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Karen K. Lewis and Edith X. Liu

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How Can Asset Prices Value Exchange Rate Wedges?*

Karen K. Lewis[†] Edith X. Liu[‡]

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Abstract

When available financial securities allow investors to optimally diversify risk across countries, standard theory implies that exchange rates should reflect this behavior. However, exchange rates observed in the data deviate from these predictions. In this paper, we develop a framework to value the welfare costs of these exchange rate wedges, as disciplined by asset returns. This framework applies to a general class of asset pricing and exchange rate models. We further decompose the value of these wedges into components, showing that the ability of goods markets to respond to financial markets through exchange rate adjustment has significant implications for welfare.

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[†]University of Pennsylvania, NBER, and CEPR; Email: lewisk@wharton.upenn.edu

[‡]Federal Reserve Board of Governors; Email: edith.x.liu@frb.gov

Resource dislocations across countries generated by recent events ranging from the Covid pandemic to increasing inflation rates raise important concerns about how individuals have been impacted. In principle, financial markets can mitigate these concerns by allowing for efficient sharing of risks. However, a long line of research has demonstrated that investors are not sufficiently diversified across countries.[1] Furthermore, disparities in inflation rates across countries point to the importance of exchange rates adjustment as a mechanism to reallocate resources. Thus, whether financial markets provide channels to minimize potential losses due to adverse exchange rate movements is a significant and enduring economic issue.

In this paper, we confront this important issue by asking: can asset prices be used to value the differences between observed real exchange rates and those implied by optimally diversified financial markets? If so, how? To answer these questions, we develop a general framework that conveniently nests a key building block in many macroeconomic and financial models. Thus, the framework provides an approach for using asset pricing data to value the potential under-diversified risk due to exchange rate misalignment.

The essential insight combines two well-known features. First, for a general class of preferences, welfare can be uniquely measured from the value of currrent wealth and consumption. Moreover, this value is implied by the budget constraint on lifetime consumption disciplined by asset return observations. Second, marginal utilities of consumption across individuals are equalized when available financial market securities span all of the economically important sources of risk; that is, when asset markets are complete. By contrast, when financial markets are incomplete, individuals value returns according to their own distinct intertemporal marginal utilities, or "stochastic discount factors." Combining these two features implies that the welfare differentials perceived by individuals across countries may be valued using asset returns together with standard techniques.

These two features have roots in two research traditions, in turn. The first tradition uses asset markets to uncover the implied "costs" of aggregate risk. This tradition has a long history including Lucas (1987), Alvarez and Jermann (2004), and others. The second tradition focuses upon international investor asset price deviations, or "wedges," in the presence of incomplete financial markets. This wedge was identified by D. Backus, Foresi,

[1] We discuss these papers in the Related Literature below.

and Telmer (2001) as the difference between the exchange rate in the data and the exchange rate implied by the ratio of stochastic discount factors across countries.[2]

To motivate our analysis, we briefly summarize here this measure of the international valuation wedge from the second literature as a starting point. For simplicity, assume there are two representative investors; a home investor with stochastic discount factor at state-time t defined as M_t and a foreign investor with counterpart $\widetilde{M}_t$. Further, denote S_t as the real exchange rate given by the relative price of foreign goods in units of domestic goods. Then, denoting with asterisk $*$ the complete markets counterparts to the variables above, the relationship between stochastic discount factors and prices can be written:[3]

$$M^*_{t+1}\left(S^*_{t+1}/S^*_t\right) = \widetilde{M}^*_{t+1}. \tag{1}$$

Analyzing this relationship requires data counterparts for these variables. While exchange rates are observed directly in the data, counterparts for stochastic discount factors are typically inferred indirectly using Euler equations disciplined with asset return data. It will be useful to distinguish these data-inferred variables from their complete markets counterparts with a "D" superscript. Using this notation, the complete markets relationship above may be rewritten as a deviation or "wedge" from the unique complete markets equation as:

$$\frac{S^D_{t+1}}{S^D_t}\frac{M^D_{t+1}}{\widetilde{M}^D_{t+1}} = \frac{S^*_{t+1}}{S^*_t}\frac{M^*_{t+1}}{\widetilde{M}^*_{t+1}}\exp(\eta_{t+1}) = \exp(\eta_{t+1}), \tag{2}$$

where the last equality follows from the identity in equation (1). Note that the variable η captures the difference between incomplete markets valuations across investors.

This paper combines these insights from the literature to consider asset market-implied costs of the deviation from a counterfactual equilibrium of complete markets. We begin by considering the value of the η wedge. For this purpose, we compare the two investor stochastic discount factors as in equation (2) to compare the valuations of a return implied by the data. Given that wealth is a sufficient statistic for welfare, we choose the return on wealth, defined

[2] The positive implications of these wedges have been examined by Sandulescu, Trojani, and Vedolin (2021), Lustig and Verdelhan (2019) and Bakshi, Cerrato, and Crosby (2018), as described in Related Literature below.

[3] On this equivalence between price-adjusted marginal utilities under complete markets, see Debreu (1959).

as R_c, as a candidate asset return to value across international investors. To illustrate the value of this return described in detail below, Table 1 considers a simple two-country example of this valuation using financial and macro data moments for two countries with close financial and trade ties: the U.S. and Canada. Given these close connections, the means and volatilities of consumption growth are similar across countries and their cross country correlations are greater than one half, as shown in Panel A. Despite these similarities, Panel B shows that the valuations of the η wedge, assuming that consumption growth in each country is identically and independently distributed (i.i.d.), is 31% of permanent consumption.[4] As discussed below, the size of this value is a reflection of the low correlation across countries in stochastic discount factors, a feature familiar from standard international diversification puzzles.

The η wedge provides a valuation comparison across investors under incomplete markets, but is silent on any deviation from complete markets. Therefore, it clearly does not answer the important question of how well asset markets may help buffer international shocks. We next illustrate this point by using the same data moments to infer the complete financial markets solution under the lens of three canonical exchange rate views: Non-tradeables vs Tradeables prices, Home Bias in preferences, and Sticky prices. We choose these three exchange rate examples to highlight a range of assumptions about how well goods markets function, ranging from fully flexible to fixed prices. While these cases provide useful examples, the framework can be applied more generally to other exchange rate approaches as well.[5] Calculating the value of the wedge compared to this counterfactual complete markets value of wealth implies more modest values of 1.76% and 1.93%.

Why are the η wedges in this example large when the wedges derived relative to complete markets are not? The η wedge definition in equation (2) shows that this wedge measures the difference in valuations between the two stochastic discount factors, M and $\widetilde{M}$, once converted into a common price as given by the data. Thus, given the imperfect correlation between consumption and prices in Table 1, even for two relatively integrated countries such as the U.S. and Canada, the derived benefit to a domestic investor of hypothetically

[4] Details of this simple example are described in Section 1.5 below as well as Appendix A.

[5] In Section 4 and Appendix D.2, we discuss these broader applications.

consuming the foreign wealth return corresponds to a less volatile consumption profile with the same growth rate, thereby generating large valuation differences. By contrast, when asset markets are complete, resources are redistributed across countries so that investors instead pool their wealth returns. They therefore internalize the effect of pooling that is absent in the η valuation. As such, these complete markets wedges generate valuation differences closer to business cycle costs.

Moreover, in contrast to values of the η wedge, the complete market wedges are dependent upon how exchange rates are determined. Intuitively, the value of completing financial markets depends on any goods market constraints because they impact the ability for prices to adjust to redistributed consumption. For example, when some goods are nontradeable, then a redistribution of tradeables across countries due to greater access to tradeable securities will alter the relative price of nontradeables and the exchange rate.

While i.i.d consumption provides a useful initial example, disciplining the stochastic discount factors with asset returns requires some persistence in investor risk. For example, this persistence may arise due to fears of a disaster (e.g., Barro and Ursúa (2008), Nakamura, Steinsson, Barro, and Ursúa (2013)), habit persistence (Campbell and Cochrane (1999)), and long run risk (Bansal and Yaron (2004)), to name a few. Although all of these models could be used in our framework, for parsimony we show the basic approach using only the long run risk version. As an illustration of this approach, Panel C reports the results of fitting the financial data moments for the two countries to a long run risk version using Simulated Method of Moments. In particular, the panel provides financial data moments such as real equity returns and risk-free rates for the two countries measured in real domestic price units.[6] To discipline measures of the stochastic discount factors with asset returns, we report our valuations throughout the rest of the paper using these implied measures.[7]

Our paper provides another important contribution by showing how the wedges can be decomposed. For example, as equation (2) highlights, the η wedge not only depends upon the exchange rates, but also upon the ratio of stochastic discount factors across countries. Thus, η may be comprised of three different wedges corresponding to each variable derived

[6] Appendix A describes these data and the Simulated Method of Moments matching approach.

[7] For this simplified simulation, we treat volatility as homoskedastic, although including time-varying volatility would improve the fit further.

from the data. More generally, as a novel implication of our framework, we decompose the value of the wedges into the components due to exchange rates, and to the stochastic discount factors of domestic versus foreign investors. Using only standard preferences and asset pricing assumptions, our simple decomposition example suggests that much of the value of wealth under complete markets relative to data measures comes from wedges in stochastic discount factors, instead of the exchange rate.

Finally, this wedge decomposition also provides another insight that is novel to this paper. That is, we show that the returns on wealth are equalized under complete markets once adjusted by the impact of exchange rates. Therefore, we can solve for the lifetime value of the effects of exchange rates on this common complete markets wealth return. We term this new wedge the "Total S-Wedge." Using our same data moments, this wedge implies that exchange rates have a negative impact on welfare.

The format of the paper is as follows. Section 1 sets up the valuation framework for the real costs of η and also shows that this wedge framework matches various exchange rate puzzles that are not targeted by our approach. Section 2 describes how the same framework can be used to evaluate the complete markets wedge and its decomposition into exchange rates and stochastic discount factors. Section 3 shows a new complete markets relationship implied by wealth returns and its Total S-Wedge effect. Section 4 describes various generalizations of the approach including multiple countries, alternative exchange rate views, and asset pricing models. Concluding remarks are in Section 5.

Related Literature: Since this paper is related to a number of important literatures in macroeconomics and finance, we mention only representative papers within each literature.

First, our paper is related to the literature on consumption, exchange rates, and complete markets. Brandt, Cochrane, and Santa-Clara (2006) use the η wedge to illustrate the risk-sharing puzzle implied by consumption and exchange rate data. Lustig and Verdelhan (2019) analyze risk-free rates to consider implications of the wedge for exchange rates while Bakshi et al. (2018) evaluate exchange rates and a portfolio of international returns. Sandulescu et al. (2021) demonstrate that the ratio of stochastic discount factors do not correspond to the equilibrium exchange rate in complete financial markets. Burnside and Graveline (2020) show that evaluating the costs from imperfect financial market risk-sharing inherent in these data requires an economic framework that depends upon goods markets, but do not specify that framework. Instead, our approach provides a framework that connects the implicit goods market conditions to financial markets.[8] Moreover, in contrast to the literature, our approach further allows a decomposition of these wedges into their respective components.

Second, this paper relates to the growing international asset pricing literature based upon complete markets that identifies the exchange rate with the ratio of stochastic discount factors. A number of studies have used this identity including, among others, Colacito and Croce (2011) and Colacito, Croce, Gavazzoni, and Ready (2018) with long run risk, Farhi and Gabaix (2016) with disaster risk, and Lustig, Stathopoulos, and Verdelhan (2019) for the term structure of exchange rate returns. In contrast with these papers, this paper begins with the presumption that markets may be incomplete and uses data to value its importance.

Third, our paper is related to the literature on consumption insurance and its welfare costs. These studies include household level analysis as in Mace (1991) and Cochrane (1991), and the welfare costs of business cycles as in Lucas (1987), and Alvarez and Jermann (2005). It is also related to the implications of that consumption insurance across countries noted by Obstfeld (1994), Tesar (1995), and Kalemli-Ozcan, Sørensen, and Yosha (2003) as well as the financial market integration literature including Bekaert, Harvey, Lundblad, and Siegal (2011) and Carrieri, Chaieb, and Errunza (2013). However, these papers do not consider goods market price effects on the international risk-sharing.

Finally, this paper is connected to the literature that examines the general connection between exchange rates and consumption aggregators used in international macroeconomics and finance. In highlighting the role of exchange rates, these papers range from D. K. Backus, Kehoe, and Kydland (1992), Coeurdacier and Rey (2012), and Berka, Devereux, and Engel (2018) in macroeconomics to Pavlova and Rigobon (2007), Verdelhan (2010), and Ready, Roussanov, and Ward (2017) in financial economics. This literature focuses upon understanding or explaining regularities in the data. By contrast, we provide a framework to ask what these models would imply about the costs of international financial market wedges.

[8] This aspect of the framework therefore builds on the seminal work by Cole and Obstfeld (1991).

1 Valuing the η Wedge

We now describe the general framework for valuing the Table 1 costs using standard Euler equation solutions in a model-free environment. Specifically, we show how to calculate the cost of the η wedge as well as the implied cost of a total wedge deviation from complete markets. For now, we assume that there exist data measures of all key economic variables, although the end of this section describes identifying data assumptions.

1.1 Domestic and Foreign Investor Valuations

To understand the valuation approach, we briefly summarize the relationship described in D. Backus et al. (2001), hereafter BFT, who pointed out a connection between the standard Euler equation based upon complete markets and its counterpart used in empirical analysis. This connection arises when a domestic investor who consumes in local goods units evaluates an asset with return payouts that are denominated in the units of a foreign good. Without loss of generality, we will treat the foreign price level as the numeraire and define the return of any asset that provides payouts denominated in those units at time t as $\widetilde{R}_{a,t}$. Then, the return on this foreign-denominated asset measured in units of domestic goods is: $\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)$ where S_t is the real exchange rate, defined by the price of foreign goods in units of domestic goods. Then the Euler equation for that foreign asset return valued by domestic investors and by foreign investors, respectively, can be written as:

$$E_t\left\{\widetilde{M}_{t+1}\widetilde{R}_{a,t+1}\right\} = 1 \tag{3}$$

$$E_t\left\{M_{t+1}\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)\right\} = 1 \tag{4}$$

Given these valuations, BFT combine two relationships. The first is that the stochastic discount factors measured in common price units are the same under complete markets as in equation (1). In this case, the above two Euler equations (3) and (4) are equivalent. The second relationship is that these complete markets identities can be rewritten as deviations from their counterparts in the data, if markets are not complete. Rewriting the data-implied counterparts for exchange rates and stochastic discount factors relative to the complete markets alternative defines η as in equation (2). Although these relationships have been combined with various asset returns to infer properties of η, we will focus on the implied wealth returns to measure the welfare implications of incomplete markets.

1.2 Wedges and Life-Time Consumption

The η wedge valuation above can be recast in welfare terms by replacing the return on any asset, R_a, with the return on wealth.[9] Specifically, defining C_t^D as consumption measured in the data, the value of wealth or lifetime consumption as measured by data, W_t^D, can be written as: $W_t^D \equiv C_t^D + \Gamma_t^D$ where Γ_t^D is the present value of future consumption discounted by the stochastic discount factors in the data.[10] Moreover, it will be useful to define the price ratio of this future wealth to consumption as: $Z_t \equiv \Gamma_t / C_t$. Therefore, wealth in the data can be written with this definition as:

$$W_t^D \equiv C_t^D (1 + Z_t^D) \tag{5}$$

Using this definition, the value of the future consumption sequence may then be calculated using the Euler equations above by treating the realization of consumption as the return on an asset R_c^D for the domestic investor and $\widetilde{R}_c^D$ for the domestic investor given as:

$$R_{c,t+1}^D \equiv (C_{t+1}^D / C_t^D)(1 + Z_{t+1}^D)/Z_t^D;\ \widetilde{R}_{c,t+1}^D \equiv (\widetilde{C}_{t+1}^D / \widetilde{C}_t^D)(1 + \widetilde{Z}_{t+1}^D)/\widetilde{Z}_t^D \tag{6}$$

We call these variables the wealth returns because they measure the per-period value of payouts on a claim to future lifetime consumption to each investor. The value of wealth can then be priced with data on consumption and the stochastic discount factors using the Euler equation for the wealth return. Specifically, substituting equation (6) into:

$$E_t \left\{ M_{t+1}^D R_{c,t+1}^D \right\} = 1 \tag{7}$$

[9] The value of lifetime consumption is equal to wealth through the intertemporal constraint. See for example, Cochrane (2005) for financial economics and Rogoff and Obstfeld (1996) for international finance. Campbell (1993) uses this condition to replace consumption with wealth.

[10] In particular, $\Gamma_t^D \equiv E_t \sum_{\tau=1}^{\infty} Q_{t+\tau}^D C_{t+\tau}^D$ where the discount factors are the intertemporal stochastic discount factors between $t+1$ and future periods $t+\tau$. That is, $Q_{t+\tau}^D \equiv \Pi_{j=1}^{\tau} M_{t+j}^D$ and $Q_{t+\tau}^* \equiv \Pi_{j=1}^{\tau} M_{t+j}^*$.

and solving for Z_t^D provides the value to the domestic investor of the future wealth-to-consumption ratio. For simplicity, throughout the rest of the paper, we refer to this ratio as the "price ratio". Further substituting Z_t^D into the wealth definition in equation (5) and repeating the process for the foreign investor provides the values of wealth, W_t^D and $\widetilde{W}_t^D$.

We can then use this insight to measure the value of the wedges. For example, the η wedge relative to wealth is captured in the two valuations given in the equations (3) and (4) by setting the return $\widetilde{R}_{a,t+1} = \widetilde{R}_{c,t+1}$ and denoting the variables with "D", implying the set of Euler equations:[11]

$$E_t\left\{\widetilde{M}_{t+1}^D \widetilde{R}_{c,t+1}^D\right\} = 1 \quad (8)$$

$$E_t\left\{M_{t+1}^D \widetilde{R}_{c,t+1}^D \left(S_{t+1}^D / S_t^D\right)\right\} = 1 \quad (9)$$

Clearly, solving equation (8) for $\widetilde{Z}^D$ in $\widetilde{R}_{c,t+1}^D$ gives the foreign investor's internal valuation of their own wealth price ratio as described above. However, solving equation (9) for the value of that same foreign wealth payout to the domestic investor implies a different price ratio, when markets are incomplete. We define this price ratio from domestic investors as $\widetilde{Z}^\eta$ and next connect these different values of foreign wealth to welfare measures.[12]

1.3 Certainty Equivalent Consumption Wedges

We can now consider the relative value to an investor consuming their own home country wealth relative to a foreign investor's valuation of that same wealth payout, but measured in foreign price units. For this purpose, consider a general utility function $U(C_t)$ for an agent with a set of resources that determines an intertemporal budget constraint. The optimization of preferences given this constraint then implies a sequence of lifetime consumption $\{C_t\}$ and a value function, $V(W_t)$. Assuming that preferences are homogeneous, this value function can be written as: $V(W_t) = C_t V(W_t/C_t)$.

To measure the relative welfare cost of an η wedge, we consider the value to a domestic agent investor of the foreign investor's lifetime wealth, a measure defined as: $W_t^\eta \equiv$

[11] We could also calculate the value of domestic wealth return from the point of view of each investor, but since we assume symmetry in most of the paper only one view is reported for parsimony. Section 4 describes non-symmetric settings.

[12] Appendix B.3 describes the connection between these and other price ratios in more detail.

$C_t^D(1+Z_t^\eta)$. This measure provides the implied wealth faced by the domestic investor using lifetime consumption measures in equation (7) given by W_t^D relative to an alternative lifetime consumption of the foreign wealth implied by W_t^η.

