Inflation Measured Every Day Keeps Adverse Responses Away:
Temporal Aggregation and Monetary Policy Transmission

Margaret M. Jacobson, Christian Matthes, and Todd B. Walker

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INFLATION MEASURED EVERY DAY KEEPS ADVERSE RESPONSES AWAY: 
TEMPORAL AGGREGATION AND MONETARY POLICY TRANSMISSION*

Margaret M. Jacobson†  Christian Matthes‡  Todd B. Walker§

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Abstract

Using daily inflation data from the Billion Prices Project [Cavallo and Rigobon (2016)], we show how temporal aggregation biases estimates of monetary policy transmission. We argue that the information mismatch between private agents and the econometrician — the source of temporal aggregation bias—is equally important as the more studied mismatch between private agents and the central bank (the “Fed information effect”). In impulse responses from both local projections and an unobserved components model of inflation dynamics, we find that the response of daily inflation to high-frequency monetary policy shocks is largely in line with theoretical predictions. If there is an adverse response such that inflation increases in response to a contractionary monetary policy shock, it is only short-lived and temporary. To reconcile how one can obtain a sizable adverse response with monthly or quarterly data when only a limited adverse response exists at a higher frequency, we appeal to a simple monetary policy model and show how temporal aggregation bias can exacerbate initial impulse response functions. Because our modeling results are generic and macroeconomic indicators are published with a lag, we argue that temporal aggregation bias will be a key feature of the nascent field of high-frequency macroeconomics.

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†Federal Reserve Board; Margaret.M.Jacobson@frb.gov
‡Department of Economics, Indiana University, matthesc@iu.edu
§Department of Economics, Indiana University, walkertb@indiana.edu
1 Introduction

This paper revisits a fundamental question of monetary economics: What is the transmission of monetary policy to the economy? Empirical work often finds that macroeconomic variables respond to monetary policy shocks in the opposite direction of what standard theory predicts. Researchers trace these adverse responses to information issues, with existing solutions consisting of either adding more information [Sims (1992)] or emphasizing information mismatches between central banks and private sector agents as a “Fed information effect” [Campbell et al. (2012) and Nakamura and Steinsson (2018a)].

We propose temporal aggregation bias as a new information-based explanation for the adverse transmission of monetary policy shocks. When using the daily CPI from the Billion Prices Project [Cavallo and Rigobon (2016)] as a temporally disaggregated macroeconomic indicator, we find that the adverse response of inflation is short-lived, if it is present at all. We argue that existing work on monetary policy transmission finds an adverse response because of the frequency mismatch between the information sets of the econometrician and private agents. A temporally disaggregated measure of inflation overcomes this mismatch by better aligning the frequencies of shocks and dependent variables.

To understand how one can obtain a sizable adverse response to monetary policy shocks with monthly or quarterly data when only a limited adverse response actually exists, we combine informal and formal empirical evidence with a simple model of temporal aggregation bias. We first establish that temporally aggregated high-frequency measures of inflation correlate well with official lower-frequency measures (e.g. monthly CPI) over our sample period (July 2008 to August 2015). Our empirical tests corroborate the claim that the high-frequency measure of inflation is “good at anticipating major changes in inflation trends,” [emphasis added, Cavallo and Rigobon (2016)].

Our main finding—the adverse response of inflation is short-lived, if it is present at all—is obtained from the local projection specification advocated by Nakamura and Steinsson (2018b).
The monetary policy shocks are identified via high-frequency variation in asset prices around monetary policy meetings, as is standard in the literature [Kuttner (2001), Gürkaynak et al. (2005), Campbell et al. (2012), Nakamura and Steinsson (2018a), Bu et al. (2021)]. We thus align the frequency of our variable of interest (inflation) more closely to the frequency of variation used to identify shocks. Impulse response functions show the positive response of inflation to a contractionary monetary policy shock is only significant for a few weeks.

To study the transmission of monetary policy shocks to daily inflation more systematically, we then build an unobserved components model that adds the daily CPI and daily break-even inflation rates as well as possible effects of monetary policy shocks into a state-space model of inflation dynamics along the lines of Stock and Watson (2016) and Nason and Smith (2020). These impulse responses corroborate our local projection results by showing a muted and short-lived adverse transmission of monetary policy shocks. Finally, we use a well-known model from the monetary policy literature consisting of an Euler equation and a monetary policy rule to show how temporal aggregation can exacerbate initial impulse response functions.

Our contribution of temporal aggregation bias as an explanation for the transmission of monetary policy shocks provides further support for the ongoing claim, dating back to at least Kuttner (2001), that monetary policy needs to be studied in a high-frequency environment. Even though high-frequency economic indicators and temporal aggregation theory have been available for decades, we are the first—to our knowledge—to apply them to the study of monetary policy transmission.¹ By pairing high-frequency shocks with high-frequency response variables, our work follows existing specifications that estimate the transmission of monetary policy shocks to financial indicators.² Financial indicators, however, may not be as susceptible to temporal aggregation bias as macroeconomic indicators because the former are observable at high frequencies. By contrast, economic indicators are accumulated over a fixed time interval and published with a lag, resulting in aggregation bias from potentially mismatched info-

¹See Shapiro et al. (2022), Aruoba et al. (2009), Lewis et al. (2020) for other high frequency economic indicators.
²See Golez and Matthies (2021), Andrade and Ferroni (2021), Nakamura and Steinsson (2018a), Bauer and Swanson (2022), Gürkaynak et al. (2022), and Gürkaynak et al. (2021).
tion sets between private agents observing high-frequency indicators and an econometrician relying on official releases.\textsuperscript{3}

Because our temporal aggregation results are generic, and macroeconomic indicators are published with a lag, we argue that temporal aggregation bias is neither limited to monetary policy transmission nor prices and will be a key feature of the nascent field of high-frequency macro [Baumeister et al. (2021), Lewis et al. (2021)]. In a macroeconomic environment characterized by fast-moving turning points, such as the Great Financial Crisis or the COVID-19 recession, estimates of policy effects may be sensitive to the sampling frequency of economic response variables. Although high-frequency observables may be susceptible to measurement noise because they are only proxies of their lower frequency official counterparts, frameworks like our state space model allow for measurement error. We thus argue that measurement noise is not necessarily more important than the bias induced by temporal aggregation.

1.1 Connection to Literature While Campbell et al. (2012) and Nakamura and Steinsson (2018a) find adverse responses when estimating the transmission of high-frequency monetary policy shocks to lower frequency forecasts of macroeconomic aggregates, subsequent work finds that properly accounting for information delivers results that are either ambiguous or in line with structural predictions.\textsuperscript{4}

Closest to our specification of high-frequency inflation indicators responding to high-frequency monetary policy shocks are specifications that rely on high-frequency expected inflation (TIPS) [Nakamura and Steinsson (2018a)] or commodity prices [Velde (2009)]. Relative to these previously used proxies, we argue that the Billion Prices Project daily CPI is a relatively more complete measure of inflation and hence better suited to assess the transmission of monetary policy shocks. Expected and realized inflation may have different sensitivities to monetary policy shocks because the former tends to be anchored while the latter is more prone to fluctuations.\textsuperscript{5}

\textsuperscript{3}For example, Stock and Watson (2007) note that time series estimates of the CPI are susceptible to temporal aggregation bias.

\textsuperscript{4}Uribe (2022) takes a contrasting stance and argues that monetary policy shocks may actually be neo-Fisherian.

\textsuperscript{5}Common specifications that rely on the change in Blue Chip forecasts may thus be understating the transmission of monetary policy shocks to inflation because they capture changes in expected rather than current inflation.
Similarly, commodities are known to be more volatile than measures of inflation which may result in different sensitivities to monetary policy shocks.

