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Equilibrium Yield Curves with Imperfect Information*

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Abstract

I study the dynamics of default-free bond yields and term premia using a novel equilibrium term structure model with a New-Keynesian core and imperfect information about productivity. The model generates term premia that are on average positive with sizable countercyclical variation that arises endogenously. Importantly, demand shocks, in addition to supply shocks, play a key role in the dynamics of term premia. This is in sharp contrast to existing DSGE term structure models with perfect information, which tend to rely on large supply shocks to generate time-variation in yields and term premia. With imperfect information, a shock to productivity is a supply shock, while a shock to signals about productivity that do not lead to actual changes in productivity acts as a demand shock. Nevertheless, an increase in economic activity generates more information about productivity, regardless of which type of shock it arises from. Moreover, a decrease in economic uncertainty leads to a decline in term premia as longer-term bonds are risky on average. This feature helps reconcile the empirical evidence that term premia have been on average positive and countercyclical, with numerous studies pointing to demand shocks as being an important driver of business cycles over the last few decades.

JEL: D83, E12, E32, E43, E44, E52, G12

Keywords: Term Premium, Term Structure of Interest Rates, Yield Curve, DSGE Model, Imperfect Information, Learning.

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1 Introduction

Understanding the economic determinants of term premia is an important topic for both academics and policymakers. Its implications span beyond default-free bonds, since the term premium is a function purely of the term structure of stochastic discount factors, which is the foundation for pricing financial assets in general. In addition, extracting market expectations accurately from bond prices by adjusting for term premia is a critical component of conducting monetary policy.[1]

In this paper, I propose a structural explanation of the term structure of interest rates, with a primary focus on the (counter)cyclicality of longer-maturity term premia. A number of studies have shown empirical evidence of countercyclical term premia, consistent with the ample evidence of countercyclical risk premia across multiple asset classes. Meanwhile, a large part of the macroeconomic literature based on dynamic stochastic general equilibrium (DSGE) models or structural vector autoregressions has found that "demand" shocks—shocks that move inflation and aggregate quantities such as output in the same direction—have been an important source of business cycle fluctuations in recent decades, after the great inflation period.[2] In addition, these demand shocks are typically shocks to the *level* of a variable, rather than to the second moments such as the variance, or higher-order moments.[3] Relatedly, multiple studies have begun to document the correlation of consumption growth and inflation turning positive, as the correlation of stock and bond returns has turned negative since around the beginning of 2000.[4] Taken together, these findings would suggest that demand shocks should play an important role in making term premia countercyclical.

Nonetheless, studies that use DSGE models to analyze yield dynamics, such as Rudebusch and Swanson (2012), have typically considered supply shocks to play a dominant role in generating the empirical pattern of yields and term premia.[5] This is because supply shocks can generate inflation risk premia that are on average positive, which helps explain the significantly positive nominal term premia observed over the last several decades. While term premia can be countercyclical in that setting, the mechanism seems at odds with the aforementioned empirical support for demand shocks in explaining business cycles.

To explain how the countercyclicality of term premia can endogenously arise from demand (level) shocks, I explore a novel channel in which imperfect information about productivity plays a crucial role. As I explain in more detail below, this is an environment of rational learning about the unobservable states of productivity, where the precision of the signal is increasing in

[1] For example, Atkeson and Kehoe (2008) advocate models of monetary policy that explicitly considers macroeconomic risk (premia).

[2] Similarly, I refer to "supply" shocks as shocks that move inflation and quantities in the opposite direction. This is consistent with the "traditional interpretation" (Blanchard (1989)) also used in Smets and Wouters (2007) among others. Baumeister and Hamilton (2015) and Bekaert, Engstrom, and Ermolov (2021) use this interpretation for econometric identification.

[3] See, for example, Justiniano, Primiceri, and Tambalotti (2010), Blanchard, L'Huillier, and Lorenzoni (2013) Chahrour and Jurado (2018), Christiano, Motto, and Rostagno (2014).

[4] See, for example, Campbell, Sunderam, and Viceira (2013), Song (2017), Campbell, Pflueger, and Viceira (2020).

[5] See the following literature review for more examples.

economic activity. A number of studies, such as recent work by Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) have shown that this particular form of imperfect information can explain key aspects of the business cycle, such as countercyclical uncertainty.[6] The contribution of this paper is to show that this framework can be embedded in a nonlinear DSGE term structure model in a tractable manner, and generate meaningful demand-side effects that can be effective in explaining the dynamics of the term structure of interest rates.

There are, in fact, few studies that systematically analyze the cyclicality of longer-maturity term premia including the post-financial crisis sample. Hence, before I present the model, I review the empirical evidence of the countercyclicality of the 5-to-10 year forward term premia by regressing three well-known measures of term premia onto a number of business cycle indicators. I find that although the evidence varies somewhat depending on the term premium measure and the economic indicator, there is, overall, statistically significant countercyclicality over the sample period from the beginning of 1990 to the end of 2019, which appears to further strengthen after 2000—the timing which coincides with the change in correlation of stock and bond returns mentioned above.

Motivated by the empirical findings, I first build a simple equilibrium term structure model with imperfect information to clarify the intuition for why a shock to productivity as well as a shock to signals about productivity that do not lead to actual changes in productivity (or "noise shocks") can generate countercyclical term premia. The model consists of a state space model of productivity, a consumption rule, and an Euler equation. Productivity is the sum of a persistent and a transitory component, both unobservable. Agents (rationally) infer levels of the unobservables from productivity itself and a noisy public signal about the persistent component of productivity. A key feature of the signal is that its precision is increasing in economic activity, which, in turn, is increasing in productivity or the signal. Intuitively, more economic activity leads to further information about productivity via social learning, and hence the collective signal becomes more precise. In other words, uncertainty about productivity is countercyclical. Since term premia are on average positive in the model, this endogenous countercyclicality of uncertainty leads to the countercyclicality of term premia. In contrast, under perfect information, term premia are still positive but roughly constant. Importantly, this mechanism works through both the persistent productivity shock and the noise shock, though each shock has distinct impacts on productivity and consumption—only the former shock actually affects productivity.

The model is nonlinear, but simple enough to be solved without approximation, and allows for some analytical characterizations of the term premium. However, it is limited in certain aspects. Notably, since the model does not feature inflation, it is difficult to interpret the shocks as "demand" or "supply" shocks. In addition, the signal is not completely linked to equilibrium variables within the model and is not suited for quantitative evaluations.

To address these issues, I build a DSGE term structure model with imperfect information by embedding the productivity structure and the learning process of the simple model into an otherwise standard New-Keynesian setup. The New-Keynesian model allows me to analyze the economic

[6] See the following literature review for more examples.

determinants of both nominal and real yields in a more realistic setting where inflation is determined endogenously through nominal price rigidities and monetary policy. In this model, the intermediate goods firms must infer the states of productivity from a public signal that becomes more informative as output increases. While models with information frictions can face computational challenges and be hard to solve without linearization, my particular specification remains relatively tractable. I solve the model using a high-order perturbation method to properly account for time-varying uncertainty and term premia.

I calibrate the model to US data and show that the mechanism that generated the countercyclicality of term premia in the simple model carries over to the DSGE model in a quantitatively meaningful way. In addition, the noise shock can be clearly interpreted as a demand shock in the DSGE model. This is because a noise shock motivates consumption without the actual increase in the capacity of supply, causing upward pressure on prices. I show that incorporating imperfect information significantly increases average nominal term premia, and amplifies the countercyclicality of term premia through *both* demand and supply shocks, such that the model-implied term premium dynamics become more in line with empirical estimates. In other words, *noisy news* about productivity works as a *demand* shock, but affects term premia through its effect on the *belief* about *supply* (productivity), in a quantitatively relevant manner. I also show that imperfect information amplifies the volatility and countercyclicality of term premia due to *supply* shocks through an intuitive mechanism of countercyclical uncertainty, without resorting to exogenous shocks to the volatility of productivity.

The information friction in the model is governed by two key parameters—the volatility of the productivity signal and the mass of signals produced per unit of output. I show that both parameters have distinct implications for nominal and real term premia, and the latter parameter, which controls the cyclicality of signal precision, is crucial in generating significant countercyclical variation in term premia. Importantly, the model shows that the amplification of the average and volatility of nominal term premia is largely coming from real term premia. The emphasis of the "real" channel in explaining nominal yield dynamics is consistent with recent studies such as Duffee (2018), which argues that only a relatively small portion of the variation in news about yields can be explained by the variation in news about expected inflation. This paper can be viewed as providing an economic interpretation of such empirical results based on imperfect information.

The structure of the paper is as follows. Following a literature review, Section 2 presents empirical evidence on the countercyclicality of term premia that motivates this paper. Section 3 presents a simple term structure model with imperfect information that helps understand the key mechanism of the DSGE term structure model, which is analyzed in Section 4. Section 5 offers concluding remarks.

Literature Review

This work is related to a few strands of the literature. First, it is related to studies that analyze models of the aggregate economy with imperfect information, which dates back to the seminal work of Kydland and Prescott (1982) that launched the real business cycle literature. Various forms of imperfect information have been examined in the context of macroeconomic models, but the specific form of imperfect information in my model builds on work that assume agents form homogeneous expectations rationally and learn about unobservable state variables in a Bayesian fashion. Studies that embed such a structure into an RBC model include Bomfim (2001), Edge, Laubach, and Williams (2007), Boz, Daude, and Durdu (2011), and more recent studies that analyze the implications using DSGE (New-Keynesian) models include Lorenzoni (2009), Blanchard, L'Huillier, and Lorenzoni (2013) and Faccini and Melosi (2022). However, a key departure from this body of work in terms of the information friction is the feature of procyclical signal precision, bringing my work closer to a smaller set of papers such as Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) and Ilut and Saijo (2021). While these papers focus on how the information friction helps explain the business cycle, my paper argues that the friction has implications beyond that and is useful in understanding term structure dynamics as well.

Second, it is related to studies that analyze asset pricing models with imperfect information. This literature is also large, and difficult to cite exhaustively. Collin-Dufresne, Johannes, and Lochstoer (2016) and Johannes, Lochstoer, and Mou (2016) are recent examples of endowment economies. Ai (2010), Ai, Croce, Diercks, and Li (2018), Winkler (2020) and Bianchi, Lettau, and Ludvigson (2022) are examples of production-based economies, but these papers do not focus on Treasury yields and term premia, as I do.

Third, it is related to studies on equilibrium term structure models. Models based on endowment economies have been studied as early as Campbell (1986) and more recently by Piazzesi and Schneider (2007), but work using production-based models, and in particular, DSGE models have picked up only in the last decade or so. Examples include Rudebusch and Swanson (2008), Doh (2011), Andreasen (2012a), Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), Chen, Cúrdia, and Ferrero (2012), Rudebusch and Swanson (2012), Dew-Becker (2014), Kung (2015), Lopez, Lopez-Salido, and Vazquez-Grande (2015), Carlstrom, Fuerst, and Paustian (2017), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018), Andreasen and Jørgensen (2019), Swanson (2019), Gourio and Ngo (2020) and Hsu, Li, and Palomino (2021). A common assumption across these papers is perfect information, and the implications of imperfect information on the yield curve remains largely unexplored, especially in the context of macroeconomic models. By contrast, the key feature of my DSGE term structure model is imperfect information. This feature leads to endogenous heteroskedasticity in the stochastic discount factor and provides a deeper microfoundation to studies that incorporate forms of exogenous stochastic volatility, such as Andreasen (2012b), Nakata and Tanaka (2016), Bianchi, Kung, and Tirskikh (2018), and Bretscher, Hsu, and Tamoni (2020). My work is consistent with recent papers by Duffee (2018) and Chernov, Lochstoer, and Song (2021) in emphasizing the real term structure in explaining the nominal term structure of interest rates. However, I provide a novel explanation that works through imperfect information.

2 Empirical Motivation

In this section, I discuss the empirical pattern of term premia that motivates the paper. More specifically, I provide some new evidence on the countercyclicality of nominal term premia over the recent few decades up to the end of 2019. I also show suggestive evidence of countercyclical macroeconomic uncertainty as well as procyclical production of information about technology, which further motivates an explanation of term premium dynamics based on a framework with imperfect information. I focus on the sample period before 2020, since the unprecedented economic impact of the pandemic calls into question the suitability of the standard statistical methods I use for the analysis.

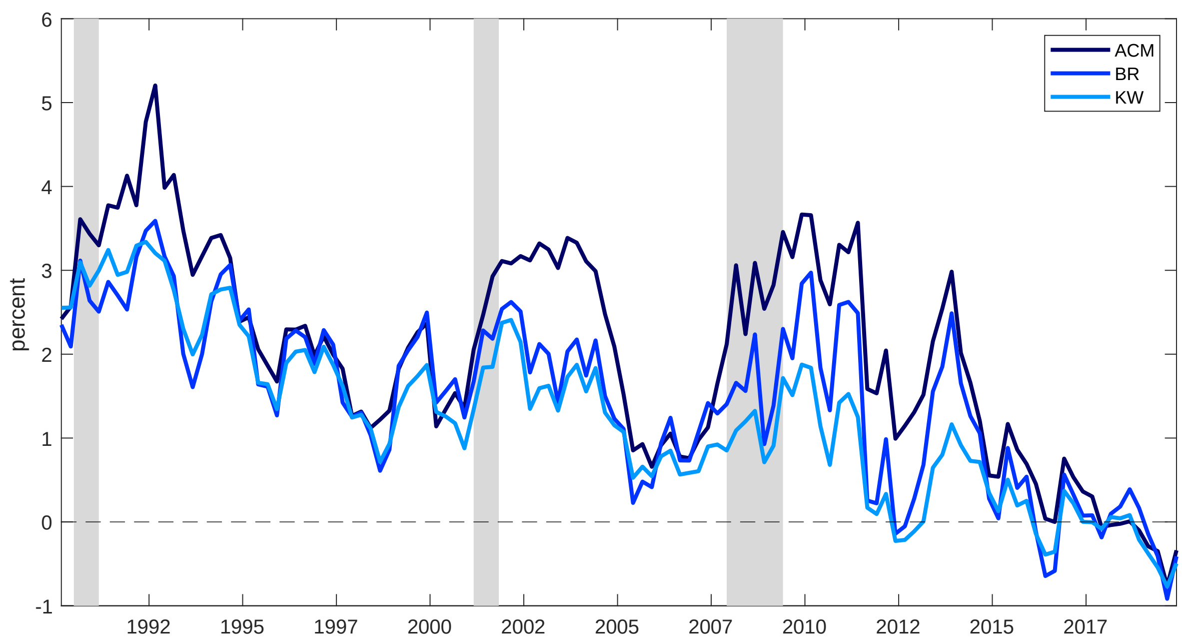

Figure 1: **5-to-10-year Forward Term Premium**

Notes: Quarterly time series from 1990.Q1 to 2019.Q4. ACM is the term premium of Adrian, Crump, and Moench (2013b). BR is a the term premium estimated from a version of the model by Bauer and Rudebusch (2020). KW is the term premium of Kim and Wright (2005). Shaded grey areas correspond to NBER recession periods.

There is ample evidence on the countercyclicality of risk premia across a broad set of asset classes, including equity, corporate bonds, and currency.[7] In line with other assets, the countercyclicality of nominal term premia has also been documented in a number of empirical studies. However, analyses that include the post-financial crisis period are somewhat limited in the scope of the term premium measures and the business cycle indicators they consider.[8] Therefore, I revisit

[7]See Cochrane (2011) for a summary.

[8]Cochrane and Piazzesi (2005), Ludvigson and Ng (2009) and Piazzesi and Swanson (2008) are examples using data before the financial crisis. Note Piazzesi and Swanson (2008) focus on excess returns of fed funds futures up to

this issue by conducting a regression analysis using a different set of term premium estimates and a wider range of business cycle indicators.

As the dependent variable, I consider three well-known estimates of the nominal term premium—first, from the model of Adrian, Crump, and Moench (2013b) (ACM), second, from the model of Kim and Wright (2005) (KW), and third, from a version of the model of Bauer and Rudebusch (2020) (BR).[9] While all three models assume yields and term premia are driven by multiple latent factors and use no-arbitrage restrictions to help identification, the three models differ significantly in other aspects. Importantly, only KW uses surveys (Treasury yield forecasts from the Blue Chip surveys) to further assist identification. Meanwhile, only BR allows for a non-stationary factor in the model, which may better account for the downward trend in interest rates than strictly stationary models.

I focus on the 5-to-10 year forward measure to mitigate potential measurement issues stemming from the effective lower bound (ELB).[10] Figure 1 shows the three term premium series since the beginning of 1990. As documented in the literature, term premia have generally been positive but trending down in recent years and were close to zero by the end of 2019. Meanwhile, the rise in term premia in the last three recessions before 2020 is visually evident, although the magnitude differs across the series—KW rising the least, while ACM rises the most.

The independent variables are nonfarm payroll, industrial production, real GDP (all year-on-year changes), GDP gap, unemployment gap, and capacity utilization. The details of the data are relegated to Appendix A. I regress each term premium series on to each of the independent variables separately with a constant. I also regress changes in term premia onto changes in the independent variables to account for the persistence of the variables. The results are summarized in Table 1. Each number corresponds to the regression coefficient associated with the business cycle indicator. The number is shaded in dark blue if the coefficient is negative and statistically significant, and is shaded in light blue if the coefficient is statistically insignificant yet still negative, consistent with countercyclicality.