In permanent consumption units, what is the value to such an agent of consuming not their own wealth, but that of the foreign agent? Clearly, the values to the agents of the two economies are equivalent when markets are financially complete because the returns on lifetime wealth are equalized.[13] However, when markets are incomplete, the value of lifetime wealth by the domestic agent valuing foreign wealth relative to their own wealth will differ by $\Delta_{\eta,D}$ in the expression:

$$(1-\Delta_{\eta,D}) = \frac{V(W_t^D/C_t^D)}{V(W_t^\eta/C_t^D)} = \frac{V(1+Z^D)}{V(1+Z^\eta)} \tag{10}$$

In other words, $\Delta_{\eta,D}$ measures the certainty equivalent (CE) difference in permanent consumption of the hypothetical value to domestic households of consuming the foreign wealth return relative to value of their own domestic wealth return. We use this insight next.

1.4 Quantifying η Wedges

As the discussion above shows, the consumption processes in each country can be used to construct the relative value of wedges given preferences that are homogeneous in wealth. As an example, we consider Epstein-Zin-Weil recursive preferences for the value function as it conveniently nests common preferences used in macro-finance, including Constant-Relative Risk Aversion (CRRA). We also assume that the two countries have identical preferences over time and aggregate consumption. Thus, for the domestic investor, the utility at time t over the general consumption basket can be written:

$$U(C_t, U_{t+1}) = \left\{ C_t^{\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}\right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{11}$$

[13] When countries are not symmetric in initial wealth, the complete markets solution of the social planner must compensate some investors with differential initial levels of consumption. In this section, we only consider symmetric countries for simplicity but return to this issue in Section 4 below.

where U_{t+1} is the utility function at $t+1$; $0 < \beta < 1$ is the time discount rate; $\gamma \geq 1$ is the risk-aversion parameter; $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution; and where $\mathrm{E}_t(\cdot)$ is the expectation operator conditional on the information set at time t.[14] The foreign country preferences are identical with variables $\widetilde{C}$ and $\widetilde{W}$. However, we will allow below for country-specific preferences in individual goods within the consumption aggregates.

To compare these consumption processes in welfare units requires the value function solution for each country. Using the value function from Epstein and Zin (1991) and Weil (1990), equation (11) can be written with the future wealth ratio Z_t as:

$$V(C_t, W_t) = (1 + Z_t)^{\Psi} C_t \tag{12}$$

where W_t is the present value of all future expected consumption and where $\Psi \equiv \psi/(\psi - 1)$. Therefore, rewriting the solution for the cost of wedges in equation (10) using equation (12) provides a general form for the Certainty Equivalent value of the η wedge given as $\Delta_{\eta,D}$ in:

$$(1 - \Delta_{\eta,D}) = \frac{V(W^D/C^D)}{V(W^\eta/C^\eta)} = \left\{\frac{1 + Z^D}{1 + Z^\eta}\right\}^{\Psi} \tag{13}$$

Clearly, valuing the difference in lifetime CE units for investors facing the η wedge depends upon the price-ratios Z given by the consumption processes.

1.5 A Simple Two-Country Example: η Wedge Explained

We now explain the η valuation calculations in Table 1, relegating details to Appendix A. Specifically, Panel B reports the results of $\Delta_{\eta,D}$ for the symmetric example of two countries, solved in equation (13). The required price ratios, Z^D and Z^η, are respectively determined by the valuation of the foreign wealth return by foreign investors in (8) and by domestic investors in (9). Note that the foreign wealth return, $\widetilde{R}_c$, is inferred from the consumption process of the foreign country. Thus the Euler equations represent internal investor valuations of consumption claims that in general need not correspond to an actual traded security. Observed asset return data are therefore used to determine these internal

[14] Specifically, we follow the form in Epstein and Zin (1989), equation (5.3) which specializes to standard time-additive CRRA preferences when $\gamma = \frac{1}{\psi}$.

valuations of wealth returns through the price ratios.

For the purpose of solving for these Z price ratios, we use the data moments in Panel A for the United States and Canada first assuming joint log normal i.i.d consumption growth. As detailed in Appendix A, calculating the relevant Euler equations then only requires the means, variances and co-variances of the two consumption and real exchange rate growth processes. Feenstra, Inklaar, and Timmer (2015) argue that for welfare comparisons, consumption and relative prices must be comparable across countries and time as in the Penn World Tables (PWT). Therefore, we discipline the valuations using PWT 9 data for the U.S. and Canada, and further extend to the United Kingdom and Australia in Section 4.

For the simple i.i.d consumption case reported in Table 1B, each country follows identical consumption processes given here for the domestic country along with the exchange rate as:

$$\ln(C^D_{t+1}/C^D_t) = \mu + \sigma_c \nu_{t+1} \ ; \ \ln(S^D_{t+1}/S^D_t) = \sigma_s \nu^s_{t+1} \tag{14}$$

where $\nu_t \sim N(0,1)$, $\nu^s_t \sim N(0,1)$, and where the cross-country correlations of consumption growth rates and exchange rates are defined as: $Corr(c, \widetilde{c}) \equiv Corr(\nu_t, \widetilde{\nu}_t)$ and $Corr(c, s) \equiv Corr(\nu^s_t, \nu_t)$, respectively. This simple example assumes random walk processes for consumption and the exchange rate, although we modify this assumption below.[15] To focus upon the implications of diversification, we assume that the two country mean growth rates, μ, and the standard deviations, σ_c, in Table 1 A are the same across the two countries.

Then, why is the value of the η-Wedge reported in Table 1 so large? Comparison of the two investor Euler equations (8) and (9) of the same foreign wealth return $\widetilde{R}_c$ in equation (6) helps answer that question. Noting that the stochastic discount factors, M and $\widetilde{M}$, depend upon each country's respective consumption growth rate, then the $\widetilde{Z}^D$ priced by equation (8) depends only on the variance of the local foreign consumption growth rate, σ_c. By contrast, when the domestic investor values foreign wealth return in equation (9), the price ratio Z^η depends on the second moments of M, $\widetilde{M}$, and S. These moments include the cross-country consumption correlations, $Corr(c, \widetilde{c})$, the correlation between consumption

[15]These assumptions are made only for simplicity. Alternatively, these processes could include a cointegrating error-correction term as in Colacito and Croce (2013). We discuss this possibility in Section 4, as well as alternative exchange rate processes that allow for long term mean reversion in the real exchange rate.

and exchange rates, $Corr(c, s)$, and the variance of the exchange rate. Clearly, if stochastic discount factors were equalized as under complete markets in equation (1), then either the correlations of consumption would be equal to one if the $\sigma_s = 0$ or else exchange rate movements would explain the observed consumption correlations, a point made by Brandt et al. (2006). However, the observed correlations of consumption across these two countries at 0.57 and of consumption with the exchange rate at -0.18 are relatively low. Therefore, the value to domestic investors of the foreign consumption process implies a lower variability, and thereby a high value of the η wedge in this example.[16] The high valuation reflects the well-known regularity that investors do not sufficiently diversify internationally, a phenomenon also captured in low consumption growth correlations.[17]

1.6 Matching Asset Return and Exchange Rate Regularities

The specification of i.i.d consumption growth as in Table 1B is inconsistent with asset return behavior.[18] Therefore, in Table 1C, we include a persistent risk component in consumption. SMM is then used to estimate the persistent "long run risk" component that provides the best fit for the reported equity and the risk-free rate moments. For parsimony, we do not include stochastic volatility and therefore the asset returns are somewhat less volatile than the data. However, the fit does a reasonable job of matching return levels. We therefore report results throughout the rest of the paper using this asset-return disciplined consumption process. This analysis also matches exchange rate regularities discussed in recent papers such as Lustig and Verdelhan (2019) and Jiang, Krishnamurthy, Lustig, and Jialu (2022). We therefore highlight them here.

A. Volatility The analysis in Table 1 is calibrated to the volatility of the real exchange rate and is therefore matched by construction. Similarly, the cross-country consumption correlations and standard deviations and asset returns are fit to data moments.

B. Foreign Exchange Risk Premium We do not target the foreign exchange return volatility. Nevertheless, the implied volatility at 1.40% matches the volatility in the data of

[16] Sufficiently high levels for exchange rate volatility can significantly reduce the value of η and even drive it negative. The range of plausible η values for our sample of OECD countries was positive both with i.i.d. and persistent consumption.

[17] See, e.g., Bekaert et al. (2011), Carrieri et al. (2013) for financial and Coeurdacier and Rey (2012) for macro variables.

[18] For example, the equity premium is too low and the risk-free rate is constant (e.g., Mehra and Prescott (1985), Weil (1990)).

2.77% reasonably well. As with the other asset returns, stochastic volatility would improve this fit further and could readily be input into the wedge analysis.

C. Cyclicality Another exchange rate puzzle is the low correlation between consumption and exchange rates, as pointed out by D. Backus and Smith (1993). Indeed, the correlation between exchange rates and consumption growth is -0.18 in Table 1, a number that is within the range reported by Lustig and Verdelhan (2019). Thus, overall, the wedge analysis below incorporates the essential exchange rate puzzles from the literature.

2 Valuing the Complete Financial Markets Wedge

The prior section described a framework for measuring the valuation wedge between investors across countries using data measures of returns. However, this η wedge does not capture any deviation from complete markets if observed markets are truly incomplete. As a result, the deviation of the exchange rate from its complete market counterpart cannot be directly observed. Therefore, we now show how the same data can be used to infer the value of the wedge deviation from incomplete markets. Details are relegated to Appendix B.2.

2.1 Certainty Equivalent Costs of Incomplete Financial Markets

To motivate this investigation, we return to the insight in the previous section that the value of lifetime wealth can be implied by the Euler equation (7). By this reasoning, we define the difference in the valuation of the state price of current wealth relative to an alternative complete markets wealth signified as $*$ as:

$$M_{t+1}^{D} R_{c,t+1}^{D} \equiv \zeta_{t+1} M_{t+1}^{*} R_{c,t+1}^{*} \equiv \zeta_{t+1} \widetilde{M}_{t+1}^{*} \left(S_{t+1}^{*} / S_{t}^{*} \right)^{-1} R_{c,t+1}^{*} \tag{15}$$

where the second identity follows from the complete markets relationship between stochastic discount factors in equation (1). This ζ wedge measures the state price deviation of the return on wealth relative to its complete markets alternative. In this case, the value of the deviation ζ could be measured by solving for the return in Euler equation (7) replacing the consumption process in the data with a consumption process for an investor with access to complete financial markets. Defining the domestic consumption process for an investor facing complete markets as C^* and the wealth price-to-consumption ratio as Z^*, then the implied value of wealth under complete financial markets is:

$$W_t^* \equiv C_t^*(1 + Z_t^*) \tag{16}$$

Then the certainty equivalent loss for the domestic country, $\Delta_{D,*}$, is given by the following:

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi} \tag{17}$$

where we have used the fact that under symmetry, $C^D = C^*$.[19] Since this cost compares wealth in the data to its counterpart under complete markets, it must be non-negative.

Measuring the counterfactual consumption and prices under complete markets requires deriving the resource allocations implied by spanning a set of Arrow-Debreu securities. These allocations can be derived as the outcome of a planner's problem facing any relevant goods market constraints. To see that outcome in the general context, first define the world aggregate consumption and world wealth both measured in the foreign numeraire country price units as: $\widetilde{C}_t^w \equiv S_t^* C_t^* + \widetilde{C}_t^*$ and $\widetilde{W}_t^{w*} \equiv S_t^* W_t^* + \widetilde{W}_t^*$ where $S_t^* \equiv (P_t^*/\widetilde{P}_t^*)$ is the real exchange rate between the domestic and foreign country when markets are complete. Then optimal consumption for the domestic and the foreign investors are given by a sharing rule:

$$C_t^* = \omega_t \widetilde{C}_t^w ;\ \widetilde{C}_t^* = (1 - \omega_t)\widetilde{C}_t^w \ . \tag{18}$$

where $\widetilde{C}_t^w$ is aggregate world consumption measured in numeraire units and where ω_t is the share for the domestic country implied by complete asset market spanning.

This sharing rule depends upon any goods market restrictions that impact the ability of the real exchange rates to adjust. Therefore, these total ζ wedge valuations differ by exchange rate view because the shares include relative goods price effects generated by consumption reallocations. The implications for these differences were reported in Table 1 Panel C for certainty equivalent consumption based upon three different exchange rate views discussed

[19] These consumption levels will differ when countries have asymmetric resource processes, as described in Section 4.

below: Nontradeables price, Home Bias, and Sticky Prices. As these numbers showed the deviations are around 2% of permanent consumption, which are far lower than the implied numbers for the η wedge. We next describe these examples to illustrate the role of goods market constraints in impacting the costs of financial market frictions.

2.2 The Role of Goods Market Frictions

We choose these three examples as representatives of a range of goods market responses. The "Sticky Price" case considers a counterfactual extreme in which prices do not adjust to reallocations in consumption, and thereby provides a metric to compare other endogenous exchange rate versions. By contrast, the "Home Bias" case represents a different extreme in which goods markets adjust frictionlessly to clear commodity prices. In this case, completing financial markets can impact exchange rates by reallocating resources across investors who have different preferences for goods. Finally, the "Non-Tradeables" case provides an intermediate goods market version in which prices adjust to clear the market for tradeable goods, but not for nontradeable goods. Greater access to securities that reallocate tradeables then impacts the real exchange rate through the relative price of nontradeables.[20]

To understand the effects of the different exchange rate views, we describe next how the complete markets exchange rates can be recovered from the data under the lens of these three goods market views. Moreover, viewing the data through the goods market equilibrium condition disciplines the quantity of resources that the planner can reallocate. We briefly summarize in turn each of these solutions as they relate to data measures.

A general feature of this approach can be seen by rewriting the relationship between exchange rates and stochastic discount factors in equation (1) using the solution for the stochastic discount factor for Epstein-Zin preferences given as: $M_{t+1} = \beta^{\theta}(C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)}(R_{c,t+1})^{\theta-1}$. Thus, combining this complete markets condition with the sharing rule, the relationship

[20] Appendix C details the implications for the sharing rule ω for these different views of exchange rates determination, which is based upon allowing the planner to reallocate consumption although we can in principle based our analysis on reallocations of income as described in Section 4.

between stochastic discount factors can be restated as:

$$\frac{\widetilde{M}^*_{t+1}}{M^*_{t+1}} = \left(\frac{\widetilde{C}^*_{t+1}/\widetilde{C}^*_t}{C^*_{t+1}/C^*_t}\right)^{-\gamma}\left(\frac{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}{(1+Z^*_{t+1})/Z^*_t}\right)^{(\theta-1)} = \frac{S^*_{t+1}}{S^*_t}. \tag{19}$$

We use this first-order condition to the planner's optimization to illustrate how the goods market restrictions impact the cases below. In each case, we infer the existing quantity of commodities to be reallocated by financial markets using data on exchange rates and consumption aggregates.[21]

Sticky Prices This version provides a simple benchmark to consider the case when complete financial markets cannot alter the exchange rate. The social planner is then constrained to take prices as given by the data. Thus, calculating the complete markets shares of aggregate consumption for each country simply requires reallocating aggregate consumption at the given prices. Based upon this assumption, the aggregate resource constraint is just given by the aggregate world consumption in numeraire units at the data exchange rate; that is, $\widetilde{C}^w_t \equiv S^*_t C^*_t + \widetilde{C}^*_t = S^D_t C^D_t + \widetilde{C}^D_t$. Since this case solves for financial market adjustment without any goods market response, by construction, the exchange rate in the data is the same under complete financial markets so that: $S^* = S^D$.

The optimal consumption growth in this case has an intuitive form. For example, in the case where consumption growth is i.i.d. and the countries are symmetric, then, equation (19) can be rewritten in log growth terms to imply:

$$g^*_{c,t+1} = \widetilde{g}^*_{c,t+1} + \frac{1}{\gamma} g^*_{s,t+1} = \frac{1}{2} g^{wD}_{c,t+1} - \frac{1}{2}(1-\frac{1}{\gamma}) g^D_{s,t+1}. \tag{20}$$

Thus, the complete markets domestic consumption growth rate is an equal share of aggregated world consumption adjusted by the intertemporal effect of the exchange rate, both measured from aggregate consumption and exchange rate growth in the data.

Home Bias This version assumes that goods markets function frictionlessly so that prices clear individual commodity markets. According to this view, the real exchange rate

[21] We treat consumption allocations in the data as the outcome of an unspecified incomplete markets equilibrium and therefore are agnostic about the nature of financial market integration in the data. As a result, the analysis is isomorphic to the "portfolio autarky" condition studied in i.i.d. by Cole and Obstfeld (1991) and in a long-run risk setting by Colacito and Croce (2013).

varies due to a greater preference for the home goods produced in each respective country.[22] In this case, suppose there are two goods, indexed by 1 for the good produced in the domestic country and 2 for the good produced in the foreign country. Following much of the literature, we assume that the consumption aggregator over these two goods is Cobb-Douglas. Then, since the domestic investor prefers their own good, the domestic and foreign consumption aggregators can be written as:

$$C_t = (C_{1,t})^a \, (C_{2,t})^{1-a} \, ; \; \widetilde{C}_t = \left(\widetilde{C}_{1,t}\right)^{1-a} \left(\widetilde{C}_{2,t}\right)^a \tag{21}$$

where $a > 1/2$. In this case, the real exchange rate implied by goods market equilibrium is: $S_t = (P_t/\widetilde{P}_t) = (P_{1,t}/P_{2,t})^{2a-1}$ where $P_{i,t}$ is the price of good i. Thus, as the planner reallocates domestic and foreign goods across countries, the exchange rate under complete markets will generally differ from the exchange rate data.

To quantify the goods that can be distributed, we follow the assumption of this exchange rate view that goods market equilibrium holds within any given period. By contrast, if financial markets are incomplete, intertemporal consumption decisions will not be optimized in general. In this case, we can uncover the relative consumption in the domestic country for each good depending on its relative price in the data, a decision that in turn depends on the observed exchange rate. Specifically, this first-order condition can be written:

$$\left(C_{1,t}^D / C_{2,t}^D\right) \; = a(1-a)^{-1} \left(S_t^D\right)^{(1-2a)}$$

A similar relationship holds for the foreign country but with preferences for good 2. Using these conditions allows us to measure the global resource constraint for each good as a function of observed exchange rates and consumption aggregates. The implied complete markets solutions generate sharing rules that allocates the world supplies of each good to optimize risk-sharing over time. The complete markets exchange rate, S_t^*, will be changed as a result of these reallocations and, consequently, will differ from the exchange rate in the data.

[22] Studies that have used this approach to explain exchange rate behavior include Colacito and Croce (2011), Coeurdacier and Rey (2012), and Stathopoulos (2021), among many others.

Non-Tradeables While these two extreme perspectives on goods markets provide useful benchmarks, the standard non-tradeables view of exchange rates treats goods markets somewhere in between. That is, only the traded goods market is assumed to clear internationally while the nontradeables markets do not.[23] Continuing the two country Cobb Douglas example and defining $C_{T,t}$ and $C_{N,t}$ as the consumption of tradeables, T, and nontradeables, N, respectively, the consumption aggregator for both countries would be:

$$C_t \equiv C(C_{T,t}, C_{N,t}) = (C_{T,t})^{\alpha} (C_{N,t})^{1-\alpha} \tag{22}$$

Thus, defining $P_{T,t}$ as the price of tradeables and $P_{N,t}$ and $\widetilde{P}_{N,t}$ as the price of nontradeables in the domestic and foreign countries, respectively, the real exchange rate becomes $S_t = (P_{N,t}/\widetilde{P}_{N,t})^{1-\alpha}$, the relative price of non-tradeables across countries.

