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Endogenous Bargaining Power and Declining Labor Compensation Share

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Abstract:

Workhorse search and matching models assume constant bargaining weights, while recent evidence indicates that weights vary across time and in cross section. We endogenize bargaining weights in a life-cycle search and matching model by replacing a standard Cobb-Douglas (CD) matching function with a general constant elasticity of substitution (CES) matching function and study the implications for the long-term labor share and bargaining power in the U.S. The CES model explains 64 percent of the reported decline in the labor share since 1980, while the CD model explains only 28 percent of the decline. We then use the model to recover changes in bargaining power and find that workers'

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bargaining power has declined 11 percent between 1980 and 2007 because of a decline in tightness.

Keywords: Labor share, Endogenous bargaining power, Search and matching, CES matching function

JEL Codes: E25, J30, J50

1 Introduction

Both academics and the public have raised concerns about workers' declining bargaining power in the United States. Evidence from recent decades points in that direction: Labor's share of income has declined, median wage growth has been sluggish, profitability of firms has risen, and both union membership and coverage have dropped drastically (Stansbury and Summers, 2020). While workhorse search and matching models, like the Diamond-Mortensen-Pissarides (DMP) model, include bargaining between firms and workers, these models typically assume constant bargaining weights, making them unable to capture long-term changes in workers' bargaining power.

We address this discrepancy by endogenizing bargaining weights in a life-cycle search and matching model. Instead of assuming that the matching function takes the standard Cobb-Douglas (CD) form, we rely on a more general constant elasticity of substitution (CES) matching function. This approach endogenizes the matching elasticity and, conditional on the Hosios condition holding (Hosios, 1990), bargaining weights. We calibrate both models using the Current Population Survey (CPS) data and historical vacancy data from Petrosky-Nadeau and Zhang (2021) to match job-finding rates, wages, and tightness rates. We find that the CES model explains about 64 percent of the observed 4-percent decline in the labor share in the U.S. between two business cycle peaks, 1980 and 2007, while the CD model explains only 28 percent of the decline. Thus, the CES model with endogenous matching elasticity and bargaining power performs drastically better at capturing the decline in the labor share, indicating that the bargaining power channel is a key ingredient for the model to match the data. This is the first main contribution of this paper.

The workers' bargaining weight equals the matching elasticity with respect to job seekers when the Hosios condition holds. Under the CES matching function and reasonable parameter values, the bargaining weight increases with labor market tightness, indicating that a greater demand for labor translates into higher bargaining power of workers. Intuitively, when labor markets are tight and jobs are abundant, workers have more bargaining power over wages and other terms of employment. Our model thus implies that changes in market conditions, reflected in declined tightness, can generate a reduction in workers' bargaining power—even in the absence of institutional changes. This is the second main contribution of the paper.

We use the CES model to recover workers' efficient bargaining power and study how it has changed over time. Our results suggest that the aggregate bargaining power has declined about 11 percent between 1980 and 2007 because of a decline in labor market tightness. Previous literature has found that bargaining power varies among different demographic groups[1]. So we also include disaggregated results of bargaining power trends for four groups: males and females with at least some college education, as well as males and females without a college education. We find that the bargaining power of males has decreased more than that of females, leading to a decrease in the gender bargaining power

[1] Recent literature has documented a gender bargaining power gap, and the gap can explain a fraction of the gender wage gap (Biasi and Sarsons, 2021; Blau and Kahn, 2017; Card et al., 2016; Harding et al., 2003). Some literature also uses bargaining power differences to explain a wage gap between older workers and young workers (Farmand and Ghilarducci, 2019 and Glover and Short, 2020). The literature thus suggests that assuming a constant bargaining power across groups is problematic and that accounting for the noted differences in bargaining power is important to understand the dynamics of the labor market.

gap. The bargaining power of both college and non-college males has declined around 17 percent, while the declines have been 1 percent for college-educated females and 6 percent for non-college females. Lastly, while the gender bargaining power gap has diminished, the opposite is true for the education gap—especially for females—as college-educated workers' bargaining power has decreased less relative to non-college workers.

We highlight that the CES matching function has desirable properties over the CD matching in our exercise and in general. First, the CES function is theoretically sounder, as the CD introduces discontinuities and requires truncation. Second, it generates intuitively sensible matching elasticities. Specifically, when vacancies v and job seekers u are complements in a matching process, matching elasticity with respect to job seekers is increasing in tightness. Intuitively, the number of successful matches is more sensitive to the number of job seekers when there are many available vacancies relative to job seekers. Third, we show that the CES matching function with complementarity between v and u is consistent with micro-evidence that shows that groups with weaker labor markets (for example, women) have lower bargaining power. Fourth, the CES matching function is consistent with casual evidence that workers' bargaining power increases with labor scarcity, such as during and after the COVID-19 pandemic. As we show, these properties turn out to be quantitatively important in explaining the long-run decline in the labor share.

Our model includes human capital accumulation through learning-by-doing and a non-participation state. We include these features in the model for the following reasons. First, we are interested in studying how bargaining power evolves over the life cycle; bargaining power likely evolves with human capital—a more experienced, skilled worker has higher bargaining power compared with a less experienced worker, all else being equal. And second, many studies, such as Choi et al. (2015) and Veracierto (2008), have pointed out that the nonparticipation state is important in understanding labor market dynamics. In our setting, nonparticipation matters, especially for studying gender differences in bargaining power, as females are traditionally more likely to experience nonparticipation periods over the life cycle. For that reason, we extend the model to include nonparticipation.

As our results rely on the Hosios condition holding, we show that it holds in our extended model. The original condition states that an equilibrium allocation in a search and matching model is constrained efficient when the workers' bargaining weight equals the elasticity of the vacancy-filling rate with respect to labor market tightness. We show that the condition holds in our model despite the life-cycle dynamics, human capital accumulation, and nonparticipation whenever there is enough segmentation in labor markets.[2]

Finally, we decompose the decline in bargaining power. We conclude that an increase in κ, the relative vacancy-posting cost, has driven the decline in tightness, and thus bargaining

[2]This result contrasts with Laureys (2021), who builds a DMP model with similar human capital accumulation. Her model assumes integrated labor markets for workers with different human capital levels and that the decentralized labor market is inefficient because of a labor composition externality. We prove that there is a more generalized Hosios condition that guarantees efficiency in decentralized labor markets when labor markets are segmented. Moreover, we show that the Hosios condition holds endogenously in our model if we follow the competitive search theory literature (see Wright et al., 2021) and assume that firms post a menu of bargaining powers and that workers choose to apply to jobs that offer bargaining power that maximizes their utility. In that case, bargaining power works as a price device that guarantees that the decentralized allocation is constrained efficient.

power. While our calibration results point to an improved matching efficiency and higher returns to experience, increasing tightness for all groups, we find that vacancy-posting costs have risen. This rise is necessary for the model to match the observed decline in tightness along with the observed employment and wage trends.

Relation to the literature. On the theory side, the most related paper is Mangin and Sedláček (2018). They study business cycle fluctuations of the labor share by building a search and matching model where heterogeneous firms compete over workers and in which the division of output between firms and workers in endogenous. Specifically, a tighter labor market increases labor's share of output—a mechanism that is like the one in our model. However, Mangin and Sedláček (2018) focus on explaining the business cycle dynamics of the labor share, while our focus is on longer-term changes in bargaining power and the labor share.

We are not the first to use the CES matching function in search and matching models—den Haan et al. (2000) are. They use the CES matching function and highlight the preferable properties of CES matching function that guarantee matching probabilities between zero and one. A CES matching function is also used, for example, by Hagedorn and Manovskii (2008) and Petrosky-Nadeau et al. (2018). Stevens (2007) microfounds a matching function by showing that a "telephone line" Poisson queuing process implies a CES matching function. Recently, Bernstein et al. (2022) studied how a CES matching function and cyclicality of matching efficiency affect nonlinear business cycle properties of search and matching models and found quantitatively important effects. While these papers allow variation in matching elasticities, they assume constant bargaining weights.

Our paper also relates to the literature that studies workers' bargaining power—both the long-run trends and the differences among different worker groups—and the relationship between bargaining power and the labor share.[3] We contribute to the efforts to measure changes in bargaining power by indirectly inferring changes in efficient bargaining power for different demographic groups using a general equilibrium model with endogenous bargaining power. Consistent with the previous literature, we find that bargaining power has declined in the past four decades and that there are gaps in bargaining power across gender, education, and age.

While previous literature has focused on studying a decline bargaining power arising from changes in labor market institutions (Stansbury and Summers, 2020 and Ratner and Sim, 2022), we focus on studying changes in bargaining power arising from labor demand. Stansbury and Summers (2020) argue that three factors have caused the decline in worker power in the U.S. over recent decades: (1) institutional changes like decreased unionism, (2) within-firm changes like an increase in shareholder power that has led to pressure to cut labor costs, and (3) changes in economic conditions—like increased globalization and technology— that have improved employers' outside options. While Stansbury and Summers (2020) focus on studying and presenting supporting evidence for the first two factors, we complement their work by focusing on the third.

We thus bring a new angle to the literature: We argue that bargaining power has decreased

[3]See for example, Bental and Demougin (2010); Biasi and Sarsons (2021); Blau and Kahn (2017); Card et al. (2016); Farmand and Ghilarducci (2019); Glover and Short (2020); Ratner and Sim (2022); Roussille (2022); Stansbury and Summers (2020).

as an equilibrium reaction to a decrease in labor demand. If labor demand has decreased, workers face a trade-off between choosing a lower wage (and implicitly lower bargaining power) or a higher unemployment. In addition, the declining unionization documented by Stansbury and Summers (2020) can also reflect the described response. Workers may be less inclined to join unions for fear of jobs disappearing quickly.

Charles et al. (2021) find a similar result when looking at a decline in unionization. They estimate the causal effect of increased import competition from China on the accelerated decline in the rate of union elections between 1990 and 2007. They find that the "China shock" contributed to 4.5 percent of the decline among workers in directly exposed industries, while the shock contributed to 8.8 percent of the decline among workers indirectly exposed through weaker local relative labor demand. In other words, workers in industries that were not directly exposed to the China shock unionized less because the shock weakened their outside employment options in the face of a job loss.

We find a similar mechanism using a structural general equilibrium model, but we focus on studying the effect on bargaining power. An increased vacancy cost, which can capture the China shock, reduces rents from any match and leads to weaker labor demand via lower tightness. This increases the cost of job loss because workers' job-finding rate goes down. We also show that workers' efficient bargaining power decreases. The decreases in the job-finding rate and bargaining power then lead to a decrease in the labor share.

The rest of the paper is organized as follows. Section 2 summarizes the properties of the CD versus CES matching functions. Section 3 introduces the model, and Section 4 describes

(1) how tightness, the labor share, and other outcomes have evolved between 1980 and 2007; (2) how we parameterize the model; and (3) the calibration results. We then move on to reporting model-generated changes in efficient bargaining power (Section 5) and counterfactuals (Section 6). Section 7 concludes the paper.

2 The Properties of the CD and the CES Matching Functions

This section highlights some properties of the CES matching function that speak to its use. First, under the CES matching function, matching probabilities are between zero and one (den Haan et al., 2000 and Petrosky-Nadeau et al., 2018). Define the CES matching function as

$$M(u,v) = \left\{ \begin{array}{c} A\left(\alpha u^{\rho} + (1-\alpha)\, v^{\rho}\right)^{1/\rho} \text{ if } \rho \leq 1, \rho \neq 0 \\ A u^{\alpha} v^{(1-\alpha)} \text{ if } \rho = 0 \end{array} \right\}, \quad (1)$$

where u refers to job seekers, v refers to vacancies, $\alpha \in (0,1)$ is the share parameter, and A is the matching efficiency. The elasticity of substitution is $\sigma \equiv \frac{1}{1-\rho} \in (0,\infty)$. The case of the CD matching function with $\rho = 0$ implies that $\sigma = 1$. The value of ρ is negative when u and v are complements.

Given the matching function, a firm's vacancy-filling rate $q(\theta) = M(u,v)/v = M(1/\theta, 1)$ is given by

$$q(\theta) = \left\{ \begin{array}{c} A\left(\alpha \theta^{-\rho} + (1-\alpha)\right)^{1/\rho} \text{ if } \rho \leq 1, \rho \neq 0 \\ A\theta^{-\alpha} \text{ if } \rho = 0 \end{array} \right\}. \quad (2)$$

Notice that $q'(\theta) < 0$. Furthermore,

$$q(0) = \left\{ \begin{array}{c} A\left(1-\alpha\right)^{1/\rho} \text{ if } \rho < 0 \\ \\ \infty \text{ if } \rho \geq 0 \end{array} \right\},\ q(\infty) = \left\{ \begin{array}{c} 0 \text{ if } \ \rho < 0 \\ \\ A(1-\alpha)^{1/\rho} \text{ if } \ \rho > 0 \\ \\ 0 \text{ if } \rho = 0 \end{array} \right\}.$$

Therefore, the probability of filling a vacancy is well behaved when $\rho < 0$ if $1 \geq A(1-\alpha)^{1/\rho}$. When that is the case, $q(\theta) \in [0,1]$ for all $\theta \geq 0$. In contrast, $q(\theta)$ is not well behaved when $\rho \geq 0$, as $q(0) = \infty$.

In a similar manner, a job seeker's job-finding rate $f(\theta) = M(u,v)/u = M(1,\theta)$ is

$$f(\theta) = \left\{ \begin{array}{c} A\left(\alpha + (1-\alpha)\,\theta^{\rho}\right)^{1/\rho} \text{ if } \rho \leq 1, \rho \neq 0 \\ \\ A\theta^{1-\alpha} \text{ if } \rho = 0 \end{array} \right\}. \tag{3}$$

A job-finding rate is increasing in tightness, $f'(\theta) > 0$ and,

$$f(0) = \left\{ \begin{array}{c} A\alpha^{1/\rho} \text{ if } \rho > 0 \\ \\ \text{if } \rho \leq 0 \end{array} \right\},\ f(\infty) = \left\{ \begin{array}{c} A\alpha^{1/\rho} \text{ if } \ \rho < 0 \\ \\ \infty \text{ if } \rho \geq 0 \end{array} \right\}.$$

Therefore, the job-finding probability is well behaved when $\rho < 0$ if $1 \geq A\alpha^{1/\rho}$. In that case, $f(\theta) \in [0,1]$ for all $\theta \geq 0$. When $\rho \geq 0$, $f(\theta)$ is not well behaved, as $f(\infty) = \infty$.

To conclude, the CES matching function produces sensible job-finding and vacancy-filling probabilities when $1 \geq max[A(1-\alpha)^{1/\rho}, A\alpha^{1/\rho}]$. This is not the case with the CD matching function.

Second, we show how the CES matching function generates intuitively reasonable matching elasticities. Note first that with the CES matching function, $q'(\theta) = -A\alpha[\alpha\theta^{-\rho} + (1-$

$\alpha)]^{\frac{1}{\rho}-1}(\theta^{-\rho-1})$. Therefore, we can write the matching elasticity with respect to job seekers as a function of θ:

$$M_u(u,v)\frac{u}{M} = \alpha(\theta) = -\frac{q'(\theta)\theta}{q(\theta)} = \frac{\alpha}{\alpha + (1-\alpha)\theta(x)^{\rho}}. \tag{4}$$

This expression includes the CD result with $\rho = 0$. As is well known, the CD matching function implies a constant matching elasticity α. Moreover, notice that $\alpha'(\theta) > 0$ whenever $\rho < 0$—that is, the matching elasticity is increasing in tightness when v and u are complements in the matching process. As $\alpha(\theta)$ represents the elasticity of the matching function with respect to u, a lower θ means that there are relatively more job seekers compared with vacancies. This means that the number of successful matches is less sensitive to the number of job seekers, a result that highlights the complementarity of job seekers and vacancies in the matching process.

Third, we show that the CES matching function generates efficient bargaining power dynamics consistent with both micro-evidence and macro-evidence. The well-established Hosios condition (Hosios, 1990) states that the decentralized solution of the standard DMP model is constrained efficient as long as the elasticity of the matching function with respect to the number of job seekers equals the bargaining weight ϕ of the worker,

$$\phi(\theta) = \alpha(\theta) = \frac{\alpha}{\alpha + (1-\alpha)\theta(x)^{\rho}}. \tag{5}$$

This simple formulation shows that the bargaining power of workers is increasing in α, but more importantly, bargaining power is increasing in the endogenous tightness rate θ whenever ρ is negative. We now have efficient bargaining power that depends on labor market tightness.

Proposition 1. *Under the CES matching function $M(u,v) = A(\alpha u^\rho + (1-\alpha)v^\rho)^{1/\rho}$, the efficient bargaining power of workers decreases with labor market tightness if $\rho > 0$; the efficient bargaining power of workers increases with labor market tightness if $\rho < 0$.*

From the point of view of the social planner, proposition 1 means that the large relative number of job seekers reduces the potential for forming a match because of the complementarity of u and v. To increase the number of vacancies, it is optimal to reduce the surplus share of workers to spur vacancy creation.

It is also easy to see that the expression nests the Cobb-Douglas case: When $\rho = 0$, the bargaining power is exactly α. The Cobb-Douglas case also means that the bargaining power of workers does not depend on workers' characteristics or labor market conditions, contrasting with both micro-evidence as well as casual observations, as noted in the introduction.[4]

Let's further derive the bargaining power elasticity with respect to tightness θ:

$$\begin{aligned}\varepsilon_{\phi,\theta} = \frac{\partial\phi}{\partial\theta}\frac{\theta}{\phi} &= -\alpha[\alpha + (1-\alpha)\theta^\rho]^{-2} \times [\rho(1-\alpha)\theta^{\rho-1}] \times \frac{\theta[\alpha + (1-\alpha)\theta^\rho]}{\alpha} \\ &= -[\alpha + (1-\alpha)\theta^\rho]^{-1} \times [\rho(1-\alpha)\theta^\rho] = -\rho\frac{(1-\alpha)\theta^\rho}{\alpha + (1-\alpha)\theta^\rho}.\end{aligned} \tag{6}$$

The above expression implies that the elasticity of bargaining power with respect to tightness is positive whenever $\rho < 0$, and that bargaining power increases with tightness more

[4]Intuitively, when $\rho < 0$, bargaining power responds to labor demand and supply. Labor market tightness reflects the relative demand for labor. As a larger θ means a shift in the demand curve to the right, the labor market endogenously gives a larger production share to workers through larger bargaining power ϕ. This relationship is also in line with the finding of Fortin (2006), who shows that the college wage premium is negatively related to the supply of highly educated workers.

whenever ρ gets smaller and the complementarity between vacancies and job seekers in the matching process gets higher. Again, the above formula includes the Cobb-Douglas case: When $\rho = 0$, $\varepsilon_{\phi,\theta} = 0$.

