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Beliefs, Aggregate Risk, and the U.S. Housing Boom*

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Abstract

Endogenously optimistic beliefs about future house prices can account for the path and standard deviation of house prices in the U.S. housing boom of the 2000s. In a general equilibrium model with incomplete markets and aggregate risk, agents form beliefs about future house prices in response to shocks to fundamentals. In an income expansion with looser credit conditions, agents are more likely to underpredict house prices and revise up their beliefs. Matching the standard deviation and steady rise in house prices results in homeownership becoming less affordable later in the boom as well as consumption dynamics that match the data.

Keywords: housing boom; aggregate risk; heterogeneous agents; incomplete information
JEL Codes: E20, E3, C68, R21

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1 Introduction

Loose credit conditions pertaining to mortgage financing and optimistic beliefs about future house prices are the most common explanations for the previous record increase in aggregate house prices attained during the 1999-2007 U.S. housing boom.[1] Kaplan et al. (2020) show that shifts in beliefs are quantitatively more important than shifts in credit conditions when both are modeled as exogenous orthogonal shocks. How often beliefs shift is therefore an important determinant of the frequency of housing booms and a low transition probability in their calibration suggests that booming house prices are unlikely to occur more than once in a generation. Aggregate house prices, however, rebounded in the 2010s to a new record high in 2022 along with heightened measures of house price optimism. This experience suggests that revisiting the modeling of optimistic beliefs is important to understand house price determination when booms are no longer tail events.

While Kaplan et al. (2020) model beliefs as exogenous, I propose a framework where beliefs instead arise endogenously in response to economic fundamentals and incomplete information. I show that house prices under endogenous beliefs better match the empirical time path, standard deviation, and autocorrelation observed throughout the 2000s. Because empirical aggregate house prices in the 21st century are nearly twice as volatile as their counterparts from the 20th, matching house price statistics other than the level increase is important for understanding the determinants of housing demand and hence house prices. More volatile house prices result in 1) homeownership becoming riskier and less affordable as prices rise and 2) a time path for consumption that more closely matches that of the data.

This paper asks the question of why there was a shift in beliefs and proposes incomplete information about the evolution of house prices in an economic expansion with unprecedented looser credit conditions as an explanation.[2] When agents lack full information about the evolution of house prices in a state of the economy with little historical precedent and higher housing demand, they underpredict house prices and revise up their beliefs resulting in optimistic expected future house prices. Persistently positive forecast errors give rise to booming house prices and are consistent with both an empirical proxy I construct from the University of Michigan Survey of Consumers and evidence from Kindermann et al.'s (2022)

[1]See Favilukis et al. (2017), Greenwald (2018), Arslan et al. (2022), Lind (2021), Johnson (2019), Landvoigt (2016), Kermani (2012), Justiniano et al. (2019), Mian and Sufi (2017) and Di Maggio and Kermani (2017) for the role of loose credit conditions in the housing boom. See Kaplan et al. (2020), Burnside et al. (2016), Piazzesi and Schneider (2009), Gelain and Lansing (2014), Adelino et al. (2018), Nathanson and Zwick (2018), Glaeser and Nathanson (2017), and Foote et al. (2012) for the role of optimistic expectations.

[2]Glaeser et al. (2013) ask, "Why were buyers so optimistic about prices? Irrational expectations are surely not exogenous, so what explains them?" Howard and Liebersohn (2022) address the same question and propose regional divergence as the fundamental shock underlying shifts in beliefs when studying house prices and rents more generally over the last 80 years.

survey that households underpredict house prices in booms.

Under endogenous beliefs, house prices from the model match the time path, 84 percent of the level increase, 97 percent of the autocorrelation, and 95 percent of the volatility of their empirical counterparts for the 1999 to 2007 period. By contrast, the exogenous beliefs framework of Kaplan et al. (2020) can match upwards of 80 percent of the level increase, but with values for the autocorrelation and volatility that are only 71 and 16 percent of the data, respectively.[3] The closer match to the data under endogenous beliefs can be attributed to house prices persistently rising throughout the boom rather than abruptly jumping in a single period as under exogenous beliefs.

By developing a framework that links optimistic beliefs and loose credit conditions, this paper also complements growing efforts to unify the two most common explanations of the U.S. housing boom.[4] Credit conditions have only a muted direct effect on house prices but are important for matching the dynamics of homeownership, mortgage leverage, and foreclosures throughout the housing boom and bust. Under endogenous beliefs, an unprecedented shift in credit conditions takes on the indirect yet important role of triggering incomplete information about the evolution of house prices and hence optimistic beliefs. By allowing for an unprecedented shift in fundamentals to be the source of optimistic beliefs, this framework can generalize to settings other than credit conditions and the 2000s housing boom which is important for generating optimistic beliefs more often than the once in a generation frequency under exogenous beliefs.

The economy-wide nature of income fluctuations and shifts in mortgage finance in the late 1990s motivate aggregate shocks and household heterogeneity. Without household heterogeneity, changes in credit conditions would either affect all or no households which is at odds with empirical evidence. An empirically plausible distribution of wealth is therefore important to discipline the response of house prices to looser credit conditions. In summary, households face housing market frictions and idiosyncratic income shocks as they make consumption, savings, borrowing, and housing tenure decisions subject to housing adjustment costs and borrowing constraints. Markets are incomplete so that households can only partially insure against both idiosyncratic and aggregate shocks.

[3] See Piazzesi and Schneider (2016) for a discussion of why full information rational expectations models give rise to a house price volatility puzzle in both the housing boom and other economic cycles.

[4] Johnson (2019), Cox and Ludvigson (2021), Chodorow-Reich et al. (2021), and Dong et al. (2022) show that the interaction of beliefs and fundamentals are important for understanding housing boom dynamics. Howard and Liebersohn (2022) show this importance more generally when explaining house prices and rents over the past 80 years. Low interest rates have also been studied as a source of housing booms. Although Jordà et al. (2015) link loose monetary policy to high house prices in 140 years of data spanning 14 advanced countries, research is mixed on the specific role of low interest rates in the housing boom of the 2000s. Even though Adam et al. (2012) and Garriga et al. (2019) successfully link low interest rates to high house prices, Dokko et al. (2011) and Glaeser et al. (2013) struggle to find a similar connection.

With incomplete markets and aggregate risk, agents must keep track of the potentially infinitely dimensional distribution over individual states and its law of motion to determine prices. Computational tractability can be achieved by solving for a Krusell and Smith (1998) approximate equilibrium where agents instead track house price forecasts directly. Agents thus form boundedly rational expectations with full knowledge of exogenous shocks but beliefs about endogenous house prices. Underlying this well-established state space reduction technique is the assumption that agents maintain access to the entire history of endogenous variables when forming expectations.

This paper is one of the first to allow for endogenous beliefs via adaptive learning in a general equilibrium model with incomplete markets and aggregate risk.[5] Adaptive learning about house price forecasts relaxes the assumption that agents know the entire evolution of house prices in each aggregate state of the economy. Agents fix the form of the forecasting rule and update its parameter values in response to incoming data. How closely their beliefs about future house prices align with the actual evolution depends on the amount of historical data available for a particular aggregate state. Because the economy-wide changes to mortgage finance observed throughout the 1990s/2000s had no historical precedent, it is reasonable to assume that agents lacked a sufficient data sample to know the evolution of house prices. The resulting learning dynamics correspond to the U.S. housing boom and generate house prices that better match the path, autocorrelation, and standard deviation compared to other frameworks where agents know the evolution of house prices and beliefs are exogenous.

2 Related Literature

Methodologically, this paper is most similar to Favilukis et al. (2017), Kaplan et al. (2020), and Hoffman (2016) who also develop quantitative frameworks with incomplete markets and aggregate risk to study the U.S. housing boom. While Favilukis et al. (2017) do not include shifts in beliefs as a possible explanation, Kaplan et al. (2020) find that exogenous beliefs are quantitatively important. Hoffman (2016) allows for endogenous beliefs and, like this paper, can also match the volatility of aggregate house prices. Although he also embeds adaptive learning into a framework with incomplete markets and aggregate risk, he focuses on the effects of income fluctuations across several economic cycles instead of the interaction of beliefs with credit conditions as in this paper.

Structural models with learning can successfully account for house price and mortgage market dynamics throughout the housing boom. Chodorow-Reich et al. (2021) show that learning about local fundamentals under over-reactive diagnostic expectations explains regional house prices throughout the 2000s and 2010s in a spatial equilibrium model that

[5] Others include Hoffman (2016) and Giusto (2014).

abstracts away from beliefs about future house prices, household heterogeneity, and why there was a shift in beliefs. Work by Adam et al. (2012), Boz and Mendoza (2014), Kuang (2014), and Caines (2020) similarly relies on representative agents which may result in an overstated direct effect of looser credit conditions on higher house prices.

By assuming that shifts in aggregate credit conditions along with fluctuations in aggregate income trigger adaptive learning about house price forecasts, this framework takes a stance that beliefs respond to credit conditions. Although some evidence suggests the opposite direction of causality, identification remains debated and difficult to disentangle.[6]

In line with the results of Kiyotaki et al. (2011) and Kaplan et al. (2020), this paper shows that looser credit conditions have a muted quantitative effect on house prices. This finding contrasts that of Favilukis et al. (2017) who also include aggregate risk and incomplete markets, but rely on a potentially counterfactually high risk aversion, a less simplified financial sector, and influx of foreign borrowers. Subsequent work suggests that the quantitative link between credit conditions and house prices depends not only on modeling assumptions regarding household heterogeneity, but also those on market segmentation of renters and homeowners [Greenwald and Guren (2021)], the type of credit conditions [Greenwald (2018) and Justiniano et al. (2019)], and the feedback between households and the financial sector [Arslan et al. (2022)]. Even though these additional channels could result in credit conditions having a larger quantitative effect on house prices, they may still fall short of matching house price statistics other than the level increase as is the goal of this paper.

Households who increase housing demand when credit conditions loosen are the closest parallel to subprime borrowers in this paper. While the work of Mian and Sufi (2009, 2017) and Griffin et al. (2021) find that the expansion of mortgage debt to subprime borrowers is key for understanding the dynamics of the housing boom and bust, the findings of Adelino et al. (2018), Albanesi et al. (2017), Foote et al. (2018) suggest that wealthier prime households played a more central role. To reconcile the borrowing and housing expenditure patterns of wealthier households, Adelino et al. (2018) and Foote et al. (2012) propose an optimistic shift in beliefs as an alternative explanation of the U.S. housing boom.

Even though explicit measures of house price expectations are only available post 2007, empirical evidence in this paper and others supports optimistic beliefs during the 2000s. First, Case and Shiller (1988, 2004) and Case et al. (2012) survey several cities and find that home buyers have higher expectations for house price growth during local booms. Second, Piazzesi and Schneider (2009) note that households perceived housing as too expensive later

[6] While Mian and Sufi (2009, 2017) find that changes in credit conditions increased expected future house prices, Adelino et al. (2018) and Foote et al. (2012) suggest the opposite direction of causality. Cox and Ludvigson (2021) find contemporaneous correlation between beliefs and credit conditions.

in the boom but were still optimistic that house prices would continue to grow. Third, Soo's (2018) housing sentiment index shows house price optimism peaking around 2004-2005 just before house prices. Finally, Ben-David et al.'s (2019) VAR estimates link innovations in expectations to the housing boom, but abstract away from endogenous expectation formation as in this paper.

Because the exact form and evolution of house price expectations is still debated, adaptive learning is one of many forms that endogenizes beliefs. Nonetheless, adaptive learning has the advantages of tractability when embedded in a detailed quantitative model and well-studied equilibrium convergence properties.[7] Adaptive learning also tracks several properties found in empirical evidence on house price expectations. First, Kindermann et al. (2022) find that adaptive learning can account for households underpredicting house price growth in booms and then updating their expectations accordingly. Second, De Stefani (2021) and Armona et al. (2019) find that households' house price expectations underestimate short-run momentum and neglect medium-run mean reversion.[8]

Heterogeneous beliefs can account for the housing boom in structural models where relative optimists push up house prices.[9] Although heterogeneous beliefs are consistent with the survey evidence of Kindermann et al. (2022), they require additional state variables which would complicate both computation and interpretation.[10] My paper is one of few that departs from full information in a model with aggregate risk, homogeneous expectations allow for a simpler mechanism to endogenize beliefs.

3 Model

3.1 Environment

The model introduces incomplete information about future house prices into the framework of Kaplan et al. (2020) to compare housing booms dynamics under exogenous and endogenous beliefs. Time is discrete and the economy is populated by a continuum of measure one finitely lived households who are heterogeneous in ages and idiosyncratic income endowments. These households trade goods and services with a lending sector, a rental housing sector, housing construction firms, and final goods firms. Lowercase letters denote

[7] See Evans and Honkapohja (2001) for adaptive learning convergence to rational expectations equilibria. Although alternatives such as extrapolative, diagnostic, or natural expectations may be equally suitable for explaining the housing boom, their equilibrium properties in macroeconomic models are less well developed relative to those for adaptive learning. See Bianchi et al. (2022) and L'Huillier et al. (2022) for examples of diagnostic expectations in general equilibrium.

[8] Niu and van Soest (2014) find short-run momentum but argue in favor of mean reversion.

[9] Piazzesi and Schneider (2009), Burnside et al. (2016), Glaeser and Nathanson (2017), Dong et al. (2022).

[10] Porapakkram and Young (2007) develop a Krusell and Smith (1998) economy where households' information sets give rise to heterogeneous expectations. Kübler and Scheidegger (2021) develop a computational method that allows for different forecasts and expectations in an economy with aggregate risk.

individual household quantities and uppercase letters denote aggregates.

Preferences: Households are active in economic life for $j = 1, \ldots, J$ periods where they work from $j = 1$ to J^{ret-1} and retire at J^{ret} until they exit the economy with certainty at J. All j subscripts have been dropped unless needed. Households have preferences over final goods consumption and housing expenditures $\{c_j, s_j\}_{j=1}^{J}$ with final goods consumption as the numeraire. The utility function is given as:

$$u_j(c,s) = e_j \frac{\left[(1-\phi)c^{1-\gamma} + \phi s^{1-\gamma}\right]^{\frac{1-\sigma}{1-\gamma}} - 1}{1-\sigma}$$

Where ϕ is the taste for housing relative to goods consumption, $1/\gamma$ is the elasticity of substitution between housing expenditures and goods consumption, and σ is the intertemporal elasticity of substitution. A deterministic equivalence scale $\{e_j\}_{j=1}^{J}$ adjusts consumption for changes in household size over the life-cycle. Expected life-time utility is given as:

$$\mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} u_j(c,s) + \beta^J v(\flat + \underline{\flat}) \right]$$

Where the warm-glow bequest motive at the end of life J follows the functional form of De Nardi (2004) which modifies that of Carroll (2002):

$$v(\flat) = \psi \frac{(\flat + \underline{\flat})^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

The bequest motive prevents households from counterfactually ramping up debt and drawing down housing when they exit the economy. The strength of the bequest motive is regulated by ψ and the extent to which bequests are luxuries is given by $\underline{\flat}$.

Income endowments: While working, households receive an income endowment comprised of an aggregate stochastic endowment $\Theta(Z)$, a deterministic life-cycle profile that varies by age χ_j, and an idiosyncratic stochastic endowment $\epsilon_j(z)$ that follows a Markov process.

$$\log y = \log \Theta(Z) + \chi_j + \epsilon_j(z), \quad \text{when} \quad j < J^{ret}$$

The transition matrix for earnings $\epsilon_{j+1}(z') \sim \Upsilon_{j+1|j}(\epsilon_j(z))$ is age dependent which helps account for rising income volatility throughout working life.

In retirement, households receive a fraction ρ_{SS} of their last working period income with aggregate income averaged across states $\bar{\Theta}$. Heterogeneity in retirement income helps preserve the wealth distribution of retired agents.

$$y = \rho_{SS}(\log \bar{\Theta} + \chi_{J^{ret}-1} + \epsilon_{J^{ret-1}}), \quad \text{when} \quad J^{ret} \leq j \leq J \tag{2}$$

Income tax follows the functional form of Heathcote et al. (2017) where τ_y^0 sets the average level of taxation and τ_y^1 sets the degree of tax progressivity. To capture the tax benefits of homeowning, households can deduct ϱ fraction of the interest paid on their mortgage $r_m m$.

$$\mathcal{T}(y, m) = y - \tau_y^0(\max\{y - \varrho r_m m, 0\})^{1-\tau_y^1} \tag{3}$$

If a household does not have a mortgage, the expression simplifies to, $\mathcal{T}(y) = y - \tau_y^0(y)^{1-\tau_y^1}$.

Liquid savings: Households can save in one-period bonds b at the exogenous risk free rate q_b. Unsecured borrowing is prohibited and households cannot trade among themselves. Households transact bonds with risk neutral non-modeled foreign agents with deep pockets.

Housing: Households can either rent or own houses. To capture the lumpy dynamics of housing over the life-cycle—as described by Chambers et al. (2009)—housing units are indivisible and available in discrete fixed sizes $\mathcal{H} \in \{h^0, \ldots, h^N\}$ for homeowners and $\tilde{\mathcal{H}} \in \{\tilde{h}^0, \ldots, \tilde{h}^N\}$ for renters. If housing units were instead a continuum of sizes, households would make a counterfactually large number of adjustments to their square footage over the life-cycle since they would be able to upsize or downsize by a fraction of a room.

Markets for rental and owner-occupied housing are both frictionless and competitive where the law of one price holds and selling is instantaneous.[11] Rental units cost $\rho(\mu, Z)$ and owner-occupied units cost $p(\mu, Z)$. Renting generates housing services equal to the size of the housing unit, $s = \tilde{h}$ while homeownership offers additional utility $s = \omega h$ for $\omega \geq 1$. Homeowners pay per-period taxes and maintenance costs $(\delta_h + \tau_h)p(\mu, Z)h$ where maintenance offsets the depreciation of the housing unit. When selling, homeowners pay a transaction cost $\kappa p(\mu, Z)h$ that is linear in housing value. Renters do not pay these costs and can adjust the size of their housing unit without transaction costs.

Mortgages: Homeowners finance housing purchases with multi-period defaultable mortgages subject to a fixed origination cost $\kappa^m(Z)$. These mortgages are amortized over the life of the household at interest rate $r^m = (1 + \iota)r_b$ which is equal to the risk-free rate scaled by an intermediation wedge.[12] Default risk is thus priced by the mortgage pricing function

[11] See Hedlund (2016) for the role of search frictions such as time delays in housing market dynamics.

[12] Allowing homeowners to choose the length of mortgages with borrower-specific interest rates would add more realism at the cost of more state variables. As explained by Arslan et al. (2022), tractability is assured by 1) making mortgages due at the deterministic end of economic life and by 2) assuming amortization at the common mortgage interest rate r^m.

$q_j(\mathbf{x}', y; Z, \mu)$ rather than the interest rate. This function depends on variables that predict future repayment which include the age j of borrowing households, individual state variables $\mathbf{x}' = \{b', h', m'\}$, individual income y, and aggregate states Z and μ.