Given that the goods market does not equilibrate for nontradeables goods, the financial market preserves this condition as a resource constraint. In order to identify the quantity of tradeables implied by goods market equilibrium, we again use the insights of the goods market condition implying that the domestic country Tradeables and Nontradeables consumption can be measured as:

$$\left(C^D_{T,t}/C^D_{N,t}\right) = \alpha(1-\alpha)^{-1}\widetilde{\rho}_{N,t}\left(S^D_t\right)^{(1-\alpha)^{-1}}$$

where $\widetilde{\rho}_{N,t}$ is the relative price on nontradeables in the foreign country. A similar counterpart relationship holds for the foreign country. Using this condition with consumption allows the aggregate world tradeables to be inferred from the data. Optimizing the sharing rule for tradeables across investors implies that the planner will redistribute according to nontradeables shocks as well as tradeables. As such, the real exchange rate will again be different under complete markets then the exchange rate implied by the data.

[23] See for example Cole and Obstfeld (1991), Engel (1999), and Asea and Mendoza (1994).

2.3 The Exchange Rate Wedge and Decomposition

Since complete markets implies a reallocation of existing consumption, the implied real exchange rate in general differs from the exchange rate in the data, as noted above. We therefore call this deviation the S-Wedge, as given by the exchange rate wedge between the data and its complete markets alternative as:

$$\left(\frac{S^D_{t+1}}{S^D_t}\right) \equiv \zeta_{S,t+1}\left(\frac{S^*_{t+1}}{S^*_t}\right) \tag{23}$$

The approach to value wealth inferred from the data and under complete markets suggest that this S-Wedge may be valued in the same way. In particular, we can compare the value of complete markets wealth distorted by the exchange rate in the data as the wealth price ratio-consumption implied by solving the Euler equation:

$$E_t\left\{\widetilde{M}^*_{t+1}R^*_{c,t+1}\left(S^D_{t+1}/S^D_t\right)\right\} = 1 \tag{24}$$

where Z^S is the wealth-consumption ratio that solves equation (24) for the consumption process under the risk-sharing rule in equation (18). Given this price ratio, we can again consider the welfare loss for an investor with lifetime consumption under complete markets relative to an investor with consumption distorted by the exchange rate wedge:

$$1-\Delta_{S,*} = \left\{\frac{1+Z^S}{1+Z^*}\right\}^{\Psi} \tag{25}$$

where $\Delta_{S,*}$ is the certainty equivalent cost of consuming wealth under complete markets but distorted by the exchange rate in the data.

The two wedges given by η in equation (13), and by the S Wedge in equation (25) suggests a general decomposition of the total welfare costs relative to complete markets given by ζ in equation (17). Specifically, defining the value of future wealth-to-consumption of any two arbitrary sets of lifetime consumption streams labeled X and Y as, respectively, Z^X and Z^Y, the certainty equivalent cost of an investor consuming wealth X instead of Y is given by:

$$1 - \Delta_{X,Y} \equiv \left\{ \frac{1 + Z^X}{1 + Z^Y} \right\}^{\Psi} \tag{26}$$

Then the wedges for η and for the exchange rate S Wedge can be used to decompose the effect on the overall cost Δ. That is, since the total wedge can be rewritten according to this decomposition as: $1-\Delta_{D,*} = (1 - \Delta_{D,\eta})(1 - \Delta_{\eta,S})(1 - \Delta_{S,*})$, and because $\ln(1-\Delta) \approx -\Delta$, then the impact of a particular wedge relative to other wedges can be decomposed as:

$$\Delta_{D,*} = \Delta_{D,\eta} + \Delta_{\eta,S} + \Delta_{S,*} \tag{27}$$

Figure 1 illustrates the structure of this decomposition, subsuming the time subscripts in the labels for clarity. The top row illustrates the total cost, $\Delta_{*,D}$ between Complete Markets and the Data Inference. This cost compares the difference between the price ratios that value the complete markets wealth, M^*R^* and the standard data-inferred wealth, M^DR^D. However, the vertical arrows provide an "under-the-hood" evaluation of the impact of, alternatively, the S-Wedge and the η-Wedge. That is, the first vertical column demonstrates the relative valuations of wealth to an investor under complete markets M^*R^* relative to one distorted only by the effect of exchange rates, $\widetilde{M}^*R^*S^D$. This comparison is measured by $\Delta_{*,S}$. By contrast, the impact of the η-Wedge compares two valuations inferred directly from data as the difference in valuations of M^DR^D and $\widetilde{M}^DR^DS^D$. The remaining term, $\Delta_{S,\eta}$, compares the value of these distortions on the measured wealth returns.

2.4 Exchange Rate Wedge Decomposition Quantified

Table 2 shows the effects of the costs of incomplete markets for the three goods market conditions along with their decompositions for the i.i.d. case in Panel A and the persistent case in Panel B. The total cost measure, $\Delta_{\eta,S}$, is around 2% for the i.i.d. case, but near 1% for the persistent consumption case. This pattern reflects the assumption that the cross-country correlation of the persistent "long run risk" component is assumed to be equal to one, in order to provide a conservative case.[24] The next two columns report the costs of the wedges given by the S-Wedge certainty equivalent wedges, $\Delta_{D,\eta}$ and $\Delta_{S,*}$, respectively. By construction, the "Sticky Price" version has no S-Wedge and we therefore refer to this case as also the "No S-Wedge" case below. By contrast, the Non-Tradeables case implies that the cost of distorting the exchange rate as in the data rather than complete markets is about 3% and 2.5% of permanent consumption for the i.i.d. and persistent case, respectively.

Table 2 also reports the decompositions as noted in equation (27). While the total cost measure $\Delta_{D,*}$ is positive, as noted earlier, the components need not be. Therefore, to highlight the relative absolute contribution of each component to the total, we follow Nagel and Xu (2022) in measuring the shares in absolute values. These shares are reported under "Absolute Shares." To focus on the comparisons to either the Data or Complete markets, we report $\Delta_{D,\eta}$ and $\Delta_{S,*}$, subsuming the other component, $\Delta_{D,\eta}$, as the residual. As the shares show, the distortions implied by the exchange rate wedges are relatively small compared to the implicit wedges due to the other components of η.

Finally, under "Returns", we report the implied wedges in return form. Specifically, we use the wealth return as implied by the consumption processes in the data from Table 1 adjusted by a leverage parameter used to match equity returns. This calibration provides an estimated return of 7.8% for the i.i.d. case and 9.74% for the persistent risk case. The next two columns present differentials for hypothetical assets that pay out the value of the η wedge under the column headed $E[r^D - r^\eta]$, and the $S - Wedge$ under $E[r^D - r^S]$. Since the value of η is high, the η returns are lower than the consumption asset ranging from 32 to 23 basis points. By contrast, the $S - Wedge$ for Nontradeables commands a premium of 1 to 3 basis points while the No $S - Wedge$ and Home bias cases suggest a discount.

The small relative share of the S Wedge relative to η as a contribution of welfare raises questions about the other components of η. Therefore, the next section examines the impact of the implicit wedges in these components more carefully.

[24]For example, Lewis and Liu (2015) show that the benefits of risk-sharing are minimized when the correlation of persistent risk is equal to one.

3 Valuing the Total Exchange Rate Wedge

So far, we have focused upon the role of the exchange rate in impacting welfare through the stochastic discount factors. However, the deviation of wealth under complete markets compared to standard data measures in equation (15) depends critically on the return to future wealth itself. This section therefore also considers the role of wedges in the return on future wealth, implying a new relationship between wealth returns under complete markets. This observation highlights a new wedge between how the exchange rate impacts the total history of future returns compared to wealth returns under complete markets.

3.1 The Stochastic Discount Factor versus the Return on Wealth

As noted above, the ζ wedge defined in equation (15) captures the deviation in state prices between their data-inferred values and complete markets counterparts through the lens of different exchange rate models. These state prices depend upon the data-implied returns on wealth for the domestic and foreign investor, given as R_c^D and $\widetilde{R}_c^D$, respectively. Therefore, to look at the potential contribution of each component, we define partial wedges for each of these components as:

$$\begin{aligned} M_{t+1}^D &\equiv \zeta_{M,t+1} M_{t+1}^*; \; \widetilde{M}_{t+1}^D \equiv \zeta_{\widetilde{M},t+1} \widetilde{M}_{t+1}^* \\ R_{c,t+1}^D &\equiv \zeta_{R_c,t+1} R_{c,t+1}^*; \; \widetilde{R}_{c,t+1}^D \equiv \widetilde{\zeta}_{R_c,t+1} \widetilde{R}_{c,t+1}^* \end{aligned} \tag{28}$$

Following our terminology above, we call $\zeta_{M,t+1}$ the "M Wedge" and $\zeta_{R_c,t+1}$ the "R Wedge", and similarly for the foreign investor. Using these definitions to replace the state price of domestic wealth in the data implies that the complete markets wedge in equation (15) can be written as:

$$M_{t+1}^D R_{c,t+1}^D \equiv \zeta_{M,t+1} \zeta_{R_c,t+1} M_{t+1}^* R_{c,t+1}^*$$

Moreover, these two partial wedges may be valued separately following the same approach as above, calculating the price ratios Z that value the wealth returns at the wedge-distorted levels. Using the general definition of the value of these wedges in equation (26), we can similarly decompose the total cost as in equation (27), replacing η and S with the M-Wedge and R-Wedge, respectively.[25] Appendix B describes the implied valuations for each version.

Table 3 reports the certainty equivalent costs to a domestic investor of consuming wealth measured in the data compared to complete markets implied by the exchange rate model examples. Under "Wedges", the first column reports the cost to a domestic investor who values their wealth at the state price value $M_t^D R_{c,t}^D$ relative to the complete markets alternative only distorted by the M Wedge, implied by $M_t^D R_{c,t}^*$. Following the terminology above, we define this certainty equivalent difference as: $\Delta_{D,M}$. As this first column shows, the results are quite different from those of the η wedge reported in Table 2. In contrast to the η wedge, the size and even the sign of the M Wedge cost depends upon the exchange rate view. When consumption is i.i.d. as shown in Panel A, the size varies from 2.88% for the "No S-Wedge" case when prices are fixed to -1.00% when prices adjust as in the Nontradeable case. Indeed, the negative sign implies that maintaining the stochastic discount factor in the data to value future wealth implied by complete markets would detract from welfare.

This varied impact of the stochastic discount factor together with the overall benefits of complete markets implies that the effects of the return to wealth, the R Wedge, must be important. Thus, the second column under Wedges reports the "cost" relative to distorting complete markets by the return in the data R^D, rather than the counterfactual complete markets return, R^*. In all exchange rate versions, this "cost" is negative, meaning that investors would prefer the return in the data compared to complete markets. This finding is not surprising because, as shown in Table 1, the value of returns in the data provide a better diversification benefit than the one implied by pooling total world resources. Indeed, the next column under "Wedges" reports the certainty equivalent gains of the returns in the data relative to this R-Wedge, $\Delta_{D,R}$, showing the sizes are indeed large in all cases. By contrast, the effects of the wedge on M as measured by $\Delta_{M,*}$ are more modest.

This pattern is highlighted by the absolute value shares reported under the columns with heading "Absolute Shares." As the first decomposition shows, the R-Wedge relative to the data-inferred wealth accounts for over 40% of the share, while the deviation due to the M-Wedge from complete markets only measures 12% or less. Similarly, the last two columns show that same pattern when the R-Wedge is compared to the data and the M-Wedge to

[25] That is, $\Delta = \Delta_{D,M} + \Delta_{M,R} + \Delta_{R,*}$ where "M" and "R" denote the state price distorted by respective M and R Wedges.

complete markets.

Overall, the impact of the M Wedge is lower than the counterparts from the R Wedge. This finding suggests that the wedge due to future wealth return is more important than the current valuations represented in the standard η wedge given in equation (2). If so, the effects of the current exchange rate wedge described in Table 2 may be missing important impacts on *future* returns. We consider this possibility next.[26]

3.2 A New Complete Markets Condition and The Total S Wedge

The results in Table 3 suggest that future wealth may be a primary component of the overall costs from incomplete markets. By contrast, the analysis above has focused only upon the current period first-order condition relating exchange rates and stochastic discount factors in equation (1). However, when markets are complete, the domestic and foreign investors share the world consumption aggregates and thereby a common component of wealth returns. In the special case of recursive preferences used in this paper the relationship across countries under complete markets has a convenient form as shown in Appendix C.[27] That is,

$$R^*_{c,t+1} = \left(\frac{\omega_{t+1}C^w_{t+1}}{\omega_t C^w_t}\right)\left(\frac{S^*_{t+1}}{S^*_t}\right)^{-1}\left[\frac{1+Z^*_{t+1}}{Z^*_t}\right];\ \widetilde{R}^*_{c,t+1} = \left(\frac{\widetilde{\omega}_{t+1}C^w_{t+1}}{\widetilde{\omega}_t C^w_t}\right)\left[\frac{1+\widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t}\right]$$

where as above, ω is the domestic country share of world consumption and $\widetilde{\omega} = 1-\omega$.

Moreover, when countries are symmetric and consumption is i.i.d., this relationship simplifies to: $R^*_{c,t+1} = \widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{\frac{1}{\gamma}}$. Combining this observation with the standard complete markets exchange rate relationship in equation (1) leads to a new complete markets relationship between the wealth returns across countries. From equation (1) and the above, we have:

$$M^*_{t+1}R^*_{c,t+1} \equiv \widetilde{M}^*_{t+1}R^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}(S^*_{t+1}/S^*_t)^{-(1-\frac{1}{\gamma})} \tag{29}$$

[26]The effect of the M Wedge relative to the R Wedge on welfare also depends critically on the assumption of the intertemporal elasticity of substitution. In this section, we have maintained the assumption that this parameter is greater than one in order to match asset pricing relationships. However, as described in the appendix and in the next section, this pattern may be reversed if the intertemporal elasticity of substitution is less than one.

[27]In particular, $\frac{R^*_{c,t+1}}{\widetilde{R}^*_{c,t+1}} = \left(\frac{S^*_{t+1}}{S^*_t}\right)^{\frac{1}{\gamma}}\left[\frac{(1+Z^*_{t+1})/Z^*_t}{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}\right]^{\frac{\theta}{\gamma\psi}}$.

Note that this relationship implies that under complete markets, wealth returns are equalized except for the impact of the exchange rate. This observation leads to measurement of a different exchange wedge that considers the impact of relative prices alone on future wealth. That is, we can consider the deviation from complete markets by replacing the right-hand side variables in equation (29) with their data counterparts to define a total wedge, η^T, as:

$$M^*_{t+1} R^*_{c,t+1} \equiv \widetilde{M}^D_{t+1} \widetilde{R}^D_{c,t+1} (S^D_{t+1}/S^D_t)^{-(1-\frac{1}{\gamma})} \eta^T \tag{30}$$

Furthermore, separating the effect of the exchange rate wedge and using the notation for the S Wedge in equation (23), we can rewrite the relationship in equation (29) as:

$$M^*_{t+1} R^*_{c,t+1} = \widetilde{M}^*_{t+1} \widetilde{R}^*_{c,t+1} \left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})} \left(\zeta_{S,t+1}\right)^{-(1-\frac{1}{\gamma})} \tag{31}$$

Therefore, we can examine the effects of an exchange rate wedge on the return to future wealth through this "Total S-Wedge", denoted as S^T.

Table 4 shows these measures using the same parameters and assumptions as in Table 2 and therefore the "Total Cost" measures and S-Wedge measures are repeated for reference in the first three columns. The next column under "Total S" labeled Δ_{D,η^T} reports the value for certainty equivalent differences due to η^T as in equation (30) relative to the basic value of wealth in the data. This relationship compares the hypothetical value of the foreign wealth in the data but valuing all future wealth at the exchange rate in the data. According to complete markets, domestic and foreign consumption growth rates are equalized except for these exchange rate valution effects. Therefore, foreign consumption no longer provides diversification to domestic investors, but instead these investors now face exchange rate risk. As such, the benefit of investing in this foreign wealth that only differs by the exchange rate will be slightly negative at -0.05% and -0.02% of permanent consumption for i.i.d. and persistent risk, respectively.

The next column compares the effect of the Total S-Wedge on complete markets. In all cases, the effects are closer to zero than for the S-Wedge itself. However, given that the overall impact of the difference in valuation of relative wealth measured in Δ_{D,η^T} is negative, the relative contribution of the Total S-Wedge is larger as can be seen by comparing the

Absolute Shares in the final four columns. For example, in Panel A, the Absolute Shares of Δ_{D,S^T} range from 5.3% to 9% compared to 1.5% and 4.6% and a similar pattern holds in Panel B. Thus, overall, the relative contribution of the Total S-Wedge has a larger impact that the current S-wedge.

4 Exchange Rate Wedge Generalizations

Many of the insights so far are based upon risk and intertemporal parameters that were seen to fit the asset returns in Table 1. The importance of the future wealth relative to the current stochastic discount factors therefore depends on these parameters. Furthermore, the analysis so far has focused upon a two country case to highlight the basic features of measuring exchange rate wedges in a parsimonious way. However, this framework can easily be formulated to consider the impact of different international valuations in other models and applications. Therefore, in this section, we illustrate the approach with various generalizations. First, we consider how the basic patterns above are altered with different assumptions about exchange rate adjustment and risk preferences. Second, we show that the framework may be extended naturally to multiple countries, illustrated with data for four countries. Third, we point to a number of other applications including different asset pricing models, exchange rate processes and production economies.

4.1 Valuing Wedges with Different Preference Parameters

The analysis above depended upon two sets of preference parameters. One set impacts risk assessment and intertemporal decisions as characterized by the intertemporal elasticity of substitution ψ, risk aversion γ, and the time discount factor β. The second set determines the goods market clearing, including the preference for Tradeables relative to Nontradeables α, and the preference for Home Goods relative to Foreign Goods a. We find that our results are most sensitive to variations in IES and in the value of Tradeables, so we focus on these parameters as examples.

Table 5 then repeats the analysis in Table 2 for different measures of α in Panel A and of ψ in Panel B taking the i.i.d. consumption case as a benchmark. As Panel A shows, the overall costs of incomplete versus complete markets decline with the preference for tradeables. Intuitively, as α approaches 0.5, the two goods become perfect substitutes and since the planner can only reallocate tradeables, any benefits from this potential reallocation declines.

By contrast, Panel B shows that the intertemporal elasticity of substitution, ψ, has a strong impact on the η wedge. When ψ is great than 1 as typically assumed to fit asset return behavior, investors easily substitute current for future consumption. Therefore, relative valuations by domestic investors of the foreign wealth become very appealing. However, when ψ is less than one, these investors prefer current consumption and the possible returns on foreign wealth become less attractive.

4.2 The Costs of Wedges with Multiple Countries

It is straightforward to generalize the approach above to a set of countries $j = 1, ..., J$. Appendix D.1 describes this extension along with basic data moments that include the United Kingdom (UK) and Australia (Aus). We now denote the variables specific to a given country j with a superscript. Arbitrarily choosing the last country J to be the numeraire, the aggregate consumption level and real exchange rate for country j can be rewritten as C_t^j and $S_t^j = (P_t^j/\widetilde{P}_t^J)$, respectively. This multi-country version requires extending the resource constraints to J countries so that world consumption units measured in the numeraire country must be written as a sum across countries. As an example, for the Sticky Price version, this resource constraint becomes: $\sum_{j=1}^{J} S_t^{jD} C_t^{j,D} = \widetilde{C}_t^w$ and the optimal consumption policy for each country is a sharing rule that depends upon the exchange rate versions that can be written in general form as: $C_t^{j*} = \omega_t^j \widetilde{C}_t^w$ where ω_t^j is country j share of world wealth depending upon the goods market restriction as seen above.

