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Robert Kurtzman and David Zeke

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Aggregate Implications of Deviations from Modigliani-Miller: A Sufficient Statistics Approach*

Robert Kurtzman[†1] and David Zeke [‡1]

[1]Federal Reserve Board of Governors

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Abstract

A few sufficient statistics can identify the aggregate effects of distortions to firm investment in a class of general equilibrium models that can accommodate rich general equilibrium effects including endogenous firm entry. This result does not depend on the microfoundation of the distortion; one can generate inferences about aggregate effects that apply for multiple microfoundations or in cases where a fully specified model is difficult to solve. To demonstrate the relevance of the methodology, we use it to quantify the aggregate consequences of costly external equity financing and a manager-shareholder friction, relying on estimates from the corporate finance literature to identify the sufficient statistics. The results elucidate differences between partial and general equilibrium findings and demonstrate how labor supply elasticities, complementarities in production, and firm entry interact with the different firm-level distortions.

Keywords: Heterogeneous Firms, General Equilibrium, Firm Entry, Agency Costs, Costly External Finance, Sufficient Statistics
JEL Classification: E22, E23, G39

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†Federal Reserve Board of Governors, Division of Research and Statistics, 20th and Constitution NW, Washington, D.C. 20551; email: robert.j.kurtzman@frb.gov.

‡Federal Reserve Board of Governors, Division of Research and Statistics, 20th and Constitution NW, Washington, D.C. 20551; email: davidzeke@gmail.com; corresponding author.

1 Introduction

There is little debate in corporate finance that firms behave differently than implied by the Modigliani and Miller (1958) theorem or other frictionless capital markets benchmarks. Such deviations have been shown theoretically and empirically to have real effects; external financing, information, and agency frictions can distort capital structure and investment decisions away from what is privately optimal (Stein, 2003). While there are many studies that carefully estimate the extent of these frictions at the micro-level, there is often a lack of consensus regarding the appropriate model for the microfoundations of a given friction. In this paper, we derive a methodology for examining the aggregate consequences of such distortions to firm-level investment *without* having to specify the microfoundations of the friction. Given the production environment and model parameters, our methodology infers these consequences from a small set of sufficient statistics on the effects of the distortions. In turn, the method is easy to implement, especially compared with solving microfounded general equilibrium (GE) models of heterogeneous firms subject to distortions.

The class of models to which our main results apply feature investment by heterogeneous firms that produce differentiated varieties of goods using capital and labor. Although our methodology applies to more general environments, in the baseline model in which we demonstrate the sufficient statistics result, firms are exposed to idiosyncratic shocks, both random walk and i.i.d. (to capital quality and productivity, respectively), but are not exposed to aggregate shocks.[1] Firms are also heterogeneous in their capital stock, productivity, and the distortions they face (and thus any state variables that govern the evolution of those distortions). The model nests—at the micro level—the canonical Q-theory of investment in the face of convex capital adjustment costs where firm revenues exhibit homogeneity of degree one in the factors of production following Hayashi (1982).

Our approach is unique in that we do not choose a particular firm-level distortion to investment to model. Instead, we show that many distortions can be represented as the difference between the actual investment rate of a firm and the investment choice an otherwise identical firm (absent distortions) in the same environment would make to maximize the value of profits less input costs (labor and investment). This difference is closely related to empirical estimates of the causal effects of distortions on rates of firm investment.

We use the model to examine the following counterfactual: starting from an equilibrium with firm-level distortions, how do aggregates change if we eliminate the distortions and adjust taxes to keep the present value of net government revenues from corporate taxation constant?[2] Our main result, Proposition 1, shows that by simply knowing the production and preference

[1] We show how these assumptions can be relaxed in various extensions.

[2] This assumption is imposed to allow distortions to have rich tax consequences, as, for example, some capital structure distortions have. If entry is exogenous or, alternatively, the distortion has no additional tax consequences, then this assumption of revenue neutrality is not required.

parameters in this model, along with two sufficient statistics—the capital-weighted mean and mean-squared distortions to investment rates—we can obtain bounds on how aggregate objects (output, consumption, welfare, etc.) behave in this counterfactual, in both the long run and the transition path from the equilibrium with firm-level distortions to the one without them. This inference is exact in special cases in which entry is exogenous or there is no time discounting. Importantly, distortions to investment rates due to a given friction are exactly what many corporate finance papers, such as Hennessy et al. (2007), estimate.[3] In turn, given this assumption, in estimating our sufficient statistics, we can take many estimates from the corporate finance literature that also rely on Q-theory approaches "off the shelf."

The intuition for this result is as follows: the aggregate equilibrium conditions can be simplified so that firm-level distortions appear in just three equations: (1) aggregate capital accumulation, (2) aggregate investment clearing, and (3) the expected present value of an entering firm. Aggregate capital accumulation intuitively depends on the distortion to the capital-weighted investment rate. The amount of resources devoted to investment depends on both the aggregate investment rate and the misallocation of investment across firms, which, with quadratic adjustment costs, are simply a function of our two sufficient statistics. That the sufficient statistics allow us to characterize the entering firm's value is less intuitive. A key insight in our proof is that although the distortions can hit a firm at any point in its lifetime, one can bound the effect of distortions on firm value by using moments of the cross-sectional distribution of firms that are reflected in the sufficient statistics. We can thus bound (or, in special cases, exactly identify) the aggregate effects of the distortion even with endogenous entry.

We further generalize Proposition 1 by relaxing the more stringent assumptions made in the model in a number of corollaries, as our approach can be extended to a wider class of models, though often at the cost of modified or additional sufficient statistics. In particular, our extensions include allowing for more general investment cost functions, relaxing the assumption of homogeneity of degree one of firm revenues, allowing for aggregate shocks, working in more general aggregation environments, and allowing for further dimensions of firm heterogeneity. Further, our generalizations can allow for distortions to impart deadweight or financial effects on the firm above and beyond those caused by distortions to investment.

We use our sufficient statistics approach in the baseline model to quantify the aggregate consequences of distortions to investment due to external financing and a shareholder-manager friction using estimates from the corporate finance literature. The external financing friction is one of underinvestment, while the manager-shareholder friction is one of overinvestment. We calibrate the sufficient statistics to corporate finance papers that estimate the effects of these

[3] That firm revenues exhibit homogeneity of degree one in the factors of production in the baseline model is key to why the sufficient statistics are able to be expressed in terms of distortions to investment rates. We assume that distortions affect firm value only through their effects on firm investment and taxation, though we relax this assumption in an extension. Specifically, in the extension, we allow distortions to also lead to deadweight losses via the destruction of capital in bankruptcy.

distortions on firm investment, and we calibrate the other model parameters to standard values in the literature on macroeconomic models with firm heterogeneity.[4]

Our quantitative results yield several insights about the macroeconomic costs of these frictions and the value of our methodology. First, we show that the bounds generated by our method are tight, especially for the change in output and welfare. Second, the model features both GE dampening (labor supply, entry) and amplifying forces (complementarities in production) that operate through aggregate prices. Quantitatively, the dampening forces dominate: For both external financing costs and managerial miscalibration, the effect on aggregate output is an order of magnitude smaller than what a partial equilibrium (PE) counterfactual (in which entry and all aggregate prices are fixed) would suggest.

Third, we show that the presence of aggregate inefficiencies can meaningfully change the welfare effects of resolving these distortions. We consider positive corporate taxation and a monopoly markup distortion. Absent these inefficiencies, resolving the distortion leads to the planner's equilibrium, and thus the welfare gains from resolving the distortion are positive. However, in the presence of these inefficiencies, we find that eliminating managerial miscalibration reduces aggregate welfare in our counterfactuals because of the interaction of the inefficiencies with the firm-level distortion. Eliminating external financing frictions increases welfare, even in the presence of these aggregate inefficiencies.[5]

Fourth, the effect of reallocation of investment can account for a meaningful share of the effect of distortions on aggregates. When we decompose changes in output into changes in the expenditures on investment and the efficiency of investment, we find that the efficiency of investment can account for in excess of one-third of the change in output.

An advantage of our methodology is that if we change the calibration of a given parameter, we do not need to alter the calibration of the sufficient statistic, and thus it is easy to perform robustness with respect to the value of the sufficient statistics and model parameters. We assess the effects of resolving the distortion across large ranges for the sufficient statistics and model parameters and show that the qualitative takeaways are strikingly robust across these different robustness exercises. When we alter the modelling assumptions to have fixed instead of free entry or to allow productivity to follow a Markov process, the results are again qualitatively consistent with our baseline results.

Our work is related to the growing literature studying policies with sufficient statistics ap-

[4] We rely on an estimate of the effects of external financing costs on investment from Hennessy, Levy, and Whited (2007). The authors write down a dynamic microeconomic model with investment and capital structure and estimate it to examine the effects of external financing and agency frictions on investment. We rely on an estimate of the effect of a manager-shareholder friction on investment from Ben-David et al. (2013). The authors examine managers' ability to predict the potential outcomes for the S&P 500. Broadly, the authors find that managers are indeed miscalibrated and show that their long-term miscalibration measure is correlated with overinvestment.

[5] The monopoly markup inefficiency leads to inefficiently low production by firms, while positive (net) corporate taxation distorts firm entry. Removing the firm-level distortions changes the amount of entry and mass of firm capital in equilibrium, which interacts with these inefficiencies.

proaches within macroeconomic models.[6] In a closely related paper, Sraer and Thesmar (2021), develop a sufficient statistics approach to assess the macroeconomic implications of policy experiments that affect misallocation. Our paper differs in its ability to accommodate endogenous entry and in the class of models and microeconomic evidence it can use to derive aggregate implications. In particular, our methodology is targeted to models where firm revenues exhibit constant returns to scale, where estimates of the effect of distortions on investment *rates* (which are widely estimated in corporate finance) can be used to infer the sufficient statistics. Their methodology is targeted at the misallocation literature, where firm revenues or value added exhibits decreasing returns to scale and the effect of policies on allocations (output-to-capital ratios, in particular) is informative about aggregate effects.[7] Baqaee and Farhi (2020) develop a flexible and general theory of aggregation in distorted economies. There are key differences in our approach: First, "wedges" in investment rates (the difference between the realized and first-best investment rate) are endogenous in our model due to GE effects. Second, our approach allows us to construct bounds on the transition path, and—given our sufficient statistics—bounds on the aggregate consequences; we are thus solving the full non-linear model for our exercises as opposed to relying on second-order approximations or making the assumption that distortions are small. Atkeson and Burstein (2019) develop a sufficient statistics approach to study the implications of innovation policy for macroeconomic and welfare dynamics in a broad class of innovation models.[8] Our paper differs in that we focus on the effect of *heterogeneous* distortions to firm investment and the statistics needed to assess the macroeconomic implications of removing those distortions. Recent work by Iachan, Silva, and Zi (2022) also use a sufficient statistic approach to answer questions at the intersection of macroeconomics and finance. They assess the welfare implications of under-diversification on aggregate welfare in a framework with endogenous investment. Their methodology differs in that it focuses on this particular distortion and uses sufficient statistics estimated from asset prices.

Our work is also related to the literature examining how information and financing frictions matter for the aggregate economy using moments from microeconomic data in general equilibrium macro models. In a few important recent examples, Catherine et al. (2022) quantify the

[6]This literature includes both papers that require only reduced-form elasticities, following Chetty (2009), and those that combine sufficient statistics with some macroeconomic parameters. Examples include approaches to misallocation of inputs (Hsieh and Klenow 2009; David and Venkateswaran 2019; Baqaee and Farhi 2020), transmission of monetary policy with heterogeneous households (Auclert, 2019), and aggregating technology and other shocks (Baqaee and Farhi, 2019).

[7]The misallocation in our model is of investment rates, not of the level of inputs. We show our method can be extended to a model with decreasing returns to scale: relative to Sraer and Thesmar (2021), this extension of our method has the benefit of accommodating endogenous entry at the cost of less flexible counterfactuals and some additional assumptions.

[8]While our paper is not an innovation model, our model and methodology can be recast as such, where firms choose investment in firm *productivity*. We choose the formulation as a model of capital investment because of the breadth and quality of estimates of distortions on firm capital investment. While the theory in Atkeson and Burstein (2019) allows for meaningful gains from effectively reallocating investment from less to more productive uses, their measurement exercise applies to policies that affect firm investment proportionally.

aggregate effects of collateral constraints, and Terry (Forthcoming) studies how short-termism affects economic growth. We view our approach as complementary to these approaches. One relative benefit of our method is that it does not rely on precisely specifying the microfoundations of the distortion; a traditional model which is calibrated to match the sufficient statistics from the micro data will, as we prove in our main proposition, provide an estimate within the bounds that our approach generates. A second relative benefit is the ease of implementation: our method allows for much simpler and computationally faster implementation than solving a heterogeneous-firm GE model with micro-founded distortions, especially those that include realistic assumptions on the dynamics of firm financing or agency problems. However, the traditional structural approach allows for more counterfactuals, such as merely reducing the extent of the friction rather than eliminating it completely.[9] Furthermore, the traditional structural approach also allows for production environments that do not satisfy the assumptions required for our approach.

The rest of the paper follows as such. Section 2 presents the baseline model. Section 3 outlines the sufficient statistics result and the extensions. Section 4 presents our calibration and the quantitative results. Section 5 assesses the robustness of the results. Section 6 concludes.

2 Model

In this section, we describe the model. For ease of exposition, we set up the production environment as it will be implemented in the quantitative analysis, rather than outlining the most general class of models for which our approach can be implemented. In Section 3.3 we discuss the broader class of models for which our sufficient statistics approach holds and additional extensions to models with richer heterogeneity, aggregate shocks, and firm revenues that exhibit decreasing returns to scale in inputs, or endogenous growth.

We start with the production environment and the problem of entering firms. We then review the household's problem and define an equilibrium. In the model, time is discrete and indexed as $t = 0,\ 1,\ 2,\ ....$

2.1 Production Environment

A continuum of intermediate good firms, indexed by j, produce output, y. Intermediate good firm output is combined into the final good, Y, using a constant elasticity of substitution (CES) production function:

$$Y_t = \left(\int_j y_{j,t}^{\frac{\rho-1}{\rho}} dj \right)^{\frac{\rho}{\rho-1}}, \tag{1}$$

[9]In this vein, in Kurtzman and Zeke (2018), we build a macroeconomic model with heterogeneous firms with a microfoundation for why firms issue debt, an explicitly defined debt contract (with strong assumptions to retain tractability), and assumptions on the bankruptcy process. We use this model to examine how changes in tax policy interact with the debt overhang problem.

where $\rho > 0$. Each firm produces output using capital, k, and labor, l, with production function

$$y_{j,t} = A_t z_{j,t} k_{j,t}^{\alpha_1} l_{j,t}^{\alpha_2}, \tag{2}$$

where A is a common level of productivity across firms; z is firm idiosyncratic productivity which is *i.i.d.* across firms; and the production function parameters are $\{\alpha_1 > 0, \alpha_2 > 0\}$.[10] Cost minimization by final good firms allows us to obtain intermediate good firm revenues:

$$y_{j,t} p_{j,t} = P_t Y_t^{\frac{1}{\rho}} (A_t z_{j,t} l_{j,t}^{\alpha_2} k_{j,t}^{\alpha_2})^{\frac{\rho-1}{\rho}}, \tag{3}$$

where p_j is the price of firm $j's$ output and P is the price of the final good. Firms choose labor within each period to maximize static operating profits (revenues less labor costs Wl_j). We assume that $\alpha_1 = \frac{\rho}{\rho-1} - \alpha_2$; therefore, firm revenues are homogeneous of degree one in the factors of production, as in Hayashi (1982). Because revenues are homogenous of degree one, firm labor demand, revenues, and operating profits (π_j) can be expressed as

$$\begin{aligned} l_{j,t} &= k_{j,t} \left(\alpha_2 (\frac{\rho-1}{\rho})(A_t z_{j,t})^{\frac{\rho-1}{\rho}} \left(\frac{W_t}{P_t}\right)^{-1} Y_t^{\frac{1}{\rho}} \right)^{\alpha_c}, \\ y_{j,t} p_{j,t} &= k_{j,t} \frac{z_{j,t}^{\frac{\rho-1}{\rho}\alpha_c} \Pi_t}{1 - \alpha_2 \frac{\rho-1}{\rho}}, \\ \pi_{j,t} &= z_{j,t}^{\frac{\rho-1}{\rho}\alpha_c} k_{j,t} \Pi_t, \end{aligned} \tag{4}$$

respectively, where $\alpha_c = (1/(1 - \alpha_2 \frac{\rho-1}{\rho}))$ and Π_t is a measure of profits per unit of effective capital that can be written as a function of parameters and aggregates:

$$\Pi_t = P_t \left(A_t^{\frac{\rho-1}{\rho}} Y_t^{\frac{1}{\rho}} \left(\frac{W_t}{P_t}\right)^{-\alpha_2(\frac{\rho-1}{\rho})} \left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})} \right)^{\alpha_c} \left(1 - \alpha_2(\frac{\rho-1}{\rho})\right). \tag{5}$$

Capital Investment Capital investment at time t increases the capital stock at time $t+1$, and capital depreciates at rate δ. Each firm's capital stock is exposed to random walk shocks, modelled as *i.i.d* shocks ϵ^k (capturing capital quality or exogenous exit) with mean κ to the capital accumulation equation. Formally, the capital accumulation equation is

$$k_{j,t} = \epsilon_{j,t}^k \left(k_{j,t-1}(1-\delta) + I_{j,t-1}\right), \tag{6}$$

where $I_{j,t-1}$ is the amount of investment by firm j. In the spirit of the Q-theory of investment (Hayashi, 1982), there are adjustment costs in investment: investment rate $\frac{I}{k}$ costs

[10]We normalize z so that its convexity-adjusted expectation, $\mathbb{E}\left[z_{j,t}^{\frac{\rho-1}{\rho}\alpha_c}\right]$, is equal to unity. We show how the sufficient statistics change when we relax the *i.i.d.* assumption in Section 3.3.

$k\left(\frac{I}{k}+\frac{\theta}{2}\left(\frac{I}{k}-\tilde{\theta}_c\right)^2\right)$ units of the final good for some parameter $\tilde{\theta}_c$ (which is typically set to 0 or δ). The expected gross growth rate of the firm (before capital quality shocks), $i_{j,t-1}$, is

$$i_{j,t-1}=k_{j,t-1}\left(1-\delta\right)+\frac{I_{j,t-1}}{k_{j,t-1}}. \tag{7}$$

This definition reduces the length of some expressions; note that $i_{j,t-1}$ is proportional in the investment rate of the firm. We can therefore express the cost of this growth rate i as $k\phi(i)$, where $\phi(i)=i-(1-\delta)+\frac{\theta}{2}\left(i-\theta_c\right)^2$ and $\theta_c=\tilde{\theta}_c+(1-\delta)$.

2.2 Firm Problem and Distortions

Firms' investment choices may be subject to distortions. Without loss of generality, we represent distortions relative to the investment of an undistorted firm.

2.2.1 Undistorted Firm Problem

We begin by modelling the problem of an undistorted ("Modigliani-Miller") firm that has value $\widetilde{V^{\mathrm{MM}}}$. This is a firm that does not face firm-specific distortions to its investment choice and expects that it never will. Note that this firm's value and choices depend on aggregate state prices, and the level of those aggregate state prices may be influenced by distortions in the economy. We define the undistorted firm's value function as the present discounted value of revenues less labor and investment expenses:

$$V_{j,t}^{\mathrm{MM}}(k,z)=\max_i k_{j,t}\Pi_t\left(z^{\frac{\rho-1}{\rho}\alpha_c}\right)-k_{j,t}\phi(i)+\kappa\mathbb{E}_t[\Lambda_{t+1}V_{j,t+1}^{\mathrm{MM}}(k',z')], \tag{8}$$

where Λ_t is the household's stochastic discount factor. The solution to this maximization problem yields the investment choice of the firm and the value of such an undistorted firm. Since the firm value function is homogeneous of degree one in the firm's capital stock, we can write the value function as $V_{j,t}^{\mathrm{MM}}(k,z)=k_{j,t}q_t^{\mathrm{MM}}(z)$, where $q_t^{\mathrm{MM}}(z)$ is the marginal value of capital to an undistorted firm. Additionally, we can write q_t^{MM} as the value function for a firm whose idiosyncratic shock is equal to the average level:

$$q_t^{\mathrm{MM}}=\max_i \Pi_t-\phi(i)+i\kappa\mathbb{E}_t[\Lambda_{t+1}q_{t+1}^{\mathrm{MM}}]. \tag{9}$$

By solving (9), we can also characterize the undistorted optimal investment rate choice, $i_{\mathrm{MM},t}$, which is common across firms regardless of their level of capital stock or realization of their idiosyncratic shock:

$$\phi'\left(i_{\mathrm{MM},t}\right)=\kappa\mathbb{E}_t[\Lambda_{t+1}q_{t+1}^{\mathrm{MM}}]. \tag{10}$$

Entry and Taxation (without firm-level distortions) There is an equilibrium condition that governs entry, $\Theta\left(M_t^{\mathrm{E}}, V_t^{\mathrm{E}}\right) = 0$, where M_t^{E} is the mass of entry and V_t^{E} is the value of an entering firm with one unit of capital. This is a general specification that nests both free entry and exogenous entry. For instance, in the case of free entry, this condition is

$$\Theta(M^{\mathrm{E}}, V^{\mathrm{E}}) = \begin{cases} V^E - c_e & \text{if } V^E \geq c_e \\ M_e & \text{if } V^E < c_e. \end{cases}$$

In other words, firms enter if the value of entering exceeds the cost; therefore, firm value must be equal to the cost if firms enter in equilibrium. The condition is even simpler in the case of exogenous entry: $\Theta(M^{\mathrm{E}}, V^{\mathrm{E}}) = M^{\mathrm{E}} - \overline{M^{\mathrm{E}}}$, where $\overline{M^{\mathrm{E}}}$ is a non-negative constant.

We assume that firm taxes or subsidies may be on revenues less expenses, lump sum to all firms, or lump sum on entry (and this schedule is identical across firms); therefore, they do not directly affect the undistorted investment problem. However, taxation can affect the value of an entering firm and can therefore distort entry.[11]

2.2.2 Distorted Firm Problem

We define $\Delta i_{j,t} = i_{j,t} - i_{\mathrm{MM},t}$ as the difference between a firm's investment rate and the undistorted firm's actual investment rate, *holding all aggregate prices fixed*. This is thus a PE counterfactual: it measures how a firm's investment choice is affected by its distortion, holding the choices of all other firms and aggregates fixed.[12] This distortion may depend on a vector of firm variables, which we represent by χ. For example, χ may represent capital structure factors, real or financial shocks to the firm, or other state variables relevant to agency problems within the firm. Thus, we can write $i_{j,t} = i_t\left(\chi_{j,t}\right)$, and express the (pre-tax) value of the distorted firm as the discounted present value of revenues less expenses:

$$V_{j,t}^F(k,\chi) = z_{j,t}^{\frac{\rho-1}{\rho}\alpha_c} k_{j,t}\Pi_t - k_{j,t}\phi(i_t(\chi_{j,t})) + \kappa\mathbb{E}_t[\Lambda_{t+1}V_{t+1}^F(k',\chi')]. \tag{11}$$

Examples of Distortions To provide intuition for the representation of distortions to firm investment, we show how distortions in standard models can be expressed as $\Delta i_{j,t}$.

Example 1: Costly External Financing Consider a specification of distortions where firms may face costs of financing with external equity, as in Hennessy et al. (2007). Let X denote net equity payouts; C, firm cash holdings; R_C, the interest rate on cash holdings; $G(X)$, the cost

[11] With free entry, if positive (in present value terms) revenue is raised on firm taxation less subsidies, there is an effective tax on entry (reducing entry in equilibrium). The entry subsidy that would offset the effect of positive taxes on the firm would lead the net revenue raised to be zero in present value terms.

[12] This specification is important for measurement. The difference between the investment of a firm and the investment it would make were it to have a different distortion, holding what other firms do fixed, is exactly the sort of counterfactual that empirical studies try to measure. By contrast, the difference between a firm's investment and what it would invest in the first-best equilibrium (with different prices) is not generally observable.

of external financing; and let $\Phi(X<0)$ be an indicator variable that is positive if firms issue equity. The optimization problem that characterizes firm investment can be written as follows:

$$
\begin{aligned}
V_t(z,k,c_{-1}) &= \max_{I,X,c} X + \mathbb{E}_t\left[\Lambda_{t+1}V_{t+1}(z',k',c)\right] \text{ s.t.} \\
k' &= \epsilon^k\left(k(1-\delta)+I\right) \\
c &= \Pi_t k z^{\frac{\rho-1}{\rho}\alpha_c} - \left(I + \frac{\theta_I}{2}\left(\frac{I}{k}-\theta_c\right)^2\right) - X - kG\left(\frac{X}{k}\right)\Phi(X<0) + c_{-1}R_C.
\end{aligned}
$$

The resulting equilibrium condition for investment in terms of i_t (gross capital growth rate) is

$$i_t(z,k,c_{-1}) = (1-\delta)+\theta_c-\frac{1}{\theta}+\frac{1}{\theta}\widehat{q}_t\left(1+G'(x)\Phi(x<0)\right),$$

where $\widehat{q}_t = \mathbb{E}_t\left[\Lambda_{t+1}\epsilon^k\frac{\partial V_{t+1}}{\partial k'}(z',k',c)\right]$ is the forward-looking (expected) discounted marginal value of capital to the firm and $x=\frac{X}{k}$ is the net equity payout scaled by the capital stock. We can thus write the distortion to the investment rate, $\Delta i = i - i^{\text{MM}}$, as

$$\Delta i_t(z,k,c_{-1}) = \frac{1}{\theta}\left(\left(\widehat{q}_t-\widehat{q}_t^{\text{MM}}\right)+\widehat{q}G'(X)\Phi(x<0)\right), \tag{12}$$

where $\widehat{q}_t^{\text{MM}} = \kappa\mathbb{E}_t[\Lambda_{t+1}q_{t+1}^{\text{MM}}]$ is the forward-looking (expected) discounted marginal value of capital to an undistorted firm in this equilibrium (e.g., if there was one firm that did not suffer from external financing costs).[13] Note that all terms in (12) are taken holding aggregate prices fixed in the equilibrium with distortions. Thus, this is a cross-sectional measure: how do the investments of a distorted and undistorted firm, in the same equilibrium, differ from each other? This distortion can occur either via distortions that affect a firm's investment Euler equation (e.g., $G'(X)\Phi(x<0)$) or via distortions that affect firm q (e.g., expectation of future costly external financing), though this latter effect can be shown to be second order.[14]

Example 2: Manager Miscalibration Consider distortions to managers' expectations of the future value of capital. That is, the true problem has value

$$V_t(z,k,m) = \Pi\left(z^{\frac{\rho-1}{\rho}\alpha_c}\right)K - K\phi(i(z,k,m)) + \mathbb{E}_t\left[\Lambda_{t+1}V_{t+1}(z',ki\epsilon^k,m')\right], \tag{13}$$

and thus the forward-looking (expected) discounted marginal value of firm capital is $\widehat{q}_t = \mathbb{E}_t\left[\Lambda_{t+1}\epsilon^k\frac{\partial V_{t+1}}{\partial k'}(z',k',m')\right]$. However, managers' expectations are distorted and they make choices given the following distorted Euler equation:

$$\phi'(i(z,k,m)) = \widehat{q}_t + m, \tag{14}$$

[13]The κ must adjust the realized marginal q for the expectation of capital quality shocks (which also nest exogenous exit shocks).