When a household aged j obtains a mortgage with principal balance m' they receive $q_j(\mathbf{x}', y; Z, \mu)m'$ from the lender where $q_j(\mathbf{x}', y; Z, \mu) < 1$. The down payment at origination is thus $p(\mu, Z)h' - q_j(\mathbf{x}', y; Z, \mu)m'$. Households make $J - j$ mortgage payments until the mortgage is fully repaid, $\pi_j^{min}(m) \leq m(1 + r_m) - m'$. The minimum payment is determined by constant amortization:

$$\pi_j^{min}(m) = \left[\frac{r_m(1 + r_m)^{J-j+1}}{(1 + r_m)^{(J-j+1)} - 1}\right] m \tag{4}$$

A loan-to-value (LTV) constraint limits mortgage size to a fraction of the value of housing collateral $m' \leq \theta^{LTV}(Z)ph'$ and the payment-to-income constraint (PTI) caps the minimum mortgage payment at a fraction of household income $\pi_j^{min}(m') \leq \theta^{PTI}(Z)y$. Because mortgages are multi-period debt, household are only subject to these constraints in the period when mortgages are originated. If mortgages were instead one-period debt, households would be subject to these constraints in every period. One-period debt would make housing a riskier asset because a house price decline or negative income shock would tighten these constraints to the extent that some homeowners would delever or default. To avoid these potential negative outcomes, homeowners would take on less leverage than what is observed in the data. Multi-period debt is therefore important for both a more realistic household problem and matching the empirical cross section of household mortgage leverage.

HELOCs: Home equity lines of credit (HELOCs) allow homeowners to borrow one-period, non-defaultable debt up to a fraction of the value of housing collateral $-b' \leq \theta^{HELOC} p(\mu, Z)h$ with an interest rate equal to that on mortgages r_m. Although only a small fraction of homeowners have HELOCs, including these debt instruments is important for matching targeted net worth moments in the calibration of the cross-sectional distribution of households.[13]

Shocks: Z indexes an aggregate state where income $\Theta(Z)$ fluctuates via a two state Markov chain with transition matrix $Z' \sim \Gamma_Z(Z)$ and credit conditions $\mathcal{C}(Z)$ loosen via a one-time unanticipated shock as in Favilukis et al. (2017). Because agents expect permanently looser credit conditions in the housing boom, the tightening of credit conditions in the housing bust is also a one-time unanticipated shock.

[13]Evidence is mixed as to whether or not HELOCS account for mortgage debt dynamics in the U.S. housing boom. Chen et al. (2020) find an important role while Kim (2021) argue that it is more negligible.

Although there is no definitive convention on the timing and relationship of aggregate shocks, Kaplan et al.'s (2020) assumption of independent shocks to aggregate income and credit conditions has three shortcomings. First, fluctuations in productivity and credit conditions may be correlated instead of independent. Guler (2015), Johnson (2022), and Foote et al. (2020) argue that technological innovations to mortgage underwriting are the source of relaxed mortgage lending standards and hence looser credit conditions in the 1990s. Second, specifying a Markov chain instead of unanticipated shocks assumes that the evolution of house prices in past episodes of looser credit conditions contains enough information to predict house prices in subsequent loosening episodes. If credit conditions only loosen once a generation, then the evolution of house prices during the nation-wide housing boom of the 1920s corresponds to that of the 2000s even though mortgage and housing markets underwent significant changes in the eighty years in between.[14] Finally, embedding time-varying beliefs into the Krusell and Smith (1998) solution method faces computational challenges with more than two aggregate states as would be the case if credit conditions also fluctuated as a Markov chain.[15]

The two aggregate states of the economy are a high and low state ($Z = high, low$). In the housing boom, income and credit conditions simultaneously attain their high state values while the subsequent bust is thus a contraction back to their low state values.

Individual income $\epsilon_j(z)$ follows an AR(1) process with persistence ρ and an age-dependent standard deviation σ_{ε_j} resulting in an age dependent transition matrix $\epsilon_{j+1}(z') \sim \Upsilon_{j+1|j}(\epsilon_j(z))$

$$\epsilon_j(z) = \rho\epsilon_{j-1}(z_{-1}) + \varepsilon_j(z), \quad \varepsilon_j(z) \sim \mathcal{N}(0, \sigma^2_{\varepsilon_j}) \tag{5}$$

Aggregate state space: Incomplete markets and aggregate risk make the distribution of agents across individual household states μ a necessary state variable for agents to correctly forecast next period prices. $\Gamma_\mu(\mu; Z, Z')$ is the equilibrium law of motion of the measure of agents such that $\mu' = \Gamma_\mu(\mu; Z, Z')$. The aggregate state space of the economy is thus the distribution over individual states and the aggregate shock $\{\mu, Z\}$. Let $\mathcal{X}^h = \mathcal{B} \times \mathcal{H} \times \mathcal{M} \times \mathcal{E} \times \mathcal{J}$ denote the set of individual states for homeowners and $\mathcal{X}^r = \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ that of renters with measure $\int_{\mathcal{X}^h} \mu^h d\mu^h + \int_{\mathcal{X}^r} \mu^r d\mu^r = 1$.

Beliefs: I develop a framework where agents form boundedly rational expectations with respect to fully known shocks $\{Z, \epsilon_j(z)\}$ and only form beliefs about house prices $p(\mu, Z)$ which are a subset of endogenous prices. I solve for a Krusell and Smith (1998) equilib-

[14]See White (2014) for a comparison of the 1920s and 2000s housing booms/busts. See Chambers et al. (2009, 2012) for an overview of changes to U.S. mortgage markets throughout the 20th century.

[15]See Appendix B for an expanded discussion.

rium by approximating the potentially infinitely dimensional distribution μ and its law of motion $\mu' = \Gamma_\mu(\mu; Z, Z')$ with lower dimensional vectors containing sufficient information to predict prices. Under my contribution of endogenous beliefs, agents know the form of the approximated law of motion but do not know its parameter values. Section 3.6 discusses in more detail the equilibrium properties of the model and Appendix B discusses the solution method with endogenous beliefs.

Government: The sum of income tax revenues $\mathcal{T}(y, m)$ less mortgage interest rate deductions, property tax revenues $p(\mu, Z)\tau_h \int_{\mathcal{X}^h} h d\mu^h$, and land permit revenues $p(\mu, Z)H_h - w(\mu, Z)N_h$ net of pension outlays $\int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}^{ret}}$ are always positive and spent on government services G that are not valued by households and thus discarded.

3.2 Households' Problem

See Appendix A.1 for the full recursive households' problems.

Renters are endowed with savings $b > 0$ and income y. They must choose to stay a renter or purchase a house and become a homeowner solving the problem:

$$V_j^r(b, y; \mu, Z) = \max\{V_j^{rent}(b, y; \mu, Z), V_j^{own}(b, y; \mu, Z)\} \tag{6}$$

Newly originated mortgages are subject to loan-to-value and payment to income constraints:

$$m' \leq \theta^{LTV}(Z) p_h(\mu, Z) h' \tag{7}$$

$$\pi_j^{min}(m') \leq \theta^{PTI}(Z) y \tag{8}$$

Relative to renting, homeowning offers households the benefits of extra utility, tax deductible mortgage interest payments, and collateral for borrowing liquid assets in the form of HELOCs. Renting has the advantage of shielding households from capital gains fluctuations due to movements in house prices. When model simulated house prices match the empirical standard deviation as is the case under endogenous beliefs, renting becomes relatively more advantageous due to more volatile house prices.

Homeowners have four options: sell their house to purchase a new house or become a renter, default and become a renter, stay in their house and pay their existing mortgage, or stay in their house and refinance a new mortgage.

$$V_j^h(\mathbf{x}, y; \mu, Z) = \max\{V_j^r(b^n, y; \mu, Z), V_j^{default}(b, y; \mu, Z), V_j^{stay,pay}(\mathbf{x}, y; \mu, Z), V_j^{stay,refi}(\mathbf{x}, y; \mu, Z)\} \tag{9}$$

3.3 Lending Sector

The lending sector originates mortgages m' to households aged j at price $q_j(\mathbf{x}', y; \mu, Z) \leq 1$.[16] The mortgages market clears loan-by-loan with the pricing of mortgage points depending on individual future default probabilities and collateral of foreclosed homes.

$$q_j(\mathbf{x}', y; \mu, Z) = -\zeta(Z) + \mathbb{E}_{Z',\epsilon'} \begin{cases} 1 & \text{sell/refi} \\ \dfrac{(1 - \delta_h^d - \tau_h - \kappa_h) p'(\mu', Z') h'}{(1 + r_m) m'} & \text{default} \\ \dfrac{(1 + r_m) m' - m'' + q_{j+1}(\mathbf{x}'', y'; \mu', Z') m''}{(1 + r_m) m'} & \text{pay} \end{cases} \tag{10}$$

If a homeowner sells or refinances, they repay the full balance of their mortgage so that the lender receives the principal plus interest less the intermediation cost $\zeta(Z)$. If the household defaults, the lender forecloses and sells the house to recover the market value of the house as a fraction of the original mortgage. If the homeowner pays their existing mortgage, the lender values the contract as the next period mortgage payment $(1 + r_m)m' - m''$ plus the continuation value of the contract $q_{j+1}(\mathbf{x}'', y'; \mu', Z')m''$. By conditioning on age, the pricing function can be solved backwards from the end of economic life J rather than as a fixed point as is often the case in other settings.

3.4 Final Goods and Construction Firms

See Appendix A.2 for the full recursive problems of final goods and housing construction firms. The competitive final goods sector has a linear constant returns to scale technology $Y = \Theta(Z)N_c$ with inelastic labor supply N_c. Profit maximization delivers an aggregate wage equal to aggregate productivity:

$$w(\mu, Z) = \Theta(Z) \tag{11}$$

A competitive construction sector produces houses with technology $H_h = (\Theta N_h)^\alpha \bar{L}^{1-\alpha}$ where N_h is labor services and $\bar{L}$ is the amount of newly available buildable land. Using the equilibrium wage in equation (11) housing supply is:

$$H_h = [\alpha p(\mu, Z)]^{\frac{\alpha}{1-\alpha}} \bar{L} \tag{12}$$

The profit maximization of the construction sector thus pins down a single price for housing $p(\mu, Z)$ via aggregate housing supply H_h and the aggregate productivity shock $\Theta(Z)$.

[16] Although lenders are competitive, uninsurable aggregate risk may induce profits and losses along the equilibrium path. Lenders are owned by non-modeled foreign agents with deep pockets who receive these profits and losses as net exports.

3.5 Rental Sector

The rental rate is pinned down by a user-cost formula that is a function of current and future house prices along with a per-period operating cost Ξ as shown in Appendix A.2. The competitive rental sector frictionlessly buys and sells housing units to convert them into rental housing which incurs the same depreciation and taxes as owner-occupied housing.

$$\rho(\mu, Z) = \Xi + p(\mu, Z) - (1 - \delta_h - \tau_h)\mathbb{E}_{Z',\epsilon'|Z,\epsilon}[q_b p'(\mu', Z')] \tag{13}$$

By including heterogeneous landlords with disperse ownership costs, Greenwald and Guren (2021) argue that looser credit conditions can quantitatively account for the drop in the rent-price ratio observed throughout the housing boom. To tractably introduce convertibility frictions into frameworks like this one, both Kaplan et al. (2020, Appendix D) and Arslan et al. (2022) include costs on converting newly purchased housing units into rentals. The former's linear costs result in no boom in house prices and the latter's quadratic costs have little effect on their results. Landlord heterogeneity underlying Greenwald and Guren's (2021) finding may therefore require more nuance when embedded into macro housing frameworks with heterogeneous households and an aggregate rental sector.

3.6 Computation of Equilibrium

See Appendix A.3 for the definition of the recursive competitive equilibrium and Appendix B for the computational algorithm and its goodness of fit. The solution method I develop is a variation of the Krusell and Smith (1998) algorithm where aggregate risk and incomplete markets rule out an equilibrium distribution over individual states μ corresponding to a steady state. To determine prices, agents must then keep track of a potentially infinitely dimensional object to compute its equilibrium law of motion $\Gamma_\mu(\mu; Z, Z')$.[17] An approximate equilibrium thus achieves computational tractability by tracking house prices directly and updating them with a forecasting rule for each combination of current and future aggregate states, $\mathcal{Z} = \{Z, Z'\}$.[18]

$$p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \quad \iff \quad \mu' = \Gamma_\mu(\mu; \mathcal{Z}) \tag{14}$$

[17] Ahn et al.'s (2018) alternative to the Krusell and Smith (1998) method first solves for a stationary equilibrium with only idiosyncratic shocks and then linearizes around the steady state of aggregate shocks. Linearization may only be suitable if the wealth distribution remains close to the aggregate steady state and this may not be the case when allowing for time-varying beliefs and hence booming house prices. The Krusell and Smith (1998) method may thus be more robust than linearization in the setting of this paper.

[18] A single housing price $p(\mu, Z)$ is the only price determined via market clearing. The lending sector, rental sector, final goods firms, and construction firms are all perfectly competitive with linear objective functions. The number of prices is thus reduced from four to one.

The log-linear AR(1) forecasting rule for house prices in equation (14) is standard in macro housing applications and assumes that the conditional sample log means of house prices $\mu^p_{\mathcal{Z}} = a^0_{\mathcal{Z}}/(1 - a^1_{\mathcal{Z}})$ are sufficient statistics to accurately predict future prices.[19] Although additional statistics, lags, or forms of a forecasting rule may be used, Krusell and Smith (1998) find that more complexity only incrementally improves accuracy. Moreover, Pancrazia and Pietrunti (2019) find that models relying on simple forecasting rules are best at fitting the path house prices throughout the housing boom.

The standard Krusell and Smith (1998) solution method assumes that agents have full knowledge of the forecasting coefficients in equation (14) and solves for these coefficients as fixed point. To compute an approximate equilibrium, one first guesses values for these coefficients $\boldsymbol{a}_{\mathcal{Z}} = (a^0_{\mathcal{Z}}, a^1_{\mathcal{Z}})'$ and then solves and simulates the model. Using the time series of simulated market clearing prices p_t, one estimates new coefficients via an ordinary least squares regression shown in equation (15). These steps are repeated until the new coefficients are close to the originals, $\boldsymbol{a}_{\mathcal{Z}} \approx \boldsymbol{a}^{new}_{\mathcal{Z}}$.

$$\log p_{\mathcal{Z}_{t+1}} = \underbrace{a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{\mathcal{Z}_t}}_{\boldsymbol{x}'_{\mathcal{Z}_t}\boldsymbol{a}_{\mathcal{Z}}} + e_{\mathcal{Z}_{t+1}}, \qquad \boldsymbol{a}^{new}_{\mathcal{Z}} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t} \right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \log p_{\mathcal{Z}_{t+1}} \tag{15}$$

Embedding adaptive learning into the Krusell and Smith (1998) solution method creates a convenient framework to endogenize beliefs about future house prices. Rather than solving for converged forecasting coefficients as a fixed point, agents instead update the coefficients each period with a combination of past coefficients and weighted lagged forecast errors. Adaptive learning in this application thus relaxes the assumption that agents know the true values of the forecasting coefficients in states of the economy lacking a historical precedent.

As agents learn the true values of the forecasting coefficients, beliefs about future house prices may not yet correspond to the actual evolution of prices. The resulting temporary equilibria may be self-referential where optimism leads to relatively higher market clearing realizations or pessimism leads to relatively lower.[20] Following the terminology of Adam and Marcet (2011), agents are internally rational because they form dynamically consistent beliefs about the future absent full information.[21]

[19]While the aggregate capital stock in Krusell and Smith (1998) is predetermined, aggregate housing is not and requires explicit market clearing. The computational algorithm in this paper is therefore closer to that with risk-free bonds or endogenous labor supply described in Krusell and Smith (2006).

[20]See Kübler and Scheidegger (2021) for a generalization of the existence and computation of an approximate equilibrium with self-referential beliefs.

[21]While most adaptive learning applications assume agents have imperfect information about model quantities, Adam and Marcet (2011) provide microfoundations for learning about market clearing prices as is the case in this paper. With imperfect information about prices, beliefs play a role in determining prices because agents do not necessarily know the mapping between prices and fundamentals.

Solving for an approximate equilibrium under either the standard known coefficients or adaptive learning with unknown coefficients relies on the convergence of temporary equilibria. Ljungqvist and Sargent (2018, p. 228-229, 825) explain that coefficients obtained via a fixed point can be "justified formally using lessons learned from the literature on convergence of least squares learning to rational expectations in self-referential environments." The key difference is that the fixed-point method disregards temporary equilibria and the time-varying method tracks them throughout simulation.

The recursive learning mechanism detailed in equation (16) shows that current period forecasting coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ are a combination of past coefficients $\boldsymbol{a}_{\mathcal{Z}_{t-1}}$ and weighted lagged forecast errors e_{t-1}.[22] How the values of time-varying coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ differ from their near-rational known counterparts $\boldsymbol{a}_{\mathcal{Z}}$ depends on three key parameters: the gain parameter g_t governing the speed of updating, the normalization matrix $\boldsymbol{R}_{\mathcal{Z}_t}$ governing the direction of updating, and the initial coefficients $\boldsymbol{a}_0$. Let $\boldsymbol{x}_{t-2} = (1, \log p_{t-2})'$.

$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t \boldsymbol{R}_{\mathcal{Z}_t} \boldsymbol{x}_{t-2} \underbrace{(\log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{\mathcal{Z}_{t-1}})}_{e_{t-1}} \tag{16}$$

Following the convention of Evans and Honkapohja (1999), agents form beliefs at time t using information available at $t-1$. A forecasting rule like that in Malmendier and Nagel (2016) where time t information is included in current period beliefs $\boldsymbol{a}_{\mathcal{Z}_t}$ via the current period forecast error $e_t = \log p_t - \boldsymbol{x}'_{t-1} \boldsymbol{a}_{\mathcal{Z}_{t-1}}$ would induce a simultaneity problem in this general equilibrium setting as agents would be determining house prices p_t while also using them to form beliefs.

Given that there are the many forms of expectations that allow for incomplete information, this paper does not claim that the adaptive learning rule given by equation (16) is uniquely suitable for endogenizing beliefs to quantitatively account for booming house prices. In fact, the solution method in Appendix B is generalizable to many forms of time-varying beliefs. As one of the first papers to allow for time-varying beliefs in in a framework with incomplete markets and aggregate risk, I rely on adaptive learning for its simplicity, equilibrium convergence properties, and backwards looking components that are consistent with evidence on house price data.[23]

[22]There are several alternative forms of (16) such as using the lagged forecast error from a particular aggregate state $e_{\mathcal{Z}_{t-1}} \equiv \log p_{\mathcal{Z}_{t-1}} - \boldsymbol{x}'_{\mathcal{Z}_{t-2}} \boldsymbol{a}_{\mathcal{Z}_{t-1}}$ or the realized forecast error $e_{t-1} = \log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{\mathcal{Z}_{t-2}}$. The former would assign a larger than desirable weight to past observations and the latter would preclude a recursive structure as is the case when (16) is re-arranged as $\boldsymbol{a}_{\mathcal{Z}_{t-1}}(1 - g_t \boldsymbol{R}_{\mathcal{Z}_t} \boldsymbol{x}'_{t-2} \boldsymbol{x}_{t-2}) + g_t \boldsymbol{R}_{\mathcal{Z}_t} \boldsymbol{x}'_{t-2} \log p_{t-1}$. The recursive structure is useful to help assure equilibrium convergence.

[23]Gelain et al. (2016), Glaeser and Nathanson (2017), Case and Shiller (1989), Pancrazia and Pietrunti (2019), and Granziera and Kozicki (2015) all find that house price forecasts that contain some backward looking component best fit house price data.