Rather than following much of the empirical monetary policy transmission literature and focusing on information refinements to possible explanatory variables, we instead follow Bauer and Swanson (2020) and contribute refinements to the less-studied measurement of response variables. Many studies find predictability and or bias in standard high-frequency monetary policy shocks such as those estimated by Nakamura and Steinsson (2018a). These studies mainly focus on the response of GDP and argue that the adverse sign disappears once the shocks are either orthogonalized [Karnaukh and Votra (2022), Bauer and Swanson (2022)] or conditioned on missing information [Caldara and Herbst (2019), Sastry (2021), Miranda-Agrippino and Ricco (2021), Bauer and Swanson (2020)].

Many studies account for the adverse transmission of high-frequency monetary policy shocks by appealing to Romer and Romer’s (2000) “Fed information effect” which argues that central banks have an information advantage over private agents. Private agents thus revise up their forecasts of inflation in response to tighter monetary policy because they perceive a signal that the central bank has relatively optimistic non-public information. However, several recent papers explicitly test for a central bank information advantage and find no evidence [Sastry (2021), Bundick and Smith (2020) and Bauer and Swanson (2020)]. Other papers take the information advantage as given, control for it directly, and find that it either changes the transmission of monetary policy shocks [Lunsford (2020), Hoesch et al. (2021), Cieslak and Schrimpf (2019)] or eliminates the adverse transmission entirely [Miranda-Agrippino and Ricco (2021), Jarocinski and Karadi (2020)].

In contrast to existing work, we do not explicitly test or model how the different sensitivities of expectations and actual indicators is less of an issue for the transmission of monetary policy shocks to GDP.

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6 Bauer and Swanson’s (2020) survey finds that Blue Chip forecasters rarely change their estimates of economic indicators in response to monetary policy announcements which calls into question the suitability of these forecasts as response variables.

7 Faust et al. (2004) find that the adverse response of inflation disappears once the Volcker disinflation is excluded from Romer and Romer’s (2000) study.

8 Lewis (2020) and Acosta (2022) specifically identify a Fed information effect shock and find evidence that is either mixed or against adverse transmission of monetary policy shocks.
information sets of central banks and private agents affect monetary policy transmission. We instead focus on the less-studied information mismatch between private agents and the econometrician and how this biases estimates.

Decades of work supports our claim that temporal aggregation bias can affect both the direction and magnitude of monetary policy transmission. We follow Marcet (1991) to show how quantitative bias arises from autoregressive processes taking on a moving average component when temporally aggregated. This moving average component, coupled with a larger initial variance, results in upwardly biased impulse response coefficients. Furthermore, Amemiya and Wu (1972) and Stram and Wei (1986) confirm the finding that temporal aggregation preserves the order of autoregressive processes, but attenuates the autocorrelation coefficients. To account for qualitative bias, we simulate a simple model to show how temporal aggregation preserves the sign of the initial innovation, but pushes subsequent shocks of the opposite sign further into the past than they actually are.

2 Data

2.1 Daily Inflation Data Our daily index is from the Billion Prices Project (BPP), which is constructed from over five million online prices from 300 retailers in 50 countries webscraped daily. While we provide a brief overview here, a meticulous description of the data is provided in Cavallo and Rigobon (2016). Our data consists of (publicly available) observations from 2008 to 2015. Advantages of the data are [i.] the higher frequency (daily) vis-a-vis the CPI (monthly or bi-monthly) or scanner data (weekly); and [ii.] the number of prices collected far exceeds the CPI (500k vs. 80k). The disadvantages are [i.] prices are only collected from online retailers and therefore the sample is not representative of all consumer prices; specifically, the sample contains no pricing from the services sector.\footnote{Although comparing the BPP to a version of the CPI with the same coverage of categories would be an ideal exercise, we are limited by data availability. We have instead repeat the calculations of this section using sub-indices of the CPI and the results are broadly similar. These sub-indices include the commodity price index, the commodity plus shelter index, the official index less energy, and the official index less medical services.} According to Cavallo and Rigobon (2016), the
data contain at least 70 percent of the weights in Consumer Price Index (CPI) baskets of roughly 25 countries; [ii.] Because prices are webscraped, the data does not contain information on quantities sold. Thus, online prices must be coupled with weights from consumer expenditure surveys or other sources to yield expenditure-weighted data.10

2.2 CONNECTION TO CPI To alleviate concerns that BPP data may not align well with the US CPI, we now conduct several tests to show that the BPP is effective at anticipating changes in inflation, a fact that we will exploit in our econometric analysis.

Panel 1a plots the percentage change of the monthly CPI and the BPP daily index; Panel 1b

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10The BPP only discloses weights pooled across all countries where they collect data. They do not disclose country specific weights. See https://www.pricestats.com/approach/data-composition.
Figure 2: Nowcast of CPI using monthly aggregated BPP, monthly percentage change. P-values in parenthesis on Panel 2b. For month $T$, $\Delta CPI_T = 100 \times (\log CPI_T - \log CPI_{T-1})$ and for day $t$ and month $T$, $\Delta BPP_T = \frac{1}{m} \sum_{t=1}^{m} 100 \times (\log BPP_t - \log BPP_{t-31})$ for $t = 1, \ldots, m$ days in month $T$.

plots the percentage change of the monthly CPI against the aggregated monthly BPP. While the correlation of the two series plotted in Panel 1b is only 0.62, several studies have shown that the BPP index is particularly adept at picking up turning points in the CPI, which leads to improved forecasts [Cavallo and Rigobon (2016), Aparicio and Bertolotto (2020) and Harchaoui and Janssen (2018)]. To show this result holds over our sample period, we use the monthly aggregated BPP series to conduct a Nowcast of the CPI by estimating, $\Delta CPI_T = \beta_0 + \beta_1 \Delta BPP_T + e_T$. Despite both indices being denoted with subscript $T$, the CPI at date $T$ is announced with a slight delay as shown by Table 1, which documents the summary statistics of release delays in days (e.g., June 2008 CPI was released July 16). Given that our interest lies in high-frequency changes in inflation, the slight difference in timing is relevant as one can use the monthly average of the BPP to predict that month's CPI number. A coefficient equal to unity ($\beta_1 = 1$) suggests the BPP perfectly predicts the CPI. The estimated value is 0.91 with an R-squared of 0.58, implying predictive power but not achieving statistical significance at the 0.05 level, see Panel 2b. Panel 2a plots the in-sample predicted values against the realized values.

Given the persistence of inflation, we address the following question: Is there any additional
predictive power of the BPP beyond that contained in past values of the CPI? Table 2 compares
the Nowcast to an autoregressive representation of the CPI. Column one reports the AR(1) spec-
ification results. Columns two and three condition only on past values of the BPP, and show a
substantial increase in the R-squared value when conditioning on the contemporaneous BPP,
while the lagged BPP has less predictive content than last month’s CPI. Columns four and five
demonstrate an affirmative answer to the question of additional predictive power of the BPP:
The coefficients on the contemporaneous BPP are positive and statistically significant. The R-
squared value is twice as high as the autoregressive specification.\(^{11}\)

<table>
<thead>
<tr>
<th>(\Delta CPI_T)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta CPI_{T-1})</td>
<td>0.558***</td>
<td>0.184*</td>
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<td></td>
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<tr>
<td></td>
<td>(0.143)</td>
<td>(0.107)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta BPP_T)</td>
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<td>0.864***</td>
<td>0.809***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.093)</td>
<td>(0.102)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta BPP_{T-1})</td>
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<td>0.09</td>
<td>−0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.242)</td>
<td>(0.191)</td>
<td>(0.218)</td>
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</tr>
<tr>
<td>(R^2)</td>
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<td>0.58</td>
<td>0.23</td>
<td>0.6</td>
<td>0.61</td>
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<td>0.31</td>
<td>0.58</td>
<td>0.22</td>
<td>0.59</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. *\((p < .10)\), **\((p < .05)\), ***\((p < .01)\)

Table 2: Nowcast of BPP vs. autoregressive CPI. For month \(T\), \(\Delta CPI_T = 100 \times (\log CPI_T - \log CPI_{T-1})\) and for day \(t\) and month \(T\), \(\Delta BPP_T = \frac{1}{m} \sum_{t=1}^{m} 100 \times (\log BPP_t - \log BPP_{t-31})\) for \(t = 1, \ldots, m\) days in month \(T\).