[14]We discuss these effects and their quantification in Section 4.1.

where m indexes the distortion to their expectations. Plugging in for our assumptions of a quadratic adjustment costs yields the following forms for i and Δi:

$$\begin{aligned} i_t(z,k,m) &= (1-\delta)+\theta_c-\frac{1}{\theta}+\frac{1}{\theta}\widehat{q}_t+\frac{m}{\theta}, \\ \Delta i_t(z,k,m) &= i-i^{\mathrm{MM}}=\frac{1}{\theta}\left(\left(\widehat{q}-\widehat{q}^{\mathrm{MM}}\right)+m\right). \end{aligned} \tag{15}$$

Taxation (with distortions) The distortions to investment can have tax implications. The following identity disaggregating the value of the entering firm with one unit of capital, which isolates the tax and investment distortions, is particularly useful:

$$V_t^{\mathrm{E}} = q_t^{\mathrm{MM}} + V_t^{\mathrm{D}} + V_t^{\mathrm{Taxes}}, \tag{16}$$

where q_t^{MM} is the present discounted value of firm profits (revenues less expenses on labor and capital investment) of an undistorted firm, $V_t^{\mathrm{D}} < 0$ is the present discounted value of the effect of the distortions on firm profits, and V_t^{Taxes} is the discounted present value of subsidies paid to the firm less taxes.[15]

2.3 Households

Households own all firm liabilities and provide labor for production. Each period, after shocks are realized, households make a consumption decision, C_t, and a labor supply decision, L_t, to maximize their utility function, $\sum_{t=0}^{t=\infty}\beta^t U(C_t, L_t)$, where β is the household's discount factor. They maximize their utility subject to a budget constraint:

$$P_t C_t \leq W_t L_t + F_t + T_t,$$

where F_t are the net profits paid by firms less entry costs and T_t are transfers from the government.[16] The household's problem results in the standard labor-leisure condition of

$$\frac{W_t}{P_t}U_{C,t} = -U_{L,t}, \tag{17}$$

and a stochastic discount factor of

$$\Lambda_t = \beta\frac{U_{C,t}}{U_{C,t-1}}. \tag{18}$$

[15] For tractability we consider a model with exogenous exit, but our method could be generalized to allow for some types of endogenous exit, such as the specification in Gomes et al. (2016). However, it would need an additional sufficient statistic with information about how the distortion affects exit, as in our extension with deadweight losses described in Section 3.3.

[16] These transfers are equal to the present value of any taxes raised so that the budget constraint and consumption market clearing are jointly satisfied.

2.4 Clearing and Equilibrium

Capital and labor clearing imply $K_t = \int_j k_{j,t}dj$ and $L_t = \int_j l_{j,t}dj$; therefore, equation (4) implies

$$L_t = K_t \left(\alpha_2 (\frac{\rho - 1}{\rho}) A_t^{\frac{\rho-1}{\rho}} \left(\frac{W_t}{P_t} \right)^{-1} Y_t^{\frac{1}{\rho}} \right)^{\alpha_c}. \tag{19}$$

Aggregate capital accumulation becomes

$$K_t = M_t^{\mathrm{E}} + \kappa \int_j \left(k_{j,t} \left(i_{j,t} - \xi_{j,t} \right) \right) dj. \tag{20}$$

Define aggregate investment as the sum of incumbent investment and entry costs:

$$I_t = \int_j k_{j,t} \phi \left(i_{j,t} \right) dj + M_t^{\mathrm{E}} c_e. \tag{21}$$

Final good clearing thus implies

$$C_t = Y_t - I_t. \tag{22}$$

Plugging (4) into (3) and integrating over firms, we can obtain aggregate output as

$$Y_t = K_t \frac{\Pi_t}{P_t \left(1 - \alpha_2 \frac{\rho - 1}{\rho} \right)}. \tag{23}$$

The distribution of operating firms at time t, $\Gamma_t(k, \chi)$, evolves over time as a function of the exogenous exit rate, κ; endogenous capital destruction, $\xi_t(k, \chi)$; the investment choices of $i_t(k, \chi)$ by incumbent firms; and the mass of entering firms each period, M_t^{E}. It is useful to define the capital-weighted steady-state distribution of the capital structure state vector in the stationary equilibrium as $\gamma^K(\chi)$. Formally,

$$\gamma^K(\chi) = \frac{\int_k k\Gamma(k, \chi)}{\int_k \int_x k\Gamma(k, x)}. \tag{24}$$

We define an equilibrium in Appendix A.1.

3 Sufficient Statistics Results

This section outlines and proves the key result in the paper: we can either exactly characterize or bound the aggregate effects of a revenue-neutral policy counterfactual in which the firm-level distortions to investment and capital destruction are undone with a simple set of sufficient statistics and the parameters of the model. We first define the counterfactual exercise of interest, state the main proposition, and sketch its proof. We then outline the broader class of models to which our result applies and discuss extensions.

3.1 Proposition and Discussion

We define a revenue-neutral counterfactual as one in which the distortions to investment, indexed by $\Delta i(k,\chi)$, are removed and taxes on firms are adjusted so that—for each generation of firms that enter—the total present value of net tax revenue that firms generate is unchanged.[17]

Proposition 1. *Assume we know the following:*

(i) Average capital-weighted distortion to firm investment: $i_\Delta = \int_\chi \Delta i(\chi)\gamma^K(\chi)\,d\chi$

(ii) Average capital-weighted squared distortion to firm investment:
$i_{\Delta^2} = \int_\chi (\Delta i(\chi))^2\gamma^K(\chi)\,d\chi$

(iii) Present value of taxes net of subsidies per period

(iv) Model parameters: β, κ, α_1, α_2, θ, ρ, *and the parameters that enter the utility function*

Then our environment leads to a system of equations that identifies bounds on the change in aggregate quantities and prices $\left\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\right\}$ *when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long run and along the transition path.*

If, in addition, either $\beta \to 1$ *or entry is exogenous, then the system of equations identifies the exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.*

Proof. See Appendix A.2. □

This proposition clarifies the set of sufficient statistics that can characterize the revenue-neutral welfare costs of a firm-level distortion. These sufficient statistics are not abstract objects; they can be mapped to estimates widely used in the literature. For example, in the case of costly external financing, Hennessy et al. (2007) provide estimates of the effect of equity financing on firm investment (adjusting for average Q). This elasticity can be combined with firm-level information on equity financing and Q to yield estimates of the (capital-weighted) mean underinvestment due to costly equity financing and the mean-squared underinvestment due to costly equity financing. The value of (iii) can easily be calibrated to aggregate statistics from the national income and product accounts (NIPA) on the total profits lost to taxes net of subsidies as a fraction of corporate profits. The remaining parameters (iv) are standard in the macro literature.

[17] In turn, in steady state, the present value of revenue raised from firm taxation is constant.

3.2 Sketch of Proof of Proposition

Though we leave the formal proof to Appendix A, we walk through the key steps of the proof here, as they are useful for developing intuition about the result. We do this in two parts. First, in the case with exogenous entry, we show that all of the aggregate equations except for the capital accumulation and aggregate investment equations rely only on aggregates and the problem of a representative undistorted firm. In steady state, the capital accumulation and aggregate investment equations rely on aggregates and our sufficient statistics (i) and (ii). Therefore, we can evaluate aggregate counterfactuals exactly in steady state. We then show that changing the entry condition specification to endogenous entry implies that the value of an entering firm enters the equilibrium system; in this case, the sufficient statistics imply bounds for aggregates in equilibrium, as we will detail below.

If the mass of entry is assumed to be exogenous, one can characterize the stationary steady-state equilibrium as a function of sufficient statistics (i), (ii), and technology and preference parameters (iv). To see this point, note that the aggregate capital stock and investment equations, (20) and (21), in steady state can be written as

$$\begin{aligned} K &= M^{\mathrm{E}} + K\kappa\left(i_{\mathrm{MM}} + i_{\Delta}\right), && (25) \\ I &= M^{\mathrm{E}} c_e + K\phi(i_{\mathrm{MM}} + i_{\Delta}) + K\frac{\theta}{2}\left(i_{\Delta^2} - (i_{\Delta})^2\right). && (26) \end{aligned}$$

Capital accumulation depends on entry, undistorted firms' choices, the mean capital quality shock κ, and a notion of the average distortion to investment, i_{Δ}. The resources spent on investment depend on the investment cost function $\phi()$; undistorted firms' investment decisions; the mean distortion to investment; and the "misallocation" of investment, which, given quadratic adjustment costs, can be inferred from model parameters and the sufficient statistics. Therefore, taking entry as given, (25), (26), and a set of equations that are a function only of aggregates and a representative undistorted firm—(5), (9), (10), (17), (18), (19), (22), and (23) jointly evaluated in steady state—characterize the remaining steady-state values (K, I, C, L, Y, i_{MM}, q^{MM}, Π, $\frac{W}{P}$, and Λ).

The mechanics of endogenous entry are more nuanced, as the amount of entry is endogenous and is related to the value function of the firm. Because of the fact that firm revenues are homogeneous of degree one in capital, and aggregate output is a function of aggregate firm revenues, the sum of the expected (not-time-discounted) stream of firm revenues less expenses can be exactly characterized by aggregates and sufficient statistics in our model. However, what matters for firm value upon entry is the time-discounted stream of these profits to firms; our sufficient statistics do not tell us the average duration of these profits or distortions. While the average duration of these profits or distortions is not pinned down by our sufficient statistics, it is constrained by them.

More precisely, recall from (16) that the value of a firm entering with one unit of capital,

V^{E}, can be decomposed into the undistorted (pre-tax) firm's value, the effects of the distortion on (pre-tax) revenues, and the present-discounted value of subsidies less taxes: $V^{\mathrm{E}} = q^{\mathrm{MM}} + V^{\mathrm{D}} + V^{\mathrm{Taxes}}$. The undistorted firm's value is given by (9) and depends only on aggregates and sufficient statistics. The present value of subsidies less taxes must be equal to some fixed level for the counterfactual to be revenue neutral. In turn, we can show that the effect of the distortion on firm (pre-tax) discounted cash flows, V^{D}, is bounded. That is,

$$\overline{V^{\mathrm{D}}} \leq V^{\mathrm{D}} \leq 0, \tag{27}$$

where $\overline{V^{\mathrm{D}}}$ is the sum of the expected (not-time-discounted) losses in firm profits. Intuitively, these costs should be greater in magnitude than the not-time-discounted losses. We can write this object as a function of sufficient statistics:

$$\overline{V^{\mathrm{D}}} = \frac{\frac{\phi_2}{2} i_{\Delta^2}}{1 - \kappa \left(i_{\mathrm{MM}} + i_{\Delta}\right)}. \tag{28}$$

As $\beta \to 1$, V^{D} converges to $\overline{V^{\mathrm{D}}}$ as the time-discounted and not-time-discounted streams of firm profits converge. Further, by definition, the undistorted problem maximizes the present discounted value of profits; therefore, $V^{\mathrm{D}} < 0$. This approach implies that we can write $V^{\mathrm{D}} = \eta^D \overline{V^{\mathrm{D}}}$, where $\eta^D \in [0, 1]$.[18] Given each value of η^D, (16) and the entry condition $V^{\mathrm{E}} = c_e$ close the equilibrium system (in addition to the equations listed when taking entry as given above). We thus have a continuum of "candidate equilibria"—equilibria not ruled out by the sufficient statistics. We can solve the model and find the implied equilibrium aggregates for each value of η^D.[19]

Evaluating the counterfactual without the distortion is straightforward and can be performed by setting i_{Δ}, i_{Δ^2}, and V^{D} all to zero and evaluating the equilibrium. The effect of the distortion on each aggregate object of interest (output, consumption, investment, labor supply, entry, and welfare) in steady state can thus be bounded by computing both the maximum and minimum of each aggregate of interest in the continuum of candidate equilibria with the distortion and subtracting the level of the aggregate in the counterfactual without the distortion.

Further, since the equilibrium system without distortions is just a function of parameters we assume are known, we can compute transition dynamics where we start in each of the candidate equilibria and then remove the distortion, allowing for welfare counterfactuals that

[18]There is also a tighter bound than 0 for V^{D}, which we denote $\underline{V^{\mathrm{D}}}$, so $V^{\mathrm{D}} \leq \underline{V^{\mathrm{D}}} \leq 0$. There is no nice analytical expression for $\underline{V^{\mathrm{D}}}$—rather, it is the solution to a certain maximization problem: how to allocate (i) the distortion to investment, (ii) the mean-squared distortion to investment, and (iii) the loss of capital over a firm's life cycle so that the losses to firm value are as small as possible (losses occur as far after entry as possible) subject to constraints due to the sufficient statistics and equilibrium conditions. This solution means that the lower bound of η^D may be above zero.

[19]As the resulting system of equations is nonlinear, existence and uniqueness of equilibria are not given; our approach can easily accommodate multiple equilibria for each value of η^D.

incorporate dynamic considerations. The upper and lower bound for welfare are the maximum and minimum welfare gain across candidate equilibria.

3.3 Model Extensions

We now demonstrate how modifications of some of the key assumptions of our environment change the sufficient statistics required for our method and to what extent the method is still applicable in such extensions. We review several extensions: (1) adding richer firm heterogeneity, (2) allowing for aggregate shocks, (3) relaxing the assumption of homogeneity of degree one, (4) changing the process for firm productivity to be a Markov process, and (5) putting the economy on an (endogenous) balanced growth path. In all of these cases, there is an analogue of our main methodology under some conditions, but additional or modified sufficient statistics are required for its implementation.

Richer General Equilibrium Environment Our baseline model is kept deliberately simple to make the intuition for our approach simpler, but Proposition 1 can be extended to richer environments. Corollary 1.1 in Appendix A.3 extends Proposition 1 to a broader class of models and allows for deadweight losses as an additional consequence of the distortions. In particular, we allow for: (1) a more general class of production functions (for example, CES production) subject to the restriction that firm revenues exhibit homogeneity of degree one in inputs, (2) increasing and convex adjustment costs instead of quadratic adjustment costs, (3) a more general entry condition, and (4) for the distortion to impart deadweight losses.

This generalization changes the second sufficient statistic and introduces additional sufficient statistics. For increasing and convex (but not quadratic) adjustment costs, the second sufficient statistic is not the mean-squared distortion to investment but rather a more general measure of the resources lost due to "misallocation" of investment across firms. If the distortion imparts deadweight losses, we require the average amount of (per-period) capital destroyed due to distortion as an additional statistic. We also require additional statistics if firm revenues do not exhibit homogeneity of degree one in inputs or when we have a more general entry condition.

Richer Firm Heterogeneity Our approach can be extended to allow for firms that are heterogeneous for reasons other than there being heterogeneity in their capital stocks, realized shocks, or distortions. For example, we can allow for industries differing in their production functions. The main limitation to this approach is that it requires additional sufficient statistics for each category of firms, and thus its application must be driven by the availability of the relevant estimates. In Appendix A.4, we demonstrate exactly how this extension can be implemented. To briefly summarize, versions of our sufficient statistics (i_Δ, i_{Δ^2}) are needed for each group of firms, and the general proof is then similar with exogenous entry or no time discounting. The proof is also similar with endogenous entry as long as firms do not know their type before they enter.

Aggregate Shocks We can extend our approach to allow for aggregate shocks if entry is exogenous; in this case, the full state-conditionality of the sufficient statistics must be known.[20]

Define time-varying functions of our sufficient statistics as the (capital-weighted) mean distortion to investment and mean-squared investment at each point in time:

$$\begin{aligned} i_{\Delta t} &= i_{\Delta}(S_t) = \int_{\chi} \Delta i(\chi)\, \gamma_t^k(\chi)\, d\chi \\ i_{\Delta^2 t} &= i_{\Delta^2}(S_t) = \int_{\chi} (\Delta i(\chi))^2\, \gamma_t^k(\chi)\, d\chi, \end{aligned} \tag{29}$$

where S_t denotes the set of model state variables. Note that, in general, S_t may be infinite-dimensional, but there are special cases where this object is finite dimensional, or, at least, can be well-approximated by a finite-dimensional measure of aggregates, à la Krussel and Smith (1998). If S_t is known and its transition is characterized by the equilibrium conditions of the model (or any additional ones we impose), Corollary 1.2 in Appendix A.5 shows that we can characterize the effects of the distortion on macroeconomic aggregates in a world with aggregate shocks if we know state-contingent sufficient statistics $i_{\Delta}(S_t)$ and $i_{\Delta^2}(S_t)$. Note that these "sufficient statistics" are functions which are typically not estimated by researchers. However, if one were to assume a given cyclicality of a distortion, our method could be used to compute the implied consequences of the distortion.

Decreasing Returns to Scale in Profits Consider a modification to the model we introduced in Section 2 where (1) firm revenues exhibit decreasing returns to scale, $\frac{\rho-1}{\rho}(\alpha_1+\alpha_2) < 1$; and (2) there are no adjustment costs in capital. In this environment, instead of representing the distortion as a change in the investment rate, we represent the distortion in terms of the (log) distortion in the quantity of capital as compared with an undistorted firm (holding aggregate prices fixed):

$$K_{j,t} = \frac{K_{j,t}^*}{1-\Delta k_{j,t}}, \tag{30}$$

where $K_{j,t}^*$ is the capital choice firm j would have made, holding aggregate prices fixed. Corollary 1.3 in Appendix A.6 shows that our methodology can be extended to this environment, where the key sufficient statistics are the average (capital-weighted) distortion and a concavity-adjusted mean distortion. With decreasing returns to scale, the second sufficient statistic thus relies on the parameters of the model but is still easily computable.

[20]The derivations with aggregate shocks can also be extended to an environment in which entry is endogenous, but only to the extent that entry depends only on aggregates and not directly on the distortion; for example, entry can depend on the value of the undistorted firm for this method to work.

Markov Process for Productivity Our baseline model assumes that productivity shocks are *i.i.d.*. This assumption can be generalized to an arbitrary finite-state Markov process at the cost of requiring knowing the sufficient statistics conditional on z. Corollary 1.4 in Appendix A.7 shows that with this information, we can apply an analogue of our methodology. In Section 5, we use this method to examine how assumptions on the productivity process affect our quantitative results.

Balanced Growth Path We can consider specifications of our model in which the economy is on a balanced growth path. In Appendix A.8, we show that there are parameterizations of the model we introduce in Section 2 that allow for an endogenous balanced growth path. Given the production and preference parameters, an analogue of our sufficient statistics approach using the same sufficient statistics can be used to characterize the transition from a balanced growth path with firm-level distortions to one without the firm level distortions. We therefore can characterize the effect of distortions not only on the level, but also the growth rate of economic activity.

4 Quantitative Results

In this section, we present quantitative estimates of the aggregate consequences of external financing frictions and manager-shareholder frictions by calibrating the sufficient statistics to estimates from the corporate finance literature. The first subsection presents the calibration. The second subsection presents some model-based counterfactual measures that are useful in explaining the quantitative results. The third subsection presents the quantitative results in the limit as $\beta \to 1$ and discusses the key mechanisms driving the results. In the final subsection, we present results under the more standard case where $\beta < 1$.

4.1 Calibration of Sufficient Statistics and Parameters

We begin this section by discussing the baseline calibration of the sufficient statistics for the external financing friction and the manager-shareholder friction. We then discuss the baseline calibration for the remaining statistics and parameters of the model. The values for the statistics and parameters are listed in Table 1; in the table, we also note whether we perform the robustness on the statistic or parameter in the main text or in the Online Appendix.

Calibration of Sufficient Statistics Our sufficient statistics are closely related to estimates of the effect of distortions on firm investment. In particular, for some measured distortion effect X, many papers will estimate linear regression specifications of the form:

$$\frac{I_{jt}}{K_{jt}} = \gamma_0 + \gamma_X X_{jt} + \epsilon_{jt}.$$

We can derive the following closed-form approximation to our sufficient statistics:[21]

$$\begin{aligned} i_{\Delta} &= \gamma_X \mu_X \\ i_{\Delta^2} &= \gamma_X \left(\mu_X^2 + \sigma_X^2 \right), \end{aligned}$$

where μ_X, σ_X^2 are the mean and variance (across firms) of distortion X, respectively. These statistics are commonly reported in the summary statistics sections of papers which estimate effects of distortions on firm investment.

To calibrate the sufficient statistics in the case of external financing frictions, we rely on the estimates of Hennessy et al. (2007) (hereafter, HLW). As part of a broader examination of how different frictions dynamically interact with investment decisions, HLW estimate the effect on investment of external financing frictions conditional on Tobin's Q. We work with the ordinary least squares (OLS) specification estimates in HLW's Table 2 that include cash flow (which have values of negative 0.0005).[22] Given the percentage of firms that issue equity (0.179), average values of Q for equity issuers (5.874), and the average equity issuance amount of issuers to the capital stock (0.254) reported in HLW, we obtain an estimate for mean underinvestment of negative 0.00134. For computing mean-squared underinvestment, we compute firm Q and equity issuance to firm capital stock using Compustat over the years 1968-2003 (to match the years used by HLW).[23] We then compute the squared value of the interaction term times Q times equity issuance for each firm and average across firms, resulting in a mean-squared investment distortion of 0.00019.

For the manager-shareholder conflict, to obtain the mean distortion across firms, we rely on estimates from Ben-David et al. (2013) (hereafter, BGH). BGH study the extent to which managers are miscalibrated by examining manager ability to estimate the potential range of returns for the S&P.[24] The authors find that miscalibration of long-term returns leads to overinvestment. Because the authors report the miscalibration variable as a probability distribution, we can obtain the mean and mean-squared overinvestment across firms with this information and the effect of miscalibration on investment.[25] From the values reported in their paper, we obtain

[21]In the Online Appendix (Section O2.2), we demonstrate how an approximation of (12) leads to this result. Approximations play a role in two ways: first, future distortions may distort today's Tobin's q, which is commonly a control in such regressions; we show that this is a second-order effect as compared to the direct effect of the distortion on investment. Second, our method requires the capital-weighted mean and variance, rather than the unweighted version, which is less commonly reported. In some cases one or both of these approximations are not required.

[22]There are some estimates in HLW's robustness section that are around an order of magnitude higher; we will present results on this statistic up to an order of magnitude in our robustness section.

[23]We discuss the cleaning and data construction details in the Online Appendix (Section O2.2). Note that we find a similar estimate for the mean underinvestment of negative 0.00239 with this sample. Also note that our method technically requires us to use the capital-weighted values. This requirement would likely lead us to having smaller estimates and would make it such that our mean estimate does not derive from estimates in HLW; recall that we vary the values of our sufficient statistics over large ranges for robustness.

[24]Stein (2003) discusses how such an overconfidence problem is an example of an agency friction.

[25]Given that we do not have information on the capital stocks of the participants, we are unable to capital-weight

Table 1: Sufficient Statistics and Parameters

Sufficient Statistic or Parameter	Value	Where in text varied for robustness?
Sufficient Statistics - External Financing		
Average underinvestment across firms	-0.00134	Section 5
Mean-squared underinvestment across firms	0.00019	Section 5
Sufficient Statistics - Manager Decisions		
Average overinvestment across firms	0.00188	Section 5
Mean-squared overinvestment across firms	0.00004	Section 5
Additional Statistics and Parameters		
Average effective taxes	0.13	Section 5
Discount rate, β	0.98	Section 4
Investment of Modigliani-Miller firm, i_{MM}	0.9859	Online Appendix
Exogenous exit rate, $(1-\kappa)$	0.024	Online Appendix
Adjustment cost parameter, θ	1.1	Online Appendix
Depreciation rate, δ	0.1	Online Appendix
Production function parameter, α_2	2/3	Online Appendix
CES parameter, ρ	4	Online Appendix
IES parameter, γ	1	Online Appendix
Labor disutility parameter, φ	0.276	Online Appendix

Note: This table presents values for the sufficient statistics and parameters used in the quantitative analysis of the model in the case where $\beta < 1$. In the third column, we report whether we perform robustness on the sufficient statistic or parameter in Section 4, Section 5, or in the Online Appendix (Section O2.2).

the statistic for mean overinvestment to be 0.00188 and that for mean-squared overinvestment to be 0.00004.[26]

Calibration of Parameters To calibrate the tax term, we require the fraction of firm value lost to taxes (net of subsidies). We find the share of corporate profits lost to taxation in the NIPA by taking before-tax profits less after-tax profits relative to before-tax profits.[27] We use the value for 2021:Q4, which rounds to 13%.

We consider two cases for β: $\beta \to 1$ and $\beta < 1$. In the latter case, we choose β of 0.98, giving an annualized interest rate of 2%.[28] We choose the value of 2.4% for κ to match the employment-weighted firm death rate from 1980 through 2018 found in Crane et al. (2022). We choose i_{MM} to match the growth rate of firms in the economy with inelastic process innovation in Atkeson and Burstein (2010); this choice implies a value of 0.9859.[29] We also follow Atkeson and Burstein (2010) in choosing ρ to be 4 in our baseline calibration. We choose standard values for labor's share of income: $\alpha_2 = \frac{2}{3}$.

We choose the adjustment cost parameter θ to be 1.1 as in the second column of Table IV

our measures. We vary the values of these sufficient statistics over large ranges for robustness.

[26] We provide additional details on how we translate the reported values in BGH to our sufficient statistics in the Online Appendix (Subsection O2.3).

[27] We use the series without inventory valuation or capital-consumption adjustments. The two series can be found at `https://fred.stlouisfed.org/series/CP` and at `https://fred.stlouisfed.org/series/A053RC1Q027SBEA`, respectively.

[28] This is a standard value used in the literature—for example, in Gertler and Karadi (2013).

[29] All firms in the Atkeson and Burstein (2010) model with inelastic process innovation make the same investment decision as the Modigliani-Miller firms make in our model.

in Whited (1992). We choose the depreciation rate to be 10%, which matches the annualized value of the quarterly rate of 2.5% assumed in Gertler and Karadi (2013). We assume constant relative risk aversion (CRRA) household utility:

$$U(C_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\gamma}. \tag{31}$$

We set the risk aversion parameter to $\gamma = 1$ following Chetty (2006). We choose an inverse Frisch elasticity parameter φ of 0.276 following Gertler and Karadi (2013). Finally, we assume free entry in our baseline quantitative results.