Overall, I find notable evidence of countercyclicality of the 5-to-10-year nominal term premium. Over the sample period from the beginning of 1990 to the end of 2019, regressions based on the level of term premia (upper left block of Table 1) show that ACM has the strongest countercyclicality which is statistically significant, for a number of business cycle indicators.[11] The statistical significance using BR is weaker, although the negative sign of the coefficients is consistent with

6-months maturity. Further evidence of countercyclical term premia including the post-financial crisis observations is provided by Adrian, Crump, and Moench (2013a), Wright (2011), Bauer, Rudebusch, and Wu (2014), Bauer and Rudebusch (2020). Econometric analysis that include observations beyond 2009 is relatively scarce, with an exception being Bekaert, Engstrom, and Ermolov (2021), who find term premia implied from the Blue Chip surveys show countercyclicality with respect to a recession dummy.

[9] I use an estimate of the trend real interest rate that is different from BR, constructed from publicly available data. The estimate is somewhat smoother than BR. Otherwise, the model is identical to their "observed shifting endpoint" model. I find that the correlation between the term premia from BR and my estimates over the sample period of BR (1971.Q4 to 2018.Q1) is nearly perfect, with a coefficient of over 0.98.

[10] The results are similar for the 10-year term premium.

[11] The countercyclicality is also highlighted in Adrian, Crump, and Moench (2013a) although they do not provide a regression analysis.

countercyclicality. The evidence based on KW is the weakest, although most indicators still have negative coefficients. In any case, it appears hard to find strong evidence of procyclical term premia. Furthermore, regressions based on quarterly differences of term premia (bottom left block) show statistically significant countercyclicality across all measures of term premia.

Table 1: **Regression of the 5-to-10-year Forward Nominal Term Premium on Business Cycle Indicators**

	1990.Q1 - 2019.Q4			2000.Q1 - 2019.Q4		
Level	ACM	BR	KW	ACM	BR	KW
NFP	-36.3***	-18.4**	-12.8	-49.8***	-32.8***	-26.0***
IP	-3.0	1.1	1.5	-5.8*	-2.8	-2.9*
GDP	-16.6**	-2.6	0.1	-29.8***	-16.0**	-11.0*
GDP gap (level)	-37.2***	-16.4*	-11.0	-38.3***	-18.8**	-9.7
UE gap (negative of level)	-35.5***	-14.5	-4.6	-36.8***	-18.5*	-7.7
CU (level)	-22.8***	-11.6**	-8.6	-27.2***	-17.0***	-13.7***
Difference						
NFP	-4.5***	-2.9**	-2.4***	-4.7***	-4.1**	-2.8**
IP	-2.3***	-2.1***	-1.9***	-2.6***	-2.6***	-2.2***
GDP	-4.2**	-1.0	-2.3**	-5.2***	-3.0	-3.8***
GDP gap	-5.8***	-1.5	-3.2***	-5.3***	-2.3	-3.7**
UE gap (negative)	-11.9***	-9.3***	-6.6***	-10.1***	-8.8***	-5.9***
CU	-4.1***	-3.5***	-3.1***	-3.7***	-3.3***	-2.8***

Notes: ***,**,* indicate 1%, 5%, 10% significance based on Newey-West standard errors, respectively. Dark blue shades indicate significantly negative coefficients. Light blue shades indicate negative coefficients that are statistically insignificant. Frequency is quarterly. Regressors are year-on-year changes unless otherwise stated. The regression coefficients are in basis point units per one percentage point change in the regressor.

As shown in the two right blocks of Table 1, the evidence of countercyclicality is even stronger since the beginning of 2000, where, in particular, both BR and KW show statistically significant countercyclicality against a larger number of indicators. What is especially interesting about this period is that it also coincides with a period in which past studies have found that the correlation between inflation and consumption growth turned from negative to positive (e.g. Song (2017)) while the correlation between returns on Treasury bonds and equity turned from positive to negative (e.g. Campbell, Pflueger, and Viceira (2020)), which could be interpreted as increased relevance of demand shocks in explaining the business cycle over the period.[12]

[12]The subsample from 2000 puts more emphasis on the post-financial crisis period in which unconventional monetary policy such as large scale asset purchases was implemented. Countercyclical term premia can still arise in such an environment if asset purchases are more of an exogenous shock, decreasing term premia and increasing economic activity at the same time, as shown by Chen, Cúrdia, and Ferrero (2012) and Carlstrom, Fuerst, and Paustian (2017).

Given these results, a natural question would be to ask what are the potential macroeconomic drivers behind the countercyclicality. To think about this issue, it is useful to note that DSGE models, specifically designed to analyze the source(s) of macroeconomic dynamics, have generally implied that, particularly since the mid-1980s (i.e., the start of the "great moderation"), demand shocks have been an important driver of the business cycle. In addition, these demand shocks are typically shocks to the *level* of a variable (or "first-moment" shocks), rather than shocks to second-moments such as variance, or higher-order moments.[13]

Most DSGE models have obtained these results without considering implications for Treasury yields and term premia, but taken at face value, the findings imply that demand shocks should be a predominant driver of countercyclical term premia. Nevertheless, studies that use DSGE models to analyze yield dynamics, such as Rudebusch and Swanson (2012), have typically considered supply shocks to play a dominant role.[14] This is because supply shocks can generate inflation risk premia that are on average positive, which helps explain the positive nominal term premia observed over the last several decades. Exceptions to this approach have emerged only recently, and rely on exogenous shocks to volatility.[15]

To understand the dynamics of yields and term premia in a world of significant demand shocks, I propose an explanation based on imperfect information. As discussed in the rest of the paper, the crucial mechanism is the link between countercyclical term premia and countercyclical macroeconomic uncertainty (conditional volatility), which in turn is caused by procyclical production of information about productivity. Indeed, numerous papers have documented evidence of countercyclical uncertainty. For completeness, I plot a few relevant measures of uncertainty in Figure 2 (left panel), which confirms the finding in the literature.[16] Meanwhile, the relation between countercyclical uncertainty and procyclical information production has been analyzed by a strand of macroeconomic studies as early as Van Nieuwerburgh and Veldkamp (2006) and more recently by Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), among others. These studies provide a

Splitting the sample periods further generally results in reduced statistical significance due to the smaller sample size, but evidence of countercyclicality can still be observed for the subsamples from 1990.Q1 to 1999.Q4, from 2000.Q1 to 2008.Q4, and from 2009.Q1 to 2019.Q4, respectively.

[13]The number of relevant studies is too large to list here in full. Some examples are Smets and Wouters (2007), Justiniano, Primiceri, and Tambalotti (2010), Christiano, Motto, and Rostagno (2014), Blanchard, L'Huillier, and Lorenzoni (2013), Gust, Herbst, López-Salido, and Smith (2017). Obviously, DSGE models are not the only models that can identify demand and supply shocks. For example, Bekaert, Engstrom, and Ermolov (2021) is a recent study that uses a less structural model and highlights the role of demand shocks in the last few recessions before 2020.

[14]Other examples include Rudebusch and Swanson (2008), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) and Swanson (2019). These papers also find supply shocks make term premia countercyclical.

[15]Bretscher, Hsu, and Tamoni (2020) show that a shock to the volatility of government spending is a demand shock, and generates countercyclical term premia. Bianchi, Kung, and Tirskikh (2018) show that a shock to the volatility of time preference/TFP growth acts as a demand shock, which generates countercyclical term premia. These papers are consistent with macroeconomic studies that highlight uncertainty shocks as important drivers of the business cycle, such as Basu and Bundick (2017).

[16]In addition to two well-known measures of uncertainty: (1) the macroeconomic uncertainty index by Jurado, Ludvigson, and Ng (2015) and (2) the VIX, I show (3) a measure of GDP growth uncertainty based on the Survey of Professional Forecasters, which is used to calibrate the term structure model in Section 4, and (4) the conditional volatility of TFP growth from a GARCH(1,1) model similar to Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018). For surveys on economic uncertainty, see for example Bloom (2014) and Cascaldi-Garcia, Sarisoy, Londono, Rogers, Datta, RT Ferreira, Grishchenko, Jahan-Parvar, Loria, Ma, et al. (2020).

compelling framework of rational learning with imperfect information which I build on, but they do not necessarily show direct evidence of information production. Hence, I offer one suggestive evidence based on the growth rate of total patent applications in the U.S. (Figure 2, right panel). It is evident that the growth rate of applications tends to fall sharply around recessions and recover afterwards, implying that information about new technology appears to be procyclical.[17]

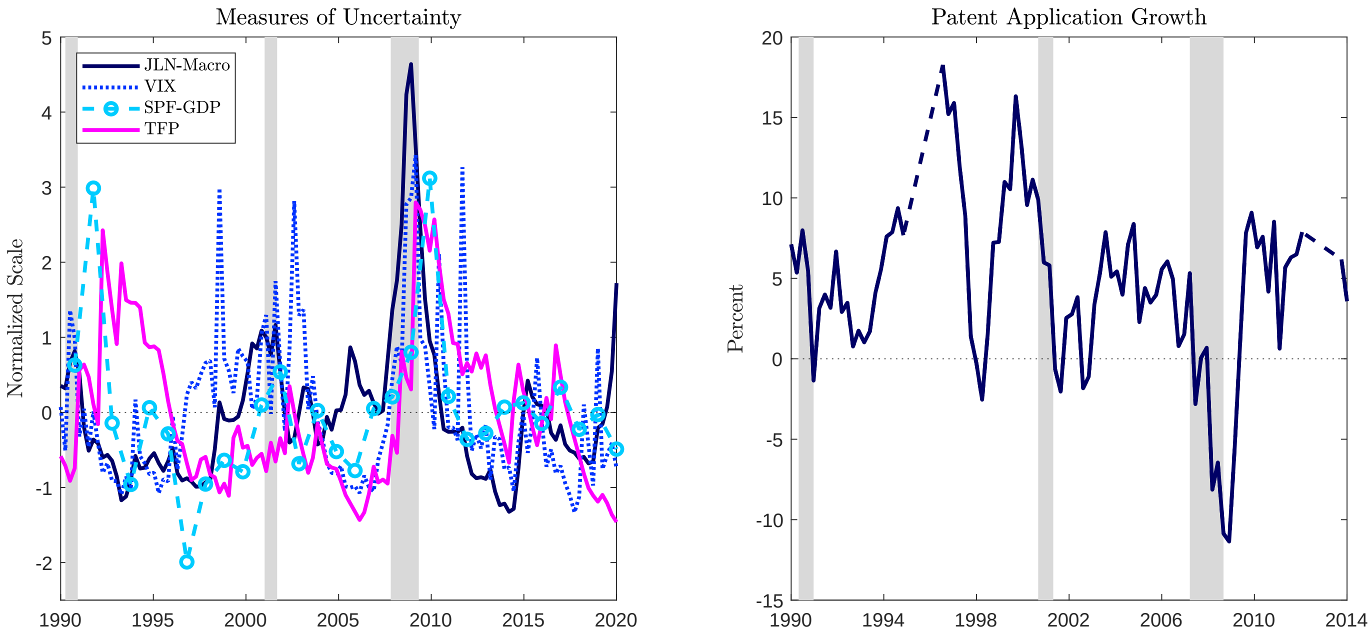

Figure 2: **Measures of Uncertainty and Information**

Notes: “JLN-Macro” is the macro uncertainty index by Jurado, Ludvigson, and Ng (2015). “VIX” is the option-based measure of the 30-day expected volatility of the S&P 500 Index. “SPF-GDP” is the standard deviation of the 1-quarter-ahead average forecast distribution of real GDP growth from the Survey of Professional Forecasters. “TFP” is the conditional standard deviation of TFP growth estimated from a GARCH(1,1) model, using the unadjusted TFP growth data by Fernald (2014). The uncertainty series are quarterly from 1990.Q1 to 2019.Q4 except for “SPF-GDP”, which is measured for the last quarter of each year. All uncertainty series are normalized such that they have zero mean and one standard deviation. Patent application is from the dataset compiled by Marco, Carley, Jackson, and Myers (2015) and the annual growth rate is computed at the quarterly frequency from 1990.Q1 to 2014.Q4. The dashed line portions indicate omitted data corresponding to periods of idiosyncratic volatility due to regulatory changes in 1995.Q2 and 2013.Q1. Shaded grey areas correspond to NBER recession periods.

By embedding into a term structure model information frictions which past macroeconomic studies have found relevant in explaining business cycles, I complement analyses using exogenous shocks to uncertainty and offer a deeper microfoundation of term structure dynamics that is consistent with the macroeconomic literature. It also seems useful to clarify the implications of imperfect

[17]See Appendix A for details about the data. Applications of this dataset appear limited in the finance/macroeconomics literature, with a notable exception of Bluwstein, Hacioglu Hoke, and Miranda-Agrippino (2020) who use it to identify technology news shocks. An alternative measure of innovation news that accounts for stock valuation was proposed by Kogan, Papanikolaou, Seru, and Stoffman (2017), which also shows procyclicality. However, such a measure could be biased if stocks are subject to mispricing (Haddad, Ho, and Loualiche (2022)).

information on Treasury yields and term premia, which remain largely unexplored in the literature.[18]

3 Simple Term Structure Model with Imperfect Information

3.1 Model

To build intuition for how imperfect information affects term premia, I analyze a simple equilibrium term structure model with imperfect information. The model consists of three components: (1) a state space system of productivity where an agent learns about the unobservable components of productivity from observable signals using a Kalman filter, (2) an optimal consumption rule that is linear in productivity, and (3) a consumption Euler equation that prices the term structure of default-free interest rates. Apart from transparency, another advantage of its simplicity is that, despite being a nonlinear model, it can be solved easily in a sequential fashion, without applying any approximation methods. In Appendix B, I show that this model is consistent with a set of equilibrium conditions from a stylized real business cycle model without capital.

State space system of productivity: Productivity z_t consists of a "persistent" component a_t and a "transitory" component $\varepsilon_{z,t}$:

$$z_t = a_t + \sigma_z \varepsilon_{z,t}. \tag{1}$$

Note z_t is observable to the agent, whereas its components a_t and $\varepsilon_{z,t}$ are *unobservable*.[19] a_t follows an AR(1) process with a zero mean:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}, \tag{2}$$

where $0 < \rho_a < 1$. Both $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ are i.i.d. standard normal.

In addition to productivity z_t itself, the agent observes a collection (continuum) of noisy signals $s_{j,t}$ about the persistent component of productivity, which is generated by economic activity. These signals can be interpreted intuitively as "data" (following the interpretation of Farboodi, Mihet, Philippon, and Veldkamp (2019)) or "news". Note $j \in [0, J_t]$ where J_t captures the total mass of signals. Each signal is characterized as:

$$s_{j,t} = a_t + \sigma_{s,j} \varepsilon_{s,j,t}, \tag{3}$$

where $\varepsilon_{s,j,t}$ is i.i.d standard normal with respect to j and t, and has a mean of zero and a variance

[18] To be clear, despite the evidence of countercyclicality, this does not necessarily mean that mechanisms which lead to procyclical term premia are irrelevant, and my analysis does not rule them out. For example, if Treasury bonds carry a premium for safety and liquidity (Krishnamurthy and Vissing-Jorgensen (2012)), they could become more valuable during a recession. See also, Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) and Nakata and Tanaka (2016) for alternative channels that lead to procyclical term premia.

[19] The parameter σ_z is observable, as well as all other model parameters that follow.

of $1/\Delta_j$. Δ_j is the (small) mass of signals represented by $s_{j,t}$.[20] $\varepsilon_{s,j,t}$ as well as a_t are unobservable, but each signal is publicly available and hence common knowledge across all agents. This allows abstraction from heterogeneous expectations and higher-order beliefs, making the model highly tractable.

A sufficient statistic for the collection of signals is the "aggregate" signal s_t, defined as:

$$s_t \equiv \frac{1}{J_t}\int_0^{J_t} s_{j,t}dj = a_t + \sigma_{s,t}\varepsilon_{s,t}, \tag{4}$$

where $\varepsilon_{s,t}$ is an i.i.d standard normal shock. This shock can be considered a "noise" shock in line with the literature, since it is the component of s_t which is orthogonal to productivity. In addition, $\sigma^2_{s,t} = \sigma^2_{s,j}(J_t^{-1})$, as a result of the aggregation of signals.[21] This formulation is similar to Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017). However, while they link the mass of signals J_t specifically to the number of firms entering into production, I interpret the source of J_t more broadly and assume it is an increasing function of an (observable) measure of economic activity from the previous period H_{t-1}:

$$J_t = \phi(H_{t-1}), \tag{5}$$

where $\phi' > 0$. Combined with the fact that $\sigma_{s,t}$ is inversely related to J_t, economic activity increases the amount of information about the persistent component of productivity, and makes the aggregate signal s_t more precise. Such a mechanism may be understood intuitively as the process where firms, by producing more goods, disseminate noisy information about aggregate productivity in the form of data or news from various media outlets, and in turn, learning about productivity more precisely among themselves as economic activity of others increase (Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) refer to this mechanism as "social learning").[22]

For simplicity, I assume ϕ is quadratic:

$$\phi \equiv \left[\xi(H_{t-1} - \bar{H}) + \bar{H}\right]^2,$$

where $\xi > 0$ controls the rate of signal production by H_{t-1} and $\bar{H} > 0$ is the (non-stochastic) steady state of H_t. H_t is specified as an AR(1) process:

$$H_t = (1-\rho_H)\bar{H} + \rho_H H_{t-1} + \sigma_{H,a}\varepsilon_{a,t} + \sigma_{H,s}\varepsilon_{s,t}, \tag{6}$$

where $\sigma_{H,a} \geq 0$ and $\sigma_{H,s} \geq 0$, i.e., H_t loads positively on the (persistent) productivity shock $\varepsilon_{a,t}$

[20] As discussed in Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), adjusting the variance of $\varepsilon_{s,j,t}$ by Δ_j is necessary to prevent the signals from perfectly revealing a_t when the number of signals is large and the mass of signals represented by each $s_{j,t}$ becomes infinitesimal.