To illustrate the implications of this generalization, we focus on the No S-Wedge case. We also extend the data series from the United States (US) and Canada (Can) to include the United Kingdom (UK) and Australia (Aus). To focus upon the impact of multiple countries, we only report the total cost $\Delta_{D,*}$. When countries have non-symmetric consumption and price processes as implied by these data, then the value of future wealth differs. Therefore, the planner reallocates initial consumption across countries to incentivize countries with better wealth returns to be willing to pool their assets in complete markets. These reallocations are given in the table as "Weights" given by the ratio of initial C^*/C^D.

To illustrate the impact of exchange rate variability, we first assume counterfactually that the exchange rate volatility is zero; that is, $g_{s,t}^{j,D} = 0$. Table 6 Panel A reports the results from these calculations using the correlations for real consumption growth. These levels are shown for a base case set of typical CRRA macro parameters of $\gamma = 2$ and $\psi = 0.5$, as well as the parameters used to match asset returns above of $\gamma = 10$ and $\psi = 1.5$. In all cases, the benefits of risk-sharing imply positive certainty equivalent consumption wedge $\Delta_{D,*}$. Moreover, these costs increase with risk aversion and the intertemporal elasticity of substitution, IES. By contrast, Panel B shows the impact of exchange rate volatility when the variance of $g_{s,t}^{j,D}$ is given by the price data. Comparing the final two rows of Table 6 reports the results with "Asset Pricing" parameters with exchange rate volatility. These rows show a benefit of 1.28% of permanent consumption for the Canadian investor who is willing to give up some initial consumption level at 0.985 of their initial level in order to participate in a better lifetime consumption growth path. After this initial compensation, all countries have non-negative welfare gains.

4.3 Other Extensions

Our framework is general and applicable to a number of other assumptions and models that we only briefly highlight here. Appendix D.2 provides more details for each of these cases.

Alternative Asset Pricing Models To illustrate how our framework can use consumption and exchange rates that match asset price data, we chose a long run risk process as an example of persistent risk. However, the approach could be also consider alternative consumption processes that match asset returns. For example, a number of papers have shown the importance of disaster risk in matching financial markets including Barro (2009), Wachter (2013), and D. Backus, Chernov, and Martin (2011). Moreover, the importance of disaster risk in global valuations has been shown in Nakamura et al. (2013) and Gourio, Siemer, and Verdelhan (2013) and Lewis and Liu (2017). Much of this literature uses recursive preferences with consumption processes disciplined by disaster events, an approach conveniently nested in our framework.

Persistence in Exchange Rates The endogenous price versions of the exchange rate that we have studied above require assumptions about its process in the data. This process is then used to uncover the commodity quantities that are implicit within the consumption data aggregate. In the analysis above, we have taken the simplifying assumption that the real exchange rate in the data is a random walk. However, studies of the longer run behavior of the real exchange rate suggest that prices cannot diverge indefinitely. Thus, while the random walk assumption provides convenient closed form solutions in order to investigate the basic wedge relationships, a more realistic approach would allow for persistence in these relative prices. Again, this approach can be readily incorporated by amending the implied persistence in relative goods supplies across countries.

The Costs of Wedges using Production The analysis in this paper has focused upon using consumption data because it is the driver in many asset pricing models, and is related to a large literature on consumption risk-sharing. Nevertheless, our approach can alternatively accommodate the implications of inefficient allocations in production. As described in Feenstra et al. (2015), the PWT data provide country output and absorption price measures that can be used for international comparisons. Thus, these data can be used along with our framework to consider the reallocation due to asset markets that span production risks rather than consumption risks.

One approach would be to suppose that production is linear in technology. For example, consider a model in which output in each firm is produced with linear technology: $y_t(z) = Y_t z l_t(z)$ where $l_t(z)$ is the amount of labor employed by the firm and where Y_t is a stochastic process generating aggregate productivity. In this case, if domestic consumption depends upon claims to this output across countries, the total world consumption, C_t^w would be replaced to total world output, Y_t^w in the data. In this way, the same analysis of the wedges due to real price differences across countries can be calculated for production-side risks. Rather than reallocating consumption, the complete markets solution would then reallocate output and consumption would become endogenous.

Alternative Exchange Rate Versions In this paper, we focused upon three different examples of exchange rate approaches. However, our approach is general enough to allow for other determination models. For example, Obstfeld and Rogoff (2001) suggested that transactions costs could potentially explain the disconnect between exchange rates and fundamentals. They use a proportional transactions cost specification with a homogeneous consumption aggregator that directly nests within our approach above. More recently, Itskhoki and Mukhin (2021) show that a friction process outside of fundamentals that they call "financial shocks" is needed to explain exchange rates. They parameterize these shocks with interest parity, similar to our foreign exchange returns. Therefore, these measured returns could be included as alternative measures of exchange rate wedges using our approach.

5 Concluding Remarks

How much financial market frictions affect the welfare of economic agents across countries is clearly an important question. In this paper, we develop an approach to help answer this question by connecting standard frameworks that address both exchange rates and asset return behavior. These connections provide four main contributions.

First, we show how financial and macroeconomic data can be used to measure the welfare costs implied by the wedge in global investor valuations of returns. The implied cost of this wedge is independent of the exchange rate determination view. However, this wedge only compares incomplete market valuations and cannot evaluate optimality.

Therefore, as a second contribution, we show how the same data can be used to consider the welfare value of financial market completeness. We demonstrate how to measure this wedge using examples of several off-the-shelf exchange rate approaches, although the setting is general enough to consider other such exchange rate views.

Third, our paper provides a framework to decompose the valuation of the implied wedge from complete markets into individual components, such as wedges in the exchange rate and the stochastic discount factors of investors. For parameters that match standard asset pricing moments, we find that the effect of the exchange rate wedge is small relative to the stochastic discount factors of investors.

Fourth, we highlight a new complete markets relationship across countries based upon the wealth returns. Using this insight, we construct a "Total" exchange rate wedge that measures the impact of exchange rates over future wealth.

In exploring these new relationships, we simplify much of our analysis by considering a two country symmetric framework with stylized assumptions. However, we illustrate how our framework is sufficiently rich to allow for further work and extensions to multiple countries, a stationary long run real exchange rate, production risk, and broader asset pricing models. Thus, the paper provides an important step toward connecting the behavior of asset returns, exchange rates, and the value of deviations from optimal financial market risk insurance.

References

Abel, A. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, *43*, 3-33.

Alvarez, F., & Jermann, U. J. (2004). Using asset prices to measure the cost of business cycles. *Journal of Political Economy*, *112*(6), 1223-1256.

Alvarez, F., & Jermann, U. J. (2005). Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica*, *73*(6), 1977-2016.

Asea, P. K., & Mendoza, E. G. (1994). The balassa-samuelson model: A general-equilibrium appraisal. *Review of International Economics*, *2*(3), 244–267.

Backus, D., Chernov, M., & Martin, I. (2011). Disasters implied by equity index options. *The Journal of Finance*, *66*(6), 1969–2012.

Backus, D., Foresi, S., & Telmer, C. (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance*, *56*(1), 279-304.

Backus, D., & Smith, G. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *35*(3-4), 297-316.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International real business cycles. *The Journal of Political Economy*, *100*(4), 745-775.

Bakshi, G., Cerrato, M., & Crosby, J. (2018). Implications of incomplete markets for international economies. *Review of Financial Studies*, *31*(10), 4017-4062.

Bansal, R., & Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, *59*(4), 1481–1509.

Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review*, *99*(1), 243–64.

Barro, R. J., & Ursúa, J. F. (2008). *Macroeconomic crises since 1870* (Tech. Rep.). National Bureau of Economic Research.

Bekaert, G., Harvey, C. R., Lundblad, C. T., & Siegal, S. (2011). What segments equity markets? *Review of Financial Studies*, *24*(12), 3841-3890.

Berka, M., Devereux, M. B., & Engel, C. (2018). Real exchange rates and sectoral productivity in the eurozone. *American Economic Review*, *108*(6), 1543-1581.

Brandt, M. W., Cochrane, J. H., & Santa-Clara, P. (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics*, *53*(6), 671-698.

Burnside, A. C., & Graveline, J. J. (2020). On the asset market view of exchange rates. *The Review of Financial Studies*, *33*(1), 239-260.

Campbell, J. Y. (1993). Intertemporal asset pricing without consumption data. *American Economic Review*, *83*(3), 487-512.

Campbell, J. Y., & Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, *107*(2), 205-251.

Carrieri, F., Chaieb, I., & Errunza, V. (2013). Do implicit barriers matter for globalization? *Review of Financial Studies*, *26*(7), 1694-1739.

Cochrane, J. H. (1991). A simple test of consumption insurance. *The Journal of Political Economy*, *99*(5), 957-976.

Cochrane, J. H. (2005). *Asset pricing: Revised edition.* Princeton University Press.

Coeurdacier, N., & Rey, H. (2012). Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, *51*(1), 63–115.

Colacito, R., & Croce, M. M. (2011). Risks for the long run and the real exchange rate. *Journal of Political Economy*, *119*(1), 153-181.

Colacito, R., & Croce, M. M. (2013). International asset pricing with recursive preferences. *The Review of Finance*, *68*(6), 2651-2686.

Colacito, R., Croce, M. M., Gavazzoni, F., & Ready, R. (2018). Currency risk factors in a recursive multicountry economy. *The Journal of Finance*, *73*(6), 2719-2756.

Cole, H., & Obstfeld, M. (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of Monetary Economics*, *28*, 3-24.

Debreu, G. (1959). *Theory of value.* Yale University Press: New Haven and London.

Engel, C. (1999). Accounting for u.s. real exchange rate changes. *Journal of Political Economy*, *107*(3), 507-538.

Epstein, L. G., & Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, *57*(4), 937-969.

Epstein, L. G., & Zin, S. E. (1991). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy*, *99*(2), 263–286.

Farhi, E., & Gabaix, X. (2016). Rare disasters and exchange rates. *The Quarterly Journal of Economics*, *131*(1), 1–52.

Feenstra, R. C., Inklaar, R., & Timmer, M. P. (2015). The next generation of the penn world table. *American economic review*, *105*(10), 3150–82.

Gourio, F., Siemer, M., & Verdelhan, A. (2013). International risk cycles. *Journal of International Economics*, *89*(2), 471–484.

Itskhoki, O., & Mukhin, D. (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, *129*(8), 2183–2232.

Jiang, Z., Krishnamurthy, A., Lustig, H. N., & Jialu, S. (2022). Beyond incomplete spanning: Convenience yields and exchange rate disconnect. *SSRN*, *3852298*.

Kalemli-Ozcan, S., Sørensen, B. E., & Yosha, O. (2003). Risk sharing and industrial specialization: Regional and international evidence. *American Economic Review*, *93*(3), 903-918.

Lewis, K. K., & Liu, E. X. (2015). Evaluating international consumption risk sharing gains: An asset return view. *Journal of Monetary Economics*, *71*, 84 - 98.

Lewis, K. K., & Liu, E. X. (2017). Disaster risk and asset returns: An international perspective. *Journal of International Economics*, *108*, S42-S58.

Lucas, R. E. (1987). Models of business cycles. Oxford: Blackwell.

Lustig, H., Stathopoulos, A., & Verdelhan, A. (2019). The term structure of currency carry trade risk premia. *American Economic Review*, *109*(12), 4142-77.

Lustig, H., & Verdelhan, A. (2019). Does incomplete spanning in international financial markets help to explain exchange rates? *American Economic Review*, *109*(6), 2208-44.

Mace, B. J. (1991). Full insurance in the presence of aggregate uncertainty. *Journal of Political Economy*, *99*(5), 928-956.

Mehra, R., & Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of monetary Economics*, *15*(2), 145–161.

Nagel, S., & Xu, Z. (2022). *Dynamics of subjective risk premia* (Tech. Rep. No. 29803).

Nakamura, E., Steinsson, J., Barro, R., & Ursúa, J. (2013). Crises and recoveries in an empirical model of consumption disasters. *American Economic Journal: Macroeconomics*, *5*(3), 35–74.

Obstfeld, M. (1994). Risk-taking, global diversification, and growth. *The American Economic Review*, *84*(5), 1310–1329.

Obstfeld, M., & Rogoff, K. (2001). The six major puzzles in international macroeconomics: Is there a common cause? In B. Bernanke & K. Rogoff (Eds.), *Nber macroeconomics annual* (Vol. 15, p. 339-390). University of Chicago Press.

Pavlova, A., & Rigobon, R. (2007). Asset prices and exchange rates. *The Review of Financial Studies*, *20*(4), 1139–1180.

Ready, R., Roussanov, N., & Ward, C. (2017). Commodity trade and the carry trade: A tale of two countries. *The Journal of Finance*, *72*(6), 2629–2684.

Rogoff, K., & Obstfeld, M. (1996). *Foundations of international macroeconomics.* Cambridge, Massachusetts: MIT Press.

Sandulescu, M., Trojani, F., & Vedolin, A. (2021). Model-free international stochastic discount factors. *The Journal of Finance*, *76*(2), 935–976.

Stathopoulos, A. (2021). Portfolio home bias and external habit formation. *SSRN Working Paper*.

Tesar, L. L. (1995). Evaluating the gains from international risksharing. In *Carnegie-rochester conference series on public policy* (Vol. 42, pp. 95–143).

Verdelhan, A. (2010). A habit-based explanation of the exchange rate risk premium. *The Journal of Finance*, *65*(1), 123–146.

Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, *68*(3), 987–1035.

Weil, P. (1990). Nonexpected utility in macroeconomics. *The Quarterly Journal of Economics*, *105*(1), 29–42.

Table 1: **Data Moments and Wedges Example**

Panel A: Data Moments

	Consumption Growth Mean	Consumption Growth Std Dev	Consumption Growth Correlation	Exchange Rate Std Dev	Exchange Rate Corr with Consn
US	1.91	1.56	0.574	N/A	N/A
Canada	1.89	1.52	0.574	1.12	-0.18

Panel B: Data η Wedge and Total Complete Markets Wedge for US and Canada

	Data η Wedge	Total Wedge
Non-Tradeable ($\alpha = 0.7$)	31.34	1.76
Home Bias (a = 0.9)	31.34	1.93
Sticky Prices	31.34	2.10

Panel C: Asset Return Data and Model Moments

	Equity Mean	Equity Vol	Risk Free Mean	Risk Free Vol	Equity Premium
(i) Data Moments					
US	8.32	18.62	2.88	1.48	5.44
Canada	7.65	21.22	0.74	2.06	6.91
(ii) SMM Model Moments					
US	7.13	17.99	2.17	0.47	4.96
Canada	8.47	21.12	1.55	0.67	6.92

Notes: Panel A under "Data Moments" gives the sample moments of real consumption growth and the relative price levels for the U.S. and Canada measured in local prices using the expenditure benchmark in Feenstra et al. (2015). Panel B provides the percentage deviation in certainty equivalent units for the i.i.d. case of the η wedge deviation in equation (2) and the total wealth deviation to be described in Section 2. Panel C under "Data Moments" reports the sample moments of real equity returns and the risk free rate measured in local prices with the same deflator as Panel A. "SMM Model Moments" gives the implied Simulated Method of Moments (SMM) counterparts assuming perfectly correlated long run risk consumption growth across countries, as detailed in Appendix A. Estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5.

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Table 2: **Wedge Value Decomposition:** S **Wedge**

Panel A: Wedge Decomposition for I.I.D. Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	$\Delta_{S},*$	Absolute Shares D,η	$S,*$	Levered Returns $E[r^D]$	$E[r^D - r_\eta]$	$E[r^D - r_S]$
Non Tradeable $\alpha = 0.7$	1.76	31.34	3.07	46.7%	4.6%	7.80	-0.32	0.01
Home Bias a = 0.9	1.93	31.34	-0.89	51.6%	1.5%	7.80	-0.32	-0.03
Sticky Price No S-Wedge	2.10	31.34	0.00	51.7%	0.0%	7.80	-0.32	-0.02

Panel B: Wedge Decomposition for Persistent Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	$\Delta_{S,*}$	Absolute Shares $\Delta_{D,\eta}$	$\Delta_{S,*}$	Returns $E[r^D]$	$E[r^D - r_\eta]$	$E[r^D - r_S]$
Non Tradeable $\alpha = 0.7$	0.78	13.41	2.49	43.2%	8.0%	9.74	-0.23	0.03
Sticky Price No S-Wedge	0.87	13.41	0.00	51.7%	0.0%	9.74	-0.23	-0.02

Notes: Panel A under "Wedge Decomposition for IID Consumption" reports certainty equivalent costs (Δ) when consumption growth is IID, while Panel B under "Wedge Decomposition for Persistent Consumption" displays results when consumption growth contains a persistent risk. "Total costs" measures the consumption equivalent loss from consuming the Data-implied domestic wealth (subscript D) instead of the complete markets wealth (subscript *). The columns under "Wedges" report the decomposition of the total cost into components of η and S-Wedge (subscript S) following Equation 27 with $\Delta_{\eta,S}$ excluded as the residual. "Absolute Shares" are the absolute values of the shares of contributions to total costs in equation (27); that is, for general contribution of Δ_i, the share is: $Abs(\Delta_i)/\Sigma_i^3 Abs(\Delta_i)$ where i indexes the Δ_i combinations that sum to $\Delta_{D,*}$. Levered returns are computed as the log of the return defined in Equation 6 multiplied by a leverage parameter of 3, as in Abel (1999). Estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5. The consumption growth parameters are calibrated at the annual mean of 1.7%, annual standard deviation of 1.52%, correlation of consumption growth across countries of 0.574, and correlation of exchange rate growth and consumption growth of -0.18. The correlation of the persistent consumption risk across countries in Panel B is set to 1 following Colacito and Croce (2011).

Table 3: **Wedge Value Decomposition in Local Prices: M-Wedge, R - Wedge**

Exchange	Total Cost	Wedges				Absolute Shares			
Rate View	$\Delta_{D,*}$	$\Delta_{D,M}$	$\Delta_{R,*}$	$\Delta_{D,R}$	$\Delta_{M,*}$	$\Delta_{D,M}$	$\Delta_{R,*}$	$\Delta_{D,R}$	$\Delta_{M,*}$
Non Tradeable $\alpha = 0.7$	1.76	-1.00	-6.31	7.88	2.76	6.2%	39.9%	40.4%	13.9%
Home Bias a = 0.9	1.93	0.94	-9.20	10.73	0.99	4.8%	47.3%	49.9%	4.6%
Sticky Price No S-Wedge	2.10	2.88	-12.21	13.62	-0.79	11.5%	48.4%	54.2%	3.2%

Notes: This table reports certainty equivalent consumption loss or wedges (Δ) from the stochastic discount rate (subscript M) and return to wealth (subscript R) in local prices. "Total costs" measures the consumption equivalent loss from the data economy (subscript D) to the risk sharing economy (subscript *). The columns under "Wedges" report the decomposition of the total cost wedge into components of M and R-Wedges with $\Delta_{M,R}$ excluded as the residual. "Absolute Shares" and parameter estimates are described in Table 2 notes.