4.2 Counterfactuals and Other Measures

PE Counterfactuals In addition to our model counterfactuals, we report PE counterfactuals. This specification is a special case of our model where all aggregate prices are fixed ($\rho = \infty$, $\gamma = \varphi = 0$, and entry is exogenous), but it is more easily understood as a simple back-of-the envelope calculation in which the effects of the distortion to aggregate investment, holding all else fixed, are translated into implications for the aggregate capital stock:

$$K = \frac{1}{1 - \kappa(i_{\mathrm{MM}} + i_{\Delta})}. \tag{32}$$

Note that in this PE counterfactual, i_{MM} is fixed and does not change with i_{Δ}, as aggregate prices (and thus optimal investment) do not respond. Because of the homogeneity of degree one in production, aggregate labor and output respond proportionally to the aggregate capital stock in such a PE world.

Decomposition of Output Note that in our production environment, changes in aggregate output can be expressed as a function of changes in input usage (total investment and labor) and changes in investment efficiency ($\frac{K}{I}$):

$$\Delta log(Y) = \alpha_2 \Delta log(L) + \alpha_1 \Delta log(I) + \alpha_1 \Delta log\left(\frac{K}{I}\right).$$

Thus, the fraction of changes in output explained by changes in investment efficiency can be written as $\frac{\alpha_1 \Delta log\left(\frac{K}{I}\right)}{\Delta log(Y)}$.

4.3 Baseline Results as $\beta \to 1$

We can obtain exact solutions for the welfare costs of the external financing and manager-shareholder frictions as $\beta \to 1$, so this special case is a useful point of departure for our analysis. We consider two cases: (1) a case where there is a subsidy that undoes the monopoly markup inefficiency and there is zero corporate taxation in both steady states, and (2) the baseline case with a monopoly markup inefficiency and positive corporate taxation in both steady states. The first case is a useful benchmark; here, resolving the external financing or manager-shareholder friction leads to the planner's problem.[30] The second case is the more quantitatively relevant one. We present the results in Table 2.

Removing the external financing friction leads to an increase of 3.35% of baseline output in the PE counterfactual in both panels, as it does not depend on the GE environment.[31] External financing reduces investment relative to the optimal investment decision. By contrast, the manager-shareholder conflict increases output in PE as managers, on average, are biased toward having beliefs that are too optimistic. Output falls 5.11% when the friction is removed in the PE counterfactual. That removing an overinvestment distortion would increase output by such a significant amount further emphasizes the value of a GE environment for counterfactuals.

Starting with panel A, we see that in an environment with a subsidy that undoes the monopoly markup inefficiency and zero corporate taxation in both steady states, resolving each distortion (which leads to the first-best case) induces positive but modest gains. The model features both GE dampening and amplifying effects, so it is not ex-ante obvious that the gains would be smaller. The most intuitive GE dampening effect is the increase in wages in response to the removal of the distortion. Removing the distortion increases aggregate labor demand which raises wages and thus reduces average operating profits. This result reduces the aggregate profit scaling factor Π, which in turn reduces the investment of undistorted firms. We analytically show this force in a simplified version of the model in the Online Appendix (Subsection O1.7). Specifically, we show that the derivative of the capital stock with respect to the mean distortion to investment i_{Δ} is decreasing in the inverse Frisch elasticity. In other words, if the equilibrium wage is more sensitive to aggregate labor supplied, the sensitivity of the aggregate capital stock to the mean distortion decreases. Our calibration uses a relatively conservative calibration for the inverse Frisch elasticity; that is, the strength of this dampening force in our calibration is on the weaker side of empirically relevant calibrations.[32]

The most intuitive GE amplifying force is the effect of complementarities on capital investment. With a CES aggregator, a lower ρ indicates greater complementarities in production.[33] Put differently, the value of a given firms' output is increasing in other firm's output. Thus, if the distortion is resolved, holding all other GE forces constant, this force increases the aggregate profit scaling factor and investment by undistorted firms. In the Online Appendix (Subsection O1.7), we show that the derivative of the capital stock with respect to the mean distortion to investment i_{Δ} is decreasing in ρ. In other words, if complementarities are larger, the response

[30] We present further details on the first-best case in the Online Appendix (Subsection O1.8).

[31] PE holds fixed prices, and the mass of entry and welfare is determined as the change in the capital stock. The percentage change in output is equal to the percentage change in capital.

[32] See Chetty et al. (2011) and Peterman (2016) for reviews of the literature estimating Frisch elasticities.

[33] Given our assumption that $\alpha_1 = \frac{\rho}{\rho-1} - \alpha_2$ and thus firm revenues are homogeneous of degree one, the substitutability of output between firms is fixed, and the substitutability of investment is governed by the properties of the investment cost function. Thus, in our calibration, changing the CES parameter affects complementarities between firms in production but not substitutability between firms.

Table 2: Effects of Resolving the External Financing and Manager-Shareholder Frictions

Panel A: Subsidy $\tau^s = \frac{\rho}{\rho-1}$ and zero corporate taxation ($\tau = 0$)

	External Financing Friction			**Manager-Shareholder Friction**		
	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value
Variable						
PE Capital stock or Output	3.35			-5.11		
GE Investment (I)	0.16	3.39	3.39	0.04	3.68	3.68
GE Capital stock (K)	0.24	25.6	25.66	0.05	27.83	27.85
GE Investment efficiency (K/I)	0.08	7.56	7.56	0.02	7.56	7.56
GE Output (Y)	0.16	5.08	5.09	0.04	5.52	5.53
GE Welfare	0.16			0.04		
GE Consumption (C)	0.16	1.69	1.7	0.04	1.84	1.84
GE Relative wage ($\frac{W}{P}$)	0.16	7.57	7.58	0.04	7.9	7.9
GE Labor supply (L)	0	0.45	0.45	0	0.47	0.47
GE Mass of entry (M_e)	2.75	1	1.03	6.23	1	1.07
GE Capital stock per entering firm ($\overline{K}$)	-2.58	25.6	24.95	-6.58	27.83	26.12
GE Investment of unlevered firm (i_{MM})	-0.24	0.99	0.98	-0.06	0.99	0.99
GE Aggregate profit scaling factor (Π)	-0.08	0.13	0.13	-0.02	0.13	0.13

Panel B: No subsidy ($\tau^s = 0$) and positive corporate taxation ($\tau = 0.13$)

	External Financing Friction			**Manager-Shareholder Friction**		
	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value
PE Capital stock or Output	3.35			-5.11		
GE Investment (I)	0.06	3.24	3.24	-0.18	3.54	3.53
GE Capital stock (K)	0.09	25.6	25.62	-0.26	27.83	27.76
GE Investment efficiency (K/I)	0.02	7.9	7.9	-0.08	7.87	7.86
GE Output (Y)	0.06	6.77	6.78	-0.18	7.37	7.35
GE Welfare	0.05			-0.17		
GE Consumption (C)	0.06	3.53	3.53	-0.17	3.83	3.82
GE Relative wage ($\frac{W}{P}$)	0.06	4.92	4.92	-0.17	5.13	5.12
GE Labor supply (L)	0	0.69	0.69	-0.01	0.72	0.72
GE Mass of entry (M_e)	-1.19	1	0.99	-2.5	1	0.98
GE Capital stock per entering firm ($\overline{K}$)	1.26	25.6	25.92	2.18	27.83	28.45
GE Investment of unlevered firm (i_{MM})	-0.08	0.99	0.99	0.27	0.99	0.99
GE Aggregate profit scaling factor (Π)	-0.03	0.13	0.13	0.08	0.13	0.13

Note: This table shows the percentage change in PE and GE objects from the baseline steady state to the steady state without the external financing and the manager-shareholder friction when $\beta \to 1$ in two cases: (1) when there is zero corporate taxation and the subsidy $\tau^s = \frac{\rho}{\rho-1}$ in both steady states (shown in Panel A), and (2) when $\tau = 0.13$ and the subsidy $\tau^s = 0$ in both steady states (shown in Panel B). The no-friction case shown in Panel A is the first-best. We also show the percentage changes in the GE objects associated with the GE welfare calculation, as well as their values in both the baseline steady state and the counterfactual steady state without the frictions. s.s. is steady state.

of the aggregate capital stock to the mean distortion increases in magnitude.

In our quantitative results in Panel A, the dampening forces indeed dominate. Removing the distortion makes entry more attractive (holding aggregate prices fixed), as entering firm value is maximized when future investment is not distorted. The capital stock and output thus rise, and so does the wage, which reduces the profitability of a unit of capital (the aggregate profit scaling factor). Intuitively, the investment decision and value of the Modigliani-Miller firm are increasing in the aggregate scaling factor. Therefore, with lower aggregate profits, the Modigliani-Miller firm's investment and value fall. In response, there is a lower capital stock per entering firm. These effects dampen the effect on output. The mass-of-entry effect dominates the lower capital-stock-per-entering-firm effect, leading to a larger overall capital stock. Therefore, output, consumption, and welfare all rise. Further, about one-third of the increase in output is explained by higher investment efficiency $\frac{K}{I}$, rather than by greater investment or labor utilization.[34] Investment efficiency rises for two reasons. First, the distortion leads to inefficient allocation of investment among incumbent firms. Second, the distortions reduce the value of an entering firm (holding aggregate prices fixed) and thus distort the relative amount of investment by incumbents versus entrants. Removing the distortion—by eliminating both of these forces—increases investment efficiency.

Panel B features our baseline calibration, in which there are aggregate inefficiencies due to corporate taxes and the monopoly markup. In such a calibration, resolving firm-level distortions does not necessarily increase welfare, as the distortions may interact with the aggregate inefficiencies. We begin with the case of resolving external financing frictions. In this case, GE welfare and output increase by around one-third of their magnitudes absent aggregate inefficiencies. The main force reducing the welfare and output gain is that, in equilibrium, reducing the friction leads aggregate entry to fall. The presence of net positive corporate taxation implies entry is inefficiently low in these equilibrium, and resolving the distortion leads to inefficiently low entry. Resolving the distortion increases the investment of incumbent firms, which reduces the aggregate scaling factor and investment by undistorted firms. Further, as the number of entrants falls, generating the same present value of taxation requires a higher tax burden as a percentage of firm value. These forces outweigh the benefit of eliminating distortions on firm value and lead to less entry in equilibrium. Labor supply barely changes despite the increased capital stock for two reasons. First, the decline in the aggregate profit scaling factor reduces the value of a marginal unit of labor; second, the wage rises because of consumption by households increases (recall with CES utility that the equilibrium wage rises in labor supply and consumption). Investment efficiency increases because distortions are eliminated, but by less than in the

[34]Labor supply does not move much in this and several other of our counterfactuals. To understand this outcome, first note that with CRRA utility, in log changes the labor-leisure condition and aggregate demand equation combine to yield $\Delta l = \frac{1}{1+\varphi}\left(\Delta y - \gamma \Delta c\right)$. Thus, since $\gamma = 1$, if consumption and output rise proportionately then the increasing marginal productivity of labor is exactly offset by the lower marginal value of wages, and therefore labor supply is unchanged.

case without the aggregate inefficiencies. Resolving the distortions eliminates misallocation of investment across incumbent firms, which increases investment efficiency; however, in equilibrium, aggregate entry (which is inefficiently low) falls, which reduces investment efficiency. Still, investment efficiency explains about one-fourth of the increase in output, while changes in aggregate resources devoted to investment explain the rest.

In the case of the manager-shareholder overinvestment friction, removing the friction reduces welfare in the presence of aggregate inefficiencies. This loss in welfare is due to the interaction of this firm-level distortion with the aggregate inefficiencies (the monopoly markup inefficiency and corporate taxation). The baseline steady-state equilibrium features too little output and too little entry as compared with the planner's problem. Resolving the manager-shareholder friction reduces both of these quantities even further, reducing welfare. In PE (holding prices fixed), reducing the friction reduces incumbent investment, as firms, on average, have expectations that are calibrated too high. In GE, this force, all else being equal, reduces demand for labor and thus leads the relative wage to fall and the aggregate profit scaling factor to rise, which increases the value and investment of the Modigliani-Miller firm. There is therefore greater investment per incumbent firm, and entry falls in response. The mass of entry falls by enough that the capital stock, output, and consumption also fall: welfare decreases even as labor supply decreases modestly (given the lower level of demand for investment). The efficiency of investment falls, as the effect of declining investment via entry (which is too low in equilibrium as compared with the planner's problem) dominates the effect of resolving misallocation among incumbent firms. This fall in investment efficiency explains two-thirds of the decline in output.

Altogether, the GE change in output and welfare is quantitatively lower than the PE change in output. With corporate taxation and an inefficiency due to the monopoly markup, removing the distortion is not necessarily positive for welfare and depends on the extent to which GE dampening effects, such as labor demand and entry, offset the removal of the distortion to incumbent investment. Further, changes in investment efficiency, due to reallocation of investment, account for a significant share of the change in output.

4.4 Baseline Results if $\beta < 1$

Our sufficient statistics approach provides us with bounds on the welfare costs of external financing and agency frictions if $\beta < 1$. Recall from Section 3.2 that when $\beta \to 1$, the duration of when distortions affect firms over their life is irrelevant for its valuation upon entry, and thus our sufficient statistics approach can yield an exact solution for the counterfactual of interest. However, if $\beta < 1$, distortions that affect firm value earlier in its lifetime are more costly, and our sufficient statistics do not exactly identify the counterfactual, though we can identify bounds on each aggregate variable.

Table 3 presents the upper bound and lower bound results under our calibration for β. The

Table 3: Effects of Resolving External Financing and Agency Frictions when $\beta = 0.98$

	External Financing		**Manager-Shareholder**	
	Upper bound $\%\Delta$ between steady states	**Lower Bound** $\%\Delta$ between steady states	**Upper bound** $\%\Delta$ between steady states	**Lower Bound** $\%\Delta$ between steady states
Variable				
PE Capital stock or Output	3.35	3.3	-5.11	-5.11
GE Investment (I)	0.05	0.08	-0.33	-0.3
GE Capital stock (K)	0.17	0.11	-0.37	-0.38
GE Investment efficiency (K/I)	0.12	0.03	-0.04	-0.06
GE Output (Y)	0.1	0.07	-0.27	-0.27
GE Welfare	0.13	0.04	-0.13	-0.15
GE Consumption (C)	0.13	0.07	-0.22	-0.24
GE Relative wage $(\frac{W}{P})$	0.12	0.07	-0.23	-0.24
GE Labor supply (L)	-0.03	0	-0.03	-0.03
GE Mass of entry (M_e)	1.3	-1.15	-1.87	-2.44
GE Capital stock per entering firm $(\overline{K})$	-1.14	1.25	1.47	2.01
GE Investment of unlevered firm (i_{MM})	-0.18	-0.09	0.24	0.26
GE Aggregate profit scaling factor (Π)	-0.08	-0.04	0.1	0.11

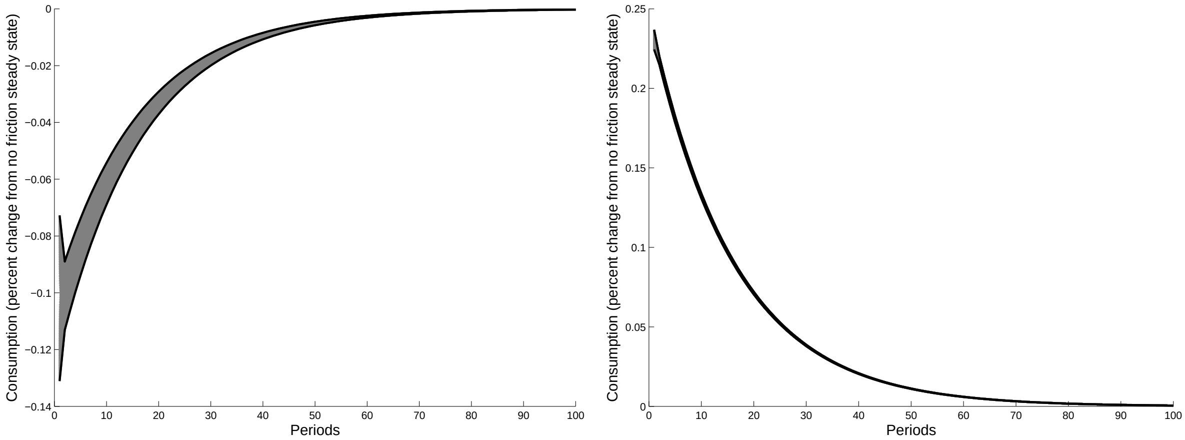
Note: This table shows the percentage change in PE and GE objects from the baseline steady state to the steady state without the external financing (top panel) or manager-shareholder (bottom panel) friction when $\beta = .98$, the subsidy $\tau^s = 0$, and $\tau = 0.13$. s.s. is steady state.

PE counterfactual is the same as in the $\beta = 1$ case, as it does not depend on aggregates, just the distortion and the mean growth rate of firms. Importantly, the GE counterfactuals differ not only in that β is lower, but also that our welfare measure accounts for the entire transition dynamic. Nonetheless, the GE welfare results are directionally similar to those for the $\beta \to 1$ case. The bounds for our welfare measures are tighter in the case of the manager-shareholder friction at 2 basis points but are still different by only 9 basis points for the external-financing friction. Furthermore, output, capital, and consumption are directionally similar to the $\beta \to 1$ case, and a similar logic holds for the drivers of the changes in aggregates between the cases as in the discussion in the last subsection. Our approach generates bounds for each aggregate variable along the transition dynamic; to get a sense of what the transition looks like, in Figure 1, we show the transition path for the percentage change in consumption relative to its level in the Modigliani-Miller steady state for each of the two frictions.

Importantly, the results also demonstrate how some of the changes in the counterfactual objects are sensitive to the parameterization. For example, a few of the counterfactual aggregates change directions in the counterfactual for the external financing friction between the lower bound and the upper bound case. The mass of entry falls in the lower bound case, as it did in the $\beta \to 1$ case, while it rises in the upper bound case. Both aggregates are sensitive to the changes in the distortions and the change in the investment of the Modigliani-Miller firm. Cap-

Figure 1: Bounds on the Transition Path after Resolving Firm-level Distortions

(a) External Financing Friction (b) Manager-Shareholder Friction



Note: Each subfigure shows the bounds on the percentage change in GE consumption relative to its level after removing the external financing friction (left panel) or the manager-shareholder friction (right panel) when $\beta = 0.98$ under our baseline calibration. s.s. is steady state.

ital demand rises by enough in the upper bound case that we observe a noticeable (albeit small) change in labor supply in the table.[35] Nonetheless, the narrative around the output and welfare results is still similar between the cases: more investment by incumbents leads to higher capital demand, which pushes down wages and thus average profits, dampening the gains from resolving the distortion. Therefore, even though the distortion has significant effects in PE, we observe only modest changes in the capital stock, output, consumption, and welfare in GE.

For further robustness, we present the welfare results (that account for the transition dynamic) for the two frictions for values of β between $\beta = 0.95$ and $\beta = 1$ in Online Appendix Figure O4.1. As expected, the figure shows that bounds widen for lower values of β; nonetheless, the welfare results are qualitatively similar to those presented in Table 3.

5 Quantitative Robustness

In this section, we test the robustness of our results as we vary sufficient statistics, parameters, the modelling of free vs. fixed entry, and the process for firm productivity.

Varying the Sufficient Statistics i_Δ, i_{Δ^2}, and $\overline{V^{\text{Taxes}}}$ Figure 2 plots the welfare gain from resolving the external financing or manager-shareholder friction as we vary the sufficient statistics i_Δ, i_{Δ^2}, and $\overline{V^{\text{Taxes}}}$.[36] The vertical lines in the figure indicate the values for the bounds on the welfare statistics under our baseline calibration.

Panels (a) and (b) show the gain in welfare from resolving the distortion ("welfare cost") as

[35]Note that labor supply falls in both cases when we do not round at the second decimal.
[36]We do so under the calibration where $\beta = 0.98$; there are thus upper and lower bounds and the welfare gains account for the transition dynamic.

we vary i_Δ by an order of magnitude; the welfare gain retains the same sign for a remarkably large range of values.[37] The magnitude of the impact on welfare is increasing in the magnitude of i_Δ locally around our estimate. The difference in the bounds is generally consistent across the range, at between 9 and 10 basis points across the external financing friction values, and around 2 basis points across the manager-shareholder friction values.

Panels (c) and (d) show the gain in welfare from resolving the distortion as we vary i_{Δ^2} by an order of magnitude. As the mean squared dispersion grows, so does the difference between the upper and lower bounds. The intuition for this result is the mean squared distortion to investment has a first order effect on firm value, while the mean distortion only has a second order effect on firm value. Because we cannot fully identify the duration of these distortions, the larger these distortions to firm value, the larger the bounds. For the external financing friction, if the mean squared distortion were several times our calibrated value, the bounds would be large enough that both welfare gains and welfare losses would be in the plausible range of equilibria identified by our bounds.[38] However, in the case of managerial miscalibration, even varying the mean squared distortion by an order of magnitude does not change the sign of the effect on welfare.

Panels (e) and (f) present the welfare gains from resolving the distortions across a range of the tax rate sufficient statistics in the bottom panels of Figure 2. We consider a range from 0.09 to 0.17, i.e., 30% in either direction from our baseline calibration. We find that the direction of the welfare gains matches those of the baseline calibration. For the external financing friction, the bounds tighten as the tax rate increases, going from 13 basis points apart at the lower end of the range to 7 basis points apart at the upper end of the range. Similarly, for the manager-shareholder friction, the bounds are 3 basis points apart at the lower end of the range and 1.5 basis point apart at the high end of the range. Altogether, the results are similar across this wide range of tax rates.

Free versus Fixed Entry We consider a version of the model where entry is exogenous and equals $\overline{M^{\mathrm{E}}}$. As described in Proposition 1, in this case, we get a point estimate for the counterfactual (e.g. the upper and lower bounds are identical). We set $\beta = 0.98$ (solving the transition for the welfare counterfactuals) and present results in Online Appendix Table O4.1. We also report the upper bound result for the endogenous entry case when $\beta = 0.98$ for ease of comparison; we do not report the lower bound because of table space constraints.

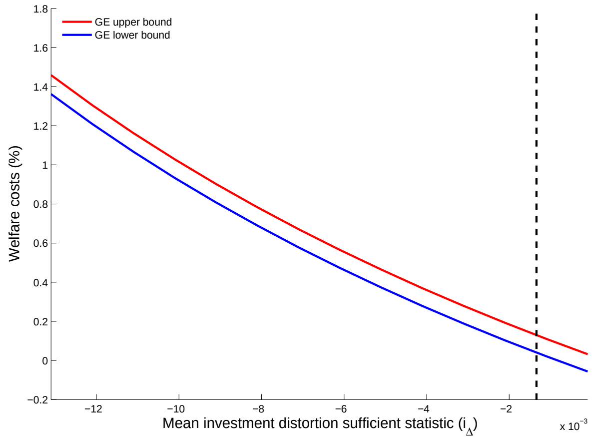
The main takeaway is that the change in consumption, output, the capital stock, the investment and value of the unlevered firm, and the aggregate profit scaling factor are all directionally similar for both frictions in the fixed-entry and free-entry models and are also similar quanti-

[37]In fact, the only place where the effect on welfare would switch signs in this range is when the mean effect of the distortion in the external financing case approaches zero, in that case the effect on the misallocation of investment dominates and thus there is a small welfare loss.

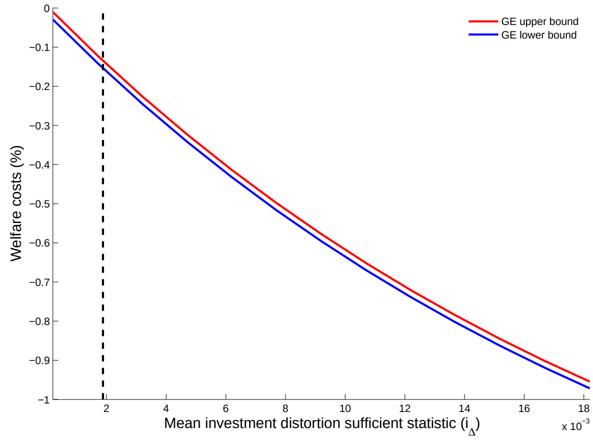
[38]If we were to provide additional data points, such as restrictions on the duration of losses after entering due to a given distortion, the bounds could be further tightened.

Figure 2: Welfare Gains from Resolving Firm-level Distortions, varying i_Δ, i_{Δ^2}, and $\overline{V^{\text{Taxes}}}$

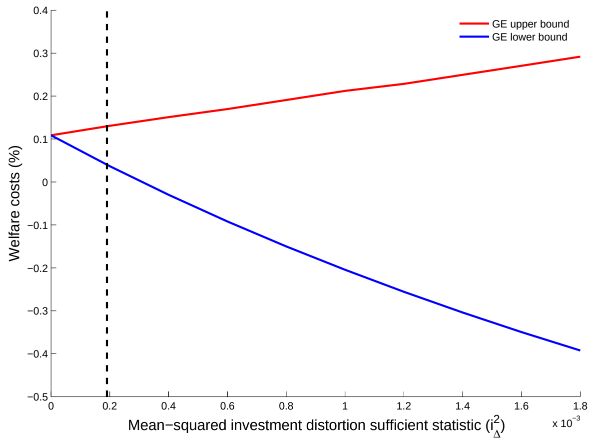
(a) External Financing Friction, varying i_Δ



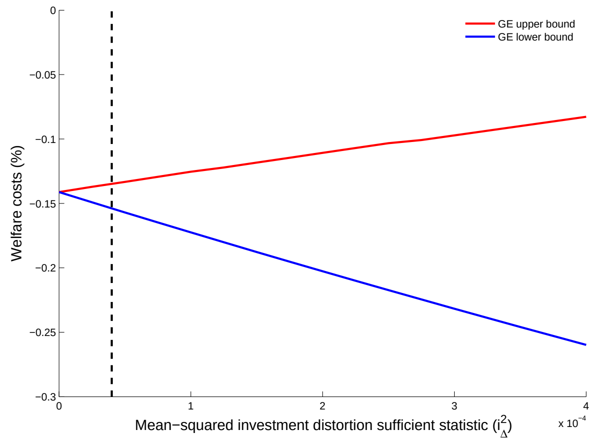
(b) Manager-Shareholder Friction, varying i_Δ



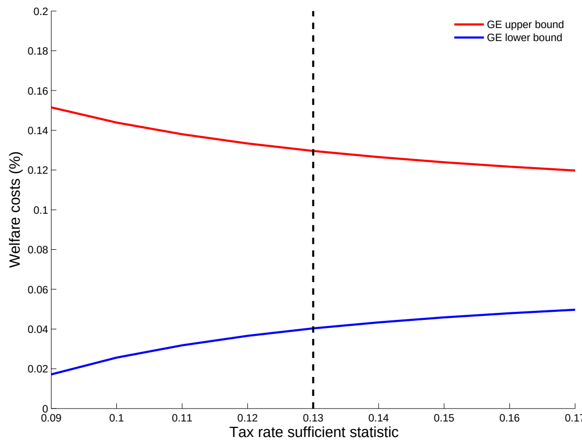
(c) External Financing Friction, varying i_{Δ^2}



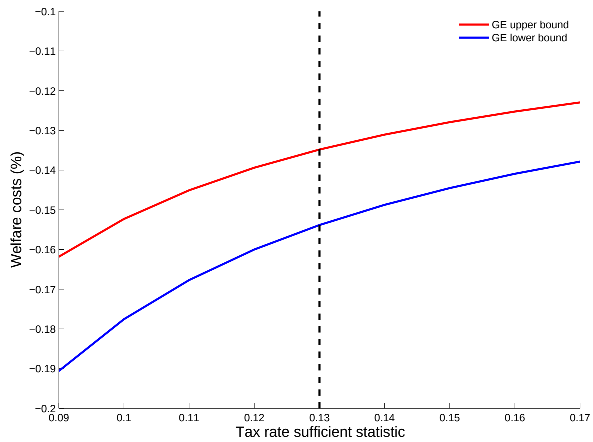
(d) Manager-Shareholder Friction, varying i_{Δ^2}



(e) External Financing Friction, varying $\overline{V^{\text{Taxes}}}$



(f) Manager-Shareholder Friction, varying $\overline{V^{\text{Taxes}}}$



Note: Each subfigure shows the bounds on the welfare change after removing the external financing friction (left panels) or the manager-shareholder friction (right panels) when $\beta = 0.98$ for a set of values of a given sufficient statistic. The vertical, dashed black line indicates the welfare values at the calibrated value of the given statistic (as shown in Table 3).

tatively. The mass of entry being fixed does affect the results for the capital stock per entering firm, which now switches directions from the free-entry case. This should not come as a surprise, as only incumbents are changing their decisions on the margin in response to the friction being removed. Labor supply still falls for the manager-shareholder friction, now by even more as incumbent capital demand decreases. The opposite holds for the external financing friction: incumbent capital and labor demand increase, raising labor supply. The resulting effects on welfare are directionally similar to our baseline model with free entry, but they are smaller in magnitude.