[21] s_t can be understood as the limiting distribution of $J^{-1}\sum_{n=1}^{N} s(m_n)\Delta_{j_n}$ as $\Delta_{j_n} \to 0$, where m_n is the midpoint of interval $[j_{n-1}, j_n] \subseteq [0, J_t]$ with length Δ_{j_n}.

[22] See also Van Nieuwerburgh and Veldkamp (2006) and Ilut and Saijo (2021) for examples of alternative mechanisms that induce signals with procyclical precision. The formulation adopted in this paper is a simple way to capture this key feature common across such models.

and/or the noise shock $\varepsilon_{s,t}$.[23] I also assume the agent only observes H_{t-1} at t, and cannot infer $\varepsilon_{a,t}$ or $\varepsilon_{s,t}$ at t to avoid the model becoming trivial. While economic activity H_t should, in principle, be explicitly linked to equilibrium variables such as consumption or output, my interpretation of H_t here is more broad and includes various types of activity that leads to information production. This specification simplifies the computation considerably while capturing the "cyclicality" of H_t and allowing for some endogeneity of the process (in the sense that H_t is not generated from independent shocks other than what are already in the model). In the DSGE model developed in Section 4, J_t will be an explicit function of equilibrium output.

The agent updates her belief about the unobservable productivity component a_t via a Kalman filter:

$$a_{t|t} \equiv \mathbb{E}_t[a_t] = \rho_a a_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}), \tag{7}$$

where $\boldsymbol{s}_t$ is the vector of signals ($\boldsymbol{s}_t \equiv [z_t, s_t]'$). $\boldsymbol{K}_t$ is the Kalman gain matrix:

$$\boldsymbol{K}_t = \left[\frac{\frac{1}{\sigma_z^2}}{\frac{\phi(H_t)}{\sigma_s^2} + \frac{1}{\sigma_z^2} + \frac{1}{\sigma_{a,t}^2}}, \quad \frac{\frac{\phi(H_t)}{\sigma_s^2}}{\frac{\phi(H_t)}{\sigma_s^2} + \frac{1}{\sigma_z^2} + \frac{1}{\sigma_{a,t}^2}} \right]. \tag{8}$$

$\sigma_{a,t}^2$ is the conditional forecast variance of a_{t+1} ($\sigma_{a,t}^2 \equiv \mathbb{V}ar_t(a_{t+1})$) and is updated according to the standard Ricatti equation:

$$\sigma_{a,t}^2 = \rho_a^2 \left(\frac{\phi(H_{t-1})}{\sigma_s^2} + \frac{1}{\sigma_z^2} + \frac{1}{\sigma_{a,t-1}^2} \right)^{-1} + \sigma_a^2. \tag{9}$$

Consumption rule: Consumption (in logs) is linear in productivity z_t:

$$c_t = \theta_c z_t. \tag{10}$$

Euler equation: The (continuously-compounded) yield of a real default-free bond of arbitrary maturity n, $r_t^{(n)}$, is priced by the Euler equation:

$$r_t^{(n)} = \bar{r} - \frac{1}{n} \ln \mathbb{E}_t \left[\exp \left(-\chi_c \sum_{i=1}^{n} \Delta c_{t+i} \right) \right]. \tag{11}$$

θ_c, $\bar{r}$ and χ_c are exogenous parameters. Of course, if derived from a fully-specified equilibrium model, the Euler equation is consistent with power utility that has a risk aversion parameter of χ_c. In addition, the Euler equation must be consistent with the consumption rule (10), which imposes a cross-restriction on θ_c and χ_c (see Appendix B). However, such a restriction will be largely irrelevant for the discussion in this section.

The term premium is constructed in a standard way.[24] First, define a hypothetical price of

[23] Technically, I choose the parameters of H_t such that it practically never falls below zero. Therefore, while H_t is assumed to be bounded above zero, it can be approximated very well by an AR(1).

[24] See for example Rudebusch and Swanson (2012).

an $n-$period bond $P_t^{\mathbb{Q}(n)}$ formed by discounting cashflows by the risk-free bond price: $P_t^{\mathbb{Q}(n)} = P_t^{(1)}\mathbb{E}_t[P_{t+1}^{\mathbb{Q}(n-1)}]$. Then the $n-$period hypothetical yield priced under risk-neutrality $r_t^{\mathbb{Q}(n)} = -\frac{1}{n}\ln P_t^{\mathbb{Q}(n)} = -\frac{1}{n}\ln \mathbb{E}_t\left[\exp\left(-\sum_{i=0}^{n-1} r_{t+i}^{(1)}\right)\right]$. The $n-$period term premium is the difference between the $n-$period yield $r_t^{(n)}$ and $r_t^{\mathbb{Q}(n)}$:

$$tp_t^{(n)} \equiv r_t^{(n)} - r_t^{\mathbb{Q}(n)}. \tag{12}$$

3.2 Results

To understand the dynamics of the yield curve under the setup with imperfect information, I plot impulse responses. Since H_t acts as a "time-varying coefficient" in the state space system, the model is nonlinear and hence standard impulse responses are neither symmetric, linear nor history independent. Nevertheless, I present standard impulse responses as it is easier to understand the mechanism. Since the ergodic mean of the system deviates from the steady state, I compute the impulse responses from the ergodic mean.[25]

3.2.1 Parameter Values

Parameter values for the model are chosen for illustrative purposes and summarized in Table 2. I choose $\rho_a = 0.99$ so the persistent component of productivity is close to a random walk—a common specification. The standard deviation of the transitory productivity shock σ_z is relatively large compared to the standard deviation of the noise shock σ_s so that learning about the persistent component of productivity is gradual, and the additional signal s_t plays a meaningful role in the learning process, and in turn, yield dynamics. The signal production rate ξ needs to be sufficiently high to generate countercyclical uncertainty of consumption and term premia, as I describe below. The values for θ_c, χ_c and $\bar{r}$ are chosen for simplicity.

Table 2: **Parameter Values for the Simple Term Structure Model**

Parameter	Description	Value
ρ_a	AR(1) coeff. of persistent TFP	0.99
ρ_H	AR(1) coeff. of economic activity	0.8
$\sigma_a \times 100$	Standard deviation of persistent TFP shock	2
$\sigma_z \times 100$	Standard deviation of transitory TFP shock	20
$\sigma_s \times 100$	Standard deviation of noise shock	2
$\sigma_H \times 100$	Elasticity of H to TFP/noise shock	0.5†
ξ	Signal production rate	50
θ_c	Consumption rule coefficient	1
χ_c	Risk aversion	1
$\bar{r}$	Steady state interest rate	0

† For each impulse response exercise, only one of $\sigma_{H,a}$ and $\sigma_{H,s}$ is assigned this parameter value while the other is set to zero.

[25]A similar approach is adopted by Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011).

The main result is that *both* the persistent productivity shock $\varepsilon_{a,t}$ and the noise shock $\varepsilon_{s,t}$ generate a decrease in conditional volatility of productivity z_t, which in turn, leads to a drop in the term premium. Importantly, this is the case although only the $\varepsilon_{a,t}$ shock has a direct effect on productivity z_t itself, and each shock leads to different dynamics of productivity and consumption.

3.2.2 Impulse Responses to a Persistent Productivity shock

Productivity/Consumption: The dark blue lines in Figure 3 show how the baseline model with imperfect information ("model-BL") responds to a positive one standard deviation shock to the (unobserved) persistent component of productivity a_t. For reference, I also plot impulse responses for a version of the model with perfect information ("model-PI", light blue lines), in which $\sigma_s = 0$, and a version with imperfect information, but when the mass of signals do not vary with respect to economic activity and thus has constant precision ("model-CP", dashed dark blue lines), in which $\xi = 0$. This last version is a popular specification of imperfect information adopted in many studies.

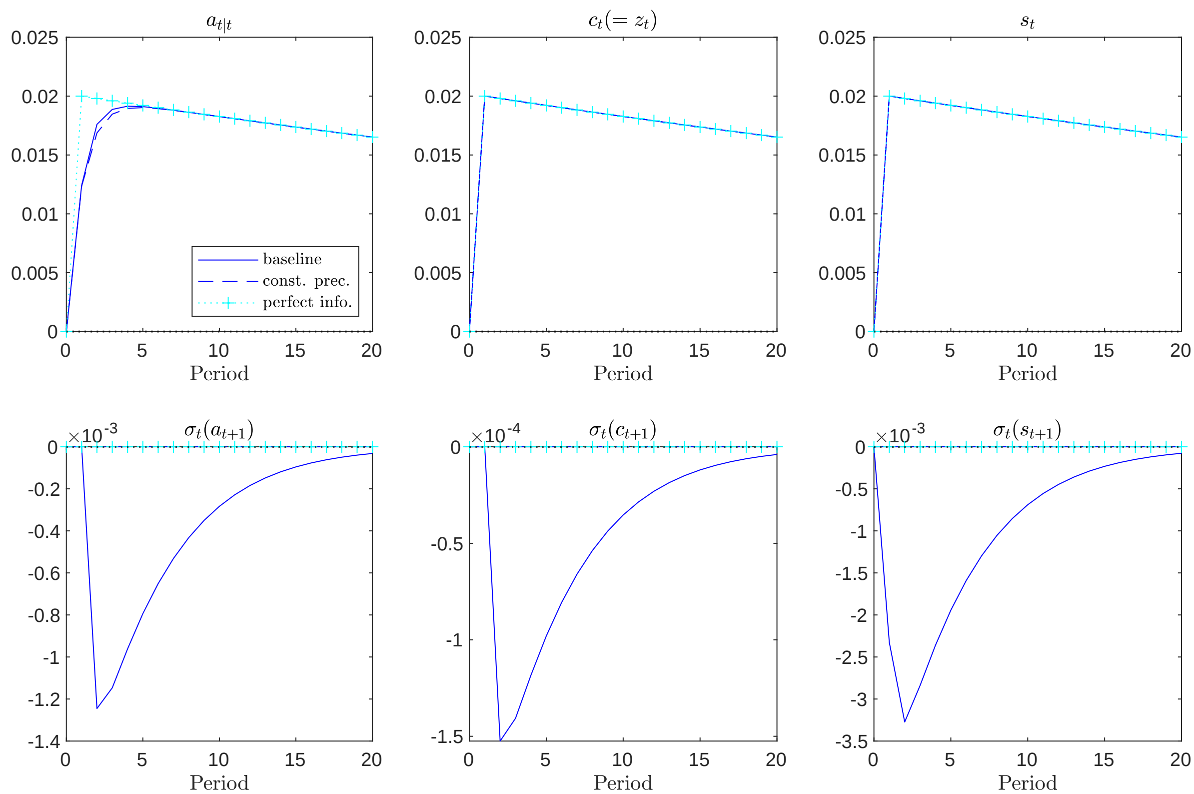

Figure 3: **Impulse Responses to $\varepsilon_{a,t}$ — productivity/consumption/signal**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

The top left panel shows the hump-shaped response of the contemporaneous belief about the persistent component of productivity a_t ($a_{t|t}$), which reflects learning about (and hence asymptotes to) the true level of a_t over time. This is in contrast to the response under perfect information, where the response of the belief completely tracks the true realization of a_t—a result documented in the literature.[26] However, compared to model-CP, the agent in model-BL learns faster as economic activity increases and more information about persistent productivity leads to an improvement in signal precision.

Indeed, as shown in the bottom left panel, the conditional forecast variance of a_{t+1} ($\sigma^2_{a,t}$) decreases in model-BL, but is unchanged in the other models. This point can be seen clearly from the recursive characterization of $\sigma_{a,t}$ (9). The sum inside the bracket in equation (9) is the precision of the belief of a_t as of time t: $(\mathbb{V}ar_t(a_t))^{-1}$. The first term inside the bracket is the contribution from the signal s_t, which, in model-BL, is increasing in H_{t-1} since $\phi'(H_{t-1}) > 0$, and hence, $\frac{\partial \sigma^2_{a,t}}{\partial H_{t-1}} < 0$.

Due to the underlying increase in its persistent component, (observed) productivity z_t and hence consumption c_t increases (top middle panel, note $c_t = z_t$ by assumption). As shown in the bottom middle panel, this does not affect the conditional volatility of consumption (productivity) in model-PI and model-CP. However, in model-BL, we see a significant *decrease* in the conditional volatility of consumption (dark blue line). In other words, model-BL can generate the well-documented empirical pattern of countercyclical uncertainty.

Term Structure of Interest Rates: Figure 4 shows corresponding impulse responses regarding the term structure of interest rates. The left and middle panel plots the responses of the 1-period risk free rate ($r_t^{(1)}$) and the n-period yield ($r_t^{(n)}$), respectively. As an example, the responses of the 5-period yield ($n = 5$) is shown. The declines in $r_t^{(1)}$ and $r_t^{(5)}$ across models are consistent with the Euler equation (11). In the models, an increase in productivity leads to an increase in consumption, but since consumption is stationary (around a deterministic trend), consumption growth is expected to decline over time. A decline in expected consumption growth can only be supported with a lower equilibrium interest rate encouraging the consumer to borrow more. The decline in rates of model-BL and model-CP are sharper compared to model-PI because when information is imperfect, the agent suspects the shock is transitory, which leads to a larger contraction in expected consumption growth. The responses of $r_t^{(5)}$ are smaller in magnitude compared to the response of $r_t^{(1)}$ due to the stationarity of interest rates, which is consistent with the empirical evidence of a downward sloping term structure of yield volatility.[27]

[26] See, for example, Edge, Laubach, and Williams (2007).

[27] See Section 4, and studies such as Cieslak and Povala (2016).

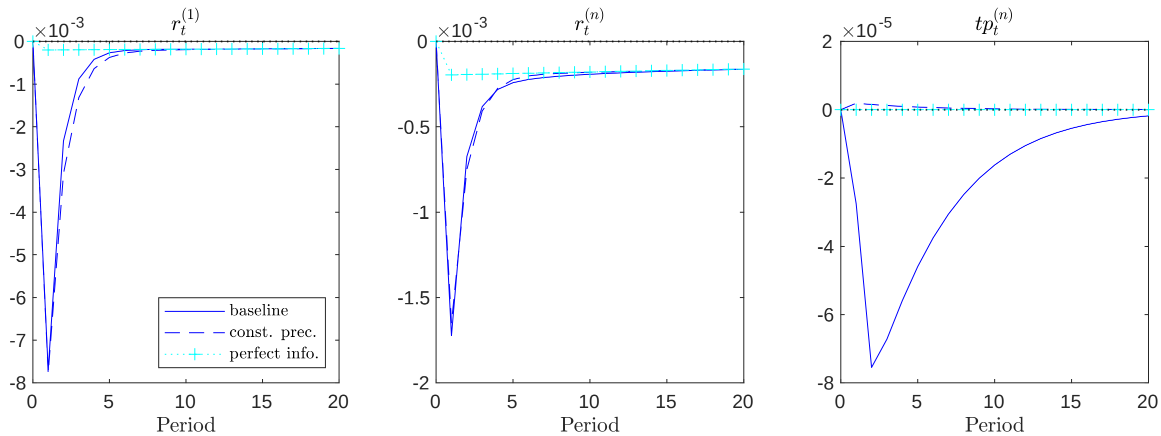

Figure 4: **Impulse Responses to $\varepsilon_{a,t}$ — term structure**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI). The yield maturity is $n = 5$.

The key result is the countercyclical drop in the term premium under model-BL, as shown in the right panel (in dark blue). One can understand this result through a combination of two largely distinct channels. The first channel is that under perfect information, the model is approximately homoskedastic, with a constant positive term premium. This is because stationarity in consumption leads to a negative autocorrelation in consumption growth, which in turn, leads to a negative autocorrelation in the stochastic discount factor. This implies that "bad (good)" times are likely to be followed by "good (bad)" times, i.e., there is a stronger demand to hedge for the near-term than for the longer-term, generating a positive term premium.[28] The second channel is the countercyclical uncertainty described above. Since the term premium is positive for a given level of consumption volatility, countercyclical consumption volatility results in countercyclical moves in the term premium as well. In other words, a rise (fall) in consumption volatility increases (decreases) the relative hedging value of shorter-term bonds even more, raising (lowering) the term premium. In Section 3.2.4, I make this point more formally through an analytical characterization of the two-period term premium.

The size of the decrease in the term premium is small compared to the decrease in yields in model-BL. However, the results of model-BL is in stark contrast with model-PI which generates little variation in the term premium. Interestingly, the fact that model-CP cannot generate meaningful variation in the term premium shows that a popular learning mechanism that assumes constant signal precision per se is not sufficient to generate variation in the term premium.

[28]This intuition has been documented in studies as early as Campbell (1986).