4 Parameterization and Calibration

4.1 Parameters

The model's parameters are similar to those of Kaplan et al. (2020) except for beliefs which are calibrated to a proxy I construct from the University of Michigan Survey of Consumers. The parameters are set to resemble the U.S. economy in the late 1990s with the cross-sectional moments from the 1998 Survey of Consumer Finances. Table (1) lists targeted moments from the model's stochastic steady state which is characterized by fluctuations in income and tight credit conditions. Table (6) of Appendix C.1 lists all parameter values.

Demographics: Each model period is equal to two years. Households begin economic life at age 21 ($j = 1$), retire at age 65 ($J^{ret} = 23$), and exit economic life at age 79 ($J = 30$).[24]

Preferences: The elasticity of substitution between goods consumption and housing expenditures $1/\gamma$ is set to 1.25 based on the estimates of Piazzesi et al. (2007). The elasticity of intertemporal substitution equals 0.5 by setting $\sigma = 2$. A McClements (1977) scale sets the consumption equivalence scale $\{e_j\}$ to match the OECD average number of children across different age groups. The discount factor β is set to replicate the 1998 ratio of aggregate net worth to annual labor income of 5.5 for which the model comes in slightly below at 4.9.

Two parameters, the strength of the bequest motive ψ and the extent to which bequests are luxuries $\underline{b}$, pin down the warm-glow bequest motive given in equation (1). The strength of bequests ψ is chosen to replicate the ratio of net worth at age 75 to age 50 of 1.55 which indicates the importance of bequests as a saving motive. The model comes in slightly below at 1.48. The luxuriousness of bequests $\underline{b}$ is chosen so that households in the bottom half of the model's wealth distribution do not leave bequests, as observed in the data.

The preference for housing relative to goods consumption ϕ pins down housing demand so that the average share of housing expenditure to total expenditures from the model is 0.16, as in the data. When beliefs are exogenous, this parameter is not fixed and instead follows a stochastic process.

Homeowners' additional utility ω sets the average homeownership rate which was 66 percent in the late 1990s and is slightly higher at 68 percent in the model. Defaulters' disutility ξ is set to match the average foreclosure rate in the late 1990s of 0.5 percent and is slightly lower at 0.1 percent in the model.

[24]Krueger and Kübler (2004) find that computational accuracy decays rapidly if the length of economic life extends beyond 30 periods when using aggregate risk in a life-cycle economy.

Income endowments: The deterministic life-cycle component of earning $\{\chi_j\}$ is from Kaplan and Violante (2014). Stochastic individual earnings $\epsilon_j(z)$ in equation (5) follow an AR(1) process in logs with an annual persistence of 0.97, annual standard deviation of 0.2, and an initial standard deviation of 0.42. The variance of log earnings rises by 2.5 between ages 21 and 64 which follows Heathcote et al. (2010). Bequested inheritances are correlated with individual income so that households with higher incomes are more likely to inherit larger quantities.

Housing: The following three parameters discipline the size of owner-occupied housing units $\mathcal{H}$: the size of the smallest unit h^0, the number of house sizes available $\#\mathcal{H}$, and the gap between housing sizes. The values of these parameters are obtained from targeting the 10th, 50th, and 90th percentiles of the distribution of the ratio of housing net worth to total net worth. The model matches the 10th percentile but comes in a bit below the 50th and 90th percentiles. The size of rental housing units $\tilde{\mathcal{H}}$ is chosen to target a ratio of median owner-occupied to rental housing of 1.5 square feet per person as detailed in Chatterjee and Eyigungor (2015) and a ratio of the average earnings of homeowners to renters equal to 2.1. The model generates moments that are mostly in line with these empirical targets.

The maintenance cost of housing that offsets depreciation δ_h replicates the empirical depreciation rate of the housing stock of 1.5 percent per year.[25] Defaulted housing has a higher depreciation rate δ_h^d equal to 0.22 to account for the loss of value from foreclosure. The linear transaction cost of housing adjustments κ_h equals 7 percent which falls within the 6-12 percent range estimated by Quigley (2002).[26] In line with the data, 9 percent of homeowners in the model buy or sell houses each year. The relative cost of renting versus owning is determined in part by the operating cost of the rental company Ξ and is particularly salient for young households. Ξ is chosen to match the 39 percent homeownership rate of households younger than 35 and comes in slightly lower at 33 percent.

Housing construction technology α is set so that the price elasticity of housing supply $\alpha/(1-\alpha)$ equals 1.5, the median among MSAs in Saiz (2010). Land permits $\bar{L}$ pin down employment in the construction sector at 5% of total employment which is consistent with the 1998 employment share of construction measured by the Bureau of Labor Statistics.

Financial instruments: The risk-free rate r is 2.5 percent per year and the lending wedge ι is 0.33 so that the interest rate on loans r_b is equal to 3.3 percent per year to replicate the

[25] Kaplan et al. (2020, p. 18) use the Bureau of Economic Analysis' Table 7.4.5 which details the consumption of fixed capital of the housing sector divided by the stock of residential housing at market value.

[26] Ghent (2012) finds a value of 13 percent and Ngai and Sheedy (2020) settle on 10 percent suggesting that 7 percent may be on the lower end of the established range.

gap between the average rate on 30-year fixed-term mortgage and the 10-year Treasury rate in the late 1990s. The maximum HELOC value θ^{HELOC} is 0.2.

Moment	Parameter	Empirical Value	Model Value
Agg. net worth/annual agg. labor income	β	5.5	4.9
Median ratio of net worth to labor income	β	1.2	1.2
Median net worth: age 75/age 50	ψ	1.55	1.48
% of bequests in bottom 1/2 of wealth dist.	$\underline{\flat}$	0	0
Housing/total cons. expenditures	ϕ	0.16	0.16
Aggregate homeownership rate	ω	0.66	0.68
Foreclosure rate	ξ	0.005	0.001
P10 housing/total net worth of owners	$\min \mathcal{H}$	0.11	0.13
P50 housing/total net worth of owners	$\#\mathcal{H}$	0.5	0.32
P90 housing/total net worth of owners	gap $\mathcal{H}$	0.95	0.76
Average sized owned/rented house	$\min \tilde{\mathcal{H}}$	1.5	1.6
Average earnings of owners to renters	$\#\tilde{\mathcal{H}}$	2.1	2.7
Annual fraction of houses sold	κ_h	0.1	0.09
Homeownership rate of < 35 y.o.	Ξ	0.39	0.33
Employment in construction sector	$\bar{L}$	0.05	0.04

Table 1: Targeted moments in calibration corresponding to model parameters.

Government: The property tax τ_h is set to 1% per year which is the median tax rate across U.S. states according to the Tax Policy Center. The income tax function in equation (3) follows the functional form of Heathcote et al. (2017). The parameter τ_y^0 indicates the average level of taxation and is set so that aggregate income tax revenues are 20% of income. The parameter τ_y^1 measures the degree of progressivity of the tax and transfer system and is set at 0.15 based on the estimates of Heathcote et al. (2017). The share of mortgage interest deducted ϱ is set at 0.75 to so that only the first \$1,000,000 of mortgage debt is deductible. Social security payments in equation (2) maintain income heterogeneity by scaling the last realization of earnings y^w_{Jret-1} by the replacement rate ρ_{ss}. Kaplan et al. (2020) compute the ratio of average benefits to average lifetime earnings and find a replacement rate equal to 0.4.

Aggregate shocks: The economy has aggregate shocks to income $\Theta(Z)$ and credit conditions $\mathcal{C}(Z) = \{\theta^{LTV}(Z), \theta^{PTI}(Z), \kappa^m(Z), \zeta^m(Z)\}$. Aggregate income follows a two-state Markov chain with values $\{\Theta(Z_{low}), \Theta(Z_{high})\}$ following a discrete approximation of an AR(1) process estimated from a linearly de-trended series of total U.S. labor productivity. The accompanying transition probabilities $\pi^{\Theta}_{Z,Z'}$ are similarly obtained from the Markov chain approximation.

Aggregate credit conditions loosen via a one-time unanticipated shift to characterize the onset of the housing boom in the 1990s. They are represented by perfectly correlated movements in the loan-to-value constraint $\theta^{LTV}(Z)$, payment-to-income-constraint $\theta^{PTI}(Z)$, the fixed mortgage origination cost $\kappa^m(Z)$, and the mortgage intermediation wedge $\zeta(Z)$. Although higher loan-to-value constraints are the most common representation of looser credit conditions in macro housing models, other innovations in mortgage finance also played a role in extending relatively more credit to households and are hence included.[27] Dokko et al. (2019) find that 60 percent of all mortgages contained at least one non-traditional feature by 2005 and Greenwald (2018) shows that payment-to-income constraints $\theta^{PTI}(Z)$ are quantitatively more important than loan-to-value constraints $\theta^{LTV}(Z)$ in accounting for mortgage debt throughout the 2000s.

The loan-to-value constraint is equal to 0.95 in the low state which is slightly higher than Duca et al.'s (2011) estimates for cumulative loan-to-value ratios of first-time home buyers in the 1990s. Following Kaplan et al. (2020), the value 0.95 replicates the 90th percentile of the loan-to-value distribution observed in the pre-boom data. The high state value of 1.1 targets a 15 percentage point rise in combined loan-to-value ratios from the late 1990s to 2006. The payment-to-income constraint values are set at $\theta^{PTI}(Z_{high}) = 0.5$ and $\theta^{PTI}(Z_{low}) = 0.25$ which are slightly lower than the values of Greenwald (2018) to account for the life time amortization of mortgages being longer than the typical 30 years. Falling financial intermediation and funding costs are represented by a decrease in the fixed mortgage origination costs $\kappa^m(Z)$ from \$2,000 to \$1,200 as described by Favra and Imbs (2015) and a decrease in the mortgage intermediation wedge $\zeta^m(Z)$ from 100 basis points to 60 capture as described by Arslan et al. (2022).

Learning: My contribution of endogenous beliefs via adaptive learning introduces three parameters that affect the formation of house price expectations.

First, the learning gain g_t assigns the speed at which agents update their beliefs with incoming information. Following Marcet and Nicolini (2003) and Milani (2014), I use a mixed gain where there is a constant gain $g_t = g$ during the housing boom state and a least-squares gain $g_t = 1/t$ in the non-boom states. In their study of hyper inflations, Marcet and Nicolini (2003) find that a mixed gain has the benefit of more accurate forecasts in boom states and dampened oscillations in non-boom states. Constant gain learning estimates coefficients like those in equation (16) via a rolling window regression that puts more weight

[27] Loan-to-value constraints also make the households' problems well defined as explained by Engelhardt (1996). Along with Kiyotaki et al. (2011), Engelhardt (1996) notes that loan-to-value constraints are a reduced-form representation of contract enforcement frictions in credit markets. Lenders require both a down payment and the house to be pledged as collateral because they fear that households might not repay mortgages. By forcing households to hold equity in their homes, loan-to-value constraints distribute some of the down-side risk of house price declines from lenders to borrowers.

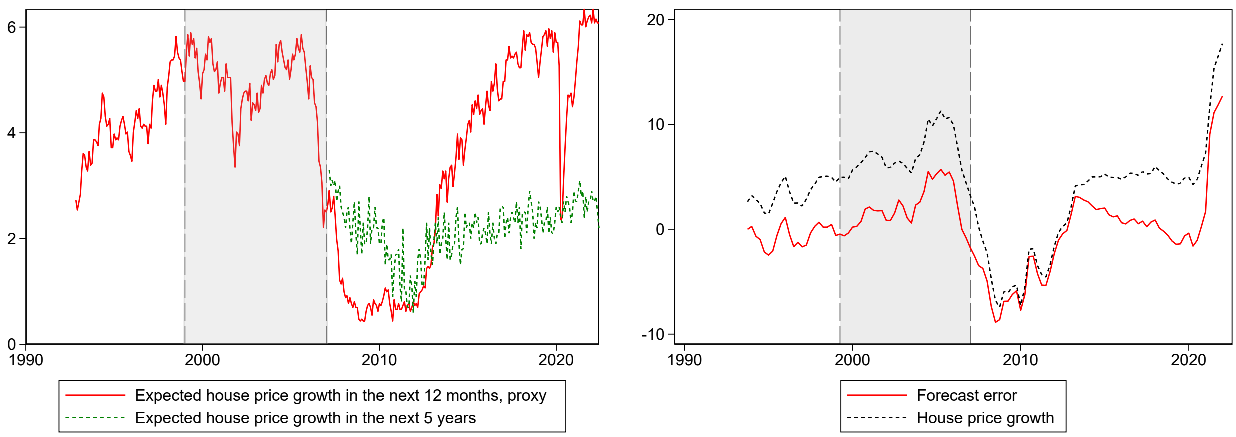
Interpretation	Parameter	Value
Aggregate income $\Theta(\mathbf{Z})$		
Aggregate income	$\{\Theta(high), \Theta(low)\}$	$\{1.035, 0.965\}$
Transition probability	$\pi^{\Theta}_{h,h} = \pi^{\Theta}_{l,l}$	0.9
Aggregate credit conditions $\mathcal{C}(Z)$		
Loan-to-value ratio	$\{\theta^{LTV}(high), \theta^{LTV}(low))\}$	$\{1.1, 0.95\}$
Payment-to-income ratio	$\{\theta^{PTI}(high), \theta^{PTI}(low)\}$	$\{0.5, 0.25\}$
Fixed origination cost	$\{\kappa^{m}(high), \kappa^{m}(low)\}$	$\{\$1,000, \$2,000\}$
Proportional origination cost	$\{\zeta(high), \zeta(low)\}$	$\{0.006, 0.010\}$
Beliefs/learning		
Constant gain	$g_t = g$	0.3557
Least-squares gain	t	100
Initial coefficients	$\boldsymbol{a}_0$	$\boldsymbol{a}_{Z_{high}, Z'_{high}}$
Normalization matrix	$R_{\mathcal{Z}_t}$	$\boldsymbol{I}$

Table 2: Parameters for aggregate shocks to income and credit conditions along with those for endogenous beliefs characterized via adaptive learning. The model period is two years and values are not annualized.

on recent observations which is useful when there are structural changes in the economy like the unprecedented shift in credit conditions in the late 1990s. Least squares learning instead assigns a declining weight to more recent observations to estimate a time-varying regression that can potentially guarantee convergence to rational expectations equilibrium.[28]

Similar to Caines (2020), I calibrate the housing boom's constant gain $g_t = g$ by minimizing the difference between mean squared house price forecast errors from the model and an empirical proxy I construct from the University of Michigan Survey of Consumers detailed in Appendix D and shown in Figure (1). Even though series of expected house price growth are only available at the end of the housing boom in 2007, the "next 12 months" expectations series is tightly correlated with a series on selling conditions that dates back to 1992. Backcasting the shorter series from the longer can deliver a proxy of house price expectations throughout the 2000s. This proxy and its counterpart of expected house price growth for the next 5 years were elevated in the 2000s housing boom as shown in Panel (1a). Because the series for the next 5 years was at an all-time high even after house price growth peaked, it is reasonable to conclude that households were optimistic about housing. The resulting forecast errors used to calibrate the constant learning gain $g_t = g$ are large and have a steady upward path following that of house prices as shown in Panel (1b).

[28] Because this paper solves for a Krusell and Smith (1998) approximate equilibrium rather than a true rational expectations equilibrium, least squares learning—like constant gain learning—may only guarantee convergence to an ergodic distribution. See Evans and Honkapohja (2001) and Marcet and Sargent (1989) for details on the learnability of rational expectations equilibria.



(a) Expectations of house price growth

(b) House price growth and forecast errors

Figure 1: 12-month percent change in expected house price growth and 4-quarter percent change in house price growth with forecast errors calculated as house price growth less the 12-month proxy. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

With an annualized value of g =0.1973 (0.3557 non-annualized), this paper's constant gain is on the larger end of values typically found elsewhere in the literature. Milani (2014) notes that when forecast errors are large, agents may be concerned that the economy is experiencing a structural break and assign a large weight to incoming information and hence a relatively higher value for the constant learning gain. Caines (2020) and Adam et al. (2012) use annualized gains near 0.06 in their applications of adaptive learning to the housing boom. These smaller values may arise from 1) agents learning about house price growth instead of house price forecasting coefficients and 2) differences in house price expectations used in calibration.[29] On the other hand, Marcet and Nicolini (2003) estimate annualized constant gains ranging from 0 to 0.5 in boom-like settings and Milani (2014) estimates constant gains ranging from 0.21 to 0.32. Taken together, this evidence suggests that an annualized constant gain of 0.1973 is high but still within an established range of values.

The least-squares gain $g_t = 1/t$ for the non-boom states is set to 200 years ($t = 100$) so that agents take into consideration historical data when forming beliefs. The results for the bust are only sensitive to small values of $t < 40$ so allowing agents to incorporate a longer or shorter horizon than 200 years is not problematic.[30]

Second, initial beliefs $\boldsymbol{a}_0$ determine the initial forecast error and there is no definitive convention, as noted by Lubik and Matthes (2016). I assume agents use the coefficients corresponding to the pre-boom economy characterized by tight aggregate credit conditions

[29]Caines (2020) uses post-boom house price expectations from Case-Shiller house price futures.

[30]In the first period of the bust $t = 45$ as there are $45 = (\pi^{\Theta}_{l,l} / \sum_{Z,Z'} \pi^{\Theta}_{Z,Z'}) \times (200/2)$ occurrences of the $\{low, low\}$ state for the frequencies $\pi^{\Theta}_{l,l} = 0.9$ and $\sum_{Z,Z'} \pi^{\Theta}_{Z,Z'} = 2$, and a model period of 2 years.

and fluctuations in aggregate income as initial beliefs for the boom state. The coefficients $\boldsymbol{a}^{tight}_{Z_{high},Z'_{high}}$ do not internalize the increase in housing demand arising from looser credit conditions which results in agents underpredicting house prices.[31] The resulting positive forecast errors are consistent with those shown in Panel (1a) and evidence from Kindermann et al.'s (2022) German housing survey.

The final learning parameter is the normalization matrix $\boldsymbol{R}_{\mathcal{Z}_t}$ which is set to the identity matrix $\boldsymbol{I}$ for all observations so that the model is solved under stochastic gradient learning. In contrast to least squares learning, stochastic gradient learning does not normalize by the inverse of the matrix of second moments, $\boldsymbol{R}_{\mathcal{Z}_t} = \boldsymbol{R}_{\mathcal{Z}_{t-1}} - g_t(\boldsymbol{x}'_{t-2}\boldsymbol{x}_{t-2} - \boldsymbol{R}_{\mathcal{Z}_{t-1}})$. Eliminating this normalization step results in a learning mechanism that is both simpler to embed in a general equilibrium framework and easier to interpret.[32] Additional advantages of stochastic gradient learning include dynamics that are 1) self-referential as noted by Evans et al. (2010) and 2) not dependent on initial estimates $\boldsymbol{R}_{\mathcal{Z}_0}$ as noted by Galimberti (2022).[33]

4.2 Calibration

Before simulating the U.S. housing boom under my contribution of endogenous beliefs, I first discuss the cross-sectional distribution of households in the model's stochastic steady state which assumes known forecasting coefficients, tight aggregate credit conditions, and fluctuations in aggregate income. Figure (2) shows that the life-cycle profiles of the means and variances of model quantities are largely consistent with a typical incomplete markets model where households succeed at some smoothing of goods consumption and housing expenditures by accumulating a buffer stock of liquid financial instruments.