Firm Heterogeneity in Idiosyncratic Productivity Our baseline model assumes that idiosyncratic productivity is *i.i.d.* across firms. In Section 3.3, we detail an extension that generalizes the framework to allow for idiosyncratic productivity to follow a Markov process and demonstrates the necessary sufficient statistics for constructing the counterfactuals of interest in this setting. We implement this extension for the external financing friction using firm-level data to understand its quantitative importance, and then vary model parameters related to this heterogeneity (and its covariance with distortions) to more generally understand in which cases this assumption may be quantitatively important. The full details of this implementation are provided in Online Appendix O3; here, we provide a summary of our approach and the results.

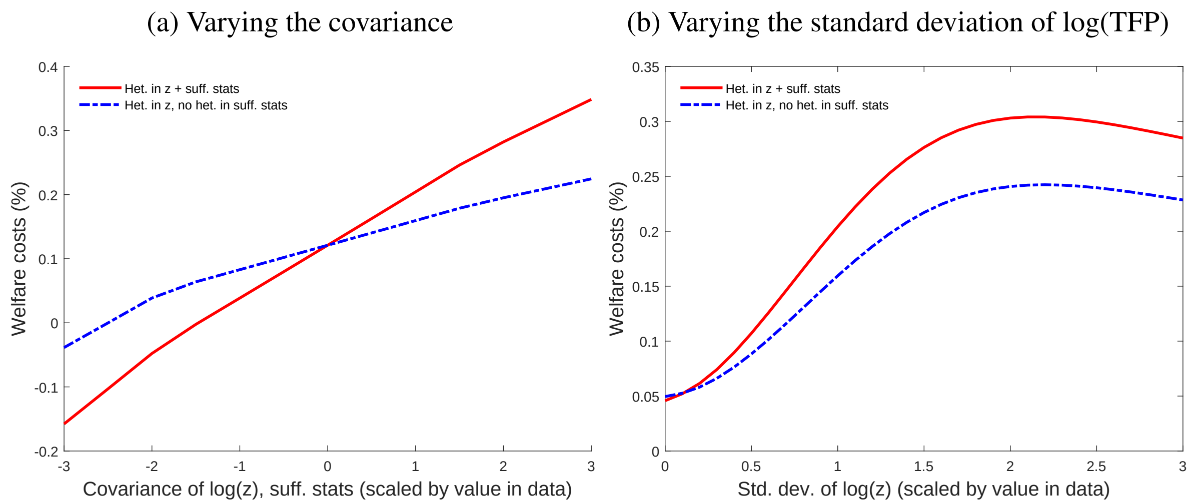
Following Proposition 1.4, to implement our approach, we need to know sufficient statistics conditional on firm productivity and the process for firm productivity. We therefore use data from Compustat and firm TFP from İmrohoroğlu and Tüzel (2014) to compute sufficient statistics analogous to those we obtained from HLW, but for each quintile of firm productivity; we further compute the transition matrix and mean levels of productivity for each group. There is considerable heterogeneity in the sufficient statistics across productivity levels, though the relationship in non-monotonic.[39] To understand the effect of heterogeneity in how distortions affect firms of different productivity levels, we consider the difference between our results in this framework and another in which productivity is heterogeneous but the distortions (and thus sufficient statistics) are independent of the level of productivity.[40] The resulting measures of the effect of resolving costly external financing are qualitatively similar with heterogeneity in sufficient statistics vs without such heterogeneity. Quantitatively, the effect on welfare/output are somewhat larger: welfare increases by 0.23% with heterogeneity in sufficient statistics instead of 0.15% without it; Online Appendix Table O3.1 presents the full set of quantitative results under the counterfactuals.

To better understand in which cases heterogeneity in firm exposure to distortions along the

[39]Online Appendix Table O3.2 presents the sufficient statistics by quantile, Online Appendix Table O3.3 displays the Markov transition matrix between productivity levels, and Online Appendix Table O3.4 presents the mean and standard deviation of log(TFP) by quantile.

[40]If the sufficient statistics are independent of the level of productivity a simple analogue of our main approach can be used with no additional sufficient statistics required. Firms with different productivities have heterogeneous elasticities of investment with respect to aggregate prices, which can change the quantitative results without any heterogeneity in how distortions affect firms.

Figure 3: Varying Heterogeneity parameters



Note: Each subfigure shows the welfare change after removing the external financing friction $\beta = 1$, but under different values of heterogeneity parameters. In the left subfigure, we vary the covariance between the sufficient statistics and heterogeneity relative to its value used in the previous subsection for the results presented in Table O3.1. In the right subfigure, we vary the standard deviation of log(TFP) relative to its value used in the previous subsection for the results presented in Table O3.1. Log is the natural logarithm. Suff. stats are the sufficient statistics. Het. is heterogeneity. Std. dev. is standard deviation

dimension of productivity matters, we simulate data varying either the covariance of firm distortions with productivity (holding firm productivity constant) or the cross-sectional standard deviation of log(TFP) (holding the correlation between distortions and productivity constant). We then compute the sufficient statistics by group and measure the effects of resolving costly external financing, both with and without heterogeneity in sufficient statistics. Figure 3 presents the results, varying the covariance in the left subfigure and the standard deviation in the right, where both are normalized by the respective measure in our data. We see that the covariance between firm productivity and the distortions can bias the results upwards or downwards, depending on the sign, though it has to be several times the value we found in the data to be qualitatively meaningful. Varying the standard deviation of firm TFP alone does not change the results qualitatively, and quantitatively maxes out at less than 10 basis points across a large range of values of the standard deviation, up to 3 times more than our assumed value. The gap shrinks if the standard deviation is smaller. Altogether, we see this exercise as demonstrating the general robustness of our qualitative results and as an illustration of how the extensions we develop to our methodology can be applied.

6 Conclusion

This paper develops a novel sufficient statistics methodology for examining the aggregate costs of firm-level distortions. Our methodology allows a researcher to evaluate the aggregate consequences of distortions without having to specify the microfoundation of the distortion.

This methodology is useful in many instances where such microeconomic dynamics are in debate (e.g., reasons for firm capital structure choice or questions about the exact microfoundation of agency frictions) or in cases where realistic modelling of the microeconomic details is sufficiently complicated to be intractable in a general equilibrium model (e.g., modelling firm liabilities with realistic covenants, convertibility, or multiple maturities). Further, the sufficient statistics needed are closely related to standard statistics reported in empirical and quantitative results in the corporate finance and firm dynamics literatures.

We use our approach to examine two cases of distortions: costly external equity financing and a manager-shareholder conflict. We show that the welfare costs of external financing frictions and manager-shareholder conflicts are significantly smaller than their PE costs, and that they can be negative in models with aggregate inefficiencies. Our approach provides us with insights into the mechanisms driving our results, as we can describe how all aggregates of the model change along the transition path after resolving the distortion of interest. We demonstrate how our approach can be extended to even richer environments, such as those with business cycles; we detail the additional sufficient statistics needed for such extensions. By providing this guidance, we hope that researchers will pursue the measurement of such statistics.

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A Theory Appendix: Proofs and Corollaries

In this section of the appendix, we first define an equilibrium (in Subsection A.1). Second, we present the proof to Proposition 1 (in Subsection A.2). Third, we outline the mathematical definitions of the corollaries (in Subsections A.3-A.8. We present proofs of the corollaries in the Online Appendix.

A.1 Definition of Equilibrium

Given the initial distribution of firms $\Gamma_0(k,\chi)$ across states, a sequential equilibrium consists of policy and value functions of firms, $\{l_t(k,\chi), i_t(k,\chi), V_t^F(k,\chi), V_t^{\mathrm{E}}, \chi_t'(\chi)\}$; household policy functions for consumption, C_t, and labor, L_t; aggregate prices, $\{W_t, R_t, P_t\}$; and a mass of new entrants, M_t^{E}, such that for all t, (i) the policy and value functions of intermediate good firms are consistent with the optimization problem (which we do not fully specify), (ii) the representative consumer's policy function is consistent with its maximization problem, (iii) the firm's value functions and decision rules are priced such that they break even in expected value, (iv) the free-entry condition holds, (v) labor, capital, and final good markets clear, and (vi) the measure of firms evolves in a manner consistent with the policy functions of firms, households, and shocks.

A stationary competitive equilibrium is an equilibrium in which all aggregates, aggregate prices, and the distribution of firms are constant over time. In such an equilibrium, we say these aggregates are in steady state. We focus only on equilibria with positive entry.

A.2 Proof of Proposition 1

Note that in steady state, the capital accumulation and investment equations, (20) and (21), can be written as the follows:

$$K = M^{\mathrm{E}} + K\kappa \int_{\chi} (i(\chi))\, \gamma^k(\chi)\, d\chi \tag{33}$$

$$I = M^{\mathrm{E}} c_e + K \int_{\chi} \phi(i(\chi))\, \gamma^k(\chi)\, d\chi. \tag{34}$$

Plugging in the sufficient statistics yields (25) and (26). These conditions are therefore functions of aggregates K, I, M^{E}, representative undistorted firm investment i_{MM}, and sufficient statistics i_{Δ}, i_{Δ^2}. Note that the remaining equilibrium conditions in steady state include the equations for the value of a firm at entry and (35)–(43):

$$
\begin{aligned}
\Pi &= \left(A^{\frac{\rho-1}{\rho}} Y^{\frac{1}{\rho}} (W)^{-\alpha_2(\frac{\rho-1}{\rho})} \left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})}\right)^{\alpha_c} \left(1-\alpha_2(\frac{\rho-1}{\rho})\right), && (35)\\
q^{\mathrm{MM}} &= \Pi - \phi(i_{\mathrm{MM}}) + i_{\mathrm{MM}}\kappa\Lambda q^{\mathrm{MM}} && (36)\\
\phi'(i_{\mathrm{MM}}) &= \kappa\Lambda q^{\mathrm{MM}} && (37)\\
\Lambda &= \beta && (38)\\
W &= \frac{-U_L(C,L)}{U_C(C,L)} && (39)\\
L &= K\left(\alpha_2(\frac{\rho-1}{\rho}) A^{\frac{\rho-1}{\rho}} (W)^{-1} Y^{\frac{1}{\rho}}\right)^{\alpha_c} && (40)\\
C &= Y - I && (41)\\
Y &= K\frac{\Pi}{\left(1-\alpha_2\frac{\rho-1}{\rho}\right)} && (42)\\
0 &= \Theta(M^E, V^{\mathrm{E}}). && (43)
\end{aligned}
$$

A.2.1 Exogenous Entry

Let us first consider the case with exogenous entry: $\Theta(M^{\mathrm{E}}, V^{\mathrm{E}}) = M^{\mathrm{E}} - \overline{M^{\mathrm{E}}}$. If we know the parameters of the model and sufficient statistics i_{Δ}, i_{Δ^2}, then the 11 unknowns K, I, C, L, Y, W, Π,M^{E}, Λ, q^{MM}, and i_{MM} are characterized by the 11 equations (25), (26), and (35)–(43). Therefore, we can solve the steady-state equilibrium both with the distortion and without it (setting i_{Δ}, i_{Δ^2} both to zero).

A.2.2 Endogenous Entry

With endogenous entry, the value of the entering firm, V_E, enters the entry equilibrium condition $\Theta(M^{\mathrm{E}}, V^{\mathrm{E}}) = 0$, which depends on the present discounted value of the stream of expected firm profits. In this case, the proof is more complex. We start by defining a number of objects that are a function of *when* the firm entered, i.e., one period ago versus two periods ago versus n periods ago. Using these objects, we then derive an expression for the key variable for our proof: the difference in firm value between the distorted and undistorted benchmark. With bounds on this object, we can then solve for the counterfactual of interest.

We use subscripts in parentheses to denote statistics of firms that entered n periods ago. $K_{(n)}$ denotes the total capital stock of firms that entered n periods ago; by definition, $K = \sum_{n=0}^{\infty} K_{(n)}$. $\overline{K_{(n)}} = \frac{K_{(n)}}{M^E}$ denotes the entry-scaled total capital stock that entered n periods ago, and $\widetilde{K_{(n)}} = \beta^n \overline{K_{(n)}}$ is the time-since-entry discounted version of $\overline{K_{(n)}}$. We define the sums of these objects as $\overline{K} = \sum_{n=0}^{\infty} \overline{K_{(n)}}$ and $\widetilde{K} = \sum_{n=0}^{\infty} \widetilde{K_{(n)}}$. Let $\Gamma_{(n)}(k, \chi)$ denote the distribution of firms across states that entered n periods ago. In turn, the capital-weighted steady-state relative distribution of the capital structure state vector of firms that entered n periods ago is

$$\gamma^K_{(n)}(\chi) = \frac{\int_k k\Gamma_{(n)}(k,\chi)}{\int_k \int_x k\Gamma_{(n)}(k,x)}. \tag{44}$$

Let $\widetilde{\gamma^K}(\chi)$ denote the time-since-entry discounted distribution of firms across states:

$$\widetilde{\gamma^K}(\chi) = \sum_{n=0}^{\infty} \frac{\widetilde{K_{(n)}}}{\widetilde{K}} \gamma^K_{(n)}(\chi). \tag{45}$$

$\int_\chi \gamma^K_{(n)}(\chi)d\chi$ and $\int_\chi \widetilde{\gamma^K}(\chi)d\chi$ can be thought of as similar to probability distributions, since

$$\int_\chi \gamma^K_{(n)}(\chi)d\chi = \int_\chi \widetilde{\gamma^K}(\chi)d\chi = 1.$$

It is also useful to define

$$\widehat{\gamma^K}(\chi) = \sum_{n=0}^{\infty} \beta^n \frac{\overline{K_{(n)}}}{\overline{K}} \gamma^K_{(n)}(\chi), \tag{46}$$

noting this object need not be equal to 1. There are two properties of $\widehat{\gamma^K}(\chi)$ worth noting. First, it can be mapped into $\widetilde{\gamma^K}(\chi)$:

$$\widehat{\gamma^K}(\chi) = \widetilde{\gamma^K}(\chi)\frac{\widetilde{K}}{\overline{\overline{K}}}. \tag{47}$$

Second, because we can write $\gamma^K(\chi) = \sum_{n=0}^{\infty} \frac{\overline{K_{(n)}}}{\overline{K}} \gamma^K_{(n)}(\chi)$,

$$\widehat{\gamma^K}(\chi) \leq \gamma^K(\chi). \tag{48}$$

Note that since, by definition, $\overline{K_{(n)}} = \widetilde{K_{(n)}} = 1$ and $\widetilde{K_{(n)}} = \beta\kappa\widetilde{K_{(n)}} \int_\chi (i(\chi))\, \gamma^K_{(n)}(\chi)$, we can derive the following expression for $\widetilde{K} = \sum_{n=0}^{\infty} \widetilde{K_{(n)}}$:

$$\widetilde{K} = 1 + \widetilde{K}\beta\kappa \left(\int_\chi (i(\chi))\, \widetilde{\gamma^K}(\chi) \right). \tag{49}$$

Additionally, note that the expected pre-tax value of an entering firm can be written as

$$V^{E,\text{notax}} = \widetilde{K}\left(\Pi_t - \sum_{n=0}^{\infty} \int_\chi \phi(i(\chi))\widetilde{\gamma^K}(\chi)d\chi \right). \tag{50}$$

We define $\widetilde{K^{MM}}$ as the time-since-entry discounted expected lifetime sum of capital stock of an undistorted firm that enters with one unit of capital; this object is related to the value of an undistorted firm:

$$\widetilde{K^{MM}} = 1 + \kappa\beta i_{\mathrm{MM}} \widetilde{K^{MM}} = \frac{1}{1 - \kappa\beta i_{\mathrm{MM}}}. \tag{51}$$

$$q^{\mathrm{MM}} = \widetilde{K^{MM}} \left(\Pi_t - \phi(i_{\mathrm{MM}})\right). \tag{52}$$

Difference between distorted and undistorted benchmark Define $V^{\mathrm{D}} = V^{E,\mathrm{notax}} - q^{\mathrm{MM}}$ as the change in (pre-tax) profits between the distorted and undistorted case. (50) and (52) yields

$$\begin{aligned} V^{\mathrm{D}} &= \tilde{K}\bigg(\Pi - \int_{\chi} \tilde{\gamma}^{K}(\chi)\phi(i(\chi))d\chi\bigg) - \tilde{K}^{\mathrm{MM}}\bigg(\Pi - \phi(i_{\mathrm{MM}})\bigg) \\ &= \bigg(\tilde{K} - \tilde{K}^{\mathrm{M\tilde{M}}}\bigg)\bigg(\Pi - \phi(i_{\mathrm{MM}})\bigg) - \tilde{K}\int_{\chi} \tilde{\gamma}^{K}(\chi)(\phi(i(\chi)) - \phi(i_{\mathrm{MM}}))d\chi. \end{aligned} \tag{53}$$

We can then combine (49) and (51) and plug this expression for $(\tilde{K} - \tilde{K}^{\mathrm{M\tilde{M}}})$ into (53) (also plugging in (47) and the undistorted firm's value function) to obtain:

$$V^{\mathrm{D}} = \bar{K}\int_{\chi} \hat{\gamma}^{K}(\chi)\bigg(\kappa\beta q^{\mathrm{MM}}(i(\chi) - i_{\mathrm{MM}}) - (\phi(i(\chi)) - \phi(i_{\mathrm{MM}}))\bigg)d\chi. \tag{54}$$

Note that the quadratic adjustment cost implies that $\phi'(i_{mm}) = 1 + \frac{\theta_2}{2}(x - \theta_c)^2$, and thus:

$$\phi(i(\chi)) - \phi(i_{mm}) = \Delta i(\chi)\,\phi'(i_{mm}) + \frac{\theta}{2}(\Delta i(\chi))^2. \tag{55}$$

Plugging (55) into (54) yields

$$V^{\mathrm{D}} = \bar{K}\int_{\chi} \hat{\gamma}^{K}(\chi)\bigg(\kappa\beta q^{\mathrm{MM}}(\Delta i(\chi)) - \Delta i(\chi)\,\phi'(i_{mm}) - \frac{\theta}{2}(\Delta i(\chi))^2\bigg)d\chi. \tag{56}$$

Given that the FOC of the undistorted problem implies $\phi'(i_{mm}) = \kappa\beta q^{\mathrm{MM}}$, (56) yields

$$V^{\mathrm{D}} = -\bar{K}\int_{\chi} \hat{\gamma}^{K}(\chi)\bigg(\frac{\theta}{2}(\Delta i(\chi))^2\bigg)d\chi. \tag{57}$$

Note that from (57), we can construct bounds for V^{D}. Since $\hat{\gamma}^{K}(\chi)$, and $\bar{K}$ are all positive, V^{D} is negative, so 0 is an upper bound. Also note that (48) implies that we can construct a lower bound if we replace $\hat{\gamma}^K$ with γ^K. Formally, define $\overline{V^{\mathrm{D}}}$ as

$$\overline{V^{\mathrm{D}}} \;=\; -\bar{K}\int_{\chi}\gamma^K(\chi)\left(\frac{\theta}{2}\left(\Delta i\left(\chi\right)\right)^2\right)d\chi = -\bar{K}\left(\frac{\theta}{2}i_{\Delta^2}\right). \tag{58}$$

Therefore, we have bounds for V^{D}: $\overline{V^{\mathrm{D}}} \leq V^{\mathrm{D}} \leq 0$. Another way of expressing this bound result is that $V^{\mathrm{D}} = \eta^D \overline{V^{\mathrm{D}}}$ for some $\eta^D \in [0,1]$. In the limit as $\beta \to 1$, $V^{\mathrm{D}} \to \overline{V^{\mathrm{D}}}$. That is, in the limit, V^{D} converges to that lower bound (larger losses). There is a tighter upper bound for V^{D} than 0, but it is a less analytically clean object. Online Appendix O1.1 derives this tighter bound.

Finishing the result Recall that we can decompose firm value at entry into the value of an undistorted firm with a unit of capital, the losses in (pre-tax) profits due to the distortion, and any tax consequences. Since our counterfactuals are revenue neutral, the present value of taxes raised must remain constant. We can thus write a system of equations for the value of the entering firm:

$$V^{\mathrm{E}} \;=\; q^{\mathrm{MM}} + V^{\mathrm{D}} + V^{\mathrm{Taxes}} \tag{59}$$

$$V^{\mathrm{Taxes}} \;=\; \frac{\overline{\mathrm{TaxPV}}}{M^E} \tag{60}$$

$$V^{\mathrm{D}} \;=\; -\eta^D\frac{K}{M^E}\left(\frac{\theta}{2}i_{\Delta^2}\right). \tag{61}$$

Thus, for each value of η^D, equations (25), (26), (35)–(43), (59)–(61) characterize unknowns K, I, C, L, Y, W, Π, M^E, Λ, q^{MM}, i_{MM}, V^{E}, V^{D}, and V^{Taxes}. Therefore, we can solve for a continuum of "candidate equilibria" for the model. We can then evaluate the maximum and minimum of each aggregate across these candidate equilibria.

We can then solve for the counterfactual where the distortion is removed. Note that this solution is achieved by setting i_Δ, i_{Δ^2}, both equal to zero. In that case, V^{D} must be exactly equal to zero. Thus, bounds for steady-state counterfactuals for any aggregate in question can be evaluated by taking the difference between the undistorted case and the maximum and minimum of the candidate equilibria.

A.2.3 Transition Dynamic

Note that without distortions, the dynamics of 12 unknowns K_t, I_t, C_t, L_t, Y_t, W_t, Π_t, M_t^E, Λ_t, q_t^{MM}, $i_{\mathrm{MM},t}$, and V_t^{E} are characterized by the following equilibrium system:

$$K_t \;=\; M_t^E + K_{t-1}\kappa i_{\mathrm{MM},t-1} \tag{62}$$

$$I_t \;=\; c_e M_t^E + K_t\phi\left(i_{\mathrm{MM},t}\right) \tag{63}$$

$$\Pi_t \;=\; \left(A^{\frac{\rho-1}{\rho}}Y_t^{\frac{1}{\rho}}\left(W_t\right)^{-\alpha_2(\frac{\rho-1}{\rho})}\left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})}\right)^{\alpha_c}\left(1-\alpha_2(\frac{\rho-1}{\rho})\right) \tag{64}$$

$$
\begin{aligned}
q_t^{\text{MM}} &= \Pi_t - \phi(i_{\text{MM},t}) + i_{\text{MM},t}\kappa\Lambda_t q_t^{\text{MM}} && (65)\\
\phi'\left(i_{\text{MM},t}\right) &= \kappa\Lambda_t q_t^{\text{MM}} && (66)\\
\Lambda_t &= \beta\frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t-1}, L_{t-1}\right)} && (67)\\
W_t &= \frac{-U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)} && (68)\\
L_t &= K_t\left(\alpha_2(\frac{\rho-1}{\rho})A^{\frac{\rho-1}{\rho}}\left(W_t\right)^{-1}Y^{\frac{1}{\rho}}\right)^{\alpha_c} && (69)\\
C_t &= Y_t - I_t && (70)\\
Y_t &= K_t\frac{\Pi_t}{\left(1-\alpha_2\frac{\rho-1}{\rho}\right)} && (71)\\
0 &= \Theta(M_t^E, V_t^{\text{E}}) && (72)\\
V_t^{\text{E}} &= q_t^{\text{MM}} + \frac{\overline{\text{TaxPV}}}{M_t^E}. && (73)
\end{aligned}
$$

Therefore, if we know the original (distorted) steady-state, we can use this system of equations to solve for the transition dynamic. In the cases where we have a continuum of counterfactual equilibria, we can solve for the transition path and evaluate aggregate objects (such as the welfare gain from the resolution of the distortions).

A.3 Generalization to a Broader Class of Models

Assume that instead of the assumptions we placed on the production environment in Section 2, we generalize the environment as follows:

(i) The output of heterogeneous firms is aggregated into the final good using a homogeneous and symmetric production function G that is continuous and differentiable; inputs are chosen to satisfy cost minimization. Each firm produces output with an identical production function $F(zk, l)$, where z is an *i.i.d* shock.

(ii) Assume that G and F are defined such that firm revenue, $p_{j,t}y_{j,t} = \frac{\partial Y_t}{\partial y_{j,t}}y_{j,t}$, can be written as a function of inputs and aggregate output: $p_{j,t}y_{j,t} = Rev\left(z_{j,t}k_{j,t}, l_{j,t}, Y_t\right)$, where Rev is continuous, differentiable, homogeneous of degree one in k and l, and satisfies the Inada conditions in the first two arguments. Additionally, the marginal revenue product of labor, $\frac{\partial Rev_t(zk,l,Y)}{\partial l}$, is invertible in l.