3 Empirical Results

3.1 Measures of High-Frequency Monetary Policy Shocks Before estimating moneta-
tary policy transmission with disaggregated inflation data, we briefly describe our choice of
monetary policy shocks and their respective timing and identification. We discuss two such

\(^{11}\)We conduct several robustness checks in Appendix A which corroborate our findings that the BPP index is
effective at predicting changes in inflation. For example, we construct alternative metrics for computing inflation
(levels, end-of-month values) and examine different types of seasonality (day-of-the-week).
constructions in detail—Nakamura and Steinsson (2018a) (NS) and Bu et al. (2021) (BRW). We focus on these shocks because they are characterized by a single factor that can be parsimoniously embedded into more complex frameworks like our state space model. Even though the NS shock is widely used, there are known concerns about predictability and bias. For this reason, we also include estimates using the BRW shock as it claims to control for some of these concerns. NS find a substantial adverse transmission of monetary policy shocks, while BRW find “scant evidence” of such dynamics. Our aggregated results replicate these findings.

NS define a “policy news shock” as the first principal component of the change in five interest rates / futures around a 30-minute window of FOMC announcements: the federal funds rate immediately following the FOMC meeting, the expected federal funds rate immediately following the next FOMC meeting, and expected 3-month Eurodollar interest rates at horizons of two, three and four quarters. The last three interest rates are meant to capture the effects of forward guidance as it impacts expectations beyond the federal funds rate. BRW use the Fama and MacBeth (1973) two-step procedure to extract unobserved monetary policy shocks from the common component of zero-coupon yields encompassing the full yield curve. The first step in the procedure estimates the sensitivity of yields of different maturity to monetary policy via standard time-series regressions. Filtering out non-monetary policy news is done through the heteroskedasticity-based estimator of Rigobon (2003) and Rigobon and Sack (2004), implemented by employing instrumental variables (IV).

Figure 3 plots the extracted shock series for each approach over our sample period. As noted in BRW, their shock series has “moderately high correlation” with that of NS in addition to those of Swanson (2021) and Jarocinski and Karadi (2020). What is evident from the figure is that the BRW shock series has much more dispersion which is likely attributed to the different frequencies, methods, tenures, and asset prices used in the construction. Despite these differences in dispersion, our empirical analysis confirms that temporal aggregation exacerbates initial impulse responses for both shock series.
3.2 Local Projections

We employ local projections using the methodology of Canova and Ferroni (2022) to estimate the impulse responses of disaggregated and aggregated inflation. Let $y_{t+h}$ be the value of daily inflation for the next $h$ days, $x_{t-1}$ be the monetary policy shock, and $z_t$ be the vector of controls which are the 31 lags of daily inflation. Given the model,

$$y_{t+h} = \alpha(h) + \beta(h)x_{t-1} + \Gamma(h)z_t + e^{(h)}_t, \quad e^{(h)}_t \sim N(0, \sigma(h))$$

estimates are computed via instrumental variables with robust heteroskedasticity and autocorrelation consistent (HAC) standard errors. We report 90% confidence bands and examine the response of inflation observed at various frequencies. We normalize the shock series to have unit variance.

Figure 4 plots the impulse response to a one-time contractionary NS monetary policy shock.
Figure 4: Impulse response of daily inflation (31-day percentage change) to a one standard deviation Nakamura and Steinsson (2018a) shock: aggregated vs disaggregated. For a given month, the aggregated series are the sum of the monetary policy shocks and the average of 31-day annualized percentage change of daily inflation,

\[ BPP_T = \frac{1200}{m} \sum_{t=1}^{m} (\log BPP_t - \log BPP_{t-31}) \]

for days \( t = 1, \ldots, m \) of month \( T \).
Panel 4a shows that at impact and for seven periods, disaggregated daily inflation responds positively with the 90% confidence band falling quickly and overlapping with zero for periods seven through 33. After roughly 30 periods (one month), the inflation response turns negative and is significantly so for the remaining periods shown. By itself this impulse response is merely suggestive. At a daily frequency, the NS shock sequence does not produce a substantial and long-lasting positive response of inflation to a contractionary monetary policy shock. The magnitude of the initial positive response is roughly half that of the negative (and much more persistent) response. Since the NS monetary policy shock is associated with initially adverse responses, feeding in this sequence gives us the best chance of recovering one.

Panels 4c and 4d aggregate the daily index to a monthly frequency. Panel 4c plots the response with one lag in the local projection specification, while Panel 4d plots the response with 12 lags. Again, the adverse response emerges but only temporarily on impact for the estimation with one lag and with 12 lags. By comparing the aggregated impulse responses in Panels 4c and 4d to the disaggregated impulse response on the same scale in Panel 4b, we show that temporal aggregation can decisively throw off magnitudes. The disaggregated initial response of the daily frequency given by Panels 4a and 4b was dominated by the larger and more significant negative response of the later time periods. When aggregated, the data suggest the alternative interpretation: the initial perverse response is quantitatively large and the only component of the impulse response function for which the confidence band does not cover zero. This figure behooves researchers to provide an explanation for this adverse response when in fact it is not the prominent feature of the data.

Figure 5 plots the impulse response of inflation to a contractionary BRW monetary policy shock. We find no significant adverse (positive) response of inflation to a contractionary shock which corroborates the findings of BRW and contrasts those of NS. Panel 5a shows that the disaggregated response of inflation is close to zero or slightly negative until about period 60 (two months) when it becomes more negative and on the margin of the confidence band. By

\footnote{We average the daily shocks for each month and then normalize the resulting shock to have unit variance, just as before.}
Figure 5: Impulse response of daily inflation (31-day percentage change) to a one standard deviation Bu et al. (2021) shock: aggregated vs disaggregated. For a given month, the aggregated series are the sum of the monetary policy shocks and the average of 31-day annualized percentage change of daily inflation, $BP_{PT} = \frac{1200}{m} \sum_{t=1}^{m} (\log BP_{Pt} - \log BP_{P_{t-31}})$ for days $t = 1, \ldots, m$ of month $T$. 
contrast, Panel 4a shows the NS shock delivering a temporary adverse response of inflation that
turns negative and substantially so after about 30 days.

Panels 5c and 5d show the response of the daily index aggregated to a monthly frequency. The former panel plots the response with one lag of the regressor and the latter panel plots the response with 12 lags. The aggregated response in both panels is largely conventional: inflation almost always decreases, with the 90 percent error band turning negative at four months. The negative response of inflation shown in Panels 5c and 5d is an exacerbation of the slightly negative impulse response observed at a disaggregated frequency as shown in Panel 5b which is a re-scaled version of Panel 5a. This temporal aggregation bias is thus consistent with that found in the impulse responses to the NS shocks (Figure 4) even though the different shocks produce different responses. In both cases, temporal aggregation significantly increases the response of inflation to a monetary policy shock.

