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LINVER: The Linear Version of FRB/US*

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Abstract

FRB/US, a large-scale, nonlinear macroeconomic model of the U.S., has been in use at the Federal Reserve Board for 25 years. For nearly as long, the FRB/US "project" has included a linear version of the model known as LINVER. A key reason that LINVER exists is the vast reduction in the computational costs that linearity confers when running experiments requiring large numbers of simulations under the assumption that expectations are model-consistent (MC). The public has been able to download FRB/US simulation code, documentation, and data from the Federal Reserve Board's website since 2014. To further expand access to and understanding of the FRB/US project, a package devoted to LINVER is now available on the website. In this paper, we provide both a general introduction to LINVER and an overview of the contents and capabilities of its package. We review the ways that LINVER has been used in past research to study key policy issues; describe the package's comprehensive set of programs for running simulations with MC expectations, with or without imposing the effective lower bound (ELB) on the federal funds rate and other nonlinear constraints; and illustrate how LINVER deterministic and stochastic simulations can be used to gauge the implications of the ELB for macroeconomic performance and to assess different strategies for mitigating its adverse effects.

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1. Introduction

FRB/US, a large-scale, nonlinear macroeconomic model of the U.S., has been in use at the Federal Reserve Board for 25 years.[1] For nearly as long, the FRB/US "project" has included a linear version of the model known as LINVER. (For clarity, we will refer to the nonlinear version as FRB/US in this note.) The public has been able to download FRB/US model code, documentation, data, and simulation programs from the Federal Reserve Board's website since 2014. To further expand access to and understanding of the FRB/US project, a package devoted to LINVER is now available to download from the website. This paper aims to provide a general introduction to LINVER and to present some examples of the types of simulation experiments that can be run with the package's programs; this information should prove useful to users who wish to run model simulations as well as readers of research papers based on LINVER. (Detailed information on running LINVER simulations can be found in the package's *LINVER User's Manual*.)

Why does LINVER exist? Linear models have simple analytic solutions, whereas solutions of nonlinear models require slower iterative numerical methods. The speed advantage of linearity is especially pronounced in models that assume at least some economic agents form accurate expectations of future outcomes as generated by simulations of the overall model.[2] Expectations of this type, which are usually described as rational or model-consistent (MC), add future-dated endogenous variables to a model. If the model is nonlinear, the presence of such leads substantially increases the computational costs of solving it. If the model is linear and satisfies a stability condition, on the other hand, the model has a unique reduced form that has no leads and which, once created, can be solved very efficiently. LINVER satisfies the stability requirement. A key reason that LINVER exists is the vast reduction in the computational costs of running experiments requiring large numbers of simulations under MC expectations.

A distinctive feature of FRB/US and LINVER is the availability of two ways that the expectations of agents in different sectors of the economy can be formed. One is the MC case just discussed. In the other option, expectations are based on the average historical dynamics of

[1] For information about FRB/US, see Brayton, Laubach, and Reifschneider (2014).
[2] In the case of the package's programs for LINVER simulations, such model-consistent expectations conform with the predictions of the overall model when simulated under the assumption that the economy will not be subject to shocks in the future. This assumption implies that agents do not have perfect foresight about the future but do have a complete understanding of how the effects of current and past shocks will play out over time.

the economy as manifested in the predictions of estimated limited-information VAR models. VAR-based expectations, because they do not add leads to either FRB/US or LINVER, do not present the computational challenges that arise in the MC case. In addition to assuming that all expectations are either entirely VAR-based or entirely MC, mixed-case simulations are permitted in which some agents have MC expectations while others do not. Two frequently used mixed cases are one in which only expectations associated with financial markets are MC, and another in which the expectations associated with both financial markets and price and wage setting are MC. The technical difficulties associated with MC expectations arise when all or any subset of expectations are of this type.

The uses of LINVER would be severely circumscribed if its simulations could not take account of the effective lower bound (ELB) on nominal interest rates that prevents central banks from reducing their policy rates materially below zero. Because the normal level of interest rates in the major developed economies has fallen markedly over the past 30 years, the ELB constraint has become a major concern of policymakers. Obviously, the ELB is a nonlinearity and solving LINVER with the ELB thus requires an iterative method. Nonetheless, it is much easier to solve LINVER when augmented to include the ELB restriction as a single nonlinearity than it is to solve FRB/US similarly augmented, especially under the assumption of MC expectations. Moreover, the computational advantages of a linear model are also preserved when additional nonlinearities designed to mitigate the adverse effects of the ELB are introduced in the policy rules (monetary and fiscal) themselves. Researchers have used LINVER and other linear models in a large number of studies to assess the implications of the ELB for macroeconomic performance and ways to mitigate its adverse effects.

The main part of the LINVER package is a comprehensive set of programs for running simulations with MC expectations, with or without the ELB and other nonlinear constraints imposed. These programs are written in Matlab for use with Dynare. The package also provides versions of these programs that run in Octave. Matlab and Octave provide the advanced matrix algebra commands that Dynare needs to create the reduced form of a linear economic model containing future-dated variables. Matlab is a commercial product; Dynare and Octave are freely available. The package also includes programs written in EViews, another commercial product. EViews is not capable of computing LINVER's reduced form, which limits its usefulness for simulations with MC expectations, but it does provide a useful platform for simulations when all expectations are assumed to be VAR-based.

The structure of the rest of the paper is as follows. First, we briefly review the ways in which previous incarnations of LINVER have been used to study a range of policy issues, including the effects of the ELB. We then turn to a number of technical issues. Sections 3 discusses the way LINVER is constructed and show that its dynamic properties are similar to those of FRB/US. Section 4 discusses a procedure known as stochastic simulation, which is commonly used to calculate the expected probability distribution of a model's endogenous variables, especially when a model is nonlinear, as is the case with LINVER and the ELB. Such estimated probability distributions, when generated using simulations conditioned on a particular monetary strategy and expected value of the ELB, can be used to gauge how that strategy might influence future macroeconomic performance. In section 5, we provide an overview of the efficient iterative solution procedures the package uses to impose the nonlinear ELB restriction on the stochastic simulations of an otherwise linear model.

When the neutral level of the nominal federal funds rate is low, adverse shocks often cause the ELB to bind for extended periods in LINVER simulations, and under some circumstances this situation can result in dynamics that moderately or even severely unstable. As discussed in section 6, the LINVER simulation programs contain several ways to make the model more stable, with a goal of preventing outcomes that would be regarded as unrealistic. One approach increases the degree to which fiscal policy is countercyclical symmetrically in periods of economic slack and strength; another increases the degree of fiscal countercyclicality but only in extremely deep recessions; and a third modifies the countercyclicality of term premiums embedded in long-term Treasury yields.[3] The default in the programs is to increase the degree of symmetric fiscal policy countercyclicality whenever all agents have MC expectations, which is the expectation setting for which stability issues are most substantial. Also, an asymmetric fiscal response to deep recession is included by default whenever the ELB is

[3] These types of stabilizing adjustments have been employed in various combinations in previous research that has used LINVER to study the implications of the ELB. Chung *et al* (2019) and Arias *et al* (2020) increase the countercyclicality of symmetric fiscal policy, while all other such research papers have employed some type of nonlinear asymmetric fiscal mechanism to deal with extreme economic contractions. The countercyclicality of term premiums is suppressed in Kiley and Roberts (2017), Bernanke, Kiley, and Roberts (2019), Bernanke (2020), Chung *et al* (2019), and Arias *et al* (2020).

imposed. Users of the simulation programs may modify these settings as well as invoke the alternative characterizations of term premiums.

The last two sections of the paper illustrate how LINVER simulations can be used to gauge the implications of the ELB for future macroeconomic performance and to assess different strategies for mitigating its adverse effects. To this end, section 7 uses a recession scenario to illustrate how the ELB constraint on monetary policy can cause economic downturns to become deeper and more protracted, as well as to provide some intuition on how policymakers may be able to mitigate the ELB's adverse effects by influencing expectations about their future policy actions. The section then provides some illustrative estimates of these adverse effects, computed using different assumptions about the way in which expectations are formed. Finally, in section 8, we illustrate how the programs and code available in the LINVER package can be used to assess different strategies for mitigating the effects of the ELB.

2. Previous Research using LINVER

Since the first version of LINVER was created in the late 1990s, it has played a prominent role in two strands of research that have come out of the FRB/US project. The first concerns the "optimal" way to conduct monetary policy under a wide range of economic conditions, ignoring the ELB constraint. The second, in contrast, is focused explicitly on the ELB and seeks to quantify its macroeconomic costs and to evaluate possible ways to mitigate them. Because both types of analyses typically assume that at least some economic agents have MC expectations, they are not computationally feasible to carry out using a large non-linear model like FRB/US.

Much of the LINVER-based research on optimal monetary policy assumes that monetary policy can be reasonably approximated by a simple, generic rule for setting the value of the nominal federal funds rate, $R_t = \alpha R_{t-1} + (1-\alpha)[r^* + \pi_t + \beta(\pi_t - \pi^*) + \gamma Y_t]$. In this rule, the primary short-run determinants of R are the deviation of inflation from its target ($\pi - \pi^*$) and the output gap (Y), whose respective coefficients, β and γ, are expected to be positive. The funds rate adjusts gradually to these determinants if $0 < \alpha < 1$. In equilibrium with inflation at target and an output gap of zero, the nominal funds rate equals the sum of the neutral value of the real funds (r^*) and the target rate of inflation. Using this generic characterization of monetary policy, the researcher then examines what specific parameterization of the rule would minimize some loss function when embedded in LINVER, ignoring the constraint imposed by the ELB.

Following this approach, Williams (2003) uses LINVER to calculate the optimal values of α, β, and γ under various assumptions about policymakers' preferences. Because such calculations are sensitive to the specific structure of the model used in the analysis, Levin, Wieland, and Williams (1999) use LINVER and two other models to investigate the performance of an "optimal" rule computed using one model when applied to another. Levin, Wieland, and Williams (2003) extend this robustness analysis to policy rules in which the prescribed setting of the federal funds rate depends on expected future macroeconomic conditions. In a somewhat different vein, Finan and Tetlow (1999) demonstrate the feasibility of computing a policy rule for LINVER that sets the federal funds rate in a way that makes optimal use of the information in all the model's endogenous variables, while Tetlow (2008) analyzes policy rules in which the inflation target is a band rather than a single value.