We use the expression for bargaining power to discuss bargaining power gaps documented in the literature. What can explain the lower bargaining power of females? Females generally have higher separation rates compared with males over the life cycle (Choi et al., 2015 and Córdoba et al., 2021). Does that imply that females will be in a weaker position when bargaining with firms? The answer depends on the bargaining power elasticity. We derive the effect of a separation rate, π_{EN}, on the efficient bargaining power:

$$\frac{\partial \phi}{\partial \pi_{EN}} = \frac{\partial \phi}{\partial \theta} \frac{\partial \theta}{\partial \pi_{EN}} = \frac{-\alpha(1-\alpha)\rho\theta^{\rho-1}}{[\alpha + (1-\alpha)\theta^{\rho}]^2} \times \frac{\partial \theta}{\partial \pi_{EN}} = \varepsilon_{\phi,\theta} \times \frac{\phi}{\theta} \times \frac{\partial \theta}{\partial \pi_{EN}}.$$

The sign of $\frac{\partial \phi}{\partial \pi_{EN}}$ depends on the signs of ρ and $\frac{\partial \theta}{\partial \pi_{EN}}$. $\frac{\partial \theta}{\partial \pi_{EN}}$ is negative, and the intuition for $\frac{\partial \theta}{\partial \pi_{EN}}$ being negative is that hiring workers with higher separation rates will lower the match continuation value, and the lower continuation value needs to be compensated by a higher chance of successfully hiring such workers. Then the sign of $\frac{\partial \phi}{\partial \pi_{EN}}$ can be fully pinned down by ρ. When $\rho < 0$, groups with higher separation rates have lower bargaining power compared with groups with lower separation rates, all else equal.

To conclude, we argue that there are four reasons why the CES matching function with $\rho < 0$ is a sounder choice for a matching function: (i) it is theoretically sounder as the CD introduces discontinuities and requires truncation; (ii) it generates intuitively sensible matching elasticities and efficient bargaining power; (iii) it is consistent with micro-evidence that shows that groups with weaker labor markets (for example, women) have lower bargaining power; (iv) it is consistent with casual evidence—for example during and after COVID-19 pandemic—of workers' bargaining power increasing with labor scarcity.

3 Dynamic Search and Matching Model

3.1 *Basic Environment*

The model presented here closely follows the life-cycle search and matching model presented in Córdoba et al. (2021). The model features a life cycle with a finite horizon, human capital accumulation, and a nonparticipation state. Time is discrete, and age is denoted by a, where $a \in A \equiv [\underline{a}, \bar{a}]$. The model focuses on working years, which are the years between one's education and retirement.

Workers. Workers enter the labor market at age $\underline{a}$, retire at age a_R, and die at age $\bar{a}$, where $\bar{a} > a_R$. At any point in time, a worker is either employed, $\overline{E}$, unemployed, $\overline{U}$, or not participating in the labor force, $\overline{N}$. Let $s \in S \equiv \{\overline{E}, \overline{U}, \overline{N}\}$ denote the labor market status of a worker. Workers enter the labor market without work experience and gain experience by working. Let $e \in [0, a_R - \underline{a}]$ denote the years of experience. Experience increases by one unit during each period of employment, $e_{a+1} = e_a + 1$, and we assume that experience stays constant during nonemployment, $e_{a+1} = e_a$. Workers also differ in terms of their demographics like gender (female or male) and level of schooling (college or non-college), which is captured by i. Let I denote the set of demographic groups. A worker is fully identified by her years of experience (e), age (a), labor market status (s), and demographic group (i). Denote the state or the type of a worker by $x = (e, a, s, i)$, where $x \in [0, \bar{a} - \underline{a}] \times [\underline{a}, a_R] \times S \times I$, and let $x' = (e + 1, a + 1, s', i)$. The state of a retiree is defined as $x_R = (e, a_R, \overline{N}, i)$.

Let $m(x)$ be the mass of workers of type x. The initial mass distribution, $m_i^s(0, \underline{a})$, is taken as given for all s and i. Workers transition into unemployment and nonparticipation at exogenous rates $\pi_{EU}(x)$, $\pi_{EN}(x)$, $\pi_{UN}(x)$, and $\pi_{NU}(x)$, and into employment at endogenous rates $f_i\left(e, a, \bar{U}\right)$ and $f_i\left(e, a, \bar{N}\right)$. Workers seek to maximize their expected present value of consumption. They are risk neutral and discount the future according to the discount factor $\beta \in (0, 1)$. There are no savings. Wages of employed workers are determined by Nash bargaining between workers and firms, while consumption of non-employed workers and retirees is given by $\overline{c}(x)$, an exogenous parametric form. For simplicity, we do not explicitly describe the domain of each function whenever it is clear. For example, $w(x)$ refers only to the wages of the employed workers, $x = \left(e, a, \bar{E}, i\right)$.

Human Capital: Each worker is endowed with one unit of labor, but workers differ in terms of their labor productivity. We refer to the productivity of a worker as human capital, $h(x)$, and it is of the general type. We assume the following functional form for the human capital:

$$h(x) = y_i \exp\left(r(x)\, e\right), \tag{7}$$

where y_i is the baseline level of human capital that a member of a group i has when entering the labor market, and $r(x)$ is type-specific returns to experience. Both y_i and $r(x)$ are exogenous. We interpret y_i as education-related human capital—the human capital of a new worker for whom $e = 0$.

Firms and Labor Markets: The continuum of infinitely lived firms seek to maximize their expected present value of profits net of hiring costs. Firms are risk neutral and discount the future at the same rate as workers do. Labor markets are assumed to be perfectly segmented across worker types. Firms can freely enter any segmented markets. Firms post vacancies for long-term positions at a cost of $\kappa(x)$ per vacancy, a cost that may depend on a worker's type. Once a firm is matched with a worker, a worker produces $h(x)$ units of output per period, while gross per-period profits of the firm are $h(x) - w(x)$. A match is destroyed exogenously at a rate of $d(x) = \pi_{EU}(x) + \pi_{EN}(x)$.

Matching Technology*:* A worker and a firm with a vacant position are randomly matched in each submarket according to the matching technology $M(u(x), v(x); x)$, where $u(x)$ and $v(x)$ are the masses of workers and firms, respectively, searching in a labor market. We assume that (1) all unemployed workers search for a job, (2) employed workers do not search, and (3) a fraction $\psi(x) \leq 1$ of nonparticipants search. Thus, the mass of workers searching at a given employment status can be defined as follows:

$$u(x) \equiv \left\{ \begin{array}{c} m(x), \text{ if } s = \overline{U}, \\ \\ \psi(x)\, m(x), \text{ if } s = \overline{N}. \end{array} \right\} \tag{8}$$

Labor market tightness for each market x is defined as $\theta(x) \equiv v(x)/u(x)$, the vacancy-filling rate as $q(\theta(x)) = M(u(x), v(x))/v(x)$, and the job-finding rate as $f(\theta(x)) = \theta(x)q(\theta(x))$. Once a match is formed, the match output is distributed according to a Nash bargaining solution in which a worker's bargaining power is $\phi(x)$.

Labor Flows. Given the initial distribution of workers, $m_i^s(0, \underline{a})$, and job-finding rates $f(x)$ for all x, the subsequent distribution of workers $m(x)$ can be calculated assuming a law of large numbers. See details in Appendix C.

3.2 *Value Functions of Firms and Workers*

Firms. Let $\bar{V}$ be the value of a firm without a worker and $J(x)$ be the value of a firm with a worker of type $x = \left[e, a, \overline{E}, i\right]$. The value of a firm posting a vacancy in market x is

$$V\left(x\right) = \max\left\{-\kappa\left(x\right) + \beta\left[q\left(x\right) J_i\left(e, a+1\right) + \left(1 - q\left(x\right)\right)\bar{V}\right], 0\right\}.$$

The maximum value of posting a vacancy in any labor market is then given by $\bar{V} = \underset{x}{max}\left\{V\left(x\right), 0\right\}$. Free entry of firms into any labor market guarantees that the values of unfilled vacancies must all be equal to zero: $V\left(x\right) = 0$ for all feasible x. As a result, the maximum value of posting a vacancy must be zero as well: $\bar{V} = 0$. Active firms are thus indifferent to which type of worker to hire and to which segmented markets they operate in.

The problem of a firm with a worker is then

$$J(x) = \left\{\begin{array}{ll} h\left(x\right) - w\left(x\right) + \beta\left(1 - d\left(x\right)\right) J\left(x'\right) & \text{if } \underline{a} \leq a < a_R - 1 \end{array}\right\}, \tag{9}$$

which states that the value of a firm with a worker is the flow of gross profits plus the discounted continuation value of the match. The firm value right before a worker's retirement, $a = a_R - 1$, is $J(x) = h\left(x\right) - w\left(x\right)$.

The value of firms that post vacancies simplifies to

$$\kappa\left(x\right) = \beta q\left(x\right) J_i\left(e, a+1\right) = \beta f\left(x\right)\theta\left(x\right)^{-1} J_i\left(e, a+1\right) \text{ for } \underline{a} \leq a < a_R - 1. \tag{10}$$

The last equation states that the expected present value of filling a vacancy must be just enough to recover the costs of posting the vacancy.

Workers. Now consider the value functions of an employed worker, E, an unemployed worker, U, a nonparticipating worker, N, and a retired worker, R. The expected present value of consumption of a newly retiree simply satisfies

$$R\left(x_R\right) = \sum_{i=a_R}^{\overline{a}} \beta^{i-a_R} \overline{c}(x_R) = \frac{1-\beta^{\overline{a}-a_R-1}}{1-\beta} \overline{c}(x_R), \tag{11}$$

where $\overline{c}(x_R)$ is consumption of a retiree of type x_R. The corresponding value functions E, U, and N can then be written recursively as

$$E(x) = \left\{ w(x) + \beta \begin{bmatrix} \pi_{EU}(x)U(x') + \pi_{EN}(x)N(x') \\ +(1-\pi_{EU}(x) - \pi_{EN}(x))E(x') \end{bmatrix}, \text{if } \underline{a} \leq a < a_R - 1 \right\}, \tag{12}$$

$$U(x) = \left\{ \overline{c}(x) + \beta \begin{bmatrix} f(x)E_i(e, a+1) + \pi_{UN}(x)N_i(e, a+1) \\ +(1 - f(x) - \pi_{UN}(x)U_i(e, a+1) \end{bmatrix}, \text{if } \underline{a} \leq a < a_R - 1 \right\}, \tag{13}$$

$$N(x) = \left\{ \overline{c}(x) + \beta \begin{bmatrix} f(x)E_i(e, a+1) + \pi_{NU}(x)U_i(e, a+1) \\ +(1 - f(x) - \pi_{NU}(x))N_i(e, a+1) \end{bmatrix}, \text{if } \underline{a} \leq a < a_R - 1 \right\}, \tag{14}$$

while $E(x) = w(x) + \beta R_i\left(e+1, a_R, \overline{N}\right)$, $U(x) = \overline{c}(x) + \beta R_i\left(e, a_R, \overline{N}\right)$, and $N(x) = \overline{c}(x) + \beta R_i\left(e, a_R\right.$ if $a = a_R - 1$. An employed worker consumes her wage $w(x)$ each period. A match between a worker and a firm can be destroyed in two ways: (1) with probability $\pi_{EU}(x)$, a worker becomes unemployed, and- (2) with probability $\pi_{EN}(x)$, the worker becomes a nonparticipant. The worker continues producing with probability $1 - \pi_{EU}(x) - \pi_{EN}(x)$ and stays in the employment state. At the beginning of each period, an unemployed worker consumes $\overline{c}(x)$. During the next period, she finds a job with probability $f(x) = f_i(e, a, \bar{U})$, in which case she moves to the employment state. A worker may also move to the nonparticipation state with probability $\pi_{UN}(x)$; otherwise, she will stay unemployed. A similar interpretation holds for the value function of a nonparticipating worker.

3.3 *Nash Bargaining*

3.3.1 Bargaining Solution

Wages are negotiated through Nash bargaining. A firm and a worker share the match surplus $S(x) = S_{Es}(x) + J(x)$, $s \in \{E, U, N\}$, $S_{EU}(x) = E(x) - U(x)$ for an unemployed, and $S_{EN}(x) = E(x) - N(x)$ for a nonparticipant. Given the bargaining weights $\phi(x)$ for the worker and $1 - \phi(x)$ for the firm, the maximization problem is written as:

$$\underset{S_{Es},J}{max}(S_{Es}(x))^{\phi(x)} J(x)^{1-\phi(x)} \text{ subject to } S(x),$$

and the solution for each labor market satisfies

$$J(x) = \Theta(x) \times (S_{Es}(x)) \text{ where } \Theta(x) = \frac{1-\phi(x)}{\phi(x)}. \tag{15}$$

3.3.2 The Hosios Condition under the Dynamic DMP Model

Efficiency. We prove that the Hosios condition holds in our life-cycle model with human capital accumulation and nonparticipation.

Proposition 2. *Under the DMP model with a life cycle, human capital accumulation, and nonparticipation,* $\phi_i^{\bar{U}}(e-1, a+1) = -\frac{q'(\theta_i^{\bar{U}}(e,a))\theta_i^{\bar{U}}(e,a))}{q(\theta_i^{\bar{U}}(e,a))}$ *and* $\phi_i^{\bar{N}}(e-1, a+1) = -\frac{q'(\theta_i^{\bar{N}}(e,a))\theta_i^{\bar{N}}(e,a))}{q(\theta_i^{\bar{N}}(e,a))}$ *ensure labor market efficiency.*

Proof. See Appendix A. □

In short, we show that labor markets in the model are efficient when the standard Hosios condition holds if labor markets are segmented. Note that the bargaining power is set at the time of the match—one period before the production occurs and rents are shared—which implies that both the worker's age and experience have evolved by one unit by the time of the production. In the proof, we assume that workers' experience depreciates while non-employed, but the condition also holds if we assume that experience stays constant or increases over time. For simplicity, we denote the efficient bargaining condition by $\phi(x') = -\frac{q'(\theta(x))\theta(x))}{q(\theta(x))}$.

In Appendix B, we further show that the Hosios condition arises endogenously if we follow the competitive search theory literature and assume that firms post a menu of bargaining powers and workers choose to apply jobs that offer the bargaining power that maximizes their utility.

3.4 *Characterization of the Solution*

The solution for wages, tightness rates, and job-finding rates using backward induction is characterized as in Córdoba et al. (2021). In particular, we first obtain closed-form solutions for the last period of working life, which we then use to find solutions for the previous periods. The solutions for periods $a < a_R - 1$ can be expressed in terms of workers' surpluses and value changes defined as

$$\begin{aligned} S_{EU}(x) &\equiv E(x) - U(x_i^{\bar{U}}(e,a));\ S_{EN}(x) \equiv E(x) - N(x_i^{\bar{N}}(e,a)); \\ S_{NU}(x) &\equiv N(x) - U(x_i^{\bar{U}}(e,a));\ S_{UN}(x) \equiv U(x) - N(x_i^{\bar{N}}(e,a)); \\ \Delta U(x) &\equiv U(x) - U_i(e-1,a);\ \Delta N(x) = N(e,a) - N_i(e-1,a). \end{aligned} \tag{16}$$

The solutions for $w(x)$, $\theta(x)$, and $f(x)$, for $0 \le a < a_R - 1$, then satisfy

$$w(x) = \frac{h(x) + \Theta(x)\left[\bar{c}(x) + \beta\Omega(x)\right]}{1 + \Theta(x)}, \tag{17}$$

$$\kappa(x) = \beta A(x)(\alpha\theta(x)^{-\rho} + (1-\alpha))^{1/\rho} J_i^s(e, a+1), \text{ and} \tag{18}$$

$$f(x) = A(x)(\alpha + (1-\alpha)\theta(x)^{\rho})^{1/\rho}, \text{ where} \tag{19}$$

$$J_i^s(e, a+1) = \Theta(x) S_{Es}^i(e, a+1), \text{ and} \tag{20}$$

$$\Omega(x) = \left\{ \begin{array}{c} f_i^{\bar{U}}(e,a) S_{EU}^i(e, a+1) + \pi_{UN}(x) S_{NU}^i(e, a+1) + \\ \pi_{EN}(x)\left[S_{EN}(x') - S_{EU}(x')\right] - \Delta U^i(e+1, a+1), \text{if } s = U \\ f_i^{\bar{N}}(e,a) S_{EN}^i(e, a+1) - \pi_{NU}(x) S_{NU}^i(e, a+1) + \\ \pi_{EU}(x)\left[S_{EU}(x') - S_{EN}(x')\right] - \Delta N^i(e+1, a+1), \text{if } s = N \end{array} \right\}. \tag{21}$$

3.5 *Labor Share*

The labor share is defined as a share of output $h(x)$ that goes to the worker, as measured by the wage $w(x)$,

$$\begin{aligned} s_L(x) &= \frac{w(x)}{h(x)} = \frac{\frac{h(x)+\Theta(x)[\bar{c}(x)+\beta\Omega(x)]}{1+\Theta(x)}}{h(x)} \\ &= \frac{\phi(x)h(x) + (1-\phi(x))[\bar{c}(x) + \beta\Omega(x)]}{h(x)} \\ &= \phi(x) + (1-\phi(x))\frac{[\bar{c}(x) + \beta\Omega(x)]}{h(x)}. \end{aligned} \tag{22}$$

We further study how the derived labor share responds to changes in labor tightness:

$$\frac{ds_L(x)}{d\theta(x)} = \underbrace{\left\{1 - \left[\frac{\bar{c}(x) + \beta\Omega(x)}{h(x)}\right]\right\} \times \frac{d\phi(x)}{d\theta(x)}}_{\textit{bargaining channel}} + \underbrace{[1-\phi(x)] \times \frac{\beta}{h(x)} \times \frac{d\Omega(x)}{d\theta(x)}}_{\textit{outside option channel}}. \tag{23}$$

The above equation shows that the effect of tightness on the labor share can be divided into two parts: a bargaining channel and an outside option channel.[5] The bargaining channel measures the effect of tightness on the labor share that runs through the changes in workers' bargaining power, weighted by $\left[1 - \frac{\bar{c}(x)+\beta\Omega(x)}{h(x)}\right]$. With the CD matching function, the bargaining channel disappears, as $\frac{d\phi(x)}{d\theta(x)} = 0$. With the CES matching function and $\rho < 0$, bargaining power increases with tightness, so the bargaining channel is positive if the weight is positive.

The outside option channel measures the changes that run through the changes in $\Omega(x)$. Intuitively, and as equation (21) shows, higher tightness increases workers' outside options by increasing their job-finding rate. Thus, tightness increases the labor share via the outside option channel by increasing the outside option value, and this effect is weighted by $[1 - \phi(x)] \times \frac{\beta}{h(x)}$.

4 Parameterization

4.1 *Stylized Facts*

In this section, we investigate the relationship between the long-term evolution of the labor share, wage growth, and labor market tightness in the U.S. Earlier literature has linked the labor share decline with a decline in bargaining power (Bental and Demougin, 2010; Glover and Short, 2020; Stansbury and Summers, 2020). If tightness serves as a proxy for workers' bargaining power, and changes in bargaining power drive the decline in the labor share and wage growth, we expect these series to move in tandem.

[5] As noted earlier in this Section, we assume that $\bar{c}(x)$ is exogenous and thus will not vary with $\theta(x)$.

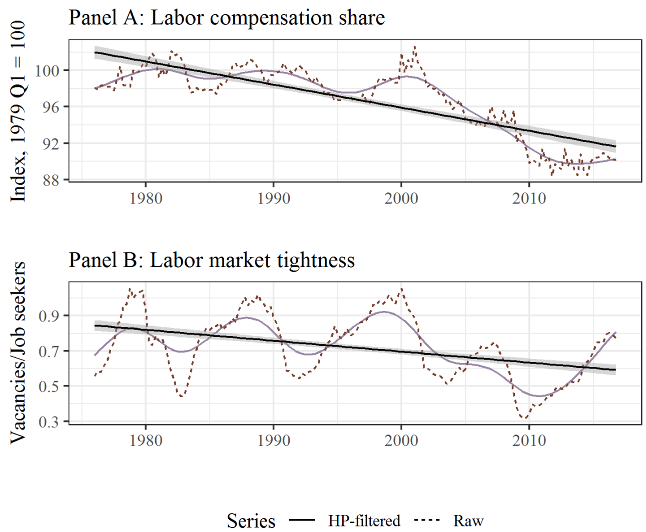

Figure 1. Labor market tightness and labor compensation share, 1976–2016

Note: Panel A shows the quarterly, seasonally adjusted labor share for all employed persons in the nonfarm business sector, and panel B shows the quarterly labor market tightness series, normalized to 1 in 1979:Q1. The dashed lines plot the raw series, while the purple solid lines plot the Hodrick-Prescott (HP)-filtered series with lambda set to 1,600. Both figures include a linear trend with 95 percent confidence bounds. Source: Bureau of Labor Statistics; Authors' calculations based on Petrosky-Nadeau and Zhang (2021) and IPUMS-CPS.