3.3 Unobserved Components Model To study the response of high-frequency inflation to a monetary policy shock more systematically, we now introduce an unobserved components model. We employ this methodology for several reasons. First, the trend-cycle decompositions cast in state space form have proven very useful for inflation at lower frequencies [Stock and Watson (2020)]. Second, there is transparency in modeling assumptions. Relative to the local projections methodology, which relies on IV and HAC errors, the modeling assumptions here are more straightforward. This allows us to take a more definitive stance on our result of temporal aggregation, as opposed to disentangling how temporal aggregation might interact with say our IV estimation. Third and relatedly, the model specification is parsimonious. Finally and most importantly, the state space / estimation methodologies allow us to more easily handle data observed at different frequencies and with observations missing at different dates—we use daily inflation data, data on break-even inflation rates that is available daily except for holidays and weekends, infrequent monetary policy shocks, and monthly inflation rates.

Our model consists of the following state equations: Unobserved daily CPI inflation, \( \pi_t = \tau_t + g_t + e^\pi_t \), broken down into trend \( \tau \), a cyclical component \( g \), and shock \( e^\pi \). The trend
and cyclical components follow, \( \tau_t = \tau_{t-1} + \sum_{k=0}^{K} \theta_k^r m_{t-k} + e_t^r \) and \( g_t = \rho g_{t-1} + \sum_{j=0}^{J} \theta_j m_{t-j} + e_t^g \), respectively.\(^{13}\) Trend inflation allows for a unit-root specification and a sequence of monetary policy shocks for 60 periods \((K = J = 60)\). The cyclical component permits auto-correlation and the same number of monetary policy shocks. We assume monetary shock dynamics \( m_t = e_t^m \) with all shocks \( e \) being i.i.d. Gaussian. The observation equations are the monthly observation of CPI (real-time vintages): \( \pi_t^m = \alpha^m + \pi_{t-p} + e_t^{monthly} \), where \( p \) is publication lag mentioned in Section 2 (which can vary over time as shown in Table 1). At higher frequencies, we use the daily measure of monthly (31-day) inflation: \( \pi_t^{daily} = \alpha^{daily} + \pi_t + e_t^{daily} \), and the 10-year break-even rates: \( \pi_t^{BE,h} = \alpha^{BE} + E_t \pi_{t+h} + e_t^{BE} \). We assume that the monetary policy surprise is a noisy measurement of the true monetary policy shock: \( m_t^{obs} = m_t + e_t^{m,obs} \), along the lines of Caldara and Herbst (2019). Note that the model implies \( E_t \pi_{t+h} = E_t(\tau_{t+h} + g_{t+h}) = \tau_t + \rho^h g_t \approx \tau_t \), where the last approximation is imposed on the estimation procedure (our prior imposes that the daily persistence of the cyclical component \( |\rho| < 1 \), and \( h \) represents the 10 year horizon).

The estimation is Bayesian with the likelihood function evaluated using the Kalman filter. To effectively explore the posterior distribution, a sequential Monte Carlo algorithm is implemented [Herbst and Schorfheide (2016)]. We use 15,000 particles with 200 steps to go from the prior to the full posterior and five Metropolis Hastings steps per iteration of the algorithm. Table 3 reports our prior distributions, which are largely taken from the literature.

Panel 6a plots the overall impulse response function of inflation to a contractionary monetary policy shock, while Panels 6b-6c plot the response of the cyclical and trend components, respectively. Darker shaded error bands are 68th percentiles, while lighter shades are 90th. The initial observation is that inflation—at a daily frequency—does not contain an adverse response. The initial reaction of inflation to a one standard deviation monetary policy shock is negative, even at the 90th percentile, followed by an increase and an error band that contains

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\(^{13}\)In contrast to previous work using state space models to describe inflation dynamics, we explicitly incorporate a role for monetary policy shocks. We allow these shocks (which are measured with error) to affect both cyclical and permanent components of inflation. This is important because movements in inflation that might seem permanent at the daily frequency can correspond to persistent, but non-permanent components at a lower frequency.
Table 3: Prior Specification

zero over the remaining horizon. The trend response of Panel 6c shows that the standard and theory-consistent response of inflation is present in our daily data. These results further corroborate our findings from the local projections; namely, that the positive reaction of inflation to a monetary policy shock is muted substantially at the daily frequency. By decomposing into trend and cycle, we are able to parse the short-lived adverse response as a purely cyclical response. Most importantly, the cyclical response is shown to be quantitatively small relative to trend.14

The variance decomposition, plotted in Figure 7, shows that the lion’s share of volatility is explained by the trend component of inflation as opposed to the cyclical component. This highlights the importance of understanding trend dynamics, which do not contain a perverse response to monetary policy. Taken together, these figures suggest that methodologies that de-trend inflation prior to analysis could cause amplifications of adverse responses. It is the trend component of our analysis that is dominant and responds to monetary policy shocks in a theory-consistent manner. More germane to our argument, the cyclical component when evaluated at daily frequencies does not display a substantial adverse response despite the fact that

14Appendix B shows that our results are robust to less shrinkage of the estimators.
Figure 6: Impulse responses to a one standard deviation Nakamura and Steinsson (2018a) shock
the shocks fed into the system generate substantial adverse responses at much lower (monthly) frequencies.

Panels 8a-8c show the impulse responses to the monetary policy shocks of BRW. The results are similar to those of NS shown in Figure 6.

4 A Few Properties of Temporal Aggregation

We employ stylized models of monetary policy in order to establish properties of temporal aggregation designed to shed light on the empirical results of Section 3. Specifically, we demonstrate how temporal aggregation can cause substantial bias in the initial values of moving-average representations (i.e. bias in impulse response functions) consistent with Figures 4 and 5. We also show how a false adverse response can be generated by temporal aggregation—an averaged contractionary monetary policy shock generates an increase in inflation in the aggregate case, while no such dynamics hold in the disaggregated data. We keep the models sufficiently simple in order to provide clear intuition, acknowledging that these are examples as opposed to theorems. However, we conjecture robustness of our results by appealing to a broader literature.

4.1 Biased Moving-Average Representation

Consider a nominal bond that costs $1 at date $t$ and pays off $(1 + i_t)$ at date $t + 1$. The asset-pricing equation for this bond can be written in log-linearized form as a Fisher equation,

$$i_t = r + E[\pi_{t+1}|I_t]$$

(1)

where the real interest rate is assumed to be constant and $E[\pi_{t+1}|I_t]$ is the private agents’ expectation of next period’s $(t + 1)$ inflation. Monetary policy follows a Taylor rule, adjusting the
Figure 7: Variance Decomposition associated with Monetary Policy Shock as a fraction of total variance.
Figure 8: Impulse responses to a one standard deviation Bu et al. (2021) shock.
nominal interest rate in response to inflation,

\[ i_t = r + \phi [\pi_t|I_t] + x_t \]  \hspace{1cm} (2)

\[ x_t = \rho x_{t-1} + \varepsilon_t \]  \hspace{1cm} (3)

where \( \rho \in (0, 1) \), \( \varepsilon_t \) is Gaussian with mean zero and variance \( \sigma^2_{\varepsilon} \). The innovation \( \varepsilon_t \) represents the monetary policy shock and the information set of the monetary authority is consistent with private agents’ \( I_t \). The unique equilibrium rate of inflation is well known and follows from implementing the Taylor principle \( (\phi > 1) \),

\[ \pi_t = -\frac{x_t}{\phi - \rho} = \rho \pi_{t-1} + w_t \]  \hspace{1cm} (4)

where \( w_t = -\varepsilon_t / (\phi - \rho) \).