Table 4: **Wedge Value Decomposition: Total S-Wedge**

Panel A: Wedge Decomposition for I.I.D. Consumption

Exchange	Total Cost	Wedges: S		Wedges: Total S		Absolute Shares: S		Absolute Shares: Total S	
Rate View	$\Delta_{D,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	Δ_{D,η^T}	$\Delta_{S^T,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	Δ_{D,η^T}	$\Delta_{S^T,*}$
Non Tradeable $\alpha = 0.7$	1.76	31.34	3.07	-0.05	0.17	46.7%	4.6%	2.8%	9.0%
Home Bias a = 0.9	1.93	31.34	-0.89	-0.05	-0.12	51.6%	1.5%	2.3%	5.3%

Panel B: Wedge Decomposition for Persistent Consumption

Exchange	Total Cost	Wedges: S		Wedges: Total S		Absolute Value Shares: S		Absolute Value Shares: Total S	
Rate View	$\Delta_{D,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	Δ_{D,η^T}	$\Delta_{S^T,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	Δ_{D,η^T}	$\Delta_{S^T,*}$
Non Tradeable $\alpha = 0.7$	0.78	13.41	2.49	-0.02	0.07	43.2%	8.0%	2.8%	9.0%

Notes: Panel A under "Wedge Decomposition for IID Consumption" reports certainty equivalent costs (Δ) when consumption growth is IID, while Panel B under "Wedge Decomposition for Persistent Consumption" displays results when consumption growth contains a small and persistent risk. "Total costs" measures the consumption equivalent loss from consuming wealth as inferred in the data (subscript D) instead of as in complete markets (subscript *). The columns under "Wedges" reports the decomposition of the total cost wedge into components of η^T and Total S-Wedge (subscript S^T) based on Equations 30 and 31 with Δ_{η^T,S^T} excluded as the residual. "Absolute Shares" and parameter estimates are described in Table 2 notes.

Table 5: **S-Wedge Value Decomposition: Varying Parameters**

Panel A: Non Tradeable (IID), Varying α

Exchange	Total Cost	Wedges		Absolute Shares	
Rate View	$\Delta_{D,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$
$\alpha = 0.8$	1.99	31.34	4.83	44.6%	6.6%
$\alpha = 0.7$	1.76	31.34	3.07	46.7%	4.6%
$\alpha = 0.6$	1.23	31.34	6.44	42.2%	8.7%

Panel B: Non Tradeable (IID), Varying IES

Exchange	Total Cost	Wedges		Absolute Shares	
Rate View	$\Delta_{D,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$	$\Delta_{D,\eta}$	$\Delta_{S,*}$
IES = 1.5	1.76	31.34	3.07	46.7%	4.6%
IES = 0.5	0.58	-3.36	-0.45	41.0%	5.5%

Notes: The table reports the certainty equivalent percentage difference between permanent consumption that equalizes welfare between the Data-inferred and Complete Markets along with the decompositions for different parameters. Panel A considers variations in the preference parameter α for tradeables given by the utility function in equation (22). Panel A estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5. Panel B results assume $\alpha = 0.7$ and Risk Aversion parameter of 10. All results assume symmetric countries in the data with i.i.d. consumption processes and parameters decribed in Table 2 notes.

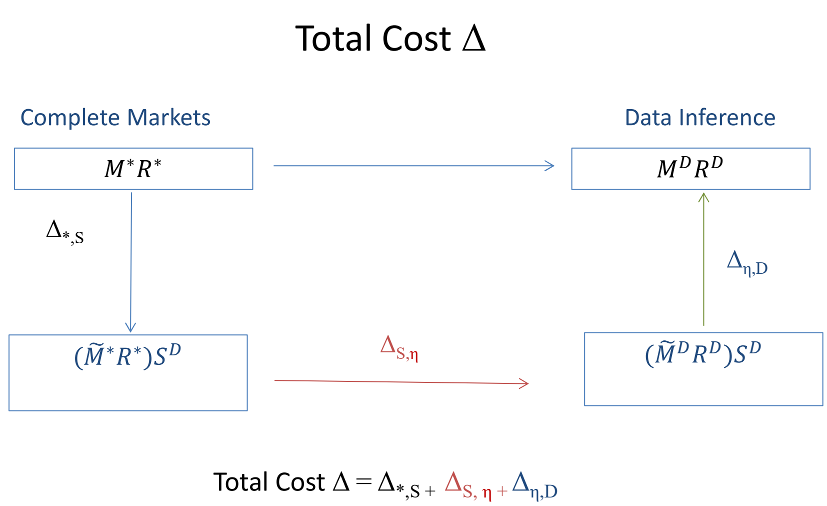
Table 6: **The Total Costs for the No S-Wedge Case: Multiple Countries**

$\Delta_{D,*}$ Allowing Asymmetric Wealth: $(C^*/C^D) \equiv$ "$Weights$"

Version		ConstantWeight Estimates			
A. Without Exchange Rate Volatility	Parameters	US	Can	UK	Aus
CRRA	$\gamma = 2, \psi = 0.5$	0.10	0.14	0.22	0.33
	Weights	1.001	1.000	1.000	0.999
Asset Pricing	$\gamma = 10, \psi = 1.5$	3.53	4.05	5.17	6.54
	Weights	1.003	0.993	1.006	0.998
B. With Exchange Rate Volatility					
CRRA	$\gamma = 2, \psi = 0.5$	0.00	-0.05	-0.01	0.23
	Weights	1.001	1.000	1.000	0.999
Asset Pricing	$\gamma = 10, \psi = 1.5$	2.52	1.28	-1.03	1.54
	Weights	0.991	0.985	1.017	1.007

Notes: This table reports certainty equivalent consumption loss ($\Delta_{D,*}$) for the wealth returns as described in Appendix D.1. Initial wealth values or "Weights" are reallocated so that all costs are non-negative. Parameter estimates are described in Table 2 notes.

Figure 1: Exchange Rate Wedge Decomposition



On-Line Appendices: How Can Asset Prices Value Exchange Rate Wedges?

Karen K. Lewis* Edith X. Liu†

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A Data and Simulated Method of Moments

In this appendix, we describe the data measurement, Simulated Method of Moments (SMM) approach used in Table 1 and the implied consumption processes for the two-country example in Tables 2, 3, 4, and 5. Appendix D.1 describes the data moments used in the SMM extension to multiple countries.

A.1 Data Measurement

The welfare costs measurement requires real consumption data for each country. Feenstra, Inklaar, and Timmer (2015) describe issues related to the comparability of prices and consumption over countries and time, arguing that welfare valuations require measures that allow direct international comparison. We therefore follow their suggested comparison, using consistent real expenditure consumption series and prices in the Penn World Tables (PWT) Version 9.1 released in 2021. This version provides data on consumption and local prices as well as prices against a US numeraire through 2017.[1] The data series measure the relative price of goods indices in non-US countries relative to the value of the US con-

*University of Pennsylvania, NBER, and CEPR
†Federal Reserve Board of Governors
[1]Alternative data sources such as the PWT output data and price are described in Appendix D.2.

sumption goods index measured in US dollars at the base year. The consumption for each country j is measured in local goods prices, denoted here as $\hat{P}^j$. However, for international comparison, the relative prices differ due to exchange rate variations. Then, for international comparison, we use real domestic consumption measured in units of the U.S. as numeraire. That is: $\widetilde{C}^{j,D}_{t+1} = S^j_{t+1} C^j_{t+1}$ where $S^j_{t+1} = e^j_t \hat{P}^j_t / P^J_t$ where J is the US (numeraire country), and e^j_t converts the nominal value of country j into US dollars. All prices and exchange rates are measured relative to the index in the base year. In the text, we use direct measures of real consumption in local prices, C^j_t, and real exchange rates across countries involving international prices. Therefore, we subsume the nominal exchange rate in the definition of the real exchange rate measure used in the international price numbers. That is, $S^j_{t+1} = e^j_t \hat{P}^j_t / P^J_t = P^j_t / P^J_t$ where P^j measures the prices level of country j at numeraire prices. Moreover, in our two country example, the total number of countries is 2 so that we ignore j for parsimony until we consider the multi-country extension in Section 4 and Appendix D.1.

Our analysis also requires data for asset returns and dividends. For risk-free rates, we use the annual average 90 day government bill rate from the OECD. For equity returns and dividends, quarterly data from the Total Market Indices in Datastream-Thomson Financial are targeted. Nominal equity returns are computed from 1970 to 2018, while dividend growth data moments are computed up to 2009. To match the annual level consumption data, we deflate nominal equity returns and dividend growth with the same consistent price deflators from the Penn World Tables. These real annual equity returns, risk-free rates, and dividend growth rates provide asset return moments to be matched by the simulated method of moments described below.

A.2 Simulated Method of Moments Approach

To solve for the consumption process parameters used in Table 1, we fit key target annual moments from a reduced SMM analysis described next.

A.2.1 Consumption Processes

We begin by defining the consumption growth process measured in the data as: $g_{c,t+1} \equiv \ln(C^D_{t+1}/C^D_t)$ for the domestic country, and $g_{\widetilde{c},t+1} \equiv \ln(\widetilde{C}^D_{t+1}/\widetilde{C}^D_t)$ for the foreign country. For the two country example, these countries follow symmetric processes and therefore we describe only the process for the domestic country for parsimony.

For the i.i.d. consumption version, this consumption process is given by:

$$g_{c,t+1} = \mu + \sigma\nu_{t+1} \tag{A.1}$$

where $\nu_{t+1} \backsim N(0,1)$ and where the correlation between the two countries is given by $Corr(\nu_{t+1}, \widetilde{\nu}_{t+1})$.

Alternatively, for the persistent consumption version we allow for an autoregressive "long run risk" processes as in:

$$\begin{aligned} g_{c,t+1} &= \mu + x_t + \sigma\nu_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1} \end{aligned} \tag{A.2}$$

where $\nu_{t+1}, e_{t+1} \backsim N(0,1)$ and where ν_{t+1}, e_{t+1} are mutually independent. To fit the asset return series in the simulation, we substitute these candidate consumption processes into the Euler equation in order to value the returns. We choose these processes due to their simplicity and common usage only. For example, as described in Section 4, other candidate processes can be used such as disaster risk and habit persistence.

A.2.2 Asset Returns and Consumption

In order to match the asset return moments to consumption as reported in Table 1, we require the asset return solutions for equity and for the risk-free rate. For the i.i.d. case, these solutions are provided in classic references such as Mehra and Prescott (1985), while the asset return solutions for the consumption process in equation (A.2) are detailed in Bansal and Yaron (2004). Below we detail the solutions for the persistent consumption process in equation (A.2) only since the corresponding solution for the i.i.d. version in equation (A.1) is nested by assuming the persistent consumption variance term, $\varphi_e = 0$.

The stochastic discount factor (SDF) is the underlying variable that values asset returns. Under Constant Relative Risk Aversion preferences, this SDF can be measured directly with consumption growth as: $(C_{t+1}/C_t)^{-\gamma}$. However, more generally, with recursive preferences, this SDF is determined by the Euler equation that values the claim on lifetime consumption; i.e. the return on wealth. The return on this process as given by consumption in the data as:

$$R^D_{c,t+1} \equiv \left(\frac{C^D_{t+1}}{C^D_t}\right)\left(\frac{1+Z^D_{t+1}}{Z^D_t}\right) \tag{A.3}$$

where Z^D_t is the ratio of the time t value of an asset paying out the consumption process divided by current consumption. The Campbell-Shiller approximation then implies that the natural logarithm of this return, $r^D_c \equiv \ln R^D_c$ is a function of the logarithm of the value of wealth-to-consumption ratio, $z^D_t \equiv \ln(Z^D_t)$, and of consumption growth, $g^D_{c,t+1}$, specified by:

$$r^D_{c,t+1} = k_0 + k_1 z^D_{t+1} - z^D_t + g^D_{c,t+1} \tag{A.4}$$

where k_0 and k_1 represent approximating constants.[2]

The SDF determined by this return on wealth then provides a valuation of equity as a dividend-paying asset. Defining the price-to-dividend ratio as: $z_{m,t} \equiv \ln(P_t/D_t)$, and dividend growth as $g_{d,t+1} \equiv \ln(D_{t+1}/D_t)$, the log equity returns follow:

$$r^D_{m,t+1} = k_{m,0} + k_{m,1} z^D_{m,t+1} - z^D_{m,t} + g^D_{d,t+1} \tag{A.5}$$

where $k_{m,0}$ and $k_{m,1}$ are approximating constants based upon the Campbell-Shiller approximation. For this process, we specify the process as:

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma \nu_{t+1} \tag{A.6}$$

The risk-free rate, $R^D_{f,t}$, is the return on an asset purchased at time t that pays one unit of domestic consumption with certainty at $t+1$. To value this return, we again follow the

[2]The constants are $k_1 = \frac{exp(\bar{z})}{1+exp(\bar{z})}$ and $k_0 = log(1+exp(\bar{z})) - k_1\bar{z}$, where $\bar{z}$ is the steady state log price to consumption ratio.

literature to apply the Euler equation of the domestic agent to value this risk-free rate.[3] Since the risk-free rate is known at time t, the Euler equation can be written as:

$$E_t \left\{ M^D_{t+1} \right\} R_{f,t} = 1 \tag{A.7}$$

So far, these asset returns are given as general solutions that do not rely on preferences. However, to conduct SMM on these returns, we require a view on preferences. Epstein-Zin utility provides a convenient benchmark for these purposes since it maps directly to many of the approaches in the literature cited in the text. Under these preferences, the SDF measured with data is:

$$M^D_{t+1} \equiv \beta^\theta (C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)} (R^D_{c,t+1})^{(\theta-1)} \tag{A.8}$$

where $\theta = (1-\gamma)/(1-\psi^{-1})$ and where R^D_c is determined by the Z^D_t in equation (A.3) that solves for the value of the wealth return:

$$E_t \left\{ \beta^\theta (C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)} (R^D_{c,t+1})^\theta \right\} = 1 \tag{A.9}$$

Therefore, in our analysis below, we will require measures of only three preference parameters: intertemporal elasticity of substitution (IES) given by ψ, the relative risk aversion coefficient γ, and the time discount factor β. Throughout the paper, we assume these parameters are the same across countries.

A.2.3 Simulations Approach

Matching the consumption processes to asset returns requires fitting the consumption payout process in the wealth return equation (A.4) jointly with the asset return equations (A.5) and (A.7). We begin with preference estimates that have been found to fit asset returns best in the US data from Bansal and Yaron (2004). Specifically, we take the parameters of $\psi = 1.5$, $\gamma = 10$, and annualized $\beta = .985$ or monthly $\beta = .998$. The difference in β depends upon

[3] As described in the text, if markets are incomplete then at most one investor's Euler equation will be marginal in the data. Here we follow the standard approach in the consumption-based asset pricing literature that the domestic agent is the marginal investor for domestic assets. For an alternative assumption that assumes foreigners price the domestic risk-free rate, see Lustig and Verdelhan (2019).

whether we simulate the model at the annual level or simulate at the monthly level and then time-aggregate to annual moments. For our example in Table 1, we use monthly data to provide the best fit, although throughout the rest of the paper we alternatively use an annual model following Colacito and Croce (2013).

To generate the parameter values, we first calibrate the monthly growth rates μ and μ_d to the annual means of consumption growth and dividend growth. To simplify, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, this calibration assumption makes little difference in the estimates of the remaining parameters but greatly decreases the computation time.

We then use the reduced SMM to fit the remaining parameters for each country:$[\sigma, \varphi_e, \rho, \phi, \varphi_d]$ to match dividend behavior for the equity return in equation (A.5). Implementing the SMM procedure involves two main steps. First, for every set of parameter values, solve the model using the analytical solutions for returns using a set of targeted moments to best represent both consumption and asset pricing data.[4] Second, compute a weighted difference between the targeted set of model-generated moments and the data moments using a weighting matrix. To treat all targets equally, we report the estimates using the identity matrix.[5] The set of parameter values that minimizes this difference is the SMM estimate.

For the time aggregated case, we compute the growth rate between the levels at t and $t+12$, given the realizations of 12 monthly growth rates.[6] To match our annual consumption, dividend growth and asset return moments, we then time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

For the asset return model, we choose the following set of target data moments for each country as reported in Table 1: the standard deviation of log consumption growth, the first order auto-correlation of log consumption growth, the mean equity premium, the mean risk-free rate, the standard deviation of the market return, and the standard deviation of the risk-free rate. Using these six moments per country, we estimate the three consumption growth

[4]See Gallant and Tauchen (1999) for a discussion on efficient method of moments and problems with moment selection.

[5]We also implemented the reduced SMM procedure using a diagonal matrix with typical components equal to the sample variance. This procedure gave qualitatively similar results.

[6]By comparison, we multiply monthly rates times 12 when we annualize as opposed to time aggregate.

parameters capturing the transitory risk, σ, persistent risk, φ_e, and degree of persistence, ρ. As a practical matter, we find that fitted values of ρ are quite similar across countries so we equate them in the analysis.

To accommodate dividend growth in the model, we augment the six moments basic moments described above to include the standard deviation of dividend growth, and the first order auto-correlation of log dividend growth. We therefore include the standard deviation of real dividend growth of 13.01% and 13.6% for Canada and the United States respectively, as well as the first order auto-correlation of log dividend growth of 0.45 for Canada and 0.08 for the United States. Using these eight moments per country, we fit the same three consumption parameters along with the two new parameters, the relative volatility of monthly dividend growth φ_d, and the consumption leverage parameter, ϕ. In fitting these two parameters, we impose bounds from the literature. Specifically, Abel (1999) suggests the leverage parameter can be guided by the ratio of the standard deviation of dividend growth over that of two variables: income growth and consumption growth. These measures calibrated in Abel (1999) using U.S. data suggested a range of $\phi \in \{2.5, 3.6\}$. For comparison, we also consider a version of the model restricting $\phi = 3$ for all countries. Similarly, we impose a range on $\varphi_d \in \{1, 6\}$ allowing for significantly higher volatility in dividends than consumption, which includes the calibrated value of $\varphi_d = 4.5$ in Bansal and Yaron (2004).

Table 1C shows the results of this fit. The mean estimated parameters from the Simulated Method of Moments are: $[\rho = 0.988, \sigma = 0.0074, \phi = 3.4, \phi_e = 0.0387, \phi_d = 5.8]$ for Canada and $[\rho = 0.986, \sigma = 0.0068, \phi = 2.9, \phi_e = 0.0448, \phi_d = 5.95]$ for the Untied States. Both sets of parameters are fairly consistent with previous estimates and calibration in the literature. While the variability of asset returns are generally lower than in the data, the match can be improved by introducing time-varying volatility as in Wachter (2013) and Bansal and Yaron (2004). We instead maintain this more parsimonious model to illustrate how asset returns can be used to discipline the valuation of wedges.

A.3 Cross Country Moments

The SMM approach described above disciplines the consumption process within countries based upon a common assumption in the literature that domestic investors price their own assets. Notably, we did not target any of the cross-country moments. However, valuing the cross-country valuations requires assumptions about the international correlations of consumption and exchange rates.