(iii) The average firm entry cost, c_e, is a function of the quantity of entry (or other aggregates such as output) but does not otherwise depend on the allocation of resources across firms.

(iv) Similarly, the equilibrium entry condition is a function of the value of entry, the mass of entry, and other aggregates: $M\left(V_{e,t}, M_t^{\text{E}}, ...\right) = 0$.

(v) The investment cost function, $\phi(i)$, is increasing and convex.

These assumptions not only nest the specification we consider in Section 2, but also can accommodate other commonly used production functions (e.g., CES at the firm level), adjustment costs, and specifications for entry (e.g., exogenous entry or entry with a positive but bounded elasticity of entry to firm value). Our approach still works under these assumptions for the following reasons: first, homogeneity of degree one of firm revenues in terms of output ensures that the undistorted firm's problem is homogeneous of degree one and, thus, undistorted investment rates are equal across firms. Second, homogeneity of the aggregation technology means that we can write total output as a function of the sum of total firm revenues, which implies a relationship between aggregate output and firm profits (and thus value functions). Third, the remaining assumptions imply that other forces, such as entry, depend only on aggregates and not on the distribution of firms.

Additionally, we allow for the distortion to have exogenous deadweight losses as well, modelled as destruction of capital (for example, through bankruptcy or as a consequence of agency problems).[41] Formally, we assume that the distortion may also destroy capital—let $\xi_{j,t} = \xi_t(\chi_{j,t})$ represent the fraction of capital destroyed due to the distortion. In the baseline model, the (pre-tax) value of the distorted firm as the discounted present value of revenues less expenses becomes

$$V_{j,t}^F(k,\chi) = z_{j,t}^{\frac{\rho-1}{\rho}\alpha_c} k_{j,t}\Pi_t - k_{j,t}\phi(i_t(\chi_{j,t})) + \kappa\,(1-\xi_t(\chi))\,\mathbb{E}_t[\Lambda_{t+1}V_{t+1}^F(k',\chi')]. \tag{74}$$

Corollary 1.1 shows that an analogue of our sufficient statistics result applies to this more general setup. The main differences as compared with Proposition 1 are that (1) we include an additional sufficient statistic, the average amount of (per-period) capital destroyed due to distortion; and (2) the second sufficient statistic is not the mean-squared distortion to investment but rather a more general measure of the resources lost due to "misallocation" of investment across firms (unless the investment adjustment cost is quadratic, in which case it maps into the mean-squared distortion).

Corollary 1.1. *Assume we know the following:*

(i) Average (capital-weighted) distortion to firm investment, $i_\Delta = \int_\chi \Delta i(\chi)\gamma^K(\chi)\,d\chi$

(ii) Average (capital-weighted) "investment misallocation,"
$\widehat{\phi} = \int_\chi \left(\phi\left(i\left(\chi\right)\right) - \phi\left(i_{\mathrm{MM}} + i_\Delta\right)\right)\gamma^K(\chi)\,d\chi$

(iii) Average share of capital destroyed due to the distortion, defined as
$\overline{\gamma}^K_{\mathrm{Kloss}} = \int_\chi \xi(\chi)\gamma^K(\chi)\,d\chi.$

(iv) Present-value of taxes net of subsidies raised by a generation of firms

[41] This feature of the model could be further generalized to allow for the distortion to merely affect firm value (but not the capital stock).

(v) *Model parameters::* β, κ, α_1, α_2, θ, *and* ρ, *as well as parameters that enter the utility function*

Then our environment leads to a system of equations that identifies bounds on the change in aggregate quantities and prices $\left\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\right\}$ *when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long-run and along the transition path.*

If, in addition, either $\beta \to 1$ *or entry is exogenous, then the system of equations identifies the exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.*

Proof. See Online Appendix O1.2. □

Note that the second sufficient statistic is now a more general notion of "investment misallocation" instead of the mean-squared underinvestment. This notion of investment misallocation, $\widehat{\phi}$, measures how many fewer units of the final good could have been spent to achieve the same average growth rate of firms, if investment had been optimally allocated across firms. With quadratic adjustment costs, this object is characterized by the mean and mean-squared underinvestment.

A.4 Model Extension with Ex-ante Firm Heterogeneity

With additional data moments, it is possible to extend our result to environments where there is richer heterogeneity across firms. In this subsection of the appendix, we demonstrate the sufficient statistics needed to use an analog of our approach for a model with heterogeneous sectors. For the sake of brevity, we do not include the full system of equations as in the previous sections, but rather, informally discuss how our approach can be applied to richer environments and the additional data moments that would be needed.

Consider an environment with nested CES in which n sectors produce differentiated output:

$$Y_t = \left(\sum_n \left(Y_t^n\right)^{\frac{\varrho-1}{\varrho}}\right)^{\frac{\varrho}{\varrho-1}} \tag{75}$$

$$Y_t^n = \left(\int_{j\in J^n} \left(y_{j,t}\right)^{\frac{\rho^n-1}{\rho^n}}\right)^{\frac{\rho^n}{\rho^n-1}}. \tag{76}$$

Within each sector, firms produce output with technology $F^n\left(z_{j,t}k_{j,t}, l_{j,t}\right)$, which is homogeneous of degree $\frac{\rho^n}{\rho^n-1}$ so that firm revenues are homogeneous of degree one in inputs:

$$Rev^n\left(z_{j,t}k_{j,t}, l_{j,t}, Y_t\right) = Y_t^{\frac{1}{\rho}}\left(F^n\left(z_{j,t}k_{j,t}, l_{j,t}\right)\right)^{\frac{\rho^n-1}{\rho^n}}. \tag{77}$$

Each sector also has its own convex investment cost function $k\phi^n(i)$ and may vary in the extent of the distortion. The properties of our production function imply that we can write aggregate output as follows:

$$Y_t = \sum_n K_t^n Rev^n \left(1, \Upsilon^n \left(W_t, Y_t\right), Y_t\right). \tag{78}$$

Therefore, we need to keep track of the accumulation of each sector's capital:

$$\begin{aligned} K_t^n &= M_t^{n,E} + K_{t-1}^n \kappa^n \int_{j\in J^n} i_t^n\left(\chi\right) - \xi_t^n\left(\chi\right) dj & (79)\\ I_t &= \sum_n c_e^n M_t^{n,E} + \int_{j\in J^n} K_t^n \phi\left(i_t^n\left(\chi\right)\right) dj & (80)\\ \Pi_t^n &= Rev^n\left(1, \Upsilon^n\left(W_t, Y_t\right), Y_t\right) - W_t \Upsilon^n\left(W_t, Y_t\right). & (81)\end{aligned}$$

Exogenous entry

If entry is exogenous, then to characterize the steady-state equilibria both with and without the distortion, we need to know equivalents for the three sufficient statistic moments for each sector: $i_\Delta^n, \widehat{\phi^n}$, and $\gamma_{\text{Kloss}}^{n,K}$. We, of course, also need the production and preference parameters. The above system of equations thus becomes

$$\begin{aligned} K^n &= M^{n,E} + K^n \kappa^n \left(i_{\text{MM}}^n + i_\Delta^n - \gamma_{\text{Kloss}}^{n,k}\right) & (82)\\ I &= \sum_n c_e^n M^E + \int_{j\in J^n} K^n \left(\phi\left(i_{\text{MM}}^n + i_\Delta^n\right) + \widehat{\phi^n}\right) & (83)\\ \Pi^n &= Rev^n\left(1, \Upsilon^n\left(W_t, Y\right), Y\right) - W\Upsilon^n\left(W, Y\right) & (84)\\ V^{\text{MM},n} &= \Pi - \phi(i_{\text{MM}}^n) + i_{\text{MM}}^n \kappa \Lambda V^{\text{MM},n} & (85)\\ \phi'\left(i_{\text{MM},n}\right) &= \kappa \Lambda V^{\text{MM},n}, & (86)\end{aligned}$$

with the rest of the equilibrium equations consisting of (1-29)–(1-32) and (78).

Endogenous entry

There are now many types of firms. Assume that entering firms are identical, with an exogenous probability of becoming each type of firm. Note that we can decompose the entering firm's problem as before:

$$V^E = V^{\text{MM},n} + \sum_n V^{D,n} + V^{\text{Taxes},n}. \tag{87}$$

As before, $V^{\text{Taxes},n}$ is pinned down by calibrated moments and the assumption of revenue-neutrality of counterfactuals, and $V^{\text{MM},n}$ depends on a system of aggregates and known parameters and sufficient statistics. For each sector, we can bound $V^{D,n}$ by following the steps in the proof of proposition 1, where there is a bound $\overline{V^{D,n}}$ such that there is some $\eta^{D,n} \in [0,1]$ such that $V^{D,n} = \eta^{D,n}\overline{V^{D,n}}$. Thus, the set of candidate equilibrium now depend on the exact combination of $\left\{\eta^{D,n}\right\} \in [0,1]^N$. For each aggregate, the maximum and minimum value in the steady-state with distortions, less the value in the equilibrium without distortions, characterizes the steady-state effect of the distortion.

A.5 Corollary 1.2: Aggregate Shocks

Corollary 1.2. *Consider the model introduced in Section 2. Also assume that*

- *parameters A, δ, and κ may be time-varying and stochastic (aggregate shocks)*
- *we know the functions $i_\Delta(S_t)$ and $i_{\Delta^2}(S_t)$*
- *entry is exogenous ($\Theta(M^{\mathrm{E}}, V^{\mathrm{E}}) = M^{\mathrm{E}} - \overline{M^{\mathrm{E}}}$)*
- *any additional equilibrium equations for the evolution of S_t (besides those already specified) are known and depend only on aggregates*

Then our environment leads to a system of equations that identifies bounds on the change in aggregate quantities and prices $\left\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\right\}$ when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long run and along the transition path.
If, in addition, either $\beta \to 1$ or entry is exogenous, then the system of equations identifies the exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.

Proof. See Online Appendix O1.3. □

A.6 Corollary 1.3: Decreasing Returns to Scale

Corollary 1.3. *Assume we know the following:*
(i) Average (capital-weighted) distortion, defined as $k_\Delta = \int_\chi (\Delta k(\chi))\, \gamma^K(\chi)\, d\chi$
(ii) The concavity-adjusted mean distortion, defined as
$k_{\Delta^\alpha} = \int_\chi \left(1 - (1 - \Delta k(\chi))^{\frac{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\right) \gamma^K(\chi)\, d\chi$
(iii) Present-value of taxes net of subsidies raised by a generation of firms, i.e., $\overline{\mathrm{TaxPV}}$
(iv) Model parameters: β, κ, α_1, α_2, θ, and ρ, as well as the parameters that enter the utility function
Then our environment leads to a system of equations that identifies bounds on the change in aggregate quantities and prices $\left\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\right\}$ when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long run and along the transition path.
If, in addition, either $\beta \to 1$ or entry is exogenous, then the system of equations identifies the

exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.

Proof. See Online Appendix O1.4. $\square$

A.7 Corollary 1.4: More General Firm Productivity Shocks

Assume that firm productivity z follows a Markov process with transition matrix $T(z, z')$, has entry distribution $T^E(z)$, and has a finite number of possible realizations. Let us define $\gamma^K(\chi, z)$ as the capital-weighted distribution of vector χ (state variables that summarize distortion) conditional on firm productivity z.

Corollary 1.4. *Assume we know the following:*

(i) Average (capital-weighted) distortion to firm investment, conditional on z: $i_\Delta(z) = \int_\chi \Delta i(\chi)\gamma^K(\chi, z)\, d\chi$

(ii) Average (capital-weighted) squared distortion to firm investment:
$i_{\Delta^2}(z) = \int_\chi (\Delta i(\chi))^2 \gamma^K(\chi, z)\, d\chi$

(iii) Present-value of taxes net of subsidies per period

(iv) Model parameters: β, κ, α_1, α_2, θ, ρ, $T(z, z')$, and T^E, as well as the parameters that enter the utility function

Then our environment leads to a system of equations that identifies bounds on the change in aggregate quantities and prices $\left\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\right\}$ when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long run and along the transition path.
If, in addition, either $\beta \to 1$ or entry is exogenous, then the system of equations identifies the exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.

Proof. See Online Appendix O1.5. $\square$

A.8 Balanced Growth Path

Consider a parameterization of our baseline model where $1 + \phi = (\phi + \gamma)\,\alpha_2(\rho - 1)$. If we want labor supply to be constant over time and not exhibit any growth, we must also have $\alpha_2(\rho - 1) = 1$. Further, let us assume that the revenue raised from corporate taxes less subsidies on the firms that enter each period growth proportionally in output. Corollary 1.5 shows that, in such an environment, the same sufficient statistics characterize (bound or exactly identify) how both the level and growth rate of aggregate variables change in response to eliminating the distortion.

Corollary 1.5. *Assume that the economy with distortions is an equilibrium with a balanced growth path and we know the following:*

(i) Average capital-weighted distortion to firm investment: $i_{\Delta} = \int_{\chi} \Delta i(\chi)\gamma^{K}(\chi)\, d\chi$

(ii) Average capital-weighted squared distortion to firm investment:
$i_{\Delta^2} = \int_{\chi} (\Delta i(\chi))^2 \gamma^{K}(\chi)\, d\chi$

(iii) Present value of taxes net of subsidies as a proportion of output

(iv) Model parameters: β, κ, α_1, α_2, θ, ρ, *and the parameters that enter the utility function*

Then our environment leads to a system of equations that identifies bounds on the change in ***both the level and growth rates*** *of aggregate quantities and prices* $\{C, Y, K, I, W, \Lambda, \Pi, M^{\mathrm{E}}, V^{\mathrm{E}}, q^{\mathrm{MM}}, i_{\mathrm{MM}}\}$ *when firm-level distortions are removed in a revenue-neutral counterfactual, in both the long run and along the transition path.*
If, in addition, either $\beta \to 1$ *or entry is exogenous, then the system of equations identifies the exact change in these aggregate quantities and prices, rather than bounds, both in the long run and along the transition path.*

Proof. See Online Appendix O1.6. □

Online Appendix

The Online Appendix contains four subsections. In Subsection O1, we outline proofs to the corollaries and present additional theoretical results discussed in the text. In Subsection O2, we present additional information on the quantitative results. In Subsection O3, we present further details on the results when TFP follows a Markov process. Last, in Subsection O4, we present additional figures and tables referenced in the text.

O1 Proofs to Corollaries and Additional Results

In this section of the Online Appendix, we first outline the tighter bound than zero for the welfare change in the counterfactual described in Proposition 1 (in Subsection O1.1). We then outline the proofs to the corollaries (in Subsections O1.2-O1.6). Next, we present the derivation of the derivative of capital with respect to i_Δ (in Subsection O1.7). Last, we outline the planner's problem (in Subsection O1.8) and the conditions on ρ and γ required for the problem to be well defined (in Subsection O1.9).

O1.1 A Tighter Bound than Zero for Losses

Define $\underline{V^{\mathrm{D}}} \leq 0$ as the solution to the following maximization problem:

$$\underline{V^{\mathrm{D}}} = \max_{i_\Delta^{(n)}, i_{\Delta^2}^{(n)}, K_{(n)}} -\sum_{n=0}^{\infty} \beta^n K_{(n)} \left(\frac{\theta}{2} i_{\Delta^2}^{(n)} \right), \tag{1-1}$$

subject to the following constraints:

$$\begin{aligned}
i_{\Delta^2}^{(n)} &\geq \left(i_\Delta^{(n)} \right)^2 && \text{(1-2)}\\
K_{(0)} &= 1 && \text{(1-3)}\\
K_{(n)} &= K_{(n-1)} \kappa \left(i_{\mathrm{MM}} + i_\Delta^{(n-1)} \right) && \text{(1-4)}\\
K &= \sum_{n=0}^{\infty} K_{(n)} && \text{(1-5)}\\
i_\Delta &= \sum_{n=0}^{\infty} \frac{K_{(n)}}{K} i_\Delta^{(n)} && \text{(1-6)}\\
i_{\Delta^2} &= \sum_{n=0}^{\infty} \frac{K_{(n)}}{K} i_{\Delta^2}^{(n)} && \text{(1-7)}\\
0 &\leq \left(i_{\mathrm{MM}} + i_\Delta^{(n)} \right), && \text{(1-8)}
\end{aligned}$$

where $i_{\Delta}^{(n)}, i_{\Delta^2}^{(n)}$, measure how many periods since entry the effects of the distortions captured in the sufficient statistics occurred:

$$i_{\Delta}^{(n)} = \int_{\chi} \Delta i\left(\chi\right)\gamma^{K}(\chi)_{(n)} d\chi \tag{1-9}$$

$$i_{\Delta^2}^{(n)} = \int_{\chi} \left(\Delta i\left(\chi\right)\right)^{2}\gamma^{K}(\chi)_{(n)} d\chi \tag{1-10}$$

Note that (1-2) comes from Jensen's inequality, (1-3)–(1-5) describe the capital stock distributed over the periods since entry, (1-6)–(1-7) impose that the distribution of the sufficient statistics over the time since entry do indeed aggregate up to the sufficient statistics. Equation (1-8) imposes that the capital stock can never turn negative. If one wished to do so, one could also impose additional restrictions, such as requiring the distortion to always reduce investment ($i_{\Delta}^{(n)} \leq 0$), or limited liability, which would make the bound tighter.

Since the allocation implies that V^{D} must satisfy these constraints, $V^{\mathrm{D}} \leq \underline{V^{\mathrm{D}}}$. Furthermore, since $i_{\Delta^2}^{(n)} \geq 0$, it must be the case that $\underline{V^{\mathrm{D}}} \leq 0$. Therefore, the set of bounds can be tightened to $\eta^{D} \in [\underline{V^{\mathrm{D}}}/\overline{V^{\mathrm{D}}}, 1]$.

O1.2 Proof to Corollary 1.1: More General Class of Models

The proof is largely analogous to that for Proposition 1. Here, we outline the additional elements needed: aggregation in the more general environment and the characterization of firm value with a more general investment cost function.

O1.2.1 Aggregation

Firms choose labor to maximize their revenues less labor expenses:

$$\max_{l} Rev\left(zk, l, Y\right) - W_t l.$$

Homogeneity of degree one implies we can write the maximization problem in terms of $\hat{l} = \frac{l}{zk}$:

$$\max_{\hat{l}} zk\left(Rev\left(1, \hat{l}, Y\right) - W_t \hat{l}\right). \tag{1-11}$$

Labor is thus paid its marginal product:

$$W_t = MPL_t(\hat{l}), \tag{1-12}$$

where

$$MPL_t(x) = \frac{\partial Rev\left(1, x, Y_t\right)}{\partial x}. \tag{1-13}$$

Define $\Upsilon(W_t, Y_t)$ as the inverse of the marginal product of labor. Formally,

$$W_t = \frac{\partial Rev\left(1, \Upsilon, Y_t\right)}{\partial x}. \tag{1-14}$$

We can then write aggregate labor demand, aggregate firm revenues, and firm revenues less expenses as

$$\begin{aligned} L_t &= K_t \Upsilon(W_t, Y_t) && \text{(1-15)} \\ \int_j p_{j,t} y_{j,t} dj &= K_t Rev\left(1, \Upsilon(W_t, Y_t), Y_t\right) && \text{(1-16)} \\ \Pi_t &= Rev\left(1, \Upsilon(W_t, Y_t), Y_t\right) - W_t \Upsilon(W_t, Y_t) && \text{(1-17)} \\ p_{j,t} y_{j,t} - W_t l_{j,t} &= z_{j,t} k_{j,t} \Pi_t. && \text{(1-18)} \end{aligned}$$

Recall that we assumed the aggregation technology is homogeneous in its inputs. Let us denote the degree of homogeneity as $\hat{\rho}$. We can then obtain output as a function of firm revenues, the capital stock, and the degree of homogeneity:

$$Y_t = \frac{1}{\hat{\rho}} \int_j p_{j,t} y_{j,t} dj = \frac{1}{\hat{\rho}} K_t Rev\left(1, \Upsilon(W_t, Y_t), Y_t\right). \tag{1-19}$$

O1.2.2 Loss to Firm Value Term V^{D}

Following the same steps we followed to obtain (54) in the proof for Proposition 1, we can show that V^{D} in this more general setting is

$$V^{\mathrm{D}} = \bar{K} \int_\chi \hat{\gamma}^K(\chi) \Big(\kappa \beta q^{\mathrm{MM}} (i(\chi) - i_{\mathrm{MM}} - \xi(\chi)) - (\phi(i(\chi)) - \phi(i_{\mathrm{MM}})) \Big) d\chi. \tag{1-20}$$

Additionally,

$$\Big(\kappa \beta q^{\mathrm{MM}} (i(\chi) - i_{\mathrm{MM}} - \xi(\chi)) - (\phi(i(\chi)) - \phi(i_{\mathrm{MM}})) \Big) < 0. \tag{1-21}$$

So the value of $i(\chi)$ that maximizes this expression is $i(\chi) = i_{\mathrm{MM}}$.

Also,

$$\Big(\kappa \beta q^{\mathrm{MM}} (i(\chi) - i_{\mathrm{MM}} - \xi(\chi)) - (\phi(i(\chi)) - \phi(i_{\mathrm{MM}})) \Big) \leq -\kappa \beta q^{\mathrm{MM}} \xi(\chi) \leq 0. \tag{1-22}$$

It thus follows that $V^{\mathrm{D}} \leq 0$ and as $\beta \to 1$, $\hat{\gamma}^K \to \gamma^K$. Therefore,

$$V^{\mathrm{D}} \;\to\; \bar{K} \int_\chi \gamma^K(\chi) \Big(\kappa \beta q^{\mathrm{MM}} (i(\chi) - i_{\mathrm{MM}} - \xi(\chi)) - (\phi(i(\chi)) - \phi(i_{\mathrm{MM}})) \Big) d\chi$$

$$= \bar{K}\left(\kappa\beta q^{\mathrm{MM}}(i_\Delta - \gamma^K_{\mathrm{Kloss}}) - \left(\widehat{\phi} + \phi(i_{\mathrm{MM}} + i_\Delta) - \phi(i_{\mathrm{MM}})\right)\right). \tag{1-23}$$

O1.2.3 Steady-State System of Equations

Given $\eta^D \in [0,1]$, the equilibrium system given by K, I, C, L, Y, W, Π, M^E, Λ, q^{MM}, i_{MM}, V^{E}, V^{D}, and V^{Taxes} is characterized by the following equations:

$$\begin{aligned}
K &= M^{\mathrm{E}} + K\kappa\left(i_{\mathrm{MM}} + i_\Delta - \overline{\gamma}^K_{\mathrm{Kloss}}\right) && \text{(1-24)}\\
I &= M^{\mathrm{E}} c_e(M^E, V^{\mathrm{E}}, ...) + K\phi(i_{\mathrm{MM}} + i_\Delta) + K\widehat{\phi} && \text{(1-25)}\\
\Pi &= Rev(1, \Upsilon(W,Y), Y) - W\Upsilon(W,Y) && \text{(1-26)}\\
q^{\mathrm{MM}} &= \Pi - \phi(i_{\mathrm{MM}}) + i_{\mathrm{MM}}\kappa\Lambda q^{\mathrm{MM}} && \text{(1-27)}\\
\phi'\left(i_{\mathrm{MM}}\right) &= \kappa\Lambda q^{\mathrm{MM}} && \text{(1-28)}\\
\Lambda &= \beta && \text{(1-29)}
\end{aligned}$$

$$\begin{aligned}
W &= \frac{-U_L\left(C,L\right)}{U_C\left(C,L\right)} && \text{(1-30)}\\
L &= K * MPL^{-1}\left(W\right) && \text{(1-31)}\\
C &= Y - I && \text{(1-32)}\\
Y &= K * Rev(1, \Upsilon(W,Y), Y)\frac{1}{\hat{\rho}} && \text{(1-33)}\\
0 &= \Theta(M^E, V^{\mathrm{E}}, ...) && \text{(1-34)}\\
V^{\mathrm{E}} &= q^{\mathrm{MM}} + V^{\mathrm{D}} + V^{\mathrm{Taxes}} && \text{(1-35)}\\
V^{\mathrm{Taxes}} &= \frac{\overline{\mathrm{TaxPV}}}{M^E} && \text{(1-36)}\\
V^{\mathrm{D}} &= -\eta^D\frac{K}{M^E}\left(\kappa\beta q^{\mathrm{MM}}(i_\Delta - \gamma^K_{\mathrm{Kloss}}) - \left(\widehat{\phi} + \phi\left(i_{\mathrm{MM}} + i_\Delta\right) - \phi(i_{\mathrm{MM}})\right)\right., && \text{(1-37)}
\end{aligned}$$

where the functions Υ and Rev come from the production function.

If the entry condition does not depend on V^{E} (for example, entry is exogenous), then equations (1-24)–(1-34) characterize the relevant aggregates. If not, it is the case that as $\beta \to 1$, $V^{\mathrm{D}} \to \frac{K}{M^E}\left(\kappa\beta V^{MM}(i_\Delta - \gamma^K_{\mathrm{Kloss}}) - \left(\widehat{\phi} + \phi\left(i_{\mathrm{MM}} + i_\Delta\right) - \phi(i_{\mathrm{MM}})\right)\right.$, and the system of equations (1-24)–(1-37) exactly characterizes the steady-state equilibrium. We thus have a similar bound result to that shown in the proof of Proposition 1. In turn, the steady-state counterfactuals can be performed as outlined in the proof of Proposition 1.

O1.2.4 Transition Dynamics

Note that without distortions, the dynamics of 12 unknowns K_t, I_t, C_t, L_t, Y_t, W_t, Π_t, M_t^E, Λ_t, q_t^{MM}, $i_{\mathrm{MM},t}$, and V_t^{E} are characterized by the following equilibrium system:

$$
\begin{aligned}
K_t &= M_t^E + K_{t-1}\kappa i_{\mathrm{MM},t-1} &\quad (1\text{-}38)\\
I_t &= c_e M_t^E + K_t \phi\left(i_{\mathrm{MM},t}\right) &\quad (1\text{-}39)\\
\Pi_t &= Rev\left(1, \Upsilon(W_t, Y_t), Y_t\right) - W_t \Upsilon(W_t, Y_t) &\quad (1\text{-}40)\\
q_t^{\mathrm{MM}} &= \Pi_t - \phi(i_{\mathrm{MM},t}) + i_{\mathrm{MM},t}\kappa\Lambda_t q_t^{\mathrm{MM}} &\quad (1\text{-}41)\\
\phi'\left(i_{\mathrm{MM},t}\right) &= \kappa\Lambda_t q_t^{\mathrm{MM}} &\quad (1\text{-}42)
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_t &= \beta \frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t-1}, L_{t-1}\right)} &\quad (1\text{-}43)\\
W &= \frac{-U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)} &\quad (1\text{-}44)\\
L_t &= K_t MPL_t^{-1}\left(W_t\right) &\quad (1\text{-}45)\\
C_t &= Y_t - I_t &\quad (1\text{-}46)\\
Y_t &= \frac{1}{\hat{\rho}} K_t Rev\left(1, \Upsilon(W_t, Y_t), Y_t\right) &\quad (1\text{-}47)\\
0 &= \Theta(M_t^E, V_t^{\mathrm{E}}) &\quad (1\text{-}48)\\
V_t^{\mathrm{E}} &= q_t^{\mathrm{MM}} + \frac{\overline{\mathrm{TaxPV}}}{M_t^E}. &\quad (1\text{-}49)
\end{aligned}
$$

Therefore, transition dynamics in which we begin in the steady-state with the distortion and then remove it may be computed as outlined in the proof of Proposition 1.