We assume the econometrician observes realizations of the equilibrium processes at a lower frequency than private agents. Specifically, let \( t = mT \) and define the temporally aggregated inflation process as

\[ \Pi_T = \left( \frac{1}{m} \right) \left( \sum_{j=0}^{m-1} L^j \right) \pi_{mT} = \left( \frac{1}{m} \right) (\pi_{mT} + \pi_{mT-1} + \cdots + \pi_{mT-m-1}) \quad T = 1, 2, 3, \ldots \]  \hspace{1cm} (5)

For example, if \( t \) is a month, then \( m = 3 \) and \( T \) is a quarter. \( \pi_t \) is interpreted as a monthly year-over-year percentage change, and the three-month non-overlapping arithmetic mean is the appropriate aggregated time series, which is consistent with our empirics of Section 3. Alternatively, we could assume to observe month-over-month inflation and the direct summation yields quarterly inflation. Our analysis below is robust to these alternative aggregation methods.

Appendix C shows that temporally aggregating the AR(1) inflation process given by (4) yields
Table 4: Estimates of the ARMA(1,1) (15) using temporally aggregated observations of (4). We match moments of the aggregated inflation series to the ARMA(1,1) process as derived in Appendix C. Note that for \( m = 1 \) (no temporal aggregation), \( \sigma^2_u = \sigma^2_w \).

<table>
<thead>
<tr>
<th></th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 5 )</th>
<th>( m = 10 )</th>
<th>( m = 20 )</th>
<th>( m = 30 )</th>
<th>( m = 40 )</th>
<th>( m = 50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.990</td>
<td>0.980</td>
<td>0.951</td>
<td>0.904</td>
<td>0.818</td>
<td>0.740</td>
<td>0.669</td>
<td>0.605</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.000</td>
<td>0.171</td>
<td>0.250</td>
<td>0.264</td>
<td>0.265</td>
<td>0.266</td>
<td>0.266</td>
<td>0.267</td>
</tr>
<tr>
<td>( \sigma^2_u )</td>
<td>0.028</td>
<td>0.041</td>
<td>0.085</td>
<td>0.160</td>
<td>0.288</td>
<td>0.391</td>
<td>0.476</td>
<td>0.542</td>
</tr>
<tr>
<td>( \sigma^2_{\Pi} )</td>
<td>1.397</td>
<td>1.389</td>
<td>1.374</td>
<td>1.351</td>
<td>1.307</td>
<td>1.266</td>
<td>1.226</td>
<td>1.186</td>
</tr>
</tbody>
</table>

An ARMA(1,1) representation,

\[
(1 - \rho^m L) \Pi_T = u_t + \theta u_{t-1} \qquad u_t \sim N(0, \sigma^2_u) \tag{6}
\]

where, for lag operator \( L \), the autocorrelation coefficient is raised to the power of \( m \) (the number of aggregate components), and the estimated shocks \( u_t \) will be fundamental for the \( \Pi_t \) process (Amemiya and Wu (1972)). The last fact ensures that an autoregressive (or VAR) representation will accurately estimate the ARMA process. An analytical mapping between the aggregated inflation process and the ARMA(1,1) parameters is not feasible but Table 4 provides estimates of the parameters for various values of \( m \) using simulated data. We set \( \rho = 0.99, \phi = 1.05, \sigma^2_\varepsilon = 0.01 \), and use one million disaggregated observations.

The estimates of Table 4 reveal important properties of the mapping between an AR(1) process and its temporally aggregated ARMA(1,1) counterpart: [i.] the autocorrelation coefficient decays exponentially at rate \( m \); [ii.] the variance of the aggregate inflation process,

\[
\sigma^2_{\Pi} = \frac{\sigma^2_\pi}{m^2} \left( m + 2[(m-1)\rho + (m-2)\rho^2 + \cdots + \rho^{m-1}] \right) \tag{7}
\]

declines multiplicatively in \( m \) (see Appendix C for derivation). Taken together, [i] and [ii] imply that the variance of the innovation process \( \sigma^2_u \) and the moving average parameter \( \theta \) must compensate for the faster decline in the autocorrelation coefficient. Table 4 shows that the variance of the innovation \( \sigma^2_u \) increases 46% for \( m = 2 \) and by a factor of ten for \( m = 20 \), and the
moving-average parameter also increases with $m$.

The increase in the estimated variance will translate into a more pronounced initial impact of the impulse response of inflation to a monetary policy shock. Figure 9a plots the impulse response functions to a one-standard deviation shock ($\sigma_u$) for various levels of aggregation. The disaggregated “true” impulse response ($m = 1$) shows an inflation process that is substantially mitigated relative to the temporally aggregated responses. This is entirely consistent with our empirical findings in Section 3, see Figures 4 and 5. This result is not an artifact of specific assumptions underlying our model but is due to the more generic properties of temporal aggregation.15 Figure 9b plots the moving-average filter $\left(\frac{1}{m}\right) \left(\sum_{j=0}^{m-1} L^j\right)$ in the frequency domain over the range of 0 to $\pi$. The figure shows that a MA filter is a low-pass filter because it allows lower frequencies to pass through while attenuating medium and higher frequencies. What is critical for temporal aggregation is how the reallocation of the spectrum is distributed across various parameters of the estimated ARMA(1,1) process. Lower frequencies are preserved when aggregation occurs despite the decline in the autocorrelation coefficient (from $\rho$ to $\rho^m$). Amemiya and Wu (1972) show that, for any stationary AR(p) representation, temporal aggregation pre-

15This finding appears in other guises in the temporal aggregation literature. For example, Marcet (1991) finds “a systematic effect of time aggregation is to increase the absolute size of the first few coefficients of the MAR (moving-average representation).”
serves the order of the autoregressive process (i.e., an AR(p) becomes an ARMA(p,q))\(^{16}\) with the autoregressive roots all raised to the power \(m\) (Amemiya and Wu (1972) Theorem 1). These seemingly conflicting properties—a decline in the value of the (positive) autocorrelation roots coupled with no subsequent change in the low frequency properties of the time series process—leads to the large initial impulse response coefficients through an increase in the variance of the innovation process and appearance of positive moving-average parameters. We state this explicitly as our first result.

**Result 1.** Temporal aggregation can exacerbate initial impulse response parameters consistent with Figures 4 and 5 of Section 3.

While Figure 9a is a simulation, it reveals the extent to which temporal aggregation can alter the magnitude of impulse response functions. Figures 4 and 5 suggest moving from daily to monthly frequency can cause a 10-fold decline in the initial impulse response coefficients. While Figure 9a does not deliver quite a 10-fold decline, it is substantial (impact response increases from -0.65 to -0.18), and slight changes to parameters—like a slight increase in the variance of the monetary policy shock—can generate quantitative changes on par with our empirical results.