For the i.i.d consumption growth examples in Tables 1 to 4, the cross-country correlations are given by $Corr(\nu_{t+1}, \widetilde{\nu}_{t+1})$ as specified following equation (A.1). We calibrate these moments with the simple correlations of consumption growths across countries. Thus, for the US and Canada in Table 1A, this correlation is calibrated in the observed data to be 0.574. To consider how exchange rate variability impacts the value of foreign wealth returns, we also require a measure of the correlation between consumption and exchange rates in the data. Assuming the exchange rate growth is also i.i.d., this growth follows a process:

$$g_{s,t+1} = \ln(S^D_{t+1}/S^D_t) = \sigma_s \nu^s_{t+1} \tag{A.10}$$

where $\nu^s_t \backsim N(0,1)$ and where the correlation between domestic consumption growth and the exchange rate is given by $Corr(\nu_{t+1}, \nu^s_{t+1})$. We calibrate this value to the measure in the data of -0.18. Note that, as pointed out by Backus and Smith (1993), the consumption and exchange rate growth rates are relatively uncorrelated.

The case of persistent consumption growth in equation (A.2) additionally requires an assumption about the correlation of the persistent "long run risk" component, e_t. Colacito and Croce (2011) show that a correlation near one matches exchange rates under complete markets. Furthermore, Lewis and Liu (2015) show that a high correlation across countries is required to simultaneously explain the high correlations of asset returns but more modest correlation of consumption. They also show that the gains from international diversification are minimized when the persistent risk correlation is high. Therefore, to provide a conservative lower bound on the costs of wedges in our persistent risk examples, we assume that $Corr(e_t, \widetilde{e}_t) = 1$ throughout the text. Appendix D.2 describes more general versions of the exchange rate process in which this correlation may be more important.

In addition, while we do not target the variability of the foreign exchange risk premium, this return is implied by the returns. For example, the return on borrowing at the domestic rate, investing in the foreign risk free rate and converting back into domestic consumption units at the end of the period is:

$$g_{fx,t+1} = \ln(S_{t+1}^D / S_t^D) + \widetilde{r}_{f,t} - r_{f,t} \tag{A.11}$$

This process therefore includes both the volatility in the exchange rates and the risk-free rates implied by our SMM results above.

B Wedge Valuation Details

This appendix summarizes the wedge valuations.

B.1 Calculating the Value of the η Wedge

As discussed in the text, valuing the η Wedge requires calculating the price ratio for the foreign investor valuation of the domestic wealth return payouts as inferred by consumption data. To illustrate, we take the Euler Equation of the foreign investor for this identification.[7] In this case, the valuation to the foreign investor of the domestic wealth return is given by:

$$E_t \left\{ \widetilde{M}_{t+1}^D R_{\eta,t+1}^D \left(S_{t+1}^D / S_t^D \right)^{-1} \right\} = 1 \tag{B.1}$$

where S_{t+1}^D / S_t^D is the change in exchange rate that converts the domestic consumption payouts into foreign goods and $R_{\eta,t+1}$ is the return on a claim that domestic consumption, valued by the foreign investor. That is, the return is:

$$R_{\eta,t+1}^D = (C_{t+1}^D / C_t^D)(1 + Z_{t+1}^\eta)/Z_t^\eta. \tag{B.2}$$

where Z_t^η is the price that solves the Euler equation (B.1) given the domestic consumption process. Substituting the stochastic discount factor, $\widetilde{M}_t^D$, implied by Epstein-Zin-Weil

[7] A symmetric relationship holds for the domestic investor valuation of claimes on lifetime foreign consumption.

preferences given in equation (A.8) into equation (B.1) gives:

$$E_t \left\{ \beta^{\theta} (\widetilde{C}^D_{t+1}/\widetilde{C}^D_t)^{(-\frac{\theta}{\psi})} (\widetilde{R}^D_{c,t+1})^{\theta-1} R^D_{\eta,t+1} (S^D_{t+1}/S^D_t)^{-1} \right\} = 1. \tag{B.3}$$

Note that the return on domestic wealth to the foreign investor, R_η differs from the return to the domestic investor, R_c, due to the difference in price ratio of Z^η in equation (B.2) versus Z^D. Nevertheless, since they both are claims on domestic consumption, for expositional simplicity we denote both measures of consumption assets as R_c in the text with some abuse of notation.

To solve this Euler equation, we assume joint-log normality following much of the literature, as well as the SMM identification in Appendix A. Then, taking logs of both sides of equation (B.3), and using the notation $g_{b,t}$ to refer to the time t log growth rate of any variable b implies:

$$E_t[\theta \ln \beta - \frac{\theta}{\psi} \tilde{g}^D_{c,t+1} + (\theta-1)\tilde{r}^D_{c,t+1} + r^D_{c,t+1} - g^D_{s,t+1}] + \frac{1}{2} Var[\theta \ln \beta - \frac{\theta}{\psi} \tilde{g}^D_{c,t+1} + (\theta-1)\tilde{r}^D_{c,t+1} + r^D_{c,t+1} - g^D_{s,t+1}] = 0 \tag{B.4}$$

We consider two examples of consumption growth: (1) i.i.d.; and (2) persistent long run risk.

(1) i.i.d consumption For the i.i.d. symmetric consumption case, each country has identical consumption processes given by equation (A.1). This assumption treats the processes of each country with their own random walk component. Alternatively, these processes could include a cointegrating error-correction term as in Colacito and Croce (2013). We discuss this possibility in Appendix D.2.

Furthermore, we require measurement of prices in the data in order to value relative consumption expenditures. For this purpose, we assume that the exchange rate in the data also follows the random walk process as in equation (A.10), although Appendix D also describes alternative exchange rate processes that allow for long term mean reversion in the real exchange rate. To illustrate the random walk base case, however, substituting equations (A.1) and (A.10) into equation (B.4) and solving for Z^η gives the wealth price for this investor.

To compare the value of the same lifetime consumption process to the domestic investor, we follow similar steps but use the domestic wealth return Euler equation.

$$E_t \left\{ M^D_{t+1} R^D_{c,t+1} \right\} \quad = \quad 1 \tag{B.5}$$

Solving this equation for Z^D provides the comparison to calculate the η wedge in the text, repeated here.

$$(1 - \Delta_{D,\eta}) = \frac{V(W^D/C^D)}{V(W^\eta/C^\eta)} = \left\{ \frac{1 + Z^D}{1 + Z^\eta} \right\}^\Psi \tag{B.6}$$

(2) With persistent risk: For persistent consumption, we repeat these steps but assume instead that the consumption process includes a persistent long run risk component given by equation (A.2). To value the wealth series, we therefore substitute this process for $g^D_{c,t+1}$ into the Euler equations (B.4) as well as the log counterpart to equation (B.5). In this persistent consumption case, the price ratios, Z^D and Z^η, are time varying.. Therefore, we follow the same approach as described in the SMM analysis of Appendix A. That is, we assume $z_{\eta,t} = \bar{A}_{\eta,0} + \bar{A}_{\eta,1} x_t + \bar{A}_{\eta,2} \tilde{x}_t$ and use the Campbell-Shiller approximation to solve returns as in equation (A.4). This assumption applied to the two Euler equations provides the measure of Z^D and Z^η needed to calculate the certainty equivalent measure $\Delta_{D,\eta}$ as above.

B.2 Calculating the Value of the Total ζ Wedge

In order to calculate the value of the total deviation from complete markets, $\Delta_{D,*}$, we follow similar steps to solve for the Euler equations as above. First, we use the solution for the price ratio Z^D as described in B.1, using Euler equation (B.5). Second, we require the solution to the complete markets consumption process depending on the view of the exchange rate determination described in Appendix C. Using this solution, we follow the same steps to solve for Z^* for returns in the Euler equation given by:

$$E_t \left\{ \beta^\theta (C^*_{t+1}/C^*_t)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^\theta \right\} = 1. \tag{B.7}$$

The wealth ratio Z^* that solves for this equation provides the comparison for calculating the total cost $\Delta_{D,*}$ in the text, repeated here.

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi}$$

Appendix C below describes the solutions to these complete markets price ratios, Z^*, with each depending upon exchange rate views.

B.3 Calculating the Value of the Total S Wedge

We next describe the relationship between wealth returns under complete markets as well as its implied "Total" exchange rate wedge.

B.3.1 Complete Markets Wealth Returns Relationship

To see the connection between wealth returns under complete markets, note that these returns can be rewritten as a sharing rule for aggregate world resources in numeraire prices, denoted Y_t^w. In this case, the complete markets wealth return for the domestic economy can be written:

$$\begin{aligned} R^*_{c,t+1} &= \left(\frac{C^*_{t+1}}{C^*_t} \right) \left(\frac{1 + Z^*_{t+1}}{Z^*_t} \right) \\ &= \left(\frac{\omega_{t+1} Y^w_{t+1}}{\omega_t Y^w_t} \right) \left(\frac{S^*_{t+1}}{S^*_t} \right)^{-1} \left(\frac{1 + Z^*_{t+1}}{Z^*_t} \right) \end{aligned} \tag{B.8}$$

where, as in the text, ω is the share of world resources consumed by the domestic agent, S^* is the exchange rate, and Z^* is the wealth price-to-consumption ratio, all implied by the complete markets allocation. Similarly, for the foreign numeraire country, the wealth return is:

$$\begin{aligned} \widetilde{R}^*_{c,t+1} &= \left(\frac{\widetilde{C}^*_{t+1}}{\widetilde{C}^*_t} \right) \left(\frac{1 + \widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t} \right) \\ &= \left(\frac{\widetilde{\omega}_{t+1} Y^w_{t+1}}{\widetilde{\omega}_t Y^w_t} \right) \left(\frac{1 + \widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t} \right) \end{aligned} \tag{B.9}$$

where $\widetilde{\omega}$ is the share of resources consumed by the foreign household.

Next, combine this relationship with the fact that price-adjusted stochastic discount factors are equalized as given in equation (1) in the text, repeated here:

$$M^*_{t+1}\left(S^*_{t+1}/S^*_t\right) = \widetilde{M}^*_{t+1} \ . \tag{B.10}$$

Recall that the stochastic discount factor for Epstein-Zin preferences is:

$$\beta^\theta (C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)} (R_{c,t+1})^{(\theta-1)} \ . \tag{B.11}$$

Then substituting this expression into equation (B.10) implies:

$$\beta^\theta (C^*_{t+1}/C^*_t)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^{(\theta-1)} \left(S^*_{t+1}/S^*_t\right) = \beta^\theta (\widetilde{C}^*_{t+1}/\widetilde{C}^*_t)^{\left(-\frac{\theta}{\psi}\right)} (\widetilde{R}^*_{c,t+1})^{(\theta-1)} \tag{B.12}$$

or, using the risk-sharing share of global resources yields:

$$\beta^\theta \left(\frac{\omega_{t+1} Y^w_{t+1}}{\omega_t Y^w_t}\right)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^{(\theta-1)} \left(\frac{S^*_{t+1}}{S^*_t}\right) = \beta^\theta \left(\frac{\widetilde{\omega}_{t+1} Y^w_{t+1}}{\widetilde{\omega}_t Y^w_t}\right)^{\left(-\frac{\theta}{\psi}\right)} (\widetilde{R}^*_{c,t+1})^{(\theta-1)} \tag{B.13}$$

Then, substituting the domestic and foreign wealth returns in equations (B.8) and (B.9) into equation (B.13), and using the fact that $\omega_t + \widetilde{\omega}_t = 1$ implies:

$$\frac{\widetilde{\omega}_{t+1}}{\widetilde{\omega}_t} = \left[1 + \left(\frac{S^*_{t+1}}{S^*_t}\right)^{1-\frac{1}{\gamma}} \left(\frac{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}{(1+Z^*_{t+1})/Z^*_t}\right)^{\left(\frac{1-\theta}{\gamma}\right)}\right] \tag{B.14}$$

Substituting these solutions for the growth rate in shares implies:

$$M^*_{t+1} R^*_{c,t+1} = \widetilde{M}^*_{t+1} \widetilde{R}^*_{c,t+1} \left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})} \left(\frac{(1+Z_{t+1})/Z_t}{(1+\widetilde{Z}_{t+1})/\widetilde{Z}_t}\right)^{\frac{1}{\gamma}\frac{\theta}{\psi}} \tag{B.15}$$

When the consumption process is i.i.d., then $Z_t = Z$, a constant for all t. Thus, this relationship becomes:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})}\left(\frac{(1+Z)/Z}{(1+\widetilde{Z})/\widetilde{Z}}\right)^{\frac{1}{\gamma}\frac{\theta}{\psi}} \tag{B.16}$$

Furthermore, when the countries are symmetric in local price units, then $Z = \widetilde{Z}$ and therefore,

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})} \tag{B.17}$$

B.3.2 Total S-Wedge Decomposition

We can then write the "Total" S-Wedge as the wedge deviation between the state price of wealth returns under complete markets, distorted only by the exchange rate in the data as:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})}\left(\zeta_{s,t+1}\right)^{-(1-\frac{1}{\gamma})} \tag{B.18}$$

Relating this relationship to all of the data-inferred measures of wealth implies:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^D_{t+1}\widetilde{R}^D_{c,t+1}\left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})}\left(\eta^T_{t+1}\right) \tag{B.19}$$

Thus, for symmetric countries, this η^T wedge measures the difference in valuations between complete markets and wealth in the data recognizing that countries value their lifetime consumption with their own stochastic discount factors, but differ in exchange rate valuations only.

B.4 Summary of Wedge Decompositions

This approach can be used more generally to value other distortions away from complete markets.

B.4.1 General Wedges Decomposed

As described in the discussion above, we can value various deviations from complete markets using this framework. For example, we can decompose the components of the ζ wedge in local price units into proportional "wedges" through the following identities:

$$
\begin{aligned}
M_{t+1}^{D} &\equiv \zeta_{M,t+1} M_{t+1}^{*};\ \widetilde{M}_{t+1}^{D} \equiv \zeta_{\widetilde{M},t+1} \widetilde{M}_{t+1}^{*} \\
\left(\frac{S_{t+1}^{D}}{S_{t}^{D}}\right) &\equiv \zeta_{S,t+1} \left(\frac{S_{t+1}^{*}}{S_{t}^{*}}\right) \\
R_{a,t+1}^{D} &\equiv \zeta_{R_a,t+1} R_{a,t+1}^{*};\ \widetilde{R}_{a,t+1}^{D} \equiv \widetilde{\zeta}_{R_a,t+1} \widetilde{R}_{a,t+1}^{*}
\end{aligned}
\tag{B.20}
$$

Since these values are unique only for the complete markets components, they provide an alternative, decomposed view of the total cost ζ according to:

$$
M_{t+1}^{*} R_{c,t+1}^{*} = M_{t+1}^{D} R_{c,t+1}^{D} \left(\zeta_{M,t+1} \zeta_{R,t+1}\right)^{-1} \tag{B.21}
$$

where, as given in equation (B.20), $\zeta_{M,t+1}$ and $\zeta_{R,t+1}$ are the wedges between complete and incomplete markets for, respectively, the stochastic discount factors and the consumption asset both in domestic goods units.

B.4.2 Decomposed Wedge Valuations

In order to illustrate how each of these values are determined, consider the effect of the wedge on stochastic discount factors from $\zeta_{M,t+1}$ to a complete markets investor. Applying this wedge to equation (B.21) implies that the state price becomes: $M_{t+1}^{*} R_{c,t+1}^{*} \zeta_{M,t+1} = M_{t+1}^{D} R_{c,t+1}^{*}$. Thus, the value of this wedge can be measured in certainty equivalent units by calculating the price-to-consumption ratio Z_M that solves the Euler equation:

$$
E_t \left\{ M_{t+1}^{D} R_{c,t+1}^{*} \right\} = 1. \tag{B.22}
$$

for $R_{M,t+1}^{*} \equiv (C_{t+1}^{*}/C_{t}^{*})(1 + Z_{t+1}^{M})/Z_{t}^{M}$. That is, this counterfactual return is the value of a perpetual claim on domestic consumption under complete markets but valued by a domestic agent with an SDF distorted by the M-Wedge. This value can be solved using the Euler equation that considers the data-implied investor's valuation of the complete markets consumption pay-off as in:

$$E_t \left\{ \beta^{\theta} (C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^{\theta-1} R^*_{c,t+1} \right\} = 1 \tag{B.23}$$

More generally, Table B1 summarizes the different wedges and Euler equations used to identify their wealth price ratios, Z. Given these price ratios, the implied welfare deviation from complete markets can be measured as the certainty equivalent consumption difference. Panel A sets up the structure by noting that when markets are complete, the wedge is zero. By contrast, Panel B shows the comparison of the wedges when the domestic investor is valuing wealth. Under the "Data counterpart" columns in the far left hand columns, the "Wedges" can be either the product of the M and of the R Wedges, implying the overall cost of ζ as given in equation (B.21). The successive rows show the effects of partial wedges applying only to M and to R, respectively. The next two columns under "General Pricing Relationships" provide the state price that is solved in the Euler equation to deliver a price, Z. For example, valuing the M-Wedge requires using the "data" stochastic discount factor, M^D, to value the wealth return under complete markets, R^*, as described in equation (B.23). The final two columns give the Certainty Equivalent Cost measures as compared by the price ratios, Z. Panel C lists the same information when the foreign investor is valuing the domestic wealth return, while Panel D shows the same structure for the Total S-Wedge.