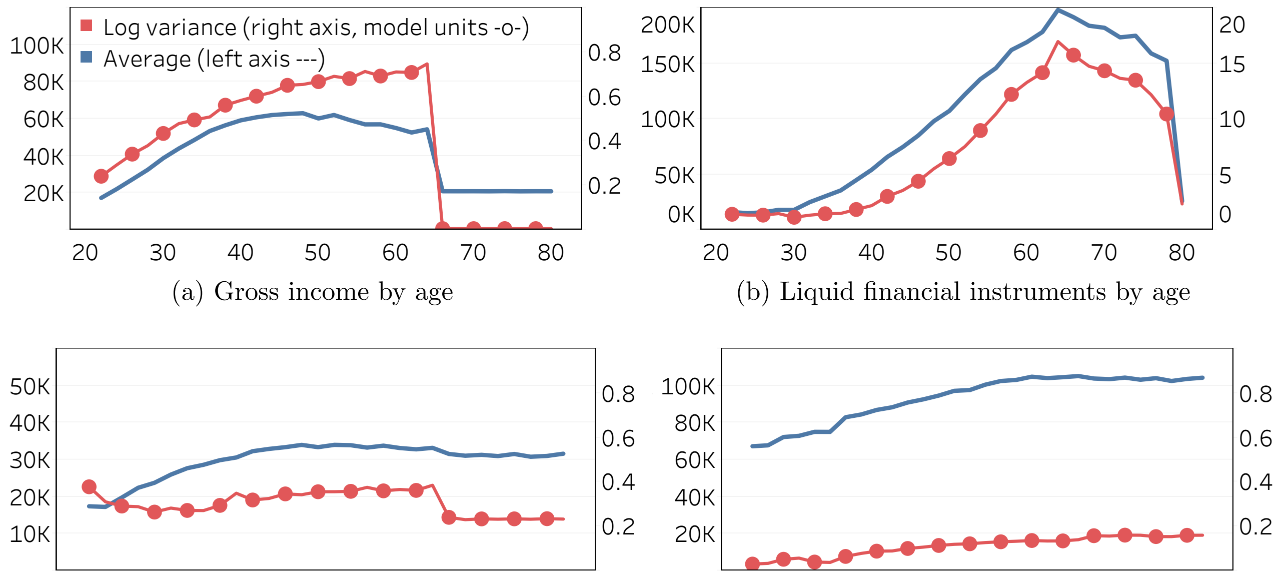
Panel (2a) shows that the average and log variance of income rise over the life-cycle consistent with the data from the 1998 Survey of Consumer Finances. The non-stationary age-dependent variance for idiosyncratic income helps match the more volatile earnings in the pre-retirement phase as is observed in the data.

Panel (2b) shows that households accumulate liquid financial instruments, on average, as a buffer against both idiosyncratic shocks and the decrease in earnings at retirement. The pronounced hump-shape profile in both the average and variance (level, not log) arises from a desire to smooth both consumption and housing expenditures in retirement. Average liquid financial instruments drop at the end of economic life because of bequest heterogeneity

[31] Setting $\boldsymbol{a}_0$ to the most recent past values $\boldsymbol{a}_{Z_{low},Z'_{high}}$ or $\boldsymbol{a}_{Z_{low},Z'_{low}}$ would lead to larger forecast errors.

[32] Stochastic gradient learning has different convergence criteria than least squares learning as shown by Barucci and Landi (1997). Although Giannitsarou (2005) argues that rational expectations equilibrium are not learnable under stochastic gradient learning, this is not problematic in this setting because coefficients can only converge to a Krusell and Smith (1998) approximate equilibrium.

[33] Least squares learning typically uses a training sample to estimate $\boldsymbol{R}_{\mathcal{Z}_0}$ which is problematic in this setting because there is no historical precedent for the housing boom state.



(a) Gross income by age

(b) Liquid financial instruments by age

(c) Good consumption by age

(d) Housing expenditures by age

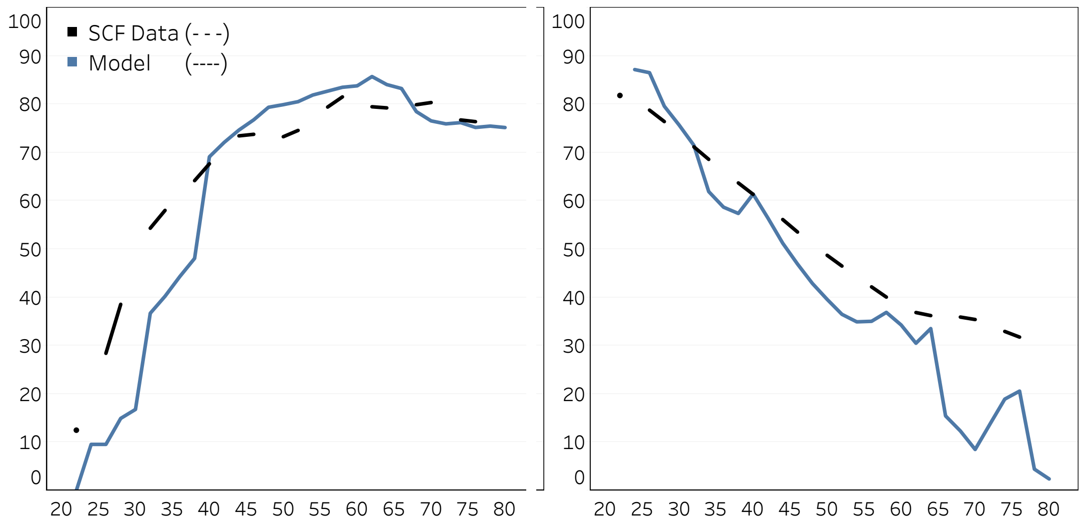
Figure 2: Life-cycle statistics from the model in 1998 dollars.

—only households in the top half of the wealth distribution leave inheritances.

The relatively flat profiles of both goods consumption and housing expenditures over the life-cycle shown in Panels (2c)-(2d) suggest that households partially self-insure against idiosyncratic shocks and retirement. Housing illiquidity and the availability of liquid financial assets are also both key for generating the flat profile of average housing expenditures which is in turn key for matching the data and solving for a Krusell and Smith (1998) approximate equilibrium. Iacoviello and Pavan (2007) explain that illiquid housing results in households accumulating a buffer stock of wealth in liquid financial instruments instead of housing since households cannot adjust housing without paying additional transaction costs. If liquid financial instruments were not available, the average housing expenditure profile would look like that of liquid financial instruments with the hump-shape indicating more variation in marginal propensities to consume within and across households. The simple AR(1) forecast given by equation (14) would then be insufficient to approximate the evolution of house prices and would generate worse goodness of fit measures than those in Appendix B.

An empirically consistent distribution of housing is key for pinning down aggregate house prices throughout the housing boom and Figure (3) shows the model also matches the home-ownership rate and mortgage leverage ratio over the life-cycle. Young households are particularly important because they are less likely to own homes and more likely to have maximum leverage which makes them the most sensitive to looser credit conditions.

Panel (3a) shows that households remain renters, on average, in the first part of economic



(a) Homeownership rate by age, % (b) Mortgage leverage by age, % loan to value

Figure 3: Housing statistics over the life-cycle, 1998 dollars.

life in order to accumulate enough wealth and earnings to overcome the loan-to-value and payment-to-income constraints. The size of inheritances and initial income endowments determine the age in which households can afford to purchase housing.[34] Consistent with the data, the worse off households can never afford housing and remain life-long renters.

Panel (3b) shows that mortgage leverage—homeowners' ratio of mortgage debt to housing value—is highest at age 22 and declines thereafter as household amortize mortgage balances. Model simulated leverage is slightly below but mostly in line with its empirical counterpart until age 65. Thereafter, it takes on counterfactual oscillations and low values due to assumptions on mortgages. In the model, homeowners can take out a mortgage that lasts for only a few years while in practice mortgages may only be available longer tenures.

Table (3) shows a comparison of untargeted moments from the model to their empirical counterparts. The model matches the distributions of homeownership and mortgage debt shown in the top panel and the bottom 90th percentile of the wealth distribution shown in the bottom panel. As is common in models with heterogeneous households, the top 10th percentile of the model's wealth distribution is far below that of the data because wealthy households can only hold housing or liquid financial instruments and thus accumulate a counterfactually high share.[35] Appendix C.2 discusses the assumption on the partial segmentation of housing unit sizes by renters and homeowners.

[34]See Brandsaas (2021) for an overview of how parental transfers help young households overcome mortgage constraints. Including intervivos transfers is beyond the scope of this paper.

[35]Although allowing for risky assets with heterogeneous returns is a solution proposed by Cioffi (2021) and Xavier (2021), it is beyond the scope of this paper as it would complicate how a shift in beliefs affects house prices.

Parameter	Empirical Value	Model Value
Fraction of homeowners w/mortgage	0.66	0.64
Fraction of homeowners w/HELOC	0.06	0.02
Aggr. mortgage debt/housing value	0.42	0.46
P10 LTV ratio for mortgages	0.15	0.01
P50 LTV ratio for mortgages	0.57	0.48
P90 LTV ratio for mortgages	0.92	0.84
Share of NW held by bottom quintile	0	0
Share of NW held by middle quintile	0.05	0.09
Share of NW held by top quintile	0.81	0.67
Share of NW held by top 10 percent	0.7	0.42
Share of NW held by top 1 percent	0.46	0.06
P10 house value/earnings	0.9	0.93
P50 house value/earnings	2.1	1.8
P90 house value/earnings	5.5	4.1

Table 3: Untargeted moments in calibration.

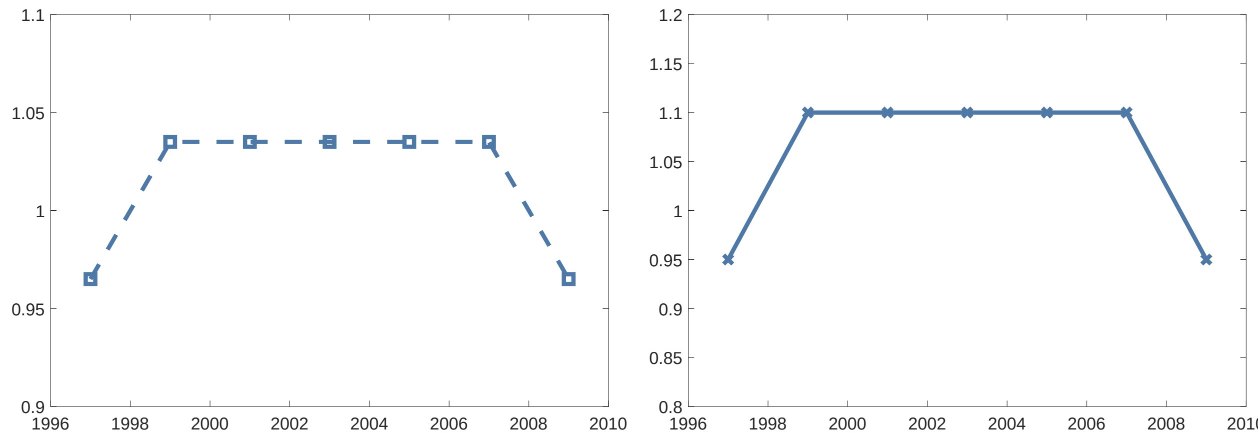
5 Results

This section shows the responses of prices and quantities to a sequence of aggregate shocks with endogenous beliefs represented by time-varying house price forecasting coefficients. The results under endogenous beliefs are compared to counterparts from the data and the exogenous belief specification of Kaplan et al. (2020). Appendix E.1 shows additional counterfactual simulations that parse the contributions of beliefs and credit conditions.

Figure (4) displays the sequence of shocks to aggregate income and credit conditions driving the U.S. housing boom and bust. In 1999, aggregate income transitions from the low state to the high state and aggregate credit conditions unexpectedly loosen. After remaining in this boom state for 5 periods (8 years), aggregate income transitions back to the low state and aggregate credit conditions unexpectedly tighten back to their pre-boom values. The housing boom is thus a period where agents do not anticipate looser credit conditions, but then expect them to stay in place permanently. Appendix E.1 shows that these shocks have little direct effect on house prices without an accompanying shift in beliefs.

Kaplan et al. (2020) simulate the housing boom with a similar sequence of shocks, fixed and known forecasting coefficients, and an additional shock to housing preferences representing an exogenous shift in beliefs.[36] Their housing preference shock follows a Markov chain with three states where two states have a low value $\phi(Z) = 0.12$ and the third a high value $\phi(Z) = 0.20$. The transition probabilities are set so that there is zero probability of housing preferences directly shifting from their low to high value. To transition, housing preferences

[36]Credit conditions attain the same values in both papers, but transition one period later in Kaplan et al. (2020). In their framework, credit conditions follow a two-state Markov chain that is independent from both aggregate income and beliefs with transition probabilities of remaining in the same state equal to 0.98.



(a) Income/productivity, $\Theta(Z)$

(b) Loan-to-value constraint, $\theta^{LTV}(Z)$

Figure 4: Aggregate shocks corresponding to values in table (2) for 1997-2009.

must first enter an intermediate "news" state where they remain at their low value but have non-zero probability of transitioning to their high value.[37] A shift in beliefs is thus the possibility of a shift towards higher housing preferences which then brings to fruition an actual increase in housing demand.

Under endogenous beliefs, a shift in beliefs similarly arises from self-fulling dynamics—albeit from incomplete information and shocks to fundamentals rather than a direct shock to housing demand. Optimism results in agents making decisions that then bring to fruition higher house prices until income contracts and credit conditions tighten.[38]

Figure (5) shows that under either exogenous or endogenous beliefs, house prices peak more than 30 percent above their pre-boom values which is close to the near 35 percent peak observed in the data. Under exogenous beliefs, house prices jump to their peak in a single period, counterfactually decline from 2003 to 2007, and then drop below their pre-boom values as income and credit conditions contract. Under this paper's endogenous beliefs, house prices more closely match the data by steadily rising throughout the boom and peaking closer to 35 percent in 2007 before declining rapidly in the bust. The better match to the path of house prices under endogenous beliefs results in an autocorrelation closer to the data, as shown in Table (4). From 1999 to 2007, endogenous beliefs account for 97 percent of the

[37]The transition to the news state occurs in 2003 which is one quarter after the transition to looser credit conditions and two quarters after the expansion in income. The transition probabilities are 0.04 from the low value to the news state, 0.8 from the news state to the high value, and 0.12 from the news state back to the low value. The intermediate news state is key for representing an exogenous shifts in beliefs because a direct transition of housing preferences between values can match the increase in house prices, but at the cost of counterfactual movements in the rent-price ratio and goods consumption.

[38]Absent the economy reverting back to the non-boom state, optimism would fade as beliefs converge to values near the true boom state coefficients. Appendix E.2 shows that beliefs do eventually converge to an ergodic distribution in a longer simulation with a counterfactual recurrence of the housing boom state.

empirical autocorrelation while exogenous beliefs only account for 71 percent.

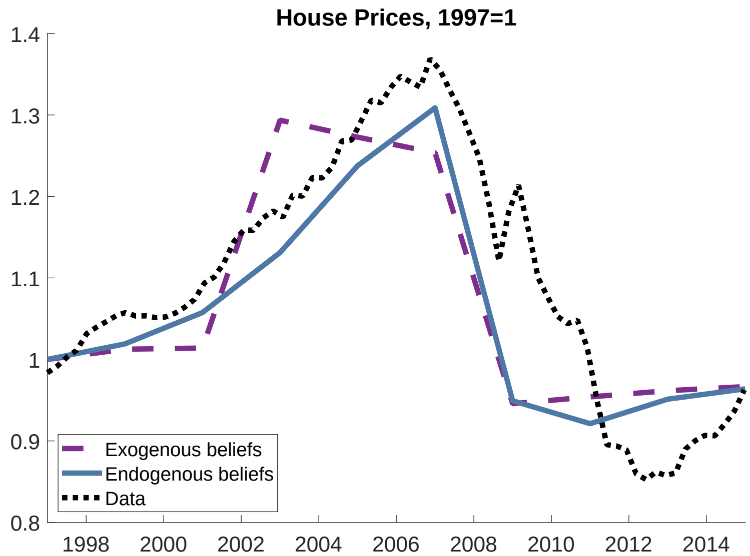


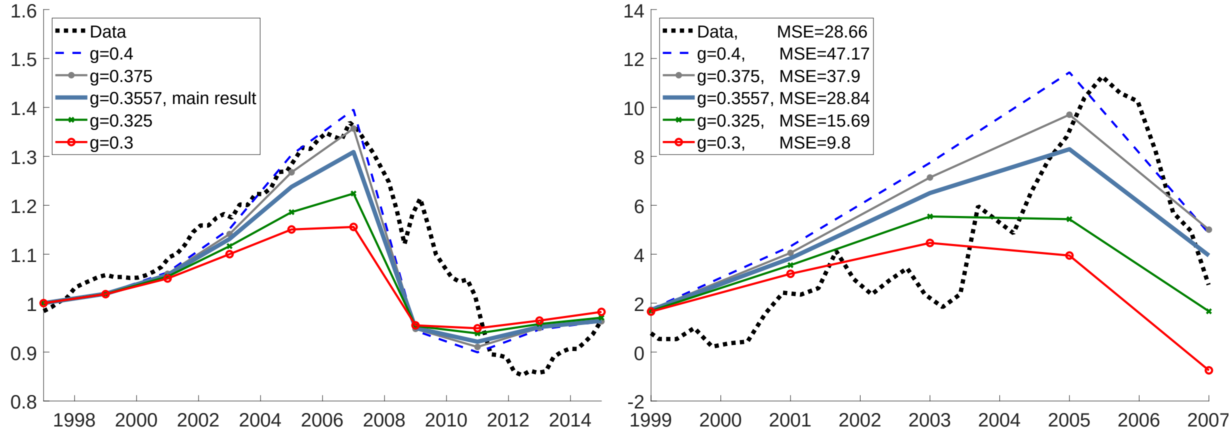
Figure 5: House prices from the data and the model solved under endogenous and exogenous beliefs.

	St. dev., σ_{p_t}	$\frac{\sigma_{p_t}}{\sigma_{data}}$	Autocorr., $\rho(p_t, p_{t-1})$	$\frac{\rho(p_t,p_{t-1})}{\rho_{data}}$
Exogenous beliefs, 2003-2007	0.020	16%	0.325	71%
Exogenous beliefs, 1999-2007	0.152	115%	0.225	39%
Endogenous beliefs, 1999-2007	0.121	95%	0.443	97%
Data, 1999-2007	0.127	100%	0.458	100%

Table 4: Bi-annual house prices statistics from the data and model for the U.S. housing boom.

Table (4) shows that house prices under endogenous beliefs match 95 percent of the empirical bi-annual standard deviation of 0.13 in addition to the time path and autocorrelation. Exogenous beliefs either severely undershoot or overshoot the data's standard deviation with either a 16 percent match if transition periods are excluded or a 115 percent match if they are included. House prices under endogenous beliefs generate an empirically consistent standard deviation because they allow for a persistent rise over several periods rather than a counterfactual jump in a single period.

The sensitivity of house prices and forecast errors to the value of the constant learning gain g is shown in Figure (6) and is key for understanding why endogenous beliefs generate empirically consistent house prices. Under all constant gain values shown, agents underpredict house prices because their initial beliefs do not internalize the increase in housing demand from looser credit conditions. Since beliefs are a combination of previous realizations and forecast errors, beliefs are revised up albeit slowly. Small upward revisions and large positive forecast errors thus result in continued underprediction of house prices.