O1.3 Proof to Corollary 1.2: Aggregate Shocks

Proof: Note that we can write the aggregate system of equations as follows:

$$
\begin{aligned}
K_t &= M_t^E + K_{t-1}\kappa_t\left(i_{\mathrm{MM},t-1} + i_{\Delta t-1}\right)\\
I_t &= c_e M_t^E + K_t\left(\phi\left(i_{\mathrm{MM},t} + i_{\Delta t}\right) + \frac{\theta}{2}\left(i_{\Delta^2 t} - (i_{\Delta t})^2\right)\right)\\
\Pi_t &= \left(A_t^{\frac{\rho-1}{\rho}} Y_t^{\frac{1}{\rho}} \left(W_t\right)^{-\alpha_2(\frac{\rho-1}{\rho})}\left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})}\right)^{\alpha_c}\left(1-\alpha_2(\frac{\rho-1}{\rho})\right)\\
q_t^{\mathrm{MM}} &= \Pi_t - \phi(i_{\mathrm{MM},t}) + i_{\mathrm{MM},t}\kappa\Lambda_t q_t^{\mathrm{MM}}\\
\phi'\left(i_{\mathrm{MM},t}\right) &= \kappa\Lambda_t q_t^{\mathrm{MM}}
\end{aligned}
$$

$$\begin{aligned}
\Lambda_t &= \beta \frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t-1}, L_{t-1}\right)} \\
W_t &= \frac{-U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)} \\
L_t &= K_t \left(\alpha_2 (\frac{\rho-1}{\rho}) A^{\frac{\rho-1}{\rho}} \left(W_t\right)^{-1} Y^{\frac{1}{\rho}} \right)^{\alpha_c} \\
C_t &= Y_t - I_t \\
Y_t &= K_t \frac{\Pi_t}{\left(1-\alpha_2 \frac{\rho-1}{\rho}\right)} \\
M_t^E &= \overline{M^E},
\end{aligned}$$

where $i_{\Delta t}$ and $i_{\Delta^2 t}$ are defined in (29).

Let $S_t^A = K_t, I_t, \Pi_t, q_t^{\mathrm{MM}}, i_{\mathrm{MM},t}, \Lambda_t, W_t, L_t, C_t, Y_t, \delta_t, A_t, \kappa_t$ denote aggregates and exogenous variables, and let S_t^X denote any additional state variables that may affect the distortions at the firm level. Letting $S_t = S_t^A, S_t^X$, note that if we know the functions $i_\Delta\left(S_t\right), i_\Delta^2\left(S_t\right)$, as well as the equilibrium condition for other state variables $F\left(S_t^A, S_t^X, S_{t-1}^X, \mathbb{E}_t\left[S_t^X\right]\right) = 0$, then the following equations characterize the equilibrium system as a function of parameters and the process of exogenous shocks for δ_t, A_t, κ_t:

$$\begin{aligned}
K_t &= M_t^E + K_{t-1}\kappa_t \left(i_{\mathrm{MM},t-1} + i_\Delta\left(S_{t-1}\right)\right) \\
I_t &= c_e M_t^E + K_t \left(\phi_t \left(i_{\mathrm{MM},t} + i_\Delta\left(S_t\right)\right) + \frac{\theta}{2}\left(i_\Delta^2\left(S_t\right) - \left(i_\Delta\left(S_t\right)\right)^2\right) \right) \\
\Pi_t &= \left(A_t^{\frac{\rho-1}{\rho}} Y_t^{\frac{1}{\rho}} \left(W_t\right)^{-\alpha_2(\frac{\rho-1}{\rho})} \left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})} \right)^{\alpha_c} \left(1-\alpha_2(\frac{\rho-1}{\rho})\right) \\
q_t^{\mathrm{MM}} &= \Pi_t - \phi_t(i_{\mathrm{MM},t}) + i_{\mathrm{MM},t}\kappa \Lambda_t q_t^{\mathrm{MM}} \\
\phi'\left(i_{\mathrm{MM},t}\right) &= \kappa \Lambda_t q_t^{\mathrm{MM}}
\end{aligned}$$

$$\begin{aligned}
\Lambda_t &= \beta \frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t-1}, L_{t-1}\right)} \\
W_t &= \frac{-U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)} \\
L_t &= K_t \left(\alpha_2 (\frac{\rho-1}{\rho}) A^{\frac{\rho-1}{\rho}} \left(W_t\right)^{-1} Y^{\frac{1}{\rho}} \right)^{\alpha_c} \\
C_t &= Y_t - I_t \\
Y_t &= K_t \frac{\Pi_t}{\left(1-\alpha_2 \frac{\rho-1}{\rho}\right)} \\
M_t^E &= \overline{M^E} \\
0 &= F\left(S_t^A, S_t^X, S_{t-1}^X, \mathbb{E}_t\left[S_t^X\right]\right).
\end{aligned}$$

O1.4 Proof to Corollary 1.3: Decreasing Returns to Scale

The logic of the proof is similar to that of Proposition 1, but the derivations with decreasing returns are different. The problem of an undistorted firm is as follows:

$$\begin{aligned} V_t\left(A_{j,t}, K_{j,t-1}\right) &= \max_{K_{j,t}, L_{j,t-1}} \left(A_{j,t} K_{j,t-1}^{\alpha_1} L_{j,t}^{\alpha_2}\right)^{\frac{\rho-1}{\rho}} Y_t^{\frac{1}{\rho}} - W_t L_t + \left(\left(1-\delta\right) K_{j,t-1} - K_{j,t}\right) \\ &+ E_t\left[\kappa \Lambda_{t+1} V_{t+1}\left(A_{j,t+1}, K_{j,t}\right)\right]. \end{aligned} \tag{1-50}$$

Let $\omega_{j,t} = \frac{1}{1-\Delta k_{j,t}}$ denote the gross distortion to investment (for shorter notation). Define expected profits from investment as

$$Profits_{j,t-1} = -K_{jt} + \kappa\beta E_t\left[P_{jt}Y_{jt} - W_t L_{jt} + \left(1-\delta\right) K_{jt} P_K\right].$$

Further, let $X_{j,t}$ denote the expectation of (convexity-adjusted) productivity, and $\mathcal{E}_{j,t}$ the unexpected shock to productivity:

$$\begin{aligned} X_{j,t-1} &= E_{t-1}\left[\left(A_{jt}^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}}\right]^{\frac{1-\alpha_2\frac{\rho-1}{\rho}}{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}} \\ \mathcal{E}_{j,t} &= \frac{\left(A_{jt}^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}}}{E_{t-1}\left[\left(A_{jt}^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}}\right]}. \end{aligned}$$

We can then write firm capital, labor, revenue, and profits in steady state (with the distortion) as

$$\begin{aligned} K_{jt} &= \Pi\left(\frac{\rho-1}{\rho}\alpha_1\frac{\kappa\beta}{r^k}\right)\omega_{jt-1}X_{j,t-1} \\ L_{jt} &= \left(\omega_{jt-1}\right)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\frac{\rho-1}{\rho}\alpha_2\frac{1}{W}\Pi X_{j,t-1}\mathcal{E}_{j,t} \\ P_{jt}Y_{jt} &= \left(\omega_{jt-1}\right)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\Pi X_{j,t-1}\mathcal{E}_{j,t} \\ E_{t-1}\left[Profits_{jt}\right] &= \kappa\beta\Pi X_{j,t-1}\left(\left(\omega_{jt-1}\right)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\left(1-\frac{\rho-1}{\rho}\alpha_2\right) - \omega_{jt-1}\frac{\rho-1}{\rho}\alpha_1\right), \end{aligned}$$

where r^k is a measure of the cost of capital and Π is an aggregate scaling factor:

$$\begin{aligned} r^k &= 1-\kappa\beta\left(1-\delta\right) \qquad (1\text{-}51) \\ \Pi &= \left(Y^{\frac{1}{\rho}}\right)^{\frac{1}{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}}\left(\frac{\rho-1}{\rho}\right)^{\frac{(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}}\left(\frac{\alpha_1}{r^k}\right)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}}\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2\frac{\rho-1}{\rho}}{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}}. \end{aligned}$$

Note that setting $\omega_{j,t} = 1$ results in the choices and profits of an undistorted firm (holding aggregates prices as given). Since there are no aggregate shocks and all idiosyncratic shocks are assumed to be uncorrelated with previous state variables, we can write aggregate output, capital, and labor as:

$$\begin{aligned}
K &= \Pi\left(\frac{\rho-1}{\rho}\alpha_1\frac{\kappa\beta}{r^k}\right)\sum_j \omega_j X_j \\
L &= \frac{\rho-1}{\rho}\alpha_2\frac{1}{W}\Pi\sum_j (\omega_j)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}} X_j \\
Y &= \Pi\sum_j (\omega_{jt-1})^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}} X_j,
\end{aligned}$$

which can be expressed also as a function of sufficient statistics:

$$K = \Pi\left(\frac{\rho-1}{\rho}\alpha_1\frac{\kappa\beta}{r^k_{t-1}}\right)\frac{\bar{X}}{(1-k_\Delta)} \tag{1-52}$$

$$Y = \frac{K}{\left(\frac{\rho-1}{\rho}\alpha_1\frac{\kappa\beta}{r^k}\right)}(1-k_{\Delta^\alpha}) \tag{1-53}$$

$$W = \frac{Y}{L}\frac{\rho-1}{\rho}\alpha_2, \tag{1-54}$$

where $\bar{X} = \int_j X_j dj$ is an aggregate index of (expected) firm productivity.

Note that, using the expected profits function, we find that the value of a firm that enters with no capital and expected productivity measure $X_{j,t}$ and distortion $\omega_{j,t}$ has expected value:

$$V(X_{jt},\omega_{jt}) = \kappa\Lambda\Pi X_{jt}\left((\omega_{jt})^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\left(1-\frac{\rho-1}{\rho}\alpha_2\right) - \omega_{jt}\frac{\rho-1}{\rho}\alpha_1\right) + \kappa\beta\mathbb{E}_t\left[V(X_{jt+1},\omega_{jt+1})\right].$$

Thus, a firm entering with capital $K_{j,t}$, current productivity $A_{j,t}$, expected productivity measure $X_{j,t}$ and distortion $\omega_{j,t}$ has expected value:

$$V(X_{jt},\omega_{jt}) + \left(A_{jt}^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}}\left(\left(K_{jt}^{\alpha_1}\right)^{\frac{\rho-1}{\rho}}Y^{\frac{1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}}\left(\frac{\alpha_2\frac{\rho-1}{\rho}}{W}\right)^{\frac{\alpha_2\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\left(1-\alpha_2\frac{\rho-1}{\rho}\right).$$

As in the proof of Proposition 1, we can decompose the value of an entering firm as $V^{\mathrm{E}} = V^{\mathrm{MM,E}} + V^{\mathrm{D}} + V^{\mathrm{Taxes}}$.

The (pre-tax) value of an undistorted firm entering with a unit of capital can be written as

$$\begin{aligned} V^{\mathrm{MM,E}} &= \mathbb{E}\left[q^{\mathrm{MM}}\left(X_{jt}\right) + \left(A_{jt}^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}} \left(Y^{\frac{1}{\rho}}\right)^{\frac{1}{1-\alpha_2\frac{\rho-1}{\rho}}} \left(\frac{\alpha_2\frac{\rho-1}{\rho}}{W}\right)^{\frac{\alpha_2\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}} \left(1-\alpha_2\frac{\rho-1}{\rho}\right)\right] \\ q^{\mathrm{MM}}\left(X_{jt}\right) &= \kappa\Lambda\Pi X_{jt}\left(\left(1-\frac{\rho-1}{\rho}\alpha_2\right)-\frac{\rho-1}{\rho}\alpha_1\right)+\kappa\beta\mathbb{E}_t\left[q^{\mathrm{MM}}\left(X_{jt+1}\right)\right]. \end{aligned}$$

Additionally, V^{D} can be expressed as

$$V^{\mathrm{D}} = \kappa\Lambda\Pi\mathbb{E}\left[\sum_{\tau}\left(\beta\kappa\right)^{t}X_{j,\tau}\left(\left(\left(\omega_{j,\tau}\right)^{\frac{\alpha_1\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}-1\right)\left(1-\frac{\rho-1}{\rho}\alpha_2\right)-\left(\omega_{j,\tau}-1\right)\frac{\rho-1}{\rho}\alpha_1\right)\right].$$

As firms are ex-ante homogeneous before entering, the distortions can be written as a function of the entry-discounted distribution of firms:

$$\begin{aligned} V^{\mathrm{D}} &= \kappa\Lambda\Pi\frac{\bar{X}}{(1-k_{\Delta})}\int_j \hat{\gamma}^{K}\Big(\chi\Big)\Bigg(\Bigg(\Delta k(\chi) \\ &\quad -\left(1-\left(1-\Delta k(\chi)\right)^{\frac{1-(\alpha_1+\alpha_2)\frac{\rho-1}{\rho}}{1-\alpha_2\frac{\rho-1}{\rho}}}\right)\Bigg)\Bigg)\left(1-\frac{\rho-1}{\rho}\alpha_2\right)-\Delta k(\chi)\frac{\rho-1}{\rho}\alpha_1\Bigg). \end{aligned}$$

Analogously to what we showed in the proof of Proposition 1, the effect of the distortion on pre-tax firm value is always negative (from the FOC of the capital choice). Additionally, since it is a function of the time-discounted stream of losses, the losses cannot be as great as the not-time-discounted value of losses (which are finite if a firm's lifetime profits are finite):

$$\overline{V^{\mathrm{D}}} \leq V^{\mathrm{D}} \leq 0,$$

where the lower bound $\overline{V_D}$ can be written as a function of our sufficient statistics:

$$\overline{V^{\mathrm{D}}} = \kappa\Pi\frac{\bar{X}}{(1-k_{\Delta})}\left(\left(k_{\Delta}-k_{\Delta^{\alpha}}\right)\left(1-\frac{\rho-1}{\rho}\alpha_2\right)-k_{\Delta}\frac{\rho-1}{\rho}\right)\alpha_1. \tag{1-55}$$

Note that if $\beta=1$, then $V^{\mathrm{D}}=\overline{V^{\mathrm{D}}}$; if the distortions are all zero, then $V^{\mathrm{D}}=0$.

Let us thus again define $\eta\in[0,1]$, and note that we can write $V^{\mathrm{D}}=\eta\overline{V^{\mathrm{D}}}$. η indexes a continuum of "possible" equilibria not ruled out by the sufficient statistics when distortions are present.

The equilibrium system is thus characterized by (1-51)–(1-63) and the following aggregate equations, as well as the (exogenous) process for firm productivity, which defines $\bar{X}$ (possibly influenced by entry):

$$\begin{aligned}
\Lambda &= \beta && (1\text{-}56)\\
W &= \frac{-U_L(C,L)}{U_C(C,L)} && (1\text{-}57)\\
C &= Y - I && (1\text{-}58)\\
I &= M^E c_e + \delta K && (1\text{-}59)\\
0 &= \Theta(M^E, V^{\mathrm{E}}) && (1\text{-}60)\\
V^{\mathrm{E}} &= V^{\mathrm{MM,E}} + V^{\mathrm{D}} + V^{\mathrm{Taxes}} && (1\text{-}61)\\
V^{\mathrm{Taxes}} &= \frac{\overline{\mathrm{TaxPV}}}{M^E} && (1\text{-}62)\\
V^{\mathrm{D}} &= \eta \overline{V^{\mathrm{D}}}. && (1\text{-}63)
\end{aligned}$$

O1.5 Proof to Corollary 1.4: More General Firm Productivity Shocks

Exogenous entry If entry is exogenous, the equilibrium system is characterized by parameters and productivity-dependent sufficient statistics $i_{\Delta}(z)$ and $i_{\Delta^2}(z)$:

$$\begin{aligned}
\Pi &= \left(A^{\frac{\rho-1}{\rho}} Y^{\frac{1}{\rho}} (W)^{-\alpha_2(\frac{\rho-1}{\rho})} \left(\alpha_2(\frac{\rho-1}{\rho})\right)^{\alpha_2(\frac{\rho-1}{\rho})}\right)^{\alpha_c} \left(1-\alpha_2(\frac{\rho-1}{\rho})\right)\\
q^{\mathrm{MM}}(z) &= \Pi z - \phi(i_{\mathrm{MM}}(z)) + i_{\mathrm{MM}}(z)\,\kappa\Lambda \sum_{z'} T(z,z')\, q^{\mathrm{MM}}(z')\\
\phi'(i_{\mathrm{MM}}) &= \kappa\Lambda \sum_{z'} T(z,z')\, q^{\mathrm{MM}}(z')\\
\Lambda &= \beta\\
W &= \frac{-U_L(C,L)}{U_C(C,L)}\\
L &= K\left(\alpha_2(\frac{\rho-1}{\rho}) A^{\frac{\rho-1}{\rho}} (W)^{-1} Y^{\frac{1}{\rho}}\right)^{\alpha_c}\\
C &= Y - I\\
Y &= \frac{\Pi}{\left(1-\alpha_2\frac{\rho-1}{\rho}\right)} \sum z K(z)\\
0 &= \Theta(M^E, V^{\mathrm{E}}).\\
K(z') &= M^{\mathrm{E}} T^E(z') + \sum_z K(z)\, T(z,z')\, \kappa\left(i_{\mathrm{MM}}(z) + i_{\Delta}(z)\right)\\
I &= M^E c_E + \sum_z K(z)\left(\phi(i_{\mathrm{MM}}(z) + i_{\Delta}(z)) + \frac{\theta}{2}\left(i_{\Delta^2}(z) - (i_{\Delta}(z))^2\right)\right).
\end{aligned}$$

Adding endogenous entry This derivation follows the proof of Proposition 1. It requires defining several objects.

Notation and preliminaries Define the distribution of firms n periods after entry as

$$\gamma_{(n)}^{K}(\chi, z) = \frac{\int_k k\Gamma_{(n)}(k, \chi, z)\, dk}{\int_{\chi,k,z} k\Gamma_{(n)}(k, \chi, z)\, dk d\chi dz}.$$

Further, define entry-scaled versions of the capital stock measures:

$$\overline{K}(z) = \frac{K(z)}{M_e}.$$

Note that this expression can be split up into the time-since-entry versions, $\overline{K_{(n)}}(z)$, where $\overline{K_{(0)}}(z) = T^E(z)$ is just a function of entry probabilities, and for $n > 0$:

$$\overline{K_{(n)}}(z) = \kappa \sum_{z_{-1}} \left(\int \gamma_{(n)}^{K}(\chi, z)\, i(\chi, z_{-1})\, d\chi \right) \overline{K_{(n-1)}}(z_{-1})\, T(z_{-1}, z).$$

We can further define a time-discounted version of $\overline{K_{(n)}}(z)$, which we call $\widetilde{K}_{(n)}(z)$:

$$\widetilde{K}_{(n)}(z) = \kappa\beta \sum_{z_{-1}} \left(\int \gamma_{(n)}^{K}(\chi, z)\, i(\chi, z_{-1})\, d\chi \right) \widetilde{K}_{(n-1)}(z_{-1})\, T(z_{-1}, z).$$

Note that the aggregate-time discounted capital stock is then:

$$\widetilde{K}(z) = \sum_{n=0}^{\infty} \widetilde{K}_{(n)}(z) = \sum_{n=0}^{\infty} \beta^n \overline{K_{(n)}}(z).$$

We can therefore also define analogous versions of these objects as if choices were made only by Modigliani-Miller firms (e.g., if $i(\chi, z_{-1}) = i_{\mathrm{MM}}(z)$), yielding $\widetilde{K}_{(n)}^{\mathrm{MM}}(z)$ and $\widetilde{K}^{\mathrm{MM}}(z)$. Define $\Delta\widetilde{K}(z) = \widetilde{K}(z) - \widetilde{K}^{\mathrm{MM}}(z)$; this expression implies:

$$\Delta\widetilde{K}(z) = \sum_{z} \widetilde{K}(z)\, T(z, z')\, \beta\kappa\widetilde{i}(z) - \sum_{z} \widetilde{K}^{\mathrm{MM}}(z)\, T(z, z')\, \beta\kappa\, (i_{\mathrm{MM}}(z)),$$

where

$$\widetilde{i}(z) = \left(\frac{\widetilde{K_{(n)}}(z)}{\widetilde{K}(z)} \int \gamma_{(n)}^{K}(\chi, z)\, i(\chi, z_{-1})\, d\chi \right).$$

This can be re-written as

$$\Delta\widetilde{K}(z) = \sum_{z} \widetilde{K}(z)\, T(z, z')\, \beta\kappa \left(\widetilde{\Delta i}(z) \right) + \sum_{z} \left(\Delta\widetilde{K}(z) \right) T(z, z')\, \beta\kappa\, (i_{\mathrm{MM}}(z)),$$

where $\left(\widetilde{\Delta i}(z) \right) = \widetilde{i}(z) - i_{\mathrm{MM}}(z)$.

Then, in matrix notation, note that we can write:

$$\begin{aligned}\Delta\widetilde{K} &= \beta\kappa\widetilde{K}\text{diag}\left(\widetilde{\Delta i}\right)T+\beta\kappa\Delta\widetilde{K}\text{diag}\left(i_{\text{MM}}\right)T\\ &= \beta\kappa\widetilde{K}\text{diag}\left(\widetilde{\Delta i}\right)T\left(I-\beta\kappa\text{diag}\left(i_{\text{MM}}\right)T\right)^{-1}.\end{aligned}$$

And the Modigliani-Miller firm's value can be written as

$$\begin{aligned}q^{\text{MM}} &= \left(\Pi z-\phi\left(i_{\text{MM}}\right)\right)+\kappa\beta\text{diag}\left(i_{\text{MM}}\right)Tq^{\text{MM}}\\ &= \left(I-\kappa\beta\text{diag}\left(i_{\text{MM}}\right)T\right)^{-1}\left(\Pi z-\phi\left(i_{\text{MM}}\right)\right).\end{aligned}$$

Firm value lost due to distortions The firm value lost to distortions can be written as the difference in discounted cash flows:

$$V^{D}=\sum_{n,z}\widetilde{K}_{(n)}\left(z\right)\left(\Pi z-\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\phi\left(i\left(\chi,z\right)\right)d\chi\right)-\sum_{n,z}\widetilde{K_{(n)}^{MM}}\left(z\right)\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right),$$

which can be simplified to yield:

$$\begin{aligned}V^{D} &= \sum_{n,z}\widetilde{K}_{(n)}\left(z\right)\left(\phi\left(i_{\text{MM}}\left(z\right)\right)-\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\phi\left(i\left(\chi,z\right)\right)d\chi\right)+\sum_{n,z}\left(\widetilde{K}_{(n)}\left(z\right)-\widetilde{K_{(n)}^{MM}}\left(z\right)\right)\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right)\\ V^{D} &= \sum_{n,z}\widetilde{K}_{(n)}\left(z\right)\left(\phi\left(i_{\text{MM}}\left(z\right)\right)-\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\phi\left(i\left(\chi,z\right)\right)d\chi\right)+\sum_{z}\left(\widetilde{K}\left(z\right)-\widetilde{K^{MM}}\left(z\right)\right)\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right)\\ V^{D} &= \sum_{n,z}\widetilde{K}_{(n)}\left(z\right)\left(\phi\left(i_{\text{MM}}\left(z\right)\right)-\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\phi\left(i\left(\chi,z\right)\right)d\chi\right)+\sum_{z}\left(\Delta\widetilde{K}\left(z\right)\right)\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right).\end{aligned}$$

Note that this second term can be written in matrix form as $\left(\Delta\widetilde{K}\right)\text{diag}\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right)$. Plugging in for the matrix form of $\Delta\widetilde{K}$ yields:

$$\left(\Delta\widetilde{K}\right)\text{diag}\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right) = \beta\kappa\widetilde{K}\text{diag}\left(\widetilde{\Delta i}\right)T\left(I-\beta\kappa\text{diag}\left(i_{\text{MM}}\right)T\right)^{-1}\text{diag}(\left(\Pi z-\phi\left(i_{\text{MM}}\right)\right).$$

Plugging in for the matrix form of q^{MM} yields:

$$\left(\Delta\widetilde{K}\right)\text{diag}\left(\Pi z-\phi\left(i_{\text{MM}}\left(z\right)\right)\right) = \widetilde{K}\text{diag}\left(\widetilde{\Delta i}\right)T\text{diag}\left(\beta\kappa q^{\text{MM}}\right).$$

Plugging this equation into V^{D} yields

$$\begin{aligned}V^{D} &= \sum_{z,n}\widetilde{K}_{(n)}\left(z\right)\left(\phi\left(i_{\text{MM}}\left(z\right)\right)-\left(\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\phi\left(i\left(\chi,z\right)\right)d\chi\right)\right)\\ &+ \sum_{z,n}\widetilde{K}_{(n)}\left(z\right)\left(\left(\int_{\chi}\gamma_{(n)}^{K}\left(\chi,z\right)\left(i\left(\chi,z\right)\right)d\chi\right)-i_{\text{MM}}\left(z\right)\right)\sum_{z'}T\left(z,z'\right)\beta\kappa q_{\text{MM}}\left(z'\right),\end{aligned}$$

which can be written in the form:

$$V^D = \sum_{z,n} \widetilde{K}_{(n)}(z) \int_\chi \gamma^K_{(n)}(\chi, z) F(\chi, z) \, d\chi,$$

where

$$F(\chi, z) = (\phi(i_{\text{MM}}(z)) - \phi(i(\chi, z))) + ((i(\chi, z)) - i_{\text{MM}}(z)) \sum_{z'} T(z, z') \beta\kappa q_{\text{MM}}(z').$$

Note that $F(\chi, z) \leq 0$, as the problem of maximizing $i(\chi, z)$ results in the same optimality condition as for the Modigliani-Miller firm's investment. Therefore, $F(\chi, z)$ is maximized if $i(\chi, z) = i_{\text{MM}}(z)$, in which case $F(\chi, z) = 0$.

It follows immediately that $V^D \leq 0$.