The second strand of LINVER-based research involves running stochastic simulations of the model conditional on different characterizations of monetary policy, with and without the ELB imposed. This approach, by simulating the behavior of the economy under a wide range of disturbances, enables researchers to estimate the degree to which the ELB is likely to hurt future macroeconomic performance—a major concern in recent years because of a secular decline in the "normal" level of short-term interest rates that has greatly reduced how much central banks can cut their policy rates to fight recessions. The stochastic-simulation approach also allows researchers to evaluate the potential ability of different monetary policy strategies to mitigate the cost of the ELB relative to that generated under some benchmark policy, such as the Taylor rule.[4]

In an early example of this type of analysis, Reifschneider and Williams (2000) use LINVER stochastic simulations to quantify the macroeconomic costs of the ELB as well as to explore how these costs can be mitigated. Among other things, they find that a "make-up" strategy—specifically, one that credibly pledges to hold the federal funds rate at the ELB for longer than the Taylor rule would otherwise prescribe, in order to make up for the cumulative

[4] Several versions of the Taylor rule are in general use. The original rule proposed by Taylor (1993) sets α to zero and both β and γ to 0.5, while a later version discussed by Taylor (1999) raises γ to 1.0. In subsequent research, such as Bernanke (2020), the latter more activist version of the Taylor rule is often made inertial by setting α to 0.85.

shortfall of policy from the unconstrained Taylor rule during the ELB episode—improves simulated outcomes for inflation and output in LINVER. By providing extended forward guidance about *future* monetary policy, such a make-up strategy enables central banks to provide additional stimulus through expectational effects even when *current* policy is constrained by the ELB. Several subsequent studies have used LINVER to explore other rule-based strategies for mitigating the ELB through extended forward guidance (explicit or implicit), including Kiley and Roberts (2017), Bernanke, Kiley, and Roberts (2019), Chung *et al* (2019), Bernanke (2020), and Arias *et al* (2020). Among the strategies considered by these studies are price-level targeting; symmetric and asymmetric average inflation targeting; rules that respond nonlinearly to changes in economic conditions; change rules that, like the make-up strategy, keep track of an unconstrained shadow rate; and the establishment of threshold conditions for unemployment and/or inflation that must first be satisfied before liftoff from the ELB is allowed to occur.[5]

The programs and code included in the LINVER package allow users to run stochastic simulations conditioned on many of the policy strategies proposed by these researchers for mitigating the adverse effects of the ELB, such as the Reifschneider-Williams make-up rule, the Kiley-Roberts change rule, the average inflation targeting rules discussed by Arias et al, and the use of unemployment and inflation thresholds to delay liftoff from the ELB. To illustrate this capability, the final section of the paper presents stochastic simulation results generated using some selected policy rules and threshold settings under different expectational assumptions. Although these illustrative results are limited in scope and do not constitute a comprehensive evaluation of the effectiveness of different strategies for mitigating the effects of the ELB, they generally accord with the findings of previous researchers in this area.

3. **Linearization Methodology and a Comparison of LINVER and FRB/US IRFs**

The process of creating LINVER has three steps. First, variables are classified by whether they will be linearized in levels or logs. Each variable's classification is based its treatment in FRB/US, which generally uses a log specification for trending variables and

[5] Kiley (2018), Chung *et al* (2019), and Bernanke (2020) also use stochastic simulations to examine the ability of large-scale asset purchases to provide additional stimulus during economic downturns when the ELB constraint binds. These simulations are carried out using a modified version of LINVER in which increases in the stock of longer-term securities held by the Federal Reserve puts downward pressure on the term premiums embedded in bond yields.

specifies other variables in levels. Second, substitutions are made for variables that would be candidates for log linearization if their values were always positive. Examples include replacing the government budget surplus with the difference between the separate variables for government receipts and outlays, the net foreign asset position with the difference between the separate variables for gross assets and liabilities, and inventory investment with the difference between the current and lagged stocks of inventories. The final step creates a symbolic linearization that is evaluated with data that are roughly consistent with an equilibrium for key macroeconomic variables, such as the federal funds rate, inflation, the unemployment rate, and the output gap, and that at the same time are not too far from recent values of household net worth and stocks of other financial assets and physical capital. The current version of LINVER is linearized around average conditions in the 2018-2019 period, when the economy was close to full employment and inflation and interest rates were fairly stable. Although economic models are typically linearized around their steady states, following this practice in FRB/US would require a full stock equilibrium that, because the model's measures of financial and physical stocks adjust only slowly to their equilibrium levels, may be far from their current positions.

As long as the ELB does not bind, FRB/US is not especially nonlinear and thus the dynamics of LINVER are similar to those of FRB/US. This similarity is illustrated in the panels of Figure 1, which compare IRFs for both models for key shocks. In these simulations, financial market participants and wage-price setters are assumed to have a complete understanding of the implications of the shocks for the future evolution of the economy—that is, their expectations are fully model consistent. In contrast, expectations in other sectors, such as those for future household income, are based on less information as reflected in the predictions of small, estimated VAR models; as a result, these expectations are broadly consistent with the average historical behavior of the economy. Monetary policy sets the federal funds rate according to the inertial form of the Taylor rule in the simulations. The figure plots the responses of the output gap, four-quarter core inflation, and the federal funds rate to separate shocks to the equations for the policy rate, consumption, core consumer prices, and the rate of growth of trend multifactor productivity. Even though the shocks are specified as one-time events, the dynamic nature of each of these equations is such that, even when viewed in isolation from the rest of the model, the effects on a specific variable of a direct shock to that variable, such as the federal funds rate, would fade away gradually.

The top row of panels presents the effects of a 100 basis point positive shock to the monetary policy rule. The simulated responses of the output gap, core inflation, and the federal funds rate are nearly identical in LINVER and FRB/US. Confirming this similarity, a simple metric of the degree to which these LINVER responses correspond to the FRB/US ones—specifically, one less the ratio of the sum of squared differences between the LINVER and FRB/US responses to the sum of squares of the FRB/US responses—shows that the fit is essentially perfect for this shock, with values equal to 1.000 for the output gap and federal funds rate and 0.999 for inflation (Table 1). The correspondence of the two models' pairs of responses is similarly close for the other shocks, as shown in the lower rows of the figure and by response goodness of fit statistics that range from 0.995 to 1.000. The other shocks are a one percent increase in the level of consumption and one percentage point increases in the annualized rates of change of each of core consumer prices and trend multifactor productivity.

As mentioned earlier, an important feature of FRB/US and LINVER is that both can be run under different assumptions for the manner in which expectations are formed in different sectors. In addition to the assumption used to generate Figure 1, the model can be run assuming that: all expectations are VAR-based; only financial market participants have MC expectations; and all expectations are fully MC. As indicated by the goodness of fit values reported in Table 1, the dynamic responses of FRB/US and LINVER are very close under all four expectations assumptions when the ELB is not imposed, with the exception of the response of inflation to a productivity shock when all expectations are MC, where goodness of fit is only 0.890.

4. Historical Variability and Simulated Statistics

As noted earlier, LINVER stochastic simulations can be used to estimate the distribution of possible future outcomes for real activity, inflation, and interest rates, conditional on specific assumptions for the future conduct of monetary and fiscal policy. One way to gauge the reliability of these estimates is to compare the simulated distributions of real activity, inflation, and interest rates with those seen on average over the past 50 years. One difficulty in making such a comparison, however, is that stochastic simulations condition on a fixed characterization of monetary policy, whereas monetary policy in fact evolved through several phases since the late 1960s. One way to address the resulting apples-to-oranges comparison problem is to generate historical variability statistics using not only actual data but also "adjusted" data that take account of how real activity, inflation, and interest rates might have evolved over the past 50 years if monetary policymakers had always followed the same interest rate rule used in the stochastic simulations. Specifically, the federal funds rate never deviates in the counterfactual simulation (and in the stochastic simulations to which it is compared) from the unconstrained prescriptions of the inertial Taylor (1999) rule in which the inflation target is fixed at 2 percent.[6]

Aside from standardizing on the monetary policy rule, the counterfactual simulation also removes the effects of historical innovations to long-run inflation expectations; this assumption is consistent both with the use of a fixed policy rule that would presumably have acted to keep inflation near 2 percent over the past 50 years, and with the exclusion of expectational shocks from the stochastic simulations.[7] In addition, neither the counterfactual simulation nor the stochastic simulations impose the effective lower bound on nominal interest rates. Although unrealistic, this conditioning assumption is needed when comparing adjusted history to the stochastic simulations because it minimizes differences arising from the secular decline in the equilibrium real interest rate since the 1960s. This decline has the effect of causing the ELB constraint, when imposed, to be much more binding in the stochastic simulations than it has been on average in the past.

The first two columns of Table 2 report historical volatility statistics for real activity, inflation, and interest rates for the period 1970 to 2019, computed using first actual quarterly data and then data generated from the counterfactual historical simulation. In the latter, agents are assumed to base their expectations on the predictions of small-scale VAR models estimated using data from approximately the same historical period. In light of the marked decline in inflation volatility since the mid-1980s, the next two columns of the table report these volatility statistics using actual and adjusted historical data beginning in 1983.

[6] The specific version of the inertial Taylor rule used in both the counterfactual historical simulation and later in the stochastic simulations is $R_t = 0.85R_{t-1} + 0.15[0.5 + \pi_t + 0.5(\pi_t - 2) + 1.0Y_t]$, where R is the federal funds rate, π is the four-quarter rate of core PCE inflation, and Y is the output gap, where the gap is expressed as the log difference between actual and potential real GDP multiplied by 100.

[7] Innovations to long-run inflation expectations were marked during the 1970s and 1980s as a result of first a persistently easy stance of monetary policy and then a subsequent successful effort to re-establish price stability. Since the mid-to-late 1990s, the survey measures used in the model to measure such expectations over history have been remarkably stable. To capture this stability, the counterfactual simulation (which begins in 1969Q4) assumes that long-run inflation expectations in late 1968 were equal to 2 percent. Thereafter, these expectations (denoted as *PTR* in the model) evolve according to the expression $PTR_t = .9PTR_{t-1} + .05PICX4_{t-1} + .05PITARG_{t-1}$, where *PICX4* is the 4-quarter rate of core inflation and *PITARG* is the Federal Reserve's inflation target. This updating formula is the same as the one used in the LINVER simulations reported in this paper.

A comparison of the first two columns of the table reveals that the historical variability of real activity does not change appreciably when the data are adjusted by conditioning on a standard policy rule. This similarity between the actual and adjusted data also holds when the standard deviations and 95 percent ranges are computed using the truncated sample that begins in 1983. In contrast, conditioning on a standardized monetary policy rule with a fixed inflation target markedly reduces the adjusted historical variability of inflation and interest rates for both sample periods. Finally, a comparison of results for the full sample and the truncated sample demonstrates that the variability of inflation and interest rates has declined appreciably since the mid-1980s, while the variability of real activity has remained roughly unchanged.