Panel A in figure 1 shows the evolution of the quarterly labor share for the U.S. nonfarm business sector between 1976 and 2016 (U.S. Bureau of Labor Statistics, 2022). As previously documented, the U.S. labor share has declined in the long run: Between 1976 and 2016, there has been about a 7.9 percent decline in the Bureau of Labor Statistics (BLS) data. Between two five-year periods that preceded to business cycle peaks, 1976–80 and 2003–07, the decline is 4.0 percent. However, the majority of the decline has occurred after 2000, and, overall, the labor share shows significant variation over time and across business cycles.

We use historical vacancy-rate data from Petrosky-Nadeau and Zhang (2021) and unemployment and nonparticipation data from the CPS from the Integrated Public Use Microdata Series (IPUMS-CPS) between 1976 and 2016 to construct a quarterly rate of labor market tightness. Specifically, our tightness measure includes the number of unemployed workers and nonparticipants between the ages of 25 and 64 in the denominator.[6] The underlying data series are reported on a monthly basis, so we calculate quarterly rates by averaging monthly values. Panel B in figure 1 shows the results. Similar to the labor share, labor market tightness has trended downward.[7] The decline between the two periods 1976–80 and 2003–07 is 21.5 percent. Compared with the labor share series, labor market tightness shows stronger business cycle fluctuations, but otherwise the two series exhibit similar trends: Neither show a strong downward trend between 1976 and 2000, so the decline in both occurs mostly after 2000. Also, both fluctuate similarly over time, with changes in tightness seeming to lead changes in the labor share.

Panel A in figure 2 plots the quarterly nominal wage growth of workers between 1983 and 2016, which is calculated by averaging the raw monthly growth rates from the Atlanta Fed's Wage Growth Tracker.[8] Following the trends in both the labor share and tightness, wage

[6]We rely on this measure because the same measure is used in the model calibration. The measure is also consistent with current literature that notes that vacancies over unemployment may not be the best approximation of tightness because it ignores large employment flows from nonparticipation and between jobs (see, for example, Abraham et al., 2020 and Hall and Schulhofer-Wohl, 2018). Our tightness measure is also closely positively correlated with a tightness rate constructed using as the denominator the Hornstein-Kudlyak-Lange Nonemployment Index (Hornstein-Kudlyak-Lange Non-Employment Index, 2023) from the Richmond Fed. The correlation coefficient is .97 when using data from 1994 to 2016. Index starts in 1994.

[7]Hall (2017) also documents the declining trend in the rate of labor market tightness.

[8]The Atlanta Fed's Wage Growth Tracker uses microdata from the CPS. The wage growth rate is calculated

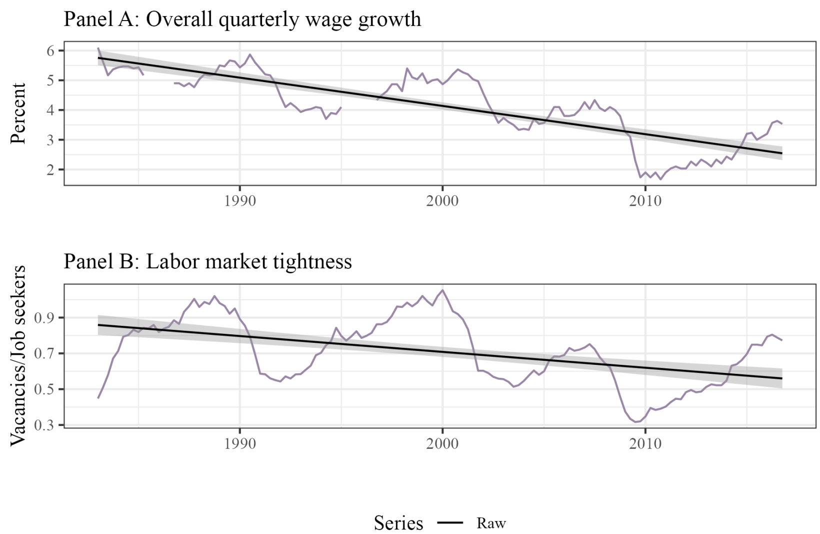

Figure 2. Labor market tightness and wage growth, 1983–2016

Note: Panel A includes the quarterly wage growth rate from the Atlanta Fed, and panel B shows the quarterly labor market tightness series. We plot the series starting from 1983, the first available year of the Atlanta Fed Wage Growth Tracker data. The purple solid lines plot the raw series. Both figures include a linear trend with 95 percent confidence bounds.

Source: Federal Reserve Bank of Atlanta; Authors' calculations based on Petrosky-Nadeau and Zhang (2021) and IPUMS-CPS.

growth rates have trended down. Moreover, wage growth follows closely the business cycle variation in both tightness and the labor share.

We further report the Pearson cross-correlation between tightness and both wage growth and the labor share. The correlation coefficient between the tightness rate and labor share series is .502 and is statistically significant, indicating a strong positive correlation between the series. The correlation between tightness and wage growth series is .693, again

as the median percent change in the hourly wage of individuals observed 12 months apart. See details at https://www.atlantafed.org/chcs/wage-growth-tracker.

indicating a strong positive correlation between the series.[9]

We check the robustness of the trends and correlations using a standard measure of labor market tightness. The results are reported in Appendix D.2., and those confirm the trends and correlations reported in this section.

4.2 *Preliminaries*

We calibrate both the CD and CES models to the same targets and compare the models' performance in generating a decline in the labor share. We calibrate the models using data from two periods, $t \in \{1976–80, 2003–07\}$. Both periods reflect the peak of the business cycle, and we use the data for five-year periods to have a sufficient sample to estimate life-cycle labor market flows for our disaggregated groups. We refer to these two periods using simply the years 1980 and 2007.

4.3 *Constant Parameters*

We calibrate the life-cycle trends at a quarterly frequency and set the discount rate β equal to 0.9902, which implies that the real interest rate equals 4 percent annually. We assume that workers work between the ages of 25 and 64. After that, workers retire and live until the age of 80. This gives us $\underline{a} = 0$(age 25), $a_R = 163$ (age 65), and $\bar{a} = 319$ (age 80). We assume that unemployed workers and retired workers consume a fixed fraction of their human capital:

[9] We also look at the cross-correlations between the labor share and time-lagged tightness series. The correlation coefficient increases when looking at the cross-correlation between the labor share and lagged labor market tightness. For example, the value of the correlation coefficient between the labor share and the tightness rate lagged by three quarters equals .557, indicating that the labor share responds with a delay to changes in labor market tightness.

$\bar{c}(x) = \gamma \cdot h_i(e, a)$, and $\bar{c}(x_R) = \gamma^R \cdot h_i(e, a_{R-1})$, respectively. We set the replacement-rate parameters for unemployed and retired workers to $\gamma = 0.35$ and $\gamma^R = 0.33$. Under these parameter values, in the model, the average consumption during unemployment is about 40 percent of the average consumption of the employed, and the average consumption during retirement is about 50 percent of the average human capital at the retirement age. The parameter α in both the CD and CES models is 0.5, following den Haan et al. (2000), and we set the matching elasticity ρ in the CES model to -0.3, which is close to values in Stevens (2007), with ρ of -0.3, Blanchard and Diamond (1989), with ρ of -0.35, and Hagedorn and Manovskii (2008), with ρ of -0.4.

4.4 *Time-Specific Parameters*

Life-Cycle Parameters. We assume that at age 25 the initial mass one of workers, $m_t^s(i, \underline{a})$, is divided between employment, unemployment, and nonparticipation such that the values match the average values for each group i observed in the IPUMS-CPS data in each t.

The calibration of life-cycle outcomes of workers follows closely Córdoba et al. (2021). We observe average labor market flows and average wages for every age but not for every level of experience in the data. For that reason, we use the model's analytical averages over experience to match the corresponding data. We directly estimate the exogenous flows $(\pi_{EU}^t(i,a), \pi_{EN}^t(i,a), \pi_{UN}^t(i,a), \pi_{NU}^t(i,a))$ for each i, a, and t using IPUMS-CPS data.[10]

We use the model solution, equations (17)–(21), to recover the matching efficiency and human capital parameters for each demographic group i. We assume that the human

[10]See Appendix D.1. for details.

capital of a worker i of age a and experience e is $h_i(e,a) = y_i e^{r_i(a)e}$. Specifically, we calibrate the initial human capital y_i^t to match the wage rate of i at age 25. Then returns to experience $r_i^t(a)$ are calibrated using equation (17) such that we exactly match the life-cycle profile of wages observed in the Center for Economic and Policy Research (CEPR) data (Center for Economic and Policy Research, Center for Economic and Policy Research (CEPR)) for each i.

We allow different matching efficiency $A_i^t(a)$, $A_{i,s}^t(a)$ for job-seekers from the unemployment pool and the nonparticipation pool. Specifically, using equation (19), we recover the life-cycle profiles of the matching efficiencies for the unemployed by matching their job-finding rates we estimate from the IPUMS-CPS data. Then we calibrate $\psi_i^t(a)$ such that we match the unexplained differences in job-finding rates between unemployed and nonparticipants of otherwise similar workers. Hence, $A_{i,s}^t(a) = \psi_i^t(a) A_i^t(a)$.

We set the consumption for nonparticipants $\bar{c}(x) = \gamma_i^t(e,a) \cdot h_i(e,a)$ and calibrate the replacement rate $\gamma_i^t(e,a)$ for each group such that the wage rates for workers coming from unemployment versus nonparticipation are the same in the model.

Vacancy-Posting Cost: κ_i^t. Hall (2017) suggests that the cost of posting a new vacancy is a constant share of the worker's productivity. This insight reflects the idea that the investment needed to create jobs increases with potential revenues. We follow Hall (2017) and set $\kappa_i^t(e,a) = \bar{\kappa}_i^t \times h_i^t(e,a)$.

We separately calibrate the values of $\bar{\kappa}_i^t$ for each gender–education group. Specifically, we set the $\bar{\kappa}_i^t$ to 0.33 for non-college males between 1976 and 1980, set $\bar{\kappa}_{MNC}^{1980}$ to 0.33—a standard value in the literature—and set $\bar{\kappa}_i^{1980}$ for other groups such that the average tightness gaps between non-college males and other groups are matched. We then calibrate the $\bar{\kappa}_i^t$ for all groups such that the changes in tightness rates between 1980 and 2007 are matched. In particular, the tightness target in the data that we use for each group is vacancies per group over the non-employed between the ages of 25 and 64 (unemployed + nonparticipants), a target that can easily be mapped to our model.

Constructing the group-specific vacancies is not straightforward because vacancy postings are not targeted to specific demographic groups. We rely on a simple assumption that current employment shares of each group in each year provide an estimate of the number of vacancies available for each group. Thus, we calculate group-specific vacancies v_i^t for each group by multiplying the number of vacancies with the employment share of group i at time t. The tightness-rate denominators for each group are simply their unemployment levels at t. These data are directly observed in IPUMS-CPS data. Group-specific measures of tightness are then

$$\theta_i^t \equiv \frac{s_{E,i}^t \times v^t}{u_i^t + n_i^t}, \tag{24}$$

where v_i^t is the number of vacancies, u_i^t is the number of unemployed, n_i^t is the number of nonparticipants, and $s_{E,i}^t$ is the share of group i of the total employment at t.

Labor market tightness has decreased for all groups, but specifically for males (see table 3 for details). For both college and non-college males, tightness in 2007 was about one-third of the tightness in 1980. The decline for females was more subdued: The labor market tightness has decreased about 14 percent for non-college females and about 8 percent for college-educated females.

Table 1. Parameter values for 1976–80 and 2003–07 steady states—common parameters across the CD and CES models

Parameter	Explanation	Value	Source
β	Discount factor	.9902	Quarterly rate
γ	Replacement rate	.35	Average consumption during unemployment
γ^R	Replacement rate: retired	.33	Average consumption during retirement
α	Matching function: share	.5	den Haan et al. (2000)
ρ	Matching elasticity: CES model	-.3	Stevens (2007)
$\pi^t_{EU}(i,a)$	Quarterly separation rate	See figure D.4, App D.4	Authors' estimation using IPUMS-CPS data
$\pi^t_{EN}(i,a)$	Quarterly separation rate	See figure D.4, App D.4	Authors' estimation using IPUMS-CPS data
$\pi^t_{UN}(i,a)$	Quarterly flow rate: U to N	See figure D.5, App D.4	Authors' estimation using IPUMS-CPS data
$\pi^t_{NU}(i,a)$	Quarterly flow rate: N to U	See figure D.5, App D.4	Authors' estimation using IPUMS-CPS data

4.5 *Results*

Tables 1 and 2 sum up the model parameters and how they are set. We focus next on describing the changes in the main parameters: matching efficiencies, returns to experience, nonparticipant consumption, and vacancy-posting costs.

We find that life-cycle returns to experience have increased for all groups between 1980 and 2007 (figure 3), and the trends are similar in both the CD and the CES models. The increase in the returns to experience has been most pronounced for females aged 35 and older, which has led to a convergence between the returns to experience of males and females within the education groups. The only group for which returns to experience have

Table 2. Calibrated parameter values for 1976–80 and 2003–07 steady states

Parameter	Explanation	Value: CD	Value: CES
$y_{MC,80}$	Initial human capital	1.28	1.22
$y_{MC,07}$	Initial human capital	1.27	1.29
$y_{MNC,80}$	Initial human capital	.97	1.12
$y_{MNC,07}$	Initial human capital	.99	1.03
$y_{FC,80}$	Initial human capital	1.16	1.07
$y_{FC,07}$	Initial human capital	1.17	1.18
$y_{FNC,80}$	Initial human capital	.85	.94
$y_{FNC,07}$	Initial human capital	.90	.93
$r_i^t(a)$	Returns to experience	See figure 3	See figure 3
$A_i^t(a)$	Matching efficiency: $\bar{U}$	See figure 4	See figure 4
$\psi_i^t(a)$	Search effort: $\bar{N}$	See figure D.6, App D.4	See figure D.6

Source: Authors' estimations.

declined is males younger than 35 without a college education. As shown in table 1, the calibration results also indicate that the initial human capital of non-college males has declined from 1980 to 2007.

These results reflect the observed changes in wage trends during the same period. First, an increase in returns to experience reflects the fact that real wages have increased for all groups except non-college males. Second, the decline in the gender gap in returns to experience goes hand in hand with the significant decline in the gender wage gap during the same period, as documented in Blau and Kahn (2017) and shown in figure D.2 in

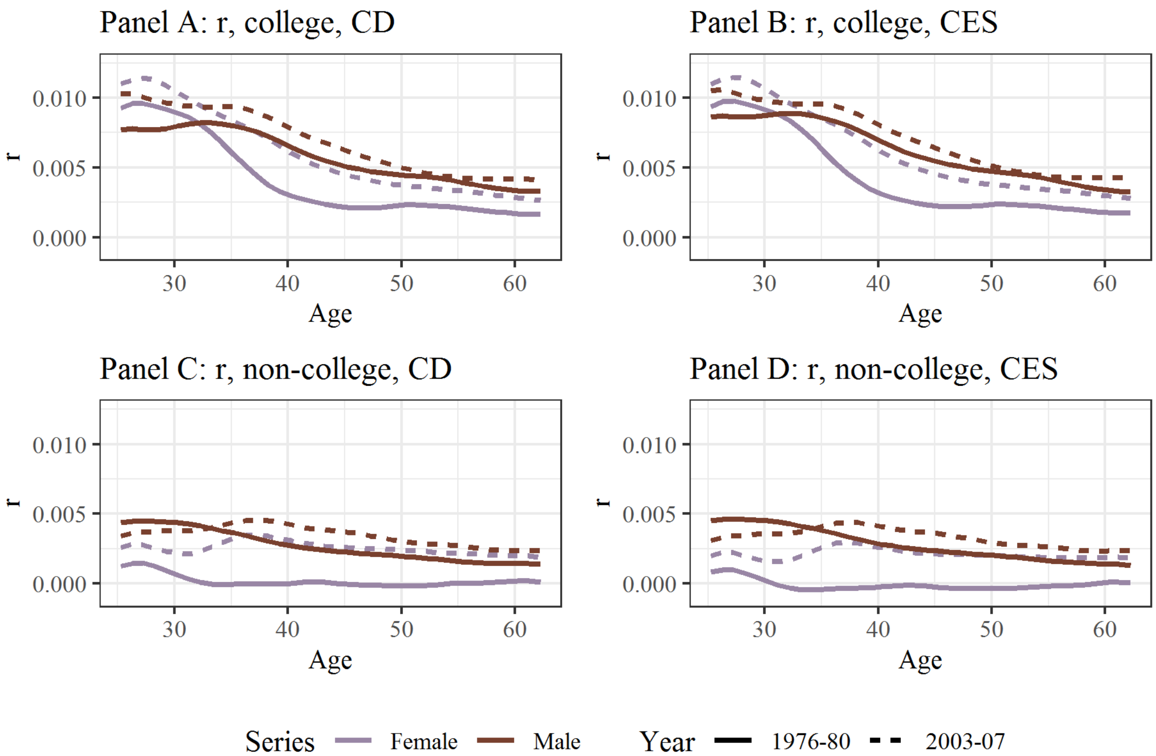

Figure 3. Workers' life-cycle returns to experience in 1980 and 2007, by gender and education
Note: Panels A and C plot the simulated life-cycle returns to experience for workers with and without a college education, respectively, by gender in the CD model. Panels B and D show the same simulation results in the CES model.

Source: Authors' estimations.

Appendix D.4.

Our calibration results show that for all groups matching efficiency is higher in 2007 than in 1980 (figure 4). Matching efficiencies are consistently higher in the CES model compared with the CD model, reflecting the differences in matching functions, but the trends align. Men have higher matching efficiency than women within each education group except for college-educated women, who had higher matching efficiency in 1980 compared with college-educated men. Likewise, younger workers have higher matching efficiency compared with older workers.

A validation of our calibration results for matching efficiency between the late 1970s and 2000s comes from the corresponding shifts in the Beveridge curve.[11] Shifts in the Beveridge curve are typically interpreted as changes in matching efficiency, with outward shifts reflecting a lower matching efficiency. Michaillat and Saez (2021) and Diamond and Şahin (2015) study long-term movements of the Beveridge curve in the U.S., and their findings show that the Beveridge curve in the late 1970s was located to the right of the curve in the late 2000s. This result supports the finding that matching efficiency was lower in the 1970s.

Figure 5 shows the replacement rates for nonparticipants over the life cycle, $\gamma_i^t(a)$. First, the calibrated values are higher in the CES model compared with the CD model. The bargaining power channel in the CES model amplifies the wage gap between the unemployed and nonparticipants whenever tightness rates are different for these groups. The consumption of nonparticipants needs to be higher for wages to be set equal between these groups.

