagnation hypothesis.

Even though growth recovered in the major developed economies after the Great Recession, the pace of global economic activity remains sluggish. Moreover, the structural forces that explain the steady decline in interest rates over the past decades are still at play (e.g. population ageing). Close to the ZLB the effectiveness of conventional monetary policy in counteracting negative demand shocks is limited. In this paper, I show how sufficiently safe bubbles provide a mechanism to sustain aggregate demand and a higher level of employment in such an economy. Moreover, I discuss how fiscal policy compares to, and potentially complements, bubbles in a secular stagnation. While bubbles can be expansionary, in principle they can also push the economy into a slump when they burst, implying that policy may be desirable to deal with the drawbacks of this instability. Further research on the interactions between asset bubbles, demand and economic activity will be useful to shed light on this question.

Appendix A Deterministic bubbles

A.1 Flexible wages: bubbles and welfare

In this section I compare the two steady states of the flexible wage economy — depicted in figure (3)— and analyze the welfare implications of asset bubbles. I compare lifetime utility in both steady states — X and Y — and show that, whenever a positive bubble is possible, it is welfare reducing. I focus on the case where $\frac{\beta}{1+\beta}(Y^f - D) > D$ so that bubbles are possible. Otherwise, the only possible equilibrium would be the bubbleless one.

Let us start by considering the bubbleless equilibrium — point X. Here, $B^* = 0$ and each generation's steady state consumption is given by

$$C^y = \frac{\beta}{1+\beta}(Y^f - D) \tag{49}$$

$$C^m = \frac{1}{1+\beta}(Y^f - D) \tag{50}$$

$$C^o = D \tag{51}$$

Given the agents' preferences, the lifetime utility associated with this consumption pattern is given by

$$U^X = log\Big[\frac{\beta}{1+\beta}(Y^f - D)\Big] + \beta \cdot log\Big[\frac{1}{1+\beta}(Y^f - D)\Big] + \beta^2 \cdot log(D) \tag{52}$$

Let us now turn to equilibrium Y with a positive steady state bubble given by $B^* = \frac{\beta}{1+\beta}(Y^f - D) - D$ and $r^* = 0$. In this equilibrium, each generation's consumption is given by

$$C^y = D \tag{53}$$

$$C^m = \frac{1}{1+\beta}(Y^f - D) \tag{54}$$

$$C^o = \frac{\beta}{1+\beta}(Y^f - D) \tag{55}$$

Note that, compared to equilibrium X, the middle-aged consume the exact same amount, but the young agent swaps their consumption level with the old individual. Expected lifetime utility is given by

$$U^Y = log(D) + \beta \cdot log\Big[\frac{1}{1+\beta}(Y^f - D)\Big] + \beta^2 \cdot log\Big[\frac{\beta}{1+\beta}(Y^f - D)\Big] \tag{56}$$

By comparing (52) and (56) we can investigate which equilibrium yields higher lifetime utility. Whenever $\frac{\beta}{1+\beta}(Y^f - D) > D$, the bubbleless equilibrium is welfare superior to the bubble one.[19]

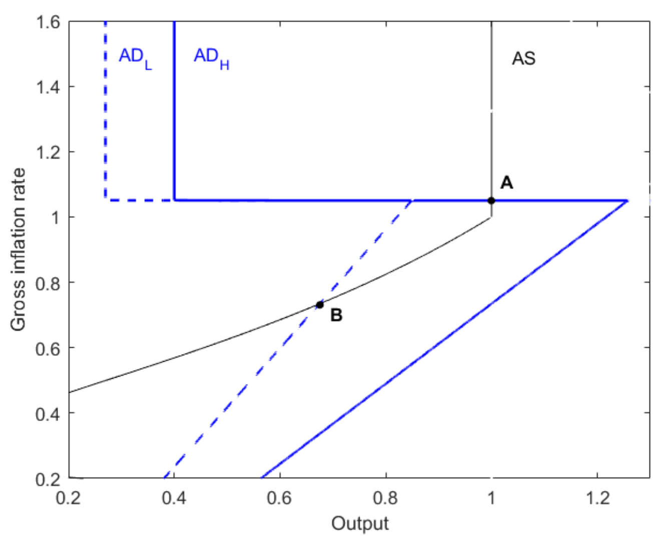
$$U^X - U^Y = (1-\beta^2) \cdot log\left[\frac{\frac{\beta}{1+\beta}(Y^f - D)}{D}\right] > 0 \tag{57}$$