The last three columns of Table 2 present stochastic simulation results. In all three cases, the assumptions for expectations and monetary policy are the same as those used in the historical counterfactual simulation. Statistics are computed using 5000 simulated outcomes, each of 200 quarters length; prior to calculating standard deviations and 95 percent bounds, the first 100 quarters of simulated data are discarded to minimize the effects of starting conditions.[8] All the stochastic simulations take account of the same range of potential macroeconomic shocks, including ones to different types of consumption and investment as well as exports and imports; government expenditures and tax rates; multifactor productivity and other supply-side factors; term and risk premiums on loans, bonds, equity, and the foreign exchange value of the dollar; foreign activity; and wages and prices. The results reported in the last three columns differ, however, in the specific sampling method used to generate random shocks consistent with the disturbances seen over the past 50 years.[9]

The first sampling method shown is standard bootstrapping. Under this approach, the shocks applied in the stochastic simulations are drawn directly from the historical set of equation residuals seen from 1970Q1 to 2019Q4, after subtracting their sample means. To preserve contemporaneous correlations, the procedure randomly draws quarters from the historical sample and then applies the demeaned residuals for that quarter as a set. In principle, bootstrapping should help to preserve non-normal features of the historical equation residuals, although it will

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[8] Expanding the length of each stochastic simulation to 300 quarters, so that the simulated statistics are computed using 200 quarters of simulated data (the same number of quarters in the non-truncated historical sample), would have only a trivial effect on the results reported in the table, as would increasing the number of simulated outcomes beyond 5000.
[9] All the variants shown in the table are standard options that the user can select by setting various parameter values in the main stochastic simulation script. See the *LINVER User's Manua*l for further details.

not capture any serial correlation or cyclicality that may exist.[10] As can be seen, the simulated standard deviations for the output gap, inflation, and the federal funds rate generated under this approach are broadly similar to those seen from 1983 to 2019, while those for the unemployment gap and 10-year Treasury yields appear to be somewhat on the low side. Notably, however, this sampling procedure fails to replicate the considerable skewness in outcomes seen historically. Relatedly, other tests reveal that stochastic simulations run using standard bootstrapping fail to replicate the frequency and magnitude of downturns seen historically.[11]

To help correct these deficiencies, González-Astudillo and Vilán (2019) propose a state-contingent sampling procedure in which the random shocks hitting the simulated economy depend on its cyclical state. The cyclical state is randomly determined at each point in time using a Markov-switching model with three states—normal, mild slump, and severe slump. In the normal state, shocks are bootstrapped by applying historical residuals drawn from randomly selected non-recessionary quarters between 1970 to 2019. In the mild slump state, shocks equal the residuals that occurred in one of the randomly selected recessions that occurred between 1970 and 2001, with the sequence of shocks matching that seen historically. Finally, shocks in the severe slump state equal the sequence of residuals that occurred during the Great Recession.[12] As shown in the next to last column of the table, this approach does a much better job of replicating the skew in 95 percent bounds seen in the historical data. Other tests (not shown) demonstrate that this sampling procedure also generates recessions with a depth and duration closer to that seen over the past 50 years relative to that obtained under standard bootstrapping, similar to what González-Astudillo and Vilán find using a previous version of LINVER and a somewhat shorter sample period.

One issue with the state-contingent sampling procedure is that it generates simulated standard deviations for inflation that are elevated relative to those seen since the mid-1980s, especially when the latter are computed using adjusted data. Correspondingly, simulated 95

[10] The procedures used to estimate the FRB/US model ensure for the most part that equation residuals are serially uncorrelated; that said, the residuals of some equations display modest serial correlation. In addition, some residuals appear to have a cyclical component while others are non-normally skewed. Finally, the residuals of some equations display significant shifts in means and variances between the pre-1983 and post-1983 periods.

[11] The stochastic simulation procedures also provide the user with the option of drawing shocks randomly from a multivariate normal distribution with mean zero and a variance-covariance matrix estimated using the historical equation residuals. This sampling procedure yields simulated statistics similar to those generated with standard bootstrapping.

[12] The transition probabilities and steady-state frequencies of the three states are calibrated to match that seen since 1970. In addition, prior to sampling the historical residuals are adjusted by removing their 1970-2019 means.

percent intervals are relatively wide compared to that seen in the past few decades. In part, these results reflect the fact that the variances of the residuals of the main wage-price equations prior to 1983 are considerably higher than the variances seen thereafter. One way to address this issue is to rescale the pre-1983 wage-price residuals so that their standard deviations match those seen from 1983 on. When rescaling is combined with state-contingent sampling (last column of the table), simulated inflation variability is somewhat reduced. In light of this improvement in the comparability of simulated statistics to history, this sampling procedure is the one employed in the rest of this paper.[13]

An important feature of both the FRB/US and LINVER models is that users can run the simulations assuming model-consistent (MC) expectations in some or all sectors of the economy. Among other things, this capability allows expectations formation and the overall dynamics of the economy to incorporate the effects of permanent changes in the conduct of monetary and fiscal policy, as would be expected to occur once sufficient time has passed. Table 3 illustrates the sensitivity of simulated standard deviations and 95 percent ranges to different expectational assumptions by reporting stochastic simulation results for four different cases: VAR-based expectations in all sectors; MC expectations in financial markets and VAR-based expectations elsewhere; MC expectations in financial markets and wage-price setting; and MC expectations in all sectors. Other stochastic simulation assumptions are the same as those used for Table 2 except that in all cases state-contingent sampling is used with rescaling of wage-price shocks. For the purposes of comparison, Table 3 also reports the statistics computed using adjusted historical data previously discussed. The main take-away from these results is that simulated standard deviations and 95 percent ranges differ only modestly across the four expectational cases. As will be documented below, however, this general similarity does not hold when the ELB constraint binds frequently or when monetary policy is based on an alternative to the inertial Taylor rule.

Although these results suggest that stochastic simulations are a useful way to gauge future uncertainly, there are several caveats to this conclusion. An economic model is only a simplified description of the real-world behavior it attempts to describe. Moreover, the accuracy

[13] In the suite of LINVER stochastic simulation routines, the default option is state-contingent sampling from the 1970-2019 set of equation residuals with rescaling of the wage-price shocks. As discussed in the *LINVER User's Manual*, however, users can opt to use other sampling procedures and other historical periods for drawing shocks. Users can also choose to exclude some types of shocks, if desired.

of this approximation may diminish as the monetary policy rules used in simulation experiments diverge more substantially from the way policy was conducted on average in the past. Important, unforeseen changes in the structure of the economy may diminish the accuracy of a model's ability to explain how the economy will behave in the future. We cannot rule out such changes in the future, as they have clearly taken place in the past—for example, the flattening of the slope of the Phillips curve over the past 30 years for reasons that are still not clearly understood. Finally, the shocks included in the LINVER stochastic simulations do not encompass all the sources of variation seen historically, such as various slow-moving secular forces that have changed the value of the equilibrium real interest rate over time.

5. Solving LINVER Subject to Nonlinear Policy Constraints

One advantage of using LINVER in place of FRB/US is that the former's linearity permits the use of simple matrix operations to solve the model, even when some or all agents have model-consistent expectations. However, analyzing the effects of the ELB constraint is inherently a nonlinear problem: Past some point, the constraint prevents conventional monetary policy from easing further in response to economic downturns but puts no limit on the ability of monetary policy to tighten in response to an overheating economy and rising inflation. Fortunately, the nonlinear ELB constraint can be imposed on the solutions of a linear model in an expeditious way by applying positive additive adjustments to the future path of the federal funds rate expected at each point in time, thereby preventing any ELB violations.

These ELB-related additive adjustments cannot be directly computed because they depend on the paths of inflation, resource utilization and interest rates expected by agents, where those paths in turn depend on those adjustments. Reflecting this simultaneity, iterative search procedures are used to solve for the current and future adjustments that, at each point in time, ensure both that the expected path of the federal funds rate for N quarters into the future does not violate the ELB constraint and that other variables are fully consistent with the expected policy path. (The stochastic simulation routines set N equal to 60 quarters by default, although the user has the option of setting N to a different value if desired.) The methodology used to compute these adjustments exploits the procedure employed by Bodenstein, Guerrieri, and Gust (2013) to impose the nonlinear ELB constraint on a linear model in a parsimonious manner.[14]

The LINVER stochastic simulation routines are also able to solve the model when the ELB constraint is augmented with the requirement that unemployment and/or inflation cross certain thresholds before the federal funds rate is allowed to lift off from the ELB. In addition, solutions are available when the monetary policy rule itself responds nonlinearly to economic conditions in certain specified ways, such as occurs under the asymmetric average inflation targeting rule discussed in Arias *et al* (2020). As with the effective lower bound, the constraints implied by nonlinear rules and threshold conditions can be imposed on the solutions of the linear model through additive adjustments to the current value and, in the MC case, the expected future path of the federal funds rate. (See the appendix to the *LINVER User's Manual* for more information.)

The stochastic simulation routines are reasonably fast. Using a run-of-the-mill laptop, it takes about seven minutes to simulate 5000 stochastic outcomes, each of 200 quarters length, when some or all agents are assumed to have MC expectations and a frequently binding ELB constraint is imposed on the prescriptions of a linear monetary policy rule for 15 years into the future. As might be expected, employing a nonlinear monetary policy rule increases execution time somewhat, to around 15 minutes, while imposing threshold conditions on a linear policy rule increases running times to around 30 minutes. But in the slowest case, in which all agents have MC expectations and policymakers employ the asymmetric inflation targeting rule subject to threshold conditions for liftoff, it takes about an hour and a half to generate these many stochastic outcomes. These speeds are much faster than those of the solution procedure used in

[14] In this procedure, the equation for the federal funds rate in the model is written as $R_t = RULE_t + ERADD_t$. Here, $RULE_t$ denotes the unconstrained prescriptions of the linear policy rule embedded in the model and $ERADD_t$ is an additive adjustment defined to ensure that $R_t = max[ELB, RULE_t]$. To impose the ELB constraint for M quarters into the future on the expectations of MC agents made at time t, the equations $ERADD_t = ERADD1_{t-1} + E0_t$, $ERADD1_t = ERADD2_{t-1} + E1_t$, $ERADD2_t = ERADD3_{t-1} + E2_t$, …, $ERADDM_t = EM_t$ are first appended to the model. The solution procedure then uses OLS in conjunction with the precomputed derivatives of the expected future values of the funds rate with respect to the shocks $\{E0_{t,}, E1_t, \ldots, EM_t\}$ to estimate the values of the shocks required at time t to force the projected path of the federal funds rate to the ELB in those future quarters that have been provisionally identified as requiring non-zero adjustments. Because the adjustments alter the projected paths of real activity and inflation, and thus the projected path of $RULE_t$, an iterative procedure is used to repeatedly re-estimate $\{E0_{t,}, E1_t, \ldots, EM_t\}$ until the minimum value of the federal funds rate along its projected path that is no more than 1 basis point below the ELB and the nonzero values of the shocks are all positive..

some previous studies, such as Kiley and Roberts (2017) and Bernanke (2020). Moreover, the solution procedures we use are quite robust, being able to impose the ELB constraint and other nonlinearities without fail on both actual conditions and agents' expectations, subject to the caveat that in some cases special adjustments are needed to ensure that the model is dynamically stable. These are discussed in the next section.

6. The ELB Constraint and Model Stability

As previously discussed in section 4, when the ELB is not imposed on monetary policy, LINVER stochastic simulations generate probability ranges for the output gap that are in line with both the actual and adjusted cyclical experience of 1970 to 2019. On the other hand, when stochastic simulations are run with an ELB equal to zero and a longer-run neutral value of the nominal federal funds rate equal to 2 or 3 percent (in line with current estimates), LINVER predicts that monetary policy will be frequently constrained and thus that the simulated frequency, depth, and persistence of recessions will increase relative to that seen on average over the past 50 years. Holding the model's structure constant, this effect is more pronounced when some sectors have model consistent expectations, and especially so when all do.[15] In fact, under some conditions, overall stability may deteriorate to the point that the projected paths of real activity, inflation, and interest rates begin to oscillate violently, causing the volatility of stochastic simulation results to explode. Although this oscillatory behavior rarely if ever arises when only financial market participants and wage-price setters have MC expectations, it occurs frequently under many monetary policy rules when all agents have MC expectations.