3.2.3 Impulse Responses to a Noise Shock

I now discuss the impulse responses to a noise shock $\varepsilon_{s,t}$. For ease of comparison with the impulse responses to $\varepsilon_{a,t}$, the (positive) shock is of the same size as $\varepsilon_{a,t}$, which results in the same response to H_t. The top left and middle panels of Figure 5 show distinct responses of $a_{t|t}$ and $c_t(= z_t)$ compared to the responses to $\varepsilon_{a,t}$. Since the shock does not impact persistent productivity itself, there is no response to $a_{t|t}$ under perfect information, while the agent learns that the signal was actually false only gradually under imperfect information. Similar to the impulse responses to $\varepsilon_{a,t}$, model-BL leads to faster learning about a_t compared to model-CP. Since the shock has no impact on actual productivity, there is no impact on consumption by definition.

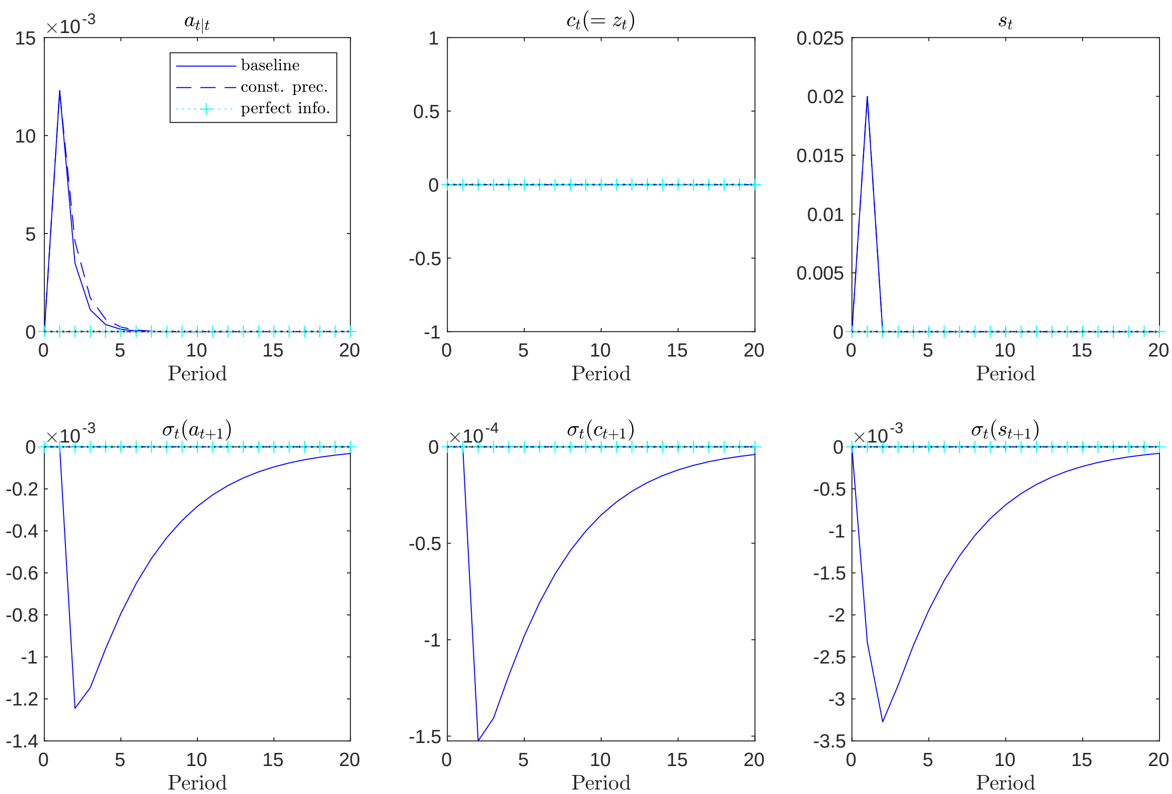

Figure 5: **Impulse Responses to $\varepsilon_{s,t}$ — productivity/consumption/signal**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

However, although the responses of $a_{t|t}$ and $c_t(= z_t)$ differ significantly from the responses to $\varepsilon_{a,t}$, the increase in H_t leads to an increase in precision of $a_{t|t}$ and a decrease in conditional uncertainty about consumption, similar to the case of a positive $\varepsilon_{a,t}$ (bottom left and middle panels).

As the first two panels of Figure 6 shows, yields rise in response to the shock since the shock has no effect on current consumption, but yet increases beliefs about future consumption under imperfect information (model-BL and model-CP). However, consumption beliefs are still stationary, and the countercyclical uncertainty generated from the $\varepsilon_{s,t}$ shock then leads to a countercyclical term premium for model-BL, as shown in the right panel of Figure 6. In contrast, both model-PI and model-CP cannot generate meaningful variation in term premia, similar to the responses to $\varepsilon_{a,t}$.

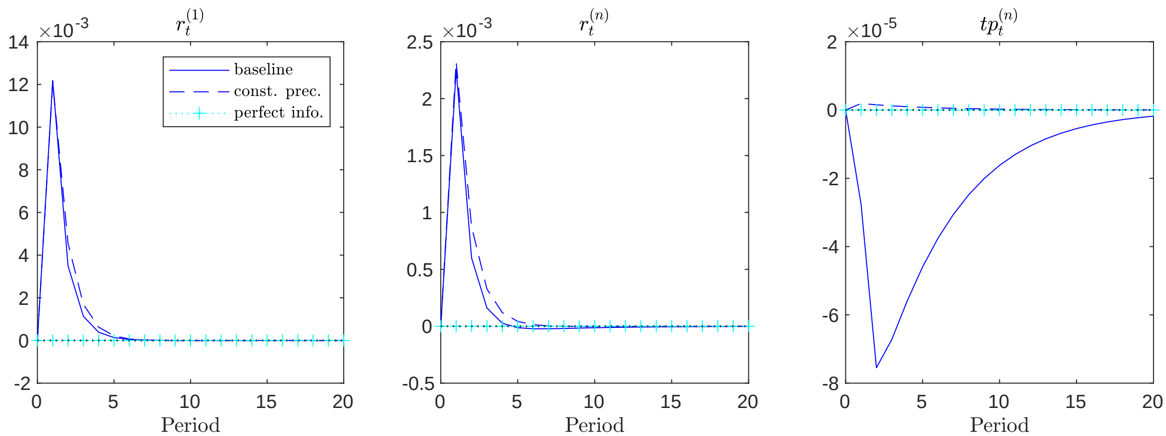

Figure 6: **Impulse Responses to $\varepsilon_{s,t}$ — term structure**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).The yield maturity is $n = 5$.

3.2.4 An Analytical Characterization

As I explained thus far, the model is stylized enough to make the key mechanism of countercyclical term premia transparent. Furthermore, I can show that the two-period term premium has a simple analytical expression that confirms the intuition explained in the previous sections. The conditional log-normality of consumption implies that the term premium of a two-period real bond is:

$$\begin{aligned} tp_t^{(2)} \equiv r_t^{(2)} - r_t^{(2)\mathbb{Q}} &\propto \mathbb{C}ov_t(m_{t+1}, r_{t+1}^{(1)}) \\ &= -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}^{(1)}) \\ &= (1-\rho_a)\sigma_{a,t}^2 + \sigma_z^2, \end{aligned} \tag{13}$$

where m_{t+1} is the (real) stochastic discount factor (See Appendix C for details of the derivation).[29] $\sigma_{a,t}^2$ is the uncertainty about the persistent component of productivity characterized by (9). Since

[29]For simplicity, I assume $\chi_c = \theta_c = 1$, as in the calibration. If the term premium was defined as the one-period excess return of the two-period bond (with a Jensen's correction), the final expression holds exactly, and not as a proportionality.

$\rho_a \in (0, 1)$, this expression shows that the two-period term premium is always positive. Intuitively, due to the (trend) stationarity of a_t, the one-period bond provides a better hedge against economic fluctuations than the two-period bond.

Recall from (9), $\frac{\partial \sigma^2_{a,t}}{\partial H_{t-1}} < 0$. This inequality and (13) further imply $\frac{\partial tp^{(2)}_t}{\partial H_{t-1}} < 0$. Therefore, the term premium is decreasing in H and hence in both ε_a and ε_s, i.e., the term premium is countercyclical.

Note that a_t is observable under perfect information, in which case:

$$tp^{(2)}_t \propto (1 - \rho_a)\sigma^2_a + \sigma^2_z \tag{14}$$

Therefore, the term premium is constant. In contrast, imperfect information leads to a time-varying term premium from underlying shocks that are homoskedastic. Moreover, although the term premium is still positive under perfect information, it is smaller compared to the term premium with imperfect information since $\sigma^2_{a,t} > \sigma^2_a$ from equation (9).

3.2.5 Summary—Towards a DSGE Term Structure Model

In sum, the analysis of the simple term structure model shows that *both* the persistent productivity shock $\varepsilon_{a,t}$ and the noise shock $\varepsilon_{s,t}$ generate a decrease in conditional volatility of productivity z_t, which in turn, leads to a drop in, or the countercyclicality of, the term premium. Importantly, this is the case although only the $\varepsilon_{a,t}$ shock has a direct effect on productivity z_t itself, and each shock leads to very different dynamics of productivity and consumption.

However, the model is limited in several ways. First, since the model does not feature inflation, the mechanism solely relies on the "real" channel. In particular, it is difficult to interpret the shocks in the model as the arguably more intuitive "demand" or "supply" shocks. Second, the microfoundation of H_t is somewhat unclear and the exogeneity of H_t eliminates some interesting channels where beliefs about technology affect the dynamics of H_t, and hence, consumption. Third, the model is not suited to assess the quantitative relevance of the featured mechanism of countercyclical term premia. In an attempt to address these issues, I analyze a DSGE term structure model with imperfect information that embeds the core of the simple model.

4 DSGE Term Structure Model with Imperfect Information

4.1 Model

I construct a DSGE term structure model with a mostly standard New-Keynesian core that features nominal price rigidities and a Taylor-type monetary policy rule. The key departure from the standard structure is the inclusion of imperfect information. While features such as Epstein-Zin preferences with habit formation are added to improve the quantitative performance of the model, I deliberately keep the rest of the model relatively simple and close to canonical models such as Rudebusch and Swanson (2012), so that the impact of imperfect information remains transparent.

4.1.1 Households

The representative household has Epstein-Zin (EZ) preferences (Epstein and Zin (1989)). Its value function V_t takes the following recursive form:

$$V_t = \begin{cases} U_t(C_t, N_t) + \beta \left\{ \mathbb{E}_t \left[V_{t+1}^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \geq 0 \\ U_t(C_t, N_t) - \beta \left\{ \mathbb{E}_t \left[(-V_{t+1})^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \leq 0. \end{cases} \tag{15}$$

β is the time discount rate. The period utility function $U_t(C_t, N_t)$ takes a standard form with external habits and separable labor disutility:

$$U_t(C_t, N_t) = \frac{\left(C_t - \chi_h \tilde{C}_{t-1} \right)^{1-\chi_c}}{1 - \chi_c} - G_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{1 + \chi_n}, \tag{16}$$

where $\chi_c > 0$ captures the attitude towards intertemporal substitution of consumption (net of habits) and $\chi_n > 0$ is the inverse Frisch elasticity. G_t is a deterministic trend in total factor productivity (TFP) which I return to later in the section. The scaling of labor disutility by G_t ensures the existence of a balanced growth path in equilibrium.

C_t is the household's aggregate consumption of final goods based on a CES aggregator of intermediate goods $C_t \equiv \left(\int_0^1 C_{i,t}^{1-\frac{1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of demand for the intermediate goods. $\tilde{C}_{t-1}$ is aggregate consumption in the previous period which is taken as given by the household (external habits). $N_t = \int_0^1 N_{i,t} di$ denotes the household's total supply of labor, which is the integral of labor $N_{i,t}$ supplied to each intermediate good producer $i \in [0, 1]$ in a perfectly competitive labor market. The household takes nominal wage W_t as given. $\tilde{\gamma}$ parameterizes the household's risk aversion. $\tilde{\gamma} = 0$ corresponds to the special case of power utility. Note that a larger $\tilde{\gamma}$ implies higher risk aversion when $U \geq 0$ and lower risk aversion when $U \leq 0$.

The household maximizes (15) by choosing state contingent paths for C_t, N_t and asset holdings subject to its initial wealth and the following sequence of flow budget constraints:

$$P_t C_t + \mathbb{E}_t \left[M_{t+1} \mathcal{W}_{t+1} \right] \leq W_t N_t + \mathcal{W}_t + D_t, \tag{17}$$

where the aggregate price level of the consumption basket $P_t \equiv \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is implied by the household's cost minimization problem (or equivalently, the optimization of a perfectly competitive representative final good producer combining intermediate goods). Assuming complete financial markets, $\mathcal{W}_{t+1}$ is the household's wealth portfolio of state contingent claims chosen by the end of period t. These claims are priced by the unique nominal pricing kernel M_{t+1} implied by the household's problem (for $U_t \geq 0$):

$$M_{t+1} = \beta \left(\frac{U_{C,t+1}}{U_{C,t}} \right) \left[\frac{V_{t+1}}{\left[\mathbb{E}_t \left[V_{t+1}^{1-\tilde{\gamma}} \right] \right]^{\frac{1}{1-\tilde{\gamma}}}} \right]^{-\tilde{\gamma}} \frac{1}{\Pi_{t+1}}, \tag{18}$$

where $U_{C,t} = \left(C_t - \chi_h \tilde{C}_{t-1} \right)^{-\chi_c}$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the (gross) aggregate inflation rate. The term with squared brackets is the additional term that appears by assuming EZ preferences, implying that the household is sensitive to the distribution of future consumption (and labor supply) in addition to current consumption growth. $D_t = \int_0^1 D_{i,t} di$ is aggregated firms' dividends rebated back to the household.

4.1.2 Firms

There are a continuum of intermediate goods producers (firms) indexed by $i \in [0, 1]$ who are monopolistically competitive and maximize their equity value. Each firm faces nominal rigidities in the form proposed by Calvo (1983) where a firm can reoptimize the price of its good with only a fixed probability $1 - \varphi$ in each period. Firm i's equity value $V_{i,t}^f$ is then:

$$V_{i,t}^f = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \varphi^\tau M_{t+\tau} \left\{ P_{i,t} \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right) Y_{i,t+\tau} - W_{t+\tau} N_{i,t+\tau} \right\} \right]. \tag{19}$$

When a firm cannot optimize its price, it indexes the price to a weighted average of inflation in the previous period and steady state inflation $\bar{\Pi}$. Each firm i is also subject to the demand and production functions for its own good $Y_{i,t}$:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t \tag{20}$$

$$Y_{i,t} = K_t^{1-\alpha} (G_t Z_t N_{i,t})^\alpha, \tag{21}$$

where Y_t is aggregate output taken as given by each firm and $K_t = \bar{K} G_t$ is the level of capital that grows deterministically with G_t (see next paragraph).

Productivity with imperfect information: Productivity (TFP) consists of two observable components G_t and Z_t. G_t is a deterministic trend component which grows at a rate of ζ (i.e., $\zeta = \frac{G_t}{G_{t-1}} - 1$). Z_t is a stationary component. Similar to the simple term structure model, it is composed of a "persistent" component A_t and a "transitory" component e_t:

$$Z_t = A_t + e_t. \tag{22}$$

A_t and e_t are *unobservable* and follow independent AR(1) processes:

$$A_t = (1-\rho_a)\bar{A} + \rho_a A_{t-1} + \sigma_a \varepsilon_{a,t} \tag{23}$$

$$e_t = \rho_e e_{t-1} + \sigma_e \varepsilon_{e,t}, \tag{24}$$

where $\varepsilon_{a,t}$ and $\varepsilon_{e,t}$ are i.i.d standard normal shocks that are also unobservable. I assume $1 > \rho_a \gg \rho_e \geq 0$.[30] Note the state space system in the simple term structure model was a special case of this system where the transitory component was assumed to be i.i.d.

As in the simple term structure model, firms also observe a continuum of noisy signals about the persistent component of productivity $s_{j,t}$, where $j \in [0, J_t]$. The mass of signals J_t is increasing in output ($J_t \equiv \phi(Y_{t-1})^2$ and $\phi' > 0$) and can be aggregated to a noisy public signal S_t.[31] Thus, S_t can be characterized as:

$$S_t = A_t + \frac{\sigma_s}{\phi(Y_{t-1})}\varepsilon_{s,t}, \tag{25}$$

where $\varepsilon_{s,t}$ is i.i.d standard normal and unobservable to the firms. I assume J_t is quadratic, and hence $\phi(Y_{t-1})$ is linear:

$$\phi(Y_{t-1}) \equiv \xi(Y_{t-1} - \bar{Y}) + \bar{Y}, \tag{26}$$

where $\xi > 0$ is now the parameter that controls the amount of productivity signals generated by the output gap $Y_{t-1} - \bar{Y}$.[32] Note the mass of signals (J_t) is increasing with respect to an endogenous variable (output), in contrast to the simple term structure model, in which J_t was largely exogenous.