Note that since $\widetilde{K}_{(n)}(z) = \overline{K}_{(n)}(z) \beta^n$ and $F(\chi, z) \leq 0$,

$$V^D = \sum_{z,n} \overline{K}_{(n)} \beta^n (z) \int_\chi \gamma^K_{(n)}(\chi, z) F(\chi, z) \, d\chi \geq \sum_{z,n} \overline{K}_{(n)}(z) \int_\chi \gamma^K_{(n)}(\chi, z) F(\chi, z) \, d\chi.$$

Define this not-time-discounted object as $\overline{V^D}$, and note that it can be simplified to be expressed as a function of our sufficient statistics:

$$\begin{aligned}
\overline{V^D} &= \sum_{z,n} \overline{K}_{(n)}(z) \int_\chi \gamma^K_{(n)}(\chi, z) F(\chi, z) \, d\chi \\
&= \sum_z \frac{K(z)}{M^{\text{E}}} \left(\phi(i_{\text{MM}}(z)) - \phi(i_{\text{MM}}(z) + i_\Delta(z)) - \frac{\theta}{2} \left(i_{\Delta^2}(z) - (i_\Delta(z))^2 \right) \right) \\
&+ \sum_z \frac{K(z)}{M^{\text{E}}} i_\Delta \sum_{z'} T(z, z') \beta\kappa q_{\text{MM}}(z').
\end{aligned}$$

Note that $\overline{V^D}$ is thus a function of sufficient statistics and aggregates from our system of equations. Thus, we can follow the proof of Proposition 1, noting that we can write $V^D = \eta^D \overline{V^D}$ for some $\eta^D \in [0, 1]$. There is thus a continuum of these "possible equilibria", and the maximum and minimum of each aggregate endogenous variable across these possible equilibria can be used to construct bounds. If there is no time discounting, then $\eta^D = 1$.

O1.6 Proof to Corollary 1.5: Balanced Growth Path

The proof is analogous to the proof of Proposition 1. For brevity here we only highlight the additional steps required. The key insight is to note that we can write the set of equilibrium equations as

$$\Pi = \tau_g \left(\left(\overline{Y}\right)^{\frac{1}{\rho}} \left(\overline{W}\right)^{-\alpha_2(\frac{\rho-1}{\rho})} \left(\tau_g \alpha_2 (\frac{\rho-1}{\rho}) \right)^{\alpha_2(\frac{\rho-1}{\rho})} \right)^{\alpha_c} \left(1 - \alpha_2 (\frac{\rho-1}{\rho}) \right) \tag{1-64}$$

$$
\begin{aligned}
\overline{Y} &= \frac{\overline{K\Pi}}{\left(1-\alpha_2 \frac{\rho-1}{\rho}\right)\tau_g} && \text{(1-65)}\\
\overline{C} &= \overline{Y}+\overline{I} && \text{(1-66)}\\
\overline{I} &= \overline{K}\left(\phi\left(\overline{i^{MM}}+i_\Delta\right)+\frac{\theta}{2}\left(i_{\Delta^2}-(i_\Delta)^2\right)\right)+c_e\overline{M_E} && \text{(1-67)}\\
\overline{W} &= \tau_g\alpha_2(\frac{\rho-1}{\rho})\frac{\overline{Y}}{\overline{L}} && \text{(1-68)}\\
\overline{K}g_K &= \overline{K}\kappa\left(\overline{i^{MM}}+i_\Delta\right)+g_K\overline{M_E} && \text{(1-69)}\\
\overline{W} &= \left(\overline{C}\right)^\gamma\left(\overline{L}\right)^\varphi && \text{(1-70)}\\
\overline{\Lambda} &= \beta\left(g_C\right)^{-\gamma} && \text{(1-71)}\\
q^{MM} &= \Pi-\phi(i^{MM})+i^{MM}\kappa\mathbb{E}_t[\Lambda q^{MM}] && \text{(1-72)}\\
\phi'(i^{MM}) &= \kappa\mathbb{E}_t[\Lambda q^{MM}] && \text{(1-73)}\\
V_E &= q^{MM}-\frac{\overline{Y}\tau}{\overline{\overline{M_E}}}-V_D && \text{(1-74)}\\
V_E &= c_e && \text{(1-75)}\\
V_D &= -\eta^D\frac{1}{1-\kappa\beta\left(g_C\right)^{-\gamma}\left(i_{MM}+i_\Delta\right)}\left(\frac{\theta}{2}i_{\Delta^2}\right) && \text{(1-76)}\\
g_K &= \{g_I,g_C,g_M,g_Y\} && \text{(1-77)}\\
g_W &= \left(g_Y\right)^{\frac{1}{\alpha_2(\rho-1)}} && \text{(1-78)}\\
g_L &= \frac{g_Y}{g_W}, && \text{(1-79)}
\end{aligned}
$$

where in our notation, an endogenous variable X can be written as $X_t=\bar{X}g_x^t$. Given an initial capital stock, the sufficient statistics and parameters thus characterize an equilibrium path. The growth path with the distortion resolved can be computed by setting i_Δ and i_{Δ^2} equal to zero and using the initial capital stock from the equilibrium with distortions. The value of a unit of capital and undistorted firm q is constant along the balanced growth path, so the solution to the value of an entering firm and the bounds are unchanged from those derived in the proof to Proposition 1.[42]

O1.7 Derivative of Capital with respect to i_Δ

With the functional form assumptions in the model in Section 2, if we assume that utility is linear in consumption but has disutility in leisure $\frac{L^{1+\varphi}}{1+\varphi}$, we can simplify steady-state expressions for K,Π,i_{MM} as follows:

[42]There are additional conditions that need to be satisfied for a balanced growth rate to exist. For instance, we have a condition on the equilibrium growth rate of capital: $g_K=\frac{\kappa\left(i^{MM}+i_\Delta\right)}{1-\frac{M_E}{K}}>1$ (otherwise we are in a stationary equilibrium). Further, for investment to be finite, we must have Tobin's q be finite. Therefore, we must have $\kappa\beta g_K^{-\gamma}i^{MM}<1$. Similarly, for the loss to the distortion to be finite, we must have $1\geq\kappa\beta\left(g_C\right)^{-\gamma}\left(i_{MM}+i_\Delta\right)$.

$$\Pi = K^{\frac{\frac{1}{\rho-1}(1+\varphi)-\varphi\alpha_2}{(1-\alpha_2+\varphi)}} \left((A)^{\frac{\rho-1}{\rho}}\right)^{\frac{(1+\varphi)}{\frac{\rho-1}{\rho}(1-\alpha_2+\varphi)}} \left(\alpha_2\frac{\rho-1}{\rho}\right)^{\frac{\alpha_2}{(1-\alpha_2+\varphi)}} \left(1-\alpha_2\frac{\rho-1}{\rho}\right) \tag{1-80}$$

$$\phi'\left(i_{\mathrm{MM}}\right) = \beta\kappa\frac{\Pi-\phi\left(i_{\mathrm{MM}}\right)}{1-\kappa\beta i_{\mathrm{MM}}} \tag{1-81}$$

$$K = \frac{M_e}{1-\kappa\left(i_{\mathrm{MM}}+i_\Delta\right)}. \tag{1-82}$$

If we then take the derivatives of these expressions with respect to i_Δ, we obtain:

$$\frac{\partial\Pi}{\partial i_\Delta} = \frac{\partial K}{\partial i_\Delta}\frac{\frac{1}{\rho-1}\left(1+\varphi\right)-\varphi\alpha_2}{\left(1-\alpha_2+\varphi\right)}\frac{\Pi}{K} \tag{1-83}$$

$$\beta\kappa\frac{\partial\Pi}{\partial i_\Delta} = \left(\phi''\left(i_{\mathrm{MM}}\right)\left(1-\kappa\beta i_{\mathrm{MM}}\right)\right)\frac{\partial i_{\mathrm{MM}}}{\partial i_\Delta} \tag{1-84}$$

$$\frac{\partial M_e}{\partial i_\Delta} = \frac{\partial K}{\partial i_\Delta}\left(1-\kappa\left(i_{\mathrm{MM}}+i_\Delta\right)\right) - K\kappa\left(\frac{\partial i_{\mathrm{MM}}}{\partial i_\Delta}+1\right). \tag{1-85}$$

Combining these expressions yields the following expression for $\frac{\partial K}{\partial i_\Delta}$:

$$\frac{\partial K}{\partial i_\Delta} = \frac{K\kappa + \frac{\partial M_e}{\partial i_\Delta}}{1-\kappa\left(i_{\mathrm{MM}}+i_\Delta\right) - \kappa\Pi\frac{\varphi\alpha_2-\frac{1}{\rho-1}(1+\varphi)}{(1-\alpha_2+\varphi)}\frac{\beta\kappa}{(\phi''(i_{\mathrm{MM}})(1-\kappa\beta i_{\mathrm{MM}}))}}. \tag{1-86}$$

If entry is exogenous, we can further show the dampening effect of the inverse Frisch φ and CES term ρ:

$$\frac{\partial\frac{\partial K}{\partial i_\Delta}}{\partial\varphi} = -\frac{K\kappa^2\Pi\frac{\left(1-\alpha_2+\frac{1}{\rho-1}\alpha_2\right)}{(1-\alpha_2+\varphi)^2}\frac{\beta\kappa}{(\phi''(i_{\mathrm{MM}})(1-\kappa\beta i_{\mathrm{MM}}))}}{\left(1-\kappa\left(i_{\mathrm{MM}}+i_\Delta\right)+\kappa\Pi\frac{\varphi\alpha_2-\frac{1}{\rho-1}(1+\varphi)}{(1-\alpha_2+\varphi)}\frac{\beta\kappa}{(\phi''(i_{\mathrm{MM}})(1-\kappa\beta i_{\mathrm{MM}}))}\right)^2} < 0 \tag{1-87}$$

$$\frac{\partial\frac{\partial K}{\partial i_\Delta}}{\partial\rho} = -\frac{K\kappa^2\Pi\frac{(1+\varphi)}{(\rho-1)^2(1-\alpha_2+\varphi)}\frac{\beta\kappa}{(\phi''(i_{\mathrm{MM}})(1-\kappa\beta i_{\mathrm{MM}}))}}{\left(1-\kappa\left(i_{\mathrm{MM}}+i_\Delta\right)+\kappa\Pi\frac{\varphi\alpha_2-\frac{1}{\rho-1}(1+\varphi)}{(1-\alpha_2+\varphi)}\frac{\beta\kappa}{(\phi''(i_{\mathrm{MM}})(1-\kappa\beta i_{\mathrm{MM}}))}\right)^2} < 0. \tag{1-88}$$

O1.8 Planner's Problem

The model in Section 2 has two additional distortions outside of the firm-level distortions to investment: corporate taxation and a distortion due to the monopoly markup. These distortions can be undone by setting the corporate tax rate to zero and including a subsidy, $\tau^s = \frac{\rho}{\rho-1}$, in the steady-state system equations (40) and (42) as follows

$$L = K\left(\tau^s\alpha_2\left(\frac{\rho-1}{\rho}\right)A^{\frac{\rho-1}{\rho}}\left(W\right)^{-1}Y^{\frac{1}{\rho}}\right)^{\alpha_c} \tag{1-89}$$

$$Y = K\frac{\Pi}{\tau^s\left(1-\alpha_2\frac{\rho-1}{\rho}\right)}.$$

With the taxation and monopoly markup distortions removed, the "no friction" case that removes the investment distortions is equivalent to the planner's problem. In Table 2, we present results when $\beta \to 1$ from the planner's problem (labelled as the "no friction" case) compared with the case with investment distortions under a zero corporate tax rate and the subsidy incorporated as above.

Unlike the results presented in Section 4, the gains from resolving the distortion are always positive no matter if it is an overinvestment or underinvestment distortion. This result is to be expected, as the investment distortions take us away from the first-best case. We see the GE welfare effects of the friction are still modest compared with the PE effects when we are comparing with the case where there is also zero taxation and no subsidy. If we were to compare the effects to the case where there is taxation and no subsidy, then the GE welfare gains would be large.

O1.9 Conditions on ρ and γ

Here, we show that certain combinations of ρ, γ, and other parameters lead to the problem being undefined, as there is explosive growth. Consider the utility function in our quantitative calibration in Section 4. In an equilibrium with explosive growth, there will be no entry, so we will have the following block of conditions:

$$\begin{aligned}
\xi C_t^{\gamma} L_t^{\varphi} &= \alpha_2\frac{\rho-1}{\rho}\frac{Y_t}{L_t}\\
Y_t &= AK_t^{\frac{\rho}{\rho-1}-\alpha_2}L_t^{\alpha_2}\\
C_t &= Y_t - K_t\phi\left(i_t\right).
\end{aligned}$$

We can reduce this block to

$$L_t = \left(\frac{\alpha_2\frac{\rho-1}{\rho}}{\xi\left(Y_t-K_t\phi\left(i_t\right)\right)^{\gamma}}Y_t\right)^{\frac{1}{1+\varphi}}. \tag{1-90}$$

If we then plug this equation into output and rearrange, we obtain

$$\left(\frac{Y_t}{K_t}\right)^{1-\frac{\alpha_2}{1+\varphi}}\left(\frac{Y_t}{K_t}-\phi\left(i_t\right)\right)^{\gamma\frac{\alpha_2}{1+\varphi}} = AK_t^{\frac{\rho}{\rho-1}-\alpha_2+\alpha_2\frac{1-\gamma}{1+\varphi}-1}\left(\frac{\alpha_2\frac{\rho-1}{\rho}s_t}{\xi}\right)^{\frac{\alpha_2}{1+\varphi}}.$$

This expression implies $\frac{\rho}{\rho-1} - \alpha_2 + \alpha_2 \frac{1-\gamma}{1+\varphi} - 1 \leq 0$ as to prevent unstable equilibria. So we have a condition on ρ of

$$1 + \frac{1}{\alpha_2} \frac{1+\varphi}{\varphi+\gamma} \leq \rho. \tag{1-91}$$

O2 Additional Information on the Quantitative Results

In this section of the Online Appendix, we first present the mapping between our model and that of Stein (2003) (in Subsection B.1). Second, we present further details on how we obtain the sufficient statistics from Hennessy et al. (2007) and Ben-David et al. (2013) (in Subsections O2.2 and O2.3). Last, we present further details on the robustness of the quantitative results to the model parameters (in Subsection O2.4).

O2.1 Mapping between our Model and Stein (2003)

Stein (2003) sets up a simple problem of distorted investment due to external equity (equation (2) in his paper), which we reproduce below:

$$\max_I \frac{f(I)}{1+r} - I - \theta C(e). \tag{1-92}$$

Here, $I = e + w$ (where I is investment, e is external finance raised externally, and w is wealth or internal resources). The solution to this problem has FOC:

$$\frac{f'(I)}{1+r} = 1 + \theta C'(I - w).$$

Note that we could change this notation—let $i = \frac{f(i)}{\kappa V}$ and $\phi(x) = f^{-1}(x\kappa V)$. Then we could write this problem in our notation:

$$\max_I iV \frac{\kappa}{1+r} - \phi(i) - \theta C(e).$$

In this case, V and κ are arbitrary constants, but in our model, they correspond to the shadow value of capital and the mean "capital quality shock." In this notation, we get our familiar FOC (noting that $\beta = \frac{1}{1+r}$):

$$\beta\kappa V = \phi'(i) + \theta C'(e) \frac{\partial e}{\partial i}.$$

Therefore, the FOC could be distorted by external financing consistent with Stein (2003).

Additionally, equation (3) in Stein (2003) adds an agency conflict, which could imply over-investment:

$$\max_I \frac{f(I)}{1+r}(1+\gamma) - I - \theta C(e), \tag{1-93}$$

where γ is a parameter that governs the intensity of agency conflicts. In our notation, this becomes:

$$\max_{I} i\kappa\beta V\left(1+\gamma\right)-\phi\left(i\right)-\theta C\left(e\right),$$

and thus has FOC:

$$\beta\kappa V\left(1+\gamma\right)=\phi'\left(i\right)+\theta C'\left(e\right)\frac{\partial e}{\partial i}.$$

So, this expression now has distortions that can encourage too much investment.

Altogether, our framework has a natural mapping to the distortions considered in Stein (2003).

O2.2 Obtaining Sufficient Statistics using Hennessy et. al. (2007)

O2.2.1 Mapping between Frameworks

Note that the environment introduced in example 1 in Section 2.2.2 is a discrete-time version of the simplified model in Hennessy et al. (2007). We can thus take the expression for Δi_t we derive, plug in the quadratic functional form of external financing costs they consider (which imply $G'(x)=\theta_x$), and take a first-order approximation (with respect to current and future equity issuance x_t or cash holdings) to yield the following:[43]

$$\Delta i_t\left(z,k,c_{-1}\right)=\theta_x\widehat{q}x\Phi\left(x<0\right). \tag{1-94}$$

This is exactly the functional form they consider and estimate, and in their continuous time environment, there is no difference between q this period and expected discounted q next period.[44] Therefore, in a regression of the form

$$\frac{I_{jt}}{K_{jt}}=\theta_x\left(\widehat{q}_{jt}x_{jt}\Phi\left(x_{jt}<0\right)\right)+controls+error.$$

Note that we can map i_Δ into the regression coefficient times the mean of the observable independent variable on which the regression coefficient loads:

$$i_\Delta=\int_j\Delta i_j\frac{K_j}{K_j}dj\approx\theta_x mean\left(\widehat{q}x\Phi\left(x<0\right)\right).$$

[43]Because the undistorted firm maximizes firm value, the effect of any distortions on investment do not have a first-order effect on the value of the firm, since the derivative of firm value with respect to investment is 0 absent distortions. Thus, the effect of distortions on firm value is second order, and only their squared value enters.

[44]How the physical cost of external finance affects firm value upon entry is a second-order effect that we assume away. This assumption is consistent with external financing costs being purely an asymmetric information problem (rather than a true cost), so, on average, the value of issued equity is the fair value, but the marginal cost of issuing equity is distorted by adverse selection. Our approach can accommodate such costs (as in Corollary 1.1) with a sufficient statistic capturing them. Inferring the effect of such a cost as a percentage of firm value using the estimates in Hennessy and Whited (2007) yields small values for such a statistic, as the mean-squared issuance is small.

Similarly,

$$i_{\Delta^2} = \int_j (\Delta i_j)^2 \frac{K_j}{K_j} dj \approx \theta_x^2 \left(mean\left(\widehat{q}x\Phi\left(x<0\right)\right)^2 + \sigma^2\left(mean\left(\widehat{q}x\Phi\left(x<0\right)\right)\right)\right).$$

O2.2.2 Data

We download the Compustat Annual file in July 2022.[45] We select on the years 1968–2003 to match the years in Hennessy et al. (2007). We also follow their criteria for selecting on SIC codes (dropping firms with a one-digit SIC of 6 or 9 or two-digit SIC of 49). We then compute Tobin's Q as the market value of equity (computed as the price times shares outstanding) less the book value of equity less deferred taxes plus the book value of assets, all scaled by the book value of assets. We compute equity issuance as the sale of less the purchase of common and preferred stock. We drop observations with missing values for the SIC code, total assets, sales, gross PP&E, capital expenditures, sales of property, plant, and equipment, the objects enter Q and equity issuance, and the additional variables that go into the Kaplan and Zingales (1997) index (cash flow computed as income plus depreciation, dividends computed as dividends of common and preferred shares, and long-term debt). We also drop non-positive values of sales, total assets, and gross PP&E. Additionally, we keep only firms that report in U.S. dollars and with U.S. headquarters. The capital stock used in capital weighting is in 1997 dollars. We winsorize the (inflation-adjusted) capital-stock, Tobin's Q, and equity issuance at the 1% level. As noted in the main text, with this approach, we find a similar non-capital-weighted mean underinvestment value to that using the reported values from HLW (negative 0.00239 in our case versus negative 0.00134 in HLW).

To perform the measurement exercise described in Section 5, we merge in the TFP data from İmrohoroğlu and Tüzel (2014), keeping only the observations that uniquely merge.[46] In Online Appendix Table O3.4, we present information on the mean, median, and standard deviation across the sample and by quantile.

O2.3 Obtaining Sufficient Statistics from Ben-David et. al. (2013)

O2.3.1 Mapping between Frameworks

We can show that a first-order approximation of investment with respect to current and (possibly future) distortions implies that the distortion to investment is identical to the causal effect of miscalibrated expectations today:

$$\Delta i_t\left(z,k,m\right) \approx m_t,$$

[45] We download the Compustat Annual file from the WRDS database in July 2022.

[46] Therefore, we follow the cleaning steps above including restricting our data to nonfinancial firms from pre-2003, among the other cleaning steps, and also drop observations with missing TFP.

which is precisely the object estimated by BGH. The proof is analogous to the proof in Appendix O2.2.

O2.3.2 Data

In the survey used by BGH, managers are asked to report their projections for the S&P 500. Managers are also asked for the values above and below which there is a 1 in 10 chance for the actual return. They perform this exercise for the year-ahead and next-10-year returns. BGH convert these 90-10 percentiles for returns into imputed-individual probabilities.

We focus on the estimate of calibration of long-term returns on investment from Table VII (of 0.6)—call it β^{misc}. Given reported values in their paper, we can recover the mean miscalibration adjusted for the actual return—call it μ^{misc}. From Table I, Panel A, the average and standard deviations of the imputed individual volatilities are 11.4% and 9.25%, respectively. From Table II, Panel B, the 10-year realized annual volatility of S&P 500 returns is 14.3%. So we obtain μ^{misc} as $-\frac{11.4-14.3}{9.25}$. We can write the predicted effect of miscalibration on investment as $\beta^{\text{misc}}\mu^{\text{misc}}$, which is 0.00188. Similarly, we can obtain the mean-squared effect on investment as $E\left[\left(\beta^{misc}\mu_{misc}\right)^2\right] = \left(\beta^{misc}\right)^2\left(\mu_{misc}^2+1\right) = 0.00004$.

O2.4 Robustness Across the Model Parameters

The remaining parameters of the model are the investment of the Modigliani-Miller firm i_{MM}, the exogenous exit rate κ, the adjustment cost parameter θ, the production function parameter α_2, the depreciation rate, δ, the CES parameter ρ, the IES parameter γ, and the labor disutility parameter φ. In this subsection, we vary each of these parameters and show how the bounds on welfare change (under the calibration when $\beta = 0.98$) in response to removing the external financing and manager-shareholder investment distortions.

We present the results for i_{MM} and κ in Appendix Figure O4.2. These two parameters affect the change in PE output from resolving the friction, so we present those changes on the figures.[47] We vary both parameters from 0.9 to 0.995 (the baseline calibrations of i_{MM} and κ are 0.9859 0.976, respectively). Although the parameters do not necessarily vary linearly across the ranges, the results are directionally consistent with our baseline cases for both of the bounds.

We present the results for θ, α_2, and δ in Appendix Figure O4.3. We vary θ from 1 to 1.5 (the baseline calibration is 1.1), we vary α_2 from 0.6 to 0.8 (the baseline calibration is 2/3), and we vary δ from 0.07 to 0.13 (the baseline calibration is 0.1). Though there is variation in the GE welfare losses, the results are again directionally consistent with our baseline cases for both bounds.

[47]The remaining parameters considered in this section do not affect the PE losses, so we show only the GE welfare losses in the remaining figures.

Lastly, we present results for the macro parameters φ, ρ, and γ in Appendix Figure O4.4. We consider a wide range for the Frisch elasticity parameter φ, spanning from 0.2 to 3 (the baseline calibration is .0276). We also consider a wide range for ρ, which we vary from 4 (our baseline calibration) to 30. Finally, we vary γ from 0 to 5 (the baseline calibration is 1). To keep from having to change ρ as we vary γ, we need to set ρ to be higher (we set it to 10), as the problem is undefined for certain combinations of low ρ and low γ.[48] Across cases, the results are again directionally consistent with our baseline cases for both bounds. In summary, the welfare results are directionally similar across all cases of the parameters we consider.

[48]In particular, we have condition (1-91), which we derive in Appendix O1.9. Note that because ρ is higher in this case the baseline welfare numbers are different.

O3 TFP follows Markov Process

In this section of the Online Appendix, we provide more detail on the results when TFP follows a Markov process.

Results when TFP Follows a Markov Process First, we perform a measurement exercise to demonstrate how (a) one can implement our approach with productivity heterogeneity, and (b) study the potential influence of this heterogeneity on our baseline results.

For this exercise, we need a firm-level measure of idiosyncratic productivity, which we obtain from Selale Tuzel's website that has the data constructed following İmrohoroğlu and Tüzel (2014).[49] We then compute the sufficient statistics by group of firms. We clean the data following HLW; the cleaning details are described in Appendix O2.2. We set there as being five quantiles for firm-level TFP. We then need two sets of moments for our exercise.[50] First, we need the values of the sufficient statistics by quantile, which we present in Table O3.2.[51] Here, we see that the values of the sufficient statistics are different enough for the highest TFP firms that it is worth considering how the results might change if we consider heterogeneity. Second, we need the transitions between quantiles, which we present in Table O3.3. This table shows that there is much "switching" in our data between quantiles, which intuitively should limit the importance of heterogeneity.

We perform two sets of counterfactuals that help us to understand the role of persistent heterogeneity in firm productivity. First, we study the case where there is heterogeneity in idiosyncratic productivity and in the sufficient statistics for each type. In this case, we solve for the counterfactuals following Corollary 1.4. Second, we study the case where there is heterogeneity in idiosyncratic productivity, but the sufficient statistics do not differ across types. This case preserves "independence" of the sufficient statistics, so here we can solve for our counterfactuals using an analogue of our main proposition (Proposition 1). Note that our baseline results presented in Section 4 reflect a third case where idiosyncratic productivity does not follow a Markov process and where there is no heterogeneity in the sufficient statistics. The natural comparison is between the two cases with heterogeneity in idiosyncratic productivity, as heterogeneity can also interact with the aggregate distortions to affect the overall welfare results.[52]

In Table O3.1, we show the results from the two cases (along with the values from the no friction case) when $\beta \to 1$.[53] We see that the results between the first and second cases are

[49] We obtained data from Selale Tuzel's website (https://sites.google.com/usc.edu/selale-tuzel/home) in December 2022.

[50] We also need the mean TFP by quantile, which we present in Table O3.4. along with other summary statistics.

[51] Note that we only look at equal-weighted statistics as in our baseline calibration; the capital-weighted statistics are closer to 0 across quantiles.

[52] The elasticities of the growth rates of the firms with different types are going to be heterogeneous across productivity levels. This implies that resolving the distortion will have heterogeneous effects across types; therefore, changes in state prices can lead to changes in the allocation of resources across firms of different types.