(a) House prices, 1997=1

(b) House price forecast errors, percent

Figure 6: House prices and forecast errors from the data and model solved under endogenous beliefs. The model's forecast error is, $\log p_t - \mathbb{E}_{Z_t}[a^0_{\mathcal{Z}_{t-1}} + a^1_{\mathcal{Z}_{t-1}} \log p_{t-1}]$.

Panel (6a) shows that although house prices steadily rise under all values of g shown, a smaller g results in a more muted housing boom while a larger g results in one that is more pronounced. There is a positive correlation between g and house price growth because g determines the speed at which agents incorporate new information into their beliefs —a larger value of g thus updates beliefs more aggressively.

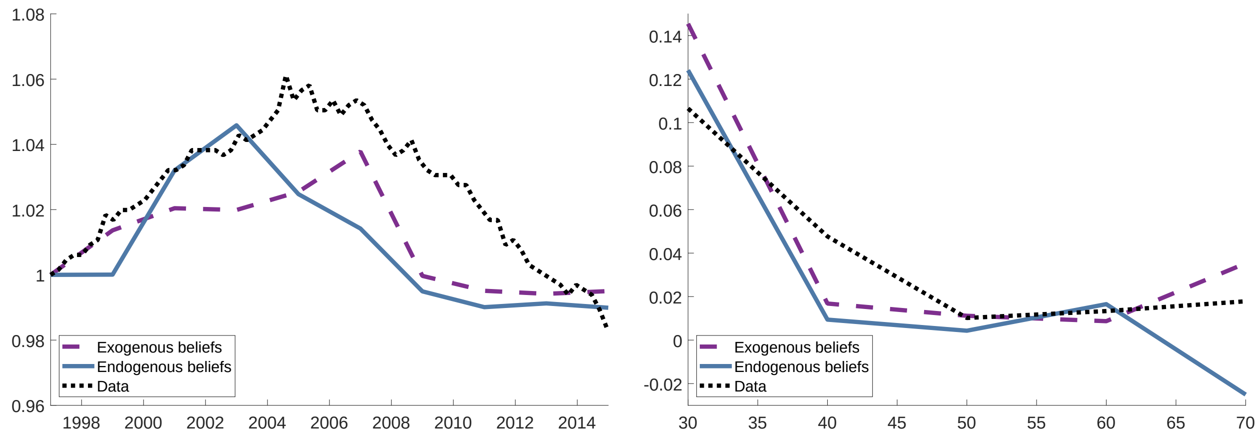
Self-referential dynamics and lack of historical experience in the boom state account for larger values of g resulting in larger forecast errors as shown in Panel (6b). Because agents do not know the evolution of house prices in the boom, they revise up their beliefs absent incoming information suggesting too much optimism. Under all values of g, forecast errors match the upward trend observed in the data, but with less volatility due to their observation at a lower two year frequency.[39] At 28.66, the mean squared error for $g = 0.3557$ is the closest fit to the data and hence used in the main results.

Given that endogenous beliefs generate house prices that better match the path, autocorrelation, and standard deviation of the data, this paper next investigates their effect on the homeownership rate which is a key determinant of housing demand and hence house prices.

Figure (7) shows that the homeownership rate largely matches its empirical counterpart over time and in the cross section. Panel (7a) shows that the peak in the aggregate homeownership rate under endogenous beliefs is higher than under exogenous beliefs and matches about 75 percent of the data's peak. However, the timing of these peaks differs —under exogenous beliefs the peak is too late relative to the data while under endogenous beliefs it is too early. The jump in house prices in 2003 under exogenous beliefs makes homeowning

[39]Because it is unclear if it is better to aggregate via end of period observations or averages, I do not transform the quarterly data to a 2-year frequency.

less affordable and leads to a temporary leveling off of the homeownership rate. Thereafter, the homeownership rate ticks up due to the counterfactual decline in house prices from 2003 to 2007. By contrast, the more gradual rise in house prices under endogenous beliefs leads to the homeownership rate tracking the data well until about 2003. The continued rise in house prices thereafter makes homeownership less affordable and riskier.



(a) Homeownership rate, 1997=1

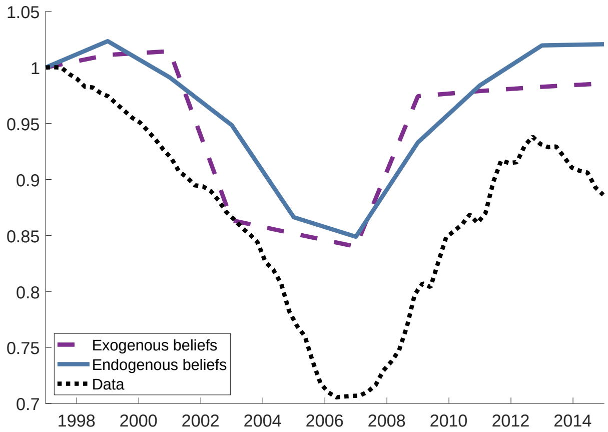
(b) Log change in homeownership by age

Figure 7: The homeownership rate from the data and the model solved under endogenous and exogenous beliefs.

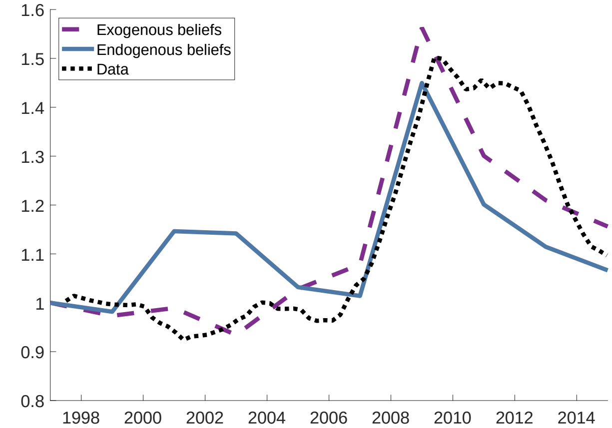
The increase in the homeownership rate in both the model and the data is mostly driven by households under the age of 40 purchasing homes as shown in Panel (7b). Looser credit conditions allow a larger fraction of young households to overcome payment-to-income and loan-to-value constraints earlier in life than under tighter credit conditions. New homeowners tend to purchase the same sized housing unit they were previously renting which results in more demand for housing, but not necessarily an increase in house prices. Higher house prices can mostly be attributed to unconstrained household upsizing their housing units in response to wealth effects from optimistic future house prices.

Figure (8) shows that other prices and quantities similarly match the data under either exogenous or endogenous beliefs. Taking into account the volatility of house prices under endogenous beliefs leads to aggregate consumption better matching the path observed in the data which is useful for understanding the consumption response to house prices.

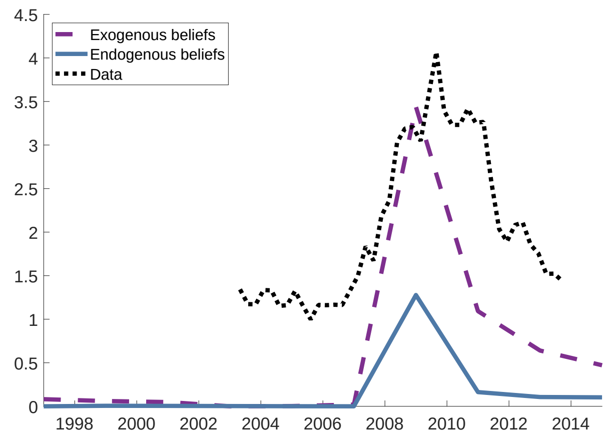
Panel (8a) shows that the decline in the rent-price ratio under either exogenous or endogenous beliefs matches about half of the decline observed in the data. Although macro housing models succeed at generating an increase in prices and homeownership, it is often at the expense of counterfactually rising rents. The rental rate in equation (13) shows that high expected future house prices—like those from a shift in beliefs—are key to generating a decrease in rents and thus a drop in the rent-price ratio. Greenwald and Guren (2021),



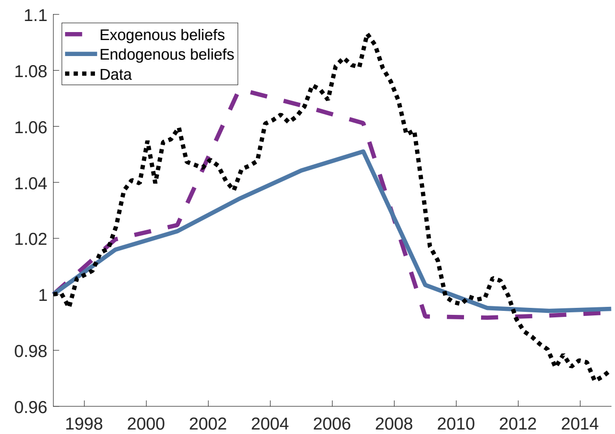
(a) Rent-price ratio, 1997=1



(b) Mortgage leverage, 1997=1



(c) Foreclosure rate, percent



(d) Goods consumption, 1997=1

Figure 8: Housing boom prices and quantities from the data and the model solved under endogenous and exogenous beliefs.

however, argue that credit conditions rather than beliefs are the main drivers of the drop in the rent-price ratio in the housing boom. Similar to Kaplan et al. (2020), their housing boom shocks generate a jump in house prices rather than an empirically consistent gradual increase like that delivered by this paper's endogenous beliefs.

Panel (8b) shows that mortgage leverage under endogenous beliefs rises in the early part of the boom before dropping back towards its pre-boom values. By contrast, mortgage leverage under exogenous beliefs is flat or declines. This difference arises because rising mortgage debt from looser credit conditions is offset by house prices peaking relatively earlier under exogenous beliefs. Under endogenous beliefs, however, a later peak in house prices results in a relatively smaller offset. As in the data, mortgage leverage under either belief specification spikes in the early periods of the bust as house prices fall.

The model's foreclosure rate shown in Panel (8c) spikes in 2009 when income contracts and credit conditions tighten in the bust. Under endogenous beliefs, the spike is below that of the data because of a potentially counterfactually high default cost.[40]

Finally, panel (8d) shows that optimistic beliefs and higher house prices lead to an empirically consistent increase in goods consumption throughout the housing boom. In line with a housing wealth effect, the paths of consumption under either exogenous or endogenous beliefs show a strong correlation with the respective paths of house prices.[41] The model under endogenous beliefs can thus better match the data's steady rise in the path of goods consumption due to its corresponding steady rise in house prices.

6 Conclusion

This paper addresses *why* beliefs about future house prices shifted in the late 1990s to push up house prices throughout the 1999-2007 U.S. housing boom. Agents are more likely to underpredict future house prices and update their time-varying beliefs in the direction of these positive forecast when they lack a historical precedent for the evolution of house prices in an economic expansion accompanied by looser credit conditions. The resulting endogenously optimistic beliefs help match the time path, autocorrelation, and standard deviation of empirical aggregate house prices in the 2000s.

This paper endogenizes a shift in beliefs in a general equilibrium model with incomplete markets and aggregate risk. Aggregate fluctuations in income and unprecedented looser credit conditions represent the nation-wide shift in the economy in the late 1990s that trigger adaptive learning about unknown house price forecasting coefficients. By linking optimistic beliefs and loose credit conditions, this framework complements growing efforts to unify the most common explanations of the U.S. housing boom. Even though looser credit conditions have a muted direct quantitative effect on house prices, they still maintain an important indirect role by triggering optimistic beliefs and determining homeownership decisions.

Finally, this paper's endogenous beliefs can generalize beyond adaptive learning and the study of the 2000s to allow for more frequent housing booms than once in a generation as calibrated under exogenous beliefs. An unexpected shift to an economic state without precedent could describe events such as the onset of the COVID-19 pandemic in 2020. Although credit conditions did not loosen, other unprecedented shocks could result in incomplete information about the evolution of future house prices and provide insights on house price determination and consumption dynamics.

[40] In Appendix E.1, a lower default cost results in a counterfactually high spike in foreclosures in 2009.

[41] The size of the housing wealth effect remains debated. While Berger et al. (2018) find large consumption responses to house price movements, Guren et al. (2020) note that responses in the 2000s are smaller than those of the 1980s.

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A Appendix: Recursive Problems

A.1 Recursive Households' Problem

Households are either renters r or homeowners h with distributions $\mu^r + \mu^h = \mu = 1$. In the final period of economic life, $(j = J)$, all households solve the bequest problem.

Renters' Problem (j < J)

Renters have two choices: stay renters or purchase a house and become homeowners.

$$V_j^r(b, y; \mu, Z) = \max\{V_j^{rent}(b, y; \mu, Z), V_j^{own}(b, y; \mu, Z)\} \tag{6}$$

Rent to rent: if renters choose to stay renters, they solve

$$\begin{aligned} V_j^{rent}(b, y; \mu, Z) = \max_{\{b', \tilde{h}', c\}} &\left\{U_j(c, s) + \beta \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_{j+1}^r(b', y'; \mu', Z')]\right\} \qquad (6.a) \\ s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}' =& y - \mathcal{T}(y) + b \\ 0 \leq& b' \\ s =& \tilde{h}' \in \tilde{\mathcal{H}} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ \epsilon'_{j+1}(z') \sim& \Upsilon_{j+1|j}(\epsilon_j(z)) \end{aligned}$$

Liquid financial instruments b, income y, and age j are individual state variables. Rental housing $\tilde{h}'$ costs $\rho(\mu, Z)$ and enters the budget constraint as a cost of foregone consumption.
Rent to own: if renters choose to purchase a house and become homeowners, they solve

$$V_j^{own}(b, y; \mu, Z) = \max_{\{b', h', c\}} \left\{U_j(c, s) + \beta \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_{j+1}^h(b', h', m', y'; \mu', Z')]\right\} \tag{6.b}$$

$$\begin{aligned} s.to. \quad c + p(\mu, Z)h' + q_b b' =& y - \mathcal{T}(y) + b + q_j(b', h', m', y; \mu, Z)m' - \kappa^m(Z) \\ 0 \leq& b' \\ 0 \leq m' \leq& \theta^{LTV}(Z) p_h(\mu, Z) h' \\ \pi_j^{min}(m') \leq& \theta^{PTI}(Z) y \\ s =& \omega h', \quad h' \in \mathcal{H} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ \epsilon'_{j+1}(z') \sim& \Upsilon_{j+1|j}(\epsilon_j(z)) \end{aligned}$$

Where the minimum mortgage payment is defined in (4) as:

$$\pi_j^{min}(m) = \left[\frac{r_m(1+r_m)^{J-j+1}}{(1+r_m)^{(J-j+1)}-1}\right] m \tag{4}$$

Renters who purchase a house have the homeowners' continuation value $V_{j+1}^h(b', h', m', y'; \mu, Z)$ which has a state space that also includes the newly originated mortgage m' and the purchased house h'.

Homeowners' Problem $(j < J)$

Homeowners have five options: sell their house and purchase a new house, sell their house and become a renter, default and become a renter, stay in their house and pay their existing mortgage, or stay in their house and refinance a new mortgage. Let $\mathbf{x} \equiv \{b, h, m\}$.

$$V_j^h(\mathbf{x}, y; \mu, Z) = \max \begin{cases} V_j^{sell,buy}(b^n, y; \mu, Z) \\ V_j^{sell,rent}(b^n, y; \mu, Z) \\ V_j^{default}(b, y; \mu, Z) \\ V_j^{stay,pay}(\mathbf{x}, y; \mu, Z) \\ V_j^{stay,refi}(\mathbf{x}, y; \mu, Z) \end{cases} \tag{9}$$

Sell to rent or buy: homeowners who sell their house solve the renters' problem (6) with financial assets b^n equal to one-period liquid financial instruments b plus the equity from the sale of the house —the sale price less housing costs and the outstanding mortgage balance.

$$b^n = b + (1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m$$

Default: homeowners who default become renters, incur a utility penalty ξ, and live in the smallest rental housing unit $\tilde{h}^0$. Their property is foreclosed and the lender receives the proceeds from the sale of the housing unit less property taxes and depreciation. In default, depreciation is higher $\delta_h^\delta > \delta_h$.[42] Defaulters are not subject to recourse meaning that the lender cannot lay claim to other assets should the housing collateral not cover all of the defaulted mortgage balance.

[42] As studied by Gabriel et al. (2021), not all defaults lead to foreclosure and there is sometimes a time lag between the two events. A default is failure to meet the terms of mortgage contract and a defaulted mortgage is foreclosed when the homeowner's rights are to the property are eliminated. See Fannie Mae's glossary: https://www.knowyouroptions.com/find-resources/information-and-tools/glossary.

$$V_j^{default}(b,y;\mu,Z) = \max_{\{b',c\}} \Big\{ U_j(c,\tilde{h}^0) - \xi + \beta \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[V_{j+1}^r(b',y';\mu',Z')] \Big\} \tag{9.a}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu,Z)\tilde{h}^0 =& y - \mathcal{T}(y) + b \\ 0 \leq& b' \\ \tilde{h}^0 =& \min \tilde{\mathcal{H}} \\ \mu' =& \Gamma_\mu(\mu;Z,Z') \\ Z' \sim& \Gamma_Z(Z) \\ \epsilon' \sim& \Upsilon_{j+1|j}(\epsilon) \end{aligned}$$

Stay and pay: homeowners who stay in their house and pay their existing mortgage solve

$$V_j^{stay,pay}(\mathbf{x},y;\mu,Z) = \max_{\{b',m',c\}} \big\{ U_j(c,s) + \beta \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[V_{j+1}^h(\mathbf{x}',y';\mu',Z')] \big\} \tag{9.b}$$

$$\begin{aligned} s.to. \quad c + q_b b' + (\delta_h + \tau_h) p(\mu,Z) h + m(1+r_m) =& y - \mathcal{T}(y,m) + b + m' \\ \pi_j^{min}(m) \leq& (1+r_m)m - m' \\ 0 \leq& m' \\ -b' \leq& \theta^{HELOC} p(\mu,Z) h \\ s =& \omega h, \quad h' = h \\ \mu' =& \Gamma_\mu(\mu;Z,Z') \\ Z' \sim& \Gamma_Z(Z) \\ \epsilon'_{j+1}(z') \sim& \Upsilon_{j+1|j}(\epsilon_j(z)) \end{aligned}$$

Because these homeowners stay in the same house $h' = h$, there is no housing choice. Instead, these homeowners pay the housing depreciation costs and tax $(\delta_h + \tau_h)p(\mu,Z)h$ and a mortgage payment greater than the minimum payment $\pi_j^{min}(m)$ described in equation (4). These homeowners can also borrow against the value of their house in the form of HELOCs.