Various simulation options provide users with several ways to address this instability problem and ensure that stochastic simulations deliver reasonable estimates of future uncertainty; some of these options are invoked by default unless overridden by the user. One involves increasing the stability of the model under full MC expectations by making fiscal policy at all times more strongly countercyclical than it is estimated to have been historically. To do this, tax_gamma, a parameter that governs the (symmetric) countercyclicality of fiscal policy in the

[15] The ELB imposes what are essentially positive shocks to MC agents' expectations for the future path of the federal funds rate, which in turn have an adverse influence on their expectations for future economic conditions more broadly. When all agents have MC expectations, and not just those in financial markets or involved in setting wages and prices, the contractionary effect on current economic activity of these expected shocks increases substantially for shocks projected to occur well into the future.

linear model, is set by default to .00130 under full MC expectations, whereas it is left at its estimated historical value (.00075) when another expectational assumption is selected.[16]

Raising tax_gamma to .00130 does not by itself guarantee that LINVER is always stable under full MC expectations and all monetary policy rules; to do that would require setting this parameter to .00150 or higher. Instead, the default program settings ensure sufficient stability by additionally employing an extreme-case fiscal stabilization (ECFS) option. When this option is used, positive shocks are applied to federal government purchases and transfer payments in the current quarter to prevent economic downturns in simulations from becoming excessively deep. Specifically, these ECFS fiscal shocks are scaled to keep the minimum projected gap within a two-year evaluation window to a specified floor; by default, this floor is set to -16 percent unless a different value is chosen by the user. Importantly, this asymmetric emergency response is in addition to the whatever symmetric countercyclical fiscal actions embedded in the linear model itself.

In addition to its stabilizing effects, we believe that the ECFS mechanism also increases the realism of LINVER simulations. Even when some or all agents have VAR expectations, downturns as severe or worse than the Great Depression can occasionally occur in stochastic simulations of the standard model. But in reality, fiscal policy would probably respond more aggressively than predicted by the model's linear tax and spending equations in such extreme situations. The downturns included in the pre-2020 historical data used to estimate these equations were much less severe, in part because monetary policy usually had considerable room to fight recessions. Moreover, the government's massive response to the recent Covid-induced recession, which was much larger than LINVER would have predicted even when conditioned on the massive declines in employment and output that occurred, arguably illustrates the type of asymmetric fiscal action ECFS is intended to capture. For these reasons, ECFS is used under all expectational and monetary policy assumptions whenever the ELB constraint is imposed unless explicitly turned off. (See the **Appendix** for further information about the ECFS mechanism and its economic effects in simulations.)

Users also have the option of increasing the stability of the model by modifying the cyclicality of the term premiums embedded in yields on Treasury securities with maturities of five, ten, and thirty years. In the standard version of the model, these term premiums are

[16] Specifically, tax_gamma defines the coefficient on the output gap in the equation for the trend personal tax rate.

estimated to move inversely with the average level of resource utilization expected over the life of the security. As a result, when adverse shocks push the economy into what is expected to be a severe and persistent slump, current term premiums and long-term yields rise, exacerbating the downturn. One option for short-circuiting this potentially destabilizing mechanism is to suppress the cyclical term premium effects altogether, an approach that has been used in some previous studies. A second option instead replaces the standard equations with ones estimated using an alternative specification, in which term premiums depend on the current and lagged output gap, not on the average cyclical state of the economy expected well into the future. An advantage of the second option over the first is that the alternative equations, like the standard ones, are consistent with the historical countercyclical behavior of term premiums reported in the empirical finance literature.[17]

7. The ELB, Policy Expectations, and Macroeconomic Performance

Based on both theory and empirical evidence, economists and central bankers believe that expectations play a critical role in determining the effectiveness of monetary policy and shaping macroeconomic performance more generally, especially when policy is constrained by the ELB. In LINVER (as in FRB/US), monetary policy influences real activity and inflation primarily by affecting long-term interest rates, equity prices, and the exchange rate via expectations for the future path of the federal funds rate and other factors. As a result, economic conditions today are importantly influenced by anticipated policy responses to projected movements in real activity and inflation, including responses expected to occur years into the future. If the ELB constraint materially limits the actual and, more importantly, the expected future response of monetary policy to adverse shocks, recessions will be deeper and more prolonged. Symmetrically, if policymakers can convince financial market participants that they will keep interest rates lower for longer than would otherwise be the case, the adverse expectational effects of the ELB constraint will be diminished.

These expectational effects are illustrated by the recession scenario shown in Figure 2, run under three different assumptions about the monetary policy response to the downturn. (The

[17] With either term premium modification, the model becomes sufficiently stable that it can be reliably simulated with tax_gamma equal to 0.00075 even when the ELB is expected to bind for many years into the future. However, the ECFS option is still needed to prevent excessively deep and persistent recessions from occurring in stochastic simulations and distorting the volatility estimates.

baseline is calibrated to match the medians of the longer-run projections recently made by FOMC participants.) In these simulations, all agents have model consistent expectations, implying that everyone has a full understanding of both how monetary policymakers will respond to the recession over time and the implications of their response for future income, sales, inflation, and interest rates. Three policies are considered: the inertial Taylor rule without the ELB imposed (black lines); the inertial Taylor rule subject to an ELB constraint that prevents the FOMC from cutting the nominal federal funds rate by more than 250 basis points (blue lines); and a strategy that delays liftoff from the ELB relative to the inertial Taylor rule for about five quarters (red lines). As can be seen by comparing the black and blue lines, imposing the ELB constraint markedly exacerbates the depth and duration of the recession, causing the peak unemployment rate to rise an additional 1½ percentage point and delaying the subsequent recovery of the labor market. As a result, the disinflationary effects of the downturn more than double. But as illustrated by the red lines, policymakers can substantially offset the adverse consequences of the ELB if they can credibly pledge to delay liftoff until labor market conditions have become robust and inflation has fully recovered.

Both the effects of the ELB constraint and the effectiveness of counteractive forward guidance depend importantly on the expectational assumptions used in LINVER simulations. This dependence is illustrated by Figure 3, which reports results for the same recession scenario and policy responses but now assuming that only financial market participants have MC expectations. Under this assumption, households, non-financial firms, and wage-price setters are slow to recognize how severely the downturn will play out over time, and because they are thus less pessimistic initially about the future, the downturn is less deep. In addition, because they anticipate that monetary policy in the future will respond to movements in real activity and inflation as it did on average in the past and so don't recognize the implications of a lower bound on nominal interest rates, the imposition of the ELB constraint on the monetary policy response is less contractionary. Correspondingly, pledging to delay liftoff provides less stimulus and is less effective in checking disinflation.[18]

[18] The dynamics shown in Figures 2 and 3 are a general feature of macro models with forward-looking "rational" agents and are not unique to LINVER and FRB/US. In fact, as documented by Chung (2015), the effects of anticipated policy actions (and hence the ELB) tend to be less pronounced in FRB/US and LINVER because, relative to other models, households discount the future more heavily, firms face larger adjustment costs, and the Phillips curve is flatter.

Table 4 illustrates the implications of the ELB constraint for simulated estimates of the future volatility of the economy and average macroeconomic performance under different assumptions for the nominal value of the neutral federal funds rate and the formation of expectations. In addition to reporting simulated means, standard deviations, and 95 percent intervals, the table also reports the frequency of ELB episodes and a loss statistic that measures overall macroeconomic performance as the average squared deviations of the output gap and headline inflation from zero. These statistics are based on 5000 stochastic-simulated outcomes, with shocks drawn from the 1970-2019 period using state-contingent sampling with rescaling of the wage-price residuals. In addition to showing simulation results without the ELB imposed, the table reports results generated using three different assumptions about the value of the neutral federal funds rate: 4 percent, 3 percent, and 2 percent, with the ELB assumed to equal zero. Monetary policy follows the prescriptions of the inertial Taylor rule. In the simulations where the ELB constraint is imposed on the inertial Taylor rule, the constraint is applied to the current quarter as well as the first 60 quarters of the future path of the federal funds rate expected by MC agents at each point in time.[19]

Three general results from these simulations are worth highlighting. First, under all expectational assumptions, the average levels of resource utilization and inflation are predicted to become lower and lower as the neutral rate declines and increasingly limits the actual and expected ability of monetary policy under the inertial Taylor rule to ease in response to economic weakness; correspondingly, simulated standard deviations become larger and larger and 95 percent ranges become wider and wider. Second, the estimated frequency of constrained monetary policy rises appreciably as the ELB constraint becomes more binding, rising to roughly a quarter of the time. And third, the extent to which macroeconomic performance worsens as the ELB binds more frequently is sensitive to the expectational assumption used in the stochastic simulations. The deterioration is greatest when both financial market participants and wage-price setters have MC expectations and smallest when all agents have VAR expectations.

[19] Unless otherwise noted, all the simulation results reported Table 4 and in the rest of the paper are generated using the standard term premium equations. In addition, the ECFS option is turned on whenever the ELB is imposed, with shocks applied to prevent the minimum level of the output gap expected over the next two years from falling below -16 percent.

8. Monetary Policy Strategies and Macroeconomic Performance

As noted in the introduction, LINVER can be used to help gauge the potential effectiveness of possible strategies for stabilizing the economy and mitigating the adverse effects of the ELB constraint. Specifically, by running stochastic simulations under different policy assumptions, one can see how the strategies affect the simulated average level and variability of resource utilization, inflation, and other conditions, as well as the estimated likelihood of extreme conditions. To this end, the programs and code included in the LINVER package allow the user to choose among a variety of rules for setting the federal funds rate that have been discussed in the research literature. To illustrate these capabilities, Table 5 presents stochastic simulation results generated using the following rules for setting the federal funds rate:[20]

1. *Inertial Taylor rule*. The non-inertial form of this rule, which differs from the original Taylor rule, was first discussed by Taylor (1999) and in its inertial form was labeled the balanced-approach rule by former Federal Reserve Chair Yellen (2012). It is specified as $R_t = \max\{0.85R_{t-1} + 0.15[1.5\pi_t + 1.0Y_t],\ ELB\}$, where *R* is the federal funds rate, π is the four-quarter rate of core PCE inflation, and *Y* is the output gap.
2. *Average inflation targeting.* As specified by Arias *et al* (2020), this rule takes the form $R_t = \max\{0.85R_{t-1} + 0.15[\pi_t + 1.0Y_t + 8.0\pi_t^{32}],\ ELB\}$, where π^{32} is the average annual rate of inflation over the preceding 32 quarters. It differs from the inertial Taylor rule by not letting bygones be bygones with respect to past deviations of inflation from target.
3. *Asymmetric average inflation targeting.* This rule, also discussed in Arias *et al* (2020), is $R_t = \max\{0.85R_{t-1} + 0.15[1.5\pi_t + 1.0Y_t] + \varepsilon_t,\ ELB\}$, where $\varepsilon_t = 0.15[8\pi_t^{32} - 0.5\pi_t]$ if $\pi_t^{32} < 0$ and zero otherwise. In contrast to average inflation targeting, this rule only acts to reverse past shortfalls of average inflation relative to target, not past overshooting.
4. *RW make-up rule.* This rule, originally proposed by Reifschneider and Williams (2000), is $R_t = \max\{\pi_t + Y_t + CSF_t,\ ELB\}$, where *CSF* represents the cumulative shortfall of

[20] The rule specifications shown here omit terms that are held constant in the simulations, such as the target rate of inflation and the long-run equilibrium real federal funds rate. In addition to these five rules, the user has the option of choosing several others, including the noninertial version of the Taylor rule, rules that target the price level, and the nonlinear rule proposed by Chung *et al* (2019) that responds asymmetrically to movements in unemployment. See the *LINVER User's Manual* for further details.

the federal funds rate from the unconstrained prescriptions of the non-inertial Taylor rule. Specifically, $CSF_t = CSF_{t-1} + 1.5\pi_t + 1.0Y_t - R_t$. This updating formula implies that the CSF term becomes increasingly negative whenever the unconstrained Taylor rule calls for R_t to be less than the ELB. But once the economy recovers sufficiently to cause the unconstrained Taylor rule prescriptions to be greater than the ELB, the CSF term acts to keep the federal funds rate at the ELB until the cumulative shortfall falls to zero, at which point the make-up rule is the same as the non-inertial Taylor rule.