Firms form beliefs about the unobservable components A_t and e_t by learning from observations on Z_t and S_t via a Kalman filter. In other words, the beliefs are updated through the following system of equations (27) through (29):

$$\boldsymbol{X}_{t|t} \equiv \mathbb{E}_t[\boldsymbol{X}_t] = \boldsymbol{\rho}\boldsymbol{X}_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{S}_t - \boldsymbol{S}_{t|t-1}) \tag{27}$$

$$\boldsymbol{K}_{t-1} = \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}_{s,t-1}')^{-1} \tag{28}$$

$$\boldsymbol{V}_{t+1|t} = \boldsymbol{\rho}(\boldsymbol{V}_{t|t-1} - \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}_{s,t-1}')^{-1}\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}')\boldsymbol{\rho}' + \boldsymbol{\Sigma}_x\boldsymbol{\Sigma}_x', \tag{29}$$

where $\boldsymbol{X}_t = [\hat{A}_t, e_t]'$, $\boldsymbol{S}_t = [\hat{Z}_t, \hat{S}_t]'$, $\boldsymbol{\rho} = [\rho_a, 0; 0, \rho_e]$, $\boldsymbol{\Psi} = [1, 1; 1, 0]$, $\boldsymbol{\Sigma}_x = [\sigma_a, 0; 0, \sigma_e]$, $\boldsymbol{\Sigma}_{s,t} = [0, 0; 0, \sigma_s/\phi(Y_t)]$. The "hat" variables indicate the demeaned versions. $\boldsymbol{K}_t$ is the Kalman gain matrix and $\boldsymbol{V}_{t|t-1}$ is the forecast variance matrix of $\boldsymbol{X}_t$.

[30] While the assumption that A_t is stationary is entirely standard, it is also important in obtaining an upward sloping real term structure. Various studies of the yield curve based on macroeconomic models therefore adopt this feature, such as Rudebusch and Swanson (2012). Nevertheless, I calibrate the process of A_t to be close to a random walk, consistent with empirical evidence. Note that although I assume TFP does not have a stochastic trend, it still allows for a deterministic trend.

Separately, the process of A_t is characterized in levels, but the realized values practically never fall below zero in the simulations.

[31] The assumption that J_t is a function of previous period output makes the model more tractable.

[32] Assuming J_t is linear in Y_{t-1} leads to broadly similar results, except that the time-variation in consumption growth and term premia is smaller.

The timeline of events within a period for each firm is summarized in Figure 7. After the unobserved shocks $\varepsilon_{a,t}$ and $\varepsilon_{s,t}$ are realized, firms update their beliefs about productivity based on signals $\boldsymbol{S}_t$, the precision of which is affected by output in the previous period Y_{t-1}. Then goods are produced based on the beliefs and dividends are paid out to the household.

Figure 7: **Timeline of Events for the Firm**

4.1.3 Monetary Policy

The central bank sets the (gross) nominal one-period interest rate, $R_t^{(1)}$, following a standard Taylor rule:

$$R_t^{(1)} = \left(R_{t-1}^{(1)}\right)^{\rho_r} \left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\Pi} \left[\frac{Y_t}{\bar{Y}Z_t}\right]^{\phi_Y}\right)^{1-\rho_r}, \tag{30}$$

where $\bar{R}$ and $\bar{Y}$ denote the steady state of $R_t^{(1)}$ and normalized output $\hat{Y}_t \equiv \frac{Y_t}{Z_t}$, respectively. I abstract from monetary policy shocks for simplicity.

4.1.4 Market Clearing

In equilibrium, the goods market, labor market, and asset market must clear at all dates and states. The clearing condition for final goods is:

$$Y_t = C_t + (\zeta + \delta)\bar{K}Z_t. \tag{31}$$

I focus on a symmetric equilibrium. Aggregating the supply of intermediate goods by integrating each producer's supply leads to:

$$Y_t = \frac{1}{\Delta_t} K_t^{1-\alpha}(G_t Z_t N_t)^{\alpha}, \tag{32}$$

where $\Delta \equiv \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} di$ is price dispersion, which follows:

$$\Delta_t = (1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + \varphi\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\theta}\Delta_{t-1}, \tag{33}$$

where P_t^* is the price set by the optimizing firm. For the asset market, I make a standard assumption that state contingent claims are in zero net supply.

4.1.5 Term Structure of Interest Rates

Given the equilibrium under complete markets, the price of a $n-$period zero-coupon nominal bond that pays one dollar at maturity $P_t^{(n)}$ can be derived recursively using the nominal stochastic discount factor (18):

$$P_t^{(n)} = \mathbb{E}_t[M_{t+1} P_{t+1}^{(n-1)}], \tag{34}$$

where $P_t^{(0)} = 1$ for $\forall t$. The continuously compounded yield to maturity of this bond follows directly from its price: $r_t^{(n)} = -\frac{1}{n} \ln P_t^{(n)}$. Note $r_t^{(1)}$ is the one-period nominal risk-free rate and $R_t^{(1)} = \exp(r_t^{(1)})$ in the monetary policy rule.

As described in Section 3.1, the $n-$period term premium $tp_t^{(n)}$ is computed from equation (12) with the risk neutral yield $r_t^{\mathbb{Q}(n)}$ computed by recursively discounting cashflows using the nominal risk-free rate (instead of the real risk-free rate). As also shown in Section 3.1, the yield to maturity and the term premium of a $n-$period zero-coupon real bond can be derived analogously, by simply replacing the nominal stochastic discount factor and the nominal one-period interest rate used for discounting the risk-neutral prices with their real counterparts.

4.1.6 Equilibrium Characterization

Given the initial condition $\{R_{-1}^{(1)}, Y_{-1}, \boldsymbol{X}_{-1|-1}, \boldsymbol{V}_{0|-1}\}$ and the exogenous processes $\{G_t, A_t, e_t, \varepsilon_{s,t}\}_{t \geq 0}$, a monopolistically competitive rational expectations equilibrium is defined in a standard way as a set of stochastic processes for quantities and prices such that (1) households maximize utility, (2) firms set prices and maximize profits identically (symmetry), (3) the central bank conducts monetary policy according to the interest rate rule, and (4) goods, labor and asset markets clear. Conditional expectations $\mathbb{E}[\cdot|\mathcal{I}_t]$ are defined over the information set $\mathcal{I}_t$ common across households, firms and the central bank, which precludes $\{A_{t-\tau}, e_{t-\tau}, \varepsilon_{s,t-\tau}\}_{\tau \geq 0}$.

To obtain a stationary equilibrium I follow the standard procedure of normalizing all relevant variables by the (deterministic) trend growth G_t. Equilibrium conditions are summarized in Appendix D.

4.1.7 Calibration

I calibrate the model to fit key moments of macroeconomic variables and moments of the term structure of interest rates using U.S. data. In particular, I target the first and second moments of quarterly data on consumption growth, inflation and interest rates covering a sample period from the beginning of 1990 to the end of 2008.[33] The end point is a conservative choice to avoid

[33] The data source is mostly standard. See Appendix A for details.

complications due to the ELB.[34] The calibrated parameter values are summarized in Table 3.

Table 3: **Parameter Values for the DSGE Term Structure Model**

Parameter	Description	Value	Parameter	Description	Value
Household			Monetary Policy		
$\tilde{\beta}$	Time discount rate	0.99	ϕ_π	Inflation gap coeff.	2.1
χ_c	1/IES	4	ϕ_y	Output gap coeff.	0.02
χ_h	External habit	0.3	ρ_r	Interest-rate smoothing coeff.	0.5
χ_n	1/Frisch elasticity	3			
RRA	Risk aversion	52	Exogenous Processes		
			ρ_a	AR(1) of persistent TFP	0.99
Firm			ρ_e	AR(1) of transitory TFP	0.7
θ	Demand elasticity	6	$\sigma_a \times 100$	Std of persistent TFP	0.41
φ	1 - price adjust. freq.	0.8	$\sigma_e \times 100$	Std of transitory TFP	0.41
ι_p	Indexation weight	0.5	$\sigma_s \times 100$	Std of noisy signal	1.9
α	Labor share in prod.	0.67			
δ	Capital depreciation rate	0.02			
ξ	Signal prod.	65			

χ_c, the inverse of the elasticity of intertemporal substitution (without habits) is set to 4. Accounting for external habits, the elasticity is 0.18, which is much smaller than 1, consistent with many macroeconomic studies.[35] The degree of habit persistence (χ_h) is set to 0.3, within the range found in previous studies.[36] χ_n is set such that the Frisch elasticity of labor supply is 1/3, in line with estimates from microeconomic studies. The risk aversion parameter ($\tilde{\gamma}$) is set to -21, which implies a relative risk aversion of 52, based on the measure proposed by Swanson (2018) that accounts for the household's ability to hedge risk by adjusting its labor supply.[37] While this value appears high relative to what is typically used in the macroeconomic literature, a number of models with EZ preferences aimed to price the yield curve requires a comparable or even higher level of risk aversion to fit the data. For example, the value I use is significantly lower compared to Rudebusch and Swanson (2012), which reports a risk aversion of 110 in their canonical DSGE term

[34] For an analysis on how the ELB affects yields and term premia using a DSGE term structure model, see for example, Nakata and Tanaka (2016).

[35] Log-linearizing the Euler equation implies an elasticity of $(1 - \chi_h(1+\zeta)^{-1})/\chi_c$.

[36] See, for example, Del Negro, Giannoni, and Schorfheide (2015).

[37] Swanson's risk aversion measure for recursive utility with external habit can be computed as:

$$RRA = \frac{\chi_c}{1-\chi_h(1+\zeta)^{-1}} \frac{1}{1+\frac{\chi_c \bar{W}\bar{N}}{\chi_n(1-\chi_h(1+\zeta)^{-1})\bar{C}}} + \frac{\tilde{\gamma}(1-\chi_c)}{1-\chi_h(1+\zeta)^{-1}} \frac{1}{1+\frac{(\chi_c-1)\bar{W}\bar{N}}{(1+\chi_n)(1-\chi_h(1+\zeta)^{-1})\bar{C}}},$$

where $\bar{X}$ is the steady state value of normalized X_t, i.e., X_t/Z_t. Intuitively, the first and third terms on the right hand side constitute the traditional measure of relative risk aversion abstracting from the flexible labor margin, while the second and fourth terms scale down that measure when labor supply is determined endogenously.

structure model.[38] The deterministic trend growth in TFP is set to 2.1 percent per year, which aligns the model-implied average consumption growth rate to the data. The time discount rate $\tilde{\beta}$ is set to 0.99, which implies an average 1-quarter real interest rate of 2.0 percent taking into account the trend growth rate.

The parameters that characterize the firms' problem are set to values fairly standard in the literature; the elasticity of substitution among intermediate goods (θ) is set to 6, and the probability with which a firm cannot readjust its price each period (φ) is set to 0.8. The firm uses a price indexation scheme where it places 50 percent weight (ι_p) on previous period inflation and another 50 percent weight on steady state inflation. The Cobb-Douglas parameter for the labor share (α) is set to 0.67 and the capital depreciation rate (δ) is set to 0.02. The choices of the steady state capital stock ($\bar{K}$) and persistent component of technology ($\bar{A}$) largely determine the capital-output ratio of 2.5, the value also targeted by Rudebusch and Swanson (2012).

For the parameters of the monetary policy rule, I set the coefficient on inflation (ϕ_π) and the output gap (ϕ_y) to be 2.1 and 0.02 respectively. These values are fairly standard—for example, they are in line with the parameters estimated for several variants of the Smets-Wouters model in Del Negro, Giannoni, and Schorfheide (2015). The interest-rate-smoothing parameter (ρ_r) is set to 0.5, a value somewhat smaller than what is reported in Del Negro, Giannoni, and Schorfheide (2015). However, as shown by Rudebusch (2006), policy inertia could be overestimated, and in fact, the value I use is close to the estimate in Blanchard, L'Huillier, and Lorenzoni (2013). The steady state inflation target rate is set such that it implies an average annual inflation of 2.0 percent, which is lower than the average core CPI inflation, but very close to the average core PCE inflation over the sample period.

ξ controls the amount of signals generated from output, and is a key parameter that affects the conditional uncertainty of macroeconomic variables and term premia, as we discuss further below. Since there is no clear empirical counterpart to the signals in the model, I discipline the parameter based on measures of conditional uncertainty about GDP growth from the Survey of Professional Forecasters (SPF), similar to Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017). Specifically, I take the standard deviation of the average forecast distribution of GDP growth in the SPF (reported each quarter) as a measure of conditional uncertainty about GDP growth, and set ξ such that the model is in line with the (time-series) property of this standard deviation.[39]

I set $\rho_a = 0.99$, so that a_t is close to a random walk, but still stationary. The rest of the parameters characterizing the exogenous processes (ρ_e, $\sigma_{\{a,e,s\}}$) are determined such that the model fits the volatility of consumption growth and inflation, as well as the moments of the term structure.

While models with information frictions can face computational challenges and be hard to solve without linearization, my particular specification remains relatively tractable. I solve the model

[38] As explored in Andreasen and Jørgensen (2019), an extension that explicitly models the timing attitude of consumers may reduce the risk aversion even further.

[39] From the SPF, I take each survey in the fourth quarter and compute the standard deviation of the distribution for the current year forecast of year-over-year GDP growth. See Figure 2 for a visual of the series. I consider this standard deviation as a proxy for conditional volatility of year-over-year GDP growth 2-quarters ahead in the model, taking into account the uncertainty from future revisions of GDP data.

using a third-order perturbation method to properly account for time-variation in volatility and term premia. The state-space system is pruned based on the method of Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018).

4.2 Results

4.2.1 Moments

Table 4 summarizes the quantitative performance of the model by comparing model-implied moments with those of the data. The second column reports the moments from the data, and the third column reports the moments of the simulated data from the model with imperfect information. For reference, the last column reports the moments from the model with perfect information, which sets the standard deviation of the noise shock (σ_s) to zero while keeping the rest of the parameters unchanged from the model with imperfect information.

Table 4: **Selected Moments**

	Data	Model (Imperfect Info.)	Model (Perfect Info.)
Macro Variables			
$\mathbb{E}[\Delta c]$	2.12	2.09	2.09
$\mathbb{E}[\pi]$	2.70	2.02	2.22
$\sigma[\Delta c]$	1.39	1.33	1.10
$\sigma[\pi]$	0.81	0.91	0.84
$\rho[\Delta c, \pi]$	-0.24	-0.07	-0.14
$\mathbb{E}[\sigma[\Delta c]]$	0.66	0.70	0.67
$\sigma[\sigma[\Delta c]]$	0.18	0.10	0.06
Yields			
$\mathbb{E}[r^{(1)}]$	4.00	4.08	4.51
$\mathbb{E}[r^{(20\to40)}]$	6.44	6.07	6.16
$\mathbb{E}[r^{r,(20\to40)}]$	2.97	2.94	2.98
$\mathbb{E}[tp^{(20\to40)}]$	2.03	1.99	1.66
$\sigma[r^{(1)}]$	1.81	1.70	1.63
$\sigma[r^{(20\to40)}]$	1.28	1.25	0.88
$\sigma[r^{r,(20\to40)}]$	0.72	0.55	0.36
$\sigma[tp^{(20\to40)}]$	0.85	0.60	0.30

Notes: Data is quarterly and in annualized percent. Sample period is from 1990Q1 to 2008Q4.

In terms of the macro variables, the averages and standard deviations of consumption growth and inflation from the model with imperfect information are in line with the data. The model also captures the negative correlation between consumption growth and inflation, although the magnitude is relatively smaller. The model with perfect information generates a somewhat reduced standard deviation of consumption growth and inflation. This is intuitive since imperfect information adds uncertainty to the economy by introducing a shock to the signal. The correlation between consumption growth and inflation is more negative for the model with perfect information, but the mechanism is not completely obvious, a point which I revisit later in Section 4.2.3.

The table also reports the mean ($\mathbb{E}[\sigma[\Delta c]]$) and standard deviation ($\sigma[\sigma[\Delta c]]$) of conditional volatility of consumption (= GDP) growth 2-quarters ahead. The mean of conditional volatility is a bit larger in the model with imperfect information compared to the model with perfect information, but both are broadly consistent with the data. In terms of the standard deviation of conditional volatility, both models fall short of fitting the data, but the model with imperfect information generates a significantly larger variation in conditional volatility, nearly doubling what is implied by the model with perfect information. This ability to generate sizable time-varying volatility in macro variables translates to better performance in fitting the term structure of interest rates, as I explain below.

The model with imperfect information can fit several important moments of the yield curve well. As a proxy for long-term yields, I focus on the 5-to-10-year forward rate so the results are easily comparable with the empirical analysis in Section 2. As shown by the fit to the average 1-quarter nominal rate ($\mathbb{E}[r^{(1)}]$) and the average 5-to-10 year nominal forward rate ($\mathbb{E}[r^{(20\to40)}]$), the model can match the average level and (upward) slope of the nominal yield curve data. The model can also fit the unconditional standard deviations of nominal yields well, as indicated by its fit to the standard deviations of the 1-quarter nominal rate ($\sigma[r^{(1)}]$) and the 5-to-10 year forward rate ($\sigma[r^{(20\to40)}]$). Thus, the model-implied yield volatility curve is downward sloping, as suggested by the data.

While the model with perfect information can also replicate the qualitative pattern of an upward sloping yield curve and downward sloping yield volatility curve, the average slope is significantly smaller, and the yield volatility curve is significantly more downward sloping than what the data and the model with imperfect information suggest.