A.2 Nominal wage rigidities

A.2.1 Aggregate demand and aggregate supply curves

Figure 4 depicts the aggregate demand and supply curves. The solid black line is the AS curve with a kink at $\Pi = 1$. The two demand curves, AD_H and AD_L, depict aggregate demand for two different sets of parameters. In particular, the value of D is larger for the AD_H curve.[20] Aggregate demand has a kink at the point in which $i = 0$, below which the ZLB binds. The upward sloped segment of the AD curve reflects the fact that, for $i = 0$, the real interest rate falls with inflation, stimulating consumption.

Figure 4: Steady state aggregate demand and supply curves



[19] Assuming $\beta \in (0, 1)$.
[20] To produce figure 4, ϕ_π is set to 5000, as an approximation to $\phi_\pi \to \infty$.

Point A depicts the equilibrium of the economy when D is *high*. In this equilibrium, the natural interest rate – the rate at which the demand and the supply of savings equalize at full employment – is positive. Moreover, it features positive inflation, determined by the inflation target, and zero unemployment. In contrast, point B represents an equilibrium under tight financing conditions (low D). This is an equilibrium with permanent unemployment and a binding ZLB (stagnation trap).

A.2.2 Bubbles and welfare

In the bubble equilibria, output is at full employment, Y^f, and each generation's consumption is given by

$$C^y = D \tag{58}$$

$$C^m = \frac{1}{1+\beta}(Y^f - D) \tag{59}$$

$$C^o = \frac{\beta}{1+\beta}(Y^f - D) \tag{60}$$

In the stagnation equilibrium, output falls below full employment. Recall that in this equilibrium, output is given by $Y = \left(\frac{1-\gamma}{1-\gamma\Pi^{-1}}\right)^{\frac{\alpha}{\alpha-1}} Y^f$ and $\Pi < 1$. Since, at this equilibrium, $(1+r) = \Pi^{-1}$, it must also be that $(1+r) > 1$. Each generation's consumption is given by

$$C^y = \frac{D}{1+r} < D \tag{61}$$

$$C^m = \frac{1}{1+\beta}(Y - D) < \frac{1}{1+\beta}(Y^f - D) \tag{62}$$

$$C^o = D \tag{63}$$

where I used the fact that $(1+r) = \frac{D}{\frac{\beta}{1+\beta}(Y-D)}$ to get (47). Since I am considering the case $\frac{\beta}{1+\beta}(Y^f - D) > D$, it follows that all generations consume more in the bubble equilibria than in the stagnation one. The bubble equilibria are, thus, associated with higher lifetime utility than the stagnation equilibrium.

Appendix B Stochastic bubbles

B.1 Bubbles, risk and welfare

Consider the set of bubbles that are feasible and consistent with full employment (bubble) equilibria. In steady state, the consumption of each generation is given by (29)-(31). Given households' preferences, the lifetime utility associated with this consumption profile is given by

$$U(\lambda) = log\Big[\lambda\frac{\beta}{1+\beta}(Y^f - D) + D\Big] + \beta \cdot log\Big[\frac{1}{1+\beta}(Y^f - D)\Big] + \beta^2 \cdot log\Big[(1-\lambda)\frac{\beta}{1+\beta}(Y^f - D)\Big] \tag{64}$$

Taking the derivative of (64) with respect to λ yields

$$\frac{\partial U(\lambda)}{\partial \lambda} = \frac{\beta}{1+\beta}(Y^f - D)\left[\frac{1}{C^y} - \frac{\beta^2}{C^o}\right]. \tag{65}$$

This expression is positive whenever $\beta^2 C^y < C^o$, which is equivalent to the condition that $\lambda < \frac{1}{1+\beta^2}\left[1 - \beta(1+\beta)\frac{D}{Y^f - D}\right]$.