The calibration results also indicate that the replacement rates have increased between 1980 and 2007 for females and college males but stayed fairly constant for non-college males. For college males, there is a clear drop in the replacement rate in 1980 between the ages of 30 and 40, while the drop largely disappears by 2007. This could capture the higher likelihood of fathers engaging in child-rearing activities in the latter period.

[11] The Beveridge curve reflects the negative relationship between unemployment and vacancy rates over the business cycle (Beveridge, 1944).

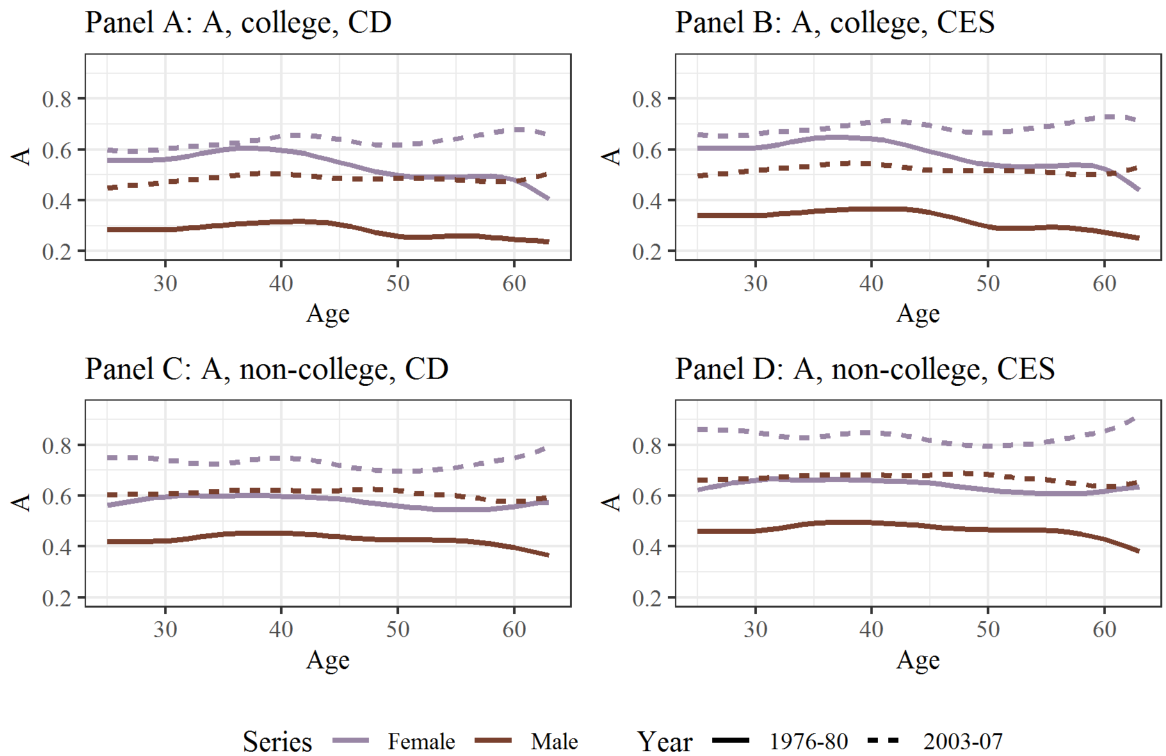

Figure 4. Unemployed workers' life-cycle matching efficiencies in 1980 and 2007, by gender and education

Note: Panels A and C plot the simulated life-cycle matching efficiencies for workers with and without a college education, respectively, in the CD model by gender. Panels B and D show the same simulation results in the CES model.

Source: Authors' estimations.

All the calibrated values of $\bar{\kappa}_i^t$ are shown in table 3, along with the calibration targets. First, the results show that we can closely match the tightness targets from data. Second, we find that vacancy costs vary by gender and education, likely capturing the differences in representative occupations and industries for each group. More interestingly, we find that $\bar{\kappa}$ has increased for every group between 1980 and 2007. First, we find that $\bar{\kappa}$ for both male groups has more than doubled, indicating a large increase in vacancy costs. This increase

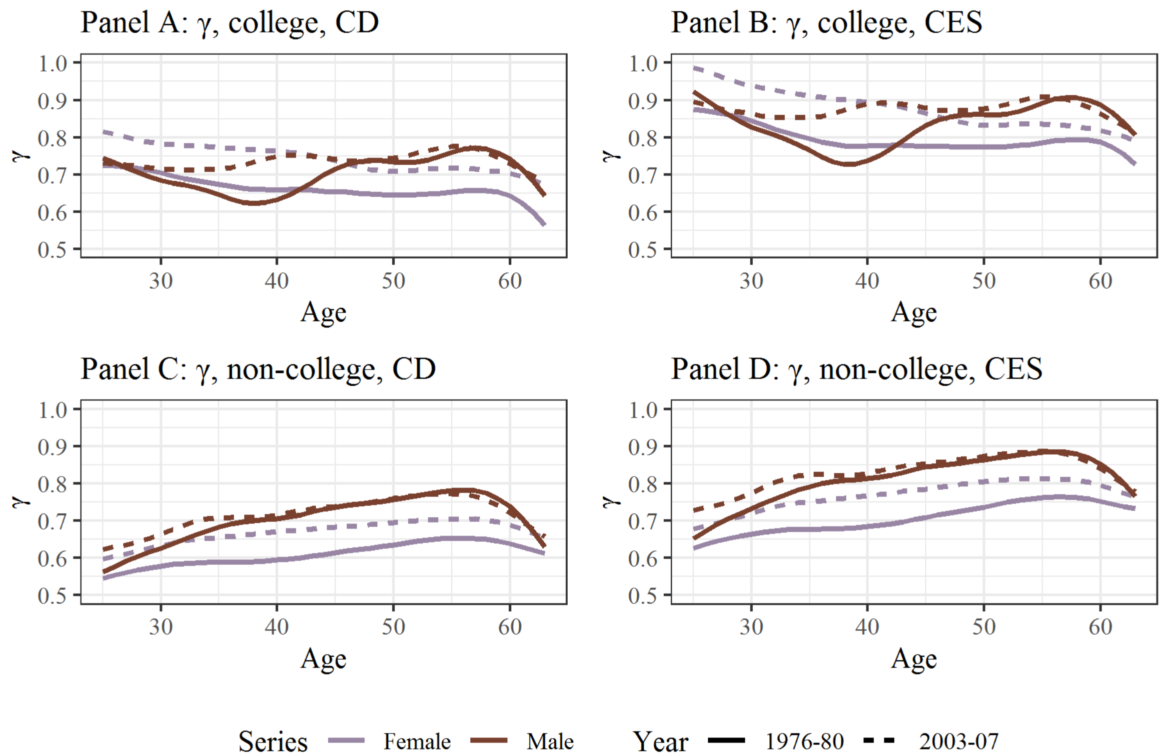

Figure 5. Replacement rates of nonparticipants over the life cycle in 1980 and 2007, by gender and education

Notes: Panels A and C plot the simulated life-cycle replacement rates for nonparticipants with and without a college education, respectively, in the CD model by gender. Panels B and D show the same simulation results in the CES model.

Source: Authors' estimations.

is also reflected in the decreased tightness rates. Second, while $\bar{\kappa}$ for non-college females has almost doubled, the increase has been more moderate for college-educated females. The $\bar{\kappa}$ for college-educated females has increased by a factor of 1.5.

What are the potential reasons for the increased $\bar{\kappa}$, and why has this increase varied significantly between groups? We interpret the changes in the vacancy posting costs to

Table 3. Calibrated $\bar{\kappa}$ by gender and education: 1980 and 2007 steady states

	1980					2007				
Group	θ, data	θ_{CD}	θ_{CES}	$\bar{\kappa}_{CD}$	$\bar{\kappa}_{CES}$	θ, data	θ_{CD}	θ_{CES}	$\bar{\kappa}_{CD}$	$\bar{\kappa}_{CES}$
Male, college	2.37	2.37	2.36	.20	.16	.72	.72	.72	.66	.73
Female, college	.35	.35	.36	.73	.90	.31	.32	.32	1.05	1.34
Male, non-college	1.00	1.00	1.00	.33	.33	.37	.36	.36	.86	1.10
Female, non-college	.20	.20	.20	.78	.99	.15	.15	.15	1.56	2.21

Note: We normalize the tightness rate of non-college males in 1980 to 1 and set their $\bar{\kappa}$ to 0.33. We then calibrate the remaining $\bar{\kappa}$ to match the relative tightness rates of other groups.

Source: Authors' estimations.

broadly reflect the changes in relative costs of creating jobs for certain groups.[12] Any outside factor that raises the relative cost of opening a vacancy in the U.S., given the vacancy value in the U.S., will be captured by $\bar{\kappa}$.

Hence, natural candidates are automation, increased globalization, and import competition. As described in Section 4.1, a large share of the drop in both the labor share and tightness occurred after 2000. The sluggish employment growth in the U.S. in the 2000s is tightly linked to increased import competition (Acemoglu et al., 2016; Charles et al., 2019). While import competition has directly depressed employment in the most affected industries, these effects have transmitted to other industries through input-output and aggregate demand linkages, further elevating employment losses (Acemoglu et al., 2016; Autor et al.,

[12]Another way to interpret the cost of posting a vacancy is to interpret it as a fixed entry cost, either in the units of capital or labor, as in Mangin and Sedláček (2018).

2016). Moreover, there is no strong evidence of offsetting employment gains in other industries in the long-term: Out-migration from the most affected local labor markets has been modest, manufacturing job losses have translated to declines in employment-to-population ratios, and the negative effects of the China shock have persisted until the late 2010s (Autor et al., 2021). In our model, these negative employment effects are captured by $\bar{\kappa}$, leading to lower vacancy posting and a drop in labor demand.

Moreover, evidence shows that the described negative employment effects from rising import competition have had heterogeneous effects on different gender and education groups, potentially explaining the heterogeneous changes in the calibrated vacancy costs. First, while both female- and male-dominated manufacturing industries have faced negative employment and wage consequences from import competition, a larger share of males work in manufacturing, leading to a larger effect on men (Autor et al., 2019). Second, the negative effect of trade exposure on employment have concentrated in local labor markets with a smaller share of college-educated workers (Autor et al., 2021). The lower decline in college-educated females' $\bar{\kappa}$ could arise from the fact that, compared with males, college-educated females are more often working on health-care- and education-related occupations, which are less affected by import competition.

Finally, the increase in the cost of creating a new job also lines up with the findings of Wolcott (2020). She concludes that the decline in employment rates of U.S. male workers, especially those without a college education, since the late 1970s is driven by demand factors rather than supply factors.

To sum up, we find that in order to match the observed wage trends (increases for other groups except non-college males), the increases in job-finding rates for females and the decreases for males, and the decreases in tightness rates, there must be counteracting forces that can jointly generate these trends. First, increased real wage rates indicate an increase in the productivity captured by human capital parameters. Second, while tightness rates have decreased, job-finding rates either have decreased less or have increased, meaning that matching efficiencies must have increased. Both factors increase demand for workers by increasing the value of opening a vacancy, leading to higher tightness rates. To match the declines in tightness rates, vacancy costs have grown, capturing the fact that while the vacancy value has also grown, there has been a counter force that has lowered labor demand.

Changes in the Labor Share in the CD and the CES Models. We find that the CES model generates a 2.6 percent decline in the labor share between 1980 and 2007, while the CD model generates a notably smaller decline of 1.1 percent (table 4). The labor share has dropped 4.0 percent during the same period.[13] The CES model thus explains about two-thirds (65 percent) of the decline in the labor share, while the CD model explains only 28 percent of the decline.[14] This result implies that the bargaining power channel is quantitatively important in generating the labor share decline in the model.[15]

[13]The decline is calculated by comparing the average labor shares in the periods of 1976–80 and 2003–07.

[14]Note that we do not target the labor share decline in our calibration.

[15]We further report model-generated changes in the labor shares for different groups in Appendix D.5.

Table 4. Decrease in the labor share from 1980 to 2007

	Data	CD model	CES model
Percent decline in labor share	4.0	1.1	2.6
Percent of decline in data	100.0	27.5	65.0

Note: Table 4 presents the decrease in the labor share in the data, in the CD model, and in the CES model.

Source: Bureau of Labor Statistics; Authors' estimations.

5 Changes in Efficient Bargaining Power

We study changes in efficient bargaining power of workers between 1980 and 2007 by gender and education (table 5). Our calibration results suggest a decline in bargaining power, consistent with previous empirical findings. First, we find that average bargaining power over the life cycle has decreased for all groups. The decrease has been larger for males and for workers without a college education. Bargaining power has dropped by 16.8 percent for college-educated males and 1.2 percent for college-educated females. At the same time, bargaining power has decreased by 17 percent for non-college males and by 5.5 percent for non-college females. Using employment shares of each group as weights, we find that aggregate bargaining power has declined by 11.1 percent.

Relying on the life-cycle feature of the model, we further investigate the life-cycle trends of bargaining power. Figure 6 plots the life-cycle bargaining power patters for all the groups between the ages of 25 and 64. When looking at male groups, bargaining power slowly declines over the life cycle. The life-cycle trends in bargaining power are somewhat different

Table 5. Model-implied efficient bargaining power of workers by gender and education: 1980 and 2007 steady states

Group	1980	2007	Percent change
Male, college	.563	.467	-16.8
Female, college	.403	.398	-1.2
Male, non-college	.501	.415	-17.0
Female, non-college	.365	.345	-5.5
Weighted average	.467	.415	-11.1

Source: Authors' estimations.

for women compared with men, following an S-shape. Bargaining power starts at a higher level, decreases at a faster pace up to a woman's mid-30s, increases again until age 45, and then decreases until retirement. However, the life-cycle levels and trends of males and females have converged over time, with the trends of females more closely reflecting those of males during the latter period. This is expected, given the convergence in other labor market outcomes, like wages and labor market flows.

By using equation (5), the calibrated parameter values for α and ρ, and observed time series of aggregate tightness, we construct an aggregate bargaining power series between 1976 and 2016. Based on the aggregate data, we find that the bargaining power of workers has decreased around 8 percent between the two business cycle peaks of 1979 and 2007 (panel B in figure 7). The decline in bargaining power is smaller than the weighted average from table 5, indicating that relying on aggregate tightness can lead to underestimating the overall decline.

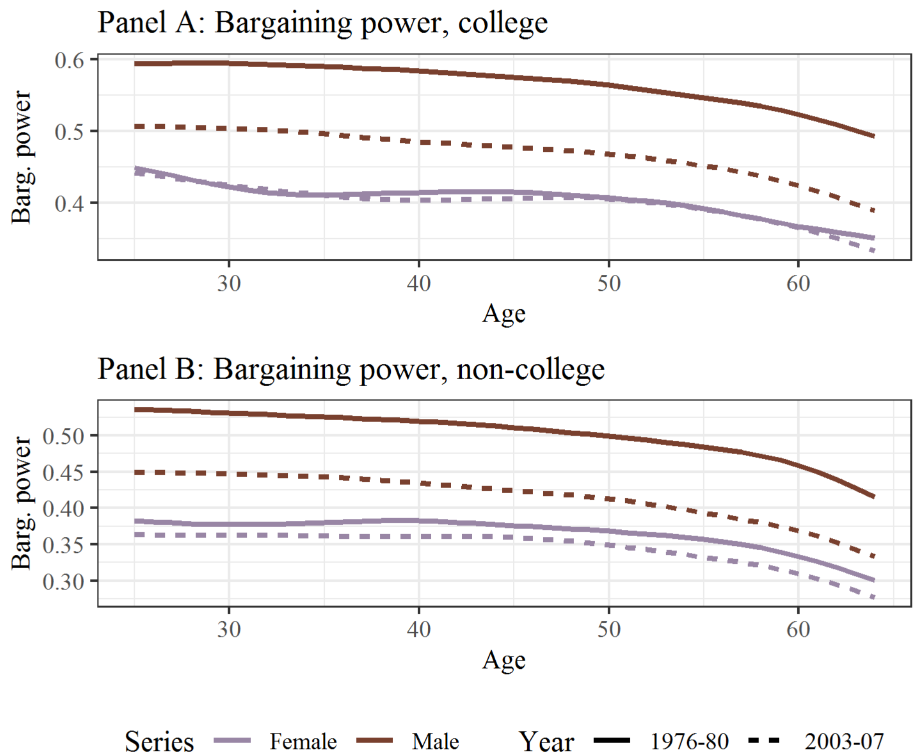

Figure 6. Workers' efficient life-cycle bargaining power in 1980 and 2007, by gender and education

Note: Panels A and B plot the simulated life-cycle bargaining power of workers with and without a college education, respectively, by gender.

Source: Authors' estimations.

When looking at the variation in bargaining power between 1976 and 2016, we find that bargaining power decreased notably between 2000 and 2010, consistent with the declines in tightness and the labor share. However, bargaining power recovered strongly after 2010, while the labor share increased slightly.

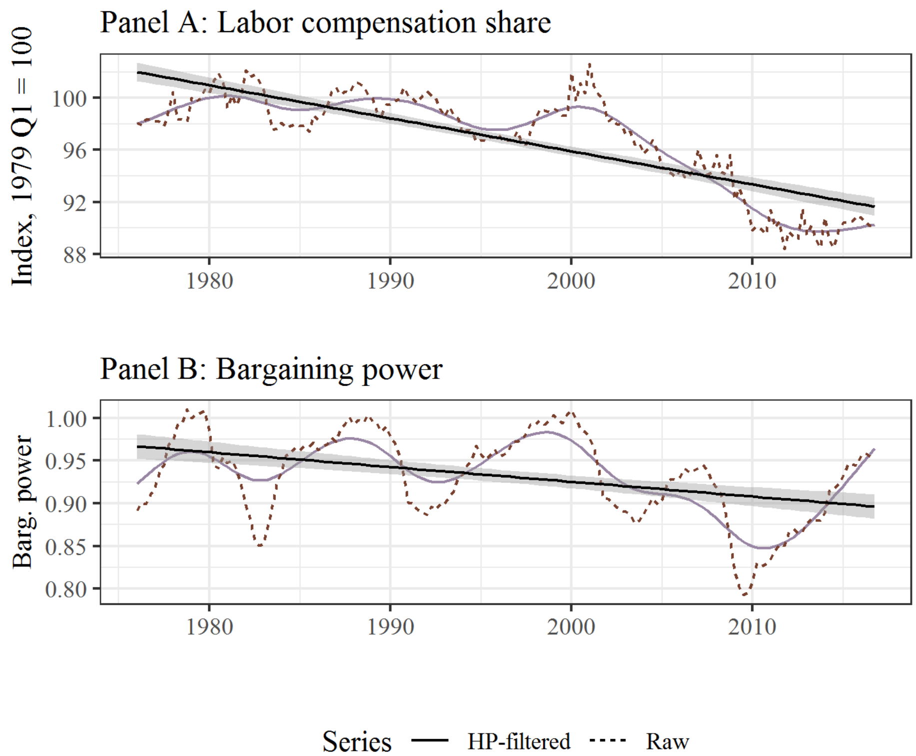

Figure 7. Efficient bargaining power has declined between 1979 and 2007, along with the labor share

Note: Panel A includes the quarterly, seasonally adjusted labor share for all employed persons in the nonfarm business sector, and panel B shows the quarterly bargaining power series normalized to 1 in 1979:Q1. The dashed lines plot the raw series, while the purple solid lines plot the HP-filtered series with lambda set to 1,600. Both figures also include a linear trend line with 95 percent confidence bounds.

Source: Bureau of Labor Statistics; Authors' estimations.

6 Counterfactual Analysis

We assess the effect of each exogenous parameter on the model-generated changes in bargaining power. We do that by giving each parameter its 1980 value one at a time and by keeping all the other parameters at their 2007 levels. Table 6 shows the results.