### 4.2 A False Adverse Response

The previous example demonstrated how temporal aggregation can lead to significant quantitative errors; we now show how it can cause qualitative errors. Consider the data-generating process of inflation,

\[
\pi_t = \sum_{j=0}^{59} \Theta_j \epsilon_{t-j}^m + u_t \\
u_t = \rho_u u_{t-1} + \epsilon_t^u
\]

where \(\epsilon_t^m \sim N(0, 1)\) is uncorrelated with the persistent shock \(u_t \sim N(0, \sigma_u^2)\). We assume that one shock occurs every 30 days and is zero otherwise. The parameters are given by \(\Theta_i = 1\) for

\(^{16}\)Stram and Wei (1986) show this condition holds as long as the AR roots are distinct from the MA roots.
\( i = 0, \ldots, 9 \) and \( \Theta_i = -1 \) for \( i = 10, \ldots, 59 \). Our parameterization accomplishes two tasks: first, it approximates the adverse response, as the initial positive response of inflation to a monetary policy shock. Second, the average effect over the 30-day period is negative, so the adverse response should not materialize in the aggregate (monthly) data.

To approximate population moments, we simulated three million “daily” observations, taking 30-day averages of shocks and the inflation process (8) to obtain corresponding “monthly” data. Local projections were used to estimate “monthly” responses of inflation to the monetary policy shock, controlling for lagged inflation outcomes. We set \( \rho_u = 0.99 \) and \( \sigma_u = 1 \) to capture the idea that other shocks are just as important as monetary policy for the evolution of inflation at the daily frequency. Table 5 shows results for three local projection specifications. Figure 10 shows that the qualitative patterns are similar for less volatile and less persistent \( u_t \) processes.

Table 5: Regression results - three million simulated daily data points, aggregated to monthly (30 day) frequency.

<table>
<thead>
<tr>
<th>Left-hand side variable</th>
<th>Right-hand side variables</th>
<th>( \varepsilon_t^{\text{monthly}} )</th>
<th>( \pi_{t-1}^{\text{monthly}} )</th>
<th>( \varepsilon_{t-1}^{\text{monthly}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_t^{\text{monthly}} )</td>
<td></td>
<td>0.29</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>( \pi_t^{\text{monthly}} )</td>
<td></td>
<td>0.38</td>
<td></td>
<td>-0.85</td>
</tr>
<tr>
<td>( \pi_{t+1}^{\text{monthly}} )</td>
<td></td>
<td>-0.92</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>

The econometrician would find a positive short-term response of inflation to a monthly monetary policy shock, which dissipates after one month, as seen by the one step ahead local projection results. To make sure the results do not hinge on the specifics of local projection setup, we also estimate an alternative specification where we control for lagged shocks instead. We again find that on impact there is an adverse response in monthly data. These results qualitatively confirm our empirical findings—a short-lived adverse response at daily frequency is exacerbated at monthly frequency.

In order to understand this result, assume three periods as opposed to 30. The shock is as-
Figure 10: Impact IRF estimated via local projections as a function of Autocorrelation ($\rho^u$) and Standard Deviation ($\sigma^u$)
sumed to enter the first period positively and the last two periods negatively, so that, on average, the shock sequence will be negative. The DGP of (8) becomes

\[ y_t = (1 - L - L^2)\varepsilon_t^m + u_t \]
\[ u_t = \rho u_{t-1} + \varepsilon_t^u \]

Now temporally aggregate such that \( Y_t = y_t + y_{t-1} + y_{t-2} = (1 + L + L^2)y_t \) and \( U_t = u_t + u_{t-1} + u_{t-2} = (1 + L + L^2)u_t \). (Note that taking the average \( Y_t = (1/3)(y_t + y_{t-1} + y_{t-2}) \) will not materially change the analysis.) The aggregated DGP becomes

\[ Y_t = (1 - L - L^2)(1 + L + L^2)\varepsilon_t^m + U_t = (1 - L^2 - 3L^3)\varepsilon_t^m + U_t \]
\[ U_t = \rho u_{t-1} + \varepsilon_t^u + \theta \varepsilon_t^u \]

From (9), we see that temporal aggregation preserves the initial positive innovation, but pushes the negative shocks further into the past \((-L^2 - 3L^3)\). This can explain why the local projection using the NS shocks at horizon 0 yields an adverse response (Panel 4a). As discussed in the previous section, temporally aggregating an AR(1) yields an ARMA(1,1) with a positive MA coefficient (assuming a positive autocorrelation coefficient). Therefore the positive initial innovation will not be overturned by the aggregated AR process of (10) but will be made larger by the positive MA coefficient. This is confirmed by Figure 10. The positive impact of the IRF is monotonically increasing in the autocorrelation parameter \( (\rho^u) \) and standard deviation \( (\sigma^u) \).

As noted above and from (11), the variance of the aggregated process approaches the variance of the disaggregated process as \( \rho \to 1 \), even though the autocorrelation coefficient of the aggregated process declines exponentially. Thus, the impact IRF will be increasing in both the autocorrelation parameter \( (\rho^u) \) and standard deviation \( (\sigma^u) \) as shown in Figure (10). We summarize this as:

**Result 2. False Appearance or Magnification of an Adverse Response.** A short-lived adverse
response of inflation to a monetary policy shock is exacerbated by temporal aggregation.

Our example shows that even when an overwhelming majority of the observations point to a decline of inflation in response to a contractionary monetary policy shock, an econometrician that temporally aggregates can find the opposite—an increase in inflation—in aggregate data.

5 Concluding Thoughts

This paper revisits a fundamental question of monetary economics: What is the transmission of monetary policy to the economy? We introduce temporal aggregation bias as a new information-based explanation for the adverse transmission of monetary policy shocks. When using the daily CPI from the Billion Prices Project as a temporally disaggregated macroeconomic indicator, we find that the adverse response of inflation is short-lived, if it is present at all. To understand how one can obtain a sizable adverse response to monetary policy shocks with monthly or quarterly data when only a limited adverse response actually exists, we combine informal and formal empirical evidence with a simple model of temporal aggregation bias. Because our temporal aggregation results are generic, and macroeconomic indicators are published with a lag, we argue that temporal aggregation bias is not limited to our study of monetary policy transmission and will likely be a key feature of the nascent field of high-frequency macro.
REFERENCES


JACOBSON, MATTHES & WALKER: HIGH FREQUENCY


A Appendix: BPP Robustness Checks

A.1 Alternative Constructions of BPP Inflation

This section shows an alternative version of figure 2.

Figure 11: Nowcast of CPI using end of month values of the BPP, monthly and 31-day percentage change. For month $T$, $\Delta CPI_T = 100 \times (\log CPI_T - \log CPI_{T-1})$ and for day $m$ of month $T$, $\Delta BPP_T = 100 \times (\log BPP_m - \log BPP_{m-31})$. 
Figure 12: Nowcast of CPI using aggregated monthly values of the BPP, index. For month $T$, $CPI_T = \log CPI_T$ and for day $t$ of month $T$, $\Delta BPP_T = \sum_{t=1}^{m} \log BPP_t$ for $t = 1, \ldots, m$ days in month $T$. 
A.2 Seasonality

\[ BPP_t = \text{trend}_t + \sum_j \alpha_j^{\text{day}} \mathbb{1}_{\text{day of week}} + \epsilon_t \]

Figure 13: Day of week effects of the Billion Prices Project daily inflation.
B Appendix: Impulse Response Functions with less Shrinkage

This Appendix shows the impulse responses from the state space model under the assumption of less shrinkage.

Figure 14: Impulse response of inflation ($\pi_t$) to a one standard deviation Nakamura and Steins-son (2018a) monetary policy shock
Figure 15: Impulse response of the cyclical component of inflation ($g_t$) to a one standard deviation Nakamura and Steinsson (2018a) monetary policy shock.