Stay and refi: homeowners who stay in their house and refinance a new mortgage solve:

$$V_j^{stay,refi}(\mathbf{x}, y; \mu, Z) = \max_{\{b',m',c\}} \left\{U_j(c,s) + \beta \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[V_{j+1}^h(\mathbf{x}', y'; \mu', Z')]\right\} \tag{9.c}$$

$$\begin{aligned} s.to. \quad c + q_b b' + (\delta_h + \tau_h)p(\mu, Z)h + m(1 + r_m) =& y - \mathcal{T}(y, m) + q_j(\mathbf{x}', y; \mu, Z)m' - \kappa^m(Z) \\ 0 \leq m' \leq& \theta^{LTV}(Z)p_h(\mu, Z)h' \\ \pi_j^{min}(m') \leq& \theta^{PTI}(Z)y \\ -b' \leq& \theta^{HELOC}p(\mu, Z)h \\ s =& \omega h, \quad h' = h \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ \epsilon'_{j+1}(z') \sim& \Upsilon_{j+1|j}(\epsilon_j(z)) \end{aligned}$$

These homeowners stay in their house h and pay housing maintenance costs $(\delta_h + \tau_h)p(\mu, Z)$. They pay off their existing mortgage m with the proceeds of a new mortgage m' that is subject to points $q_j(\mathbf{x}', y; \mu, Z) \leq 1$, the fixed mortgage origination cost $\kappa^m(Z)$, and loan-to-value and payment-to-income constraints described in equations (7)-(8).

Homeowners can thus extract equity from HELOCS or mortgage refinancing which is essentially a cash-out equity extraction because all mortgages amortize at the same rate r_m.[43] Because mortgage refinancing requires the payment of fixed origination cost $\kappa^m(Z)$ and mortgage points $q_j(\mathbf{x}', y; \mu, Z)$, it is preferred to HELOCS when homeowners are extracting relatively large amounts of equity or have a higher income such that $q_j(\mathbf{x}', y; \mu, Z)$ is closer to 1. Otherwise, HELOCs may be preferred.

Bequest Problems ($\mathbf{j = J}$)

When households exit economic life, they solve the following problems with the bequest motive $V_{J+1} = v(\flat + \underline{\flat})$ described in equation (1).

Renters' Bequest Problem

Renters must stay renters and cannot become homeowners in the final period of economic life. Their value function is thus $V_J^r(b, y; \mu, Z) = V_J^{rent}(b, y; \mu, Z)$.

[43]Wong (2019) relaxes the fixed interest rate assumption when modeling mortgage refinancing and finds that younger households or those with larger mortgages are more likely to refinance.

$$V_J^{rent}(b, y; \mu, Z) = \max_{\{b', \tilde{h}', c\}} \{U_j(c, s) + \beta v(\flat + \underline{\flat})\} \tag{17}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}' &= y - \mathcal{T}(y) + b \\ \flat &= b' \\ 0 &\leq b' \\ s &= \tilde{h}' \in \tilde{\mathcal{H}} \end{aligned}$$

Homeowners' Bequest Problem

Homeowners have three choices in the final period of economic life. They can sell their houses and become renters, default and become renters, or stay in their house and leave it as a bequest after paying off the residual mortgage.[44] Homeowners can neither purchase a new house nor refinance a new mortgage in the final period of economic life.

$$V_J^h(\mathbf{x}, y; \mu, Z) = \max \begin{cases} V_J^{sell,rent}(b^n, y; \mu, Z) \\ V_J^{default}(b, y; \mu, Z) \\ V_J^{stay,pay}(\mathbf{x}, y; \mu, Z) \end{cases} \tag{18}$$

Sell to rent: when homeowners sell their house and choose to become renters, they solve the renter's bequest problem (17) with liquid financial instruments b^n.

$$b^n = b + (1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m$$

Default: homeowners who default in the last period of economic life solve:

$$V_J^{rent}(b, y; \mu, Z) = \max_{\{b', c\}} \left\{U_j(c, \tilde{h}^0) - \xi + \beta v(\flat + \underline{\flat})\right\} \tag{18.a}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}^0 &= y - \mathcal{T}(y) + b \\ 0 &\leq b' \\ \flat &= b' \end{aligned}$$

[44] Strategic bequests and dynasties are beyond the scope of this paper.

Stay and pay: homeowners who pay off their mortgage and leave their house in bequest solve:

$$V_J^{pay,stay}(\mathbf{x}, y; \mu, Z) = \max_{\{b',c\}} \left\{U_j(c, s) + \beta \mathbb{E}_{Z'|Z}[v(\flat + \underline{b})]\right\} \tag{18.b}$$

$$\begin{aligned} s.to. \quad c + q_b b' + (\delta_h + \tau_h) p(\mu, Z) h + (1 + r_m) m' =& y - \mathcal{T}(y, m) + b \\ 0 \leq& b' \\ \flat =& b' + (1 - \kappa_h) p'(\mu'; Z, Z') h \\ s =& h\omega, \quad h \in \mathcal{H} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \end{aligned}$$

A.2 Final Goods, Construction, and Rental Firms' Problems

Final goods firms: V_c denotes the value function of final goods producing firms.

$$\begin{aligned} V_c(N_c; \mu, Z) =& \max_{N_c} \{Y - w(\mu, Z) N_c\} \\ s.to. \qquad Y =& \Theta(Z) N_c \\ 0 \leq& N_c \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \end{aligned}$$

Because final goods consumption C is the numeraire, the price for final goods has been normalized to one. Taking the first order condition with respect to labor N_c pins down the aggregate equilibrium wage:

$$w(\mu, Z) = \Theta(Z) \tag{11}$$

Housing construction firms: V_h denotes the value function of housing construction firms.

$$\begin{aligned} V_h(N_h; \mu, Z) =& \max_{N_h} \{p(\mu, Z) H_h - w(\mu, Z) N_h\} \\ s.to. \qquad H_h =& [\Theta(Z) N_h]^\alpha \bar{L}^{1-\alpha} \\ 0 \leq& N_h \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \end{aligned}$$

Taking the first order condition with respect to labor N_h and using the above equilibrium expression for aggregate wage $w(\mu, Z)$ yields:

$$\alpha \Theta(Z) p(\mu, Z) [\Theta(Z) N_h]^{\alpha - 1} \bar{L}^{1-\alpha} = \Theta(Z)$$

Re-arranging delivers the expression for housing supply:

$$\underbrace{[\Theta(Z)N_h]^{\alpha}\bar{L}^{1-\alpha}}_{\equiv H_h} = [\alpha p(\mu, Z)]^{\frac{\alpha}{1-\alpha}}\bar{L} \tag{12}$$

Rental housing sector: V_r denotes the value function of the rental housing sector with the stock of housing units owned by the rental company given as $\tilde{H}'$ and the rental ready housing units given as X'.

$$\begin{aligned}
V_r(\tilde{H}, X; \mu, Z) =& \max_{\tilde{H}', X'} \big\{ [\rho(\mu, Z) - \Xi]X' - p(\mu, Z)[\tilde{H}' - (1 - \delta_h - \tau_h)\tilde{H}] \ldots \\
& \ldots + q_b \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_r(\tilde{H}', X'; \mu', Z')] \big\} \\
s.to. \qquad X' \leq& \tilde{H}' \\
0 \leq& \tilde{H}', X' \\
\mu' =& \Gamma_\mu(\mu; Z, Z')
\end{aligned}$$

First order and envelope conditions yield the rental housing pricing equation:

$$\rho(\mu, Z) = \Xi + p_h(\mu, Z) - (1 - \delta_h - \tau_h) q_b \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[p'(\mu', Z')] \tag{13}$$

See Appendix D of Kaplan et al. (2020) for a more general version that allows for financial and convertibility frictions.

A.3 Recursive Competitive Equilibrium

A *recursive competitive equilibrium* consists of:

- A sequence of income endowments y to households.
- Prices for owner-occupied housing $p(\mu, Z)$, rental housing $\rho(\mu, Z)$, wages $w(\mu, Z)$, and mortgages $q_j(\mathbf{x}', y; \mu, Z)$ where $\mathbf{x} = \{b, h, m\}$.
- Government parameters for constraints on loan-to-value $\theta^{LTV}(Z)$ and payment-to-income $\theta^{PTI}(Z)$, land permits $\bar{L}$, taxes $\mathcal{T}(y, m)$, and social security payments ρ_{SS}.
- Perceived laws of motion for the state space $\mu = \Gamma_\mu(\mu; Z, Z')$ where $\mu = \mu^r + \mu^h$ is the measure over the set of individual states for renters r and homeowners h. Let $\mu = 1$ and the respective state spaces be defined as $\mathcal{X}^h = (\mathcal{B} \times \mathcal{H} \times \mathcal{M} \times \mathcal{E} \times \mathcal{J})$ and $\mathcal{X}^r = (\mathcal{B} \times \mathcal{E} \times \mathcal{J})$.
- Value functions for renters $V_j^r(b, y; \mu, Z)$ with policy functions for consumption, liquid financial instruments, owner-occupied housing, and rental housing $\{c, b', h', \tilde{h}'\}$ solve the renters' problem. Value functions for homeowners $V_j^h(\mathbf{x}, y; \mu, Z)$ with policy functions for consumption, liquid financial instruments, owner-occupied housing, rental housing, and mortgages $\{c, b', h', \tilde{h}', m'\}$ solve the homeowners' problem.

Markets clear:

- Demand for liquid financial instruments in the form of savings and HELOCs is supplied by foreign agents at price q_b.

$$\textstyle\int_{\mathcal{X}^h} b' d\mu^h + \int_{\mathcal{X}^r} b' d\mu^r = B'$$

- The lending sector maximizes profits and the mortgage market clears loan-by-loan with homeowner specific pricing functions $q_j(\mathbf{x}', y; \mu, Z)$.

$$\textstyle\int_{\mathcal{X}^h} m' d\mu^h = M'$$

- The rental sector maximizes profits with policy function $\tilde{H}'$ at price $\rho(\mu, Z)$ to supply rental housing.

$$\underbrace{\textstyle\int_{\mathcal{X}^r} \tilde{h}' d\mu^r}_{\text{renters}} + \underbrace{\textstyle\int_{\mathcal{X}^h} \tilde{h}' d\mu^h}_{\substack{\text{sellers} \\ \& \\ \text{defaulters}}} = \tilde{H}'$$

- Housing construction firms maximize profits with policy functions for labor demand N_h and the supply of new housing units H_h. There is a single housing price $p(\mu, Z)$ that clears the housing market so that housing outflows (LHS) equal inflows (RHS).

$$\underbrace{\tilde{H}' - (1-\delta_h)\tilde{H}}_{\substack{\text{rental sector}\\\text{housing purchases}}} + \underbrace{\textstyle\int_{\mathcal{X}^r} h' d\mu_r}_{\substack{\text{purchases}\\\text{by renters}}} + \underbrace{\textstyle\int_{\mathcal{X}^h} h' d\mu_h}_{\substack{\text{purchases}\\\text{by owners}}}$$

$$= \underbrace{H_h - \delta_h \textstyle\int_{\mathcal{X}^h} h d\mu^h}_{\substack{\text{new construction}\\\text{less depreciation}}} + \underbrace{\textstyle\int_{\mathcal{X}^h} h[\mathbb{1}_{sell} + \mathbb{1}_{default}(1-\delta_h^d + \delta_h)]d\mu^h}_{\text{sales by owners \& sales of foreclosures}} + \underbrace{\textstyle\int_{\mathcal{X}^h} \mathbb{1}_{bequest} h' d\mu^h}_{\text{estate sales}}$$

- Final goods firms maximize profits so that the labor market clears at $\Theta(Z) = w(\mu, Z)$ with the total labor supply from housing construction and final goods firms $N_h + N_c$ normalized to 1.

$$\textstyle\int_{\mathcal{X}} \exp(\chi_j + \epsilon_j(z) d\mu_{\mathcal{J}_{work}} = \underbrace{N_h + N_c}_{1}$$

- The government collects revenue from income taxes $\mathcal{T}(y, m)$, property taxes τ_h, and the sale of land permits $[p(\mu, Z)H_h - w(\mu, Z)N_h]$ to finance expenditures on social security income for retirees y_{ret} and non-valued government spending G:

$$\mathcal{T}(y, m) + \tau_h p(\mu, Z) \textstyle\int_{\mathcal{X}^h} h d\mu^h + [p(\mu, Z)H_h - w(\mu, Z)N_h] = \rho_{ss} \textstyle\int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}_{ret}} + G$$

- Profits or losses from the financial and rental sectors are expressed as net exports:

$$NX = \underbrace{\textstyle\int_{\mathcal{X}^r} [b - q_b b'] d\mu^r + \int_{\mathcal{X}^h} [b - q_b b' \mathbb{1}_{[b'>0]} - \big(r_b(1+\iota)\big)^{-1} b' \mathbb{1}_{[b'<0]}] d\mu^h}_{\text{lenders' net expenses from liquid financial instruments}}$$

$$\ldots + \underbrace{\textstyle\int_{\mathcal{X}^h} [(1+r_m)m + q_j(\mathbf{x}', y; \mu, Z)m'] d\mu^h}_{\text{lenders' net revenue from mortgages}}$$

$$\ldots + \underbrace{(\rho(\mu, Z) - \Xi)\tilde{H}'}_{\substack{\text{rental sector revenue}\\\text{less operating costs}}} - \underbrace{p(\mu, Z)[\tilde{H}' - (1-\delta_h - \tau_h)\tilde{H}]}_{\substack{\text{rental sector housing}\\\text{purchases}}}$$

- The aggregate resource constraint is satisfied where household consumption C, government spending G, net exports NX equal output Y less housing and mortgage adjustment costs.

$$\textstyle\int_{\mathcal{X}^h} c d\mu^h + \int_{\mathcal{X}^r} c d\mu^r + G + NX + \Xi \tilde{H}'$$
$$= Y - \kappa p(\mu, Z) \textstyle\int_{\mathcal{X}^h} h(\mathbb{1}_{sell} + \mathbb{1}_{default}) d\mu^h - \iota r_b \int_{\mathcal{X}^h} (m + b\mathbb{1}_{\{b<0\}}) d\mu^h$$
$$-(\zeta + \kappa^m) \textstyle\int_{\mathcal{X}^h} m'(\mathbb{1}_{buy} + \mathbb{1}_{refi}) d\mu^h$$

- Consistency is satisfied and perceived laws of motion of the state space $\mu' = \Gamma_\mu(\mu, Z, Z')$ is consistent with individual behavior

B Appendix: Computational Algorithm

The standard algorithm with known coefficients is adapted from Kaplan et al. (2020) and Favilukis et al. (2017) who use a variation of the Krusell and Smith (1998) algorithm. The beliefs algorithm with unknown coefficients is drawn from Giusto (2014) and Hoffman (2016) and can applied to any form of recursively updating beliefs, not just adaptive learning.

The algorithms with either fixed known or time-varying unknown forecasting coefficients rely on the assumption that agents keep track of house prices via a log-linear forecasting rule for each combination of current and future aggregate states $\mathcal{Z} = \{Z, Z'\}$ as an approximation of the entire distribution over individual states μ and its law of motion $\Gamma_\mu(\mu; \mathcal{Z})$.

$$p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \qquad \Longleftrightarrow \qquad \mu' = \Gamma_\mu(\mu; \mathcal{Z})$$

I first solve the model under known coefficients that can be obtained as a fixed point. I then solve the model with beliefs where time-varying coefficients are tracked throughout the simulation. Because beliefs require as many additional grids as there are aggregate states, the households' problem must be solved $3^2 = 9$ time for two belief grids with 3 points.[45] Fortunately, the increase in dimensionality is not a computational burden because the belief grids can be parallelized.[46] The model has been solved on the high-throughput computing clusters at Indiana University (Karst and Carbonate), the University of Texas at Dallas (BigTex), and the Federal Reserve Board.

1. Define grids over ages $j = 1, \ldots, J$, liquid financial instruments b, liquid financial instrument choices b', rental housing choices $\tilde{h}'$, owner-occupied housing h, owner-occupied housing choices h', mortgage balances m, mortgage choices m', aggregate income $\Theta(Z) \in \{\Theta(Z_{high}), \Theta(Z_{low})\}$, aggregate credit conditions $\mathcal{C}(Z) \in \{\mathcal{C}(Z_{high}), \mathcal{C}(Z_{low})\}$, and house prices p.
2. Define coefficients:
   - Fixed: to pin down next period house prices, guess coefficients $\boldsymbol{a}_{\mathcal{Z}}$ for each $\mathcal{Z}$ for a total of $\#Z^2 = 4$ vectors of coefficients.[47]

$$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z)$$

[45] Experiments with 5 grid points instead of 3 show that although individual aggregate housing demand may vary, aggregate demand is roughly similar.

[46] Allowing independent aggregate shocks to credit conditions and income would require defining four belief grids instead of two. With 3 points for each grid, the households' problem would then need to be solved $3^4 = 81$ times instead of $3^2 = 9$ times. Increasing the dimensions of the arrays requires large amounts of virtual memory ranging from 150 to 200 gigabytes and interpolation in two additional dimensions. Computation times increase drastically as a result of these additional grids.

[47] The forecasting equation is $\log p_{\mathcal{Z}_{t+1}} = \mathbb{1}_{\mathcal{Z}} a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{Z_t}$ for $\mathcal{Z} \equiv \{Z, Z'\}$ which is slightly different, but practically equivalent to equation (14), $\log p_{\mathcal{Z}_{t+1}} = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{\mathcal{Z}_t}$.

- Beliefs: define additional grids over future house prices in each future state $\{p'(Z'_{low}), p'(Z'_{high})\}$ for a total of $\#Z' = 2$ additional grids.

With a risk-neutral lending sector, the mortgage price $q_j(\mathbf{x}', y; p(Z), Z)$ is pinned down by the lenders' mortgage pricing function in equation (10). The prices for financial assets q_b is taken as given. The price of rental housing $\rho(\mu, Z)$ is pinned down by the rental sector's maximization problem (13) and aggregate wages $w(\mu, Z)$ are pinned down by the final goods firms' maximization problem (11).

3. Solve the individual households' problem at each point on the house price grid p (and at each point on the two belief grids if time-varying). I use value function iteration with grid search for housing and mortgages and a golden section solver for liquid financial instruments.

4. Simulate a long time series of aggregate states Z_t for $t = 1, \ldots, T$ where T =5000 with a burn in period of 300.

5. Fix an initial distribution of liquid financial instruments, idiosyncratic incomes, and ages assuming that all households are initially renters $\mu_t \in \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ for $t = 1$. Beginning realizations $\{\mathcal{B}, \mathcal{E}, \mathcal{J}\}$ denote liquid financial instruments, income realizations, and ages, respectively. There are N =150000 households.

   Some notes on initialization:
   - endowments of liquid financial instruments equal zero for all households at $t = 1$.
   - because all agents are renters at $t = 1$, there is no initial housing endowment
   - initial household ages are drawn from a uniform distribution[48] $j \sim [1, \ldots, J]$
   - when household i reaches the final period of economic life, $j_{i,t} = J$, a new household replaces them with $j_{i,t+1} = 1$ as renters with no mortgage debt $m_{i,t} = 0$. These new households inherit a random draw of liquid financial instruments $b_{i,t}$ that are correlated with individual income so that household with a higher income are more likely to inherit larger quantities.
   - if solving with beliefs, assign coefficients $\boldsymbol{a}_0$ and a normalization matrix $\boldsymbol{R}_{\mathcal{Z}_t}$.

6. Simulate
   - Fixed: coefficients are already set at $\mathbf{a}_{\mathcal{Z}}$ at the beginning of the problem.
   - Beliefs: compute period t coefficients to pin down $p'(p(Z); \mathcal{Z})$. Coefficients for each future aggregate state $\boldsymbol{a}_{Z_t, Z'_{low}}$ and $\boldsymbol{a}_{Z_t, Z'_{high}}$ are necessary because the value of Z' is not yet known. For this reason, time-varying beliefs require as many additional grids as there are aggregate states.

[48] This assumption follows Kaplan and Violante (2014). Matching the distribution of ages would introduce generational demographic changes into equilibrium dynamics which is beyond the scope of this paper.