[53] The change in welfare is higher from removing the friction under the second case (where the sufficient statis-

Table O3.1: Effects of Resolving External Financing Frictions with Markov Idiosyncratic Productivity

External Financing	**Heterogeneity in z and suff. stats**			**Heterogeneity in z but not in suff. stats**		
	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value
Variable						
PE Output	2.37			2.07		
GE Investment (I)	0.68	0.36	0.36	2.28	0.36	0.37
GE Capital stock (K)	0.34	10.3	10.34	0.29	10.28	10.31
GE Investment efficiency (K/I)	-0.35	28.48	28.38	-2.03	28.48	27.91
GE Output (Y)	0.28	2.78	2.78	0.18	2.77	2.78
GE Welfare	0.23			0.15		
GE Consumption (C)	0.25	1.52	1.53	0.16	1.52	1.52
GE Labor supply (L)	0.02	0.6	0.6	0.01	0.59	0.59
GE Mass of entry (M_e)	0.11	1	1	-0.47	1	1
GE Capital stock per entering firm ($\overline{K}$)	0.23	10.3	10.32	0.76	10.28	10.36
GE Aggregate profit scaling factor (Π)	-0.12	0.18	0.18	-0.08	0.18	0.18

Note: This table shows the percentage change in PE and GE objects from the baseline steady state to the steady state without the external financing friction for the model with where idiosyncratic productivity follows a Markov process and there is heterogeneity in the sufficient statistics (left two columns) versus a case where productivity similarly follows a Markov process but there is no heterogeneity in the sufficient statistics (right columns). We also show the percentage changes in the GE objects associated with the GE welfare calculation, as well as their values in both the baseline steady state and the counterfactual steady state without the friction. Here, we assume $\beta \to 1$. z is firm productivity. suff. stats is sufficient statistics. s.s. is steady state.

directionally similar and the change in welfare differs by about 8 basis points. The reason the change in welfare is higher with both forms of heterogeneity is that the change in output is higher, as the higher TFP firms benefit more by having their distortions removed and matter more for aggregate output given their different levels of the sufficient statistics. Nonetheless, the mechanisms for why welfare increases by so much less than output does in PE—which is around 2 percentage points in this case—are similar to what we described in Section 4, as is clear from the similar directional movements in aggregates presented in the table.[54] Altogether, we view the results as being qualitatively similar to our baseline results.[55]

tics are not heterogeneous by type) when we compare it to the change in welfare in our baseline model. This is to be expected that the results will differ, as the elasticities of the growth rates of the firms with different types are going to be heterogeneous across productivity levels just taking the sufficient statistics as given. Importantly, if we wanted to exactly match the levels of aggregates in our baseline case, we could, but this would require us to adjust the values of our sufficient statistics to account for there being heterogeneity in firm TFP. We decided that using the baseline values of the sufficient statistics was more transparent for this exercise.

[54] Note that the change in PE output is not the same as partial equilibrium capital in this case; we only report the change in partial equilibrium output (the change in PE capital is modestly lower than the change in PE output in each version of the model with heterogeneity).

[55] With fixed entry, the results (not shown) are still qualitatively similar and even closer between the first and second cases).

Table O3.2: External Financing Friction Sufficient Statistic Estimates

Full Sample and by TFP Quantile

Mean

	Value
i_Δ: From paper	-0.00134
i_Δ: Replication	-0.00146
i_Δ: TFP Q1: Replication	-0.00144
i_Δ: TFP Q2: Replication	-0.00049
i_Δ: TFP Q3: Replication	-0.00056
i_Δ: TFP Q4: Replication	-0.00101
i_Δ: TFP Q5: Replication	-0.00379

Mean-squared

	Value
i_{Δ^2}: Replication	0.00023
i_{Δ^2}: TFP Q1: Replication	0.00035
i_{Δ^2}: TFP Q2: Replication	0.000028
i_{Δ^2}: TFP Q3: Replication	0.000040
i_{Δ^2}: TFP Q4: Replication	0.000097
i_{Δ^2}: TFP Q5: Replication	0.00066

Notes: This table reports values for the mean (i_Δ) or mean-squared (i_{Δ^2}) external financing friction sufficient statistics. The first row reports the statistic derived from what is reported in HLW2007, while the other rows rely on the authors' calculations in replicating the statistics (all on an equal-weighted basis). We also calculate versions of the statistics by quantile of TFP, where Q1 is the highest quantile and Q5 the lowest. *Source:* Authors' calculations using Compustat and TFP data from Selale Tuzel's website (constructed following İmrohoroğlu and Tüzel (2014)), downloaded in December 2022.

Table O3.3: Transition Matrix for TFP by Quantile

	TFP quantile[t+1]											
TFP quantile[t]	**1**		**2**		**3**		**4**		**5**		**Total**	
	No.	Row %	No.	Row %	No.	Row %	No.	Row %	No.	Row %	No.	Row %
1	5,678	67	1,960	23	514	6	209	2	112	1	8,473	100
2	2,200	23	4,642	48	2,131	22	550	6	139	1	9,662	100
3	717	7	2,265	23	4,534	46	2,063	21	366	4	9,945	100
4	334	3	685	7	2,156	21	5,229	52	1,656	16	10,060	100
5	309	3	261	3	469	5	1,731	18	7,033	72	9,803	100
Total	9,238	19	9,813	20	9,804	20	9,782	20	9,306	19	47,943	100

Notes: This table displays the transition matrix for firms between TFP quantiles in our sample. We show both the cell counts and the row percentages. *Source:* Authors' calculations using Compustat and TFP data from Selale Tuzel's website (constructed following İmrohoroğlu and Tüzel (2014)), downloaded in December 2022.

Table O3.4: Log TFP Summary Statistics by Quantile

	Mean	Median	S.D.
Whole sample	-0.336	-0.318	0.376
Q1	-0.866	-0.751	0.312
Q2	-0.467	-0.463	0.051
Q3	-0.318	-0.318	0.039
Q4	-0.175	-0.178	0.046
Q5	0.144	0.068	0.219

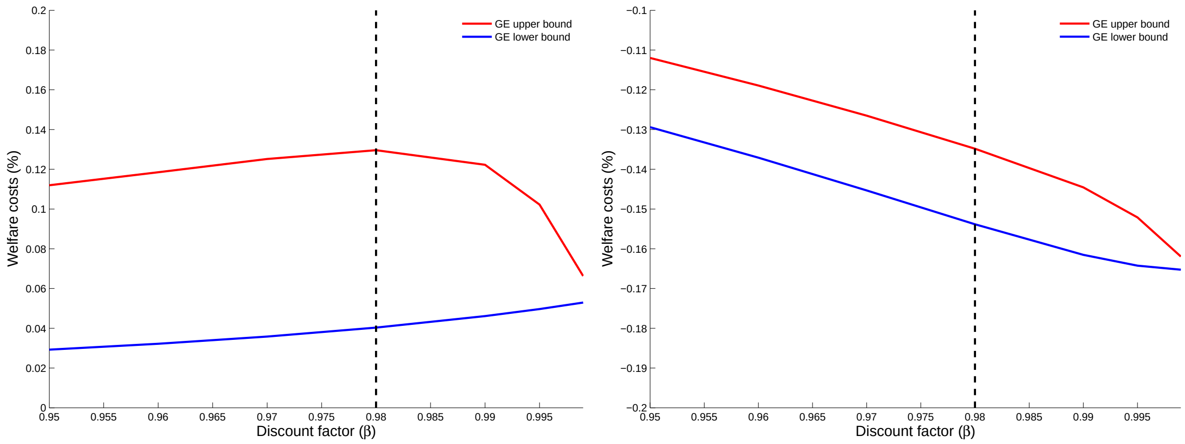
Notes: This table reports the value for the mean, median, and standard deviation (SD) of the natural logarithm of TFP for the overall (cleaned) sample and by quantile of TFP. Log is the natural logarithm.
Source: Authors' calculations using Compustat and TFP data from Selale Tuzel's website (constructed following İmrohoroğlu and Tüzel (2014)), downloaded in December 2022.

O4 Additional Figures and Tables

In this section of the Online Appendix, we present additional figures and tables referenced in the text.

Figure O4.1: Welfare Gains from Resolving Firm-level Distortions varying β

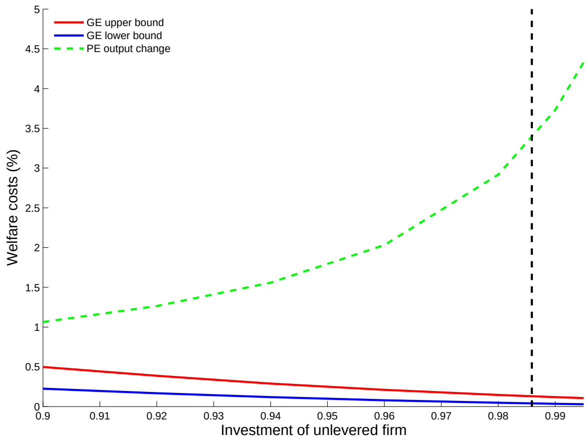
(a) External Financing Friction (b) Manager-Shareholder Friction



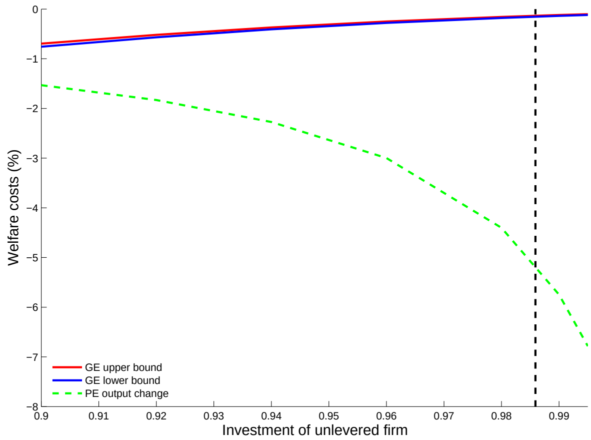
Note: The subfigures show the bounds on the welfare change after removing the external financing friction (left) and manager-shareholder friction (right) across a range of values for β (from 0.95 and 1). The vertical, dashed black line indicates the welfare values at the calibrated value of β (as shown in Table 3).

Figure O4.2: Welfare Gains from Resolving Firm-level Distortions varying i_{MM} and κ

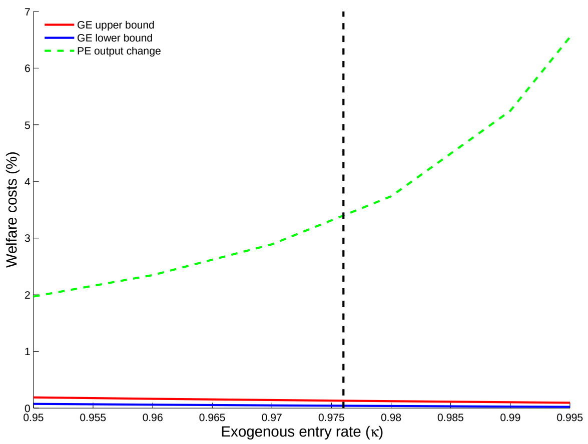
(a) External Financing Friction, varying i_{MM}



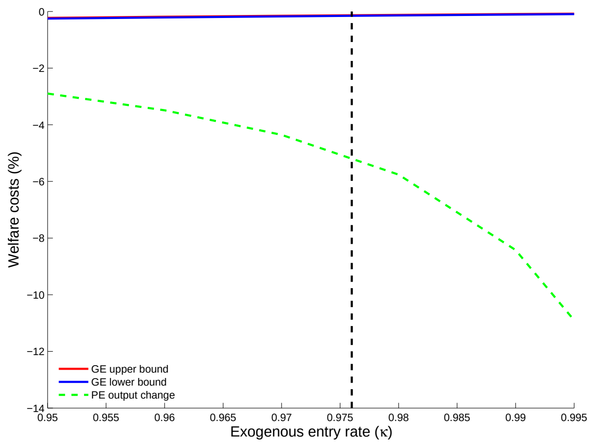
(b) Manager-Shareholder Friction, varying i_{MM}



(c) External Financing Friction, varying κ



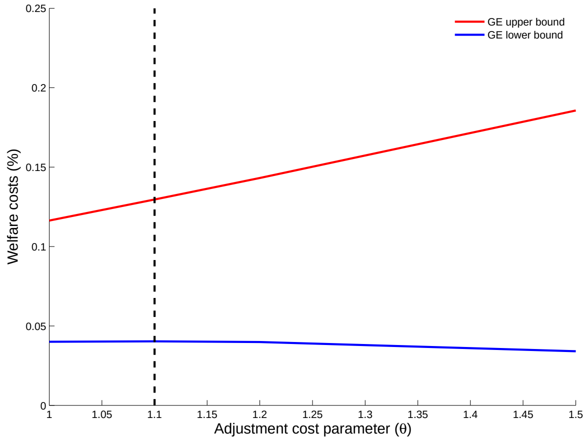
(d) Manager-Shareholder Friction, varying κ



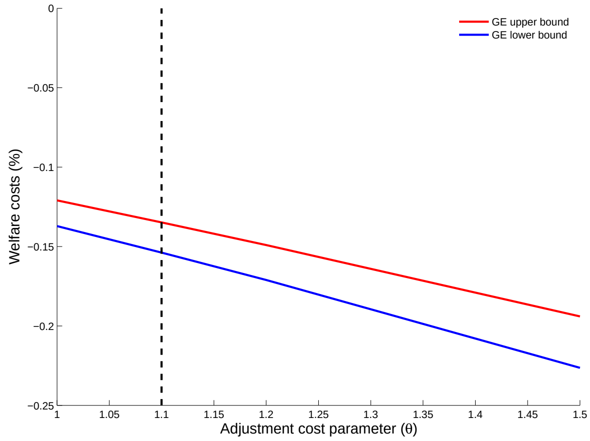
Note: Each subfigure shows the PE output change and the bounds on the welfare change after removing the external financing friction (left panels) or the manager-shareholder friction (right panels) when $\beta = 0.98$ for a set of values of a given parameter: investment of the Modigliani-Miller firm parameter i_{MM} (top panels) and exogenous exit rate parameter κ (bottom panels). The vertical, dashed black line indicates the welfare values at the calibrated value of the given parameter (as shown in Table 3).

Figure O4.3: Welfare Gains from Resolving Firm-level Distortions varying θ, α_2, and δ

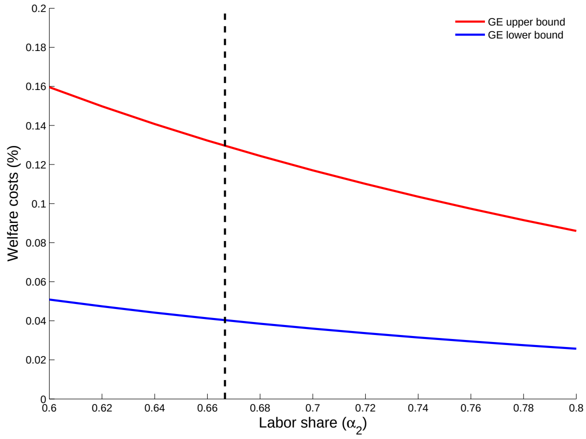
(a) External Financing Friction, varying θ



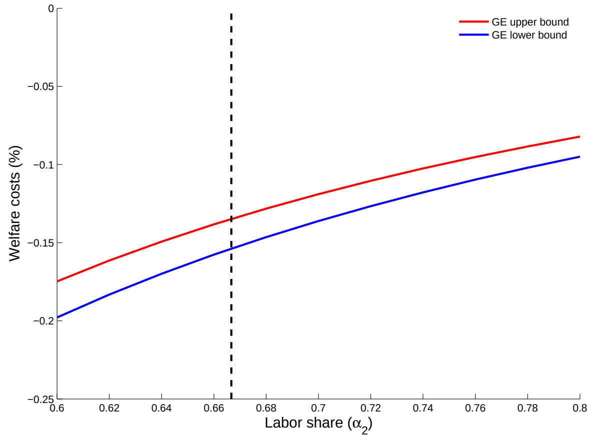
(b) Manager-Shareholder Friction, varying θ



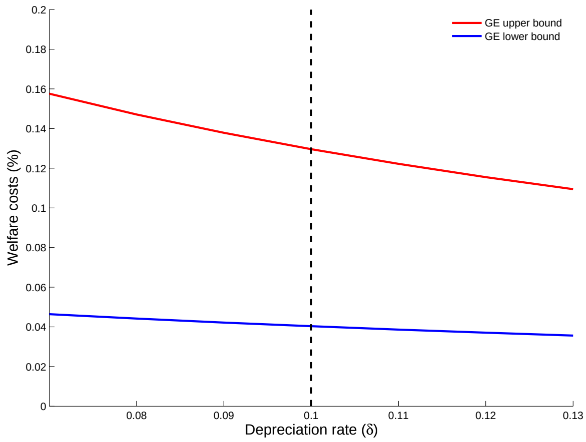
(c) External Financing Friction, varying α_2



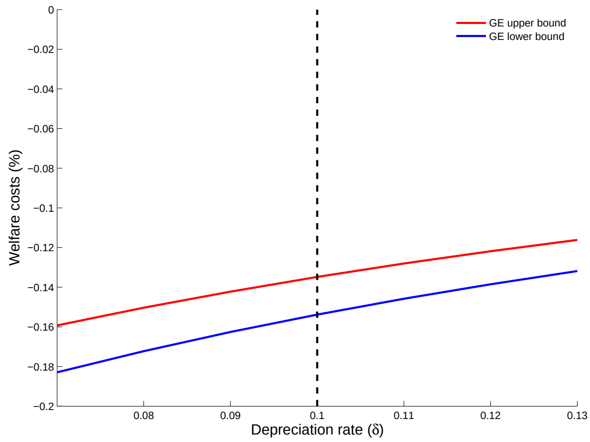
(d) Manager-Shareholder Friction, varying α_2



(e) External Financing Friction, varying δ



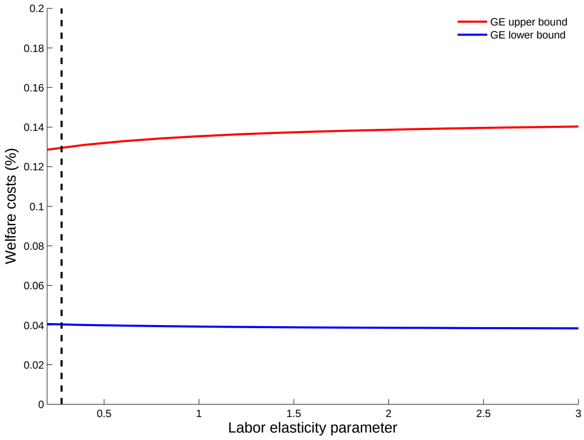
(f) Manager-Shareholder Friction, varying δ



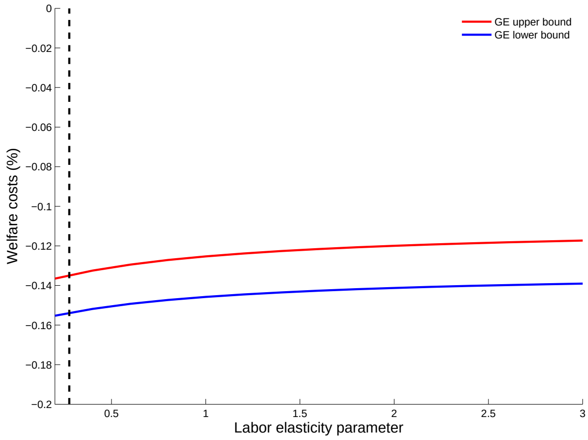
Note: Each subfigure shows the bounds on the welfare change after removing the external financing friction (left panels) or the manager-shareholder friction (right panels) when $\beta = 0.98$ for a set of values of a given parameter: adjustment cost parameter θ (top panels), production function parameter α_2 (middle panels), and depreciation rate parameter δ (bottom panels).

Figure O4.4: Welfare Gains from Resolving Firm-level Distortions varying φ, ρ, and γ

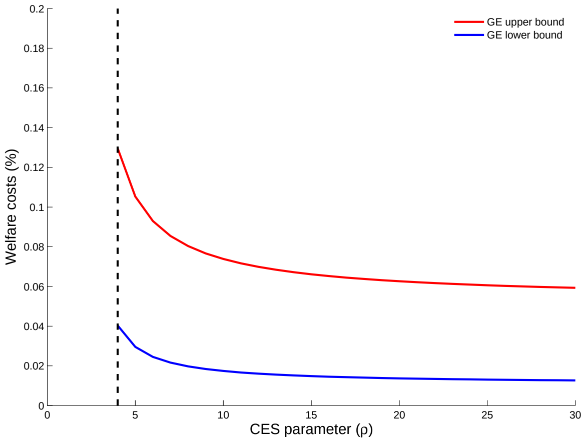
(a) External Financing Friction, varying φ



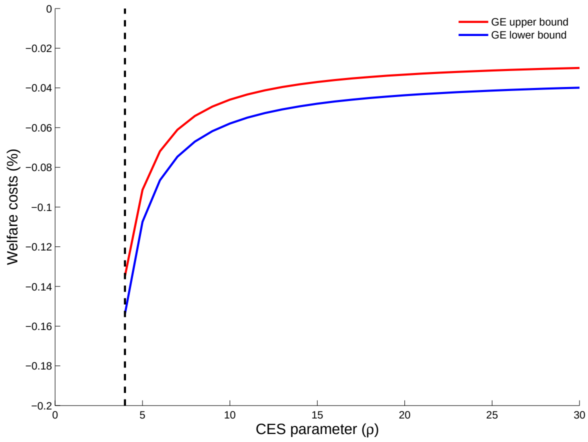
(b) Manager-Shareholder Friction, varying φ



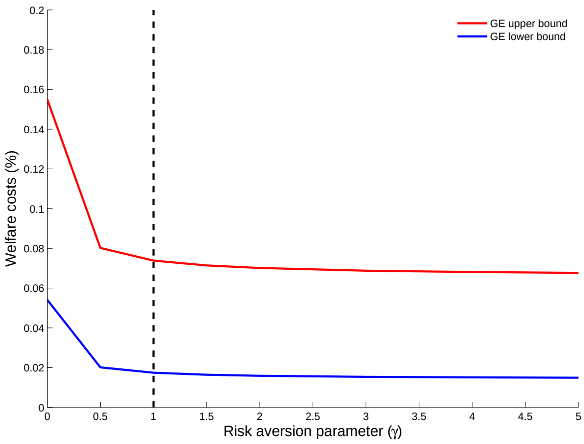
(c) External Financing Friction, varying ρ



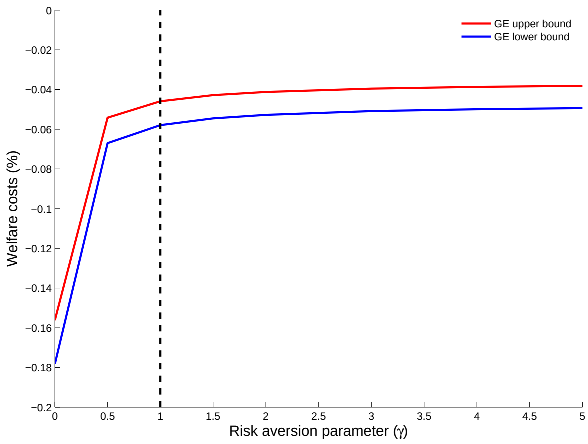
(d) Manager-Shareholder Friction, varying ρ



(e) External Financing Friction, varying γ



(f) Manager-Shareholder Friction, varying γ



Note: Each subfigure shows the bounds on the welfare change after removing the external financing friction (left panels) or the manager-shareholder friction (right panels) when $\beta = 0.98$ for a set of values of a given parameter: Frisch elasticity parameter φ (top panels), CES parameter ρ (middle panels), and IES parameter γ (bottom panels). Except in the plots for γ, the vertical, dashed black line indicates the welfare values at the calibrated value of the given parameter (as shown in Table 3). When varying the IES parameter, we set ρ to 10 as the problem becomes undefined in our baseline calibration of ρ for certain values of γ (see Section O1.9); in turn, the vertical, dashed black line corresponds to the welfare value with a γ of 1 and a ρ of 10.

Table O4.1: Effects of Resolving Firm-level Distortions: Fixed versus Free Entry

External Financing

	Fixed Entry			Free entry U.B.		
	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value
Variable						
PE Capital stock or Output	3.35			3.35		
GE Investment (I)	0.15	2.27	2.27	0.05	3.26	3.26
GE Capital stock (K)	0.18	25.6	25.64	0.17	25.58	25.62
GE Investment efficiency (K/I)	0.03	11.29	11.29	0.12	7.84	7.85
GE Output (Y)	0.12	7.94	7.95	0.1	7.94	7.95
GE Welfare	0.08			0.13		
GE Consumption (C)	0.11	5.68	5.68	0.13	4.68	4.69
GE Relative wage $(\frac{W}{P})$	0.12	4.54	4.55	0.12	4.54	4.54
GE Labor supply (L)	0.01	0.87	0.87	-0.03	0.88	0.87
GE Mass of entry (M_e)	0	1	1	1.3	0.98	0.99
GE Capital stock per entering firm $(\overline{K})$	0.18	25.6	25.64	-1.14	26.21	25.92
GE Investment of unlevered firm (i_{MM})	-0.13	0.99	0.98	-0.18	0.99	0.99
GE Aggregate profit scaling factor (Π)	-0.05	0.16	0.16	-0.08	0.16	0.16

Manager-Shareholder

	Fixed Entry			Free entry U.B.		
	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value	$\%\Delta$ between steady states	Baseline s.s. value	No-friction s.s. value
Variable						
PE Capital stock or Output	-5.11			-5.11		
GE Investment (I)	-0.59	2.56	2.55	-0.33	3.56	3.55
GE Capital stock (K)	-0.39	27.83	27.73	-0.37	27.83	27.73
GE Investment efficiency (K/I)	0.19	10.87	10.89	-0.04	7.82	7.82
GE Output (Y)	-0.32	8.64	8.61	-0.27	8.64	8.62
GE Welfare	-0.08			-0.13		
GE Consumption (C)	-0.21	6.08	6.06	-0.22	5.08	5.07
GE Relative wage $(\frac{W}{P})$	-0.23	4.73	4.72	-0.23	4.73	4.72
GE Labor supply (L)	-0.09	0.91	0.91	-0.03	0.91	0.91
GE Mass of entry (M_e)	0	1	1	-1.87	0.99	0.98
GE Capital stock per entering firm $(\overline{K})$	-0.39	27.83	27.73	1.47	27.99	28.4
GE Investment of unlevered firm (i_{MM})	0.18	0.99	0.99	0.24	0.99	0.99
GE Aggregate profit scaling factor (Π)	0.07	0.16	0.16	0.1	0.16	0.16

Note: This table shows the percentage change in PE and GE objects from the baseline steady state to the steady state without the external financing (top panel) or manager-shareholder (bottom panel) friction for the model with fixed entry versus upper bound of the baseline model with free entry when $\beta = 0.98$. We also show the percentage changes in the GE objects associated with the GE welfare calculation, as well as their values in both the baseline steady state and the counterfactual steady state without the friction. s.s. is steady state. U.B. is upper bound.