Figure 16: Impulse response of the trend component of inflation ($\tau_t$) to a one standard deviation Nakamura and Steinsson (2018a) monetary policy shock.
C  Appendix: Temporal Aggregation

Theorem 1. The temporally aggregated inflation process given by (5) and (4) satisfies the following two properties:

1. The temporally aggregated inflation series, $\Pi_T$, follows an ARMA(1,1) process.

\[(1 - \rho^m L)\Pi_T = \epsilon_t - \theta \epsilon_{t-1} \quad (11)\]

2. The innovation of the ARMA(1,1) process (11) is fundamental for the temporally aggregated inflation sequence, $\Pi_T$.

This theorem is well known and dates back to at least to Amemiya and Wu (1972); thus, we do not offer a complete proof but provide intuition and references. To understand part (1), let $\pi_t = \rho \pi_{t-1} + w_t$, where $w_t$ is Gaussian with mean zero and variance $\sigma_w^2 = \sigma^2 / (\phi - \rho)^2$, and note

\[
\gamma(0) = \text{Var}(\Pi_T) = \frac{\sigma^2}{m^2} (m + 2[(m-1)\rho + (m-2)\rho^2 + \cdots + \rho^{m-1}]) \quad (12)
\]

\[
\gamma(s) = \text{Cov}(\Pi_t, \Pi_{t-s}) = \frac{\sigma^2}{m^2} \rho^m [s-1] + (1 + \rho + \rho^2 + \cdots + \rho^{m-1})^2 \quad s \neq 0 \quad (13)
\]

\[
\gamma(s) = \rho^m \gamma(s-1) \quad |s| \geq 2 \quad (14)
\]

where $\sigma^2 = \sigma_w^2 / (1 - \rho^2)$, see Wei and Ahsanullah (1984). The intuition of (12)–(13) comes from the correlation structure of an autoregressive process, where all elements are multiplied by $\sigma^2 / m^2$. Thus, there are $(m-1)$ “neighbors”, $(m-2)$ elements two periods removed, etc. Given the strength of the autocorrelation of many macro aggregates, the following limits are useful. As $\rho \to 1$, the term in brackets in (12) converges to $m(m-1)/2$ and therefore, $\text{Var}(\Pi_T) \to \sigma^2_\pi$ and $\text{Var}(\Pi_T) \in (0, \sigma^2_\pi)$. Further, the parenthetic term in (13) converges to $m$ as $\rho \to 1$, and $\text{Cov}(\Pi_t, \Pi_{t+s}) \to \sigma^2_\pi$.

The covariance difference equation (14) identifies the autocorrelation coefficient of the $\Pi_T$
process as $\rho^m$. We can then multiply $[(1 - \rho^m L)/(1 - \rho L)] \sum_{j=0}^{m-1} L^j$ to both sides of $\pi_t$ to give,

$$\left(\frac{(1 - \rho^m L)(1 - \rho L) \sum_{j=0}^{m-1} L^j}{1 - \rho L}\right) \pi_t = \left(\frac{(1 - \rho^m L) \sum_{j=0}^{m-1} L^j}{1 - \rho L}\right) w_t$$

$$\pi_t = \sum_{j=0}^{m-1} (\rho L)^j w_t = \epsilon_t - \theta \epsilon_{t-1}$$

where $\epsilon_t \sim N(0, \sigma^2)$. The errors defined by the $m$ moving-average terms $\sum_{j=0}^{m-1} (\rho L)^j w_t$ are correlated and therefore cannot be used to obtain the Wold innovations associated with predicting $\Pi_T$ linearly from its past. Theorem 1 of Amemiya and Wu (1972) proves that with $m \geq 2$, then the moving-average terms are at most of order one, which establishes the final equality.

The proof of part 2. also relies on arguments in Amemiya and Wu (1972). In order for the process to be fundamental, one must show that the roots of $1 - \theta z$ lie outside of the unit circle (i.e., $|\theta| < 1$). Given that the initial AR(1) process is positive definite ($r \ho \in (0, 1)$), then it has a positive spectral density. As shown in Amemiya and Wu (1972), temporal aggregate maintains the positive definite structure and hence the roots of the moving-average representation must lie outside the unit circle.

C.1 Moving-Average Filters Suppose we have a stationary stochastic process $x_t$ that is aggregated according to

$$X_T = \left(\frac{1}{m}\right) \left(\sum_{j=0}^{m-1} L^j\right) x_{mT} = \left(\frac{1}{m}\right) (x_{mT} + x_{mT-1} + \cdots + x_{mT-m+1})$$

where $\epsilon_t \sim N(0, \sigma^2)$. The errors defined by the $m$ moving-average terms $\sum_{j=0}^{m-1} (\rho L)^j w_t$ are correlated and therefore cannot be used to obtain the Wold innovations associated with predicting $\Pi_T$ linearly from its past. Theorem 1 of Amemiya and Wu (1972) proves that with $m \geq 2$, then the moving-average terms are at most of order one, which establishes the final equality.

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$$X_T = \left(\frac{1}{m}\right) \left(\sum_{j=0}^{m-1} L^j\right) x_{mT} = \left(\frac{1}{m}\right) (x_{mT} + x_{mT-1} + \cdots + x_{mT-m+1})$$

where $\epsilon_t \sim N(0, \sigma^2)$. The errors defined by the $m$ moving-average terms $\sum_{j=0}^{m-1} (\rho L)^j w_t$ are correlated and therefore cannot be used to obtain the Wold innovations associated with predicting $\Pi_T$ linearly from its past. Theorem 1 of Amemiya and Wu (1972) proves that with $m \geq 2$, then the moving-average terms are at most of order one, which establishes the final equality.

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Note that $1 + L + L^2 + \cdots + L^{m-1} = (1 - L^m)/(1 - L)$. Thus, the covariance generating function of $X_T$ is related to $x_t$ by

$$g_X(z) = \frac{1}{m^2} \left( \frac{1 - z^m}{1 - z} \right) \left( \frac{1 - z^{-m}}{1 - z^{-1}} \right) g_X(z)$$

(17)

In the frequency domain ($z = e^{-i\omega}$),

$$g_X(e^{-i\omega}) = \frac{1}{m^2} \left( \frac{1 - e^{-i\omega m}}{1 - e^{-i\omega}} \right) \left( \frac{1 - e^{i\omega m}}{1 - e^{i\omega}} \right) g_X(e^{-i\omega})$$

$$= \frac{1}{m^2} \left( \frac{1 - \cos(\omega m)}{1 - \cos(\omega)} \right) g_X(e^{-i\omega})$$

(18)

where $(1 - e^{-i\omega m})(1 - e^{-i\omega m}) = 2 - (e^{i\omega m} + e^{-i\omega m}) = 2 - 2\cos(\omega m) = 2(1 - \cos(\omega m))$ because $e^{i\omega m} = \cos(\omega m) + i \sin(\omega m)$ and $e^{-i\omega m} = \cos(\omega m) - i \sin(\omega m)$. Plotting this function over the range of $[0, \pi]$ gives Figure 9b.
D Appendix: Data

This section lists the source and description of each series used in this paper.

Official CPI Index Analysis in section 2.2 use the BLS’ seasonally adjusted Consumer Price Index (FRED: CPIAUCSL) at a monthly frequency. Results in section (3) use the seasonally adjusted (PCPI) and not seasonally adjusted (CPIN) real-time Consumer Price Index which is accessed via the Real-time Data Research Center at the Federal Reserve Bank of Philadelphia.\textsuperscript{17} In each real-time spreadsheet, the columns are the date of the vintage and the rows are the time series for that vintage. We then construct a time series by computing the annualized monthly percentage change for the last two entries for each vintage.

Daily CPI The Billion Prices Project publicly available daily inflation index can be obtained via Cavallo and Rigobon (2016) for July 2008 through August 2015.\textsuperscript{18} The index is obtained by webscraping prices from multichannel retailers that sell both online and offline.