It is worth emphasizing the model's fit to empirical estimates of the term premium.[40] In particular, the model with imperfect information can generate a sizable average term premium for the nominal 5-to-10-year rate ($\mathbb{E}[tp^{(20\to40)}]$). The nominal term premium is positive because, in the model, (1) average real term premium is positive due to the (trend) stationarity of consumption, and (2) average inflation risk premium is positive due to the negative correlation between (longer-run) consumption growth and inflation. Additionally, while the term premium is not as volatile as what an average of the empirical estimates suggests, it still explains about 70 percent of the standard deviation of its empirical counterpart ($\sigma[tp^{(20\to40)}]$). This is in clear contrast to the model with

[40]The values for the mean and standard deviation of the 5-to-10-year nominal term premium listed in the data column are the averages over the three term premium estimates analyzed in Section 2.

perfect information, in which both the average and volatility of term premia are significantly smaller, and the model-implied volatility is only about a third of its empirical counterpart. Importantly, the increase in the volatility of term premia by incorporating imperfect information is not simply a reflection of higher volatility in consumption growth and inflation. For example, the ratio of term premium volatility to consumption growth (inflation) volatility is 45 (66) percent for the model with imperfect information, while 27 (36) percent for the model with perfect information. In other words, imperfect information is a useful channel to explain the large variation in term premia compared to variation in macro variables, which is harder to explain with a model with perfect information.

Lastly, the model with imperfect information fits the average 5-to-10-year real rate based on TIPS data ($\mathbb{E}[r^{r,(20\to40)}]$). Given the model also fits the average 1-quarter nominal rate and one-quarter inflation, the model can fit the average real yield curve, which is upward sloping in the data, but more mildly so, compared to the nominal yield curve. While the model-implied volatility of the 5-to-10 year real rate ($\sigma[r^{r,(20\to40)}]$) is smaller than the data, the volatility is larger compared to the model with perfect information.[41]

The reason for the large increase in the volatility of the term premium is its countercyclical variation generated through imperfect information. To understand the role of imperfect information further, I next turn to impulse responses, which isolate the contribution of each shock to the dynamics of macro variables and the term structure of interest rates.

4.2.2 Impulse Responses

Figures 8 through 11 show impulse responses of the DSGE term structure model. The format closely follows that of the impulse responses of the simple term structure model in Section 3, but here I plot the generalized impulse responses of Koop, Pesaran, and Potter (1996), which are the nonlinear responses to the average path without the shock. The dark blue lines show responses of the baseline DSGE model with imperfect information ("model-BL"). For reference, I also plot impulse responses of the model with perfect information ("model-PI", light blue lines) in which $\sigma_s = 0$, and of a version with imperfect information, but when the mass of signals do not vary with respect to output and thus has constant precision ("model-CP", dashed dark blue lines) in which $\xi = 0$.

The main result is that consistent with the findings from the simple term structure model with imperfect information, *both* the persistent shock to productivity and the shock to the signal about

[41] Duffee (2018) proposes an alternative way to assess a macro-finance term structure model's fit to the data, by looking at the ratio of the variance of news about expected inflation to the variance of yield shocks. Specifically, the inflation variance ratio for yield maturity m is computed as $\frac{\mathbb{V}\mathrm{ar}[\eta^{(m)}_{\pi,t}]}{\mathbb{V}\mathrm{ar}[\tilde{r}^{(m)}_t]}$, where $\eta^{(m)}_{\pi,t} \equiv \mathbb{E}_t\left[\frac{1}{m}\sum_{i=1}^{m}\pi_{t+i}\right] - \mathbb{E}_{t-1}\left[\frac{1}{m}\sum_{i=1}^{m}\pi_{t+i}\right]$ and $\tilde{r}^{(m)}_t \equiv r^{(m)}_t - \mathbb{E}_{t-1}[r^{(m)}_t]$. Duffee argues that at a quarterly frequency, this ratio is between 10 to 20 percent in the data. It turns out that the model with imperfect information implies a variance ratio of 16 and 18 percent at the 5- and 10-year maturity, respectively, which is consistent with Duffee's observation and my finding that the model reasonably fits other moments of the data. Interestingly, the model with perfect information under my specification implies a similar variance ratio, suggesting that adding imperfect information does not necessarily seem to improve the model fit based on this particular measure.

productivity significantly amplify the countercyclicality of term premia. However, in contrast to the simple model, the DSGE model allows the shocks to be interpreted intuitively as a "supply shock" and a "demand shock", respectively.

Impulse Responses to a Persistent Productivity shock

Figure 8 shows responses of various macroeconomic variables to an (unobserved) positive persistent TFP shock ($\varepsilon_{a,t}$). The responses of model-BL show that consumption (top left) rises as inflation drops (top middle). The opposite response of consumption and inflation allows the shock to be interpreted as a "supply shock", a feature which is common across all model specifications. Consumption shows a hump-shaped response which is well-documented in the literature to be empirically plausible. This is partly due to habit formation, which is why the hump-shape can be seen across all specifications. More importantly, the hump-shape is also generated from incorporating imperfect information through a gradual learning of the persistence in the TFP shock. This can be seen from the more delayed response in model-BL and model-CP compared with the response in model-PI. Furthermore, consumption in model-BL shows a somewhat faster increase compared with consumption in model-CP. The mechanism is similar to that in the simple model; the TFP shock increases consumption (output), which generates more signals about the persistent component of TFP. The information is shared among the agents via social learning, and the signals become more precise in the aggregate. Indeed, model-BL shows a decrease in the volatility (or increase in the precision) of beliefs about the persistent component of TFP (top right) as well as volatility of TFP per se (bottom right). Such a response cannot be observed in model-CP and model-PI. These results confirm that the mechanism of the simple model in Section 3 holds more generally in the DSGE model.[42]

Compared with model-PI, inflation in model-BL decreases more, reflecting the slower pickup in consumption (demand). The decrease in inflation is most pronounced for model-CP, since household's learn about the persistent productivity the slowest in this model, and thus demand is held back the most. Labor hours (bottom left) decrease upon the shock for all specifications, which is consistent with some leading DSGE models (with perfect information) such as Smets and Wouters (2003).[43] Real wages (bottom middle) generally rise as productivity increases, but the rise is slower for model-BL and model-CP, in which it takes time to learn that the increase in productivity will persist.

Figure 9 shows the responses of interest rates and term premia. The 1-quarter nominal rate (policy rate) declines in model-BL (top left), which is similar across other specifications, and is a standard response to a positive supply shock as the central bank accommodates to alleviate deflationary pressure. The degree of accommodation is most pronounced in model-CP, which is

[42]In the simple model, this mechanism was seen in the belief of the persistent component of productivity as opposed to consumption since the latter was equal to productivity in that model.

[43]Labor hours can be made to respond positively by using a utility function proposed by Greenwood, Hercowitz, and Huffman (1988). I find that the impulse responses for the other variables are qualitatively similar with this modification.

followed by model-BL, reflecting the magnitude of the decline in inflation. The relatively large drop in the policy rate compared to longer-term yields leads to an increase in the nominal yield spread in model-BL and model-CP (top middle). These responses of the yield spread are consistent with studies that use similar models with perfect information.[44] The 1-quarter real rate (bottom left) and the real yield spread (bottom middle) show a qualitatively similar pattern as their nominal counterparts.

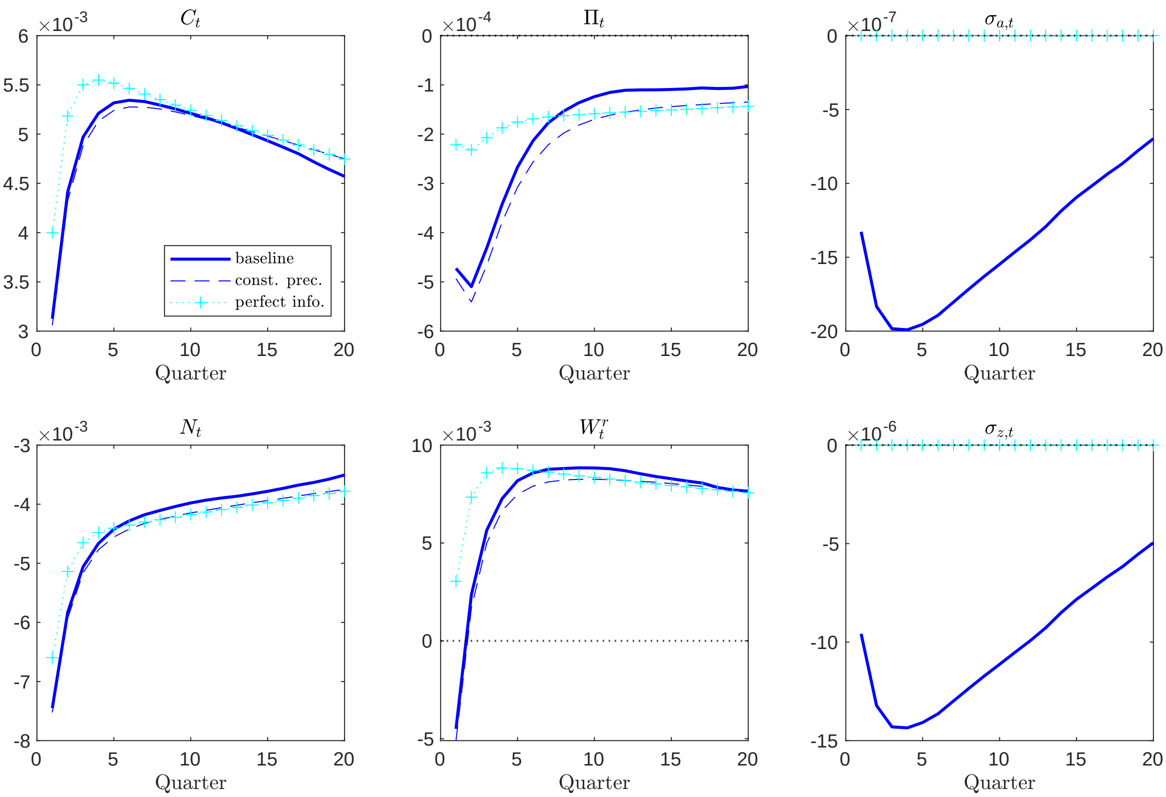

Figure 8: **Impulse Responses to $\varepsilon_{a,t}$ — macro variables**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

The key feature of model-BL that differentiates it from the other specifications is the response of the term premium. In model-BL, both the nominal (top right) and real (bottom right) term premia fall in response to the TFP shock. The countercyclicality of real term premia follows from countercyclical uncertainty about productivity, and hence other macro variables, as explained in Section 3 using the simple term structure model. The (trend) stationarity of consumption implies a positive term premium on average, and the countercyclicality of consumption volatility means

[44]See, for example, Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018).

that the level of the term premium is compressed due to a decrease in the quantity of risk when the economy is booming. Such a mechanism is non-existent in model-PI and model-CP, and term premium variation in both specifications is significantly smaller.

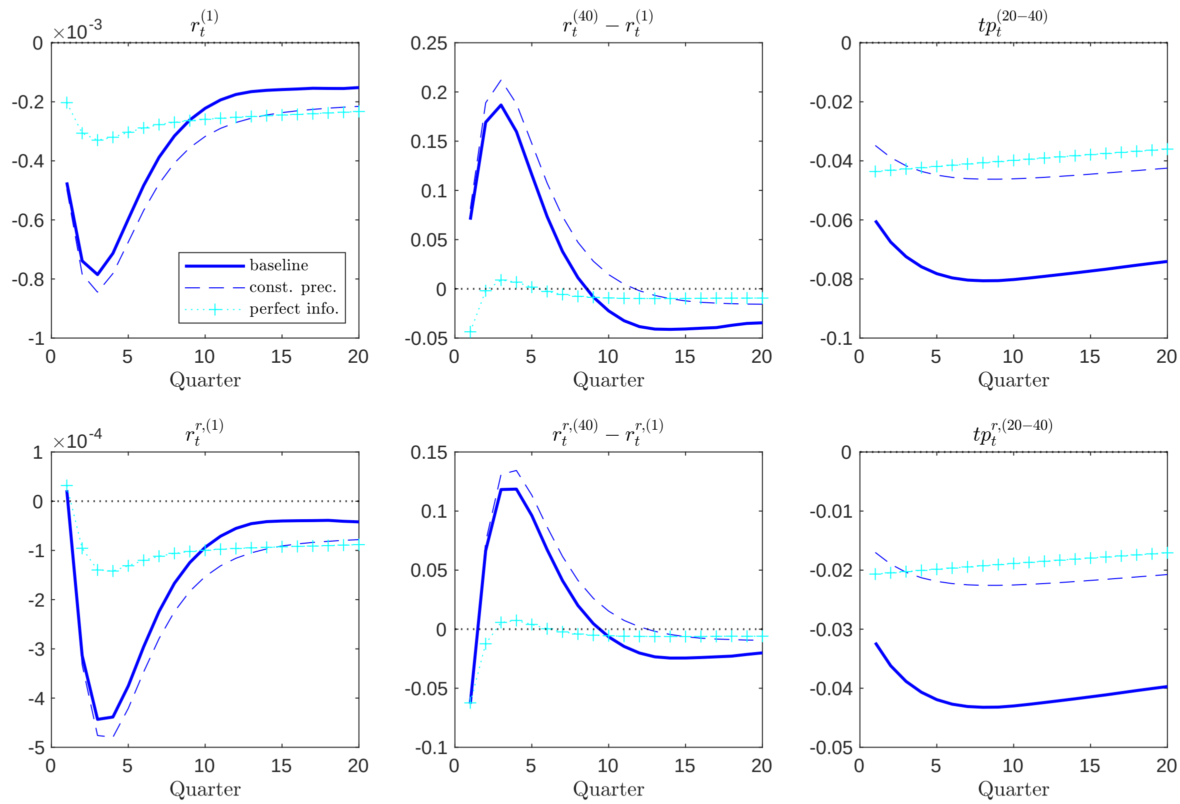

Figure 9: **Impulse Responses to $\varepsilon_{a,t}$ — term structure**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

Furthermore, the nominal term premium shows a similar countercyclical pattern, but with a larger magnitude. This is because the inflation risk premium is also countercyclical in the models. In particular, to understand the significant countercyclicality in model-BL, recall that the correlation between consumption growth and inflation is negative, and hence the inflation risk premium is positive. In addition, since inflation risk premium increases with productivity uncertainty which is countercyclical, inflation risk premium is also countercyclical.[45]

The fact that the response of the term premium in model-CP is as small as that in model-PI shows that time-varying signal precision is crucial in generating variation in term premia, and

[45]See Section 4.2.3 for further discussion on the inflation risk premium. Also, note what matters for the positive longer-term inflation risk premium is the correlation of consumption growth and inflation over multiple periods, as opposed to the one-period correlation.

not imperfect information *per se*. While other studies discuss different mechanisms that result in countercyclical term premia due to supply shocks, my model with imperfect information offers a complementary mechanism that is intuitive and consistent with the data.[46] In addition, my model does not require an independent shock to the volatility of TFP, providing a deeper microfoundation to setups that use exogenous stochastic volatility.

In the context of matching the data on consumption and the term structure jointly, the fact that consumption dynamics feature an endogenous hump-shaped response due to learning is an appealing feature, and complements other mechanisms such as habit formation. While the highly persistent, but trend stationary feature of consumption ensures the (real) term premium to be positive on average, the gradual increase in consumption after the response leads to a positive autocorrelation of consumption growth in the near-term, consistent with the data.

Impulse Responses to a Noise Shock

In this section, I analyze the impulse responses to an (unobserved) positive noise shock ($\varepsilon_{s,t}$). As in Figure 8, Figure 10 collects the responses of key macroeconomic variables. Since this shock plays no role under model-PI, the relevant comparison with model-BL will only be model-CP.

A positive noise shock to the signal leads to increases in both consumption (top left) and inflation (top middle) in model-BL and model-CP. This is because under imperfect information, a positive noise shock makes the consumer believe her present value of income has increased due to a persistent increase in productivity, which raises consumption. However, since productivity did not actually increase, supply cannot increase in tandem, creating upward pressure on inflation. The initial impact of a one standard deviation noise shock is about a third of the impact of a TFP shock of the same magnitude, and the impact on inflation is about two thirds of a TFP shock in model-BL. The positive correlation between consumption and inflation suggests that the shock can be clearly interpreted as a "demand shock", which confirms the results of related studies.[47]

In addition to consumption and inflation, labor hours (bottom left) and real wages (bottom middle) rise following the shock, for both model specifications. Similar to when a positive persistent TFP shock hits the economy, the increase in output leads to a decrease in the volatility (or increase in the precision) of beliefs about the persistent component of TFP (top right) as well as volatility of TFP per se (bottom right). Again, such a countercyclical response of volatility cannot be seen in the other models.

Figure 11 shows the responses of interest rates and term premia. Both the 1-quarter nominal rate (policy rate, top left) and to a lesser extent, the 1-quarter real rate (bottom left) increases similarly for both model-BL and model-CP. This is a standard response to a positive demand shock as the central bank tightens monetary policy to reign in economic activity. The relatively large increase in the policy rate compared to longer-term yields leads to a decrease in the nominal yield

[46] See, for example, Rudebusch and Swanson (2012), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018), and Swanson (2019).

[47] See, for example, Lorenzoni (2009) and Blanchard, L'Huillier, and Lorenzoni (2013). Note these models fall into the class of model-CP, as they do not exhibit time-varying signal precision.

spread in both models (top middle). As the decrease in the yield spread is followed by a decrease in consumption and inflation, such a response is largely consistent with strong empirical evidence of the slope of the yield curve being a leading indicator of the business cycle. The results show that noise shocks help explain the empirical pattern.