Table B1: **Wedge Cost Valuation Summary**

<table>
<tr><td colspan="6">A. Complete Markets State Price Benchmark</td></tr>
<tr><td colspan="2">Wedge</td><td colspan="2">General Pricing Relationships</td><td colspan="2">CE Cost relative to:</td></tr>
<tr><td colspan="2"></td><td>Data State Price</td><td>Price Ratio</td><td>Complete Mkt</td><td>Data</td></tr>
<tr><td colspan="2">0</td><td>$M^*_t R^*_{c,t}$</td><td>Z^*</td><td>0</td><td>$\Delta_{D,*}$</td></tr>
<tr><td colspan="6">B. Variables for Cost Decomposition in Local Prices</td></tr>
<tr><td colspan="2">Data counterpart</td><td colspan="4">$M^*_t R^*_{c,t} = M^D_t R^D_{c,t} \left(\zeta_{M,t}\zeta_{R,t}\right)^{-1}.$</td></tr>
<tr><td>Wedge</td><td>Notation</td><td colspan="2">General Pricing Relationships</td><td colspan="2">CE Cost relative to:</td></tr>
<tr><td></td><td></td><td>Data State Price</td><td>Price Ratio</td><td>Risk-Share</td><td>Data</td></tr>
<tr><td>$Total$</td><td>$\zeta_{M,t}\zeta_{R,t}$</td><td>$M^D_t R^D_{c,t}$</td><td>Z^D</td><td>$\Delta_{D,*}$</td><td>0</td></tr>
<tr><td>M</td><td>$\zeta_{M,t}$</td><td>$M^D_t R^*_{c,t}$</td><td>Z_M</td><td>$\Delta_{M,*}$</td><td>$\Delta_{D,M}$</td></tr>
<tr><td>R</td><td>$\zeta_{R,t}$</td><td>$M^*_t R^D_{c,t}$</td><td>Z_{R_c}</td><td>$\Delta_{R,*}$</td><td>$\Delta_{D,R}$</td></tr>
<tr><td colspan="6">C. Variables for Cost Decomposition relative to Foreign Prices</td></tr>
<tr><td colspan="2">Data counterpart</td><td colspan="4">$M^*_t R^*_{c,t} = \widetilde{M}^D_t R^D_{c,t} \left(\frac{S^D_t}{S^D_{t-1}}\right)^{-1} \left(\zeta_{\widetilde{M},t}\zeta_{R,t}\right)^{-1} (\zeta_{s,t})$</td></tr>
<tr><td>Wedge</td><td>Notation</td><td colspan="2">General Pricing Relationships</td><td colspan="2">CE Cost relative to:</td></tr>
<tr><td></td><td></td><td>Data State Price</td><td>Price Ratio</td><td>Risk-Share</td><td>Data</td></tr>
<tr><td>η</td><td>η_t</td><td>$\widetilde{M}^D_t R^D_{c,t}(\frac{S^D_t}{S^D_{t-1}})^{-1}$</td><td>$Z^\eta$</td><td>$\Delta_{\eta,*}$</td><td>$\Delta_{D,\eta}$</td></tr>
<tr><td>S</td><td>$\zeta_{s,t}$</td><td>$\widetilde{M}^*_t R^*_{c,t}(\frac{S^D_t}{S^D_{t-1}})^{-1}$</td><td>$Z_S$</td><td>$\Delta_{S,*}$</td><td>$\Delta_{D,S}$</td></tr>
<tr><td colspan="6">D. Variables for Cost Decomposition relative to Foreign Returns (iid only)</td></tr>
<tr><td colspan="2">Data counterpart</td><td colspan="4">$M^*_t R^*_{c,t} = \widetilde{M}^D_t \widetilde{R}^D_{c,t} \left(\frac{S^D_t}{S^D_{t-1}}\right)^{-(1-\frac{1}{\gamma})} (\zeta_{s,t})^{(1-\frac{1}{\gamma})} \left(\zeta_{\widetilde{M},t}\zeta_{\widetilde{R},t}\right)^{-1}$</td></tr>
<tr><td>Wedge</td><td>Notation</td><td colspan="2">General Pricing Relationships</td><td colspan="2">CE Cost relative to:</td></tr>
<tr><td></td><td></td><td>Data State Price</td><td>Price Ratio</td><td>Risk-Share</td><td>Data</td></tr>
<tr><td>η^T</td><td>η^T_t</td><td>$\widetilde{M}^D_t \widetilde{R}^D_{c,t}(\frac{S^D_t}{S^D_{t-1}})^{-(1-\frac{1}{\gamma})}$</td><td>$Z^{\eta^T}$</td><td>$\Delta_{\eta^T,*}$</td><td>$\Delta_{D,\eta^T}$</td></tr>
<tr><td>S^T</td><td>$\zeta_{s,t}^{(1-\frac{1}{\gamma})}$</td><td>$\widetilde{M}^*_t \widetilde{R}^*_{c,t}(\frac{S^D_t}{S^D_{t-1}})^{-(1-\frac{1}{\gamma})}$</td><td>$Z_{S^T}$</td><td>$\Delta_{S^T,*}$</td><td>$\Delta_{D,S^T}$</td></tr>
<tr><td colspan="6">Notes:Recursive Value function determined by $V(W) = (1+Z)^\Psi$.</td></tr>
</table>

C Complete Markets Allocations and Data Mapping

In this appendix, we highlight the solutions for the complete markets allocations based upon some standard views of exchange rate determination. These views provide examples of the applications of the framework in the text and as such are only meant to illustrate the general approach. Other applications are described briefly in Section 4 and in Appendix D.2.

C.1 General Approach

Before providing the specific examples, it is useful to describe the general approach applied to each exchange rate version. This approach follows two steps: (a) Complete Markets allocations given theoretical resources; and (b) Mapping of resources implied by the data. We therefore describe these two general steps before considering the specific examples.

C.1.1 Complete markets allocations given resources:

The general problem can be viewed as the problem of a utilitarian social planner allocating resources between consumers in two countries. Defining $Q_{t+\tau}$ and $\widetilde{Q}_{t+\tau}$ as the endogenous state price discount rates for the domestic and foreign country, respectively, between t and $t+\tau$ and Y_t and $\widetilde{Y}_t$ as the consumption-weighted index of their respective resources per period, the problem is:

$$\underset{\{C_t,\widetilde{C}_t\}}{Max}\left(U(C_t,U_{t+1})+U(\widetilde{C}_t,\widetilde{U}_{t+1})\right) \tag{C.1}$$

for

$$U(C_t,U_{t+1})=\left\{C_t^{\frac{1-\gamma}{\theta}}+\beta E_t\left[(U_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}};\ U(\widetilde{C}_t,\widetilde{U}_{t+1})=\left\{\widetilde{C}_t^{\frac{1-\gamma}{\theta}}+\beta E_t\left[\left(\widetilde{U}_{t+1}\right)^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}$$

such that wealth constraints hold:

$$E_t\sum_{\tau=0}^{\infty}Q_{t+\tau}P_{t+\tau}C_{t+\tau}=E_t\sum_{\tau=0}^{\infty}Q_{t+\tau}P_{t+\tau}Y_{t+\tau}\equiv W_t, \tag{C.2}$$

$$E_t\sum_{\tau=0}^{\infty}\widetilde{Q}_{t+\tau}\widetilde{P}_{t+\tau}\widetilde{C}_{t+\tau}=E_t\sum_{\tau=0}^{\infty}\widetilde{Q}_{t+\tau}\widetilde{P}_{t+\tau}\widetilde{Y}_{t+\tau}\equiv\widetilde{W}_t.$$

and such that goods resources clear:

$$P_tC_t + \widetilde{P}_t\widetilde{C}_t \quad = \quad P_tY_t + \widetilde{P}_t\widetilde{Y}_t \equiv \widetilde{P}_t\widetilde{Y}_t^W, \forall t \tag{C.3}$$

where $\widetilde{Y}_t^W = S_tY_t + \widetilde{Y}_t$ for $S_t \equiv \left(P_t/\widetilde{P}_t\right)$. The wealth constraints in equation (C.2) ensure that each country's agent cannot consume more than the present value of lifetime resources. Furthermore, the within-period resource constraint in equation (C.3) is written as a general constraint on the index of resources, summarized in price indices P and $\widetilde{P}$. However, the specific constraints within this general relationship depend upon the particular view of the exchange rate - Nontradeables, Home Bias, and Sticky prices - as described below.

F.O.C. with respect to wealth: The first-order condition with respect to the wealth constraint in equation (C.2) implies that the state price discount rates are equalized according to: $((Q_{t+1}P_{t+1})/(Q_tP_t)) = ((\widetilde{Q}_{t+1}\widetilde{P}_{t+1})/(\widetilde{Q}_t\widetilde{P}_t))$. Since the state price factor is the product of stochastic discount rates and prices across countries as in $Q_t \equiv \prod_{\tau=0}^{t} M_\tau\lambda_0$, this relationship gives the familiar complete markets condition given as equation (1) in the text:

$$M_{t+1}^* \left(S_{t+1}^*/S_t^*\right) = \widetilde{M}_{t+1}^*. \tag{C.4}$$

F.O.C. with respect to resources: The resource constraint in equation (C.3) gives the relationship between prices and thereby exchange rates to determine this equilibrium relationship. In turn, this condition summarizes auxiliary resources constraints depending upon the three different exchange rate views. Therefore, we describe them more fully here although we provide more details about their measures for each case later in this section.

1. Home Bias: In this case, the consumption aggregator for each country is:

$$C_t = \left(C_{1,t}\right)^a \left(C_{2,t}\right)^{1-a}; \ \widetilde{C}_t = (\widetilde{C}_{1,t})^{1-a}(\widetilde{C}_{2,t})^a \tag{C.5}$$

where $a > 1/2$ and where good 1 is preferred by domestic residents while good 2 is preferred by the foreign households. Defining the total amount of output in each good at time t as $Y_{i,t}^W$ and the allocations of resources of each good by domestic and foreign consumers as, respectively, $Y_{i,t}$ and $\widetilde{Y}_{i,t}$ for $i = 1, 2$, then the quantities embedded in the resource constraint equation (C.3) can be rewritten:

$$
\begin{aligned}
C_{1,t} + \widetilde{C}_{1,t} &= Y_{1,t} + \widetilde{Y}_{1,t} = Y^{W}_{1,t} \\
C_{2,t} + \widetilde{C}_{2,t} &= Y_{2,t} + \widetilde{Y}_{2,t} = Y^{W}_{2,t}
\end{aligned}
\tag{C.6}
$$

The intratemporal first-order conditions of this constraint imply standard relative price relationships. For example, the relative quantities of goods held by domestic consumers is:

$$
\begin{aligned}
(C_{1,t}/C_{2,t}) \; &= a(1-a)^{-1}\,(S_t)^{(1-2a)} \\
\text{for } S_t \equiv (P_t/\widetilde{P}_t) &= (P_{1,t}/P_{2,t})^{2a-1}
\end{aligned}
\tag{C.7}
$$

with the inverse relationship holding for foreign consumers.

2. Non-Tradeables and Tradeables: Now consider the resource constraints when there are Nontradeable goods that are not exchanged in international markets. In this case, the consumption aggregator for each country is:

$$
C_t \equiv (C_{T,t})^{\alpha}\,(C_{N,t})^{1-\alpha} \;\; ; \widetilde{C}_t \equiv \left(\widetilde{C}_{T,t}\right)^{\alpha}\left(\widetilde{C}_{N,t}\right)^{1-\alpha} \tag{C.8}
$$

As above, defining the total amount of output in each good at time t as $Y^{W}_{i,t}$ and the allocations of resources of each good by domestic and foreign consumers as, respectively, $Y_{i,t}$ and $\widetilde{Y}_{i,t}$ for $i = N, T$, then the resource constraint in equation (C.3) can be rewritten:

$$
\begin{aligned}
C_{T,t} + \widetilde{C}_{T,t} &= Y_{T,t} + \widetilde{Y}_{T,t} = Y^{W}_{T,t} \\
C_{N,t} = Y_{N,t} = Y^{W}_{N,t} \; ; \; \widetilde{C}_{N,t} &= \widetilde{Y}_{N,t} = \widetilde{Y}^{W}_{N,t}
\end{aligned}
\tag{C.9}
$$

The first-order conditions of this constraint imply standard relative price relationships for this case as well. For example, the relative quantities of goods held by domestic consumers is:

$$
C_{T,t} = C_t\left(\rho_{N,t}\frac{\alpha}{(1-\alpha)}\right)^{(1-\alpha)} \;\; ; \widetilde{C}_{T,t} = \widetilde{C}_t\left(\widetilde{\rho}_{N,t}\frac{\alpha}{(1-\alpha)}\right)^{(1-\alpha)}
$$

where $\rho_{N,t} \equiv (P_{N,t}/P_{T,t})$, the relative price of non-tradeables in the domestic country and similarly for $\widetilde{\rho}_{N,t}$, the relative price of non-tradeables in the foreign country. Then, according to this view, the exchange rate is:

$$S_t = \frac{(P_{T,t})(\rho_{N,t})^{1-\alpha}}{\left(\widetilde{P}_{T,t}\right)(\widetilde{\rho}_{N,t})^{1-\alpha}} = \frac{(\rho_{N,t})^{1-\alpha}}{(\widetilde{\rho}_{N,t})^{1-\alpha}} \tag{C.10}$$

where we have used the fact that $P_{T,t} = \widetilde{P}_{T,t}$ since the law of one price holds for tradeables.

3. Sticky Prices: In the case of sticky prices, the planner takes the price process as constraints. We define the "sticky" price processes that are given by this constraint as: $\overline{P}_t$ and $\overline{\widetilde{P}}_t$, for the domestic and foreign price levels, respectively. Therefore, the planner can only reallocate the resource constraints recognizing the prices are given. In this case, the resource constraint in equation (C.3) becomes:

$$\overline{P}_t C_t + \overline{\widetilde{P}}_t \widetilde{C}_t = \overline{P}_t Y_t + \overline{\widetilde{P}}_t \widetilde{Y}_t \equiv \overline{\widetilde{P}}_t \widetilde{Y}_t^W, \forall t \tag{C.11}$$

where $\widetilde{Y}_t^W \equiv \overline{S}_t Y_t + \widetilde{Y}_t$ for:

$$S_t = \overline{S}_t = \left(\frac{\overline{P}_t}{\overline{\widetilde{P}}_t}\right) \tag{C.12}$$

Clearly, this assumption represents an extreme version of sticky prices since goods markets do not adjust at all. As such, this case represents an upper bound of incomplete financial markets costs because goods markets are restricted from adjusting.

C.1.2 Mapping resources to the data

The first step described above illustrates standard complete markets allocations given three different views of exchange rate determination as examples. These solutions require knowledge of the available resources to be distributed, summarized in the variables $Y_{i,t}$ and $\widetilde{Y}_{i,t}$. Therefore, the next step is to provide a mapping from the data in order to measure the welfare costs. For this purpose, we discipline the analysis with the goods markets constraints that are presumed in each of the exchange rate views. That is, we treat the aggregate data "as if" agents are facing the same goods market conditions underlying each exchange rate determination model. In doing so, we treat aggregate consumption observations in the data as though they are the outcome of prior intertemporal financial decisions. No conditions are imposed on where these initial consumption allocations come from and therefore we do not take a stand on the degree of market completeness observed in the data. We do, however, treat the intratemporal allocations among commodities as representing the goods market conditions according to a given exchange rate view.

Take the Non-tradeables view in the data, for example. In this case, the demand for consumption of tradeables versus nontradeables is determined as a share of aggregate consumption and the relative price of nontradeables in equation (C.10). Moreover, these prices are related across countries by the relative price according to this exchange rate view as in equation (C.10). Then, *taking the consumption levels and prices from the data as given*, the commodity-level resources of tradeables and non-tradeables can be inferred by replacing aggregate consumption and prices in these equations and solving for the implied resources. As in the text, denoting measures from the data as "D", domestic tradeable consumption would be:

$$Y_{T,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\widetilde{\rho}^{D}_{N,t}\right)^{1-\alpha} C^{D}_{t} S^{D}_{t}. \tag{C.13}$$

and combining with a similar expression for the foreign country, implies that aggregate world tradeables can be inferred from the data as:

$$\widetilde{Y}^{W}_{T,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\widetilde{\rho}^{D}_{N,t}\right)^{1-\alpha} \left(C^{D}_{t} S^{D}_{t} + C^{D}_{t}\right) \tag{C.14}$$

Similarly, the level of non-tradeables in the data for the domestic household can be derived by substituting the solution for tradeables in equation (C.13) into the consumption aggregator in equation (C.8) and solving for $Y_{N,t}$. These substitutions also yield solutions of non-tradeables in terms of observed consumption and prices:

$$Y_{N,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{-\alpha} \left(\widetilde{\rho}^{D}_{N,t}\right)^{-\alpha} C^{D}_{t} \left(S^{D}_{t}\right)^{-\frac{\alpha}{1-\alpha}} \ , \tag{C.15}$$

with a similar solution holding for foreign non-tradeables, $\widetilde{Y}_{N,t}$. These equations then identify the consumption of non-tradeables in the data using only observed exchange rates and aggregate consumption.

By similar reasoning for the Home Bias and Sticky Price versions, we can back-out the implied levels of each good using only aggregate consumption and exchange rates. We next describe the specific solutions using these two main steps below.

C.2 Framework with Home Bias

We now describe in more detail the two-step solution for the complete markets consumption growth rates for the domestic and foreign investor.

A. Theoretical Solution of Complete Markets We first solve for the complete market shares of domestic and foreign consumption of good i, $\omega_{i,t}$ and $\widetilde{\omega}_{i,t}$, respectively. Thus, the optimal consumption allocations may be written as:

$$C_t^* = \left(\omega_{1,t} Y_{1,t}^W\right)^a \left(\omega_{2,t} Y_{2,t}^W\right)^{1-a} \tag{C.16}$$

$$\widetilde{C}_t^* = \left(\widetilde{\omega}_{1,t} Y_{1,t}^W\right)^a \left(\widetilde{\omega}_{i,t} Y_{2,t}^W\right)^{1-a} \tag{C.17}$$

We first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor for Epstein-Zin preferences in equation (B.11), and the fact that: $\omega_{i,t} + \widetilde{\omega}_{i,t} = 1$. Then using these solutions for the shares and defining $g_{yi,t+1} \equiv ln(Y_{i,t+1}^W / Y_{i,t+1}^W)$, the solution to the complete markets domestic and foreign consumption growth based upon these resource growth rates reduce to:

$$g_{c,t+1}^* \equiv ln(C_{t+1}^* / C_{i,t}^*) = \frac{B_2 - B_0 B_1}{1 - (B_0)^2}\, g_{y1,t+1} + \frac{B_1 - B_0 B_2}{1 - (B_0)^2}\, g_{y2,t+1} \tag{C.18}$$

$$g_{\widetilde{c},t+1}^* \equiv ln(\widetilde{C}_{t+1}^* / \widetilde{C}_{i,t}^*) = \frac{B_1 - B_0 B_2}{1 - (B_0)^2}\, g_{y1,t+1} + \frac{B_2 - B_0 B_2}{1 - (B_0)^2}\, g_{y2,t+1} \tag{C.19}$$

where the coefficients are given by:

$$B_0 \equiv \frac{2a(1-a)(1-\gamma)}{1 - 2a(1-a)(1-\gamma)} \;;\; B_1 \equiv \frac{(1-a)}{1 - 2a(1-a)(1-\gamma)} \;;\; B_2 \equiv \frac{a}{1 - 2a(1-a)(1-\gamma)}$$

Note that these consumption shares depend both upon intratemporal goods allocations, given by a, and also intertemporal consumption allocation preferences captured by γ.

B. Mapping to the Data To measure the welfare cost in the data, the next step requires solving for the available resources by viewing the consumption and exchange rate data through the lens of the model. Below, we outline a mapping to the data that applies to observed consumption growth rate processes for the two countries as well as their exchange rates. Note that equation (C.7) shows how aggregate consumption would split between the two goods depending upon the observed exchange rate. Using this condition, we can then back out the growth rate of the available commodity supplies as:

$$\ln\left(\frac{Y^W_{1,t+1}}{Y^W_{1,t}}\right) = g_{y1,t+1} \approx \left(\frac{2a(1-a)}{1-2a}\right) g^D_{s,t+1} + a g^D_{c,t+1} + (1-a) g^D_{\tilde{c},t+1} \tag{C.20}$$

$$\ln\left(\frac{Y^W_{2,t+1}}{Y^W_{2,t}}\right) = g_{y2,t+1} \approx -\left(\frac{2a(1-a)}{1-2a}\right) g^D_{s,t+1} + (1-a) g^D_{c,t+1} + a g^D_{\tilde{c},t+1} \tag{C.21}$$

Unlike the complete markets solution in equations (C.18) and (C.19), these measures only reflect the relative demand across goods, captured by the preference parameter a.

C. Combining Optimal Allocations with Data Measures Substituting the consumption growth rates in the data given by equations (C.20) and (C.21) into the risk-sharing consumption solution in (C.18) and (C.19) imply that the complete markets consumption growth rates in terms of the data measures as:

$$g^*_{c,t+1} = [aD_2 + (1-a)D_1]\ g^D_{c,t+1} + [(1-a)D_2 + aD_1]\ g^D_{\tilde{c},t+1} + (D_2 - D_1)D_3\ g^D_{s,t+1} \tag{C.22}$$

$$g^*_{\tilde{c},t+1} = [aD_1 + (1-a)D_2]\ g^D_{c,t+1} + [(1-a)D_1 + aD_2]\ g^D_{\tilde{c},t+1} - (D_2 - D_1)D_3\ g^D_{s,t+1} \tag{C.23}$$

where the coefficients are given by:

$$D_1 \equiv \frac{B_1 - B_0 B_2}{1-(B_0)^2}\ ;\ D_2 \equiv \frac{B_2 - B_0 B_1}{1-(B_0)^2}\ ;\ D_3 \equiv \frac{2a(1-a)}{1-2a}$$

These processes are used to calculate both the optimal stochastic discount rates, M^* and $\widetilde{M}^*$, as well as the exchange rate under complete markets, S^*.