5. *KR change rule.* This rule, proposed by Kiley and Roberts (2017), differs from the others by defining a "virtual" interest rate RV whose change from quarter to quarter depends on the level of inflation and the output gap, $RV_t = RV_{t-1} + 0.4\pi_t + 0.4Y_t$. Among other things, this specification implies that RV is not constrained by the ELB from below but can become increasingly negative over time if economic conditions are persistently weak. The unbounded virtual rate, in turn, is used to set the actual value of the federal funds rate subject to the ELB constraint; that is, $R_t = \max\{RV_t,\ ELB\}$.

For each rule, stochastic simulations are run first without the ELB imposed and then with an ELB equal to zero and a neutral federal funds rate equal to 2 percent. In both cases, two alternative assumptions are made about expectations formation: only financial market participants have MC expectations, with others basing theirs on the predictions of limited-information VAR models; and both financial market participants and wage-price setters have MC expectations. Relative to the first assumption, the latter increases the ability of monetary policy to influence actual inflation through expectational effects. In both expectational cases, MC agents (correctly) anticipate that monetary policymakers will follow the prescriptions of the assumed policy rule without fail, subject to the ELB constraint. VAR agents, in contrast, expect the federal funds rate to respond in an unconstrained way to movements in economic conditions in line with that seen on average historically, regardless of the actual constrained rule employed in the simulation.[21]

Subject to the disclaimer that these simulation results are only illustrative and do not constitute a comprehensive analysis of the potential effectiveness of the five rules, the results

[21] Reported results are based on 5000 simulated outcomes, with shocks drawn from the 1970-2019 period using state-contingent sampling and rescaling of wage-price residuals.

reported in Table 5 are generally consistent with previous research. First, as many studies have found, macroeconomic performance deteriorates markedly as the ELB constraint on policy becomes more severe, with loss statistics increasing appreciably and adverse tail conditions becoming more extreme under all five rules and the two expectational assumptions. Second, consistent with Arias *et al* (2020), the results suggest that average inflation targeting, whether implemented in a symmetric or asymmetric fashion, is likely to improve macroeconomic performance modestly relative to that obtained under the inertial Taylor rule when the neutral federal funds rate is low and the ELB frequently binds. Third, these results are in line with the finding of Bernanke, Kiley, and Roberts (2019) that the RW make-up rule and especially the KR change rule appear to be much more effective in stabilizing the economy than the inertial Taylor rule.[22]

The LINVER routines also give users the option of imposing threshold conditions for unemployment and inflation that, once adverse conditions force the federal funds rate to the ELB, must later be satisfied before tightening is allowed to occur. As a result, thresholds tend to delay liftoff from the ELB relative to what the policy rule would otherwise prescribe, thereby providing additional stimulus through expectational effects. In LINVER, thresholds can be defined in terms of the unemployment gap, a selected measure of inflation, or both.[23] When imposed, liftoff is not allowed until the following conditions are all projected to be met in both the liftoff quarter and the next three quarters: the unemployment gap is below its specified threshold level, the selected inflation measure is above its threshold level, and the unconstrained policy rule prescribes setting the federal funds rate above the ELB. This forward-average definition of threshold satisfaction prevents liftoff from occurring in situations where conditions are expected to be only momentarily back to acceptable levels for initiating policy tightening. If

[22] It is important to note, however, that the second and third conclusions hinge on the assumption that at least financial market participants have MC expectations. Without this accurate knowledge, as is the case when all agents have VAR expectations, the performance of the pair of average inflation targeting rules, the RW make-up rule, and the KR change rule is either about the same or worse than the inertial Taylor rule. The relative performance of the various policy rules is based on their "loss" statistic, as defined in Table 5 and reported there first with MC expectations only in financial markets, and then with MC expectations in wage-price setting as well. Loss values when all agents have VAR expectations (not reported in the table) and the ELB is imposed are: inertial Taylor rule, 16.3; average inflation targeting rule, 25.7; asymmetric average inflation targeting rule, 16.2; RW make-up rule, 16.2; and KR change rule, 211.4.

[23] Inflation thresholds can be defined in terms of headline or core PCE inflation, averaged over the previous four, twelve, or twenty quarters.

a threshold is imposed for labor utilization but not inflation, or vice-versa, only the operative threshold condition must be satisfied for liftoff to occur.[24]

Table 6 presents stochastic simulation results when unemployment and inflation threshold conditions for liftoff are combined with the inertial Taylor rule, first with MC expectations only in financial markets, and then with MC expectations in wage-price setting as well. In these simulations, the unemployment threshold is satisfied when the unemployment gap is persistently at or below its LINVER baseline value (zero), while the inflation threshold when imposed is satisfied when either 4-quarter or 20-quarter core PCE inflation is persistently at or above its baseline value (also zero). MC agents anticipate that monetary policymakers will delay liftoff for as long as necessary to satisfy the threshold commitment. In the simulations, the neutral federal funds rate always equals 2 percent and the ELB equals zero, while other simulation assumptions are the same as those used to generate the results reported in Table 5. Consistent with previous studies, the results reported in Table 6 suggest that setting threshold conditions for liftoff can be modestly effective in improving macroeconomic performance.[25] As with the preceding rule analysis, however, these results are only meant to illustrate the capabilities of the LINVER routines, as they fall far short of a comprehensive analysis of the effectiveness of thresholds.

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[24] In addition to delaying liftoff from the ELB, the procedure used to impose threshold constraints in LINVER simulations also influences how quickly the federal funds rate falls to the ELB in response to adverse shocks. Specifically, whenever the future path of the federal funds rate conditional on no thresholds is projected to become constrained by the ELB at some point in the future, policymakers are assumed to immediately lower the federal funds to the ELB. As discussed in the *LINVER User's Manual*, combining such an immediate-drive-to-the-ELB (IDE) strategy with threshold constraints simplifies the iterative search process used to solve for the appropriate liftoff point along the expected path of the federal funds rate.

[25] Table 6 also shows that the frequency of quarters with the federal funds rate at the ELB increases markedly with the imposition of thresholds. Additional analysis (not shown) reveals that the increased ELB frequency partially reflects the IDE component of the threshold strategy, which by itself turns out to provide little improvement in macroeconomic performance. Two effects of an IDE policy are at work here. First, when policymakers only gradually reduce the federal funds rate to the ELB in response to contractionary shocks, as occurs under the inertial Taylor rule, the probability that the federal funds rate will be at the ELB at time t or sometime later in the future is higher than the likelihood of it actually being at the ELB at time *t*. Under an IDE strategy, in contrast, the two probabilities are necessarily the same and equal the higher figure. This distinction is important because, when the IDE strategy is not imposed, some future ELB episodes projected at time *t* will never actually occur. Second, an IDE strategy does not appreciably increase the average number of ELB quarters along the projected RFF path *expected* by agents at each point in time. Although agents with MC expectations recognize that monetary policy will ease more quickly on average under this strategy, they also anticipate that liftoff will occur somewhat sooner on average because of the additional stimulus provided by the faster initial cut in interest rates. Thus, the *expected* duration of future ELB episodes is little changed relative to what would occur if the federal funds rate were not immediately driven to the ELB in response to contractionary shocks, causing the initial decline in bond yields to be about the same with and without an IDE policy, and thus the initial response of real activity.

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Appendix. Extreme Case Fiscal Stabilization (ECFS)

As discussed in section 6, the solution routines include an ECFS mechanism that supplies emergency fiscal stimulus whenever the output gap would otherwise fall to an extremely low level, thereby preventing downturns from becoming unrealistically severe in stochastic simulations. In addition, the mechanism helps to prevent explosive oscillations from occurring, a problem that can occur with some frequency when all sectors of the economy have MC expectations. ECFS is used by default under all expectation settings whenever the ELB is imposed, although users have the option of turning it off if desired.

The emergency fiscal stimulus is applied through non-negative shocks to the variable $FISCAL$, which evolves according to the equation $FISCAL_t = .97FISCAL_{t-1} + \varepsilon_t$. When needed, the fiscal shock ε_t is set at a value that keeps the minimum value of the output gap expected between quarters *t* and t+7 from falling below a specified floor within plus-or-minus one percentage point. The default setting for this floor is such that the ECFS mechanism prevents the near-term expected value of the output gap from falling below -16 percent. (Users have the option of setting the floor at a different value if desired.)

To illustrate the properties of the ECFS mechanism, Figure A1 shows the direct effects of a one-time fiscal shock on federal outlays and aggregate spending, in a partial equilibrium approach that holds employment, production, inflation, and interest rates constant. As shown in the upper-left panel, the shock is scaled so that FISCAL initially increases by 1. Because the shock is not repeated, FISCAL thereafter slowly falls back towards zero. In response, federal government purchases of goods and services and federal transfer payments expand steadily for four years, peaking at around 6½ percent above baseline. Thereafter, federal outlays decline slowly but are still noticeably above baseline after twenty years.

The lower-left panel shows the response of consumption and residential investment to the ECFS-induced increase in transfer payments, again holding employment, production, inflation and constant. When households have VAR-based expectations, they do not immediately recognize the longer-term implications of the fiscal shock for household income.[26] As a result of this recognition lag as well as significant adjustment costs, household spending increases only gradually to the initial boost in transfer income and takes eight years to reach its peak. When

[26] Note that households have VAR-based expectations in three of the four expectations cases described in this paper. Households do not have VAR-based expectations only when all sectors have MC expectations.

households have MC expectations, however, there is no recognition lag and so consumer spending adjusts more quickly to the fiscal shock, peaking at a somewhat higher level after only three years and thereafter fading away more rapidly. At the same time, households' better understanding of the longer-run income implications of the fiscal shock under MC expectations results in a more muted response of residential investment because (unlike consumption), the category of spending primarily depends on homebuyers' perceptions of long-run conditions.