7. Compute the aggregate demand schedule for housing at each point on the house price grid p from the housing policy functions for renters $\tilde{h}'$ and homeowners h'. Interpolate over $b_{i,t}$ for renters and $\{b_{i,t}, h_{i,t}, m_{i,t}\}$ for homeowners. Average by housing type:

$$\begin{aligned} \text{Renters:} \quad \tilde{H}_{t+1}(p, \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^r} \tilde{h}'(b_{i,t}, y_{i,t}, j_{i,t}; p, \boldsymbol{X}_t) \\ \text{Homeowners:} \quad H_{t+1}(p, \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^h} h'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p, \boldsymbol{X}_t) \end{aligned}$$

The number of aggregate states in the vector $\boldsymbol{X}_t$ depends on the algorithm:
   - Fixed: $\boldsymbol{X}_t = \{Z_t\}$
   - Beliefs: $\boldsymbol{X}_t = \{Z_t, p'(Z'_{low}), p'(Z'_{high})\}$ which requires additional interpolation over $p'(Z'_{low})$ and $p'(Z'_{high})$ to incorporate the effects of time-varying beliefs.

8. Compute excess demand for aggregate housing at each point on the house price grid p and recover the equilibrium house price $p_t^*(Z_t)$ from the market clearing condition:

$$H_{t+1}(p_t^*(Z_t), \boldsymbol{X}_t) + \tilde{H}_{t+1}(p_t^*(Z_t), \boldsymbol{X}_t) = H_h + (1 - \delta_h)[H_t + \tilde{H}_t]$$

9. Interpolate the individual policy functions b', h', $\tilde{h}'$, and m' at the equilibrium house price $p_t^*(Z_t)$ and then average to calculate the aggregate equilibrium quantities.

$$\begin{aligned} B_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N} b'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\ \tilde{H}_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^r} \tilde{h}'(b_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\ H_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^h} h'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\ M_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N} m'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \end{aligned}$$

Again, the number of aggregate states $\boldsymbol{X}_t$ depends on the algorithm:
   - Fixed: $\boldsymbol{X}_t = \{Z_t\}$
   - Beliefs: $\boldsymbol{X}_t = \{Z_t, p'(Z'_{low}), p'(Z'_{high})\}$

$$\boldsymbol{p'}(p_t^*(Z_t)) = \begin{pmatrix} \exp\{a^0_{Z_t, Z'_{low}} + a^1_{Z_t, Z'_{low}} \log(p_t^*(Z_t))\} \\ \exp\{a^0_{Z_t, Z'_{high}} + a^1_{Z_t, Z'_{high}} \log(p_t^*(Z_t))\} \end{pmatrix}$$

10. Simulate for $t = 1, \ldots, T$ periods by repeating steps (6)-(9).

11. Compare coefficients:
   - Fixed: partition the time series of market clearing house prices $\{p_t^*(Z_t)\}_{t=burn}^{T}$ by $\mathcal{Z} = \{Z, Z'\}$ to generate $\#Z^2 = 4$ sub-samples. Estimate new forecasting

coefficients for each sub-sample $\mathcal{Z}$ via ordinary least squares regression:

$$\boldsymbol{a}_{\mathcal{Z}}^{new} = \left(\sum_{t=burn}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t} \right)^{-1} \sum_{t=burn}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \log p_{\mathcal{Z}_{t+1}}$$

Repeat steps (2) - (11) until coefficients converge, $\boldsymbol{a}_{\mathcal{Z}}^{new} \approx \boldsymbol{a}_{\mathcal{Z}}$

- Beliefs: adjust belief parameters to verify that the time-varying coefficients $\{\boldsymbol{a}_{\mathcal{Z}_t}\}_{t=burn}^{T}$ from step (9) converge to their near-rational counterparts $\boldsymbol{a}_{\mathcal{Z}}$.

Chipeniuk et al. (2022) propose a solution check called auctioneer iteration to avoid relying on potentially misleading R^2 statistics as measures of accuracy. Their method adds the following steps after coefficients are obtained via a fixed point:

12. Use converged $\boldsymbol{a}_{\mathcal{Z}}$ and a price grid that is a subset of simulated prices, $p =\subset \{p_t^*(Z_t)\}_{t=1}^{T}$.
13. Repeat steps (1)-(11) to obtain new converged coefficients and new prices $\{p_t^{new}(Z_t)\}_{t=1}^{T}$.
14. Stop if $\{p_t^{new}(Z_t)\}_{t=1}^{T} \approx \{p_t^*(Z_t)\}_{t=1}^{T}$, otherwise go to step (1).

B.1 Computational Details

The known forecasting coefficients $\boldsymbol{a}_{\mathcal{Z}}$ shown in Table (5) are solved as a fixed point in the model's stochastic steady state characterized by tight aggregate credit conditions and fluctuations in aggregate income. The coefficients have similar values in all combinations of current and future income states which is consistent with Kaplan et al. (2020) and suggests that shocks to aggregate income do not have much effect on house price forecasts. Approximating the sample mean of house prices $\bar{p}_{\mathcal{Z}}$ via the coefficients points to slightly more variation in the highest and lowest values attained in the transitions between income states.

Income states	$a_{\mathcal{Z}}^0$	a_Z^1	$\bar{p}_{\mathcal{Z}} \approx \frac{a_{\mathcal{Z}}^0}{1-a_Z^1}$	R^2	Den Haan	Chipeniuk et. al
High, High	−0.070	0.888	0.53			
High, Low	−0.084	0.888	0.47	0.9999	0.01	0.01
Low, High	−0.059	0.889	0.59			
Low, Low	−0.073	0.889	0.52			

Table 5: Converged forecasting coefficients under tight aggregate credit conditions.

Like Kaplan et al. (2020) and Favilukis et al. (2017), the model's R^2 statistic is close to 1 which suggests adequate goodness of fit standards. Because R^2 statistics can be misleading gauges of accuracy, Den Haan (2010) and Chipeniuk et al. (2022) propose alternative tests which both come in sufficiently low at 0.01.

C Appendix: Calibration

C.1 Parameter Values

Interpretation	Parameter	Value	Annualized
Demographics			
Maximum age	J	30	N
Retirement age	J^{ret}	23	N
Preferences			
Inverse elasticity of substitution	γ	0.8	N
Risk aversion	σ	2	N
Discount factor	β	0.967	Y
Strength of bequest motive	ψ	100	N
Extent of bequests as a luxuries	$\underline{b}$	7.7	N
Taste for housing	ϕ	0.13	N
Additional utility from owning	ω	1.015	N
Utility cost of foreclosure	ξ	0.8	N
Individual income			
Deterministic income	$\{\chi_j\}$	Kaplan & Violante (2014)	N
Annual persistence, ind. income	ρ_ϵ	0.97	Y
Annual st. dev., ind. income	σ_ϵ	0.20	Y
Initial st. dev., ind. income	σ_{ϵ_0}	0.42	Y
Distribution of bequest to new hhs	$b_{j=1} = b'_{j=J}$	Kaplan & Violante (2014)	N
Housing			
Owner-occupied housing unit sizes	$\mathcal{H}$	$\{1.5, 1.92, 2.46, \ldots$ $\ldots 3.15, 4.03, 5.15\}$	N
Rental housing unit sizes	$\tilde{\mathcal{H}}$	$\{1.125, 1.5, 1.92\}$	N
Depreciation rate of housing	δ_h	0.015	Y
Housing loss in foreclosure	δ_h^d	0.22	Y
Housing transaction cost	κ_h	0.07	N
Operating cost of rental company	Ξ	0.003	N
Housing supply elasticity	$\alpha/(1-\alpha)$	1.5	N
New land permits	$\bar{L}$	0.311	N
Financial instruments			
Risk-free interest rate	r	0.025	Y
Interest rate wedge on borrowing	ι	0.33	N
Maximum HELOC	θ^{HELOC}	0.2	N
Government			
Property tax on housing	τ_h	0.01	Y
Income tax function	τ_y^0, τ_y^1	0.75,0.151	N
Mortgage interest deduction	ϱ	0.75	N
Social Security replacement rate	ρ_{SS}	0.42	N

Table 6: Parameter values. A period is two years and annualized values are noted in the final column with a Y. 1=\$52,000 which is the average value of income in the 1998 SCF.

C.2 Segmentation of Housing Unit Sizes

This Appendix evaluates the assumptions on the partial segmentation of housing unit sizes and shows that the model's distributions match those of the data minus a few discrepancies. Because housing size enters the households' utility function, empirically consistent partial segmentation can be represented by certain sizes of housing units only being available to a subset of households.[49] To best fit the pre-boom homeownership rate of 66 percent, I follow Kaplan et al. (2020) and assume that the smallest housing unit cannot be owned and the larger housing units cannot be rented. Table (7) shows that in the data only 9 percent of homeowners occupy the smallest sized unit and only 10 percent of renters occupy the largest sized unit which suggests that the partial segmentation assumption only restricts the model from matching a fraction of the overall size distribution. The model's distributions of owner-occupied and rental units overstate the demand for the smallest available units but match the rest of the distributions relatively well. Alternative segmentation assumptions shown in Table (7) provide small fixes to these shortcomings at the expense of distorting other targeted moments such as the homeownership rate or the housing expenditure share as shown in Table (8).

House Size	Data Owners	Benchmark Model	No seg.	Full seg.	Partial seg.	Smaller size 1	Larger size 1
1	9		19		19		
2	24	55	23		22	46	49
3	25	9	23		24	17	14
4	18	13	13	89	12	13	13
5	10	17	16	2	16	18	18
6	9	4	5	9	5	5	4
7	6	2	1	0	1	1	1
House Size	**Data Renters**	**Benchmark Model**	**No seg.**	**Full seg.**	**Partial seg.**	**Smaller size 1**	**Larger size 1**
1	51	79	76	75	76	73	77
2	28	14	14	12	14	18	14
3	11	7	6	13	10	9	9
4	5		2				
5	2		2				
6	1		1				
7	2		0				

Table 7: Distribution of housing unit sizes in percentage points by segmentation assumptions. Source: American Housing Survey, Kaplan et al. (2020).

[49] Landvoigt et al.'s (2015) empirical evidence points to partial segmentation of housing markets by quality of units rather than size which is beyond the scope of most macro housing models. Absent a standard segmentation convention, Arslan et al. (2022) rely on rental sector convertibility frictions and Greenwald and Guren (2021) assume separate housing stocks similar to the full segmentation shown in Table (7).

Moment	Empirical Value	Benchmark Model	No seg.	Full seg.	Smaller size 1	Larger size 1
Housing/total cons. expenditures	0.16	0.16	0.15	0.18	0.15	0.16
Aggregate home-ownership rate	0.66	0.68	0.7	0.54	0.71	0.68
Av. sized owned/rented house	1.5	1.6	1.6	2	1.6	1.7
Av. earnings of owners/renters	2.1	2.7	2.6	2.6	2.6	2.7
Homeownership rate of $<$ 35 y.o.	0.39	0.33	0.38	0.26	0.38	0.36

Table 8: Moments by segmentation assumptions. Source: American Housing Survey, Kaplan et al. (2020).

The segmentation assumptions are:

- **Benchmark**: renters can only choose the smallest three housing unit sizes and owners cannot choose the smallest size.
- **No segmentation**: renters and owners can choose all available housing unit sizes.
- **Full segmentation**: homeowners can only choose the largest four housing unit sizes so that there is no overlap between the units available to renters and homeowners.
- **Partial segmentation**: benchmark where homeowners can choose size 1.
- **Smaller unit 1**: size 1 is decreased to 1.07 (1.125 in the benchmark).
- **Larger unit 1**: size 1 is increased to 1.17 (1.125 in the benchmark).

D Appendix: Learning Gain Calibration

When constant, the learning gain g is set to minimize the mean squared errors of house price forecasts from the model relative to an empirical proxy constructed from the University of Michigan Surveys of Consumers. Because the Michigan Survey's questions about future house prices are only available starting in 2007, there are no direct measures of house price expectations from 1999 to 2007.[50] To construct a house price expectations proxy, I exploit the 0.92 correlation between expected house price growth in the next 12 months (2007 start) and whether or not it is a relatively good time to sell a house (1992 start).[51]

Figure 9: Responses from the University of Michigan Surveys of Consumers, percent of respondents who say it is a good time to sell a house less those who say it is a bad time plus 100 and the median expected house price growth in the next 12 months expressed as 12-month percentage change. The shaded bar between the dashed vertical lines denotes the U.S. housing boom (1999 to 2007).

Given the tight correlation between the two series, I create a backcast of the expected change in house prices going back to 1992 by projecting expectations y_t onto selling conditions x_t for $t = \{\text{Jan. } 2007, \ldots, \text{Jun. } 2022\}$.

$$y_t = \beta_0 + \beta_1 x_t$$

[50] Kuchler et al. (2022, Table 2) shows limited availability of house price expectations prior to 2007.

[51] The `Relative` column of Table 43 is the `Good time to sell` column minus the `Bad time to sell` column plus 100. The survey asks, "Generally speaking, do you think now is a good time or a bad time to sell a house?" The `Median` column of Table 46 is the median response to the question, "By about what percent do you expect prices of homes like yours in your community to go (up/down), on average, over the next 12 months?"

Using the estimated coefficients $\hat{\boldsymbol{\beta}}$, I then backcast the expectations series $\hat{y}_t$ from the selling conditions series x_t when the former is not yet available, $t = \{\text{Jan. } 1992, \ldots, \text{Dec. } 2006\}$.

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_t$$

Figure (10) shows that the backcasted empirical proxy has a good in sample fit with an r^2 near 1. Although the higher frequency movements of the two series differ, the backcast captures the path of expectations which is key for the study of the U.S. housing boom.

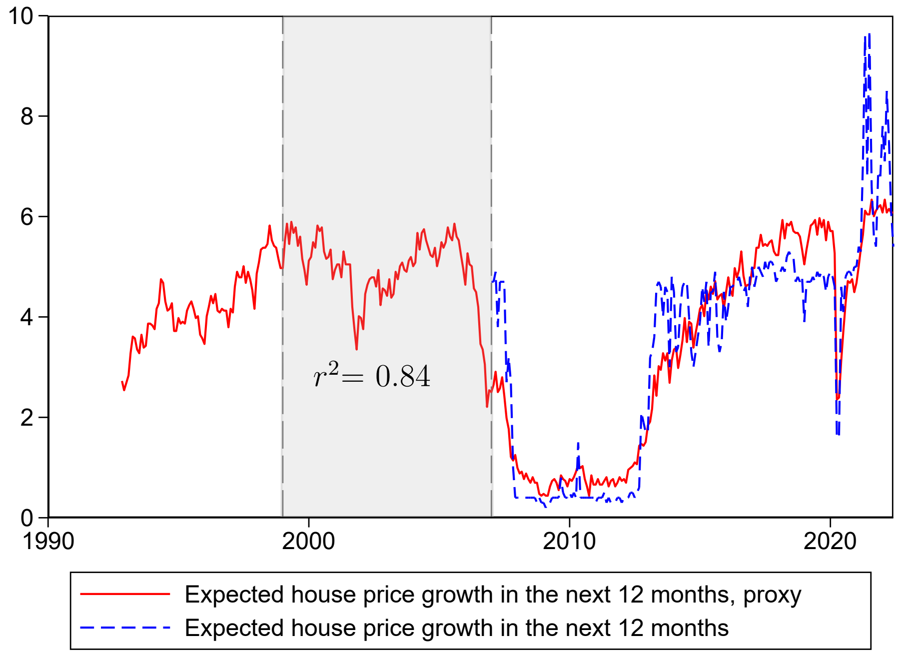


Figure 10: Median expected house price growth in the next 12 months from the University of Michigan Survey of Consumers, 12-month percentage change. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

House price expectations from the model and the empirical proxy have four key differences that affect their comparability. Those from the model are for 2-year real detrended house prices in levels while those from the data are for nominal expected 12-month growth rates. Calculations below discuss which differences affect the mean squared forecast error used to calibrate the constant learning gain.

Growth rates vs. levels: Forecast errors from growth rates and levels are roughly equivalent as shown by equations (19) and (20). Assuming the current month's house prices p_τ are known to survey participants, the h period ahead forecast error e_τ^{survey} for month τ can be defined as:

$$e_\tau^{survey} \equiv \log p_{\tau+h} - \log F_\tau p_{\tau+h}$$

Noting that the Michigan survey provides an expected 12-month growth rate $F_\tau g_{\tau+h}$ for $h = 12$, the above definition becomes:

$$e_\tau^{survey} \equiv \log p_{\tau+h} - \log\big(p_\tau(1 + F_\tau g_{\tau+h})\big)$$

Re-arranging and letting $\log(1 + F_\tau g_{\tau+h}) \approx F_\tau g_{\tau+h}$, the above expression becomes:

$$e_\tau^{survey} \equiv \log\big(p_{\tau+h}/p_t\big) - F_\tau g_{\tau+h} \tag{19}$$

In the model, the f period ahead forecast for period t in aggregate state Z_t with $\mathcal{Z}_t = \{Z_t, Z_{t+1}\}$ is given as:

$$e_t^{model} \equiv \log p_{t+f} - \mathbb{E}_{Z_{t+f}}[a^0_{\mathcal{Z}_t} + a^1_{\mathcal{Z}_t} \log p_t] \tag{20}$$

Where the period t is two years and $f = 1$ so that the forecast is one period ahead.

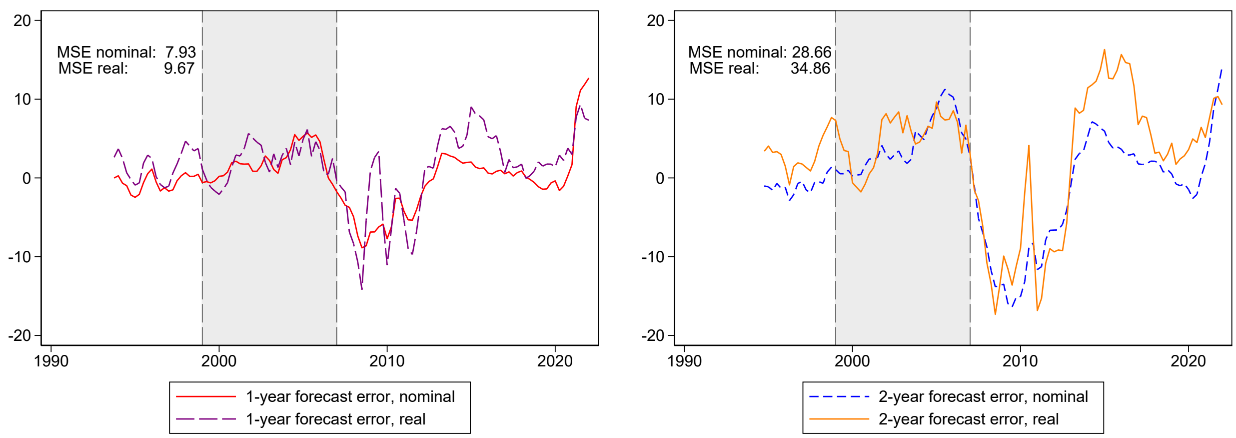
Forecast horizon: To compare the empirical proxy's 12-month forecast horizon to the 2-year forecast horizon from the model, I assume that survey respondents expect house price growth for the next 12 months to remain for the following 12 months. The imputed forecast for the next $2 \times h = 24$ can be written as:

$$\begin{aligned} e_\tau^{survey} \equiv & \log p_{\tau+2\times h} - \log F_\tau p_{\tau+2\times h} \\ & \log p_{\tau+2\times h} - \log\big(p_\tau(1 + F_\tau g_{\tau+h})^2\big) \\ & \log(p_{\tau+2\times h}/p_\tau) - 2 \times F_\tau g_{\tau+h} \end{aligned} \tag{21}$$

Imputing the forecast for the next 2 years via compounding may overstate expected house price growth in the next 2 years, $2 \times F_\tau g_{\tau+h} > F_\tau g_{\tau+2\times h}$. Although Figure (11) shows that the forecast errors of house price growth at the 12-month and 2-year horizons (equations 19 and 21) are quite similar throughout the housing boom, they differ in the later part when the 2-year series is higher resulting in a larger mean squared error. However, because expected house prices for the next 5 years are at their all-time high at the end of the housing boom as shown in Panel (1a) in the main text, house price expectations for the next 2 years may have indeed been higher than those for the next 12 months.[52]

[52] The expectations for the next 5 years is the `median` column of Table 47, "Expected Change in Home Values During the Next 5 Years." The survey asks, "By about what percent per year do you expect prices of homes like yours in your community to go (up/down), on average, over the next 5 years or so?" Two problems prevent interpolation of the 2-year expectations from the 1- and 5-year expectations. First, the 1- and 5-year expectations do not have a monotonic ordering. Survey respondents expected higher house price growth in the next 5 years than in the next 1 year from early 2007 to mid-2013, but the opposite ordering from mid-2013 throughout the remainder of the period shown in Panel (1a). Second, there is no tight correlation of 5-year series with other series in the Michigan survey thereby precluding a backcasting exercise to obtain proxies for the pre-2007 period.