C.3 Framework with Non-tradeable goods

We now put together the same two approaches for an alternative exchange rate view that exchange rates are determined as the relative price of tradeables to nontradeables.

A. Theoretical Solution of Complete Markets We first solve for the complete market shares of domestic and foreign consumption of the tradeable good, $\omega_{T,t}$ and $\widetilde{\omega}_{T,t}$, respectively, where $\omega_{T,t} + \widetilde{\omega}_{T,t} = 1$. Only tradeables can be reallocated in this case, so that the optimal consumption allocations may be written as:

$$C_t^* = \left(\omega_{T,t} Y_{T,t}^W\right)^\alpha \left(Y_{N,t}\right)^{1-\alpha} \tag{C.24}$$

$$\widetilde{C}_t^* = \left(\widetilde{\omega}_{T,t} Y_{T,t}^W\right)^a \left(\widetilde{Y}_{N,t}\right)^{1-\alpha} \tag{C.25}$$

As with the home bias case, we first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor in equation (B.11). Defining $g_{y,t+1}^T \equiv ln(Y_{T,t+1}^W / Y_{T,t+1}^W)$, the solution to the complete markets domestic and foreign consumption growth based upon these resource growth rates are:

$$g_{c,t+1}^* = \ln(C_{t+1}^*/C_t^*) = \frac{1}{2}\left(g_{y,t+1}^T - (1+\frac{1}{\gamma})g_{s,t+1}^*\right) + \frac{1}{\gamma}g_{s,t+1}^* \tag{C.26}$$

$$\widetilde{g}_{c,t+1}^* = \ln(\widetilde{C}_{t+1}^*/\widetilde{C}_t^*) = \frac{1}{2}\left(g_{y,t+1}^T - (1+\frac{1}{\gamma})g_{s,t+1}^{1*}\right) \tag{C.27}$$

B. Mapping to the Data We next connect these theoretical resource measures for g_y^T and the equilibrium Complete Markets exchange rate g_s^* to the data. The world tradeables supplies is inferred using equation (C.14) while nontradeables are measured with equation (C.15). For the i.i.d. case, substituting these solutions into the optimal allocations above imply:

$$g_{s,t+1}^* = (1-\alpha)\chi(g_{c,t+1}^D - \widetilde{g}_{c,t+1}^D) - \alpha\chi g_{s,t+1}^D \tag{C.28}$$

where $\chi \equiv ((1-\alpha)\frac{1}{\gamma} - \alpha)^{-1}$. In the case of persistent risk, this relationship also depends upon the processes of price ratios, Z_t and $\widetilde{Z}_t$.

Clearly, according to the Nontradeables exchange rate view, there is an S-Wedge because a reallocation of tradeables consumption impacts the relative valuation of tradeables to nontradeables goods within each country. Specifically, the complete markets exchange rate in equation (C.28) has two terms that relate to the data. The difference between domestic and foreign consumption $g^D_{c,t+1} - \widetilde{g}^D_{c,t+1}$ that affects the equilibrium according to the share of consumption allocated to non-tradeables, $(1-\alpha)$. The quantities of this expenditure is offset by the relative price of non-tradeables observed in the data, $g^D_{s,t+1}$ according to expenditures on tradeables through α. The effects are amplified by $((1-\alpha)\frac{1}{\gamma} - \alpha)^{-1}$ because the planner cannot reallocate nontradeables. Indeed, when preferences for nontradeables are sufficiently high, the planner may not be able to reallocate tradeables in a welfare-improving way if over-all consumption is positively correlated across countries. For this reason, we assume $\alpha > 1/2$ in our examples in the text.

C.4 Framework with Sticky Prices

We now consider the benchmark when there is no exchange rate adjustment as consumption reallocates. In this case, the exchange rate process is given to the planner by equation (C.12).

A. Theoretical Solution of Complete Markets In this case, prices do not adjust within a consumption index and therefore it is isomorphic to a model with one consumption good. We therefore first solve for the complete market shares of domestic and foreign consumption bundle ω_t and $\widetilde{\omega}_t$, respectively, where $\omega_t + \widetilde{\omega}_t = 1$. In this case, the consumption allocations can be written as:

$$C^*_t = \left(\omega_t \widetilde{Y}^W_t\right) \; ; \widetilde{C}^*_t = \left(\widetilde{\omega}_t \widetilde{Y}^W_t\right) \tag{C.29}$$

where now $\widetilde{Y}^W_t = \left(\overline{S}_t Y_t + \widetilde{Y}_t\right)$. As with the earlier cases, we first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor in equation (B.11). The solution to the complete markets domestic consumption growth based upon these resource growth rates are:

$$g^*_{c,t+1} = \frac{1}{2} g^w_{y,t+1} - \frac{1}{2}(1 - \frac{1}{\gamma}) g^*_{s,t+1} \tag{C.30}$$

where $g^W_{y,t+1} \equiv ln(\widetilde{Y}^W_{t+1}/\widetilde{Y}^W_t)$ and where in this case, $g^*_{s,t+1} = g_{\overline{s},t+1} \equiv ln(\overline{S}_{t+1}/\overline{S}_t)$.

B. Mapping to the Data Then, under the lens of this extreme sticky price assumption, connecting these variable to the data is straightforward. Since the exchange rates are observable, the given exchange rate process must correspond to the data so that:

$$g^*_{s,t+1} = g_{\overline{s},t+1} = g^D_{s,t+1} \ . \tag{C.31}$$

Furthermore, the growth rate of world resources in numeraire prices are:

$$g^*_{y,t+1} \approx g^D_{\widetilde{c},t+1} + g^D_{c,t+1} + g^D_{s,t+1} \tag{C.32}$$

and therefore, defining $g^{wD}_{c,t+1} = g^D_{c,t+1} + \widetilde{g}^D_{c,t+1}$, we have:

$$g^*_{c,t+1} = \frac{1}{2}(g^{wD}_{c,t+1} + g^D_{s,t+1}) - \frac{1}{2}(1 - \frac{1}{\gamma})g^D_{s,t+1} \tag{C.33}$$

D Generalizations

In addition to the cases described in the text, our framework can generalize in a number of ways. Some of them we discuss in Section 4 of the text. Details are provided in this appendix.

D.1 Multiple Country Example

Incorporating more than two countries is a straightforward extension of the problem described in Appendix C. Here we describe the general framework of the extension and then provide an example based upon Sticky Prices as highlighted in the text.

General Framework with Multiple Countries: Indexing each country with superscript j, the planner now seeks to allocate resources across $j = 1, ..., J$ representative households in each country according to the problem:

$$\underset{\substack{\{C^j_t\} \\ \forall t}}{Max} \sum_{j=1}^{J} U(C^j_t, U^j_{t+1}) \tag{D.1}$$

for

$$U(C_t^j, U_{t+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \ j = 1, ..., J. \tag{D.2}$$

such that

$$E_t \sum_{\tau=0}^{\infty} Q_{t+\tau} P_{t+\tau}^j C_{t+\tau}^j = E_t \sum_{\tau=0}^{\infty} Q_{t+\tau} P_{t+\tau}^j Y_{t+\tau}^j \equiv W_t^j, \forall j \tag{D.3}$$

$$\sum_{j=1}^{J} P_t^j C_t^j = \sum_{j=1}^{J} P_t^j Y_t^j \equiv \widetilde{P}_t \widetilde{Y}_t^W, \forall t \tag{D.4}$$

where $\widetilde{Y}_t^W \equiv \sum_{j=1}^{J} S_t^j Y_t^j$ and where $S_t^J = 1$ for numeraire country J. As in the two country example, the solution to this problem leads to the complete markets exchange rate first-order condition as in equation (C.4). Furthermore, specific form of the goods market constraint in equation (D.4) depends upon the exchange rate view, as described above for Home Bias, Nontradeables, and Sticky Prices.

The multiple country extension requires two main modifications. First, the aggregate "world" variables are now aggregated over multiple countries and, as such, the optimized consumption growth rates incorporate the processes of all the countries. This extension can be handled in a straightforward manner by rewriting world resources available in numeraire prices as: $\widetilde{Y}_t^W$ defined above. Second, in a global economy, countries will have asymmetrical resource processes that will consequently make securities with payouts in their countries either more or less valuable relative to others. In this case, the value of wealth under complete markets will not be equal across countries.

How does this asymmetry impact the cost measures? Note that from Appendix C, the growth rates of Q, the state price discount factor, are equalized once converted into common goods prices. However, as noted there, $Q_t \equiv \prod_{\tau=0}^{t} M_\tau \lambda_0$ and therefore also depends upon an initial valuation of the wealth constraint measured in λ_0. Under our two-country symmetric model, these initial planner weights are equalized. However, more generally, the planner will allocate initial consumption to a country with more valuable wealth in order to jointly optimize global welfare. Therefore, the Certainty Equivalent costs measured in the text will have to be modified according to:

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi} \left(\frac{C_0^D}{C_0^*} \right) \tag{D.5}$$

where $C_0^* = \omega_0(\widetilde{Y}_0^W)$ and where $\omega_0 = \lambda_0 / \sum_{j=1}^{J} \lambda_0^j$. That is, the planner redistributes initial consumption to compensate countries with more valuable wealth.

Sticky Price Example: The text provides an example of this extension for four countries. For parsimony, we only report the Sticky-Price example as it provides an upper bound to the value of the complete markets wedges.

Table D2 reports basic relationships in the data moments for the U.S., the U.K., Canada, and Australia for the real growth rate of consumption provided in the PWT. We choose these four countries because their growth rates are similar, allowing the analysis to instead focus upon differences in second moments. Indeed, as the first line of Panel A shows, the annualized mean growth rates are all near 1.9% with Australia being on the high end during the sample. In our analysis, we set these means, captured by μ to be equal to their average. The next two lines report the standard deviations of the local prices as well as the world prices that include exchange rate variability. For these prices, Australia and the UK exhibit more volatility. Although not used in our analysis, we also report the standard deviation of prices measured at output prices for the economy in the last row of Panel A. The availability of these data provide opportunities for other extensions of our framework using production-based models, as described in more details below. Panel B of D2 gives the consumption correlation matrix. The US and Canada are clearly the most correlated, providing the rationale for our focus in the text.

Table 6 in the text then provides the consumption equivalent results for equation (D.5). In the rows, we report the results assuming first that $C_0^D = C_0^*$ with the compensation due to ω reported as "weights" underneath the row.

D.2 Other Extensions

A. Alternative Asset Pricing Models To illustrate how our framework can use consumption and exchange rates that match asset price data, we chose a long run risk process

Table D2: **Multi-Country Example Data Summary**

A. Consumption Growth				
	US	Can	UK	Aus
Mean	1.91%	1.89%	1.90%	1.96%
St Dev at Local P	1.56%	1.52%	1.76%	1.82%
St Dev at World P	1.96%	2.16%	2.82%	3.14%
St Dev at Output P	1.84%	1.97%	2.74%	2.69%
B. Consumption Correlation in Local Prices				
	US	Can	UK	Aus
US	1.00	0.57	0.45	0.08
Can	0.57	1.00	0.18	-0.02
UK	0.45	0.18	1.00	0.29
Aus	0.08	-0.02	0.29	1.00

Notes: Panel A reports the annualized means and standard deviations of consumption growth for the United States (US), Canada (Can), the United Kingdom (UK), and Australia using the Penn World Tables 9.1 through 2019. The first two lines give the means and standard deviations for consumption expenditures measured in local prices (Local P). The third and fourth rows report the same standard deviations measured at world prices (World P) and based upon country output prices (Output P), respectively. Panel B gives the cross-country correlation matrix for consumption at local prices. All prices are benchmarked to the PWT 9.1 base year.

as an example of persistent risk. However, as noted in the introduction, there are clearly other consumption-based approaches that match asset return behavior. Thus, in principle, any asset pricing model that connects returns to consumption processes can be handled in this framework.

For example, a number of papers have shown the importance of disaster risk including Barro (2009) and Gabaix (2012). Moreover, Wachter (2013) shows that consumption-measured disasters based upon Barro and Ursúa (2008) data can match U.S. asset moments in U.S. data while Backus, Chernov, and Martin (2011) show these effects using options data. Nakamura, Steinsson, Barro, and Ursúa (2013) show how disasters in different countries can be used to measure the U.S. market risk. Furthermore, Gourio, Siemer, and Verdelhan (2013) and Lewis and Liu (2017) show how disaster risk are important for explaining international risks and asset returns. Therefore, the approach in this paper could be used to consider an alternative consumption process, For example, as shown by Lewis and Liu (2017), a disaster risk process that matches cross-country asset return behavior is:

$$g^j_{c,t+1} = \mu + \sigma^j \eta^j_{t+1} + e^{\nu^j_t} N^j_t$$

where N is a Poisson jump process, η_t is i.i.d normally distributed, and ν is a negative random number that measures the size of the disaster. Moreover, much of this literature is based upon recursive preferences, and therefore naturally nests within our framework.

B. Persistence in Exchange Rates The endogenous price versions of the exchange rate that we have studied above require measures of the quantities within the consumption aggregate. These quantities are used to identify the resource constraints available to be redistributed through complete financial markets. In the analysis above, we have taken the simplifying assumption that the real exchange rate in the data is a random walk. However, a large literature has studied longer run behavior of the real exchange rate finding that prices do not diverge indefinitely across countries. That is, there is a stationary long run exchange rate. Thus, while the random walk assumption provides convenient closed form solutions in order to investigate the basic wedge relationships, a more realistic approach would allow for persistence in these relative prices.

To illustrate a straightforward extension to introduce this persistence in the current setting, suppose that instead of the i.i.d. exchange rate growth process in equation (C.10), this process instead contains a persistent processas in the consumption example above. That is,

$$
\begin{aligned}
g^D_{s,t+1} &= x_{s,t} + \sigma \nu_{s,t+1} \\
x_{s,t+1} &= \rho_s x_{s,t} + \varphi_s e_{s,t+1}
\end{aligned}
$$

where all random variables are normally distributed. Putting these processes together with the implied commodity allocation ratios for Home versus Foreign goods in equation (C.6) or Tradeables versus Nontradeables in equation (C.13) and (C.15) shows that the ratios of these goods will be persistent. This assumption then impacts the sharing rules and hence the long run real exchange rate under complete markets. Again, our framework is rich enough to consider these generalizations.

C. The Costs of Wedges using Production The analysis in this paper has focused upon using consumption data because it is the driver in many asset pricing models and also is an important block in most macroeconomic models. Moreover, it relates to a large literature on consumption risk-sharing. Nevertheless, the approach can easily be generalized to consider the implications of inefficient allocations in production. As described in Feenstra et al. (2015) and highlighted in the output price data in Table D2, the PWT data set provides country output and absorption price measures that can be used for international comparisons. Therefore, one could allow asset markets to span production risk rather (or in addition to) consumption risk.

One approach would be to suppose that production is linear in technology. For example, consider a model in which output in each firm is produced with linear technology:

$$
y_t(z) = Y_t z l_t(z)
$$

where $l_t(z)$ is the amount of labor employed by the firm and where Y_t is a stochastic process generating aggregate productivity. In this case, if domestic consumption depends upon claims to this output across countries, the total world consumption, C_t^w would be replaced by total world output in the data. In this way, the same analysis of the wedges due to real price differences across countries can be calculated for production-side risks. Rather than reallocations of consumption, the complete markets solution would instead reallocate output.

D. Alternative Exchange Rate Versions In this paper, we focused upon three different examples of exchange rate approaches. However, our approach is general enough to allow for other determination models. For example, Obstfeld and Rogoff (2001) suggested that transactions costs could potentially explain the disconnect between exchange rates and fundamentals. Given that this transactions cost approach nests within our consumption aggregator above, we can use it to provide solutions based on this exchange rate view. Specifically, they consider the aggregator for the home consumer as:

$$C_t = \left(C_{1,t}^{\Theta} + C_{2,t}^{\Theta}\right)^{\Theta^{-1}}$$

where good 1 is the domestic good and good 2 is the foreign good, as in the Home Bias case above. However, in this case, goods markets are not frictionless as consumers face iceberg shipping costs. Indeed, if these costs are proportional, the relative consumption of consumption across the two good differ according to:

$$\frac{C_{1,t}}{C_{2,t}} = \frac{\widetilde{C}_{1,t}}{\widetilde{C}_{2,t}}(1-\vartheta)^{\xi}$$

where ϱ is the proportional transaction cost and ξ is a parameter that depends upon preference parameter Θ. Since the consumption aggregator is homothetic and the transactions costs apply to goods markets, this type of goods market restriction can be readily incorporated into the framework above.

An alternative exchange rate version would be to embed the model on financial frictions directly. For example, Itskhoki and Mukhin (2021) have recently shown that a friction process outside of fundamentals are needed to explain exchange rates, a process they term "financial shocks." Given that these shocks are measured as deviations from uncovered interest parity deviations, or in our language in this paper, the foreign exchange returns, then these frictions could also potentially be treated as a measure of market incompleteness.

References

Abel, A. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, *43*, 3-33.

Backus, D., Chernov, M., & Martin, I. (2011). Disasters implied by equity index options. *The Journal of Finance*, *66*(6), 1969–2012.

Backus, D., & Smith, G. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *35*(3-4), 297-316.

Bansal, R., & Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, *59*(4), 1481–1509.

Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review*, *99*(1), 243–64.

Barro, R. J., & Ursúa, J. F. (2008). *Macroeconomic crises since 1870* (Tech. Rep.). National Bureau of Economic Research.

Colacito, R., & Croce, M. M. (2011). Risks for the long run and the real exchange rate. *Journal of Political Economy*, *119*(1), 153-181.

Colacito, R., & Croce, M. M. (2013). International asset pricing with recursive preferences. *The Review of Finance*, *68*(6), 2651-2686.

Feenstra, R. C., Inklaar, R., & Timmer, M. P. (2015). The next generation of the penn world table. *American economic review*, *105*(10), 3150–82.

Gabaix, X. (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly journal of economics*, *127*(2), 645–700.

Gallant, A. R., & Tauchen, G. (1999). The relative efficiency of method of moments estimators. *Journal of Econometrics*, *92*(1), 149-172.

Gourio, F., Siemer, M., & Verdelhan, A. (2013). International risk cycles. *Journal of International Economics*, *89*(2), 471–484.

Itskhoki, O., & Mukhin, D. (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, *129*(8), 2183–2232.

Lewis, K. K., & Liu, E. X. (2015). Evaluating international consumption risk sharing gains: An asset return view. *Journal of Monetary Economics*, *71*, 84 - 98.

Lewis, K. K., & Liu, E. X. (2017). Disaster risk and asset returns: An international perspective. *Journal of International Economics*, *108*, S42-S58.

Lustig, H., & Verdelhan, A. (2019). Does incomplete spanning in international financial markets help to explain exchange rates? *American Economic Review*, *109*(6), 2208-44.

Mehra, R., & Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of monetary Economics*, *15*(2), 145–161.

Nakamura, E., Steinsson, J., Barro, R., & Ursúa, J. (2013). Crises and recoveries in an empirical model of consumption disasters. *American Economic Journal: Macroeconomics*, *5*(3), 35–74.

Obstfeld, M., & Rogoff, K. (2001). The six major puzzles in international macroeconomics: Is there a common cause? In B. Bernanke & K. Rogoff (Eds.), *Nber macroeconomics annual* (Vol. 15, p. 339-390). University of Chicago Press.

Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, *68*(3), 987–1035.