All told, the direct impetus to total spending from a one-time unit shock to FISCAL peaks at close to1 percent of GDP after three to six years, depending on how households form their expectations. Thereafter, the direct stimulus to spending fades away but even after twenty years remains substantial. These persistent effects mean that an ECFS-induced fiscal shock implicitly increases the effective level of the neutral interest rate for an extended period in the simulations, thereby mitigating the expected adverse effects of the ELB constraint. Moreover, they also mean that the ECFS mechanism can have an important general influence on stochastic simulation results even when such fiscal shocks are applied only infrequently. Under the inertial Taylor rule, for example, only 1½ percent of quarters receive ECFS shocks when all agents have MC expectations, while less than ½ percent do when MC expectations are confined to the financial sectors. Without the persistent stimulus provided by these relatively infrequent shocks, however, the simulated volatility of the economy under this policy rule would be noticeably higher when only some agents have MC expectations and would be explosively so when all agents do.

Table A1 illustrates the general importance of the ECFS mechanism by reporting the direct fiscal impetus (FI) provided by current and past fiscal shocks in stochastic simulations run under different assumptions. The neutral interest rate is always assumed to equal 2 percent. FI effects are expressed as a percent of GDP. The upper portion of the table presents results for stochastic simulations run using the inertial Taylor and the standard term premium equations. When only the financial sector has MC expectations, the FI effect averaged across all simulation quarters is almost 1 percent of GDP, and 5 percent of all quarters have an FI effect is greater than 5.9 percent of GDP. When all agents have MC expectations and tax_gamma = .00130, the average FI effect increases to 2¾ percent of GDP and 5 percent of quarters have effects greater than 14.8 percent of GDP.

When the standard term premium equations are overridden by ones estimated using the alternative more-stable specification, severe downturns become both less frequent and less extreme in stochastic simulations. Accordingly, and as documented in the next block of results reported in the table, the importance of FI effects falls appreciably under both expectational assumptions. If, however, users take advantage of the increased stability provided by the alternative term premium equations under full MC expectations to set tax_gamma equal to its lower estimated historical value (.00075), ECFS-induced FI effects again play an important role in stochastic simulations.

In general, the magnitude of FI effects is relatively insensitive to the monetary policy rule used in stochastic simulations. The Reifschneider-Williams and the Kiley-Roberts rules, however, are exceptions. Because both rules are so effective in mitigating the adverse effects of the ELB, they greatly reduce the need to use emergency fiscal stimulus to stabilize the economy and prevent deep recessions. As a result, almost no quarters ever receive fiscal shocks and thus ECFS has essentially no influence on stochastic simulation results. This effect is documented in the lower half of the table, which shows that average FI effects are essentially zero under the RW rule.

Table 1. Correspondence of LINVER and FRB/US Dynamics: IRF Goodness of Fit Statistics

Shock variable	Response Variable		
	Output Gap	4-qtr Core PCE Inflation	Federal Funds Rate
VAR-based expectations			
Federal funds rate	0.998	0.999	0.999
Consumption	0.999	0.996	0.999
Price level	0.995	1.000	0.998
Trend MFP growth	0.996	0.999	0.997
MC expectations in financial markets			
Federal funds rate	0.998	1.000	1.000
Consumption	1.000	0.998	1.000
Price level	0.996	1.000	1.000
Trend MFP growth	0.998	0.998	0.977
MC expectations in financial markets and wage-price setting			
Federal funds rate	1.000	0.999	1.000
Consumption	1.000	0.997	1.000
Price level	1.000	0.999	0.999
Trend MFP growth	0.997	0.999	0.995
MC expectations in all sectors			
Federal funds rate	0.998	0.993	1.000
Consumption	0.999	0.980	0.998
Price level	0.999	0.999	1.000
Trend MFP growth	0.997	0.890	0.997

Notes: Goodness of fit statistic is one less the ratio of the sum of squared differences between the LINVER and FRB/US responses to the sum of squares of the FRB/US responses calculated over the first forty quarters of the IRFs. No ELB is imposed in the simulations.

Table 2. Historical Variability of Real Activity, Inflation, and Interest Rates Compared to Stochastic Simulation Estimates (expectations are assumed to be VAR-based in model simulations)

	History, 1970-2019		History, 1983-2019		Stochastic Simulation Estimates		
						State-contingent sampling	
	Actual	Adjusted	Actual	Adjusted	Boot-strapping	No shock rescaling	Rescaled wage-price shocks
Standard deviation							
Output gap	2.5	2.6	2.5	2.8	2.6	3.1	3.0
Unemployment gap	1.5	1.6	1.5	1.7	1.2	1.5	1.4
PCE inflation (4-qtr)	2.5	2.1	1.1	0.8	1.1	1.3	1.1
Core PCE inflation (4-qtr)	2.2	1.7	1.1	0.4	1.0	1.3	1.0
Federal funds rate	3.9	2.8	3.0	2.3	2.3	2.6	2.6
10-year Treasury yield	3.0	2.0	2.8	1.7	1.5	1.6	1.5
Upper bound of 95% interval (less sample mean for history)							
Output gap	4.1	4.6	3.9	4.5	5.1	5.2	5.1
Unemployment gap	3.5	3.7	3.4	4.1	2.5	3.4	3.2
PCE inflation (4-qtr)	7.1	5.5	2.2	1.8	2.3	3.0	2.2
Core PCE inflation (4-qtr)	5.6	4.1	2.2	0.9	2.2	3.0	2.1
Federal funds rate	9.4	4.6	5.6	3.3	4.5	4.6	4.5
10-year Treasury yield	6.8	3.7	6.1	3.5	2.9	2.9	2.9
Lower bound of 95% interval (less sample mean for history)							
Output gap	-6.3	-7.2	-6.5	-7.7	-5.3	-7.1	-6.8
Unemployment gap	-2.3	-3.2	-2.5	-3.1	-2.4	-2.5	-2.4
PCE inflation (4-qtr)	-3.3	-2.8	-2.6	-2.0	-2.2	-2.5	-2.2
Core PCE inflation (4-qtr)	-2.3	-1.7	-1.5	-0.8	-2.0	-2.1	-1.9
Federal funds rate	-5.1	-6.5	-3.8	-5.5	-4.7	-6.0	-5.9
10-year Treasury yield	-4.6	-4.3	-3.9	-3.6	-2.9	-3.3	-3.2

Notes: Adjusted historical statistics are based on a counterfactual simulation from 1970 to 2019 in which the federal funds rate follows the unconstrained inertial Taylor rule and historical shocks to long-run inflation expectations are excluded. Stochastic simulation estimates are based on 5000 simulated paths, each of 200 quarters length; the first 100 quarters are discarded prior to computing statistics. In the stochastic simulations, monetary policy is based on the inertial Taylor rule without an ELB imposed and shocks are based on demeaned 1970-2019 equation residuals. All simulations are run using the standard term premium equations.

Table 3. Stochastic-Simulation Estimates Derived Under Different Expectational Assumptions Compared to Adjusted History

	Adjusted History		Stochastic Simulation Estimates Using State-Contingent Sampling and Rescaled Wage-Price Shocks			
	1970-2019	1983-2019	VAR-based Expectations in All Sectors	MC Expectations in Financial Markets Only	MC Expectations in Fin. Markets and Wage-Price Setting Only	MC Expectations in All Sectors
Standard deviation						
Output gap	2.6	2.8	3.0	3.1	2.9	3.2
Unemployment gap	1.6	1.7	1.4	1.5	1.4	1.7
PCE inflation (4-qtr)	2.1	0.8	1.1	1.1	1.7	1.8
Core PCE inflation (4-qtr)	1.7	0.4	1.0	1.1	1.6	1.7
Federal funds rate	2.8	2.3	2.6	3.1	3.4	3.8
10-year Treasury yield	2.0	1.7	1.5	1.3	1.6	1.8
Upper bound of 95% interval (less sample mean for history)						
Output gap	4.6	4.5	5.1	5.4	4.9	5.5
Unemployment gap	3.7	4.1	3.2	3.4	3.2	3.7
PCE inflation (4-qtr)	5.5	1.8	2.2	2.3	3.5	3.6
Core PCE inflation (4-qtr)	4.1	0.9	2.1	2.2	3.4	3.5
Federal funds rate	4.6	3.3	4.5	5.6	6.0	6.7
10-year Treasury yield	3.7	3.5	2.9	2.5	3.0	3.3
Lower bound of 95% interval (less sample mean for history)						
Output gap	-7.2	-7.7	-6.8	-7.0	-6.7	-7.1
Unemployment gap	-3.2	-3.1	-2.4	-2.7	-2.4	-2.9
PCE inflation (4-qtr)	-2.8	-2.0	-2.2	-2.3	-3.5	-3.5
Core PCE inflation (4-qtr)	-1.7	-0.8	-1.9	-2.0	-3.2	-3.3
Federal funds rate	-6.5	-5.5	-5.9	-6.9	-7.4	-8.2
10-year Treasury yield	-4.3	-3.6	-3.2	-2.8	-3.4	-3.6

Notes: Adjusted historical statistics are based on a counterfactual simulation from 1970 to 2019 in which the federal funds rate follows the unconstrained inertial Taylor rule, historical shocks to long-run inflation expectations are excluded, and expectations are VAR-based. Stochastic simulation estimates are based on 5000 simulated paths, each of 200 quarters length; the first 100 quarters are discarded prior to computing statistics. In the stochastic simulations, monetary policy is based on the inertial Taylor rule without an ELB imposed and shocks are based on demeaned 1970-2019 equation residuals. All simulations are run using the standard term premium equations.

Table 4. Stochastic Simulation Results Under Different Assumptions for the Neutral Federal Funds Rate and Expectations (monetary policy follows the prescriptions of the inertial Taylor rule)

	Mean		Standard Deviation		Lower and Upper 95 Percent Bounds		ELB Frequency	Loss
	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation		
VAR expectations in all sectors								
ELB not imposed	-0.1	0.0	3.0	1.0	-6.8, 5.1	-1.9, 2.1	—	10.0
Neutral rate = 4 percent	-0.3	-0.1	3.2	1.0	-7.8, 5.1	-2.1, 2.1	7.9	11.6
Neutral rate = 3 percent	-0.4	-0.1	3.4	1.1	-8.7, 5.1	-2.2, 2.1	14.3	13.2
Neutral rate = 2 percent	-0.8	-0.2	3.8	1.1	-10.0, 5.0	-2.5, 2.0	25.3	16.3
MC expectations in financial markets								
ELB not imposed	-0.1	0.0	3.1	1.1	-7.0, 5.4	-2.0, 2.2	—	11.0
Neutral rate = 4 percent	-0.2	-0.1	3.5	1.1	-8.7, 5.4	-2.2, 2.2	9.9	13.5
Neutral rate = 3 percent	-0.4	-0.1	3.7	1.1	-10.1, 5.5	-2.3, 2.2	15.9	15.6
Neutral rate = 2 percent	-0.7	-0.2	4.2	1.2	-11.7, 5.6	-2.6, 2.2	24.7	19.8
MC expectations in financial markets and wage-price setting								
ELB not imposed	-0.1	0.0	2.9	1.6	-6.7, 4.9	-3.2, 3.4	—	11.2
Neutral rate = 4 percent	-0.3	-0.1	3.3	1.7	-8.6, 5.0	-3.5, 3.4	11.2	14.2
Neutral rate = 3 percent	-0.5	-0.2	3.7	1.8	-10.2, 5.0	-3.9, 3.4	17.4	17.1
Neutral rate = 2 percent	-0.9	-0.4	4.2	2.0	-11.9, 5.2	-5.1, 3.3	26.7	23.0
MC expectations in all sectors								
ELB not imposed	-0.1	0.0	3.2	1.7	-7.1, 5.5	-3.3, 3.5	—	13.1
Neutral rate = 4 percent	-0.3	-0.1	3.7	1.7	-10.0, 5.6	-3.4, 3.5	12.6	17.3
Neutral rate = 3 percent	-0.3	0.0	3.9	1.7	-10.5, 5.8	-3.4, 3.5	17.1	18.7
Neutral rate = 2 percent	-0.2	0.0	4.1	1.8	-10.6, 6.1	-3.5, 3.5	22.2	20.1