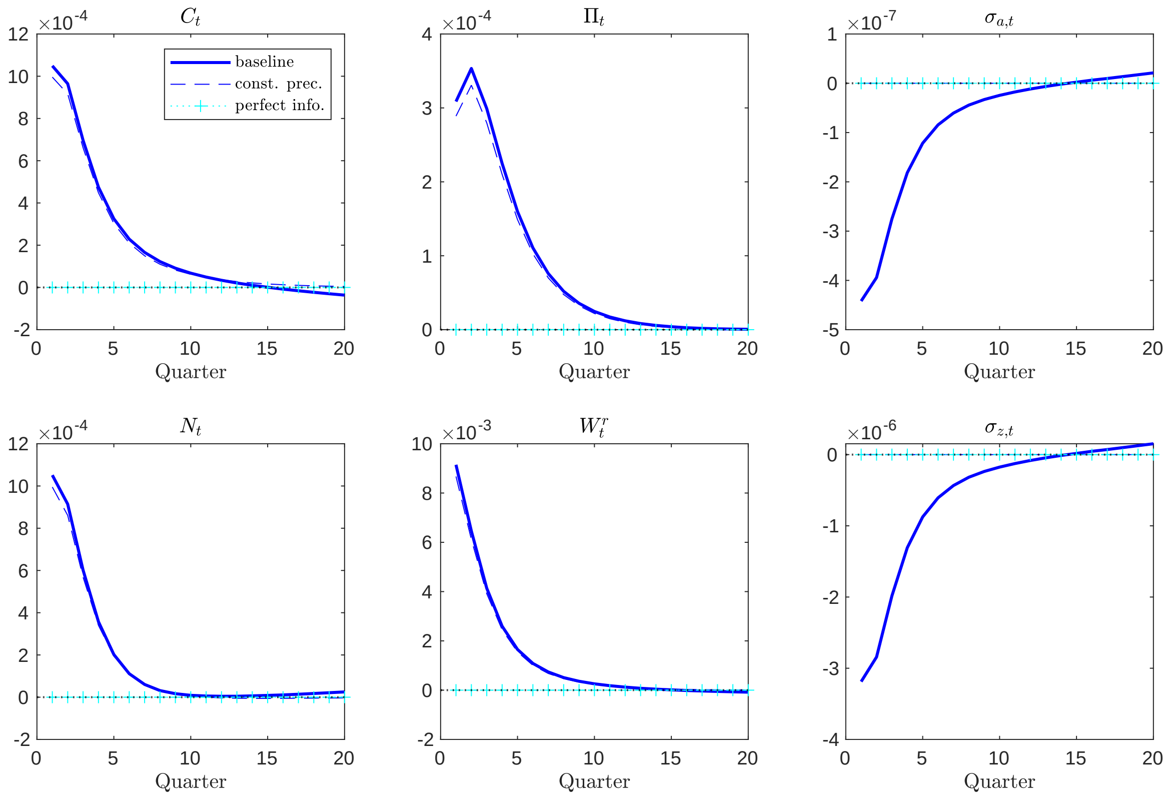

Figure 10: **Impulse Responses to $\varepsilon_{s,t}$ — macro variables**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

Similar to the case of a positive persistent TFP shock, the countercyclical response of volatility to a signal shock leads to a fall in both the nominal and real term premia in model-BL (top and bottom right). The key difference is that the drop in term premia is now associated with a demand shock that has distinct effects on the macroeconomy compared to a supply shock. While term premia in model-CP also show a countercyclical decline, the magnitude is significantly less than in model-BL. In sum, I show that the noise shock about productivity—despite being a *demand* shock—becomes an important driver of the term premium through changes in the perceived properties of productivity, i.e., *supply*.

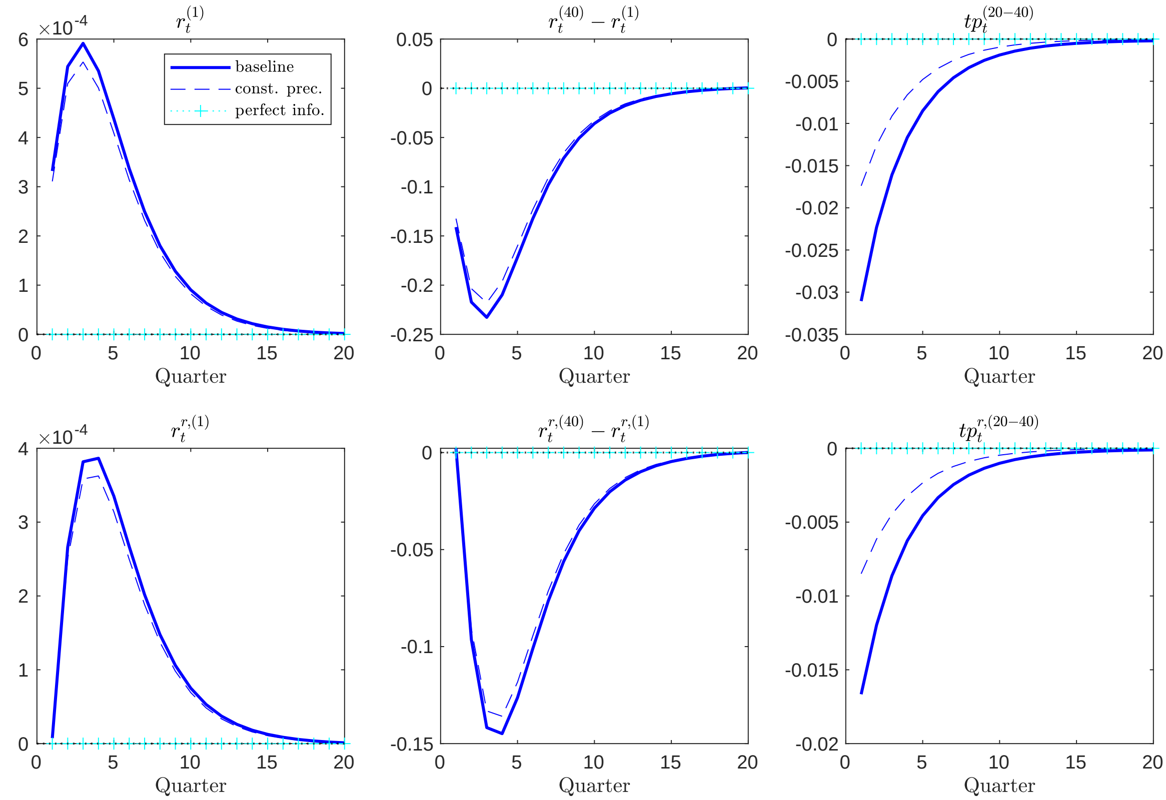

Figure 11: **Impulse Responses to $\varepsilon_{s,t}$ — term structure**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

4.2.3 Further Discussion on the Effect of Imperfect Information

In this section, I provide further discussion on how imperfect information affects the model dynamics. In particular, I isolate the roles of the two key parameters that characterize the information friction—the volatility of the noise shock (σ_s), and the parameter that controls the rate of signal production (ξ), and provide a more detailed study about their impact on term premia. I also use the comparative statics to offer some thoughts on the increase in the correlation of consumption growth and inflation observed over the last few decades and its relation to term premia.

The effect of imperfect information on term premia: Figure 12 shows how the signal specification affects nominal term premia, by plotting the model-implied average (left panel) and volatility (right panel) of the 5-to-10 year nominal term premium with respect to σ_s, and for different values of ξ. σ_s and ξ each leads to an increase in both the average term premium and the volatility of term premium, but the way in which each parameter affects term premia is quite different. The average of the 5-to-10 year nominal term premium ($\mathbb{E}[tp^{(20\to 40)}]$) monotonically increases with respect to σ_s. The panel also shows that ξ has a negligible impact on $\mathbb{E}[tp^{(20\to 40)}]$. In contrast, the volatility of the 5-to-10 year nominal term premium ($\sigma[tp^{(20\to 40)}]$) has a hump-shape with respect to σ_s, which becomes more pronounced as ξ increases. Also, a sufficiently large ξ is key in generating time variation in the term premium.

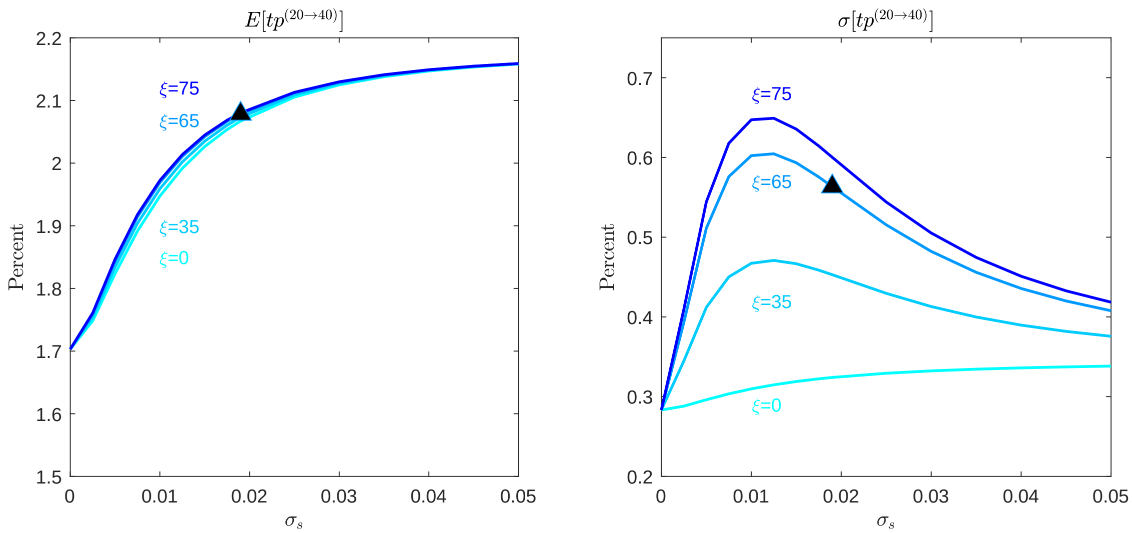

Figure 12: **Effect of the Noisy Signal on Nominal Term Premia**

Notes: Each line corresponds to the moments generated from the indicated value of ξ. The black triangle indicates the moment from the baseline model.

To better understand the result, I further decompose the effect on the nominal term premium into the effect on the inflation risk premium and the real term premium. Figure 13 shows that in terms of averages (left panel), the real term premium (solid lines) increases with respect to σ_s but is largely insensitive to ξ, and these effects are comparable to the effects on the inflation risk premium (dashed lines). However, in terms of volatilities (right panel), both parameters have outsized effects on the real term premium.

The intuition for the effects of σ_s and ξ on the real term premium can be understood clearly by revisiting the analytical expression for the two-period real term premium (13) in Section 3.2.4. From equations (13) and (9), it is immediate that the average real term premium is (monotonically) increasing in σ_s through an increase in $\sigma_{a,t}$, i.e., a noisier signal increases uncertainty about the persistent component of TFP, and leads to an increase in the term premium. In contrast, the volatility of the real term premium is monotonically increasing in σ_s when $\xi = 0$, but is hump-shaped when $\xi > 0$, and the hump becomes more pronounced as ξ increases. When $\xi > 0$, the term ϕ/σ_s^2 in (9) can become an important source of time-variation in the term premium and ξ increases its variability, all else fixed. However, the effect of a time-varying ϕ disappears either as $\sigma_s \to 0$ (since $\phi/\sigma_s^2 \to \infty$, and hence $\sigma_{a,t}^2 \to \sigma_a^2$ (constant)), or as $\sigma_s \to \infty$ (since $\phi/\sigma_s^2 \to 0$, and hence

$\sigma_{a,t}^2 \to \bar{\sigma_a}^2$ (another constant), where $\bar{\sigma_a}$ solves $\bar{\sigma_a}^2 = \rho_a^2 \left(\sigma_z^{-2} + \bar{\sigma_a}^{-2}\right)^{-1} + \sigma_a^2$).

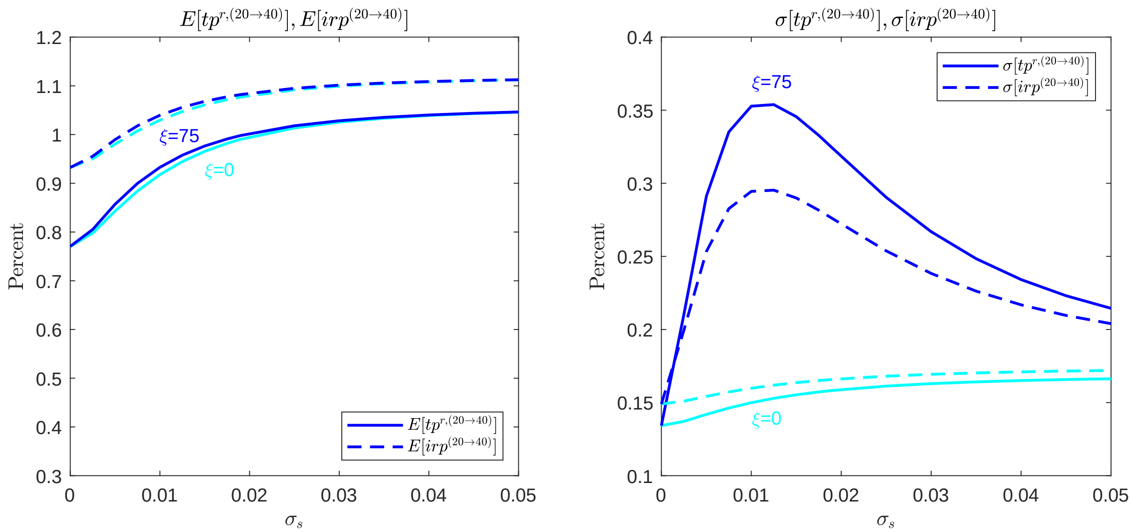

Figure 13: **Effect of the Noisy Signal on Inflation Risk Premia and Real Term Premia**

Notes: The solid lines are the moments of the real term premium ($tp^{r,(20\to40)}$). The dashed lines are the moments of the inflation risk premium ($irp^{(20\to40)}$). Different colors correspond to the moments generated from different values of ξ.

The effects of σ_s and ξ on the inflation risk premium turns out to be qualitatively similar, and the mechanism is analogous to the effect on the real term premium. To guide intuition, consider a simple extension of the two-period term premium analysis in Section 3.2.4, where I add inflation π_t specified as $\pi_t = -\theta_\pi z_t$, where $\theta_\pi > 0$ is an exogenous parameter. Then, the two-period inflation risk premium ($irp_t^{(2)}$) is:

$$irp_t^{(2)} \propto \mathbb{C}ov_t(m_{t+1}, \pi_{t+2}) \propto \theta_\pi \rho_a \sigma_{a,t}^2. \tag{35}$$

In other words, the inflation risk premium is an increasing function of $\sigma_{a,t}$ just like the real term premium. Hence, the effects of σ_s and ξ work through $\sigma_{a,t}$ analogously.[48]

Imperfect information offers a channel that can increase both the average and the volatility of nominal term premia by largely impacting real term premia. This emphasis on real term premia is a notable departure from the literature that stresses positive inflation risk premia as the primary factor behind positive nominal term premia. The result can also be seen as lending theoretical support to studies such as Duffee (2018), which aruges that a relatively small portion of the variation in yield news can be explained by the variation in news about expected inflation.

Term premia and the changing correlation of consumption growth and inflation: While a detailed study is out of the scope of this paper, the comparative statics can offer some insight into

[48] θ_π is obviously affected by σ_s and ξ in the DSGE model, and in particular, θ_π can decrease as consumption and inflation becomes more positively correlated under imperfect information. However, the simple example highlights the channel through the effect on $\sigma_{a,t}$, which is useful to understand the overall effect of the change in parameters.

the determinants of a longer-term relationship between term premia and the increasing correlation of consumption growth and inflation that has been documented in the literature. The left panel of Figure 14 shows the model-implied correlation of consumption growth and inflation with respect to σ_s and ξ. Recall a noise shock can be interpreted as a demand shock, in the sense that it moves consumption and inflation in the same direction, as shown by the impulse responses. Hence, it may be natural to expect an *increase* in the correlation of consumption and inflation by adding a noise shock. However, the figure shows that this is not necessarily the case. In particular, when the signal precision is constant ($\xi = 0$), the correlation becomes more *negative* as σ_s increases from the perfect information case when there are only TFP shocks ($\sigma_s = 0$). Correlation can increase only when the signal precision is sufficiently time-varying, i.e., ξ is large, but even in this case, the correlation is not monotonically increasing with respect to σ_s.

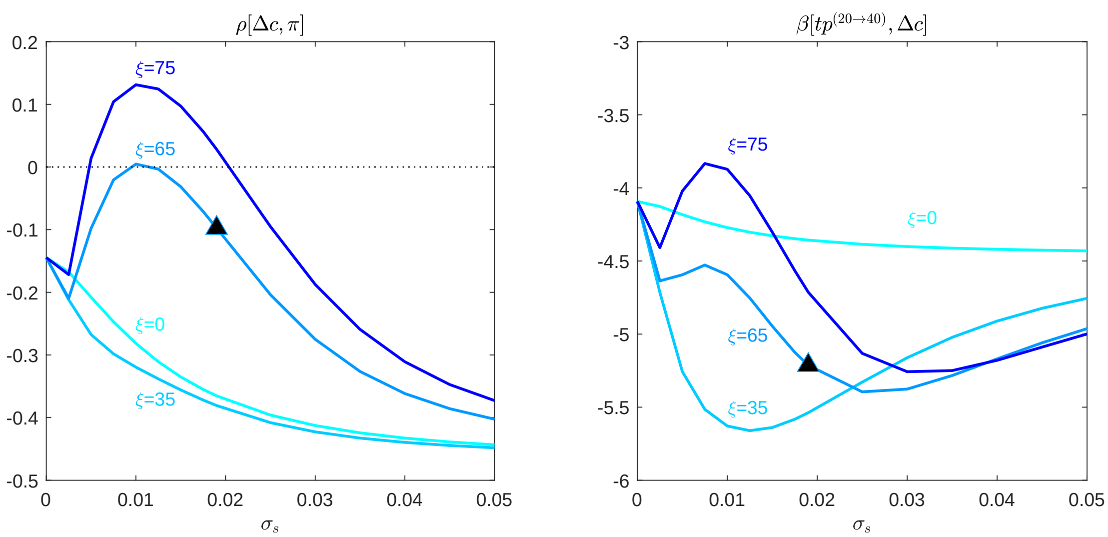

Figure 14: **Effect of the Noisy Signal on the Correlation of Δc and π and the Regression Coefficient of $tp^{(20\to40)}$ on to Δc**

Notes: Each line corresponds to the correlations (left panel) and regression coefficients (right panel) generated from the indicated value of ξ. The black triangle indicates the statistic from the baseline model.