Break-even Inflation Rates 10 year spot breakeven inflation rates are the daily 10-year treasury yield at constant maturity (FRED: BC\_10YEAR) less the daily 10-year TIPS at constant maturity (FRED: TC\_10YEAR). These rates are obtained from the U.S. Treasury Department via FRED.

Zero-coupon Treasury Yields Continuously compounded zero-coupon yields (mnemonic: SVENYXX) are obtained via the Federal Reserve Board.\textsuperscript{19}

\textsuperscript{17}We thank Tom Stark for help obtaining these series. https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/real-time-data-set-full-time-series-history
\textsuperscript{18}Series indexCPI for country==USA in spreadsheet pricestats_bpp_arg_usa.csv in folder all\_files\_in\_csv\_format.zip at website https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi\%3A10.7910\%2FDVN\%2F6RQCRS. Alternatively, the data are also available from the pricestats\_bpp\_ar\_usa.dta file in the RAWDATA folder on the website, https://www.openicpsr.org/openicpsr/project/113968/version/V1/view.
\textsuperscript{19}See https://www.federalreserve.gov/data/yield-curve-tables/feds200628\_1.html or as a csv file.
High-frequency monetary policy shocks are originally available from 1995 to 2014.\textsuperscript{20} We extend this shock series from 1990 to present using the authors’ code and futures tick data accessed via CME Group Inc. DataMine (\url{https://datamine.cmegroup.com/}) at the Federal Reserve Board.\textsuperscript{21} The construction of the shock series follows that of Gürkaynak et al. (2005) and relies on five short-term interest rate futures. Let $t$ index the current FOMC meeting and the changes in the five interest rate futures be given as:

- $mp1_t$: change in federal fund futures expiring at the end of the month of the FOMC meeting
- $mp2_t$: change in federal funds futures expiring in three months,
  - i.e. expected change in the federal funds rate at the next FOMC meeting
- $ed2_t$: change in eurodollar futures expiring in 1.5 quarters (called 2nd contract)
- $ed3_t$: change in eurodollar futures expiring in 2.5 quarters (called 3rd contract)
- $ed4_t$: change in eurodollar futures expiring in 3.5 quarters (called 4th contract)

The calculations underlying the above series are given below. Let $s$ index the current month of the FOMC announcement. For example, for the July 29, 2020 FOMC announcement $s = \text{July 2020}$ and $s + 1 = \text{August 2020}$. We define $t$ more precisely as 20 minutes after the FOMC announcement. $t - \Delta t$ is defined as 10 minutes before the FOMC announcement. For the July 29, 2020 FOMC announcement which occurred at 14:00, $t = \text{July 29, 2020 14:20}$ and

\textsuperscript{20}Series FFR\textunderscore shock from the spreadsheet PolicyNewsShocksWeb.xlsx
\textsuperscript{21}\url{https://eml.berkeley.edu/~jsteinsson/papers/realratesreplication.zip}
$t - \Delta t = \text{July 29, 2020 13:50.}$

\[
mp1_t = \frac{D1}{D1 - d1} (ff1_{s,t} - ff1_{s,t-\Delta t}) \tag{19}
\]

\[
mp2_t = \frac{D2}{D2 - d2} \left[ (ff2_{s+2,t} - ff2_{s+2,t-\Delta t}) - \frac{d2}{D2} mp1_t \right] \tag{20}
\]

\[
ed1_2 = ed1_{s+3.5,t} - ed1_{s+3.5,t-\Delta t} \tag{21}
\]

\[
ed1_3 = ed1_{s+7.5,t} - ed1_{s+7.5,t-\Delta t} \tag{22}
\]

\[
ed1_4 = ed1_{s+11.5,t} - ed1_{s+11.5,t-\Delta t} \tag{23}
\]

$D1$: number of days in month of monetary policy announcement

$d1$: day of monetary policy announcement

$ff1_{s,t-\Delta t}$: current month federal funds futures 10 minutes before monetary policy announcement

$ff1_{s,t}$: current month federal funds futures 20 minutes after monetary policy announcement

$mp2_t$: next monetary policy surprise

$D2$: number of days in the month of the next monetary policy announcement

$d2$: day of the next monetary policy announcement

$ff2_{s+2,t-\Delta t}$: 3-month ahead federal funds futures 10 minutes before monetary policy announcement

$ff2_{s+2,t}$: 3-month ahead federal funds futures 20 minutes after monetary policy announcement

$ed2_{s+3.5,t-\Delta t}$: 2nd expiring eurodollar futures contract 10 minutes before

$ed2_{s+3.5,t}$: 2nd expiring eurodollar futures contract 20 minutes after

$ed3_{s+7.5,t-\Delta t}$: 3rd expiring eurodollar futures contract 10 minutes before

$ed3_{s+7.5,t}$: 3rd expiring eurodollar futures contract 20 minutes after

$ed4_{s+11.5,t-\Delta t}$: 4th expiring eurodollar futures contract 10 minutes before

$ed4_{s+11.5,t}$: 4th expiring eurodollar futures contract 20 minutes after
If the FOMC announcement occurs in the last 7 days of the month, for example at 14:00 on July 29, 2020, then the scaling is not used and the next month’s futures contract is used:

\[ mp_1^t = (ff_{1s+1,t} - ff_{1s+1,t-\Delta t}) \] (1.a)
\[ mp_2^t = (ff_{2s+3,t} - ff_{2s+3,t-\Delta t}) \] (2.a)

Following the Appendix of Nakamura and Steinsson (2018a), only trades on the trading day in question and until noon the next day are considered. If there are no eligible trades, the change is set to zero (i.e., we interpret no trading as no price change).\(^{22}\)

The monetary policy shock is then the first principal component of expressions (19)-(23) scaled so that its effect on one-year nominal Treasury yields is equal to one.

**Bu et al. (2021) Monetary Policy Shock** Daily monetary policy shock are available from 1994 to 2020.\(^{23}\) This shock series is constructed by a Fama and MacBeth (1973) two-step procedure that extracts unobserved monetary policy shocks \(\Delta i_t\) from the common component of the change in zero-coupon yields \(\Delta R_{j,t}\).

1. estimate sensitivity of yields with maturity \(j = 1,\ldots,30\) to monetary policy via time-series regressions

\[ \Delta R_{j,t} = \alpha_j + \beta_j \Delta i_t + \epsilon_{j,t} \]

assume \(\Delta i_t\) is one-to-one with two-year yield \(\Delta R_{2,t}\) to allow for normalization\(^{24}\)

\[ \Delta R_{j,t} = \theta_j + \beta_j \Delta R_{2,t} + \epsilon_{j,t} - \beta_j \epsilon_{2,t} \]
\[ \xi_{j,t} \]

\[ \text{corr}(\Delta R_{j,t}, \xi_{j,t}) \] due to \(\beta_j \epsilon_{2,t}\) reconciled w/ IV or Rigobon (2003) het. estimator

\(^{22}\)See the online Appendix of Nakamura and Steinsson (2018a).


\(^{24}\)Our results are robust to scaling by one-year yields so that the BRW shocks have the same scaling as the NS shocks.
2. recover aligned monetary policy shock $\Delta i_t^{aligned}$ form cross-sectional regressions of $\Delta R_{j,t}$ on the sensitivity index $\hat{\beta}_j$ for each FOMC announcement $t$

\[
\Delta R_{j,t} = \alpha_j + \Delta i_t^{aligned} \hat{\beta}_j + v_{j,t}, \quad t = 1, \ldots, T
\]