Notes: Stochastic simulation estimates are based on 5000 simulated paths, each of 200 quarters length. The first 100 quarters are discarded prior to computing statistics. Shocks are drawn from the 1970-2019 period using state-contingent sampling and rescaled wage-price shocks. All simulations are run using the standard term premium equations. In simulations with the ELB imposed, a lower bound for the federal funds rate equal to zero is imposed on the projected path of the federal funds rate expected by MC agents at each point in time for 60 quarters into the future. The ECFS option is invoked whenever the ELB is imposed to prevent the project output gap from falling below -16 percent. ELB frequency is the percent of the time that the federal funds rate is less than 1 basis point. Loss is the average squared deviation of the output gap from zero plus the average squared deviation of headline inflation from its target rate.

Table 5. Macroeconomic Performance Under Different Monetary Policy Rules, With and Without the ELB Binding

	Mean		Standard Deviation		Lower and Upper 95 Percent Bounds		ELB Freq	Loss
	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation		
MC expectations in financial markets, no ELB								
Inertial Taylor rule	-0.1	0.0	3.1	1.1	-7.0, 5.4	-2.0, 2.2	—	11.0
Average inflation targeting rule	0.0	0.0	3.8	0.9	-8.3, 6.7	-1.7, 2.0	—	15.3
Asymmetric average inflation targeting	0.8	0.2	3.1	1.0	-6.0, 6.6	-1.6, 2.3	—	11.7
RW make-up rule	-0.1	0.0	3.1	1.1	-6.9, 5.5	-2.0, 2.2	—	11.0
KR change rule	0.0	0.0	1.8	0.9	-4.1, 3.3	-1.7, 2.1	—	4.2
MC expectations in financial markets, neutral rate = 2 percent								
Inertial Taylor rule	-0.7	-0.2	4.2	1.2	-11.7, 5.6	-2.6, 2.2	24.7	19.8
Average inflation targeting rule	-0.5	-0.1	4.3	1.0	-10.3, 6.8	-2.2, 2.0	27.7	19.6
Asymmetric average inflation targeting	-0.3	-0.1	3.8	1.1	-10.6, 5.6	-2.3, 2.2	22.4	16.2
RW make-up rule	0.3	0.1	3.6	1.1	-8.2, 6.4	-2.0, 2.3	40.0	14.1
KR change rule	0.0	0.0	2.8	1.0	-6.2, 5.8	-1.9, 2.1	46.0	8.9
MC expectations in financial markets and wage-price setting, no ELB								
Inertial Taylor rule	-0.1	0.0	2.9	1.6	-6.7, 4.9	-3.2, 3.4	—	11.2
Average inflation targeting rule	0.0	0.0	3.5	1.3	-8.2, 6.1	-2.3, 2.8	—	14.3
Asymmetric average inflation targeting	0.5	0.4	2.9	1.4	-6.0, 5.7	-2.2, 3.6	—	11.1
RW make-up rule	-0.1	-0.1	2.8	1.7	-6.3, 4.8	-3.3, 3.5	—	10.9
KR change rule	0.0	0.0	1.9	1.3	-4.4, 3.5	-2.4, 3.1	—	5.7
MC expectations in financial markets and wage-price setting, neutral rate = 2 percent								
Inertial Taylor rule	-0.9	-0.4	4.2	2.0	-11.9, 5.2	-5.1, 3.3	26.7	23.0
Average inflation targeting rule	-0.5	-0.1	4.0	1.4	-10.0, 6.1	-2.8, 2.8	27.5	18.6
Asymmetric average inflation targeting	-0.5	-0.1	3.9	1.9	-11.0, 5.2	-4.5, 3.4	24.2	18.9
RW make-up rule	0.1	0.3	3.4	1.5	-8.6, 5.6	-2.5, 3.6	42.0	13.9
KR change rule	-0.2	0.1	2.7	1.4	-6.4, 4.9	-2.3, 3.2	45.2	9.2

Notes: Stochastic simulation estimates are based on 5000 simulated paths, each of 200 quarters length. The first 100 quarters are discarded prior to computing statistics. Shocks are drawn from the 1970-2019 period using state-contingent sampling and rescaled wage-price shocks. All simulations are run using the standard term premium equations. When an ELB is imposed, ECFS is used to prevent the projected output gap from falling below -16 percent, and a lower bound of zero is imposed on the projected path of the federal funds rate expected by MC agents for 60 quarters into the future. The ELB frequency is the percent of the time that the federal funds rate is less than 1 basis point. Loss is the average squared deviation of the output gap from zero plus the average squared deviation of headline inflation from its target rate.

Table 6. Macroeconomic Performance under the Inertial Taylor Rule when Combined with Unemployment and Inflation Threshold Conditions for Liftoff, with the Neutral Federal Funds Rate Equal to 2 Percent

	Mean		Standard Deviation		Lower and Upper 95 Percent Bounds		ELB Freq	Loss
	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation	Output Gap	Core PCE Inflation		
MC expectations in financial markets								
No thresholds	-0.7	-0.2	4.2	1.2	-11.7, 5.6	-2.6, 2.2	24.7	19.8
Unemployment threshold	-0.4	-0.1	4.0	1.1	-11.1, 5.5	-2.4, 2.2	25.0	17.4
Core inflation threshold (4-qtr average)	-0.1	0.0	4.1	1.1	-10.6, 6.5	-2.3, 2.2	30.0	17.8
Core inflation threshold (20-qtr average)	0.2	0.1	4.3	1.1	-10.4, 7.8	-2.2, 2.3	36.8	19.8
Both thresholds (4-qtr inflation average)	0.2	0.1	4.3	1.1	-10.4, 7.9	-2.1, 2.3	29.8	19.5
Both thresholds (20-qtr inflation average)	0.5	0.1	4.7	1.1	-10.2, 10.3	-2.0, 2.3	35.1	23.3
MC expectations in financial markets and wage-price setting								
No thresholds	-0.9	-0.4	4.2	2.0	-11.9, 5.2	-5.1, 3.3	26.7	23.0
Unemployment threshold	-0.6	-0.2	4.0	2.0	-11.5, 5.1	-4.8, 3.4	26.6	20.7
Core inflation threshold (4-qtr average)	-0.4	-0.1	3.9	1.9	-10.7, 5.7	-4.4, 3.4	26.7	19.3
Core inflation threshold (20-qtr average)	-0.1	0.0	4.1	1.8	-10.6, 6.9	-4.3, 3.5	35.9	20.8
Both thresholds (4-qtr inflation average)	-0.2	0.1	3.9	1.9	-10.6, 5.7	-4.3, 3.5	26.2	18.8
Both thresholds (20-qtr inflation average)	0.0	0.2	4.1	1.9	-10.5, 7.1	-4.2, 3.6	31.1	20.6

Notes: Stochastic simulation estimates are based on 5000 simulated paths, each of 200 quarters length. The first 100 quarters are discarded prior to computing statistics. Shocks are drawn from the 1970-2019 period using state-contingent sampling and rescaled wage-price shocks. All simulations are run using the standard term premium equations. MC agents expect liftoff to be delayed until the unemployment gap is projected to fall persistently below baseline and the inflation measure to rise persistently above baseline, for up to 15 years into the future. ECFS is used to prevent the projected output gap from falling below -16 percent and the projected path of the federal funds rate expected by MC agents for 15 years into the future is constrained from falling below zero. ELB frequency is the percent of time the federal funds rate is less than 1 basis point. Loss is the sum of the average squared deviations of the output gap and headline inflation from its target rate.

Table A1. ECFS-Induced Fiscal Impetus in Stochastic Simulations Run Under Different Assumptions for Expectations, Monetary Policy, and Cyclical Term Premium Effects
(fiscal impetus expressed as a percent of GDP)

	Average for All Quarters	5 Percent Upper Bound	1 Percent Upper Bound
Inertial Taylor rule, standard term premium equations			
MC expectations in financial markets	0.9	5.9	13.5
MC expectations in all sectors, tax_gamma = .00130	2.7	14.8	27.1
MC expectations in all sectors, tax_gamma = .00075	unstable	unstable	unstable
Inertial Taylor rule, alternative term premium equations			
MC expectations in financial markets	0.1	0.0	1.6
MC expectations in all sectors, tax_gamma = .00130	0.3	1.7	7.8
MC expectations in all sectors, tax_gamma = .00075	4.3	28.2	65.9
RW rule, standard term premium equations			
MC expectations in financial markets	0.1	0.0	1.4
MC expectations in all sectors, tax_gamma = .00130	0.0	0.0	1.1
MC expectations in all sectors, tax_gamma = .00075	0.1	0.3	3.8
RW rule, alternative term premium equations			
MC expectations in financial markets	0.0	0.0	0.0
MC expectations in all sectors, tax_gamma = .00130	0.0	0.0	0.1
MC expectations in all sectors, tax_gamma = .00075	0.1	0.0	2.5

Notes. Fiscal impetus equals the direct boost to aggregate spending from current and past ECFS fiscal shocks to government purchases and transfer payments, holding employment, output, inflation, and interest rates constant. The fiscal shocks come from stochastic simulations in which the neutral interest rate equals 2 percent with a zero ELB constraining monetary policy.