The reason why a larger volatility of the noise shock can lead to a (further) negative correlation between consumption and inflation is that a noise shock changes how a TFP shock affects consumption and inflation by influencing agents' beliefs about productivity. For instance, under imperfect information, an increase in TFP due to an increase in the persistent component of TFP is perceived to be partly driven by an increase in the transitory component. Hence, compared with the case under perfect information, the increase in consumption is dampened, exacerbating the deflationary pressure of the TFP shock and resulting in a stronger negative correlation.[49]

[49] By assuming no habit formation, no monetary policy reaction to the output gap, $\rho_a = 1$ and $\rho_e = 0$ for the TFP process, the linearized solution of the DSGE model admits an analytical expression where inflation is orthogonal to ε_a under perfect information, but negatively correlated with ε_a under imperfect information.

The time-variation in signal precision is critical in reversing this mechanism. If signal precision is procyclical, this dampening effect is offset as the increase in consumption leads to an increased flow of information.[50] This result shows that the increase in the correlation may not be simply due to an increase in the volatility of a demand shock, but rather, the origin may be traced to a combination of moves in σ_s and ξ. This point can actually be made without considering the term structure of interest rates, but it is an interesting byproduct of introducing time-varying signal precision to improve the DSGE model's fit to the term structure, and a point that appears overlooked in the literature.

Meanwhile, the right panel of Figure 14 shows the model-implied slope coefficient of a regression of the 5-to-10 year nominal term premium on year-over-year consumption growth. The empirical counterpart of this regression was discussed in Section 2. Consistent with my analysis thus far, the slope coefficient becomes significantly more negative, i.e., the term premium becomes more countercyclical, when signal precision is procyclical ($\xi > 0$). Interestingly, the effect of σ_s is nonlinear, and the slope coefficient is most negative under intermediate values of σ_s, because that is when ξ has the most impact on the volatility of the term premium, as explained above.

As discussed in Section 2, the countercyclicality of term premia did not necessarily decrease across the two sample periods, while there was a notable decline in the level of term premia and increase in the correlation of consumption growth and inflation. Through the lens of my model, this phenomenon would be qualitatively consistent with some decrease in σ_s from an intermediate level in an economy where ξ is sufficiently positive. Alternatively, and perhaps more realistically, the mechanism through which imperfect information affects the term premium can be combined with other possible explanations in the literature. For instance, Nakata and Tanaka (2016) show that an increase in the volatility of a preference shock depresses term premia and increases the correlation of consumption growth and inflation. However, term premia end up being procyclical. Imperfect information could reverse this procyclicality, offering an explanation that brings the model more in line with the data.

5 Conclusion

In this paper, I studied the dynamics of default-free bond yields and term premia using a novel equilibrium term structure model which combined a New-Keynesian core with imperfect information about the persistence of shocks to productivity. The model generated term premia that are on average positive with sizable countercyclical variation that arose endogenously. Importantly, demand shocks, in addition to supply shocks, played a key role in the dynamics of term premia by creating countercyclical uncertainty over the aggregate variables. This is in sharp contrast to existing DSGE term structure models with perfect information, which tend to rely on large supply shocks to generate time-variation in yields and term premia. I argued that incorporating imperfect

[50] As $\sigma_s \to \infty$, the correlation asymptotes to a unique level that is more negative than the case with perfect information, regardless of ξ. This is because, as s_t becomes completely uninformative, the only effective signal is productivity itself, the precision of which does not vary over time.

information in a DSGE model helps reconcile the empirical evidence that term premia have been on average positive and countercyclical, with numerous studies pointing to demand shocks as being an important driver of business cycles over the last few decades. While the focus on a specific form of information friction proved to be tractable and effective in understanding some important features of the yield curve, other, perhaps more elaborate variants could be explored that could explain more aspects of agents' beliefs. I leave such an investigation for future research.

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Appendix

A Data

The macroeconomic data source used for the regression analysis in Section 2 is as follows. Nonfarm payroll, industrial production, capacity utilization, and real GDP data are taken from the FRED database. GDP gap is the CBO measure of the output gap, from the Haver Analytics database and the unemployment gap is the Civilian Unemployment Rate: 16 yr + (seasonally adjusted) minus the CBO measure of the natural rate of unemployment, also from Haver. I remove a linear trend from capacity utilization, estimated from monthly observations from January 1990 to December 2019.

Additional data is used for calibrating the DSGE term structure model. For the short-term nominal interest rate, I use the 3-month T-bill rates from the Federal Reserve Board's H.15 statistical release. For nominal yields of 5-, and 10-year maturities, I use the zero-coupon yields from Gürkaynak, Sack, and Wright (2007). For 5- and 10-year real yields, I use zero-coupon yields interpolated from TIPS by Gürkaynak, Sack, and Wright (2010). These yield data are widely used in the literature.

For consumption data, I compute per capita consumption from personal consumption expenditures (nondurables + services, seasonally adjusted). I use the quarterly change in the core CPI as a measure of inflation. Both measures are taken from Haver.

I use the Survey of Professional Forecasters (SPF), published by the Federal Reserve Bank of Philadelphia, to construct a measure of conditional uncertainty about GDP growth. In each quarterly survey, the SPF includes average forecast distributions of year-over-year GDP growth for the current year (and the next). Assuming the probability assigned to each bin represents the probability of the mid-point of that bin, I compute the standard deviation of the average forecast distribution. The nature of the data is somewhat disconnected before and after 1992.Q1; the forecasts are based on real GNP before 1992.Q1 and on real GDP since then. The bins also vary across the two periods. I construct a time series using only surveys in the fourth quarter so that the forecast horizon is effectively constant at about a quarter.

The patent data used in Figure 2 is compiled by Marco, Carley, Jackson, and Myers (2015) and available on the website of the U.S. Patent and Trademark Office. I sum the monthly total application series for each quarter and compute the annual growth rate for each quarter. I do not show growth rates that include data for 1995.Q2, 2013.Q1 and 2013.Q2 since there were large swings in applications due to regulatory changes, as described in Marco, Carley, Jackson, and Myers (2015).

B The Simple Term Structure Model as an RBC Model

In this section, I show that the simple term structure model in Section 3.1 can be derived from a stylized real business cycle model without capital.

The representative household maximizes lifetime expected utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right) \right], \tag{B.1}$$

subject to its budget constraint:

$$C_t + \mathbb{E}_t\left[M_{t+1}\mathcal{W}_{t+1}\right] \leq W_t N_t + \mathcal{W}_t. \tag{B.2}$$

C_t is consumption, N_t is labor, W_t is (real) wages. Assuming complete financial markets, $\mathcal{W}_{t+1}$ is the household's wealth portfolio of state contingent claims chosen by the end of period t. These claims are priced by the unique (real) stochastic discount factor $M_{t+1} \equiv \beta(C_{t+1}/C_t)^{-\chi_c}$ implied by the household's optimizing behavior. Assets are in zero net supply.

The perfectly competitive firm with a production function $Y_t = Z_t N_t$ maximizes its profits $Y_t - W_t N_t$ each period. Z_t is an exogenous productivity process. Market clearing imposes $C_t = Y_t$.

The model is simple enough to solve analytically. The equilibrium condition from the labor market implies:

$$C_t^{\chi_c} N_t^{\chi_n} = (Z_t N_t)^{\chi_c} N_t^{\chi_n} = Z_t. \tag{B.3}$$

Solving for N_t:

$$n_t = \frac{1-\chi_c}{\chi_c+\chi_n} z_t, \tag{B.4}$$

where small-case variables correspond to their log counterparts e.g. $n_t \equiv \ln(N_t)$. The decision rules for consumption immediately follows from the production function:

$$c_t = \frac{1+\chi_n}{\chi_c+\chi_n} z_t, \tag{B.5}$$

which corresponds to the consumption rule (10) in Section 3.1 with $\theta_c = \frac{1+\chi_n}{\chi_c+\chi_n}$.

Since this solution holds for any arbitrary process z_t, it must also hold for the state space system characterized by equations (1) through (6) in Section 3.1. Lastly, the stochastic discount factor of the household implies that default-free bonds are priced according to the Euler equation (11) where $\bar{r} = -\ln\beta$.

C Derivation of the Two-period Term Premium

This section shows the derivation of the two-period term premium with imperfect information (equation (13) in Section 3.2.4).

$$\begin{aligned} tp_t^{(2)} \equiv r_t^{(2)} - r_t^{(2)\mathbb{Q}} &= -\frac{1}{2}\mathbb{C}ov_t(m_{t+1}, \mathcal{R}_{t+1}^{(2)}) \\ &= -\frac{1}{2}\mathbb{C}ov_t(-\chi_c\Delta c_{t+1}, \mathbb{E}_{t+1}[-\chi_c\Delta c_{t+2}] + \frac{1}{2}\mathbb{V}ar_{t+1}[\chi_c\Delta c_{t+2}]) \\ &= -\frac{1}{2}\chi_c^2\mathbb{C}ov_t(\Delta c_{t+1}, \mathbb{E}_{t+1}[\Delta c_{t+2}]) \\ &= -\frac{1}{2}\chi_c^2\theta_c^2\mathbb{C}ov_t(\Delta z_{t+1}, \mathbb{E}_{t+1}[\Delta z_{t+2}]) \\ &\propto -\mathbb{C}ov_t(\Delta z_{t+1}, \mathbb{E}_{t+1}[\Delta z_{t+2}]), \end{aligned} \tag{C.1}$$

where $\mathcal{R}_{t+1}^{(2)} = p_{t+1}^{(1)} - p_t^{(2)}$ is the (log) return from holding a two-period bond for one-period. The second equality follows from $m_{t+1} \equiv -\bar{r} - \chi_c\Delta c_{t+1}$, and $p_t^{(1)} = -r_t^{(1)} = \mathbb{E}_t[m_{t+1}] + \frac{1}{2}\mathbb{V}ar_t[m_{t+1}]$. The third equality uses the fact that $\mathbb{V}ar_{t+1}[\chi_c\Delta c_{t+2}]$ is measurable at time t. The fourth equality follows from $c_t = \theta_c z_t$.

z_t follows:

$$z_t = \rho_a a_{t-1|t-1} + (\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{C.2}$$

$$a_{t|t} = \rho_a a_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}), \tag{C.3}$$

where $\boldsymbol{K}_t$ is the Kalman gain matrix and $\boldsymbol{s}_t \equiv [z_t, s_t]'$.

Generalizing Lemma 2 in Blanchard, L'Huillier, and Lorenzoni (2013) to the case of time-varying coefficients, C.3 and C.2 are observationally equivalent to the system:

$$z_t = \rho_a \tilde{a}_{t-1} + \boldsymbol{\Sigma}_{s,t-1}\tilde{\varepsilon}_t \tag{C.4}$$

$$\tilde{a}_t = \rho_a \tilde{a}_{t-1} + \boldsymbol{K}_{t-1}\boldsymbol{\Sigma}_{s,t-1}\tilde{\varepsilon}_t, \tag{C.5}$$

where $\tilde{a}_t$ and $\tilde{\varepsilon}_t$ are observable, $\tilde{\varepsilon}_t$ are mutually independent, i.i.d. standard normal shocks, and $\boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}'_{s,t-1} = \mathbb{V}ar_{t-1}[\boldsymbol{s}_t]$. Substituting C.5 and C.4 into C.1 and after some algebra, I obtain (13) in the main text.

D Equilibrium Conditions of the DSGE Term Structure Model

In this section, I list the equilibrium conditions of the DSGE term structure model in full (excluding the equations for the term structure to save space). Defining the normalized variables using hats (e.g. $\hat{C}_t \equiv \frac{C_t}{G_t}$), the normalized equilibrium conditions, are (for $U_t \leq 0$):

$$\hat{V}_t = \frac{\hat{X}_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} - \tilde{\beta}\zeta\left\{\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right\}^{\frac{1}{1-\tilde{\gamma}}} \tag{D.1}$$

$$M_{t+1} = \tilde{\beta}\left(\frac{\hat{X}_{t+1}}{\hat{X}_t}\right)^{-\chi_c}\left[\frac{-\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right]^{\frac{1}{1-\tilde{\gamma}}}}\right]^{-\tilde{\gamma}}\frac{1}{\Pi_{t+1}} \tag{D.2}$$

$$\hat{W}_t^r = N_t^{\chi_n}\hat{X}_t^{\chi_c} \tag{D.3}$$

$$\mathbb{E}_t\left[M_{t+1}R_t^{(1)}\right] = 1 \tag{D.4}$$

$$\hat{F}_t = \frac{\theta}{\theta-1}\hat{\lambda}_t^r\hat{Y}_t + \mathbb{E}\left[\xi_p M_{t+1}\frac{\Pi_{t+1}}{\Pi_t^{\iota_p}\bar{\Pi}^{1-\iota_p}}\hat{F}_{t+1}\zeta\right] \tag{D.5}$$

$$\hat{H}_t = \hat{Y}_t + \mathbb{E}\left[\xi_p M_{t+1}\frac{\Pi_{t+1}}{\Pi_t^{\iota_p}\bar{\Pi}^{1-\iota_p}}\hat{H}_{t+1}\zeta\right] \tag{D.6}$$

$$(P_t^*)^{1+\frac{\theta(1-\alpha)}{\alpha}} = \frac{\hat{F}_t}{\hat{H}_t} \tag{D.7}$$

$$\hat{\lambda}_t^r = \frac{\hat{W}_t^r}{\alpha\bar{K}^{1-\alpha}Z_t N_t^{\alpha-1}} \tag{D.8}$$

$$P_t^* = \left(\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}} \right)^{-\frac{\theta}{\alpha}}}{1 - \xi_p} \right)^{\frac{1}{1-\theta}} \tag{D.9}$$

$$\Delta_t = (1-\varphi) \left(\frac{P_t^*}{P_t} \right)^{-\theta} + \varphi \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}} \right)^{\theta} \Delta_{t-1} \tag{D.10}$$

$$R_t^{(1)} = \left(R_{t-1}^{(1)} \right)^{\rho_r} \left(\bar{R} \left[\frac{\Pi_t}{\bar{\Pi}} \right]^{\phi_\Pi} \left[\frac{\hat{Y}_t}{\bar{Y}} \right]^{\phi_Y} \right)^{1-\rho_r} \tag{D.11}$$

$$\hat{Y}_t = \frac{1}{\Delta_t} \bar{K}^{1-\alpha} Z_t N_t^{\alpha} \tag{D.12}$$

$$\hat{Y}_t = \hat{C}_t + (\zeta + \delta) \bar{K} \tag{D.13}$$

$$\boldsymbol{S}_t = \boldsymbol{\Psi} \boldsymbol{\rho} \boldsymbol{X}_{t-1|t-1} + (\boldsymbol{S}_t - \boldsymbol{S}_{t|t-1}) \tag{D.14}$$

$$\boldsymbol{X}_{t|t} \equiv \mathbb{E}_t[\boldsymbol{X}_t] = \boldsymbol{\rho} \boldsymbol{X}_{t-1|t-1} + \boldsymbol{K}_{t-1} (\boldsymbol{S}_t - \boldsymbol{S}_{t|t-1}) \tag{D.15}$$

$$\boldsymbol{K}_{t-1} = \boldsymbol{V}_{t|t-1} \boldsymbol{\Psi}' (\boldsymbol{\Psi} \boldsymbol{V}_{t|t-1} \boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1} \boldsymbol{\Sigma}_{s,t-1}')^{-1} \tag{D.16}$$

$$\boldsymbol{V}_{t+1|t} = \boldsymbol{\rho} (\boldsymbol{V}_{t|t-1} - \boldsymbol{V}_{t|t-1} \boldsymbol{\Psi}' (\boldsymbol{\Psi} \boldsymbol{V}_{t|t-1} \boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1} \boldsymbol{\Sigma}_{s,t-1}')^{-1} \boldsymbol{\Psi} \boldsymbol{V}_{t|t-1}') \boldsymbol{\rho}' + \boldsymbol{\Sigma}_x \boldsymbol{\Sigma}_x', \tag{D.17}$$

where $\hat{X}_t \equiv \hat{C}_t - \frac{\chi_h}{\zeta} \hat{C}_{t-1}$, $\hat{V}_t \equiv \frac{V_t}{G_t^{1-\chi_c}}$, $\tilde{\beta} \equiv \beta \zeta^{-\chi_c}$, and all other variables are defined in the main text.