Given the concerns about imputing the house price expectations for the next 2 years, one could instead calibrate the learning gain to the 1-year forecast error. The resulting housing boom simulations would be like those shown in Figure (6) with a constant gain of $g = 0.3$ instead of g=0.3557. With the lower value of the learning gain, the mean squared forecast error is closer to the 7.93 value of the 1-year horizon rather than the 28.66 value at the 2-year horizon. House prices still boom, but to a peak that is lower than the 84 percent match to the data observed in the main specification.



(a) 1-year nominal and real forecast errors

(b) 2-year nominal and real forecast errors

Figure 11: Panel (a): Real house price growth is the FHFA house price index scaled by the price index for non-durable consumption. Expected house price growth for the next 12 months is the empirical proxy constructed from the University of Michigan Survey of Consumers and is scaled by responses for expected inflation from the survey. Panel (b) is same as Panel (a), but with house price and inflation expectations both imputed to a 2-year forecast horizon. The panels are given in the quarterly average of the 12-month percentage change. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

Nominal vs. real: The real house price forecast error can be calculated by dividing both realized and expected house prices by actual and expected inflation. Let $\pi_{\tau+h}$ be 1-year realized inflation and $F_\tau \pi_{\tau+h}$ be its expected value for month τ. The real forecast error is:

$$\begin{aligned} e_\tau^{survey,real} \equiv & \log\left(\frac{p_{\tau+h}}{1+\pi_{\tau+h}}\right) - \log\left(\frac{p_\tau(1+F_\tau g_{\tau+h})}{(1+F_\tau \pi_{\tau+h})}\right) \\ & \underbrace{\log(p_{\tau+h}/p_\tau) - F_\tau g_{\tau+h}}_{e_\tau^{survey}} - \left(\pi_{\tau+h} - F_\tau \pi_{\tau+h}\right) \end{aligned} \tag{22}$$

The real house price forecast error is the forecast error of nominal house price growth less the forecast for the inflation rate.[53] Figure (11) shows that the real and nominal house

[53]The realized inflation series is the PCE price index for durables less consumption which is what is used elsewhere in the paper. Results are similar if the headline PCE price index is used instead.

price forecasts are roughly similar for both the 1-year expectations and the 2-year imputed expectations throughout the housing boom.[54] Within each forecast horizon, there is little difference between the real or nominal mean squared errors at the one year horizon. At the 2-year horizon, the real error is larger than the nominal error and corresponds most closely to the value of 37.88 obtained under a constant gain of $g = 0.375$ shown in Figure (6). The larger constant gain results in a boom that matches almost all of the empirical increase instead of the 84 percent match under the main specification with g=0.3557 and mean squared error of 28.66.

De-trending: The empirical real house price forecast errors are not affected by subtracting out a time trend like the one used to construct house prices in the model. The predicted linear time trend $\hat{\Gamma} p_{\tau+h}$ cancels from both sides of equation (22):

$$\log\left(p_{\tau+h}(1-\Gamma)/(1+\pi_\tau)\right) - \log\left(p_\tau(1-\Gamma)(1+F_\tau g_{\tau+h})/(1+F_\tau \pi_{\tau+h})\right)$$

Additional caveats: Evidence from other surveys suggest that responses from the University of Michigan Survey of Consumers may actually underestimate expected house price growth which could lead to lower forecast errors than those presented above.[55] For this reason, I use the median house price expectations for the next 12 months instead of the mean as the former tends to run higher thus resulting in lower forecast errors. Moreover, figure (12) shows that the median point prediction for house price growth in the next 12-month from the NY Fed Survey of Consumer Expectations is not always higher than the empirical proxy used in this paper for the period in which they are both available.

[54]The 2-year real forecast error is: $\log(p_{\tau+2\times h}/p_\tau) - 2 \times F_\tau g_{\tau+2\times h} - \left(\pi_{\tau+2\times h} - 2 \times F_\tau \pi_{\tau+2\times h}\right)$.

[55]Kuchler et al. (2022) note in footnote 5 that house price expectations from the NY Fed Survey of Consumer Expectations (2013 start) show patterns similar to those of the University of Michigan Survey of Consumers with average 1-year expectations usually about 2 percentage points higher. De Stefani (2021) notes that house prices expectations from the University of Michigan Survey of Consumers are only available for homeowners which may be problematic because Kindermann et al. (2022) find that renters looking to buy houses are the most informed about house price expectations.

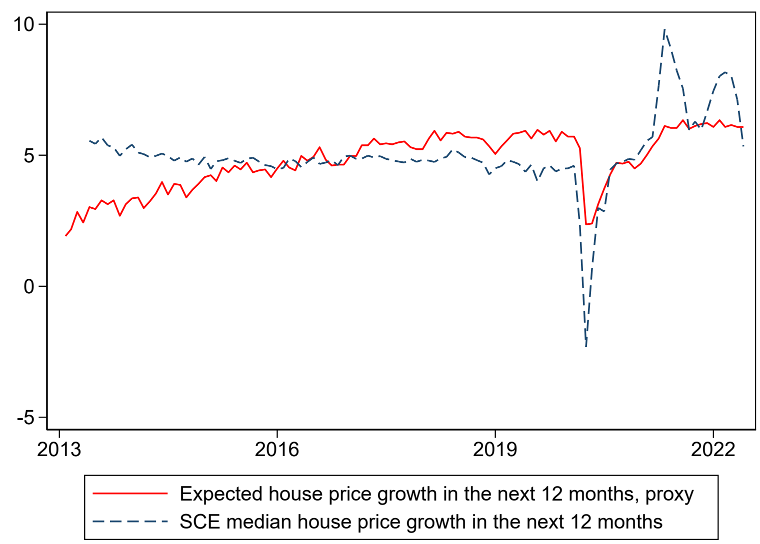


Figure 12: Expected house price growth for the next 12 months is the empirical proxy constructed from the University of Michigan Survey of Consumers and the median point prediction of expected house price growth for the next 12 months from the Federal Reserve Bank of New York Survey of Consumer Expectations, 12-month percentage change.

E Appendix: Alternative Housing Boom Simulations

E.1 Alternative housing boom simulations

Figure (13) shows counterfactual housing boom simulations that parse the contributions of beliefs and credit conditions. Panel (13a) shows that house prices are highest and thus closest to matching the data in the main specification when there is a shift in income and credit conditions accompanied by endogenously optimistic beliefs.

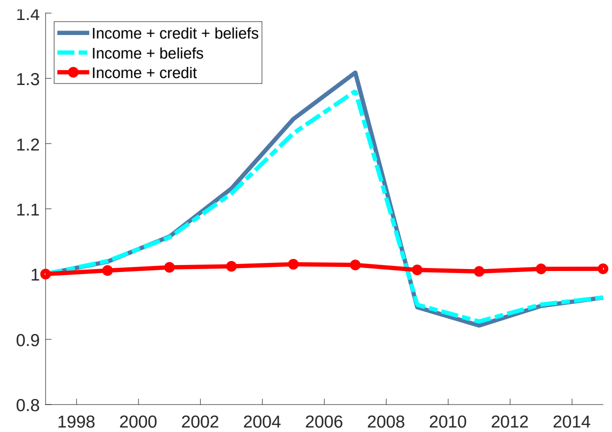
Panel (13a) shows that shutting down the shift in credit conditions, but still allowing for endogenous beliefs and the shock to income results in house prices that boom, albeit less than the main specification. This suggests that incomplete information plays an important role in generating an empirically consistent housing boom. Panels (13c)-(13e) show that without the loosening of credit conditions there are counterfactually low responses in the homeownership rate, mortgage leverage, and the foreclosure rate, respectively.[56]

Shutting down endogenous beliefs, but allowing for shocks to income and credit conditions results in no boom in house prices as shown in Panel (13a).[57] Although there is an increase in homeownership as shown in Panel (13c), new homeowners are mostly purchasing the same sized house as they previously rented rather than upsizing. Homeowners must upsize housing units to push up house prices and this does not occur without optimistic expectations about future house prices. The rent-price ratio (Panel 13b) remains flat, mortgage leverage counter factually rises (Panel 13d) and consumption and foreclosures remain counterfactually low (Panels 13f-13e).

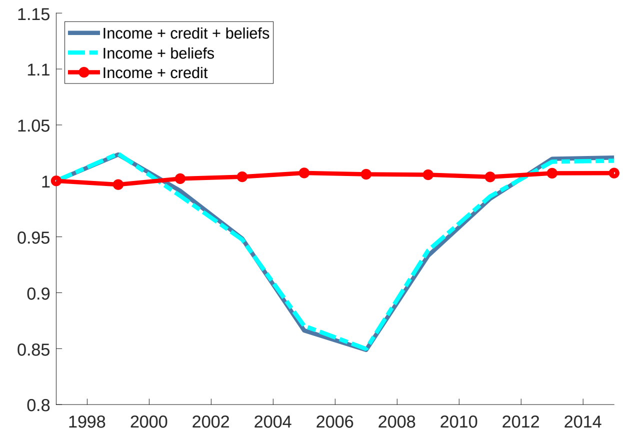
Together, these counterfactual simulations suggest that beliefs are quantitatively most important for determining booming house prices. Credit conditions, however, must loosen to account for other model housing dynamics.

[56]To show the drivers of the model's foreclosure dynamics, panel (13e) shows an additional counterfactual simulation with a lower default cost $\xi = 0.1$.

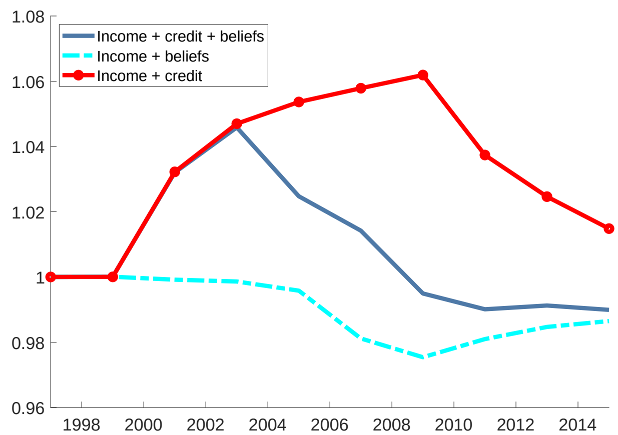
[57]For the housing boom simulation, one must first obtain the forecasting coefficients $\boldsymbol{a}_{\mathcal{Z}}^{loose}$ so that agents know how house prices evolve when credit conditions loosen. Assuming that coefficients are fixed and known, these coefficients are solved as a fixed point with fluctuations in aggregate income only and credit conditions set at their loose values. As in the main specification, the housing boom simulation then starts with the economy in the state with low aggregate income and tight aggregate credit conditions $\{\Theta(Z_{low}), \mathcal{C}(Z_{low})\}$ along with the corresponding coefficients $\boldsymbol{a}_{Z_{low},Z'}^{tight}$. As the economy transitions to the state with high aggregate income and loose credit conditions $\{\Theta(Z_{high}), \mathcal{C}(Z_{high})\}$ coefficients take on the values $\boldsymbol{a}_{Z_{high},Z'}^{loose}$.



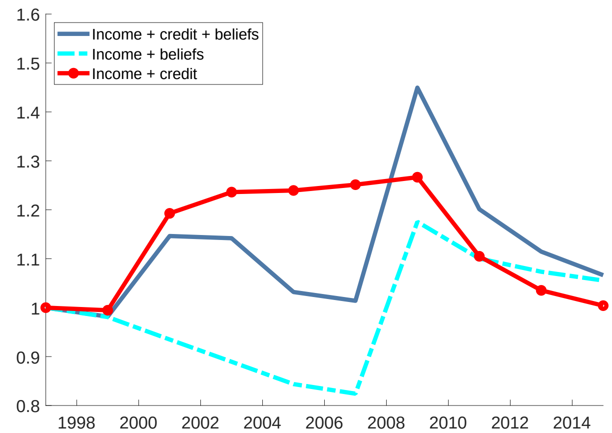
(a) House prices, 1997=1



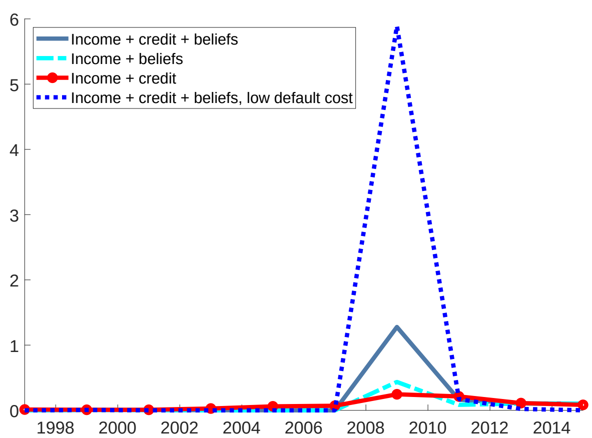
(b) Rent-price ratio, 1997=1



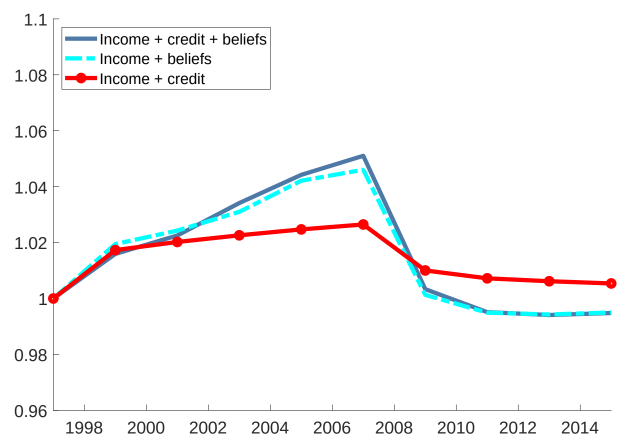
(c) Homeownership rate, 1997=1



(d) Mortgage leverage, 1997=1



(e) Foreclosure rate, percent

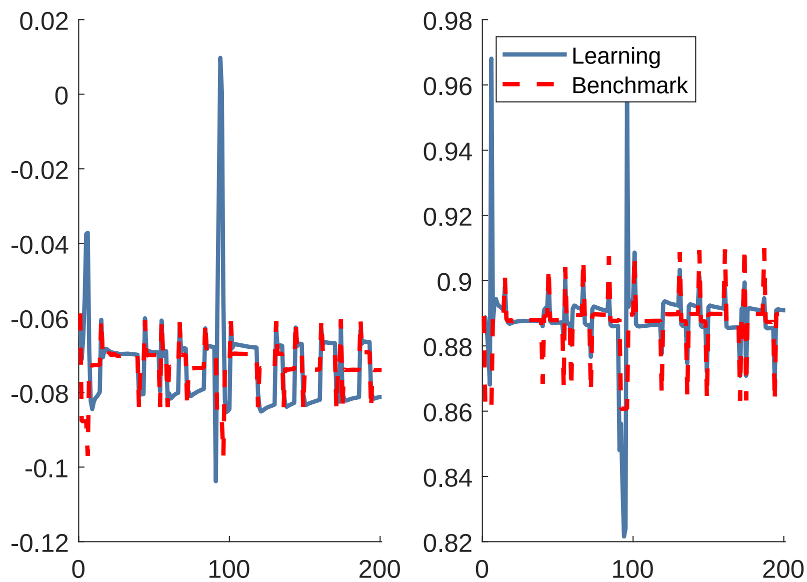


(f) Goods consumption, 1997=1

Figure 13: Counterfactual housing boom simulations from the model.

E.2 Learning Coefficients

Figure (14) shows that coefficients under adaptive learning settle at an ergodic distribution near their known counterparts. A counterfactual return to the housing boom state 100 periods (200 years) after the 2000s boom shows that coefficients adjust in response to forecast errors, but revert back to an ergodic distribution in the non-boom state.



(a) Evolution of a^0 coefficient (b) Evolution of a^1 coefficient

Figure 14: Time path of beliefs for 200 periods. Period 0 is the start of the housing boom and near period 100 there is another counterfactual unexpected loosening of credit conditions.

F Appendix: Data Definitions

The aggregate data definitions follow those in Appendix E.1 of Kaplan et al. (2020). They have been detrended using a linear time trend estimated from 1975 to 1997.

- **Consumption**: Quarterly nominal nondurable expenditures (line 8 of NIPA Table 2.3.5 Personal Consumption Expenditures by Major Type of Product) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **Homeownership**: Census Bureau U.S homeownership rate (Table 14) and by the age of household head (Table 19).
- **House Prices**: House price index for the entire United States (Federal Housing Finance Agency (`FRED: USSTHPI`)) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **Rent-Price Ratio**: Rents (Bureau of Labor Statistics Price Index for Rent of Primary Residences) divided by the FHFA house price index described above.
- **Foreclosures**: Number of consumers with new foreclosures and bankruptcies (Federal Reserve Bank of New York Quarterly Report on Household Debt and Credit) divided by the civilian noninstitutional population (`FRED: CNP16OV`).
- **Leverage**: Flow of Funds Table B.101 Balance Sheet of Households and Nonprofit Organizations. Home mortgage liabilities (FL163165505) divided by the sum of household owner occupied housing at market value (LM155035015) and nonprofit organization real estate at Market Value (LM165035005).
- **Labor Productivity**: U.S. Total Labor Productivity (`FRED: ULQELP01USQ661S`)
- **Mortgage and Treasury Interest Rates**: 30-year fixed rate mortgage (`FRED: GAGE30US`) and 10-year Treasury at a constant maturity (`FRED: GS10`)
- **House price expectations**: University of Michigan Surveys of Consumers. The construction of the empirical proxy and data definitions are discussed in detail in Appendix D. New York Fed Survey of Consumer Expectations.