We find that three parameters have driven changes in bargaining power through changes in tightness. First, the increased vacancy-posting cost $\bar{\kappa}$ can explain the majority of the bargaining power decline for all groups. Second, improved matching efficiency and higher returns to experience have mitigated the decline in the bargaining power of all groups. Third, a lower π_{EN} has mitigated the decline for females, while a higher π_{EN} has increased the decline for males.

Intuitively, better matching efficiencies and higher returns to experience increase labor market tightness by increasing a vacancy value. To match the observed *decline* in the tightness, along with the observed wages and job-finding rates, the model predicts that the vacancy-posting cost has increased. An increase in the relative cost of opening a vacancy has decreased demand for labor and the number of vacancies.

To summarize our findings, we find that κ, a proxy for a decline in labor demand, has driven the decline in tightness, and thus bargaining power, between 1980 and 2007.

7 Conclusion

In this paper, we used a life-cycle DMP search and matching model with endogenous bargaining power to study how the labor share and workers' bargaining power have changed in the past four decades. Specifically, we assumed that the matching function takes the CES form, which implies that bargaining power increases with labor market tightness whenever the Hosios condition holds and there is enough complementarity between vacancies and job seekers in the matching process.

First, we find that a DMP model with endogenous bargaining power generates a signif-

Table 6. Counterfactuals—efficient bargaining power

	Male, College	Female, College	Male, Non-college	Female, Non-college
Total change—model	-16.8	-1.2	-17.0	-5.5
Parameter—percent contribution to the change				
y	0.1	-21.2	4.6	3.5
A	-7.6	-26.2	-11.0	-58.5
ψ	1.0	4.3	-1.4	-25.9
r	-4.5	-64.5	-9.3	-90.3
π_{EU}	0.7	7.5	1.0	5.3
π_{EN}	7.8	-78.0	14.4	-52.2
π_{UN}	2.9	-8.6	6.6	-3.1
π_{NU}	2.2	-1.2	3.2	-3.1
$\bar{\kappa}$	91.8	218.4	91.3	293.4
γ_N	3.8	60.8	-1.4	27.7
Starting masses, $m_t^s(i, \underline{a})$	1.8	8.6	1.9	3.1
Total contribution	100.0	100.0	100.0	100.0

Note: The contribution of each counterfactual is scaled such that the sum of individual counterfactuals equals 100 percent.

Source: Authors' estimations.

icantly larger drop in the labor share compared with the model with fixed bargaining power. Second, our calibration results suggest that workers' efficient bargaining power has decreased about 11 percent between 1980 and 2007. This can be attributed to a higher vacancy-posting cost, which has driven down the labor demand. We also find that the decline in bargaining power has been larger for men and workers without a college education.

Overall, the decline in bargaining power based on our model has been modest. However, as we abstract away from the reported decline in both union membership and coverage, we are likely underestimating the decline in bargaining power. Consider the import-competition shock that reduces demand for U.S. labor and thus labor market tightness. Unionized workers affected by the shock now face a tradeoff: Staying unionized with higher bargaining power and better benefits while facing even higher risk of jobs moving abroad. Existing evidence points out that unionization has decreased because of the same shocks that have decreased tightness, and it is possible that tightness would have decreased more without unionization declining, leading to lower bargaining power in our model. It would be interesting to extend our model to include endogenous decisionmaking on union membership and study the dynamics of bargaining power, union membership, and tightness in the face of demand shocks for labor. This is left to future work.

Also, we leave it to future work to establish what exactly has driven the decline in tightness in the U.S.

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Appendix A: A Social Planner's Problem and Hosios Condition in a Life-Cycle DMP Model with Human Capital Accumulation, Nonparticipation, and Segmented Markets

Appendix A.1: Social Planner's Problem

The planner maximizes the sum of flows of market and home productions net of search costs. In the life-cycle framework, these flows are considered over all ages a and different time periods. There are a^R overlapping generations of different ages a working in a given period $t+a$, where t is the date of birth of a given cohort. Moreover, workers gain experience when employed and may be subject to experience depreciation when out of work. Thus, the flows are also considered over all experience levels.

The planner chooses the optimal full sequences of vacancies $\left\{v_i^{\bar{U},t}\left(e,a\right), v_i^{\bar{N},t}\left(e,a\right)\right\}$, and employment, unemployment, and nonparticipation masses $\{M_i^{\bar{E},t+1}\left(e,a\right), M_i^{\bar{U},t+1}\left(e,a\right), M_i^{\bar{N},t+1}\left(e,a\right)\}$, given $a \in A \equiv [\underline{a}, a_R]$ and $e \in A \equiv [\underline{e}, a_R]$. Denote the control variables by $X = \left\{v_i^{\bar{U},t}\left(e,a\right), v_i^{\bar{N},t}\left(e,a\right), M_i^{\bar{E},t+1}\left(e,a\right), M_i^{\bar{U},t+1}\left(e,a\right), M_i^{\bar{N},t+1}\left(e,a\right)\right\}$. We assume that labor markets are segmented for each i, e, and a, which means that the planner faces a different matching technology for workers from different statuses. This also simplifies the planner's problem: It can treat each problem as a separate optimization problem. Moreover, the planner also observes the current labor market status of a job-seeker, $\bar{U}$ and $\bar{N}$. The social planner's problem is

$$\max_{X}\sum_{t=0}^{\infty}\beta^{t}\left[\sum_{e=1}^{a^{R}-1}\sum_{a=\underline{a}}^{a^{R}-1}h_{i}\left(e,a\right)M_{i}^{\bar{E},t}\left(e,a\right)+\bar{c}_{i}^{\bar{U}}\left(e,a\right)M_{i}^{\bar{U},t}\left(e,a\right)+\bar{c}_{i}^{\bar{N}}\left(e,a\right)M_{i}^{\bar{N},t}\left(e,a\right)-\right.$$

$$\left.\kappa_{i}^{\bar{U}}\left(e,a\right)v_{i}^{\bar{U},t}\left(e,a\right)-\kappa_{i}^{\bar{N}}\left(e,a\right)v_{i}^{\bar{N},t}\left(e,a\right)\right] \tag{25}$$

subject to

$$M_{i}^{\bar{E},t+1}\left(e,a\right)=\left[1-\pi_{EU}^{i}\left(e-1,a-1\right)-\pi_{EN}^{i}\left(e-1,a-1\right)\right]M_{i}^{\bar{E},t}\left(e-1,a-1\right)+$$

$$q\left(\frac{v_{i}^{\bar{U},t}\left(e+1,a-1\right)}{M_{i}^{\bar{U},t}\left(e+1,a-1\right)}\right)v_{i}^{\bar{U},t}\left(e+1,a-1\right)+\widehat{q}\left(\frac{v_{i}^{\bar{N},t}\left(e+1,a-1\right)}{M_{i}^{\bar{N},t}\left(e+1,a-1\right)}\right)v_{i}^{\bar{N},t}\left(e+1,a-1\right), \tag{26}$$

$$M_{i}^{\bar{U},t+1}\left(e,a\right)=M_{i}^{\bar{U},t}\left(e+1,a-1\right)+\pi_{EU}^{i}\left(e-1,a-1\right)M_{i}^{\bar{E},t}\left(e-1,a-1\right)-$$

$$q\left(\frac{v_{i}^{\bar{U},t}\left(e+1,a-1\right)}{M_{i}^{\bar{U},t}\left(e+1,a-1\right)}\right)v_{i}^{\bar{U},t}\left(e+1,a-1\right)-\pi_{UN}^{i}\left(e+1,a-1\right)M_{i}^{\bar{U},t}\left(e+1,a-1\right)+$$

$$\pi_{NU}^{i}\left(e+1,a-1\right)M_{i}^{\bar{N},t}\left(e+1,a-1\right), \tag{27}$$

$$M_{i}^{\bar{N},t+1}\left(e,a\right)=M_{i}^{\bar{N},t}\left(e+1,a-1\right)+\pi_{EN}^{i}\left(e-1,a-1\right)M_{i}^{\bar{E},t}\left(e-1,a-1\right)-$$

$$\widehat{q}\left(\frac{v_{i}^{\bar{N},t}\left(e+1,a-1\right)}{M_{i}^{\bar{N},t}\left(e+1,a-1\right)}\right)v_{i}^{\bar{N},t}\left(e+1,a-1\right)-\pi_{UN}^{i}\left(e+1,a-1\right)M_{i}^{\bar{U},t}\left(e+1,a-1\right)+$$

$$\pi_{NU}^{i}\left(e+1,a-1\right)M_{i}^{\bar{N},t}\left(e+1,a-1\right), \tag{28}$$

for $a=\underline{a},\ldots,a^{R}-1,\ e=1,\ldots,a^{R}-1,$ and $t=0,\ldots,\infty,$

and given initial masses $M_{i}^{\bar{E},t+1}\left(1,\underline{a}\right)$, $M_{i}^{\bar{U},t+1}\left(1,\underline{a}\right)$, and $M_{i}^{\bar{N},t+1}\left(1,\underline{a}\right)$ and terminal conditions $M_{i}^{\bar{E},t+1}\left(e,a^{R}\right)=0$, $M_{i}^{\bar{U},t+1}\left(e,a^{R}\right)=0$, and $M_{i}^{\bar{N},t+1}\left(e,a^{R}\right)=1$. The terminal conditions capture the assumption that workers retire at age a^{R} and move to nonparticipation, which also implies that $\pi_{EN}^{i}(e,a^{R-1})=1$, $\pi_{EU}^{i}(e,a^{R-1})=0$, $\pi_{UN}^{i}(e,a^{R-1})=1$, and $\pi_{NU}^{i}(e,a^{R-1})=0$.

The first constraint represents employment dynamics between two periods, t and $t-1$ for each e and a, while the last two constraints represent the evolution of unemployment and nonparticipation masses.

As labor market tightness is defined as $\theta_i^{s,t}(e,a) = \frac{v_i^{s,t}(e,a)}{M_i^{s,t}(e,a)}$, where $s \in \bar{U}, \bar{N}$, the planner's problem can be written in terms of tightness. The Lagrangian becomes

$$
\begin{aligned}
L = \max_X & \sum_{t=0}^{\infty} \beta^t \Bigg\{ \sum_{e=1}^{a^R-1} \sum_{a=\underline{a}}^{a^R-1} h_i(e,a) M_i^{\bar{E},t}(e,a) + M_i^{\bar{U},t}(e,a) \left(\bar{c}_i^{\bar{U}}(e,a) - \kappa_i^{\bar{U}}(e,a) \theta_i^{\bar{U},t}(e,a) \right) \\
& + M_i^{\bar{N},t}(e,a) \left(\bar{c}_i^{\bar{N}}(e,a) - \kappa_i^{\bar{N}}(e,a) \theta_i^{\bar{N},t}(e,a) \right) \Bigg\} \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{e=0}^{a^R-1} \sum_{a=\underline{a}}^{a^R-1} \lambda_i^t(e,a) \Bigg\{ \left(1 - \pi_{EU}^i(e-1,a-1) - \pi_{EN}^i(e-1,a-1)\right) M_i^{\bar{E},t}(e-1,a-1) \\
& + f\left(\theta_i^{\bar{U},t}(e+1,a-1)\right) M_i^{\bar{U},t}(e+1,a-1) + \widehat{f}\left(\theta_i^{\bar{N},t}(e+1,a-1)\right) M_i^{\bar{N},t}(e+1,a-1) - M_i^{\bar{E},t+1}(e,a) \Bigg\} \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{e=0}^{a^R-1} \sum_{a=\underline{a}}^{a^R-1} \mu_i^t(e,a) \Bigg\{ (1 - f\left(\theta_i^{\bar{U},t}(e+1,a-1)\right) - \pi_{UN}^i(e+1,a-1)) M_i^{\bar{U},t}(e+1,a-1) \\
& + \pi_{EU}^i(e-1,a-1) M_i^{\bar{E},t}(e-1,a-1) + \pi_{NU}^i(e+1,a-1) M_i^{\bar{N},t}(e+1,a-1) - M_i^{\bar{U},t+1}(e,a) \Bigg\} \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{e=0}^{a^R-1} \sum_{a=\underline{a}}^{a^R-1} \eta_i^t(e,a) \Bigg\{ (1 - \widehat{f}\left(\theta_i^{\bar{N},t}(e+1,a-1)\right) - \pi_{UN}^i(e+1,a-1)) M_i^{\bar{N},t}(e+1,a-1) \\
& + \pi_{EN}^i(e-1,a-1) M_i^{\bar{E},t}(e-1,a-1) + \pi_{UN}^i(e+1,a-1) M_i^{\bar{U},t}(e+1,a-1) - M_i^{\bar{N},t+1}(e,a) \Bigg\} \\
& + \mu_0(0) \left[u_t(0) (1 - f(\theta_t(0))) - u_{t+1}(0) \right]
\end{aligned}
$$

The first order conditions with respect to $M_i^{\bar{E},t+1}(e,a)$, $M_i^{\bar{U},t+1}(e,a)$, $M_i^{\bar{N},t+1}(e,a)$, $\theta_i^{\bar{U},t}(e,a)$ and $\theta_i^{\bar{N},t}(e,a)$ are written as follows:

$$
\begin{aligned}
M_i^{\bar{E},t+1}(e,a) : & \beta^{t+1} h_i(e,a) - \beta^t \lambda_i^t(e,a) + \beta^{t+1} \lambda_i^{t+1}(e+1,a+1) \left(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right) \\
& + \beta^{t+1} \mu_i^{t+1}(e+1,a+1) \pi_{EU}^i(e,a) + \beta^{t+1} \eta_{t+1}^i(e+1,a+1) \pi_{EN}^i(e,a) = 0
\end{aligned}
$$

$$M_i^{\bar{U},t+1}(e,a): \quad \beta^{t+1}(\bar{c}_i^{\bar{U}}(e,a) - \kappa_i^{\bar{U}}(e,a)\theta_i^{\bar{U},t+1}(e,a)) + \beta^{t+1}\lambda_i^{t+1}(e-1,a+1)\, f\left(\theta_i^{\bar{U},t+1}(e,a)\right)$$

$$+ \beta^{t+1}\mu_i^{t+1}(e-1,a+1)\left[1 - f\left(\theta_i^{\bar{U},t+1}(e,a)\right) - \pi_{UN}^i(e,a)\right]$$

$$+ \beta^{t+1}\eta_i^{t+1}(e-1,a+1)\,\pi_{UN}^i(e,a) - \beta^t\mu_i^t(e,a) = 0$$

$$M_i^{\bar{N},t+1}(e,a): \quad \beta^{t+1}(\bar{c}_i^{\bar{N}}(e,a) - \kappa_i^{\bar{N}}(e,a)\theta_i^{\bar{N},t+1}(e,a)) + \beta^{t+1}\lambda_i^{t+1}(e-1,a+1)\,\widehat{f}\left(\theta_i^{\bar{N},t+1}(e,a)\right)$$

$$+ \beta^{t+1}\eta_i^{t+1}(e-1,a+1)\left[1 - \widehat{f}\left(\theta_i^{\bar{N},t+1}(e,a)\right) - \pi_{NU}^i(e,a)\right]$$

$$+ \beta^{t+1}\mu_i^{t+1}(e-1,a+1)\,\pi_{NU}^i(e,a) - \beta^t\eta_i^t(e,a) = 0$$

$$\theta_i^{\bar{U},t}(e,a): -\beta^t M_i^{\bar{U},t}(e,a)\,\kappa_i^{\bar{U}}(e,a) + \beta^t\lambda_i^t(e-1,a+1)\, f'\left(\theta_i^{\bar{U},t}(e,a)\right) M_i^{\bar{U},t}(e,a)$$

$$-\beta^t\mu_i^t(e-1,a+1)\, f'\left(\theta_i^{\bar{U},t}(e,a)\right) M_i^{\bar{U},t}(e,a) = 0$$

$$\theta_i^{\bar{N},t}(e,a): -\beta^t M_i^{\bar{N},t}(e,a)\,\kappa_i^{\bar{N}}(e,a) + \beta^t\lambda_i^t(e-1,a+1)\,\widehat{f'}\left(\theta_i^{\bar{N},t}(e,a)\right) M_i^{\bar{N},t}(e,a)$$

$$-\beta^t\eta_i^t(e-1,a+1)\,\widehat{f'}\left(\theta_i^{\bar{N},t}(e,a)\right) M_i^{\bar{N},t}(e,a) = 0.$$

In the steady state, $t = t+1$ for all t, and we can reorganize and write the following:

$$\frac{\lambda_i(e,a)}{\beta} = h_i(e,a) + \lambda_i(e+1,a+1)\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right]$$

$$+\mu_i(e+1,a+1)\,\pi_{EU}^i(e,a) + \eta_i(e+1,a+1)\,\pi_{EN}^i(e,a);$$

$$\frac{\mu_i(e)}{\beta} = (\bar{c}_i^{\bar{U}}(e,a) - \kappa_i^{\bar{U}}(e,a)\theta_i^{\bar{U}}(e,a)) + \lambda_i(e-1,a+1)\, f\left(\theta_i^{\bar{U}}(e,a)\right)$$

$$+\mu_i(e-1,a+1)\left[1 - f\left(\theta_i^{\bar{U}}(e,a)\right) - \pi_{UN}^i(e,a)\right] + \eta_i(e-1,a+1)\,\pi_{UN}^i(e,a);$$

$$\frac{\eta_i^t(e,a)}{\beta} = (\bar{c}_i^{\bar{N}}(e,a) - \kappa_i^{\bar{N}}(e,a)\theta_i^{\bar{N}}(e,a)) + \lambda_i(e-1,a+1)\,\widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right)$$
$$+\eta_i(e-1,a+1)\left[1-\widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) - \pi_{UN}^i(e,a)\right] + \mu_i(e-1,a+1)\,\pi_{NU}^i(e,a);$$

$$-\kappa_i^{\bar{U}}(e,a) + \lambda_i(e-1,a+1)\,f'\left(\theta_i^{\bar{U}}(e,a)\right) - \mu_i(e-1,a+1)\,f'\left(\theta_i^{\bar{U}}(e,a)\right) = 0;$$

$$-\kappa_i^{\bar{N}}(e,a) + \lambda_i(e-1,a+1)\,\widehat{f}'\left(\theta_i^{\bar{N}}(e,a)\right) - \eta_i(e-1,a+1)\,\widehat{f}'\left(\theta_i^{\bar{N}}(e,a)\right) = 0.$$