Appendix C Fiscal policy with risky bubbles

Given the problem described in section 5.3, optimization by the middle-aged at time t yields the following first-order conditions:

$$\frac{1}{C_t^m} = \beta(1+r_t^b)\left[\frac{\lambda(1+r_t)}{A_t^m(1+r_t) - T_{t+1}^o} + \frac{(1-\lambda)(1+r_t)}{A_t^m(1+r_t) + B_{t+1}}\right] \tag{66}$$

$$\frac{1}{C_t^m} = \frac{\beta(1-\lambda)(1+r_t^b)}{A_t^m(1+r_t) + B_{t+1}} \tag{67}$$

Rewriting equation (66) and imposing bond market clearing yields the following expression for the equilibrium bubble as function of the interest rate r,

$$B = \frac{1}{1+\beta(1-\lambda)}\left\{\beta(1-\lambda)(Y - D) - \left[1 + \frac{\beta(1-\lambda)}{1+r}\right]D\right\} \tag{68}$$

where I have used $r^b = 0$. On the other hand, equations (66) and (67) imply the following expression for the interest rate:

$$1 + r = \frac{(1-\lambda)(T^o - D)}{(1-\lambda)T^o - D - \lambda B} \tag{69}$$

Using (69) in (68) yields the expression in equation (47) for the equilibrium bubble. Finally, combining (47) and (69) allows me to obtain an expression for the risk premium as in (48).

References

Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe, "The Perils of Taylor Rules," *Journal of Economic Theory*, 2001, *96* (1), 40–69.

Benigno, Gianluca and Luca Fornaro, "Stagnation traps," *The Review of Economic Studies*, 2017, *85* (3), 1425–1470.

Biswas, Siddhartha, Andrew Hanson, and Toan Phan, "Bubbly recessions," *American Economic Journal: Macroeconomics*, 2020, *12* (4), 33–70.

Bordo, Michael D, Michael Dueker, and David C Wheelock, "Monetary Policy and Stock Market Booms and Busts in the 20th Century," *FRB of St. Louis Working Paper No. 2007-020A*, 2007.

Caballero, Ricardo and Arvind Krishnamurthy, "Bubbles and capital flow volatility: Causes and risk management," *Journal of monetary Economics*, 2006, *53* (1), 35–53.

_ **and Emmanuel Farhi**, "The safety trap," *The Review of Economic Studies*, 2017, *85* (1), 223–274.

Christiano, Lawrence, Cosmin Ilut, Roberto Motto, and Massimo Rostagno, "Monetary Policy and Stock Market Booms," Working Paper 16402, National Bureau of Economic Research September 2010.

Eggertsson, G. and M. Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *2003* (1), 139–211.

_ **and P. Krugman**, "Debt, Deleveraging, and the Liquidity Trap: a Fisher- Minsky-Koo Approach," *The Quarterly Journal of Economics*, 2012, *127* (3), 1469–1513.

Eggertsson, Gauti B, Neil R Mehrotra, and Jacob A Robbins, "A model of secular stagnation: Theory and quantitative evaluation," *American Economic Journal: Macroeconomics*, 2019, *11* (1), 1–48.

Farhi, Emmanuel and Jean Tirole, "Bubbly liquidity," *The Review of Economic Studies*, 2012, *79* (2), 678–706.

Guerrieri, Veronica and Guido Lorenzoni, "Credit crises, precautionary savings, and the liquidity trap," *The Quarterly Journal of Economics*, 2017, *132* (3), 1427–1467.

Ikeda, Daisuke, "Monetary policy, Inflation and Rational Asset Price Bubbles," *Journal of Money, Credit and Banking*, forthcoming.

Krugman, Paul, "Bubbles, regulation, and secular stagnation," *The Conscience of a Liberal*, 2013, *25.*

_ , "Secular stagnation, coalmines, bubbles, and Larry Summers," *New York Times*, 2013, *16*, 2013.

Martin, Alberto and Jaume Ventura, "Economic growth with bubbles," *American Economic Review*, 2012, *102* (6), 3033–58.

_ **and** _ , "Economic Growth with Bubbles," *American Economic Review*, 2012, *102* (6), 3033–3058.

_ **and** _ , "Managing credit bubbles," *Journal of the European Economic Association*, 2016, *14* (3), 753–789.

Rachel, Lukasz and Lawrence H Summers, "On Secular Stagnation in the Industrialized World," Technical Report, National Bureau of Economic Research 2019.

Samuelson, Paul, "An Exact Consumption-loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 1958, *6*, 467–482.

Summers, Lawrence, "Why stagnation might prove to be the new normal," *Financial Times*, 2013, *15*, 12.

Tirole, Jean, "Asset Bubbles and Overlapping Generations," *Econometrica*, 1985, *53* (6), 1499–1528.