Figure 1: LINVER (blue) versus FRB/US (red) IRFs
MC Expectations in Financial Markets and Wage-Price Setting

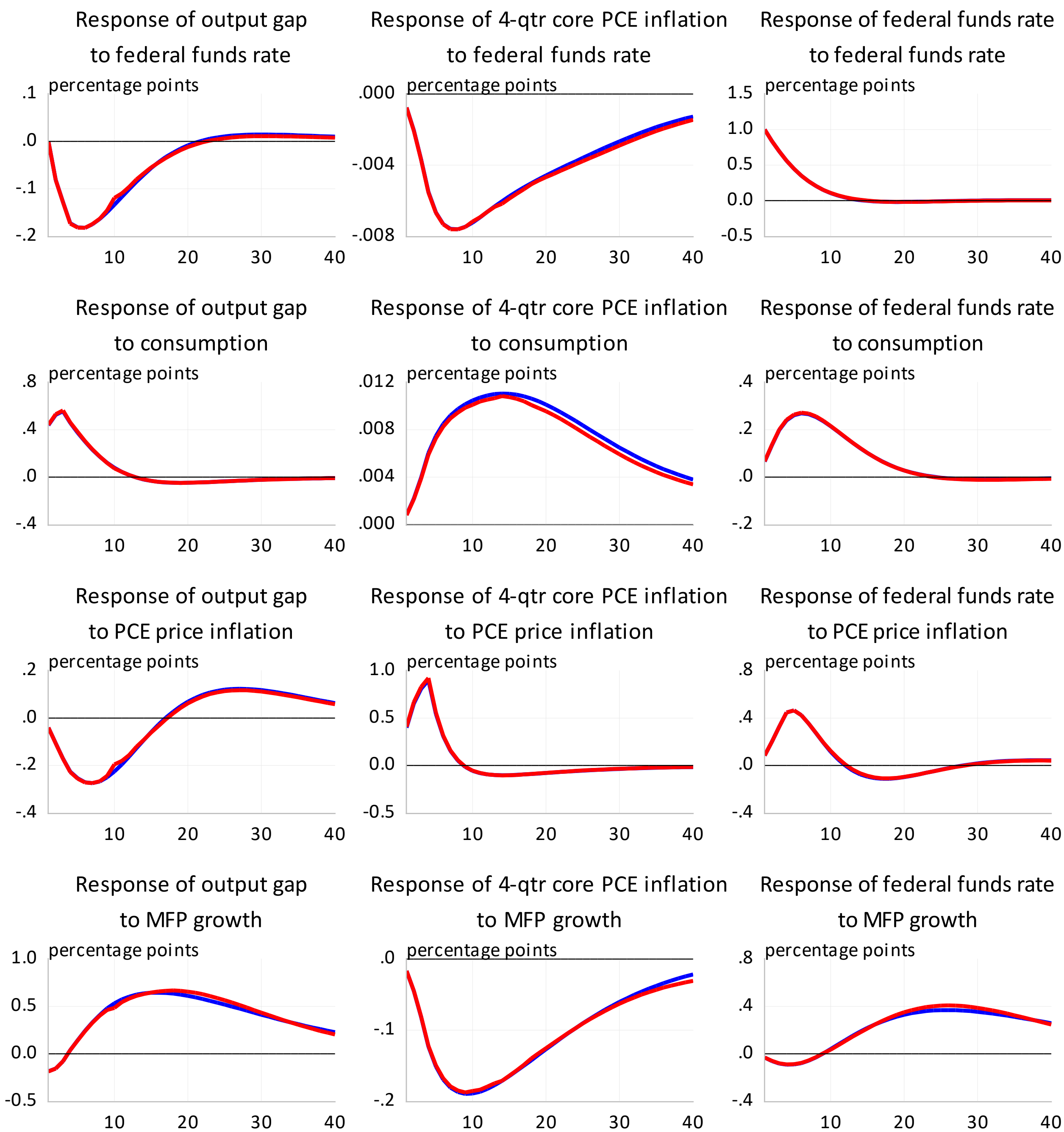

Figure 2. Implications of the ELB Constraint for the Depth and Duration of an Illustrative Recession Assuming that All Agents Have Model Consistent Expectations

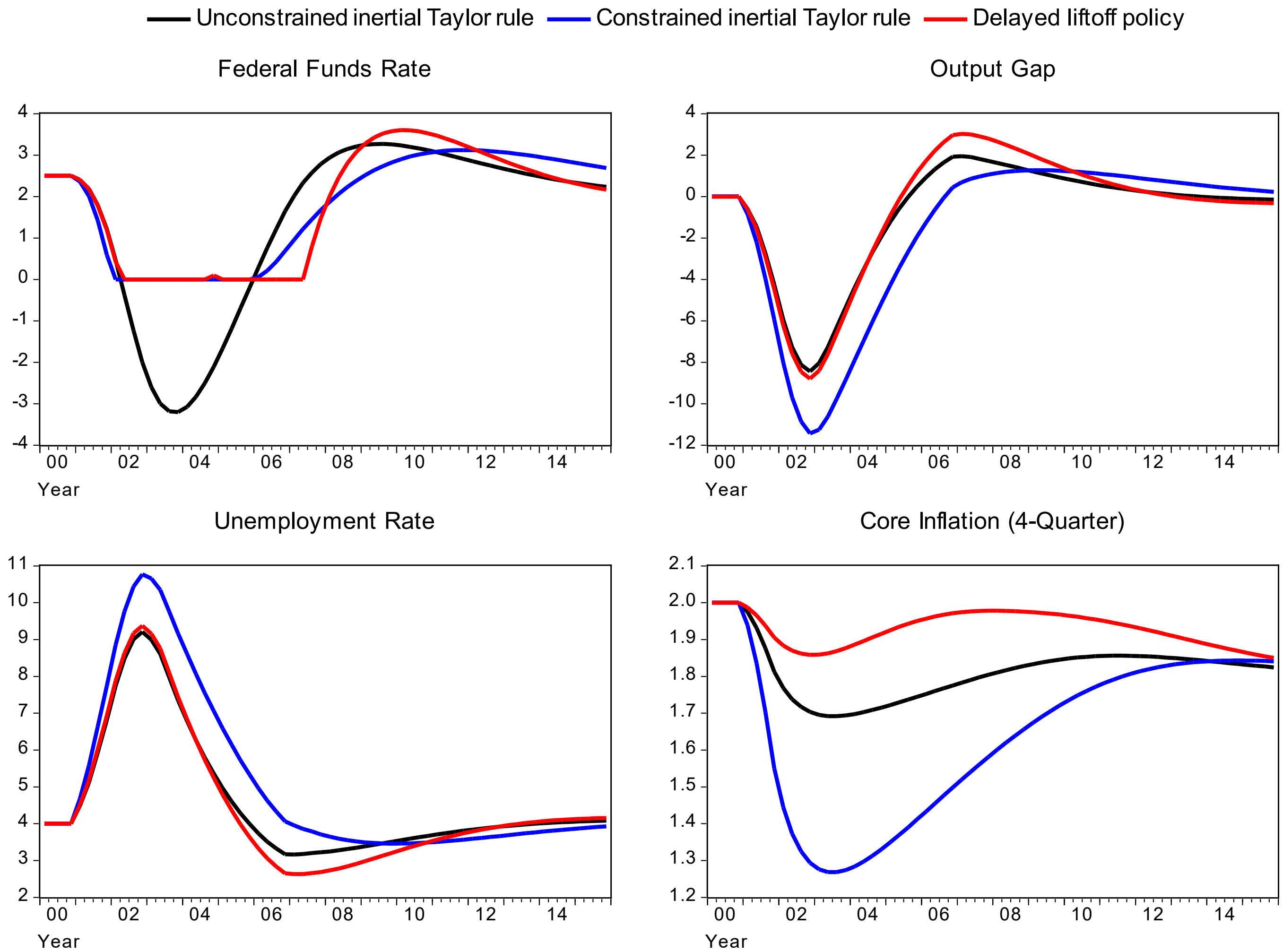

Figure 3. Implications of the ELB Constraint for the Depth and Duration of an Illustrative Recession Assuming Model Consistent Expectations in Financial Markets Alone

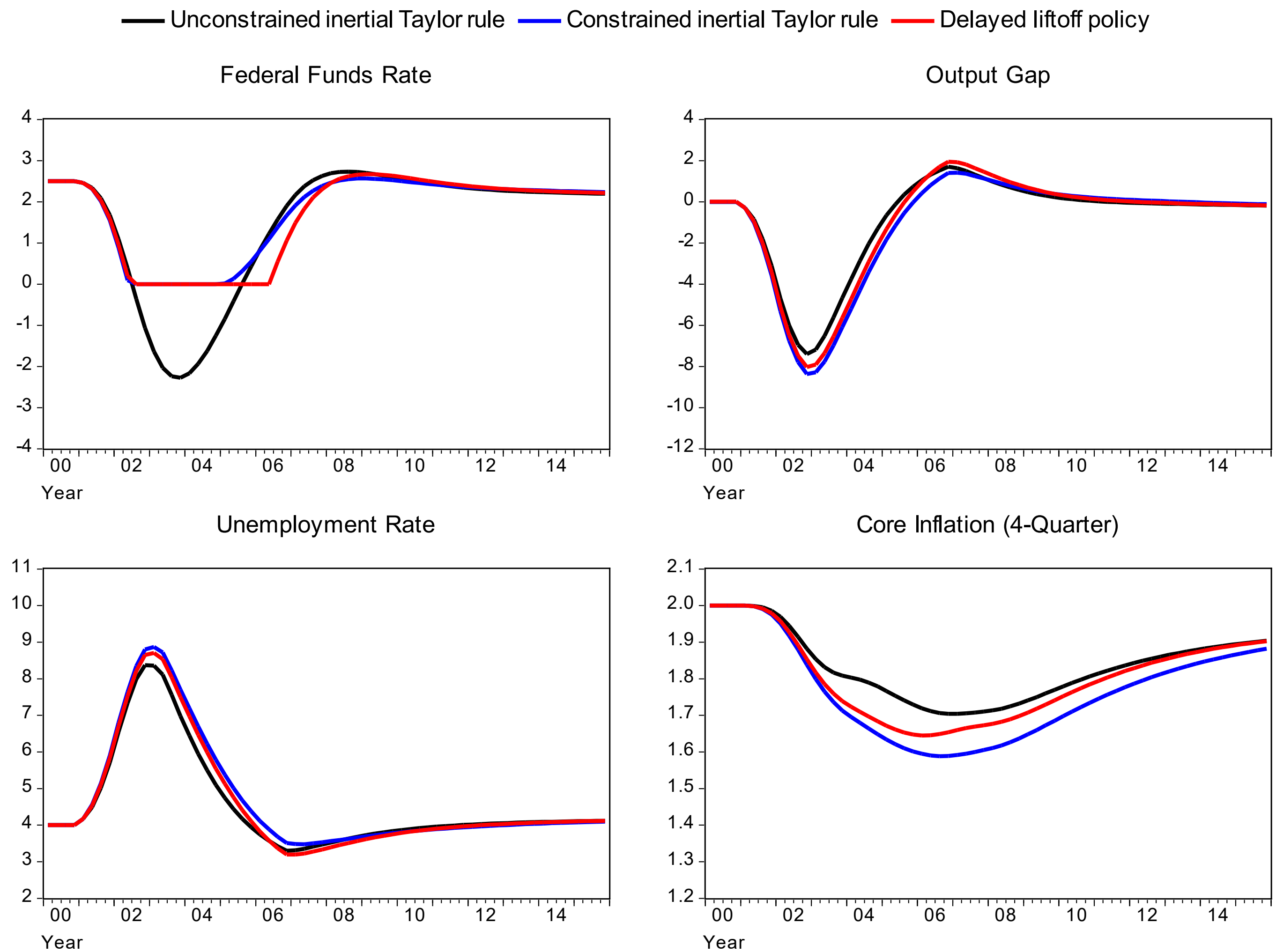

Figure A1. Impetus to Spending Provided by a One-Time ECFS Fiscal Shock
Holding Employment, Production, Inflation, and Interest Rates Constant
(in lower panels, solid lines are results for VAR expectations and dashed lines results for MC expecations)