Moreover, define $S_i^{\bar{U}*}(e,a) = (\lambda_i(e,a) - \mu_i(e,a))/\beta, S_i^{\bar{N}*}(e,a) = (\lambda_i(e,a) - \eta_i(e,a))/\beta$, and write

$$\begin{aligned}\lambda_i(e,a)/\beta = \quad & h_i(e,a) + \lambda_i(e+1,a+1) - (\pi_{EU}^i(e,a) + \pi_{EN}^i(e,a))\beta S_i^{\bar{U}*}(e+1,a+1) \\ & -\pi_{EN}^i(e,a)[\mu_i(e+1,a+1) - \eta_i(e+1,a+1)] \\ = \quad & h_i(e,a) + \lambda_i(e+1,a+1) - (\pi_{EU}^i(e,a) + \pi_{EN}^i(e,a))[\beta S_i^{\bar{N}*}(e+1,a+1)] \\ & -\pi_{EU}^i(e,a)[\eta_i(e+1,a+1) - \mu_i(e+1,a+1)];\end{aligned} \tag{29}$$

$$\begin{aligned}\mu_i(e,a)/\beta = \quad & (\bar{c}_i^{\bar{U}}(e,a) - \kappa_i^{\bar{U}}(e,a)\theta_i^{\bar{U}}(e,a)) + \beta f\left[\theta_i^{\bar{U}}(e,a)\right] S_i^{\bar{U}*}(e-1,a+1) + \mu_i(e-1,a+1) \\ & +\pi_{UN}^i(e,a))\left[\eta_i(e-1,a+1) - \mu_i(e-1,a+1)\right];\end{aligned} \tag{30}$$

$$\begin{aligned}\eta_i(e,a)/\beta = \quad & (\bar{c}_i^{\bar{N}}(e,a) - \kappa_i^{\bar{N}}(e,a)\theta_i^{\bar{N}}(e,a)) + \beta\widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) S_i^{\bar{N}*}(e-1,a+1) + \eta_i(e-1,a+1) \\ & +\pi_{NU}^i(e,a)\left[\mu_i(e-1,a+1) - \eta_i(e-1,a+1)\right];\end{aligned} \tag{31}$$

$$\kappa_i^{\bar{U}}(e,a) = \beta f'\left(\theta_i^{\bar{U}}(e,a)\right) S_i^{\bar{U}*}(e-1,a+1); \tag{32}$$

$$\kappa_i^{\bar{N}}(e,a) = \beta \widehat{f}'\left(\theta_i^{\bar{N}}(e,a)\right) S_i^{\bar{N}*}(e-1,a+1). \tag{33}$$

Then, subtract equation (30) from equation (29), and (31) from (29), and insert (32) and (33):

$$\begin{aligned}
S_i^{\bar{U}*}(e,a) \equiv\ & \frac{\lambda_i(e,a) - \mu_i(e,a)}{\beta} \\
=\ & h_i(e,a) - \bar{c}_i^{\bar{U}}(e,a) + \beta S_i^{\bar{U}*}(e-1,a+1)\left[\theta_i^{\bar{U}}(e,a) f'\left[\theta_i^{\bar{U}}(e,a)\right] - f\left[\theta_i^{\bar{U}}(e,a)\right]\right] \\
& +\mu_i(e+1,a+1) - \mu_i(e-1,a+1) - \pi_{UN}^i(e,a)\left[\eta_i(e-1,a+1) - \mu_i(e-1,a+1)\right] \\
& -\pi_{EN}^i(e,a)\left[\mu_i(e+1,a+1) - \eta_i(e+1,a+1)\right] + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{U}*};
\end{aligned} \tag{34}$$

$$\begin{aligned}
S_i^{\bar{N}*}(e,a) \equiv\ & \frac{\lambda_i(e,a) - \eta_i(e,a)}{\beta} \\
=\ & h_i(e,a) - \bar{c}_i^{\bar{N}}(e,a) + \beta S_i^{\bar{N}*}(e-1,a+1)\left[\theta_i^{\bar{N}}(e,a) \widehat{f}'\left[\theta_i^{\bar{N}}(e,a)\right] - \widehat{f}\left[\theta_i^{\bar{N}}(e,a)\right]\right] \\
& +\eta_i(e+1,a+1) - \eta_i(e-1,a+1) - \pi_{NU}^i(e,a)\left[\mu_i(e-1,a+1) - \eta_i(e-1,a+1)\right] \\
& -\pi_{EU}^i(e,a)\left[\mu_i(e+1,a+1) - \eta_i(e+1,a+1)\right] + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{N}*};
\end{aligned} \tag{35}$$

$$S_i^{\bar{U}*}(e,a) - S_i^{\bar{N}*}(e,a) = \frac{\eta_i(e,a) - \mu_i(e,a)}{\beta}. \tag{36}$$

Thus, the planner's solution is summarized by the following seven equations:

$$S_i^{\bar{U}*}(e,a) \equiv \frac{\lambda_i(e,a)-\mu_i(e,a)}{\beta}$$

$$= h_i(e,a) - \bar{c}_i^{\bar{U}}(e,a) + \beta S_i^{\bar{U}*}(e-1,a+1)\left[\theta_i^{\bar{U}}(e,a) f'\left[\theta_i^{\bar{U}}(e,a)\right] - f\left[\theta_i^{\bar{U}}(e,a)\right]\right]$$

$$+\mu_i(e+1,a+1) - \mu_i(e-1,a+1) - \pi_{UN}^i(e,a)\left[\eta_i(e-1,a+1) - \mu_i(e-1,a+1)\right]$$

$$-\pi_{EN}^i(e,a)\left[\mu_i(e+1,a+1) - \eta_i(e+1,a+1)\right] + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{U}*}; \tag{37}$$

$$S_i^{\bar{N}*}(e,a) \equiv \frac{\lambda_i(e,a)-\eta_i(e,a)}{\beta}$$

$$= h_i(e,a) - \bar{c}_i^{\bar{N}}(e,a) + \beta S_i^{\bar{N}*}(e-1,a+1)\left[\theta_i^{\bar{N}}(e,a) \widehat{f}'\left[\theta_i^{\bar{N}}(e,a)\right] - \widehat{f}\left[\theta_i^{\bar{N}}(e,a)\right]\right]$$

$$+\eta_i(e+1,a+1) - \eta_i(e-1,a+1) - \pi_{NU}^i(e,a)\left[\mu_i(e-1,a+1) - \eta_i(e-1,a+1)\right]$$

$$-\pi_{EU}^i(e,a)\left[\mu_i(e+1,a+1) - \eta_i(e+1,a+1)\right] + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{N}*}; \tag{38}$$

$$S_i^{\bar{U}*}(e,a) - S_i^{\bar{N}*}(e,a) = \frac{\eta_i(e,a) - \mu_i(e,a)}{\beta} \tag{39}$$

$$\mu_i(e,a)/\beta = \left(\bar{c}_i^{\bar{U}}(e,a) - \kappa_i^{\bar{U}}(e,a)\theta_i^{\bar{U}}(e,a)\right) + \beta f\left[\theta_i^{\bar{U}}(e,a)\right] S_i^{\bar{U}*}(e-1,a+1) + \mu_i(e-1,a+1)$$

$$+\pi_{UN}^i(e,a))\left[\eta_i(e-1,a+1) - \mu_i(e-1,a+1)\right]; \tag{40}$$

$$\eta_i(e,a)/\beta = \left(\bar{c}_i^{\bar{N}}(e,a) - \kappa_i^{\bar{N}}(e,a)\theta_i^{\bar{N}}(e,a)\right) + \beta \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) S_i^{\bar{N}*}(e-1,a+1) + \eta_i(e-1,a+1)$$

$$+\pi_{NU}^i(e,a)\left[\mu_i(e-1,a+1) - \eta_i(e-1,a+1)\right]; \tag{41}$$

$$\kappa_i^{\bar{U}}(e,a) = \beta f'\left(\theta_i^{\bar{U}}(e,a)\right) S_i^{\bar{U}*}(e-1,a+1); \tag{42}$$

$$\kappa_i^{\bar{N}}(e,a) = \beta \widehat{f}' \left(\theta_i^{\bar{N}}(e,a)\right) S_i^{\bar{N}*}(e-1,a+1). \tag{43}$$

Appendix A.2: Decentralized Problem

Next, we characterize the decentralized problem and its solution. We assume markets are segmented, which implies that firms can choose how many vacancies to post for each type of worker across i, e, a, and s.

Workers' value functions are written as follows:

A value of an employed worker from the unemployment pool is $E_i^{\bar{U}}$ and from the nonparticipation pool is $E_i^{\bar{N}}$:

$$\begin{aligned} E_i^{\bar{U}}(e,a) = \quad & w_i^{\bar{U}}(e,a) + \beta \left[\pi_{EU}^i(e,a) U_i(e+1,a+1) + \pi_{EN}^i(e,a) N_i(e+1,a+1) \right. \\ & \left. + \left(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right) E_i^{\bar{U}}(e+1,a+1)\right] \\ = \quad & w_i^{\bar{U}}(e,a) + \beta \left[E_i^{\bar{U}}(e+1,a+1) - (\pi_{EU}^i(e,a) + \pi_{EN}^i(e,a)) D_i^{\bar{U}}(e+1,a+1) \right. \\ & \left. - \pi_{EN}^i(e,a)(U_i(e+1,a+1) - N_i(e+1,a+1))\right]; \end{aligned}$$

$$\begin{aligned} E_i^{\bar{N}}(e,a) = \quad & w_i^{\bar{N}}(e,a) + \beta \left[E_i^{\bar{N}}(e+1,a+1) - (\pi_{EU}^i(e,a) + \pi_{EN}^i(e,a)) D_i^{\bar{N}}(e+1,a+1) \right. \\ & \left. + \pi_{EU}^i(e,a)(U_i(e+1,a+1) - N_i(e+1,a+1))\right], \end{aligned}$$

where $D_i^{\bar{U}}(e,a) = E_i^{\bar{U}}(e,a) - U_i(e,a)$; $D_i^{\bar{N}}(e,a) = E_i^{\bar{N}}(e,a) - N_i(e,a)$.

The value functions for unemployed and nonparticipants are the following:

$$
\begin{aligned}
U_i(e,a) =& c_i^{\bar{U}}(e,a) + \beta\Big\{ f\left(\theta_i^{\bar{U}}(e,a)\right) E_i^{\bar{U}}(e-1,a+1) + \left(1 - f\left(\theta_i^{\bar{U}}(e,a)\right) - \pi_{UN}^i(e,a)\right) U_i(e-1,a+1) \\
&+ \pi_{UN}^i(e,a) N_i(e-1,a+1)\Big\} \\
=& c_i^{\bar{U}}(e,a) + \beta\Big\{ U_i(e-1,a+1) + f\left(\theta_i^{\bar{U}}(e,a)\right) D_i^{\bar{U}}(e-1,a+1) \\
&+ \pi_{UN}^i(e,a)(N_i(e-1,a+1) - U_i(e-1,a+1))\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
N_i(e,a) =& c_i^{\bar{N}}(e,a) + \beta\Big\{ \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) E_i^{\bar{N}}(e-1,a+1) + \left(1 - \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) - \pi_{UN}^i(e,a)\right) N_i(e-1,a+1) \\
&+ \pi_{UN}^i(e,a) N_i(e-1,a+1)\Big\} \\
=& c_i^{\bar{N}}(e,a) + \beta\Big\{ N_i(e-1,a+1) + \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) D_i^{\bar{N}}(e-1,a+1) \\
&- \pi_{NU}^i(e,a)(N_i(e-1,a+1) - U_i(e-1,a+1))\Big\}.
\end{aligned}
$$

To get worker surpluses, subtract the value of unemployment (nonparticipation) from $E_i^{\bar{U}}$ ($E_i^{\bar{N}}$):

$$
\begin{aligned}
D_i^{\bar{U}}(e,a) =& w_i^{\bar{U}}(e,a) - c_i^{\bar{U}}(e,a) + \beta\Big\{(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)) D_i^{\bar{U}}(e+1,a+1) + U_i(e+1,a+1) \\
&- f\left(\theta_i^{\bar{U}}(e,a)\right) D_i^{\bar{U}}(e-1,a+1)] - \pi_{EN}^i(e,a)(U_i(e+1,a+1) - N_i\left(e+1,a+1\right)) \\
&- U_i(e-1,a+1) - \pi_{UN}^i(e,a)(U_i(e-1,a+1) - N_i(e-1,a+1))\Big\}
\end{aligned}
$$

$$
\begin{aligned}
D_i^{\bar{N}}(e,a) =& w_i^{\bar{N}}(e,a) - c_i^{\bar{N}}(e,a) + \beta\Big\{(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)) D_i^{\bar{N}}(e+1,a+1) + N_i(e+1,a+1) \\
&- \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) D_i^{\bar{N}}(e-1,a+1)] + \pi_{EU}^i(e,a)(U_i(e+1,a+1) - N_i\left(e+1,a+1\right)) \\
&- N_i(e-1,a+1) + \pi_{NU}^i(e,a)(N_i(e-1,a+1) - U_i(e-1,a+1))\Big\}.
\end{aligned}
$$

Regarding the firms' problem, firms' value functions can be written as follows. First, the value of filling the vacancy from the unemployment pool is

$$J_i^{\bar{U}}(e,a) = h_i(e,a) - w_i^{\bar{U}}(e,a) + \beta\left(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right) J_i^{\bar{U}}(e+1,a+1),$$

and from the nonparticipant pool is

$$J_i^{\bar{N}}(e,a) = h_i(e,a) - w_i^{\bar{N}}(e,a) + \beta\left(1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right) J_i^{\bar{N}}(e+1,a+1).$$

The values of unfilled vacancies are written as

$$V_i^{\bar{U}}(e,a) = max\{-\kappa_i^{\bar{U}}(e,a) + \beta[q[\theta_i^{\bar{U}}(e,a)]J_i^{\bar{U}}(e-1,a+1) + (1 - [q[\theta_i^{\bar{U}}(e,a)])\bar{V}], 0\} \quad \text{and}$$

$$V_i^{\bar{N}}(e,a) = max\{-\kappa_i^{\bar{N}}(e,a) + \beta[\widehat{q}[\theta_i^{\bar{N}}(e,a)]J_i^{\bar{N}}(e-1,a+1) + (1 - [\widehat{q}[\theta_i^{\bar{N}}(e,a)])\bar{V}], 0\}.$$

With free entry, the values of unfilled vacancies are zero, so the previous two equations simplify to

$$\kappa_i^{\bar{U}}(e,a) = \beta q[\theta_i^{\bar{U}}(e,a)J_i^{\bar{U}}(e-1,a+1) \quad \text{and}$$

$$\kappa_i^{\bar{N}}(e,a) = \beta \widehat{q}[\theta_i^{\bar{N}}(e,a)J_i^{\bar{N}}(e-1,a+1).$$

Wages are determined through Nash bargaining. The match surpluses, $E_i^{\bar{U}}(e,a) - U_i(e,a) + J_i^{\bar{U}}(e,a)$ and $E_i^{\bar{N}}(e,a) - N_i(e,a) + J_i^{\bar{N}}(e,a)$, are shared according to the Nash product:

$$\max_{E_i - U_i, J_i} (E_i^{\bar{U}} - U_i)^{\phi_i^{\bar{U}}(e,a)} J_i^{\bar{U}\,1-\phi_i^{\bar{U}}(e,a)} \text{ subject to } S_i^{\bar{U}} = D_i^{\bar{U}} + J_i^{\bar{U}} \quad \text{and}$$

$$\max_{E_i - N_i, J_i} (E_i^{\bar{N}} - N_i)^{\phi_i^{\bar{N}}(e,a)} J_i^{\bar{N}\,1-\phi_i^{\bar{N}}(e,a)} \text{ subject to } S_i^{\bar{N}} = D_i^{\bar{N}} + J_i^{\bar{N}}.$$

The solution for unemployed satisfies

$$\frac{\phi_i^{\bar{U}}(e,a)}{1-\phi_i^{\bar{U}}(e,a)} = \frac{E_i^{\bar{U}}(e,a) - U_i(e,a)}{J_i^{\bar{U}}(e,a)} \quad \text{or}$$

$$E_i^{\bar{U}}(e,a) - U_i(e,a) = \phi_i^{\bar{U}}(e,a) S_i^{\bar{U}}(e,a) \text{ and } J_i^{\bar{U}}(e,a) = \left(1-\phi_i^{\bar{U}}(e,a)\right) S_i^{\bar{U}}(e,a),$$

and for nonparticipants

$$\frac{\phi_i^{\bar{N}}(e,a)}{1-\phi_i^{\bar{N}}(e,a)} = \frac{E_i^{\bar{N}}(e,a) - N_i(e,a)}{J_i^{\bar{N}}(e,a)} \quad \text{or}$$

$$E_i^{\bar{N}}(e,a) - N_i(e,a) = \phi_i^{\bar{N}}(e,a) S_i^{\bar{N}}(e,a) \text{ and } J_i^{\bar{N}}(e,a) = \left(1-\phi_i^{\bar{N}}(e,a)\right) S_i^{\bar{N}}(e,a).$$

Thus, the decentralized solution is defined by the following six equations:

$$\begin{aligned} S_i^{\bar{U}}(e,a) \equiv & J_i^{\bar{U}} + D_i^{U}(e,a) \\ = \quad & h_i(e,a) - c_i^{\bar{U}}(e,a) + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{U}}(e+1,a+1) \\ & -\beta f\left[\theta_i^{\bar{U}}(e,a)\right] D_i^{\bar{U}}(e-1,a+1) + \beta\Big\{U_i(e+1,a+1) - U_i(e-1,a+1) \\ & - \pi_{EN}^i\left[U_i(e+1,a+1) - N_i(e+1,a+1)\right] - \pi_{UN}^i(e,a)\left[N_i(e-1,a+1) - U_i(e-1,a+1)\right]\Big\} \end{aligned} \tag{44}$$

$$\begin{aligned} S_i^{\bar{N}}(e,a) \equiv & J_i^{\bar{N}} + D_i^{N}(e,a) \\ = \quad & h_i(e,a) - c_i^{\bar{N}}(e,a) + \beta\left[1 - \pi_{EU}^i(e,a) - \pi_{EN}^i(e,a)\right] S_i^{\bar{N}}(e+1,a+1) \\ & -\beta \widehat{f}\left[\theta_i^{\bar{N}}(e,a)\right] D_i^{\bar{N}}(e-1,a+1) + \beta\Big\{N_i(e+1,a+1) - N_i(e-1,a+1) \\ & + \pi_{EU}^i\left[U_i(e+1,a+1) - N_i(e+1,a+1)\right] + \pi_{NU}^i(e,a)\left[N_i(e-1,a+1) - U_i(e-1,a+1)\right]\Big\} \end{aligned} \tag{45}$$

$$\begin{aligned} U_i(e,a) = & c_i^{\bar{U}}(e,a) + \beta\Big\{U_i(e-1,a+1) + f\left(\theta_i^{\bar{U}}(e,a)\right) D_i^{\bar{U}}(e-1,a+1) \\ & + \pi_{UN}^i(e,a)(N_i(e-1,a+1) - U_i(e-1,a+1))\Big\} \end{aligned} \tag{46}$$

$$N_i(e,a) = c_i^{\bar{N}}(e,a) + \beta \Big\{ N_i(e-1,a+1) + \widehat{f}\left(\theta_i^{\bar{N}}(e,a)\right) D_i^{\bar{N}}(e-1,a+1) \\ - \pi_{NU}^i(e,a)(N_i(e-1,a+1) - U_i(e-1,a+1)) \Big\} \tag{47}$$

$$\kappa_i^{\bar{U}}(e,a) = \beta q[\theta_i^{\bar{U}}(e,a)] J_i^{\bar{U}}(e-1,a+1), \quad \text{and} \tag{48}$$

$$\kappa_i^{\bar{N}}(e,a) = \beta \widehat{q}[\theta_i^{\bar{N}}(e,a)] J_i^{\bar{N}}(e-1,a+1). \tag{49}$$

Appendix A.3: Proof of Proposition 1

Let's now compare the social planner's solution and the decentralized solution. When comparing equations (37)–(43) and (44)–(49), we see that the two systems are equivalent when $\phi_i^{\bar{U}}(e-1,a+1) = -\frac{q'(\theta_i^{\bar{U}}(e,a))\theta_i^{\bar{U}}(e,a))}{q(\theta_i^{\bar{U}}(e,a))}$ and $\phi_i^{\bar{N}}(e-1,a+1) = -\frac{q'(\theta_i^{\bar{N}}(e,a))\theta_i^{\bar{N}}(e,a))}{q(\theta_i^{\bar{N}}(e,a))}$, when also noting that $\frac{\mu_i(e,a)}{\beta} = U_i(e,a)$ and $\frac{\eta_i(e,a)}{\beta} = N_i(e,a)$. $\square$

Appendix B: Competitive Equilibrium with Bargaining Posting in DMP Model with a Life Cycle, Human Capital Accumulation and Nonparticipation

The wage setting in the model follows the one commonly used in the competitive search theory: While competitive search theory assumes that firms directly post wage rates, we assume that firms post bargaining weights and workers direct their search towards their utility-maximizing bargaining weight. As noted in Wright et al. (2021) on page 131, these approaches are fundamentally the same. Competitive search equilibrium implies that the match surplus shares are not constant but respond to market conditions. The only exception is the special case of the Cobb-Douglas matching function, which guarantees constant surplus shares.

In our case, explicitly focusing on bargaining power posting allows us to use the model to discuss how bargaining power might have changed in response to labor market conditions, given that bargaining power is not directly observed in the data. Once a firm and a worker are matched in a submarket determined by the bargaining weight $\phi(x)$, the firm and the worker share the match surplus using the optimal bargaining weight, as in Nash bargaining. Firms and workers update wages every period. We assume that bargaining weights respond to current market conditions: Whenever market tightness changes, bargaining weights react accordingly, even if a worker and a firm are already matched.

Following Moen (1997) competitive search equilibrium, we show that the profit- and utility-maximizing behaviors of firms and job seekers determine the optimal bargaining power. While Moen (1997) shows how firms and workers optimally choose a wage from a menu of wages, we assume instead that both parties choose their optimal bargaining weights. This assumption leads to competitive allocation, which we confirm coincides with the socially optimal allocation.

Assume again the similar dynamic DMP model with nonparticipation, life cycle, and human capital accumulation as discussed before. Assume that for each labor market x there is a set of Φ equilibrium sub-labor markets, where Φ stands for all the possible workers' bargaining powers. Then the values of being either a job seeker or employed in a submarket with ϕ and state x are given as

$$E^s(x;\phi) = w(x;\phi) + \beta\left[E^s\left(x';\phi\right) - (\pi_{EU}(x) + \pi_{EN}(x))D^s(x';\phi) - \pi_{EN}(x)(U(x';\phi) - N(x';\phi))\right];$$

$$U(x;\phi) = c(x) + \beta\left[U(x';\phi) + f\left(\theta(x;\phi)\right)D^{\bar{U}}(x';\phi) + \pi_{UN}(x)(N(x';\phi) - U(x';\phi))\right];$$

$$N(x;\phi) = c(x) + \beta\left[N(x';\phi) + \widehat{f}\left(\theta(x;\phi)\right)D^{\bar{N}}(x';\phi) - \pi_{NU}(x)(N(x';\phi) - U(x';\phi))\right].$$

We can also define the value of posting a vacancy and the value of having a vacancy filled as:

$$V^{\bar{U}}(x,\phi) = \max\left\{-\kappa(x) + \beta\left[q\left(\theta\left(x;\phi\right)\right)J^{\bar{U}}(x';\phi) + \left(1 - q\left(\theta\left(x;\phi\right)\right)\right)V^{\bar{U}}(x;\phi)\right], 0\right\};$$

$$V^{\bar{N}}(x,\phi) = \max\left\{-\kappa(x) + \beta\left[\widehat{q}\left(\theta\left(x;\phi\right)\right)J^{\bar{N}}(x';\phi) + \left(1 - \widehat{q}\left(\theta\left(x;\phi\right)\right)\right)V^{\bar{N}}(x;\phi)\right], 0\right\};$$

$$J^{s}(x,\phi) = h(x) - w(x;\phi) + \beta\left\{(\phi_{EU}(x) + \phi_{EN}(x))V^{s}(x;\phi) + \left(1 - \phi_{EU}(x) - \phi_{EN}(x)\right)J^{s}(x';\phi)\right\}.$$

Because of the free-entry condition and the workers' search behaviors, we have

$$V^{s}(x;\phi) = 0 \; \forall\phi \in\phi \text{ and for any } x \,,$$

$$U(x,\phi) = \overline{U}(x) \; \forall\phi \in\phi \text{ and for any } x,$$

$$N(x,\phi) = \overline{N}(x) \; \forall\phi \in\phi \text{ and for any } x.$$

Now we differentiate the value functions $U(x;\phi)$, $N(x;\phi)$, $V^{\bar{U}}(x;\phi)$, and $V^{\bar{N}}(x;\phi)$, and we get

$$\frac{dU(x;\phi)}{d\phi} = f'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(D^{\bar{U}}(x';\phi)) + f(\theta(\phi))\frac{d(D^{\bar{U}}(x';\phi))}{d\phi}) = 0; \tag{50}$$

$$\frac{dN(x;\phi)}{d\phi} = \widehat{f}'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(D^{\bar{N}}(x';\phi)) + \widehat{f}(\theta(\phi))\frac{d(D^{\bar{N}}(x';\phi))}{d\phi}) = 0; \tag{51}$$

$$\frac{dV^{\bar{U}}(x;\phi)}{d\phi} = q'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J^{\bar{U}}(x';\phi)) + q(\theta(\phi))\frac{dJ^{\bar{U}}(x';\phi)}{d\phi}) = 0; \tag{52}$$

$$\frac{dV^{\bar{N}}(x;\phi)}{d\phi} = \widehat{q}'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J^{\bar{N}}(x';\phi)) + \widehat{q}(\theta(\phi))\frac{dJ^{\bar{N}}(x';\phi)}{d\phi}) = 0. \tag{53}$$

Let's solve the optimal bargaining power for the unemployed. The same solution applies to the optimal bargaining power for nonparticipants. Rearrange equations (50) and equation (52) and get

$$f'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}D^{\bar{U}}(x';\phi) = -f(\theta(\phi))\frac{dD^{\bar{U}}(x';\phi)}{d\phi})$$

$$q'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J(x';\phi)) = -q(\theta(\phi))\frac{dJ(x';\phi)}{d\phi})$$
$$\Rightarrow \frac{f'(\theta(x;\phi))}{q'(\theta(x;\phi))}\frac{D^{\bar{U}}(x';\phi)}{J(x';\phi)} = \frac{f(\theta(x;\phi))}{q(\theta(x;\phi))}\frac{dD^{\bar{U}}(x';\phi)/d\phi}{dJ(x';\phi)/d\phi}. \tag{54}$$

Combining the equation (54) with the fact that a surplus is $S(\phi) = J(x;\phi) + E(x;\phi) - U(x;\phi)$[16] and the sharing rule of surplus is $E(x;\phi) - U(x;\phi) = \phi S(x;\phi)$, $J(x;\phi) = (1 - \phi)S(x;\phi)$, we get

$$\frac{f'(\theta(x;\phi))}{q'(\theta(x;\phi))}\frac{\phi S(x';\phi)}{(1-\phi)S(x';\phi)} = \frac{f(\theta(x;\phi))}{q(\theta(x;\phi))} \times \frac{d\phi S(x';\phi)}{d\phi} \times \left[\frac{d(1-\phi)S(x';\phi)}{d\phi}\right]^{-1}$$
$$\Rightarrow \frac{\theta q'(\theta(\phi)) + q(\theta(\phi))}{q'(\theta(\phi))}\frac{\phi}{1-\phi} = -\theta(\phi).$$

When the equation is simplified, the solution for the system becomes

$$\phi = -\frac{q'(\theta(x;\phi))\theta(x;\phi)}{q(\theta(x;\phi))}. \tag{55}$$

Thus, equation (55) is exactly the socially efficient condition for bargaining power, and the efficient condition for bargaining power holds for any given set Φ of submarkets that exists in the equilibrium. In other words, the bargaining power ϕ serves as a price device to adjust the relative demand and supply of labor. In equilibrium, firms are picking the efficient submarkets to pursue the highest profit, and so do the utility-maximizing job seekers. The "price" of bargaining power must follow the efficient rule $\phi = -\frac{q'(\theta(\phi))\theta(\phi)}{q(\theta(\phi))}$.

The decentralized equilibrium is efficient. Equivalently, the efficient bargaining condition arises endogenously.

[16] Note here the surplus created is irrelevant of the bargaining power. The reason is that once the worker entered the bargaining process, the surplus is fixed, the bargaining is just dividing this surplus between two parties. Because of this, the surplus has already been maximized before determining the share of each party. Thus, it has no effect on the surplus.

Appendix C: DMP Model Details

Labor Flows. Given the initial distribution of workers, $m_i^s(0, \underline{a})$, and job-finding rates $f(x)$ for all x, the subsequent distribution of workers $m(x)$ can be calculated assuming a law of large numbers. The mass of individuals with experience $e \in [1, a]$ at age $a \in [\underline{a}, a_R - 2]$ is determined by

$$\begin{aligned}
m_i^{\bar{E}}(e, a+1) &= (1 - \pi_{EU}(x) - \pi_{EN}(x)) \times m_i^{\bar{E}}(e-1, a) + f_i\left(e, a, \bar{U}\right) \times m_i^{\bar{U}}(e, a) \\
&\quad + f_i\left(e, a, \bar{N}\right) \times m_i^{\bar{N}}(e, a); \\
m_i^{\bar{U}}(e, a+1) &= (1 - \pi_{UN}(x) - f_i\left(e, a, \bar{U}\right)) \times m_i^{\bar{U}}(e, a) + \pi_{NU}(x) \times m_i^{\bar{N}}(e, a) \\
&\quad + \pi_{EU}(x) \times m_i^{\bar{E}}(e-1, a); \\
m_i^{\bar{N}}(e, a+1) &= (1 - \pi_{NU}(x) - f_i\left(e, a, \bar{N}\right)) \times m_i^{\bar{N}}(e, a) + \pi_{UN}(x) \times m_i^{\bar{U}}(e, a) \\
&\quad + \pi_{EN}^i(x) \times m_i^{\bar{E}}(e-1, a).
\end{aligned} \tag{56}$$

The above equations nest the flows for individuals without experience when one sets $e = 0$ and $m_i^{\bar{E}}(0, a) = 0$.

Appendix D: Data and Detailed Calibration Results

Appendix D.1: Description of Data

We use the basic monthly CPS data from 1976 to 1980 and 2003 to 2007 (Center for Economic and Policy Research, Center for Economic and Policy Research (CEPR) and Flood et al., 2020). The data include both full- and part-time U.S. workers. We disaggregate the data based on an individual's gender (male or female) and education status (college or non-college). An individual is assigned to the college group if she has completed at least some college and to the non-college group, if her highest level of completed education is high school or less. We then calculate life-cycle trends of average wages, and employment, unemployment, and non-participation rates for each of the described demographic groups.

We rely on the hourly wage rates obtained from the CEPR (Center for Economic and Policy Research (CEPR)), while the other data are obtained from the raw CPS data files from IPUMS (Flood et al., 2020). The advantage of using the CEPR wage data instead of the raw CPS data is that the CEPR adjusts the raw CPS wage data such that the constructed wage data series are consistent and comparable over time and are especially suitable for research uses.[17]

We also estimate monthly, age-specific transition probabilities between employment ($\bar{E}$), unemployment ($\bar{U}$), and nonparticipation ($\bar{N}$) separately for each group, following the method in Choi et al. (2015). In practice, the transition probability estimates are weighted-average flows between labor market states for every age when controlling for birth cohorts. For a given cohort and survey year, we observe the fraction of individuals of a given age that transfers from one labor market state to another. Denote this variable as $\pi_{ss'}(a, c, t)$, where ss' denotes the transition from a status $s \in \left\{\bar{E}, \bar{U}, \bar{N}\right\}$ to a status $s' \in \left\{\bar{E}', \bar{U}', \bar{N}'\right\}$, a denotes an individual's age, c denotes the cohort (the birth year) an individual belongs to, and t denotes the survey year.

We obtain the estimated transition probabilities by running seemingly unrelated regressions of $\pi_{ss'}(a, c, t)$ against age dummies. The coefficient for each age dummy is the probability that a transition happens at age a. A limitation of the CPS data is that it does not contain

[17] For a detailed description, please refer to the CEPR-CPS documentation found at https://ceprdata.org/cps-uniform-data-extracts/cps-basic-programs/.

a variable capturing the work experience of individuals. As a result, only average (over experience) transition probabilities can be estimated. We denote these estimated transition probabilities, $\pi_{ss'}(e, a, s, i) \equiv \pi_{ss'}(x)$, as $\pi_{ss'}(a, s, i)$.

To remove high-frequency reversals of transitions between unemployment and nonparticipation, we follow the method suggested by Elsby et al. (2015) called "deNUNification." The key idea is to correct for a possible classification error of an individual's labor market state: An individual who moves from nonparticipation to unemployment and back to nonparticipation within a short period of time is likely to be a nonparticipant—including these high-frequency transitions between states may lead to spurious transition estimates. The correction method thus recodes the high-frequency transitions, NUN, as NNN. The same method is applied to high-frequency transitions from unemployment to non-participation and back.

The estimated flows between different labor market states are flow probabilities from employment to unemployment and to nonparticipation—$\pi_{EU}(a, s, i)$ and $\pi_{EN}(a, s, i)$, respectively; unemployment to nonparticipation, $\pi_{UN}(a, s, i)$; nonparticipation to unemployment, $\pi_{NU}(a, s, i)$; and unemployment and nonparticipation to employment—$\pi_{UE}(a, s, i) \equiv f_i\left(e, a, \bar{U}\right)$ and $\pi_{NE}(a, s, i) \equiv f_i\left(e, a, \bar{N}\right)$, respectively. As the period in our model calibration will be set to a quarter instead of a month, we calculate quarterly transition probability matrices, $\Lambda_Q(a, s, i)$, as $\Lambda_Q(a, s, i) = (\Lambda_M(a, s, i)^{\wedge 3}$, where $\Lambda_M(a, s, i)$ equals

$$\begin{pmatrix} 1 - \pi_{EU}(a, s, i) - \pi_{EN}(a, s, i) & \pi_{EU}(a, s, i) & \pi_{EN}(a, s, i) \\ \pi_{UE}(a, s, i) & 1 - \pi_{UE}(a, s, i) - \pi_{UN}(a, s, i) & \pi_{UN}(a, s, i) \\ \pi_{NE}(a, s, i) & \pi_{NU}(a, s, i) & 1 - \pi_{NE}(a, s, i) - \pi_{NU}(a, s, i) \end{pmatrix}.$$

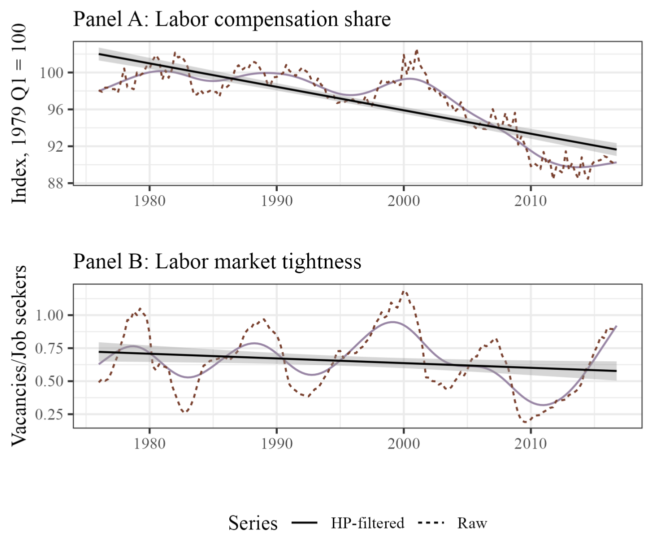

Figure D.1. Labor market tightness and labor compensation share, 1976–2016

Note: Panel A includes the quarterly, seasonally adjusted labor share for all employed persons in the nonfarm business sector, and panel B plots the quarterly labor market tightness using the number of unemployed workers over the age of 16 as the denominator. The dashed lines plot the raw series, while the purple solid lines plot the HP-filtered series with lambda set to 1,600. Both figures also include a linear trend with 95 percent confidence bounds.

Source: Bureau of Labor Statistics; Authors' calculations based on Petrosky-Nadeau and Zhang (2021) and IPUMS-CPS.

Appendix D.2: Stylized Facts—Robustness

Figure D.1 plots the labor share and the standard tightness rate (total vacancies/all unemployed people over 16 years), and table D.1 shows correlation coefficients between the labor share and both the standard and alternative tightness rates. The results confirm that the labor share and tightness are positively correlated, although the correlation is weaker between the labor share and the standard tightness rate.

Table D.1. Correlation coefficients between the labor share and different measures of labor market tightness

	Standard	Alternative 1
θ	$.353^{***}$	$.502^{***}$
θ_{-1}	$.394^{***}$	$.520^{***}$
θ_{-2}	$.432^{***}$	$.539^{***}$
θ_{-3}	$.464^{***}$	$.557^{***}$

Note: *** p< 0.001. Both tightness measures share the same numerator—all vacancies from Petrosky-Nadeau and Zhang (2021). The standard measure of tightness (column 1) uses the number of all unemployed workers over the age of 16 as the denominator. Alternative 1 (column 2) uses the number of all unemployed workers and nonparticipants between the ages of 25 and 64 as the denominator. Table shows correlation coefficients and related p-values between raw labor share (U.S. Bureau of Labor Statistics, 2022) and tightness rates and their lags. Lags are quarterly lags: for example, -1 represents a tightness series that is lagged by one quarter.
Source: Authors' calculations based on data from Bureau of Labor Statistics, Petrosky-Nadeau and Zhang (2021) and IPUMS-CPS.

Appendix D.3: Calibration Algorithm

We solve the model using backwards induction. Given the set values of β, $\bar{\gamma}$, γ_R, α, ρ, and $\pi_{ss'}(i,a)$, the calibration algorithm to recover y_i, $r_i(a)$, $A_i(a)$, $\psi_i(a)$, $\gamma_i(e,a)$, $\bar{\kappa}_i$ and bargaining weights $\phi_i^s(e,a)$ is the following:

Step 1: Make a reasonable guess of the bargaining weights of workers $\phi_i^s(e,a)$ and vacancy-posting costs $\bar{\kappa}_i$.

Step 2: At given bargaining weights, vacancy-posting costs, and other parameter values, solve the model and use model solutions to reverse engineer the group-specific human capital parameters (y_i, $r_i(a)$) and matching efficiencies ($A_i(a)$, $\psi_i(a)$) to fit the observed wage rate and job-finding rates. We obtain $\gamma_i(e, a)$ by equalizing the wage rates for the unemployed and nonparticipants.

Step 3: We use group-specific tightness rates with Proposition 2 (the Hosios condition) to update the guess of bargaining power and group-specific tightness to update guesses for $\bar{\kappa}_i$.

Step 4: We repeat steps 2 and step 3 until the bargaining power series converge and the tightness rates hit their targets.

Appendix D.4: Calibration Results

This section includes the remaining calibration targets and results.

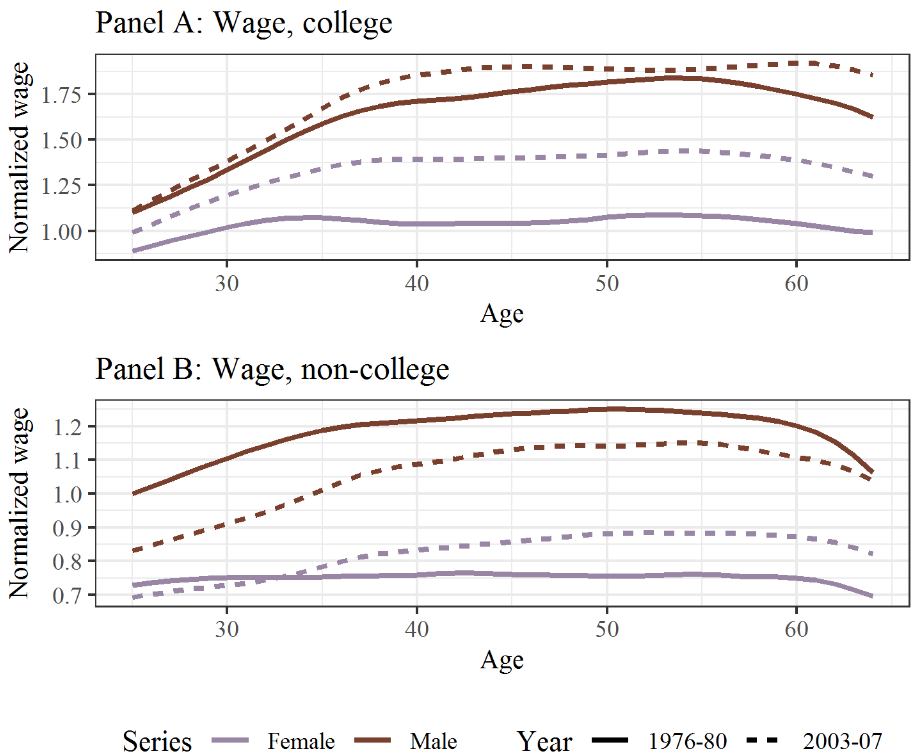

Figure D.2. Life-cycle hourly wages in 1976–80 and 2003–07, by gender and education

Note: Panels A and B plot the life-cycle wages for workers with and without a college education, respectively, by gender. All wages are shown relative to the wage rate of non-college males at age 25 in 1976–80 period.

Source: Authors' calculations based on data from CEPR.

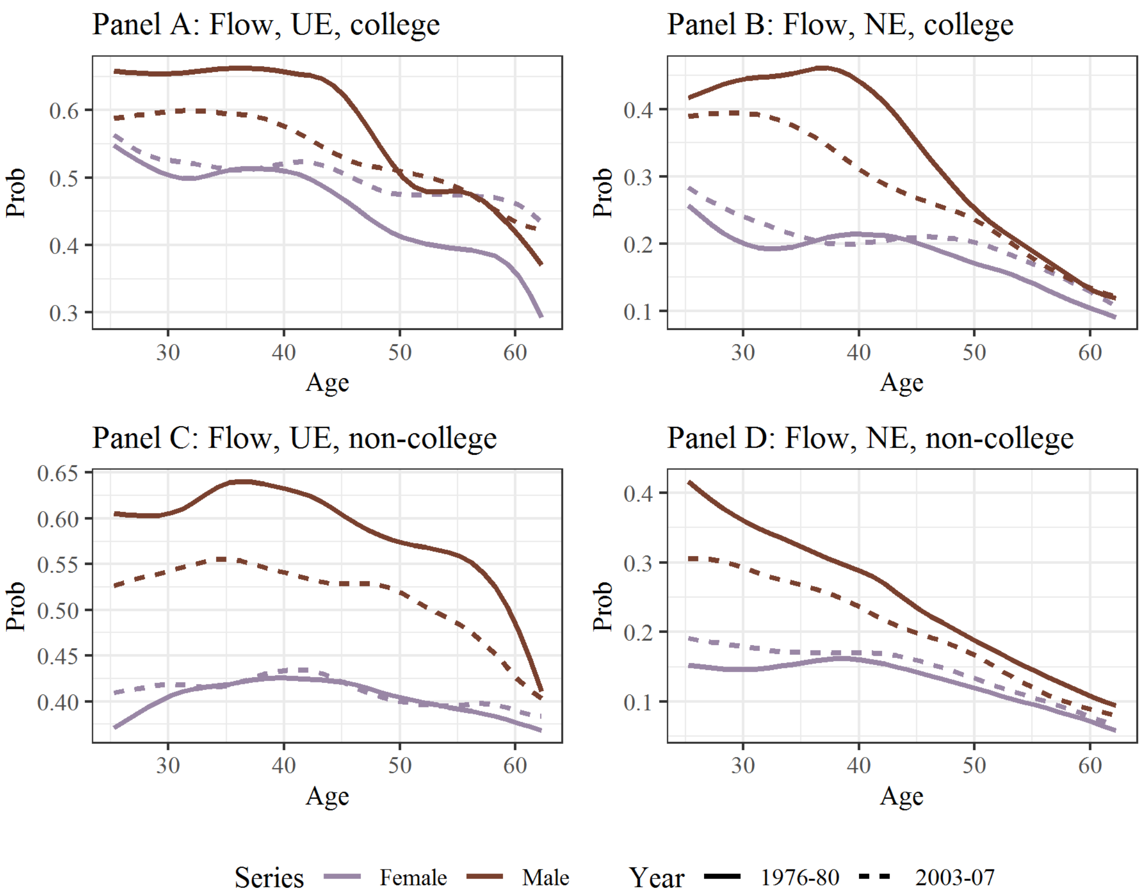

Figure D.3. Life-cycle job-finding rates in 1976–80 and 2003–07, by gender and education

Note: Panels A and C plot the life-cycle job-finding rates from unemployment for workers with and without a college education, respectively, by gender. Panels B and D plot the life-cycle job-finding rates from nonparticipation for workers with and without a college education, respectively, by gender.

Source: Authors' estimations based on data from IPUMS-CPS.

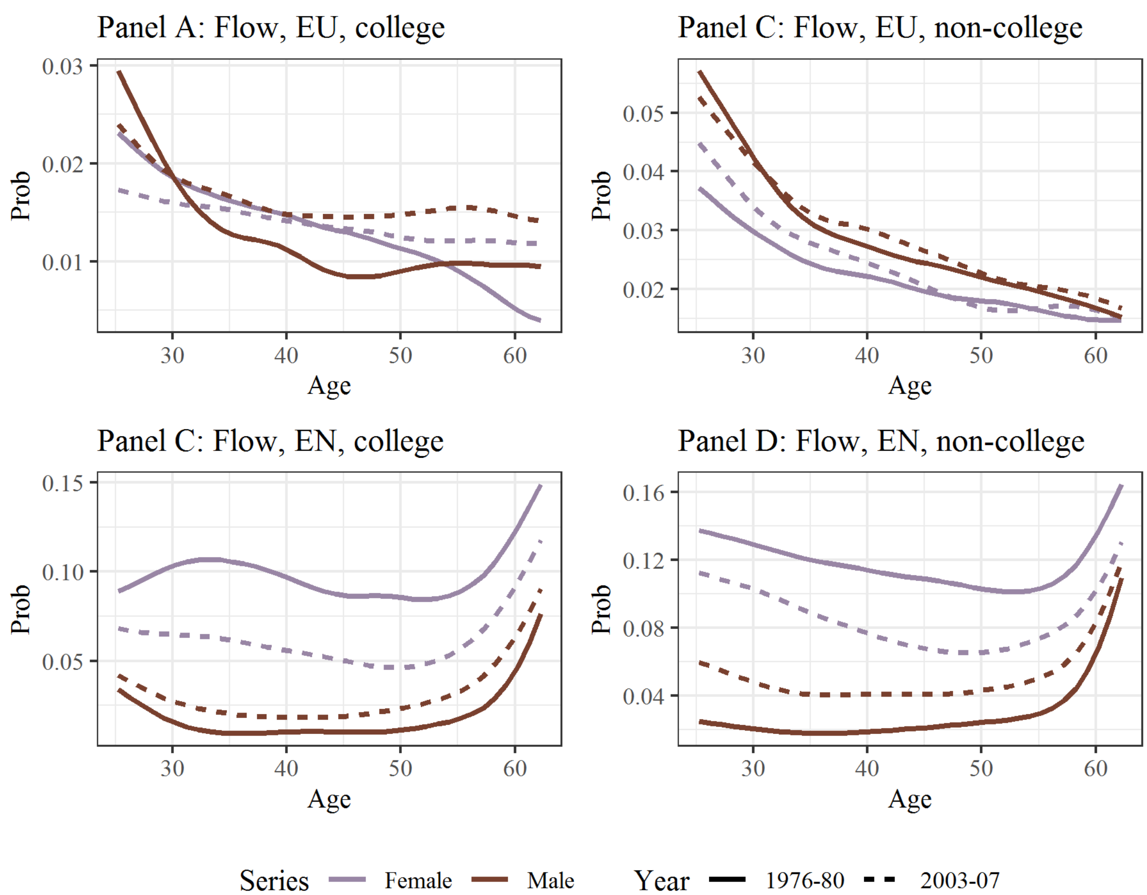

Figure D.4. Life-cycle job separation rates in 1976–80 and 2003–07, by gender and education

Note: Panels A and B plots the life-cycle separation rates into unemployment for workers with and without a college education, respectively, by gender. Panels C and D plots the life-cycle separation rates into nonparticipation for workers with and without a college education, respectively, by gender.

Source: Authors' estimations based on data from IPUMS-CPS.

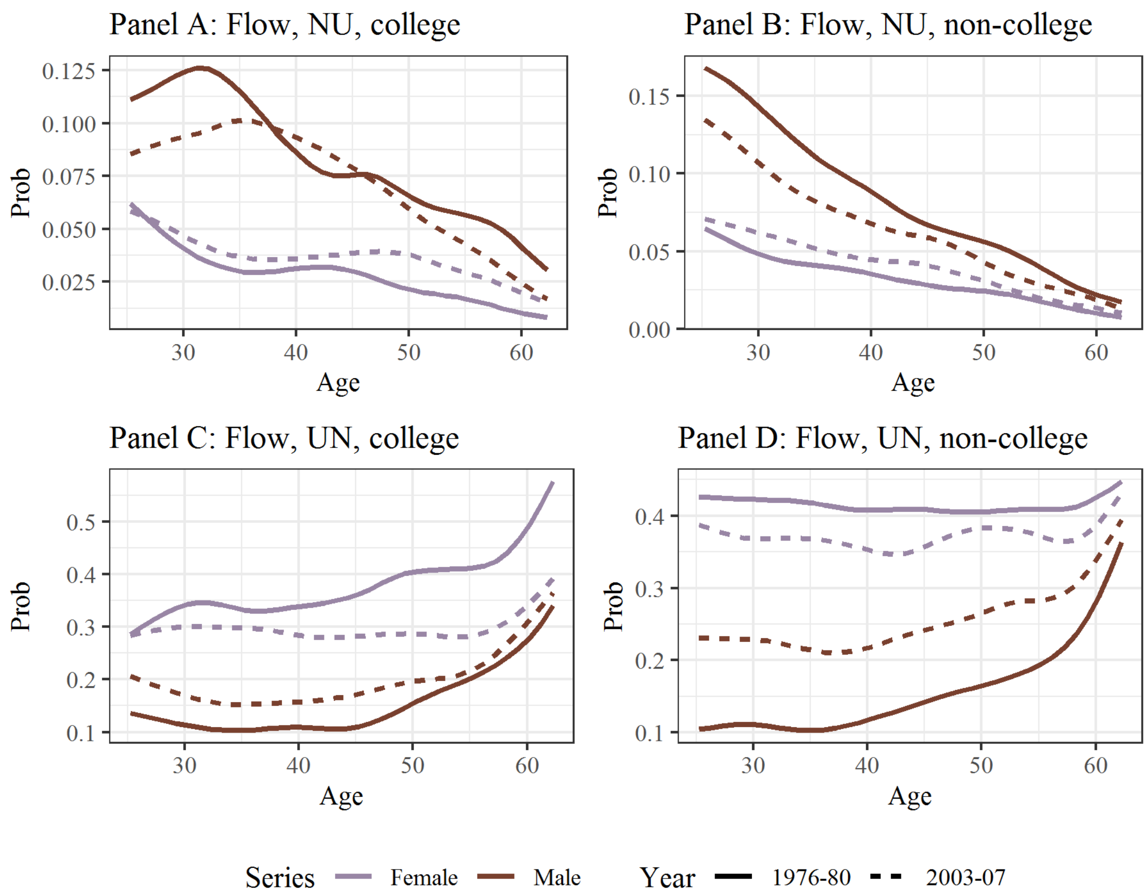

Figure D.5. Life-cycle flows between unemployment and nonparticipation in 1976–80 and 2003–07, by gender and education

Note: Panels A and B plot the life-cycle flow rates from nonparticipation to unemployment for workers with and without a college education, respectively, by gender. Panels C and D plot the life-cycle flow rates from unemployment to nonparticipation for workers with and without a college education, respectively, by gender.

Source: Authors' estimations based on data from IPUMS-CPS.

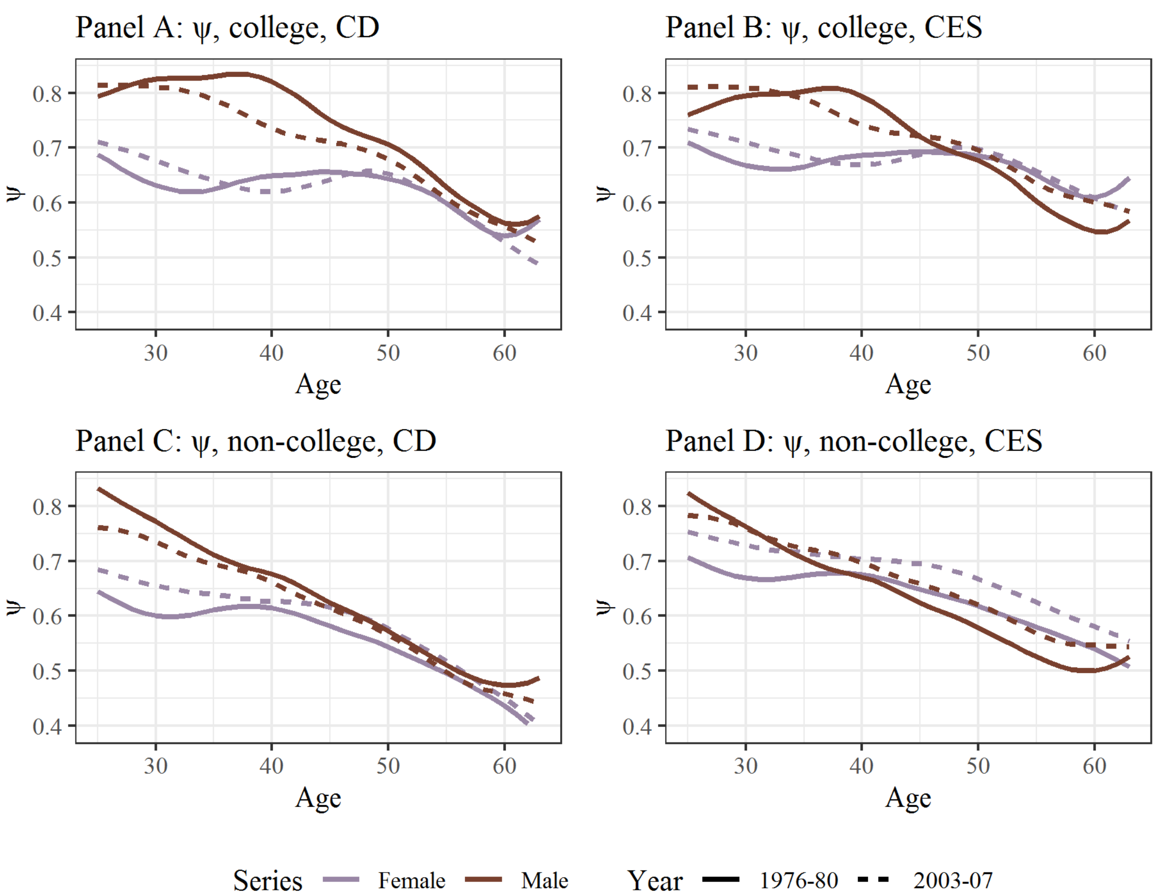

Figure D.6. The fraction of nonparticipants searching over the life cycle in 1976–80 and 2003–07, by gender and education

Note: Panels A and C shows the fraction of nonparticipants—with and without a college education, respectively—searching over the life cycle in the CD model. Panels B and D show the same results in the CES model.

Source: Authors' estimations.

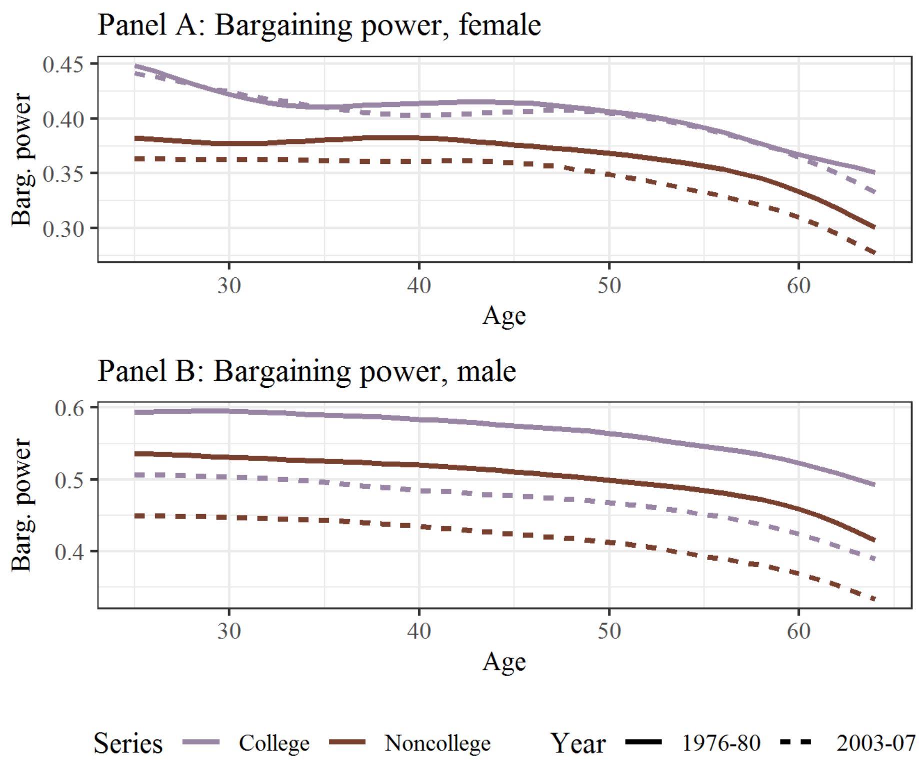

Figure D.7. Workers' life-cycle bargaining power in 1976–80 and 2003–07, by gender and education

Note: Panels A and B plot the simulated life-cycle bargaining power of female and male workers, respectively, by education group.

Source: Authors' estimations.

Appendix D.5: Changes in the Disaggregated Labor Shares

Table D.2 summarizes the model-generated changes in the labor share for different groups. First, we find that the labor share has declined for all groups in the CES model. Male workers have faced the largest decline: Non-college males experienced a 5.6 percent decline in their labor share, while college-educated males experienced a 4.2 percent decline. The labor share decreased by 2.8 percent for college-educated females and by 2.1 percent for non-college females. Consistent with the aggregate results, the labor shares have declined less in the CD model—or have even increased, as is the case for college-educated women.

Table D.2. Simulated labor compensation shares: 1976–80 and 2003–07

Group	Percent change, CD model	Percent change, CES model
Male, college	-2.7	-4.2
Female, college	3.1	-2.8
Male, non-college	-3.6	-5.6
Female, non-college	.0	-2.1

Note: Table D.2 presents the simulated change in the labor share for each group under the CD and the CES models.

Source: Authors' estimations.